hitbullseye.com

Aptitude Advanced

Geometry II

eBook 02

hitbullseye.com

Table of Contents

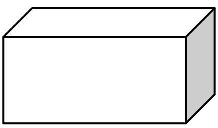
hitbullseye.com

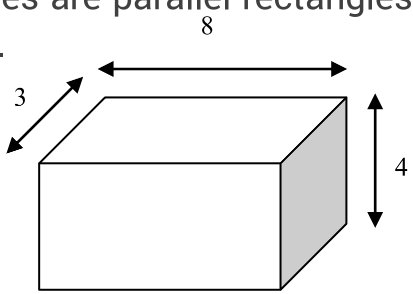
Chapter 1: Rectangular Solids and Cylinders

Solids: Solids are three – dimensional objects, bound by one or more surfaces. When plane surfaces bound a solid, they are called its faces. The lines of intersection of adjacent faces are called its edges. For any regular solid, Number of faces + Number of vertices = Number of edges + 2. This formula is called Euler's formula.

Volume: Volume of a solid figure is the amount of space enclosed by its bounding surfaces. Volume is measured in cubic units.

Cuboid: A cuboid is a three-dimensional figure formed by six rectangular surfaces, as shown below. Each rectangular surface is a face. Each solid line segment is an edge, and each point at which the edges meet is a vertex. A rectangular solid has six faces, twelve edges, and eight vertices. Edges mean sides and vertices mean corners. Opposite faces are parallel rectangles that have the same dimensions.

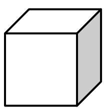




The surface area of a rectangular solid is equal to the sum of the areas of all the faces. The volume is equal to (length) × (width) × (height); in other words, (area of base) × (height).

In the rectangular solid above, the dimensions are 3, 4, and 8.
The surface area is equal to 2[(3 × 4) + (3 × 8) + (4 × 8)] = 136.
The volume is equal to 3 × 4 × 8 = 96.
Body diagonal of a cuboid = Length of the longest rod that can be kept inside a rectangular room is $= \sqrt{L^2 + B^2 + H^2}$.

Cube: A rectangular solid in which all edges are of equal length is a cube. In a cube, just like cuboid, there are six faces, eight vertices & twelve edges.
Volume = a^3.



Surface Area = $6a^2$, where a is the side of a cube.
Body Diagonal = Length of the longest rod inside a cubical room = $a\sqrt{3}$

Sphere: The set of all points in space, which are at a fixed distance from a fixed point, is called a sphere. The fixed point is the centre of the sphere and the fixed distance is the radius of the sphere.
Volume = $4/3\pi r^3$. Surface Area (curved and total) = $4\pi r^2$.

Hemisphere: A sphere cut by a plane passing through its centre forms two hemispheres. The upper surface of a hemisphere is a circular region.
Volume = $2/3\pi r^3$. Surface Area (curved) = $2\pi r^2$.
Surface Area (Total) = $2\pi r^2 + \pi r^2 \Rightarrow 3\pi r^2$.

Spherical shell: If R and r are the outer and inner radius of a hollow sphere, then volume of material in a spherical shell
= $4/3\pi (R^3 - r^3)$.

Pyramid:
A pyramid is a solid, whose lateral faces are triangular with a common vertex and whose base is a polygon. A pyramid is said to be tetrahedron (triangular base),

square pyramid, hexagonal pyramid etc, according to the number of sides of the polygon that form the base.

In a pyramid with a base of n sides, number of vertices = $n + 1$. Number of faces including the base = $n + 1$.
Surface area of lateral faces
= ½ × perimeter of base × slant height
Total surface area of pyramid
= Base area + ½ × perimeter of base × slant height
Volume of pyramid = 1/3 × Base area × height. A cone is also a pyramid.

Solved examples:

Ex. 1. A rectangle 7 cm × 5 cm is rotated about its smaller edge as axis. Find the curved surface area and volume of solid generated.

Sol. Curved Surface Area = $2\pi rh$ = 2 × 22/7 × 7 × 5 = 220 sq. cm.
Volume of solid = $\pi r^2 h$ = 22/7 × 7 × 7 × 5 = 770 cu. m.

Ex. 2. Find the volume of the largest right circular cone that can be cut out of a cube of edge 42 cm.

