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Aptitude Advanced

Number System

eBook 01

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Chapter 1: Types of Numbers

Natural Numbers: 1,2,3,4 are called natural numbers or positive integers.

Whole Numbers: 0,1,2,3 are called whole numbers. Whole numbers include “0”.

Integers: -3, -2, -1, 0, 1, 2, 3 are called integers.

Negative Integers: -1, -2, -3 are called negative integers.

Positive Fractions: ${}^{2}/_{3}$, ${}^{4}/_{5}$, ${}^{7}/_{8}$ are called positive fractions.

Negative Fractions: The numbers -${}^{6}/_{8}$,${}^{7}/_{19}$,${}^{12}/_{47}$,...are called negative fractions.

Rational Numbers: Any number which is a positive or negative integer or fraction, or zero is called a rational number. A rational number is one which can be expressed in the following format$\Rightarrow {}^{a}/_{b}$, where b $\neq$ 0 and a & b are positive or negative integers.

Irrational Numbers: An infinite non-recurring decimal number is known as an irrational number. These numbers cannot be expressed in the form of a proper fraction a/b where b $\neq$ 0., e.g. $\sqrt{2}$, $\sqrt{5}$, π, etc.

Surds: Any root of a number, which cannot be exactly found is called a surd. Essentially, all surds are irrational numbers. e.g. √2,√5 etc.

Even Numbers: The numbers, which are divisible by two are called even numbers,e.g., 4, 0, 2, 16 etc.

Odd Numbers: The numbers, which are not divisible by two are odd numbers, e.g., 7, -15, 5, 9 etc.

Prime Numbers: Those numbers, which are divisible only by themselves and 1, are called prime numbers. In other words, a number, which has only two factors 1 and itself, is called a prime number. e.g. 2, 3, 5, 7, etc.

2 is the only even prime number.

There are 25 prime numbers upto 100. These are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 & 97. These should be learnt by heart.

When two numbers HCF is 1 they are considered to be prime to each other. e.g. 5 and 21 are prime to each other. In other words, 5 and 21 are **co-prime.**

Remember: A number Z can be written as the product of two factors, which are co-prime to each other, in 2^{Y-1} ways, where Y is the number of different prime factors of Z.

E.g. $Z = 120 = 2^3 \times 3^1 \times 5^1$. Now here the number of different prime factors of 120 is 3 (2, 3 & 5).

So the value of *Y* is 3. 120 can be written as the product of two numbers which are co-prime to each other as 2^{3-1}. These are 15 × 8, 24 × 5, 40 × 3, 120 × 1

To Check whether a number is prime, e.g. 113, we do not need to check all the factors below 113. The square of 10 is 100 and that of 11 is 121. Therefore, test if any of the prime numbers less than 11 is a factor of 113. The prime numbers less then 11 are 2, 3, 5, 7, 11 and none of these is a factor of 113, so 113 is a prime number. In case it is divisible by any of those prime numbers, then the number is not prime.

Composite Number: A number, which has factors other than itself and 1, is called a composite number. e.g. 9, 16, 25....or the number which has more than two factors are called composite number. So, 4 is the first composite number.

1 is neither a composite number nor a prime number.

Consecutive Numbers: Numbers arranged in increasing order and differing by 1 is called consecutive numbers. e.g. 4, 5, 6, 7

Real Numbers: The sets of natural numbers, integers, whole numbers, rational numbers and irrational numbers constitute the set of real numbers. Points can represent every real number on a number line.

Perfect Numbers: If the sum of all the factors of a number excluding the number itself happens to be equal to the number, then the number is called a perfect number. 6 is the first perfect number. The factors of 6 are 1, 2, 3 & 6. Leaving 6 the sum of other factors of 6 are equal to 6. The next three perfect numbers after 6 are 28, 496 and 8128.

Chapter 2: Operations on Odd & Even Numbers

- Addition or subtraction of any two odd numbers will always result in an even number or zero.
  For example: $1 + 3 = 4$; $5 - 3 = 2$.
- Addition or subtraction of any two even numbers will always result in an even number or zero.
  For example: $2 + 4 = 6$; $12 - 4 = 8$.
- Addition or subtraction of an odd number from an even number will result in an odd number.
  For example: $4 + 3 = 7$; $10 - 3 = 7$.
- Addition or subtraction of an even number from an odd number will result in an odd number.
  For example: $3 + 4 = 7$; $5 - 2 = 3$.
- Multiplication of two odd numbers will result in an odd number. For example: $3 \times 3 = 9$.
- Multiplication of two even numbers will result in an even number. For example: $2 \times 4 = 8$.
- Multiplication of an odd number by an even number or vice versa will result in an even number.
  For example: $3 \times 2 = 6$.
- An odd number is raised to an odd or an even power is always odd.

