APL103 Lab

Experiment 3

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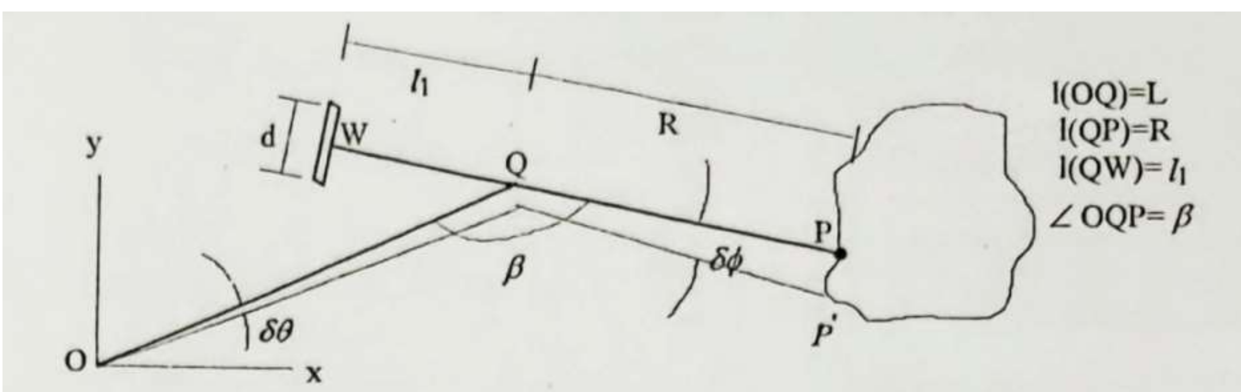
Objectives:

1. To calibrate the planimeter.

2. To determine the area of an irregular figure.

Theoretical Background:

This instrument is used to measure the area of an irregular figure. It consists of two arms OQ and QP and a wheel as shown. O is the pivot point and is fixed in space, while Q is a pin joint connecting the two arms. P is the tracing point and is used to traverse the irregular area. The wheel W is of diameter, d and is attached to arm QP and is free to rotate about its axis.



Let the tracing point P move along the periphery of area A, by a small distance to location P. During this time let arm OQ rotate by an angle $\delta\theta$ and arm QP by an angle $\delta\phi$ with respect to XY. (Note $\delta\phi = \delta\theta + \delta\beta$). Let the wheel rotate by δN revolutions.

Area traced by OQ = $\frac{1}{2}L^2\delta\theta$

Area traced by QP = $\frac{1}{2}R^2\delta\phi \quad + \quad RL\delta\theta cos(\pi - \beta)$

(Rotation of QP + Translation of QP parallel to itself)

Thus, the total area $\delta A'$, traced by the two arms is .

$$\delta A' = \frac{1}{2}L^2\delta\theta + \frac{1}{2}R^2\delta\phi + RL\delta\theta cos(\pi - \beta) \quad (3.1)$$

Now $\pi d\delta N$ = ds (distance moved by W) = $L\delta\theta cos(\pi - \beta) - l1\delta\phi$

$$\Rightarrow \delta A' = \frac{1}{2}L^2\delta\theta + (\frac{1}{2}R^2 + Rl1)\delta\phi + \pi Rd\delta N \quad (3.2)$$

Integrating over the full periphery we have,

$$\Rightarrow \delta A' = \frac{1}{2}L^2 \int \delta\theta + (\frac{1}{2}R^2 + Rl1) \int \delta\phi + \pi Rd \int \delta N \quad (3.3)$$

We now have 2 cases:

Case (1). O lies inside area A:

Here $\int d\theta = \int d\phi = 2\pi$. It is also easy to see that the area traced by the 2 arms is in fact the area enclosed by the curve traversed by P.

$$A = A' = \pi L^2 + \pi(R^2 + 2Rl1) + \pi RdN \quad (3.4)$$

N is the total number of revolutions of wheel W

Thus,

$$A = \alpha_0 + \alpha_1 N \quad (3.5)$$

where $\alpha_0 = \pi L^2 + \pi(R^2 + 2Rl1)$ and $\alpha_1 = \pi RdN$ (3.6)

Case (2). O lies outside area A:

Here $\int d\theta = \int d\phi = 0$, thus

$$A = \pi RdN \quad (3.7a)$$

and thus,

$$\alpha_0 = \pi L^2 + \pi(R^2 + 2Rl1) \quad \text{and} \quad \alpha_1 = \pi RdN \quad (3.7b)$$

Apparatus:

Drawing board, graph paper, tape, planimeter, micrometer screw gauge, and scale.

