

**Aptitude Advanced**

# Ratio and Mixture

**eBook**

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## Chapter 1: Ratio

A ratio is the relation between two quantities of the **same kind**. This relation indicates how many times one quantity is equal to the other; or in other words, ratio is a number, which expresses one quantity as a fraction of the other.

E.g. Ratio of Rs 12 to Rs 13 is  $\frac{12}{13}$  or 12 : 13.

The numbers forming the ratio are called *terms*. The numerator, "12", in this case, is known as the *antecedent* and the denominator, "13", in this case, is known as the *consequent*.

Let us say that Rs 1000 is divided between A and B in the ratio 3: 2. It means that the amount received by A is  $\frac{3}{2}$  times the amount received by B or the amount received by B is  $\frac{2}{3}$  times the amount received by A.

### 1.1 Important Properties

- The ratio between two quantities  $a$  and  $b$  is written as:

$$\frac{a}{b} \text{ or } a : b$$

$\frac{a}{b}$  is called *fractional form*.

$a : b$  is called *linear form*.

- While finding the ratio between two quantities,  $a$  and  $b$ , the units of both the quantities should be ensured to be the same.

**E.g.** The ratio between 5 cm and 30 mm is  $\frac{5}{3}$  or  $5 : 3$  since  $30 \text{ mm} = 3 \text{ cm}$

- Ratio does not have any unit. It is a mere number.
- Since a ratio between two numbers is measured as a fraction, the ratio would stay unaltered even if both the antecedent and the consequent are multiplied by the same number ( $\neq 0$ ). The ratio obtained on account of multiplying both the antecedent and the consequent by the same number is known as an *equivalent ratio*.
- If two different ratios,  $a : b$  and  $c : d$  are expressed in different units, then the two are compounded to obtain a combined ratio.

Compounding of  $a : b$  and  $c : d$  yields  $\frac{a \times c}{b \times d}$ .

**E.g.** A takes 3 days to complete a job, working 5 hours a day. B takes 5 days to complete the same job, working 4 hours a day. What is the ratio of the time taken by A and B.

Here two different units, no of days and no of hours are used. The ratio of days worked is 3 : 5 and the ratio of the number of working hours per day is 5 : 4. Compounding these two

The ratio of the time taken by A and B is  $\frac{3 \times 5}{5 \times 4} = \frac{15}{20} = 3 : 4$

➤ If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ , then each of these ratios is equal to

$$\frac{a+c+e}{b+d+f}$$

➤ If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{b}{a} = \frac{d}{c}$  (*Invertendo*)

➤ If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a}{c} = \frac{b}{d}$  (*Alterendo*)

➤ If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a+b}{b} = \frac{c+d}{d}$  (*Componendo*)

➤ If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a-b}{b} = \frac{c-d}{d}$  (*Dividendo*)

➤ If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ , (*Componendo dividendo*)

## Chapter 2: Proportion

### Definition:

If the ratio of two terms is equal to the ratio of two other terms, then these four terms are said to be in proportion.

i.e. If  $a : b = c : d$ , we read the above equality as “ $a$  is to  $b$  as  $c$  is to  $d$ ” and say that  $a, b, c, d$  are in proportion.

The principal property of proportion is if  $a : b = c : d$ , then  $ad = bc$ .

Proof:  $a : b = a/b$  and  $c : d = c/d$

if  $a : b = c : d$ , then  $a/b = c/d$  By cross multiplication, we get  $ad = bc$ .

In other words it can also be said that the product of the means ( $b, c$ ) is equal to the product of the extremes ( $a, d$ )

### Continued Proportion:

Four numbers  $a, b, c$  and  $d$  are said to be in proportion if  $a : b = c : d$ .

If on the other hand,  $a : b = b : c = c : d$ , then the four numbers are said to be in continued proportion.

## 2.1 Important Results

- Let us take the first two ratios  $a:b = b:c$ . Here  $b$  is called the mean proportional and is equal to the square root of the product of  $a$  and  $c$

$$b^2 = a \times c \Rightarrow b = \sqrt{ac}$$

- If  $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$  etc., then  $a, b, c, d$  are in geometric progression.

$$\text{Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k, \text{ then, } c = dk; b = ck \text{ and } a = bk$$

Since  $c = dk$ ,  $b = dk \times k = dk^2$  and  $a = bk = dk^2 \times k = dk^3$ , implying they are in geometric progression.