- An even number is raised to an odd or an even power is always even.

The standard form of writing a number $m \times 10^n$ where m lies between 1 and 10 and n is an integer.

For example: 0.89713 $\Rightarrow 8.9713/10^1 . \Rightarrow 8.9713 \times 10^{-1}$.

Chapter 3: Tests of Divisibility

1. **By 2** - A number is divisible by 2 when its unit's place is 0 or divisible by 2 e.g. 120, 138.
2. **By 3** – A number is divisible by 3, in case the sum of its digits is divisible by 3 e.g. 19272 is divisible by 3 as the sum of the digits of 19272 is *21,* which is divisible by 3.
3. **By 4** - A number is divisible by 4 when the last two digits of the number are 0s or are divisible by 4 e.g. 145896, 128, 18400
4. **By 5** - A number is divisible by 5 if its unit's digit is 5 or 0., e.g. 895, 100
5. **By 6** - A number is divisible by 6 if it is divisible by both 2 and 3.,i.e. the number should be an even number and the sum of its digits should be divisible by 3.
6. **By 8** - A number is divisible by 8 if the last three digits of the number are 0s or are divisible by 8. e.g. 135128, 45000
7. **By 9** - A number is divisible by 9 if the sum of its digits is divisible by 9. e.g. 810, 92754
8. **By 11** - A number is divisible by 11 if the difference between the sum of the digits at the odd places

and the sum of the digits at the even places of the number is either 0 or a multiple of 11.

e.g. 121, 65967. In the first case 1+1 - 2 = 0. In the second case 6+9+7 = 22 and 5+6 = 11 and the difference is 11. Therefore, both these numbers are divisible by 11.

9. **By 12** - A number is divisible by 12 if it is both divisible by 3 and by 4. i.e., the sum of the digits should be divisible by 3 and the last two digits should be divisible by 4. e.g. 144, 8136.

10. **By 15 –** A number is divisible by 15 if it is divisible by both 5 and 3.

11. **By 25** – 2358975 is divisible by 25 if the last two digits of 2358975 are divisible by 25 or if the last two digits are 0.

12. The number of factors of a number say 48, can be found by knowing how many prime factors it has. 48 has four 2s and one 3.

$(2^4 \times 3^1)$

So 48 has $\Rightarrow$ (4 + 1) (1 + 1) = 10.

Factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24 & 48.

If $J = p^x \times q^y \times r^z$, (p, q &r are prime) then J has $(x + 1)(y + 1)(z + 1)$ factors.

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Chapter 4: Important Results on Numbers

1. The sum of 5 successive whole numbers is always divisible by 5.

2. The product of 3 consecutive natural numbers is divisible by 6.

3. The product of 3 consecutive natural numbers, the first of which is an even number is divisible by 24.

4. The sum of a two-digit number and the number formed by reversing its digits is divisible by 11. E.g. 28 + 82 = 110, which is divisible by 11. At the same time, the difference between those numbers will be divisible by 9. e.g. 82 – 28 = 54, which is divisible by 9.

5. $\Sigma n = \frac{n(n+1)}{2}$, Σn is the sum of first n natural numbers.

6. $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$, Σn^2 is the sum of first n perfect squares.

   e.q. what is the value of P, where P = $1^2 + 2^2 + \ldots 10^2$?

You have to find the sum of first 10 perfect squares.

The above mentioned formula is to be applied.

$\Sigma 10^2 = \frac{10 \times 11 \times 21}{6} = 385.$

7. $\Sigma n^3 = \frac{n^2(n+1)^2}{4} = (\Sigma n)^2$, Σn^3 is the sum of first n perfect cubes.

8. $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}.y + x^{n-3}.y^2 - ... + y^{n-1})$ when n is odd. Therefore, when n is odd, $x^n + y^n$ is divisible by $x + y$.

e.g. $3^3 + 2^3 = 35$ and is divisible by 5.

9. $x^n - y^n = (x + y)(x^{n-1} - x^{n-2}.y + ... y^{n-1})$ when n is even. Therefore, when n is even, $x^n - y^n$ is divisible by $x + y$.

e.g $7^2 - 3^2 = 40$, which is divisible by 10.

10. $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}.y + ... + y^{n-1})$ for both odd and even n. Therefore, $x^n - y^n$ is divisible by $x - y$.

e.g. $9^4 - 2^4 = 6545$ which is divisible by 7.

