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Aptitude Advanced

Average & Decimals

eBook

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Chapter 1: Fractions

Fractions play an important role in most of the competitive exams. As the questions on fractions do appear directly and do have some indirect application in few of the questions.

Following are some definitions and fundamentals one needs to know before understanding the topic of Fractions.

- Numbers in the form of 3/4, 4/5, are called fractions. A fraction can be written as p/q where q $\neq$ 0.
- If the numerator and denominator of a fraction are multiplied/divided by the same number, then the value of the fraction does not change.
- For any positive proper fraction p/q (p<q), the value of the fraction increases when both the denominator and numerator are increased by the same positive number. e.g $\frac{3}{4}$ = 0.75, $\frac{3+1}{4+1}$ = $\frac{4}{5}$ = 0.8.
- For any positive proper fraction p/q (p<q), the value of the fraction decreases when both the numerator

and denominator are decreased by the same positive number. e.g $\frac{3}{4}$ = 0.75, $\frac{3-1}{4-1}$ = $\frac{2}{3}$ = 0.67.

- For any positive improper fraction p/q (p>q), the value of the fraction decreases when both the numerator and the denominator are increased by the same positive. e.g. $\frac{5}{4}$ = 1.25, adding 1 to the numerator and the denominator, we get $\frac{5+1}{4+1}$ = $\frac{6}{5}$ = 1.2, which is less than 1.25.
- For any positive improper fraction p/q (p>q), the value of the fraction increases when both the numerator and denominator are decreased by the same positive number. e.g. $\frac{5}{4}$ = 1.25, by subtracting 1 from both the numerator and denominator we get, $\frac{5-1}{4-1}$ = $\frac{4}{3}$ = 1.33 > 1.25.

1.1 Types of Fractions

Common Fractions: Fractions such 3/4, 32/43 etc are called common or vulgar fractions.

Decimal Fractions: Fractions whose denominators are 10, 100, 1000, ... are called decimal fractions.

Proper Fraction: A fraction whose numerator is less than its denominator is known as a proper fraction, e.g. 3/4

Improper Fraction: A fraction whose numerator is greater than its denominator is known as an improper fraction. e.g. 4/3

Mixed Fractions: Fractions which consists of an integral part and a fractional part are called mixed fractions. All improper fractions can be expressed as mixed fractions and vice versa. e.g. $1\frac{3}{4}$.

Recurring Decimals: A decimal in which a set of figures is repeated continually is called a recurring or periodic or a circulating decimal. e.g. $1/7$ = 0.142857........ the dots indicate that the figure between 1 and 7 will repeat continuously.

1.2 Addition of Fractions

The addition of mixed fractions can be carried out by adding the integral and fractional parts separately or even by converting them into improper fractions, e.g. $3\frac{4}{5} + \frac{11}{12}$ and then they can be added as either 3 + $\frac{4}{5}+\frac{11}{12} \Rightarrow 3 + \frac{4\times12+11\times5}{60}$

$\Rightarrow 3 + \frac{103}{60} \Rightarrow 3 + 1\frac{43}{60} = \frac{283}{60}$ or by converting the mixed fractions into improper fraction, $\frac{(19 \times 12 + 11 \times 5)}{60}$ $= \frac{283}{60}$.

1.3 Decimals and Fractions to be remembered

You get many questions in the exams based on Percentage, Profit, Interest etc. in which you have to calculate, say 87.5 % of 800, 58.33 % of 2400 etc. Calculating these values with the help of traditional methods is time-consuming. If you have the fraction approach, you can crack these easily, i.e., if you know that 87.5 % is just 7/8th of the number and 58.33 % is 7/12th of the number, then it becomes easy to calculate.

% age	Fraction		% age	Fraction
50 %	$1/2$		$55\,5/9$ %	$5/9$
$33\,1/3$ %	$1/3$		$77\,7/9$ %	$7/9$
$66\,2/3$ %	$2/3$		$88\,8/9$ %	$8/9$
25 %	$1/4$			
75 %	$3/4$		$9\,1/11$ %	$1/11$