Sol. The base of the cone will be circle inscribed in a face of the cube and its height will be equal to an edge of the cube.
Radius of cone = 21 cm. Height = 42 cm.
Volume of cone = $1/3\pi r^2 h$ = $1/3 \times 22/7 \times 21 \times 21 \times 42$ = 19,404 cu. cm.

Ex.3. An iron ball of diameter 6 inch is dropped into a cylindrical vessel of diameter 12 inches filled with water.Find the rise in water level.

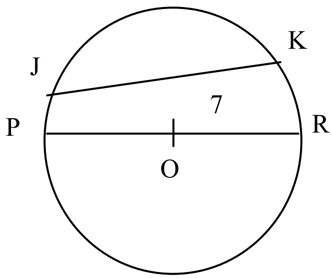
Sol. Radius of vessel = 12 inch/2 = 6 inch.
Volume of water that has risen = Volume of sphere
$\pi R^2 h = 4/3\,\pi\, r^3 \Rightarrow 6 \times 6 \times h = 4/3 \times 3^3 \Rightarrow h = 1$ inch.

Chapter 2: Circles

A circle is a set of points in a plane that are all located at the same distance from a fixed point (the center of the circle).

A **chord** of a circle is a line segment that has its endpoints on the circle. A chord that passes through the center of the circle is a diameter of the circle. A radius of a circle is a segment from the center of the circle to a point on the circle. The words "diameter" and "radius" are also used to refer to the lengths of these segments.

The **circumference** of a circle is the distance around the circle. If r is the radius of the circle, then the circumference is equal to $2\pi r$, where π is approximately 3.14. The area of a circle of radius r is equal to πr^2.



In the circle above, O is the center of the circle and JK and PR are chords. PR is a diameter and OR is a radius.

If OR = 7, then the circumference of the circle is 2π (7) = 14π, and the area of the circle is $\pi(7)^2 = 49\pi$.

Arc An arc is a part of a circle. A minor arc is an arc less than the semicircle and a major arc is an arc greater than a semicircle.

Central Angle An angle in the plane of the circle with its vertex at the centre is called a central angle.

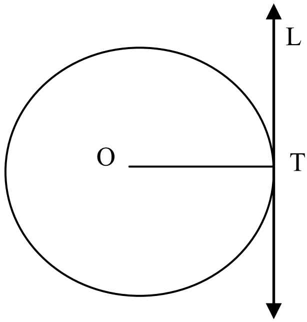
Measure of an arc:

(i) The measure of a semicircle is $180°$.
(ii) The measure of a minor arc is equal to the measure of its central angle.
(iii) The measure of a major arc = $360° -$ (measure of corresponding minor arc).

Congruent Circles: Circles with equal radii are called congruent circles.

Concentric Circles: Circles lying in the same plane with a common centre are called concentric circles.

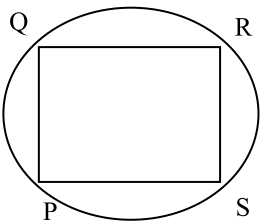
Tangent Circles: Circles lying in the same plane and having one and only one point in common are called tangent circles.

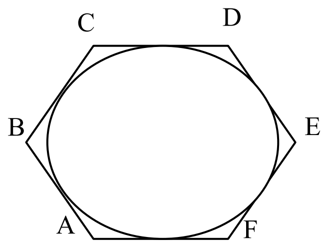


A line that has exactly one point in common with a circle is said to be tangent to the circle, and that common point is called the point of contact. A radius or diameter with an endpoint at the point of contact is perpendicular to the tangent line, and, conversely, a line that is perpendicular to a diameter at one of its endpoints is tangent to the circle at that endpoint.

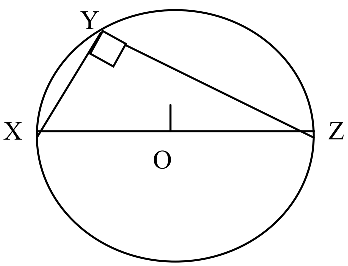
The line L above is tangent to the circle and radius OT is perpendicular to L.

If each vertex of a polygon lies on a circle, then the polygon is inscribed in a circle and the circle is circumscribed about the polygon. If each side of a polygon is tangent to a circle, then the polygon is circumscribed about the circle and the circle is inscribed in the polygon.





