Homework 3

February 23, 2022

0.1 Problem 1

0.1.1 MLP

Given MLP has - two inputs - $\boldsymbol{x}$ - 1 hidden layer - ReLU activation - $h(\boldsymbol{x}) = max(\boldsymbol{x}, \boldsymbol{0})$ - two Output nodes - (Linear) Identity activation - $h(\boldsymbol{x}) = \boldsymbol{x}$

Here,

$$x = a^{(1)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$W_1 = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$$

$$W_2 = \begin{pmatrix} 2 & 2 \\ 2 & -3 \end{pmatrix}$$

$$b_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$b_2 = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

Output at the hidden layer -

$a^{(2)} = h_2(W_1 a^{(1)} + b_1)$

First we calculate the argument for the 'h' function as follows

$$\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1+2 \\ 3-4 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$a^{(2)} = h_2(\begin{pmatrix} 4 \\ -1 \end{pmatrix}) = max(\begin{pmatrix} 4 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

Therefore, the output of the hidden layer is $\boxed{a^{(2)} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}}$. This is the input to the final output layer, which has an identity activation function, i.e, $h(x) = x$

Next we compute - $output = h_{output}(W_2 a^{(2)} + b_2)$

We again compute the argument for h_{output} as follows

$$\begin{pmatrix} 2 & 2 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 8+0 \\ 8+0 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

$\Rightarrow$

$$h_2(\begin{pmatrix} 8 \\ 4 \end{pmatrix}) = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

Therefore, the output activation of the MLP is

$$\boxed{output - activation = \begin{pmatrix} 8 \\ 4 \end{pmatrix}}$$

0.2 Problem 2

0.2.1 HD5

In this problem, I have generated 25 random binary sequences each of length 20 manually. Before generating the HD5 file, I have changed the following variable from

```
DATA_FNAME = 'brandon_franzke_hw1_1.hd5'
```

to

```
DATA_FNAME = 'Aditi_Bodhankar_EE541_Hw2Prob2.hd5'
```

And then the .hd5 file was submitted to Autolab. Here is the modified Python_code.

[ ]:
```
## copyright, Keith Chugg
##  EE599, 2020

#################################################
## this is a template to illustrate hd5 files
##
## also can be used as template for HW1 problem
```

```
##################################################

import h5py
import numpy as np
import matplotlib.pyplot as plt

DEBUG = False
DATA_FNAME = 'Aditi_Bodhankar_EE541_Hw2Prob2.hd5'

if DEBUG:
    num_sequences = 3
    sequence_length = 4
else:
    num_sequences = 25
    sequence_length = 20

### Enter your data here...
# ----------------------- MY CODE ----------------------- #
"""
    Here I create a random binary sequence generator using numpy as follows -
"""
my_list = [
    [1, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0],
    [0, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1],
    [0, 1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1],
    [1, 1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1],
    [1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0],
    [0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0],
    [1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1],
    [1, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0],
    [0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1],
    [1, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 1],
    [0, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0],
    [0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0],
    [1, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0],
    [0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 0],
    [1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1],
    [0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1],
    [0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 1, 1, 0, 1],
    [1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0],
    [0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 0],
    [1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 0, 1, 1],
    [0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0],
    [1, 0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0],
    [0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 1],
    [0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0],
    [1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1],
```

```
]

# ----------------------   END   ---------------------- #
### Be sure to generate the data by hand.  DO NOT:
###     copy-n-paste
###     use a random number generator
###
x_list = [
    [ 0, 1, 1, 0],
    [ 1, 1, 0, 0],
    [ 0, 0, 0, 1]
]

# convert list to a numpy array...
human_binary = np.asarray(my_list)

### do some error trapping:

assert human_binary.shape[0] == num_sequences, 'Error: the number of sequences␣
 ↪was entered incorrectly'
assert human_binary.shape[1] == sequence_length, 'Error: the length of the␣
 ↪seqeunces is incorrect'

# the with statement opens the file, does the business, and close it up for us...
with h5py.File(DATA_FNAME, 'w') as hf:
    hf.create_dataset('human_binary', data = human_binary)
    ## note you can write several data arrays into one hd5 file, just give each␣
 ↪a different name.

###################
# Let's read it back from the file and then check to make sure it is as we wrote.
 ↪..
with h5py.File(DATA_FNAME, 'r') as hf:
    hb_copy = hf['human_binary'][:]

### this will throw and error if they are not the same...
np.testing.assert_array_equal(human_binary, hb_copy)
```

