TURBOMACHINERY

(Code : 402043)

Semester VII- Mechanical and Automobile Engineering (Savitribai Phule Pune University)

Strictly as per the New Credit System Syllabus (2019 Course) of Savitribai Phule Pune University w.e.f. academic year 2022-2023

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We dedicate this Publication soulfully and wholeheartedly, in loving memory of our beloved founder director,

Late Shri. Pradeepji Lalchandji Lunawat, who will always be an inspiration, a positive force and strong support behind us.

"My work is my prayer to God"

- Lt. Shri. Pradeepji L. Lunawat

Soulful Tribute and Gratitude for all Your Sacrifices, Hardwork and 40 years of Strong Vision...

PREFACE

Dear Students,

*This text book on "**Turbomachinery**" covers comprehensively the latest syllabus for students of 7th Semester of Mechanical Engineering as prescribed by the Savitribai Phule Pune University. This book will also be useful for the students of other universities.*

Attempt has been made to present the subject matter in simple, lucid and precise manner. Basic aim of the author has been to clarify the important concepts and to encourage the students to develop good problem solving skills.

Unique features of the book are :

(1) Text written in simple and easy to follow language.

(2) Logical approach to text based on fundamentals of thermodynamics and fluid dynamics.

(3) Explanation of subject matter with the help of neat and self explanatory diagrams.

(4) Inclusion of large number of solved examples. Solution of problems explained step by step.

(5) A detailed summary has been added at the end of each chapter. It would help the students to revise and comprehend the basic definitions and equations just prior to examination.

(6) Including large number of solved University Numericals.

***For answers to theory questions, the related section numbers have been indicated in bracket** for the students to refer during their course of study.*

*Author dedicate this book in the loving memory of **Late. Shri. Pradeepji Lunawat,** m source of inspiration and laid a strong foundation of "**TechKnowledge Publications**". He will always be remembered in our hearts and motivate us to achieve our new milestone.*

Author is thankful to Mr. Shital Bhandari, Prof. Arunoday Kumar and Mr. Chandroday Kumar and the staff of TechKnowledge Publications, who have taken considerable pains and shown their extreme co-operation during the preparation of this book.

Any suggestions and errors from Professors and students for the improvement of the text book would be gratefully acknowledged by the authors and they would be incorporated in the next edition of the book..

-Author

SYLLABUS

Savitribai Phule Pune University

Board of Studies - Mechanical and Automobile Engineering
Final Year Mechanical Engineering (2019 pattern)

TURBOMACHINERY (402043)

Teaching Scheme		Credits		Examination Scheme	
Theory	2 Hrs./week	Theory	2	In-Semester	-
Practical	2 Hrs./week	Term Work	1	End-Semester	50 marks
				Term Work	25 marks
				Oral	25 marks

Prerequisites :

Fluid Mechanics, Thermodynamics, Heat Transfer, Engineering Mathematics

OBJECTIVE

1 *To provide the knowledge of basic principles, governing equations and applications of Turbomachines.*

2. *To provide the students with opportunities to apply basic thermos-fluid dynamics flow equations to Turbomachines.*

3. *To explain construction and working principles of Turbomachines.*

4. *To evaluate the performance characteristics of Turbomachines.*

COURSE OUTCOMES

On completion of the course the learner will be able to;

CO 1: ***VALIDATE*** *impulse moment principle using flat, inclined and curved surfaces and* ***INVESTIGATE*** *performance characteristics of hydraulic turbines.*

CO 2: ***DETERMINE*** *performance parameters of impulse and reaction steam turbine along with discussion of nozzles, governing mechanism & losses.*

CO 3: ***MEASURE*** *performance parameters of single & multistage centrifugal pumps along with discussion of cavitation and selection.*

CO 4: ***EXPLAIN*** *performance parameters of centrifugal compressor along with discussion of theoretical aspects of axial compressor.*

COURSE CONTENTS

Unit 1 : Impact of Jet and Hydraulic Turbines

Introduction and Impact of Jet : Introduction to Turbomachines (Hydraulic & Thermal), Classification of Turbo machines, Applications of Turbomachines. Impulse momentum principle and its application to fixed and moving flat, inclined, and curved plate/vanes. Velocity triangles and their analysis, work done equations, vane efficiency (No numerical) **(Refer Chapters 1 and 2)**

Hydraulic Turbines : Introduction to Hydro power plant, Classification of Hydraulic Turbines, Concept of Impulse and Reaction Turbines. Construction, Principle of Working, design aspects, velocity diagrams and its analysis of Pelton wheel, Francis, and Kaplan turbines, Degree of reaction, Draft tube: types and efficiencies, governing of hydraulic turbines, Cavitation in turbines. **(Refer Chapter 3)**

Unit 2 : Steam Turbines

Steam Nozzle : Equations for velocity and mass flow rate (No derivation, no numerical)

Steam Turbines : Construction and working of Impulse and Reaction steam turbine, velocity diagram, work done efficiencies, Multi-staging, compounding, Degree of reaction, losses in steam turbine, governing of steam turbines **(Refer Chapter 4)**

Unit 3 : Centrifugal Pumps

Introduction & classification of rotodynamic Pumps, Main Components of Centrifugal Pump, Construction and Working of Centrifugal Pump, Types of heads, Velocity triangles and their analysis, Effect of outlet blade angle, Work done and Efficiency, Series and parallel operation of pumps, Priming of pumps, specific speed **(Refer Chapter 5)**

Unit 4 : Rotary Compressors

Centrifugal Compressors : Classification of Centrifugal Compressor, construction and working, velocity diagram, flow process on T-S Diagram, Euler's work, actual work input, various losses in Centrifugal Compressor **(Refer Chapter 6)**

Axial flow compressors : Construction and working, stage velocity triangle and it's analysis, enthalpy entropy diagram, stage losses and various efficiencies of axial flow compressors, [No numerical]

(Refer Chapter 7)

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UNIT - I

Chapter 1 : Introduction to Turbomachinery 1 - 1 to 1 - 17

Syllabus : Introduction to Turbomachines (Hydraulic & Thermal), Classification of Turbo machines, Applications of Turbomachines.

Chapter 2 : Impact of Jet 2-1 to 2-23

Syllabus : Impulse-momentum principle and its application to fixed and moving flat, inclined and curved plate/vanes, Velocity triangles and their analysis, workdone equations, vane efficiency (No numerical)

Chapter 3 : Hydraulic Turbines 3-1 to 3-91

Syllabus : Introduction to Hydro power plant, Classification of Hydraulic Turbines, Concept of Impulse and Reaction Turbines, Construction, Principle of Working, design aspects, velocity diagrams and its analysis of Pelton wheel, Francis and Kaplan turbines, Degree of reaction, Draft tube, types and efficiencies, governing of hydraulic turbines, Cavitation in turbines.

UNIT III

Syllabus : Introduction & classification of rotodynamic Pumps, Main Components of Centrifugal Pump, Construction and Working of Centrifugal Pump, Types of heads, Velocity triangles and their analysis, Effect of outlet blade angle, Work done and Efficiency, Series and parallel operation of pumps, Priming of pumps, specific speed.

UNIT IV

Chapter 6 : Centrifugal Compressors 6-1 to 6-36

Syllabus : Classification of Centrifugal Compressor, construction and working, velocity diagram, flow process on T-S Diagram, Euler's work, actual work input, various losses in Centrifugal Compressor.

Chapter 7 : Axial Flow Compressors 7-1 to 7-27

Syllabus : Construction and working, stage velocity triangle and it's analysis, enthalpy entropy diagram, stage losses and various efficiencies of axial flow compressors. [No numerical]

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UNIT - 1

1 Introduction to Turbomachinery

Syllabus

Introduction to Turbomachines (Hydraulic & Thermal), Classification of Turbo machines, Applications of Turbomachines.

1.1 Introduction to Fluid Machines (Hydraulic &Thermal)

University Question

Q. Define turbo machine. Explain salient feature of turbomachine.

SPPU : April 17 (In Sem), Oct. 19(In Sem)

We are generally encountered with two types of fluid machines in practice, namely the *power producing* and *power absorbing machines.*

In case of ***power producing devices,*** the energy of the working fluid is converted into mechanical power e.g. reciprocating engines and turbines. While in ***power absorbing devices,*** the mechanical power supplied at the input shaft is utilized and transferred to the working fluid either in the form of pressure energy or kinetic energy of fluid or both e.g. pumps, fans, compressors etc.

Both type of above machines use the working fluid as liquids in case of hydraulic machines and air/gases or vapours in case of thermal machines. These are two types :

(i) Reciprocating machines

- The principle of operation of reciprocating machines is based on the positive displacement of the fixed amount of working fluid in a piston-cylinder arrangement.
- The working fluid undergoes the change in pressure due to change in volume of fluid during the reciprocating motion of the piston. In other words, the fluid is subjected to change in its state by means of moving boundary.
- The examples of power producing reciprocating machines are internal combustion engines, reciprocating steam engines **(both thermal)** and examples of power absorbing reciprocating machines are reciprocating pumps **(hydraulic)** and reciprocating compressors **(thermal)**.

(ii) Turbomachines

- These are rotary type of machines in which the energy transfer is brought about by dynamic action of rotating element.
- In these machines the fluid is not positively contained, like in reciprocating machines, but it continuously flows steadily through the machine. Since the energy transfer in these machines takes places due to dynamic effects, *these turbomachines are also called as* ***dynamic machines.***
- *Example of power producing turbomachines,* are wind, hydraulic, steam and gas turbines and the *examples of power absorbing turbomachines* are centrifugal pumps, fans, blowers and compressors etc.
- Based on the above discussion, a ***turbomachine can be defined as :***
- *It is a device in which the energy transfer occurs between the fluid and a rotating element in a steady flow process due to dynamic action and results into change in pressure and momentum of the fluid.*

Note : The positive displacement machines like gear pump, roots and vane blowers etc. though rotative are not classified as turbomachines since the energy transfer is not fundamentally due to dynamic action due to rotation.

1.2 Principle Components of Turbomachines

The principle components of a turbomachine are :

(i) ***Vanes mounted on rotating element called a impeller or rotor or runner***

Term impeller is used in case of centrifugal pumps and compressors, runner is used for radial flow hydraulic turbines and the rotor is used in case of axial flow gas turbines and steam turbines.

(ii) ***Stationary element or guide vanes/blades or nozzles***

These are used for controlling the flow direction of fluid for energy conversion process without any pressure changes except in case of nozzles. Stationary element is not a necessary component of every turbomachine e.g. in case of ceiling fan which is used to circulate air in a room.

(iii) A **shaft** either an input shaft (power absorbing devices) or an output shaft, in case of power producing devices.

However in power transmitting machines we have both input and output shaft as in case of torque converter and fluid couplings.

(iv) ***A housing or casing :***

It is not a necessary component in certain types of turbomachines. It does not play a direct part in the energy conversion process. It is only used to restrict the flow and to direct the fluid flow in particular direction in a given space.

A turbomachine with housing is said to be *enclosed* and without the housing is said to be *extended.*

(v) ***Diffuser :***

It is a passage of increasing cross-sectional area in the direction of fluid flow. It's function is to convert the K.E. of fluid into pressure energy.

e.g. in case of centrifugal compressors.

(vi) ***Draft tube :***

It is similar in action as of diffuser. It is placed at the outlet of a hydraulic reaction turbines e.g. in case of Francis and Kaplan turbines.

1.3 Classification of Turbomachines

University Questions

Q. Classify turbo machines based on any four criteria with examples. **SPPU : April 18 (In Sem)**

Q. Give classification of turbomachines. **SPPU : Oct. 19 (In Sem)**

The turbo machines are mainly classified as follows :

1. Based on working medium

(a) Using Air/gases/vapour e.g. fans, blowers, compressors, wind mills, steam and gas turbines. These are called as *thermal turbomachines.*

(b) *Using liquids* e.g. hydraulic turbines and pumps. These are called as *hydro turbomachines.*

2. According to energy conversion

(a) ***Power producing devices*** in which the energy of fluid is converted into mechanical work e.g. hydraulic, gas and steam turbines, wind mills.

(b) ***Power absorbing devices*** in which the mechanical energy is converted into pressure energy or kinetic energy of a fluid e.g. centrifugal pumps, fans, blowers and compressors.

3. According to change in pressure of fluid

(a) ***Decrease in pressure*** e.g. turbines, wind mills

(b) ***Increase in pressure*** e.g. pumps, fans, blowers, compressors.

4. According to direction of flow

(a) ***Radial flow*** e.g. centrifugal pumps and Francis turbine

(b) ***Axial flow*** e.g. axial flow pumps and turbines, wind mills.

(c) ***Mixed flow*** e.g. mixed flow pumps and turbines.

(d) ***Tangential flow*** e.g. impulse hydraulic turbine.

5. According to action of fluid on rotor vanes

(a) ***Impulse machine*** which work on principle of impulse e.g. de-laval steam turbine and Pelton wheel.

(b) ***Reaction machines*** which work on principle of reaction e.g. Francis and Kaplan hydraulic turbines and Parson's steam turbine.

Various types of turbomachines are shown in Fig. 1.3.1 with their principle components.

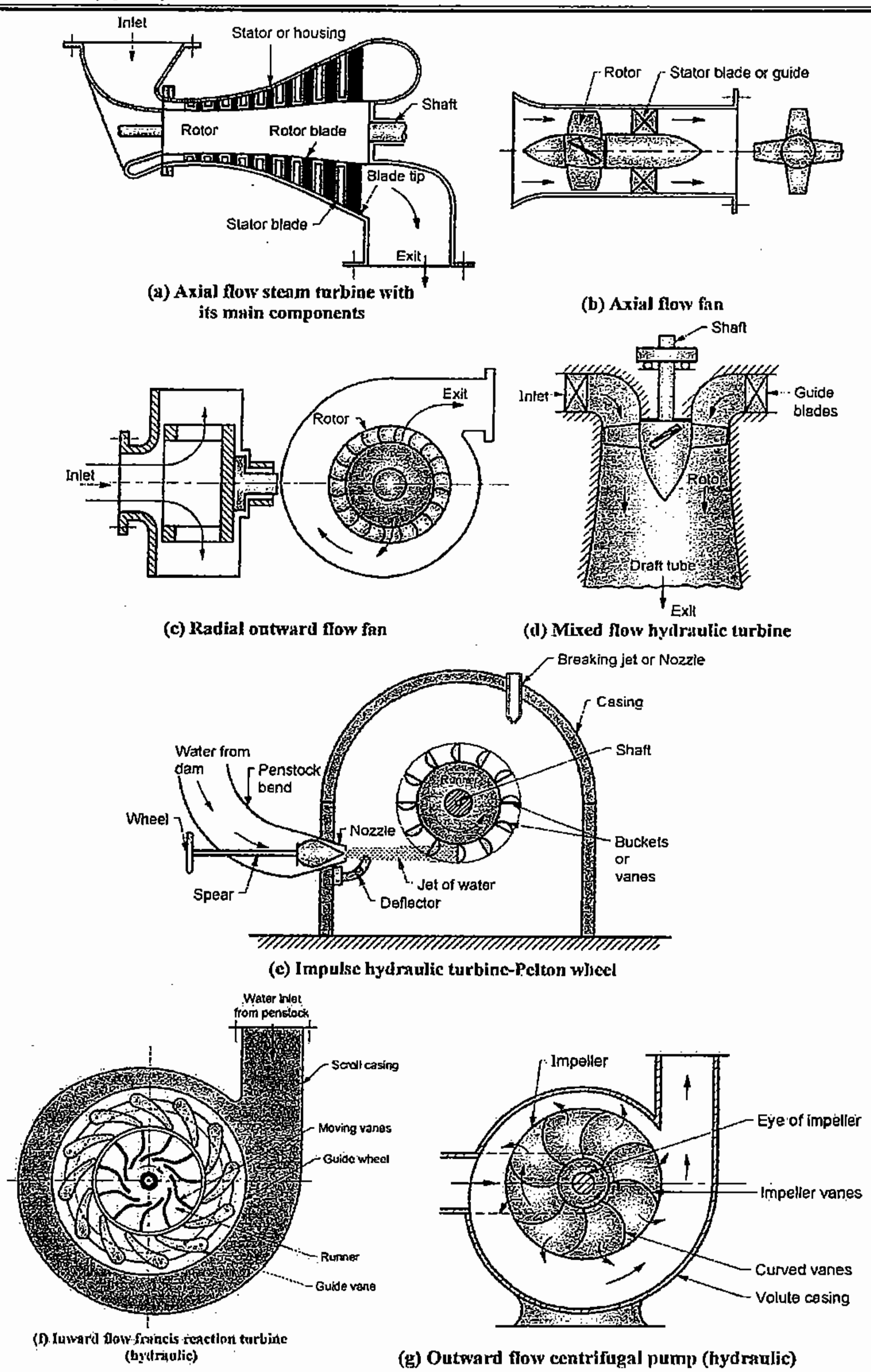


Fig. 1.3.1 : Various turbomachines showing the principal components and types

1.4 Comparison of Positive Displacement Machines and Turbo machines

Sr. No.	Positive displacement machines	Turbo machines
1.	Fluid is subjected to a change in volume resulting into change in pressure.	Fluid is subjected to pressure and momentum changes resulting into energy transfer.
2.	Fluid action is nearly static.	Fluid action is dynamic.
3.	It has pure reciprocating motion of mechanical element.	It has pure rotary motion of the mechanical element.
4.	It has unsteady flow of fluid.	It is steady flow of fluid.
5.	When the machine is stopped, it results into trapping of fluid in machine having the state of entrapped fluid different from surroundings.	When the machine is stopped the fluid state remains the same as that of the surroundings.
6.	Due to reciprocating masses, these machines have : (i) More vibrations, (ii) Needs heavy foundation. (iii) Needs to run at low speed and (iv) Mechanical design is complex.	It has rotating masses which can be balanced completely it results into : (i) Elimination of vibration (ii) Requires light foundation (iii) Can run at very high speeds and (iv) Design is simple.
7.	It has high energy conversion efficiency due to static energy transfer.	It has low efficiency due to dynamic energy transfer.
8.	Volumetric efficiency is low due to frequent opening and closing of valves.	Volumetric efficiency is almost 100% since all valves are open all the time.
9.	The rate of fluid handling capacity is low.	It can handle large volume flow rates of the fluid.
10.	These machines are not subjected to serious problems during their operation.	These machines are subjected to following problems during their operation : (i) Cavitations in hydraulic turbines and pumps. (ii) Erosion of steam turbine blades. (iii) Surging in pumps and compressors results into severe vibrations and it results into damage to machine.

1.5 Basic Definitions and Units - A review

1. *System and Boundary :* **A thermodynamic system** is defined as fixed mass under consideration to analyse a problem.

   It is surrounded by an imaginary boundary which separates the system from its surroundings.

2. **Control volume :** It is an arbitrary volume fixed in space through which the fluid flows. The surface surrounding the control volume is called **control surface** across which the energy transfers take place.

3. **Unit of force, F :** Newton's second law states that force is directly proportional to the rate of change of momentum. Thus, F = m.a

   The unit of force is **Newton (N)**.

   1 N force is defined as the force required to accelerate a mass of 1 kg by $1 m/s^2$. Thus,

$$1N = 1 \text{ kg m/s}^2$$

Weight of body, W is product of mass of body, m and local acceleration of gravity, g. Therefore,

$$W = m.g \quad ; \quad (g \simeq 9.81 \text{ m/s}^2 \text{ at sea level})$$

4. **Energy, E** is defined as the capacity to do work.

$$\text{Total energy, } E = \text{Internal energy, } U + \text{K.E.}\left(\frac{m\,c^2}{2}\right) + \text{P.E. (mgz)}$$

5. **Fluid :** A fluid is a substance which continuously deforms under the application of shear force.

A non-viscous fluid is called an **ideal fluid** and viscous fluids are called **real fluids.**

6. **Pressure, p** is defined as the normal force exerted by a system per unit area i.e. $p = F/A$
   (a) **1 bar** = 10^5 N/m^2 = 10 N/cm^2 = 100 kN/m^2
   (b) **Standard atmospheric pressure, p_o**
   p_o = 760 mm of Hg = 1.013 bar = 10.33 m of water column.

7. **Density, ρ** is defined as the mass per unit volume
   i.e. $\rho = m/V$
   Density of water, ρ_w = 1000 kg/m^3 or 1 kg / litre

8. **Specific gravity,** S of a substance is defined as the ratio of density of a substance to density of liquid.
   i.e. $S = \rho / \rho_w$

9. **Ideal gas :** An ideal gas is one which Obey's the Boyle's law and Charle's law.

   The equation of state is,

$$p.V = m\,R\,T$$

$$p = \rho\,R\,T$$

R is called characteristic gas constant and

$$R_{air} = 287 \text{ Nm/kg K}$$

$$R = \frac{\text{Universal gas constant } R^o = 8314.3 \text{ Nm / kg}_{mole}\text{ K}}{\text{Molecular mass of a gas } M, \text{ kg / kg}_{mole}}$$

10. **Specific heat of a substance, C** is defined as the amount of heat required to raise the temperature of 1 kg substance by 1 degree temperature i.e.

$$Q = m \cdot C \cdot \Delta T$$

11. **Specific heat of gases :** It has two specific heats as follows :

Specific heat at constant volume,

$$C_v = \left(\frac{du}{dT}\right)_V$$

Specific heat at constant pressure,

$$C_p = \left(\frac{dh}{dT}\right)_p$$

Ratio of specific heats,

$$\gamma = \frac{C_p}{C_v}$$

γ = 1.4 for diatomic gases like air, N_2, H_2, O_2 etc.

12. **Enthalpy, H = Internal energy, U + p .V**

Specific enthalpy,

$$h = \frac{H}{m} = C_p \cdot T$$

Specific internal energy,

$$u = \frac{U}{m} = C_v\,T$$

13. **Coefficient of isothermal compressibility, K_T** is defined as the fractional change in volume per unit change in pressure at constant temperature.

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

Coefficient of isentropic compressibility,

$$K_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_s$$

14. **Viscosity, μ**

Viscosity is defined as the property of a fluid which offers resistance to the movement of layers of fluid over another adjacent layer of the same fluid. Consider two layer at 'dy' distance apart having the velocities C and C + dC as shown in Fig. 1.5.1.

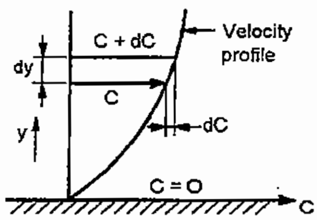


Fig. 1.5.1 : Velocity variation near a solid boundary

The top layer causes a shear stress, τ on adjacent lower layer. This shear stress is proportional to the rate of change of velocity with respect to 'y'. (Such fluids are called **Newtonian fluids**). Mathematically,

$$\tau \propto \frac{dC}{dy}$$

i.e.

$$\tau = \mu \cdot \frac{dC}{dy} \qquad ...(1.5.1)$$

where, 'μ' is the coefficient of proportionality called **dynamic viscosity.**

Units of 'μ' are N sec/m^2 or Ns/m^2

The ratio of dynamic viscosity, μ to density, ρ is called **Kinematic viscosity, ʊ** i.e.

$$\text{Kinematic viscosity } \upsilon = \frac{\text{Dynamic viscosity, } \mu}{\text{Density, } \rho} \text{ (m}^2\text{/s)}$$

1.6 Thermodynamic Laws and Relations

1.6.1 1st Law of Thermodynamics as Applied to Closed Systems

It states that if a system undergoes a cycle, then the algebraic sum of total energy transfer to it as heat and work is zero. Mathematically,

$$\oint (d'Q - d'W) = 0$$

1.6.2 1st Law as Applied to Closed System Processes

When a closed system executes a process, the net heat transfer is equal to the sum of net work transfer and the change in total energy. Mathematically,

$$Q = W_{nf} + \Delta E \quad \text{...(1.6.1)}$$

Suffix 'nf' represent the non-flow process or for closed system where,

Total energy, E = Internal energy, U + K.E. + P. E.

Usually, changes in K.E. and P.E. of a closed system are negligible. The Equation (1.6.1) reduces to,

$$Q = W_{nf} + (U_2 - U_1) \quad \text{...(1.6.2)}$$

Where, $U = m\,C_v \cdot T$ and

$$W_{nf} = \int p \cdot dV$$

1.6.3 First Law as Applied to Steady Flow Open System

Turbomachines are steady flow open systems.

(a) Mass flow balance for steady flow system (Continuity Equation).

A steady flow open system is defined as the system in which the mass flow rate into the system is equal to the mass flow rate out of the system. Accordingly,

Mass flow rate in, $\dot{m}_1$ = Mass flow rate out, $\dot{m}_2$

= A constant, say $\dot{m}$ ...(i)

$$\dot{m} = \rho \cdot A \cdot C \quad \text{...(1.6.3)}$$

$$\therefore \quad \rho_1 A_1 C_1 = \rho_2 \cdot A_2 \cdot C_2 \quad \text{...(1.6.4)}$$

where, ρ = density (kg/m³),

A = Cross sectional area of pipe (m²),

C = Velocity of fluid

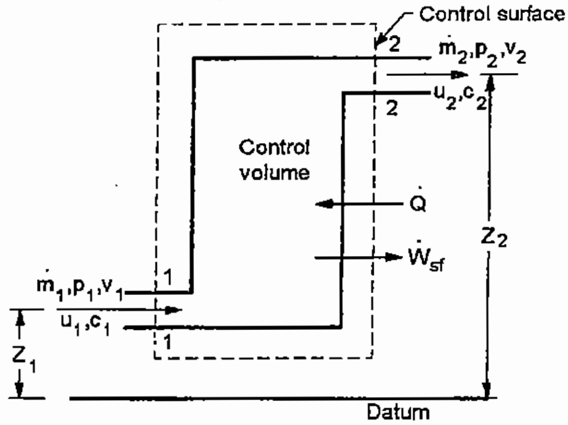


Fig. 1.6.1 : Steady flow open system

In case of liquids

Since density does not change in case of liquids, $\rho_1 = \rho_2$. Therefore, Equation (1.6.4) reduces to,

$$A_1 C_1 = A_2 C_2 \quad \text{...(1.6.5)}$$

where A.C represent the **volume flow rate** of liquid.

(b) Steady flow energy equation (S.F.E.E)

Rate of flow of, energy into system = Rate of flow of energy out of system

$$\dot{Q} + \dot{m}\left[u_1 + p_1 v_1 + \frac{C_1^2}{2} + gZ_1\right] = \dot{W}_{sf} + \dot{m}\left[u_2 + p_2 v_2 + \frac{C_2^2}{2} + gZ_2\right]$$

where, term p.v represents flow work and specific enthalpy,

$$h = u + p.v = C_p \, . \, T \quad \text{(for gases)}$$

$$\therefore \quad \dot{Q} - \dot{W}_{sf} = \dot{m}\left[(u_2 - u_1) + (p_2 v_2 - p_1 v_1) + \frac{C_2^2 - C_1^2}{2} + g(Z_2 - Z_1)\right] \quad \text{...(1.6.6)}$$

$$\dot{Q} - \dot{W}_{sf} = \dot{m}\left[(h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} + g(Z_2 - Z_1)\right] \quad \text{...(1.6.7)}$$

where, $\dot{Q} = \dot{m} \times q$

and $\dot{W}_{sf} = \dot{m} \times w_{sf}$

S.F.E.E. (1.6.7) on unit mass basis reduces to,

$$q - w_{sf} = (h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} + g(Z_2 - Z_1) \quad ...(1.6.8)$$

In case of liquids : Internal energy, u is not affected by pressures and temperatures. Therefore Equation (1.6.6) reduces to :

$$\dot{Q} - \dot{W}_{sf} = \dot{m}\left[(p_2 v_2 - p_1 v_1) + \frac{C_2^2 - C_1^2}{2} + g(Z_2 - Z_1)\right] \quad ...(1.6.9)$$

(c) Mechanical work, W_{sf}

If the changes in K.E. and P.E. are negligible, the mechanical work for steady flow open system is given by the equation,

$$W_{sf} = \int -V \cdot dp \quad ...(1.6.10)$$

1.6.4 Second Law of Thermodynamics

There are two statements of second Law as follows:

(a) Kelvin – Planck statement

It is impossible to construct a devices operating on a cycle whose sole effect is the transfer of heat energy from a single heat reservoir and its conversion into equal amount of work. Thus, Q = W is not possible and a heat engine cannot be 100% efficient.

(b) Clausius statement

It is impossible to construct a device whose sole effect is the transfer of heat from low temperature body to higher temperature body.

1.6.5 Entropy

The concept of entropy is obtained with the help of second of law of thermodynamic. Mathematically, the change in entropy of a system is defined as :

$$dS = \left(\frac{d'Q}{T}\right)_{rev}$$

$$\text{i.e. } \Delta S = \int\left(\frac{d'Q}{T}\right)_{rev} \quad ...(1.6.11)$$

$$\left.\begin{aligned}
\Delta S &= m R \log_e\left(\frac{V_2}{V_1}\right) + m C_V \log_e\left(\frac{T_2}{T_1}\right)\\
\Delta S &= m R \log_e\left(\frac{p_1}{p_2}\right) + m C_p \log_e\left(\frac{T_2}{T_1}\right)\\
\Delta S &= m C_p \log_e\left(\frac{V_2}{V_1}\right) + m C_V \log_e\left(\frac{p_2}{p_1}\right)
\end{aligned}\right\} \quad ...(1.6.12)$$

For a reversible adiabatic process, dS = 0 since d'Q = 0.

Entropy of universe in an irreversible processes increases. According to *principle increase in entropy.*

$$(\Delta S)_{universe} \geq 0 \quad ...(1.6.13)$$

where, $(\Delta S)_{universe} = (\Delta S)_{system} + (\Delta S)_{surr}$

Due to inherent irreversibility's of a system like friction viscosity etc., the entropy of system always increases.

The ideal turbine power is in an ideal reversible adiabatic or isentropic process. However, the actual power developed is less than the ideal power due to increase in entropy.

Similarly, the actual power input in case of pumps and compressors is greater than the ideal or isentropic power input due to irreversibility's involved in an actual processes.

1.7 Stagnation States

In turbomachines, the kinetic energy (K.E) of the fluid cannot be neglected while the changes in P.E. are negligible. It is frequently desirable to combine enthalpy (internal energy + flow work) and K.E. into a single term.

Consider a steady flow open system comprising of perfectly insulated horizontal duct in which the gas flows with no shaft work. Applying S.F.E.E.,

$$\dot{Q} - \dot{W}_{sf} = \dot{m}\left[(h_2 - h_1) + \frac{C_2^2 - C_1^2}{2g} + g(Z_2 - Z_1)\right] \quad ...(i)$$

But, $\dot{Q} = 0$ (System is insulated)

$\dot{W}_{sf} = 0$ (no shaft work)

and $Z_2 \simeq Z_1$.

Above Equation (i) reduces to,

$$h_1 + \frac{C_1^2}{2g} = h_2 + \frac{C_2^2}{2} \quad .... \text{A constant, } h_0$$

Thus the *summation of static enthalpy, h and K.E.* $\left(\frac{C^2}{2}\right)$ *remains constant under adiabatic conditions in the absence of shaft work, heat transfer and change in P.E.* This is defined as ***stagnation or total head enthalpy represented by h_o.***

Accordingly,

Stagnation enthalpy,

$$h_o = \text{Static enthalpy, } h + \text{K.E.}, \frac{C^2}{2}$$

$$\therefore \quad h = C_p \cdot T \quad \text{...(1.7.1)}$$

$$\therefore \quad T_o = T + \frac{C^2}{2\,C_p} \quad \text{...(1.7.2)}$$

where, $(C^2 / 2C_p)$ is called **dynamic temperature, T_o stagnation temperature and T static temperature.**

$$\textbf{Mach number, M} = \frac{\text{Actual velocity, C}}{\text{Sonic velocity, a}} \quad \text{...(1.7.3)}$$

Sonic velocity, a is defined as the propagation of waves of very small pressure disturbance through a compressible fluid. It is also called as **sound velocity** or **acoustic velocity.**

$$\text{Sonic velocity,} \quad a = \sqrt{\frac{dp}{d\rho}} = \sqrt{\gamma RT} \quad \text{...(1.7.4)}$$

Using $\quad C_p - C_v = R$

$$\text{i.e. } C_p = \left(\frac{\gamma}{\gamma - 1}\right) R \quad \text{...(i)}$$

$$C = a\,M = M \times \sqrt{\gamma RT} \quad \text{...(ii)}$$

Using above equations, the Equation (1.7.2) can be rewritten as,

$$T_0 = T + \frac{C^2}{2\,C_p} = T + \frac{M^2 \gamma RT}{2\left(\frac{\gamma}{\gamma - 1}\right) R}$$

$$T_0 = T\left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right] \quad \text{...(1.7.5)}$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma - 1)} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\gamma/(\gamma - 1)} \quad \text{...(1.7.6)}$$

1.7.1 Application of Steady Flow Energy Equation as Applied to Turbomachines

S. F. E. E. can be written as,

$$\dot{Q} - \dot{W}_{sf} = \dot{m}\left[(h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} + g(Z_2 - Z_1)\right]$$

Since *all turbomachines are high speed machines, the rate of heat transfer, $\dot{Q}$ are negligible.* Thus all turbomachines can be treated like insulated devices.

The changes in P.E. are also negligible

i.e. $g(Z_2 - Z_1) = 0$, therefore above equation reduces to ($\dot{W}_{sf} = \dot{W}$ = power),

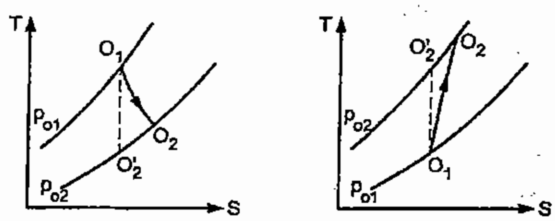
$$0 - \dot{W} = \dot{m}\left[(h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} + 0\right]$$

$$\dot{W} = \dot{m}\left[\left(h_1 + \frac{C_1^2}{2}\right) - \left(h_2 + \frac{C_2^2}{2}\right)\right]$$

$$= \dot{m}\left[h_{o1} - h_{o2}\right] \quad \text{...(1.7.7)}$$

1.7.2 Efficiency of Flow Machine

In case of actual steady flow turbomachines with gas as working substance, the work developed is less than the ideal or isentropic work in case of turbines due to various losses while, the actual work input is more than the isentropic work in case of compressor. The processes are shown in Fig. 1.7.1. The ratio of these work is defined as **isentropic efficiency.**



(a) Turbine **(b) Compressor**

Fig. 1.7.1 : Actual and ideal processes in steady flow turbo machines

For an expansion machine like turbine,

Isentropic efficiency,

$$\eta_t = \frac{\text{Actual work output, } (h_{o1} - h_{o2})}{\text{Isentropic work, } \left(h_{o1} - h_{o2}'\right)}$$

For a compression machine like compressor,

Isentropic efficiency,

$$\eta_i = \frac{\text{Isentropic work, } \left(h_{o2}' - h_{o1}\right)}{\text{Actual work input, } (h_{o2} - h_{o1})}$$

1.8 Impulse Momentum Equation

The **momentum of a fluid** is defined as the product of mass, m and its velocity, C. Therefore,

$$\text{Momentum,} \quad M = \text{Mass, m} \times \text{Velocity, C} \quad \text{...(i)}$$

According to **Newton's second law of motion**, the rate of change of momentum is proportional to the external force, F acting on it. According,

$$F \propto \frac{dM}{dt}$$

i.e. $$F = g_c \cdot \frac{dM}{dt} \quad \text{...(ii)}$$

Where, g_c is the constant of proportionality.

$$g_c = 1 \frac{N\,s^2}{kg\,m^2}$$

$$\therefore \quad F = \frac{d\,(m.C)}{dt} = \left[m \cdot \frac{dC}{dt} + c \cdot \frac{dm}{dt}\right]$$

In hydrodynamic machines, $\frac{dm}{dt} = 0$

(under steady state flow)

$$\therefore \quad F = m \cdot \frac{dC}{dt} = m \cdot a \quad \text{...(iii)}$$

where, acceleration, $a = \frac{dC}{dt}$

Above Equation (i) can be rewritten as :

$$F{\cdot}dt = g_c \cdot dM \quad \text{...(1.8.1)}$$

In Equation (1.8.1), $F{\cdot}dt$ represents the **impulse** while $g_c{\cdot}dM$ represents the **change in momentum**. Therefore, *we conclude that the impulse acting on a body is proportional to change in momentum*. This Equation (1.8.1) is known as **impulse momentum equation**.

1.9 General Theory of Rotodynamic Machines

The energy transfer between the fluid and rotodynamic machine takes place according to the impulse momentum equation discussed in section 1.8 above. This equation is based on Newton's second law of motion.

1.9.1 Components of Velocity in Rotodynamic Machines

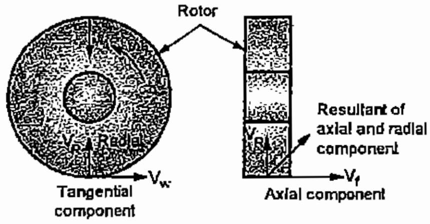
When the fluid flows in a rotodynamic machine, at an absolute velocity V, it will have three components in three mutually perpendicular directions as shown in Fig. 1.9.1.

These are :

(a) **Axial component**, V_f or C_f or V_a. It acts along the axis of rotor. The rate of change of momentum due to axial component is responsible to develop **axial force** which is taken care of by the radial thrust bearings and it is finally transferred to casing.

(b) **Radial component**, V_R or C_R. It acts along direction of rotor. The rate of change of momentum in radial velocity causes **radial force** which is taken care of by journal bearings.

(c) **Tangential or whirl component**, V_W or C_W. It acts along the tangential direction of rotor.



(a) Front view of rotor **(b) Side view of rotor**

Fig. 1.9.1 : Components of velocity of fluid in three mutually perpendicular direction

1. **Transportation of fluid**

The resultant of radial and axial components of the fluid velocity is called **meridional velocity** or **velocity of flow**. This velocity is responsible for flow of fluid through the machine. It plays no role in transfer of energy between the fluid and machine.

Thus, the transportation of fluid through the machine is the responsibility of radial and axial components of velocity.

2. **Transmission of force and moment of momentum (Angular momentum)**

Tangential component of velocity, V_w is responsible for transmission of power.

Product of mass flow rate, $\dot{m}$ and tangential component of velocity of fluid produces a **momentum in tangential direction to the rotor.**

The **moment of momentum** about the axis of rotor gives rise to angular moment producing **torque**. This torque causes the rotor to rotate, thus produces power.

1.10 Fundamental Equation for Energy Transfer Between A Fluid and A Rotor

University Questions

Q. Derive the fundamental equation of fluid machines and how it is applied to turbine and pumps. **SPPU : May 12**

Q. Define angular momentum and explain how it is used to determine the torque and work done in case of radial flow turbine runner. **SPPU : May 16, Dec. 19**

Consider a rotor rotating at an angular speed of ω rad/s as shown in Fig. 1.10.1.

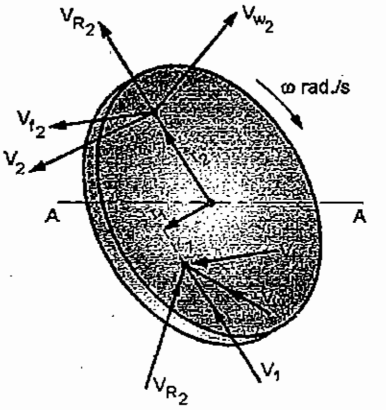


Fig. 1.10.1 : Transfer of energy between fluid and rotor

Consider two points 1 and 2 on the rotor at radii r_1 and r_2. Assume that the fluid enters at point 1 and leaves at point 2.

Assumptions

1. Flow is under steady state i.e. mass flow rate of fluid at any point on the rotor is constant.
2. The state of the fluid at any given point is constant i.e. the rate of heat and work transfer by the fluid passing on the rotor is constant.
3. There is no leakage of fluid of any point and the entire fluid is undergoing the same process.
4. There are no frictional losses as the fluid flows over the owns through the rotor.

Note that the transfer of energy between fluid and machine only depends upon the dynamic condition of fluid at points 1 and 2 and it is independent of the actual path followed by the fluid.

Let, $\dot{m}$ = mass flow rate of fluid (kg/s)

V_1 = Absolute velocity of fluid at point 1 (m/s)

V_2 = Absolute velocity of fluid at point 2 (m/s)

ω = Angular velocity of rotor (rad/s)

Resolving the absolute velocity in axial, radial and tangential directions, we can write :

$$\vec{V_1} = \overrightarrow{V_{f_1}} + \overrightarrow{V_{R_1}} + \overrightarrow{V_{w_1}} \quad ...(i)$$

$$\vec{V_2} = \overrightarrow{V_{f_2}} + \overrightarrow{V_{R_2}} + \overrightarrow{V_{w_2}} \quad ...(ii)$$

As discussed earlier, the energy transfer between the fluid and machine is only due to change in momentum caused by the tangential component of velocity. Therefore,

The rate of change of momentum between points 1 and 2,

$$= \dot{m}\ \overrightarrow{V_{w_1}} - \dot{m}\ \overrightarrow{V_{w_2}}$$

The moment of momentum at point 1 = $\dot{m}\ \overrightarrow{V_{w_1}} \cdot r_1$

The moment of momentum at point 2 = $\dot{m}\ \overrightarrow{V_{w_2}} \cdot r_2$

Since *the rate of change of moment of momentum represents the rate of change of angular momentum which is equal to torque produced on the rotor*. Hence,

Torque, T = Rate of change of angular momentum

$$T = \dot{m}\ \overrightarrow{V_{w_1}} \cdot r_1 - \dot{m}\ \overrightarrow{V_{w_2}} \cdot r_2$$

$$T = \dot{m}\,(\overrightarrow{V_{w_1}} \cdot r_1 - \overrightarrow{V_{w_2}} \cdot r_2) \quad ...(1.10.1)$$

In case of power producing machines like turbine, the torque is produced on the rotor due to change of angular momentum. While, in case of power absorbing devices like pumps and compressors, the torque given to the rotor is responsible for causing the change in tangential velocity of fluid.

Consider the machine as turbine. The torque produced by the fluid on rotor is used in producing useful work or developing power. Thus,

Rate of energy transfer, (or Power P),

$$E = \text{Torque, } T \times \text{Angular velocity of rotor, } \omega$$

i.e. $$E = T.\omega = \dot{m}\,(\overrightarrow{V_{w_1}} \cdot r_1 - \overrightarrow{V_{w_2}} \cdot r_2) \cdot \omega \quad ...(1.10.2)$$

But, peripheral velocity of fluid, ($u = \omega r$) at points 1 and 2 can be written as,

$$\vec{u_1} = \omega \cdot r_1$$

and $$\vec{u_2} = \omega \cdot r_2$$

$$\therefore\ E = \dot{m}\,(\overrightarrow{V_{w_1}} \cdot \vec{u_1} - \overrightarrow{V_{w_2}} \cdot \vec{u_2}) \quad ...(1.10.3)$$

Equation (1.10.3) represents the **general energy equation** *for transfer of energy between the fluid and machine*. Energy transfer per unit mass i.e. Workdone (W.D.) or (W).

$$E \text{ or } W = (\overrightarrow{V_{w_1}} \cdot \vec{u_1} - \overrightarrow{V_{w_2}} \cdot \vec{u_2}) \quad ...(1.10.4)$$

If H is the head on the machine, then energy transfer can be written as,

$$E = \dot{m} \cdot g \cdot H \qquad ...(1.10.5)$$

Equating Equations (1.10.3) and (1.10.5) we get,

$$H = \frac{(\overrightarrow{V_{w_1}} \cdot \overrightarrow{u_1} - \overrightarrow{V_{w_2}} \cdot \overrightarrow{u_2})}{g} \qquad ...(1.10.6)$$

Equation (1.10.2) to (1.10.6) are forms of **Euler's Equation** which are applicable to all turbomachines.

Note :

1. If $V_{w_1} \cdot u_1 > V_{w_2} \cdot u_2$, then machine is called *turbine*.
2. If $V_{w_2} \cdot u_2 > V_{w_1} \cdot u_1$, then machine is called a *pump or compressor*.
3. If $\overrightarrow{V_{w_2}}$ is in opposite direction of $\overrightarrow{V_{w_1}}$, the Equation (1.10.2) can be modified as :

$$E = \dot{m}\,(V_{w_1} u_1 + V_{w_2} u_2)$$

1.10.1 Centrifugal Head, h_c

When fluid particles rotate at high speeds, they are subjected to centrifugal forces which give rise to pressure forces. Consider an annular ring of fluid revolving at an angular velocity ω having inner and outer radius as r_1 and r_2 respectively as shown in Fig. 1.10.2.

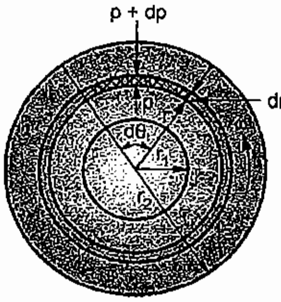


Fig. 1.10.2 : Centrifugal head

Consider a small segment of thin ring at radius r, of thickness dr which subtends an angle $d\theta$ at the centre. Let 'p' and (p + dp) be the pressures at inner and outer surface of ring due to centrifugal force of fluid flowing across the ring. Let dA the area of annular ring. Then,

Net force due to pressure difference = Change in centrifugal force

$$dp \cdot dA = dm \cdot \omega^2 \cdot r$$

$$= (\rho \cdot dA \cdot dr)\,\omega^2 \cdot r$$

$$\therefore \frac{1}{\rho} \cdot dp = \omega^2 \cdot r \cdot dr$$

In case of liquids , ρ is constant. On integrating the above equation,

$$\frac{1}{\rho}\int_1^2 dp = \omega^2 \cdot \int_1^2 r \cdot dr = \omega^2 \left[\frac{r^2}{2}\right]_1^2$$

$$\frac{p_2 - p_1}{\rho} = \omega^2 \left(\frac{r_2^2}{2} - \frac{r_1^2}{2}\right)$$

$$\text{Centrifugal head, } h_c = \frac{p}{\rho \cdot g} \quad \text{and} \quad u = \omega \cdot r$$

$$\therefore \text{Centrifugal head, } h_c = \frac{u_2^2 - u_1^2}{2g} \qquad ...(1.10.7)$$

1.11 Components of Energy Transfer

Consider the inlet and outlet velocity diagram of a generalized rotor having vanes mounted on rotor as shown in Fig. 1.11.1. The rotor rotates at N rpm.

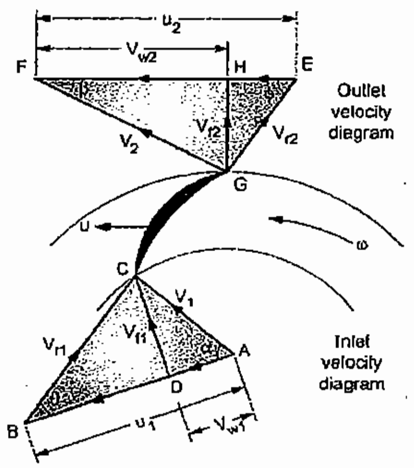


Fig. 1.11.1 : Velocity diagram of a generalized rotor

$$\omega = \frac{2\pi N}{60}$$

Vane velocity, $u = \omega \cdot r$

$\therefore$ $u_1 = \omega \cdot r_1$

and $u_2 = \omega \cdot r$

Let, V = absolute velocity of fluid

V_r = relative velocity

V_f = flow velocity

and V_w = Tangential or whirl velocity

V_f and V_w are the components of absolute velocity, V.

Note that V_f represents the radial velocity in case of radial machines and axial velocity in case of axial flow machines.

α = Exit angle of fixed vane at inlet to rotor

β = Inlet angle of fixed vane at outlet to rotor

θ = Inlet angle of moving vane

ϕ = Exit angle of moving vane

From inlet velocity triangle ABC, by cosine rule we have,

$$V_{r_1}^2 = V_1^2 + u_1^2 - 2 V_1 \cdot u_1 \cdot \cos\alpha$$

and $\quad V_1 \cos\alpha = V_{w1}$

$$\therefore V_{r1}^2 = V_1^2 + u_1^2 - 2 V_{w1} \cdot u_1$$

$$V_{w1} \cdot u_1 = \frac{V_1^2 + u_1^2 - V_{r_1}^2}{2} \quad \text{...(i)}$$

From outlet velocity triangle EFG,

$$V_{r_2}^2 = V_2^2 + u_2^2 - 2 V_2 \cdot u_2 \cdot \cos\beta \qquad (\text{But, } v_2 \cos\beta = V_{w_2})$$

$$V_{r_2}^2 = V_2^2 + u_2^2 - 2 V_{w_2} \cdot u_2$$

$$V_{w_2} \cdot u_2 = \frac{V_2^2 + u_2^2 - V_{r_2}}{2} \quad \text{...(ii)}$$

on subtracting Equation (ii) from Equation (i)

$$(V_{w_1} \cdot u_1 - V_{w_2} \cdot u_2) = \frac{V_1^2 - V_2^2}{2} + \frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}$$

The energy transfer per unit mass E from Equation (1.10.3) or (1.10.4) is given as,

$$E = V_{w_1} \cdot u_1 - V_{w_2} \cdot u_2 \quad \text{...(iii)}$$

$$\therefore \quad E = \frac{1}{2}(V_1^2 - V_2^2) + \frac{1}{2}(u_1^2 - u_2^2) + \frac{1}{2}(V_{r_2}^2 - V_{r_1}^2) \quad \text{...(1.11.1)}$$

Using Equation (1.10.6), above equation in terms of head H can be written as :

$$H = \frac{1}{2g}(V_1^2 - V_2^2) + \frac{1}{2g}(u_1^2 - u_2^2) + \frac{1}{2g}(V_{r_2}^2 - V_{r_1}^2) \quad \text{...(1.11.2)}$$

Equations (1.11.1) and (1.11.2) are the alternative forms of **Euler's equation.**

Components of energy transfer in Equation (1.11.1) are :

1. **First component** $\left(\frac{V_1^2 - V_2^2}{2}\right)$ represents the change in absolute kinetic energy or **dynamic head imported by the fluid** while flowing through the rotor. This effect is also known as **impulse effect.** The exit kinetic energy $\frac{V_2^2}{2}$ is negligible in some turbomachines, particularly in case of power producing machines.

   While, exit velocity V_2 is usually large in power absorbing machines like pump and compressors. Since, in these machines the pressure head or static head is required as useful energy, the kinetic energy at exit is converted into pressure energy by incorporating an additional device called a **scroll or diffuser.**

2. **Second component** $\left(\frac{u_1^2 - u_2^2}{2}\right)$ represents the change in fluid energy due to centrifugal force called as **centrifugal head.** This is due to change in the radius of rotation of the fluid. Due to this head, the static pressure of the fluid changes. Therefore, this component is also called as **static component.**

3. **Third component** $\left(\frac{V_{r_2}^2 - V_{r_1}^2}{2}\right)$ represents the change in K.E. due to change in relative velocity. This causes the change in static head of the fluid across the rotor.

The change in relative velocity may be caused due to following reasons :

a) Due to fluid friction

b) Due to change in area of flow passage

Fluid friction reduces the relative velocity in the direction of flow. Whereas, the relative velocity may increase or decrease depending upon the change in area in direction of fluid flow. $V_{r_2} > V_{r_1}$ in case of turbines and $V_{r_2} < V_{r_1}$ in case of pumps and compressors.

1.12 Classification of Machines

Turbomachines are classified as follows :

1. **Depending upon the fluid flow direction :**

   These are classified as :

   (a) *Axial flow machines* which have no significant change of radius during fluid flow at entry and exit i.e. $u_1 = u_2$

   (b) *Radial flow machines* in which there is substantial change in radius during fluid flow. Thus, there is change in energy due to change in radius. *These may be inward or outward flow machines.*

   (c) *Mixed flow machines* in which the fluid flow is mixed i.e. it has both axial and radial flow of fluid..

2. **Classification depending upon the degree of reaction, R (Impulse and Reaction machines)**

(i) Definition of degree of reaction, R

University Question

Q. Define the term Degree of reaction. **SPPU : Dec. 16**

Degree of reaction is defined as the ratio of static component of energy transfer to the total energy transfer in the machine i.e.

Degree of reaction,

$$R = \frac{\text{Static energy transfer}}{\text{Total energy transfer, E}} \quad \text{...(1.12.1)}$$

$$= \frac{\left(\frac{u_1^2 - u_2^2}{2}\right) + \left(\frac{V_{r_2}^2 - V_{r_1}^2}{2}\right)}{\left(\frac{V_1^2 - V_2^2}{2}\right) + \left(\frac{u_1^2 - u_2^2}{2}\right) + \left(\frac{V_{r_2}^2 - V_{r_1}^2}{2}\right)} \quad \text{...(1.12.2)}$$

(ii) Significance of degree of reaction

Method of energy transfer between fluid and machine affects the design and various features of a machine. *Based on degree of reaction, the machines can be classified as :*

(a) Impulse machines
(b) Reaction machines

(a) Impulse machines : A machine in which there is no change in static head or static pressure in the rotor is called a **impulse machine.**

Therefore, the degree of reaction of impulse machines is zero i.e. R = 0. Thus in these type of machines, $u_2 = u_1$ and $V_{r_2} = V_{r_1}$ and the energy transfer is purely due to change in absolute kinetic energy of fluid passing over the rotor. In other words, *the energy transfer in impulse machines is purely due to dynamic action of fluid.*

In impulse machines, the rotor can be of open type since there is no rise in static pressure e.g. in case of pelton turbine as shown in Fig. 1.12.1(a). In this case nozzle is fixed, thus the K. E. of the fluid is transferred to rotor purely by impulse action.

