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Aptitude Advanced

Time, Speed and Distance

eBook

Table of Contents

Chapter 1: Basics of Time, Speed and Distance

Time, speed and distance (TSD) is one of the most popular topics in most of the tests. Almost 2 to 3 questions in the paper are asked from this chapter. The concept of time, speed and distance is also used extensively for questions relating to some other areas as well . Students should try to understand the inter-relationship between the factors time, speed and distance.

The most important relationship between time, speed and distance is

Distance Travelled = Speed × Time

And the different ways of expressing the same relationship will also come in handy while solving problems based on this concept. Those are

Speed = Distance ÷ Time and

Time = Distance ÷ Speed

The problems that occur in this topic can be classified into one of the following sub headings:

(a) **Speed, Time & Distance**

(b) **Trains**

(c) **Relative Speed**

(d) **Races**

1.1 Important Distance and Time Conversions

1 km = 1000 meter

1 meter = 100 cm

1 hour = 60 min

1 min = 60 sec

1 hour = 3600 sec

1 km/hr **=** $\left(\frac{1\times 1000}{1\times 3600}\right)=\frac{5}{18}$ **m/sec**.

1.2 Solved examples on Speed, Time and Distance

Usually, problems in this subheading cover finding one of the values, when the other two are provided. The important thing to note in such problems is that the units of all three quantities used are the same, i.e. if speed in km/hr, then distance will have to be in km and time in hours.

Ex.1. What is the distance covered by a car traveling at a speed of 40 kmph in 15 minutes?

Sol: $40 \times 15/60 = 10$ km.

The important point to note is that time given was in minutes, whereas the speed was in kmph.

Therefore, either speed will have to be expressed as km/min or time will have to be expressed in hours to apply the relationship.

In this case we converted time into hours to get the answer.

Ex.2. Traveling at a speed of 50 Kmph, how long is it going to take to travel 60 km?

Sol: Distance = Speed × Time ∴ Time = Distance ÷ Speed ⇒60/50 = 1.2 hours = 1 hour and 12 minutes.

Note: While converting decimal hours into the minutes, these are to be multiplied with 60 and not by hundred.

Ex.3. A man covers 75 km in 90 minutes. What is his speed in km/h?

Sol: Speed = Distance ÷ Time.
Since, time is given in minutes and the required answer is in km/h, we need to convert time into equivalent hours.
90 minutes = 90/60 = 1.5 hours.
Therefore, speed = 75/1.5 = 50 km/h.

Ex.4. Walking $^5/_6{}^{th}$ of his usual speed, Mike reached his destination 10 minutes late. Find his usual time, and the time taken on this occasion?

Sol: Let his usual speed be *x* km/hr and his usual time be *t* hours.

His time on this occasion is $\frac{5}{6}x$,

The time taken is $\left(t+\frac{10}{60}\right)$ hours.

Since the distance traveled on both occasions is the same, $xt = \frac{5x}{6} \times \left(t+\frac{10}{60}\right)$.

Solving for *t*, we get *t* = $\frac{5}{6}$ hours

= 50 minutes, and the time taken on this occasion = 50 + 10 = 60 minutes.

Ex.4(a). If the distance traveled by Mike be 60 km, then what was his usual speed and what was the speed on this occasion?

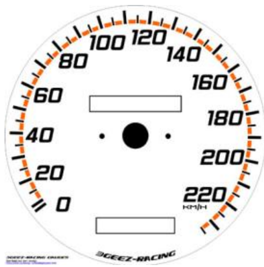
Sol: Usual time taken = 50 minutes = $\frac{5}{6}$ hours

The distance = 60 km.

Usual Speed = Distance ÷ Usual Time $\Rightarrow \frac{60}{5/6}$ = 72 kmph.

Speed on this occasion =
Distance ÷ Time on this occasion = $\frac{60}{1}$ = 60 kmph.