0.3 Problem 3

0.4 LMS

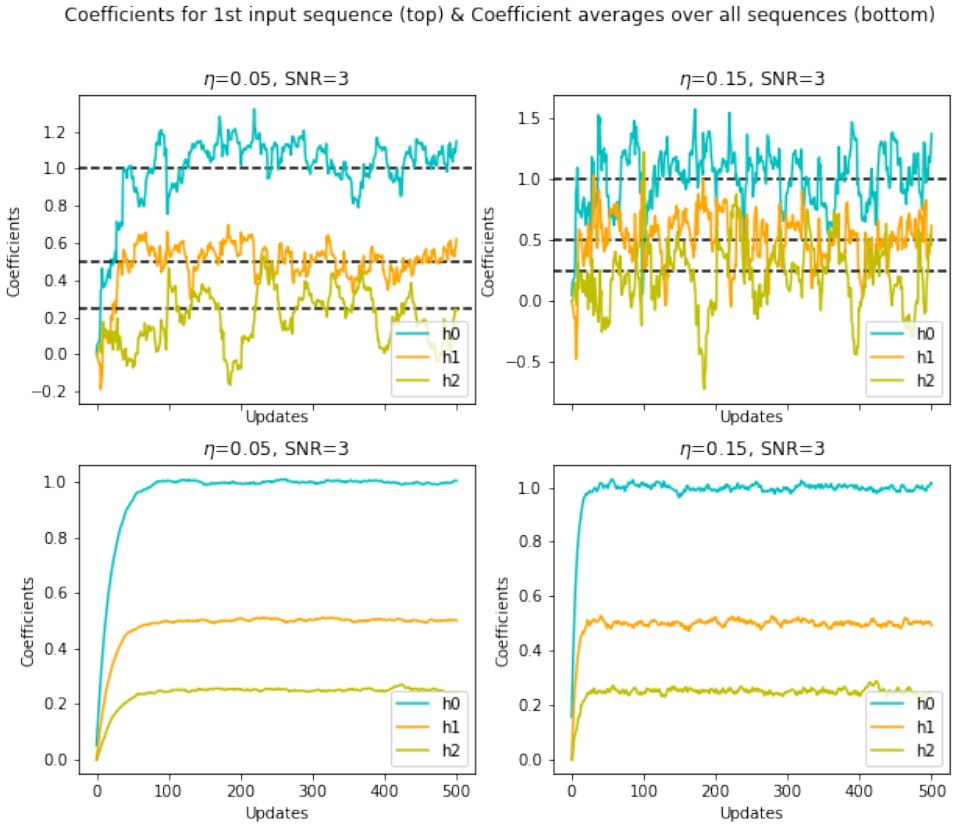
0.4.1 Solution 3(a) - Part(i) - MATCHED FILTER CASE

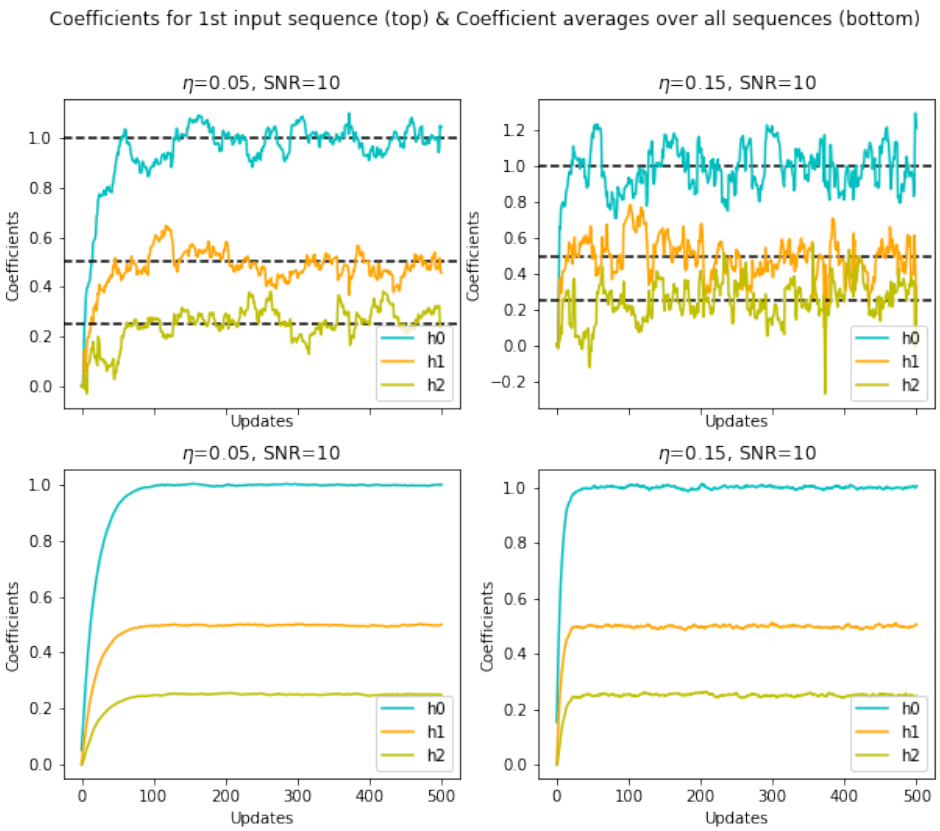
We are given three kinds of datasets - - Matched - SNR = 3dB, 10 dB - Time Varying - Mismatched

We first begin with buiding the Weiner Filter and calculate the coefficients. In the Appendix, we have created a class - LMS comprising of the method - adapt_filter which is the weiner filter. This

filter calculates the coefficients as weights, corresponding predictions of the y_n and square_error.

