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Aptitude Advanced

Time & Work, Interest

eBook

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Chapter 1: Chain Rule

The technique of chain rule is applicable in cases where two or more components are given. Each of these components has two parts. Out of these components, one component has one of its parts missing. Then in the other given part, the same component is taken as the base and is compared individually with all the other components. The following two methods are applied:

1. If the missing part should be greater than the given part, then the numerator is kept greater than the denominator.
2. If the missing part should be smaller than the given part, then the numerator is kept smaller than the denominator.

1.1 Solved Examples

Ex. 1. If 20 men working 12 hrs a day can do a piece of work in 24 days. 24 men working, 8 hrs a day can do the same piece of work in how many days?

Sol. Here the component days is having its one part missing and the given part of this i.e. 24 days, will be taken as base and will be compared individually with the other components.

When 20 men are working it takes 24 days, when 24 men will work they will take lesser days (more men lesser days). So numerator should be lesser.

When work is done 12 hrs it takes 24 days, when it is done 8 hrs the more days will be taken (less hours more days). So numerator should be more.

$24 \times \frac{20}{24} \times \frac{12}{8}$ = 30 days.

Ex. 2. A group of 600 men has provision for 39 days. After 12 days a reinforcement of 300 men comes. The food will last for how many days more?

Sol. Originally there are 600 men having provision for 39 days. Whatever happened is after 12 days, means 600 men would have eaten their share of 12 days. The remaining provision is for 39 –12 = 27 days. Now when 300 men came, the total strength becomes 600 + 300 = 900. 600 men has provision for 27 days, 900 will have for how many days. (More men lesser days) 27 × 600/900 = 18 days.

Ex.3. A contractor employed 200 men to complete a strip of highway in 200 days. After 150 days he found that only 3/5th of the strip is complete. How many additional men should be employed to complete the work on time?

Sol. men work days
200 3/5 150
? 2/5(remaining work) 50 (remaining days)

Less work is there, less men are required; rule 2.
Fewer days are remaining; more men are required; rule 1.

Total men required = $200 \times \frac{2}{5} \times \frac{5}{3} \times \frac{150}{50} = 400$.

Now 200 men are already there, so 400 – 200 = 200 additional men are required.

Ex. 4. If 8 men or 12 women can do a piece of work in 25 days, in how many days can the work be done by 6 men and 11 women working together?

Sol. $8m = 12w \Rightarrow 2m = 3w$

$\Rightarrow$ 2 men are equal to 3 women i.e. lesser men are equal to more women.

6 men will be equal to $6 \times 3/2 = 9$ women.

Second part of the problem is 6 men and 11 women.

As calculated 6 men are equal to 9 women and there are 11 women themselves.

So total number of women is $9 + 11 = 20$.

Apply the chain rule as

Women	Days
12	25
20	?

More women, so lesser days i.e. numerator should be smaller $\Rightarrow 25 \times 12/20 = 15$ days.

Ex. 5. Some men promised to do a job in 18 days. 6 of them were absent and the remaining men did the job in 20 days. What is the original number of men?

Sol. Let there be m men in the beginning, who promised to do the job in 18 days.

But 6 became absent means the remaining $m - 6$ men did the job in 20 days.

The work is same $\Rightarrow 18m = 20(m - 6)$

$\Rightarrow 2m = 120 \Rightarrow m = 60$.

Ex. 6. Lenin Square was built by 1700 men in 24 days. In how many days can 1800 men do the work if their working hours per day are reduced in the ratio 4 : 5 ?

Sol. Let the number of working hours per day be t hours.
Therefore, Lenin Square was built in $1700 \times 24 \times t$ man hours = 40,800t man hours.
If 1800 men work $\frac{4t}{5}$ hours a day they can build it in $\frac{(40{,}800 \times t)}{\left(1800 \times \frac{4t}{5}\right)}$ = 28.33 days.

Ex. 7. A group of people can do a work in 10 days. With 5 of them being absent, the work is done in 12 days. How many people are there in the original group?

Sol. Let there be x no of people who do the work in 10 days.
When there are $x - 5$, they do the work in 12 days.