Note: In general, **speed and time have an inverse relationship**. Therefore, if the speed becomes, say 0.5 times the original speed, then the time taken becomes twice as much as taken originally for the same distance. Or if the ratio of the speed of two moving objects is in the ratio of 3 : 4, the time taken by them to cover identical distance will be in the ratio of 4 : 3.



Chapter 2: Average Speed

Case 1: When the time taken is the same:
The average speed of traveling at two different speeds for the same time span is just the simple average of these two speeds. So, if two speeds are a and b

$$\textbf{Average Speed} = \frac{a+b}{2}$$

Case 2: When the distance travelled is the same:

However, the above simple average rule does not work when the time span of each of the different speeds is different and only the distance is the same. In this case, **one** should take the simple average of the inverses of the two speeds and then again inverse the speed.
Let the two speeds be *a* kmph and *b* kmph.
Let the distance traveled in each of the speeds be *x* km.
Time taken to cover *x* km at ‘*a*’ kmph= $\frac{x}{a}$ & at ‘*b*’ kmph= $\frac{x}{b}$.

Total time taken = $\frac{x}{a}+\frac{x}{b}=\frac{bx+ax}{ab}=\frac{x(b+a)}{ab}$, and the total distance covered = 2x.

Therefore, average speed = $\frac{2x}{x(a+b)/ab}=\frac{2ab}{a+b}$.

The average speed of traveling at two different speeds when the distance travelled is same, is harmonic mean of these two speeds. So, if two speeds are a and b

$$\textbf{Average Speed} = \frac{2ab}{a+b}$$

2.1 Solved examples on Average speed

Ex.5. A motorist travels one hour at an average speed of 45 kmph and the next hour at an average speed of 65 kmph. Then his average speed is

Sol: (45 + 65) ÷ 2 = 55 kmph.
The total distance traveled by the motorist in these two hours = 65 + 45 = 110 km and he has taken

two hours. Therefore, his average speed = 55 kmph.

Ex.6. On my way from the office to the Pimpri class, I drive at 30 kmph and on the return journey I drive at 45 kmph. What is my average speed of travel?

Sol: 37.5 kmph is incorrect as the time traveled is different in both the cases and only the distances are same.

Here as the distance traveled is the same at both the speeds. Thus the average speed will be 2ab/(a+b) => 2*30*45/(30+45) = 36 kmph

Chapter 3: Relative Speed and Trains

- When two objects are moving in the same direction, then their relative speed is the difference between the two speeds.

- When two objects are moving in the opposite direction, then their relative speed is the sum of the two speeds.
- When it crosses a stationary man / lamp post / sign post / pole - in all these cases the object which the train crosses is stationary - and the distance traveled is the length of the train.
- When it crosses a platform / bridge - in these cases, the object which the train crosses is stationary - and the distance traveled is the length of the train + length of the object
- When it crosses another train which is moving at a particular speed in the same / opposite direction - in these cases, the other train is also moving and the relative speed between them is taken depending upon the direction of the other train - and the distance is the sum of the lengths of both the trains

- When it crosses a car / bicycle / a mobile man - in these cases again the relative speed between the train and the object is taken depending upon the direction of the movement of the other object relative to the train - and the distance traveled is the length of the train.

Ex.7. A train traveling at 60 kmph crosses a man in 6 seconds. What is the length of the train?

Sol: Speed in m/sec = 60 × 5/18= 50/3 m/sec.
Time taken to cross the man = 6 seconds.
Therefore, distance traveled = 50/3 × 6 = 100 meters = Length of the train

Ex.8. A train traveling at 60 kmph crosses another train traveling in the same direction at 50 kmph in 30 seconds. What is the combined length of both the trains?

Sol: Speed of train A in m/sec = 60 × 5/18= 50/3m/sec
Speed of train B in m/sec = 125/9 m/sec
The relative speed is = 50/3 - 125/9= 25/9 m/sec.