- If the three ratios,  $a : b, b : c, c : d$  are known, we can find  $a : d$  by the multiplying these three ratios.

$$\frac{a}{d} = \frac{a}{b} \times \frac{b}{c} \times \frac{c}{d}$$

- If  $a, b, c$  and  $d$  are four terms and the ratios  $a : b, b : c, c : d$  are known, then one can find the ratio  $a : b : c : d$

**Illustration:** If  $a : b = 1 : 2; b : c = 3 : 4; c : d = 5 : 6$ , then the combined ratio  $a : b : c : d$  can be found out as follows:

Express the ratios  $a : b$  and  $b : c$  as equivalent ratios where the value of  $b$  is the same in both the ratios - by equating the value of  $b$  in both the ratios to the L.C.M. of the value of  $b$  in the two ratios.

$a : b = 3 : 6$  and  $b : c = 6 : 8$  (LCM of 2 and 3 is 6).

$\therefore a : b : c = 3 : 6 : 8$ .

$\Rightarrow a : b : c = 3 : 6 : 8; c : d = 5 : 6$

$\Rightarrow a : b : c = 15 : 30 : 40; c : d = 40 : 48$

(L.C.M. of 8 and 5, the value of  $c$  is 40)

$\therefore a : b : c : d = 15 : 30 : 40 : 48$ .

## 2.2 Variation

### Direct Variation (Proportion):

If two magnitudes  $a$  and  $b$  are related in such a way that for any increase or decrease in  $a$ ,  $b$  will also increase or decrease and vice versa, then the two magnitudes are in direct variation or direct proportion to each other.

Suppose that the price of coffee is Rs 200 per kg. Then the cost of 5 kg of coffee will be Rs 1000. i.e. as the quantity of coffee increases, the total cost of the coffee purchased also increases.



Similarly, if the salary of an executive for a month is Rs 21,000, then the salary of the executive for 10 days will be Rs 7,000.

In other words, the ratio of  $a$  and  $b$  is a constant. Take an example to understand the concept of direct variation.

**Ex 1.** A Precious stone weighing 35 grams worth Rs 12,250 is accidentally dropped and gets broken into two pieces having weights in the ratio of 2 : 5. If the price varies as the square of the weight then find the loss incurred.

**Sol.** The question states price is directly related with the square of the weight i.e.  $p \propto q^2$

$$\Rightarrow p = kq^2.$$

Putting the value as given in the question

$$12250 = k(35)^2 \Rightarrow 12250 = k1225 \Rightarrow k = 10.$$

Now the equation can be rewritten as  $p = 10q^2$ .

Now their weights become  $35 \times 2/7 = 10$  gm & 25 gm. Their values can be calculated in the same manner.

The value of the smaller piece is  $p = 10 \times (10)^2 = 1000$  and the value of the bigger piece is  $p = 10 \times (25)^2 = 6250$ .

The sum of these values =  $1000 + 6250 = 7250$ .

The decrease in the value is  $12250 - 7250 = 5000$ .

### **Inverse Variation (Proportion):**

If two magnitudes “ $a$ ” and “ $b$ ” are related in such a way that for any increase (or decrease) in “ $a$ ”, quantity “ $b$ ” decreases (or increases) in the same ratio, then these two magnitudes are said to vary inversely to each other and the proportion in that case is called inverse proportion or inverse variation.

Suppose 8 men can do a piece of work in 10 days, and then we know that 16 such men can do the same piece of work in 5 days.

Similarly, if 8 men can do the work in 10 days, then 4 men will take 20 days to complete the same work.

**Note:** Number of days and number of workers are inversely proportional to each other. Similarly, speed and time are inversely proportional and productivity and time taken to complete a job are also inversely proportional.

## 2.3 Solved Examples

**Ex. 2.** In a certain examination, the number of students who passed was 4 times the number of those who failed. If there had been 35 fewer candidates and 9 more had failed, the numbers would have been in the ratio of 2 : 1. Find the total number of candidates.

**Sol.** Let the total number of candidates =  $5x$  (because you have to break it in the ratio of 4 and 1). Therefore, the number of those who passed =  $4x$  and the number of those who failed =  $x$ . If the number of students had been 35 fewer =  $5x - 35$ , And 9 more had failed =  $x + 9$ , then  
The number of those who passed =  $5x - 35 - (x + 9) = (4x - 44) \Rightarrow (4x - 44) : (x + 9) = 2 : 1$   
 $\Rightarrow 4x - 44 = 2x + 18 \Rightarrow 2x = 62 \Rightarrow 5x = 155$  are the total number of students in the class.

