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Aptitude Advanced

Algebra

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Chapter 1: Algebra

Algebra plays an essential role in many competitive exams. Following are some definitions and formulae one needs to know before starting basic algebra.

Algebraic expression: An algebraic expression comprises both numbers and variables together with at least one arithmetic operation.

Constant: A fixed quantity that doesn't change is known as a constant. E.g. 2, 5/6,π.

Variable: A variable is a symbol that we assign to an unknown value. It is usually represented by letters such as *x*, *y*, or *t*.

Coefficient: The coefficient of a variable is the number that is placed in front of a variable.

Equation: An equation consists of two expressions separated by an equal sign. The expression on one side of the equal sign has the same value as the expression on the other side.

Algebraic fraction: An algebraic fraction is a fraction that contains an algebraic expression in its numerator and/or denominator. For Ex.: 4/(2x−3), (3x−5x+3)/4, (3x−5)/(x+3)

1.1 Basic Formulae

I. $(a + b)^2 = a^2 + b^2 + 2ab$.
II. $(a - b)^2 = a^2 + b^2 - 2ab$.
III. $(a^2 - b^2) = (a + b)(a - b)$.
IV. $(a + b)^2 - (a - b)^2 = 4ab$.
V. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$.
VI. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$.
VII. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$.
VIII. $(a^3 + b^3) = (a + b)(a^2 + b^2 - ab)$.
IX. $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$.
X. $(a + b + c)^2 = [a^2 + b^2 + c^2 + 2(ab + bc + ca)]$.
XI. $(a + b + c + \mathrm{d})^2 = [a^2 + b^2 + c^2 + \mathrm{d}^2 + 2a(b + c + \mathrm{d}) + 2b(c + \mathrm{d}) + 2c\mathrm{d}]$.
XII. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$.
If $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$.
XIII. $(x + a)(x + b) = x^2 + (a + b)x + ab$.
XIV. $(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$.

1.2 Solved Examples

Ex. 1: If $x + \frac{1}{x} = 7$, what is the value of $x^2 + \frac{1}{x^2}$?

Sol. $x + \frac{1}{x} = 7$ (squaring both sides)

$x^2 + \frac{1}{x^2} + 2.x.\frac{1}{x} = 49 \Rightarrow x^2 + \frac{1}{x^2} = 49 - 2 = 47.$

Ex. 2: Given $x^2 + \frac{1}{x^2} = 27$, find the value of $x - \frac{1}{x}$.

Sol. Now apply the logic that $x - \frac{1}{x}$ is the square root of

$x^2 + \frac{1}{x^2} - 2.x.\frac{1}{x}$.

$x^2 + \frac{1}{x^2} - 2 = 27 - 2$ (subtracting 2 from both sides)

$(x - \frac{1}{x})^2 = 25. \Rightarrow x - \frac{1}{x} = \pm 5.$

Ex. 3: Find the value of $m^3 + n^3$, given $mn = 600$, $(m + n) = 50$.

Sol. $m + n = 50$ (cubing both sides)

$m^3 + n^3 + 3mn(m + n) = 125000.$

$m^3 + n^3 + 3 \times 600(50) = 125000$. (Replacing the variables by two values given) $\Rightarrow m^3 + n^3 = 125000 - 90000 = 35000$.

Ex. 4: If $\left(x + \frac{1}{x}\right) = 5$, then $\left(x^2 + \frac{1}{x^2}\right)$ is equal to

1. 27 2. 23 3. 25
4. None of these

Sol. $\left(x + \frac{1}{x}\right) = 5 \Rightarrow \left(x + \frac{1}{x}\right)^2 = 5^2 \Rightarrow$

$x^2 + \frac{1}{x^2} + 2 \cdot x \cdot \frac{1}{x} = 25 \Rightarrow x^2 + \frac{1}{x^2} = 23$.

Hint: Check options, only 27 can be expressed as 3^3 (i.e. x^x)

Ex. 5: $\left(\frac{0.25 \times 0.25 - 0.09 \times 0.09}{0.25 + 0.09}\right) = ?$

1. 0.16 2. 0.34 3. 0.32
4. None of these

Sol. Use $\frac{a^2 - b^2}{a - b} = a + b$. So, Answer = 0.25 + 0.09 = 0.34

Chapter 2: Systems of equations

Consistent System: A system, which could have two or more simultaneous linear equations is known as consistent if it has at least one solution.

