Aptitude Advanced

Venn Diagram and Logs

eBook

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Chapter 1: Set Theory

A set is well–defined collection of objects. Sets are usually denoted by A, B, C, –––. The objects in the set are called 'elements' of the set.

If x is an element of the set A, we say, $x \in A$ (x belongs to A).

Sets are usually defined in two forms – the tabular form and the roster method.

Tabular form: In the tabular form, all elements of the set are enumerated or listed. e.g. The set of natural numbers: N = {1, 2, 3, 4, – –}, the set of integers: I = {– –, –3, – 2, – 1, 0, 1, 2, 3, – –}.

Roster method: Under this method, the defining property of the set is specified. If all the elements in the set have a property P, then we can define the set as A = {x : x has the property P}.

E.g. N = {x : x is a natural number}, I = {y : y is an integer}.

1.1 Types of Sets

Finite sets: A set with a finite number of elements is called a finite set. e.g. A = {a, e, i, o, u}.

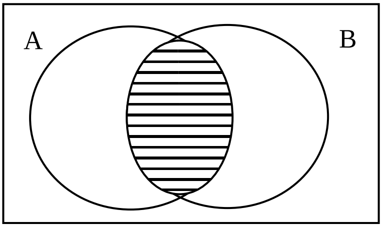
Empty set: A set which contains no elements is called an empty set or a null set. An empty set is denoted as '{}' or ϕ.

Infinite set: A set that is neither an empty set nor a finite set is called an infinite set. Such a set will contain infinitely many elements. e.g. $N = \{1, 2, 3, --\}$.

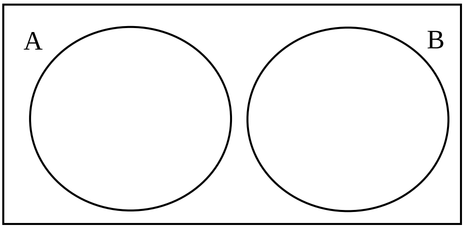
Disjoint sets: Two sets are said to be disjoint sets if they do not have any common elements. e.g. If A = {x : x is an even number} and B = {y : y is an odd number}, then A and B will be disjoint sets.

Overlapping sets: If two sets A and B have some common elements, the sets are said to be overlapping. e.g. A = {1, 2, 3, 4, 5} and B = {3, 4, 5, 6, 7}. These sets are overlapping sets as the elements 3, 4 and 5 are common to both sets. The elements common to both sets is called the intersection of the two sets and is denoted as $A \cap B$. On the other hand, the addition of two sets is called the union of two sets and is denoted by $A \cup B$.

Equal sets: Two sets are said to be equal if they contain the same elements. e.g. A = {2, 4, 6, 8}



Overlapping



Non – overlapping

B = {x : x is an even number between 1 and 9} are equal sets as they contain the same elements.

Universal set: A universal set is the set that contains the elements of all the sets under consideration. It is usually denoted by U, e.g. A = {4, 7, 8, 9}, B = {−4, −2, 0, 1, 4, 7, 10}. The set of integers, I = {− −, −2, −1, 0, 1, 2, − −} will be the universal set for A and B.

Complement of a set: Given a set A, the complement of a set is the set that contains elements not belonging to A and is denoted by Ac. The union of the given set and its complement will give the universal set i.e. U = A + Ac. e.g.
A = {x : x is a Mathematics book in the library}.
Ac = {y : y is not a Mathematics book in the library} And the universal set in this case will be
U = the set of all books in the library.

1.2 Basic Operations on Sets

1. **Intersection of two sets:** The intersection of two sets is the set of elements common to both the given sets.
   The intersection of two sets A and B is denoted as $A \cap B$.
   In notation form, we can define the intersection of two sets A and B as $A \cap B = \{x : x \in A, x \in B\}$.
   If, $A \cap B = \phi$, then A and B are disjoint sets.
   If $A \cap B \neq \phi$, then A and B are overlapping sets.