**Ex. 3.** A family divides its monthly income of Rs 7000 in the following way. Food : Shelter :: 4 : 5; and Shelter : Savings :: 10 : 7. How much does the

family spend on each of these activities in a month?

**Sol.** Food : Shelter :: 8 : 10 and Shelter : Savings :: 10 : 7.

Therefore, Food : Shelter : Savings :: 8 : 10 : 7

Therefore, Food =  $7000 \times \frac{8}{25} = 280 \times 8 = 2240$ .

Shelter =  $7000 \times \frac{10}{25} = 2800$

Savings =  $7000 \times \frac{7}{25} = 1960$

**Ex. 4.** Three partners Ram, Shiv and Giri divide their profit of Rs 1.7 lakhs in the following ratio: 2 times Ram's share = 7 times Shiv's share = 6 times Giri's share. How much did each partner get?

**Sol.** Note that in a problem of this kind, the ratios given are not direct ratios. It would be incorrect if one considers the ratio in which the profits are divided to be 2 : 7 : 6. The correct approach is to find that number which will be equal to twice the share of "a", 7 times the share of "b" and "6" times the share of "c". That number will be nothing but the L.C.M of 2, 7 and 6 which is 42. Then, divide the LCM of their

shares by their respective shares. i.e.  $\frac{42}{\text{Ram's share}} =$

$$\frac{42}{\text{Shiv's share}} = \frac{42}{\text{Giri's share}} \quad \text{Therefore, Ram : Shiv : Giri} = 21 : 6 : 7$$

$$\text{Hence, Ram's profit} = \frac{21}{34} \times 170,000 = 105,000.$$

$$\text{Shiv's profit} = \frac{6}{34} \times 170,000 = 30,000 \text{ and}$$

$$\text{Giri's profit} = \frac{7}{34} \times 170,000 = 35,000.$$

**Ex. 5.** Find two numbers “ $a$ ” and “ $b$ ” such that their mean proportional is 15 and the third proportional to “ $a$ ” and “ $b$ ” is 405.

**Sol. Case I.** When 15 is the mean proportional of “ $a$ ” and “ $b$ ”,  $a, 15, b$  are in continued proportion.

$$\text{Therefore, } \sqrt{ab} = 15$$

$$\Rightarrow ab = 225 \text{ or } a = \frac{225}{b}.$$

**Case II.** Since 405 is their third proportional,  $a, b$  and 405 are in continued proportion.

$$\text{Therefore, } 405 \times a = b^2.$$

Solving for “a” and “b” in case I and case II, we get

$$405 \times \frac{225}{b} = b^2$$

$$\Rightarrow 405 \times 225 = b^3$$

$$\Rightarrow 27 \times 15 \times 15^2 = b^3$$

$$\Rightarrow b = 45.$$

$$\text{Therefore, } a = \frac{225}{45} = 5.$$

**Ex. 6** Find the value of  $\frac{x+a}{x-a} + \frac{x+b}{x-b}$  if  $x = \frac{2ab}{a+b}$

**Sol.**  $\frac{x}{a} = \frac{2b}{a+b}; \frac{x}{b} = \frac{2a}{a+b}$

$$\text{Therefore, } \frac{x+a}{x-a} = \frac{2b+(a+b)}{2b-(a+b)} = \frac{3b+a}{b-a} \text{ -- Componendo}$$

dividendo rule

$$\text{Similarly, } \frac{x+b}{x-b} = \frac{2a+(a+b)}{2a-(a+b)} = \frac{3a+b}{a-b}$$

Therefore, the required answer is

$$\frac{3b+a}{b-a} + \frac{3a+b}{a-b} = \frac{2a-2b}{a-b} = 2.$$

**Ex .7.** The expenses at the IIM hostel are partly fixed and partly variable. The charge per head works out to Rs 100 when there are 60 students and Rs 90 per

head when the number of students is 80. Find the charge per head when there are 100 students.

**Sol.** Let the fixed component be  $F$ , a constant, and  $V$  be the variable component

Therefore when there are 60 students  $F + 60V = 6000$

and when there are 80 students,  $F + 80V = 7200$

$20V = 1200$  and  $V = 60$ , which is the variable component and the fixed component is Rs 2,400.

Therefore, the total cost when there are 100 students  $= 2400 + 100 \times 60 = 8400$

And the per student cost = Rs 84.

**Ex. 8.** The ratio of the first and second-class fares between the two stations is 3 : 1 and the number of passengers traveling by first and second-class is 1 : 30. If Rs 2200 is collected as fare, what is the amount collected from first class passengers?

