Aptitude Advanced

Clocks and Calendars

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Chapter 1: Clocks

The dial of a clock is divided into 12 spaces, which stand for the hours of the day. Each hour space is further sub divided into 5 spaces – this represents duration of 1 minute. There are typically 2 hands that rotate on the clock - The hour hand and the minute hand. The hour hand covers $\frac{1}{12}$th of the dial in one hour's time. The minute hand covers exactly 1 circumference of the dial. In terms of relative speed, the minute hand is moving at a rate equivalent to 55 minutes per hour ahead of the hour hand. So in 1 minute the minute hand covers a relative distance equivalent to $\frac{55}{60}$ or $\frac{11}{12}$ minutes.

Important Points:

- The two hands of the clock meet once every hour, but in a 12-hr period 11 times or 22 times in a day. This is also called the two hands are in a straight line in the opposite direction.
- The two hands of the clock make an angle of 180 degrees once every hour, but in a 12-hr period 11

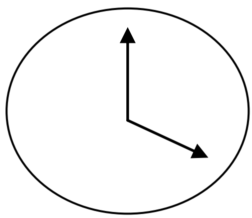
times or 22 times in a day. It is also said to be that the two hands are in a straight line in the opposite direction.

- There are two right angles in every hour, but in a 12 hour period there are 22 such angles or 44 angles in a day.
- If the hands are moving at the normal pace, they meet after every 65 5/11 min.
- The relative speed of the minute hand is 11/12 min or 11/2 degrees per minute.

1.1 Solved Examples

Ex.1. At what time between 4 and 5 are the minute and hour hand together?

Sol. At 4 O'clock the relative distance between the hour and the minute hand is 20 minutes.



To catch up with the hour hand, the minute hand has to cover a relative distance of 20 minutes at the relative speed of $\frac{11}{12}$ minutes per minute.

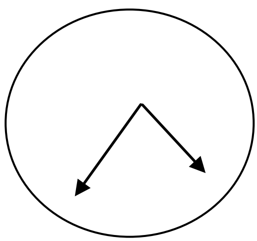
Thus time required = $\frac{20}{11/12}$ = 20 * 12/11 => 240/11

=> 21 9/11 min. .

Ex.2. At what time between 4 and 5 are the minute and hour hand are at right angles?

Sol. At 4 O'clock the relative distance between the hour and the minute hand is 20 minutes. To make a 90–degree angle with the hour hand, the minute hand has to cover a relative distance of 5 minutes at the relative speed of $\frac{11}{12}$ minutes per minute. Thus time required = 5 * 12/11 = 60/11 5 5/11 min.

Hang on, there is still one more case. When a relative distance of 35 minutes has been covered, even then the angles would be at right angles.



Time required = 35 * 12/11 = 420/11 = 38 2/11 min

Ex. 3. At what time between 4 and 5 are the minute and hour hand be in a straight line in the opposite direction?

Sol. At 4 O'clock the relative distance between the hour and the minute hand is 20 minutes.
To make a 180–degree angle with the hour hand, the minute hand has to cover a relative distance of 50 minutes at the relative speed of $\frac{11}{12}$ minutes per minute. Thus time required = 50 × 12/11 = 600/11 or 54 6/11 minutes.

1.2 Clock is losing/gaining Time

Too Fast and Too Slow:
If a watch indicates 9.20, when the correct time is 9.10, it is said to be 10 minutes too fast. And if it indicates 9.00, when the correct time is 9.10, it is said to be 10 minutes too slow.

Such kind of problems appear in exams very often, when a clock runs faster or slower than the expected pace. **There are just two possibilities:**

Clock is running fast: It is also referred to as gaining time i.e. when a normal clock covers 60 minutes, a faster clock will cover more than 60 minutes.

Clock is running slow: It is also referred to as losing time i.e. when a normal clock covers 60 minutes, a slower clock will cover less than 60 minutes.

Ex. 1. A watch gains 5 minutes in one hour and was set right at 8 AM. What time will it show at 8 PM on the same day?

Sol. A correct clock would have completed 12 hours by 8 pm. But the faster clock actually covers 5 min. extra in one hour. So, it will cover $12 \times 5 = 60$ minutes extra.

Therefore, when the correct clock would show 8 pm, the faster clock will show 60 minutes extra i.e. 9 pm.

Ex. 2. A watch loses 5 seconds in one hour and was set right at 7 am. What time will it show at 2 pm on the same day?

Sol. A correct clock would have completed 7 hours by 2 pm, whereas the slower clock looses 5 seconds per hour i.e. $5 \times 7 = 35$ seconds in 7 hours.
Therefore the slower clock shows 1:59:25 pm.

NOTE: "Even a broken clock is right twice a day." However, a clock which gains or loses a few minutes might not be right twice a day or even once a day. It would be right when it had gained/lost exactly 12 hours.

Ex. 3. A watch loses 5 minutes every hour and was set right at 6 am on a Monday. When will it show the correct time again?
Sol. For the watch to show the correct time again, it should lose 12 hours.
It loses 5 minutes in 1 hour. $\Rightarrow$ It loses 1 minute in 12 minutes

⇒ It will lose 12 hours (or 720 minutes) in 720 × 12 minutes = 8640 minutes = 144 hours = 6 days.

⇒Thus, the clock will show the correct time again at 6 am on next Sunday.

Ex. 4. The minute hand of a clock overtakes the hour hand at intervals of 63 minutes of correct time. How much does the clock lose or gain?

Sol. In a correct clock, the minute hand gains 55– minute spaces over the hour hand in 60 minutes. To be together again, the minute hand must gain 60 min over the hour hand. We know that 60 minutes are gained in $\frac{60}{55}\times 60$ minutes i.e. $65\frac{1}{11}$ minutes.