The calculated coefficients are as follows for learning rates $\boxed{\eta = 0.05, 0.15}$ and SNR $=$ 3dB and 10dB

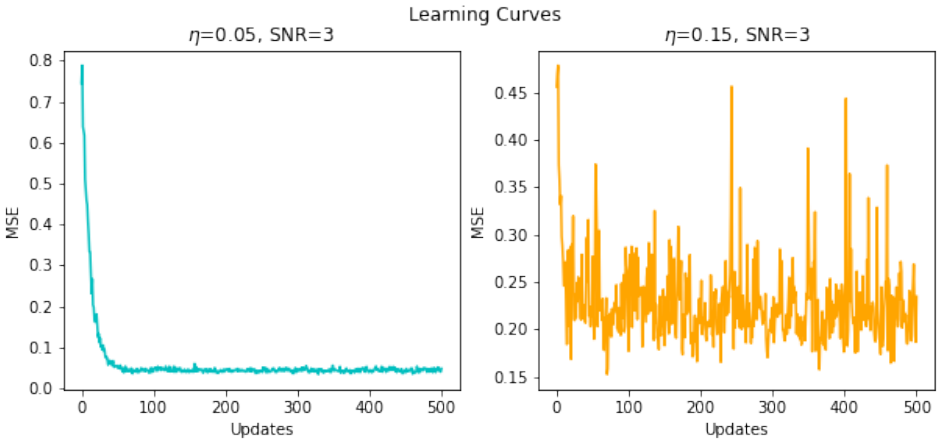


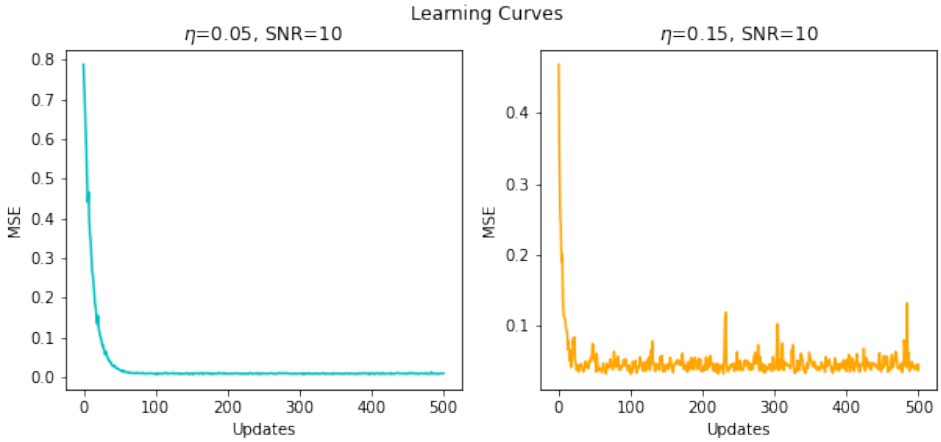


As seen from the plots, the filter predicts the coefficients approximately in the range of the actual coefficient of x_n signal generator, which aligns with the concept of 'Matched" filter.

0.4.2 Solution 3(a) - Part(ii)

Here we plot the Learning Curves in each 'SNR' case. The Learning curves are plots of MSE w.r.t. sample number and averaged over all sequences. We see that the in both the SNR cases, for a learning rate of 0.05, the filter was learning the coefficients better and the overall MSE was reduced with less fluctuations and high convergence.



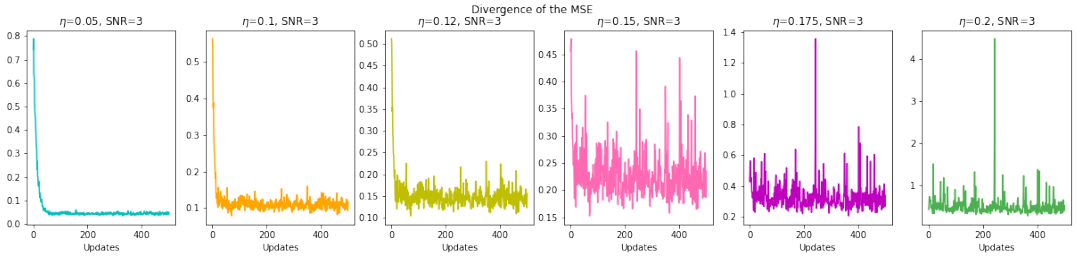


0.4.3 Solution 3(a) - Part(iii)

In order to study the divergence of the MSE in both the SNR cases, I have run the LMS model on the matched dataset for a set of 6 different learning rates = $\eta = [\mathbf{0.05, 0.1, 0.12, 0.15, 0.175, 0.2}]$, to see the behavior of the MSE and to note the largest learning rate beyond which the divergence in MSE is unavoidable.