2. **Union of two sets:** The union of two sets is the set containing the elements belonging to A and also

the elements belonging to B. The union of these sets is denoted by A ∪ B. In notation form, we can define the union of two sets as A ∪ B = {x : x ∈ A, x ∈ B, x ∈ A ∩ B}.

3. **Difference of two sets:** The difference of two sets A and B is the set of elements that belong to A but do not belong to B.
The difference of two sets is denoted by A – B. In notation form, we can define the difference of two sets as A – B = {x : x ∈ A, x ∉ B}.

SYMBOL	MEANING	EXAMPLE
{ }	is a set	$S = \{4,5\}$
∈	is an element of	$s \in S$
∉	is not an element of	$s \notin T$
⊆	is a subset of	$S \subseteq T$
⊂	is a proper subset of	$S \subset T$
∪	union	$S \cup T$
∩	intersection	$S \cap T$
∅	the empty set	$\{2,3,4\} \cap \{5,6,7\} = \varnothing$

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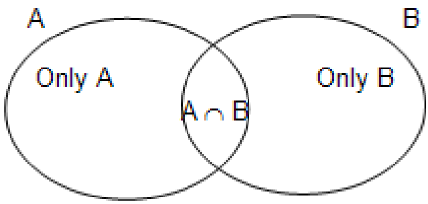
Chapter 2: Venn Diagram

Venn Diagram: A Venn diagram is a closed figure used to denote the set of all points within the figure.

Venn Diagram of two objects: If the two given objects are called 'A' & 'B'. The sum of 'A' & 'B' here is known as "A union B" and is symbolically represented as "$A \cup B$".
$A \cup B = A + B - A \cap B$.

Where "$A \cap B$" ⇒ "A intersection B" ⇒ that part which is common in A & B. "$A \cap B$" is subtracted once from the sum of A & B because it is included twice in the total, being a part of both A & B.



If the value of 'A union B' is to be calculated from the diagram, the sum of these 3 values given inside the diagram will give you "$A \cup B$". Because one part represents only 'A', other part only 'B' and the third common part "$A \cap B$".

2.1 Solved Examples

Ex.1. In a locality, 2500 persons read Times of India and 3500 persons read The Tribune. There are 250 persons who read both of these newspapers. Find the number of persons who read 1 or 2 of these newspapers.

Sol. It is very much clear after reading the question that we have to find the value of ‘$A \cup B$’.

$\Rightarrow A \cup B = 2500 + 3500 - 250 = 5750.$

Ex.2. In a class of 64 students, 50 % of the students have taken Sociology and 75 % of the students have taken Politics.

How many students have taken both the subjects?

Sol. We have $S = 64 \times \frac{50}{100} = 32$, $P = 64 \times \frac{75}{100} = 48$.

Now $(S \cup P) = S + P - (S \cap P)$

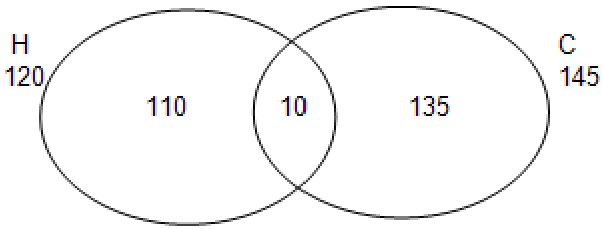
$\Rightarrow 64 = 32 + 48 - (S \cap P)$.

Therefore, $(S \cap P) = 80 - 64 = 16$.

Ex.3. In a class, 120 students play Hockey, 145 students play Cricket. There are 10 students who play both of these games. The total number of students in the class is 280. What is

a) The number of students playing 1 or more of games?
b) The number of students playing no game?
c) The number of students playing only Cricket?
d) The number of students playing only Hockey?

Sol. As we have to answer four questions, it is better to make diagram in this case.



a) The students who play one or more of these two games means the value of $H \cup C \Rightarrow 110 + 10 + 135 = 255$.
b) The total number of students in the class is given to be 280, as only 255 play one or more games, this implies the remaining students play none of these games i.e. $280 - 255 = 25$.