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20 %	$\frac{1}{5}$		$18\frac{2}{11}$ %	$\frac{2}{11}$
40 %	$\frac{2}{5}$		$27\frac{3}{11}$ %	$\frac{3}{11}$
60 %	$\frac{3}{5}$		$36\frac{4}{11}$ %	$\frac{4}{11}$
80 %	$\frac{4}{5}$		$45\frac{5}{11}$ %	$\frac{5}{11}$
			$54\frac{6}{11}$ %	$\frac{6}{11}$
$16\frac{2}{3}$ %	$\frac{1}{6}$		$63\frac{7}{11}$ %	$\frac{7}{11}$
$83\frac{1}{3}$ %	$\frac{5}{6}$		$72\frac{8}{11}$ %	$\frac{8}{11}$
$14\frac{2}{7}$ %	$\frac{1}{7}$		$81\frac{9}{11}$ %	$\frac{9}{11}$
			$90\frac{10}{11}$ %	$\frac{10}{11}$
$12\frac{1}{2}$ %	$\frac{1}{8}$			
$37\frac{1}{2}$ %	$\frac{3}{8}$		$8\frac{1}{3}$ %	$\frac{1}{12}$
$62\frac{1}{2}$ %	$\frac{5}{8}$		$41\frac{2}{3}$ %	$\frac{5}{12}$
$87\frac{1}{2}$ %	$\frac{7}{8}$		$58\frac{1}{3}$ %	$\frac{7}{12}$
			$91\frac{2}{3}$ %	$\frac{11}{12}$
$11\frac{1}{9}$ %	$\frac{1}{9}$		$6\frac{2}{3}$ %	$\frac{1}{15}$
$22\frac{2}{9}$ %	$\frac{2}{9}$		$6\frac{1}{4}$ %	$\frac{1}{16}$
$44\frac{4}{9}$ %	$\frac{4}{9}$		5 %	$\frac{1}{20}$

- Anything doubles to increase by 100 % and becomes 200 %.
- Anything trebles to increase by 200 % and becomes 300 %.

1.4 Solved Examples

Ex. 1: One-quarter of one-seventh of land is sold for a total of Rs.30,000. What would be the value of eight thirty-fifths of the land?

Sol: One-Quarter of one-seventh = $\frac{1}{4} \times \frac{1}{7} = \frac{1}{28}$. Now, $\frac{1}{28}$ of land will cost = Rs.30,000

$\therefore \frac{8}{35}$ of the land will cost $\frac{30{,}000 \times 28 \times 8}{35}$ = Rs.1,92,000

Ex. 2: After taking out of a purse $\frac{1}{5}$ of its contents, $\frac{1}{12}$ of the remainder was found to be Rs7.40. What sum did the purse contain at first?

Sol: After taking out $\frac{1}{5}$ of its contents, the purse remains with $\frac{4}{5}$ of its contents.

Now $\frac{1}{12}$ of $\frac{4}{5}$ = Rs. 7.40 or $\frac{1}{15}$ = Rs. 7.40 $\therefore$ 1 = Rs.111.

Ex. 3: A sum of money increased by its seventh part amounts to Rs. 40. Find the sum.

Sol: As per the question, S+ $\frac{S}{7}$ = Rs.40

$\Rightarrow \frac{8S}{7}$ = Rs. 40 $\Rightarrow$ S = Rs. 35.

Ex. 4: A train starts full of passengers. At the first station, it drops one-third of passengers and takes in 96 more. At the next, it drops one – half of the new total and takes in 12 more. On reaching the next station, there are found to be 248 left. With how many passengers did the train start?

Sol: Let the train start with x passengers. After dropping one –third and taking in 96 passengers, the train has x - $\frac{x}{3}+96=\frac{2x}{3}+96$ = $\frac{2x+288}{3}$ passengers.

At the second station, the number of passengers $= \frac{2x+288}{6}$ +12. Now, $\frac{2x+288}{6}$ +12=248

Or 2x + 288 = 1416 $\therefore$ x = 564.

Ex. 5: A motorcycle, before overhauling, requires 5/6 hour service time every 90 days, while after overhauling. It requires 5/6 hour service time every 120 days. What fraction of the pre-overhauling service time is saved in the latter case?

Sol: LCM of 90 and 120 = 360. So, in 360 days, the pre-overhauling service time= $\frac{5}{6} \times \frac{360}{90} = \frac{10}{3}$ hrs.

And after overhauling, the service time $= \frac{5}{6} \times \frac{360}{120} = \frac{5}{2} hrs$. Time Saved = $\frac{10}{3} - \frac{5}{2} = \frac{5}{6} hrs$ hrs

$\therefore$ The required answer = $\frac{\frac{5}{6}}{\frac{10}{3}} = \frac{5}{6} \times \frac{3}{10} = \frac{1}{4}$

Chapter 2: Square Root

In order to find the square root of a number, there are two methods available. The first method is prime factorisation and the second is the conventional square root method. Factorization is suitable only when the numbers are relatively small and their factors can be easily found. Considering the kind of questions which appear in the competitive exams, firstly we are going to learn the conventional square root calculation method. In this, firstly the number is divided into pairs from the right-hand side. If in the beginning there is a pair, then the starting is done with that pair, and if there is a single digit number, then that would be the starting point.