Inconsistent System: A system of two simultaneous linear equations is said to be inconsistent if it has no solution at all.

To get the number of solutions a set of two equations have, the following rules can be applied. The equations are of the form of $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

- The equations have a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.
- The equations have infinitely many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.
- The equations have no solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Chapter 3: Linear Algebra

Linear Equations with one unknown

To solve a linear equation with one unknown (that is, to find the value of the unknown that satisfies the equation), the unknown should be isolated on one side of the equation. This can be done by performing the same mathematical operations on both sides of the equation. Remember that if the same number is added to or subtracted from both sides of the equation, this does not change the equality; likewise, multiplying or dividing both sides by the same nonzero number does not change the equality. For Ex., to solve the equation $\frac{5x-6}{3}$ = 4 for *x*, x can be isolated using the following steps:

$\Rightarrow$ $5x - 6 = 12$ (multiplying by 3)

$5x = 12 + 6 = 18$ (adding 6)

$x = \frac{18}{5}$ (dividing by 5)

Linear Equations with two unknowns

There are several methods of solving two linear equations in two unknowns. With any method, if a contradiction is reached, then the equations have no solution; if a trivial equation such as 0 = 0 is reached, then the equations are equivalent and have infinitely many solutions. Otherwise, a unique solution can be found.

One way to solve for the two unknowns is to express one of the unknowns in terms of the other using one of the equations, and then substitute the expression into the remaining equation for obtaining an equation with one unknown. This equation can be solved and the value of the unknown substituted into either of the original equations to find the value of the other unknown. For example, the following two equations can be solved for x and y.

(1) $3x + 2y = 11$
(2) $x - y = 2$

In equation (2), $x = 2 + y$. Substitute $(2 + y)$ in equation (1) for x:

$3(2 + y) + 2y = 11$
$\Rightarrow 6 + 3y + 2y = 11$
$\Rightarrow 6 + 5y = 11$
$\Rightarrow 5y = 5$
$\Rightarrow y = 1.$
If $y = 1$, then $x = 2 + 1 = 3$.

There is another way to solve for x and *y* by eliminating one of the unknowns. This can be done by making the coefficients of one of the unknowns the same (disregarding the sign) in both equations and either adding the equations or subtracting one equation from the other. For Ex., to solve the equations

(1) $6x + 5y = 29$
(2) $4x – 3y = -6$

by this method, multiply equation (1) by 3 and equation (2) by 5 to get
$18x + 15y = 87$
$20x – 15y = -30$.
Adding the two equations eliminates *y*,
Yielding $38x = 57$, or $x = \frac{57}{38} = \frac{3}{2}$.
Finally, substituting $\frac{3}{2}$ for *x* in one of the equations gives $y = 4$.

3.1 Examples based on Linear equations

Ex.1: $2x–3y=7,\ 3x+y=5$

Sol. $2x - 3y = 7$ .....(i), $3x + y = 5$ ......(ii)

(i) x 3 - (ii) x 2 gives

$6x - 9y = 21$ (i)

$6x + 2y = 10$ (ii)

Subtracting the (ii) equation from (i) equation

$-11y = 11$

$\Rightarrow y = -1, \Rightarrow 3x - 1 = 5$ [from (ii)], $\Rightarrow x = 2$.

Ex.2: $\frac{2x-1}{3} + \frac{y+2}{4} = 4, \frac{x+3}{2} - \frac{x-y}{3} = 3$.

Sol. $\frac{2x-1}{3} + \frac{y+2}{4} = 4$

$\Rightarrow 4(2x-1) + 3(y+2) = 48,$

$\Rightarrow 8x - 4 + 3y + 6 = 48 \Rightarrow 8x + 3y = 46$----(i)

$\frac{x+3}{2} - \frac{x-y}{3} = 3$

$\Rightarrow 3(x+3) -2(x-y) = 18,$

$\Rightarrow 3x + 9 - 2x + 2y = 18, x + 2y = 9$---(ii)

From (i) $8x + 3y = 46$, $8x + 16y = 72$ (multiplying the second equation by 8)

$\Rightarrow -13y = -26$

$\Rightarrow y = 2,$

Substituting in (i) $\frac{2x-1}{3} + 1 = 4$

$\Rightarrow x = 5.$

Ex.3: When the first of the two numbers is added to twice the second the result is 21, but when the second number is added to twice the first, the result is 18. Find the two numbers.