**Sol.** Ratio of the amounts collected from 1<sup>st</sup> and 2<sup>nd</sup> class =  $(3 \times 1) : (1 \times 30) = 1 : 10$ .

$\therefore$  Amount collected from 1<sup>st</sup> class passengers =  $2200 \times \frac{1}{11} = 200$ .

**Ex. 9.** 8 men and 2 women earn Rs 70 a day. 6 men and 8 women earn Rs 72 a day. How much does a man and a woman earn a day?

**Sol.**  $8M + 2W = 70$  ..... (1)

$6M + 8W = 72$  ..... (2)

Solving for  $M$ , we get  $M = 8$ .

Therefore,  $W = 3$ .

Hence, the salary earned by a man and a woman in a day is Rs 11.

**Ex. 10.** There are 40 members of a student's council in a school and ratio of the number of boy to the number of girls is 3 : 1. How many more girls be added to the council to make the ratio of boys to girls 3 : 2?

**Sol.** Dividing 40 members in the ratio of 3 : 1 we get boys = 30 and girls = 10

Let the new girls to be added =  $x$ , then

$$\frac{B}{G+x} = \frac{3}{2} \Rightarrow \frac{30}{10+x} = \frac{3}{2} \Rightarrow x = 10$$



**Ex. 11.** The monthly incomes of two persons are in the ratio of 4 : 5 and their monthly expenditures are in the ratio of 7 : 9. If each saves Rs. 50 a month, then what are their monthly incomes?

**Sol.** Let the incomes of the two persons be 4I and 5I respectively. Let the expenditure of the two persons be 7E and 9E respectively.

$$\therefore 4I - 7E = 50 \text{ and } 5I - 9E = 50$$

. Now multiply the first equation by 5 and the second by 4 and then subtract.

. You get  $E = 50$  and  $I = 100$   $\therefore$  Monthly incomes of the two persons are Rs 400 and Rs 500 respectively.

**Ex. 12.** Rs. 675 was divided between A, B and C. If each of them had received Rs. 5 less, their shares would have been in the ratio of 1 : 2 : 3. How much did B receive?

**Sol.** Rs.  $(675 - 3 \times 5)$  is divided in the proportion 1:2:3,  
B's share in this  $= 660 \times \frac{2}{6} = \text{Rs. } 220$   
 $\therefore$  B's share in total = Rs. 220 + Rs. 5 = Rs. 225

**Ex. 13.** Rs. 120 are divided among X, Y and Z so that X's share is Rs. 20 more than Y's share and Rs. 20 less than Z's share. What is the Y's share?

**Sol.**  $X + Y + Z = 120$      $X = Y + 20 \Rightarrow Y = X - 20$ ,  
    $X = Z - 20 \Rightarrow X + 20 = Z$ .  
 $\Rightarrow X + (X - 20) + (X + 20) = 120$   
 $\Rightarrow 3X = 120 \Rightarrow X = 40$  and  $Y = 40 - 20 = 20$ .

**Ex. 14.** A bag contains 50 paise, 25 paise and 10 paise coins in the proportion of 3: 4 : 5. The total value of these coins is Rs. 12. Find the number of 25 paise coins.

**Sol.** Ratio of no of 50 p, 25 p and 10 p coins = 3 : 4 : 5.  
Ratio of value 50 p , 25 p and 10 p coins  
=  $3 \times 50 : 4 \times 25 : 5 \times 10 = 150 : 100 : 50 = 3 : 2 : 1$   
 $\therefore$  Value of 25 p coins =  $\frac{2}{6} \times 12 = 4$  Rs.  
In 4 Rs. no of 25 p coins =  $4 \times 4 = 16$ .

**Ex. 15.** The ratio between two numbers is 3 : 5 and their sum is 640. The numbers are:

**Sol.** The ratio is 3 : 5 and the sum is 640. If the sum is 3 + 5 = 8, then the numbers would 3 and 5.  
Therefore, if the sum is 640, the numbers are  $\frac{3}{8} \times 640 = 240$  and  $\frac{5}{8} \times 640 = 400$ .

## Chapter 3: Partnership

Partnership is an association of two or more persons who put in money together in order to carry on a certain business. Partnership is of two types.

