The key idea here is "avoided pegs" at each step. For $n=3$, the moves of recursive algorithm are:

disk	from $\to$ to	avoided
1	$0 \to 2$	1
2	$0 \to 1$	2
1	$2 \to 1$	0
3	$0 \to 2$	1
1	$1 \to 0$	2
2	$1 \to 2$	0
1	$0 \to 2$	1

Notice that if i only write the avoided sequence, the moves are determined. For the non-recursive algo.), the avoided sequence is

1, 2, 0, 1, 2, 0 . . .   for n odd

2, 1, 0, 2, 1, 0 . . .   for n even (1 & 2 swapped)

We just have to prove that the recursive one also has the same sequence.

You have to prove that optimal number of moves for n disks is $2^n - 1$ and recursive one achieves this. I leave this as exercise hanoi$(n, \text{from}, \text{to})$ has the same avoided sequence as non-recursive where $0 \leftrightarrow$ from, $2 \leftrightarrow$ to, $1 \leftrightarrow$ other.

<u>Pf.</u> Use induction on n

(Base) $n=1$: Both sequences are just $1 \leftrightarrow$ other

(Inductive) $n>1$

Case1 : n is odd

The non-recursive one is 1, 2, 0, 1, 2, 0 ...

The first recursive call is hanoi (n-1, from, other)

which by IH produces

2, 1, 0, 2, 1, 0 ...

$\updownarrow$ $\updownarrow$ $\updownarrow$

to, other, from ... (as in recursive call)

$\updownarrow$ $\updownarrow$ $\updownarrow$

other, to, from ... (as in original call)

$\updownarrow$ $\updownarrow$ $\updownarrow$

1, 2, 0 ...

note we swap to <-> other in call and keep from same.

The first recursive call produces $2^{n-1}-1$ steps.

Since n is odd

$2^{n-1} \bmod 3 = 1$

$(2^{n-1})^{th}$ step.

So the first 2^{n-1} steps are: $1, 2, 0, \ldots 1, 2, 0, 1$

The second recursive call is:

hanoi(n-1, other, to)

which by IH produces

2, 1, 0, ...

to, other, from ....

to, from, other, to, from, other

2, 0, 1, 2, 0, 1 ...

So whole sequence is 1, 2, 0, 1, 2, 0 .... as required.

Notice since n is odd

$$2^n \bmod 3 = 2$$

So in the $(2^n)^{th}$ step, non-recursive algo has to avoid 2. But, since in the first $2^n - 1$ steps it followed recursive algo, all disks are in peg 2. So there is no legal move and it stops, exactly like recursive algo.

For even n, the proof is similar, I leave this as exercise. $\square$