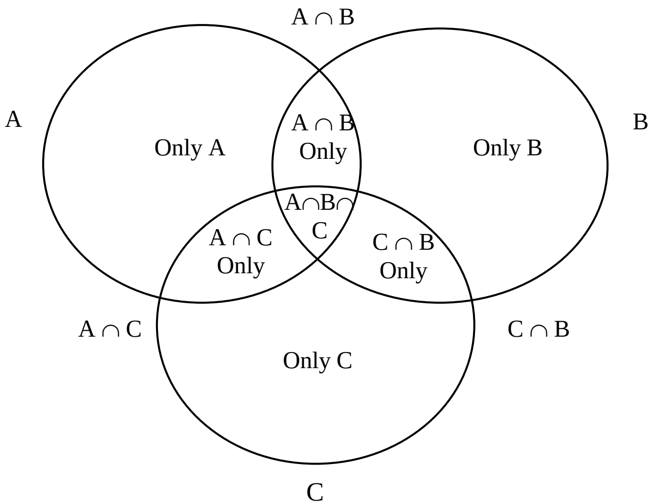
c) It can be seen from the diagram that there are 135 students who play only Cricket.

d) It can be seen from the diagram that there are 110 students who play only Hockey.

2.2 Venn Diagram of 3 objects

If the three given objects are called 'A', 'B' & 'C', the sum of 'A', 'B' & 'C' is known as "A union B union C" & is represented as "$A \cup B \cup C$".

$A \cup B \cup C = A + B + C - A \cap B - B \cap C - C \cap A + A \cap B \cap C$.



If "$A \cup B \cup C$" is to be calculated from the diagram, then all the 7 parts given inside the circle should be added i.e.

only A, only B, only C, only $A \cap B$, only $B \cap C$, only $C \cap A$ and $A \cap B \cap C$.

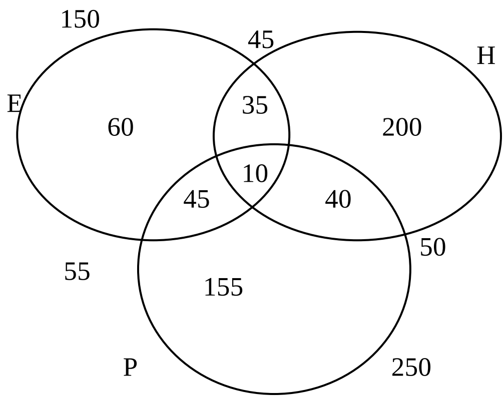
It is very important here to understand the meaning of certain terms.
At least 1: means minimum 1 i.e. 1 or more than 1.
At least 2: means minimum 2 i.e. 2 or more than 2.
At the most 2: means maximum 2 i.e. 2 or less than 2.
At the most 3: means maximum 3 i.e. 3 or less than 3.

Ex.4. In a school 150 students have English, 200 have Hindi only, 250 have Punjabi. 45 students have Hindi & English, 50 students have Hindi & Punjabi, 55 students have Punjabi & English. There are 10 students, who have all the 3 languages. Having at least one of these languages is a must. Calculate the number of students

1. In the school.
2. Having only English.
3. Having only Hindi & English.
4. Having exactly 1 language.
5. Having exactly 2 languages.
6. Having at least 1 language.

7. Having at least 2 languages.
8. Having at most 3 languages.
9. Having Hindi & one other language.
10. Having English & one or more other languages.

Sol. First of all the diagram will be made. Take care in this case, that the second sentence specifies that 200 have Hindi only, means it is to be written inside the Hindi circle.



1. Number of students in school = 60 + 200 + 155 + 35 + 45 + 40 + 10 = 545.
2. Only English = 60 (diagram).
3. Only Hindi & English = 35 (common part between H & E only)

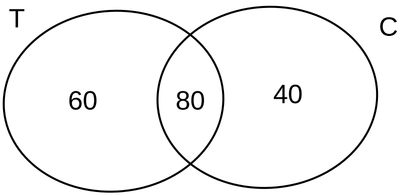
4. Exactly 1 language = 60 + 200 + 155 = 415.
5. Exactly 2 languages = 35 + 45 + 40 = 120.
6. 1 or more than 1 $\Rightarrow$ includes all 3 = 545.
7. 2 or more than 2 $\Rightarrow$ 35 + 45 + 40 + 10 = 130.
8. 3 or less than 3 $\Rightarrow$ includes all 3 = 545.
9. 40 + 35 = 75.
10. 35 + 45 + 10 = 90.