2.1 Method to find Square Root

Illustration 1: Find the square root of 64516.

Sol:

	2 5 4
2	6 45 16
	4
45	2 45
	2 25
504	20 16
	20 16
	X

In the first place, when 6 is the single number left, after making pairs from the RHS, then you should take a number, which can be multiplied by the same number itself, and the result is less than equal to 6, which is 2.

After subtracting 4 from it, the new pair 45 is taken. The number now becomes 245. The previous quotient is doubled and 4 is obtained. Then a number ‘x’ is written with 4, in such a way that the product of ‘4x’ and ‘x’ is less than or equal to 245. So the value of x is 5.

The remainder of the next step is 20 and the last pair 16 is written with it.

Then the previous quotient 25 is doubles and 50 is obtained and a number ‘y’ is written with it in such a way that the product of ‘50y’ and ‘y’ is less than or equal to 2016.

When y is substituted by 4, the product is 2016.

In this way, the final quotient 254 is the square root of 64,516.

Illustration 2: Find the square root of 328329.

Sol:

```
            5  7  3
       ----------------
  5    | 32 83 29
       |  25
       |--------
107    |  7 83
       |  7 49
1143   |--------
       |  34 29
       |  34 29
       |--------
       |      x
       |
```

Applying the square root calculation method, in the first place after making pairs from the RHS, you are left with the number 32. Now you should take a number, which can be multiplied by the same number itself, and the result is less than equal to 32, which is 5.

After subtracting 25 from it, the new pair 83 is taken. The number now becomes 783. The previous quotient is doubled and 10 is obtained. Then a number ‘x’ is written with 10, in such a way that the product of ‘10x’ and ‘x’ is less than or equal to 783. So the value of x is 7.
The remainder of the next step is 34 and the last pair 29 is written with it. Then the previous quotient 57 is doubled and 114 is obtained and a number ‘y’ is written

with it in such a way that the product of ‘114y’ and ‘y’ is less than or equal to 3429. When y is substituted by 3, the product is 3429. In this way, the final quotient 573 is the square root of 328,329.

2.2 Solved Examples based on Square Root

Ex. 1: Find the smallest number with which 60 should be multiplied so that it becomes a perfect square.

Sol: In order to answer such questions, firstly the prime factorisation of the number is done. The factors of 60 are $2 \times 2 \times 3 \times 5$.

In this, it can be seen that ‘2’ is occurring twice, but 3 and 5 are occurring only once. In order to make a number a perfect square every prime factor should be there twice or an even number of times. So a ‘5’ and ‘3’ is required, the product of which is 15. Therefore 15 is the smallest number.

Ex. 2: In a class, each of the students contributed as many paisa as there is number of students. If the total collection was Rs. 144, what is the number of students in the class?

Sol: Let the number of students in the class be x. Now each of these students contributed 'x' paisa each. So the total collection will be x^2 paisa.

Now the total collection is given to be Rs. 144, which is 14400 paisa.

As per the statement of the question $x^2 = 14400 \Rightarrow x = 120$. Thus there are 120 students.

Ex. 3: What is the value of $\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}$?

Sol: Given exp.

$$= \sqrt{10+\sqrt{25+\sqrt{108\sqrt{154+15}}}}$$

$$= \sqrt{10+\sqrt{25+\sqrt{108+\sqrt{169}}}}$$

$$= \sqrt{10+\sqrt{25+\sqrt{108+13}}} = \sqrt{10+\sqrt{25+\sqrt{121}}}$$

$$= \sqrt{10+\sqrt{25+11}} = \sqrt{10+\sqrt{36}} = \sqrt{10+6} = \sqrt{16} = 4$$

Ex. 4: What is the value of $\sqrt{175+\sqrt{2401}}$?

Sol: Given exp. = $\sqrt{176+49} = \sqrt{225} = 15$

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Ex. 5: What is the value of $\left(\frac{\sqrt{625}}{11}\times\frac{14}{\sqrt{25}}\times\frac{11}{\sqrt{196}}\right)$?

Sol: Given exp. = $\frac{25}{11}\times\frac{14}{5}\times\frac{11}{14}=5$

Ex. 6: What is the value of $\left(\sqrt{\frac{225}{729}}-\sqrt{\frac{25}{144}}\right)\div\sqrt{\frac{16}{81}}$?

Sol: Given exp. =

$$\left(\frac{\sqrt{225}}{\sqrt{729}}-\frac{\sqrt{25}}{\sqrt{144}}\right)\div\frac{\sqrt{16}}{\sqrt{81}}=\left(\frac{15}{27}-\frac{5}{12}\right)\div\frac{4}{9}=\left(\frac{15}{108}\times\frac{9}{4}\right)=\frac{5}{16}$$

Ex. 7: What is the digit in the unit's place in the square root of 15876?