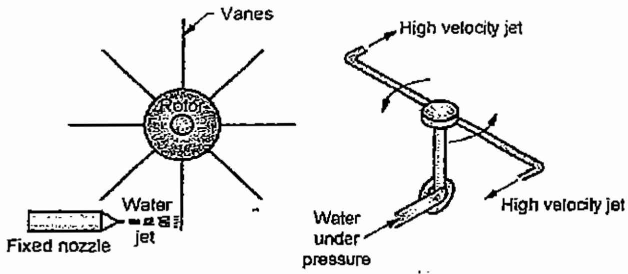
(b) Reaction machines

University Questions

Q. Explain the meaning of pure reaction and 50% reaction turbines. **SPPU : Dec. 16**

In a pure reaction machine, the degree of reaction, R = 1 unity. In such a machine the energy transfer is by virtue of change of static pressure in the rotor and essentially $V_1 = V_2$.

However, the absolute velocity at inlet and outlet may not be equal i.e. $V_1 \neq V_2$ due to which the degree of reaction turbine is usually less than 1. A reaction machine with any degree of reaction must have an *enclosed rotor* because the fluid cannot expand freely in all direction. A simple example of a reaction machine is a lawn sprinkler shown in Fig. 1.12.1(b).



(a) Impulse turbine (b) Lawn sprinkler as reaction machines

Fig. 1.12.1 : Examples of impulse and reaction machines

In this the water enters the nozzles under high pressure or high head which is converted into velocity head in the nozzles which is the part of rotor. The high velocity jet leaves the rotor in tangential direction. The change in momentum of fluid in the nozzle which is free to move, gives rise to reactive force, hence the word reaction machine follows.

1.13 Efficiencies of Turbomachines

University Question

Q. Write a note on Types of losses in Turbo machines. **SPPU : Aug.18(In Sem)**

An **ideal machine** is defined which is subjected to ideal or reversible process i.e. the processes are without friction and having no energy losses.

However, the mechanical energy transfer in real machines takes place with friction i.e. with energy loss to overcome the friction during its passages during the energy transfer.

Generally speaking, the energy losses in turbomachines are of two types :

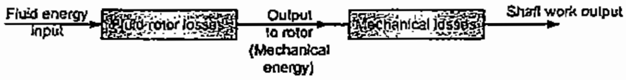
1. Energy absorbed by bearing friction, glands, couplings, windage etc. These are called ***mechanical losses.***
2. The friction between the rotor blade and the working fluid and rotor. These are called as ***fluid-rotor losses.***

Therefore, we consider three types efficiencies at present as follows :

1. **Hydraulic efficiency,** η_h between the fluid and rotor.
2. **Mechanical efficiency,** η_m between the rotor and shaft
3. **Overall efficiency,** η_o between the fluid energy and shaft work.

For power producing devices like turbines the energy transfers from fluid energy to shaft work and for power absorbing machines the energy transfer from shaft work to fluid energy can be represented as follows.

(a) Power producing devices (e.g. Turbines)



Hydraulic efficiency,

$$\eta_h = \frac{\text{Mechanical energy available at rotor}}{\text{Hydrodynamic or fluid energy input}} \quad ...(1.13.1)$$

Mechanical energy,

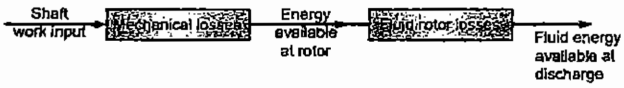
$$\eta_m = \frac{\text{Shaft work output}}{\text{Mechanical energy input to rotor}} \quad ...(1.13.2)$$

Overall efficiency,

$$\eta_o = \frac{\text{Shaft work output}}{\text{Fluid energy input}} \quad ...(1.13.3)$$

$$\therefore \ \eta_o = \eta_h \cdot \eta_m \quad ...(1.13.4)$$

(b) Power absorbing devices (e.g. Compressors and Pumps)



Hydraulic efficiency,

$$\eta_h = \frac{\text{Fluid energy available at discharge}}{\text{Mechanical energy available at rotor}} \quad ...(1.13.5)$$

Mechanical efficiency,

$$\eta_m = \frac{\text{Mechanical energy available at rotor as output}}{\text{Shaft work input}} \quad ...(1.13.6)$$

Overall efficiency,

$$\eta_o = \frac{\text{Fluid energy available at dischage}}{\text{Shaft work input}} \quad ...(1.13.7)$$

Again, $\eta_o = \eta_h \times \eta_m$ ...(1.13.8)

Apart from the efficiencies discussed above, there are many types of efficiencies which shall be discussed with reference to particular type of applications later with further development of concepts of thermodynamics and fluid flow.

1.14 Applications of Turbo Machines

Applications of turbo machines are many. These machines are widely used in generation of power in industries and fluid handling system. These are also used in other energy producing systems such as hydropower generation, wind power generation, geothermal power plants, tidal power plants etc.

Some of the major applications of turbo machines are given below :

(i) **In Steam power plants** which uses turbine to convert the thermal energy of steam generated in boiler into mechanical power to drive an electrical generator. Many pumps are used to handle water as boiler feed pump, condensate pump and cooling water circulating pumps. Blowers are used to supply air on the grate of the boiler.

(ii) **Gas turbine power plants** are used for power generation and in aircraft applications both for power generation and air conditioning of aircrafts. It uses both gas turbine for conversion of thermal energy of gas into mechanical power and compressors to raise the pressure of surrounding power and compressors to raise the pressure of surrounding air to be supplied to combustion chamber. **Turbochargers** are used for supercharging of I.C. engines.

Gas turbine power plants are also used for marine applications and military tanks.

(iii) *Various industries use pumps, blowers and compressors for fluid handling systems.* These turbo machines are used to increase the pressure of liquid in pumps and pressure of air / gas in blowers and compressors. This pressure energy is used to transport the fluids.

These are also used in petroleum industry :

(iv) *Compressors* are used to operate pneumatic tools.

(v) *Centrifugal vapour compressors* are used in large air conditioning systems.

(vi) *Compressed air* is also used to convey materials like sand and concrete in a pipe line.

(vii) *Air motors* are used to drive the mining machinery.

(viii) *Compressors / blowers* are used in blast furnaces and cleaning the surfaces by air blast.

Summary

- **Turbomachines** are rotary type of machines in which the energy transfer is brought about by the dynamic action of rotating element. In these machines the fluid flows steadily.
- **Turbomachines are two types :** 1. Power producing turbomachines e.g. hydraulic, steam and gas turbines. 2. Power absorbing turbomachines e.g. centrifugal pumps, fans, blowers, compressors.
- **Principle components of turbomachines are :**
  1. Stationary element or guide vanes/blades or nozzles
  2. A shaft either input or output shaft
  3. A housing or casing
  4. Draft tube

- **Turbomachines are classified according to :**
  1. Working medium
  2. Energy conversion
  3. Change in pressure of fluid
  4. Direction of flow
  5. Action of fluid on rotor vanes
- **Turbomachines differ from positive displacement** since **in positive displacement machines** the pressure rise is due to change in volume, fluid action is due to change in volume, fluid action is static and unsteady flow, have more vibrations, needs heavy foundation runs at low speed, has high conversion efficiency due to static energy transfer with low volumetric efficiency and low handling capacity.
- Whereas the pressure rise in **turbo machines** is due to dynamic action of fluid under steady flow, it is vibration free, needs light foundation, can run at very high speeds having low energy transfer efficiency and almost 100% volumetric efficiency with simple design and high volume flow rates.
- Turbomachines are subjected to serious problems like cavitation, erosion of blades in turbines and surging in pumps and compressors whereas positive displacement machines do not have such problems.
- **Ist law of thermodynamics** states that if a system undergoes to a cycle, then the algebric sum of total energy transfer to it as heat and work is zero. i.e. $\oint (d'Q - d'W) = 0$
- **Ist law as applied to closed system process**

$$Q - W_{nf} = E_2 - E_1$$

where, Total energy,

$$E = \text{Internal energy, } U + \text{K.E.} \left(\frac{mC^2}{2}\right) + \text{P.E. } (mgZ)$$

Ist law as applied to steady flow processes in open system

$$\dot{Q} - \dot{W}_{sf} = \dot{m} \left[(h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} + g(Z_2 - Z_1) \right]$$

- **Continuity equation** $\dot{m} = \rho AC$

Under steady state : $\dot{m}_{in} = \dot{m}_{out}$

i.e. $\rho_1 A_1 C_1 = \rho_2 \cdot A_2 C_2$

- **Second law of thermodynamics has two statements**
  1. Kelvin – Planck statement
  2. Clausius statement
- **Concept of entropy** is based on the second law of thermodynamics which states that no process is possible whose sole effect is decrease in entropy of the universe.

$\therefore \quad (\Delta S)_{universe} \geq 0$

- **Entropy is defined** as ; $ds = \int \left(\frac{d'Q}{T}\right)_{rev}$
- **Stagnation states** are defined as the property of fluid obtained when the fluid is brought to rest under reversible adiabatic conditions in a duct.

Stagnation enthalpy,

$$h_0 = \text{Static enthalpy, } h + \text{K.E.} \left(\frac{C^2}{2}\right)$$

$$T_0 = T + \frac{C^2}{2\,C_p} = T \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]$$

- **Sonic velocity, a** is defined as the propagation of waves of very small pressure disturbance through a compressible fluid. $a = \sqrt{\gamma \cdot R \cdot T}$
- **Mach number, M** $= \dfrac{\text{Actual velocity, C}}{\text{Sonic velocity, a}}$
- **Steady flow energy equation as applied to turbomachines :**

$$\dot{W} = \dot{m}\ (h_{01} - h_{02})$$

- Isentropic efficiency, η_i
  1. **Turbines :** $\eta_i = \dfrac{\text{actual work output}}{\text{isentropic work}}$
  2. **Compressors :** $\eta_i = \dfrac{\text{Isentropic work}}{\text{Actual work input}}$
- **Momentum of a fluid** is defined as the product of mass and its velocity.
- **Newtons second law of motion state** that the applied force is proportional to the rate of change of momentum i.e. $F \propto \dfrac{dM}{dt}$
- Impulse $= F \cdot dt$
- **Impulse momentum equation** states that the impulse is proportional to the change in momentum.
- **Theory of rotodynamic machines** is based on the impulse momentum equation :

$$F = C \cdot \frac{dm}{dt} = \dot{m}\, C = \rho \cdot Q\, C \text{ or } \rho\, Q\, V$$

- **Components of velocity** in rotodynamic machines are:

(i) **Axial component, V_f or V_a or C_f** which acts along the axis of rotor.

(ii) **Radial component, V_R or C_R** which acts in radial direction of rotor.

(iii) **Tangential or whirl component V_w or C_w** which acts in tangential direction of rotor.

– **Torque,** T represents the moment of momentum about the axis of rotor.

$$\text{Torque, } T = \dot{m}\,[V_{w1} \cdot R_1 - V_{w2} \cdot R_2]$$

Rate of energy transfer,

$$E = T \cdot \omega = \dot{m}\,[V_{w1} \cdot R_1 - V_{w2} \cdot R_2]\,\omega$$
$$= \dot{m}\,[V_{w1} \cdot u_1 - V_{w2} \cdot u_2]$$

– **Centrifugal head,** $h_c = \frac{u_2^2 - u_1^2}{2g}$. It is the energy per unit weight of fluid which is lost or gained due to displacement form R_1 to R_2.

– **Euler's equation is given as :**

$$H = \frac{V_{w1} \cdot u_1 - V_{w2} \cdot u_2}{g}$$

$$= \underbrace{\underbrace{\frac{V_1^2 - V_2^2}{2g}}_{\text{Dynamic head}} + \underbrace{\underbrace{\frac{u_1^2 - u_2^2}{2g}}_{\text{Centrifugal head}} + \underbrace{\frac{V_{r2}^2 - V_{r1}^2}{2g}}_{\text{Head due to change in K.E. due to relative velocity}}}_{\text{Static head}}}$$

Degree of reaction, $R = \frac{\text{Static head}}{(\text{Dynamic + Static head})}$

– **R = 0 for impulse turbines and R = 1 for reaction turbines**

– **Efficiencies of turbomachines (Power producing devices)**

(a) **Hydraulic efficiency, η_h** between fluid and rotor

$$\eta_h = \frac{\text{Mechanical energy available at rotor}}{\text{Fluid energy input}}$$

(b) **Mechanical efficiency,**

$$\eta_m = \frac{\text{Shaft work output}}{\text{Mechanical energy input ot rotor}}$$

(c) **Overall efficiency,** $\eta_o = \frac{\text{Shaft work output}}{\text{Fluid energy input}}$

(d) $\eta_o = \eta_h \times \eta_m$

– **Various applications of turbomachines are :**

1. Power generation in steam and gas turbines on large scale and for applications like marine and military tanks.
2. Use of pumps, blower and compressors for fuel handling systems and fluid handling systems.
3. Transportation of fluids.
4. Operation of pneumatic tools.
5. Rotary vapour compressors for large air conditioning systems.
6. Transportation of concrete and sand in pipes.
7. To drive mining machinery.
8. In blast furnaces and clearing of surfaces by air blast.

Exercise

Note : For answers please refer the section numbers indicated in bracket.

Q. 1 State the basic classification of fluid machines. Define turbomachines with examples. **[Section 1.1]**

Q. 2 State the major components of a turbomachine with their functions. **[Section 1.2]**

Q. 3 Discuss the detailed classification of turbomachines with examples. **[Section 1.3]**

Q. 4 Compare turbomachines with positive displacement machines. **[Section 1.4]**

Q. 5 Define stagnation enthalpy, sonic velocity, stagnation temperature and Mach number. Prove that :

$T_0 = T\,[\,1 + \left(\frac{\gamma - 1}{2}\right) M^2]$ **[Section 1.7]**

Q. 6 Define isentropic efficiency as applied to turbines and compressors. **[Section 1.7.2]**

Q. 7 Define momentum of a fluid and Newton second law of motion. Hence, derive the impulse momentum equation. **[Section 1.8]**

Q. 8 On what principle the energy transfer takes place between the fluid and rotor ? What are various components of velocity in rotodynamic machine. **[Sections 1.9, 1.9.1]**

Q. 9 Derive the fundamental energy equation for energy transfer between the fluid and rotor and show that :

$$H = \frac{(\vec{V}_{w_1} \cdot \vec{u}_1 - \vec{V}_{w_2} \cdot \vec{u}_2)}{g}$$ **[Section 1.10]**

Q.10 What is centrifugal head ? Prove that,

$h_c = \frac{u_2^2 - u_1^2}{2g}$, where notations have their usual meaning **[Section 1.10.1]**

Q.11 What are various components of energy transfer ? State their significance. **[Section 1.11]**

Q.12 Define degree of reaction. Show that the degree of reaction of an impulse machine is zero and that for reaction turbine is 1. **[Section 1.12(2)]**

Q.13 What are generally the energy losses in turbomachines ? Hence, define hydraulic, mechanical and overall efficiencies. **[Section 1.13]**

Q.14 Write a short note on "Applications of turbo machines". **[Section 1.14]**

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2

UNIT III

Impact of Jet

Syllabus

Impulse momentum principle and its application to fixed and moving flat, inclined and curved plate/vanes, Velocity triangles and their analysis, workdone equations, vane efficiency (No numerical)

2.1 Introduction to Impact of Jets

University Question

Q. What are the applications of impulse momentum principle? **SPPU : May 18**

Water flows in the pipe line under pressure head. If a nozzle is fitted at the end of the pipe, the water comes out at high velocity in the form of a jet. This high velocity jet is used in various applications as follows :

(i) In case the jet strikes a fixed or moving plate (flat or curved), the energy of jet gets converted to produce a force on the plate. This force exerted on the plate is called **impact of jet** and the magnitude of force, is according to the second Newton's law of motion. This force is then used to obtain workdone as in case of turbines.

(ii) When the jet leaves the nozzle, it exerts the reactive force on the nozzle itself according to the third Newton's law of motion according to which every action has equal and opposite reaction. Reactive force acts in the direction opposite to the direction of jet. Reactive force pushes the body attached to the nozzle in backward direction. This action is known as **jet propulsion.** This concept is used in propulsion of boats, ships, aeroplanes, lawn sprinklers etc.

In this chapter we shall investigate the impact of jet on flat and curved plates when they are fixed and moving.

2.2 Impulse-Momentum Principle

Momentum is defined as the product of mass, m and the velocity, V of the body.

$$\therefore \quad \text{Momentum} = m \cdot V$$

According to second Newton's law of motion, the magnitude of the applied force is equal to rate of change of momentum.

Force is a vector quantity which has both the magnitude and direction. Therefore, the direction of force will depend on the direction in which the change of momentum take place.

Therefore,

$$\text{Applied force,} \quad F = \frac{d}{dt}(m \cdot V) \quad \text{...(2.2.1)}$$

On differentiating the Equation (2.2.1) we have,

$$F = m \cdot \frac{dV}{dt} + V \cdot \frac{dm}{dt} \quad \text{...(i)}$$

In applied mechanics, we deal with solid bodies where the mass of body remains constant i.e. $\frac{dm}{dt} = 0$. Equation (i) as applied in mechanics reduces to :

$$F = m \cdot \frac{dV}{dt} + 0 = m \cdot a \quad \text{...(ii)}$$

Where, a represents the acceleration of body.

However, in fluid mechanics *we are concerned with constant mass flow of a continuous fluid,* therefore, $\frac{dm}{dt} = 0$. Thus the Equation (i) reduces to :

$$F = m \cdot \frac{dV}{dt} = \frac{m}{t}(V_2 - V_1) \quad \text{...(2.2.2)}$$

Where, V_2 and V_1 are the velocity of fluid at section 2 and section 1 of fluid stream respectively.

Product (F · t) is called **impulse**. In Equation (2.2.2),

$$\text{Mass flow rate of fluid, } \dot{m} = \frac{m}{t} = \rho \cdot Q \quad \text{...(iii)}$$

Where, ρ is density of fluid and Q is volume flow rate.

Therefore Equation (2.2.2) reduces to :

$$F = \rho \cdot Q (V_2 - V_1) \text{ (Newtons)} \quad \text{...(2.2.3)}$$

Equation (2.2.3) represents the force F exerted by the body on fluid. Using the third Newton's law of motion, the force exerted by the fluid on body will equal and opposite. It implies that,

Force exerted by fluid on body,

$$F = \rho \cdot Q (V_2 - V_1)$$

∴ Remember :

Forces exerted by fluid on body

= Rate of momentum in ($\dot{m} V_1$) – Rate of momentum out ($\dot{m} V_2$) ...(2.2.4)

Now, we shall evaluate the hydrodynamic force exerted by a jet on stationary and moving vanes as an application of impulse momentum equation discussed above.

2.3 Force Due to Impact of Jet on a Flat Fixed Plate

Consider a jet of water of cross-sectional area A moving with a velocity V which impinges on a fixed plate.

Jet may impinge (strike) in direction or on an inclined plate.

Then volume flow rate of fluid,

$$Q = A \cdot V \quad ...(i)$$

Mass flow rate, $\dot{m} = \rho \cdot Q = \rho \cdot A \cdot V$ ...(2.3.1)

Jet may strike the fixed plate either in normal direction or at some angle.

It is assumed that the plate is perfectly smooth i.e. friction between the jet and the plate is neglected, there is no loss of energy during impact of jet and the velocity of jet is uniform throughout.

2.3.1 When the Fixed Plate is Normal to Jet

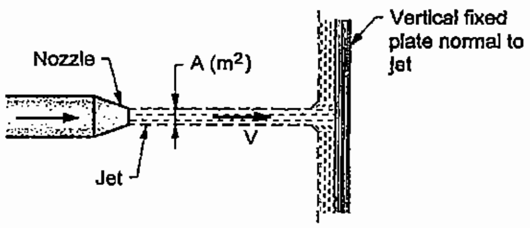


Fig. 2.3.1 : Jet of water striking flat plate fixed normal to jet

Consider a fluid jet striking a fixed plate normal to jet shown in Fig. 2.3.1 let,

A = cross-sectional area of jet, m²

V = velocity of jet, m/s

ρ = density of fluid, kg/m³

Since plate is fixed its velocity is zero.

When the jet strikes the plate, its momentum is completely destroyed by the application of force and its final velocity becomes zero. Therefore, force acting on the plate in the direction normal to plate can be determined by using impulse momentum Equation (2.3.1),

Force, $F = \dot{m} (V_1 - V_2)$

where, $\dot{m} = \rho \cdot Q = \rho \cdot A \cdot V$; $V_1 = V$; $V_2 = 0$

$$F = \rho \cdot A \cdot V (V - 0) = \rho \cdot A \cdot V^2 \quad ...(2.3.2)$$

Since the plate is stationary, therefore, workdone on plate is zero.

2.3.2 When Fixed Plate is Inclined at an Angle θ with Jet

University Question

Q. Derive the relation for force exerted by jet of water on inclined fixed plate in the direction of jet.

SPPU : April 17(In Sem)

Consider the fixed plate inclined at an angle θ as shown in Fig. 2.3.2.

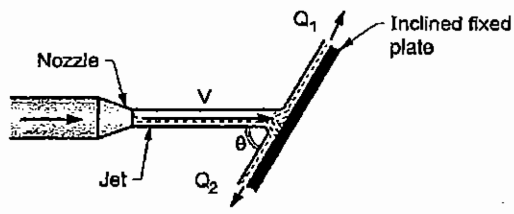


Fig. 2.3.2 : Jet striking with inclined fixed plate

V is the velocity of jet in the X-X direction.

Resolution of velocity normal and parallel to the plate is shown in Fig. 2.3.3(a).

Velocity of jet normal to plate = $V \cdot \sin\theta$

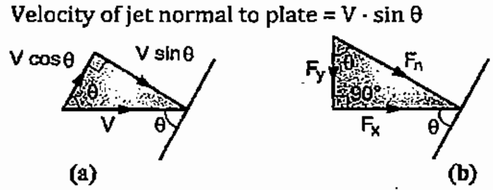


Fig. 2.3.3 : Resolution of velocity and forces

Final velocity normal to jet = 0

(a) Normal force on plate

Using impulse momentum Equation (2.3.1), normal force to jet we get,

$$F_n = \rho \cdot A \cdot V (\sin\theta - 0)$$
$$= \rho \cdot A \cdot V^2 \sin\theta \quad ...(2.3.3)$$

(b) Force on plate in X-X and Y-Y direction

By resolving the normal force in (X – X) and (Y – Y) directions as shown in Fig. 2.3.3(b) we get,

$$F_x = F_n \cdot \sin\theta$$
$$= (\rho \cdot A \cdot V^2 \sin\theta) \sin\theta$$
$$= \rho \cdot AV^2 \sin^2\theta \quad ...(2.3.4)$$
$$F_y = F_n \cdot \cos\theta = (\rho AV^2 \sin\theta)\cos\theta$$
$$= \frac{\rho \cdot AV^2 \cdot \sin 2\theta}{2} \quad ...(2.3.5)$$

In this case also, the **workdone on the plate is zero** since the plate is stationary.

(c) Ratio of discharges tangential to plate

Let Q_1 and Q_2 be volume flow rate along the plate out of the stream of jet at the rate of Q striking on the plate as shown in Fig. 2.3.2.

$$\therefore \quad Q = Q_1 + Q_2 \quad ...(i)$$

Since the resultant force in tangential direction on plate is zero. Applying impulse - momentum equation in tangential direction to plate, we get,

$$(\rho Q_1 V - \rho Q_2 V) - \rho QV\cos\theta = 0$$
$$\therefore \quad Q_1 - Q_2 - Q\cos\theta = 0 \quad ...(ii)$$

on solving Equations (i) and (ii), we get,

$$\left.\begin{aligned} Q_1 &= \frac{Q}{2}(1 + \cos\theta), \text{ and} \\ Q_2 &= \frac{Q}{2}(1 - \cos\theta) \end{aligned}\right\} \quad ...(2.3.6)$$

2.4 Force Exerted by a Jet on Stationary Curved Plate OR A Vane

2.4.1 When Jet Strikes at the Centre of a Symmetrical Curved Plate

University Question

Q. Derive an expression for the force exerted by the jet of water on the fixed curved plate jet strikes at centre of the curved plate at normally. **SPPU : May 18**

Consider a fluid jet of cross-sectional area A at velocity V striking a smooth stationary curved plate at its centre as shown in Fig. 2.4.1.

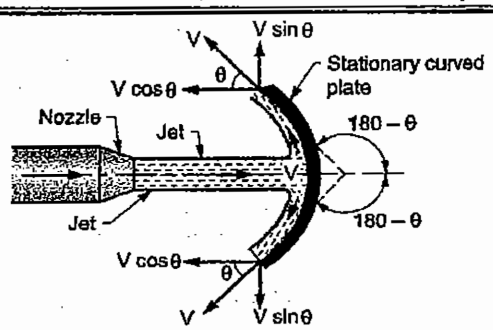


Fig. 2.4.1 : Impact of jet on stationary curved plate

The jet comes out at the same velocity V along the curved surface of the plate.

Components of velocity in (X – X) and (Y – Y) directions are V cos θ and V sin θ respectively.

Mass flow rate, $\dot{m} = \rho \cdot A \cdot V$

Initial velocity of jet in (X – X) direction,

$$V_1 = V$$

Final velocity of jet in (X – X) direction,

$$V_2 = -V\cos\theta$$

According to impulse momentum Equation (2.3.1),

$$F = \dot{m}(V_1 - V_2)$$

$\therefore$ Force in (X – X) direction

$$F_x = \rho AV[V - (V\cos\theta)]$$
$$F_x = \rho AV^2(1 + \cos\theta) \quad ...(2.4.1)$$

Similarly, force in (y – y) direction will be

$$(\because V_{1y} = 0 \text{ and } V_{2y} = V\sin\theta)$$
$$F_y = \rho AV[0 - (V\sin\theta)]$$
$$= -\rho AV^2\sin\theta \quad ...(2.4.2)$$

Negative sign indicates that the force F_y acts in downward direction.

2.4.2 When Jet Strikes an Unsymmetrical Curved Plate at One End Tangentially

Consider a jet of velocity V striking tangentially to an unsymmetrical curved plate as shown in Fig. 2.4.2.

Let the plate tip angles be θ and ϕ.

Initial velocity components at inlet are :

$$V_x = V\cos\theta, V_y = V\sin\theta$$

Final velocity components at exit are :

$$V_x = -V\cos\phi, V_y = V\sin\phi$$

and, $\dot{m} = \rho \cdot A \cdot V$

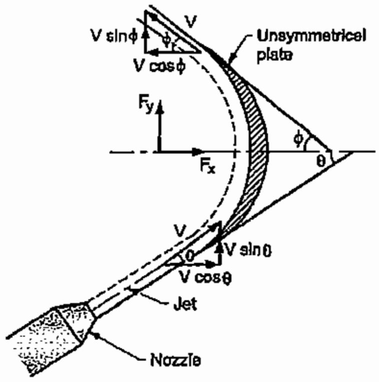


Fig. 2.4.2 : Jet striking on fixed unsymmetrical curved plate tangentially at one end

Using impulse momentum equation, the force exerted on plate in (X – X) and (Y – Y) directions will be :

$$F_x = \rho A V [V \cos\theta - (-V \cos\phi)]$$

$$F_x = \rho A V^2 (\cos\theta + \cos\phi) \quad ...(2.4.3)$$

$$F_y = \rho A V [V \sin\theta - V \sin\phi]$$

$$= \rho A V^2 (\sin\theta - \sin\phi) \quad ...(2.4.4)$$

2.4.3 For Semicircular Curved Vane or Plate

$\theta = \phi = 0$, the Equation (2.4.3) can be modified and the force in normal direction becomes,

$$F_x = \rho A V^2 (\cos 0 + \cos 0) = 2\rho \cdot A \cdot V^2 \quad ...(2.4.4(A))$$

It can be seen that this force exerted on semicircular fixed plate is twice to that of fixed vertical plate given by Equation (2.3.2).

Ex. 2.4.1 : Find the force exerted by jet of water on a stationary vertical plate if the diameter of jet is 8 cm. and its velocity is 40 m/s. Assume that the jet strikes in normal direction to plate. Also find the workdone.

Soln. : Refer Fig. 2.4.1.

Given : $d = 8 \text{ cm} = \frac{8}{100} = 0.08 \text{ m}$,

$V = 40 \text{ m/s}$

Assume, density of water,

$$\rho = 1000 \text{ kg/m}^3$$

Cross-sectional area of jet,

$$A = \frac{\pi}{4}(d^2) = \frac{\pi}{4} \times (0.08)^2$$

$$= 50.265 \times 10^{-4} \text{ m}^2$$

Force exerted by jet of water normal to stationary plate is given by Equation (2.3.2) as,

$$F = \rho A V^2$$

$$= 1000 \times (50.265 \times 10^{-4}) \times (40)^2$$

$$F = 8042.4 \text{ N} \quad ...\text{Ans.}$$

Work done is zero since the plate is stationary. **...Ans.**

Ex. 2.4.2 : An oil jet of specific gravity 0.8 of 40 mm diameter strikes a stationary plate inclined at an angle 30° with the axis of jet at a velocity of 30 m/s. Find the force exerted by the jet on the plate in the direction.

(i) Normal to plate

(ii) Along the X – axis and Y – axis

Also, find the ratio of discharge which is divided into two streams.

Soln. : Given :

Specific gravity = 0.8, $d = 40 \text{ mm} = 0.04 \text{ m}$,

$\theta = 30°$, $V = 30 \text{ m/s}$

∴ Density of oil, ρ = Specific gravity × density of water

$= 0.8 \times 1000 = 800 \text{ kg/m}^3$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.04)^2 = 12.567 \times 10^{-4} \text{ m}^2$$

(i) Force on plate in normal direction to plate; F_n

$$F_n = \rho A V^2 \sin\theta$$

... [According to Equation (2.3.3)]

$$= 800 \times (12.567 \times 10^{-4}) (30)^2 \sin 30$$

$$= 452.41 \text{ N} \quad ...\text{Ans.}$$

(ii) Force along X- and Y-axis on plate

According to Equation (2.3.4)

$$F_x = \rho A V^2 \sin^2\theta$$

$$= (800)(12.567 \times 10^{-4} \times 30^2) \times (\sin 30)^2$$

$$= 226.21 \text{ N} \quad ...\text{Ans.}$$

$$F_y = \rho A V^2 \sin\theta \cos\theta$$

...[According to Equation (2.3.5)]

$$= 800 \times (12.567 \times 10^{-4})(30^2) \sin 30 \cos 30$$

$$= 391.8 \text{ N} \quad ...\text{Ans.}$$

(iii) Ratio of streams, $\frac{Q_1}{Q_2}$,

According to Equation (2.3.6), we have,

$$\frac{Q_1}{Q_2} = \frac{\frac{Q}{2}(1+\cos\theta)}{\frac{Q}{2}(1-\cos\theta)} = \frac{1+\cos\theta}{1-\cos\theta}$$

$$= \frac{1+\cos 30}{1-\cos 30} = 13.928 \quad ...\text{Ans.}$$

Ex. 2.4.3 : Water jet from a nozzle strikes a vertical plate in horizontal direction. Nozzle operates under a head of 20 m. If the force exerted on the plate is 3 kN find.

(i) Velocity of jet, assume coefficient of velocity of nozzle as 0.9.

(ii) Cross-sectional area of jet required.

(iii) Volume flow rate in m^3/s.

Soln. :

Given : $H = 20$ m, $F = 3$ kN $= 3000$ N, $C_v = 0.9$

(i) Velocity of jet, V

$$V = C_v\sqrt{2gh} = 0.9\sqrt{2 \times 9.81 \times 20}$$

$$= \mathbf{17.83\ m/s} \quad \text{...Ans.}$$

(ii) Cross-sectional area of jet ,A

$$F = \rho \cdot A V^2$$

$$3000 = 1000 \times A \times (17.83)^2$$

$$\therefore A = \mathbf{0.0094\ m^2} \quad \text{...Ans.}$$

(iii) Volume flow rate, Q

$$Q = A \cdot V = 0.0094 \times 17.83$$

$$= \mathbf{0.1676\ m^3/s} \quad \text{...Ans.}$$

Ex. 2.4.4 : A vertical jet is issuing upward from a nozzle with a velocity of 8 m/s. The fluid is oil having density 700 kg/m^3 and the nozzle exit diameter is 7.0 cm. A flat horizontal plate bearing a total load of 40 N is supported only of the impact of the jet. Determine the equilibrium height of the plate above the nozzle exit. (Neglect all the losses).

Soln. : Refer Fig. P. 2.4.4.

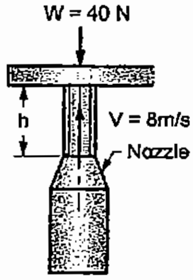


Fig. P. 2.4.4

Given : Density of oil, $\rho = 700$ kg/m^3

Diameter of nozzle, $d = 7$ cm $= 0.07$ m

Load, $W = 40$ N, Velocity of jet, $V = 8$ m/s

Let, h = height of nozzle from plate surface.

$$F = \rho \cdot A \cdot V^2 = \rho \cdot \frac{\pi}{4}(d^2) \times V^2$$

$$F = 700 \times \frac{\pi}{4}(0.07)^2 \times 8^2 = 172.41 \text{ N}$$

Upward force,

$$F = 172.41 \text{ N}$$

Downward force, $W = 40$ N, due to weight of plate

$\therefore$ Net upward force, $F_1 = F - W = 172.41 - 40 = 132.41$ N

This force will accelerate the plate with acceleration 'f'.

$$F_1 = mf$$

$$132.41 = \frac{40}{9.81} \times f \; ; \quad \left(\because m = \frac{W}{g}\right)$$

$$f = 32.47 \text{ m/s}^2$$

Under this acceleration, the plate will start moving upwards, therefore, force on the plate due to impact of jet will start reducing. Let u be the velocity of plate. Then,

$$\text{Force on moving plate} = \rho \times A \cdot (V - u)^2$$

$$40 = 700 \times \frac{\pi}{4}(0.07)^2 (8 - u)^2$$

$$\therefore (8 - u) = 3.85$$

$$u = 8 - 3.85 = 4.15 \text{ m/s}$$

Let h is the distance moved by plate. From Newton's equation of motion,

$$V^2 - u^2 = 2f \cdot h$$

$$8^2 - 4.15^2 = 2 \times 32.47 \times h$$

$$\therefore h = \mathbf{0.72\ m} \quad \text{...Ans.}$$

Ex. 2.4.5 : A jet of water of 5 cm diameter moving at a velocity of 30 m/s impinges on a symmetrical curved fixed plate at its centre.

Find the force exerted by jet in the direction of jet and normal to it the jet is deflected through an angle of 130° at the outlet of curved plate.

Soln. : Refer Fig. P. 2.4.5.

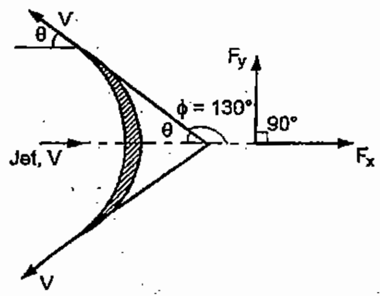


Fig. P. 2.4.5

Given : $d = 5$ cm $= 0.05$ m, $V = 30$ m/s

Area of jet, $A = \frac{\pi}{4} \cdot d^2 = \frac{\pi}{4}(0.05)^2$

$= 1.9635 \times 10^{-3}\ m^2$

Angle of deflection, $\phi = 130°$

$\therefore\ \theta = 180 - \phi = 180 - 130 = 50°$

Force exerted at the centre of plate,

$$F_x = \rho A V^2 (1 + \cos\theta)$$

$$= 1000 \times (1.9635 \times 10^{-3}) \times (30)^2 \times (1 + \cos 50)$$

$$= \mathbf{2903\ N\ or\ 2.903\ kN}$$ ...**Ans.**

Force exerted in the direction normal to direction of jet,

$$F_y = \rho A V^2 (\sin\theta - \sin\phi)$$

But $\theta = \phi$

$\therefore$ $\mathbf{F_y = 0\ N}$ ...**Ans.**

Ex. 2.4.6 : A water jet of 0.003 m^2 area moving at a velocity of 50 m/s strikes as unsymmetrical curved plate tangentially at one end at an angle of 35° with the horizontal. The jet leaves the curved plate at an angle of 25° with the horizontal. Find the force exerted by the jet in horizontal and vertical directions.

Soln. :

Given : $A = 0.003\ m^2$, $V = 50\ m/s$, $\theta = 35°$, $\phi = 25°$

According to Equation (2.4.3), *force exerted in horizontal direction,*

$$F_x = \rho A V^2 (\cos\theta + \cos\phi)$$

$$= 1000 \times 0.003 \times 50^2 (\cos 35 + \cos 25)$$

$$= \mathbf{12940.95\ N}$$ ...**Ans.**

Force exerted in vertical direction,

$$F_y = \rho A V^2 (\sin\theta - \sin\phi)$$

...[According to Equation (2.4.4)]

$$= 1000 \times 0.003 \times 50^2 (\sin 35 - \sin 25)$$

$$= \mathbf{1132.2\ N}$$ ...**Ans.**

Ex. 2.4.7 : A 15 mm diameter nozzle having C_v = 0.97, is supplied with water under a head of 30 m. The jet impinges on a fixed curved vane. Water glides on the vane tangentially and being deflected through 165°. Calculate the force on the vane in the direction of the jet, if

(i) there is no friction

(ii) the velocity of water leaving the vane is 0.8 of its impinging velocity. **SPPU : Dec. 19, 4 Marks**

Soln. : Refer Fig. P. 2.4.7

Diameter of nozzle, d = 15 mm = 0.015 m

$C_v = 0.97$, $H = 30\ m$

Angle of deflection = 165°

$\therefore\ \theta = 180 - 165 = 15°$

Velocity of jet, $V = C_v\sqrt{2gH}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 30}$

$= 23.533\ m/s$

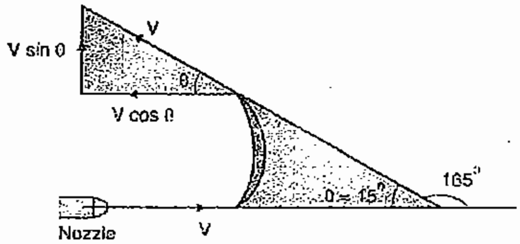


Fig. P. 2.4.7

(i) Force in the direction of jet, F_x

$$\dot{m} = \rho A V = \rho \times \frac{\pi}{4} d^2 \times V$$

$$= 1000 \times \frac{\pi}{4} (0.015)^2 \times 23.533$$

$$= 4.1586\ kg/s$$

$$F_x = \dot{m}\ [\text{Initial velocity} - \text{Final velocity}]$$

$$= \dot{m}\ [V - (-V\cos\theta)]$$

$$= \dot{m} \times V(1 + \cos\theta)$$

$$= 4.1586 \times 23.533\ (1 + \cos 15)$$

$$= \mathbf{192.4\ N}$$ ... **Ans.**

(ii) Force in direction of jet when velocity of water leaving the vane, $V_1 = 0.8\ V$

$$F_x = \dot{m}\ [V - (-V_1\cos\theta)] = \dot{m}\ [V + 0.8\ V\cos\theta]$$

$$= \dot{m}\ V[1 + 0.8\cos\theta]$$

$$= 4.1586 \times 23.533 \times (1 + 0.8 \times \cos 15)$$

$$= \mathbf{173.49\ N}$$ ... **Ans.**

Ex. 2.4.8 : A jet of water of diameter 70 mm moving with velocity 20 m/s strikes a fixed plate in such a way that the angle between jet and the plate is 60 degree. Find the force exerted by jet on plate in following cases :

(i) In the direction normal to the plate

(ii) In the direction of jet. **SPPU : Oct. 19(In Sem.), 4 Marks**

Soln. :

Refer Fig. P. 2.4.8

Diameter of jet,

$$d = 70 \text{ mm} = 0.07 \text{ m}$$

Velocity of jet, $V = 20$ m/s; $\theta = 60°$

$$\dot{m} = \rho AV = \rho \times \frac{\pi}{4} d^2 \times V$$

$$= 1000 \times \frac{\pi}{4}(0.07)^2 \times 20$$

$$= 76.969 \text{ kg/s}$$

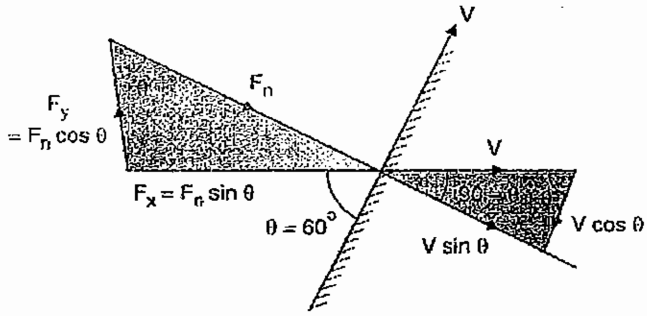


Fig. P. 2.4.8

i) **Force exerted in the direction normal to jet, F_n**

$$F_n = \dot{m}\, [V \sin \theta - 0] = 76.969 \times 20 \sin 60$$

$$= \mathbf{1333.1 \text{ N}} \qquad \textbf{Ans.}$$

ii) **Force exerted in the direction of jet, F_x**

$$F_x = F_n \sin \theta = 1333.1 \times \sin 60$$

$$= \mathbf{1154.5 \text{ N}} \qquad \textbf{...Ans.}$$

2.5 Force Exerted by a Jet on a Hinged Plate

2.5.1 When the Plate is Held Vertical by External Force

University Question

Q. Derive the expression for the force exerted by a jet of water on a hinged plate in the direction of jet.

SPPU : May 19

Consider a vertical flat plate of height h (m) hinged at the top edge 'O' as shown in Fig. 2.5.1.

When the plate is vertical a jet of area A at a velocity V strike the plate in normal direction at a distance 'y' (m) from top edge.

Due to the normal force, F_n exerted by the jet on the plate, it will tend to swing about 'O'.

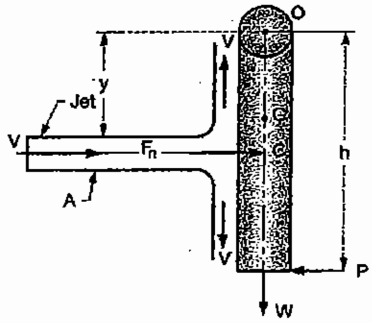


Fig. 2.5.1 : Jet impinging on hinged plate

In order to keep the plate in vertical position, a force P is applied at the bottom edge of the plate in the direction opposite to the fluid jet force.

For equilibrium of the plate, the moment of the forces about the hinge must be zero. Therefore,

$$F_n \times y = P \cdot h \qquad ...(i)$$

But the force exerted by jet,

$$F_n = \rho A V^2 \qquad ...(ii)$$

On substituting the value of F_n from Equation (ii) in (i), we get,

$$\text{Holding force, } P = F_n \times \frac{y}{h} = (\rho A V^2) \cdot \frac{y}{h} \qquad ...(2.5.1)$$

2.5.2 Angle Turned by Plate when Holding Force P is not Applied

In case the holding force P in the horizontal direction is not applied, the hinged plate will swing about O.

Let the angle turned by the plate by θ as shown in Fig. 2.5.2.

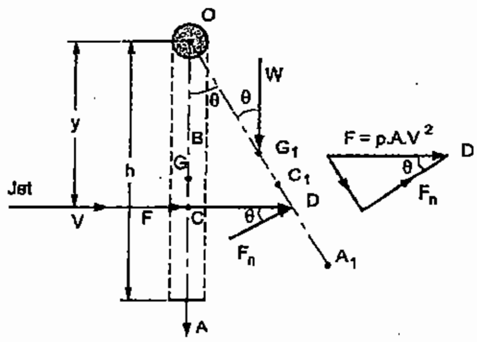


Fig. 2.5.2 : Angle turned by hinged plate due to impact of jet

The plate is under the equilibrium due to moment caused by the force of jet and the moment caused by the weight of plate, W.

Plate OGCA is moved to $OG_1C_1A_1$.

Component of fluid force F in normal direction,

$$F_n = F \sin\theta$$

$$\therefore \quad F_n = (\rho A V^2) \cos\theta \qquad ...(i)$$

For equilibrium,

$$\text{Moment of force } F_n \text{ about hinge} = \text{Moment of weight about hinge}$$

$$F_n \times OD = W \times BG_1$$

$$(\rho \cdot A V_2 \cos\theta) \times \frac{y}{\cos\theta} = W \times \left(\frac{h}{2} \cdot \sin\theta\right)$$

$$\therefore \quad \rho A V_2 y = W \times \frac{h}{2} \cdot \sin\theta$$

$$\sin\theta = \rho \cdot A \cdot V^2 \cdot \frac{2y}{W \cdot h} \qquad ...(2.5.2)$$

In case the jet strikes at the centre of plate, then,

$$y = \frac{h}{2} \quad \text{Equation (2.5.2) reduces to,}$$

$$\sin\theta = \frac{\rho \cdot A V^2}{W} \qquad ...(2.5.3)$$

From Equation (2.5.2) the angle turned by the plate 'θ' from vertical can be calculated.

Ex. 2.5.1 : A square plate weighing 115 N and of uniform thickness and 30 cm edge is hung so that a horizontal jet 2 cm diameter and having a velocity of 15 m/s impinges on the plate. The centerline of the jet is 15 cm below the upper edge of the plate and when the plate is vertical the jet strikes the plate normally and at its centre. Find what force must be applied at the lower edge of the plate in order to keep the plate vertical. If the plate is allowed to swing freely, find the inclination to the vertical. **SPPU : Aug.18(In Sem.), 6 Marks**

Soln. :

Refer Fig. P. 2.5.1

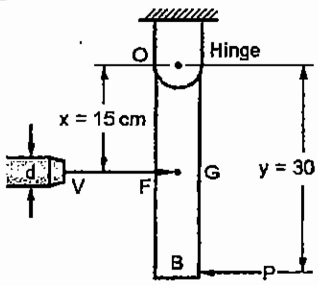


Fig. P. 2.5.1

Weight of plate, $W = 115$ N; $y = 30$ cm $= 0.3$ m

Jet diameter, $d = 2$ cm $= 0.02$ m

$V = 15$ m/s ; $x = 15$ cm $= 0.15$ m

1. Force required to keep the plate vertical P when applied at lower edge

Force on plate,

$$F = (\rho \cdot a \cdot V)\, V = \rho \times \frac{\pi}{4} d^2 \times v^2 \ \text{(N)}$$

$$F = 1000 \times \frac{\pi}{4} \times (0.02)^2 \times (15)^2 = 70.686 \text{ N}$$

Taking moment about hinge 'o',

$$F \times x = P \cdot y \,;$$

$$70.686 \times 0.15 = P \times 0.3$$

$$P = 35.343 \text{ N} \qquad \text{... Ans.}$$

2. Inclination of plate, θ if allowed to swing freely.

Refer Fig. P. 2.5.1(a)

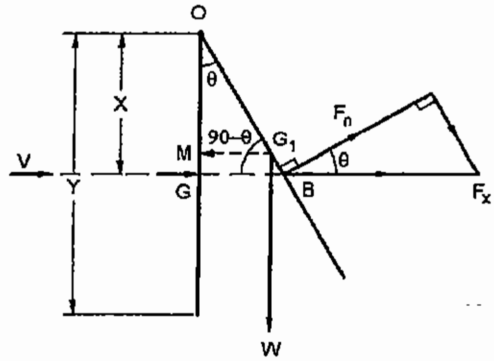


Fig. P. 2.5.1(a)

Let the inclination from vertical be θ. In that case the square plate be inclined by (90 − θ) from horizontal force exerted by jet normal to the plate,

$$F_n = F_x \cos\theta$$

$$= 70.686 \times \cos\theta$$

Taking moment about 'o',

$$F_n \times OB = W \times GM$$

$$70.686 \cos\theta \times \frac{OG}{\cos\theta} = W \times OG_1 \sin\theta \qquad (\text{But } OG = x)$$

$$70.686 \times x = W \times x \sin\theta$$

$$70.686 \times 0.15 = 115 \times 0.15 \sin\theta$$

$$\sin\theta = 0.61466$$

$$\theta = 37.93° \qquad \text{... Ans.}$$

2.6 Force Exerted by Jet on Moving Plates

No work is done when a force is applied on a stationary plates or vanes, hence, it is not of any practical consequence. However, the work and power can be produced in case the jet applies a force on moving flat and curved plates and vanes. These concepts are being discussed in this section.

2.6.1 Force on Moving Flat Plate in Direction of Jet

Fig. 2.6.1 shows a horizontal fluid jet striking a vertical plate moving with a uniform velocity away from jet in horizontal direction.

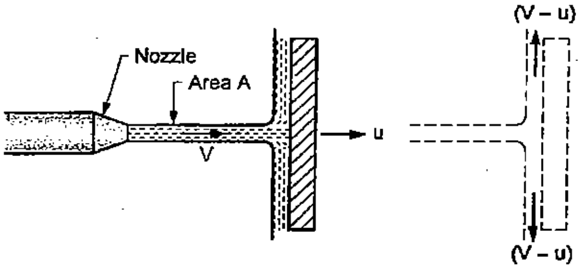


Fig. 2.6.1 : Fluid jet striking a moving plate

Let V = Velocity of jet (absolute)

A = Area of cross-section of jet

u = Velocity of flat plate (absolute)

Relative velocity of an object B relative to object A is the velocity with which B appears to move to an observer situated at A and moving with velocity V_A.

Relative velocity of jet by which it strikes the plate

$$= (V - u)$$

Mass flow rate of water striking the plate,

$$\dot{m} = \rho \cdot A(V - u)$$

$\therefore$ Force exerted by jet normal to plate,

$$F_n = \dot{m}\,[\text{Initial velocity} - \text{Final velocity}]$$

$$= \rho A (V - u)\,[(V - u) - 0]$$

$$= \rho A (V - u)^2 \qquad ...(2.6.1)$$

This case would not be possible in practice as there would be a continually lengthening of the jet i.e. the distance between the jet and plate increases by '**u**' m/s.

Rate of workdone by jet or power

In this case work will be done by jet on the plate as the plate is moving.

Rate of workdone (W.D) by jet,

$$\dot{W} = \text{Force, } F_n \times \text{Distance moved by jet u /s}$$

$$= [\rho A (V - u)^2]\, u \text{ (Nm/s or Watts)} \qquad ...(2.6.2)$$

Efficiency of system, η

Efficiency, η of the system is defined as the ratio of workdone (W.D.) by the jet to the kinetic energy (K.E.) supplied.

$$\text{K.E. of jet} = \frac{1}{2}\dot{m}\,V^2 = \frac{1}{2}(\rho A V)(V)^2 = \frac{1}{2}\rho \cdot A \cdot V^3$$

$$\therefore \eta = \frac{W.D}{K.E} = \frac{\rho A(V-u)^2 u}{\frac{1}{2}\rho \cdot AV^3}$$

$$= \frac{2}{V^3}(V^2 u - 2V \cdot u^2 + u^3) \qquad ...(2.6.3)$$

Condition for maximum efficiency

For a given jet velocity, the condition for maximum efficiency is that $\frac{d\eta}{du} = 0$. Therefore, differentiating Equation (2.6.3) w.r.t. u and equating to zero, we get,

$$\frac{d}{du}\left[\frac{2}{V^3}(V^2 \cdot u - 2V \cdot u^2 + u^3)\right] = 0$$

$$\frac{2}{V^3}[V^2 - 2V \times 2u + 3u^2] = 0$$

Since $\frac{2}{V^3} \neq 0$, it follows that,

$$V^2 - 4Vu + 3u^2 = 0$$

i.e. $$V^2 - 3Vu - Vu + 3u^2 = 0$$

$$\therefore (V - u)(V - 3u) = 0$$

or, $V = u$

or $V = 3u$

When V = u, the workdone will be zero. Therefore, the condition for maximum efficiency is,

$$V = 3u \qquad ...(2.6.4)$$

On substituting V = 3u in Equation (2.6.3) and on solving,

$$\eta_{max} = \frac{2}{V^3}\left[V^2 \cdot \frac{V}{3} - 2V \cdot \left(\frac{V}{3}\right)^2 + \left(\frac{V}{3}\right)^3\right]$$

$$= \frac{2}{V^3}\left(\frac{V^3}{3} - \frac{2V^3}{9} + \frac{V^3}{27}\right)$$

$$\eta_{max} = \frac{8}{27} \qquad ...(2.6.5)$$

2.6.2 Force on Moving Inclined Plate in the Direction of Jet

University Question

Q. Derive an expression for the force exerted by a jet of water on a moving inclined plate in the direction of plate. **SPPU : Dec.12**

Consider a jet of cross-sectional area A at velocity **V** strikes a moving inclined plate at velocity **u** as shown in Fig. 2.6.2(a). θ represents the angle between the plate and the jet.

Components of velocity in normal and tangential direction are shown in Fig. 2.6.2(b).