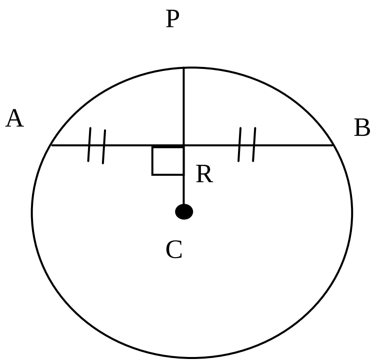
In the figure above, quadrilateral PQRS is inscribed in a circle and hexagon ABCDEF is circumscribed about a circle.



If a triangle is inscribed in a circle so that one of its sides is a diameter of the circle, then the triangle is a right triangle.
In the circle above, XZ is a diameter and the measure of angle XYZ is 90°.

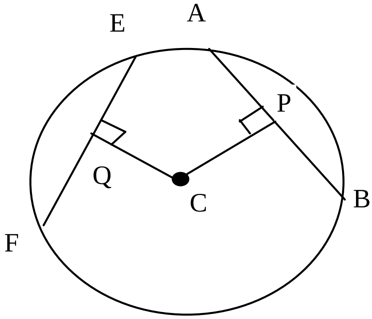
2.1 Important Properties of Circles

(i) The perpendicular from the centre of a circle to a chord of the circle bisects the chord.

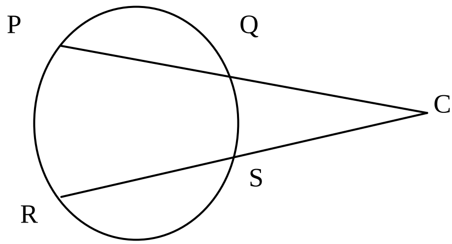


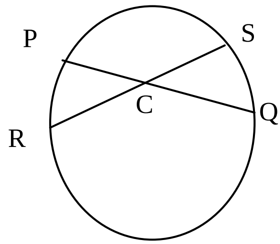
Conversely, the line joining the centre of the circle and the midpoint of a chord is perpendicular to the chord.

(ii) Equal chords of a circle or congruent circles are equidistant from the centre.

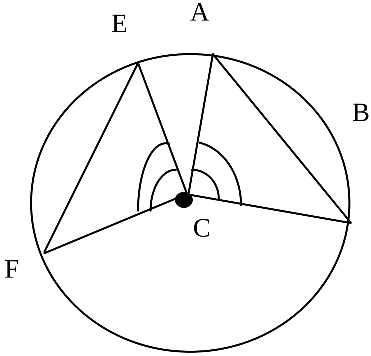


Conversely, two chords of a circle or congruent circles that are equidistant from the centre are equal.
Two chords PQ, RS intersect at a point then
CP × CQ = CR× CS



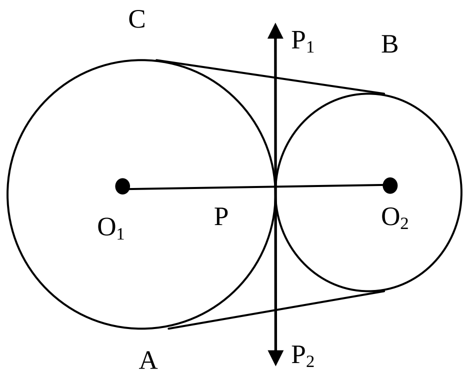


(iii) In a circle or congruent circles, equal chords subtend equal angles at the centre.

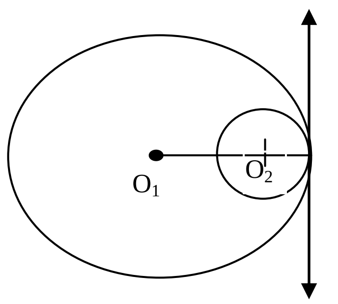


Conversely, chords, which subtend equal angles at the centre of the same or congruent circles, are equal.

(iv) If the two circles touch each other externally, distance between their centers = sum of their radii.

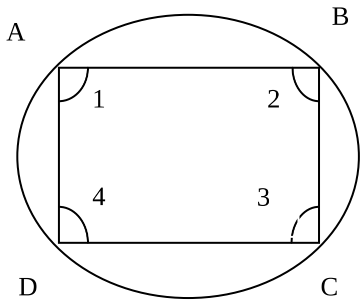


(v) If the two circles touch each other internally, distance between their centres = difference of their radii.

