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QUANT

MATHEMATICAL PUZZLES

eBook

Mathematical Puzzles

1.1 Basic Theory

Puzzles or mathematical puzzles have a frequent appearance in the campus tests. Usually, in such questions, a specific situation is given with some conditions or constraints. You have to answer the question given in the end, considering all the possibilities, conditions, directions and constraints. Sometimes in such questions, you have to apply the basic knowledge of mathematics to answer the question and sometimes the puzzle is purely logic based. Having the basic knowledge of algebra or equations will prove to be handy while solving puzzles.

Sometimes you may see a question, like an angle between the two hands of the clock at a specific time; in that case, the basic knowledge of clocks would be of great help. Similarly, so many puzzles are based on the meeting points of two people on a track; in such cases, the basic knowledge of time, speed and distance formulas would prove to be very helpful. The definitions of the various types of numbers are important as you may come across a puzzles having use of integers or natural numbers and so on. The given below solved examples will help you to understand the concepts of mathematical puzzles in a logical manner.

1.2 Solved Examples

Example 1: My watch is 10 minutes slow and I am under the impression that it is five minutes fast. Your watch is five minutes fast and you think it is 10 minutes slow. We both plan to catch a train at 4 O'clock. Who gets there first?

Sol: You get there earlier than me. As I am under the impression that my watch is five minutes fast so I will try to arrive a short time before 4: 05. But 4: 05 by my watch is in fact 4: 15. You think that your watch is 10 minutes slow. So you will try to come earlier than 3:50 as per your watch. But 3:50 by your watch is in fact 3:45, well in advance of the departure of the train.

Example 2: A group of friends and me went on a holiday to a hill station. It rained for 13 days. But when it rained in the morning the afternoon was lovely. And when it rained in the afternoon the day was preceded by a clear morning.Altogether there were 11 very nice mornings and 12 very nice afternoons. How many days did our holiday last?

Sol: Let the holiday lasted for ‘x’ days. In these ‘x’ days there will be ‘x’ mornings and ‘x’ afternoons. It is given that there were 11 mornings when it did not rain. It means that in the remaining x – 11 mornings it rained. Similarly, there were 12 afternoons when there was no rain. So in the remaining x – 12 afternoons it rained. Since in totality there were 13 days when it rained, so we have

$x - 11 + x - 12 = 13$

$\Rightarrow 2x = 36$

$\Rightarrow$ x = 18. So the vacation lasted for 18 days.

Example 3: Romila appeared for a Math exam. She was given 100 problems to solve. She tried to solve all of them correctly but some of them went wrong. Anyhow she scored 85.Her score was calculated by subtracting two times the number of wrong answers from the number of correct answers. Can you tell how many problems she solved correctly?

Sol: Let us assume that she answered W wrong answers and R right answers.

$$R - 2W = 85$$
$$R + W = 100$$
$$-3W = -15.$$

$\Rightarrow W = 5$. Now $5 + R = 100$. $\Rightarrow R = 100 - 5 = 95$.
So she solved 95 problems correctly.

Example 4: Mammu has 16 pairs of white socks and 16 pairs of black socks. She keeps them all in the same drawer. If she picks out three socks at random what is the probability she will get a matching pair?

Sol: Let in the first draw she picked a white sock. In the second she picked a black sock. Now when she will pick the third sock, it will be either white or black. So she will get one matching pair. Hence the probability of getting a matching pair is 1.

Example 5: If human beings did not die the earth would sooner or later be overcrowded with the progeny of just one couple. If death did not hinder the growth of human life, within a score of years or so our continents would be teaming with millions of people fighting each other for living space.

Let us take for instance the case of two couples today. Supposing each of these couples gave birth to 4 children and the eight children in turn form 4 couples and each couple, in turn, give birth to 4 children, and those 16 children again form eight couples and give birth to 4 children each and so on, can you tell exactly the number of progeny the two initial couples would have after 10 generations?