2.3 More Solved examples

Ex.1. In a college, 200 students are randomly selected. 140 like tea, 120 like coffee and 80 like both tea and coffee.

1. How many students like only tea?
2. How many students like only coffee?
3. How many students like neither tea nor coffee?
4. How many students like only one of tea or coffee?
5. How many students like at least one of the beverages?

Sol. The given information may be represented by the following Venn diagram, where T = tea and C = coffee.



1. Number of students who like only tea = 60
2. Number of students who like only coffee = 40
3. Number of students who like neither tea nor coffee = 20
4. Number of students who like only one of tea or coffee = 60 + 40 = 100
5. Number of students who like at least one of tea or coffee = n (only Tea)+ n (only coffee) + n (both Tea & coffee) = 60 + 40 + 80 = 180

Ex.2. A school has 65 students studying English, Maths and Physics. 31 study English, 22 Maths and 24 Physics. 10 study English and Maths, 8 study

Physics and Maths while 6 study both English and Physics. How many study all the three subjects?

Sol. Let A, B and C denotes the no of students studying English, Maths and Physics respectively.
No of students studying all the three subjects = $n(A \cap B \cap C)$
We know,
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
$65 = 31 + 22 + 24 - 10 - 8 - 6 + n(A \cap B \cap C)$
$n(A \cap B \cap C) = 65 - 53 = 12.$

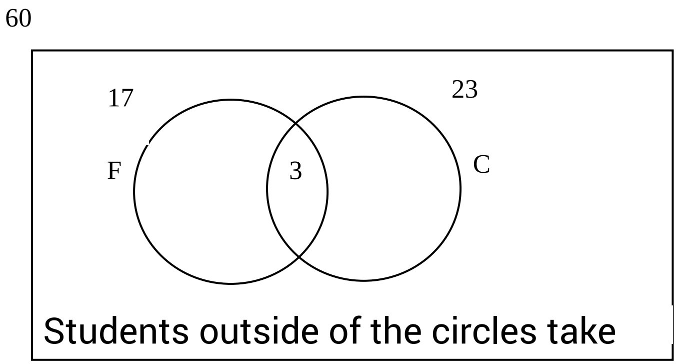
Ex.3. There are 120 students at DAV Public School. 72 students play football and 57 play hockey. How many of these students play both football and hockey?

Sol. Here, No of students who play football $n(A) = 72$
No of students who play hockey $n(B) = 57$
So, the no of students who play both football and hockey = $n(A \cap B)$
We know, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$\Rightarrow 120 = 72 + 57 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 129 - 120 = 9$

Ex.4. In a class of 60 students, 17 opt for French, 23 opt for Chinese, and 3 opt for both French and Chinese. How many students in the class are not enrolled in either of the two languages French or Chinese?

Sol.



Here, No of students who take French n(A) = 17

No of students who take Chinese n(B) = 23

So, the no of students who take both French and Chinese = $n(A \cap B) = 3$

Now, No of students who take neither of the two languages = 60 – [17 + 23 - 3] = 60 – 37 = 23.

Ex. 5. In Delhi Public School, 45 students study Biology and English both. If 60 students study biology, and 75 students study English. How many of these students study at least one of the two subjects?

Sol. Here, No of students who study Biology n(A) = 60
No of students who study English n(B) = 75
So, the no of students who study both Biology and English = $n(A \cap B) = 45$
Now, no. of students who study at least one of the two subjects = $n(A \cup B)$
So, $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 60 + 75 - 45 = 90$.

Ex. 6. In a school of 420 students, there are two clubs, dramatics club and dance club. 180 students are in dramatics club, 70 students are the members of both the clubs. How many students are involved in dance club, given every student must be a member of at least one of these two clubs ?