But we know that they are together in 63 minutes.
So gain in 63 minutes is 27/11 minutes.
Gain in 24 hours = $\frac{24\times 60}{63}\times\frac{27}{11}$ = 56 8/77 min..

Chapter 2: Calendars

In an ordinary year there are 365 days, which means 52 × 7 + 1, or 52 weeks and one day. This additional day, we call an odd day. If 1st January of this year is on Sunday, then 1st January next year will be exactly 52 full weeks and a day after that – so on a Monday. So you can already decide if the New Year hangover is going to be on a weekday or a weekend.

This is all right as long as the year is not a leap year. The Earth actually completes 1 orbit around the Sun (which approximates our calendar year) in $365\frac{1}{4}$ days, (not exactly but actually a few minutes less). A leap year occurs every 4 years to adjust for the $\frac{1}{4}$ day – so every 4th year has 366 days (2 odd days).

And as far as the few odd minutes of the orbit time are concerned, well every 100 years starting 1 AD, the year is declared to be a non–leap year, but every 4th century is a leap year. So any year divisible by 400 will be a leap year e.g. : 1200, 1600 and 2000. The years 1800, 1900 will be non-leap years.

The concept of odd days is very important in determining the days of the week. Let us look at how many odd days will there be in a century – i.e. 100 years. There will be 24 leap years and 76 non–leap years. This means that there will be $24 \times 2 + 76 \times 1 = 124$ odd days. Since 7 odd days make a week, to find out the net odd days, divide 124 by 7. The remainder is 5 – this is the number of odd days in a century.

You can remember the following points relating to the concepts of calendars.

100 years give us 5 odd days as calculated above.
200 years give us $5 \times 2 = 10 - 7$ (one week) $\Rightarrow$ 3 odd days.
300 years give us $5 \times 3 = 15 - 14$ (two weeks) $\Rightarrow$ 1 odd day.
400 years give us {5 × 4 + 1 (leap century)} – 21} (three weeks) $\Rightarrow$ 0 odd days.

Month of January gives us 31 – 28 = 3 odd days.

Month of February gives us 28 – 28 = 0 odd day in a normal year and 1 odd day in a leap year and so on for all the other months.

In total first six months i.e. January to June give us 6 odd days in a normal year and 7 – 7 = 0 odd days in a leap year. This is going to help, when you want to find a day, which is after 30th June.
In total first nine months i.e. January to September give us 0 odd days in a normal year and 1 odd day in a leap year.

When you count from the beginning i.e. 1st January, 0001
1 odd day mean – Monday
2 odd days mean – Tuesday
3 odd days mean – Wednesday and so on 6 odd days means Saturday.

2.1 Solved Examples

Ex. 1. Let us take the example of 10 May, 1999.

Sol. We know that in 1600 years there will be 0 odd days.
So start counting from here. Then from 1601 – 1900 these i.e. 300 years after that will give 1 odd day.
10 May, 1999 implies that starting from the end of 1900, 98 years, 4 months and 10 days have elapsed since then.
98 years have 24 leap years and 74 non leap years leading to 122 odd days.
Dividing by 7 and checking remainder, net odd days = 3.
In the 4 months and 10 days of 1999, there are 31 days in Jan, 28 in Feb, 31 in March, and 30 in April.
Total days elapsed in 1999 = 31 + 28 + 31 + 30 + 10 = 130. So net odd days = 4.
Adding up all the odd days we have got so far we get a total of 1 + 3 + 4 = 8.
Net odd day = 1, so May 10, 1999 was a Monday.

(The rule is that 0 odd days means the day is a Sunday, 1 means Monday, 2 means Tuesday and so on.)
Sometimes a reference date might be given to you. This makes your task easier, as you then start counting odd days only from that day only.

Ex. 2. You know that May 10, 1999 was a Monday. So what will be the day on 10Dec,–2001?

Sol. . May 10, 2001 will be a Thursday. (1999 is a non leap year so add one day and 2000 is a leap year, so add 2 odd days.). Now start counting the days from May 10, 2001 to 10–Dec–2001. Complete months in between are June, July, Aug, Sep, Oct, Nov – total days = 30 + 31 + 31 + 30 + 31 + 30 = 183 days. Besides that add 21 days in May and 10 days in Dec. So total days = 214. Net odd days = 4. So 10–Dec–2001 will be 4 days after Thursday, which is Monday.

Ex. 3. What was the day on 15th August, 1947?

Sol. Up to first 1600 years ⇒ no odd day.
From 1601 – 1900 ⇒ 1 odd day
From 1901 – 1946: ⇒ 46 + 11 (leap)
= 57 – 56 (complete weeks) ⇒ 1 odd day

First six months give = 6 odd days
July 3 = 3 odd days
Up to August 15 = 1 odd days

Total up to 15th August ⇒ 10 odd days
Total no. of odd days ⇒ 12 – 7 (complete week) = 5
Now counting from the beginning, 5 odd days, so India's first Independence Day was on a Friday.

Ex. 4. Find the day for **9 December 2001**

Sol. No. of odd days in 2000 years = 0.
Now we will calculate odd days in 2001.
As discussed in total first nine months i.e. January to September give us 0 odd days in a normal year.
After that October gives you **3** odd days.

November gives you **2** odd days.
And December, we want to go till 9, it will give us 9 – 7 = **2** odd days.
Total odd days are **3 + 2 + 2 = 7 ⇒ 0 odd days.**
As discussed earlier, 0 odd days means **Sunday**.