Sol: We start out with 2 couples, These 2 couples will give birth to 4 children each. So there will be 8 or 2^3 children. These 8 children will form 4 couples and these 4 couples will give birth to 16 or 2^4 children. So the progeny the two initial couples will be $2^3+2^4+2^5+2^6+2^7+2^8+2^9+2^{10}+2^{11}+2^{12}$, which is a G.P. with first term 8 and common difference 2.

So the required sum = $\frac{8\left(2^{10}-1\right)}{2-1} = 8 \times 1023 = 8184.$

Example 6: On a certain island, 5% of the 10000 inhabitants are one-legged and half of the others go barefoot. What is the least number of shoes needed on the island?

Sol:10,000-Surprised? Well, this is how it works out, It is really immaterial what percentage of the population is one-legged! In any case, the one-legged people will all require one shoe per head. From the remaining, half will go barefoot and therefore they need no shoes and the rest will need two shoes per head. And this works out at one shoe per person for the ‘others’. Therefore, we shall need for the whole population on the average one shoe per head.

Example 7: A friend of mine emptied a box of matches on the table and divided them into three heaps, while we stood around him wondering what he was going to do next. He looked up and said ‘well friends, we have here three uneven heaps. Of course, you know that a matchbox contains altogether 48 matches. This I don’t have to tell you. And I am not going to tell you how many there are in each heap’. ‘What do you want us to do?’ one of the men shouted. ‘Look well, and think. If I take off as many matches from the first heap as there are in the second and add them to the second, and then take as many from the second as there are in the third and add them to the third, and lastly if take as many from the third as there are in the first and add them to the first – then the heaps will all have equal number of matches.’ As we all stood there puzzled he asked,’ can you tell me how many were there originally in each heap?’ Can you?

Sol: To solve this problem, we shall have to start from the end. We have been told that after all the transpositions; the number of matches in each heap is the same. Let us proceed from this fact. Since the total number of matches has not changed in the process, and the total number being 48, it follows that there were 16 matches in each heap. And so, in the end, we have: First Heap: 16, Second Heap: 16, Third Heap: 16.Immediately before this, we have added to the first heap as many matches as there were in it, i.e. we had doubled the number. So, before the final transposition, there are only 8 matches in the first heap. Now, in the third heap, from which we took these 8 matches, there were: 16 + 8 = 24 matches. We before the last step we had the numbers as follows: First Heap: 8, Second Heap: 16, Third Heap: 24.

We know that we took from the second heap as many matches as there were in the third heap, which means 24 was double the original number. That implies the number before the double must be 12 and the additional 12 has come from the second heap. From this, we know how many matches we had in each heap after the first transposition.

First Heap: 8, Second Heap: 16 + 12 = 28, ThirdHeap: 12.

Now applying the same process in the first step, w can find that 28 is the double of 14. The additional 14 must have come from the first heap. We can draw the final conclusion that before the first transposition the number of matches in each heap was:

First Heap: 22, Second Heap: 14, Third heap: 12.

1.3 More solved Examples

Example 8: Ram, Shyam and Gumnaam are friends. Ram is a widower and lives alone and his sister takes care of him. Shyam is a bachelor and his niece cooks his food and looks after his house. Gumnaam is married to Gita and lives in a large house in the same town. Gita gives the idea that all of them could stay together in the house and share monthly expenses equally. During their first month of living together, each person contributed Rs. 25. At the end of the month, it was found that Rs 92 was the expense so the remaining amount was distributed equally among everyone. The distribution was such that everyone received a whole number of Rupees. How much did each person receive?

Sol: Let us say that Ram's sister is X and Shyam's niece is Y. So there will be six persons including Gita and Gumnaam. So they contributed 25 × 6 = Rs 150. As the expenditure was Rs 92, so the leftover amount is Rs 58 which cannot be distributed among the six persons in such a way that they get an equal whole number. The same is true if the persons are 5. Now, what if Gita is the woman who is Ram’s sister and Shyam’s niece. So there will be 4 persons and the total initial contribution is 25 × 4 = Rs 100. As the expenditure is Rs 92, So they are left with Rs 8 which they will divide among themselves and each one will get Rs 2.

Example 9: When Roni was questioned about the number of children he is having, he replied, “Each of my sons has as many sisters as brothers, whereas each of my daughters has twice as many brothers as sisters”. What is the number of sons and daughters that Roni is having?