Sol. x + 2y = 21 $\Rightarrow$ 2x + 4y = 42, ……(1)

2x + y = 18 $\Rightarrow$ 2x + y = 18,…….(2)

Subtracting (2) from (1)

$\Rightarrow$ 3y = 24, $\Rightarrow$ y = 8.

x + 2 x 8 = 21,

$\Rightarrow$ x = 5.

Ex.4: If the numerator and denominator of a certain fraction are both increased by 3, the resulting fraction equals 2/3. If however, the numerator and denominator are both decreased by 2, the resulting fraction equals half. Determine the fraction.

Sol. $\frac{p+3}{q+3} = \frac{2}{3}$

$\Rightarrow$ 3p + 9 = 2q + 6,

$\Rightarrow$ 3p - 2q = -3 -----(i)

$\frac{p-2}{q-2} = \frac{1}{2}$

$\Rightarrow$ 2p - 4 = q - 2

$\Rightarrow$ 2p - q = 2 ------(ii)

Multiplying (i) by 2 and multiplying (ii) by 3 we get

$6p - 4q = -6$

$6p - 3q = 6$

$-q = -12$

$\Rightarrow$ q = 12, $\Rightarrow$ 2p = 12 + 2 = 14, $\Rightarrow$ p = 7.

Ex.5: The sum of two numbers is 28 and their difference is 12. Find the numbers.

Sol. x + y=28,

X - y=12,

$\Rightarrow$ 2y=16,

$\Rightarrow$ y=8, thus x=20.

Chapter 4: Quadratic Equations

Solving Quadratic Equations by Factoring

The standard form for a quadratic equation is $ax^2 + bx + c = 0$, where *a, b*, and *c* are real numbers and $a \neq 0$; for example.: $x^2 + 6x + 5 = 0$, $3x^2 - 2x = 0$, and $x^2 + 4 = 0$.

Some equations can be solved by factoring. To do this, first add or subtract expressions to bring all the expressions to one side of the equation, with 0 on the other side. Then try to factor the nonzero side into a product of expressions. Each of the factors can be set equal to 0, yielding several simpler equations that possibly can be solved. The solutions of the simpler equations will be solutions of the factored equation. As an example., consider the equation $x^2 - 7x = -12$:

$x^2 - 7x + 12 = 0$ (taking all terms on one side and putting the expression equal to zero)

Now try to break b into two parts, such that the sum of those two parts = '*b*' and the product is equal to the product of 'a' and 'c'.

$x^2 - 4x - 3x + 12 = 0$

$\Rightarrow x(x - 4) - 3(x - 4) = 0$

$\Rightarrow (x - 3)(x - 4) = 0$.

Putting these separately equal to 0

$\Rightarrow x - 3 = 0, x = 3$ and $x - 4 = 0, x = 4$.

Thus the solutions of the equation are 3 and 4.

The solutions of an equation are also called the roots of the equation.

A quadratic equation has at most two real roots and may have just one or even no real root. For example., the equation $x^2 - 6x + 9 = 0$ can be expressed as $(x - 3)^2 = 0$, or $(x - 3)(x - 3) = 0$; thus the only root is 3. The equation $x^2 + 4 = 0$ has no real root; since the square of any real number is greater than or equal to zero, $x^2 + 4$ must be greater than zero.

An expression of the form $a^2 - b^2$ can be factored as $(a - b)(a + b)$.

For example ., the quadratic equation $9x^2 - 25 = 0$ can be solved as follows.

$(3x - 5)(3x + 5) = 0$

$3x - 5 = 0$ or $3x + 5 = 0$

$x = \frac{5}{3}$ or $x = -\frac{5}{3}$

Solving Equations by using the Quadratic Formula

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If a quadratic expression is not easily factored, then its roots can always be found using the quadratic formula: If $ax^2 + bx + c = 0$ $(a \neq 0)$, then the roots are $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$.

These are two distinct real numbers unless $b^2 - 4ac < 0$.

If $b^2 - 4ac = 0$; then these two expressions for x are equal to $-b/2a$ and the equation has only one root.

If $(b^2 - 4ac) < 0$, then $\sqrt{b^2-4ac}$ is not a real number and the equation has no real roots.

To solve the quadratic equation $x^2 - 7x + 8 = 0$ using the above formula, note that $a = 1$, $b = -7$, and $c = 8$, and hence the roots are

$x = \frac{7+\sqrt{17}}{2}$ = 5.6 approx. and $x = \frac{7-\sqrt{17}}{2}$ = 1.4 approx.