- (i) Simple Partnership: When all the partners start the business at the same point of time i.e. their capitals remain in the business for the same duration of time is called simple partnership. In this kind of partnership the profit is simply distributed in the ratio of their capitals.
- (ii) Compound Partnership: When the capitals of the partners are invested in the business for the different time periods the partnership is known as compound. In this kind the profit sharing ratio is calculated by multiplying the capital invested with the unit of time (mostly months).

### 3.1 Solved Examples

**Ex. 1.** X and Y are partners in a business. X invests Rs 300 for 12 months and Y invested Rs 600 for 6

months. If they gain a profit of Rs 700 at the end of one year, what is X's share?

**Sol.** X's total capital =  $300 \times 12 = 3600$ .

Y's total capital =  $600 \times 6 = 3600$ .

Profit sharing ratio =  $3600 : 3600 \Rightarrow 1 : 1$ .

The profit is given to be Rs 700

The share of X and Y each =  $700 \times \frac{1}{2} = \text{Rs } 350$ .

**Ex.2.** A, B and C hired a ground for Rs 12000. A used this ground for 8 cows for 3 weeks, B used it for 6 cows for 8 weeks and C used it for 18 cows for 4 weeks. What amount of rent should C pay?

**Sol.** A's total use =  $8 \times 3 = 24$ . B's total use =  $6 \times 8 = 48$ .

C's total use =  $18 \times 4 = 72$ .

Their expenditure ratio =  $24 : 48 : 72 \Rightarrow 1 : 2 : 3$ .

$\Rightarrow$  C should pay  $\frac{3}{6}$  of the rent

i.e.  $12000 \times \frac{3}{6} = \text{Rs } 6000$ .

**Ex.3.** Ram invested Rs 8000 for the whole year in a business. Sham joins after 4 months. How much he should invest so that the profits are distributed in the ratio of 2 : 1?

**Sol.** Ram's total capital =  $8000 \times 12 = 96000$ .

Let us take the capital of Sham = S, he invested this capital after 4 months means it remains in the business for 8 months.

Their profit sharing ratio = 2 : 1.

So the equation will be  $96000/8S = 2/1$

$\Rightarrow 16 S = 96000 \Rightarrow S = 6000$ .

So the capital of Sham is Rs 6000.

**Ex.4.** A, B and C invest Rs 400, 500 and 600 in a business respectively. A gets one-fourth of the profits as remuneration for managing the business. The rest of the profits are distributed by A, B and C in the ratio of their investments. If in a particular year, A gets Rs 10 less than B and C together, what was the total profit for that year?

**Sol.** After giving one-fourth of the total profit amount to A for managing the business, the rest three-fourth of total profit is divided among A, B and C in the ratio of their investments. The share of A, B and C in the profit will be in the ratio of 4 : 5 : 6.

Three-fourth of the total profits =  $4x + 5x + 6x = 15x$ .

$\Rightarrow$  Total profit =  $15x \times \frac{4}{3} = 20x$ .

Total share of A =  $4x + 20x/4 = 9x$ . (i)

Share of B and C =  $5x + 6x = 11x$ . (ii)

The difference in (i) and (ii) above is given to be Rs 10,  $\Rightarrow (5x + 6x) - 9x = 10 \Rightarrow 2x = 10 \Rightarrow x = 5$ .

Total profit =  $5 \times 20 = 100$ .

**Ex.5.** Bulls Eye rents a computer to facilitate its operations in the field of Business Consulting, Management Education and Product designing. Bulls Eye pays a rent of Rs 6,750 for the computer. The consultancy business uses the computer for 20 days of a month for 6 hours, while the education business uses it for 12 days for 15 hours and the product design business uses the computer for 12 days for 2 hours. How should the cost of the computer allocated between these business units?

**Sol.** The number of hours each business group uses the computer is given by compounding the two ratios.  
i.e. B Con : Mg Ed : P Dev =  $20 \times 6 : 12 \times 15 : 12 \times 2$   
=  $120 : 180 : 24 = 10 : 15 : 2$

Therefore, the cost allocation should be done as:

$$\text{B Con} = 6750 \times 10/27 = \text{Rs } 2,500$$

$$\text{Mg. Ed} = 6750 \times 15/27 = \text{Rs } 3,750$$

$$\text{P Dev} = 6750 \times 2/27 = \text{Rs } 500$$

**Ex.6.** A, B and C start a partnership by investing Rs 3000, Rs 4000 and Rs 5000 respectively. B increases his investment to Rs 5000 after 3 months and C withdraws 20 % of his contribution after 9 months. What will be ratio in which the partners share their profits?

