Summary

December 6, 2024

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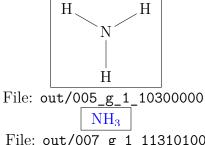
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0.1 Loaded Graphs

0.1.1g_{0}

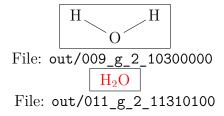
 $N \equiv C - H$ File: out/001_g_0_10300000 $N \equiv CH$ File: out/003_g_0_11310100

0.1.2 g_{1}



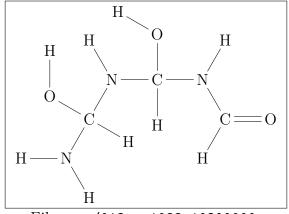
File: out/007_g_1_11310100

0.1.3g_{2}

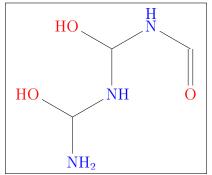


Product Graphs 0.2

0.2.1p_{0,56}



File: out/013_g_1088_10300000



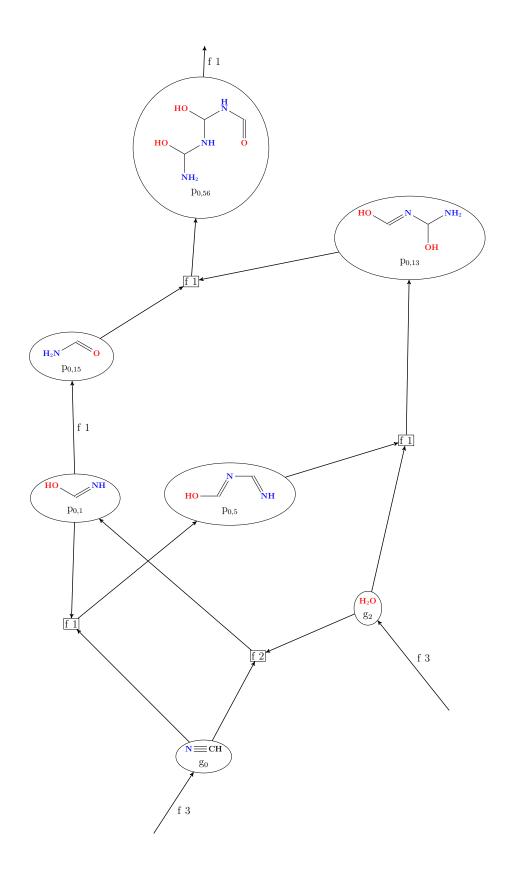
File: out/015_g_1088_11310100

0.3 Enumerated Flows

0.3.1 Solution 0

Overall Data

Objective val	(non-integral): -0.100346				
Vertex/Graph	In	Out	G	logK	t_order
g_{0}	3	0	-5.507141	-3.000000	0
g_{2}	3	0	-5.068043	-3.000000	0
p_{0,13}	0	0	-21.240635	-3.000000	64
p_{0,15}	0	0	-10.625174	-6.000000	2
p_{0,1}	0	0	-10.610101	-3.000000	1
p_{0,56}	0	1	-31.869324	-6.000000	67
p_{0,5}	0	0	-16.158998	-6.000000	63



File: out/029_dg_0_11100_f_0_0_filt

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
$\overline{g_0}$	3	0	-14458.99666	-3.0	-14468.974015	0
g_2	3	0	-13306.144961	-3.0	-13316.122316	0
$p_{0,1}$	0	0	-27856.817054	-3.0	-27866.79441	1
$p_{0,5}$	0	0	-42425.44262	-6.0	-42445.397331	63
$p_{0,13}$	0	0	-55767.279594	-3.0	-55777.256949	64
$p_{0,15}$	0	0	-27896.389963	-6.0	-27916.344673	2
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	2	-84.192417	-84.192417
14	$g_0, p_{0,1},$	$p_{0,5},$	1	-109.628906	-109.628906
36	$p_{0,1},$	$p_{0,15},$	1	-47.055925	-47.055925
123	$g_2, p_{0,5},$	$p_{0,13},$	1	-20.72598	-20.72598
169	$p_{0,13}, p_{0,15},$	$p_{0,56},$	1	-1.746218	-1.746218
Sum					-263.4583849519488