$b^2 - 4ac$ is called the discriminant and it is denoted by the symbol Δ or is represented by the letter D. Following are some of the important points relating to the discriminant and its relation with the nature of the roots.

- If $\Delta > 0$, then both the roots will be real and unequal and the value of roots will be $\frac{-b \pm \sqrt{\Delta}}{2a}$. If Δ is a perfect square, then roots are rational otherwise they are irrational.

- If $\Delta = 0$, then the roots are real, equal and rational. In this case, the value of roots will be $-b/2a$.
- If $\Delta < 0$, then the roots will be imaginary, unequal and conjugates of each other.
- If α and β are the roots of the equation $ax^2 + bx + c = 0$, then sum of the roots i.e. $\alpha + \beta = -b/a$.
- If α and β are the roots of the equation $ax^2 + bx + c = 0$, then product of the roots i.e. $\alpha\beta = c/a$.
- If α and β, the two roots of a quadratic equation is given, then the equation will be $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

The equation is x^2 – (sum of roots)x + product of roots = 0

These were some crucial points relating to the quadratic equations. The following are some properties regarding the roots of the equation.

- If in the equation $b = 0$, then roots are equal in magnitude but opposite in sign.
- If $a = c$, then roots are reciprocal of each other.
- If $c = 0$, then one of the roots will be zero.

- If one root of a quadratic equation be a complex number, the other root must be its conjugate complex number i.e. $\alpha = j + \sqrt{-k}$, then $\beta = j - \sqrt{-k} \Rightarrow \alpha = j + ik$ and $\beta = j - ik$

4.1 Solved examples of Quadratic Equations

Ex. 1: $x^2 + 6x + 5 = 0$

Sol. $(x + 5)(x + 1) = 0$
$x + 5 = 0$ or $x + 1 = 0$
$x = -5$ or $x = -1$

Ex. 2: $3x^2 - 3 = 8x$

Sol. $3x^2 - 8x - 3 = 0 \Rightarrow 3x^2 - 9x + x - 3 = 0$
$3x(x - 3) + 1(x - 3) \Rightarrow (3x + 1)(x - 3) = 0$
$3x + 1 = 0$ or $x - 3 = 0$

Ex. 3: If one root of the quadratic equation $8x^2 - 28x + z = 0$ is six times the other, find the value of z

Sol. Here in this equation $a = 8$, $b = -28$ and $c = z$.
From the formula sum of the roots = $\frac{-28}{8} = \frac{7}{2}$ (i)

If one root is α, the other root is 6α and the sum of roots will be 7α.

$7\alpha = 7/2 \Rightarrow \alpha = 1/2$. Other root will be $1/2 \times 6 = 3$.

Now the product of the roots will be $= c/a = z/8$.

Product of roots $\alpha \times 6\alpha = 6\alpha^2. \Rightarrow 6(1/2)^2$

$\Rightarrow$ Now $6/4 = z/8$.

z = 12.

Ex. 4: If $16x^2 - 24x + m = 0$ have equal roots, find the value of *m*.

Sol. Because it has equal roots, discriminant should be equal to zero.

$(-24)^2 - 4 \times 16 \times m = 0 \Rightarrow 576 = 64m$

$\Rightarrow m = 576/64 = 9$.

Ex. 5: If *p* and *q* are the roots of the equation $x^2 + px + q = 0$, then which value of p is not possible?

1. $p = 1$
2. $p = 1$ or 0 or $-1/2$
3. $p = -2$
4. $p = 1$ or 0

Sol. Since p and q are the roots of the equation $x^2 + px + q = 0$,

$\rightarrow\ p^2 + p^2 + q = 0$ and $q^2 + pq + q = 0$
$\rightarrow 2p^2 + q = 0$ and $q(q + p + 1) = 0$
$\rightarrow 2p^2 + q = 0$ and $q = 0$ or $q = -\ p - 1$
when we take $2p^2 + q = 0$ and $q = 0$, we get $p = 0$.
Or when we take $2p^2 + q = 0$ and $q = -\ p - 1$, we get $2p^2 - p - 1 = 0$, which gives us $p = 1$ or $p = -\ ^1\!/_2$
Hence there can be three values for p i.e. $p = 0$ or 1 or $-\,^1\!/_2$.
Thus 3rd option is our answer.