Sol: $\sqrt{15876}=126$. Hence unit digit in the square root of 15876 is 6.

Ex. 8: What is the value of $\sqrt{0.01}+\sqrt{0.81}+\sqrt{1.21}+\sqrt{0.0009}$?

Sol: Given exp.

$$= \sqrt{\frac{1}{100}} + \sqrt{\frac{81}{100}} + \sqrt{\frac{121}{100}} + \sqrt{\frac{9}{10000}}$$

$$= \frac{1}{10} + \frac{9}{10} + \frac{11}{10} + \frac{3}{100}$$

= 0.1 + 0.9 + 1.1 + 0.03 = 2.13

Ex. 9: What is the value of $\sqrt{0.0025} \times \sqrt{2.25} \times \sqrt{0.0001}$?

Sol:

$$= \sqrt{\frac{25}{10000}} \times \sqrt{\frac{225}{100}} \times \sqrt{\frac{1}{10000}} = \frac{5}{100} \times \frac{15}{10} \times \frac{1}{100} = \frac{75}{100000} = 0.00075$$

Ex. 10: What is the value of $\sqrt{1.5625}$?

Sol: $\sqrt{1.5625} = 1.25$

Ex. 11: If $\sqrt{0.00000676} = 0.0026$, then what is the square root of 67,60,000?

Sol:

$$\sqrt{6760000} = \sqrt{0.00000676 \times 10^{12}}$$

$$= \sqrt{0.00000676} \times \sqrt{10^{12}} = 0.0026 \times 10^{6} = 2600$$

Ex.12: If $\sqrt{18225} = 135$, then what is the value of $\left(\sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225} + \sqrt{0.00018225}\right)$?

Sol: Given exp.

$$= \sqrt{\frac{18225}{10^2}} + \sqrt{\frac{18225}{10^4}} + \sqrt{\frac{18225}{10^6}} + \sqrt{\frac{18225}{10^8}}$$

$$= \frac{\sqrt{18225}}{10} + \frac{\sqrt{18225}}{10^2} + \frac{\sqrt{18225}}{10^3} + \frac{\sqrt{18225}}{10^4}$$

$$= \frac{135}{10} + \frac{135}{100} + \frac{135}{1000} + \frac{135}{10000}$$

= 13.5 + 1.35 + 0.135 + 0.0135 = 14.9985

Ex. 13: An Armyman wants to arrange his men in the form of a perfect square, but he finds there are 52 men too many. What will be the total number of men in front row, if the total number of men with him is 14693?

Sol: Required number of men in the front row $= \sqrt{14693 - 52} = \sqrt{14641} = 121$.

Note: No perfect square ends with 2, 3, 7, 8, and an odd number of zeroes, i.e. any number, which has 2, 3, 7 and 8 at its unit's place and any number ending with an odd number of zeroes can never be a perfect square.

Chapter 3: Averages

Average means Arithmetic mean of the items and it is = $\frac{\text{Sum of Items}}{\text{Number of Items}}$

When the difference between all the items is same, then the average is equal to $\frac{n+1}{2}$ item, where n is the total number of items.

Average speed: If a man covers some journey from A to B at u km/hr. and returns back to B from A at a uniform speed of v km/hr., then the average speed during the whole journey is $\frac{2uv}{u+v}$ km/hr

- If x is added in all the items, then average increases by x.
- If x is subtracted from all the items, then average decreases by x.
- If every item is multiplied by x, then average also gets multiplied by x.
- If every item is divided by x, then average also gets divided by x.
- This means if average increases by y, it can be assumed that y is added to all the items.

3.1 Solved Examples based on Averages

Ex. 1: Find the average of first three prime numbers.

Sol: First three prime numbers are 2, 3 and 5. Their average is $\frac{2+3+5}{3} = \frac{10}{3}$ = 3.33.

Ex. 2: Find the average of 8, 11, 14, 17 and 20.

Sol: The sum of the items is 8 + 11 + 14 + 17 + 20. Average = $\frac{70}{5}$ = 14.

Ex. 3: Find the average of 18, 21, 24, 27 & 30.

Sol: Here in this question, the difference between all the items is same. Therefore average would be equal to (n + 1) ÷ 2th item. (5 + 1)/2 = 3rd item, i.e. 24 would be average of these five items.

Ex. 4: The average of seven consecutive numbers is 47. Find the smallest of these numbers.