$\frac{x}{x-5} = \frac{12}{10}$. (Note that it is not $\frac{10}{12}$, as the number of people working and the number of days it takes to finish the work are inversely proportional).

$10x = 12x - 60 \Rightarrow x = 30$.

Ex. 8. A certain number of men can complete a piece of work in 90 days. If there are 15 men less, it will take 10 days more for the work to be completed. How many men were there originally?

Sol. Let there be x men originally.

They were to complete the work in 90 days but as the number of persons is reduced to $x - 15$

$\therefore$ work takes 10 more days.

$\therefore$ $x : x - 15 = 100 : 90 \Rightarrow x = 150$.

Ex. 9. A garrison is provided with ration for 80 soldiers to last for 60 days. For how much more time would the whole ration last if 20 additional soldiers join them after 15 days.

Sol. Let the whole ration now lasts for x days.

Equating the consumption on both sides, we get

$(80 \times 60) = (80 \times 15) + (100 \times x) \Rightarrow x = 36$ days.

Chapter 2: Time and Work

In a work problem, the rates at which certain persons or machines work alone are usually given, and it is necessary to compute the rate at which they work together (or vice versa).

The basic formula for solving work problems is: $1/r + 1/s = 1/h$ where r and s are, for example, the number of hours it takes Rae and Sam, respectively, to complete a job when working alone, and h is the number of hours it takes Rae and Sam to do the job when working together. The reasoning is that in 1 hour Rae does $1/r$ of the job, Sam does $1/s$ of the job, and Rae and Sam together do $1/h$ of the job.

2.1 Solved Examples

Ex. 1. If machine X can produce 1,000 bolts in 4 hours and machine Y can produce 1,000 bolts in 5 hours, in how many hours can machines X and Y, working together at these constant rates, produce 1,000 bolts?

Sol. $1/4 + 1/5 = 1/h$

$\Rightarrow$ 9/20 = 1/h

Working together, machines X and Y can produce 1,000 bolts in 2 2/9 hours.

Ex. 2. If Art and Rita can do a job in 4 hours when working together at their respective constant rates and Art can do the job alone in 6 hours, in how many hours can Rita do the job alone?

Sol. 1/6 + 1/R = 1/4

$\Rightarrow$ 4R + 24 = 6R

$\Rightarrow$ 24 = 2R

$\Rightarrow$ 12 = R

Working alone, Rita can do the job in 12 hours.

Ex. 3. A can do a piece of work in 30 days, which B can do in 20 days. Both started the work but A left 5 days before the completion of the work. It took how many days to complete the work?

Sol. A left the job 5 days before the completion means for the last 5 days only B worked. First calculate B's five days' work, which he did alone.

In 5 days B will do 5 $\times$ 1/20 = 1/4 th of the work.

Remaining work 1- 1/4 = 3/4.
Which A and B have done together.
A and B can do 1/20 + 1/30 work in 1 day.
Their one-day's work is $\frac{1}{20}+\frac{1}{30}\Rightarrow\frac{2+3}{60}\Rightarrow\frac{5}{60}=\frac{1}{12}$.
They can finish the work in 12 days.
They would have done three-fourth of the work in $12\times$ 3/4 = 9 days.
$\Rightarrow$ Total days = 5 + 9 = 14.

Ex. 4 . A and B can do a piece of work in 18 days. B and C in 24 days. A and C can do this work in 36 days. In what time can they do it all working together?

Sol. A and B's one day's work = 1/18.
B and C's one day's work = 1/24.
C and A's one day's work = 1/36.
If we add all this it will give us the work of 2A, 2B and 2C in 1 day i.e. $\frac{1}{18}+\frac{1}{24}+\frac{1}{36}=\frac{1}{8}$.
A, B and C's one day's work $\frac{1}{2}\times\frac{1}{8}=\frac{1}{16}$.
They can complete the work in 16 days.

2.2 Alternate Days Problems

As these type of questions required a different approach. We will discuss the method through following examples.

Ex. 5. A can complete a work in 6 days B can complete the same job in 12 days. In how many days work will be complete if they work on alternate days starting with A?