 $\Delta G = -34.75418585038536$

 $\Delta E = -0.10034599928725518$

|E| = 6

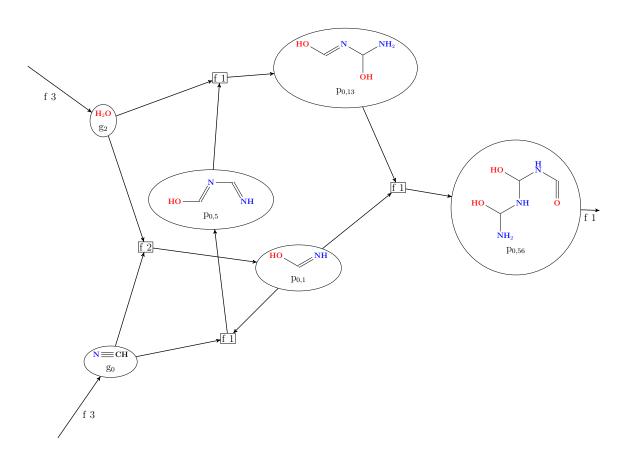
 $|\mathbf{U}| = 5$

0.3.2 Solution 1

Overall Data

Objective value (non-integral): -0.100346

Vertex/Graph	In	Out	G	logK	t_order
g_{0}	3	0	-5.507152	-6.000000	0
g_{2}	3	0	-5.068043	-6.000000	0
p_{0,13}	0	0	-21.240678	-6.000000	34
p_{0,1}	0	0	-10.610101	-3.000000	1
p_{0,56}	0	1	-31.869324	-6.000000	67
p_{0,5}	0	0	-16.158998	-6.000000	33



 $File: \verb|out/038_dg_0_11100_f_0_1_filt|$

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
g_0	3	0	-14459.025578	-6.0	-14478.980288	0
g_2	3	0	-13306.144961	-6.0	-13326.099672	0
$p_{0,1}$	0	0	-27856.817054	-3.0	-27866.79441	1
$p_{0,5}$	0	0	-42425.44262	-6.0	-42445.397331	33
$p_{0,13}$	0	0	-55767.391129	-6.0	-55787.345839	34
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1}$,	2	-69.197466	-69.197466
14	$g_0, p_{0,1},$	$p_{0,5},$	1	-102.116972	-102.116972
123	$g_2, p_{0,5},$	$p_{0,13},$	1	-20.837515	-20.837515
263	$p_{0,1}, p_{0,13},$	$p_{0,56},$	1	-41.207592	-41.207592
Sum					-263.4583849519478

 $\Delta G = -30.255700976893987$

 $\Delta E = \text{-}0.1003459992872548$

|E| = 5

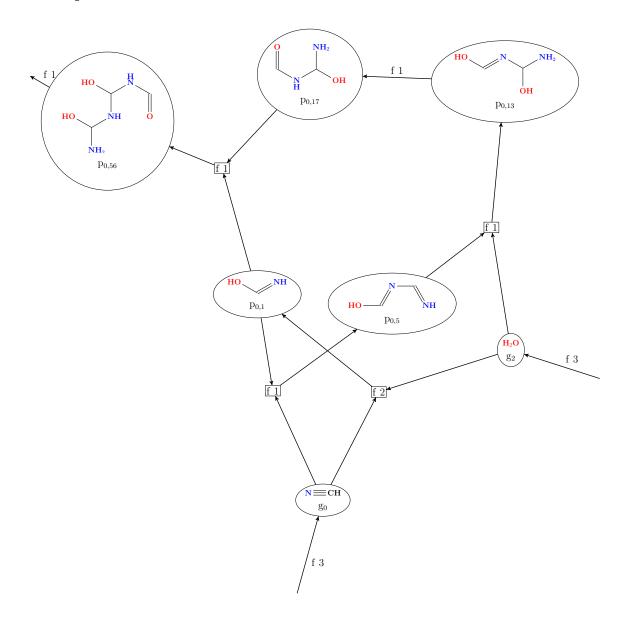
|U| = 4

0.3.3 Solution 2

Overall Data

Objective val	lue	(non-integral): -0.100346				
Vertex/Graph	In	Out	G	logK	t_order	
g_{0}	3	0	-5.507152	-3.000000	0	
g_{2}	3	0	-5.068033	-3.000000	0	
p_{0,13}	0	0	-21.240678	-3.000000	3	
p_{0,17}	0	0	-21.248887	-6.000000	65	
p_{0,1}	0	0	-10.610101	-3.000000	1	
p_{0,56}	0	1	-31.869324	-6.000000	67	
p_{0,5}	0	0	-16.158998	-6.000000	2	