Procedure:

1. Fix the graph paper to the drawing board ensuring that it has no wrinkles.

2. Choose pivot point O such that the entire graph paper can be traversed by P without causing the planimeter to jam. (Choose O to lie outside the graph paper.)

3. Fix point O and adjust P, such that wheel W touches the paper.

4. Draw a regular figure (square or rectangle) and an irregular figure on graph paper.

5. Adjust the zero of the wheel and traverse the regular shape 5 times, noting N after each traversal. (Move only clockwise). Repeat this step 4 times.

6. Traverse the irregular figure twice and again note N after each traversal. Repeat this step 4 times. You should attach the signed graph paper (or its photocopy) with the regular and irregular figures in your report.

Observations:

1. Least count of the planimeter = ___________rev

2. Least count of the scale = ___________ m

3. Least count of the micrometer = __________ m

4. Length of arm QP, R = ___________ m

5. Diameter of wheel W, d = __________ m

Table 1:

		Number of Revolutions			
Known Area Calibration	A_0	$2A_0$	$3A_0$	$4A_0$	$5A_0$
1	0.165	0.325	0.499	0.664	0.824
2	0.137	0.292	0.47	0.637	0.797
3	0.14	0.294	0.423	0.642	0.804
4	0.144	0.29	0.421	0.644	0.807
A_0 from columns:	0.1465	0.150125	0.15108333	0.1616875	0.1616
Formulae used:	(AVG/1)	(AVG/2)	(AVG/3)	(AVG/4)	(AVG/5)
Known area =	9 cm^2	N =	0.15419917	$_1$ =	58.3660742

Table 2:

Unknown Area Determination	A	2A	3A	4A	5A
1	0.333	0.671	0.944	1.423	1.736
2	0.33	0.658	1.04	1.365	1.695
3	0.303	0.637	0.934	1.24	1.57
4	0.289	0.619	0.905	1.2	1.514
A from columns:	0.31375	0.323125	0.31858333	0.32675	0.32575
Formulae used:	(AVG/1)	(AVG/2)	(AVG/3)	(AVG/4)	(AVG/5)
$_1$ =	58.3660742	N =	0.32159167	Area (cm^2) =	18.7700431

Analysis:

1. **Determine the best estimates for α_0 and α_1 from the data in table 1. Also determine the internal estimates of error in α_0 and α_1.**

From Table 1, we got $\alpha_1 = 58.3660742\ cm2/rev$, $\alpha_0 = 0$.

0.1465	61.4334471
0.150125	59.9500416
0.15108333	59.5697752
0.1616875	55.66293
0.1616	55.6930693

Internal Estimate of error⇒ $1.31749529\ cm2/rev$ (using the formula for uncertainty)

2. **Determine α_1 directly from the geometry of the planimeter.**

3. Determine the area of the irregular figure using the values α_0 and α_1 from step 1.
   For unknown area,

   $N_2 = 0.32159167\ rev$

   α_0 and α_1 is the same for both areas hence,

   $\alpha_0 = 0$

   $\alpha_1 = 58.3660742\ cm2/rev$

   $A_2 = \alpha_0 + \alpha_1 N_2$

   $= 18.7700431 cm^2$

4. **Directly measure the area of the irregular figure from the graph paper by counting squares.**
   Area from the graph for the irregular figure by counting the number of squares inside it ≈ 17.5 cm^2
   Error in the area = (18.77 - 18.25) / 18.25 * 100 = 2.84 %

DISCUSSION:

1. **Compare the values α_0 and α_1 obtained by the two methods. Are they consistent with each other?**
   - $\alpha_0 = 0$ for both cases
     From the table,
     $\alpha_1 = 58.3660742\ cm^2/rev$
     From the geometry of the planimeter,

α_1 = ___________ cm^2/rev

Deviation (with respect to geometrical calculation)

2. **Compare the internal and external estimates of error in α_1. State whether the experiment was a balanced one. If not, how would you make it more balanced?**

    - **Internal estimates of error in α_1** $\Rightarrow 1.31749529\ cm2/rev$
      External estimates of error in α_1

      The experiment was unbalanced as the measurements of N, and hence α_1, were slightly varying for the number of times we traced the figure to obtain A_0 values. We should accommodate all the errors(instrumental, procedural, human etc) to make it more balanced. Also, ensuring proper calibration by making no error during tracing and taking readings will make it more balanced.

3. **Compare the two methods of measuring the area of the irregular figure. Thus comment on the accuracy and precision of the planimeter.**

    - Relative error in the area = (18.77 - 18.25) / 18.25 * 100 ~ 2.84 %
      Accuracy = 97.16 %
      Precision = Adjusted Std Dev = 2.94600902
      Thus the planimeter has good accuracy (error within bounds) and good precision (values of N measured were close).