The following are the learning curves -

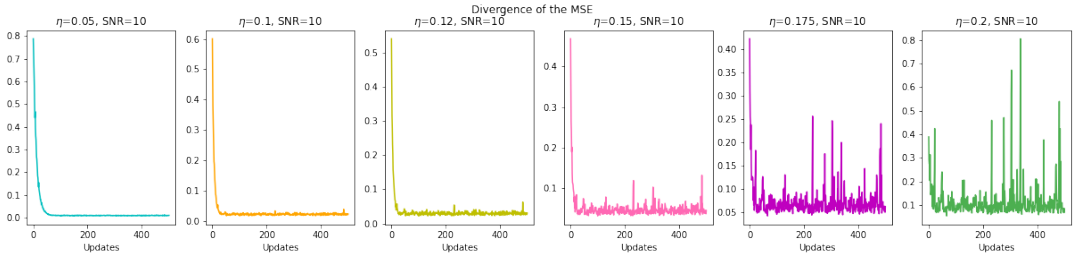
- The first one is for SNR = 3dB:



As seen in the plots, from the learning rate of 0.12, the divergence and the fluctuations in MSE with varying datasamples is quite high. Therefore, a learning rate, $\boxed{\eta_{largest}^{3dB} \approx 0.12}$ can be the largest possible value before the MSE starts to diverge.

Next,

- The second one is for SNR = 10dB:

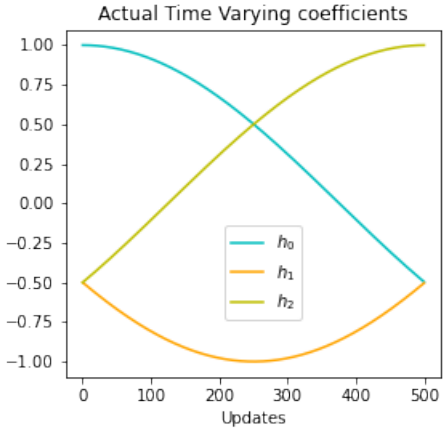


With the same set of 6 learning rates

We see that, in the case of SNR = 10dB, the divergence in MSE is significant when the Learning rate increases more than ≈ 0.15. There the largest learning rate can be recorded as, $\boxed{\eta_{largest}^{10dB} \approx 0.15}$

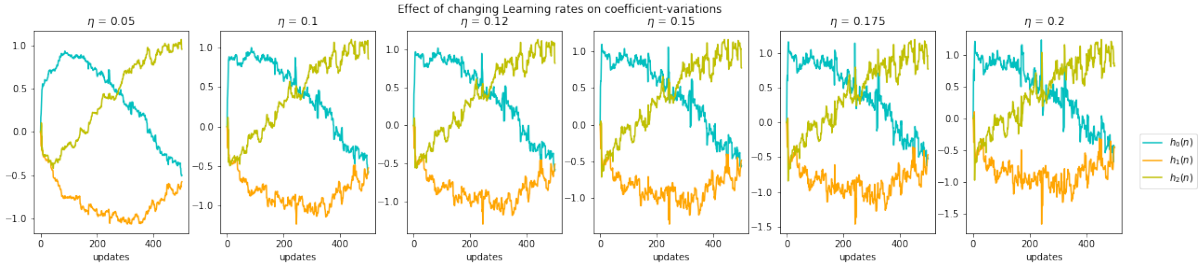
0.4.4 Solution 3(b) - TIME-VARYING FILTER CASE

In this case, we use the 'timevarying' datasets on which the LMS algorithm is trained. In this case, we are already given a set of time-varying coefficients from the signal generator, which is as follows -