Filtered Graph



 $File: \ \mathtt{out/049_dg_0_11100_f_0_2_filt}$

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
$\overline{g_0}$	3	0	-14459.025578	-3.0	-14469.002933	0
g_2	3	0	-13306.118349	-3.0	-13316.095704	0
$p_{0,1}$	0	0	-27856.817054	-3.0	-27866.79441	1
$p_{0,5}$	0	0	-42425.44262	-6.0	-42445.397331	2
$p_{0,13}$	0	0	-55767.391129	-3.0	-55777.368484	3
$p_{0,17}$	0	0	-55788.944315	-6.0	-55808.899025	65
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	2	-84.190111	-84.190111
14	$g_0, p_{0,1},$	$p_{0,5},$	1	-109.599988	-109.599988
123	$g_2, p_{0,5},$	$p_{0,13},$	1	-20.864127	-20.864127
196	$p_{0,1}, p_{0,17},$	$p_{0,56},$	1	-19.654406	-19.654406
250	$p_{0,13},$	$p_{0,17},$	1	-29.036202	-29.036202
Sum					-263.45838495194414

 $\Delta G = -34.7534937686804$

 $\Delta E = \text{-}0.1003459992872534$

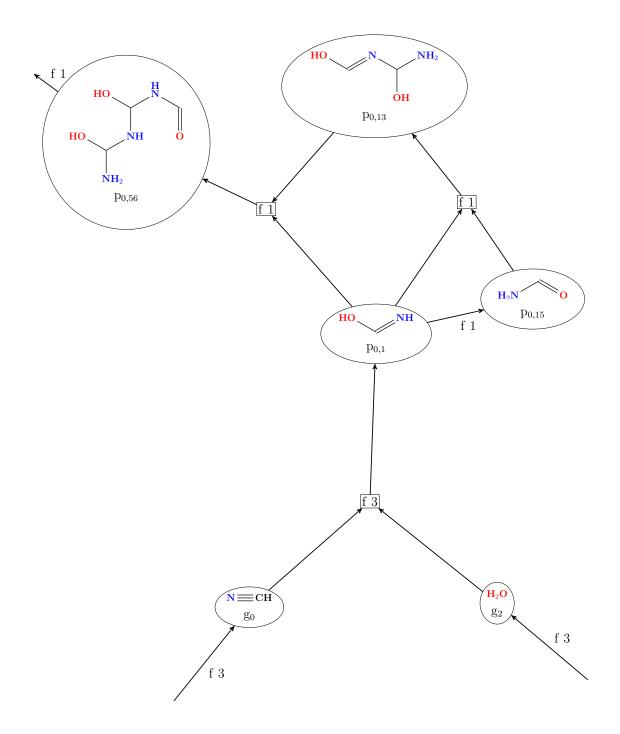
|E| = 6

|U| = 5

0.3.4 Solution 3

Overall Data

Objective value (non-integral): -0.0682899 Vertex/Graph In Out G logK t_order g_{0} 3 0 -5.507152 -6.000000 0 g_{2} 3 0 -5.068043 -6.000000 0 p_{0,13} 0 0 -21.240678 -6.000000 65 p_{0,15} -10.625174 -6.000000 33 0 0 p_{0,1} -10.610101 -5.688972 1 0 0 p_{0,56} 1 -31.869260 -3.000000 67



 $File: \verb"out/058_dg_0_11100_f_0_3_filt"$

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
g_0	3	0	-14459.025578	-6.0	-14478.980288	0
g_2	3	0	-13306.144961	-6.0	-13326.099672	0
$p_{0,1}$	0	0	-27856.817054	-5.688972	-27875.737354	1
$p_{0,13}$	0	0	-55767.391129	-6.0	-55787.345839	65
$p_{0,15}$	0	0	-27896.389963	-6.0	-27916.344673	33
$p_{0,56}$	0	1	-83672.731446	-3.0	-83682.708802	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1}$,	3	-75.904675	-75.904675
36	$p_{0,1},$	$p_{0,15},$	1	-40.348717	-40.348717
192	$p_{0,1}, p_{0,15},$	$p_{0,13},$	1	0.006113	0.006113
263	$p_{0,1}, p_{0,13},$	$p_{0,56}$,	1	-26.850022	-26.850022
Sum					-179.29506906107235