Sol. Let us take amount of work is 12 units (LCM of 6 and 12)

A can make 12/6 = 2units/day

B can make 12/12 = 1 unit/day

As A and B work on alternate days starting with A

So (A + B)' 2 days' work = (2+1)= 3 units

So 3 units can be made in = 2 days

12 units can be made in =2/3*12= 8 days (12 is multiple of 3)

Ex. 6. A can complete a work in 8 days, B can complete the same job in 12 days. In how many days work will be complete if they work on alternate days starting with A?

Sol. Let us take amount of work is 24 units (LCM of 8 and 12)
A can make 24/8 = 3 units/day
B can make 24/12 = 2 units/day
As A and B work on alternate days starting with A
So (A + B)' 2 days' work = (3+2)= 5 units
So 5 units can be made in = 2 days
20 units can be made in =2/5*20 = 8 days (20 is multiple of 5 and immediate near to 20)
Remaining work = 24 – 20 =4 units
Now next turn is of A and he will make 3 units on ninth day
Remaining units = 4-3= 1 unit
Now next turn is of B and B can make 2 units/day so he will take ½ day
So total time = 8 + 1 +0.5 = 9.5 days

2.3 Pipes and Cisterns

In pipes and cisterns problems - find out what portion of the tank each of the pipes fill or drain in unit time (say in a minute / hour / second) and then perform arithmetic operation on this value. The approach to be applied to solve *pipes and cisterns* questions is the same as that for *time and work* problems.

Ex. 7. Two pipes A and B fill a tank in 20 minutes and 40 minutes respectively. A pipe C at the bottom can empty the tank in 60 minutes. If all three pipes were open simultaneously, how long does it take to fill the empty tank?

Sol. Pipe A fills $1/20^{th}$ of the tank in a minute
Pipe B fills $1/40^{th}$ of the tank in a minute
Pipe C drains $1/60^{th}$ of the tank in a minute.
Therefore, if all three are open the net effect = $\left(\frac{1}{20}+\frac{1}{40}-\frac{1}{60}\right)th$ of the tank will be filled in a min.
i.e. $\frac{7}{120}$ th of the tank will be filled in a minute.
Therefore, the tank will be filled in $\frac{120}{7}$ minutes

Ex. 8. Two pipes A and B can fill a cistern in 20 and 24 minutes respectively. Both pipes being opened, find when the first pipe must be turned off, so that the cistern may be filled in 12 minutes?

Sol. Since the cistern is to be filled in 12 minutes, Second pipe can fill only 12/24 = ½ of the cistern in total time. This means the other half must be filled by the first pipe. The first pipe can fill the whole tank in 20 minutes, so half of the tank it can fill in half of the 20 minutes i.e. 10 minutes. Now the first pipe is opened from the beginning, it should be turned off after 10 minutes and this is the answer.

Ex. 9. A cistern is filled in 9 hours and it takes 10 hours when there is a leak in its bottom. If the cistern is full, in what time shall the leak empty it?

Sol. Cistern filled in 1 hour by the filling pipe = 1/9. Cistern filled by the leak and the filling pipe in 1 hour = 1/10. Cistern emptied by the leak in one hour = 1/9 -1/10 = 1/90. Hence the leak can empty the tank in 90 hours.

2.4 Efficiency and Wages Related Problems

Efficiency of a person is his one day's work. Efficiency is inversely proportional to number of days.

Wages are distributed among persons according to their efficiency ratio or ratio of amount of work done by them.

Ex. 10. P and Q can do a job in 10 and 15 days, if they receive RS. 2000 for the entire job. Find their respective wages ?

Sol. P's one day's work = 1/10 and Q's one day's work = 1/15

P's and Q's efficiency ratio = 1/10 :1/15 = 3 : 2

P's share = 3/5*2000 = 1200

Q's share = 2/5*2000= 800

Chapter 3: Interest

3.1 Simple Interest

If I borrow money from you for a certain time period, then at the end of the time period, I return not only the borrowed money but also some additional money. This additional money that a borrower pays is called interest. The actual borrowed money is called Principal. The interest is usually calculated as a percentage of the principal and this is called the interest rate.