**Sol.** B's capital in the venture =  $\frac{4000 \times 3 + 5000 \times 9}{12} = 4750$ .

$$\text{C's capital in the venture} = \frac{5000 \times 9 + 4000 \times 3}{12} = 4750.$$

Therefore, the capital of A, B and C are in the ratio of 3 : 4.75 : 4.75.

**Ex. 7.** A began business with Rs. 4200 and was joined afterwards by B with Rs. 7200. When did B join if the profits at the end of the year were divided equally?



**Sol.** Let B join after  $x$  months so that he remains in the business for  $(12-x)$  months

$$\Rightarrow 4200 \times 12 = 7200 \times (12-x) \Rightarrow x = 5$$

**Ex.8.** Rahul started a business with a capital of Rs. 8000. After six months, Sanjay joined him with investment of some capital. If at the end of the year each of them gets equal amount as profit, how much did Sanjay invest in the business?

**Sol.** Investment by Rahul Rs. 8000 for 12 months  
Capital of Rahul =  $8000 \times 12 =$  Rs. 96000 for 1 month. Suppose Sanjay invested Rs.  $x$  for 6 months

$\therefore$  Capital of Sanjay = Rs.  $6x$  for 1 month, Since they get equal amounts as profit

$\therefore$  Their investment will also be same

Therefore,  $6x = 96000$  Or,  $x = 96000/6 =$  Rs. 16000

Sanjay invested Rs. 16000 in the business.

**Ex.9.** A, B and C invested capitals in the ratio 3 : 5 : 9; the timing of their investments being in the ratio 2 : 3 : 1. In what ratio would their profits be distributed?

**Sol.** Ratio of capitals of A, B and C are 3:5:9. Let the capitals of A, B and C be  $3x$ ,  $5x$  and  $9x$  respectively. Ratio of timing of their investments are 2 : 3 : 1. Let A, B and C invest their capitals for  $2y$ ,  $3y$  and  $y$  months, respectively.

Then, profit of A : profit of B : profit of C

$$= 3x \times 2y : 5x \times 3y : 9x \times y = 6 : 15 : 9 \text{ or, } 2 : 5 : 3.$$

## Chapter 4: Mixtures and Alligation Problems

In mixture problems, substances with different characteristics are combined, and it is necessary to determine the characteristics of the resulting mixture.

In doing mixture problems, we can make use of the Alligation Rule. Alligation means “Linking”. The rule states, that “When different quantities of different ingredients are mixed together to produce a mixture of a mean value, the ratio of their quantities is inversely proportional to the differences in their cost from the mean value.

### 4.1 Solved Examples

**Ex. 1.** If 6 pounds of nuts that cost \$1.20 per pound are mixed with 2 pounds of nuts that cost \$1.60 per pound, what is the cost per pound of the mixture?

**Sol.** The total cost of the 8 pounds of nuts is  $6(\$ 1.20) + 2(\$ 1.60) = \$ 10.40$ .

The cost per pound is  $\$ 10.40 / 8 = \$ 1.30$ .

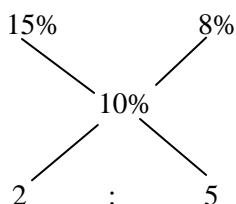
**Ex.2.** How many liters of a solution that is 15 percent salt must be added to 5 liters of a solution that is 8 percent salt so that the resulting solution is 10 percent salt?

**Sol.** Let  $n$  represent the number of liters of the 15 % solution. The amount of salt in the 15 % solution  $[0.15n]$  plus the amount of salt in the 8 % solution  $[(0.08)(5)]$  must be equal to the amount of salt in the 10 % mixture  $[0.10(n + 5)]$ .

$$\text{Therefore, } 0.15n + 0.08(5) = 0.10(n + 5) \Rightarrow 15n + 40 = 10n + 50$$

$$5n = 10 \Rightarrow n = 2 \text{ liters.}$$

Two liters of the 15% salt solution must be added to the 8% solution to obtain 10% solution. This question was solved with the help of equations. Now let us solve this question by alligation rule

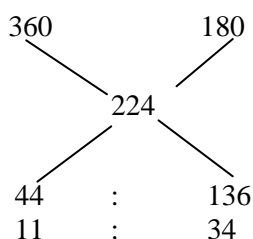


There are already 5 litres of 8 % solution, this means 2 litres of 15 % solution should be added.

**Ex.3.** In a zoo there are goats and parrots. If their heads are counted, there are 90 while their legs are 224. Find the number of parrots.