Relative velocity of jet of water striking the plate = (V – u)

$$\text{Mass flow rate, } \dot{m} = \rho \cdot A \cdot (V - u)$$

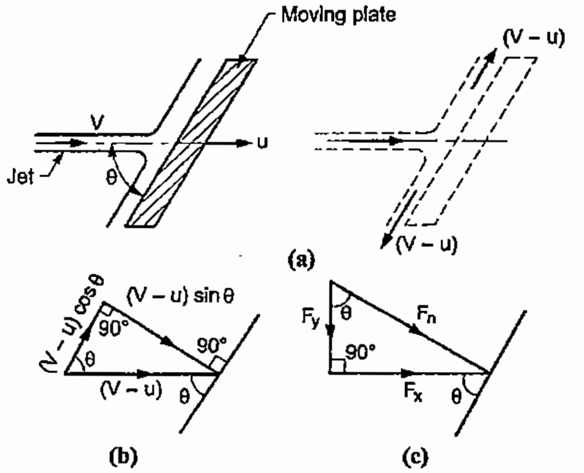


Fig. 2.6.2 : Fluid jet striking an inclined moving plate

Force of jet on plate in normal direction,

$F_n = \dot{m}$ [Initial relative velocity in normal direction – Final velocity]

$$= \rho A (V - u) [(V - u) \sin\theta - 0]$$

$$= \rho A (V - u)^2 \cdot \sin\theta \quad ...(2.6.6)$$

Component of normal force F_n (X – X) axis and (Y – Y) axis are shown in Fig. 2.6.2(c).

$$F_x = F_n \cdot \sin\theta = \rho \cdot A (V - u)^2 \cdot \sin\theta \cdot \sin\theta$$

$$= \rho A (V - u)^2 \cdot \sin^2\theta \quad ...(2.6.7)$$

$$F_y = F_n \cos\theta$$

$$= \rho \cdot A \cdot (V - u)^2 \sin\theta \cdot \cos\theta \quad ...(2.6.8)$$

Rate of workdone by jet on plate in (X – X) direction,

$$\dot{W} = F_x \cdot u = [\rho \cdot A (V - u)^2 \cdot \sin^2\theta]\, u \text{ (Nm/s)} \quad ...(2.6.9)$$

2.6.3 Impact of Jet on A Series of Flat Moving Plate

University Questions

Q. Show that when a jet of water impinging normally on a series of curved vanes, Maximum efficiency is obtained when the vane is semicircular in section and the velocity of the vane is half that of the jet. **SPPU : May 13, May 14**

Q. Show that the efficiency of a free jet striking normally on a series flat plates mounted on the periphery of a wheel can never exceed 50%. **SPPU : Dec.16**

Q. A jet has a direct impact on a series of flat plate vanes mounted over the periphery of a large wheel. Determine the force of impact and the work done per second. **SPPU : Dec.18**

As brought out in section 2.6.1, the force exerted by the jet on a single moving plate will not be of practical use since the distance between jet and the plate will keep on increasing continuously.

Instead of a single plate, consider the case of continuous series of plates at a fixed distance apart as shown in Fig. 2.6.3.

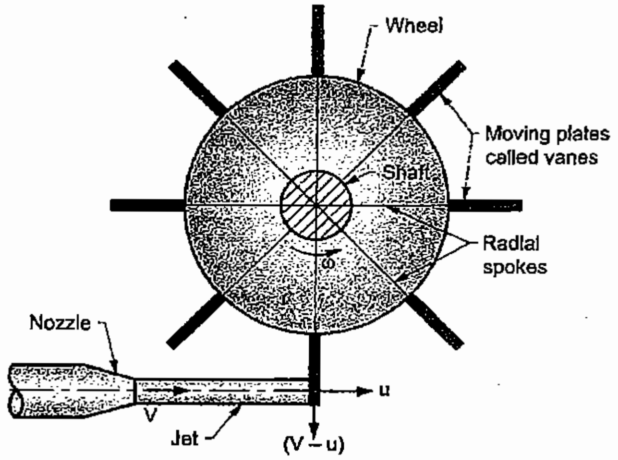


Fig. 2.6.3 : Impact of jet on a series of flat moving plate

All plates are moving in the same direction as the jet with a velocity u, then such a system can be put to practical use.

In Fig. 2.6.3, the **plates** called **vanes** are fixed on moving wheel in radial direction along its circumference. The jet strikes the vanes tangentially.

Because series of moving radial plates are available continuously to intercept the jet, the mass flow rate of water striking the plates will be,

$$\dot{m} = \rho \cdot A \cdot V$$

Force on plate,

$$F_n = \dot{m} \text{ (Initial velocity – Final velocity)}$$

$$= \rho A V [(V - u) - 0]$$

$$F_n = \rho A V (V - u) \quad ...(2.6.10)$$

Rate of workdone,

$$\dot{W} = F_n \cdot u$$

$$= \rho AV(V-u)\cdot u \qquad ...(2.6.11)$$

$$\text{K.E. supplied} = \frac{1}{2}(\rho \cdot A \cdot V)V^2 = \frac{1}{2}\rho \cdot A \cdot V^3$$

Efficiency of system,

$$\eta = \frac{\dot{W}}{K.E} = \frac{\rho AV(V-u)u}{\frac{1}{2}\rho AV^3}$$

$$= \frac{2}{V^2}(V-u)u \qquad ...(2.6.12)$$

Condition for maximum efficiency is that,

$$\frac{d\eta}{du} = 0 \quad \text{Therefore,}$$

$$\frac{d}{du}\left[\frac{2}{V^2}(V-u)u\right] = 0$$

$$\frac{2}{V^2}[V-2u] = 0$$

$$\therefore V = 2u\left(\because \frac{2}{V^2} \neq 0\right) \qquad ...(2.6.13)$$

On substituting the condition of maximum efficiency V = 2u in Equation (2.6.12) we get,

$$\eta_{max} = \frac{2}{V^2}\left(V-\frac{V}{2}\right)\cdot\frac{V}{2} = \frac{1}{2} \text{ or } 50\% \qquad ...(2.6.14)$$

Note that the flat plate used are called **vanes** and type of wheel shown in Fig. 2.6.3 is called **undershot water wheel**. The maximum efficiency obtainable from this system is 50%.

Ex. 2.6.1 : A jet of water 40 mm in diameter and moving a velocity of 30 m/s strikes normally to a plate moving at a velocity of 10 m/s. Determine :

(i) Normal thrust exerted by jet on plate
(ii) Power produced
(iii) Efficiency of the system
(iv) Maximum efficiency

Soln. :

Given : $d = 40 \text{ mm} = 0.04 \text{ m}$; $V = 30 \text{ m/s}$; $u = 10 \text{ m/s}$

Cross-sectional area of jet,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.04)^2 = 1.257 \times 10^{-3} \text{ m}^2$$

(i) Normal thrust, F_n :

According to Equation (2.6.1),

$$F_n = \rho A(V-u)^2$$

$$= 1000 \times (1.257 \times 10^{-3}) \times (30-10)^2$$

$$= \mathbf{502.7\ N} \qquad \textbf{...Ans.}$$

(ii) Power produced, $\dot{W}$:

According to Equation (2.6.2),

$$\dot{W} = F_n \cdot u = 502.7 \times 10$$

$$= \mathbf{5027\ Nm/s\ or\ W} \qquad \textbf{...Ans.}$$

(iii) Efficiency of the system, η

$$\eta = \frac{\dot{W}}{\text{K.E.supplied}}$$

$$= \frac{\rho \cdot A(V-u)^2 \cdot u}{\left(\frac{1}{2}\right)(\rho \cdot A \cdot V)V^2} = \frac{2(V-u)^2 \cdot u}{V^3}$$

$$\therefore \eta = \frac{2(30-10)^2 \times 10}{(30)^3}$$

$$= \mathbf{0.2963\ or\ 29.63\%} \qquad \textbf{...Ans.}$$

(iv) Maximum efficiency, η_{max} :

According to Equation (2.6.5),

$$\eta_{max} = \frac{8}{27} = \mathbf{0.2963\ or\ 29.63\%} \qquad \textbf{...Ans.}$$

Ex. 2.6.2 : A 7.5 cm diameter jet having velocity of 12 m/s impinges on a smooth plate at an angle of 60° to the normal to the plate. What will be the force when (i) the plate is stationary and (ii) when the plate is moving in the direction of jet at 6 m/s. Determine also the work done per second on the plate in each case.

Soln. :

Given : $d = 7.5 \text{ cm} = 0.075 \text{ m}$, $V_1 = 12 \text{ m/s}$, $\theta = 60°$

(i) Force by jet when plate is stationary, F

[Refer Fig. P. 2.6.2(a)]

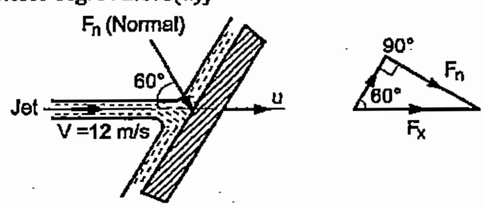


Fig. P. 2.6.2(a)

Plate velocity, $u = 0$

$$F_x = \rho \cdot A \cdot V^2$$

$$= 1000 \times \frac{\pi}{4}(0.075)^2 \times 12^2$$

$$= \mathbf{636.2\ N} \qquad \textbf{...Ans.}$$

The component of jet in normal direction,

$$F_n = F_x \cdot \cos 60$$
$$= 636.2 \times \cos 60$$
$$= \mathbf{318.1\ N} \quad \text{...Ans.}$$

Workdone by jet,

$$W = F_x \times u = 636.2 \times 0 = 0 \quad \text{...Ans.}$$

(ii) Force by jet when plate is moving with a velocity, u = 6 m/s

Refer Fig. P. 2.6.2(b).

Jet now strikes with relative velocity (V – u).

$$F_n = \rho A (V - u)\,[(V - u) \cos 60 - 0]$$
$$= \rho A (V - u)^2 \cdot \cos 60$$
$$= 1000 \times \frac{\pi}{4} (0.075)^2 \times (12 - 6)^2 \cos 60$$
$$= \mathbf{79.52\ N} \quad \text{...Ans.}$$

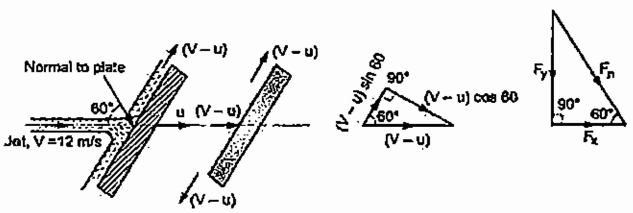


Fig. P. 2.6.2(b)

Component of force F_n in direction of plate velocity,

$$F_x = F_n \cdot \cos 60$$
$$= 79.52 \cos 60 = 39.76\ N$$

Workdone, $\quad W = F_x \cdot u = 39.76 \times 6$

$$= \mathbf{238.56\ Nm} \quad \text{...Ans.}$$

Ex. 2.6.3 : A 6mm diameter jet of water having a velocity of 30m/s strikes a flat plate, the normal from which is inclined at 33° to the axis of the jet. Find the force exerted by jet on plate in the direction of motion of plate when (i) the plate is stationary and (ii) the plate is moving with a velocity of 18m/s away from the jet. Determine the power and efficiency of the jet when the plate is moving. **SPPU - April 15 (In Sem)**

Soln. : Refer Fig. P. 2.6.3(a).

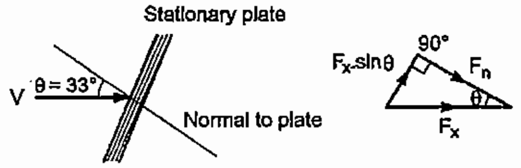


Fig. P. 2.6.3(a)

Given : $\quad d = 6\ mm = 6 \times 10^{-3}\ m,$

$V = 30\ m/s, \quad \theta = 33°$

(i) Normal force when plate is stationary, F_n :

$$F_n = F_x \cdot \cos\theta = \rho A V^2 \cdot \cos\theta$$
$$= 1000 \times \frac{\pi}{4} (6 \times 10^{-3})^2 \times 30^2 \times \cos 33$$
$$= \mathbf{21.342\ N} \quad \text{...Ans.}$$

(ii) When plate is moving with velocity 18 m/s :

Refer Fig. P. 2.6.3(b)

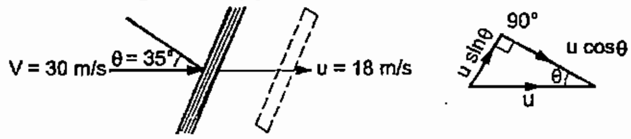


Fig. P. 2.6.3(b)

Force normal to plate,

$$F_n = \rho A (V - u)^2 \cos\theta$$
$$\therefore\ F_n = 1000 \times \frac{\pi}{4} (6 \times 10^{-3})^2 \times (30 - 18)^2 \cos 33$$
$$= \mathbf{3.41\ N} \quad \text{...Ans.}$$

Rate of Workdone or power,

$$\dot{W} = F_n \times u \cos\theta$$
$$= 3.41 \times 18 \cos 33$$
$$= \mathbf{51.48\ Nm/s\ or\ W} \quad \text{...Ans.}$$

Efficiency of jet, η

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\dot{W}}{\frac{1}{2}\dot{m} V^2}$$
$$= \frac{2\dot{W}}{(\rho \cdot A \cdot V) V^2}$$
$$= \frac{2 \times 51.48}{1000 \times \frac{\pi}{4} (6 \times 10^{-3})^2 \times (30)^3}$$
$$= \mathbf{0.1349\ or\ 13.49\ \%} \quad \text{...Ans.}$$

Ex. 2.6.4 : A 7.5 cm diameter of jet having velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to axis of jet. Find the normal force exerted on the plate, workdone and efficiency of the system. **SPPU - April 2017 (In sem), 6 Marks**

Soln. :

Jet diameter, $d = 7.5\ cm = 0.075\ m,$

Velocity of jet, $V = 30\ m/s;$

$\theta = 45°;$

$u = 10\ m/s$

Refer Fig. 2.6.2

Area of jet, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.075)^2$

$= 4.4179 \times 10^{-3}\ m^2$

Mass flow rate, $\dot{m} = \rho \cdot A \cdot (V - u)$

$= 1000 \times (4.4179 \times 10^{-3})(30 - 10)$

$= 88.358\ kg/s$

(i) Normal force, F_n

$F_n = \dot{m}$ [Initial relative velocity in normal direction – Final velocity]

$= \dot{m}\ [(V - u) \sin\theta - 0] = \dot{m}\ (V - u) \sin\theta$

$= 88.358\ (30 - 10) \sin 45$

$= \mathbf{1249.57\ N}$ **...Ans.**

(ii) Workdone, $\dot{W}$

$F_x = F_n \sin\theta = 1249.57 \sin 45 = 883.58\ N$

$\dot{W} = F_x \times u = 883.58 \times 10 = \mathbf{8835.8\ Nm/s}$ **...Ans.**

(iii) Efficiency, η

$$\eta = \frac{\dot{W}}{\text{K.E. supplied}}$$

$$= \frac{\dot{W}}{\frac{1}{2}(\rho AV)\,V^2} = \frac{2\dot{W}}{\rho AV^3}$$

$$= \frac{2 \times 8835.8}{1000 \times (4.4179 \times 10^{-3}) \times (30)^3}$$

$= \mathbf{0.1481\ or\ 14.81\%}$ **...Ans.**

Ex. 2.6.5 : Prove that the force exerted by a jet of water on a fixed semi-circular plate in the direction of the jet when the jet strikes at the centre of the plate is two times the force exerted by the jet on the fixed vertical plate.

Soln. :

(a) Force exerted on fixed plate :

Refer Fig. P. 2.6.5(a).

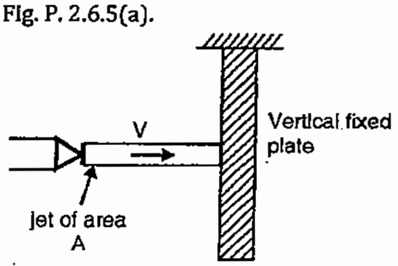


Fig. P. 2.6.5(a)

Consider the fluid of density ρ, Jet area A and velocity $V = V_1$

$F_1 = \dot{m}\ (V_1 - V_2) = \rho.AV\ (V - 0)$

$= \rho AV^2$ ...(i)

(b) Force exerted on fixed semi-circular plate :

Refer Fig. P. 2.6.6(b)

Let θ be the angle of jet leaving the circular plate.

$\dot{m} = \rho.A.V$

Initial velocity $\dot{m}$ (X–X) direction,

$V_{1x} = V$

Final velocity $\dot{m}$ (X–X) direction,

$V_{2x} = -V \cos\theta$

(But $\theta = 180°$ being semicircular plate)

$= -V \cos 180 = -V$

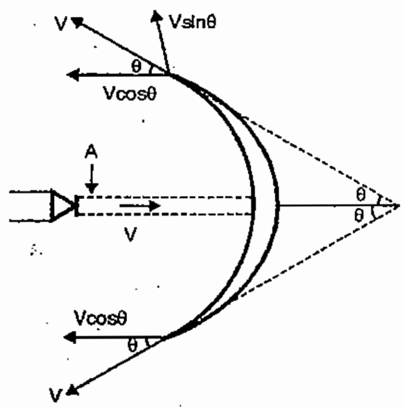


Fig. P. 2.6.5(b)

Final velocity $\dot{m}$ (Y–Y) direction,

$V_{2y} = V \sin\theta = V \sin 180 = 0$

$\therefore F_x = \dot{m}\ (V_{1x} - V_{2x})$

$= \rho AV\ [V - (-V)] = 2\rho AV^2$

$F_y = \dot{m}\ [V_{1y} - V_{2y}]$

$= \rho AV\ [0 - 0] = 0$

$\therefore$ Net force,

$F_2 = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(2\rho AV^2)^2 + 0^2}$

$= 2\rho AV^2$ ...(ii)

$$\frac{\text{Force on semicircular plate}, F_2}{\text{Force on vertical plate}, F_1} = \frac{2\rho AV^2}{\rho AV^2} = 2$$ **...Proved.**

2.7 Impact of Jet on Moving Curved Vanes

2.7.1 Curved Vane Moving in the Direction of Jet

University Questions

Q. Consider a single, symmetric 2D curved vane having centrally impinging 2D water jet. Jet cross-section area is A and density of water is ρ. Velocity of the jet is V and the vane moves at velocity u in the same direction as the jet. The turning angle of the vane on each side is θ. Derive the expression for hydraulic efficiency η of the vane in terms of the speed ratio u/V and the half angle of the vane θ.

Then derive the condition for maximum efficiency for given angle θ. Hence obtain the maximum efficiency for :

(i) A semicircular vane,

(ii) A flat plate perpendicular to the flow.

SPPU : Dec. 11

Q. A jet of water moving with V m/s strikes at the centre of a curved vane which is moving with 'u' m/s. If the outgoing jet makes an angle θ with the incoming jet, prove that,

(i) Maximum efficiency $= \eta_{max} = \frac{8}{27}(1 + \cos\theta)$

(ii) Blade speed u = V/3 **SPPU : May 15**

Consider a jet of water of area **A** at velocity **V** strikes a single symmetrical moving curved vane at velocity **u** as shown in Fig. 2.7.1.

Jet leaves the vane in tangential direction at angle θ from horizontal at relative velocity (V – u).

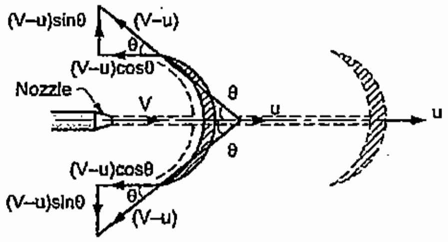


Fig. 2.7.1 : Impact of jet on single moving curved vane

Mass flow rate of jet of water striking the vane, $\dot{m} = \rho \cdot A\,(V - u)$. Component of velocity in direction of jet after striking the vane is [– (V – u) cos θ] as shown in Fig. 2.7.1.

∴ Normal force on vane,

$$F_n = \dot{m}\,[\text{Initial velocity} - \text{Final velocity}]$$

$$F_n = \rho \cdot A \cdot (V-u)\,[(V-u) - \{-(V-u)\cos\theta\}]$$

$$F_n = \rho \cdot A \cdot (V-u)^2\,(1+\cos\theta) \qquad ...(2.7.1)$$

Rate of workdone,

$$W = \text{Force} \times \text{distance travelled/second by vane}$$

$$W = \rho \times A \cdot (V-u)^2\,(1+\cos\theta) \times u \qquad ...(2.7.1(A))$$

$$\text{K.E. supplied to jet} = \frac{1}{2}\dot{m}\,V^2 = \frac{1}{2}(\rho\,A\,V)\,V^2 = \frac{1}{2}\rho \cdot A \cdot V^3$$

Efficiency of system, η

$$\eta = \frac{\text{Workdone, W}}{\text{K.E}} = \frac{\rho \cdot A\,(V-u)^2\,(1+\cos\theta)\,u}{\frac{1}{2} \cdot \rho \cdot A \cdot V^3}$$

$$= \frac{2\,(V-u)^2\,(1+\cos\theta)\,u}{V^3} \qquad ...(2.7.2)$$

Condition for maximum efficiency is, $\frac{d\eta}{du} = 0$

$$\therefore \frac{d}{du}\left(\frac{2\,(V-u)^2\,(1+\cos\theta)\,u}{V^3}\right) = 0$$

$$\frac{2\,(1+\cos\theta)}{V^3}\left(\frac{d}{du}(V^2 - 2Vu + u^2)\,u\right) = 0$$

$$\frac{2\,(1+\cos\theta)}{V^3}\,[V^2 - 2V \cdot 2u + 3u^2] = 0$$

But $\frac{2\,(1+\cos\theta)}{V^3} \neq 0$, it follows that :

$$V^2 - 4Vu + 3u^2 = 0$$

$$(V-3u)\,(V-u) = 0$$

$$\therefore V = 3u \text{ or } V = u \qquad ...(i)$$

V = **u is not possible since jet will never strike the vane.**

Hence, the system has the maximum efficiency when,

$$V = 3u \qquad ...(2.7.3)$$

On substituting the above value in Equation (2.7.2), we have,

$$\eta_{max} = \frac{8}{27}(1+\cos\theta) \qquad ...(2.7.4)$$

For semicircular vane, θ = 0, on substituting the value of θ in Equation (2.7.4), we have,

$$\therefore \eta_{max} = \frac{16}{27} = 0.5926 = 59.26\%$$

2.7.2 Impact of Jet in case of Series of Moving Vanes in Direction of Jet

The case of single moving vane system is not of any practical use since there is a continually lengthening of jet i.e. the distance between the jet and vane increases by 'u' m/s. In order that the system could be used to produce the work continuously, a series of curved vanes can be mounted at equidistance around the periphery of a wheel as shown in Fig. 2.7.2.

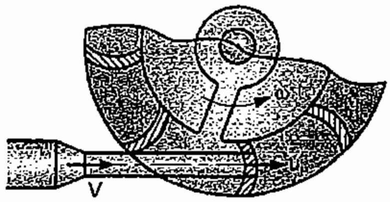


Fig. 2.7.2 : A series of moving vanes

Since series of vanes are available to intercept the jet. Therefore,

Mass flow rate striking the vanes,

$$\dot{m} = \rho \cdot A \cdot V$$

Normal force exerted on vanes,

$$F_n = \dot{m}\ [(V-u) - \{-(V-u)\cos\theta\}]$$

$$= \rho AV (V-u)(1+\cos\theta) \quad ...(2.7.5)$$

Rate of work done,

$$W = F_n \cdot u = \rho \cdot A \cdot V (V-u)(1+\cos\theta) \cdot u \quad ...(2.7.6)$$

Efficiency of system, η

$$\eta = \frac{\text{Workdone, W}}{\text{K.E.of jet, } \frac{1}{2}\dot{m}V^2} = \frac{\rho AV(V-u)(1+\cos\theta)u}{\frac{1}{2}\cdot \rho AV \cdot V^2}$$

$$\eta = \frac{2(V-u)(1+\cos\theta)u}{V^2} \quad ...(2.7.7)$$

Condition for maximum efficiency is $\frac{d\eta}{du} = 0$

$$\frac{d}{du}\left[\frac{2(V-u)(1+\cos\theta)u}{V^2}\right] = 0$$

$$\frac{2}{V^2}[(1+\cos\theta)(V-2u)] = 0$$

$$\therefore \quad V = 2u \quad ...(2.7.8)$$

Using the Equation (2.7.4), the maximum efficiency becomes,

$$\eta_{max} = \left(\frac{1+\cos\theta}{2}\right) \quad ...(2.7.9)$$

This efficiency is twice that of flat vane given by Equation (2.7.2).

For semi-circular vanes, $\theta = 0$. In this case deflection of jet becomes 180°.

$$\therefore \quad \eta_{max} = 1 \text{ i.e. } 100\%$$

2.7.3 Impact of Jet on Unsymmetrical Moving Curved Vane When Jet Strikes on One Tip of Vane Tangentially and Leaves at the Other

Case I : In case of single vane

Consider a jet of water leaving the nozzle at absolute velocity V_1 and the vane moves with a velocity u shown in Fig. 2.7.3.

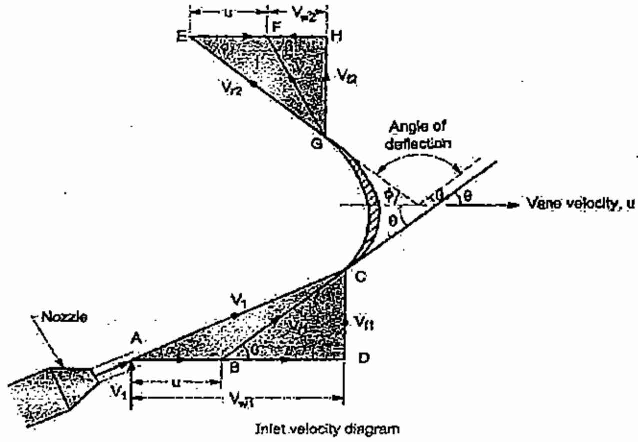


Fig. 2.7.3 : Jet striking one tip of moving vane tangentially

The jet will enter at relative velocity, $\vec{V_{r1}} = \vec{V_1} - \vec{u}$, tangentially at inlet angle of vane θ.

Let, V_1 = absolute velocity of jet;

u = vane velocity at inlet of vane

α = exit angle of nozzle

θ = inlet angle of moving vane

ϕ = exit angle of moving vane

β = angle at which the jet leaves the vane

V_{r1} = relative velocity at inlet

V_2 = absolute velocity of jet at outlet

V_{r2} = relative velocity at outlet of vane.

All angles are measured with direction of motion of vane.

V_w and V_f are the component of absolute velocity *in the direction of motion of vane and normal to it respectively.* V_ω *is called* **velocity of whirl or tangential velocity** and V_f is called **velocity of flow.** Therefore,

V_{w1} = Velocity of whirl at inlet

V_{w2} = Velocity of whirl at outlet

V_{f1} = Velocity of flow at inlet.

V_{f2} = Velocity of flow at outlet.

Inlet velocity diagram

Draw AB to represent the vane velocity u in magnitude and direction.

Draw AC to represent jet velocity V_1 at an angle α in magnitude and direction.

Then vector BC = $\vec{V}_{r1}$ which is equal to $(\vec{V}_1 - \vec{u})$ represents the relative velocity at inlet which strikes the vane tangentially at an angle θ.

V_{w1} and V_{f1} are the components of V_1 which represent whirl velocity and velocity of flow at inlet.

Whirl component of velocity, V_w is responsible to produce the driving force while the flow component of velocity, V_f is responsible for flow of water.

Outlet diagram

The water will pass over the vane and leave tangentially with relative velocity V_{r2} at an outlet angle of vane ϕ.

The absolute velocity of jet V_2 at outlet can be determined by vector sum of V_{r2} and u. It leaves the vane at angle β.

V_{w2} and V_{f2} are components of absolute velocity V_2 is the direction of motion of vane and normal to it respectively.

If friction between water and vane is neglected then :

Relative velocity at outlet,

$$V_{r2} = \text{Relative velocity at inlet, } V_{r1} \quad ...(i)$$

Mass flow rate of water striking the vane,

$$\dot{m} = \rho \cdot A \cdot V_{r1} \quad ...(ii)$$

Force exerted in direction of motion of vane, F_x

Force F_x is produced due to change in whirl velocity. Therefore,

Normal force, $$F_n = F_x = \dot{m}(\vec{V}_{w1} - \vec{V}_{w2}) \quad ...(2.7.9(A))$$

but $\vec{V}_{w2}$ is negative to $\vec{V}_{w1}$ when β < 90°. Therefore magnitude of force,

$$F_x = \dot{m}\,[V_{w1} - (-V_{w2})]$$

$$= \dot{m}(V_{w1} + V_{w2}) \quad ...(2.7.10)$$

Equation (2.7.10) above is applicable only when $\beta_2 < 90°$ as shown in Fig. 2.7.3.

When $\beta_2 = 90°, V_{w2} = 0$

When $\beta_2 > 90°$ i.e. β_2 is an obtuse angle. Then $\vec{V}_{w2}$ will be positive and in the same direction as $\vec{V}_{w1}$. Equation (2.7.10) for F_x can be written as,

$$F_x = \dot{m}(V_{w1} - V_{w2}) \quad ...(2.7.10(A))$$

$$\therefore \text{ In general, } F_x = \dot{m}(V_{w1} \pm V_{w2}) \quad ...(2.7.10(B))$$

Workdone (W)

Workdone per second is given as,

W = Force × distance moved by vane in the direction of force per second

$$= \dot{m}(V_{w1} + V_{w2}) \times u \text{ Nm/s} \quad \text{or} \quad W$$

$$= \rho \cdot A \cdot V_{r1}(V_{w1} + V_{w2})\, u \quad ...(2.7.11)$$

Workdone per unit weight of fluid,

$$W_1 = \frac{\rho A V_{r1}(V_{w1} + V_{w2})\, u}{g \cdot \rho \cdot A \cdot V_{r1}}$$

$$= \frac{(V_{w1} + V_{w2})\, u}{g} \quad ...(2.7.11(A))$$

Efficiency of jet, η

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\rho A V_{r1}(V_{w1} + V_{w2})\, u}{\left(\frac{1}{2} m V_1^2\right)}$$

Initial kinetic energy of jet,

Where, m = Rate of mass flow of water per second in the jet = ρ AV_1

$$\therefore \eta = \frac{\rho A V_{r1}(V_{w1} + V_{w2})\, u}{\frac{1}{2}(\rho \cdot A V_1) V_1^2} = \frac{2V_{r1}(V_{w1} + V_{w2})\, u}{V_1^3} \quad ...(2.7.12)$$

Case II : In case of series of curved vanes

Mass flow rate striking the vanes = Mass flow rate issued by jet = ρ · A · V_1. Equation (2.7.11) for work and Equation (2.7.12) for efficient can be modified as :

∴ Rate of workdone,

$$W = \rho A V_1 (V_{w1} + V_{w2})\, u \quad ...(2.7.13)$$

Efficiency of jet, $$\eta = \frac{2(V_{w1} + V_{w2})\, u}{V_1^2} \quad ...(2.7.14)$$

2.7.4 Force Exerted on a Series of Moving Radial Curved Vanes

University Questions

Q. Define angular momentum and explain how it is used to determine the torque and work done by a vane in case of a radial flow runner. **SPPU : Dec. 13**

Q. Prove that the work done per second on a series of moving curved vanes by a jet of water striking at one of the tips of the vane tangentially is given by workdone/sec = ρaV_1 (V_{w1} ± V_{w2}) × u. **SPPU : May 19**

For a radial curved vane, the radius of vane at inlet and outlet is different, hence, the blade velocity u will be different at inlet and outlet.

Consider a series of radial vanes mounted on a wheel as shown in Fig. 2.7.4.

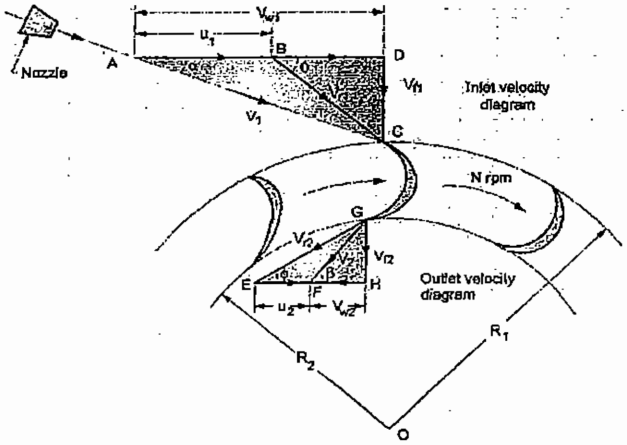


Fig. 2.7.4 : Series of radial vanes mounted on a wheel

Let, N = Speed of wheel be in r.p.m.

R_1 = Radius of wheel at inlet

R_2 = Radius of wheel at outlet

Angular speed of wheel, $w = \frac{2\pi N}{60}$ rad/s

$\therefore$ Blade velocity at inlet, $u_1 = \omega \cdot R_1$

Blade velocity at outlet, $u_2 = \omega \cdot R_2$

Water may flow radially on vanes either inwards or outwards depending upon whether the water enters the outer or inner periphery of wheel.

Velocity diagram at inlet and outlet of vane can be drawn as explained in section 2.7.3 above and it is shown in Fig. 2.7.4.

Mass of water striking the series of vanes per second,

$$\dot{m} = \rho \cdot A \cdot V_1$$

(where, A = area of jet and V_1 = velocity of jet)

Rate of momentum of water striking the vanes in tangential direction at inlet = $\dot{m}\, V_{w1}$ (where, $V_{w1} = V_1 \cos\alpha$) *and the angular momentum is the moment of the rate of momentum.*

$\therefore$ Rate of angular moment of jet at inlet

$$= \dot{m}\, V_{w1} \cdot R_1 = \rho\, A\, V_1 \times V_{w1} \times R_1 \quad ...(i)$$

Similarly, Rate of angular moment of jet at outlet,

$$= \dot{m}\, V_{w2} \cdot R_2 = \rho\, A\, V_1 \times V_{w2} \times R_2 \quad ...(ii)$$

Torque exerted on wheel, T

The rate of change of angular momentum is defined as torque. Therefore,

Torque, T = Rate of angular momentum at inlet – Rate of angular momentum at outlet

$$\therefore\ T = \rho\, A\, V_1 \times V_{w1} \times R_1 - \rho\, AV_1 \times (-V_{w2}) \times R_2$$

$$= \rho\, A\, V_1 (V_{w1} \cdot R_1 + V_{w2} \cdot R_2) \quad ...(2.7.15)$$

(a) Rate of workdone, W i.e. workdone per second

Rate of workdone,

$$W = T \times \omega = \rho\, AV_1 (V_{w1}\, R_1 \cdot \omega + V_{w2}\, R_2 \cdot \omega)$$

$$= \rho\, AV_1 (V_{w1} \cdot u_1 + V_{w2} \cdot u_2) \text{ Nm/s} \quad ...(2.7.16)$$

(b) Efficiency of radial curve vanes, η

$$\eta = \frac{\text{Rate of workdone, W}}{\text{K.E.of jet}\left(\frac{1}{2}\dot{m}\, V_1^2\right)}$$

$$= \frac{\rho \cdot A\, V_1 (V_{w1} \cdot u_1 + V_{w2} \cdot u_2)}{\frac{1}{2} \cdot \rho\, AV_1 \cdot V_1^2}$$

$$= \frac{2\,(V_{w1} \cdot u_1 + V_{w2} \cdot u_2)}{V_1^2} \quad ...(2.7.16(A))$$

(c) Blade velocity coefficient or friction factor K

When water flows over the radial vanes the relative velocity at outlet, V_{r2} is less than the relative velocity of jet V_{r1} due to energy lost in overcoming the friction. The blade velocity coefficient is defined as the ratio of relative velocity at outlet to the relative velocity at inlet.

$$\therefore\ K = \frac{V_{r2}}{V_{r1}} \quad ...(2.7.17)$$

If friction is neglected K = 1, then $V_{r2} = V_{r1}$.

(d) Axial thrust on wheel, F_a

$$F_a = \dot{m}\,(V_{f1} - V_{f2})$$

$$= \rho\, A\, V_1 (V_{f1} - V_{f2}) \quad ...(2.7.18)$$

(e) Efficiency of radial vanes when friction is neglected

If friction is neglected, there will not be any loss of kinetic energy of fluid.

In this case the, rate of workdone will also be equal to decrease in K.E. of fluid.

$$\therefore\ W = \frac{1}{2}\dot{m}\,(V_1^2 - V_2^2)$$

$$= \frac{1}{2}\rho\, AV_1 (V_1^2 - V_2^2) \quad ...(i)$$

$$\text{Efficiency, } \eta = \frac{W}{\text{K.E.supplied}}$$

$$= \frac{\frac{1}{2}\dot{m}\,(V_1^2 - V_2^2)}{\frac{1}{2}\dot{m}\,V_1^2}$$

$$\eta = 1 - \frac{V_2^2}{V_1^2} \qquad ...(2.7.19)$$

From Equation (2.7.19), it is clear that the efficiency would be maximum when V_2 is minimum.

However, V_2 cannot be made zero since the fluid entering the vane will not flow out.

Ex. 2.7.1 : A jet of water having a velocity of 40 m/s strikes a curved vane, which is moving with a velocity of 20 m/s. The jet makes an angle of 30° with the direction of vane at inlet and leaves at an angle of 90° to the direction of vane at outlet. Determine vane angles at inlet and outlet so that water enters and leaves the vane without shock.

SPPU - Feb.16 (In Sem), 6 Marks

Soln. : Refer Fig. P. 2.7.1

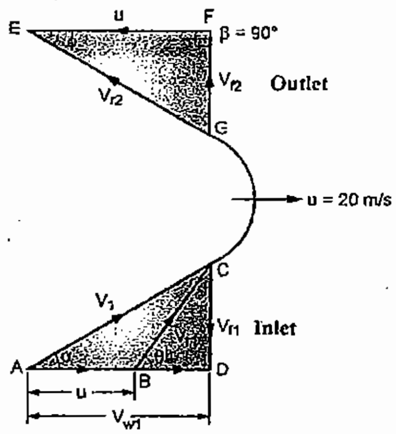


Fig. P. 2.7.1

$$V_1 = 40 \text{ m/s}, \qquad u = 20 \text{ m/s},$$

$$\alpha = 30°,$$

$$\beta = 90°, \text{ therefore } V_{f_2} = V_2$$

Vane angle at inlet θ and at outlet, ϕ

Consider inlet Δ ACD,

$$V_{w1} = V_1 \cos\alpha = 40 \cos 30 = 30.64 \text{ m/s}$$

$$V_{f1} = V_1 \sin\alpha = 40 \sin 30 = 20 \text{ m/s}$$

$$BD = AD - AB = V_{w1} - u = 30.64 - 20 = 10.64 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{BD}\right) = \tan^{-1}\left(\frac{20}{10.64}\right)$$

$$= \mathbf{61.99°} \qquad \text{...Ans.}$$

$$V_{r1} = \sqrt{V_{f_1}^2 + BD^2} = \sqrt{(20)^2 + (10.64)^2}$$

$$= 22.65 \text{ m/s}$$

$$V_{r2} = V_{r1} = 22.65 \text{ m/s}$$

From outlet Δ EFG,

$$\theta = \cos^{-1}\left(\frac{u}{V_{r2}}\right) = \cos^{-1}\left(\frac{20}{22.65}\right) = \mathbf{28°} \qquad \text{...Ans.}$$

Ex. 2.7.2 : A jet of water 8 cm in diameter and at a velocity of 20 m/s of curved vanes at the centre moving at a velocity of 7 m/s. The vanes are so arranged that each vane appears before the jet in the same position and at the same velocity. The jet is deflected through 160°. Find the normal force exerted on vanes, the workdone per second and the efficiency of the system.

Soln. :

Given : Diameter of jet, d = 8 cm;

$$V = 20 \text{ m/s}; \qquad u = 7 \text{ m/s};$$

$$\theta = 180 - 160 = 20°$$

Mass flow rate of jet,

$$\dot{m} = \rho \cdot A \cdot V = \rho \times \frac{\pi}{4} d^2 \times V$$

$$= 1000 \times \frac{\pi}{4}\left(\frac{8}{100}\right)^2 \times 20$$

$$= 100.53 \text{ kg/s}$$

(i) Normal force exerted on vanes, F_n

[Refer Equation (2.7.1)]

$$F_n = \dot{m}\,(V - u)(1 + \cos\theta)$$

$$= 100.53\,(20 - 7)\,(1 + \cos 20)$$

$$= \mathbf{2534.96 \text{ N}} \qquad \text{...Ans.}$$

(ii) Workdone/s, $\dot{W}$

$$\dot{W} = F_n \cdot u = 2534.96 \times 7$$

$$= \mathbf{17744.7 \text{ Nm/s or W}} \qquad \text{...Ans.}$$

(iii) Efficiency of system, η

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{F_n \cdot u}{\frac{1}{2}\dot{m}\,V^2} = \frac{2534.96 \times 7}{\frac{1}{2} \times 100.53 \times (20)^2}$$

$$= \mathbf{0.8826 \text{ or } 88.26\%} \qquad \text{...Ans.}$$

Ex. 2.7.3 : In Ex. 2.7.2, if the vanes are hemispherical in shape. Find the normal force, rate of workdone and efficiency of system.

Soln. :

Vanes are hemispherical in shape, therefore, the jet will be deflected through 180°. Therefore, θ = 180 − 180 = 0°

Normal force, $F_n = \dot{m}(V-u)(1+\cos\theta)$

$= 100.53\,(20-7)(1+\cos 0)$

$= \mathbf{2613.78\ N}$ ...Ans.

Rate of workdone, $\dot{W}$:

$\dot{W} = F_n \times u = 2613.78 \times 7$

$= \mathbf{18296.46\ W}$ ...Ans.

Efficiency of system, η

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\dot{W}}{\frac{1}{2}\dot{m}V^2} = \frac{18296.46}{\frac{1}{2}\times 100.53\times(20)^2}$$

$= \mathbf{0.91\ or\ 91\%}$ ...Ans.

Ex. 2.7.4 : A jet of water 5 cm diameter, moving with a velocity of 25 m/sec strikes horizontally a single moving vane, moving in the direction of jet with a velocity of 16 m/sec. The vane deflects the jet through 130°. Find the axial force exerted by the jet on the vane. Also, find the velocity and direction of the water at outlet. Neglect friction.

SPPU - May 14, 9 Marks, Oct. 19(In Sem.), 6 Marks

Soln. : Refer Fig. P. 2.7.4.

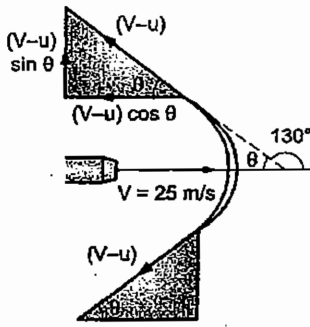


Fig. P. 2.7.4

Diameter of jet, $d = 5\text{ cm} = 0.05\text{ m}$

Jet velocity, $V = 25\text{ m/s}$

Vane velocity, $u = 16\text{ m/s}$

Jet deflection, $\theta = 180° - 130° = 50°$

1. Velocity and direction of water of outlet

Velocity of water at outlet $= V - u = 25 - 16$

$= \mathbf{9\ m/s}$ ...Ans.

Direction of water, $\theta = 180 - 130 = \mathbf{50°}$ ...Ans.

2. Axial force exerted by the jet, F_n

Cross-sectional area of jet,

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.05)^2$

$= 1.9635 \times 10^{-3}\ m^2$

Mass flow rate of water,

$\dot{m} = \rho A (V-u)$

$= 1000 \times (1.9635\times10^{-3}) \times (25-16)$

$= 17.6715\text{ kg/s}$

$F_n = \dot{m}$ [Initial velocity – final velocity]

$= \dot{m}[(V-u) - \{-(V-u)\cos\theta\}]$

$= \dot{m}(V-u)(1+\cos\theta)$

$= 17.6715\,(25-16)(1+\cos 50)$

$= \mathbf{261.275\ N}$ ...Ans.

Ex. 2.7.5 : A 2.5 cm diameter jet strikes without shock on a series of vanes. The jet velocity is 60 m/s and the vanes move with velocity of 14 m/s in the same direction as the jet. The jet is deflected through an angle of 150° and the relative velocity reduces by 10% as water flows across the vanes. Calculate power developed and Efficiency.

SPPU : Aug. 18(In Sem.), 6 Marks

Soln. : Refer Fig. P. 2.7.5

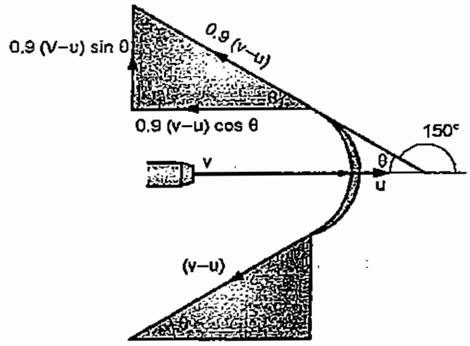


Fig. P. 2.7.5

Diameter of jet, $d = 2.5\text{ cm} = 0.025\text{ m}$

$V = 60\text{ m/s}$, Vane velocity, $u = 14\text{ m/s}$

$\theta = 180 - 150° = 30°$

Relative velocity with which the jet strikes the vane,

$= (V-u)$

Velocity of exit $= (v-u)(1 - \text{friction losses})$

$$= (V-u)\left(1-\frac{10}{100}\right) = 0.9\,(v-u)$$

$\dot{m}$ = area of jet × Relative velocity of jet × density of water

$$= \frac{\pi}{4} d^2 \times (v - u) \times \rho$$

$$= \frac{\pi}{4} (0.025)^2 \times (60 - 14) \times 1000$$

$$= 22.58 \text{ kg/s}$$

$$F_x = \dot{m} \{ (V - u) - [-0.9 \times (V - u) \cos \theta] \}$$

$$= 22.58 \{ [(60 - 14) + 0.9 (60 - 14) \cos 30] \}$$

$$= 1848.25 \text{ N}$$

1. **Power developed, P**

$$P = F_x \cdot u = 1848.25 \times 14$$

$$= \mathbf{25875.5 \text{ Nm/s}}$$ **... Ans.**

2. **Efficiency, η**

$$\eta = \frac{P}{\frac{1}{2} \dot{m} V^2} = \frac{2P}{\dot{m} V^2} = \frac{2 \times 25875.5}{22.58 \times (60)^2}$$

$$= \mathbf{0.6366 \text{ or } 63.66\%}$$ **... Ans.**

Ex. 2.7.6 : A jet discharges 0.15 m³/sec of water with a velocity of 70 m/sec impinges without shock on a series of curved vanes which move in the same direction as the jet. The shape of each vane is such that it would deflect the jet through an angle of 150°. Surface friction reduces the relative velocity by 8 percent as the water passes across the vanes and there is further windage loss equivalent to $\left[\frac{0.5u^2}{2g}\right]$ N.m per N of water, u being the vane velocity. Find

(i) The velocity on vanes corresponding to maximum efficiency

(ii) The value of this efficiency

(iii) Corresponding force on the vanes in and at right angles to the direction of their motion

(iv) The power of this arrangement

SPPU - May 12, 10 Marks

Soln. :

Refer Fig. P. 2.7.6

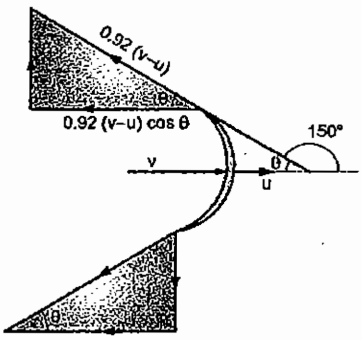


Fig. P. 2.7.6

Given : $Q = 0.15 \text{ m}^3/\text{s}$

Windage losses = $(0.5u^2/2g)$

$V = 70 \text{ m/s}$

$\theta = 180° - 150° = 30°$

Relative Velocity with which the jet strikes the vane = $(V - u)$

Velocity at exit = $(V - u)$ (1 – friction losses)

$$= (V - u)\left(1 - \frac{8}{100}\right) = 0.92 (V - u)$$

Force of water per unit mass in x - direction,

$$F_x = (V - u) - [-0.92 (V - u) \cos \theta]$$

$$= (V - u) (1 + 0.92 \cos \theta)$$

Work done / N of water, $W = F_x \cdot u$ – Windage losses

$$= \frac{(V - u)}{g} (1 + 0.92 \cos \theta) \cdot u - \frac{0.5 u^2}{2g}$$

$$\eta = \frac{W}{\left(\frac{V^2}{2g}\right)}$$

$$= \frac{1}{V^2} [2 (V - u) (1 + 0.92 \cos \theta) u - 0.5 u^2]$$

$$= \frac{1}{V^2} [2 (V \cdot u - u^2) (1 + 0.92 \cos \theta) - 0.5 u^2] \quad ...(a)$$

(i) Velocity of vanes corresponding to maximum efficiency

Condition for maximum efficiency is that

$\frac{d\eta}{du} = 0$, therefore from Equation (a),

$$0 = [2 (V - 2u) (1 + 0.92 \cos \theta) - 0.5 \times 2u]$$

On substituting the values :

$$0 = 2 (70 - 2u) (1 + 0.92 \cos 30) - u$$

$$= (70 - 2u)\, 3.5935 - u$$

$$251.544 - 7.187 u - u = 0$$

$$u = \frac{251.544}{8.187} = \mathbf{30.725 \text{ m/s}}$$ **...Ans.**

(ii) Value of maximum efficiency, η_{max}

From Equation (a) at u = 30.725 m/s we get :

$$\eta_{max} = \frac{1}{V^2} [2 (V - u) (1 + 0.92 \cos \theta) u - 0.5 u^2]$$

$$= \frac{1}{(70)^2} [2 (70 - 30.725) (1 + 0.92 \cos 30) \times 30.725 - 0.5 \times 30.725^2]$$

= 0.7886 or 78.86 %. **...Ans.**

(iii) Force on vanes in at right angles to direction of motion

$$\dot{m} = \rho.Q = 1000 \times 0.15 = 150 \text{ kg/s}$$

$$F_x = \dot{m} [(V - u) (1 + 0.92 \cos \theta)]$$

$$= 150\,[(70 - 30.725)(1 + 0.92\cos 30)]$$

$$= \mathbf{10585.06\ N} \quad \text{...Ans.}$$

$$F_y = \dot{m}\,[(V - u)(0 + 0.92\sin\theta)]$$

$$= 150\,[(70 - 30.725)(0.92\sin 30)]$$

$$= \mathbf{2709.98\ N} \quad \text{...Ans.}$$

(iv) Power developed, P

$$P = F_x \cdot u \times \eta_{max}$$

$$= 10585.06 \times 30.725 \times 0.7886\ \text{N m/s or W}$$

$$= 256479\ \text{W}$$

$$= \mathbf{256.479\ kW} \quad \text{...Ans.}$$

Summary

– According to II[nd] Newton's law, the **applied force,**

$$F = \frac{d}{dt}(m\,V) \quad ...(2.2.1)$$

$$= m \cdot \frac{dV}{dt} + V \cdot \frac{dm}{dt}$$

– **In fluid mechanics,** the mass flow rate of fluid is constant i.e. $\frac{dm}{dt} = 0$. Therefore, Equation (2.2.1) can be modified as :

$$F = m \cdot \frac{dv}{dt} = \dot{m}\,(V_2 - V_1)$$

$$F = \rho\,Q\,(V_2 - V_1);$$

where, $\dot{m} = \rho \cdot Q = \rho \cdot A \cdot V$

∴ **Force exerted by the fluid on body, according to III[rd] Newton's law will be,**

$$F = \rho\,Q\,(V_1 - V_2) = \dot{m}\,V_1 - \dot{m}\,V_2$$

– **Force exerted by jet on fixed plate**

A = area of jet, V = Velocity of jet

(a) On vertical plate :

$$F_n = F_x = \dot{m}\,V = (\rho\,A\,V)\,V = \rho \cdot A \cdot V^2$$

(b) On inclined plate at an angle θ with jet

$$F_n = \rho \cdot A \cdot V^2 \sin\theta$$

$$F_x = \rho \cdot A \cdot V^2 \sin^2\theta$$

$$F_y = \rho \cdot A \cdot V^2 \cdot \frac{\sin 2\theta}{2}$$

(c) On curved plate or vane when jet strikes at centre :

$$F_x = \rho \cdot A \cdot V^2 (1 + \cos\theta)$$

$$F_y = \rho \cdot A \cdot V^2 \sin\theta$$

(d) On curved unsymmetrical plate when jet strikes at one end tangentially :

$$F_x = \rho \cdot A \cdot V^2 (\cos\theta + \cos\phi)$$

$$F_y = \rho \cdot A \cdot V^2 (\sin\theta - \sin\phi)$$

– **Force exerted by jet on hinged vertical plate of height 'h'**

y = distance from hinge at which jet strikes.