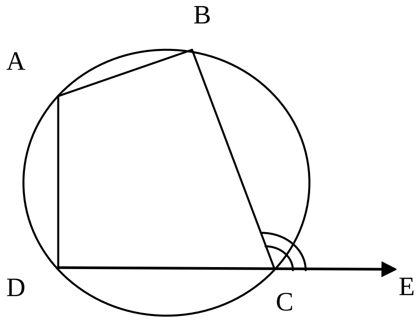


Chapter 3: Cyclic Quadrilaterals

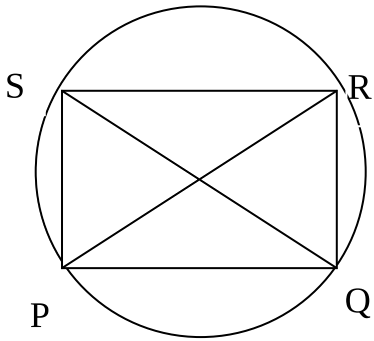
A quadrilateral is said to be cyclic if all its vertices lie on a circle. The points lying on a circle are called concyclic.



The opposite angles of a cyclic quadrilateral are supplementary.
Conversely, if the opposite angles of a quadrilateral are supplementary, then it is a cyclic quadrilateral.



An exterior angle of a cyclic quadrilateral is equal to the angle opposite to its adjacent interior angle. ∠BCE = ∠DAB.

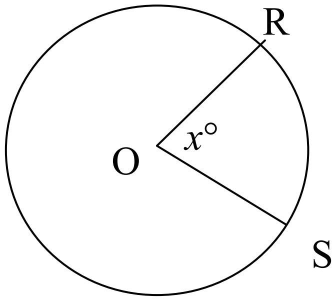


For any cyclic quadrilateral, the sum of product of two pairs of opposite sides equals the product of diagonals PQ × RS + QR × SP = PR × SQ.

Area of a cyclic quadrilateral $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$ where *s* is the semi perimeter and *a*, *b*, *c* and *d* are the sides of the quadrilateral.

Chapter 4: Sectors of a Circle

The number of degrees of arc in a circle (or the number of degrees in a complete revolution) is 360.

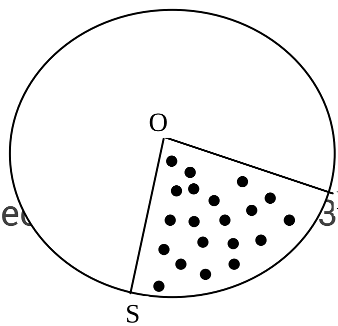


In the circle with center O above, the length of arc RS is $x/360$ of the circumference of the circle; for Ex, if $x = 60°$, then arc RS has length 1/6 of the circumference of the circle.

We can remember the following formulas:

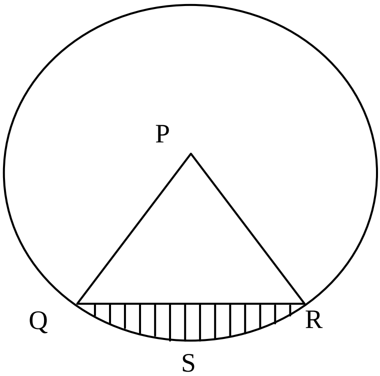
- Length of arc RS = $2\pi r \times x/360$.

  ∵ The complete circle is having 360 degrees & any part of that shall be equal to $x/360$.



- Area of Se 360.

∵ The complete circle is having 360 degrees & any part of that shall be equal to $x/360$.

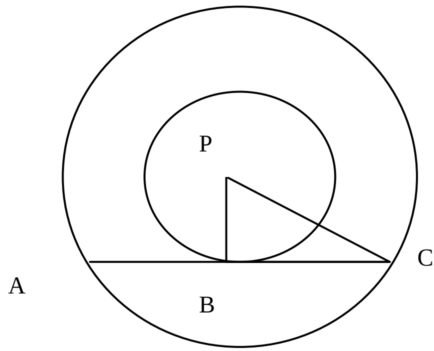


➢ Example of Circles

Ex. 1: Two concentric circles with centre P have radii 6.5 cm and 3.3 cm. Through a point A of the larger circle, a tangent is drawn to the smaller circle touching it at B. Find AC.