**Sol.** As the total heads are 90 means these are 90 in number. If all of them are goats then the total number of legs would be  $90 \times 4 = 360$ . If all of them are parrots then the total number of legs are  $90 \times 2 = 180$ .

Now apply alligation as the mean number of legs is given to be 224.



$\Rightarrow$  There are  $90 \times 34/45 = 68$  parrots.

**Ex.4.** Two vessels contain milk and water in the ratio of 3 : 1 and 5 : 3 respectively. Find the ratio in which these are to be mixed to get a new mixture in which the ratio of milk to water is 2 : 1.

**Sol.** In such questions, you should simply take one component out of the two given i.e. milk and water and then take its mean value and find the ratio.

Let us take milk. Milk is  $\frac{3}{4}^{\text{th}}$  in the first vessel and  $\frac{5}{8}^{\text{th}}$  in the second vessel and the average milk needed is  $\frac{2}{3}^{\text{rd}}$ .

$$\begin{array}{ccc}
 \frac{3}{4} & & \frac{5}{8} \\
 & \searrow \quad \nearrow & \\
 & \frac{2}{3} & \\
 & \swarrow \quad \searrow & \\
 \frac{1}{24} & : & \frac{1}{12} \\
 1 & : & 2
 \end{array}$$

⇒ Ratio is 1 : 2 and this is the answer. If 1 litre is taken from the first vessel 2 litres should be taken from the second vessel.

**Ex.5.** Indian Paints mixes cyan and magenta in the ratio of 5 : 4. However, after mixing the company finds that the resultant shade has less of magenta and removes 18 litres of the mixture and adds equivalent quantity of magenta. The shade thus obtained has cyan and magenta in the inverse ratio. How many litres of paint can the mixing tank hold?

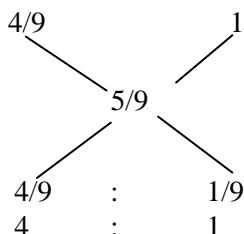
**Sol.** Let the tank hold  $9x$  litres of paint.

In the mixture of 18 litres, which was removed, there were 10 litres of cyan and 8 litres of magenta paint.

Therefore, in the new mixture amount of cyan present =  $5x - 10$  and the amount of magenta present =  $4x + 10$

$$5x - 10 : 4x + 10 = 4 : 5 \Rightarrow 25x - 50 = 16x + 40$$

⇒  $9x = 90$  litres which is the volume of the mixing tank.



Here also alligation can be applied. Firstly there was  $\frac{4}{9}^{\text{th}}$  of magenta. And added is magenta only means  $\frac{1}{1}$  of magenta and ultimately it becomes  $\frac{5}{9}^{\text{th}}$  of magenta.

⇒ If 4 parts of original are there, one part of magenta should be added, and this is known that 18 litres of magenta is added, this means there are 72 litres of original mixture. Capacity of the tank is  $72 + 18 = 90$  litres.

**Ex.6.** A tank has a mixture of milk and water in the ratio of 3 : 1. What proportion of the mixture should be withdrawn and replaced with water so that the resulting mixture has equal proportions of milk and water?



**Sol.** Water is initially  $\frac{1}{4}^{\text{th}}$  of the total and what you are mixing in it is the water. Water has how much water in it (seems odd). But true, water has 100 % water in it (in percentage) or 1 time of water (in fractions). So the rule of alligation will be applied as ,

$$\begin{array}{ccc}
 \frac{1}{4} & & 1 \\
 & \searrow \quad \swarrow & \\
 & \frac{1}{2} & \\
 & \swarrow \quad \searrow & \\
 \frac{1}{2} & : & \frac{1}{4} \\
 2 & : & 1
 \end{array}$$

Now as they are to mixed in the ratio of 2 : 1. It is also known that the proportion of water is actually by replacing the mixture, this means 1 out of a total of 3 (1 + 2) should be replaced i.e.  $\frac{1}{3}^{\text{rd}}$  of the total should be replaced. You can also solve the same question by the following equation method.