Chapter 5: Shortcuts in Multiplication & Division

1. ***To multiply by 9, 99, 999 etc.***

To multiply a number α by 9, multiply a by 10 and subtract $\alpha$from the result.

Algebraically, $\alpha \times 9 = \alpha \times (10 - 1) = 10\alpha - \alpha$.

Similarly, for 99, 999 etc multiply a by 100, 1,000 respectively.

e.g $745 \times 99 = 745 \times 100 - 745$

$= 74{,}500 - 745 = 73{,}755$.

2. ***To multiply by 5 or powers of 5***

a. To multiply by 5, multiply by 10 and divide by 2.

e.g. $137 \times 5 = \frac{1{,}370}{2} = 685$

b. To multiply by 25, multiply by 100 and divide by 4.

e.g. $24 \times 25 = 24 \times \frac{100}{4} = \frac{2{,}400}{4} = 600$

c. To multiply by 125, multiply by 1,000 and then divide by 8

e.g. 48 $\times$ 125 = 48 $\times \frac{1,000}{8} = \frac{48,000}{8}$ = 6,000

d. To multiply by 625, multiply by 10,000 & then divide by 16

e.g. 64 $\times$ 625 = 64 $\times \frac{10,000}{16}$ = 40,000

3. ***Multiplication of numbers whose unit digits add up to 10 and have the same 10th place***

Let us take an example - 47 $\times$ 43

Multiply the unit's digits = 7 $\times$ 3 = 21

Then multiply 4, which is the tenth's digit with 5 which 1 + 4 = 20. Therefore, the answer is 2021

The corollary would be finding the squares of numbers ending with 5

e.g. $35^2 \Rightarrow$ (3 $\times$ 4), 25 = 1225

4. ***Division by 5 and higher powers of 5***

To divide by 5, 25, 125, 625 etc. multiply the number by 2, 4, 8, 16 and cut off from the right 1, 2, 3, 4 digits respectively to get the quotient. To get the remainder, divide the cut off figure by 2, 4, 8, 16 respectively.

e.g. $3458 \div 25 = 3458 \times 4 / 100 = 138.32$.

Therefore, the quotient is 138 & the remainder is $32 \div 4 = 8$.

5. ***Finding the remainder when a number is divided by 9***

Add up the digits of the number rejecting 9s. Add up the digits of the resulting answer and continue this process till you get a single digit number, which will be remainder.

e.g. $3684799 \Rightarrow 3 + 6 + 8 + 4 + 7 \Rightarrow 28 \Rightarrow 2 + 8 \Rightarrow 1$.

Therefore, the remainder is 1.

Chapter 6: Solved Examples

Ex. 1. Find the greatest 5-digit number, which is a multiple of 23.

To solve such a question, take the greatest five-digit number, which is 99999.

Divide this number by 23 and get the remainder as 18. Simply because the remainder is 18 if you subtract 18 from the number the remaining number will be a multiple of 23. So the greatest such number will be 99999 – 18 = 99981.

Ex. 2. Find the smallest 7-digit number, which is a multiple of 19.

To solve such question take the smallest seven-digit number, which is 1000000.

Divide this number by 19 and get the remainder as 11.

Here if you subtract 11 from the number, no doubt you will get a multiple of 19. But because you have already taken the smallest seven-digit number, if you subtract anything from it you will get a six-digit number. Think it otherwise, that instead of subtracting you should add something.

Now, what should be added to 11(the remainder) so that it becomes a multiple of 19, i.e. 19 – 11 = 8.$\Rightarrow$ 8 should be added to the number to make it divisible by 19, i.e. 1000000 + 8 = 1000008 is the answer.

Ex. 3. Which letter should replace the \$ in the number 2347\$98, so that it becomes a multiple of 9.

As you know that if the sum of all the digits is divisible by 9, then the number is divisible by 9.

Now sum of the given digits is 2 + 3 + 4 + 7 + 9 + 8 = 33 + \$.

Now think the next multiple of 9 after 33, i.e. 36. This means you add 3 to this. The value of \$ is 3

Ex. 4. In a party there are 25 persons are present. If each of them shakes hand with all the other persons. In total how many handshakes will take place?

This question you can solve with the help of combinations.

Otherwise, you can apply other logic.

Out of 25 persons, the first person will shake hand with 24 persons.