 $\Delta G = -29.490664250894866$

 $\Delta E = \text{-}0.0682898852336476$

|E| = 6

|U| = 4

0.3.5 Solution 4

Overall Data

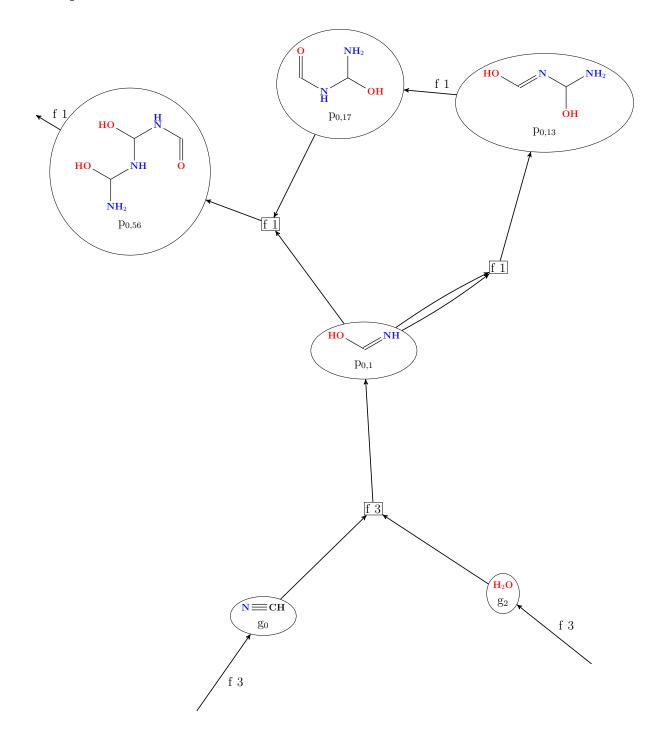
Objective value (non-integral): -0.0682899

Vertex/Graph In Out G logK t_order
g_{0} 3 0 -5.507152 -6.000000 0

g_{1} 3 0 -5.068033 -6.000000 0

g_{2} 3 0 -5.068033 -6.000000 0
p_{0,13} 0 0 -21.240635 -3.000000 3
p_{0,17} 0 0 -21.248844 -3.000000 65
p_{0,1} 0 0 -10.610080 -3.000000 1
p_{0,56} 0 1 -31.869324 -6.000000 67

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 $File: \verb"out/067_dg_0_11100_f_0_4_filt"$

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
g_0	3	0	-14459.025578	-6.0	-14478.980288	0
g_2	3	0	-13306.118349	-6.0	-13326.073059	0
$p_{0,1}$	0	0	-27856.761341	-3.0	-27866.738696	1
$p_{0,13}$	0	0	-55767.279594	-3.0	-55777.256949	3
$p_{0,17}$	0	0	-55788.832737	-3.0	-55798.810092	65
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1}$,	3	-69.168365	-69.168365
30	$p_{0,1}, p_{0,1},$	$p_{0,13},$	1	-46.273896	-46.273896
196	$p_{0,1}, p_{0,17},$	$p_{0,56},$	1	-27.304714	-27.304714
250	$p_{0,13},$	$p_{0,17},$	1	-21.553143	-21.553143
Sum					-179.29506906107076

