# Summary

December 6, 2024

# Contents

0.1	Loaded	Graphs	 	2
	0.1.1	<u>-</u> {0}	 	2
	0.1.2	_{_{1}}	 	2
	0.1.3	:_{2}	 	2
0.2	Produc	Graphs	 	2
	0.2.1	_{0,56}	 	2
0.3	Enume	ated Flows	 	3
	0.3.1	olution $0 \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	 	3
	0.3.2	olution 1	 !	5
	0.3.3	olution $2 \ldots \ldots \ldots \ldots \ldots$	 ,	7
	0.3.4	olution 3	 	9
	0.3.5	olution 4	 1	1
	0.3.6	olution 5	 13	3
	0.3.7	olution $6 \ldots \ldots \ldots \ldots \ldots \ldots \ldots$	 1	5

# 0.1 Loaded Graphs

## $0.1.1 g_{0}$

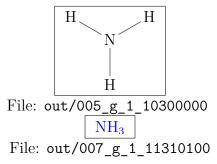
N C H

File: out/001\_g\_0\_10300000

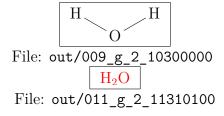
N CH

File: out/003\_g\_0\_11310100

0.1.2 g\_{1}

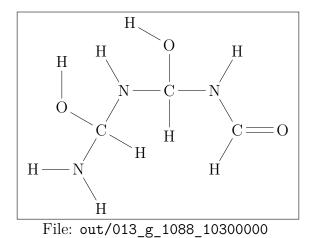


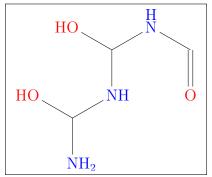
0.1.3 g\_{2}



# 0.2 Product Graphs

# 0.2.1 p\_{0,56}



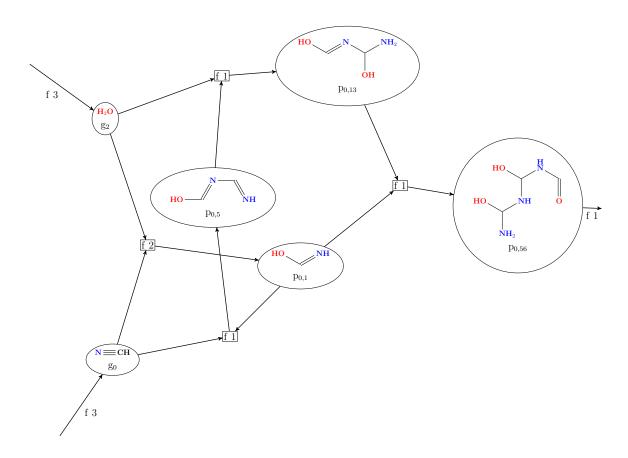


File: out/015\_g\_1088\_11310100

# 0.3 Enumerated Flows

## 0.3.1 Solution 0

Objective val	Lue	(non-integral): -0.101116			
Vertex/Graph	In	Out	G	logK	t_order
g_{0}	3	0	-5.507152	-3.000000	0
g_{2}	3	0	-5.068033	-3.000000	0
p_{0,13}	0	0	-21.240635	-3.000000	65
p_{0,1}	0	0	-10.610101	-3.000000	1
p_{0,56}	0	1	-31.869324	-6.000000	67
p_{0,5}	0	0	-16.158998	-6.000000	2



 $File: \verb"out/027_dg_0_11100_f_0_0_filt"$ 

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	$x_v^K$	$x_v^{\Delta G}$	$t_v$
$g_0$	3	0	-14459.025578	-3.0	-14469.002933	0
$g_2$	3	0	-13306.118349	-3.0	-13316.095704	0
$p_{0,1}$	0	0	-27856.817054	-3.0	-27866.79441	1
$p_{0,5}$	0	0	-42425.44262	-6.0	-42445.397331	2
$p_{0,13}$	0	0	-55767.279594	-3.0	-55777.256949	65
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	2	-84.863583	-84.863583
14	$g_0, p_{0,1},$	$p_{0,5},$	1	-109.599988	-109.599988
123	$g_2, p_{0,5},$	$p_{0,13},$	1	-22.099535	-22.099535
263	$p_{0,1}, p_{0,13},$	$p_{0,56},$	1	-48.802143	-48.802143
Sum					-265.4787993681657