Chapter 5: Progressions

Arithmetic Progression

An arithmetic progression is a sequence of numbers in which each term is derived from the preceding term by adding or subtracting a fixed number called the common difference. For example., the sequence 9,6,3,0,-3,.... is an arithmetic progression with –3 as the common difference. The progression –3, 0, 3, 6, 9 is an Arithmetic Progression (AP) with 3 as the common difference.

- The general form of an Arithmetic Progression is a, $a + d$, $a + 2d$, $a + 3d$ and so on. Thus nth term of an AP series is $T_n = a + (n - 1) d$. Where $T_n = n^{th}$ term and a = first term. d = common difference = $T_m - T_{m-1}$.
- Sometimes the last term is given and either 'd' is asked or 'a' is asked. Then formula becomes $l = a + (n - 1) d$
- There is another formula, applied to find the sum of first n terms of an AP $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$
- The sum of n terms is also equal to the formula $S_n = \frac{n}{2}(a+l)$ where l is the last term.
- When three quantities are in AP, the middle one is

called as the arithmetic mean of the other two. If *a, b* and *c* are three terms in AP then $b = \frac{a+c}{2}$.

Geometric Progression

A geometric progression is a sequence in which each term is derived by multiplying or dividing the preceding term by a fixed number called the common ratio. The sequence 4, -2, 1, $-\frac{1}{2}$,... is a Geometric Progression (GP) for which ($-\frac{1}{2}$) is the common ratio.

- The general form of a GP is a, ar, ar^2, ar^3 and so on. Thus nth term of a GP series is $T_n = ar^{n-1}$, where a = first term and r = common ratio = T_m/T_{m-1}.
- The formula applied to calculate the sum of first n terms of a GP
  $$S_n = \frac{a(r^n - 1)}{r-1} \text{ where} \rightarrow r > 1 \quad \text{and} \quad S_n = \frac{a(1-r^n)}{1-r} \text{ where} \Rightarrow r < 1$$
- When three quantities are in GP, the middle one is called as the geometric mean of the other two. If *a, b* and *c* are three quantities in GP and *b* is the geometric mean of *a* and c, i.e. $b = \sqrt{ac}$
- The sum of infinite terms of a GP series $S_\infty = \frac{a}{1-r}$

5.1 Solved examples of Progressions

Ex. 1: How many terms of the series 1, 5, 9, 13 … should be taken so that their sum is 231?

Sol. As per the formula

$S_n = \frac{n}{2}\left[2a+(n-1)d\right] \Rightarrow 231$

$= n/2\ [2.1 + (n - 1)\,4]$.

$462 = n\,[2 + 4n - 4]$

$\Rightarrow 4n^2 - 2n - 462 = 0.$

$\Rightarrow 2n^2 - n - 231 = 0.$

$2n^2 - 22n + 21n - 231 = 0$

$\Rightarrow 2n(n - 11) + 21(n - 11) = 0$

$\Rightarrow (2n + 21)\,(n - 11)$. $N = 11$ or $-21/2$,

number of terms cannot be negative, so 11 terms are needed.

Ex. 2: Find 9th term of the following series.

5, 10, 20, 40 ……

Sol. $T_9 = ar^{n-1} \Rightarrow r = 10/5 = 2 \Rightarrow T_9 = 5\,(2)^{9-1}$.

$T_9 = 5 \times 256 = 1280.$

Ex. 3: Ram gives his son Rs. 100 on one day, Rs. 50 on the second day, Rs. 25 on the third day and so on.

What will be total amount given by Ram to his son starting from the first day, if he lives forever?

Sol. In this question, every day he is giving half the amount, he has given the previous day. And he has to pay forever. This makes it an infinite GP series.

$$S_\infty = \frac{a}{1-r} \Rightarrow S_\infty = \frac{100}{1-\frac{1}{2}}$$

$\Rightarrow S_\infty = 100 \times 2 =$ Rs. 200.

Ex. 4: What is the seventh term of the sequence 1, 1/4, 1/9... Is it a progression?

Sol. The sequence appears to be a set of numbers squared. The terms can be rewritten as 1^2, $(\frac{1}{2})^2$, $(\frac{1}{3})^2$. So the seventh term is obviously $(\frac{1}{7})^2$ or 1/49. The difference between terms is not a constant number; neither is the ratio of two successive terms a constant number, so the sequence is neither an AP nor a GP.

Ex. 5: The first term of an A.P. is 5 and the common difference is 4. What is the sum of the first six terms?