Time taken for train A to cross train B = 30 seconds.
Therefore, distance traveled = (25/9) × 30 = 250/3 meters
= Combined length of two trains.

Chapter 4: Boats and Streams

Downstream movement: When the direction of the movement of a river and a boat is the same, their collective movement is known as the downstream movement. And the distance covered by boat is known as downstream distance.

Upstream movement: When the direction the movement of the river and a boat is opposite, they are said to be in upstream movement. The distance covered in this case is known as upstream distance.

If the speed of the river = R and the speed of the boat - B. then upstream speed = B - R (Conventionally the speed of boat is taken more the than speed of the river.)

In most of the cases of boats and streams. The distance covered downstream and upstream are the same. In those cases, the ratio of the time taken becomes inverse of the ratio of the speeds.

i.e. $\frac{\text{Timetakendownstream}}{\text{Timetakenupstream}} = \frac{\text{upstreamspeed}}{\text{downstramspeed}}$

So we can say if the speed of the boat in still water is say B kmph and if the speed at which the stream is flowing is W kmph, then

(i) When the boat is traveling with the stream the speed of the boat = $(B + W)$ kmph

(ii) When the boat is traveling against the stream the speed of the boat = $(B - W)$ kmph.

(iii) If the upstream is denoted as U and downstream is denoted as D then

(iv) $B = \frac{D+U}{2}$., $W = \frac{D-U}{2}$.

Hence, if downstream speed and upstream speed are given as 20 km/h and 10 km/h respectively, then the speed of the boat = 15 km/h and speed of the river = 5 km/h.

4.1 Solved Examples on Boats and Streams

Ex.9. A boat travels at a speed of 30 kmph with the stream and 18 kmph against the stream. What is the boat's speed in still water and the speed of the stream?

Sol: Speed of the boat in still water = $\frac{30+18}{2}$ = 24 kmph

and Speed of the stream = $\frac{30-18}{2}$ = 6 kmph

Ex.10. Driving 5/4th of his usual speed, David reached the destination 12 minutes earlier. What is the usual time he takes to travel?

Sol: Let *X* km/hr be the usual speed and let t hours be the usual time taken.

Speed on this occasion = $\frac{5}{4}X$ km/hr

The time taken on this occasion = $\left(t-\frac{12}{60}\right)$ hrs.

Since the distance is the same in both the cases,

$Xt = \frac{5}{4}X \times \left(t-\frac{12}{60}\right)$

Solving for *t*, *t* = 1 hour - the usual time taken.

Ex.11. In a cross-country race, a motorist averages a speed of 140 mph during the first 4 hours and then increases his average by 20 mph during the last 3 hours. What was his average speed during the entire race?

Sol: Distance traveled in first 4 hours = 140 $\times$ 4 = 560 miles

Distance traveled in next 3 hours = 160 $\times$ 3 = 480 miles

Therefore, the total distance traveled = 1040 miles.

The total time taken = 7 hours.

Therefore, the average speed = $\frac{1040}{7}$ = $148\frac{4}{7}$ mph.

Ex.12. During the onward journey from Bombay to Pune, Deccan Queen travels at an average speed of 80 kmph, while on the return journey, the train is able to average a speed of 100 kmph. What is the average speed of the train on its entire journey?

Sol: Average speed = $\frac{2ab}{a+b}$ = $\frac{2\times80\times100}{180}$

= $\frac{800}{9}$ = $88\frac{8}{9}$ km/hr

Ex.13. A train traveling at 100 km/hr crosses a bridge of half a km length completely in 30 seconds. What is the length of the train?

Sol: Speed = 100 km/hr = 100 $\times \frac{5}{18}$ = $\frac{250}{9}$ m/sec

Time taken to cross = 30 seconds.

Therefore, distance traveled = $\frac{250}{9} \times$ 30 = $\frac{2500}{3}$ m

Distance = Length of the train + length of the bridge

$\frac{2500}{3}$ = Length of the train + 500

$\Rightarrow$ Length of the train = $\frac{1000}{3}$ m

Ex.14. A train crosses a signpost in 6 seconds and a car traveling in the same direction at 50 kmph in 72 seconds. What is the length of train and the speed at which it is traveling?