 $\Delta G = -35.02288237153935$ 

 $\Delta E = -0.10111553449717699$ 

|E| = 5

|U| = 4

## **0.3.2** Solution 1

Objective val	lue	(non-integral): -0.101116			
Vertex/Graph	In	Out	G	logK	t_order
g_{0}	3	0	-5.507152	-3.000000	0
g_{2}	3	0	-5.068043	-6.000000	0
p_{0,13}	0	0	-21.240678	-6.000000	64
p_{0,15}	0	0	-10.625195	-3.000000	2
p_{0,1}	0	0	-10.610101	-3.000000	1
p_{0,56}	0	1	-31.869324	-6.000000	67
p_{0,5}	0	0	-16.158998	-3.000000	2



 $File: \ \mathtt{out/038\_dg\_0\_11100\_f\_0\_1\_filt}$ 

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	$x_v^K$	$x_v^{\Delta G}$	$t_v$
$\overline{g_0}$	3	0	-14459.025578	-3.0	-14469.002933	0
$g_2$	3	0	-13306.144961	-6.0	-13326.099672	0
$p_{0,1}$	0	0	-27856.817054	-3.0	-27866.79441	1
$p_{0,5}$	0	0	-42425.44262	-3.0	-42435.419975	2
$p_{0,13}$	0	0	-55767.391129	-6.0	-55787.345839	64
$p_{0,15}$	0	0	-27896.445756	-3.0	-27906.423111	2
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1}$ ,	2	-78.027426	-78.027426
14	$g_0, p_{0,1},$	$p_{0,5},$	1	-102.790443	-102.790443
36	$p_{0,1},$	$p_{0,15},$	1	-39.628701	-39.628701
123	$g_2, p_{0,5},$	$p_{0,13},$	1	-28.994003	-28.994003
169	$p_{0,13}, p_{0,15},$	$p_{0,56},$	1	-2.252362	-2.252362
Sum					-265.47879936816463

 $\Delta G = -32.97203528995104$ 

 $\Delta E = -0.10111553449717658$ 

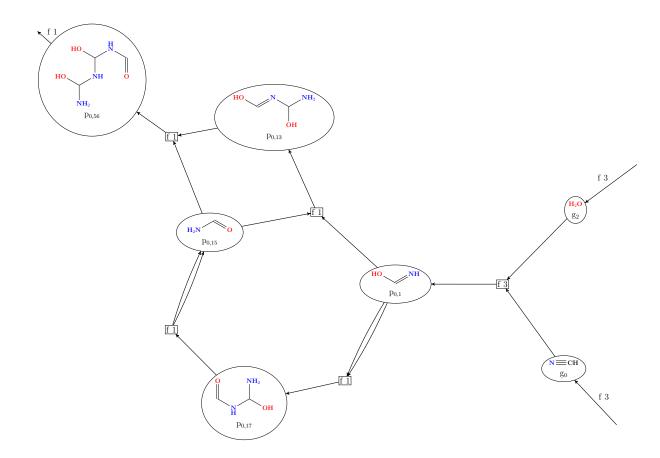
|E| = 6

|U| = 5

#### 0.3.3 Solution 2

#### Overall Data

Objective value (non-integral): -0.0688029 Vertex/Graph In Out G logK t\_order g\_{0} 3 0 -5.507152 -3.000000 0 g\_{2} 3 0 -5.068043 -3.000000 0 p\_{0,13} 0 0 -21.240635 -3.000000 34 p\_{0,15} -10.625195 -6.000000 32 0 0 p\_{0,17} -21.248887 -3.000000 3 0 0 -10.610101 -3.000000 1  $p_{0,1}$ 0  $p_{0,56}$ -31.869324 -6.000000 67



 $File: \ \mathtt{out/049\_dg\_0\_11100\_f\_0\_2\_filt}$ 

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	$x_v^K$	$x_v^{\Delta G}$	$t_v$
$g_0$	3	0	-14459.025578	-3.0	-14469.002933	0
$g_2$	3	0	-13306.144961	-3.0	-13316.122316	0
$p_{0,1}$	0	0	-27856.817054	-3.0	-27866.79441	1
$p_{0,13}$	0	0	-55767.279594	-3.0	-55777.256949	34
$p_{0,15}$	0	0	-27896.445756	-6.0	-27916.400466	32
$p_{0,17}$	0	0	-55788.944315	-3.0	-55798.92167	3
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-84.836971	-84.836971
40	$p_{0,1}, p_{0,1},$	$p_{0,17},$	1	-68.500661	-68.500661
169	$p_{0,13}, p_{0,15},$	$p_{0,56}$ ,	1	-2.363897	-2.363897
192	$p_{0,1}, p_{0,15},$	$p_{0,13},$	1	-0.397694	-0.397694
237	$p_{0,17},$	$p_{0,15}, p_{0,15},$	1	-24.375831	-24.375831
Sum					-180.6420120052131

