APL321 : Introduction to Computational Fluid Dynamics
Indian Institute of Technology Delhi
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Lab 7

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1. The given prediction equation for interior nodes is:

$$\frac{1}{Re}\nabla^2\vec{u}^* = \nabla p^* \tag{1}$$

The discretized equation for the x-component of the above equation is:

$$\frac{1}{Re}\left[\frac{u^*_{i+1,j}+u^*_{i-1,j}-2u^*_{i,j}}{\Delta^2}+\frac{u^*_{i,j+1}+u^*_{i,j-1}-2u^*_{i,j}}{\Delta^2}\right] = \frac{p^*_{i,j}-p^*_{i-1,j}}{\Delta} \tag{2}$$

$$\implies u^*_{i,j} = \frac{1}{4}[u^*_{i+1,j}+u^*_{i,j+1}+u^*_{i-1,j}+u^*_{i,j-1}+Re\Delta(p^*_{i-1,j}-p^*_{i,j})] \tag{3}$$

The above eqn(3) gives the following coefficients:
$A_P = 1, A_E = A_W = A_N = A_S = -\frac{1}{4}, Q_P = \frac{Re\Delta}{4}(p^*_{i-1,j}-p^*_{i,j})$

Similarly, we get the same equation for $v^*_{i,j}$ from the y-component of eqn (1) with a minute change of the source term $Q_P = \frac{Re\Delta}{4}(p^*_{i,j-1}-p^*_{i,j})$.

2. Given, $u'_{i,j} = u_{i,j} - u^*_{i,j}, v'_{i,j} = v_{i,j} - v^*_{i,j}$. Since $\vec{u}$ and $\vec{u}^*$ satisfy eqn (1), $\vec{u}'$ also satisfies this equation and thus, we can write:

$$A_P u'_{i,j} = \Sigma A_{nb} u'_{nb} + Q_P \tag{4}$$

Since u'_{nb} approaches zero while converging, we may write:

$$u'_{i,j} = \frac{Re\Delta}{4A_P}(p'_{i-1,j}-p'_{i,j}) \implies u_{i,j} = u^*_{i,j} + \frac{Re\Delta}{4A_P}(p'_{i-1,j}-p'_{i,j}) \tag{5}$$

$$v'_{i,j} = \frac{Re\Delta}{4A_P}(p'_{i,j-1}-p'_{i,j}) \implies v_{i,j} = v^*_{i,j} + \frac{Re\Delta}{4A_P}(p'_{i,j-1}-p'_{i,j}) \tag{6}$$

3. So far, $\vec{u}$ satisfies only the Stokes equation. We need to ensure that it also satisfies the mass conservation equation. For this, we do the following:

$$\nabla.\vec{u} = 0 \tag{7}$$

$$\implies \frac{u_{i+1,j}-u_{i,j}}{\Delta} + \frac{v_{i,j+1}-v_{i,j}}{\Delta} = 0 \tag{8}$$

Letting $Q_D = \frac{Re\Delta}{4A_P}$ and substituting from eqns 5 and 6, we get:

$$u^*_{i+1,j}+Q_D(p'_{i,j}-p'_{i+1,j})-u^*_{i,j}-Q_D(p'_{i-1,j}-p'_{i,j})+v^*_{i,j+1}+Q_D(p'_{i,j}-p'_{i,j+1})-v^*_{i,j}+Q_D(p'_{i,j-1}-p'_{i,j}) = 0 \tag{9}$$

$$\implies Q_D(p'_{i+1,j}+p'_{i,j+1}+p'_{i-1,j}+p'_{i,j-1}-4p'_{i,j}) = u^*_{i+1,j}-u^*_{i,j}+v^*_{i,j+1}-v^*_{i,j} \tag{10}$$

From the above equation, we obtain the following coefficients:
$A_P = -4, A_E = A_W = A_S = A_N = 1, Q_P = \frac{1}{Q_D}[u^*_{i+1,j}-u^*_{i,j}+v^*_{i,j+1}-v^*_{i,j}]$

