APL321 : Introduction to Computational Fluid Dynamics
Indian Institute of Technology Delhi
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Lab 6

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1. The given discretized pressure correction equations are:

$$u_{i,j} = u^*_{i,j} - \Delta(p_{i,j} - p_{i-1,j}) \tag{1}$$

$$v_{i,j} = v^*_{i,j} - \Delta(p_{i,j} - p_{i,j-1}) \tag{2}$$

The given discretized continuity equation is:

$$\frac{u_{i+1,j} - u_{i,j}}{\Delta} + \frac{v_{i,j+1} - v_{i,j}}{\Delta} = 0 \tag{3}$$

Substituting the expressions from equations 1 and 2 into 3, we get:

$$u^*_{i+1,j} - \Delta(p_{i+1,j} - p_{i,j}) - u^*_{i,j} + \Delta(p_{i,j} - p_{i-1,j}) + v^*_{i,j+1} - \Delta(p_{i,j+1} - p_{i,j}) - v^*_{i,j} + \Delta(p_{i,j} - p_{i,j-1}) = 0 \tag{4}$$

$$\implies \Delta(p_{i+1,j} + p_{i,j+1} + p_{i-1,j} + p_{i,j-1} - 4p_{i,j}) = (u^*_{i+1,j} - u^*_{i,j}) + (v^*_{i,j+1} - v^*_{i,j}) \tag{5}$$

Thus, we obtain the relation between p and u^* and v^*.

2. From equation 4, we get the following coefficients for linear equations used to solve $p_{i,j}$.

$A_E = 1, A_N = 1, A_W = 1, A_S = 1, A_P = -4$
$Q_P = \frac{u^*_{i+1,j} - u^*_{i,j}}{\Delta} + \frac{v^*_{i,j+1} - v^*_{i,j}}{\Delta}$
$= \frac{[-x_{i+1} + 0.1sin(x_{i+1}) + x_i - 0.1sin(x_i) + y_{j+1} + 0.1sin(y_{j+1}) - y_j - 0.1sin(y_j)]}{\Delta}$
$= (\frac{0.1}{\Delta})[sin(x_{i+1}) - sin(x_i) + sin(y_{j+1}) - sin(y_j)]$

Thus, the final form of the equation is
$A_E p_{i+1,j} + A_N p_{i,j+1} + A_W p_{i-1,j} + A_S p_{i,j-1} + A_P p_{i,j} = Q_P$

3. **WEST BOUNDARY**
The boundary condition given is $\frac{\partial p}{\partial x} = 0$. We consider ghost cells just adjacent to the boundary cells. This gives us the relation: $p(X_0, Y_j) = p(X_1, Y_j)$.

NORTH BOUNDARY
The boundary condition given is $\frac{\partial p}{\partial y} = 0$. We consider ghost cells just adjacent to the boundary cells. This gives us the relation: $p(X_i, Y_{N_y}) = p(X_i, Y_{N_y-1})$.

EAST BOUNDARY
The boundary condition specified is Dirichlet with $p = 0$. Using Linear Extrapolation gives us the relation: $p(X_{N_x}, Y_j) = -p(X_{N_x-1}, Y_j)$

SOUTH BOUNDARY
The boundary condition given is $\frac{\partial p}{\partial y} = 0$. We consider ghost cells just adjacent to the boundary cells. This gives us the relation: $p(X_i, Y_{-1}) = p(X_i, Y_0)$.