 $\Delta G = \text{-}35.01489822713569$ 

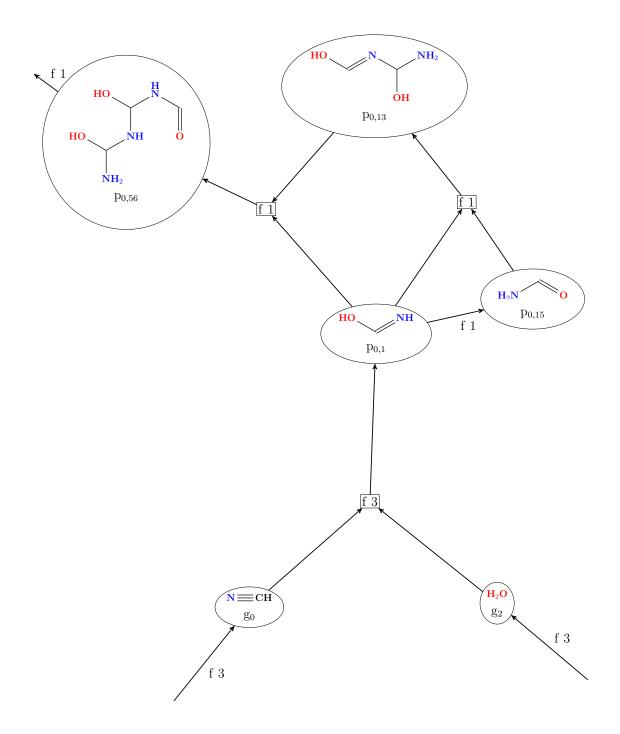
 $\Delta E = \text{-}0.06880290870692735$ 

|E| = 7

|U| = 5

## **0.3.4** Solution 3

Objective val	lue	(non-integral): -0.0688029			
Vertex/Graph	In	Out	G	logK	$t\_order$
g_{0}	3	0	-5.507152	-3.000000	0
g_{2}	3	0	-5.068033	-6.000000	0
p_{0,13}	0	0	-21.240678	-6.000000	46
p_{0,15}	0	0	-10.625174	-6.000000	22
p_{0,1}	0	0	-10.610101	-6.000000	20
p_{0,56}	0	1	-31.869260	-3.000000	67



 $File: \verb"out/058_dg_0_11100_f_0_3_filt"$ 

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	$x_v^K$	$x_v^{\Delta G}$	$t_v$
$g_0$	3	0	-14459.025578	-3.0	-14469.002933	0
$g_2$	3	0	-13306.118349	-6.0	-13326.073059	0
$p_{0,1}$	0	0	-27856.817054	-6.0	-27876.771765	20
$p_{0,13}$	0	0	-55767.391129	-6.0	-55787.345839	46
$p_{0,15}$	0	0	-27896.389963	-6.0	-27916.344673	22
$p_{0,56}$	0	1	-83672.731446	-3.0	-83682.708802	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1}$ ,	3	-84.863583	-84.863583
36	$p_{0,1},$	$p_{0,15},$	1	-39.572909	-39.572909
192	$p_{0,1}, p_{0,15},$	$p_{0,13},$	1	-0.565022	-0.565022
263	$p_{0,1}, p_{0,13},$	$p_{0,56}$ ,	1	-28.094628	-28.094628
Sum					-180.64201200521208