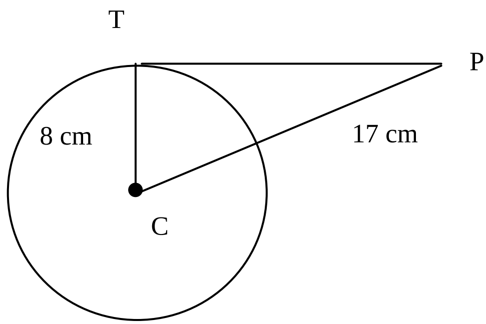
Sol. $\angle PBC = 90°$ (A tangent is perpendicular to the radius at the point of contact)

So $(6.5)^2 = (3.3)^2 + (BC)^2$. So BC = 5.6. Hence AC = 2 × 5.6 = 11.2 cm.



Ex. 2: Determine the length of the tangent to a circle of radius 8 cm from a point at a distance of 17 cm from the centre of the circle.

Sol. C is the centre of the circle P is a point outside the circle such that CP = 17 cm.



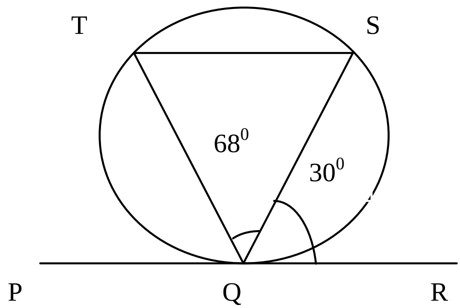
PT is a tangent from P to the circle $\therefore$ CT = 8 cm.

Also CT is perpendicular to PT (radius is always perpendicular to the tangent at the point of contact)

$\therefore$ In rt. Angled ΔCTP, $PT^2 = CP^2 - CT^2 = 17^2 - 8^2 = 15^2$

$\therefore$ PT = 15 cm

Ex. 3: In the figure $\angle SQT = 68^0$ and $\angle SQR = 30^0$ find $\angle QST$



Sol. PQR is a tangent at Q. QS is a chord through Q.

Hence $\angle SQR = \angle QTS = 30^0$

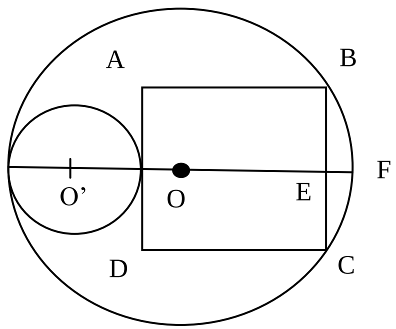
(Angle between chord and tangent is equal to the angle is the alternate segment)

$\angle SQT + \angle QTS + \angle QST = 180^0$

$\therefore \angle QST = 180^0 - (30^0 + 68^0) = 82^0$

Ex 4. In the figure O and O' are the centers of the bigger and smaller circles respectively. The smaller circle touches the square ABCD at the midpoint of side AD. The radius of the bigger circle is 15 cm and the side of the square ABCD is 18 cm. Find the radius of the smaller circle

.



1. 4.25 cm 2. 4.5 cm 3. 4.45 cm 4. 5 cm

Sol. Radius of bigger circle – *R*

Radius of the smaller circle = r

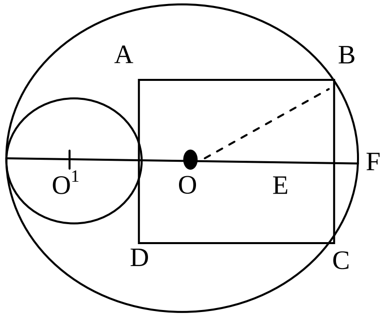
Side of square = 2*a*

OE = *R* – EF = *R* – [2*R*– (2*r* + 2*a*)] = (2a + 2r - R).