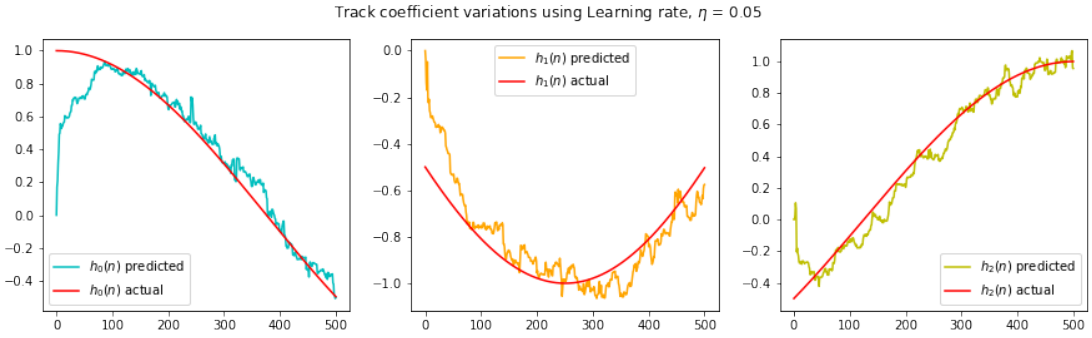


I then run the LMS algorithms on the timevarying data for different values of learning rates, η, to check the convergence and choose the best η that tracks the variations in the predicted coefficients with higher accuracy.

The following are the plots of the timevarying coefficients for the above mentioned set of 6 different learning rates -

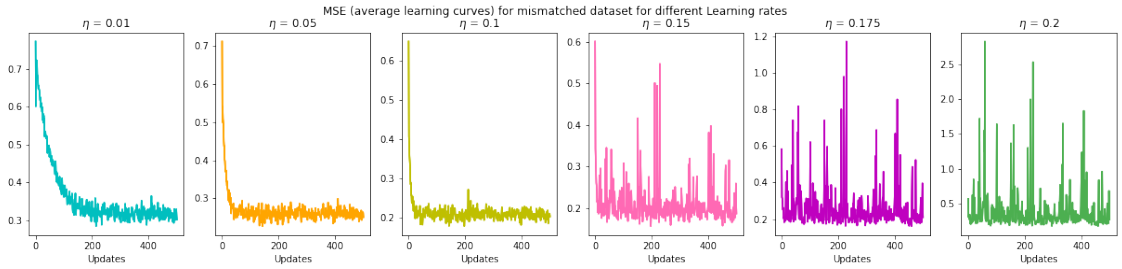


As seen from the learning rate, $\eta = 0.05$, the tracking of the coefficient variations is more accurate, with less noise. With $\eta = 0.1$, the predicted time-varying coefficients are also closely tracking the actual coefficients. But the noise in the latter case is quite higher. Therefore, we choose the smallest and approximately accurate $\boxed{\eta = 0.05}$. For better Data visualization, the actual and the predicted timevarying coefficients are plotted against each other as follows -



0.4.5 Solution 3(c) - MISMATCHED FILTER CASE

In this case, we run the LMS algorithm over the 600 samples for different learning rates and compute the Learning curves as follows -



As seen with increasing learning rate, the divergence in MSE is increasing with slight decrease in the overall MSE from 0.35 to 0.2 and again increasing to 0.5

Then we compute the Correlation matrices- $\hat{R}_v$ and $\hat{r}_{vy}$ using the following scalar from vector equations -

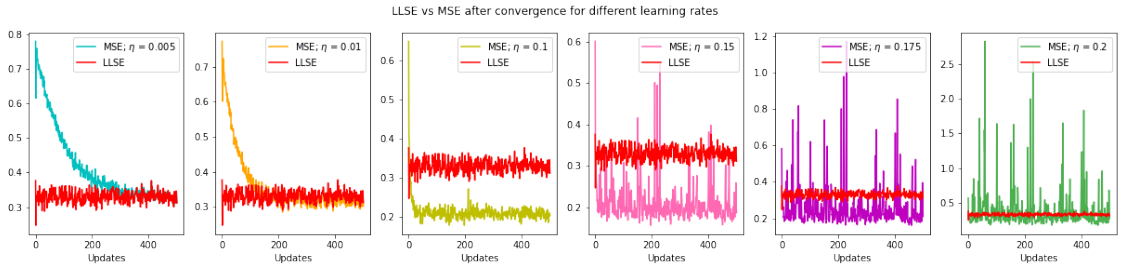
$$\hat{R}_v = \frac{1}{N}(\boldsymbol{V}^{\boldsymbol{T}}\boldsymbol{V})$$

$$\hat{r}_{vy} = \frac{1}{N}(\boldsymbol{V}^{\boldsymbol{T}}\boldsymbol{y})$$

$$\boldsymbol{w} = \hat{R}_v^{-1}\hat{r}_{vy}$$

$$\hat{\boldsymbol{y}} = \boldsymbol{V}\boldsymbol{w}$$

As seen from the code - we have the following plots



As seen from the plots, $\boxed{LLSE \approx 0.35}$ and almost same as MSE for very small learning rates, but as learning rate increases MSE reduces to - $\boxed{MSE \approx 0.20}$ and LLSE is higher. but as the learning rate increases further, LLSE is MSE is lower than LLSE and the former has more noise (fluctuations) with increasing data samples, going far away from convergence.