 $\Delta G = -30.263683808547825$

 $\Delta E = -0.068289885233647$

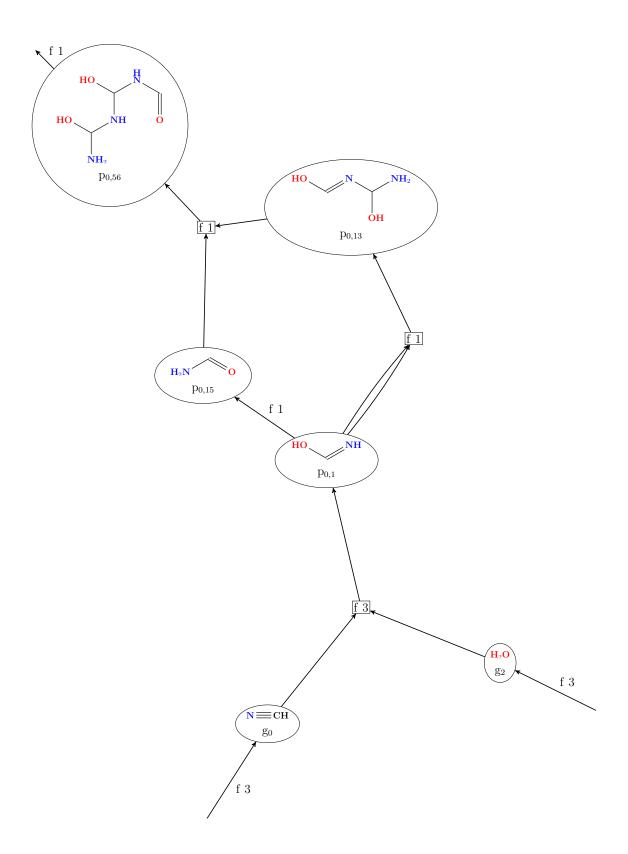
|E| = 6

|U| = 4

0.3.6 Solution 5

Overall Data

p_{0,15} 0 0 -10.625195 -3.000000 2



 $File: \ \mathtt{out/076_dg_0_11100_f_0_5_filt}$

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
g_0	3	0	-14459.025578	-6.0	-14478.980288	0
g_2	3	0	-13306.118349	-6.0	-13326.073059	0
$p_{0,1}$	0	0	-27856.817054	-3.0	-27866.79441	1
$p_{0,13}$	0	0	-55767.391129	-3.635441	-55779.481823	3
$p_{0,15}$	0	0	-27896.445756	-3.0	-27906.423111	2
$p_{0,56}$	0	1	-83672.898792	-3.0	-83682.876147	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-69.224079	-69.224079
30	$p_{0,1}, p_{0,1},$	$p_{0,13},$	1	-47.859008	-47.859008
36	$p_{0,1},$	$p_{0,15},$	1	-39.628701	-39.628701
169	$p_{0,13}, p_{0,15},$	$p_{0,56},$	1	0.006113	0.006113
Sum					-179.2950690610697

 $\Delta G = -29.515383067450603$

 $\Delta E = \text{-}0.0682898852336466}$

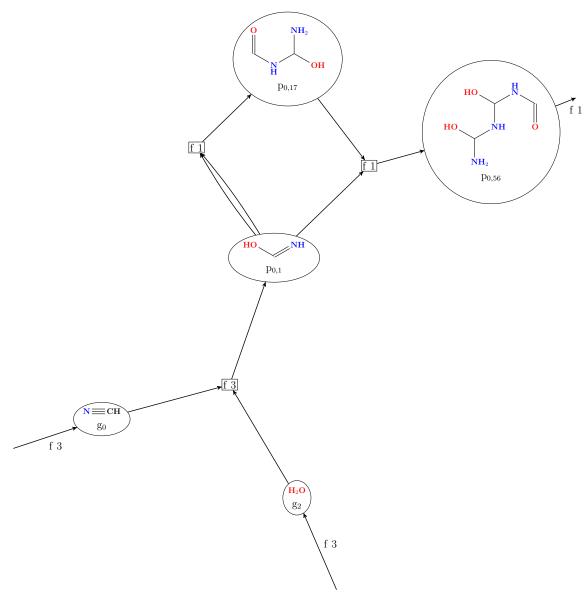
|E| = 6

|U| = 4

0.3.7 Solution 6

Overall Data

Objective val	Lue	(nor	n-integral):	: -0.068289	99
Vertex/Graph	In	Out	G	logK	t_order
g_{0}	3	0	-5.507152	-3.000000	0
g_{2}	3	0	-5.068043	-6.000000	0
p_{0,17}	0	0	-21.248844	-3.000000	3
p_{0,1}	0	0	-10.610080	-3.000000	1
p_{0,56}	0	1	-31.869324	-6.000000	67



File: out/085_dg_0_11100_f_0_6_filt

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
g_0	3	0	-14459.025578	-3.0	-14469.002933	0
g_2	3	0	-13306.144961	-6.0	-13326.099672	0
$p_{0,1}$	0	0	-27856.761341	-3.0	-27866.738696	1
$p_{0,17}$	0	0	-55788.832737	-3.0	-55798.810092	3
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-76.624769	-76.624769
40	$p_{0,1}, p_{0,1},$	$p_{0,17},$	1	-67.827039	-67.827039
196	$p_{0,1}, p_{0,17},$	$p_{0,56},$	1	-27.304714	-27.304714
Sum					-179.2950690610687

$$\begin{split} \Delta G &= \text{-}32.50060556313533 \\ \Delta E &= \text{-}0.06828988523364621 \end{split}$$

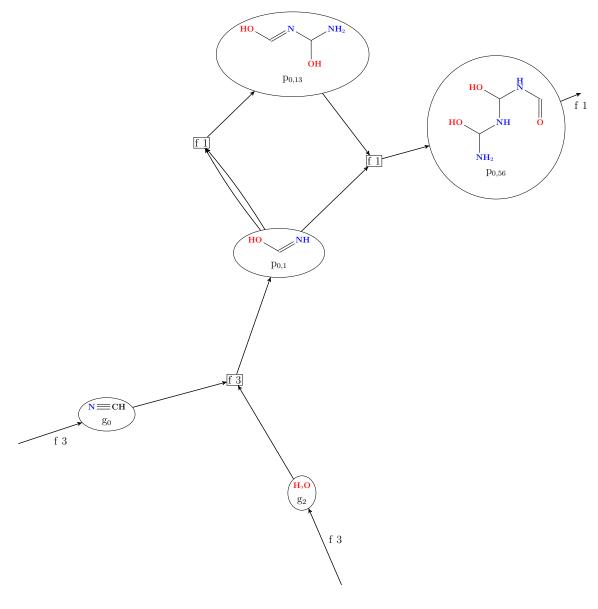
```
\begin{aligned} |E| &= 5\\ |U| &= 3 \end{aligned}
```