(a) Holding force, $P = \rho \cdot A \cdot V^2 \cdot \frac{y}{h}$

(b) Angle turned by hinged plate with no holding force

$$\sin\theta = \rho \cdot A \cdot V^2 \cdot \frac{2y}{W \cdot h}$$

where, W = weight of plate

– **Force exerted by Jet on moving plates with velocity u.**

(a) Flat plate

$$\dot{m} = \rho \cdot A\,(V - u)$$

$$F_n = \rho \cdot A \cdot (V - u)^2$$

Workdone, $W = F_n \cdot u$

$$\text{K.E. supplied} = \frac{1}{2}(\rho \cdot A\,V)\,V^2$$

Efficiency, $\eta = \frac{W}{K.E.}$

– **Condition for maximum efficiency, : V = 3 u**

$$\eta_{max} = \frac{8}{27}$$

– **On inclined plate at angle θ.**

$$F_n = \rho \cdot A\,(V - u)^2 \cdot \sin\theta$$

$$F_x = \rho \cdot A \cdot (V - u)^2 \cdot \sin^2\theta$$

$$F_y = \rho\,A\,(V - u)^2 \cdot \sin\theta\cos\theta$$

Workdone,

$$W = F_x \cdot u = [\rho \cdot A\,(V - u)^2 \cdot \sin^2\theta]\,u$$

Kinetic energy supplied (K.E.) $= \frac{1}{2} \cdot (\rho \cdot A \cdot V)\,V^2$

– **Force exerted by jet on a series of moving flat plates :** $\dot{m} = \rho \cdot A \cdot V$

(a) Jet striking tangentially on flat plate or vanes

$$F_n = \dot{m}\,(V - u) = \rho\,A\,V\,(V - u)$$

Workdone, $W = F_n \cdot u = \rho \cdot AV\,(V - u)\,u$

$$\text{K.E. supplied (K.E.)} = \frac{1}{2}\rho\,AV^3$$

$$\text{Efficiency, } \eta = \frac{2\,(V - u)\,u}{V^2}$$

- **Condition for maximum efficiency is :**
  $V = 2u$; $\eta_{max} = 50\%$
- **Force exerted by jet on single moving curved vane at velocity u in direction jet.**

$$\dot{m} = \rho A (V - u)$$

$$F_n = \rho \cdot A (V - u)^2 (1 + \cos\theta)$$

Workdone, $W = F_n \cdot u$

$$K.E. = \frac{1}{2} \rho AV (V^2)$$

Efficiency, $\eta = \frac{2 (V - u)^2 \cdot (1 + \cos\theta) u}{V^3}$

- **Condition for maximum efficiency**

$$V = 3u ; \quad \eta_{max} = \frac{8}{27} (1 + \cos\theta)$$

- **Force exerted by jet on series of moving curved vanes in direction of jet.**

$$\dot{m} = \rho \cdot A \cdot V$$

$$F_n = \rho \cdot A \cdot V (V - u) (1 + \cos\theta)$$

Workdone, $W = F_n \cdot u$

Efficiency, $\eta = \frac{W}{K.E.} = \frac{2 (V - u) (1 + \cos\theta) u}{V^2}$

Condition for maximum efficiency,

$$V = 2u ; \quad \eta_{max} = \frac{(1 + \cos\theta)}{2}$$

For semi-circular vanes : $\theta = 0°$

$\therefore \eta_{max} = 1$ or 100%

- **Force exerted by jet on moving curved vanes when jet strikes the unsymmetrical vane at one tip and leaves at the other.**

Case I : Single vanes

The force exerted on the vane and the workdone is obtained from velocity triangles at inlet (1) and outlet (2).

$$\dot{m} = \rho \cdot A \cdot V_{r1}$$

$$F_n = F_x = \dot{m} (V_{w1} \pm V_{w2})$$

Workdone, $W = F_n \cdot u = \rho \cdot A \cdot V_{r1} (V_{w1} \pm V_{w2}) u$

$$\text{Efficiency} = \frac{W}{K.E.supplied} = \frac{W}{\frac{1}{2} (\rho \cdot A \cdot V_1) V_1^2}$$

Case II : On series of curved vanes

Mass flow rate striking the vanes, $\dot{m} = \rho \cdot A \cdot V_1$

$$W = \rho \cdot A \cdot V_1 (V_{w1} + V_{w2}) u$$

$$\eta = \frac{2 (V_{w1} + V_{w2}) u}{V_1^2}$$

Force exerted on series of moving radial vanes

Mass flow rate, $\dot{m} = \rho \cdot A \cdot V_1$

Torque, $T = \rho A V_1 (V_{w1} R_1 + V_{w2} R_2)$

Rate of workdone, $W = \rho A V_1 (V_{w1} \cdot u_1 + V_{w2} \cdot u_2)$

Efficiency, $\eta = \frac{2 (V_{w1} \cdot u_1 + V_{w2} \cdot u_2)}{V_1^2}$

Exercise

Q. 1 What is meant by jet Propulsion ? Explain **[Section 2.1(ii)]**

Q. 2 Explain the impulse momentum principle. **[Section 2.2]**

Q. 3 Derive the expression for the force exerted by a jet of water of velocity V and area A on a plate when : (i) Plate is vertical (ii) Plate is inclined at θ. **[Sections 2.5.1 and 2.5.2]**

Q. 4 Prove that the force exerted by a jet of water on a fixed semi-circular plate in the direction of the jet when the jet strikes at the centre of the semi circular plate is two times the force exerted by the jet on the fixed vertical plate. **[Section 2.4]**

Q. 5 Show that the force exerted on an unsymmetrical curved plate by a jet of velocity V area is given by :

$F_x = \rho A V^2 (\cos\theta + \cos\phi)$

$F_y = \rho A V^2 (\sin\theta - \sin\phi)$

Where θ and ϕ are inlet and exit angles and the jet enters and leave the plate tangentially.

Deduce the expression for a semi-circular plate and show that it is equal to $2 \rho A V^2$. **[Sections 2.4.2 and 2.4.3]**

Q. 6 A jet at velocity V strikes the hinged plate of height h and weight W at its centre. Show that the angle turned by plate is given as,

$\sin\theta = \frac{\dot{m} V}{W}$ **[Section 2.5]**

Q. 7 Show that the power developed by a jet of area A at velocity V when strikes a moving plate at velocity u is given by : Power, $P = \rho A (V - u)^2$ ·
Show that V = 3u is the condition for maximum efficiency of the system. **[Section 2.6.1]**

Q. 8 Derive an expression for the force excreted by a jet of water on a moving inclined plate in the direction of plate. **[Section 2.6.2]**

Q. 9 Show that, in case of jet striking the flat plate mounted on wheel, efficiency shall be maximum when the tangential velocity of wheel is half that of the jet. **[Section 2.6.3]**

Q. 10 Consider a single, symmetric 2D curved vane having centrally impinging 2D water jet. Jet cross-section area is A and density of water is ρ. Velocity of the jet is V and the vane moves at velocity u in the same direction as the jet. The turning angle of the vane on each side is θ. Derive the expression for hydraulic efficiency η of the vane in terms of the speed ratio u/V and the half angle of the vane θ. Then derive the condition for maximum efficiency for given angle θ. Hence obtain the maximum efficiency for :

(i) A semicircular vane,

(ii) A flat plate perpendicular to the flow. **[Section 2.7.1]**

Q. 11 A jet of water strikes at the centre of a curved vane. The jet speed is 'V' m/s and the vane moves at 'u' m/s in the direction of the jet. If the outgoing jet makes an angle 'θ' with the entering jet, prove that : (i) for maximum efficiency

(i) $u = \frac{V}{3}$

(ii) Maximum efficiency $= \frac{8}{27}(1 + \cos\theta)$

[Section 2.7.1]

Q. 12 In Q. 11, in case the jet strikes the series of vanes show that the efficiency is,

$\eta = \frac{2(V-u)(1+\cos\theta)u}{V^2}$ and, show that maximum efficiency occurs when V = 2u **[Section 2.7.2]**

Q. 13 Show that when a jet of water impinging normally on a series of curved vanes, maximum efficiency is obtained when the vane is semicircular in section and velocity of the vane is half that of the jet. **[Section 2.7.2]**

Q. 14 Define angular momentum and explain how it is used to determine the torque and work done by a vane in case of a radial flow runner. **[Section 2.7.4]**

❑❑❑

3 UNIT - I

Hydraulic Turbines

Syllabus

Introduction to Hydro power plant, Classification of Hydraulic Turbines, Concept of Impulse and Reaction Turbines. Construction, Principle of Working, design aspects, velocity diagrams and its analysis of Pelton wheel, Francis, and Kaplan turbines, Degree of reaction, Draft tube: types and efficiencies, governing of hydraulic turbines, Cavitation in turbines.

SECTION - I : Impulse Water Turbine

3.1 Introduction

A **hydraulic turbine** is a machine which converts the pressure and kinetic energy of water called *hydraulic energy* into mechanical energy. These are also called as *water turbines*.

The mechanical energy of turbine is further converted into electric energy by an *electric generator* which is directly coupled to the shaft of hydraulic turbine. The electrical power generated is known as *hydro-electric power*.

Hydraulic turbines are efficient. These have low wear and tear and ease of maintenance. However their capital cost is high with long gestation period due to the requirement of constructing the dam across the river and laying the long pipe lines.

3.1.1 Classification of Hydraulic Turbines

University Questions

Q. Explain the classification of water turbines with examples. **SPPU : Dec. 15**

Q. Explain the classification of hydraulic turbines in detail with examples. **SPPU : Feb. 16 (In Sem)**

Hydraulic turbines are classified based on the energy available at the inlet of turbine, the availability of head, direction flow over the vanes in the runner and its specific speed.

Following is the important classification of turbines.

1. **According to the type of energy available at inlet to the turbine**

   (i) Impulse turbine
   (ii) Reaction turbines

(i) Impulse turbine

- An impulse turbine, as the name suggests, works on the principle of impulse.
- In these turbines, the head or pressure energy of water is first converted into kinetic energy by means of a nozzle or set of nozzles kept close to the runner.
- This high velocity jet produced by nozzle is allowed to impinge on the set of buckets fixed on the outer periphery of the wheel or runner. The direction of jet is changed by buckets. The change of momentum of water causes the wheel to rotate, thus produces mechanical energy.
- It should be noted that the pressure of water is atmospheric and remains constant while passing over the runner.
- Examples of important impulse turbines are *pelton wheel, Girard turbine, Turgo turbine* etc.

(ii) Reaction turbines

- In these turbines, a part of pressure energy is first converted into kinetic energy before supplied to runner.
- Turbine to run in closed passages which are completely filled with supplied to runner.
- Therefore, the water enters the runner having partly the pressure energy and partly the kinetic energy and both these energies are reduced simultaneously while passing over the runner and produce mechanical energy. Hence, these turbines work on the principle of impulse-reaction.

- The runner of these turbines being under pressure above atmospheric, it requires the blades of turbine to run is closed passages which are completely filled with water in all conditions.
- Examples of reaction turbine are Francis, Kaplan and Propeller turbines.

2. According to direction of flow though runner

(i) **Tangential flow turbines** in which water flows tangent to the runner.

(ii) **Radial flow turbines** in which the water flows in the radial direction through the runner.

(iii) **Axial flow turbines** in which the water flows through the runner along the direction parallel to the axis of rotation of runner.

(iv) **Mixed flow turbines** in which water flows in the runner in radial direction but leaves in axial direction.

3. According to the head available at inlet to the turbine

(i) Low head turbines (2 m to 15 m)

(ii) Medium head turbines (16 m to 70 m)

(iii) High head turbines (71 m and above)

4. According to the specific speed (Ns) of the turbine

(i) Low specific speed.

(ii) Medium specific speed.

(iii) High specific speed.

3.2 General Layout of a Hydro-Electric Power Plant

University Questions

Q. What do you mean by gross head and net head of turbine ? **SPPU : Dec. 12**

Q. Define the terms gross head and net head. **SPPU : Feb. 16(In Sem)**

Fig. 3.2.1 shows a general layout of a hydro-electric power plant. It has the following main components.

1. A **dam** which is constructed across a river to store water. It provides necessary potential energy to nozzles of a turbine. It acts as water reservoir. It is constructed with masonary or R.C.C. Top surface of water in dam is called **head race.**
2. The **penstock** is a large diameter pipe. It carries water under pressure from the water stored in a dam or reservoir to the turbines.

   The penstock is usually made of reinforced concrete or steel.
3. A **Water turbine** which converts hydraulic energy into mechanical energy.

   A water turbine can be set horizontal or vertical. The choice is governed by cost, type of turbine, building space and plant layout etc.

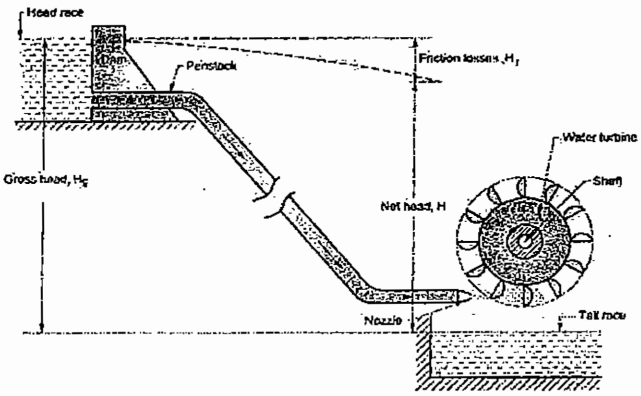


Fig. 3.2.1 : General layout of hydro-electric power plant

4. A **tail race** is a discharge canal into which water is discharged from the turbine.

   Difference of head race and tail race level is called **gross head,** H_g.

   The head of water available at turbine inlet is called **net-head,** H. It is the difference of gross head and the **frictional head,** H_f in penstock i.e.

Net head, H = Gross head H_g – Frictional head, H_f

Frictional head from Darcy Weisback equation is given as :

Frictional head, $$H_f = \frac{4 f \cdot L \cdot V^2}{d \cdot 2g}$$

where,
f = Friction factor
L = Length of pipe
V = Velocity of flow of water
d = Diameter of pipe

3.3 Pelton Wheel or Pelton Turbine

Pelton wheel is a *tangential flow impulse turbine.* It was first developed by an American Engineer Lester Allen Pelton in the year 1880.

In Pelton turbines the water strikes the buckets along the tangent of the runner or wheel.

It is used for high heads more than 100 m. of water. These turbines have been built upto a head of 1600 m.

3.3.1 Construction and Working of Pelton Wheel

University Question

Q. Explain the constructional details of pelton wheel (turbine). **SPPU : Oct. 19(In Sem)**

An arrangement of Pelton turbine is shown in Fig. 3.3.1.

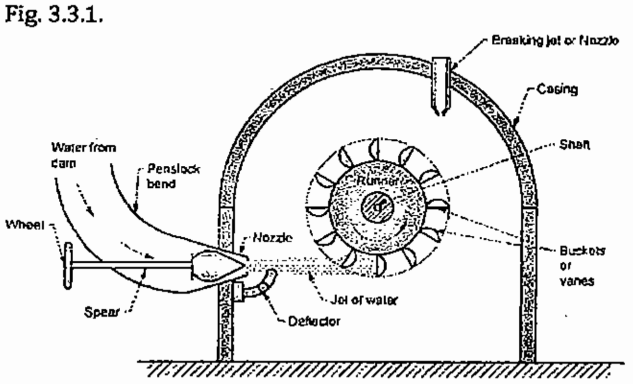


Fig. 3.3.1 : Pelton turbine

The water from reservoir flows through the penstock to the nozzle which converts the pressure energy (high head of water) into kinetic energy. The resultant high velocity jet from nozzle strikes the buckets or vanes fitted at outer periphery of runner. The main components of a pelton wheel are :

1. Nozzle and spear assembly
2. Runner and buckets
3. Casing
4. Braking jet
5. Deflector

1. Nozzle and spear assembly

- The needle spear is provided in the nozzle to regulate the water flow through the nozzle. Also, it provides the smooth flow of water with negligible loss of energy.
- A spear is a conical needle which can be moved in axial direction by operating the wheel either manually or automatically.
- When the spear is moved in forward direction into the nozzle, it reduced the nozzle exit area, hence, the quantity of water flow striking the buckets is reduced. If the spear is moved backwards, it increases the flow rate of water.
- The nozzle converts the potential energy of water into kinetic energy before jet strikes the buckets. Pressure at exit of nozzle is reduced to atmospheric pressure.

2. Runner and buckets

University Questions

Q. Sketch a Pelton Wheel bucket and explain the effect of its size , shape and number on its function. **SPPU : May 14**

Q. Explain the function of Notch of bucket. **SPPU : May 15**

Q. Sketch Pelton wheel bucket giving its approximate dimensions and answer following questions in brief: The ideal jet deflection angle is 180° however bucket deflects the jet through 160° to 165°. **SPPU: April 15(In Sem)**

- The turbine rotor called runner is a circular disc fixed with buckets. It is provided with cylindrical boss and keyed to the supporting shaft in small thrust bearings.
- The runner carries cup-shaped buckets more that 15 in number which are mounted at equidistance around its periphery. The buckets are either cast integrally with the circular disc or these are bolted individually to the runner, it helps in easy replacement of buckets when worn out.
- Buckets are made of cast iron cast steel, special steels or stainless steel with inner surface polished to reduce friction losses of water jet. Type of metal used for bucket depends on the head at turbine inlet.
- The shape of the buckets is of double hemispherical cup or bowl. Each bowl of the bucket is separated by a wall called **splitter** or a **ridge.**
- The shape and dimensions of a bucket are shown in Fig. 3.3.2.
- The commonly adopted dimensions of bucket are :

d = Diameter of jet

L = Length of height of bowl inside the rim = 2d to 3d

B = Width of bucket between the rims of bowl = 3d to 4d

T = Depth of bowl = 0.27 B to 0.32 B

M = Notch width = 1.1 d to 1.2 d

Splitter angle, ψ = 10° to 20°

– The water strikes the bucket at the splitter which splits the water into two equal streams of the hemispherical bowl. The maximum force will be obtained when the jet is deflected through 180° into exact hemispherical bowl.

– However, in practice the jet is deflected through 160° to 170° [splitter angle ψ = 10° to 20°] It avoids striking the exit jet with the back of the succeeding bucket, thus exerting a retarding force on it.

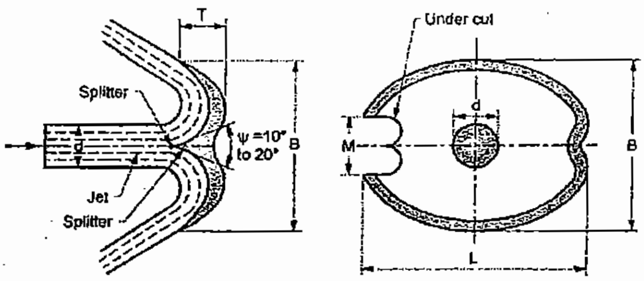


Fig. 3.3.2 : Bucket dimensions

– It would reduce the power output and the overall efficiency of turbine. This also avoids the splashing of water with a splitter. Pelton wheel is provided with two hemispherical cups since the splitter splits the jet into two equal streams, the axial component of each stream velocity is equal and opposite due to which the axial thrust on the shaft is negligible. Therefore, Pelton wheel needs very small thrust bearings.

– An undercut is provided and surface of spoons is raised so that water can be deflected back through the angle of 160° to 170° with the vertical without disturbing the incoming bucket.

3. Casing

University Question

Q. Explain the function of Casing of pelton wheel. **SPPU : May 15**

– A casing does not have any hydraulic function to perform therefore it is not actually needed in case of impulse turbines because the runner runs under atmospheric pressure.

– However, a casing is provided to prevent the splashing of water and lead the water to tail race, and to safeguard the persons against accidents. It is made of cast iron in two halves.

4. Braking Jet

– Whenever the turbine is brought to rest, the nozzle is completely closed by pushing forward the spear. However, the runner continues to rotate due to its inertia for a considerable period of time till it comes to rest.

– In order to bring the runner to stop in a shortest time, a small nozzle is provided which issues the water jet and falls on the back of buckets. It acts as a hydraulic brake for reducing the speed of runner.

5. Deflector

University Question

Q. Define Run away speed. **SPPU : May 19**

– A deflector is provided which is hinged to the casing to deflect the jet of water away from striking the buckets in case the load on turbine suddenly reduces.

– It prevents the runner of turbine attaining unsafe speeds called **runaway speed.**

– A governing mechanism is also provided to control the speed of turbine according to variation in load which shall be discussed in later chapters in detail.

3.4 Work done and Efficiency of Pelton Wheel

University Questions

Q. Explain the different efficiency of a turbine. Draw inlet and outlet velocity triangles for a pelton wheel. **SPPU : Dec. 12**

Q. Obtain an expression for the work done per second by water on the runner of Pelton Wheel and also find the relation between jet speed and bucket speed for maximum efficiency. **SPPU : May 13**

Q. Define the terms hydraulic efficiency and Mechanical Efficiency. **SPPU : Feb 16(In Sem)**

A jet of water strikes the bucket at its splitter. It splits the jet into two parts, each part of jet glides over the inner surface of the cup and leaves the outer tips of the bucket $G - G_1$.

The jet of water striking the splitter gets deflected through an angle (180 – ϕ). Inlet and outlet velocity diagrams can be drawn as explained in theory of jet and it is shown in Fig. 3.4.1 (a) for slow speed runner.

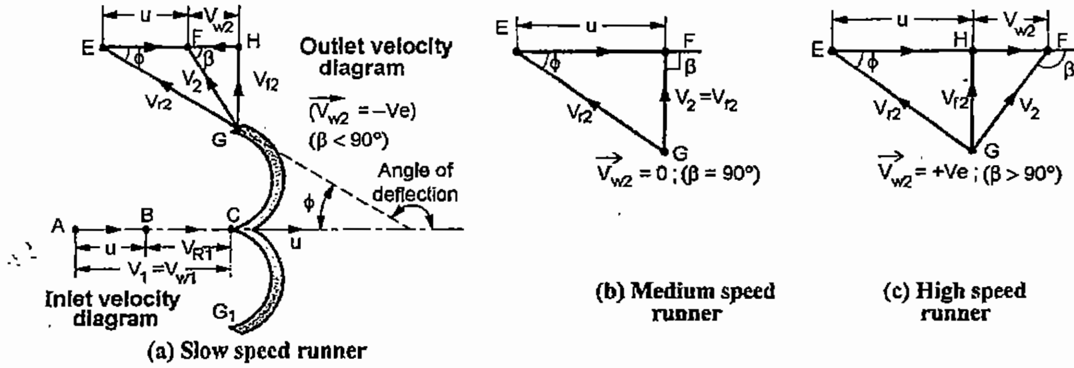


Fig. 3.4.1 : Velocity diagrams of pelton wheel

Figs. 3.4.1(b) and (c) shows that velocity diagram for medium speed and high speed runners respectively.

Let d = diameter of jet in metres.

A = cross-sectional area of jet

Q = discharge rate, m^3/s

D = mean pitch diameter of runner, m

N = speed of runner in r.p.m.

H = net head acting on pelton wheel

$\therefore$ Velocity of jet at inlet,

$$V_1 = C_v\sqrt{2gH} \quad \text{...(3.4.1)}$$

where C_v is the coefficient of velocity of jet, its value varies from 0.91 to 0.99.

$$\text{Blade velocity, } u = \frac{\pi D N}{60} \quad \text{...(3.4.2)}$$

Inlet velocity diagram

The pelton wheel is an *tangential flow turbine*. The blade speed at inlet and outlet is same.

Therefore, $u = u_1 = u_2$; $\alpha = 0$; Bucket inlet angle, $\theta = 0$.

Therefore inlet velocity diagram is a straight line where

$$V_{r1} = V_1 - u$$

$$V_{w1} = V_1 \quad \text{...(i)}$$

Outlet velocity diagram

Relative velocity at outlet $V_{r2} = K \cdot V_{r1}$ where K is called **friction factor**. It's value is slightly less than unity and it is taken as one for a highly polished surface (neglecting friction)

Depending upon the speed of runner, various outlet velocity diagrams are drawn in Fig. 3.4.1.

$$V_{w2} = V_{r2}\cos\phi - u = (KV_{r1}\cos\phi - u) \quad \text{...(ii)}$$

Mass flow rate of water,

$$\dot{m} = \rho \cdot Q = \rho A V_1 \quad \text{...(3.4.3)}$$

(Note : $\dot{m} = \rho AV_1$ since, the jet strikes the series of buckets)

(i) Force exerted by jet of water in direction of motion, F_x

$$F_x = \dot{m}(V_{w1} + \vec{V}_{w2}) \quad \text{...(iii)}$$

Considering the slow speed runner when $\beta < 90°$, $\vec{V}_{w2}$ is negative, therefore,

$$F_x = \dot{m}(V_{w1} + V_{w2}) \quad \text{...(3.4.4)}$$

(ii) Work done by jet on runner per second, W

$$W = F_x \cdot u = \dot{m}(V_{w1} + V_{w2})\,u \quad \text{...(3.4.5)}$$

(iii) Power developed, P

$$P = \frac{W}{1000}\,\text{kW} = \frac{\dot{m}(V_{w1} + V_{w2})\,u}{1000}\,\text{kW} \quad \text{...(3.4.6)}$$

(iv) Hydraulic efficiency of jet, η_h

It is defined as the ratio of power delivered by runner to the power supplied at inlet. The power supplied at inlet is also called **water power** which is equal to kinetic energy of jet supplied to runner.

$$\text{Kinetic energy supplied to jet per second} = \frac{1}{2}\dot{m}\,V_1^2$$

$\therefore$ Hydraulic efficiency,

$$\eta_h = \frac{\text{Workdone per second}}{\text{K.E supplied jet per second}}$$

$$\eta_h = \frac{\dot{m}(V_{w1} + V_{w2})u}{\dot{m} \cdot \frac{V_1^2}{2}}$$

$$\eta_h = \frac{2(V_{w1} + V_{w2})\,u}{V_1^2} \quad \text{...(3.4.7)}$$

Sometimes, the hydraulic efficiency is also called as bucket efficiency.

(v) Condition for maximum hydraulic efficiency, η_h

University Questions

Q. Show that the maximum efficiency of the Pelton wheel is given by $\frac{1 + K\cos\beta}{2}$

where K = Bucket friction factor

β = Bucket outlet angle **SPPU : May 11**

Q. Derive an expression of maximum hydraulic efficiency of Pelton Wheel **SPPU : Dec. 15**

From inlet velocity diagram

$$V_{w1} = V_1 \text{ and } V_{r1} = V_1 - u$$

From outlet velocity diagram,

$$V_{w2} = V_{r2}\cos\phi - u$$
$$= K V_{r1}\cos\phi - u$$
$$= K(V_1 - u)\cos\phi - u$$

On substituting the values of V_{w1} and V_{w2} in Equation (3.4.7) we have,

$$\eta_h = \frac{2[V_1 + K(V_1 - u)\cos\phi - u]u}{V_1^2}$$
$$= \frac{2[(V_1 - u) + K(V_1 - u)\cos\phi]u}{V_1^2}$$
$$= \frac{2(V_1 - u)(1 + K\cos\phi)u}{V_1^2} \quad ...(3.4.8)$$

The efficiency will be maximum for a given jet velocity V_1 when $\frac{d}{du}(\eta_h) = 0$.

$$\text{Therefore, } \frac{d}{du}\left[\frac{2(V_1 - u)(1 + k\cos\phi)u}{V_2^2}\right] = 0$$
$$\frac{2(1 + K\cos\phi)}{V_1^2} \cdot \frac{d}{du}(V_1 \cdot u - u^2) = 0$$
$$V_1 - 2u = 0$$
$$\therefore u = \frac{V_1}{2} \quad ...(3.4.9)$$

From Equation (3.4.9) it follows that the maximum hydraulic efficiency of pelton wheel occurs when the velocity of runner is half the velocity of jet.

On substituting the value $u = \frac{V_1}{2}$ in Equation (3.4.8), maximum hydraulic efficiency of pelton wheel becomes,

$$(\eta_h)_{max} = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1 + K\cos\phi)\frac{V_1}{2}}{V_1^2}$$

$$\text{i.e. } (\eta_h)_{max} = \frac{(1 + k\cos\phi)}{2} \quad ...(3.4.10)$$

In case bucket friction is neglected, then, friction factor k = 1, Equation (3.4.10) reduces to,

$$(\eta_h)_{max} = \left(\frac{1 + \cos\phi}{2}\right) \quad ...(3.4.11)$$

For **hemispherical bucket**, the angle of deflection is 180°, therefore, $\phi = 0$. Outlet velocity diagram will be straight line and maximum hydraulic efficiency in such a case would be 1 i.e. 100%.

(vi) Mechanical efficiency, η_m

University Question

Q. Define Mechanical efficiency **SPPU : May 19**

It is defined as the ratio of power available at turbine shaft to the power developed by runner.

Difference of these two powers is due to mechanical losses caused by friction between mating parts (e.g. in shaft and runner, bearings etc).

∴ Mechanical efficiency,

$$\eta_m = \frac{\text{Power available at turbine shaft, } P_s}{\text{Power developed by runner, } P} \quad ...(3.4.12)$$

(vii) Volumetric efficiency, η_v

It is defined as the ratio of the volume of water actually striking the buckets to the volume of water issued by the jet. Hence,

Volumetric efficiency,

$$\eta_v = \frac{\text{Actual volume of water striking the buckets, } Q_a}{\text{Volume of water issued by the jet, } Q} \quad ...(3.4.13)$$

Volumetric efficiency is less than 100% since some quantity of water misses the bucket and directly it passes to tail race without doing any useful work.

Volumetric efficiency for pelton wheel ranges from 97% to 99%.

(viii) Overall efficiency, η_o

It is defined as the ratio of power available at turbine shaft to the power supplied by the water jet. Accordingly,

Overall efficiency,

$$\eta_o = \frac{\text{Power available at turbine shaft, } P_s}{\text{Power available the water jet, } P_j} \quad ...(3.4.14)$$

where, $P_j = \rho \cdot g \cdot Q \cdot H$

$$\therefore \eta_o = \eta_v \times \eta_h \times \eta_m \quad ...(3.4.15)$$

(ix) Plant efficiency, η_p

If a generator of efficiency η_g is connected to turbine in hydro-electric plant. Then,

Plant efficiency,

$$\eta_p = \frac{\text{Power output from generator, } P_g}{\text{Power supplied by the jet, } P_i} \quad ...(3.4.16)$$

$$= \frac{P_g}{P_i} \times \frac{P_s}{P_s} = \frac{P_g}{P_s} \times \frac{P_s}{P_i} = \eta_g \times \eta_o \quad ...(3.4.17)$$

(x) Specific speed, N_s

Specific speed is the speed of a geometrically similar turbine which would produce unit power under a unit head. It is given by the expression,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \quad ...(3.4.18)$$

Where N is the speed of runner or synchronous speed.

Note : Detailed discussion on specific speed has been dealt in later chapters.

3.5 Design Aspects and Important Points to Remember for Pelton Wheel

1. **Velocity of Jet, V_1**

$V_1 = C_v\sqrt{2gH}$; ...[According to Equation (3.4.1)]

C_v = 0.97 to 0.98

H = net head on turbine

2. **Velocity of wheel, u**

$u = \frac{\pi D N}{60}$; ...[According to Equation (3.4.2)]

3. **Speed ratio, K_u**

Also we define **peripheral coefficient or speed ratio,** K_u as the ratio of runner speed to the velocity of jet.

Therefore, $u = K_u \cdot V_1$ ...(3.5.1)

The value of K_u varies from 0.43 to 0.47.

4. **Jet ratio, m**

University Question

Q. Define jet ratio. SPPU : Oct. 19(In Sem)

The ratio of mean pitch diameter of wheel, D to the jet diameter, d is called jet ratio, m. Therefore,

$$\text{Jet ratio, } m = \frac{\text{Mean pitch diameter of wheel, D}}{\text{Jet diameter, d}} \quad ...(3.5.2)$$

The value of jet ratio m varies between 11 to 15. However, *m = 12 is adopted in most cases.*

5. **Number of jets, n**

It is determined by dividing the total flow rate of water through the turbine to the rate of flow of water through a single nozzle.

Usually a single nozzle is used in pelton wheel. However, upto 4 nozzles can be used to produce more power from the same wheel. These nozzles are equispaced around the wheel. Number of jets are decided based on the total discharge and jet ratio.

6. **Angle of deflection of jet, (180 – ϕ)**

The angle of deflection of jet through the bucket varies between 160° to 170°.

However, the angle of deflection of jet can be taken as 165° if no angle of deflection is specified.

3.5.1 Number of Buckets, Z on the Periphery of Pelton Wheel

The number of buckets on the periphery of runner is decided based on the following considerations :

1. The number of buckets should be as few as possible to have the minimum friction losses.
2. Jet of water issued by nozzle is fully utilised so that no water goes as waste i.e. no water leaves without striking atleast one bucket of the runner.

Theoretical number of buckets required can be calculated as follows :

Fig. 3.5.1 shows a jet of water striking the buckets of a pelton wheel with 'O' as centre.

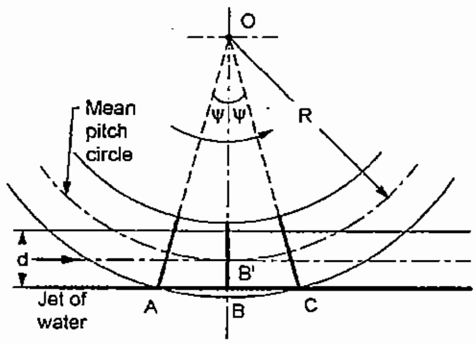


Fig. 3.5.1 : Number of buckets

The centre line of jet touches the pitch circle of bucket B and it receives the full jet. A and C are adjacent buckets.

The incoming bucket A has just started receiving the full jet and the position C of the bucket shows when it just goes out receiving the full jet.

Let, R = mean pitch circle radius of bucket

d = diameter of jet

n = theoretical number of buckets

ψ = angle subtended by adjacent buckets at the centre of runner 'O'.

h = depth of bucket which is usually taken as 1.2 d.

Consider Δ OAB′ in which,

$$OA = R+\frac{h}{2} = R+0.6.\,d$$

$$OB' = R+0.5\,d$$

$$\therefore \quad \cos\psi = \frac{OB'}{OA} = \frac{R+0.5\,d}{R+0.6\,d} \quad ...(3.5.3)$$

Jet ratio, m was defined by the Equation (3.5.2) as,

$$m = \frac{D}{d} = \frac{2R}{d};\ \text{i.e. } d = \frac{2R}{m}$$

Therefore, Equation (3.5.3) can be rewritten as :

$$\cos\psi = \frac{R+0.5\times\frac{2R}{m}}{R+0.6\times\frac{2R}{m}} = \left(\frac{1+\frac{1}{m}}{1+\frac{1.2}{m}}\right)$$

$$= \left(\frac{m+1}{m+1.2}\right) \quad ...(3.5.4)$$

Then the number of buckets may be found by the relation,

$$\text{Number of buckets, } Z = \frac{360°}{\psi°} \quad ...(3.5.5)$$

Theoretical number of buckets calculated by Equation (3.5.5) is almost twice the number of buckets used in practice since the theoretical number of buckets cannot be used due to nonavailability of space on runner. This has led to estimation of number of buckets by using certain empirical relations as follows :

Dr. H. F. Taygun suggested the following empirical relation for determination of number of buckets,

$$\text{Buckets, } Z = 15+0.5\,m = 15+0.5\times\frac{D}{d} \quad ...(3.5.6)$$

Usually the value of jet ratio, m is adopted as 12.

3.5.2 Dimensions of Bucket

Dimensions of the bucket are determined from empirical relations as follows :

Let,	d	=	diameter of the jet.
Width of bucket,	B	=	4.d
Depth of bucket,	T	=	0.3 d
Length or height of bucket,	L	=	2.5 d
Notch width,	m_w	=	1.15 d

3.6 Radial and Axial Flow Impulse Turbines

Many impulse turbines have been developed apart from pelton wheel. Working principle of some of the impulse turbines are discussed below.

3.6.1 Turgo Impulse Turbine (Axial flow turbine)

This turbine was developed by Eric Crewdson and manufactured by M/S Gilbert Gilkes and Gorden Ltd, England in 1920.

It is an axial flow impulse turbine in which jet of water strikes the buckets at an angle one side of the runner and its is discharged from the other side of the runner as shown in Fig. 3.6.1. It uses only one or two jets.

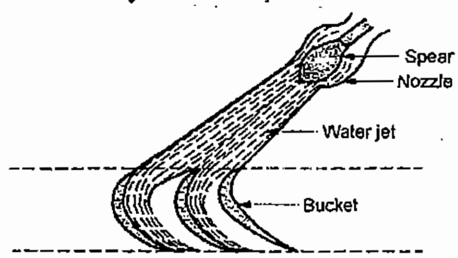


Fig. 3.6.1 : Principle of turgo turbine

These turbines have been built upto 7000 kW and for heads between 30 m to 300 m and the speed upto 2000 rpm. For the same jet diameter and discharge, the diameter of Turgo turbine is much less than the diameter of pelton wheel but its peripheral speed is higher than that of pelton wheel.

3.6.2 Banki Turbine (Radial Flow Turbine)

Banki turbine was first developed by Banki Hungarian Engineer. It is a free jet, radial flow impulse turbine as shown in Fig. 3.6.2.

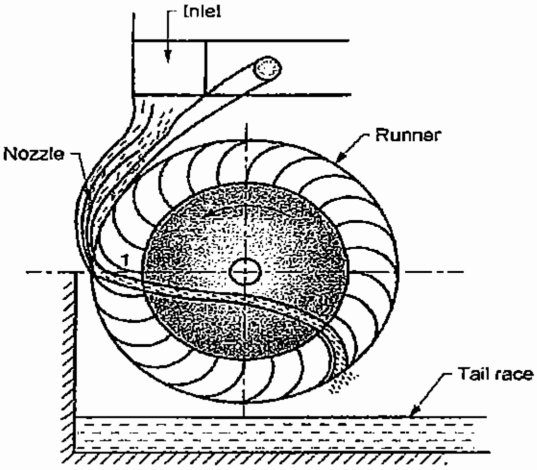


Fig. 3.6.2 : Banki turbine

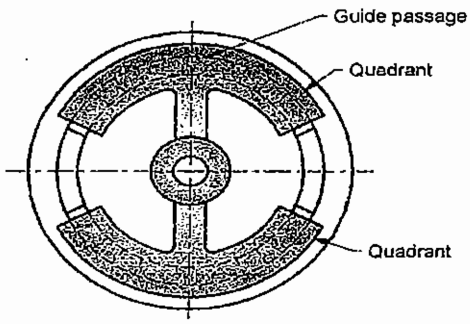


Fig. 3.6.3 : Girard impulse turbine

In this case the water jet strikes the runner blades twice at points 1 and 2 as shown in Fig. 3.6.2. Which produces the rotation of runner. Finally the water is discharged to tail race. For this reason, it is also called as cross-flow turbine. Due to utilisation of jet velocity twice, it improves the efficiency of the turbine. The runner of this turbine is drum shaped which is connected to a shaft which is always horizontal. These turbines are generally used for low heads.

3.6.3 Girard Impulse Turbine (Axial Flow Turbine)

It is an axial flow impulse turbine in which the guide vanes cover only opposite quadrants instead of the whole circumference. Therefore the guide vanes allow the water to strike through two diametrically opposite quadrants only. These turbines are suitable for generation of large power under low heads.

In case of low heads, the turbine wheel is kept horizontal while for larger heads the turbine wheel is kept vertical. This turbine has now become obsolete.

Ex. 3.6.1 : A Pelton turbine develops 3000 kW under the head of 300 m the overall efficiency of the turbine is 83%. If the speed ratio is 0.46 Coefficient of nozzle C_v = 0.98 and specific speed is 16.5 find :

i) diameter of the turbine ii) diameter of the jet.

SPPU : May 18, 6 Marks

Soln. :

Power, $P_s = 3000\text{ kW}, \quad H = 300\text{m},$

$\eta_o = 0.83$

Speed ratio, $K_u = \dfrac{u}{V_1} = 0.46, \quad C_v = 0.98,$

$Ns = 16.5$

Jet velocity, $V_1 = C_v\sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 300}$

$= 75.19\text{ m/s}$

Velocity of wheel, $u = K_u \times V_1 = 0.46 \times 75.19 = 34.59\text{ m/s}$

Specific speed, $Ns = \dfrac{N\sqrt{P}}{H^{5/4}}$

$$16.5 = \frac{N \times \sqrt{3000}}{(300)^{5/4}}\ ; \quad N = 376.1\text{ r/sm}$$

(i) Diameter of the turbine, D

$$u = \frac{\pi D N}{60}\ ;$$

$$34.59 = \frac{\pi \times D \times 376.1}{60}$$

$$\mathbf{D = 1.756m}$$ ...Ans.

(ii) Diameter of the jet, d

$$P_s = \rho \cdot g \cdot Q \cdot H \times \eta_o \times 10^{-3}\ (\text{kW})$$

$$3000 = 1000 \times 9.81 \times Q \times 300 \times 0.83 \times 10^{-3}$$

$$Q = 1.22815\ \text{m}^3/\text{s}$$

$$Q = a \times V_1 = \frac{\pi}{4} d^2 \times V_1$$

$$1.22815 = \frac{\pi}{4} \times d^2 \times 75.19\ ;$$

$$\mathbf{d = 0.1442\ m}$$ ...Ans.

Ex. 3.6.2 : Two jets strike the buckets of a pelton wheel which generates shaft power of 15450kW. The diameter of each jet is given as 200mm. If the net head on the turbine is 400 m, find the overall efficiency of the turbine. Take C_v = 1.0.

SPPU – Aug. 18 (In sem), 6 Marks

Soln. :

Shaft power, $P_S = 15450\text{ kW}\ ;$

No. of jets, $n = 2$

Diameter of each jet, $d = 200\text{ mm} = 0.2\text{ m.}\ ;$

$$H = 400 \text{ m}$$
$$C_v = 1.0$$

Velocity of jet, $V = C_v\sqrt{2gH} = 1 \times \sqrt{2 \times 9.81 \times 400}$

$$= 88.589 \text{ m/s}$$

Discharge from jets,

$$Q = \text{area of jet} \times \text{Velocity} \times \text{No. of jets}$$
$$= \frac{\pi}{4} d^2 \times V \times n = \frac{\pi}{4} (0.2)^2 \times 88.589 \times 2$$
$$= 5.5662 \text{ m}^3/\text{s}$$

Overall efficiency, η_o

$$\eta_o = \frac{\text{Output, } p_s \text{ (kW)}}{\text{Input power } (\rho \cdot g \cdot Q \times H \times 10^{-3}) \text{ (kW)}}$$
$$= \frac{15450}{1000 \times 9.81 \times 5.5662 \times 400 \times 10^{-3}}$$
$$= \mathbf{0.7074 \text{ or } 70.74\ \%} \quad \text{... Ans.}$$

Ex. 3.6.3 : A Pelton wheel is required to develop 6 MW when working under a head of 300 m. It rotates with speed of 550 rpm. Assuming jet ratio as 10 and overall efficiency as 85 %. Calculate :

i) Diameter of wheel,

ii) Quantity of water required

iii) Number of jets

Assume coefficient of velocity for nozzle = 0.96 and speed ratio = 0.46. **SPPU – Aug. 18 (In sem.), 6 Marks**

Soln. :

$$p_s = 6 \text{ MW} = 6 \times 10^3 \text{ kW},$$
$$H = 300 \text{ m}, N = 550 \text{ rpm}$$

Jet ratio, $\frac{D}{d} = 10;$

Overall Efficiency, $\eta_o = 85\% = 0.85$

$$C_v = 0.96\ ;$$

Speed ratio, $K_u = \frac{u}{V_1} = 0.46$

(i) Diameter of wheel, D

$$V_1 = C_v\sqrt{2gH} = 0.96 \times \sqrt{2 \times 9.81 \times 300}$$
$$= 73.65 \text{ m/s}$$
$$K_u = \frac{u}{V_1}\ ;$$
$$0.46 = \frac{4}{73.65}\ ;$$
$$u = 33.88 \text{ m/s}$$

Bucket velocity, $u = \frac{\pi D N}{60}$;

$$33.88 = \frac{\pi \times D \times 550}{60}$$
$$\mathbf{D = 0.243 \text{ m}} \quad \text{... Ans.}$$

(ii) Quantity of water required, Q

$$P_s = \rho \cdot g \cdot Q \times H \times \eta_o \times 10^{-3} \text{ (kW)}$$
$$6 \times 10^3 = 1000 \times 9.81 \times Q \times 300 \times 0.85 \times 10^{-3}$$
$$\mathbf{Q = 2.3985 \text{ m}^3/\text{s}} \quad \text{... Ans.}$$

(iii) Number jets, n

Discharge per jet,

$$\frac{D}{d} = 10;$$
$$d = \frac{D}{10} = \frac{0.243}{10} = 0.0243 \text{ m}$$
$$q = a \times V_1 = \frac{\pi}{4} (d)^2 \times V_1$$
$$= \frac{\pi}{4} \times (0.0243)^2 \times 73.65 = 0.03416 \text{ m}^3/\text{s}$$
$$n = \frac{\text{Total discharge, Q}}{\text{Discharge per jet, q}}$$
$$= \frac{0.243}{0.03416}$$
$$= 7.114 \text{ say 8 jets} \quad \text{... Ans.}$$

Ex. 3.6.4 : A pelton wheel has to develop 13250 kW under a net head of 800 m at a speed of 600 rpm. The coefficient of velocity for the jet is 0.97. The peripheral velocity is $0.46\sqrt{2gH}$. The diameter of the jet is not to exceed one sixteenth the wheel diameter. Find the dischatge, the wheel diameter, the jet diameter and the number of jets required. Assume the overall efficiency to be 85%. **SPPU - Dec. 18, 6 Marks**

Soln. :

Given : $P_s = 13250 \text{ kW}, \quad H = 800 \text{ m},$

$$N = 600 \text{ rpm}, \quad C_v = 0.97,$$
$$u = 0.46\sqrt{2\,gH},$$

diameter of jet, $d \leq \frac{1}{16} \times$ wheel diameter D,

$$\eta_0 = 85\% = 0.85.$$

Runner velocity,

$$u = 0.46\sqrt{2\,gH} = 0.46 \times \sqrt{2 \times 9.81 \times 800}$$
$$= 57.63 \text{ m/s}.$$

Jet velocity at inlet,

$$V_1 = C_v\sqrt{2\,gH} = 0.97 \times \sqrt{9.81 \times 800}$$

$$= 121.53 \text{ m/s}$$

1. Wheel diameter, D :

$$u = \frac{\pi D N}{60};$$

$$57.63 = \frac{\pi D \times 800}{60};$$

$$D = \mathbf{1.376\ m} \qquad \text{...Ans.}$$

2. Discharge, Q :

$$P_s = \rho \cdot g \cdot Q \cdot H \times \eta_0 \times 10^{-3} \text{ kW}$$

$$13250 = 1000 \times 9.81 \times Q \times 800 \times 0.85 \times 10^{-3}$$

$$Q = \mathbf{1.9862\ m^3/s} \qquad \text{...Ans.}$$

3. Jet diameter, d :

$$d \le \frac{1}{16} D \le \frac{1}{16} \times 1.37\ 6 \le 0.086 \text{ m}$$

4. No. of jets required, n

Discharge per jet,

$$q = \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} \times (0.086)^2 \times 121.53$$

$$= 0.706 \text{ m}^3\text{/s/jet}$$

$$n = \frac{\text{Total discharge, Q}}{\text{Discharge per jet, q}} = \frac{1.9862}{0.706}$$

$$= 2.814 \text{ say } \mathbf{3\ jets} \qquad \text{...Ans.}$$

New discharge,

$$q_1 = \frac{Q}{n} = \frac{1.9862}{3} = 0.6621 \text{ m}^3\text{/s}$$

$$q_1 = \frac{\pi}{4} d_1^2 \times V_1 \ ;$$

$$0.6621 = \frac{\pi}{4} \times d_1^2 \times 121.53;$$

$$d_1 = \mathbf{0.0833\ m} \qquad \text{...Ans.}$$

$d_1 < d$, which meets the required condition.

Ex. 3.6.5 : A double-jet Pelton Wheel has a specific speed of 14 and is required to deliver 1000 kW. The turbine is supplied through pipeline from a reservoir whose level is 400 m above the nozzles. Allowing 5% for frictional loss in the pipe, calculate :

(i) Speed in RPM

(ii) Diameter of jets

(iii) Mean Diameter of Bucket circle

Take C_v = 0.98, speed ratio = 0.46 and overall efficiency = 85%, the specific speed is based on power output per jet.

SPPU - May 19 and Oct. 19 (In sem), 6 Marks

Soln. :

No. of jets, $n = 2$;

Specific speed, $N_s = 14$;

$P_s = 1000$ kW;

Gross head, $H_g = 400$ m,

Friction Loss head, $H_f = 5\%$ of $H_g = \frac{5}{100} \times 400 = 20$ m

Net head, $H = H_g - H_f = 400 - 20 = 380$ m

$C_v = 0.98$; speed ratio,

$K_u = \frac{u}{V_1} = 0.46$;

$\eta_o = 0.85$

Let Q be discharge per jet.

$$P_s = \rho.g.Q.H \times \eta_0 \times 10^{-3} \times n \text{ (kW)}$$

$$1000 = 1000 \times 9.81 \times Q \times 380 \times 0.85 \times 10^{-3} \times 2$$

$$Q = 0.1578 \text{ m}^3\text{/s/jet}$$

Power output per jet,

$$P = \frac{P_s}{2} = \frac{1000}{2} = 500 \text{ kW}$$

(i) Speed in rpm, N

$$N_s = \frac{N\sqrt{P}}{H^{5/4}};$$

$$14 = \frac{N\sqrt{500}}{(380)^{5/4}}$$

$$N = \mathbf{1050.4\ rpm} \qquad \text{...Ans.}$$

(ii) Diameter of jets, d

Velocity of jet, $V_1 = C_v\sqrt{2gH}$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 380}$$

$$= 84.62 \text{ m/s}$$

$$Q = \text{Area of jet} \times \text{velocity of jet}$$

$$Q = \frac{\pi}{4} d^2 \times V_1;$$

$$0.1578 = \frac{\pi}{4} d^2 \times 84.62$$

$$d = \mathbf{0.0487 m = 4.87\ cm} \qquad \text{...Ans.}$$

(iii) Mean diameter of bucket circle, D

$$K_u = \frac{u}{V_1};$$

$$u = K_u \cdot V_1 = 0.46 \times 84.62 = 38.925 \text{ m/s}$$

$$u = \frac{\pi D N}{60};$$

$$38.925 = \frac{\pi \times D \times 1050.4}{60};$$

$$D = \mathbf{0.708\ m} \qquad \text{...Ans.}$$

Ex. 3.6.6 : A pelton turbine is required to work under a head of 250 m to develop 20 MW at 375 rpm. Considering speed ratio of 0.46, jet ratio of 10, mechanical efficiency of 94%, angle of deflection 165 degree and nozzle coefficient as 0.97. Determine the number of jets, diameter of runner and number of buckets. Assume bucket friction factor of 0.88.

SPPU – Oct. 19(In Sem), 6 Marks

Soln. : Refer Fig. P.3.6.6

$$H = 250 \text{ m};$$

$$P_s = 20 \text{ MW} = 20 \times 1000 \text{ kW} = 20000 \text{ kW}$$

$$N = 375 \text{ rpm};$$

$$\text{Speed ratio, } K_u = \frac{u}{V_1} = 0.46$$

$$\text{Jet ratio, } m = \frac{d}{D} = 10;$$

$$\eta_m = 94\,\% = 0.94,$$

Angle of deflection of jet = 165°

$$\therefore \phi = 180 - 165 = 15°;$$

$$C_v = 0.97;$$

$$\text{Bucket friction factor} = 0.88 \text{ i.e.}$$

$$K = \frac{V_{ro}}{V_{ri}} = 0.88$$

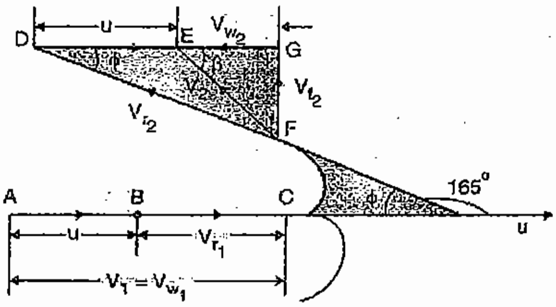


Fig. P.3.6.6

(i) Diameter of runner, D

Velocity of jet,

$$V_1 = C_v\sqrt{2gH} = 0.97 \times \sqrt{2 \times 9.81 \times 250}$$

$$= 67.93 \text{ m/s}$$

$$\therefore K_u = \frac{u}{V_1};\ 0.46 = \frac{u}{67.93}$$

$$u = 31.25 \text{ m/s}$$

$$u = \frac{\pi DN}{60};$$

$$31.25 = \frac{\pi \times D \times 375}{60}$$

$$D = \mathbf{1.5915 \text{ m}} \quad \text{... Ans.}$$

ii) No. of jets, n and No. of buckets, Z

$$m = \frac{D}{d}$$

$$10 = \frac{1.5915}{d}$$

$$d = 0.15915 \text{ m}$$

$$\text{Mass flow rate, } \dot{m} = \rho \cdot a \cdot V_1 = \rho \times \frac{\pi}{4}(d)^2 \times V_1$$

$$= 1000 \times \frac{\pi}{4}(0.15915)^2 \times 67.93$$

$$= 1351.34 \text{ kg/s per jet}$$

$$V_{w_1} = V_1 = 67.93 \text{ m/s}$$

$$V_{r_1} = V_1 - u$$

$$= 67.93 - 31.25 = 36.68 \text{ m/s}$$

$$V_{r_2} = K.V_{r_1} = 0.88 \times 36.68 = 32.28 \text{ m/s}$$

$$V_{w_2} = V_{r_2}\cos\phi - u$$

$$= 32.28 \cos 15 - 31.25 = -0.07 \text{ m/s}$$

Power developed by runner,

$$P_1 = \frac{\dot{m}(V_{w_1} + V_{w_2})u}{1000} \text{ (kW)}$$

$$= \frac{1351.34(67.93 - 0.07)\,31.25}{1000}$$

$$= 2865.62 \text{ kW}$$

Shaft power per jet,

$$P_{s1} = P_1 \times \eta_m = 2565.62 \times 0.94$$

$$= 2693.7 \text{ kW}$$

$$n = \frac{P_1}{P_{s1}} = \frac{20000}{2693.7}$$

$$= 7.425 \text{ say } \mathbf{8 \text{ jets}} \quad \text{... Ans.}$$

$$\text{No. of buckets, } Z = 15 + 0.5\,m = 15 + 0.5 \times 10$$

$$= \mathbf{20 \text{ buckets}} \quad \text{... Ans.}$$

Ex. 3.6.7 : Determine wheel diameter and jet diameter of pelton wheel which develop 1000 kW shaft power when running at 250 rev/min. Available head is 500 m. Assume velocity coefficient of 0.98, overall efficiency of 0.85, ratio of peripheral speed to jet velocity of 0.45.