 $\Delta G = -32.28232966215784$ 

 $\Delta E = -0.06880290870692696$ 

|E| = 6

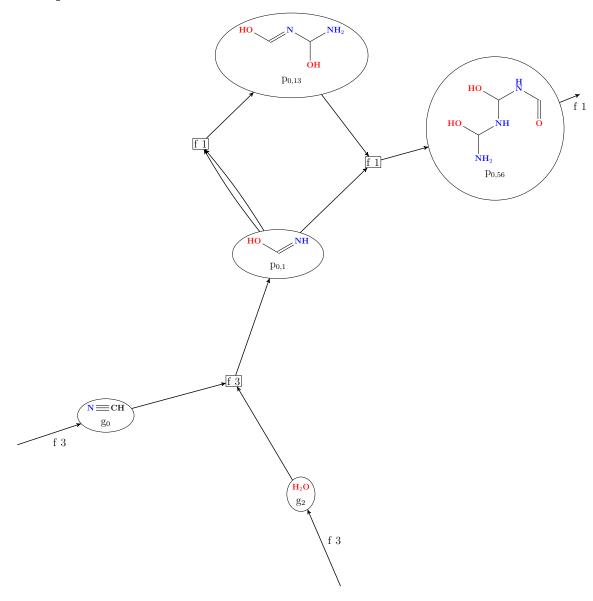
|U| = 4

#### **0.3.5** Solution 4

#### Overall Data

Objective value (non-integral): -0.0688029 Vertex/Graph In Out G logK t\_order g\_{0} 3 0 -5.507152 -6.000000 0 g\_{2} 3 0 -5.068033 -6.000000 0

p\_{0,13} 0 0 -21.240635 -3.000000 3 p\_{0,1} 0 0 -10.610080 -3.000000 1 p\_{0,56} 0 1 -31.869324 -6.000000 67



 $File: \verb"out/067_dg_0_11100_f_0_4_filt"$ 

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	$x_v^K$	$x_v^{\Delta G}$	$t_v$
$g_0$	3	0	-14459.025578	-6.0	-14478.980288	0
$g_2$	3	0	-13306.118349	-6.0	-13326.073059	0
$p_{0,1}$	0	0	-27856.761341	-3.0	-27866.738696	1
$p_{0,13}$	0	0	-55767.279594	-3.0	-55777.256949	3
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-71.18878	-71.18878
30	$p_{0,1}, p_{0,1},$	$p_{0,13},$	1	-46.947368	-46.947368
263	$p_{0,1}, p_{0,13},$	$p_{0,56},$	1	-48.857857	-48.857857
Sum					-180.642012005211

 $\Delta G = \text{-}30.93715597207012$ 

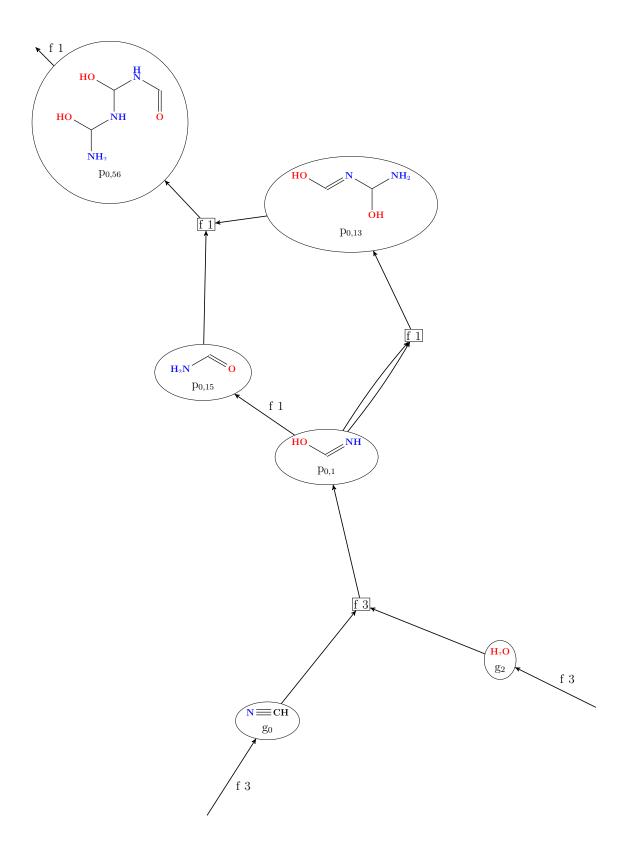
 $\Delta E = \text{-}0.06880290870692654$ 

 $\begin{aligned} |E| &= 5\\ |U| &= 3 \end{aligned}$ 

#### 0.3.6 Solution 5

#### Overall Data

Objective value (non-integral): -0.0688029 Vertex/Graph In Out G logK  $t_{order}$ g\_{0} 3 0 -5.507152 -3.000000 0 g\_{2} 3 0 -5.068043 -3.000000 0  $p_{0,13}$ 0 0 -21.240678 -6.000000 3 p\_{0,15} 0 0 -10.625195 -3.000000 2 p\_{0,1} -10.610101 -3.000000 1 0 0  $p_{0,56}$ 0 1 -31.869324 -6.000000 67