0.3.8 Solution 7

Overall Data

Objective value (non-integral): -0.0682899 Vertex/Graph In Out G logK t_order g_{0} 3 0 -5.507152 -6.000000 0 g_{2} 3 0 -5.068033 -6.000000 0 $p_{0,13}$ 0 0 -21.240635 -3.000000 45 $p_{0,1}$ 0 0 -10.610080 -3.000000 21 $p_{0,56}$ 0 1 -31.869324 -6.000000 67

Filtered Graph



File: out/094_dg_0_11100_f_0_7_filt

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
g_0	3	0	-14459.025578	-6.0	-14478.980288	0
g_2	3	0	-13306.118349	-6.0	-13326.073059	0
$p_{0,1}$	0	0	-27856.761341	-3.0	-27866.738696	21
$p_{0,13}$	0	0	-55767.279594	-3.0	-55777.256949	45
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-69.168365	-69.168365
30	$p_{0,1}, p_{0,1},$	$p_{0,13},$	1	-46.273896	-46.273896
263	$p_{0,1}, p_{0,13},$	$p_{0,56}$,	1	-48.857857	-48.857857
Sum					-179.2950690610687

 $\Delta G = -30.263683808547825$

 $\Delta E = \text{-}0.06828988523364621$

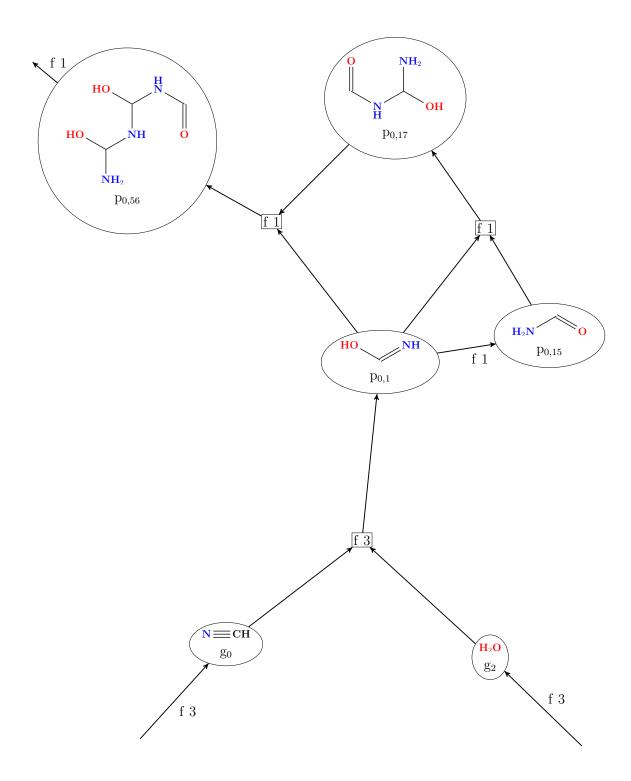
|E| = 5

|U| = 3

0.3.9 Solution 8

Overall Data

Objective value (non-integral): -0.0682899 Vertex/Graph In Out G t_order logK g_{0} 3 0 -5.507152 -6.000000 0 g_{2} 3 0 -5.068043 -6.000000 0 p_{0,15} -10.625195 -6.000000 22 -21.248844 -3.000000 44 $p_{0,17}$ 0 0 p_{0,1} 0 0 -10.610080 -3.000000 21 $p_{0,56}$ 0 1 -31.869324 -6.000000 67



 $File: \ \mathtt{out/103_dg_0_11100_f_0_8_filt}$

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
$\overline{g_0}$	3	0	-14459.025578	-6.0	-14478.980288	0
g_2	3	0	-13306.144961	-6.0	-13326.099672	0
$p_{0,1}$	0	0	-27856.761341	-3.0	-27866.738696	21
$p_{0,15}$	0	0	-27896.445756	-6.0	-27916.400466	22
$p_{0,17}$	0	0	-55788.832737	-3.0	-55798.810092	44
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-69.141753	-69.141753
36	$p_{0,1},$	$p_{0,15},$	1	-47.167431	-47.167431
183	$p_{0,1}, p_{0,15},$	$p_{0,17},$	1	-20.659608	-20.659608
196	$p_{0,1}, p_{0,17},$	$p_{0,56},$	1	-27.304714	-27.304714
Sum					-179.2950690610697