Sol. The sequence is 5,9,13,17,21,25... Summing we get the answer as 90. The summation of an AP, formula can also be applied, which is $n/2$ [2*a* + (*n* – 1) *d*]. So the summation is $6/2$ [2 $\times$ 5 + 5 $\times$ 4] = 3 [10 + 20] = 90.

Chapter 6: Miscellaneous solved examples

Ex.1: If the third term of an A.P. is 12 and its seventh term is 24, then its tenth term is

1. 48 2. 36
3. 33 4. 52

Sol. We know in A. P., the nth term is

Tn = a + (n-1) d

$\therefore$ T10 =a + 9d ....(1)

$\therefore$ T3 = a + (3-1)d = a + 2d = 12 ....(2)

$\therefore$ T7 = a + (7-1)d = a + 6d = 24 ....(3)

(3) – (2) gives 4d = 12 = d = 3

put d = 3 in (2) we get

a + 6 = 12 , $\therefore$ a = 6

$\therefore$ T10 = a + 9d = 6 + 9 $\times$ 3 = 6 + 27 = 33

Ex. 2: If the third term of G.P. is 5 then product of its first five terms is

1. 5^5 2. 5^4
3. 4^5 4. 6^5

Sol. Let a be the first term and r be common ratio of the series

$T_3 = ar^2 = 5$ ....(1) $[\therefore T_n = ar^{n-1}]$

Its 1st five terms are

a, ar, ar^3, ar^3, ar^4

and its product $= a \times ar \times ar^2 \times ar^3 \times ar^4$ ....(2)

from (1) $ar^2 = 5$ or $(ar^2)^5 = (5)^5$ or $a^5 r^{10} = 5^5$,

which is equal to (2)

Ex. 3: Find three arithmetic means (A.M.) between 6 and 14.

1. 12, 14, 16
2. 8, 12, 14
3. 8, 10, 12
4. -8, -10, -12

Sol. Let A_1, A_2, A_3 be the three A. M. between 6 and 14

$\therefore 6, A_1, A_2, A_3, 14$..........are in A.P. ....(1) where

$T_1 = 6$, $T_2 = A_1$, $T_3 = A_2$, $T_4 = A_3$, $T_5 = 14$, if d is common difference then

$T_1 = a = 6$

$T_2 = A_1 = a + d$

$T_3 = A_2 = a + 2d$

$T_4 = A_3 = a + 3d$

$T_5 = 14$ or $T_5 = 6 + 4d = 14$

$\therefore d = 2$

$\therefore A_1 = T_2 = a + d = 6 + 2 = 8$

$A_2 = T_3 = a + 2d = 6 + 4 = 10$

$A_3 = T_4 = a + 3d = 6 + 6 = 12$

Ex. 4: For what value (s) of r is $\frac{r^2+5r+6}{r+2}$ equal to 0?

1. – 2 only
2. – 3 only
3. + 3 only
4. – 2 or – 3

Sol. For $\frac{r^2+5r+6}{r+2} = 0$

$\Rightarrow r^2 + 5r + 6 = 0$

$\Rightarrow (r + 2)(r + 3) = 0$

$\Rightarrow$ r = -3 and r = -2 but r = -2 is not possible.

Hence r = - 3 only

Ex. 5: The roots of the equation $x^2 + px + q = 0$ are equal if

1. $p^2 = 2q$
2. $p^2 = 4q$
3. $p^2 = -4q$
4. $p^2 = -2q$

Sol. Here a = 1, b = p, c = q.

The roots of the equation $x^2 + px + q = 0$ equal if $b^2 - 4ac = 0$

$\Rightarrow p^2 - 4q = 0$

$\Rightarrow p^2 = 4q$.

Ex. 6: Today Ram gives Rs. 20 to Sham, then tomorrow he gives Rs. 10 to Sham. On the day after tomorrow, he gives Rs. 5 and so on. What is the total sum of money that Ram will give Sham if he keeps on paying him forever?

1. Rs. 30
2. Rs. 35
3. Rs. 40
4. None of these

Sol. Since Ram pays Rs. 20 today, Rs. 10 tomorrow and so on, using the logic of Infinite GP, using the formula of infinite terms of a GP, we get the answer as 20/ (1 – ½) = Rs. 40.

Ex. 7: If $\frac{t^2 - 1}{t - 1} = 2$, then what value (s) may have?

