Homework-4

5.7

a) $$\hat{p} = \hat{P}\{X \in A\} = \frac{\text{no. of } X_1 \dots X_n \in A}{N}$$

To estimate $P\{X > Y\}$ where X & Y are independent Poisson random variables with parameters 3 & 5 respectively.

1. Generate Random samples for the 2 Poisson distributions (Poisson (3) & Poisson(5)) from uniform random variables

$$X = \max\{k : U_1 . U_2 \dots U_k \geq e^{-3}\}$$
$$Y = \max\{k : V_1 . V_2 \dots V_k \geq e^{-5}\}$$

2. For each pair in (x,y) ~~the~~ in X & Y, check if $x > y$. IF $x > y$ our condition is met

if $x > y$

$R = 1$

else

$R = 0$

3. Count the number of times the condition ($x>y$) is met and divide it by the no. of samples generated for either distribution. This ratio is our estimate for the probability $P\{X > Y\}$

5.7 b) Royal flush (a ten, a jack, a queen, a king, an ace) of same color.

To estimate the probability of a royal flush:

1. Generate 5 numbers from a uniform (0, 1) distribution
2. Assign each card a value from 0 to 51.
3. For the 5 numbers generated, multiply each number by 52 & take its floor. By doing so each ~~card~~ number is assigned a particular card.
4. If ~~[illegible]~~ no. of unique cards is not 5, repeat steps 1 to 3.
5. If 5 unique cards are generated, check if the 5 cards ~~match the 4~~ matches with any of the 4 flushes present in the deck of cards.
   We can ~~[illegible]~~ assume cards numbered (0, 1, 2, 3, 4) (5, 6, 7, 8, 9), (10, 11, 12, 13, 14) & (15, 16, 17, 18, 19) as the 4 flushes.
6. Repeat steps 1 - 5, N no. of times & count the number of times a flush is achieved.
7. Find the ratio of success & N. This would give the estimate of the probability of achieving a royal flush.

c) $\lambda_1 = 5\,hr^{-1}$ $\lambda_2 = 20\,hr^{-1}$

$P_1 = 1/5$ $P_2 = 4/5$

Considering the probability of getting work done by mechanic 1 follows a Bernoulli distribution with probability $1/5$.
And the service time of each mechanic follows Exponential distribution with $\lambda = 5\,hr^{-1}$ & $20\,hr^{-1}$

~~To~~ To generate Random variable samples of this mode

1. If uniform random variable is less than ~~equal to~~ $1/5$ generate samples from exponential distribution with parameter 5

2. if uniform Random variable is greater than $1/5$, generate samples from exponential distribution with parameter 20.

$$\text{if } U_1 \leq 1/5$$
$$X = \frac{-1}{5} \ln(1-U_2)$$
$$\text{else}$$
$$X = \frac{-1}{20} \ln(1-U_2)$$

5. Repeat 1-2, N times and count the number of times the random variable is greater than $35/60$ ~~minutes~~ hrs ~~Take it r~~ The ratio of the count with N, gives the estimate of the probability $P\{X > 35/60\,hr\}$.

d) $\varepsilon = 0.005$ $\quad 1-\alpha = 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025$

In order to guarantee that $P\{|\hat{p} - p| > \varepsilon\} \leq \alpha$

$N \geq p^*(1-p^*)\left(\frac{z_{\alpha/2}}{\varepsilon}\right)^2$ random variables where p^* is a preliminary estimator of p.

or

$N \geq 0.25\left(\frac{z_{\alpha/2}}{\varepsilon}\right)^2$ random variables, if no such estimator is available

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For a, b, c, No estimator of p is given.

For each (a, b, c) $\quad N \geq 0.25\left(\frac{z_{\alpha/2}}{\varepsilon}\right)^2$

$$N \geq 0.25\left(\frac{z_{0.025}}{0.005}\right)^2$$

$$N \geq 0.25\left(\frac{-1.96}{0.005}\right)^2$$

$$N \geq 38416$$

$N = 38416$ should be enough to guarantee an error not exceeding 0.005 with probability $= 0.95$.