Sol. Here, No of students who participate in at least one of the two subjects $n(A \cup B) = 420$

No of students who are in dramatics club n(A) =180

No of students who participate in both the clubs $n(A \cap B) = 70$

Now, No of students who are in dance club = n(B)

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$420 = 180 + B - 70 \Rightarrow n(B) = 310$

Ex. 7. In a class there are:

- 8 students who study both Maths and English.
- 7 students who studies none of Maths and English
- 13 students who study Maths
- 19 students who study English.

How many students are there in the class?

Sol. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore n(A \cup B) = 19 + 13 - 8 = 24$

In the universal set, The elements which are not included in AUB are also added, i.e. elements not being the part of AUB. Therefore total people are = 24 + 7 = 31.

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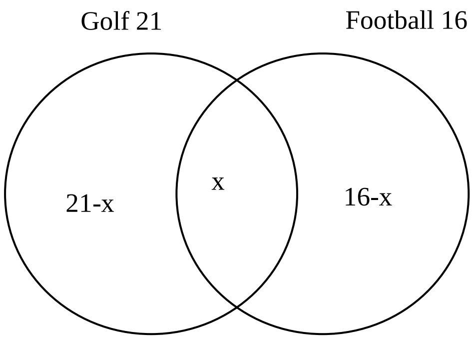
Ex. 8. In a class there are 30 students.

- 21 students like Golf
- 16 students like Football.
- 6 students like neither Golf nor Football

How many students like both Golf and Football?

Sol.



AUB = Universal set – None of them

∴ n(AUB) = 30 - 6 = 24

Further n (AUB) = n(A) + n(B) – n(A∩B)

∴ 24 = (21 - x) + (x) + (16 - x)

Solving, we get x = 13

So we can conclude that 13 students play both the games.

Ex. 9. In the class, 12 students enrolled for both Ballet dancing and Salsa. 22 enrolled for Salsa. If the students of the class enrolled for at least one of the two dance forms and there are 40 students in the class, then how many students enrolled for only Ballet dancing and not Salsa?

Sol. Let A be the set of students who have enrolled for Ballet dancing and B be the set of students who have enrolled for Salsa.

Then, (A U B) is the set of students who have enrolled at least one of the two dance forms. As the students of the class have enrolled for at least one of the two dance forms, n(A U B) = 40

We know $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ i.e., $40 = n(A) + 22 - 12$ or $n(A) = 30$ which is the set of students who have enrolled for Ballet dancing and includes those who have enrolled for both the dance forms.

However, we need to find out the number of students who have enrolled for only Ballet dancing = Students enrolled for Ballet dancing - Students enrolled for both Salsa and Ballet dancing = 30 - 12 = 18.

Chapter 3: Logarithms

The logarithms of any positive number to a given base are the index of the power to which the base must be raised in order to equal the given number. Thus $a^x = m$, x is called the logarithm of m to the base a.

e.g. since $2^4 = 16$, the logarithm of 16 to the base 2 is 4. Given are some of the important formulas relating to logs.

- $a^{\log_a N} = N$.
- $\text{Log}_a (MN) = \text{Log}_a M + \text{Log}_a N$.
- $\text{Log}_a (PQR) = \text{Log}_a P + \text{Log}_a Q + \text{Log}_a R$.
- $\text{Log}_a (M/N) = \text{Log}_a M - \text{Log}_a N$.
- $\text{Log}_a M^N = N \log_a M$.
- $\text{Log}_M M = 1$
- $\text{Log}_M N = \text{Log}_P N \div \text{Log}_P M$
- $\text{Log}_M N = \dfrac{1}{Log_N M}$
- The natural numbers 1, 2, 3,… are respectively the logarithms of 10, 100, 1000, … to the base 10.
- The logarithm of “0” and negative numbers are not defined.

Note: To every positive number there corresponds a definite value of logarithm. By taking logarithms of a certain expression and applying the properties of logarithms, the operations of multiplication, division, involution and evolution are reduced to addition and subtraction of logarithms, and their multiplication and division by a number, which is much easier to handle.