Therefore we have -

$$\boxed{LLSE \geq MSE}$$

0.5 Python Code Appendix

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import h5py
from scipy.signal import lfilter
f = h5py.File('lms_fun_v3.hdf5', 'r')

"""
Defining the LMS parameters as follows -
    L: no. of filter taps
    Learning_rate: for updating the weights
    N: Total no. of Input Sequences
    n: no. of data samples in each input sequence
"""
L = 3 # numbe of filter taps in Weiner filter
N = 600 # sequence length
n = 501 # number of samples in each sequence
learning_rate = [0.05, 0.1, 0.12,0.15, 0.175, 0.2]
#learning_rate = [0.05, 0.15] # for Problem 3(a)

# ----------------------------------------- MATCHED FILTER␣
 ↪----------------------------------------- #

## for snr = 3dB
SNR = [3,10]
matched_v = np.zeros([len(SNR),N,n,L]) # regressor
matched_x = np.zeros([len(SNR),N,n]) # inputs
matched_y = np.zeros([len(SNR),N,n]) # ground truth outputs
matched_z = np.zeros([len(SNR),N,n]) # noisy target
```

```
matched_v[0,:,:,:] =  f['matched_3_v']
matched_v[1,:,:,:] =  f['matched_10_v']

matched_y[0,:,:] = f['matched_3_y']
matched_y[1,:,:] = f['matched_10_y']

matched_x[0,:,:] = f['matched_3_x']
matched_x[1,:,:] = f['matched_10_x']

matched_z[0,:,:] = f['matched_3_z']
matched_z[1,:,:] = f['matched_10_z']

"""
    Creating an LMS class as follows -
    The adapt_filter function calculates three terms -
        1. filter output 'y'.
        2. Weights - (600, 501, 3) sized 3D array having the coefficients.
        3. w - (3,) sized 1D array calculating coefficients for each data sample␣
 ↪in each input sequence.
        4. e - error between the original labeled output and the predicted␣
 ↪output from the filter.

    The function 'adapt_filter' returns "y, e, Weights" as outputs.

    Additionally I have considered an error tolerance - 'epsilon' to avoid␣
 ↪division by zero while normalization.

"""

class LMS():
    def __init__(self, L=L, learning_rate=learning_rate):
        self.L = L # no. of filter taps
        self.learning_rate = learning_rate # Eeta
        self.w = np.zeros(self.L) # initialiing the weight vector
        self.epsilon = 1e-4 # error tolerance

    """
    BUILDING THE WEINER FILTER
    """

    def adapt_filter(self, v, x, zn, yn, N, n, normalize):
        Weights = np.zeros([N,n,3]) # N = 600, n = 501
        Y = np.zeros([N,n])
        E = np.zeros([N,n])
        for i in range(N): # for each input sequence
            self.w = np.zeros(self.L)
```

```
            for j in range(n): # for each data sample in one input sequence
                prod = np.inner(self.w, v[i,j,:]) # prediction
                if normalize:
                    self.w += self.learning_rate*(zn[i,j]-prod)*v[i,j]/(np.
 ↪inner(v[i,j],v[i,j]) + self.epsilon)
                else:
                    self.w += self.learning_rate*(zn[i,j]-prod)*v[i,j]   # each␣
 ↪data point
                Weights[i,j,:] = self.w

                y = np.inner(self.w, v[i,j]) # filter output with noise w^{T}x #␣
 ↪scalar

                e = (yn[i,j] - y)**2 # error in the prediction # scalar

                Y[i,j] = y
                E[i,j] = e

        return E, Y, Weights

# creating an LMS class instance -
def Compute_LMS(v, x, z, y, snr):

    # creating figures
    fig1,axs = plt.subplots(2,2, figsize = (10,8),sharex=True, sharey=False)
    fig2,axes = plt.subplots(1,6, figsize = (21,4), sharex = True, sharey =␣
 ↪False)

    for i in range(len(learning_rate)):
        mse = np.zeros([n])
        lr = learning_rate[i]
        lms = LMS(L=3, learning_rate=lr)

        # running the LMS algorithm
        square_error, predicted_output, weights = lms.adapt_filter(v, x, z, y,␣
 ↪N=N, n=n, normalize=False)

        # calculating mean square error over all data points
        for m in range(n): # n= 501 (600,501)
            mse[m] = np.mean(square_error[:,m], axis=0) # (501,)

        ## coefficient averages
        weights_average = np.zeros([n,3])
        for k in range(n):
            for j in range(L):
                weights_average[k,j] = np.sum(weights[:,k,j])/N
```

```
        original_weights = [1, 0.5, 0.25] # original coefficients
        labels = ["h0", "h1", "h2"]
        colors = ["c","orange","y","hotpink","m","#4CAF50"]