4. The isocontour for the pressure field $p(x, y)$ is given below:

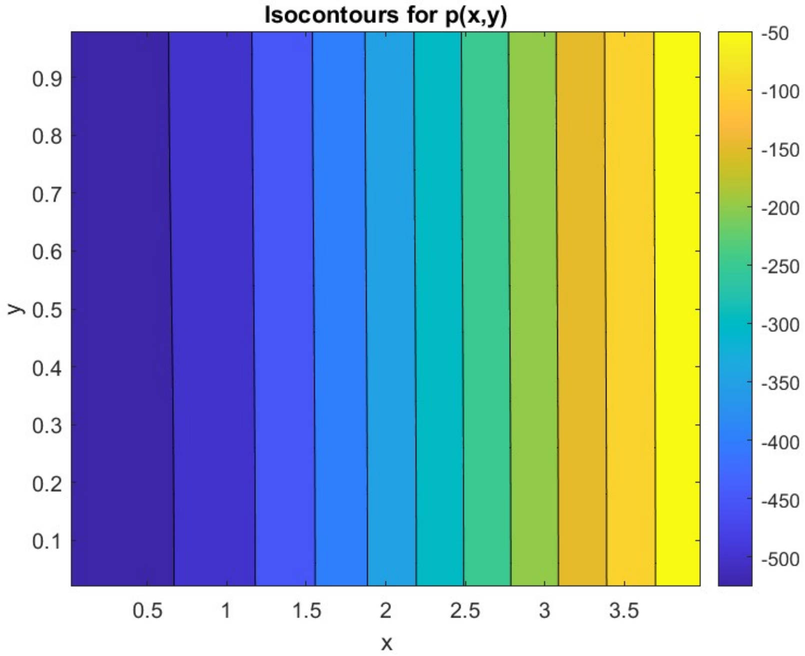

Figure 1: Isocontour for pressure ($p(x, y)$)

5. The isocontours for the corrected velocity fields are given below:

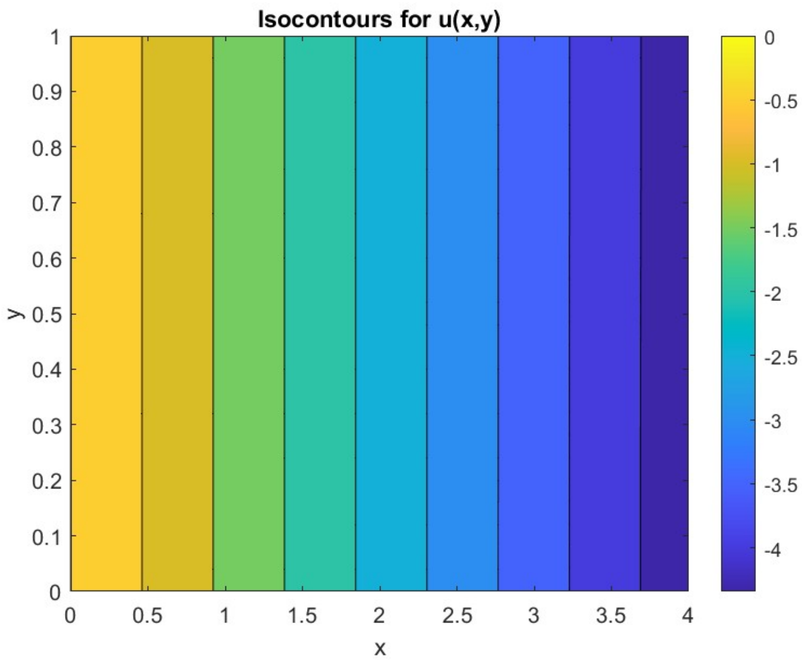

Figure 2: Isocontour for corrected velocity ($u(x, y)$)

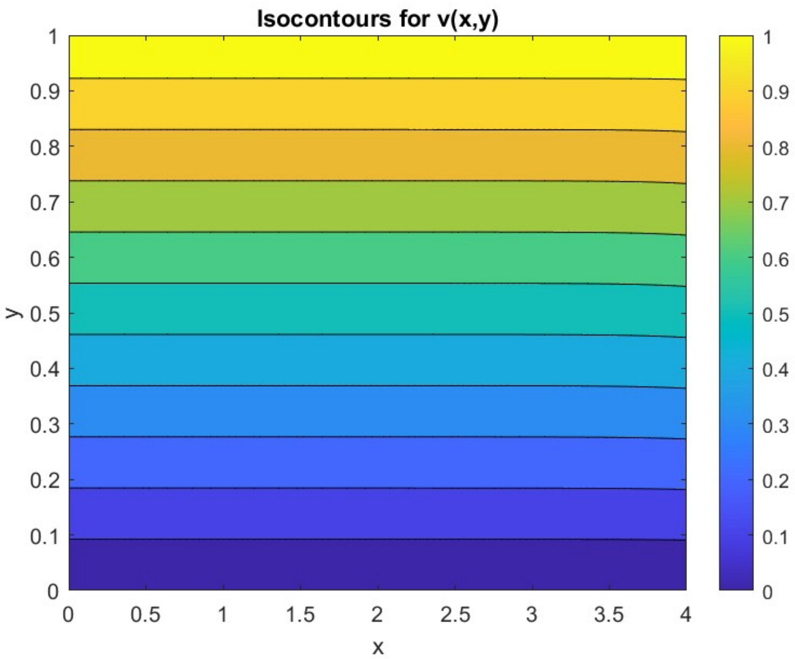

Figure 3: Isocontour for corrected velocity ($v(x, y)$)

