APL321 : Introduction to CFD

Lab assignment 4: Unsteady convection diffusion equation (Taylor Dispersion)

Total points: 10

Date of release: March 4th; Lab date: March 7th

Due date for report and code (please submit the matlab codes and short report with results on moodle only): March 13th; Evaluation date: March 14th

A chemical species with concentration field $c(x, y, t)$ is being transported in a 2D channel of length $L_x = 20$ and $L_y = 1$. The diffusivity of the species is $D = 10^{-4}$ and the imposed velocity field in the channel is steady and fully developed, so that $u(x, y, t) = U(y)$ and $v(x, y, t) = 0$, where $U(y) = 4u_0 \dfrac{y}{L_y}\left(1 - \dfrac{y}{L_y}\right)$ where $u_0 = 0.01$ is the maximum velocity in the channel. The species concentration satisfies the following 2D advection diffusion equation:

$$\frac{\partial c}{\partial t} + U\frac{\partial c}{\partial x} = D\nabla^2 c \tag{1}$$

Discretize the above equation on a uniform grid with $N_x = 400$ and $N_y = 20$ grid cells. The initial condition satisfied by c is

$$\text{c(x,y,0)}=\begin{cases} 0 & \text{for } |x - x_c| > \Delta x \\ \frac{1}{2\Delta x} & \text{for } |x - x_c| \leq \Delta x \end{cases} \tag{2}$$

where the initial concentration is centered at $x_c = L_x/4$. The boundary conditions for c have been shown in the figure below.

For spatial discretization, use first order upwind scheme for convection term and central differencing scheme for diffusion term. For time stepping scheme, use fully implicit backward Euler scheme for the convection term and test two different time schemes for diffusion term: Scheme 1: fully implicit backward Euler scheme. Scheme 2: Crank-Nicolson scheme. You may use either finite difference method or finite volume method, as per your own preference.

Q1. (1 point) Discretize Eqn 1 for both scheme 1 and scheme 2.

Q2. (2 points) Derive and state coefficients A_P, A_E, A_W, A_N, A_S for both scheme 1 and scheme 2. Also state the transformation of coefficients required at the 4 boundaries.

Q3. (4 points) Discretize and evolve Eqn 1 until $t = 500$ using time step $\Delta t = 2$ for both schemes 1 and 2. Show isocontour of $c(x, y, 500)$ for both schemes 1 and 2. Comment on any differences in the isocontours between the 2 schemes. (2 graphs)

Q4. (3 points) Conduct simulations with time steps $\Delta t = 0.5$, $\Delta t = 1$, $\Delta t = 1.5$ and $\Delta t = 2$. Compare $c(x, L_y/2, 500)$ for the solution using 4 time steps on the *same graph* and comment on the convergence of solution with respect to Δt. Perform this analysis for both scheme 1 and scheme 2. (2 graphs).

Note: If you are using SOR for iterative solver, then $\omega \approx 1.5$ should work well for relaxation factor.