        ### Plotting predicted coefficients for learning_rates = [0.05, 0.15]

        fig1.suptitle(f'Coefficients for 1st input sequence (top) & Coefficient␣
 ↪averages over all sequences (bottom)')

        for j in range(L):
            axs[0,i].axhline(y=original_weights[j], color='black',␣
 ↪linestyle='--')
            axs[0,i].plot(weights[0,:,j], color=colors[j], label=labels[j])
            axs[0,i].legend(loc="lower right")
            axs[0,i].set_xlabel('Updates')
            axs[0,i].set_ylabel('Coefficients')
            axs[0,i].set_title(f'$\eta$={lr}, SNR={snr}')
            axs[1,i].plot(original_weights[j], color=colors[j])
            axs[1,i].plot(weights_average[:,j], color=colors[j], label=labels[j])
            axs[1,i].legend(loc="lower right")
            axs[1,i].set_xlabel('Updates')
            axs[1,i].set_ylabel('Coefficients')
            axs[1,i].set_title(f'$\eta$={lr}, SNR={snr}')

        ###### Plot of Learning Curves #
        axes[i].plot(mse[:], color=colors[i])
        axes[i].set_title(f'$\eta$={lr}, SNR={snr}')
        axes[i].set_xlabel('Updates')
        #axes[i].set_ylabel('MSE')
        #axes[i].set_ylim([-0.1, 1])
        fig2.suptitle(f'Divergence of the MSE')
        #plt.savefig('Result_figures/Problem1(a-ii)_'+str(SNR[i])+'dB.png')

print("LMS successfully adapted!")
print("Weights, error and Output signal successfully predicted!")
```

```
LMS successfully adapted!
Weights, error and Output signal successfully predicted!
```

[ ]:
```
# ---------------------------- COMPUTATIONS FOR MATCHED FILTER␣
 ↪---------------------------- #

for i in range(len(SNR)):
    Compute_LMS(matched_v[i,:,:,:], matched_x[i,:,:,:], matched_z[i,:,:,:],␣
 ↪matched_y[i,:,:,:], snr=SNR[i])
    #plt.savefig('Result_figures/Problem1(a-iii)_'+str(SNR[i])+'dB.png')
```

```
# ---------------------------------- Time Varying coefficients␣
 ↪------------------------------- #

time_varying_coeff = f['timevarying_coefficents']
time_varying_v = np.reshape(f['timevarying_v'],(1,501,3))
time_varying_x = np.reshape(f['timevarying_x'],(1,501))
time_varying_y = np.reshape(f['timevarying_y'],(1,501))
time_varying_z = np.reshape(f['timevarying_z'],(1,501))
time_varying_coeff.shape # (1,501,3) --- 1 input sequence with 501 samples has 3␣
 ↪coeff changing with time.

#### Plotting actual time varying coefficients from the␣
 ↪'timevarying_coefficients' object given in the dataset

plt.figure(figsize=(4.2,4))
colors = ["c","orange","y","hotpink","m","#4CAF50"]
labels = ["$h_{0}$", "$h_{1}$", "$h_{2}$"]
for i in range(3):
    plt.plot(time_varying_coeff[:,i], color=colors[i], label=labels[i])
plt.legend(bbox_to_anchor=(0.65, 0.3),loc='center right')
plt.title(f'Actual Time Varying coefficients')
plt.xlabel('Updates')
plt.ylabel('Coefficients')
#plt.savefig('Result_figures/Problem1(b)3.png')

## creating figures

fig1,axs = plt.subplots(1,3,figsize=(15,4),sharex = True, sharey = False)
fig2,axes = plt.subplots(1,6, figsize = (21,4), sharex = True, sharey = False)
fig1.suptitle(f'Track coefficient variations using Learning rate, $\eta$ =␣
 ↪'+str(learning_rate[0]))

for i in range(len(learning_rate)):
    lr = learning_rate[i]
    lms = LMS(L=3, learning_rate=lr)

    # running the LMS algorithm
    square_error, predicted_output, weights = lms.adapt_filter(time_varying_v,␣
 ↪time_varying_x, time_varying_z,
                                                                time_varying_y,␣
 ↪N=1, n=n, normalize=False)

    labels = ['$h_{0}(n)$', '$h_{1}(n)$', '$h_{2}(n)$']
```

```
    #### Plotting predicted coefficients for different learning rates to choose␣
 ↪the best learning_rate

    for j in range(L):
        axes[i].plot(weights[0,:,j], color=colors[j], label=labels[j])
        axes[i].set_title('$\eta$ = '+str(lr))
        axes[i].set_xlabel('updates')