$OE^2 + EB^2 = OB^2$

i.e. $(2a + 2r - R)^2 + a^2 = R^2$, for $a = 9$ cm & $R = 15$ cm

$\Rightarrow r = 4.5$ cm



Chapter 5: Quadrilaterals

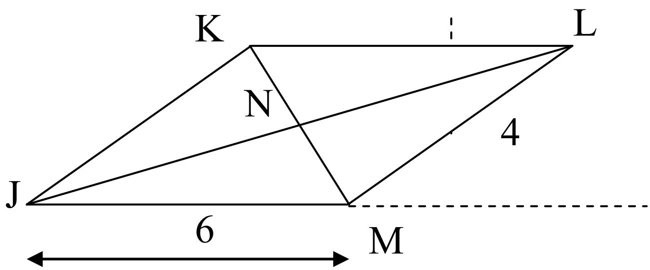
A polygon with four sides is a quadrilateral. In a quadrilateral, sum of all angles is 360°. Area of a quadrilateral = ½ × one of the diagonals × sum of the perpendiculars drawn to that diagonal from the opposite vertices.

The different kinds of quadrilaterals are parallelogram, rectangle, square, rhombus, trapezium and kite.

Parallelogram:

A quadrilateral in which both pairs of opposite sides are parallel is a parallelogram. The opposite sides of a parallelogram also have equal length. In a parallelogram opposite sides are parallel and equal. Opposite angles are equal. Diagonals bisect each other. Sum of any 2 adjacent angles = 180°.

A parallelogram inscribed in a circle is always a rectangle. Parallelogram circumscribed about a circle is always a Rhombus.

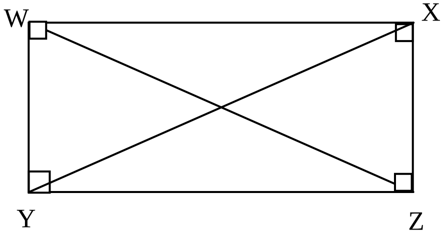


In parallelogram JKLM, JK || LM and JK = LM, KL || JM and KL = JM, The diagonals of a parallelogram bisect each other that is, KN = NM and JN = NL.

The area of a parallelogram is equal to: Base × height
The area of JKLM is equal to 4 x 6 = 24. Every diagonal of a parallelogram divides it into two triangles of equal area. Parallelograms that lie on the same base and between the same parallels are equal in area.

Rectangle:
A parallelogram with right angles is a rectangle. In a rectangle, each pair of opposite sides is parallel and equal. Diagonals are equal and bisect each other, but not at right angles. A parallelogram is a rectangle if its diagonals are equal.
Perimeter of rectangle = 2(L + B), where L = Length, B = Breadth
Area of rectangle = LB.
Area is written in the square units of sides.
$\text{Diagonal}^2 = L^2 + B^2$
The perimeter of WXYZ = 2[3 + 4] = 14 and the area of WXYZ is equal to 3 × 4 = 12.

The diagonals of a rectangle are equal; therefore WY = XZ = √(9 + 16) = √25 = 5



Square:

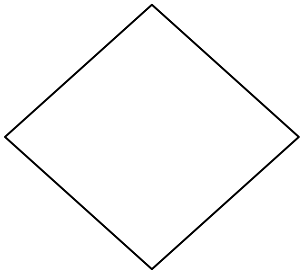
A rectangle with all sides equal is known as square. In a square, all 4 sides are equal. All the 4 angles are equal & each angle is equal to 90°. Diagonals are equal and bisect each other at right angles. The perimeter of a square is ‘4a’ and the area of the square is ‘a^2’, where ‘a’ is the side of the square. Every square is a rhombus, rectangle and parallelogram.

When a square is inscribed in a circle, the diagonal is equal to the diameter of the circle. When a circle is inscribed in a square, side of the square is equal to the diameter of the circle.

Rhombus:

In a rhombus all the sides are equal and all the angles are not equal. In a rhombus, the two pairs of opposite sides are parallel. Diagonals are not equal but they bisect each other at right angles.

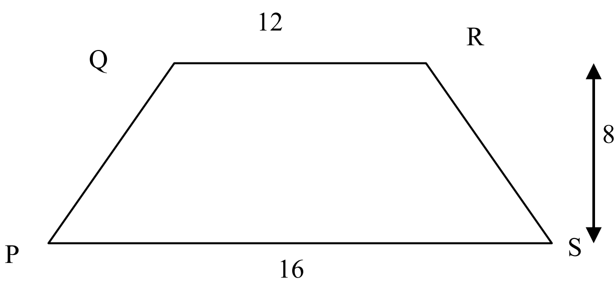
Opposite angles are equal.