There is a well-accepted norm about the interest rate. It is always assumed to be per annum, i.e. for a period of one year, unless stated otherwise.

Interest can be computed in two basic ways. The first way, with simple annual interest, the interest computed on the principal only and is equal to (principle) x (rate) x (time)/100.

Where rate is taken as percent per annum and time is taken in years. Sometimes the interest is given and time or one of other two items is missing. Then the formula for calculating the time becomes (interest ×

100)/(principal × rate). And similarly the rate/principal can be calculated.

Ex. 1. If Rs 8,000 is invested at 6 percent simple annual interest, how much interest is earned after 3 months?

Sol. Since the annual interest rate is, 6 %, the interest for 3 months is $\frac{8,000 \times 6 \times 3}{100 \times 12}$ = Rs. 120.

Ex. 2. A sum was put at simple interest at a certain rate for 4 years. Had it been put at 2 % higher rate, it would have fetched Rs 56 more. Find the sum.

Sol. Let *P* be the principal and x be the original % rate of interest. The interest for 4 years will amount to 4px/100. If the rate is increased by 2 %, the new interest then becomes 4p(x+2)/100. The difference between the two is

$\Rightarrow$ [4p(x+2)/100] – 4px/100 = 4p*2/100.

This is equal to Rs 56. So *P* = $\frac{56 \times 100}{4 \times 2}$ = Rs 700.

Ex. 3. A sum of Rs 3,800 is lent out in two parts in such a way that the interest on one part at 8 % for 5 years is equal to that on another part at $\frac{1}{2}$ % for 15 yrs. Find the sum lent out at 8 %.

Sol. Let P be the sum let out at 8% and Q be the sum let out at $\frac{1}{2}$ %. So $P \times 8 \times 5 = Q \times \frac{1}{2} \times 15$.

Therefore $P : Q = \frac{15}{2 \times 8 \times 5} = 3 : 16$.

The total sum of Rs. 3,800 is to be divided in the ratio of 3 : 16. This way the first sum is $3800 \times \frac{3}{19} =$ Rs. 600.

Ex. 4. Sham rao deposits Rs 2,000 in his savings account at Bank of Maharashtra at 4 % and Rs 3,000 in US – 64 at 14 % p.a. Find the rate of interest for the whole sum.

Sol. Sham rao earns an interest of Rs 0.04×2000 = Rs. 80 in the Bank of Maharashtra account.

He earns an interest of Rs. 0.14×3000 = Rs. 420 in US – 64.

⇒ Total interest income = 80 + 420 = 500.

⇒ Total principal = 5000.

So rate on total amount = $^{500}\!/_{5000}$ = 10 %.

Sometimes instead of interest, the amount is given, then either you need to add the above simple interest formula in the principal again and then solving the equation. Otherwise the following straight formula can be applied.

$$\text{Principal} = \frac{\text{Amount} \times 100}{100 + RT}.$$

Sometimes, the amount is given, but instead of principal, the simple interest is asked in the question. And the interest can be calculated with the help of the following straight formula

Simple interest = $\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + RT}$.

Ex. 5. A sum of money lent out at simple interest amounts to Rs. 720 after 2 years and Rs. 1020 after a further period of 5 years. Find the rate percent.

Sol. Interest for 5 years = 1020 – 720 = 300

$\therefore$ Interest for 2 years = 120. Hence P = 600.

So 120 = 600*R*2/100 $\Rightarrow$ R = 10 %.

Ex. 6. The simple interest on a sum of money is 1/9 of the principal and the number of years is equal to the rate % p.a. The rate % p.a. is

Sol. $\frac{P}{9} = \frac{P \times R \times R}{100} \Rightarrow R^2 = \frac{100}{9} \Rightarrow R = \frac{10}{3} = 3\frac{1}{3}\%$.

Ex. 7. If Rs. 450 amounts to Rs. 540 in 4 years what will it amount to in 6 years at the same rate %?

Sol. Interest for 4 years = 540 – 450 = 90

$\therefore$ Interest for 6 years = $\frac{90}{4} \times 6 = 135$

$\therefore$ 450 will become 450 + 135 = 585 after 6 years.