SPPU : Dec. 19, 6 Marks

Soln. :

$$P_s = 1000 \text{ kW}, \quad N = 250 \text{ rpm},$$

$$H = 500 \text{ m}, \quad C_v = 0.98,$$

$$\eta_o = 0.85; \quad \frac{u}{V_1} = 0.45$$

(i) Wheel diameter, D

$$\text{Jet velocity,} \quad V_1 = C_v\sqrt{2gH}$$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 500} = 97.06 \text{ m/s}$$

$$\because \frac{u}{V_1} = 0.45;$$

$$u = 0.45\, V_1 = 0.45 \times 97.06 = 43.68 \text{ m/s}$$

$$u = \frac{\pi D N}{60};$$

$$43.68 = \frac{\pi \times D \times 250}{60};$$

$$D = \mathbf{3.337\ m} \quad \text{... Ans.}$$

(ii) Jet diameter, d

$$P_s = \rho \cdot g \cdot Q \cdot H \times \eta_o \times 10^{-3} \text{ (kW)}$$

$$1000 = 1000 \times 9.81 \times Q \times 500 \times 0.85 \times 10^{-3}$$

$$Q = 2.3985 \text{ m}^3/\text{s}$$

$$\text{Discharge,} \quad Q = \text{Area of jet} \times \text{Jet velocity}$$

$$Q = \left(\frac{\pi}{4} \times d^2\right) V_1$$

$$2.3985 = \frac{\pi}{4} \times d^2 \times 97.06$$

$$d = \mathbf{0.1774\ m = 17.74\ cm} \quad \text{... Ans.}$$

Ex. 3.6.8 : A Pelton wheel is supplied with water under a effective head of 97 m at the rate of 1020 liters/s. The buckets deflects the jet through an angle of 163°. The mean speed of the bucket is 23 m/s. Find : (i) Mass flow rate of water (ii) Velocity of jet (iii) Hydraulic power output, assume coefficient of velocity of jet, C_v = 1 and friction factor of bucket is k = 1.

Soln. :

$$H = 97 \text{ m}$$

$$Q = 1020 \text{ liters/s} = \frac{1020}{1000} = 1.02 \text{ m}^3/\text{s}$$

$$\phi = 180 - 163 = 17°$$

$$u = 23 \text{ m/s}$$

$$C_v = 1$$

$$k = \frac{V_{r_2}}{V_{r_1}} = 1$$

Refer Fig. P. 3.6.8(a)

(i) Mass flow rate of water, $\dot{m}$

$$\dot{m} = \rho Q = 1000 \times 1.02$$

$$= \mathbf{1020\ kg/s} \quad \text{...Ans.}$$

(ii) Velocity of jet, V_1

$$V_1 = C_v\sqrt{2gH}$$

$$= 1\sqrt{2 \times 9.81 \times 97}$$

$$= \mathbf{43.625\ m/s} = V_{w_1} \quad \text{...Ans.}$$

(iii) Hydraulic power output, P_h:

From inlet velocity diagram,

$$V_{r_1} = V_1 - u = 43.625 - 23$$

$$= 20.625 \text{ m/s}$$

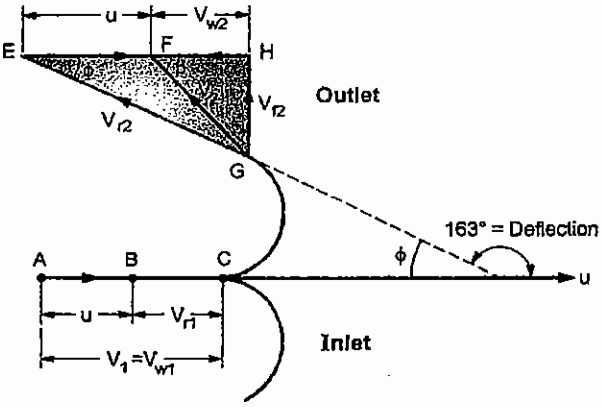


Fig. P. 3.6.8(a)

$$\because \text{Friction factor, } k = 1 = \frac{V_{r_2}}{V_{r_1}}$$

$$\text{i.e. } V_{r_2} = V_{r_1} = 20.625 \text{ m/s}$$

$$V_{w_2} = V_{r_2}\cos\phi - u$$

$$= 20.625 \cos 17 - 23$$

$$= -3.276 \text{ m/s}$$

$\because$ V_{w_2} is negative, the modified outlet diagram is shown in Fig. P. 3.6.8(a)

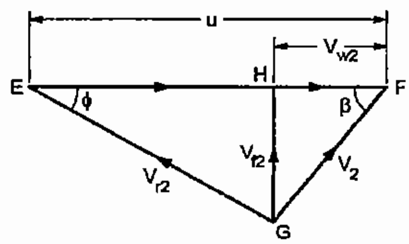


Fig. P. 3.6.8(b)

$$P_h = \frac{\dot{m}\,(V_{w_1} + V_{w_2})\,u}{1000} \text{ kW}$$

$$= \frac{1020\,(43.625 - 3.276)\,23}{1000}$$

$$= \mathbf{946.588\ kW} \quad \text{...Ans.}$$

Ex. 3.6.9 : The following data refers to a Pelton turbine. It drives a 15 MW generator. The effective head is 310 m. The generator and turbine efficiencies are 95% and 86% respectively. The speed ratio is 0.46. Jet ratio is 12. Nozzle velocity coefficient is 0.98. Determine the jet and runner diameters, the speed and specific speed of the runner.

SPPU - Dec. 13, 10 Marks

Soln. :

Given : Generator power, $P_g = 15\ MW = 15000\ kW$

Effective head, $H = 310\ m$, $\eta_{gen} = 0.95$

$$\eta = 0.86, \quad \frac{u}{V_1} = 0.46$$

$$\text{Jet ratio, } m = \frac{\text{Diameter of wheel, D}}{\text{Jet diameter, d}} = 12$$

$$C_v = 0.98$$

$$\text{Velocity of jet, } V_1 = C_v\sqrt{2\,g\,H}$$

$$= 0.98\sqrt{2 \times 9.81 \times 310}$$

$$= 76.43\ m/s$$

$$\text{Velocity of wheel, } u = 0.46\,V_1 = 0.46 \times 76.43$$

$$= 35.16\ m/s$$

$$\text{Hydraulic power, } P_i = \text{Generator power, } P_g \times \frac{1}{\eta_{gen}} \times \frac{1}{\eta_t}$$

$$P_i = \frac{15000}{0.95 \times 0.86} = 18359.8\ kW$$

$$P_i = \frac{1}{2}\dot{m} \cdot V_1^2$$

$$18359.8 = \frac{1}{2} \times \dot{m} \times (76.43)^2 \times \frac{1}{1000}\ kW$$

$$\dot{m} = 6285.94\ kg/s$$

1. **Diameter of jet, d and runner diameter, D**

$$\dot{m} = \rho \cdot A \cdot V_1$$

$$6285.94 = 1000 \times \frac{\pi}{4} \cdot d^2 \times 76.43$$

$$d = \mathbf{0.3236\ m} \qquad \text{...Ans.}$$

$$D = 12\,d = 12 \times 0.3236$$

$$= \mathbf{3.8832\ m} \qquad \text{...Ans.}$$

2. **Speed, N**

$$u = \frac{\pi D N}{60}$$

$$35.16 = \frac{\pi \times 3.8832 \times N}{60}$$

$$N = \mathbf{172.93\ rpm} \qquad \text{...Ans.}$$

3. **Specific speed, N_s**

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$\text{where, } P = \frac{P_g}{\eta_g} = \frac{15000}{0.95} = \mathbf{15789.5\ kW} \qquad \text{...Ans.}$$

$$N_s = \frac{172.93 \times \sqrt{15789.5}}{(310)^{5/4}} = \mathbf{16.7} \qquad \text{...Ans.}$$

Ex. 3.6.10 : A Pelton turbine develops 3000 kW under a head of 400 m. The overall efficiency of the turbine is 87%. If the speed ratio is 0.48 and the coefficient of velocity is 0.96 and specific speed 18. Find (i) Diameter of the turbine (ii) Diameter of the jet.

SPPU - Dec. 16, 6 Marks

Soln. :

Given data : Power, $P_s = 3000\ kW$

Head, $H = 400m$

Overall efficiency $\eta_o = 0.87$

$$\text{Speed ratio, } K_u = \frac{u}{V_1} = 0.48$$

Coefficient of velocity $C_v = 0.98$

Specific speed, $N_S = 18$

$$\text{Jet velocity, } V_1 = C_v\sqrt{2gH}$$

$$= 0.96 \times \sqrt{2 \times 9.81 \times 400}$$

$$= 85.05\ m/s$$

$$\text{Velocity of wheel, } u = K_u \times V_1 = 0.48 \times 85.05$$

$$= 40.82\ m/s$$

$$N_S = \frac{N\sqrt{P}}{H^{5/4}};$$

$$18 = \frac{N\sqrt{3000}}{(400)^{5/4}}$$

$$N = 587.88\ rpm$$

(a) Diameter of the turbine, D

$$u = \frac{\pi D N}{60};$$

$$D = \frac{60 \times u}{\pi N}$$

$$D = \frac{60 \times 40.82}{\pi \times 587.88} = \mathbf{1.326\ m} \qquad \text{...Ans.}$$

(b) Diameter of the jet (d)

$$P_S = \rho \cdot g \cdot Q\,H \times \eta_o \times 10^{-3}\ (kW)$$

$$3000 = 1000 \times 9.81 \times Q \times 400 \times 0.87 \times 10^{-3}$$

$$Q = 0.879\ m^3/s$$

$$Q = a \times V_1 = \frac{\pi}{4} d^2 \times V_1$$

$$0.879 = \frac{\pi}{4} \times d^2 \times 85.05$$

$$d = \mathbf{0.1147\ m} \qquad \text{...Ans.}$$

Ex. 3.6.11 : A Pelton wheel operates with a jet of 15 cm in diameter under a head of 500 m. Its mean runner diameter is 2.25 m and it rotates with the speed of 375 rpm. The outlet angle of bucket tip is 15°, the coefficient of velocity as 0.98, mechanical efficiency as 97 % and buckets are smooth. Determine the shaft power, hydraulic efficiency and power lost in bucket. **SPPU - May 16, 6 Marks**

Soln. :

Refer Fig. P. 3.6.11

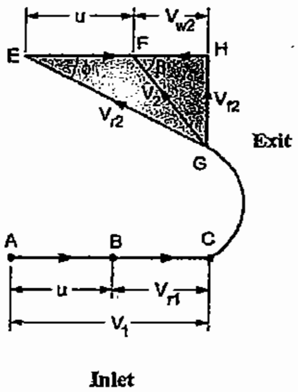


Fig. P. 3.6.11

$$\text{Jet diameter, } d = 15 \text{ cm} = 0.15 \text{ m}$$

$$H = 500 \text{ m}$$

$$\text{Runner diameter, } D = 2.25 \text{ m}$$

$$N = 375 \text{ rpm}, \qquad \phi = 15°$$

$$C_v = 0.98, \qquad \eta_{mech} = 0.77$$

$$u = \frac{\pi DN}{60}$$

$$= \frac{\pi \times 2.25 \times 375}{60} = 44.18 \text{ m/s}$$

$$V_1 = C_v\sqrt{2gH}$$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 500}$$

$$= 97.06 \text{ m/s} = V_{w1}$$

From inlet velocity ΔABC :

$$V_{r_1} = V_1 - u$$

$$= 97.06 - 44.18 = 52.88 \text{ m/s}$$

$$V_{r_2} = V_{r_1} = 52.88 \text{ m/s}$$

(since buckets are smooth)

Consider exit velocity Δ EGH

$$V_{w_2} = EH - EF = V_{r_2} \phi - u$$

$$= 52.88 \cos 15 - 44.18$$

$$= 6.90 \text{ m/s}$$

$$\dot{m} = \rho A V_1$$

$$= 1000 \times \frac{\pi}{4} \times (0.15)^2 \times 97.06$$

$$= 1715.2 \text{ kg/s}$$

Power developed by runner,

$$P = \frac{\dot{m}\,(V_{w_1} + V_{w_2})\, u}{1000}$$

$$= \frac{1715.2\,(97.06 + 6.9)\, 44.18}{1000}$$

$$= 7877.8 \text{ kW}$$

(1) Shaft power, P_s

$$P_s = \text{Power developed } (P) \times \text{Mechanical efficiency } (\eta_m)$$

$$= \mathbf{7877.8 \times 0.77 = 6065.9 \text{ kW}}$$ **...Ans.**

(2) Hydraulic efficiency, η_h

$$\eta_h = \frac{V_{w_1} + V_{w_2}}{\left(V_1^2/2\right)} = \frac{2 \times (97.06 + 6.9) \times 44.18}{(97.06)^2}$$

$$= \mathbf{0.9751 \text{ or } 97.51\ \%}$$ **...Ans.**

(3) Power lost in buckets $(P_i - P)$

Input power,

$$P_i = \rho g Q H \times 10^{-3} \text{ (kW)}$$

$$= \rho . g \times \left(\frac{\pi}{4} d^2 \times V_1\right) H \times 10^{-3}$$

$$= 1000 \times 9.81 \times \frac{\pi}{4} (0.15)^2 \times 97.06 \times 500 \times 10^{-3}$$

$$= 8413 \text{ kW}$$

$$\text{Power lost in buckets} = P_i - P = 8413 - 7877.8$$

$$= \mathbf{535.2 \text{ kW}}$$ **...Ans.**

Ex. 3.6.12 : A Pelton wheel is working under a gross head of 400 m. The water is supplied through penstock of diameter 1 m and length 4 km from reservoir to the Pelton wheel. The coefficient of friction for the penstock is given as 0.008. The jet of water of diameter 150 mm strikes the buckets of the wheel and gets deflected through an angle of 165°. The relative velocity of water at outlet is reduced by 15% due to friction between inside surface of the bucket and water. If the velocity of the buckets is 0.45 times the jet velocity at inlet and mechanical efficiency as 85%. Determine :

i) Power given to the runner

ii) Shaft power

iii) Hydraulic efficiency and overall efficiency. **SPPU - May 13, 8 Marks**

Soln. :

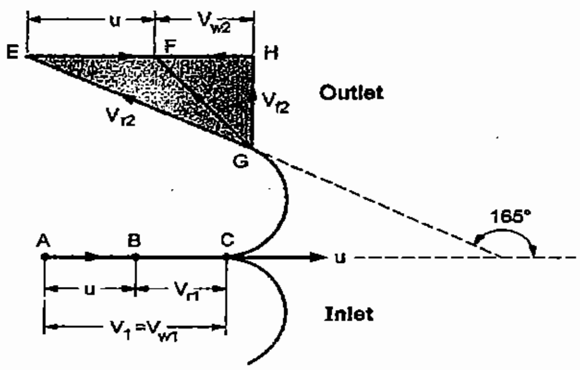


Fig. P. 3.6.12

Gross-head, $H_g = 400$ m

Penstock diameter, $D_1 = 1$ m

Length, $L = 4 \text{ km} = 4000$ m

$f = 0.008$

Diameter of jet, $d = 150 \text{ mm} = 0.15$ m

$\phi = 180 - 165 = 15°$

$V_{r2} = (1 - 0.15)\, V_{r1} = 0.85\, V_{r1}$ ...(i)

$u = 0.45\, V_1$ ... (ii)

$\eta_m = 85\% = 0.85$

Let $V_1' =$ Mean velocity of water in penstock and

$V_1 =$ Velocity of jet

From continuity equation,

$$Q = AV \text{ we have}$$

$$A_1 V_1' = a \times V_1$$

$$\frac{\pi}{4} D^2 \times V_1' = \frac{\pi}{4} d^2 \times V_1$$

i.e. $$D^2 \cdot V_1' = d^2 \cdot V_1$$

$$1^2 \times V_1' = (0.15)^2 \times V_1$$

$$V_1' = 0.0225\, V_1 \quad \text{...(iii)}$$

Gross head,

$$H_g = \text{Head lost in friction in penstock} + \frac{V_1^2}{2g}$$

$$H_g = \frac{4fL(V_1')^2}{D \cdot 2g} + \frac{V_1^2}{2g}$$

$$400 = \frac{4 \times 0.008 \times 4000 \times (0.0225\, V_1)^2}{1 \times 2 \times 9.81} + \frac{V_1^2}{2 \times 9.81}$$

$$V_1 = 85.85 \text{ m/s}$$

Bucket velocity,

$$u = 0.45\, V_1 = 0.45 \times 85.85 = 38.63 \text{ m/s}$$

From inlet velocity triangle, ABC :

$$V_{w1} = V_1 = 85.85 \text{ m/s}$$

$$V_{r1} = V_1 - u = 85.85 - 38.63 = 47.22 \text{ m/s}$$

$$V_{r2} = 0.85\, V_{r1} = 0.85 \times 47.22 = 40.14 \text{ m/s}$$

From outlet velocity Δ EGH,

$$EH = V_{r2} \cos\phi = 40.14 \cos 15 = 38.77 \text{ m/s}$$

$$\therefore V_{w2} = EH - u = 38.77 - 38.63 = 0.14 \text{ m/s}$$

$$\dot{m} = \rho \cdot a \cdot V_1 = 1000 \times \left(\frac{\pi}{4}\right) \times (0.15)^2 \times 85.85 = 1517.1 \text{ kg/s}$$

1. **Power given to runner in kW, P**

$$P = \frac{\dot{m}\,(Vw_1 + Vw_2)\, u}{1000} \text{ kW} = \frac{1517.1}{1000}(85.85 + 0.14) \times 38.63$$

$$= \mathbf{5039.48 \text{ kW}} \quad \text{...Ans.}$$

2. **Shaft power, P_S**

$$P_S = \eta_m \times P = 0.85 \times 5039.48 = \mathbf{4283.55 \text{ kW}} \quad \text{...Ans.}$$

3. **Hydraulic efficiency, η_h**

$$\eta_h = \frac{2\,(Vw_1 + Vw_2)\, u}{V_1^2} = \frac{2 \times (85.85 + 0.14)\, 38.63}{(85.85)^2}$$

$$= \mathbf{0.9014 \text{ or } 90.14\%} \quad \text{...Ans.}$$

4. **Overall efficiency, η_o**

$$\eta_o = \eta_h \cdot \eta_m = 0.9014 \times 0.85 = \mathbf{0.7662 \text{ or } 76.62\ \%} \quad \text{...Ans.}$$

Ex. 3.6.13 : A twin jet Pelton wheel has a mean runner diameter of 1.68 m and runs at 500 rpm, each jet is of diameter 152 mm. The net head is 510 m. Bucket turns the jet through 165° and relative velocity is reduced by 12% when it flows over the bucket. Windage and mechanical losses are 3% of the water power supplied. Nozzle co-efficient = 0.98. Find :

(i) Water power supplied

(ii) Brake power

(iii) Force of each jet on bucket.

(iv) Overall efficiency.

Soln. :

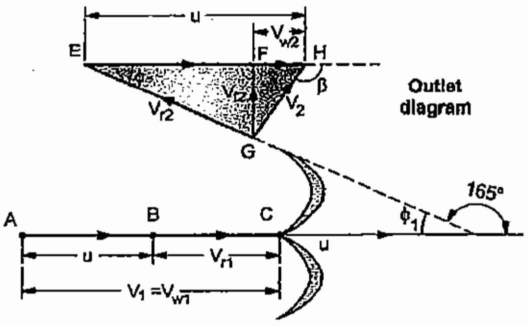


Fig. P. 3.6.13

Given

Number of jets = 2, D = 1.68 m

$N = 500$ RPM, $d = 152$ mm

$H = 510$ m, $\phi = 15°$

$K = \frac{V_{r2}}{V_{r1}} = 0.88, \quad C_v = 0.98$

$$V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2\times 9.81\times 510}$$

$$V_1 = 98.03 \text{ m/s}$$

$$\vec{V_1} = \vec{V}_{r1} + \vec{u}$$

$$V_{r1} = V_1 - u = 98.03 - u$$

$$u = \frac{\pi DN}{60} = \frac{\pi\times 1.98\times 500}{60} = 51.84 \text{ m/s}$$

$$\therefore V_{r1} = 98.03 - 51.84 = 46.19$$

$$V_{r2} = K\cdot V_{r1} = 0.88\times 46.19 = 40.65 \text{ m/s.}$$

$$V_{w2} = V_{r2}\cos\phi - u = 40.65\cos 15 - 51.84$$

$$= -12.58 \text{ m/s}$$

$$Q_1 = \frac{\pi}{4}d^2\times V_1 = \frac{\pi}{4}\times 0.152^2\times 98.03$$

$$= 1.78 \text{ m}^3\text{/s per jet}$$

Total discharge $= 2\times Q = 2\times 1.78 = 3.56 \text{ m}^3/\text{s}$

(i) Water power supplied

$$P = 2\rho\cdot Q\frac{V_1^2}{2}\times 10^{-3} \text{ kW}$$

$$= 2\times 10^3\times 1.78\times\frac{98.03^2}{2}\times 10^{-3}$$

$$= 17.105\times 10^3 \text{ kW}$$

$$= \mathbf{17.105 \text{ MW}}$$ ...Ans.

(ii) Brake power

Power developed $= 2\times\rho Q(V_{w1} - V_{w2})\,u$

$$= 2\times 10^3\times 1.78\,(98.03 - 12.58)\times 51.84$$

$$= 15.77\times 10^6 \text{ W} = 15.77 \text{ MW}$$

Windage and Mechanical losses

= 3 % of water power developed.

$\therefore$ Brake power $= 0.97\times 15.77$

$$= \mathbf{15.297 \text{ MW}}$$ ...Ans.

(iii) Force of each jet on bucket

$\therefore$ Force of each jet $= \rho Q(V_{w1} - V_{w2})$

$$= 10^3\times 1.78\,(98.03 - 12.58)$$

$$= 152.1\times 10^3 \text{ N}$$

$$= \mathbf{152.1 \text{ kN}}$$ ...Ans.

(iv) Overall Efficiency, η_o

$$\eta_o = \frac{\text{Brake power}}{\text{Water power supplied}}$$

$$= \frac{15.297}{17.105}$$

$$= \mathbf{0.8943 \text{ or } 89.43\ \%}$$ ...Ans.

Ex. 3.6.14 : A single jet Pelton wheel is supplied with water from a reservoir 300 m above the center of nozzle through a pipe of 600 mm diameter and 5 km long. The friction factor for the pipe is 0.032. The jet has diameter of 80 mm and its velocity coefficient is 0.97, the wheel bucket speed is 0.47 times the jet speed and deflects the water through 160°, the relative velocity at outlet is 80% of that at inlet. The mechanical efficiency of the wheel is 80%. Determine (i) Shaft power of the wheel (ii) Hydraulic efficiency and (iii) Overall efficiency. **SPPU - May 14,10 Marks**

Soln. : Refer Fig. P. 3.6.14.

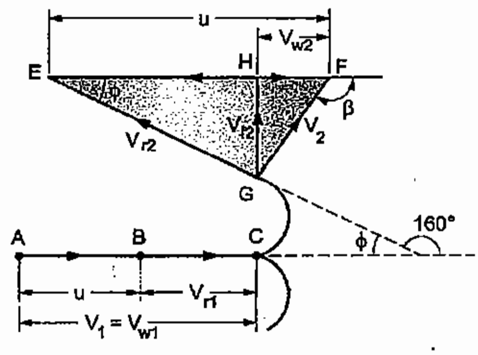


Fig. P. 3.6.14

Gross head, H_g = 300 m

Pipe diameter, D = 600 mm = 0.6 m

L = 5 km = 5000 m

friction factor, $f' = 4f = 0.032$

Jet diameter, $d = 80$ mm $C_v = 0.97$;

$u = 0.47 V_1$ $\phi = 180 - 160 = 20°$

$V_{r2} = 0.8 V_{r1}$, $\eta_m = 0.8$

Let V be the velocity of water in pipe and

V_1 be the velocity of jet.

Using continuity equation,

$$Q = AV \text{ we have,}$$

$$\text{Area of pipe} \times V = \text{Area of jet} \times V_1$$

$$\frac{\pi}{4} D^2 \times V = \frac{\pi}{4} d^2 \times V_1$$

$$V = \left(\frac{d}{D}\right)^2 \cdot V_1 = \left(\frac{0.08}{0.6}\right)^2 \cdot V_1$$

$$= 0.01778 \times V_1 \quad \text{...(i)}$$

Friction loss in pipe,

$$h_f = \frac{f' L (V)^2}{D \cdot 2g}$$

$$= \frac{0.032 \times 5000 \times V^2}{0.6 \times 2 \times 9.81}$$

$$= 13.5916 V^2 \quad \text{...(ii)}$$

Gross head,

H_g = Head lost in friction + Velocity head at entry to nozzle

$$H_g = h_f + \frac{V^2}{2g \times C_v^2}$$

$$300 = 13.5916 V^2 + \frac{V_1^2}{2 \times 9.81 \times (0.97)^2}$$

$$300 = 13.5916 \times (0.01778 V_1)^2 + 0.05417 V_1^2$$

$$V_1 = 71.63 \text{ m/s}$$

(i) Shaft power, P_s :

Consider inlet velocity Δ ABC

$$u = 0.47 V_1 = 0.47 \times 71.63$$

$$= 33.67 \text{ m/s}$$

$$V_{w1} = V_1 = 71.63 \text{ m/s}$$

$$V_{r1} = V_1 - u = 71.63 - 33.67$$

$$= 37.96 \text{ m/s}$$

$$V_{r2} = 0.8 V_{r1} = 0.8 \times 37.96$$

$$= 30.37 \text{ m/s}$$

Consider outlet velocity Δ EFG

$$V_{w2} = u - V_{r2} \cos \phi$$

$$= 33.67 - 30.37 \cos 20 = 5.13 \text{ m/s}$$

Discharge,

$$Q = \text{area of jet} \times \text{jet velocity}$$

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(0.08)^2 \times 71.63$$

$$= 0.36 \text{ m}^3/\text{s}$$

Mass flow rate,

$$\dot{m} = \rho \cdot Q = 1000 \times 0.36$$

$$= 360 \text{ kg/s}$$

Net head available at runner,

$$H = \frac{V_1^2}{2g} = \frac{(71.63)^2}{2 \times 9.81}$$

$$= 261.5 \text{ m}$$

$$\text{Water power (W.P.)} = \rho \cdot g \cdot Q \cdot H \times 10^{-3} \text{ kW}$$

$$= 1000 \times 9.81 \times 0.36 \times 261.5 \times 10^{-3}$$

$$= 923.5 \text{ kW}$$

Power developed by runner,

$$P = \frac{\dot{m}(V_{w1} - V_{w2}) u}{1000} \text{ (kW)}$$

$$= \frac{360 (71.63 - 5.13) 33.67}{1000}$$

$$= 806.06 \text{ kW}$$

$$P_s = \eta_m \times P = 0.8 \times 806.06$$

$$= \mathbf{644.85 \text{ kW}} \quad \text{...Ans.}$$

(ii) Hydraulic efficiency, η_h

$$\eta_h = \frac{2(V_{w1} - V_{w2}) u}{V_1^2}$$

$$= \frac{2(71.63 - 5.13) 33.67}{(71.63)^2}$$

$$= \mathbf{0.8728 \text{ or } 87.28\%} \quad \text{...Ans.}$$

(iii) Overall efficiency, η_o

$$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} = \frac{644.85}{923.5}$$

$$= \mathbf{0.6983 \text{ or } 69.83\%} \quad \text{...Ans.}$$

Ex. 3.6.15 : A Pelton wheel develops 750 kW under ahead of 200 m. Its nozzle has a diameter of 10 cm and losses in the pipeline due to friction amount to 90 Q^2, where Q is discharge in cubic meters per second. Assuming the gross head and efficiency of the wheel to be constant and C_v for the nozzle as 0.98. Find the discharge and overall efficiency. If the power developed is reduced to 650 kW by (i) Operating the needle in the nozzle and (ii) Closing the valve provided in the main. Determine the discharge in either case.

Soln. :

Given : $P = 750$ kW,

Gross head, $H_g = 200$ m., $d = 10$ cm $= 0.1$ m;

$h_f = 90\,Q^2$ (Q is in m^3/s),

$C_v = 0.98$

(i) Discharge at P = 750 kW

$$\text{Net head, } H = H_g - h_f = 200 - 90\,Q^2$$

$$\text{Jet velocity, } V = C_v\sqrt{2gH}$$

$$= C_v\sqrt{2g\,(H_g - 90\,Q^2)}$$

$$\text{Discharge, } Q = A \cdot V$$

$$= \frac{\pi}{4}d^2 \times C_v\sqrt{2g\,(H_g - 90\,Q^2)}$$

$$\therefore\ Q^2 = \left(\frac{\pi}{4}\times d^2\right)\times C_v^2 \times 2g\,(H_g - 90\,Q^2)$$

$$Q^2 = \left(\frac{\pi}{4}\times 0.1^2\right)^2 \times 0.98^2 \times 2 \times 9.81 \times (200 - 90\,Q^2)$$

$$Q^2 = 0.23247 - 0.10461\,Q^2$$

$$Q = \mathbf{0.4588\ m^3/s} \quad \text{...Ans.}$$

Theoretical power,

$$P_i = \rho\,Q\,g\,H$$

$$= \frac{1000 \times 0.4588 \times 9.81 \times 200}{1000}\,\text{kW}$$

$$= 900.2\ \text{kW}$$

Overall efficiency,

$$\eta_o = \frac{P}{P_i} = \frac{750}{900.2}$$

$$= \mathbf{0.8332\ or\ 83.32\%} \quad \text{...Ans.}$$

(ii) Discharge Q_1 and overall efficiency η_{o1} when power is reduced to P_1 = 650 kW by operating needle in nozzle

Net head, $H_1 = 200 - 90\,Q_1^2$

and $P = \rho\,Q_1\,g\,H_1$

$$650 \times 10^3 = 1000 \times Q_1 \times 9.81 \times (200 - 90\,Q_1^2)$$

$$200\,Q_1 - 90\,Q_1^3 = 66.26$$

$$Q_1 - 0.45\,Q_1^3 = 0.3313$$

$$\text{By hit and trial, } Q_1 = \mathbf{0.352\ m^3/s} \quad \text{...Ans.}$$

$$\text{Overall efficiency, } \eta_{o1} = \frac{P}{\rho\,Q_2 \cdot gH}$$

$$= \frac{650 \times 10^3}{1000 \times 0.352 \times 9.81 \times 200}$$

$$= \mathbf{0.9412\ or\ 94.12\%} \quad \text{...Ans.}$$

(iii) To find discharge Q_2 When the valve in the mains closed and power is reduced to P_2 = 650 kW

since, Power ∝ Head × Discharge

$$\therefore\ P = K \times (H_g - h_f) \times Q$$

$$750 = K \times (200 - 90\,Q^2) \times Q \quad \text{...(i)}$$

$$650 = K \times (200 - h_{f2}) \times Q_2 \quad \text{...(ii)}$$

where, $90\,Q^2 = h_f$

$$Q_2 = \frac{\pi}{4}d^2 \times V_1 = \frac{\pi}{4}d^2 \times C_v\sqrt{2g\,(H - h_f)}$$

$$= \frac{\pi}{4}\times 0.1^2 \times 0.98\sqrt{2 \times 9.81(H - h_{f2})}$$

$$= 0.0341\sqrt{H - h_f}$$

$$\therefore\ 650 = K \times (200 - h_{f2}) \times 0.0341\sqrt{200 - h_f}$$

On dividing Equations (i) and (ii)

$$\frac{750}{650} = \frac{[200 - (0.4588)^2 \times 90]\,0.4588}{0.0341\,(200 - h_{f2})^{3/2}}$$

$$\therefore\ (200 - h_{f2})^{3/2} = 211.2\ ;\ \text{i.e. } 200 - h_{f2} = 164.67$$

$$h_{f2} = 35.43\ \text{m}$$

$$\therefore\ Q_2 = 0.0341\sqrt{200 - 35.43}$$

$$= \mathbf{0.4375\ m^3/s} \quad \text{...Ans.}$$

Overall efficiency,

$$\eta_{o2} = \frac{P_2}{\rho\,Q_2\,g\,H} = \frac{650 \times 10^3}{1000 \times 9.81 \times 0.4375 \times 200}$$

$$= \mathbf{0.7572\ or\ 75.72\%} \quad \text{...Ans.}$$

Additional friction loss,

$$h_{f2} - h_f = 35.43 - 90\,Q^2$$

$$= 35.43 - 90 \times (0.4588)^2$$

$$= 35.43 - 18.94$$

$$= \mathbf{16.49\ m} \quad \text{...Ans.}$$

Ex. 3.6.16 : A single jet Pelton turbine operates a 10,000 kW generator. The generator efficiency is 93%, turbine efficiency is 86%, turbine head is 350 m, coefficient of nozzle velocity is 0.98, speed ratio is 0.46 and the jet ratio is approximately 12. Find the size of the jet, mean diameter of runner, synchronous speed, specific speed of turbine and bucket dimensions.

Soln. :

Given : Generator power, $P_g = 10000$ kW,

$\eta_g = 93\% = 0.93$, $\eta_h = 86\% = 0.86$,

$H = 350$ m, $C_v = 0.9$, speed ratio,

$K_u = 0.46$, Jet ratio, $m = 12$

(i) Size of jet, d

Jet velocity $V_1 = C_v\sqrt{2gH}$

$= 0.98\sqrt{2 \times 9.81 \times 350}$

$= 81.17$ m/s

Wheel velocity, $u = K_u \cdot V_1 = 0.46 \times 81.17$

$= 37.34$ m/s

Input power, $P_i = \rho \cdot Q \cdot g \cdot H$

$\therefore$ Generator power,

$P_g = P_i \times$ Hydraulic efficiency, $\eta_h \times$ generator efficiency, η_g

i.e. $P_g = \rho Q g H \times \eta_h \times \eta_g$

$10000 \times 10^3 = 1000 \times Q \times 9.81 \times 350 \times 0.86 \times 0.93$

$\therefore$ Discharge, $Q = 3.642$ m³/s

But, Q = Jet area × Jet velocity $= \frac{\pi}{4} d^2 \times V_1$

$3.642 = \frac{\pi}{4} \times d^2 \times 81.17$

$\therefore$ Jet diameter, **d = 0.239 m** ...**Ans.**

(ii) Mean diameter of runner, D :

Jet ratio, $m = \frac{D}{d}$

$\therefore D = m \cdot d = 12 \times 0.239$

$=$ **2.868 m** ...**Ans.**

(iii) Synchronous speed, N

$u = \frac{\pi DN}{60}$

$\therefore N = \frac{60 \times 4}{\pi D} = \frac{60 \times 37.34}{\pi \times 2.868}$

$=$ **248.65 rpm** ...**Ans.**

(iv) Specific speed, N_s

$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{248.65\sqrt{10000}}{(350)^{5/4}}$

$=$ **16.425 rpm** ...**Ans.**

Ex. 3.6.17 : A Pelton wheel develops 8421 kW of shaft power under a head of 320 m. Speed of rotation of wheel is 700 rpm and its overall efficiency is 87%. Assuming the coefficient of velocity of jet as 0.98, speed ratio as 0.45 and jet ratio as 6, find :
(i) Wheel Diameter
(ii) Jet diameter
(iii) Number of jets required.

SPPU - April 2017 (In sem.), 6 Marks

Soln. :

Shaft power, $P_s = 8421$ kW, $H = 320$ m

$N = 700$ rpm, $\eta_o = 87\% = 0.87$

$C_v = 0.98$

Speed ratio, $K_u = \frac{u}{V_1} = 0.45$,

Jet ratio, $m = \frac{D}{d} = 6$

(i) Wheel diameter, D

Velocity of jet, $V_1 = C_v\sqrt{2gh}$

$= 0.98\sqrt{2 \times 9.81 \times 320}$

$= 77.65$ m/s

$K_u = 0.45 = \frac{u}{V_1}$

$u = 0.45 \times V_1$

$= 0.45 \times 77.65$

$= 34.943$ m/s

But, $u = \frac{\pi D N}{60}$

$34.943 = \frac{\pi \times D \times 700}{60}$

D = 0.9534 m ...**Ans.**

(ii) Jet diameter, d

$\because m = \frac{D}{d}$

$6 = \frac{0.9534}{d}$

d = 0.1589 m ...**Ans.**

(iii) Number of jets required, n

Discharge per jet,

$q = \frac{\pi}{4} d^2 \times V_1$

$= \frac{\pi}{4} \times (0.1589)^2 \times 77.65$

$= 1.5399$ m³/s

$P_s = \rho \times g \times Q \times H \times \eta_0 \times 10^{-3}$ (kW)

$8421 = 1000 \times 9.81 \times Q \times 320 \times 0.87 \times 10^{-3}$

$\therefore$ Total discharge,

$Q = 3.08434$ m³/s

No. of jets, $n = \frac{\text{Total discharge, Q}}{\text{Discharge / Jet, q}}$

$= \frac{3.0834}{1.5399} = 2$ ...**Ans.**

Ex. 3.6.18 :

(a) Derive an expression for net work done per unit weight of flow of water of a Pelton wheel in terms of jet velocity 'V_1' bucket velocity 'u' and outlet bucket angle ϕ if the loss due to friction in buckets is expressed as $\frac{K_1 (V_1 - u)^2}{2g}$ and the other mechanical friction losses are expressed as $\frac{K_2 \cdot u_2}{2g}$, where K_1 and K_2 are constants. Show that the maximum efficiency based on jet energy occurs when $\frac{u}{V_1} = \frac{(1 - \cos\phi) + K_1}{2(1 - \cos\phi) + K_1 + K_2}$

(b) If for this Pelton wheel, gave a maximum efficiency of 82% based on energy of jet, having the speed ratio as 0.46. Find the constants K_1 and K_2 for bucket outlet angle of 16%.

Soln. : Refer Fig. P. 3.6.18.

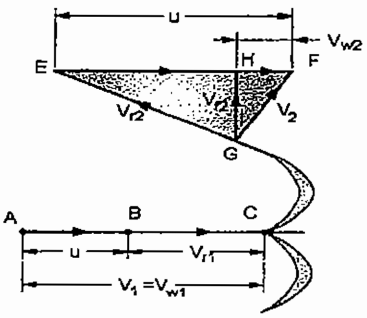


Fig. P. 3.6.18

From velocity diagram,

(a)
$$V_{w1} = V_1 \quad \text{...(i)}$$
$$V_{r1} = (V_1 - u)$$
$$V_{w2} = u - V_{r2} \cos\phi$$
$$V_{w2} = u - V_{r1} \cos\phi \quad \text{(if friction is neglected)}$$
$$V_{w2} = u - (V - u)\cos\phi \quad \text{...(ii)}$$

Therefore work developed by runner without any losses per unit weight

$$W_1 = \frac{(V_{w1} - V_{w2})\, u}{g}$$
$$= \frac{[V_1 - \{u - (V_1 - u)\cos\phi\}]\, u}{g}$$
$$= \frac{(V_1 - u)(1 + \cos\phi)\, u}{g} \quad \text{...(iii)}$$

Considering the bucket friction and other mechanical losses, the net work produced becomes,

W = W_1 – Friction losses in bucket – Mechanical losses

$$W = \frac{(V_1 - u)(1 + \cos\phi)\, u}{g} - \frac{K_1 (V - u)^2}{2g} - \frac{K_2 \cdot u^2}{2g} \quad \text{...(iv)proved.}$$

Maximum efficiency will occur when net work done is maximum. The condition is,

$$\frac{dW}{du} = 0$$
$$\therefore\ 0 = \frac{d}{du}\left[\frac{(V_1 - u)(1 + \cos\phi)\, u}{g} - \frac{K_1 (V_1 - u)^2}{2g} - \frac{K_2 \cdot u^2}{2g}\right]$$
$$0 = \frac{1}{g}\left[(V_1 - 2u)(1 + \cos\phi) - \frac{K_1}{2} \times 2(V_1 - u)(-1) - \frac{K_2}{2} \times 2u\right]$$
$$0 = \frac{1}{g}\left[(V_1 - 2u)(1 + \cos\phi) + K_1 (V_1 - u) - K_2 \cdot 4\right]$$
$$0 = V_1 (1 + \cos\phi) - 2u(1 + \cos\phi) + [K_1 V_1 - K_1 u - K_2 u]$$
$$0 = V_1 [(1 + \cos\phi) + K_1] - u[2(1 + \cos\phi) + K_1 + K_2]$$
$$\therefore\ \frac{u}{V_1} = \frac{(1 + \cos\phi) + K_1}{2(1 + \cos\phi) + K_1 + K_2} \quad \textbf{Proved ...(v)}$$

(b) **Given :** $\phi = 16°$; $\eta_{max} = 0.82$; $\frac{u}{V_1} = 0.46$

From Equation (iv),

$$0.46 = \frac{(1 + \cos 16) + K_1}{2(1 + \cos 16) + K_1 + K_2}$$

On solving, $K_2 = 1.174\, K_1 + 0.341$ ...(vi)

Hydraulic efficiency,

$$\eta_h = \frac{W}{V_1^2/2g}$$
$$\therefore\ \eta_h = \frac{2\left[(V_1 - u)(1 + \cos\phi)\, u - \frac{K_1 (V_1 - u)^2}{2} - \frac{K_2 \cdot u^2}{2}\right]}{V_1^2}$$
$$0.82 = \frac{2\left[(V_1 - 0.46\, V_1)(1 + \cos 16)\, 0.46\, V_1 - \frac{K_1}{2}(V_1 - 0.46\, V_1)^2 - \frac{K_2}{2}(0.46\, V_1)^2\right]}{V_1^2}$$
$$0.41 = 0.4872 - K_1 \times 0.1458 - 0.1058\, K_2$$
$$K_2 = 0.7297 - 1.3781\, K_1 \quad \text{...(vii)}$$

On solving Equations (vi) and (vii),

K_1 = **0.389** and K_2 = **0.194** **...Ans.**

Ex. 3.6.19 : A single jet pelton wheel is supplied from a reservoir 300 m above the centre of nozzle. The supply pipe is 0.7 m in diameter and 5.6 km long. The friction factor for the pipe is 0.03. The jet has a diameter of 0.1 m. The coefficient of velocity for nozzle is 0.97. The velocity of buckets is 0.47 of the jet speed. The outlet vane angle for the buckets is 15°. The relative velocity of water is reduced by 15% in passing over the buckets. If the mechanical efficiency is 88%, find the water power, power developed by turbine, hydraulic efficiency and the overall efficiency.

Soln. :

Given : $H = 300$ m.

Supply pipe diameter, $D_1 = 0.7$ m.

Length of pipe, $L = 5.6$ km $= 5600$ m.

Friction factor, $4f = 0.03$

Jet diameter, $d = 0.1$ m ; $C_v = 0.97$

Speed ratio, $K_v = \frac{u}{V_1} = 0.47$;

$$\phi = 15°$$

$$V_{r2} = \left(1 - \frac{15}{100}\right) V_{r1} = 0.85\, V_{r1}$$

$$\eta_m = 88\,\% = 0.88$$

Let mean velocity of water in supply pipe be V and V_1 be the jet velocity be V_1

Using continuity equation,

$$Q = A_1 \cdot V = \text{Area of jet} \times V_1$$

$$\frac{\pi}{4} D_1^2 \times V = \frac{\pi}{4} d^2 \times V_1$$

$$\frac{\pi}{4} \times 0.7^2 \times V = \frac{\pi}{4} \times 0.1^2 \times V_1$$

$$V = 0.02041\, V_1 \quad \text{...(i)}$$

Let V_1' be the theoretical velocity of jet of water,

$$\text{where} \quad V_1' = \frac{V_1}{C_v} = \frac{V_1}{0.97} \quad \text{...(ii)}$$

Applying Bernoullis equation to free surface of water in reservoir and at exit of nozzle we get,

Gross head, H_g = Head lost (in pipe + in nozzle) due to friction + Kinetic head of water

$$H_g = \frac{4 f L V^2}{D_1 \cdot 2g} + \left(\frac{V_1'^2 - V_1^2}{2 \cdot g}\right) + \frac{V_1^2}{2g}$$

$$300 = \frac{0.03 \times 5600 \times (0.02041\, V_1)^2}{0.7 \times 2 \times 9.81} + \frac{\left(\frac{V_1}{0.97}\right)^2 - V_1^2}{2 \times 9.81} + \frac{V_1^2}{2 \times 9.81}$$

$$300 = 0.005096\, V_1^2 + 0.054170\, V_1^2 - \frac{V_1^2}{2 \times 9.81} + \frac{V_1^2}{2 \times 9.81}$$

$$V_1 = 71.15 \text{ m/s}$$

$$\text{Flow rate, } Q = \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} \times 0.1^2 \times 71.15$$

$$= 0.5588 \text{ m}^3/\text{s}$$

1. **Water power, P_i**

$$P_i = \rho \cdot g \cdot QH$$

$$= 1000 \times 9.81 \times 0.5588 \times 300 \text{ W}$$

$$= 1644.55 \times 10^3 \text{W}$$

$$= \mathbf{1644.55 \text{ kW}} \quad \text{...Ans.}$$

2. **Power developed by turbine, P**

Refer Fig. P. 3.6.19.

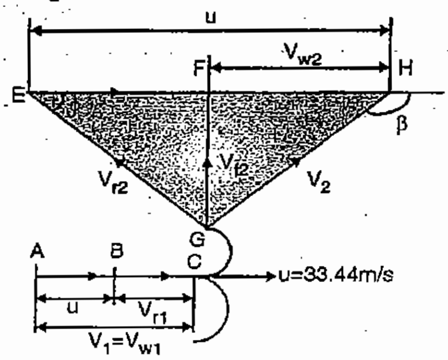


Fig. P. 3.6.19

Velocity of bucket,

$$u = 0.47 \times \text{Velocity of jet}$$

$$= 0.47 \times 71.15 = 33.44 \text{ m/s}$$

$$V_{r1} = V_1 - u = 71.15 - 33.44 = 37.71 \text{ m/s}$$

$$V_{r2} = 0.85\, V_{r1}$$

$$= 0.85 \times 37.71 = 32.05 \text{ m/s}$$

$$V_{w2} = V_{r2} \cos \phi - u$$

$$= 32.05 \cos 15 - 33.44 = -2.48 \text{ m/s}$$

V_{w2} is in the same direction as V_{w1}, accordingly the velocity diagram has been shown in Fig. P. 3.6.19.

Mass flow rate,

$$\dot{m} = \rho \cdot Q = 1000 \times 0.5588 = 558.8 \text{ kg/s}$$

$$P = \frac{\dot{m}\,(V_{w1} + V_{w2})\, u}{1000} \text{ kW}$$

$$= \frac{558.8\,(71.15 - 2.48) \times 33.44}{1000}$$

$$= \mathbf{1283.2 \text{ kW}} \quad \text{...Ans.}$$

3. **Hydraulic efficiency, η_h**

$$\eta_h = \frac{2\,(V_{w1} - V_{w2})\, u}{V_1^2} = \frac{2 \times (71.15)\, 33.48}{71.15^2}$$

$$= \mathbf{0.9728 \text{ or } 97.28\%} \quad \text{...Ans.}$$

4. **Overall efficiency, η_o**

Shaft power,

P_s = Power developed, P × Mechanical efficiency, η_m

$$= 1283.2 \times 0.88 = 1129.2 \text{ kW}$$

$$\eta_o = \frac{\text{Shaft power, } P_s}{\text{Water power, } P_i} = \frac{1129.2}{1644.55}$$

$$= 0.6866 \text{ or } 68.66\% \quad \text{...Ans.}$$

Ex. 3.6.20 : A Pelton wheel is to work at the foot of the dam whose reservoir level is 220 m. The full opening of the turbine nozzle is 200 mm and the coefficient of velocity is 0.98. The turbine is to operate at 250 rpm and develops a power of 3.75 MW. Assuming the blade to jet speed ratio of 0.46. Determine the wheel diameter at the pitch circle of the bucket. The bucket outlet angle is 16°, calculate the bucket efficiency, turbine efficiency and mechanical efficiency. If the bucket develop roughness causing bucket friction coefficient as 0.9, determine the change in the bucket efficiencies.

Soln. :

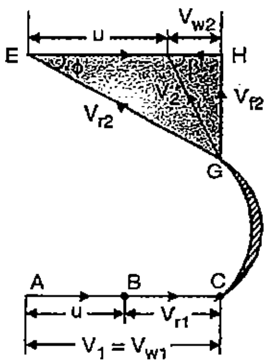


Fig. P. 3.6.20

$$H_g = 220 \text{ m}$$

Diameter of nozzle, $d = 200 \text{ mm} = 0.2 \text{ m}$

$$C_v = 0.98$$

$$N = 250 \text{ rpm}$$

$$P_s = 3.75 \text{ MW} = 3.75 \times 10^3 = 3750 \text{ kW}$$

$$\frac{u}{V_1} = 0.46 ; \quad \phi = 16°$$

Refer Fig. P. 3.6.20.

Velocity of jet, $V_1 = C_v\sqrt{2 \times g \times H}$

$$= 0.98\sqrt{2 \times 9.81 \times 220}$$

$$= 64.39 \text{ m/s} = V_{w1}$$

$$\therefore u = 0.46 \times V_1 = 0.46 \times 64.39$$

$$= 29.62 \text{ m/s}$$

$$V_{r1} = V_1 - u = 64.39 - 29.62$$

$$= 34.77 \text{ m/s}$$

$$V_{r2} = V_{r1} = 34.77 \text{ m/s}$$

(when friction is neglected)

1. Wheel diameter, D

$$u = \frac{\pi DN}{60}$$

$$D = \frac{60 \times u}{\pi N} = \frac{60 \times 29.62}{\pi \times 250}$$

$$= \mathbf{2.263 \text{ m}} \quad \text{...Ans.}$$

2. Bucket efficiency, η_b

$$V_{w2} = V_{r2} \cos \phi - u$$

$$= 34.77 - 29.62 = 3.8 \text{ m/s}$$

$$\eta_h = \frac{2(V_{w1} + V_{w2})\, u}{V_1^2}$$

$$= \frac{2\,(64.39 + 3.8)\, 29.62}{(64.39)^2}$$

$$= \mathbf{0.9743 \text{ or } 97.43\ \%} \quad \text{...Ans.}$$

3. Turbine efficiency, η_t

Discharge, $Q = AV_1 = \frac{\pi}{4} \times 0.2^2 \times 64.39$

$$= 2.0229 \text{ m}^3/\text{s}$$

Power developed, $P = \frac{\rho\, Q\, (V_{w1} + V_{w2})\, u}{1000}$ kW

$$= \frac{1000 \times 2.0229\, (64.39 + 3.8)\, 29.22}{1000}$$

$$= 4085.8 \text{ kW}$$

$$\therefore \eta_t = \frac{P}{\rho \cdot g\, QH}$$

$$= \frac{4085.8 \times 10^3}{1000 \times 9.81 \times 2.0229 \times 220}$$

$$= \mathbf{0.9358 \text{ or } 93.58\%} \quad \text{...Ans.}$$

4. Mechanical efficiency, η_m

Given shaft power, $P_s = 3750$ kW

$$\therefore \eta_m = \frac{P_s}{P} = \frac{3750}{4085.8} \times 100\ \%$$

$$= \mathbf{91.78\%} \quad \text{...Ans.}$$

5. Bucket efficiency, η_{h1} of roughness, $K = \frac{V_{r2}}{V_{r1}} = 0.9$

$$V_{r2} = 0.9 \times 34.77 = 31.29 \text{ m/s}$$

$$V_{w2} = V_{r2} \cos \phi - u$$

$$= 31.29 \cos 16 - 29.62$$

$$= 0.46 \text{ m/s}$$

$$\eta_{h1} = \frac{2(V_{w1} + V_{w2})\, u}{V_1^2}$$

$$= \frac{2 \times (64.39 + 0.46)\, 29.62}{(64.39)^2}$$

$$= \mathbf{0.9266 \text{ or } 92.6\ \%} \quad \text{...Ans.}$$

Ex. 3.6.21 : The water from a reservoir is supplied to a Pelton wheel having a gross head of 450 m through the penstock of diameter 1.1 m and length 4 km. The coefficient of friction for penstock is 0.0075. Water from penstock is discharged through two nozzles of 100 mm diameter each and strikes the bucket having an exit angle of 15°. The friction factor for buckets is 0.87. Assuming the speed ratio of 0.47 and mechanical efficiency of 85%, find :

(i) Mean velocity of water in penstock and jet velocity
(ii) Net head available of runner.
(iii) Bucket velocity
(iv) Power given to runner in kW
(v) Shaft power
(vi) Hydraulic efficiency
(vii) Overall efficiency

Soln. : Refer Fig. P. 3.6.21.

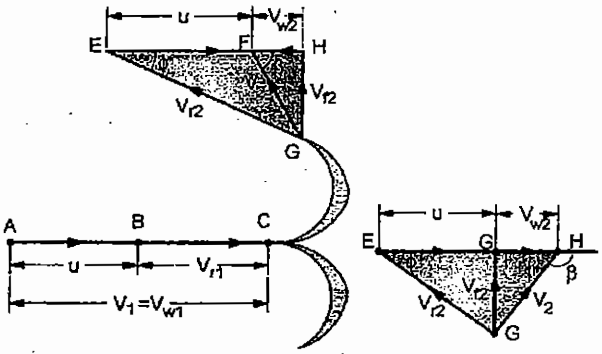


Fig. P. 3.6.21

Given :

Gross head, $h_g = 450$ m,

Penstock diameter, $D_1 = 1.1$ m,

$L = 4$ km $= 4000$ m

Number of nozzles, $n = 2$;

$f = 0.0075$,

$d = 100$ mm $= 0.1$ m

$K = \frac{V_{r2}}{V_{r1}} = 0.87$

Speed ratio, $K_v = \frac{u}{V_1} = 0.47$

$\eta_m = 85\% = 0.85$

(i) Velocity of water in Penstock and jet velocity

Let : V_1' be the mean velocity of water in penstock and V_1 be the jet velocity.