Area = ½ d_1d_2, where d_1 & d_2 are two diagonals of a rhombus. $(\text{side})^2 = (\text{½ one diagonal})^2 + (\text{½ other diagonal})^2$ Every rectangle, square and rhombus is a parallelogram.

Trapezium:

A quadrilateral with two sides that are parallel but the other two sides are not parallel, as shown below is a trapezoid.



The area of trapezoid PQRS may be calculated as follows:
½ × sum of parallel sides × height = ½ × (QR + PS)(8) = ½ × (28 × 8) = 112.

A trapezium inscribed in a circle is an isosceles trapezium. In an isosceles trapezium, the oblique sides (the sides which are not parallel) are equal. Angles made by each parallel side with the oblique side are equal.

5.1 Solved examples of Quadrilaterals

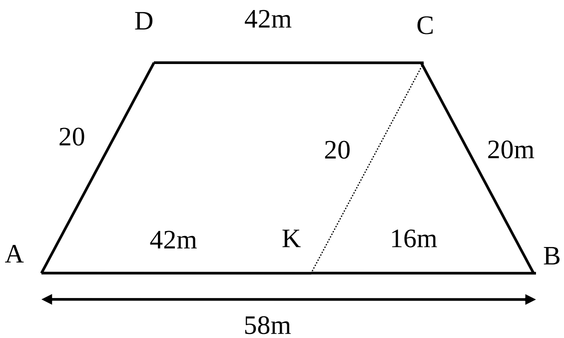
Ex 1. The parallelogram ABCD is composed of four congruent triangles, such that each triangle has two sides common with two of the other triangles. E is the point of intersection of diagonals. If BE is 3 & CE is 4, what is the perimeter of the entire parallelogram?

1. 15 2. 40 3. 20
4. 30 5. 50

Sol. As all the four triangles are congruent and the middle point where they all meet is consisting of 4 angles and each angle being equal, each angle becomes 90°.

The value of BE and CE is given to be 3 and 4 cm, thus the side of this rhombus becomes 5. The perimeter of it becomes 5 × 4 = 20,

Ex 2. Find the area of a trapezium whose parallel sides are of lengths 58 metres and 42 m and whose non-parallel sides are equal; each being 20 m.

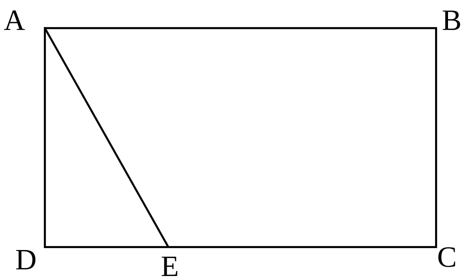


1. 840 sq. m
2. 1000 sq. m.
3. 916.50 sq. m
4. 800 sq. m

Sol. In triangle *CKB* height, $h = \sqrt{(20^2 - 8^2)} = 4\sqrt{21}$.
Area of trapezium = ½ × (sum of || sides)

(perp. Distance) = $\frac{1}{2} \times (42 + 58)\ (4\sqrt{21}) = 200\sqrt{21}$ sq. cm.

Ex 3. In the diagram below, there is a rectangle ABCD. The area of isoceles triangle ADE is 7 sq cm. Also EC = 3(DE). What is the area of rectangle ABCD?



1. 21 2. 28 3. 42 4. 56

Sol. $1/2 \times DE \times AD = 7$.

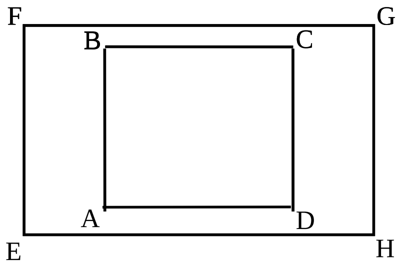
So $DE \times AD = 14$.

Area (ABCD) = $CD \times AD$.

Now CD = 4 DE.

Thus area will be $4\ DE \times AD \Rightarrow 4 \times 14 = 56$.

Ex. 4. ABCD is a square, EFGH is a rectangle. AB = 3, EF = 4, FG = 6. The area of the region outside of ABCD and inside EFGH is



1. 15 2. 12 3. 9 4. 6

Sol. Area outside sq ABCD and inside square EFGH = 24 – 9 = 15 sq units.