The second person will shake hand with 23 persons (because he has already shaken hand with the first person). The third person with 22 persons and so on. The second last person shakes hand with only one person. And last will shake hand with none (because he has already shaken hand with all persons). Net you have to add all the natural numbers from 1 to 24, i.e.Σ 24. $\Sigma 24 = 24 \times 25/2 =$ 300 handshakes.

Ex. 5. Find the prime factors of 1500.

The prime factor of 1500 are $2 \times 2 \times 3 \times 5 \times 5 \times 5$. So the answer is $2^2 \times 3^1 \times 5^3$. So it has three different prime factors, i.e. 2, 3 & 5.

Ex. 6. What will be the number of zeroes at the end of the product of the first 100 natural numbers?

In this kind of a question, you need to find greatest power of 5, which can divide the product of the first 100 natural numbers ∵ a multiple of 5 multiplied by any even number, gives you a zero. Now divide 100 by 5 and take 20 as a quotient. Then divide 20 (the quotient) by 5 and get the new quotient 4, which further cannot be divided by 5. The sum of all such quotient gives you the greatest power of 5, which can divide that number. The sum is 24 and this is the number of zeroes at the end of the product of the first 100 natural numbers.

Chapter 7: Tables, Squares and Cubes to be remembered

As you have decided to improve your quantitative skills, but keep in mind you cannot be good at Math unless you are good at calculations. Take this as the starting point and make it the most important part of your preparation.

Tables:

Learn all these tables by heart and see how you improve your calculation speed.

T×1	*T×2*	*T×3*	*T×4*	*T×5*	*T×6*	*T×7*	*T×8*	*T×9*	*T×10*
12	24	36	48	60	72	84	96	108	120
13	26	39	52	65	78	91	104	117	130
14	28	42	56	70	84	98	112	126	140
15	30	45	60	75	90	105	120	135	150
16	32	48	64	80	96	112	128	144	160
17	34	51	68	85	102	119	136	153	170

18	36	54	72	90	108	126	144	162	180
19	38	57	76	95	114	133	152	171	190
21	42	63	84	105	126	147	168	189	210
23	46	69	92	115	138	161	184	207	230
24	48	72	96	120	144	168	192	216	240
27	54	81	108	135	162	189	216	243	270
29	58	87	116	145	174	203	232	261	290
37	74	111	148	185	222	259	296	333	370

Squares:

Z	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Z^2	1	4	9	16	25	36	49	64	81	100
Z	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Z^2	121	144	169	196	225	256	289	324	361	400
Z	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Z^2	441	484	529	576	625	676	729	784	841	900
Z	**31**	**32**	**33**	**34**	**35**					

Z^2	961	1024	1089	1156	1225					

Cubes:

Y	1	2	3	4	5	6	7	8
Y^3	1	8	27	64	125	216	343	512
Y	9	10	11	12	13	14	15	16
Y^3	729	1000	1331	1728	2197	2744	3375	4096
Y	17	18	19	20	21	22		
Y^3	4913	5832	6859	8000	9261	10648		

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Chapter 8: Unit Digit of a Number

The concept that revolves around finding the unit digit of a number uses the basics of the number system. Learning this concept means you have strengthened your basic concepts.

The concept of the unit digit can be learned by figuring out the unit digits of all the single digit numbers from 0 - 9 when raised to certain powers. The first learning in that for you will be that these numbers can be broadly classified into three categories for this purpose:

Digits 0, 1, 5 & 6:

When we observe the behaviour of these digits, they all have the same unit's digit as the number itself when raised to any power, i.e. $0^n = 0$, $1^n = 1$, $5^n = 5$, $6^n = 6$. So, it becomes simple to understand this logic.

e.g. Finding the Unit digit of following numbers:

$185^{563} = 5$; $271^{6987} = 1$; $156^{25369} = 6$; $190^{654789321} = 0$.

Digits 4 & 9:

Both these numbers are perfect squares and also have the same behaviour with respect to their unit digits, i.e. they have a cyclicity of only two different digits as their unit's digit.

Have a look at how the powers of 4 operate:

$4^1 = 4$, $4^2 = 1\underline{6}$, $4^3 = 6\underline{4}$ and so on
Hence, the power cycle of 4 contains only 2 numbers 4 & 6, which appear in case of odd and even powers respectively.
Likewise $9^1 = 9$, $9^2 = 8\underline{1}$, $9^3 = 72\underline{9}$ and so on.
Hence, the power cycle of 9 also contains only 2 numbers 9 & 1, which appear in case of odd and even powers respectively.
So broadly these can be remembered in even and odd only, i.e. $4^{odd} = 4$ and $4^{even} = 6$ and likewise $9^{odd} = 9$ and $9^{even} = 1$.

e.g. Finding the Unit digit of following numbers:
$189^{562589743} = 9$ (since power is odd); $279^{698745832} = 1$(since power is even);

$154^{258741369}$ = 4 (since power is odd); $194^{65478932}$ = 6 (since power is even).