The logarithm of a number to the base "10" is known as common logarithm and the logarithm of a number to the base "*e*" is known as natural logarithm.

Characteristic and Mantissa of Common Logarithms:

The integral part of the common logarithm of a number $x > 0$ is called the ***Characteristic*** and the fractional part is called the ***mantissa***.

e.g. the logarithm of 2 to the base 10 is 0.3010, where 0 is the characteristic and 3010 is the mantissa.

Any positive number x can be written in the form $x = a10^n$, where $1 < a < 10$ and n is an integer. The number n is called the order of the number x. e.g. 30 can be written as 3×10^1 and similarly 300 can be written 3×10^2. The same rule applies to fractions as well where the value of n will be negative.

The characteristic of the logarithm of the given number *x* will be *n* and the mantissa will be the logarithm of *a*. Therefore, while log 2 = 0.3010, log 20 will be 1.3010 as *n* in this case is 1.

Thus, the value of the characteristic of the logarithm of a number will help determine the number of integral digits the number has = characteristic + 1.

3.1 Solved examples

Ex.1. Find the logarithm of 64 to the base of 2.

Sol. You are supposed to find $\log_2 64$. Let it be equal to x.

Now $\log_2 64 = x$. $\Rightarrow 2^x = 64 \Rightarrow 2^x = 2^6 \Rightarrow x = 6$.

Ex.2. If the value of $\log_4 y^3 = 3/2$, find the value of y.

Sol. Now this can be written as $(4)^{3/2} = y^3 \Rightarrow (2^2)^{3/2} = y^3$.

$2^3 = y^3 \Rightarrow 2 = y$ and this is the value of y.

Ex.3. Find the algebraic sum of logarithms $\text{Log}\,[(x^3y^2)/z^5]$

Sol. Now this expression can be written as

$\text{Log}\,x^3 + \log y^2 - \log z^5$.

$\Rightarrow$ 3 log x + 2 log y – 5 log z and this will be the answer.

Ex.4. Find the value of x from the equation $a^x \times c^{-2x} = b^{3x+1}$.

Sol. Taking logarithms of both sides, you have

$x \log a - 2x \log c = (3x + 1) \log b$;

$\Rightarrow x \log a - 2x \log c = 3x \log b + \log b$

$\Rightarrow x \log a - 2x \log c - 3x \log b = \log b$;

$x (\log a - 2 \log c - 3 \log b) = \log b$;

$\Rightarrow x = \dfrac{\log b}{\log a - 2\log c - 3\log b}$.

Ex. 5. If log 2 = 0.3010, then how many digits are contained in the number 2^{80}?

Sol. $\log 2^{80} \Rightarrow 80 \log 2 = 80 \times 0.3010 = 24.08$.

The characteristic of the log of 2^{80} = 24.

Therefore, the number of digits the number has = 25.

Do not think that it has been rounded off.

Whether the number obtained is 24.15 or 24.85 the number of digits will be 25 only i.e. the smallest integer greater than the value calculated.

Ex. 6. Solve for x: $(\log x)^2 - \log x^3 + 2 = 0$.

Sol. This is the same as $(\log x)^2 - 3 \log x + 2 = 0$.
Substituting log x with a, we get $\Rightarrow a^2 - 3a + 2 = 0$.
Solving for a, we get $a = 2$ or $a = 1$.
i.e. $\log x = 2$ or $\log x = 1$. Therefore, $x = 100$ or $x = 10$.

Ex. 7. Solve for $\frac{1}{\log_p QR + 1} + \frac{1}{\log_Q PR + 1} + \frac{1}{\log_R PQ + 1}$.

Sol. $\frac{1}{\log_P QR + 1} = \frac{1}{\log_P QR + Log_P P} = \frac{1}{\log_P PQR} = \log_{PQR} P$.

Similarly from the second and third part you get Log $_{PQR}$ Q and Log $_{PQR}$ R respectively. Now the equation becomes

$$\log_{PQR} P + \log_{PQR} Q + \log_{PQR} R \Rightarrow \log_{PQR} PQR = 1.$$