Using continuity equation,

$$Q = A_1 V_1' = n \times \text{Area of jet} \times V_1$$

$$\frac{\pi}{4} D_1^2 V_1' = 2 \times \frac{\pi}{4} d^2 \times V_1$$

i.e.

$$D_1^2 V_1' = 2d^2 \times V_1$$

$$1.1^2 \times V_1' = 2 \times (0.1)^2 \times V_1$$

$$V_1' = 0.01653\, V_1$$

According to Bernoullis equation as applied to free surface of water in the reservoir and at exit of nozzle Gross head, H_g = Head lost in friction in Penstock, $H_f + \eta \times \frac{V_1^2}{2g}$

$$H_g = \frac{4fL(V_1')^2}{D_1 \times 2g} + \frac{V_1^2}{2g}$$

$$450 = \frac{4 \times 0.0075 \times 4000 \times (0.01653\, V_1)^2}{1.1 \times 2 \times 9.81} + \frac{V_1^2}{2 \times 9.81}$$

$\therefore$ Velocity of jet $V_1 = \mathbf{92.6\ m/s}$ ...Ans.

Velocity of water in penstock,

$V_1' = 0.01653 \times 92.6 = \mathbf{1.65306\ m/s}$ ...Ans.

(ii) Net head available at runner

$$H = H_g - H_f = \frac{V_1^2}{2g}$$

$= \frac{(92.6)^2}{2 \times 9.81} = 437.04$ m ...Ans.

(iii) Bucket velocity, u

u = speed ratio, $K_u \times$ Jet velocity,

$V_1 = 0.47 \times 92.6 = \mathbf{43.6\ m/s}$ ...Ans.

(iv) Power given to runner in kW, P

From velocity Δs shown in Fig. P. 3.6.15 we have;

$$V_{w1} = V_1 = 92.6\ \text{m/s};$$

$$V_{r1} = V_1 - u = 92.6 - 43.6 = 49.0\ \text{m/s}$$

$$V_{r2} = K \cdot V_{r1} = 0.87 \times 49 = 42.63\ \text{m/s}$$

$$V_{w2} = V_{r2} \cos\phi - u$$

$$= 42.63 \cos 15 - 49 = -7.82\ \text{m/s}$$

Modified velocity diagram is shown in Fig. P. 3.6.15 since V_{w2} is negative.

$$P = \frac{\text{Number of nozzeles} \times \rho \cdot A V_1 (V_{w1} - V_{w2})\, u}{1000} \text{kW}$$

$$= \frac{2 \times 1000 \times \frac{\pi}{4} \times (0.1)^2 \times 92.6\,(92.6 - 7.82)\,49}{1000}$$

$= \mathbf{6042.6\ kW}$ ...Ans.

(v) Shaft power, P_s

$$P_s = \eta_m \times \text{Power given to runner}$$

$= 0.85 \times 6042.6 = \mathbf{5136.2\ kW}$ **...Ans.**

(vi) Hydraulic efficiency, η_h

$$\eta_h = \frac{2(V_{w1} - V_{w2})u}{V_1^2} = \frac{2 \times (92.6 - 7.82)\,49}{(92.6)^2}$$

$= \mathbf{0.9689 \text{ or } 96.89\%}$ **...Ans.**

(vii) Overall efficiency, η_o

$$\eta_o = \eta_m \times \eta_h = 0.85 \times 0.9689$$

$= \mathbf{0.8236 \text{ or } 82.36\%}$ **...Ans.**

Ex. 3.6.22 : A Pelton wheel develops 870 kW power under a head of 230 m. The nozzle diameter is 10 cm and frictional losses in the pipe line are 90 Q^2 where 'Q' is discharge in m^3/s. Assuming the gross head and efficiency of wheel to remain constant, estimate the discharge when the power developed is reduced to 700 kW.

(i) By operating spear regulation in the nozzle.

(ii) By closing the regulating valve provided in the mains.

What is the additional loss of head in case (ii) ?

Soln. :

Given : $P = 870$ kW;

Gross head, $H_g = 230$ m,

Nozzle diameter, $d = 10$ cm $= 0.1$ m

Friction losses $H_f = 90\,Q^2$

Net power, $P_1 = 700$ kW

(i) Case I : To find new discharge, Q_1 when power is reduced by spear regulation in nozzle

Spear regulations means that power reduction is obtained by reduction in discharge from nozzle.

Discharge through nozzle, $Q = A \cdot V_1$

But, jet velocity $V_1 = C_v\sqrt{2gH}$ (assuming $C_v = 1$)

$\therefore$ Net head, H = Gross head, H_g – Friction head H_f

$$H = H_g - 90\,Q^2$$

$$\therefore\ Q = \frac{\pi}{4} \cdot d^2 \times \sqrt{2g\,(H_g - 90\,Q^2)}$$

$$= \frac{\pi}{4} \times 0.1^2 \times \sqrt{2 \times 9.81 \times (230 - 90\,Q^2)}$$

On squaring both sides,

$$Q^2 = \left(\frac{\pi}{4} \times 0.1^2\right)^2 \times 2 \times 9.81\,(230 - 90\,Q^2)$$

$$Q^2 = 0.27836 - 0.108923\,Q^2$$

$$\therefore\ Q = \sqrt{\frac{0.27836}{1 + 0.108923}} = 0.501\ m^3/s$$

$$H_f = 90\,Q^2 = 90 \times 0.501^2 = 22.6\ m$$

$\therefore$ Net Head, $H = H_g - H_f = 230 - 22.6 = 207.4$ m

$$\text{Efficiency, } \eta = \frac{P}{\rho g Q H}$$

$$= \frac{870 \times 10^3}{1000 \times 9.81 \times 0.501 \times 207.4}$$

$= \mathbf{0.8535 \text{ or } 85.35\%}$ **...Ans.**

Let Q_1 be the discharge when power is reduced to 700 kW with the same efficiency. Therefore,

$$\eta = \frac{P_1}{\rho g Q_1 H}$$

where, $H = 230 - Q_1^2$

$$\therefore\ 0.8535 = \frac{700 \times 10^3}{1000 \times 9.81 \times Q_1 \times (230 - Q_1^2)}$$

$230\,Q_1 - Q_1^3 = 83.6$; Since Q_1^3 is negligible

therefore, $230\,Q_1 = 83.6$

or, $Q_1 = \mathbf{0.3635\ m^3/s}$ **...Ans.**

(ii) Case II : To find new discharge Q_2 and additional loss of power when power is reduced by regulating valve provided in the mains :

Given : $P = 870$ kW, $Q = 0.501\ m^3/s$; $H = 207.4$ m

$P_2 = 700$ kW at $Q = Q_2$ and $H_2 = 230 - H_{f2}$

Since power is proportional to head it implied.

Since, $P = \rho QgH$, therefore, $P \propto QH$. Hence,

$$\frac{P}{P_2} = \frac{Q \times H}{Q_2 H_2} = \frac{Q \times H}{Q_2\,(230 - H_{f2})}$$

$$\frac{870}{700} = \frac{0.501 \times 207.4}{Q_2 \times (230 - H_{f2})} \quad \text{...(i)}$$

But, $Q_2 = A \cdot V_2 = \frac{\pi}{4} d^2 \sqrt{2gH_2}$

$$Q_2 = \frac{\pi}{4} \times 0.1^2 \times \sqrt{2 \times 9.81 \times (230 - H_{f2})}$$

$$= 0.03479\sqrt{(230 - H_{f2})} \quad \text{...(ii)}$$

On substituting the value of Q_2 from Equation (ii) in Equation (i) we get,

$$\frac{870}{700} = \frac{0.501 \times 207.4}{0.03479\sqrt{(230 - H_{f2})} \times (230 - H_{f2})}$$

$$\therefore\ (230 - H_{f2})^{3/2} = 2403.1$$

$$\therefore\ 230 - H_{f2} = (2403.1)^{2/3} = 179.41$$

$$H_{f2} = 230 - 179.41 = 50.59\ m$$

Discharge, Q_2

From Equation (ii),

$$Q_2 = 0.03479\sqrt{230 - 50.59}$$

$= \mathbf{0.466\ m^3/s}$ **...Ans.**

Additional friction loss

$$= H_{f2} - H_f = 50.59 - 22.6$$

$$= \mathbf{27.99\ m} \quad \text{...Ans.}$$

Ex. 3.6.23 : A single jet Pelton wheel is supplied with water from a reservoir 300 m above the centre of the nozzle, through a pipe of 600 mm diameter and 5 km long. The friction factor for the pipe is 0.032. The jet has diameter of 80 mm and its velocity coefficient of 0.97. The wheel bucket has a speed of 0.47 of the jet speed and deflects the water through 160°, the relative velocity at outlet is 80% of that at inlet. The mechanical efficiency of the wheel is 80%. Determine

(i) Shaft power of the wheel
(ii) Hydraulic efficiency
(iii) Overall efficiency

Use $h_f = \frac{f \cdot l \cdot V^2}{2gD}$ with usual notations.

Soln. :

Refer Fig. P. 3.6.23.

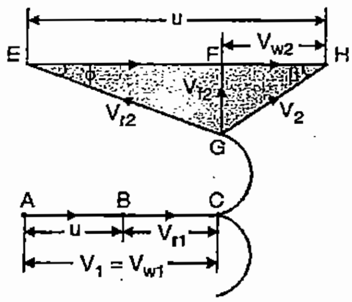


Fig. P. 3.6.23

$$H_g = 300\ \text{m}$$

Pipe diameter, $D = 600\ \text{mm} = 0.6\ \text{m}$

$l = 5\ \text{km} = 5000\ \text{m}$

$f = 0.032$

Jet diameter, $d = 80\ \text{mm} = 0.08\ \text{m}$

$C_v = 0.97$

Wheel bucket speed,

$u = 0.47 \times \text{jet speed}, V_1$

i.e. $u = 0.47\ V_1$

$\phi = 180° - \text{angle of deflection}$

$= 180 - 160 = 20°$

friction factor, $\frac{V_{r2}}{V_{r1}} = 0.8$

$\eta_m = 80\,\% = 0.8$

Let : V = Velocity of water in pipe

and V_1 = Velocity of jet of water at nozzle

From continuity equation,

$$Q = A \cdot V$$

$$\therefore \frac{\pi}{4} \cdot D^2 \times V = \frac{\pi}{4} \times d^2 \times V_1$$

$$\text{i.e. } D^2 \times V = d^2 \times V_1$$

$$V = \frac{d^2}{D^2} \times V_1 = \frac{(0.08)^2}{(0.6)^2} \times V_1$$

$$= 0.01778\ V_1 \quad \text{...(i)}$$

Let V_1' = Theoretical velocity of jet of water

$$= \frac{V_1}{C_v} = \frac{V_1}{0.97} \quad \text{...(ii)}$$

From Bernoullis equation of free surface of reservoir and exit of nozzle we have,

Gross head,

H_g = Head lost in pipe + Head lost in nozzle + Kinetic head of water at jet

$$H_g = \frac{flV^2}{2gD} + \frac{V_1'^2 - V_1^2}{2g} + \frac{V_1^2}{2g}$$

$$300 = \frac{0.032 \times 5000 \times (0.017778\ V_1)^2}{2 \times 9.81 \times 0.6} + \frac{1}{2 \times 9.81}\left(\frac{V_1^2}{0.97^2} - V_1^2\right) + \frac{V_1^2}{2 \times 9.81}$$

$$V_1 = 71.63\ \text{m/s}$$

$$\therefore Q = \frac{\pi}{4} d^2 V_1 = \frac{\pi}{4} \times (0.08)^2 \times 71.63 = 0.36\ \text{m}^3/\text{s}$$

$$u = 0.47\ V_1 = 0.47 \times 71.63 = 33.67\ \text{m/s}$$

From Fig. P. 3.6.23

We have, $V_{r1} = V_1 - u = 71.63 - 33.67 = 37.96\ \text{m/s}$

$V_{r2} = 0.8\ V_{r1} = 0.8 \times 37.96 = 30.37\ \text{m/s}$

$V_{w2} = V_{r2} \cos\phi - u$

$= 30.37 \cos 20 - 33.67 = -5.13\ \text{m/s}$

V_{w2} is in same direction as V_{w1}, accordingly the velocity diagram has been drawn as shown in Fig. P. 3.6.23.

Mass flow rate, $\dot{m} = \rho \cdot Q = 1000 \times 0.36$

$= 360\ \text{kg/s}$

(i) Shaft power, P_s

Power developed,

$$P = \frac{(V_{w1} - V_{w2})\, u}{V_1^2} \text{ kW}$$

$$= \frac{360\,(71.63 - 5.13)\, 33.67}{1000}$$

$$= 806.06 \text{ kW}$$

$$P_s = \eta_m \cdot P = 0.8 \times 806.06$$

$$= \mathbf{644.85 \text{ kW}} \qquad \text{...Ans.}$$

(ii) Hydraulic efficiency, η_h

$$\eta_h = \frac{2(V_{w1} - V_{w2})\, u}{V_1^2}$$

$$= \frac{2\,(71.63 - 5.13)\, 33.67}{(71.63)^2}$$

$$= \mathbf{0.8727 \text{ or } 87.27\%} \qquad \text{...Ans.}$$

(iii) Overall efficiency, η_o

$$\eta_o = \frac{P_s}{\rho Q g H}$$

$$= \frac{806.06 \times 1000}{1000 \times 0.36 \times 9.81 \times 300}$$

$$= \mathbf{0.7608 \text{ or } 76.08\%} \qquad \text{...Ans.}$$

Ex. 3.6.24 : A horizontal single jet Pelton wheel works at a hydro-electric power station where the head race level is vertically 450 m above the nozzle centerline. The length of the penstock is 5 km and its diameter is 1 m. Friction factor for the penstock may be taken to be 0.032. Velocity coefficient for the nozzle is 0.97 and other head losses (in fittings and bends) up to the nozzle exit amount to 10 m of water. Relative flow speed at the bucket exit is 90% of that at the bucket inlet on account of bucket friction. Clearance angle is 15 degrees and speed ratio is 0.47. Average velocity of water in the penstock is 2 m/s and the density of water is 1000 kg/m³. Mechanical efficiency accounting for the mechanical friction losses in bearings is 95% and the generator efficiency is unknown. If the turbine develops an electrical power output of 5 MW while rotating at 375 rpm, determine :

(i) Rate of work done by the jet on the wheel.

(ii) Mean diameter of the bucket pitch circle.

(iii) Hydraulic efficiency.

(iv) Generator efficiency. **SPPU – May 11, 8 Marks**

Soln. : Refer Fig. P. 3.6.24

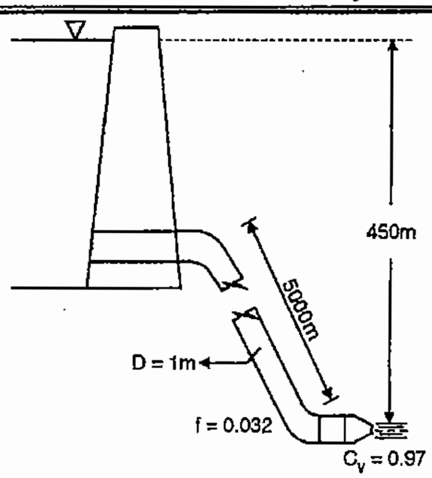


Fig. P. 3.6.24

Gross head, $H_g = 450\text{m}.$

Length of penstock, $L = 5 \text{ km} = 5000 \text{ m}$

$D_p = 1\text{m}$

friction factor $f = 0.032.$

$C_v = 0.97$, other head Losses, $h = 10\text{m}$

$V_{r_2} = 0.9\, V_{r_1}$

$\phi = 80\text{–Clearance angle } (15°) = 165°.$

Speed ratio, $K_u = \dfrac{u}{V_1} = 0.47;$

Velocity in penstock, $V = 2\text{m/s}$, $\rho = 1000\text{kg/m}^3$, $\eta_m = 0.95$

Generator efficiency $= \eta_g$

Electrical power, $P_e = 5\text{MW}$, $N = 375$ rpm

(i) Rate of workdone by jet on wheel. i.e. water power (W. P)

$$\text{Friction loss, } h_f = \frac{f\,L\,V^2}{D_p \cdot 2g}$$

$$= \frac{0.032 \times 5000(2)^2}{1 \times 2 \times 9.81} = 32.62 \text{ m}$$

$$\text{Net lead, } H = H_g - h_f - h$$

$$= 450 - 32.62 - 10$$

$$= 407.38 \text{ m}$$

$$\text{Jet velocity, } V_1 = C_v \sqrt{2gH}$$

$$= 0.97 \times \sqrt{2 \times 9.81 \times 407.38}$$

$$= 86.72 \text{ m/s}$$

$$K_u = \frac{u}{V_1} = 0.47$$

$$u = 0.47 \times 86.72 = 40.758 \text{ m/s}$$

$$u = \frac{\pi D N}{60}$$

Bucket diameter,

$$D = \frac{60 \times u}{\pi N}$$

$$= \frac{60 \times 40.758}{\pi \times 375} = 2.076 \text{ m}$$

Assuming $\frac{d}{D} = 10$

Jet diameter, $d = \frac{D}{10} = \frac{2.076}{10} = 0.2076 \text{ m}$

Area of jet, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.20762)^2$

$$= 0.03385 \text{ m}^2$$

Discharge, $Q = A \times V_1 = 0.03385 \times 86.72$

$$= 2.9354 \text{ m}^3/\text{s}$$

Water Power $= \rho \cdot g \cdot Q H \times 10^{-3}$ (kW)

$$= 1000 \times 9.81 \times 2.9354 \times 407.38 \times 10^{-3}$$

$= 11731$ kW $=$ **11.731 MW** **...Ans.**

(ii) Mean diameter of the bucket pitch circle, D

$D =$ **2.076 m** (As calculated above) **...Ans.**

(iii) Hydraulic efficiency, η_h

Refer Fig. P. 3.6.24(a)

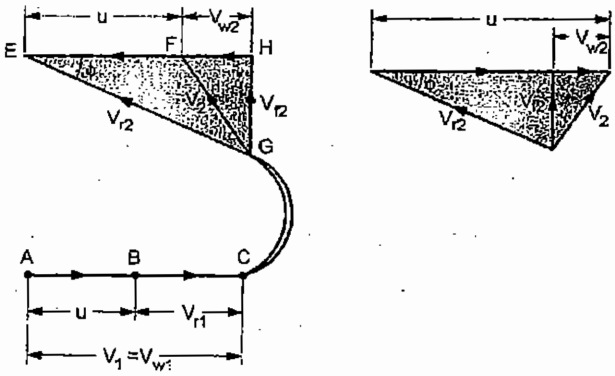


Fig. P. 3.6.24(a)

$$V_{w_1} = V_1 = 86.72 \text{ m/s}$$

$$V_{r_1} = V_1 - u = 86.72 - 40.785$$

$$= 45.935 \text{ m/s}$$

$$V_{r_2} = K.V_{r_1} = 0.9 \times 45.962$$

$$= 41.34 \text{ m/s}$$

$$V_{w_2} = V_{r_2} \cos\phi - u$$

$$= 41.34 \cos 15 - 40.785$$

$$= -0.854 \text{ m/s}$$

Hydraulic power,

$$\dot{W} = \frac{\rho Q (V_{w_1} - V_{w_2}) u}{1000} \text{ (kW)}$$

$$= \frac{1000 \times 2.9354 (86.72 - 0.854) 40.785}{1000}$$

$=$ **10279.9 kW = 10.28 MW** **...Ans.**

Hydraulic efficiency,

$$\eta_h = \frac{\dot{W}}{W.P} = \frac{10.28}{11.731}$$

$=$ **0.8763 or 87.63 %** **...Ans.**

(iv) Generate efficiency, η_g

Refer Fig. P. 3.6.24(b)

Fig. P. 3.6.24(b)

Shaft power,

$$P_S = \eta_m \times \dot{W} = 0.95 \times 10.28 = 9.766 \text{ MW}$$

$$\eta_g = \frac{\text{Electrical power}, p_e}{\text{shaft power}, p_s} = \frac{5}{9.766}$$

$=$ **0.512 or 51.2 %** **...Ans.**

Ex. 3.6.25 : Design a Pelton wheel which working under a head of 80 m at 300 rpm. It produces a power of 125 kW with an overall efficiency of 85%. The speed ratio for the turbine is 0.45 and the coefficient of velocity of nozzle is 0.98.

Soln. :

Given : $H = 80$ m ; $N = 300$ rpm ; $P_s = 125$ kW; $\eta_o = 85\% = 0.85$, $K_u = 0.45$; $C_v = 0.98$.

(i) Diameter of wheel, D

Velocity of jet, $V_1 = C_v\sqrt{2gH}$

$$= 0.98\sqrt{2 \times 9.81 \times 80}$$

$$= 38.83 \text{ m/s}$$

Speed ratio, $K_u = \frac{u}{V_1}$

$\therefore$ Velocity of wheel, $u = K_u \times V_1 = 0.45 \times 38.83$

$$= 17.47 \text{ m/s}$$

But, $u = \frac{\pi DN}{60}$

i.e. $17.47 = \frac{\pi \times D \times 300}{60}$

$\therefore$ $D =$ **1.112 m** **...Ans.**

(ii) Diameter of jet, d

$$\text{Overall efficiency, } \eta_o = \frac{\text{Shaft power, } P_s}{\text{Water power, } P_i} = \frac{P_s}{\rho g Q H}$$

$$0.85 = \frac{125 \times 10^3}{1000 \times 9.81 \times Q \times 80}$$

$$\therefore \text{ Discharge, } Q = 0.1874 \text{ m}^3/\text{s}$$

$$\text{But, } Q = \text{Area of jet} \times \text{jet velocity}$$

$$= \frac{\pi}{4} d^2 \times V_1$$

$$0.1874 = \frac{\pi}{4} d^2\, 38.83$$

$$\therefore \text{ Jet diameter, d} = 0.0784 \text{ m} = \mathbf{78.4 \text{ mm}}$$ ...Ans.

(iii) Number of buckets, Z

$$Z = 15 + \frac{D}{2d} = 15 + \frac{1.112}{0.0784 \times 2}$$

$$= 15 + 7.092 = 22.092$$

$$= \text{say, } \mathbf{23 \text{ buckets}}$$ ...Ans.

(iv) Width of buckets :

$$B = 4d = 4 \times 78.4$$

$$= \mathbf{313.6 \text{ mm}}$$ ...Ans.

(v) Depth of buckets

$$T = 0.3B = 313.6 \times 0.3$$

$$= \mathbf{94.08 \text{ mm}}$$ ...Ans.

(vi) Length or height of buckets

$$L = 2.5d = 2.5 \times 78.4$$

$$= \mathbf{196 \text{ mm}}$$ ...Ans.

(vii) Notch width

$$m = 1.15d = 1.15 \times 78.4$$

$$= \mathbf{90.16 \text{ mm}}$$ ...Ans.

Ex. 3.6.26 : A Pelton turbine is delivered water to jet from a reservoir under a gross head of 500 m through penstock 20 km long. The transmission efficiency of penstock is 85% and the coefficient of friction for pipe used is 0.006. The turbine develops a power of 5000 kW, find the net head available water power supplied to turbine, discharge rate, diameter of penstock, velocity and diameter of jet and diameter of buckets. Assume, $C_v = 0.98$, $K_u = 0.46$, hydraulic efficiency = 85%, mechanical efficiency = 95%; Speed of turbine = 800 rpm.

Soln. :

Given : Gross head, $H_g = 500$ m;

Length of penstock, $L = 20 \text{ km} = 20 \times 10^3$m

Transmission efficiency of penstock,

$\eta_t = 85\% = 0.85$; $f = 0.006$,

Shaft power, $P_s = 5000$ kW;

$C_v = 0.98$; $K_u = \frac{u}{V_1} = 0.46$,

$\eta_h = 85\% = 0.85$, $\eta_m = 95\% = 0.95$;

$N = 800$ rpm.

(i) Net head, H and water power supplied to turbine, P_i

Transmission efficiency,

$$\eta_t = \frac{\text{Net head, H}}{\text{Gross head, } H_g}$$

$$0.85 = \frac{H}{500}$$

$$\therefore H = \mathbf{425 \text{ m}}$$ ...Ans.

$\therefore$ Friction head in penstock,

$$H_f = H_g - H$$

$$= 500 - 425 = \mathbf{75 \text{ m}}$$ ...Ans.

Shaft power,

$$P_s = \text{Water power, } P_i \times \text{Mechanical efficiency, } \eta_m \times \text{Hydraulic efficiency, } \eta_h$$

$$5000 = P_i \times 0.95 \times 0.85$$

$$P_i = \mathbf{6191.95 \text{ kW}}$$ ...Ans.

(ii) Discharge rate, Q

$$P_i = \rho \cdot g \cdot QH$$

$$\therefore Q = \frac{P_i}{\rho \cdot g \cdot H} = \frac{6191.95 \times 10^3}{1000 \times 9.81 \times 425}$$

$$= \mathbf{1.4852 \text{ m}^3/\text{s}}$$ ...Ans.

(iii) Diameter of penstock, D_p

Velocity of water in penstock,

$$V_p = \frac{Q}{AP} = \frac{Q}{\frac{\pi}{4} D_1^2} = \frac{4Q}{\pi D_p^2} \quad \text{...(i)}$$

Friction loss in penstock,

$$H_f = \frac{4f \cdot L \cdot V_p^2}{D_p \times 2g};$$

On substituting the value of V_p from Equation (i),

$$H_f = \frac{4f \cdot L \cdot (4Q/\pi D_p^2)^2}{D_p \times 2g} = \frac{4fL \times 16 Q^2}{D_p \times 2g \times \pi^2 \times D_p^4}$$

$$\therefore 75 = \frac{4 \times 0.006 \times (20 \times 10^3) \times 16 \times (1.4852)^2}{D_p^5 \times 2 \times 9.81 \times \pi^2}$$

$$\therefore D_p = \mathbf{1.0313 \text{ m}}$$ ...Ans.

(iv) Velocity V_1 and diameter of jet, d

Velocity of jet,

$$V_1 = C_v\sqrt{2gH}$$

$$= 0.98\sqrt{2 \times 9.81 \times 425}$$

$$= \mathbf{89.49\ m/s} \quad \text{...Ans.}$$

$$Q = \frac{\pi}{4} d^2 \times V_1$$

$$\therefore \quad d = \sqrt{\frac{4Q}{\pi \cdot V_1}} = \sqrt{\frac{4 \times 1.4852}{\pi \times 89.49}}$$

$$= 0.1454\ m = \mathbf{145.4\ mm} \quad \text{...Ans.}$$

(v) Diameter of bucket, D

Speed ratio, $$K_u = \frac{\text{Bucket speed, u}}{\text{Jet velocity, } V_1}$$

$$\therefore \quad u = K_u \cdot V_1 = 0.46 \times 89.49$$

$$= 41.17\ m/s$$

But, $$u = \frac{\pi DN}{60}$$

$$\therefore \quad D = \frac{60 \times u}{\pi N} = \frac{60 \times 41.17}{\pi \times 800}$$

$$= \mathbf{0.9828\ m} \quad \text{...Ans.}$$

3.7 Performance of Water Turbines

3.7.1 Introduction

Water turbines are designed to work under given head, discharge and output at certain speed. However, the turbine may be needed to work under such operating conditions which are different than designed values. Therefore, the performance testing of turbines becomes very essential to evaluate its characteristics. Also, it is essential to know the variation of its performance on all dependent parameters.

Since the manufacturing of actual machine is highly costly and to the predict the performance of its actual machine, the performance tests are carried out on its prototypes and models.

3.7.2 Standard Characteristics Curves

University Question

Q. What do you mean by characteristic curves of a turbine? Why these are important ? **SPPU : Dec. 18**

The important parameters on which the performance of a turbine depends upon are head (H), discharge (Q), speed (N), power (P) and efficiency (η). In addition, the other factors which affect the performance of a turbine are runner blade geometry and dimensions of the turbine.

By taking into account all these parameters for study of characteristics of a turbine, following standard characteristics are plotted :

1. Main characteristics or constant head characteristics curves.
2. Operating characteristics or constant speed characteristic curves.
3. Constant efficiency or Iso efficiency curves.

In order to compare the performance of two similar machines i.e. the prototype and its model but of different specifications, the concept of unit and specific quantities is a must to evaluate the performance of hydraulic machines.

3.7.3 Unit Quantities

University Questions

Q. Explain the terms unit speed, unit discharge and unit power and derive the expressions for the same. **SPPU : May 11**

Q. Derive an expression of unit quantities. **SPPU : Dec. 15**

Q. Derive an expression for unit speed and unit discharge. **SPPU : Feb. 16 (In Sem)**

Q. Define
i) Unit speed
ii) Unit discharge
iii) Unit power. State its significance. **SPPU : May 18**

Q. Derive an expression for Unit Discharge and Unit power and state their significance. **SPPU : Aug. 18 (In Sem), Oct. 19 (In Sem), Dec. 19**

Q. Define unit quantities for the turbines. **SPPU : May 19**

Discharge, speed, power etc. of a hydraulic machine are all functions of head. Therefore to compare and evaluate the performance of various hydraulic machines based on experimental results, each parameter is reduced to unit head.

These reduced quantities are called as **unit quantities.**

These unit quantities are :

1.	Unit speed, N_u.
2.	Unit discharge, Q_u.
3.	Unit power, P_u.

Thus the **unit quantity** can be defined as the value of parameter which will be obtained when a hydraulic machine operates under a head of 1 m.

These unit quantities are useful in comparing the performance of machines under different operating conditions if characteristic curves are plotted using unit quantities. The meaning and significance of these quantities are as follows :

1. Unit speed (N_u)

Unit speed of a turbine is defined as its speed while operating under unit head of 1 m.

The relationship between speed and unit speed can be obtained as follows :

Since, Fluid velocity, $V = \sqrt{2gH}$ thus, $V \propto \sqrt{H}$

$\therefore$ Runner velocity, $u \propto \sqrt{H}$ ...(i)

Also, runner velocity, $u = \frac{\pi D N}{60}$

For given diameter of turbine, $u \propto N$ ...(ii)

On combining the Equations (i) and (ii).

We can write,

$$N \propto H; \text{ or, } \frac{N}{\sqrt{H}} = \text{a constant, } k_1 \quad ...(iii)$$

By definition, when H = 1 m, $N = N_u$.

$$\therefore \frac{N}{\sqrt{H}} = \frac{N_u}{\sqrt{1}}$$

or,

$$N_u = \frac{N}{\sqrt{H}} \quad ...(3.7.1)$$

Therefore, for similarly designed turbines working under different heads, we can write,

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} \quad ...(3.7.1(A))$$

2. Unit discharge, Q_u

It is defined as the discharge of a turbine when working under a unit head of 1 m.

Discharge, $Q = A \cdot V$ i.e. $Q \propto V$

But $V \propto \sqrt{H}$

$\therefore$ $Q \propto \sqrt{H}$;

$$\text{or } \frac{Q}{\sqrt{H}} = \text{a constant, } k_2 \quad ...(i)$$

By definition, when H = 1 m, $Q = Q_u$

$$\therefore \frac{Q}{\sqrt{H}} = \frac{Q_u}{\sqrt{1}}$$

$$\text{Thus, unit discharge, } Q_u = \frac{Q}{\sqrt{H}} \quad ...(3.7.2)$$

For similarly designed turbines we can write,

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}} \quad ...(3.7.2(A))$$

3. Unit power, P_u :

Unit power is defined as the power developed by a turbine while working under a unit head of 1 m.

Since, $P \propto QH$ ...(i)

And, $Q \propto \sqrt{H}$ ...(ii)

On combining the Equations (i) and (ii),

$$P \propto \sqrt{H}\, H \text{ i.e. } P \propto H^{3/2}$$

$$\therefore \frac{P}{H^{3/2}} = \text{a constant, } k_3 \quad ...(iii)$$

Now, by definition, $P = P_u$ at H = 1 m,

$$\therefore \frac{P}{H^{3/2}} = \frac{P_u}{1^{3/2}}$$

$$\text{i.e. } P_u = \frac{P}{H^{3/2}} \quad ...(3.7.3)$$

For similarly, designed turbines we can write,

$$P_u = \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}} \quad ...(3.7.3(A))$$

3.7.4 Main Characteristics

University Question

Q. Discuss main characteristics curves for hydraulic turbine. **SPPU : May 16**

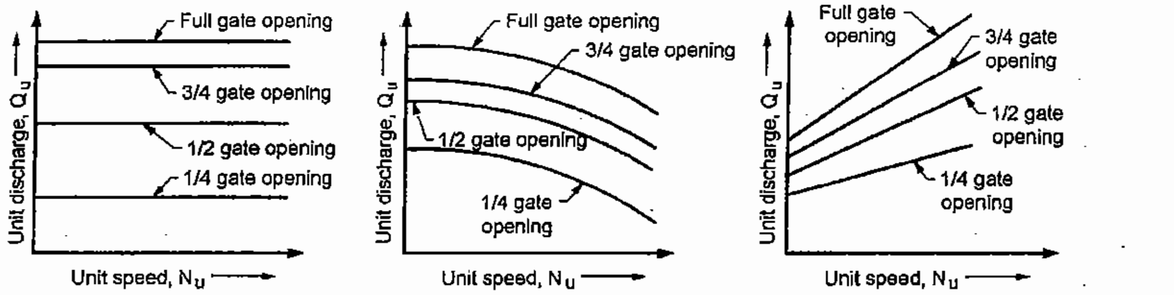
These characteristics are obtained by testing the turbine by keeping the supply head and gate opening constant and varying the flow through the turbine. The speed of the turbine is changed by varying the load on the turbine. The corresponding speed and power is measured and the efficiency is calculated.

Following characteristic curves are plotted under constant head by using unit parameters instead of absolute parameter.

(a) Unit speed Vs unit discharge.
(b) Unit speed Vs unit power.
(c) Unit speed Vs unit efficiency.

(a) Unit speed (N_u) Vs unit discharge curve (Q_u)

It could be seen that the characteristic curve between N_u Vs Q_u as shown in Fig. 3.7.1(a) is a straight line parallel to X-axis since the water comes out as a free jet from casing speed of the jet, hence the discharge is not affected by the speed of rotor.



(a) Pelton wheel (b) Francis turbine (c) Kaplan turbine

Fig. 3.7.1 : Main characteristic curves of a pelton wheel

Characteristic curve between N_u Vs P_u is shown in Fig. 3.7.1(b). It could be noted that the power first increases to a maximum with increase in speed and thereafter the power decreases with further increase in speed.

Runaway speed

Consider that a turbine is running at normal speed developing maximum power. Suppose the load on the alternator is suddenly reduced, the turbine also get unloaded. The speed of turbine will keep on increasing since the governor mechanism will take some time to control the speed due to inertia effect of the mechanism. This high speed beyond the normal speed will develop high stresses in the rotor and other components of the turbine alongwith severe vibrations. Thus these speeds beyond 10% of rated speed are undesirable.

In Fig. 3.7.1(b), at zero power, there are two extreme speeds. One is zero speed and the other is non-zero speed which is called **runaway speed.**

The characteristic curve N_u Vs η_o (overall efficiency) is shown in Fig. 3.7.1(c). The pattern of curves are similar to N_u Vs P_u curves shown in Fig. 3.7.1(b).

The efficiency is zero at zero speed and power. Maximum efficiency corresponds to design speed of the turbine. **Important point to be noted from the curves is that maximum efficiency occurs at full gate opening at their rated speed.**

3.7.5 Operating Characteristics or Constant Speed Characteristics

University Question

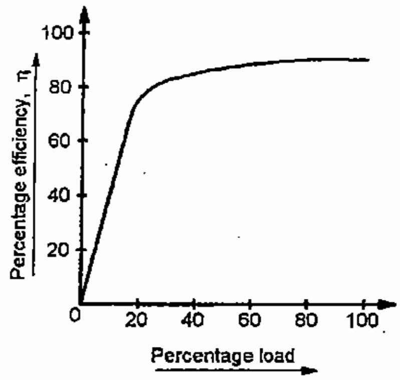
Q. Discuss Operating characteristics curves for hydraulic turbine. SPPU : May 16

Operating characteristics curves are plotted by testing the turbines at constant speed and the head is generally kept constant since it is difficult to change the head under actual working conditions.

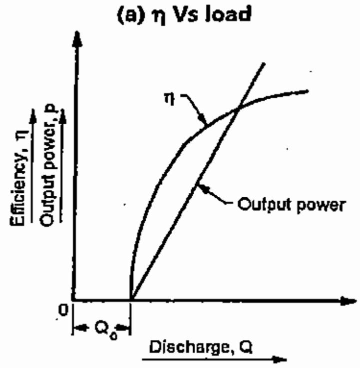
In order to maintain the constant speed, the gate openings are adjusted with the change in load on the turbine which is achieved by governing mechanism. By measuring input and output, the efficiency of turbine is calculated. Following curves are plotted (since H and N are constant) :

1. Load Vs efficiency
2. Discharge Vs power
3. Discharge Vs efficiency.

The operating characteristic curves are shown in Fig. 3.7.2(a) and (b).



(a) η Vs load



(b) η Vs Q and p V_s Q

Fig. 3.7.2 : Operating characteristics of pelton turbine at constant speed and head

Fig. 3.7.2(a) shows the load Vs efficiency curve in which load is represented as percentage of full load for which a turbine is designed. Maximum efficiency occurs at about 20 to 30 percent load and remains almost constant thereafter.

Discharge Vs efficiency and discharge Vs power curves are shown in Fig. 3.7.2(b). Efficiency of turbine increases with increase in discharge parabolically while the output varies linearly with discharge. Note that power output is zero for certain value of discharge, Q_o due to the fact that initially the power developed by the turbine is lost in overcoming the friction losses and inertia effect of rotating parts of the turbine.

3.7.6 Iso-Efficiency Curves

The iso-efficiency curves are also called as constant efficiency curves or universal characteristic curves of a turbine. These curves are also known as *hodographs* or *Muschaels maps.*

For plot of iso-efficiency curves, we make use of N_u Vs η curves and N_u Vs Q_u curves shown in Fig. 3.7.1.

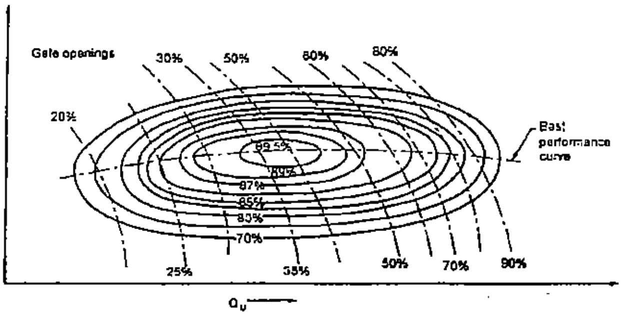


Fig. 3.7.3 : Iso-efficiency curves for turbines

For a given value of N_u, there shall be two values of efficiency, η corresponding to two values of discharge, Q_u. These two values can be obtained from N_u Vs Q_u curves.

On similar lines, for different values of N_u and Q_u, various values of efficiency could be obtained. The plot of these values on speed, N_u and discharge, Q_u is shown in Fig. 3.7.3 as Iso-efficiency curves.

The salient points which can be observed from Iso-efficiency curves are :

(i) The contour line are obtained by joining the points of equal efficiency.

(ii) Iso-efficiency contours of higher efficiency value are the inner contours while those having low efficiency are the outer contours.

(iii) Within the region enclosed by one contour, the efficiency will have a particular value.

(iv) Line joining the points where the iso-efficiency curve changes their shape is called the best performance curve. These points correspond to maximum efficiency.

3.8 Specific Speed of Turbines

University Questions

Q. Derive an expression for the specific speed of a hydraulic turbine. **SPPU : May 12, April 15(In Sem)**

Q. What is specific speed of a turbine? State its significance and derive an expression for the same. **SPPU : May 13, April 17(In Sem.)**

Q. Explain the factor which decides the speed of Pelton turbine used for Electrical power generation and discuss the relation between the specific speed and jet ratio. **SPPU : Dec. 13**

Q. Explain the term Specific speed. **SPPU : Dec.15**

Q. Define specific speed of turbine and state its significance. **SPPU : Dec.16**

Q. Define following terms and explain their importance in selection and design of hydraulic turbines.

(i) Specific speed **SPPU : Dec. 19**

Specific speed represents the speed of a turbine which is identical in shape, geometrical dimensions, blade angles and gate openings etc. to actual turbine will develop unit power of 1 kW when working under unit head of 1 m.

Significance of specific speed

Specific speed is a tool to compare different types of turbine since each turbine has a different specific speed and it helps in selecting the type of turbine. It also helps in predicting the performance of a turbine if its specific speed is known.

Derivation of specific speed

Overall efficiency, η_o of any turbine is the ratio of shaft power to hydraulic or water power i.e.

$$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} = \frac{P\ (kW)}{\left(\frac{\rho \cdot g \cdot QH}{1000}\right)(kW)}$$

$$\therefore \quad P = \eta_o \cdot \frac{\rho \cdot g \cdot QH}{1000}$$

Since η_o, ρ and g are constants we can write,

$$P \propto Q \cdot H \qquad \text{...(i)}$$

Absolute velocity, $V \propto$ blade velocity, u

$$\because \quad V = \sqrt{2gH}$$

i.e. $V \propto \sqrt{H}$

Therefore, $$V \propto u \propto \sqrt{H} \quad \text{...(ii)}$$

But, $$u = \frac{\pi D N}{60}$$

i.e. $$u \propto D \cdot N \quad \text{...(iii)}$$

From Equations (ii) and (iii),

$$\sqrt{H} \propto D \cdot N \, ;$$

$$D = \frac{\sqrt{H}}{N} \quad \text{...(iv)}$$

Discharge, Q from the turbine is given as :

$$Q = \text{Area} \times \text{Velocity}$$

$$= (B \times D) \times \sqrt{2gH} \qquad (\text{But } B \propto D)$$

$$\therefore Q \propto D^2 \cdot \sqrt{H}$$

On substituting the value of D from Equation (iv) above equation can be rewritten as :

$$Q \propto \left(\frac{\sqrt{H}}{N}\right)^2 \times \sqrt{H} \propto \frac{H^{3/2}}{N^2} \quad \text{...(v)}$$

On putting the value of Q in Equation (i),

$$P \propto \frac{H^{3/2}}{N^2} \times H \propto \frac{H^{5/2}}{N^2}$$

$$P = K \cdot \frac{H^{5/2}}{N^2}$$

(where K is the constant of (vi) proportionality)

When, $P = 1 \text{ kW}, \; H = 1 \text{ m}$

and $N = N_s$ (specific speed), then we get

$$1 = K \cdot \frac{(1)^{5/2}}{N_s^2} \text{ or } N_s^2 = K \quad \text{...(vii)}$$

From Equation (vi) and (vii),

$$P = N_s^2 \cdot \frac{H^{5/2}}{N^2}$$

$$N_s = \frac{N\sqrt{P}}{H^{(5/4)}} \quad \text{...(3.8.1)}$$

3.9 Selection of Turbines

Basically the turbines are classified as impulse and reaction turbines as discussed earlier. However, their further classification and selection is based on several ground data including the availability of head, discharge, specific speed etc as follows :

1) According to Head and Quantity of Water Available

Table 3.9.1 : Types of turbines based on Head and Discharge

Sr. No.	Head based turbine	Head (m)	Discharge	Type of turbine
1.	Low head	2 - 15 m	High	Kaplan or Propeller
2.	Medium head	16 - 70 m	High or Medium	Kaplan or Francis
3.	High head	71 - 500 m	Medium or Low	Francis or Pelton
4.	Very high head	> 500 m	Low	Pelton

2) According to Specific Speed, Ns

Specific Speed, Ns of a turbine is defined as the speed of a geometrically similar turbine or family of turbine which produces 1 kW power under a head 1 m. It is given as :

$$Ns = N\frac{\sqrt{P}}{H^{5/4}}$$

The total range of specific speed of turbine is from 5 to 1100 and its expression includes all basic parameters speed (N), power (P) and head (H), thus it becomes an important parameter in selection of turbines. Note that higher the specific speed, higher will be the speed of the runner.

The selection of turbines based on specific speed is as follows :

Table 3.9.2 : Types of Runner based on specific speed

Specific speed	Runner		
	Pelton	Francis	Kaplan
Low	5 - 15	60 - 150	300 - 450
Medium	16 - 30	151 - 250	451 - 700
High	31 - 70	251 - 400	701 - 1100

SECTION II : REACTION WATER TURBINE

3.10 Introduction to Reaction Turbines

We have already discussed that in case of impulse turbines the total head available is first converted into velocity head in nozzles before the water enters the runner. Whereas, in case of reaction turbines only a part of total available head is converted into velocity head while passing over the fixed guide vanes before it enters the runner.

Therefore in reaction turbine the water enters the runner under pressure having some velocity head. While the water passes over the runner, its pressure is gradually converted into velocity head until its pressure is reduced to atmospheric pressure along with the change in kinetic energy based on its absolute velocity.

The reaction due to pressure difference and the impulse action is responsible for rotation of runner and producing the mechanical work. *Therefore these turbines are basically impulse-reaction turbines called reaction turbines. Pure reaction turbines are not built.*

Since the water flows under pressure over the runner, above atmospheric it is necessary that the runner must run full of water. The water from runner is discharged into tail race through a closed tube of gradually increasing cross-sectional area called a draft tube. Also, the cross-sectional of flow through the passages of runner must gradually increase to accommodate the change in static pressure of water.

The reaction turbines are suitable for low and medium heads ranging from 30 m to 250 m of head some of the important reaction turbines are : Fourneyron, Francis, Kaplan and propeller turbines.

3.11 Classification of Reaction Turbines

The reaction turbines may be classified as

1. Based on direction of flow of water through the runner

These are classified as :

- (a) Radial flow turbines
- (b) Axial flow turbines
- (c) Mixed flow turbines

(a) Radial flow turbines

In a **radial flow turbine** the water flows along the radial direction in the runner. It may be an *inward flow type or outward flow type.*

In an **inward flow turbine,** the water enters at the outer periphery of the runner and flows radially inwards towards the centre of runner e.g. old Francis turbine, Girard radial flow turbine etc.

In an **outward flow turbine** the water enters at the centre of runner and flows radially outwards towards the outer periphery of the runner. e.g. the Fourneyron turbine.

(b) Axial flow turbines

Axial flow turbines are also called as *parallel flow* turbines. In these turbines the water enters and leaves the runner along the direction parallel to the axis of the shaft.

Same **examples of axial flow turbine** are Kaplan and propeller turbines, Girard axial flow turbine, Jonal turbine etc.

(c) Mixed flow turbines

In mixed flow turbines the water enters at the other periphery of runner in radial direction and leaves at the centre in the direction parallel to the axis of rotation of runner.

Modern Francis turbine is the example of mixed flow turbine.

2. Based on available head and discharge

Impulse turbines are used for high heads above 250 m whereas the reaction turbines are used for low and medium heads. Therefore reaction turbines based on head and discharge are classified as :

(a) Medium head turbines

Which operate under the head ranging from 60 m to 250 m requiring medium flow rates. Modern Francis turbine may be considered as medium head turbine.

(b) Low head turbines

Which operate under a head less than 60 m. These turbines require relatively large flow rates. Axial flow Kaplan and propeller turbines belong to this category.

3. Based on specific speed, N_s

Specific speed is defined as the speed of a geometrically similar turbine that would develop unit power under a head of 1 metre. It is given by the relation,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}},$$

where, N is the speed in rpm, P is power in kW and H is head in m.

Value of specific speed for various turbines are depending upon slow, medium and high speed runners are :

(a) Pelton turbine : $N_s = 9$ to 35

(b) Francis turbine : $N_s = 50$ to 250

(c) Kaplan turbine : $N_s = 250$ to 850

4. Based on disposition of shaft

Reaction turbines based on disposition of shaft can be classified as :

(a) Vertical shaft type.

(b) Horizontal shaft type.

3.12 Constructional Features of Reaction Turbines (Francis Turbine)

University Questions

Q. Describe with help of neat sketch the main components of Francis turbine.

SPPU : May 11, May 14

Q. Describe with a neat sketch the construction of Francis turbine.

SPPU : May 15

A Francis reaction turbine is shown in Fig. 3.12.1.

The main components of a reaction turbine are :

1. Scroll casing
2. Guide mechanism
3. Runner and shaft
4. Draft tube

1. Scroll casing

- Water from penstock flows into the outer scroll casing of turbine which is made in spiral shape,the casing surrounds the guide vanes, the runner, the shaft etc.
- The casing and runner always run full of water. The cross section of the casing is made of decreasing area so that the water is evenly distributed around the circumference of runner almost at constant velocity and pressure.
- The material used for casing of a reaction turbine depends on the head it works. The casing is usually made of concrete for less than 30 m head, of welded rolled steel plates upto 100 m head and of cast steel for more than 100 m head. In large units, stay vanes are provided inside the casing to support it.
- Stay vanes also help in guiding the water from casing to guiding vanes so that water is equally distributed around the periphery without formation of any eddies.

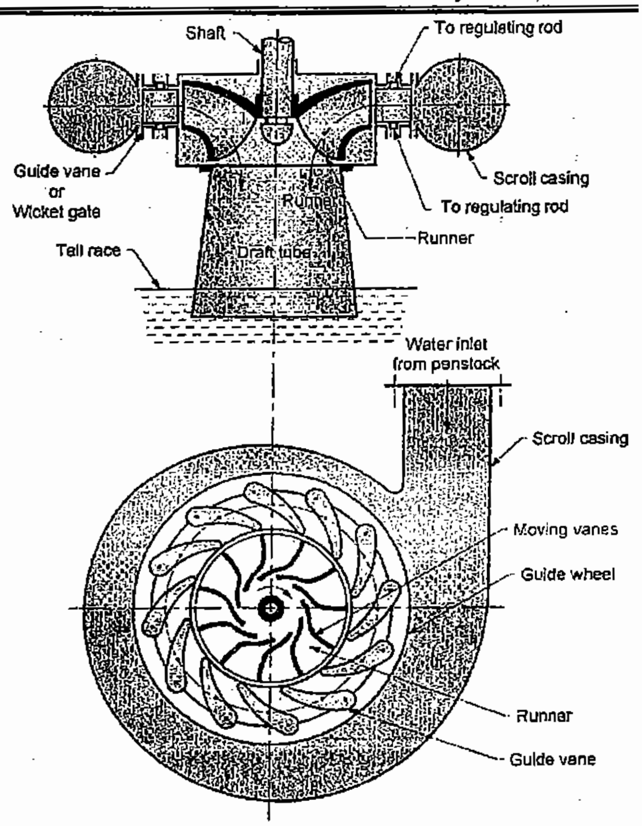


Fig. 3.12.1 : Francis reaction turbine

2. Guide Mechanism

- It consists of a stationary ring in the form of a wheel called guide wheel. It surrounds the outer periphery of runner and is fixed to inner surface of casing.
- In between the outer and inner ring of guide wheel, it carries a series of **guide vanes** or **wicket gates** of aerofoil section. These vanes form the number of passages between the casing and runner blades. Though these vanes are fixed in position but they can be rotated about their respective pivots.
- The guide vanes have the following functions to perform :
  (i) To direct the water from casing to moving vanes of runner at inlet without shock. In order to achieve smooth and shockless entry to runner, the relative velocity of water must be kept tangent to inlet tip of runner.
  (ii) To regulate the discharge according to load on the turbine.
- It is achieved by swinging the guide vanes about their own pivots by opening or closing the guide vane passages.

- Swinging action of guide vanes is obtained by the regulating rod connected to regulating shaft operated by Servo-motor governing mechanism.

3. Runner and shaft

- The runner is keyed to the main shaft of the turbine. Shaft may be horizontal or vertical, accordingly the turbines are called horizontal turbines and vertical turbines.
- The shaft is made of steel and it is supported in thrust bearings. The runner consists of suitable designed blades of aerofoil section so that the water enters and leaves the blades without shock. The number of blades usually vary between 16 to 24.The surfaces of moving vanes are made very smooth to reduce friction losses. The runners are made of cast iron for low head turbines and it is made of stainless steel for high head turbines.
- A runner for a mixed flow turbine is shown in Fig. 3.12.2.

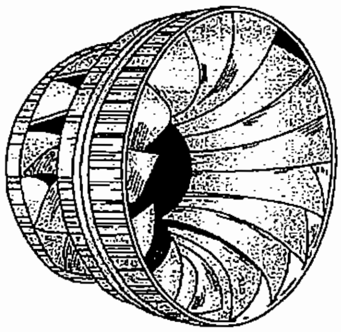


Fig. 3.12.2 : Runner of mixed flow reaction turbine

4. Draft Tube

- Water passing over the runner blades is discharged to tail race through a gradually increasing area, called draft tube.
- The small end of draft tube is fixed to outer opening of the casing and its bigger end is deeply submerged into tail race by atleast 1 m depth from tail race level.
- It is evident from discussion that the entire passages of a reaction turbine from head race to tail race are totally enclosed and it does not communicate with the atmosphere. It is so necessary since reaction turbines run under pressure.
- The functions of a draft tube are :
  (i) It converts the kinetic energy of water at exit of runner into useful pressure energy and the water discharges into tail race at very low velocity. Therefore, increases the pressure head.
  (ii) It increases the head on the turbine by an amount equal to the height of runner outlet above the tail race.
  (iii) It improves the efficiency of the turbine.