Sol: As the numbers given are consecutive, their difference would always be equal. Average would be equal to (7+1) ÷ 2th item means the 4^{th} item, which is given to be 47. This means 47 is the 4^{th} number, 46 is the 3^{rd} number, 45 is the 2^{nd} number, 44 is the 1^{st} and the smallest number.

Ex. 5: The average temperature for Monday, Tuesday and Wednesday was 39.8°C. The average for Tuesday, Wednesday and Thursday were 40.8°C. If the temperature on Monday is 38°C, find the temperature on Thursday.

Sol: In this question, the sum of three days Monday, Tue and Wed is 39.8 × 3 = 119.4°
Similarly, the sum of Tuesday, Wednesday, and Thursday is 40.8 × 3 = 122.4°
Here whatever is the difference between the two totals it must be due to Monday and Thursday because the remaining two days are common in both of these cases.
The difference is 3°, which means Thursday must be 3 more than Monday because total in case of Thursday is more. Now the temperature on Monday is given to be 38°. Therefore the temperature on Thursday is 38 + 3 = 41°.

But there is no need to do all this; straight logic can be applied, which says if average has increased by 1°, it can be assumed that 1 is added in all the items and there are 3 items. This means 3 is added to the total, which must be the difference between Monday and Thursday (the uncommon days). Even in this manner, the same difference of 3 can be calculated.

Then similarly 3 would be added to the temperature of Monday to calculate temperature on Thursday.

Ex. 6: The average of 30 boys in a class is 14 yrs. However when the age of the class teacher is included the average becomes 15 yrs. Find the age of the class teacher.

Sol: Total ages of 30 boys = $14 \times 30 = 420$ yrs. Total of age when class teacher is included = $15 \times 31 = 465$ yrs. $\therefore$ Age of class teacher = $465 - 420 = 45$ yrs.

Ex. 7: The average weight of 4 men is increased by 3 kg when another man replaces one of them whose weight is 120 kg. What will the new man's weight be?

Sol: When the average weight goes up by 3 kg, then the sum of the weights is increased by $3 \times 4 = 12$ kg.
This increase in weight is due to the inclusion of new person and the extra weight that come with them.
$\therefore$ Weight of new man = $120 + 12 = 132$ kg.

Ex. 8: The average of marks obtained by 120 candidates in a certain examination is 35. If the average marks of passed candidates are 39 and the average marks for the failed candidates are 15, what will be the number of candidates who passed the examination?

Sol: Let the number of passed candidates be x. Then total marks = $120 \times 35 = 39x + (120 - x) \times 15$
Or, $4200 = 39x + 1800 - 15x$ or, $24x = 2400 \therefore x = 100 \therefore$ number of passed candidates = 100.

Ex. 9: The average of 11 results is 50. If the average of first six results is 49 and that of last six is 52, find the sixth result.

Sol: The total of 11 results = 11 × 50 = 550. The total of first 6 results = 6 × 49 = 294
The total of last 6 results = 6 × 52 = 312. The 6^{th} result is common to both;
∴ Sixth result = 294 + 312 – 550 = 56

Ex. 10: The average age of 8 persons in a committee is increased by 2 years when two women substitute two men aged 35 yrs and 45 yrs. What is the average age of the two women?

Sol: The total age of two women = 2 × 8 + (35 + 45) = 16 + 80 = 96 yrs.
∴ Average age of two women = $\frac{96}{2} = 48\text{yrs.}$

3.2 Average related to speed

- For example, if an individual travels a certain amount of distance at a speed of x km/hr and then travels the same distance at a speed of y km/hr, Then the whole journeys average speed is calculated by $\frac{2xy}{x+y}$ km/hr.

Or,

- If half of the journey is travelled at a speed of x km/hr and the next half at a speed of y km/hr, then the average speed during the whole journey is $\frac{2xy}{x+y}$ km/hr.

Or,

- If a man goes to a particular place at a speed of x km/hr and returns to the original place at a speed of y km/hr, then the average speed during up-and-down journey is $\frac{2xy}{x+y}$ km/hr.

Ex. 11: A train travels from A to B at the rate of 20 km per hour and from B to A at the rate of 30 km/hr. What is the average rate for the whole journey?

Sol: Required Average speed = $\frac{2 \times 20 \times 30}{20+30} = 24$ km/hr.

Ex. 12: The average salary of the entire staff in an office is Rs 120 per month. The average salary for officers is Rs 460 and that of non-officers is Rs 110. If the number of officers is 15, then find the number of non-officers in the office.

Sol. Let the required number of non-officers = x.
Then, $110x + 460 \times 15 = 120\,(15 + x)$
$\therefore x = 15 \times 34 = 510.$