 $File: \ \mathtt{out/076\_dg\_0\_11100\_f\_0\_5\_filt}$ 

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	$x_v^K$	$x_v^{\Delta G}$	$t_v$
$g_0$	3	0	-14459.025578	-3.0	-14469.002933	0
$g_2$	3	0	-13306.144961	-3.0	-13316.122316	0
$p_{0,1}$	0	0	-27856.817054	-3.0	-27866.79441	1
$p_{0,13}$	0	0	-55767.391129	-6.0	-55787.345839	3
$p_{0,15}$	0	0	-27896.445756	-3.0	-27906.423111	2
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-84.836971	-84.836971
30	$p_{0,1}, p_{0,1},$	$p_{0,13},$	1	-53.75702	-53.75702
36	$p_{0,1},$	$p_{0,15},$	1	-39.628701	-39.628701
169	$p_{0,13}, p_{0,15},$	$p_{0,56},$	1	-2.252362	-2.252362
Sum					-180.64201200520992

 $\Delta G = \text{-}35.014898752235624$ 

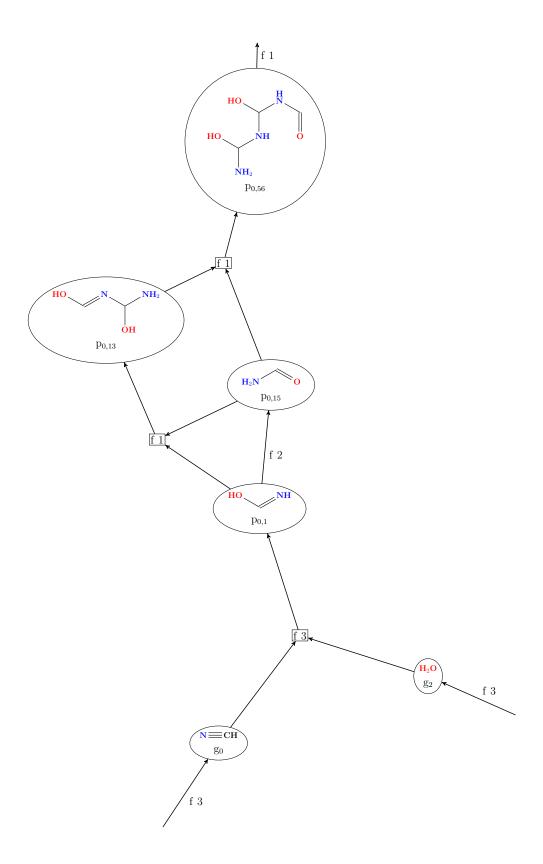
 $\Delta E = \text{-}0.06880290870692614$ 

|E| = 6

|U| = 4

## 0.3.7 Solution 6

Objective val	(non-integral): -0.0537092				
Vertex/Graph In		Out	G	logK	t_order
g_{0}	3	0	-5.507152	-3.000000	0
g_{2}	3	0	-5.068043	-3.000000	0
p_{0,13}	0	0	-21.240678	-6.000000	64
p_{0,15}	0	0	-10.625174	-3.000000	2
p_{0,1}	0	0	-10.610080	-3.000000	1
p_{0,56}	0	1	-31.869324	-6.000000	67



 $File: \ \mathtt{out/085\_dg\_0\_11100\_f\_0\_6\_filt}$ 

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	$x_v^K$	$x_v^{\Delta G}$	$t_v$
$\overline{g_0}$	3	0	-14459.025578	-3.0	-14469.002933	0
$g_2$	3	0	-13306.144961	-3.0	-13316.122316	0
$p_{0,1}$	0	0	-27856.761341	-3.0	-27866.738696	1
$p_{0,13}$	0	0	-55767.391129	-6.0	-55787.345839	64
$p_{0,15}$	0	0	-27896.389963	-3.0	-27906.367318	2
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-84.781257	-84.781257
36	$p_{0,1},$	$p_{0,15},$	2	-39.628622	-39.628622
169	$p_{0,13}, p_{0,15},$	$p_{0,56},$	1	-2.308155	-2.308155
192	$p_{0,1}, p_{0,15},$	$p_{0,13},$	1	-14.239825	-14.239825
Sum					-141.01338990141696

 $\Delta G = \text{-}35.014898752235624$ 

 $\Delta E = \text{-}0.05370916368868583$ 

 $\begin{aligned} |E| &= 7 \\ |U| &= 4 \end{aligned}$