 $\Delta G = -30.255700189244095$

 $\Delta E = \text{-}0.0682898852336466}$

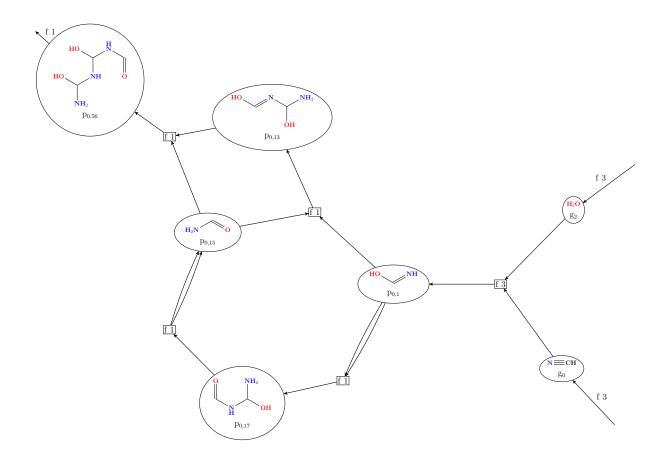
|E| = 6

|U| = 4

0.3.10 Solution 9

Overall Data

Objective val	Lue	(nor	(non-integral): -0.0682899					
Vertex/Graph	In	Out	G	logK	t_order			
g_{0}	3	0	-5.507152	-3.000000	0			
g_{2}	3	0	-5.068043	-6.000000	0			
p_{0,13}	0	0	-21.240678	-4.984418	39			
p_{0,15}	0	0	-10.625174	-4.673390	27			
p_{0,17}	0	0	-21.248887	-6.000000	26			
p_{0,1}	0	0	-10.610101	-6.000000	11			
p_{0,56}	0	1	-31.869324	-6.000000	67			



File: out/112_dg_0_11100_f_0_9_filt

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
$\overline{g_0}$	3	0	-14459.025578	-3.0	-14469.002933	0
g_2	3	0	-13306.144961	-6.0	-13326.099672	0
$p_{0,1}$	0	0	-27856.817054	-6.0	-27876.771765	11
$p_{0,13}$	0	0	-55767.391129	-4.984418	-55783.968232	39
$p_{0,15}$	0	0	-27896.389963	-4.67339	-27911.932655	27
$p_{0,17}$	0	0	-55788.944315	-6.0	-55808.899025	26
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-84.163499	-84.163499
40	$p_{0,1}, p_{0,1},$	$p_{0,17},$	1	-60.344173	-60.344173
169	$p_{0,13}, p_{0,15},$	$p_{0,56}$,	1	0.006113	0.006113
192	$p_{0,1}, p_{0,15},$	$p_{0,13}$,	1	0.006113	0.006113
237	$p_{0,17},$	$p_{0,15}, p_{0,15},$	1	-12.183616	-12.183616
Sum					-179.29506906106667

 $\Delta G = \text{-}32.50060530058536$

 $\Delta E = \text{-}0.06828988523364543}$

|E| = 7

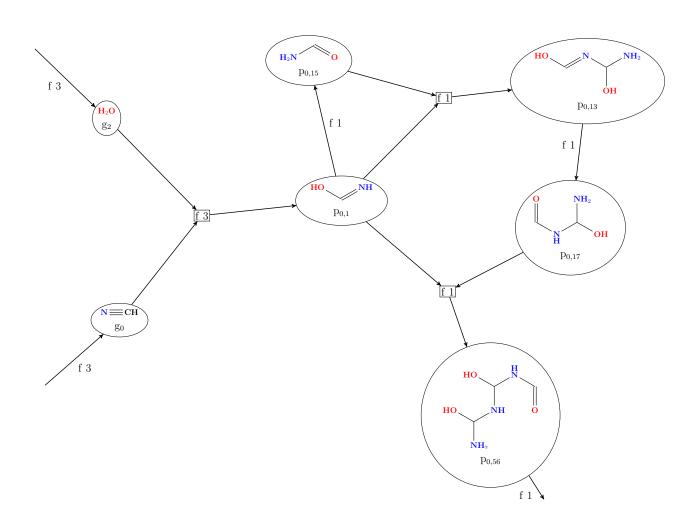
|U| = 5

0.3.11 Solution 10

Overall Data

Objective val	lue	(non-integral): -0.0682899				
Vertex/Graph	In	Out	G	logK	t_order	
g_{0}	3	0	-5.507152	-6.000000	0	
g_{2}	3	0	-5.068043	-6.000000	0	
p_{0,13}	0	0	-21.240635	-3.000000	4	
p_{0,15}	0	0	-10.625174	-5.666593	2	
p_{0,17}	0	0	-21.248887	-3.000000	65	
p_{0,1}	0	0	-10.610080	-3.00000	1	
p_{0,56}	0	1	-31.869324	-6.000000	67	