    ######## Plotting predicted coefficients against actual coeffficients for␣
 ↪the best learning_rate from above
    #learning_rate =[0.1] best learning_rate

    labels_actual = ['$h_{0}(n)$ actual', '$h_{1}(n)$ actual', '$h_{2}(n)$␣
 ↪actual']
    labels_predicted = ['$h_{0}(n)$ predicted', '$h_{1}(n)$ predicted',␣
 ↪'$h_{2}(n)$ predicted']
    axs[0].plot(weights[0,:,0], color='c', label=labels_predicted[0])
    axs[0].plot(time_varying_coeff[:,0], color='r', label=labels_actual[0])
    axs[0].legend(loc='lower left')
    axs[0].set_xlabel()

    axs[1].plot(weights[0,:,1], color='orange', label=labels_predicted[1])
    axs[1].plot(time_varying_coeff[:,1], color='r', label=labels_actual[1])
    axs[1].legend(loc='upper center')

    axs[2].plot(weights[0,:,2], color='y', label=labels_predicted[2])
    axs[2].plot(time_varying_coeff[:,2], color='r', label=labels_actual[2])
    axs[2].legend(loc='lower right')


fig2.suptitle(f'Effect of changing Learning rates on coefficient-variations')
axes[i].legend(bbox_to_anchor=(1.3, 0.2), loc='lower center')
plt.savefig('Result_figures/Problem1(b)2.png')

# ------------------------------------ MISMATCHED FILTER CASE ␣
 ↪------------------------------------ #

mismatched_v = f['mismatched_v']
mismatched_x = f['mismatched_x']
mismatched_y = f['mismatched_y']

# running the LMS algorithms on different learning rates
learning_rate =[0.01, 0.05, 0.1, 0.15, 0.175, 0.2]

#fig1,axs = plt.subplots(1,3,figsize=(15,4),sharex = True, sharey = False)
fig2,axes = plt.subplots(1,6, figsize = (21,4), sharex = True, sharey = False)
```

```
for i in range(len(learning_rate)):
    mse = np.zeros([n])
    lr = learning_rate[i]
    lms = LMS(L=3, learning_rate=lr)
    square_error, predicted_output, weights = lms.adapt_filter(mismatched_v,␣
 ↪mismatched_x, mismatched_y,
                                                               mismatched_y,␣
 ↪N=N, n=n, normalize=False)

    for m in range(n):
            mse[m] = np.mean(square_error[:,m], axis=0)

    labels = ['$h_{0}(n)$', '$h_{1}(n)$', '$h_{2}(n)$']

    for j in range(L):
        axes[i].plot(mse[:], color=colors[i], label=labels[j])
        axes[i].set_title('$\eta$ = '+str(lr))
        axes[i].set_xlabel('Updates')

fig1.suptitle(f'Track coefficient variations using Learning rate, $\eta$ =␣
 ↪'+str(learning_rate[0]))
fig2.suptitle(f'MSE (average learning curves) for mismatched dataset for␣
 ↪different Learning rates')
#plt.savefig('Result_figures/Problem1(c)1.png')
```

[ ]:
```
# ------------------------------------  R_v, r_vy AND LLSE COMPUTATIONS ␣
 ↪---------------------------------- #

LLSE = np.zeros([N,n]) # LLSE vector
RV = np.zeros([N,3,3]) # Ccorrelation vector
Rvy = np.zeros([N,3,1]) # correlation vector
W = np.zeros([N,3,1]) # weights

for i in range(N):
    # ------ calculating R_v ----- #
    prod1 = np.matmul(np.transpose(mismatched_v[i,:,:]),mismatched_v[i,:,:])/n
    R_v = prod1
    RV[i,:,:] = R_v

    # ------ calculating r_vy ----- #
    prod2 = np.matmul(np.transpose(mismatched_v[i,:,:]),mismatched_y[i,:])
    r_vy = prod2/n
    Rvy[i,:,:] = r_vy.reshape((3,1))

    # ------ computing weights ----- #
    weight = np.matmul(np.linalg.inv(R_v),r_vy)
```

```
    W[i,:,:] = weight.reshape((3,1))

    # ------- predict target signal ----- #
    y = np.matmul(mismatched_v[i,:,:], weight)

    llse =  (mismatched_y[i,:] - y)**2
    llse.shape
    LLSE[i,:] = llse # each for each sequence

## Plots

LLSE_average = np.mean(LLSE, axis=0) # over all sequences
learning_rate =[0.01, 0.05, 0.1, 0.15, 0.175, 0.2]
fig3, axes1 = plt.subplots(1,6, figsize = (21,4), sharex = True, sharey = False)
for i in range(len(learning_rate)):
    lr = learning_rate[i]
    label1 = 'MSE; $\eta$ = '+str(lr)
    axes1[i].plot(MSE[i,:], color=colors[i], label=label1)
    axes1[i].set_xlabel('Updates')
    axes1[i].plot(LLSE_average, color='red', label='LLSE')
    axes1[i].legend()
fig3.suptitle(f'LLSE vs MSE after convergence for different learning rates')
#plt.savefig('Result_figures/Problem1(c)2.png')
```