4. **WEST BOUNDARY**
The boundary condition given is $u(0, y) = u_0 exp(-(y - y_0)^2/w^2)$. Thus, $u^*_{0,j} = u_0 exp(-(y(j) - y_0)^2/w^2)$. Changes for u are:
$A_E = A_W = A_S = A_N = 0; A_P = 1; Q_P = u_0 exp(-(y(j) - y_0)^2/w^2)$
For v, $v(0, y) = 0$ is given and thus after using extrapolation, we get the following expression:
$v^*_{-1,j} = -v^*_{0,j}$. Thus, the changes are:
$A_E = A_W = A_S = A_N = 0; A_P = -1; Q_P = -v^*_{-1,j}$

NORTH BOUNDARY
$v(x, Ly) = u(x, Ly) = 0$. Thus, the changes for v are:
$A_E = A_W = A_S = A_N = 0; A_P = 1; Q_P = 0$
For u, we use extrapolation to get:
$u^*_{i,Nx+2} = -u^*_{i,Nx+1}$. Thus, the changes are:
$A_E = A_W = A_S = A_N = 0; A_P = -1; Q_P = -u^*_{i,Nx+2}$

EAST BOUNDARY
At $x = Lx$, $p = 0$. Thus, using extrapolation, we get:

$$u^*_{Nx+1,j} = \frac{(u^*_{Nx+2,j} + u^*_{Nx,j})}{2} \quad u^*_{Nx+2,j} = 2u^*_{Nx+1,j} - u^*_{Nx,j}$$

When substituted into the equations, we get the following changes in the coefficients:
For u, $A_E = 0; A_P \rightarrow A_P + 2A_E; A_W \rightarrow A_W - A_E; A_N = A_S = 0$
For v, $A_E = 0; A_P \rightarrow A_P - A_E; A_W \rightarrow A_W + 2A_E; A_N = A_S = 0)$

SOUTH BOUNDARY
$v(x, 0) = u(x, 0) = 0$. Thus, the changes for v are:
$A_E = A_W = A_S = A_N = 0; A_P = 1; Q_P = 0$
For u, we use extrapolation to get:
$u^*_{i,-1} = -u^*_{i,0}$. Thus, the changes are:
$A_E = A_W = A_S = A_N = 0; A_P = -1; Q_P = -u^*_{i,-1}$

5. (a) Upon using the relaxation factor of 0.001 for all the three values, the number of iterations doesn't affect the velocity fields much. Just the pressure values depend heavily on the number of iterations while the residual remains of the same order.

   (b) The isocontours of u,v and p are given below:

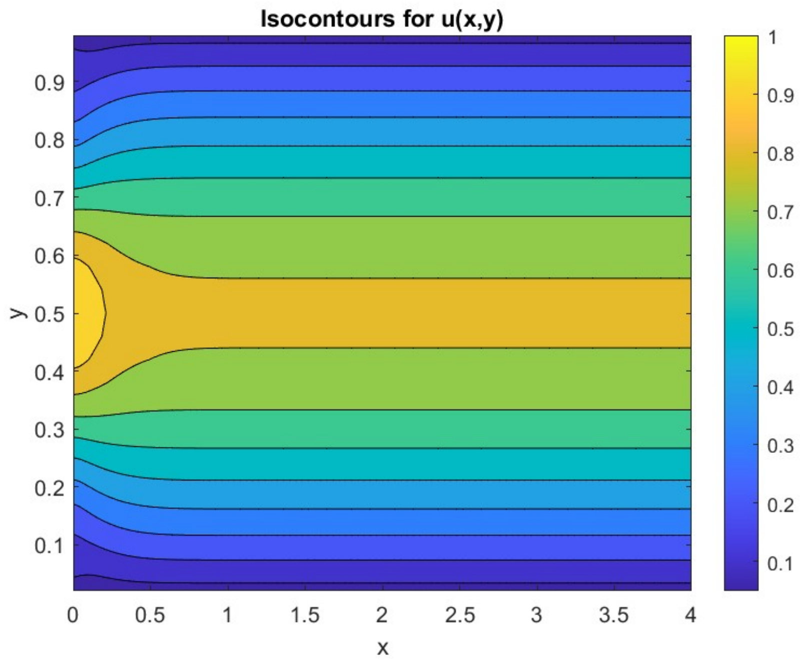

Figure 1: Isocontour for $u(x, y)$

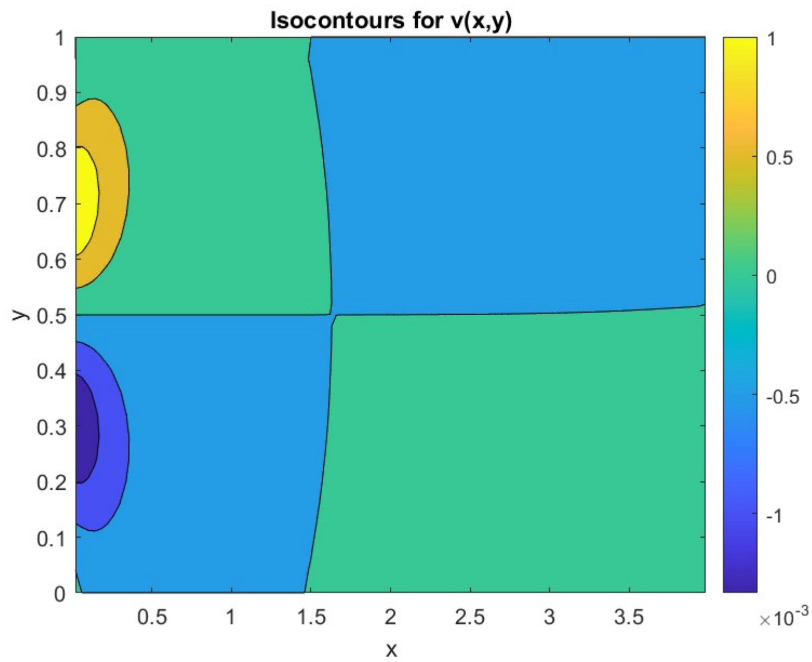

Figure 2: Isocontour for $v(x, y)$

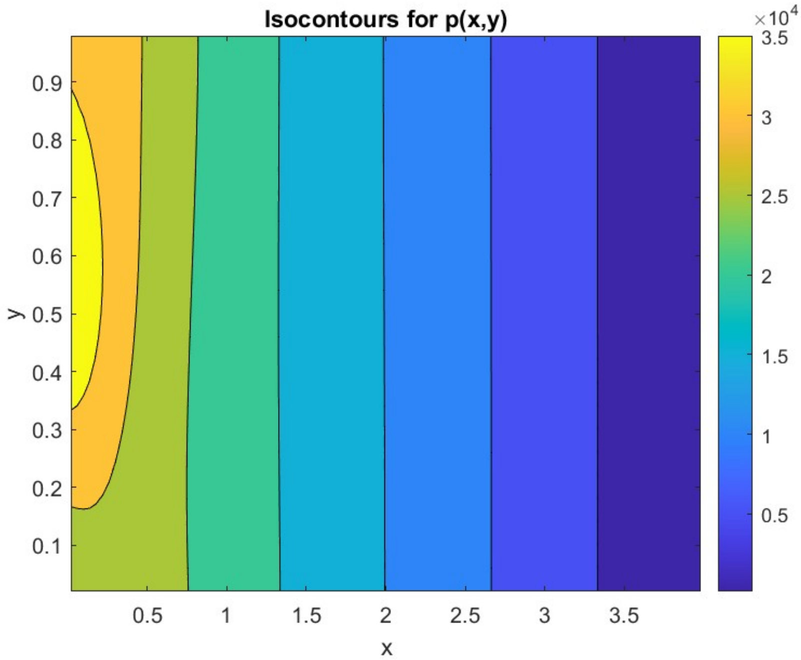

Figure 3: Isocontour for pressure ($p(x, y)$)

