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Aptitude Advanced

# LCM & HCF

eBook 02

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# Chapter 1: LCM & HCF

## 1.1 Least Common Multiple - LCM

The least common multiple (LCM) of two or more numbers is the smallest number, which is exactly divisible by each of them.

e.g. Consider two numbers: 12 and 15

Multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, ....

Multiples of 15 are: 15, 30, 45, 60, 75, 90, 105, 120, 135, ......

The common multiples of both 12 and 15 are 60, 120, 180,...

The least common multiple is 60.

How to find the LCM of two or more numbers?

The LCM of two numbers can be found by the product of the factors of the two numbers after eliminating repetition of the common factors.

In the above example, the common factor for 12 and 15 are 3, which should be taken once and then the uncommon factors i.e. 4 & 5 are to be taken

Therefore, the LCM will be 3*4*5 = 60.

Alternatively, LCM is the product of all prime factors of the given numbers, the common factors among them being in their highest degree. e.g., The LCM of $5x^2y^3z^5$ and $3xy^2z^7$ will be $5*3*x^2y^3z^7 = 15x^2y^3z^7$, where x, y and z are the prime factors.

## 1.2 Greatest Common Divisor (GCD)/Highest Common Factor (HCF):

The highest common factor of two or more numbers is the greatest number, which divides each of those numbers an exact number of times. e.g. HCF of 24 and 36 is 12.

*How to find the HCF of two or more numbers?*

a) Express the two numbers as the product of prime numbers separately.

b) Take the product of prime numbers common to both numbers.

## Chapter 2: LCM and HCF of Fractions

LCM of fractions = $\dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$;

e.g. LCM of $\dfrac{3}{4}$ and $\dfrac{1}{2}$ = $\dfrac{3\ (\text{LCM of numerators})}{2\ (\text{HCF of denominators})}$

HCF of fractions = $\dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$

e.g. HCF of $\dfrac{3}{4}$ and $\dfrac{1}{2}$ = $\dfrac{1(\text{HCF of numerators})}{4(\text{LCM of denominators})}$

Note that the product of the two fractions is always equal to the product of LCM and HCF of the two fractions.

The product of the two fractions = $\dfrac{3}{4}\times\dfrac{1}{2} = \dfrac{3}{8}$.

The product of the LCM and HCF = $\dfrac{3}{2}\times\dfrac{1}{4} = \dfrac{3}{8}$.

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## Chapter 3: Important Points in LCM and HCF

- In case of HCF, if some remainders are given, then first those remainders are subtracted from the numbers given and then their HCF is calculated.

- In case of LCM, if a single remainder is given, then firstly the LCM is calculated and then that single reminder is added in the LCM.

- In case of LCM, if for different numbers different remainders are given, then the difference between the number and its respective remainder will be equal. In that case, firstly the LCM is calculated, then that common difference between the number and its respective remainder is subtracted from that.

## Chapter 4: Solved Examples based on LCM and HCF

**Ex. 1.** Find the greatest number which when divides 259 and 465 leaves remainders 4 and 6 respectively.

**Sol:** Here the numbers 259 and 465 leave the remainders 4 and 6 respectively. So the required number will be obtained by finding the H.C.F. of 259 – 4 = 255 and 465 – 6 = 459. The HCF of 255 and 459 is 51, which is the answer.

**Ex. 2.** Find the least number which when divided by 6, 14, 18 and 22 leaves remainder 4 in each case.

**Sol:** The LCM of 6, 14, 18 and 22 is 1386. In order to get remainder 4 in each case, we will add 4 to the LCM. So the number is 1386 + 4 = 1390.

**Ex. 3.** Find the least number which when divided by 8, 12, 20 and 36 leaves remainders 6, 10, 18 and 34 respectively.

**Sol:** Here the numbers are 8, 12, 20 and 36 and the respective remainders are 6, 10, 18 and 34. The difference between numbers and the respective remainders is equal to 2. So, first of all, find the LCM of 8, 12, 20 and 36 which is 360. The required number is 360 – 2 = 358.

**Ex. 4.** Find the greatest number, which when 41, 71 and 91 leave the same remainder in each case.

**Sol:** Take the difference between all the three pairs of numbers and their HCF will be the answer

i.e. 91 – 41 = 50, 71 – 41 = 30, 91 – 71 = 20. Now the HCF of 50, 30 and 20 is 10.

Sometimes in such questions, the common remainder can also be asked.

You can divide any of the numbers given by HCF (91 ÷ 10) and find the remainder to be equal to 1.

**Ex. 5.** Find the L.C.M of $\frac{2}{5}, \frac{3}{10}$ and $\frac{6}{25}$.

**Sol:** L.C.M. of $\frac{2}{5}, \frac{3}{10}$ and $\frac{6}{25} = \frac{\text{L.C.M. of } 2,3 \text{ and } 6}{\text{H.C.F. of } 5,10 \text{ and } 25}$

L.C.M. of 2, 3 and 6 = 6; H.C.F. of 5, 10 and 25 = 5. Thus the LCM of these three fractions will be 6/5.

**Ex. 6.** How often will five bells toll together in one hour if they start together and toll at intervals of 5, 6, 8, 12, 20 seconds, respectively?

**Sol:** The time after which the bells will ring together is the L.C.M. of 5, 6, 8, 12 and 20 seconds, i.e. 120 seconds. The number of times they will toll together in one hour = (3600 ÷ 120) = 30. Thus they will toll together 30 times in an hour.

⇒Sometimes the question is how many times they toll together in the first hour, in that case after finding the answer like above, you need to add 1 for a start together as well, i.e. in the first hour it is 1 more than the usual number of times.