Sol: *Case I:*

Let *X* km/hr be the speed of the train. = $X \times \frac{5}{18}$ m/sec

Time taken to cross a signpost = 6 seconds.

Therefore distance traveled = $X \times \frac{5}{18} \times 6$ = $\frac{5X}{3}$ meter

= length of the train

Case II:

The speed of the car = 50 km/hr.
Relative speed of the train with respect to car = (X – 50) km/hr
= $(X - 50) \times \frac{5}{18}$ m/sec.
Time taken to cross the car = 72 seconds.
Therefore, distance traveled = $(X - 50) \times \frac{5}{18} \times 72$

= 20 (X-50) m = length of train
Equating length of the train in Case I and Case II, we get $\frac{5X}{3}$ = 20 (X – 50).
Solving for X, we get $X = \frac{600}{11}$ km/hr
and the length = $\frac{5}{3} \times \frac{600}{11} = \frac{1000}{11}$ m

Ex.15. Traveling at 6 km/hr, I reach my office 20 minutes late. Traveling at 8 km/hr I reach my office 30 minutes early. What is my usual speed and time taken to reach my office?

Sol: Let my usual speed be S km/hr and my usual time be$t$ hours.

Therefore, $6\times\left(t+\frac{20}{60}\right)=8\times\left(t-\frac{30}{60}\right)$. Solving for t, we get *t* = 3 hours.

Since the usual time taken = 3 hours, usual distance traveled = 3S kms,

Equating distance traveled usually, with distance traveled at any of the other two speeds, we get $6\times\left(3+\frac{20}{60}\right)=$ 3*S*. Therefore, S = $6\frac{2}{3}$ km/hr.

Chapter 5: Concept of Circular Motion

Let us explore the concept of circular motion. It involves analyzing problems which involve circular tracks. People generally move around a track clockwise or anti clockwise at different speeds. The basic objective of this concept is generally to calculate:

I. The time of meeting of people (running around the track) at the starting point again after they started.
II. Meeting for the first time anywhere on the track.
III. At how many distinct points do people meet while running on the circular track.

Time to meet at the Starting point: The concept here is that firstly the time taken by each of the persons to run one complete round at their respective speeds and then take the L.C.M of these times. This LCM gives the time of their meeting again at the starting point.

Illustration: There is a track with a length of 120 mtrs and 2 people, A & B, are running around it at 12 m/min and 20 m/min respectively in the same direction.

Now, the time of their meeting again at the starting point will be the LCM of 120/12 & 120/20 i.e. 10 & 6, which is

30 mins. So, after 30 mins these people will be together at the starting point.

You can also check this as after 30 mins. A would have taken 3 rounds and B would have taken 30/6 = 5 rounds. So after completing 3 & 5 rounds they will be at the starting point.

Time to meet anywhere: Now to meet for the first time the faster person has to complete one full round extra over the slower person. The faster person is ahead of the slower one right from the first minute only due to his speed being higher than the speed of the other and they both are moving in the same direction. It can be said that when the faster is ahead of the slower by one full track length, he will be overtaking the slower person from behind. Now, at this very moment these people meet.

In order to calculate the time we can say that time of meeting = $\frac{\text{track length}}{\text{relative speed}}$.

Taking the previous data example's data to find when will B overtake A,

Time of meeting = $\frac{120}{20-12} = \frac{120}{8} = 15\text{min}$.

In order to visualize we can say that B covers 8 mtr/min extra over A. So when B covers 120 mtrs. extra he will overtake A from the behind and hence they both will meet.

No. of points: Let's suppose the question now is at how many different points do people running around a circular track meet.

The logic that operates behind this problem: if we divide the time of their first meeting at the starting point with the time of their first meeting anywhere on the track, we get the number of points at which these people would meet including the starting point.