3.13 Comparison Between Impulse Turbine and Reaction Turbine

University Question

Q. Compare Impulse and Reaction Turbines. SPPU : Aug. 18(In Sem)

Sr. No.	Impulse turbine	Reaction turbine
1.	It works on the principle of impulse. Therefore, it follows the impulse momentum equation.	It works on the principle of impulse and reaction. Therefore, it follows the law of angular momentum.
2.	All the available head is first converted into kinetic energy in nozzles.	Only part of available head is converted into kinetic energy in guide vanes.
3.	Water from nozzles comes out in the form of jet which impinges on the buckets of runner.	Water is guided by the guide blades to flow over the moving vanes.
4.	Flow of water over the runner is at constant atmospheric pressure.	Flow of water over the runner is under pressure which gradually decreases from inlet to outlet.
5.	The runner of turbine need not run full and the water may be admitted over the full or part of periphery of runner.	The runner needs to run full of water since under pressure all the time. Water is admitted all over the circumference of the runner.
6.	Casing is not a must. It is provided to collect the water discharged from the turbine and directs it to tail race. It also avoids splashing of water.	Casing is essential since water flows from inlet to outlet under pressure.

Sr. No.	Impulse turbine	Reaction turbine
7.	The work is done due to change in kinetic energy of jet.	Most of the work is done due to change in pressure head and very small of work is due to change in kinetic energy.
8.	It is possible to regulate the flow of water without loss.	It is not possible to regulate the flow of water without loss.
9.	Suitable for high heads.	Suitable for low and medium heads.
10.	Head loss due to installation of turbine above tail race cannot be recovered.	Head loss due to installation above tail race can be recovered by using draft tube.

3.14 General Layout of a Reaction Turbine Plant

Fig. 3.14.1 shows the general layout of a hydro-electric power plant using reaction turbine.

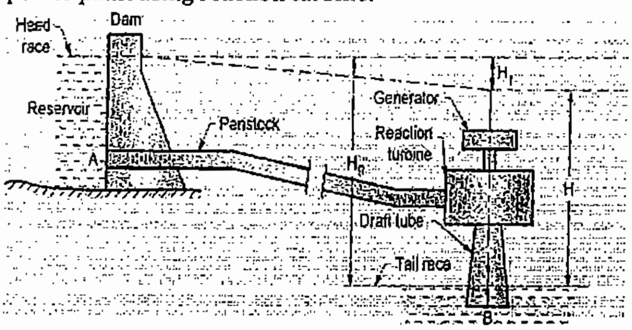


Fig. 3.14.1 : General layout of hydro-electric plant using reaction turbine

Let, H_g = Gross head on turbine. It represents the difference of level between head race and tail race.

H_f = Friction loss head in penstock.

H = Net available head at turbine = $H_g - H_f$

Net head, H is also called as *working head or operational head.* It can be determined by applying general energy equation between the total energy at exit of penstock and the total energy at the exit of draft tube as follows :

$$H = \left(\frac{p}{w}+\frac{V^2}{2g}+z\right)_{penstock} - \left(\frac{p}{w}+\frac{V^2}{2g}+Z\right)_{draft\ tube} \quad ...(3.14.1)$$

where, w = specific weight of water, N/m^3 = $\rho \cdot g$

ρ = density of water, kg/m^3

If the draft tube exit is at tail race level and it is assumed as datum, with velocity V_d as discharge then,

$$H = \left(\frac{p}{w}+\frac{V^2}{2g}+Z\right)_{penstock} - \frac{V_d^2}{2g}$$

Head when discharge velocity is negligible, hence on neglecting,

$$H = \left(\frac{p}{w}+\frac{V^2}{2g}+Z\right) \quad ...(3.14.2)$$

3.15 Calculation of Work, Power and Efficiencies of Inward Radial Reaction Turbines

In case of inward radial flow reaction turbines the water enters the runner at outer periphery of wheel and flows towards the centre of runner.

The velocity diagram of an inward flow reaction turbine is shown in Fig. 3.15.1.

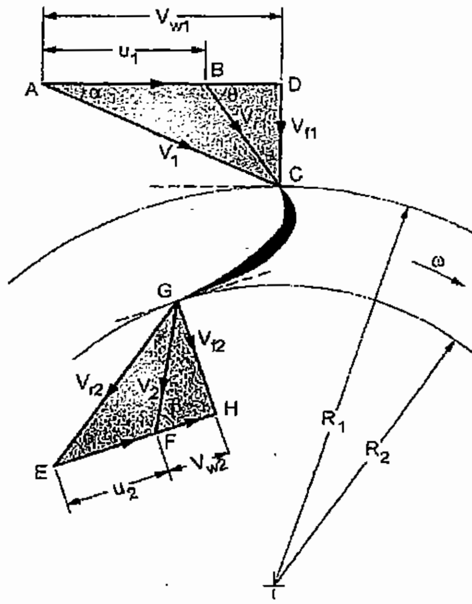


Fig. 3.15.1 : Velocity diagram of an inward flow reaction turbine

Let, D_1 and D_2 the outer and inner diameters of the runner rotating at N rpm. Then,

$$u_1 = \frac{\pi D_1 N}{60} \quad \text{and} \quad u_2 = \frac{\pi D_2 N}{60}$$

(i) **Workdone :** Workdone by the water on runner per second called runner power, P from Euler's momentum equation can be written as :

Workdone/s,

$$W = \rho Q (V_{w1} \cdot u \pm V_{w2} \cdot u_2)$$

$$= \frac{w}{g} Q (V_{w1}\ u_1 \pm V_{w2} u_2) \text{ or Runner power, P} \quad ...(3.15.1)$$

where, Q = discharge through the runner in m^3/s

Workdone per second per unit weight of water per second,

$$w = \frac{(V_{w1} \cdot u_1 \pm V_{w2} \cdot u_2)}{g} \quad ...(3.15.1(A))$$

In Equation (3.15.1), positive sign is taken when β is less than 90° and negative sign is taken when β is more than 90°.

Maximum work occurs when $\beta = 90^\circ$ and the absolute velocity at discharge is radial i.e. V_2 is minimum and $V_{w2} = 0$ Therefore, the

Equation for work reduces to :

$$W = \rho Q V_{w1} \cdot u_1 \quad ...(3.15.2)$$

Under these condition :

$$H - \frac{V_2^2}{2g} = V_{w1} \cdot u_1 \quad ...(3.15.2(A))$$

(ii) Hydraulic efficiency, η_h

If H is the net head available on turbine, then Power input or water power,

$$P_i = \rho \cdot gQH = wQH \quad ...(3.15.3)$$

$$\text{Hydraulic efficiency, } \eta_h = \frac{\text{Power developed by runner, P}}{\text{Power input}}$$

$$= \frac{\rho Q (V_{w1} \cdot u_1 \pm V_{w2} \cdot u_2)}{\rho gQH}$$

$$= \frac{V_{w1} \cdot u_1 \pm V_{w2} \cdot u_2}{gh} \quad ...(3.15.4)$$

(iii) Mechanical efficiency, η_m

$$\eta_m = \frac{\text{Shaft power}, P_s}{\text{Power developed by runner}, P} \quad ...(3.15.5)$$

(iv) Overall efficiency, η_o

$$\eta_o = \frac{\text{Shaft power}, P_s}{\text{Input power}, P_i} = \frac{P_s}{\rho gQH} = \eta_h \times \eta_m \quad ...(3.15.6)$$

Hydraulic efficiency of a radial flow inward reaction turbine ranges from 80% to 90%.

3.15.1 Working Proportions of a Radial Flow Reaction Turbine

(i) Ratio of width B_1 to diameter, D_1

Let, B_1 = Width of runner blades at inlet ;

D_1 = Diameter of runner at inlet

The ratio width to diameter is represented by n. Value of n ranges from 0.1 to 0.45.

$$\text{Thus, } n = \frac{B_1}{D_1}$$

(ii) Speed ratio, K_u

Speed ratio represents the ratio of peripheral velocity at inlet to runner, blades, u_1 to the theoretical jet velocity called spouting velocity $\sqrt{2gH}$. Thus,

Speed ratio,

$$K_u = \frac{u_1}{\sqrt{2gH}}; \quad \text{value of } K_u \text{ ranges from 0.6 to 0.9}$$

(iii) Flow ratio, K_f

Flow ratio, K_f is defined as the ratio of velocity of flow at inlet, V_{f1} to the spouting velocity $\sqrt{2gH}$. Thus,

Flow ratio,

$$K_f = \frac{V_{f1}}{\sqrt{2gH}}; \text{ Value of } K_f \text{ ranges from 0.15 to 0.30}$$

(iv) Discharge of turbine, Q

The discharge through a reaction radial flow turbine neglecting thickness of blade is given by,

$$Q = \text{Area of flow } (\pi DB) \times \text{Velocity of flow } (V_f)$$

$$Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2} \quad ...(3.15.7)$$

Suffix-1 represents the conditions at inlet of runner blade and suffix-2 at exit of runner blades.

If blade thickness is considered,

$$Q = (\pi D_1 - n_1 \times t) B_1 \cdot V_{f1} \quad ...(3.15.8)$$

where, n_1 is the number of runner blades and t is the thickness of blades.

3.15.2 Degree of Reaction, R

University Questions

Q. Define and explain the degree of reaction. **SPPU : Dec. 13, Dec. 16**

Q. What do you mean by Degree of Reaction ? Explain with significance. **SPPU : Aug. 18(In Sem)**

Q. Define degree of reaction. **SPPU : Oct. 19(In Sem)**

Degree of reaction, R of a runner is defined as the ratio of pressure energy change inside the runner to the total energy change inside the runner. Accordingly,

$$R = \frac{\text{change in pressure energy inside the runner, } H_p}{\text{change in total energy inside the runner, } H_t} \quad ...(3.15.9)$$

But, total energy change inside runner is equal to workdone per unit weight of water given by Equation (3.15.1(A)).

$$\therefore H_t = \frac{1}{g}(V_{w1} \cdot u_1 \pm V_{w2} \cdot u_2) \quad ...(i)$$

From inlet velocity diagram shown in Fig. 3.15.1, we get,

$$V_{w1} = AB + BD = u_1 + \sqrt{V_{r1}^2 - V_{f1}^2}$$

$$= u_1 + \sqrt{V_{r1}^2 - \left(V_1^2 - V_{w1}^2\right)}$$

$$\therefore V_{w1} - u_1 = \sqrt{V_{r1}^2 - \left(V_1^2 - V_{w1}^2\right)}$$

On squaring both sides and solving,

$$(V_{w1} - u_1)^2 = V_{r1}^2 - \left(V_1^2 - V_{w1}^2\right)$$

$$\therefore V_{w1}^2 + u_1^2 - 2V_{w1} \cdot u_1 = V_{r1}^2 - V_1^2 + V_{w1}^2$$

$$u_1^2 + V_1^2 - V_{r1}^2 = 2V_{w1} \cdot u_1$$

$$\therefore V_{w1} \cdot u_1 = \frac{1}{2}[u_1 + V_1^2 - V_{r1}^2] \quad ...(ii)$$

Similarly, from outlet velocity diagram,

$$V_{w2} = EH - EF = \sqrt{EG^2 - GH^2} - EF$$

$$= \sqrt{V_{r2}^2 - V_{f2}^2} - u_2$$

$$\text{i.e. } V_{w2} + u_2 = \sqrt{V_{r2}^2 - (V_2^2 - V_{w2})^2}$$

On squaring and solving the above equation we get,

$$V_{w2}^2 + u_2^2 + 2V_{w2} \cdot u_2 = V_{r2}^2 - V_2^2 + V_{w2}^2$$

$$V_{w2}\, u_2 = \frac{1}{2}\left[V_{r2}^2 - V_2^2 - u_2^2\right] \quad ...(iii)$$

On substituting the values from Equations (ii) and (iii) in Equation (i),

Change in total energy in runner,

$$H_t = \frac{1}{g}\left[\frac{1}{2}(u_1^2 + V_1^2 - V_{r1}^2) + \frac{1}{2}(V_{r2}^2 - V_2^2 - u_2^2)\right]$$

$$= \frac{V_1^2 - V_2^2}{2g} + \frac{u_1^2 - u_2^2}{2g} - \frac{V_{r2}^2 - V_{r1}^2}{2g} \quad ...(3.15.10)$$

Various changes in energy in the above expression represent the following :

$\frac{V_1^2 - V_2^2}{2g}$ = Decrease in kinetic energy of water per unit weight

$\frac{u_1^2 - u_2^2}{2g}$ = Decrease in energy due to centrifugal action per unit weight of water which is converted into pressure energy. Therefore this energy is available to turbine shaft.

$\frac{V_{r2}^2 - V_{r1}^2}{2g}$ = Change in static pressure energy per unit weight

$\therefore$ Change in pressure energy is caused due to centrifugal action,

$$h = \frac{\Delta p}{\rho \cdot g} = \frac{u_2^2 - u_1^2}{2g} \quad ...(3.15.11)$$

Hence, the change in pressure energy per unit weight inside the runner is due to centrifugal action and due to change in static pressure energy.

$\because$ Change in pressure energy in runner,

$$H_p = \frac{u_1^2 - u_2^2}{2g} + \frac{V_{r2}^2 - V_{r1}^2}{2g} = H_t - \left(\frac{V_2^2 - V_1^2}{2g}\right) \quad ...(3.15.12)$$

On substituting the value of H_p from Equation (3.15.12) and value of H_t from Equation (3.15.10) in Equation (3.15.9),

$$R = \frac{\frac{u_1^2 - u_2^2}{2g} + \frac{V_{r2}^2 - V_{r1}^2}{2g}}{\frac{V_1^2 - V_2^2}{2g} + \frac{u_1^2 - u_2^2}{2g} + \frac{V_{r2}^2 - V_{r1}^2}{2g}}$$

$$= \frac{(u_1^2 - u_1^2) + V_{r2}^2 - V_{r1}^2}{(V_1^2)} - V_2^2 + (u_1^2 - u_2^2) + V_{r2}^2 - V_{r1}^2 \quad ...(3.15.13)$$

$$R = 1 - \frac{V_2^2 - V_1^2}{2g \cdot H_t} \quad ...(3.15.13(A))$$

3.15.3 Degree of Reaction of Pelton Wheel

For a Pelton turbine, $u = u_1 = u_2$ and $V_{r2} = V_{r1}$ (when friction is neglected). It follows that :

$$H_p = \frac{u_1^2 - u_2^2}{2g} + \frac{V_{r2}^2 - V_{r1}^2}{2g} = 0$$

$$\therefore R = \frac{H_p}{H_t} = \frac{0}{H_t} = 0$$

3.15.4 Effect of Variation of Vane Shapes and Hydraulic Efficiency with Variation in Specific Speed

University Question

Q. How does the number of vanes, vane shape and hydraulic efficiency of any hydraulic machine vary with increasing specific speed ? **SPPU : May 16**

Specific Speed, N_s

Specific Speed, N_s of a turbine is defined as the speed of a geometrically similar turbine or family of turbine which produces 1 kW power under a head 1 m. It is given as :

$$N_s = N\frac{\sqrt{P}}{H^{5/4}}$$

The total range of specific speed of turbine is from 5 to 1100 and its expression includes all basic parameters speed (N), power (P) and head (H), thus it becomes an important parameter in selection of turbines. Note that higher the specific speed, higher will be the speed of the runner. Depending on the speed of the rotor, the runner speed at inlet and exit changes resulting into changes in whirl component, work output and hydraulic efficiency. Increased speed increases the discharge, the power output and reduces the vane angle.

3.16 Outward Radial Flow Reaction Turbines

Fig. 3.16.1 shows the sketch of an outward radial flow reaction turbine and Fig. 3.16.2 shows the inlet and exit velocity diagrams for such a turbine.

In this turbine the water, from casing enters on the guide vanes which directs the water to enter at the inlet tip of runner blades without shock. A part of pressure energy is converted into kinetic energy in guide vanes. The water flows over moving vanes of the runner and discharges in radial direction at outer periphery of the runner.

In this turbine the inlet diameter D_1 is less than the outer diameter of runner, therefore, blade velocity $u_1<u_2$.

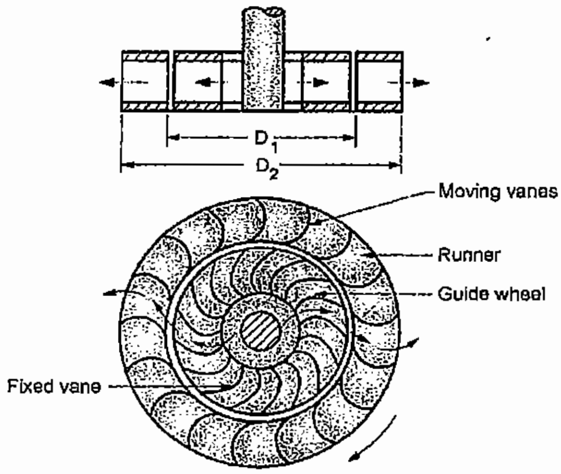


Fig. 3.16.1 : Outward radial flow reaction turbine

An outward radial flow reaction turbine suffers from the following disadvantages :

1. There is a formation of eddies due to increasing area of flow from inlet to outlet. It causes the loss of hydraulic energy.
2. Since $u_2 > u_1$ the pressure head increases due to centrifugal action of water (Refer Equation 3.15.11). It increases the relative velocity of water at outlet and consequently tends to increase the quantity of water passing through the wheel.

   It decreases the power developed by the turbine.
3. If there is slight reduction in load, the speed of the turbine tends to increase. It increases the centrifugal force which tends to increase the discharge rate thus the **wheel tends to race.** For above reasons, the outward flow reaction turbines have become obsolete. Calculation of work and efficiency is same as discussed in inward radial flow reaction turbines.

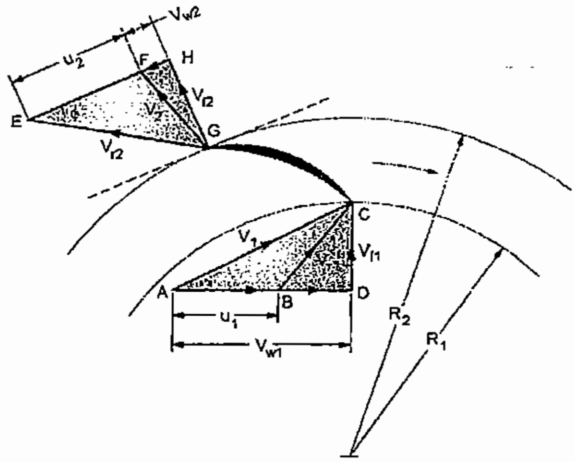


Fig. 3.16.2 : Velocity diagram for outward radial flow reaction turbine

3.17 Comparison Between Inward and Outward Radial Flow Reaction Turbines

Following are the differences between the inward and outward flow reaction turbines :

Sr. No.	Inward flow reaction turbine	Outward flow reaction turbine
1.	Water enters at outer periphery of runner and flows radially inwards towards the centre.	Water enters at the centre of runner and flows radially outwards and discharges at the outer periphery of runner.

Sr. No.	Inward flow reaction turbine	Outward flow reaction turbine
2.	The centrifugal head, $h = \frac{u_2^2 - u_1^2}{2g}$, $u_2 < u_1$ imparted to water during flow on runner is negative since $u_2 < u_1$.	The centrifugal head imported to water during flow on runner is positive since $u_2 > u_1$
3.	Negative head due to centrifugal action reduces the relative velocity at outlet since $\frac{V_{r2}^2 - V_{r1}^2}{2g} = \frac{u_2^2 - u_1^2}{2g}$	Positive head due to centrifugal action increases the relative velocity at outlet.
4.	As the speed increases, it reduces the discharge due to negative centrifugal head on water. Therefore, it provides a self governing effect on turbine.	As the speed increases, it increases the discharge rate due to positive centrifugal head. It further increases the power and speed, hence the turbine tends to race.

3.18 Francis Reaction Turbine

A Francis reaction turbine is an inward flow reaction turbine with radial discharge named after its inventor J.B. Francis, an American engineer. However, the modern Francis turbine is a mixed flow reaction turbine in which the water enters the turbine in radial direction at outer periphery of runner and leaves in axial direction.

Therefore, the vanes are suitably designed to have radial entry of water to the runner with axial discharge at its outlet. Since the discharge is axial, it follows that ; $\beta = 90°$, $V_{w2} = 0$.

Accordingly, the velocity diagram of an inward flow reaction turbine shown in Fig. 3.15.1 can be modified.

The workdone per second per unit weight of water per second from Equation (3.15.1A) can be written as ($V_{w2} = 0$)

$$W = \frac{V_{w1} \cdot u_1}{g} \qquad ...(3.18.1)$$

Ex. 3.18.1 : An outward flow reaction turbine has internal and external diameters of runner as 0.6 m and 1.2 m respectively. The guide blade angle is 15° and velocity of flow through runner is constant and equal to 4 m/sec. If the speed of the turbine 200 rpm, head on the turbine is 10 m and discharge at outlet is radial, determine :
i) Runner vane angle at inlet and outlet.
ii) Work done by water on runner.
iii) Hydraulic efficiency. **SPPU : May-18, 7 Marks**

Soln. :

Refer Fig.P.3.18.1

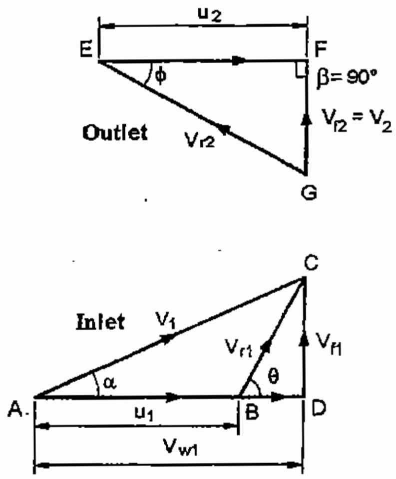


Fig. P.3.18.1

$D_1 = 0.6$ m, $D_2 = 1.2$ m,

$\alpha = 15°$, $V_{f_1} = V_{f_2} = 4$ m/s

$N = 200$ rpm, $H = 10$ m,

Discharge at outlet is radial i.e $\beta = 90°$,

$$V_{w_2} = 0$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.6 \times 200}{60} = 6.283 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.566 \text{ m/s}$$

from inlet Δ ABC

$$V_1 = \frac{V_{f_1}}{\sin\alpha} = \frac{4}{\sin 15} = 15.455 \text{ m/s}$$

$$V_{w_1} = \frac{V_{f_1}}{\tan\alpha} = \frac{4}{\tan 15} = 14.928 \text{ m/s}$$

(i) Runner vane angle at inlet (θ) and at outlet (ϕ)

$$\theta = \tan^{-1}\left(\frac{V_{f_1}}{BD}\right) = \tan^{-1}\left(\frac{V_{f_1}}{V_{w_1} - u_1}\right)$$

$$= \tan^{-1}\left(\frac{4}{14.928 - 6.283}\right)$$

$$= \mathbf{24.83°} \qquad \text{...Ans.}$$

from outlet Δ EFG

$$\phi = \tan^{-1}\left(\frac{V_{f_2}}{u_2}\right)$$

$$= \tan^{-1}\left(\frac{4}{12.566}\right) = 17.66° \qquad \text{...Ans.}$$

(ii) Workdone by water on runner, W

$$W = V_{w_1} \cdot u_1 = 14.928 \times 6.283$$

$$= \mathbf{93.793\ Nm/kg/s} \qquad \text{...Ans.}$$

(iii) Hydraulic efficiency, η_h

$$\eta_h = \frac{V_{w_1} \cdot u_1}{g H} = \frac{14.928 \times 6.283}{9.81 \times 10}$$

$$= \mathbf{0.9561\ or\ 95.61\%} \qquad \text{...Ans.}$$

Ex. 3.18.2 : Two inward flow turbine runners having the same diameter of 0.5 m have the same efficiency and work under the same head. Both the turbine has same velocity of flow 5.6 m/s. If one of the runners A runs at 525 rpm and has inlet blade angle of 65° and other runner B has inlet blade angle of 110°. What should be the speed of runner B ? Both turbine discharges radially at outlet.

SPPU - May 12, May 18, 6 Marks

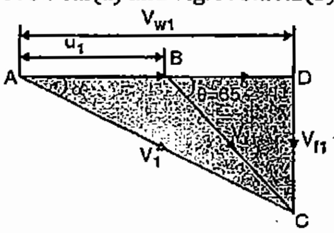
Soln. :

Let suffix 1 refer to turbine 1 and suffix 1′ to turbine 2.

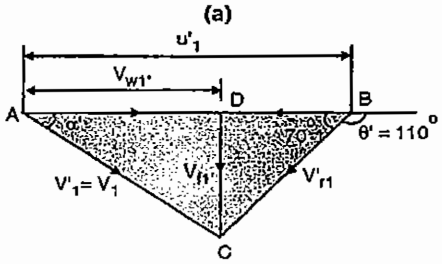
Given : $D_1 = D_1' = 0.5$ m ; $\eta_1 = \eta_1'$; $H_1 = H_2$; $V_{f1} = V_{f1}' = 5.6$ m/s ; $N_1 = 525$ rpm ; $\theta_1 = 65°$; $\theta_1' = 110°$

Discharge at outlet is radial, therefore, V_{w2} at exit is zero for both the turbines.

Inlet velocity diagram for turbine 1 and turbine 2 is shown in Fig. P. 3.18.2(a) and Fig. P. 3.18.2(b) respectively.



(a)



(b)

Fig. P. 3.18.2

$$u_1 = \frac{\pi D_1 N_1}{60} = \frac{\pi \times 0.5 \times 525}{60}$$

$$= 13.744 \text{ m/s}$$

$$\frac{V_{f1}}{BD} = \tan\theta$$

$$BD = \frac{V_{f1}}{\tan\theta} = \frac{5.6}{\tan 65} = 2.611 \text{ m/s}$$

$$V_{w1} = u_1 + BD = 13.744 + 2.611$$

$$= 16.355 \text{ m/s}$$

$$\text{since, } \eta_1 = \eta_1'$$

$$\therefore \frac{V_{w1} \cdot u_1}{V_1^2/2} = \frac{V_{w1}' \cdot u_1'}{(V_1')^2/2} \qquad (\text{But, } V_1 = V_1')$$

$$\therefore V_{w1} \cdot u_1 = V_{w1}' \cdot u_1'$$

$$V_{w1}' \times u_1' = 16.355 \times 13.744 = 224.78 \quad \text{...(i)}$$

Refer Fig. P. 3.18.2(b) :

$$\frac{V_{f1}'}{BD} = \tan 70$$

$$BD = \frac{V_{f1}'}{\tan 70} = V_{f1}' \cot 70$$

$$V_{w1}' = u_1' - V_{f1}' \cot 70$$

$$V_{w1}' = u_1' - 5.6 \tan 70 = u_1' - 2.038 \quad \text{...(ii)}$$

From Equations (i) and (ii),

$$(u_1' - 2.038)\, u_1' = 224.78$$

$$u_1'^2 - 2.038\, u_1' - 224.78 = 0$$

$$u_1' = \frac{2.038 \pm \sqrt{2.038^2 + 4 \times 224.78}}{2}$$

$$= 16.0463 \text{ m/s}$$

$$\text{But, } u_1' = \frac{\pi D_1' N_1'}{60}$$

$$16.0463 = \frac{\pi \times 0.5 \times N_1'}{60}$$

$$N_1' = \mathbf{612.92\ rpm} \qquad \text{...Ans.}$$

Ex. 3.18.3 : A reaction turbine works at 450 r.p.m. under a head of 120 m. Its diameter at inlet is 1.2 and the flow area is 0.4 m². The angles made by absolute and relative velocities at inlet are 20 degree and 60 degree respectively with the tangential velocity. Determine :

(i) The volume flow rate.

(ii) The power developed.

(iii) The hydraulic efficiency.

SPPU - Dec. 12, Aug. 18 (In Sem) 8 Marks

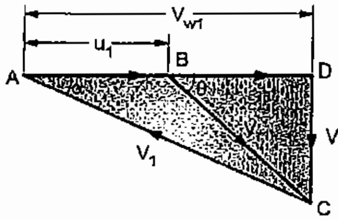
Soln. : Refer Fig. P. 3.18.3.

$N = 450$ rpm, $H = 120$ m,

$D_1 = 1.2$ m, $A_{f1} = 0.4\ m^2$,

$\alpha = 20°$, $\theta = 60°$

Assuming radial discharge i.e. $\beta = 90°$ and $V_{w2} = 0$



Inlet velocity diagram

Fig. P. 3.18.3

(i) Volume flow rate, Q

Blade velocity,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60}$$

$$= 28.27 \text{ m/s}$$

$$\tan \alpha = \frac{V_{f1}}{V_{w1}}$$

$$\tan 20 = \frac{V_{f1}}{V_{w1}}$$

$$V_{f1} = 0.364\, V_{w1}$$

$$\tan \theta = \frac{V_{f1}}{BD} = \frac{V_{f1}}{V_{w1} - u_1}$$

$$\tan 60 = \frac{V_{f1}}{V_{w1} - 28.27}$$

$$(V_{w1} - 28.27) \tan 60 = V_{f1}$$

$$1.732\,(V_{w1} - 28.27) = 0.364\, V_{w1}$$

$$V_{w1} = 35.79 \text{ m/s}$$

$$\therefore\ V_{f1} = 0.364\, V_{w1}$$

$$= 0.364 \times 35.79 = 13.03 \text{ m/s}$$

$$Q = A_{f1} \cdot V_{f1} = 0.4 \times 13.03$$

$$= \mathbf{5.212\ m^3/s}$$ ...**Ans.**

(ii) Power developed, P

$$P = \rho\, Q\, (V_{w1} \cdot u_1 \pm V_{w2} \cdot u_2) \times 10^{-3} \text{ kW}$$

$$P = 1000 \times 5.212\, (35.79 \times 28.27 \pm 0) \times 10^{-3}$$

$$= \mathbf{5273.4\ kW}$$ ...**Ans.**

(iii) Hydraulic efficiency, η_h

$$\eta_h = \frac{\text{Power developed}}{\text{Power supplied } (\rho\, g\, Q\, H \times 10^{-3})}$$

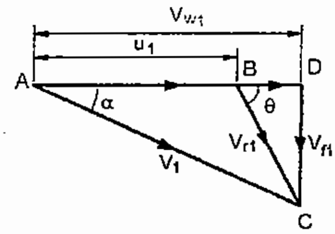
$$= \frac{5273.4}{1000 \times 9.81 \times 5.212 \times 120 \times 10^{-3}}$$

$$= \mathbf{0.8595 \text{ or } 85.95\%}$$ ...**Ans.**

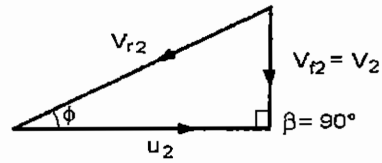
Ex. 3.18.4 : Find the main dimensions and blade angles for an inward flow reaction turbine to the following data :
Velocity of flow through runner is constant.
Speed = 950 rpm, Head on the turbine = 105 m,
Output = 800 kW, guide blade angle at inlet = 18°,
Inner diameter = 0.6 Outer diameter,
Hydraulic efficiency = 88%, Overall efficiency = 80%,
Breadth at inlet = 0.10 times the inlet diameter,
6% of the circumferential area of the runner is blocked by vane thickness the turbine discharges radially at outlet.

SPPU : Dec.-18, 6 Marks

Soln. :



Inlet velocity diagram



Outlet velocity diagram

Fig. P.3.18.4

Refer Fig. P.3.18.4

Given : $V_{f1} = V_{f2} = V_f$, $N = 950$ rpm

$H = 105$ m, $P_s = 800$ kW,

$\alpha = 18°$

Inner diameter, $D_2 = 0.6 \times$ Outer diameter, D_1

$\eta_h = 0.88$, $\eta_0 = 0.8$,

$B_1 = 0.1\, D_1$, $K_f = 6\%$

$A_f = 0.06 \times A_f$, $\beta = 90°$

i.e. $V_{w2} = 0$

$$P_s = \rho \cdot g \cdot Q \cdot H \times \eta_0 \times 10^{-3} \text{ kW}$$

$$800 = 1000 \times 9.81 \times Q \times 105 \times 0.8 \times 10^{-3}$$

Discharge, $Q = 0.9708 \text{ m}^3/\text{s}$

1. Dimension of Turbine :

$$\eta_h = \frac{V_{w1} \cdot u_1}{gH};$$

$$0.88 = \frac{V_{w1} \cdot u_1}{g \times 105}$$

$$\frac{V_{w1} \cdot u_1}{g} = 92.4 \qquad ...(i)$$

Also,
$$H = \frac{V_{w1} \cdot u_1}{g} + \frac{V_2^2}{2g};$$

$$105 = 92.4 + \frac{V_2^2}{2g}$$

$$V_2 = 15.72 \text{ m/s} = V_{f2} = V_{f1}$$

From Δ ACD ;
$$V_{w1} = \frac{V_{f1}}{\tan \alpha} = \frac{15.72}{\tan 18} = 48.38 \text{ m/s}$$

$$V_1 = \frac{V_{f1}}{\sin \alpha} = \frac{15.72}{\sin 18} = 50.87 \text{ m/s}$$

Q = (Area of flow – Area of flow due to blade Thickness) $\times V_{f1}$

$$Q = (1 - K_f)\, \pi D_1 B_1 V_{f1};$$

$$0.9708 = (1 - 0.06)\, \pi \times D_1 \times 0.1 \times B_1 \times 15.72$$

$$D_1 = \mathbf{0.4573\ m} \qquad ...\textbf{Ans.}$$

and
$$B_1 = 0.1 D_1 = 0.1 \times 0.4573 = \mathbf{0.04573\ m} \qquad ...\textbf{Ans.}$$

$$D_2 = 0.6\, D_1 = 0.6 \times 0.4573 = \mathbf{0.2744\ m} \qquad ...\textbf{Ans.}$$

$$\pi D_1 B_1 V_{f1} = \pi D_2\ B_2 V_{f2} \text{ (But } V_{f1} = V_{f2})$$

$$D_1 B_1 = D_2 B_2;$$

$$0.4573 \times 0.04573 = 0.2744 \times B_2$$

$$B_2 = \mathbf{0.0762\ m} \qquad ...\textbf{Ans.}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.4573 \times 950}{60} = 22.75 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.2744 \times 950}{60} = 13.65 \text{ m/s}$$

2. Blade angles at inlet, θ and at outlet, φ :

$$\theta = \tan^{-1}\left(\frac{BD}{V_{f1}}\right) = \tan^{-1}\left(\frac{V_{w1} - u_1}{V_{f1}}\right)$$

$$= \tan^{-1}\left(\frac{48.38 - 22.75}{15.72}\right) = \mathbf{58.48^\circ} \qquad ...\textbf{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right) = \tan^{-1}\left(\frac{15.72}{13.65}\right) = \mathbf{49.03^\circ} \qquad ...\textbf{Ans.}$$

Ex. 3.18.6 : Design a Francis turbine runner with the following data: Net Head 68 m, Speed 750 rpm, Power output 330 kW, Hydraulic efficiency 94%, overall efficiency 85%, Flow ratio 0.15, Ratio of breadth to diameter 0.1, Inner diameter of the runner is 1/2 outer diameter of the runner, 6% of circumferential area of the runner is occupied by the thickness of the vanes. Assume velocity of flow remains constant and flow is radial at exit.

SPPU - Dec. 13, 12 Marks

Soln. :

Given : $H = 68$ m, $N = 750$ rpm, $P_s = 330$ kW, $\eta_h = 94\% = 0.94$, $\eta_0 = 85\% = 0.85$, $K_f = 0.15$;

$$\frac{B_1}{D_1} = 0.1$$

Inner diameter,

$$D_2 = \frac{1}{2} \times \text{Outer diameter, } D_1$$

Area occupied by blades,

$$n_1 t B_1 = 6\% \text{ of } \pi D_1 B_1$$

$$= 0.06\, \pi D_1 B_1 \quad \text{i.e. } n_1 \cdot t = 0.06\ \pi \cdot D_1$$

$$V_{f1} = V_{f2}\ ; \quad \beta = 90^\circ \text{ (radial flow)}$$

Flow ratio,

$$K_f = \frac{V_{f1}}{\sqrt{2gH}} \quad \text{i.e.} \quad V_{f1} = K_f \times \sqrt{2gH}$$

$$\therefore \quad V_{f1} = 0.15\sqrt{2 \times 9.81 \times 68} = 5.48 \text{ m/s} = V_{f2}$$

Overall efficiency,

$$\eta_0 = \frac{\text{output power, } P_s}{\text{Input power }, \rho \cdot gQH}$$

$$\therefore \text{ Discharge, } Q = \frac{P_s}{\eta_0 \times \rho \times g \times H}$$

$$= \frac{330 \times 10^3}{0.85 \times 1000 \times 9.81 \times 68}$$

$$= 0.582 \text{ m}^3\text{/s}$$

But,
$$Q = (\pi D_1 - n_1 \cdot t)\, B_1 \cdot V_{f1}$$

$$0.582 = (\pi D_1 - 0.06\, \pi D_1)\, 0.1\, D_1 \times 5.48$$

$$D_1 = \mathbf{0.6\ m}$$

$$B_1 = 0.1\, D_1 = \mathbf{0.06\ m} \qquad ...\textbf{Ans.}$$

∴ Outlet diameter,

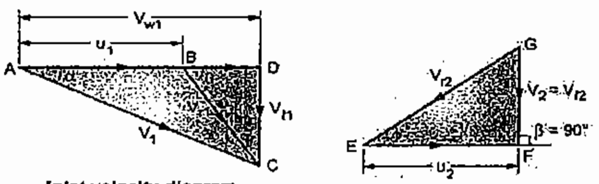
$$D_2 = \frac{D_1}{2} = \frac{0.6}{2} = \mathbf{0.3}$$

Since $V_{f1} = V_{f2}$ and discharge is constant,

Width at outlet,

$$B_2 = \frac{B_1 D_1}{D_2} = \frac{0.6 \times 0.06}{0.3} = \mathbf{0.12\ m} ...\textbf{Ans.}$$

Velocity diagram at inlet and exit is shown in Fig. P. 3.18.5.



Inlet velocity diagram

Fig. P. 3.18.5

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.6 \times 750}{60} = 23.56 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 750}{60} = 11.78 \text{ m/s}$$

Hydraulic efficiency,

$$\eta_h = \frac{V_{w1} \cdot u_1}{g \cdot H}$$

$$V_{w1} = \frac{\eta_h \times g \times H}{u_1} = \frac{0.94 \times 9.81 \times 68}{23.56} = 26.62 \text{ m/s}$$

From inlet velocity diagram,

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} = \frac{5.48}{26.62} = 0.2059$$

∴ Guide blade angle,

$$\alpha = \mathbf{11.63^\circ} \qquad \text{...Ans.}$$

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{5.48}{26.62 - 23.56} = 1.7908$$

∴ Inlet angle of moving vane,

$$\theta = \mathbf{60.82^\circ} \qquad \text{...Ans.}$$

From outlet velocity diagram,

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{5.48}{11.78} = 0.4652$$

∴ Exit angle of moving vane,

$$\phi = \mathbf{24.95^\circ} \qquad \text{...Ans.}$$

Summary of design

Inlet diameter of runner,	$D_1 = 0.6$ m
and outer diameter,	$D_2 = 0.3$ m
Width of blades at inlet,	$B_1 = 0.06$ m
and at outlet,	$B_2 = 0.03$ m
Inlet angle of moving vane,	$\theta = 60.82^\circ$
and outlet angle,	$\phi = 24.95^\circ$
Exit angle of guide vanes,	$\alpha = 11.63^\circ$.

Ex. 3.18.6 : The velocity of whirl at inlet to the runner of an inward flow reaction turbine is 3.42 H m/s where H is the head on the turbine in meters and the velocity of flow at the inlet is 1.14 m/s. The velocity of whirl at exit is 0.24 H m/s in the same direction as at inlet and the velocity of flow at exit is 0.90 H m/s the inner diameter of the runner is 0.6 times the outer diameter. The hydraulic efficiency is 80%. Draw the velocity triangle at inlet and exit and calculate the runner vane angles and the guide blade angles.

Soln. :

Given : $V_{w1} = 3.42$ H ; $V_{f1} = 1.14$ H

$V_{w2} = 0.24$ H ; $V_{f2} = 0.9$ H

$D_2 = 0.6\, D_1$; $\eta_h = 80\% = 0.8$

Velocity Δs at inlet and outlet are shown in Fig. P. 3.18.6.

Since V_{w2} is in the same direction as V_{w1}, the velocity diagram at outlet will be as shown in Fig. P. 3.18.6.

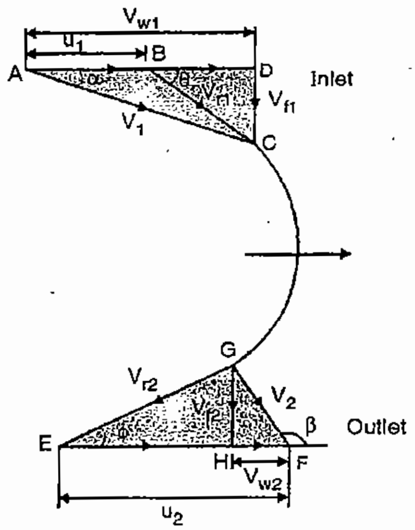


Fig. P. 3.18.6

1. Guide vane angle, at inlet, α

From inlet velocity diagram ACD we get

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} = \frac{1.14\,H}{3.42\,H} = 0.3333$$

$$\alpha = \mathbf{18.435^\circ} \qquad \text{...Ans.}$$

$$V_1 = \sqrt{V_{w1}^2 + V_{f1}^2}$$

$$= \sqrt{(3.42\,H)^2 + (1.14\,H)^2} = 3.605\,H$$

2. Runner blade angle at inlet, θ

$$\because CD = V_1 \sin \alpha = V_{r1} \sin \theta = V_{f1}$$

$$\therefore V_{r1} \sin \theta = 1.14\,H \qquad \text{...(i)}$$

$$u_1 = \frac{\pi D_1 N}{60} \quad \text{and} \quad u_2 = \frac{\pi D_2 N}{60}$$

$$\therefore \frac{u_1}{u_2} = \frac{D_1}{D_2} = \frac{1}{0.6}$$

$$u_2 = 0.6\, u_1 \qquad \text{...(ii)}$$

Hydraulic efficiency

$$\eta_h = \frac{V_{w1} \cdot u_1 - V_{w2} \cdot u_2}{(V_1^2/2)}$$

$$0.8 = \frac{(3.42\,H \times u_1 - 0.24\,H \times 0.6\,u_1)\,2}{(3.605\,H)^2}$$

$$\frac{0.8 \times 3.605^2 \times H^2}{2} = (3.24 - 0.144)\,H \times u_1$$

$$\therefore\ u_1 = 1.587\,H \qquad \text{...(iii)}$$

$$\therefore\ u_2 = u_1 \times 0.6 = 1.587 \times 0.6\,H = 0.952\,H \qquad \text{...(iv)}$$

$$\therefore\ BD = V_{w1} - u_1 = 3.42\,H - 1.587\,H = 1.833\,H$$

$$\tan\theta = \frac{V_{f1}}{BD} = \frac{1.14\,H}{1.833\,H} = 0.6219$$

$$\therefore\ \theta = \mathbf{31.88^\circ} \qquad \text{...Ans.}$$

3. Outlet runner angle, ϕ and guide vane angle, β

Consider outlet triangle GEF

$$EH = u_2 - V_{w2} = 0.952\,H - 0.24\,H = 0.712\,H$$

$$\tan\phi = \frac{V_{f2}}{EH} = \frac{0.9\,H}{0.712\,H} = 1.264$$

$$\phi = \mathbf{51.65^\circ} \qquad \text{...Ans.}$$

$$\tan(180 - \beta) = \frac{V_{f2}}{V_{w2}} = \frac{0.9\,H}{0.24\,H} = 3.75$$

$$\therefore\ (180 - \beta) = 75.07^\circ$$

$$\beta = 180 - 75.07 = \mathbf{104.93^\circ} \qquad \text{...Ans.}$$

Ex. 3.18.7 : Following data is available related to Francis turbine. Shaft power = 14990 kW, runner speed = 275 rpm, net head = 110 m, diameter at inlet = 1.8 times diameter at outlet, axial length of the blade at inlet = 0.15 times diameter at inlet, flow ratio = 0.2, hydraulic efficiency = 90%, overall efficiency = 85%, velocity of flow at inlet = velocity of flow at outlet determine :

(i) Inlet and outlet diameters

(ii) Guide blade angles

(iii) Runner vane angles

Assume radial flow at exit. **SPPU - May 16, 5 Marks**

Soln. : Refer Fig. P. 3.18.7

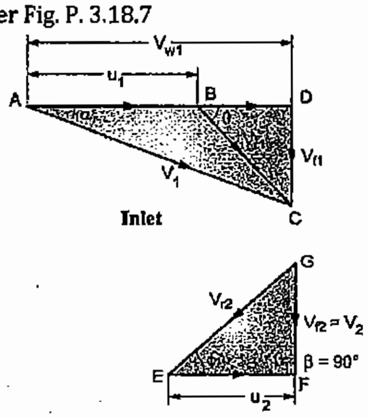


Fig. P. 3.18.7

$P_s = 14990$ kW, $\quad n = 275$ rpm,

$H = 110$ m, $\quad D_1 = 1.8\,D_2$,

$B_1 = 0.15\,D_1$, $\quad K_f = 0.2$,

$\eta_h = 0.5$, $\quad \eta_o = 0.85$,

$V_{f_1} = V_{f_2}$

(1) Inlet diameter, D_1 and outlet diameter, D_2

$$\therefore\ K_f = \frac{V_{f_1}}{\sqrt{2gH}}$$

$$0.2 = \frac{V_{f_1}}{\sqrt{2 \times 9.81 \times 110}}$$

$$V_{f_1} = 9.29\ \text{m/s} = V_{f_2}$$

$$\eta_o = \frac{\text{Shaft power, } P_s}{\text{Input power, } \rho.gQH}$$

$$0.85 = \frac{14990 \times 10^3}{1000 \times 9.81 \times Q \times 110}$$

$$Q = 16.3426\ \text{m}^3/\text{s}$$

$$Q = \pi D_1 B_1 V_{f_1}$$

$$16.3426 = \pi \times D_1 \times 0.15\,D_1 \times 9.29$$

$$\therefore\ D_1 = \mathbf{1.932\ m} \qquad \text{...Ans.}$$

$$D_2 = \frac{D_1}{1.8} = \frac{1.932}{1.8} = \mathbf{1.073\ m} \qquad \text{...Ans.}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.932 \times 275}{60} = 27.82\ \text{m/s}$$

(2) Guide blade angle, α

$$\eta_h = \frac{V_{w_1} \cdot u_1}{g\,H}$$

$$0.9 = \frac{V_{w_1} \times 27.82}{9.81 \times 110}$$

$$V_{w_1} = 34.91\ \text{m/s}$$

Consider inlet velocity Δ ACD

$$\alpha = \tan^{-1}\left(\frac{V_{f_1}}{V_{w_1}}\right) = \tan^{-1}\left(\frac{9.29}{34.91}\right) = \mathbf{14.9^\circ} \text{ ...Ans.}$$

(3) Runner Vane angle i.e. inlet angle, θ and exit angle, ϕ

$$V_1 = \frac{V_{w_1}}{\cos\alpha} = \frac{34.91}{\cos 14.9} = 36.12\ \text{m/s}$$

$$BD = V_{w_1} - u_1 = 34.91 - 27.82 = 7.09\ \text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_{f_1}}{BD}\right) = \tan^{-1}\left(\frac{9.29}{7.09}\right) = \mathbf{52.65^\circ} \text{ ...Ans.}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.073 \times 275}{60} = 15.45\ \text{m/s}$$

From exit ΔEFG :

$$\phi = \tan^{-1}\left(\frac{V_{f_2}}{u_2}\right) = \tan^{-1}\left(\frac{9.29}{15.45}\right) = \mathbf{31.02^\circ} \text{...Ans.}$$

Ex. 3.18.8 : The external and internal diameter of an inward flow reaction turbines are 1.20 m and 0.6 m respectively. The head on the turbine is 22 m and velocity of flow through the runner is constant and equal to 2.5 m/s. The guide blade angle is given as 10° and runner vanes are radial at inlet. If the discharge is radial at outlet, Determine :

(i) The speed of the turbine

(ii) The vane angle at outlet

(iii) Hydraulic efficiency. **SPPU - Feb 16 (In Sem), 6 Marks**

Soln. : Refer Fig. P. 3.18.8

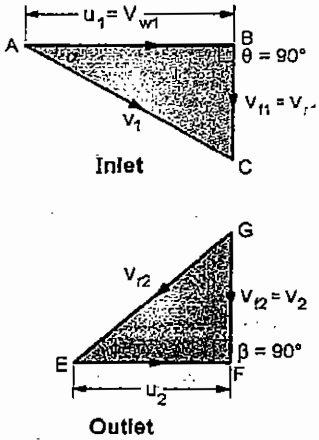


Fig. P. 3.18.8

External diameter, $D_1 = 1.2$ m

Internal diameter, $D_2 = 0.6$ m

$H = 22$ m, $\quad V_{f1} = V_{f2} = 2.5$ m/s

$\alpha = 10°$, $\quad \theta = 90°$ (radial vanes)

$\beta = 90°$ (radial discharge)

From inlet velocity Δ ABC,

$$u_1 = \frac{V_{f1}}{\tan\alpha} = \frac{2.5}{\tan 10} = 14.18 \text{ m/s} = V_{w_1}$$

$$V_1 = \frac{V_{f1}}{\sin\alpha} = \frac{2.5}{\sin 10} = 14.4 \text{ m/s}$$

(i) Speed of the turbine, N

$$u_1 = \frac{\pi D_1 N}{60} \ ;$$

$$14.18 = \frac{\pi \times 1.2 \times N}{60}$$

$$N = \mathbf{225.68 \text{ rpm}} \qquad \text{...Ans.}$$

(ii) Vane angle of outlet, ϕ

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 225.64}{60}$$

$$= 7.09 \text{ m/s}$$

From outlet Δ EFG,

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right) = \tan^{-1}\left(\frac{2.5}{7.09}\right)$$

$$\phi = \mathbf{15.945°} \qquad \text{...Ans.}$$

(iii) Hydraulic efficiency, η_h

$$\eta_h = \frac{V_{w_1} \cdot u_1}{g H} = \frac{14.18 \times 14.18}{9.81 \times 22}$$

$$= \mathbf{0.9317 \text{ or } 93.17\ \%} \qquad \text{...Ans.}$$

Ex. 3.18.9 : The outer diameter of the Francis runner is 1.4m. The flow velocity at inlet is 9.5m/s. The absolute velocity at the exit is 7 m/s. The speed of operation is 430 rpm. The power developed is 12.25 MW with a flow rate of 12 m³/s. Total head is 115 m. For shockless entry, determine the angle of the inlet guide vane. Also find the absolute velocity at entrance, the runner blade angle at inlet and the loss of head in the unit. Assume zero whirl at exit.

Soln. : Refer Fig. P. 3.18.9.

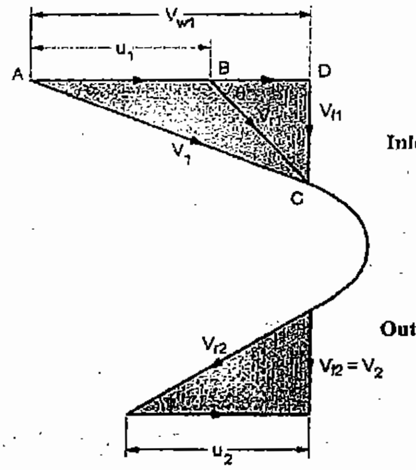


Fig. P. 3.18.9

Given : $D_1 = 1.4$ m , $\quad V_{f1} = 9.5$ m/s

$V_2 = 7$ m/s, $\quad N = 430$ rpm,

$Q = 12$ m³/s

Power developed,

$P = 12.25$ MW $= 12250$ kW

$H = 115$ m ;

$V_{w2} = 0$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.4 \times 430}{60}$$

$$= 31.52 \text{ m/s}$$

Power developed, $P = \rho Q \cdot V_{w1} \cdot u_1 \times 10^{-3}$ (kW)

$$12250 = 1000 \times 12 \times V_{w1} \times 31.52 \times 10^{-3}$$

$$V_{w1} = 32.39 \text{ m/s.}$$

1. Angle of inlet guide vane, α

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} = \frac{9.5}{32.39} = 0.2933$$

$$\alpha = \mathbf{16.35^\circ} \qquad \textbf{...Ans.}$$

2. Absolute velocity at entrance, V_1

$$V_1 = \sqrt{V_{w1}^2 + V_{f1}^2} = \sqrt{(32.39)^2 + (9.5)^2}$$

$$= \mathbf{33.76 \text{ m/s}} \qquad \textbf{...Ans.}$$

3. Runner blade inlet angle at inlet, θ

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{BD}\right) = \tan^{-1}\left(\frac{V_{f1}}{V_{w1} - u_1}\right)$$

$$= \tan^{-1}\left(\frac{9.5}{32.39 - 31.52}\right) = \mathbf{84.77^\circ} \qquad \textbf{...Ans.}$$

4. Loss of head in the unit, H_1

$$\text{Power input} = \rho \cdot g \cdot QH \times 10^{-3} \text{ kW}$$

$$= 1000 \times 9.81 \times 12 \times 115 \times 10^{-3}$$

$$= 13537.8 \text{ kW}$$

$$\text{Power loss} = \text{Power input} - \text{Power output}$$

$$= 13537.8 - 12250$$

$$= \mathbf{1287.8 \text{ kW}}$$

$$1287.8 = \rho \cdot g \cdot Q H_1 \times 10^{-3}$$

$$= 1000 \times 9.81 \times 12 \times H_1 \times 10^3$$

$$H_1 = \mathbf{10.94 \text{ m head loss}} \qquad \textbf{...Ans.}$$

Ex. 3.18.10 : An inward flow reaction turbine operates under a head of 14 m with a speed of 310 rpm. The inner and outer diameters of the runner are 510 mm and 765 mm respectively. The width of the runner at inlet is 75 mm. The blade thickness occupies 16% of the runner passage. The velocity of flow is constant throughout and flow ratio is 0.22. Water leaves the runner radially. The blade efficiency is 92%, while the overall efficiency is 85%. Calculate :

(i) Guide vane angle at inlet

(ii) Moving vane angle at inlet and outlet

(iii) Flow rate through turbine

(iv) Power developed

(v) Breadth of the runner at outlet.