Digits 2, 3, 7 & 8:

These numbers have a power cycle of 4 different numbers.

$2^1 = 2$, $2^2 = 4$, $2^3 = 8$ & $2^4 = 16$ and after that it starts repeating.
So, the cyclicity of 2 has 4 different numbers 2, 4, 8, 6.

$3^1 = 3$, $3^2 = 9$, $3^3 = 27$ & $3^4 = 81$ and after that it starts repeating.
So, the cyclicity of 3 has 4 different numbers 3, 9, 7, 1.

7 and 8 follow similar logic.
So these four digits, i.e. 2, 3, 7 and 8 have a unit digit cyclicity of four steps.

To summarise, we can say that since the power cycle of these numbers has 4 different digits, we can divide the power by 4, find the remaining power and calculate the unit's digit using that.

e.g. Find the Unit digit of 287^{562581}

The first observation for this question: the unit digit involved is 7, which has a four-step cycle. You need to divide the power by 4 and obtain the remaining power. Doing so, you get the result as 1. Now the last step is to find the unit's digit in this power of the base, i.e. 7^1 has the unit's digit as 7, which will become the answer.

Chapter 9: Some summation formulae

i) Sum of first n natural numbers:

$[1 + 2 + 3 + \dots \dots \dots + (n - 1) + n]$

1=1 ; 1+2=3 ; 1+2+3=6 ; 1+2+3+4=10

The formula applied to calculate this sum is $\Sigma n = n(n+1)/2$

Ex. 1. What would be the sum of the first 15 natural numbers?

Sol: Sum = $(15 \times 16)/2 = 120$.

ii) Sum of first n odd natural numbers:

$[1 + 3 + 5 + \dots \dots \dots + (2n-1)]$; Here, $(2n-1)$ represents an odd number where n is a natural number.

The formula to calculate the sum of the first n odd natural numbers, i.e. $\Sigma(2n - 1) = n^2$.

Ex. 2. What is the sum (1+3+5+……..+79)?

Sol: 79 can be written as (2 × 39) +1. So, 79 is the 39^{th} odd number. To, find the sum you need to find the square of 39, i.e. 1521.

iii) Sum of first n even natural numbers:

[2+4+6+………+2n]; 2n represents an even number, where n is a natural number.

The formula to calculate the sum of the first n even natural numbers, i.e.$\Sigma(2n) = n (n + 1)$.

Ex. 3. Find the sum (2+4+6+…………+88).

Sol: 88 can be written as (2 × 44). So, 88 is the 44^{th} even number. You need to find out the sum of the first 44 even numbers i.e. 44 × (44 +1) = 44 × 45 = 1980.

Chapter 10: FACTs about Factors

Factors are an essential part of Number System, and in this article, we will understand the basics of factors, and some tips related to them that can be used in the questions.

Factors of a number N refer to all the numbers which divide N completely. Those are also called divisors of a number.

Ex. 1. Find the numbers of divisors of 432?

Sol: Firstly, complete the prime factorisation of the number, i.e. $432=2^4 \times 3^3$. Now, in order to make factors of 432, either we will choose 2's power or 3's power or combination of both. The number of 2s in 432 can be chosen in 5 ways (the power of 2 can be taken from 0 till 4) and the number of 3s can be chosen in 4 ways (the power of 3 can be taken from 0 till 3). So the total number of factors of 432 is = 5× 4 = 20.

Let us learn this concept algebraically:

For any number N = $p^a q^b r^c$……..where p, q, r are distinct primes.

No. of factors = (a+1)(b+1)(c+1)

Ex. 2. Find the numbers of divisors of $N=2^3 \times 3^2 \times 5^3$.

Sol: Any combination of 2’s, 3’s or 5’s power will give the required factor. So, there are 4×3×4= 48 factors.

Ex. 3. How many factors of $N= 2^2 \times 3^3 \times 5^1$ are odd?

Sol: In this question, we only need to determine the odd factors of the given number. The factors will be odd only, if they do not have any 2 in it or those are not divisible by 2. So all you need to do is neglect the powers of 2 and rest of the steps remain the same as in above examples . The power of 3 can be selected in 4 ways and the power of 5 can be selected in 2 ways. Hence the total number of factors is 4 × 2 = 8.