Sol: He says that each of his sons has as many brothers as sisters. This tells you that the number of sons must be one more than a number of daughters. So the equation will be S = D + 1. Secondly, he says, that each of his daughters has twice as many brothers as sisters. So the equation will be (D – 1) × 2 = S. These two equations can be solved together and the answer is 3 daughters and 4 sons.

Example 10: In an isolated island there live two tribes: SACHAS and JHUTAS. The Sachas always speak the truth, whereas the Jhutas never do.

One day Sunil, a stranger in the island, met four tribesmen. He asked one of them whether the second was a Sacha or a Jhuta. The reply was ‘Jhutas’ Similarly, the second tribesman said that the third was a Jhuta and the third said the same about the fourth, When he asked the fourth as to what would the first tribesman have said about the third, the reply once again was ‘Jhuta’. Who’s who?

Sol: Before tackling the problem let us grasp the essentials of the situation. If the question ‘Are you a Sacha or a Jhuta?’ is asked to a tribesman from that island, his reply would always be ‘Sacha’ irrespective of the tribe he belongs to, because if he were a Sacha he must speak the truth and if he were a Jhuta he must lie about his actual tribe and, consequently, say that he is a Sacha. Similarly, if two tribesmen, A and B, belong to the same tribe (either both Sachas or both Jhutas) and if the question ‘Is B a Sacha or a Jhuta? ’is thrown at A the reply will always be ‘Sacha’. However, if the two belong to different tribes the answer will always be- ‘Jhuta’ for it is either a Sacha speaking the truth about a Jhuta or a Jhuta lying about a Sacha. As the first tribesman told Sunil that the second was a Jhuta we deduce that those two tribesmen belonged to different tribes. Similarly, the second and third tribesmen were from different tribes; so, the first and third belonged to the same tribe and the second belonged to the other. Moreover, as the third and fourth tribesmen too were from different tribes, we infer that the second and fourth belonged to the same tribe. Thus, either the first and third tribesmen were both Sachas and the other two both Jhutas, and vice-versa. In the light of the above deduction it is easy to see that the first tribesman, if asked whether the third was a Sacha or a Jhuta, would have replied 'Sacha' because the two belong to the same tribe. So, the fourth tribesman, according to whom the reply of the first would have been to the contrary, must be a Jhuta.

Hence, the first and third tribesmen were both Sachas and the other two Jhutas.

Example 11: There are 3 triplet brothers. They look identical. The eldest is John and he always speaks the truth. The second is Jack and he always tells lies. The third is Joe, he either tells the truth or a lie.

Jimmie Dean went to visit them one day. He was wondering who was who. So he asked each person a question.
He asked the one who was sitting on the left: "Who is the guy sitting in the middle?" The answer was "He is John."
He asked the one who was sitting in the middle: "What is your name?".The answer was "I am Joe."
He asked the one who was sitting on the right: "What is the guy sitting in the middle?" The answer was "He is Jack."
Jimmie Dean got really confused. Basically, he asked 3 same questions, but he got 3 different answers. Would you find out ‘who is who’ for Jimmie?

Sol: Since John always tells the truth, we can find him first by false logic. If the one sitting on the left were John, he would not say (when asked who the guy in the middle was), "He is John". So he cannot be John. If the one in the middle is John, he should say "I am John" (not, I am Joe). So he cannot be John. John is, therefore, the one on the right. His statement was true. Therefore the middle one is Jack. That leaves Joe on the left.

Example 12: There are 100 glasses. A servant has to supply these glasses to a person. If he supplies the glasses without any damage he will get 3 paise per glass otherwise he will lose 3 paise per glass. At the end of supplying 100 glasses if he gets 270 paise. How many glasses were supplied safely?

Sol: Apply a very simple logic, that if he supplies all the glasses in the proper condition, he will get 300 paise. If he delivers one glass in a wrong condition, his loss will be 6 paise (3 paise that he will not get and 3 paise that he will have to pay). His total loss is 300 – 270 = 30 paise. The loss of one glass is 6 paise, so 30 paise must be the loss of 5 glasses. This implies that 5 glasses were broken and rest 95 was in the proper condition.