**Ex.7.** A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is:

**Sol.** Let C.P. of 1 litre milk be Re. 1

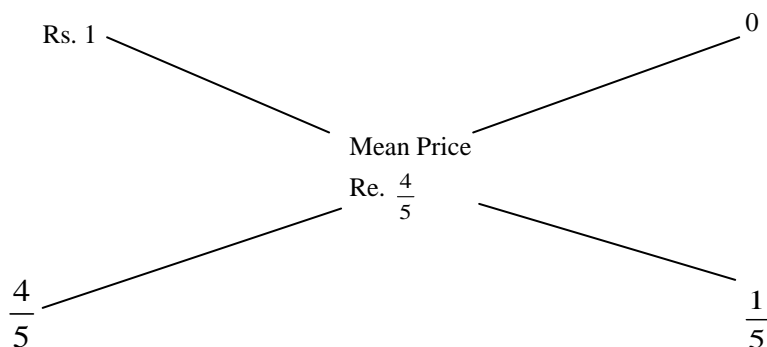
Then, S.P. of 1 litre of mixture = Re. 1, Gain = 25%.

$$\text{C.P. of 1 litre mixture} = \text{Re. } \left( \frac{100}{125} \times 1 \right) = \frac{4}{5}$$

By the rule of alligation, we have:

C.P. of 1 litre of milk

C.P. of 1 litre of water



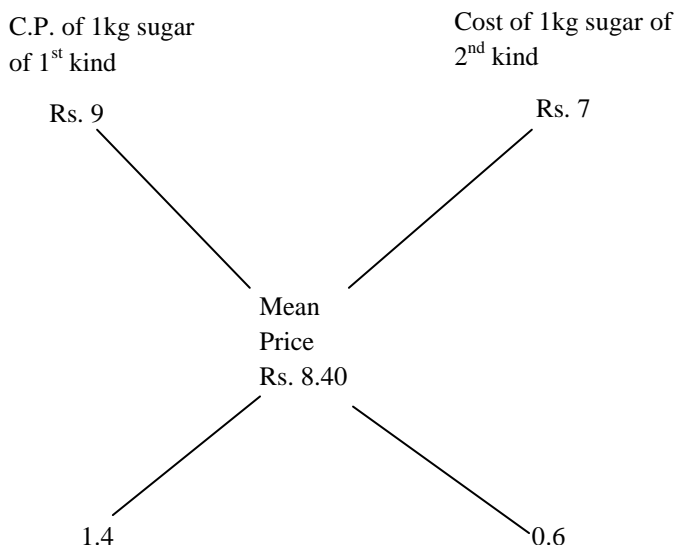
$$\text{Ratio of milk to water} = \frac{4}{5} : \frac{1}{5} = 4 : 1.$$

Hence, percentage of water in the mixture =  $100 \times \frac{1}{5} = 20\%$ .

**Ex.8.**How many kilogram of sugar costing Rs. 9 per kg must be mixed with 27 kg of sugar costing Rs. 7 per kg so that there may be a gain of 10% by selling the mixture at Rs. 9.24 per kg?

**Sol.** S.P. of 1 kg of mixture = Rs. 9.24, Gain 10%.  
∴ C.P. of 1 kg of mixture =  $\text{Rs. } 9.24 \times \frac{100}{110} = \text{Rs. } 8.40$

By the rule of allegation, we have:



∴ Ratio of quantities of 1<sup>st</sup> and 2<sup>nd</sup> kind = 14 : 6 = 7 : 3.

Let x kg of sugar of 1<sup>st</sup> be mixed with 27 kg of 2<sup>nd</sup> kind.

Then, 7 : 3 = x : 27

$$\Rightarrow x = \left( \frac{7 \times 27}{3} \right) = 63 \text{ kg.}$$

## 4.2 Repeated dilution of a Mixture

One characteristic case in mixtures is repeated dilution of a mixture with one of the ingredients, by removing, say n litres of the mixture and replacing it with n litres of one of the ingredients. Say there are m litres of water initially. We now remove n litres of the water and replace it by n litres of wine. This operation is done t times. Then

$$\text{Quantity of water left in the vessel} = m \left( 1 - \frac{n}{m} \right)^t$$

**Where**       $m$  = total quantity

$n$  = quantity drawn every time

$t$  = no. of times

**Ex.9.** In a vessel, there are 80 litres of water. 8 litres of water are withdrawn and replaced with milk, then 8 litres of mixture are withdrawn and replaced with

milk. What is the quantity of water left in the mixture?

**Sol.** Quantity of water left in the vessel =  $80(1 - \frac{8}{80})^2$ .  
 $\Rightarrow 80 \times \frac{9}{10} \times \frac{9}{10} = 64.8$  litres

**Ex. 10.** A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?

**Sol.** Amount of milk left after 3 operations =  $\left[ 40 \left( 1 - \frac{4}{40} \right)^3 \right]$

litres

$$= \left( 40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \right) = 29.16 \text{ litres.}$$