6. The isocontours for divergence of corrected and predicted velocity fields are given below:

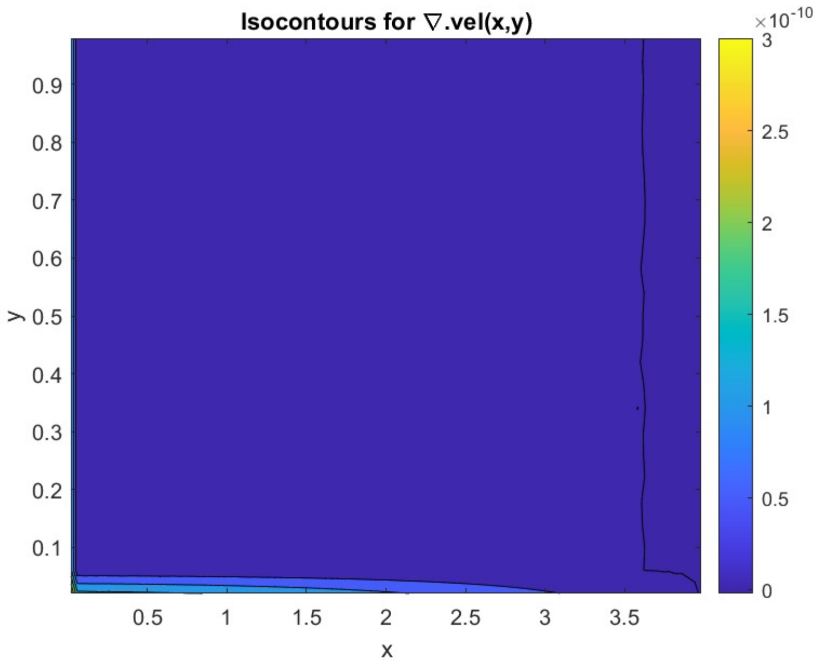

Figure 4: Isocontour for divergence of corrected field ($\nabla.u$)

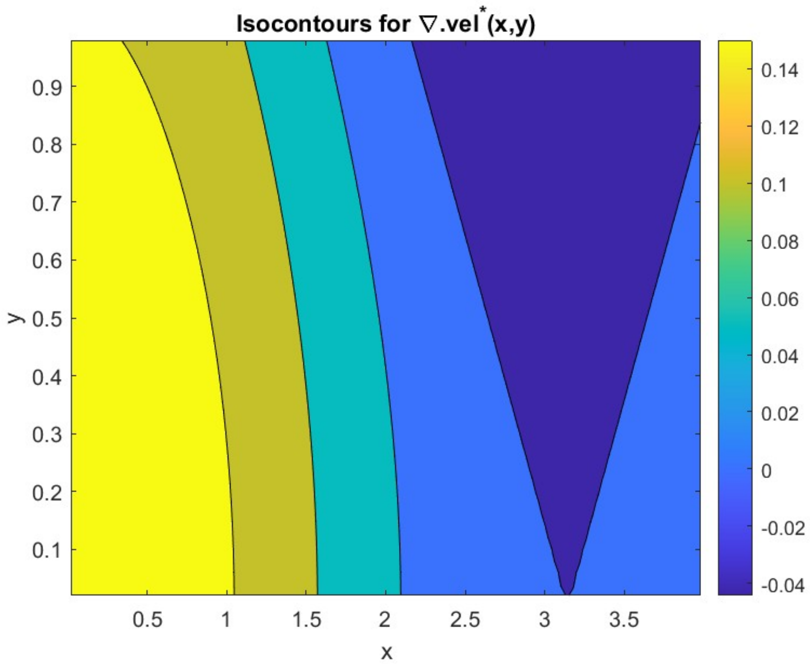

Figure 5: Isocontour for divergence of predicted field ($\nabla.u^*$)

The isocontours of the divergence of the predicted velocity field are very high in magnitude (order of 10^{-2}) which suggests that the predicted field does not satisfy the continuity equation well enough. The final corrected values are highly accurate since their divergence is as low as 10^{-10}. The isocontours are fairly uniform indicating that the corrections are valid throughout the flow domain. There are slightly higher values near the boundaries due to the boundary conditions imposed at the ends.