Example 13: I have a horse. Do you know what colour it is?
Allan said, "I guess it is not black".
Brian said, "It is either brown or grey".
Charlie said, "I know it is brown".
I said, "At least one of you is right and at least one of you is wrong." What is the colour of my horse?

Sol: If the horse is brown then everyone is right. This is not the answer. If the horse is black, then everyone is wrong. This is not the answer either. Therefore, the horse is grey. To verify the answer, Allan was right, Brian was right, but Charlie was wrong.

Example 14: The investigator asked Jaclyn about her children. Jaclyn said "I have 3 daughters, Alice, Betty, and Cindy. The product of their ages is 36. The sum of their ages is the same as the street number of our next door neighbour." The investigator went next door and came back and said: "Still not enough information". Jaclyn said: "Oh, I forgot to tell you that my oldest daughter is now in school". The investigator found out the ages of her daughters immediately. Do you know their ages?

Sol: Let us try to make the possible cases.

Alice	Betty	Cindy	Sum
36	1	1	38
18	2	1	21
12	3	1	16
9	4	1	14
9	2	2	13
6	6	1	13
6	3	2	11
4	3	3	10

Now let us take the case where the number of the house of the next door neighbor is 14. So the investigator will find that the ages of the daughters are 9, 4 and 1 year. But since the investigator said that the information is not complete, so the house number must be 13. The investigator is not sure that whether the ages are 9, 2 and 2 or 6, 6 or 1. Now the Jaclyn said that her oldest daughter is now in school, so it means that there is one daughter who is oldest. So we are left with only one case i.e. the ages of the daughters are 9, 2 and 2 as in the other case there will be two daughters of the same age and will be oldest.

Example 15: There are 3 boxes. One box is having only oranges, the second box is having only the apples and the third box is having both apples and oranges. But all 3 boxes with labels are COMPLETELY mislabeled. You will be allowed to take only one piece of fruit from one of the boxes to examine it and then tell which fruit is in which box. Which box would you choose? How do you correctly re-label all three boxes?

Sol: This puzzle is very simple if you list out all the cases. Remember that ALL the boxes are wrongly labelled.

1. Orange box (box 1) can have apples or "apples and oranges".

2. Apple box (box 2) can have oranges or "apples and oranges".

3. Mixed box (box 3) can have either apples or oranges but not both.

Let us try the orange box (box 1) first. If you are lucky, you pick up an orange, so you can be sure you also have an apple in the box (mixed). In this case, the problem is solved. Unfortunately, you have only a 25% chance of getting an orange. If you pick up an apple instead, you cannot tell if there may also be oranges in the box. If you select the Apple box (box 2), it will be the same as selecting the orange box. The only choice is to select from the box labelled "mixed" (box 3).

We know it is mislabeled so that it CANNOT contain mixed fruit. If you pick up an orange, then you know that the box labelled "apples" must contain the mixed fruits. (If it had just apples in it, it would not be mislabeled!) The orange box can have apples only. If you pick up an apple, the apple box can have oranges only. The orange box will then be the mixed fruit.

Example 16: Mohan has a certain number of apples with him. He gives half of the apples and then one apple to Sohan. Then he gives half of the remaining apples and then one apple to Rohan. And in the same way, he gives to Kohan. After giving it to Kohan he has 7 apples. What is the number of apples that Mohan was having in the beginning?

Sol: After giving apples to Kohan, Mohan is left with 7 apples. Let the apples remained after giving to Rohan be 'x'. So Mohan divided the apples into two equal parts i.e. $x/2$ apples in each part. Then he gave one part to Kohan and 1 apple from the second part to Kohan. The remaining apples are 7. So we have

$x/2 - 1 = 7 \Rightarrow x/2 = 8 \Rightarrow x = 16$. So the apples left after giving to Rohan are 16. In the same way, the apples after giving to Sohan are $(16 + 1) \times 2 = 34$ (as in the last case).

So the apples initially were $(34 + 1) \times 2 = 70$.