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HOMEWORK 3

Q1. $\frac{\partial u(x,t,\omega)}{\partial t} + u(x,t,\omega)\frac{\partial u(x,t,\omega)}{\partial x} = 0$, $t \in [0,L]$
$x \in [0,1]$

$u(0,t) = u(1,t)$; $u(x,0) = \frac{1}{4}\sin(2\pi x)$

For PCE:

$$u(x,t,\omega) = \sum_{i=0}^{P} u_i(x,t)\Phi_i(\omega)$$

$\Phi_i(\xi)$ is the set of basis functions of Hermite polynomials

$$\therefore \sum_i \dot{u}_i \Phi_i + \sum u_j \Phi_j \sum \frac{\partial u_i}{\partial x}\Phi_i = 0$$

$$\Rightarrow \sum \dot{u}_i \Phi_i + \sum_i \sum_j u_j \frac{\partial u_i}{\partial x}\Phi_i \Phi_j = 0$$

$$\Rightarrow \langle \dot{u}_i \Phi_i, \Phi_k \rangle + \langle \sum\sum_j u_j \frac{\partial u_i}{\partial x_i}\Phi_i \Phi_j, \Phi_k \rangle = 0$$

$$\Rightarrow \dot{u}_k + u_j \frac{\partial u_i}{\partial x} C_{ijk} = 0 \qquad \{C_{ijk} = \langle \Phi_i \Phi_j, \Phi_k \rangle\}$$

Using CDS for discretization:

$$\dot{u}_k = -u_j(x,t)C_{ijk}\left[\frac{u_i(x+\Delta x,t) - u_i(x-\Delta x,t)}{2\Delta x}\right]$$

Using Euler time stepping scheme:

$$u_k(x,t+\Delta t) = u_k(x,t) - \Delta t\, C_{ijk}\, u_j(x,t) \times \frac{[u_i(x+\Delta x,t) - u(x-\Delta t)]}{2\Delta x}$$

Also, initial condition: $u_k(x,0) = \langle \frac{1}{4}\sin(2\pi x), \Phi_k \rangle$