Filtered Graph



 $File: \ {\tt out/121_dg_0_11100_f_0_10_filt}$

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
g_0	3	0	-14459.025578	-6.0	-14478.980288	0
g_2	3	0	-13306.144961	-6.0	-13326.099672	0
$p_{0,1}$	0	0	-27856.761341	-3.0	-27866.738696	1
$p_{0,13}$	0	0	-55767.279594	-3.0	-55777.256949	4
$p_{0,15}$	0	0	-27896.389963	-5.666593	-27915.235834	2
$p_{0,17}$	0	0	-55788.944315	-3.0	-55798.92167	65
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-69.141753	-69.141753
36	$p_{0,1},$	$p_{0,15},$	1	-46.280009	-46.280009
192	$p_{0,1}, p_{0,15},$	$p_{0,13},$	1	0.006113	0.006113
196	$p_{0,1}, p_{0,17},$	$p_{0,56},$	1	-27.193136	-27.193136
250	$p_{0,13}$,	$p_{0,17},$	1	-21.664721	-21.664721
Sum					-179.29506906106408

 $\Delta G = -30.255700189244095$

 $\Delta E = -0.06828988523364445$

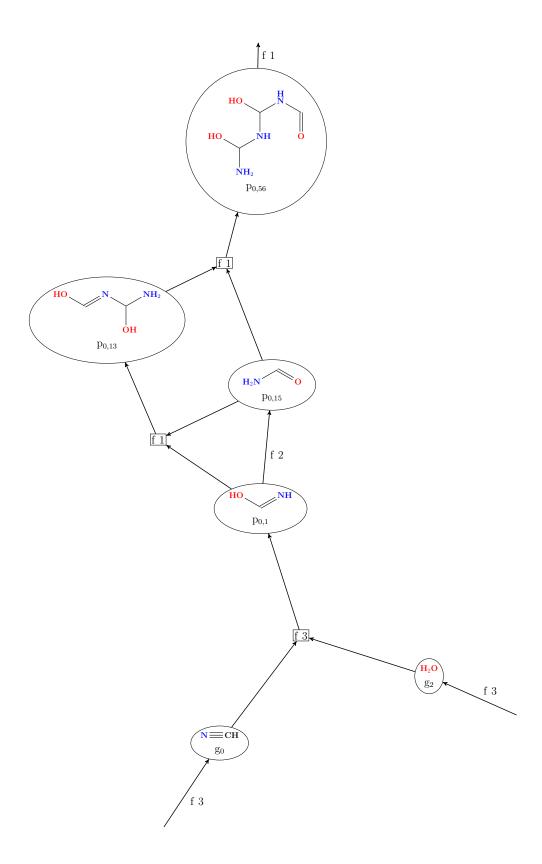
|E| = 7

|U| = 5

0.3.12 Solution 11

Overall Data

Objective value (non-integral): -0.0531961 Vertex/Graph In Out G logK t_order g_{0} 3 0 -5.507152 -3.000000 0 g_{2} 3 0 -5.068043 -3.000000 0 p_{0,13} 0 0 -21.240678 -6.000000 44 p_{0,15} -10.625174 -3.000000 22 0 0 p_{0,1} -10.610080 -3.000000 21 0 0 -31.869324 -6.000000 67 $p_{0,56}$



 $File: \ \mathtt{out/130_dg_0_11100_f_0_11_filt}$

Vertex	InFlow	OutFlow	$x_v^{\Delta G^0}$	x_v^K	$x_v^{\Delta G}$	t_v
g_0	3	0	-14459.025578	-3.0	-14469.002933	0
g_2	3	0	-13306.144961	-3.0	-13316.122316	0
$p_{0,1}$	0	0	-27856.761341	-3.0	-27866.738696	21
$p_{0,13}$	0	0	-55767.391129	-6.0	-55787.345839	44
$p_{0,15}$	0	0	-27896.389963	-3.0	-27906.367318	22
$p_{0,56}$	0	1	-83672.898792	-6.0	-83692.853502	67

Hyperedge	Source	Target	Flow	$x_e^{\Delta G}$	$\overline{x}_e^{\Delta G}$
6	$g_0, g_2,$	$p_{0,1},$	3	-84.107786	-84.107786
36	$p_{0,1},$	$p_{0,15},$	2	-39.628622	-39.628622
169	$p_{0,13}, p_{0,15},$	$p_{0,56},$	1	-1.634684	-1.634684
192	$p_{0,1}, p_{0,15},$	$p_{0,13},$	1	-14.239825	-14.239825
Sum					-139.66644695727257

 $\Delta G = -34.74551014937667$

 $\Delta E = \text{-}0.053196140215404704$

 $\begin{aligned} |E| &= 7 \\ |U| &= 4 \end{aligned}$