3.2 Compound Interest

Going back to the case where I had borrowed money from you. Now at the end of the time period, I am not able to make the payment of interest. In the entire duration of the time period, the principal amount, which is used as the basis for calculation of the interest, has remained constant. After I default, you add the unpaid interest to the principal (compounding matters for me) and we now have agreed to compound the interest.

We need not wait till the end of the time period but can compound the principal in between also. Assume that in the initial agreement that we had I was supposed to pay you every quarter of a year. I have of course defaulted on that, so you can keep on adding the interest amount every quarter and change the principal. There is a well-accepted norm about compounding. It is always assumed to be annually, i.e. after a period of one year, unless stated otherwise. As is the case in the compounded growth problem stated earlier in the chapter, the formula for the amount A at the end of T years for an initial amount of P and at an interest rate of r% per annum is $A = P\left(1+\frac{R}{100}\right)^T$

Ex. 8. If Rs 10,000 is invested at 10 percent annual interest, compounded semiannually, what is the balance after 1 year?

Sol. The balance after the first 6 months would be 10,000 + (10,000)(0.05) = 10,500 Rupees.
The balance after one year would be 10,500 + (10,500)(0.05) = Rs. 11025.

Note that the interest rate for each 6 – month period is 5 %, which is half of the 10 % annual rate. The balance after one year can also be expressed as

$10,000\ [(1 + 10/200)^2] = $ Rs. 11025.

Ex. 9. What sum of money lent out at compound interest will amount to Rs. 968 in 2 years at 10% p.a., interest being charged annually?

Sol. An amount of x will become 1.1 x in 1 year at 10 % interest. In 2 years time it will become $1.1 \times 1.1\ x = 1.21\ x$.

So, $1.21\ x = 968$.

Therefore, $x = 968/1.21 = 800$.

Ex. 10. Find the least number of complete years in which the sum of money put out at 20 % compound interest will be more than double.

Sol. Rate of interest = 20%. So after 1 year it will become 1.2. After two years it will become 1.44 times. After three years it will be 1.728 times and after 4 years it will be 2.0736 times. This is definitely more than double after 4 years.

Ex. 11. The difference in simple interest and compound interest on a certain sum of money in 2 years at 15 % p.a. is Rs. 144. Find the principal.

Sol. If Diff. between SI & CI for 2 years is Rs. x, then Principal = x $\left(\frac{100}{r}\right)^2 \Rightarrow$ P = 144 $\times \frac{10000}{15\times15} \Rightarrow$ P = 6400.

Ex. 12. The compound interest on a certain sum of money for 2 years is Rs. 52 and the simple interest for the same time at the same rate is Rs. 50. Find the rate %.

Sol. SI for 2 years = 50, CI for 2 years = 52.

$\therefore$ As SI and CI are same for the first year.

$\therefore$ SI and CI for 1st year = Rs. 25.

So CI for 2nd year = 52 – 25 = 27 i.e. a difference of 2 on 1st year's interest of Rs. 25.

Rate of interest = 100 $\times$ 2/25 = 8 %.

Ex. 13. I invested a sum of money at compound interest. It amounted to Rs. 2,420 in 2 years and Rs. 2,662 in 3 years. Find the rate of interest.

Sol. Rate of interest = 100*(2662-2420)/2420 . = 10%

Ex. 14. The simple interest on a certain sum for 3 years in Rs. 240 and the compound interest on the same sum for 2 years is Rs. 164. Find the rate percent per annum.

Sol. SI for 3 years = 240 $\Rightarrow$ SI for 1 years = 80

$\therefore$CI for 1 year = 80. So CI for 2nd year = 84 and SI for 2nd year = 80. Difference = 4

$\therefore$rate of interest = 100*4/80 = 5%.

Ex. 15. What is the minimum number of complete years, in which a sum of money becomes more than 2 times, if invested at the rate of 30 %, compounded annually?

Sol. 30% will make it 1.3 after 1 year.

Hence after 3 years it will become $(1.3)^3$ = 2.197.

$\Rightarrow$ As it is becoming more than double hence 3 years is the required answer.