(c) The velocity vector plot is given below:

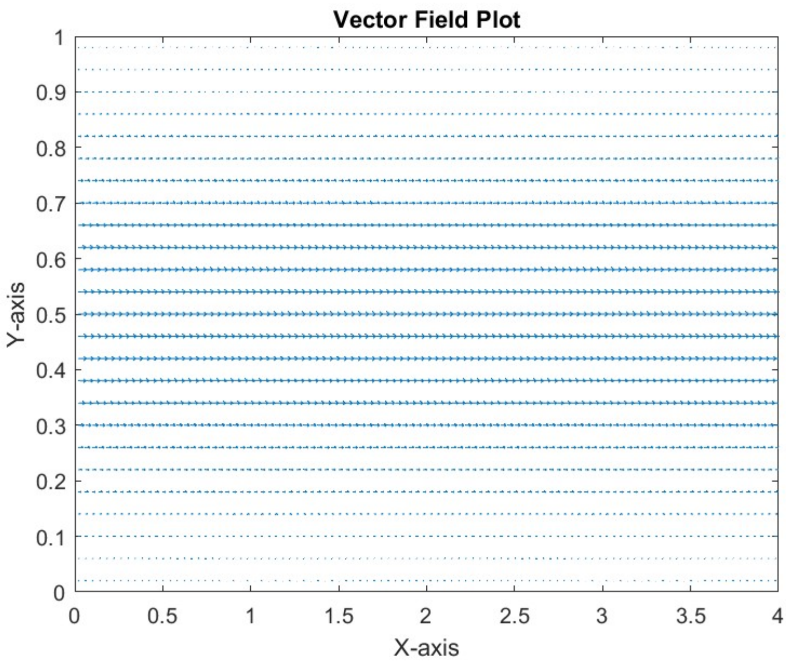

Figure 4: Velocity Vector Field

(d) The isocontour of $\nabla.\vec{u}$ is given below:

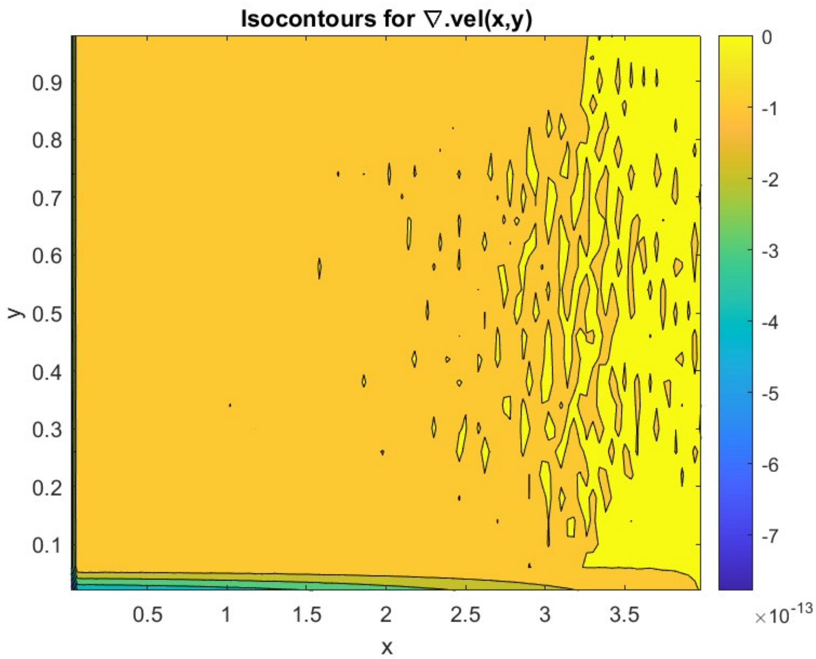

Figure 5: Isocontour for $\nabla.\vec{u}$

(d) The residual calculated is of the order of 10^{-4} even after 450 iterations.