1. 1 only
2. – 1 only
3. 1 or – 1 only
4. No values

Sol. $\frac{t^2-1}{t-1}=2$

$\Rightarrow t^2 - 1 = 2t - 2$

$\Rightarrow t^2 - 2t + 1 = 0$

$\Rightarrow (t - 1)^2 = 0 \Rightarrow t = 1$

Ex. 8 : If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is

1. 1
2. 2
3. 3
4. -1

Sol. $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3} = \left(\frac{a}{b}\right)^{-(x-3)} = \left(\frac{a}{b}\right)^{3-x}$

$\therefore$ x − 1 = 3 − x or 2x = 4

$\therefore$ x = 2

Chapter 7: Inequalities

Inequalities and functions are an integral part of advanced Algebra. They play a major role in many competitive exams. Solving inequalities is almost same as solving equations. We do most of the same things, but we must also pay attention to the direction of the inequality.

An inequality is a statement that uses one of the following symbols:

$\neq$ not equal to

$>$ greater than

$\geq$ greater than or equal to

$<$ less than

$\leq$ less than or equal to

$a \leq b$ means a is less than or equal to b.

$a \geq b$ means a is greater than or equal to b.

$0 < a < 3$ means 'a is greater than 0 but less than 3'.

$-4 < b \leq 5$ means 'b is greater than -4 but less than or equal to 5.'

Illustration 1: Solve the inequality $3x - 2 > 5$ for x.

Sol. $3x - 2 > 5$

$3x > 7$(adding 2 to both sides)

$x > 7/3$ (dividing both sides by 3)

Illustration 2: Solve the inequality $\frac{5x-1}{-2} < 3$ for *x*.

Sol. $\frac{5x-1}{-2} < 3$

$5x - 1 > -6$ ($\because$ both sides are multiplied by a −ve number, the truth of inequality changes)

$5x > -5$ (adding 1 to both sides)

$x > -1$ (dividing both sides by 5)

7.1 Absolute value inequalities

The absolute value of a quantity means the distance that the expression is having from 0 on the number line irrespective of the side, in which it is really lying. e.g. |c – d| = e

$\Rightarrow$ means (c – d) is at a distance of e units from 0, now this could be either right-hand side of 0 or left-hand side of 0. Take another example $|p - q| > r$. Here it is clear that r must be greater than zero.

Now both sides of the number line should be considered while solving this inequality.

$p - q > r$ or $p - q < -r$ (once it is taken to be greater than the +ve value of *r*, secondly it is considered to be lesser than the –ve value of r) and by combining these two the solution set of the inequality would be found.

Illustration 3: Solve this inequality $|y + 9| < 15$ for y.

Sol. Now take both the sides $y + 9 < 15$ and $y + 9 > -15$

$y < 15 - 9$ and $y > -15 - 9 \Rightarrow y < 6$ and $y > -24$.

$-24 < y < 6$.

Illustration 4: Solve this inequalities $x^2 - 7x + 16 < 4$ for x.

Sol. Firstly take all the terms given on one side.

$x^2 - 7x + 16 - 4 < 0 \Rightarrow x^2 - 7x + 12 < 0$.

$x^2 - 4x - 3x + 12 < 0 \Rightarrow (x - 4)(x - 3) < 0$.

Now because their product is less than zero, implies one of these is negative and the other is positive. Now out of the factors, it can be seen that $x - 3$ is greater than $x - 4$. So it can be concluded that $x - 3$ is positive and $x - 4$ is negative.

$x - 4 < 0$ and $x - 3 > 0 \Rightarrow x < 4$ and $x > 3$. The solution set of the inequality is $3 < x < 4$.

Illustration 5: $x^2 - 11x + 28 > 0$

Sol. $x^2 - 11x + 28 > 0. \Rightarrow (x - 7)(x - 4) > 0$.

Now their product is positive means, either both are positive, or both are negative.
Taking both to be positive $x - 7 > 0$ and $x - 4 > 0 \Rightarrow x > 7$ and $x > 4$.
Now choose that one out of the two, which includes the other also. Anything that is greater than 7 will definitely be greater than 4 also. So $x > 7$.
Now taking both to be negative $x - 7 < 0$ and $x - 4 < 0 \Rightarrow x < 7$ and $x < 4$.
Now choose that one out of these two, which includes the other also. Anything that is lesser than 4 will definitely be smaller than 7 also. So $x < 4$.
So the solution set is $x < 4$ or $x > 7$.