APL321 : Introduction to CFD
Lab assignment 6:

Total points: 10
Date of release: March 13th; Lab date: March 14th
Due date for report and code (please submit the matlab codes and short report with results on moodle only): March 20th; Evaluation date: March 21st

In this lab, we will develop only the "pressure correction" part of SIMPLE scheme on a staggered grid. The 2D flow domain is given by a rectangular box of size $L_x \times L_y$ where $L_x = 4$, $L_y = 1$. The number of grid cells are $N_x = 100$ and $N_y = 25$ in both directions. The velocity field after "predictor step" is given as:

$$u^*(x, y) = -x + 0.1 * \sin(x), \quad v^*(x, y) = y + 0.1 * \sin(y) \quad \text{for } x \in [0,\ L_x],\ y \in [0,\ L_y]$$

and pressure correction field is $p(x, y) = 0$. The variables are located on a staggered grid, where

$$X_i = i\Delta x + \Delta x/2, i = 0, 1, 2, .., N_x - 1, \qquad Y_j = j\Delta y + \Delta y/2, j = 0, 1, 2, .., N_y - 1$$

are the location of the nodes at cell centers. The cell faces are located at:

$$x_i = i\Delta x, i = 0, 1, 2, .., N_x, \qquad y_j = j\Delta y, j = 0, 1, 2, .., N_y$$

For a cell with given index (i, j), we denote $p_{i,j} = p(X_i, Y_j)$, $u^*_{i,j} = u^*(x_i, Y_j)$, $v^*_{i,j} = v^*(X_i, y_j)$. Note that the given u^*, v^* do not satisfy the incompressibility condition, i.e. $\nabla \cdot (u^*\mathbf{i} + v^*\mathbf{j}) \neq 0$. Our goal will be to find a "corrected" velocity field u, v which will satisfy the discrete version of $\nabla \cdot (u\mathbf{i} + v\mathbf{j}) = 0$ at every (X_i, Y_j).

Q1: (1 points) Let's say the discretized "pressure correction" equation is given by:

$$u_{i,j} = u^*_{i,j} - \Delta(p_{i,j} - p_{i-1,j}), \quad v_{i,j} = v^*_{i,j} - \Delta(p_{i,j} - p_{i,j-1}) \tag{1}$$

where $\Delta = \Delta_x = \Delta_y$, and the discretized continuity equation is

$$\frac{u_{i+1,j} - u_{i,j}}{\Delta} + \frac{v_{i,j+1} - v_{i,j}}{\Delta} = 0 \tag{2}$$

Derive and state the discretized equation relating the pressure correction p to u^* and v^* so that Eqns (1) and (2) are both satisfied.

Q2: (1 point) State the coefficients A_N, A_S, A_E, A_W, A_P as well as Q_P at the interior nodes for the set of linear equations being solved for $p_{i,j}$.

Q3: (1 point) The pressure correction field satisfies the boundary conditions $p = 0$ at $x = L_x$ and $\partial p/\partial n = 0$ at all the other boundaries. State the transformations that need to be made to the coefficients A_N, A_S, A_E, A_W, A_P at all the boundaries. (Hint: It helps to use a fictitious "ghost cell" for the pressure just outside the boundary, and to set the boundary condition at this "ghost cell" via linear extrapolation from the interior node and boundary condition.)

Q4: (3 points) Solve the pressure correction field p over all the pressure nodes, based on discretized equations derived in Q1-Q3. Make sure you use a very small tolerance (e.g. 10^{-10}) while solving for pressure iteratively. Show a contour plot of the pressure field after convergence (1 graph).

Q5: (2 points) Solve for the "corrected velocity" field using Eqn. (1) for u, v at all the velocity cells and show their contour plots on two separate graphs (2 graphs). (Hint: again, it helps to use the "ghost cell" values of the pressure field for evolving velocity nodes located at boundaries.)

Q6: (2 points) Calculate $\nabla \cdot \mathbf{u}^*$ and $\nabla \cdot \mathbf{u}$: show their contour plots on two separate graphs. Comment on what you are seeing. (2 graphs)