Again considering the data from the above example, we can say that number of points = 30/15 = 2 points.

This is inclusive of the starting point. There are two points, where they will meet.

Shortcut to find the number of distinct meeting points:

Step 1: Take the lowest ratio of the speeds.
Step 2: Add if objects are moving in opposite directions.

Step 3: Subtract if objects are moving in same directions.

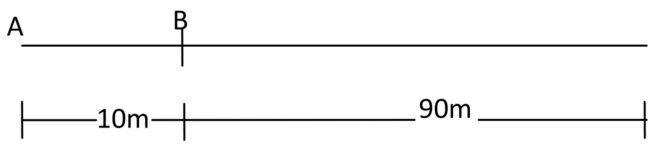
So, all the three parts can be represented in the following table:

	Same Direction	**Opposite Direction**
Time to reach initial point first time	L.C.M $\left\{\frac{x}{a}, \frac{x}{b}\right\}$	L.C.M $\left\{\frac{x}{a}, \frac{x}{b}\right\}$
Time to meet for the first time on track	$\frac{x}{\lvert a-b \rvert}$	$\frac{x}{a+b}$
No. of distinct meeting Points	Lowest ratio of a & b = c : d (say). Then No. of distinct meeting Points= |c-d|	Lowest ratio of a & b = c : d (say). Then No. of distinct meeting Points= c+d

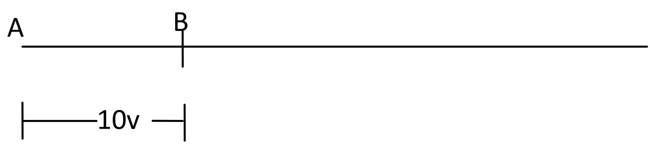
Chapter 6: Races

In a race, one can give other a head start of distance or time. When the terminologies, A gives B a start of 10 meter or A beats B by 10 meter in a 100 meter race, is used, it means that when A completes 100 meter B would have completed only 90 meter. So, we can take following two cases.

Case 1. A gives a start of 10 m to B → When B has already run 10 m, then A starts running.



Case 2. A gives a start of 10 secs to B → B has already run for 10 secs, now A starts running.



Where V m/s is the speed of B.

Ex.16. In a 400 meter race, A can give B a start of 25 meter and C a start of 50 meter. What start can B give C?

Sol: When A runs 400 meter, B runs 375 meter.
When A runs 400 meter, C runs 350 meter.
i.e. when B runs 375 meter, C runs 350 meter.
Therefore, when B runs 400 meter,
C runs 400 * 350/375 = 373.33 metres
Or B can give C a start of 26.67 meter in a 400 meter race.

Ex.17. A gives B a start of 20 meter in a 200-meter race and B gives C a start of 27 meter in a 300 meter race. How much start can A give C in a half km race?

Sol: Race 1: Length 200 meter.
When A reaches 200 meter, B reaches 180 meter.
$\Rightarrow$ When A reaches 100 meter B reaches 90 meter.
Race 2: Length 300 meter.

When B reaches 300 meter, C reaches 273 meter
$\Rightarrow$When B reaches 100 meter, C reaches 91 meter.
Therefore, in a 100 meter race, when B finishes 100 meter C finishes 90*91/100 = 81.9 metres..
In other words, A can give C a start of 18.1 meter in a 100-meter race.
Therefore, in a half a kilometre race, A can give C a start of 90.5 meter.

Ex.18. Tom can run a km in 9 minutes and 54 seconds, while Jerry can run in 10 minutes. How many metres start can Tom give Jerry?

Sol: Since, Jerry takes 6 seconds longer than Tom, the start that Tom can give Jerry is the distance that Jerry can cover in 6 s.
In 10 minutes Jerry can run 1000 metre.
Therefore, in 6 seconds he can run $1000 \times 6/600$ = 10 metre