Soln. :

Given : $H = 14$ m ; $N = 310$ rpm ;

$D_2 = 510$ mm $= 0.51$ m

$D_1 = 765$ mm $= 0.765$ m ;

$B_1 = 75$ mm $= 0.075$ m

$V_{f1} = V_{f2}$; flow ratio, $K_f = \dfrac{V_{f1}}{V_1} = 0.22$

$\beta = 90^\circ$ (radial exit) i.e. $V_{w2} = 0$

$\eta_b = 0.92$; $\eta_o = 0.85$

Area occupied by blades,

$$n_1 \cdot t \cdot B_1 = 16\% \text{ of } \pi D_1 B_1$$

$$= 0.16 \pi D_1 B_1 \qquad (n_1 = \text{no. of blades})$$

$$V_1 = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 14} = 16.57 \text{ m/s}$$

$$V_{f1} = K_f \; V_1 = 0.22 \times 16.57 = 3.65 \text{ m/s} = V_{f2}$$

Overall efficiency,

$$\eta_o = \frac{\text{Output power, } P_s}{\text{Input power, } \rho g Q H} \qquad \text{...(i)}$$

$\therefore$ Flow rate or Discharge,

$$Q = (\pi D_1 - n_1 t) B_1 \cdot V_{f1}$$

$$= (\pi D_1 B_1 - 0.16 \pi D_1 B_1) V_{f1}$$

$$= \pi \times 0.765 \times 0.075 (1 - 0.16) 3.65$$

$$= 0.5526 \text{ m}^3/\text{s}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.765 \times 310}{60} = 12.42 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.51 \times 310}{60} = 8.28 \text{ m/s}$$

Velocity diagrams are shown in Fig. P. 3.18.10(a).

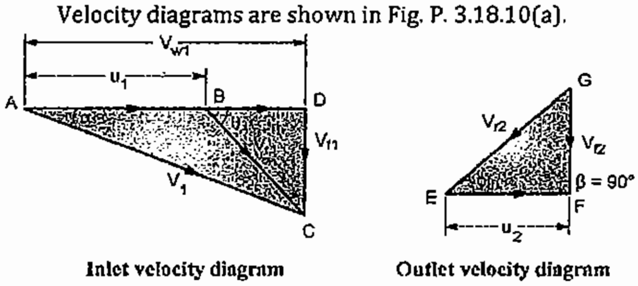


Fig. P. 3.18.10(a)

Blade efficiency,

η_b = Hydraulic efficiency, η_h (since no losses in the jet are given)

$= 0.92$

$$\eta_h = \frac{V_{w1} \times u_1}{g H}$$

$$0.92 = \frac{V_{w1} \times u_1}{9.81 \times 14}$$

$$V_{w1} \times u_1 = 126.35$$

$$V_{w1} \times 12.42 = 126.35$$

$V_{w1} = 10.17$ m/s

$\because \; V_{w1} < u_1$

The modified inlet diagram is shown in Fig. P. 3.18.10(b)

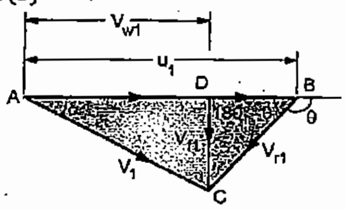


Fig. P. 3.18.10(b)

(i) Guide vane angle at inlet, α

$$\tan\alpha = \frac{V_{f1}}{V_{w1}} = \frac{3.65}{10.17} = 0.3589$$

$$\alpha = \mathbf{19.74^\circ} \quad \text{...Ans.}$$

(ii) Moving vane angles at inlet, θ and at outlet,

$\phi : \angle DBC = 180 - \theta$

$$\tan(180-\theta) = \frac{BC}{DB}$$

$$= \frac{V_{f1}}{(u_1 - V_{w1})}$$

$$= \frac{3.65}{12.42 - 10.17}$$

$$= 1.6222$$

$$180 - \theta = 58.35^\circ$$

$$\theta = \mathbf{121.65^\circ} \quad \text{...Ans.}$$

Consider outlet velocity diagram,

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right) = \tan^{-1}\left(\frac{3.65}{8.28}\right)$$

$$= \mathbf{23.79^\circ} \quad \text{...Ans.}$$

(iii) Flow rate through turbine, Q

From above, $\mathbf{Q = 0.5526\ m^3/s}$ ...Ans.

(iv) Power developed, P_s

$$\eta_o = \frac{P_s}{\rho \cdot g \cdot Q \cdot H}$$

$$P_s = \eta_o \cdot \rho \cdot g \cdot Q \cdot H \times 10^{-3}\ \text{kW}$$

$$P_s = 0.85 \times 1000 \times 9.81 \times 0.5526 \times 14 \times 10^{-3}$$

$$= \mathbf{64.5\ kW} \quad \text{...Ans.}$$

(v) Breadth of the runner of outlet, B_2

$$Q = (\pi D_2 B_2 - 0.16\,\pi D_2 B_2)\,V_{f2}$$

$$0.5526 = 0.84 \times \pi \times 0.51 \times B_2 \times 3.65$$

$$B_2 = \mathbf{0.1125\ m} \quad \text{...Ans.}$$

Ex. 3.18.11 : An inward flow reaction turbine runner has outer diameter of 1 m and breadth at the inlet is 250 mm. If the velocity of flow at inlet is 2 m/s, find the flow rate through the turbine. This turbine runs at 210 rpm. Guide blade angle at inlet is 10°. The runner diameter at outlet is 0.5 m. Determine moving blade angle at inlet and the breadth at outlet if the velocity of flow remains constant. The discharge from the runner is radial at the outlet. What is the moving blade angle at outlet ? Find the supply head, power developed and hydraulic efficiency of the turbine.

Soln. :

Given : $D_1 = 1$ m, $B_1 = 250$ mm $= 0.25$ m;

$V_{f1} = 2$ m/s; $N = 210$ rpm,

$\alpha = 10^\circ$, $D_2 = 0.5$ m;

$V_{f2} = V_{f1} = 2$ m/s; $\beta = 90$ (Discharge is radial)

(i) Flow rate through the turbine, Q :

$$Q = \pi D_1 b_1 V_{f1} = \pi \times 1 \times 0.25 \times 2$$

$$= \mathbf{1.57\ m^3/s} \quad \text{...Ans.}$$

(ii) Moving blade angles (θ and φ) and breadth at outlet, B_2

Refer Fig. P. 3.18.11

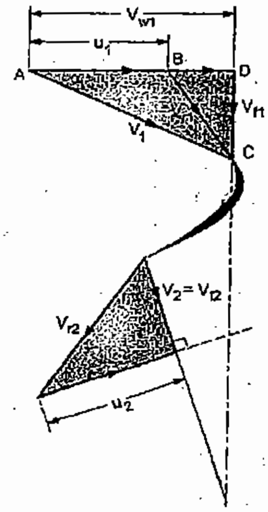


Fig. P. 3.18.11

$$\text{Blade speed, } u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1 \times 210}{60} = 11\ \text{m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 210}{60} = 5.5\ \text{m/s}$$

$$\text{Discharge, } Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$$

$$\text{But } V_{f1} = V_{f2}$$

$$\text{i.e. } D_1 B_1 = D_2 B_2$$

$$\text{Breadth at outlet, } B_2 = \frac{D_1 B_1}{D_2} = \frac{1 \times 0.25}{0.5} = \mathbf{0.5\ m} \quad \text{...Ans.}$$

Guide vane angle at inlet, α

From inlet velocity triangle :

$$\tan \alpha = \frac{V_{f1}}{V_{w1}}$$

$$\tan 10 = \frac{2}{V_{w1}}$$

$$\therefore V_{w1} = 11.343 \text{ m/s}$$

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{2}{11.343 - 11} = 5.83$$

$\therefore$ Inlet vane angle,

$$\theta = \mathbf{80.27^\circ} \quad \text{...Ans.}$$

From outlet velocity triangle,

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{2}{5.5} = 0.3636$$

$\therefore$ Vane angle at outlet,

$$\phi = \mathbf{19.98^\circ} \quad \text{...Ans.}$$

iii) Power developed, P

$$P = \rho Q V_{w1} \cdot u_1 = 10^3 \times 1.57 \times 11.343 \times 11$$

$$= 195.59 \times 10^3 \text{ W} = \mathbf{195.59\ kW} \quad \text{...Ans.}$$

(iv) Supply head, H

$$V_2 = V_{f2} = 2 \text{m/s}$$

$\because$ Power input – Kinetic energy of water at outlet

= Power developed

$$\rho Q g H - \rho \cdot Q \cdot \frac{V_2^2}{2} = \rho \cdot Q \cdot V_{w1} \cdot u_1$$

$$H = \frac{V_{w1} u_1}{g} + \frac{V_2^2}{2g}$$

$$= \frac{11.343 \times 11}{9.81} + \frac{2^2}{2 \times 9.81}$$

$$= \mathbf{12.923\ m} \quad \text{...Ans.}$$

(v) Hydraulic efficiency, η_h

$$\eta_h = \frac{\text{Power output, P}}{\text{Power input}, \rho Q gH}$$

$$= \frac{195.59 \times 10^3}{1000 \times 1.57 \times 9.81 \times 12.923}$$

$$= \mathbf{0.9827 \text{ or } 98.27\%} \quad \text{...Ans.}$$

Ex. 3.18.12 : The losses in a inward flow reaction turbine are as follows :

i) between entry to turbine and discharge from guides = 2.3 m
ii) in moving vane passage = 4.26 m
iii) in draft tube = 0.3048 m
iv) velocity head lost to tail race = 0.0765 m

The total head on the turbine is 58 m of water. The turbine axis is 3 m above the tailrace. The peripheral velocity of wheel at inlet 22.8 m/s. The velocity of flow is 7.3 m/s and is constant, throughout, find the pressure in m of water absolute

(a) At inlet to turbine (b) At exist from turbine
(c) Guide blade angle (d) Moving blade angle at inlet
(e) DOR.

Assume radial discharge from the wheel.

Soln. : Refer Fig. P. 3.18.12.

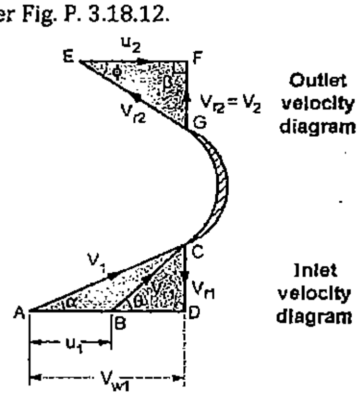


Fig. P. 3.18.12

Given : Total head on turbine, H = 58 m of water,

$u_1 = 22.8$ m/s, $V_{f1} = V_{f2} = 7.3$ m/s;

$h_s = 3$m; $\beta = 90°$ (radial discharge.)

Losses in turbine are :

a) Between entry to turbine and discharge = 2.3 m.
b) In moving vane passages = 4.26 m.
c) In draft tube = 0.3048 m
d) Velocity head in tail race = 0.0765 m.

Total losses in turbine = Loss at (a), (b) and (c)

$$= 2.3 + 4.26 + 0.3048 = 6.8648$$

Total head = W.D./kg of water + Losses in turbine + K.E. at exit

$$58 = W + 6.8648 + 0.0765$$

$$W = 51.0587 \text{ m of water head}$$

Assuming axial discharge, $\beta = 90°$, then $V_{w2} = 0$

$$W = \frac{V_{w1} \cdot u_1}{g}$$

$$51.0587 = \frac{V_{w1} \times 22.8}{9.81}$$

$$V_{w1} = 21.97 \text{ m/s}$$

$$\alpha = \tan^{-1}\left(\frac{V_{f1}}{V_{w1}}\right)$$

$$= \tan^{-1}\left(\frac{7.3}{21.97}\right) = 18.38°$$

$$V_1 = \sqrt{V_{f1}^2 + V_{w1}^2}$$

$$= \sqrt{7.3^2 + 21.97^2} = 23.15 \text{ m/s}$$

Moving blade angle, θ

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{V_{w1} - u_1}\right) = \tan^{-1}\left(\frac{7.3}{21.97 - 22.8}\right)$$

$$= -83.51°$$

$$= 180 - 83.51 = \mathbf{96.48°} \qquad \text{...Ans.}$$

(a) Absolute pressure at inlet to turbine $\frac{p_1}{w}$

Accordingly inlet velocity diagram needs to be modified. Applying Bernoulli's theorem between turbine inlet to runner,

$$H - h_s = \frac{p_1}{w} + \frac{V_1^2}{2g} + \text{Losses between turbine inlet to discharge in guide vanes, } h_{fld}$$

$$58 - 3 = \frac{p_1}{w} + \frac{(23.15)^2}{2 \times 9.81} + 2.3$$

$$\therefore \frac{p_1}{w} = \mathbf{25.385 \text{ m}} \qquad \text{...Ans.}$$

(b) Absolute pressure of exit of turbine, $\frac{p_2}{w}$

$$\frac{p_1}{w} + \frac{V_1^2}{2g} = \text{W.D.} + \frac{p_2}{w} + \frac{V_2^2}{2g} + \text{Loss of head in runner}$$

$$25.385 + \frac{23.15^2}{2 \times 9.81} = 51.0587 + \frac{p_2}{w} + \frac{7.3^2}{2 \times 9.81} + 4.26$$

$$\frac{p_2}{w} = \mathbf{-5.335 \text{ m}} \qquad \text{...Ans.}$$

Degree of reaction, (DOR)

$$\text{DOR} = \frac{\text{Total workdone W - change in dynamic velocity head}}{\text{Total workdone W}}$$

$$= \frac{W - \left(\frac{V_1^2 - V_2^2}{2g}\right)}{W} = 1 - \frac{V_1^2 - V_2^2}{2g \cdot W}$$

$$= 1 - \frac{23.15^2 - 7.3^2}{2 \times 9.81 \times 51.0587} = \mathbf{0.5182} \qquad \text{...Ans.}$$

Ex. 3.18.13 : The following data pertains to inward flow reaction turbine :

Net head	86.4 m
Speed of the runner	650 rpm
Shaft power available	397 kW
Ratio of wheel width to wheel diameter at inlet	0.1
Ratio of inner diameter to outer diameter	0.5
Flow ratio	0.17
Hydraulic efficiency	95%
Overall efficiency	85%
Flow velocity	constant
Discharge	radial

Neglecting blockage by blades, find the dimensions and blade angles of the turbine.

Soln. : Refer Fig. P. 3.18.13.

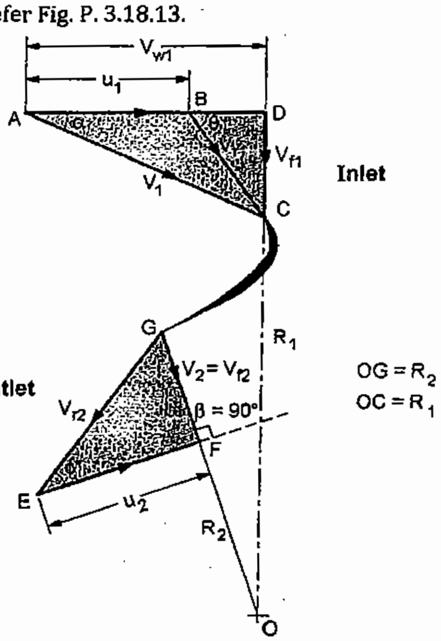


Fig. P. 3.18.13

Given : $H = 86.4$ m; $N = 650$ rpm;

$P_s = 397$ kW

$$\frac{B_1}{D_1} = 0.1; \frac{\text{Inner diameter, } D_2}{\text{Outer diameter, } D_1} = 0.5$$

Flow ratio, $$K_f = \frac{V_{f1}}{\sqrt{2gh}} = 0.17$$

$$\eta_h = 95\% = 0.95 \ ;$$

$$\eta_o = 85\% = 0.85$$

$$V_{f1} = V_{f2}$$

Discharge is radial,

i.e. $\beta = 90°$

(i) Overall efficiency,

$$\eta_o = \frac{\text{Shaft power, } P_s}{\text{Input power,} \rho g Q H}$$

$$\therefore \quad 0.85 = \frac{397 \times 10^3}{1000 \times 9.81 \times Q \times 86.4}$$

$$\text{Discharge, } Q = 0.551 \text{ m}^3/\text{s}$$

$$V_{f1} = K_f \times \sqrt{2gH}$$

$$= 0.17\sqrt{2 \times 9.81 \times 86.4} = 7 \text{ m/s}$$

$$\text{But, } Q = \pi D_1 B_1 V_{f1}$$

$$0.551 = \pi \times D_1 \times (0.1 D_1) \times 7.0$$

$\therefore$ **Diameter at inlet,**

$$D_1 = \mathbf{0.5 \text{ m}} \quad \text{...Ans.}$$

$$\text{and } B_1 = 0.1 D_1 = \mathbf{0.05 \text{ m}} \quad \text{...Ans.}$$

$$\text{Diameter at outlet, } D_2 = \frac{D_1}{2} = \mathbf{0.25 \text{ m.}} \quad \text{...Ans.}$$

Also, $Q = \pi D_2 B_2 \cdot V_{f2}$; $(V_{f2} = V_{f1} = 7 \text{ m/s})$

$$0.551 = \pi \times 0.25 B_2 \times 7$$

$\therefore$ Width at outlet, $B_2 =$ **0.1 m** ...Ans.

(ii) Blade angles of turbine

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.5 \times 650}{60} = 17.02 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 650}{60} = 8.56 \text{ m/s}$$

Hydraulic efficiency,

$$\eta_h = \frac{V_{w1} \cdot u_1}{gh}$$

$$\therefore \quad V_{w1} = \frac{\eta_h \cdot g \cdot H}{u_1}$$

$$= \frac{0.95 \times 9.81 \times 86.4}{17.02} = 47.3 \text{ m/s}$$

From inlet velocity diagram

Guide blade angle,

$$\alpha = \tan^{-1}\left(\frac{V_{f1}}{V_{w1}}\right)$$

$$\alpha = \tan^{-1}\left(\frac{7}{47.3}\right) = \mathbf{8.42°} \quad \text{...Ans.}$$

$$\tan\theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{7}{47.3 - 17.02} = 0.2312$$

$\therefore$ Inlet angle of moving vane,

$$\theta = \mathbf{13.02°} \quad \text{...Ans.}$$

From outlet velocity diagram,

Exit angle of moving vane,

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right)$$

$$= \tan^{-1}\left(\frac{7}{8.56}\right) = \mathbf{39.27°} \quad \text{...Ans.}$$

Ex. 3.18.14 : The external and internal diameters of an inward flow reaction turbine are 2 m and 1 m respectively. The head on the turbine is 60 m. The width of the vane at inlet and outlet are same and equal to 0.25 m. The runner vanes are radial are inlet and discharge is radial at outlet. The speed is 200 rpm and the discharge is 6 m³/s.

Determine :

(i) The vane angle at outlet and inlet of the runner.

(ii) The hydraulic efficiency. **SPPU - Dec. 16, 6 Marks**

Soln. :

Refer Fig. P. 3.18.14

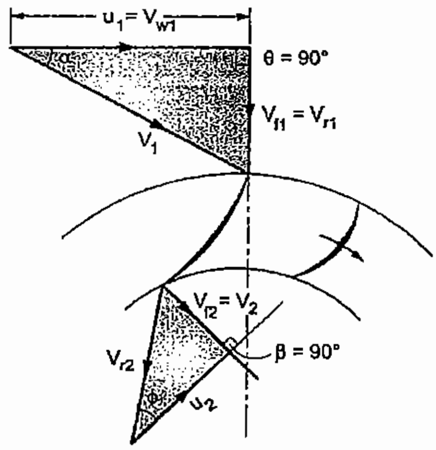


Fig. P. 3.18.14

Given :

Speed, $N = 200$ rpm ;

External diameter, $D_1 = 2.0$ m ;

Internal diameter $D_2 = 1.0$ m ;

Head, $H = 60$ m,

Discharge, $Q = 6 \text{ m}^3/\text{s}$;

$V_{f1} = V_{r1}$

$B_1 = B_2 = 0.25$ m

Runner vane angle at inlet $\theta_1 = 90°$ (radial vanes),

Radial discharge $\beta_2 = 90°$, $V_{w2} = 0$

From the inlet velocity triangle

$$u_1 = V_{w1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 2.0 \times 200}{60} = 20.95 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.0 \times 200}{60} = 10.47 \text{ m/s}$$

$$Q = \text{discharge}$$

$$= \pi D_1 B_1 V_{f1} \text{ (neglecting the thickness of vane)}$$

$$6 = \pi \times 2.0 \times 0.25 \times V_{f1}$$

$$V_{f1} = 3.82 \text{ m/s}$$

Also, $Q = \pi D_2 B_2 V_{f2}$

$$6 = \pi \times 1.0 \times 0.25 \times V_{f2}$$

$$V_{f2} = 7.64 \text{ m/s}$$

(i) Vane angle at outlet ϕ and inlet angle, α

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{7.64}{10.47} = 0.729$$

$$\phi = 36.11° \quad \text{...Ans.}$$

$$\alpha = \tan^{-1}\left(\frac{V_{f1}}{u_1}\right)$$

$$= \tan^{-1}\left(\frac{3.82}{20.95}\right) = 10.33° \quad \text{...Ans.}$$

(ii) Hydraulic efficiency (η_H)

$$\eta_H = \frac{V_{w1} u_1}{gH} = \frac{20.95 \times 20.95}{9.81 \times 60}$$

$$= 0.7456 \text{ or } 74.56\% \quad \text{...Ans.}$$

Ex. 3.18.15 : An inward flow reaction turbine develops 1200 kW power having the vane velocity at inlet as 30 m/s and the corresponding whirl velocity of 24 m/s . The ratio of outer to internal diameter is 2. The velocity of flow remains at 6 m/s throughout and discharge at exit is radial. The head available on wheel is 75 m, Find :

(i) Vane angles
(ii) Power developed by wheel per N/s of water
(iii) Discharge in m^3/s,
(iv) Hydraulic efficiency

Soln. :

Given :

$P = 1200$ kW, $u_1 = 30$ m/s,

$V_{w1} = 24$ m/s, $\frac{D_1}{D_2} = 2$,

$V_{f1} = V_{f2} = 6$ m/s,

$\beta = 90°$(discharge is radial); $H = 75$ m.

Since $u_1 > V_{w1}$, the velocity diagram at inlet and at outlet can be drawn as shown in Fig. P. 3.18.15.

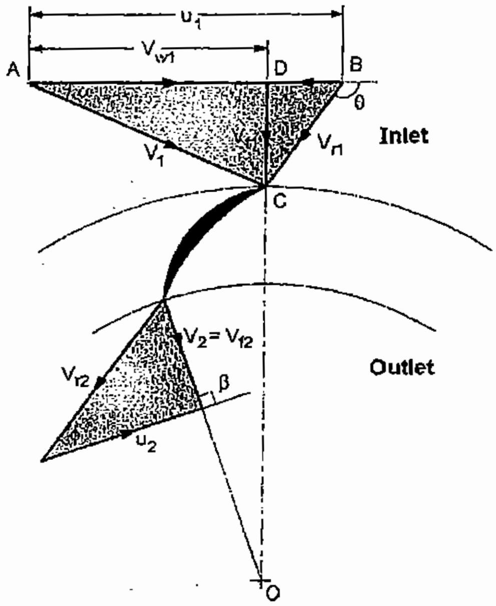


Fig. P. 3.18.15

Since, $u = \frac{\pi DN}{60}$

$$\therefore \frac{u_2}{u_1} = \frac{D_2}{D_1}$$

i.e. $u_2 = 0.5 \times 30 = 15$ m/s

(i) Vane angles :

Consider inlet velocity diagram

Guide vane angle at inlet, α

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} = \frac{6}{24} = 0.25;$$

$$\therefore \alpha = 14.036° \quad \text{...Ans.}$$

Inlet runner vane angle, θ

$$\tan(180 - \theta) = \frac{V_{f1}}{u_1 - V_{w1}} = \frac{6}{30 - 24} = 1$$

$$\therefore \theta = 135° \quad \text{...Ans.}$$

Outlet vane angle, ϕ

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{6}{15} = 0.4$$

$$\therefore \phi = 21.8° \quad \text{...Ans.}$$

(ii) Power developed by wheel per N/s of water, W

$$W = \frac{V_{w1} \cdot u_1}{g} \text{ (since, } V_{w2} = 0)$$

$$= \frac{24 \times 30}{9.81}$$

$$= 73.394 \text{Nm/N/s} \quad \text{...Ans.}$$

(iii) Discharge, Q

Power developed, $P = \rho Q (V_{w1} \cdot u_1)$

$$1200 \times 10^3 = 1000 \times Q \times 24 \times 30$$

$$\therefore \quad Q = \mathbf{1.667\ m^3/s} \quad \text{...Ans.}$$

(iv) Hydraulic efficiency, η_h

$$\eta_h = \frac{\text{Power developed, P}}{\text{Power input,}\rho QgH}$$

$$= \frac{1200 \times 10^3}{1000 \times 1.667 \times 9.81 \times 75}$$

$$= \mathbf{0.9784 \text{ or } 97.84\%} \quad \text{...Ans.}$$

Ex. 3.18.16 : In an inward radial flow turbine works under a head H. The guide vane angle at inlet is α. The flow through the turbine at inlet and outlet is radial. Show that the condition of maximum efficiency is given by the equation. $u_1 = \sqrt{\dfrac{2gH}{2 + K^2 \tan \alpha}}$ where K is the ratio of velocity of flow at outlet to inlet and u_1 is blade velocity at inlet.

Soln. :

Refer Fig. P. 3.18.16.

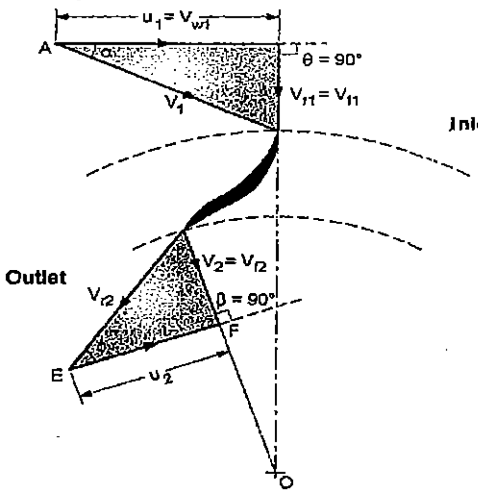


Fig. P. 3.18.16

$$\text{Friction factor, } K = \frac{V_{f2}}{V_{f1}}$$

Since flow is radial at inlet, $a = 90°$.

Hence, $V_{r1} = V_{f1} = u_1 \tan \alpha$ and $V_{w1} = u_1$

Discharge is radial at exit, therefore,

$$\beta = 90° \quad \text{and} \quad V_{w2} = 0$$

$$V_{f2} = V_2 = K \cdot V_{f1} = K \cdot u_1 \tan \alpha$$

Workdone or head utilized,

$$= \frac{V_{w1} \times u_1}{g} = \frac{u_1 \times u_1}{g} = \frac{u_1^2}{2g}$$

Head lost at exit corresponding to K.E

$$= \frac{V_2^2}{2g} = \frac{(K \cdot u_1 \tan \alpha)^2}{2g}$$

$$= \frac{K^2 \cdot u_1^2 \cdot \tan^2\alpha}{2g}$$

By energy balance,

Input head = Head utilized + Head lost at exit

$$H = \frac{u_1^2}{g} + \frac{K^2 \cdot u_1^2 \cdot \tan^2\alpha}{2g}$$

$$2\,gH = u_1^2 (2 + K^2 \tan^2\alpha)$$

$$\therefore \quad u_1 = \sqrt{\frac{2gH}{2 + K^2 \tan^2\alpha}} \quad \text{... proved}$$

$$\therefore \quad H = \frac{V_1^2 \cos^2\alpha}{g} + \frac{K^2 V_1^2 \sin^2\alpha}{2g}$$

$$2gH = 2V_1^2 \cos^2\alpha + K^2 - V_1^2 \sin^2\alpha$$

$$\therefore \quad V_1^2 (2 \cos^2\alpha + K^2 \sin^2\alpha) = 2gH$$

Ex. 3.18.17 : Water enters the radially inward flow reaction turbine, leaving the guide vane at angle of 'α' to the wheel tangent. The moving vanes are radial at inlet. The velocity of flow at exit is 'K' times that at inlet. For maximum hydraulic efficiency under a supply head of 'H' m, Prove that the blade velocity at inlet is given by $u_1 = [(2g H) / (2 + K^2 \tan^2\alpha)]^{1/2}$

Soln. :

Refer Ex. 3.18.16. We have proved that,

$$V_1^2 (2 \cos^2\alpha + K^2 \sin^2\alpha) = 2\,gH$$

$$V_1^2 \cdot \cos^2\alpha (2 + K^2 \tan^2\alpha) = 2\,gH$$

$$\text{But} \quad V_1 \cos \alpha = u$$

$$\therefore \quad u^2 (2 + K^2 \tan^2\alpha) = 2\,gH$$

$$\mathbf{u = [2gH / (2 + K^2 \tan^2\alpha)]^{1/2}} \quad \text{...Proved}$$

Ex. 3.18.18 : Show that hydraulic efficiency of Francis turbine having velocity of flow through runner as constant is given by the relation $\eta_h = 1 + \dfrac{\frac{1}{2}\tan^2\alpha}{\left(1 - \dfrac{\tan \alpha}{\tan \theta}\right)}$

where, α = guide blade angle, θ = vane angle at inlet.

The turbine is having radial discharge at outlet.

Soln. : Velocity diagram for Francis turbine are shown in Fig. P. 3.18.18

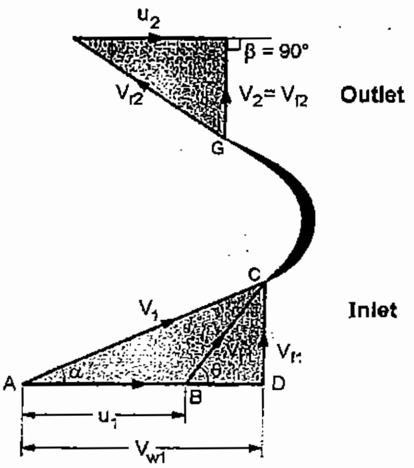


Fig. P. 3.18.18

Given : $V_{f1} = V_{f2}$, $\beta = 90°$,

therefore, $V_2 = V_{f2}$

We know that hydraulic efficiency, η_h is given as :

$$\eta_h = \frac{\text{Hydraulic power developed by runner}}{\text{Power input}}$$

$$= \frac{\rho Q (V_{w1} . u_1)}{\rho \cdot g \cdot Q \cdot H} = \frac{V_{w1} - u_1}{gH} \quad ...(i)$$

From inlet velocity triangle,

$$\tan\theta = \frac{V_{f1}}{V_{w1} - u_1}, \quad \text{i.e. } V_{w1} - u_1 = \frac{V_{f1}}{\tan\theta}$$

$$u_1 = V_{w1} - \frac{V_{f1}}{\tan\theta} \quad ...(ii)$$

$$\text{Also, } V_{f1} = V_{w1} \cdot \tan\alpha \quad ...(iii)$$

From Equations (ii) and (iii) we have,

$$u_1 = V_{w1} - \frac{V_{w1} \cdot \tan\alpha}{\tan\theta} = V_{w1}\left(1 - \frac{\tan\alpha}{\tan\theta}\right) \quad ...(iv)$$

Power developed by runner,

P = Power input – kinetic energy of water at exit

$$\rho \cdot Q \cdot V_{w1} \cdot u_1 = \rho gQH - \frac{1}{2}\rho Q V_2^2$$

$$\therefore \quad V_{w1} \cdot u_1 = g \cdot H - \frac{V_2^2}{2}$$

$$\text{i.e.} \quad H - \frac{V_2^2}{2g} = \frac{V_{w1} \cdot u_1}{g}$$

But, $V_2 = V_{f2} = V_{f1} = V_{w1} \cdot \tan\alpha$;

[Refer Equation (iii)]

$$H - \frac{V_{w1}^2 \cdot \tan^2\alpha}{2g} = \frac{V_{w1} \cdot u_1}{g}$$

On substituting the value of u_1 from Equation (iv) in the above expression,

$$H - \frac{V_{w1}^2 \cdot \tan^2\alpha}{2g} = \frac{V_{w1}}{g}\left(V_{w1} - \frac{V_{w1}\tan\alpha}{\tan\theta}\right)$$

$$\therefore \quad H = \frac{V_{w1}^2}{g}\left[\left(1 - \frac{\tan\alpha}{\tan\theta}\right) + \frac{\tan^2\alpha}{2}\right]$$

Hydraulic efficiency from Equation (i),

$$\eta_h = \frac{V_{w1} \cdot u_1}{gH} = \frac{V_{w1} \times V_{w1}\left(1 - \frac{\tan\alpha}{\tan\theta}\right)}{g \times \frac{V_{w1}^2}{g}\left[\left(1 - \frac{\tan\alpha}{\tan\theta}\right) + \frac{\tan^2\alpha}{2}\right]}$$

$$\eta_h = \frac{1 - \frac{\tan\alpha}{\tan\theta}}{1 - \frac{\tan\alpha}{\tan\theta} + \frac{\tan^2\alpha}{2}} = \frac{1}{1 + \frac{\left(\frac{1}{2}\right)\tan^2\alpha}{\left(1 - \frac{\tan\alpha}{\tan\theta}\right)}} \quad \text{...proved}$$

Ex. 3.18.19 : An experimental inward flow reaction turbine rotates at 370 rpm. The wheel vanes are radial at inlet and outlet. The inner diameter of wheel is half of the outer diameter. The constant velocity of flow in the wheel is 2 m/sec. Water enters the wheel at an angle of 30° to the tangent to the wheel at inlet. The breadth of the wheel at inlet is 75 mm and area of flow blocked by vanes is 5% of gross area of flow at inlet. Draw inlet and outlet velocity triangles for the turbine and Find :

(i) The Net available head at inlet

(ii) The wheel vane angle at outlet

(iii) Outer and inner wheel diameter

(iv) Theoretical Power developed by the turbine also find overall efficiency if the mechanical efficiency is 87%.

SPPU - May 12, 12 Marks

Soln. : Velocity diagram is shown in Fig. P. 3.18.19

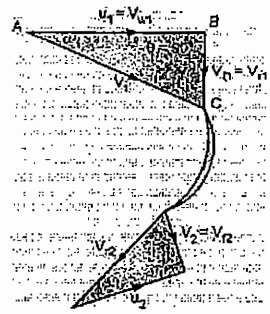


Fig. P. 3.18.19

$$D_2 = \frac{1}{2}D_1, \quad N = 370 \text{ rpm}$$

$$\theta = 90° \text{ (vanes are radial at inlet)}$$

$$\beta = 90° \text{ (vanes are radial at outlet)}$$

$$V_{r1} = V_{f1} = V_{f2} = 2 \text{ m/s} = V_2; \quad \alpha = 30°$$

$$B_1 = 75 \text{ mm} = 0.075 \text{ m};$$

Area blocked by vanes $= 5\% A_{f1} = 0.05 A_{f1}$

$$= 0.05\pi D_1 B_1$$

Mechanical efficiency,

$$\eta_m = 0.87$$

(i) Net head available at inlet, H

$$V_1 = \frac{V_{f1}}{\sin\alpha} = \frac{2}{\sin 30} = 4 \text{ m/s}; \quad \frac{V_{f1}}{u_1} = \tan\alpha$$

$$u_1 = \frac{V_{f1}}{\tan\alpha} = \frac{2}{\tan 30} = 3.464 \text{ m/s}$$

$$V_{w1} = u_1 = 3.464 \text{ m/s} \quad \text{and} \quad V_{w2} = 0$$

$$u_1 = \frac{\pi D_1 N}{60}$$

$$3.464 = \frac{\pi \times D_1 \times 370}{60}$$

$$D_1 = 0.1788 \text{ m}$$

$$D_2 = \frac{D_1}{2} = \frac{0.1788}{2} = 0.0894 \text{ m}$$

$$u_2 = \frac{\pi \times 0.0894 \times 370}{60} = 1.732 \text{ m/s}$$

$$Q = A_{f1} \cdot V_{f1} = (1 - 0.05)\,\pi D_1 B_1 \times V_{f1}$$

$$= 0.95 \times \pi \times 0.1788 \times 0.075 \times 2 = 0.08 \text{ m}^3/\text{s}$$

$$H = \frac{V_{w1} \cdot u_1}{g} + \frac{V_2^2}{2g}$$

$$= \frac{3.464 \times 3.464}{9.81} + \frac{(2)^2}{2 \times 9.81} = \mathbf{1.427 \text{ m}} \quad \text{...Ans.}$$

(ii) The wheel vane angle at outlet, ϕ

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right) = \tan^{-1}\left(\frac{2}{1.732}\right) = \mathbf{49.1°} \quad \text{...Ans.}$$

(iii) Outer and Inner Wheel Diameters

From above, Inner wheel diameter,

D_1 = 0.1788 m and outer,

D_2 = 0.0894 m ...Ans.

(iv) Theoretical power developed, P and overall efficiency, η_0

$$\dot{m} = \rho . Q = 1000 \times 0.08 = 80 \text{ kg/s}$$

$$P = \dot{m} \times V_{w1} \cdot u_1 \times 10^{-3} \text{ kW}$$

$$= 80 \times 3.462 \times 3.462 \times 10^{-3}$$

$$= \mathbf{0.9588 \text{ kW}} \quad \text{...Ans.}$$

Actual power,

$$P_S = \eta_m \times P = 0.87 \times 0.9588 = \mathbf{0.8342 \text{ kW}}$$

$$\eta_0 = \frac{P_s}{\rho . gQH} = \frac{0.8342 \times 1000}{1000 \times 9.81 \times 0.08 \times 1.427}$$

$$= \mathbf{0.7449 \text{ or } 74.49\ \%} \quad \text{...Ans.}$$

Ex. 3.18.20 : The following data is given for a Francis Turbine Net heat H = 70 m ; Speed N = 600 r.p.m ; Shaft power = 367.875 kW ; η_o = 85% ; η_h = 95% ; flow ratio = 0.25 ; breadth ratio = 0.1 ; outer diameter of the runner = 2X inner diameter of the runner. The thickness of vanes occupy 10% of circumferential area of the runner. Velocity of flow is constant at inlet and outlet and discharge is radial at outlet. Determine

(i) Guide blade angle

(ii) Runner vane angles at inlet and outlet

(iii) Diameters of runner at inlet and outlet

(iv) Width of wheel at inlets **SPPU - May 13,10 Marks**

Soln. : Refer Fig. P. 3.18.20

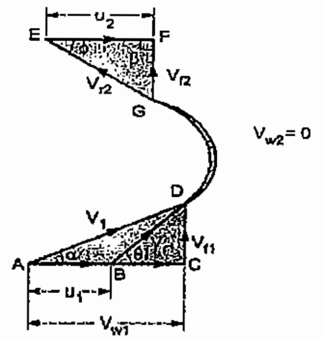


Fig. P. 3.18.20

$$H = 70 \text{ m}, \qquad N = 600 \text{ rpm},$$

$$P_s = 367.875 \text{ kW} \qquad \eta_0 = 0.85,$$

$$\eta_h = 0.95,$$

$$\text{Flow ratio,} \quad K_f = \frac{V_{f1}}{\sqrt{2\,gH}} = 0.25$$

$$\text{Breadth ratio,} \quad n = \frac{B_1}{D_1} = 0.1$$

Outer diameter, $D_2 = 2 \times$ Inner diameter, D_1

$$V_{f_1} = V_{f_2};$$

β = 90°(radial discharge) Thickness factor,

$$K_t = (1 - 0.1) = 0.9$$

(i) Guide blade angle, α

$$\eta_o = \frac{\text{Shaft power}, P_s}{\text{Input power}, \rho.g.Q.H}$$

$$\therefore \quad Q = \frac{P_s}{\eta_o \times \rho.g\,H} = \frac{367.875 \times 10^3}{0.85 \times 1000 \times 9.81 \times 70}$$

$$= 0.63025 \text{ m}^3/\text{s}$$

$$V_{f_1} = K_f \times \sqrt{2\,gH} = 0.25 \times \sqrt{2 \times 9.81 \times 70}$$

$$= 9.265 \text{ m/s}$$

$$Q = K_f \pi\, D_1 B_1 V_{f_1}$$

$$0.63025 = 0.9 \times \pi \times D_1 \times 0.1 D_1 \times 9.265$$

$$D_1 = 0.4905 \text{m}$$

$$D_2 = 2D_1 = 2 \times 0.4905 = 0.981 \text{ m}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.4905 \times 600}{60} = 15.41 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.981 \times 600}{60} = 30.82 \text{ m/s}$$

$$\eta_h = \frac{V_{w1} \cdot u_1}{gH} ;$$

$$0.95 = \frac{V_{w1} \times 15.41}{9.81 \times 70}$$

$$V_{w_1} = 42.33 \text{ m/s}$$

From Δ ACD ;

$$\alpha = \tan^{-1}\left(\frac{V_{f_1}}{V_{w_1}}\right) = \tan^{-1}\left(\frac{9.265}{42.33}\right)$$

$$= \mathbf{12.345^\circ} \quad \text{...Ans.}$$

(ii) Runner vane angle at inlet, θ and at outlet, ϕ

$$V_1 = \sqrt{V_{f1}^2 + V_{w1}^2} = \sqrt{(9.265)^2 + (42.33)^2}$$

$$= 43.33 \text{ m/s}$$

$$BC = V_{w_1} - u_1 = 42.33 - 15.41 = 26.92 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_{f_1}}{BC}\right) = \tan^{-1}\left(\frac{9.265}{26.92}\right)$$

$$= \mathbf{18.99^\circ} \quad \text{...Ans.}$$

Given : $V_{f_2} = V_{f_1} = 9.265$ m/s ; From outlet velocity

$$\phi = \tan^{-1}\left(\frac{V_{f_2}}{u_2}\right) = \tan^{-1}\left(\frac{9.265}{30.82}\right)$$

$$= \mathbf{16.73^\circ} \quad \text{...Ans.}$$

(iii) Diameter of runner at inlet (D_1) and at outlet (D_2).

From above,

D_1 = **0.4905 m** and D_2 = **0.981 m** ...Ans.

(iv) Width of wheel at inlet, B_1

$$B_1 = 0.1\,D_1 = 0.1 \times 0.4905 = \mathbf{0.04905\ m} \quad \text{...Ans.}$$

Ex. 3.18.21 : A vertical shaft mixed-flow Francis turbine (Modern Francis turbine) works between the head race and the tail race having vertical level difference of 150 meters. Frictional head losses in penstock, casing, wicket gates, runner and draft tube are 10 m, 1 m, 0.5 m, 0.5 m and 2 m of water respectively. Water discharges from the draft tube at a velocity of 2 m/s into the tail race. The turbine develops shaft power of 102.041 MW while running at a speed of 300 rpm. Mechanical efficiency of the system is 95% and the electrical generator is 98% efficient. Outer diameter and axial width of the runner at the inlet are 2.5 m and 0.625 m respectively. Blockage caused by the runner blades to the flow is negligible. Discharge from the runner is axial and whirl-free. Density of water may be taken to be 1000 kg/m³.

Determine

(i) Overall efficiency of the plant,

(ii) Discharge through the turbine in m³/s,

(iii) Runner blade angle at the inlet and

(iv) Wicket gate (or inlet guide vane) angle.

Draw neat inlet velocity triangle showing all the important details. **SPPU - Dec. 11,12 Marks**

Soln. : Refer Fig P. 3.18.21 and (a).

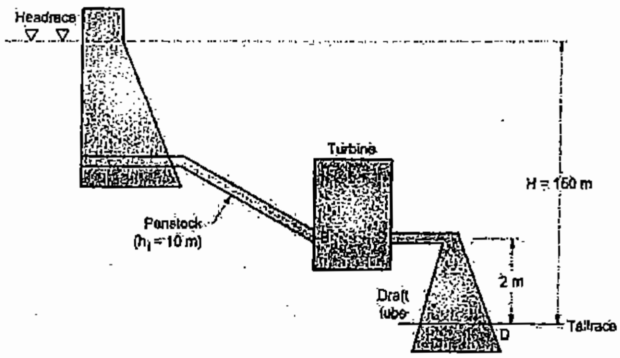


Fig. P. 3.18.21 : Schematic diagram

Total vertical head = 150 m

Friction losses are :

(i) In penstock = 10 m

(ii) In casing = 1 m

(iii) In Wicket gates = 0.5 m

(iv) In runner = 0.5 m

(v) In Draft tube = 2 m

Velocity at exit of draft tube V = 2 m/s

Shaft power, P_s = 102.041 MW = 102041 kW

N = 300 rpm,

Mechanical efficiency, η_m = 0.95

Generator efficiency, $\eta_g = 0.98$

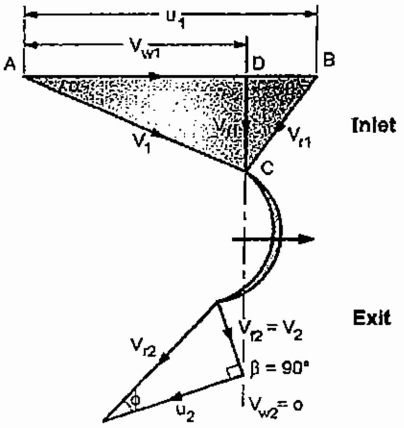


Fig. P. 3.18.21(a) : Velocity diagram

Outer diameter of runner $D_1 = 2.5\text{m}$ and $B_1 = 0.625\text{m}$

Density of water, $\rho = 1000 \text{ kg/m}^3$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 2.5 \times 300}{60}$$

$$= 39.27 \text{ m/s}$$

Velocity or kinetic head at exit of draft tube

$$= \frac{V^2}{2g} = \frac{(2)^2}{2 \times 9.81} = 0.204 \text{ m}$$

Total energy supplied to turbine (gH)

= workdone / kg of water + g × Friction losses in (penstock + casing + wicket gates + runner + draft tube + K.E. head at exit of draft tube.)

$$9.81 \times 150 = W + 9.81\,[10 + 1 + 0.5 + 0.5 + 2 + 0.204]$$

$$W = 1332.16 \text{ Nm/kg}$$

As discharge is radial.

$$W = V_{w1} \cdot u_1 \;;$$

$$1332.16 = V_{w1} \times 39.27$$

$$V_{w1} = 33.92 \text{ m/s}$$

(i) Overall efficiency of the plant, η_o

$$\eta_o = \eta_h \times \eta_m \times \eta_g = \frac{V_{w1} \cdot u_1}{gH} \times \eta_m \times \eta_g$$

$$= \frac{1332.16}{9.81 \times 150} \times 0.95 \times 0.98$$

$$= \mathbf{0.8428 \text{ or } 84.28\ \%} \qquad \text{...Ans.}$$

(ii) Discharge through the turbine in m^3/s

$$P_s = \dot{m} \times W.D \times \eta_o$$

$$102041 = 1332.16 \times 10^{-3} \times 0.8428 \times \dot{m}$$

$$\dot{m} = 90885.33 \text{ kg/s}$$

$$\dot{Q} = \frac{\dot{m}}{\rho} = \frac{90885.33}{1000}$$

$$= \mathbf{90.8853 \text{ m}^3/\text{s}} \qquad \text{...Ans.}$$

(iii) Runner blade angles at inlet, θ

Area of flow, $A_f = \pi D_1 B_1$

$$\dot{Q} = A_f \cdot V_{f1}$$

$$90.8853 = \pi \times 2.5 \times 0.625 \times V_{f1}$$

$$V_{f1} = 18.515 \text{ m/s}$$

$$\tan\theta = \frac{V_{f1}}{u_1 - V_{w1}} = \frac{18.515}{39.27 - 33.92} = 3.4607$$

$$\theta = \mathbf{73.88°} \qquad \text{...Ans.}$$

(iv) Wicket gate or inlet guide vane angle, α

$$\alpha = \tan^{-1}\frac{(V_{f1})}{(V_{w1})}$$

$$= \tan^{-1}\left(\frac{18.515}{33.92}\right)$$

$$= \mathbf{28.628°} \qquad \text{...Ans.}$$

Ex. 3.18.22 : For the Francis turbine following data is available :

Shaft power = 130 kW, Net head = 90, Speed = 120 r.p.m.

Overall efficiency = 75%, Hydraulic efficiency = 90%

Velocity of flow at inlet = $1.15\sqrt{H}$

Vane speed at inlet = $3.45\sqrt{H}$

Assume discharge radial at exit. Find :

(i) Guide blade and moving blade angle at inlet

(ii) Diameter of runner at inlet

SPPU - May 19, Oct. 19(In Sem.), 6 Marks

Soln. : Refer Fig. P. 3.18.22(a)

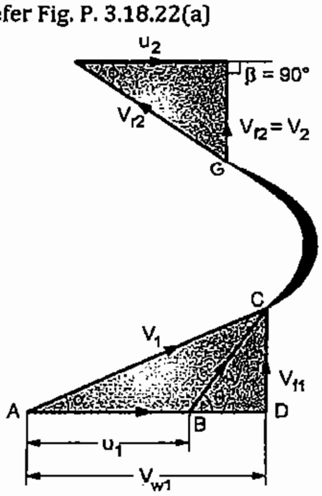


Fig. P. 3.18.22(a)

Given :

Shaft power, $P_s = 130$ kW

$H = 9$ m, $N = 120$ rpm,

$\eta_0 = 75\% = 0.75,$

$\eta_h = 90\% = 0.9;\ V_{f1} = 1.15\sqrt{H}$

$u_1 = 3.45\sqrt{H}$

$\beta = 90$ (discharge is radial)

(i) Guide blades and moving blade at inlet :

Since, overall efficiency,

$$\eta_0 = \frac{\text{Shaft power }, P_s}{\text{Input power }, \rho \cdot g \cdot Q \cdot H}$$

$$\therefore \text{ Discharge, } Q = \frac{1}{\eta_0} \times \frac{P_s}{\rho \cdot g \cdot H}$$

$$= \frac{1}{0.75} \times \frac{130 \times 10^3}{1000 \times 9.81 \times 9}$$

$$= 1.963 \text{ m}^3/\text{s}$$

$$V_{f1} = 1.15\sqrt{H} = 1.15 \times \sqrt{9} = 3.45 \text{ m/s}$$

$$u_1 = 3.45\sqrt{H} = 3.45\sqrt{9} = 10.35 \text{ m/s}$$

$$\eta_h = \frac{V_{w1}\, u_1}{gH}$$

i.e.

$$0.9 = \frac{V_{w1} \times 10.35}{9.81 \times 9}$$

$$\therefore V_{w1} = 7.68 \text{ m/s}$$

Since, u_1 is greater than V_{w1}, the modified inlet velocity diagram is shown in Fig. P. 3.18.22(b).

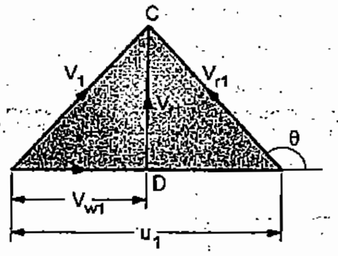


Fig. P. 3.18.22(b)

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} = \frac{3.45}{7.68} = 0.4492$$

$\therefore$ Guide vane angle,

$$\alpha = 24.19° \qquad \text{...Ans.}$$

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{3.45}{7.68 - 10.35} = -1.292$$

Inlet angle of moving vane,

$$\theta = 180 - 52.26$$

$$= 127.74° \qquad \text{...Ans.}$$

(ii) Diameter of runner at inlet, D_1

$$u_1 = \frac{\pi D_1 N}{60}$$

$$\therefore D_1 = \frac{60\, u_1}{\pi N}$$

$$= \frac{60 \times 10.35}{\pi \times 120} = \mathbf{1.6473 \text{ m}} \qquad \text{...Ans.}$$

Ex. 3.18.23 : An inward flow reaction turbine has a flow rate of 0.3 m^3/s and it works under a head of 30 m. The velocity of wheel at inlet is 20 m/s. Water discharges through a pipe at exit of 0.3 m diameter in radial direction. Determine the guide vane angle at inlet power to runner and hydraulic efficiency. velocity of flow remains constant throughout.

Soln. :

Given : $Q = 0.3 \text{m}^3/\text{s}$, $H = 30$ m;

$u_1 = 20 \text{m/s}$, $D_2 = 0.3$ m;

Discharge is radial,

therefore, $V_{w2} = 0$ and $V_2 = V_{f2} = V_{f1}$

Discharge, $Q =$ (Area of pipe × velocity of flow) at exit

$$0.3 = \left(\frac{\pi}{4} \times 0.3^2\right) \times V_{f2}$$

$$\therefore V_{f2} = V_2 = 4.244 \text{ m/s}$$

Head utilised corresponding to workdone,

$$H_1 = \frac{V_{w1} \cdot u_1}{g}$$

$$\text{Input head, } H = \text{Head utilised, } H_1 + \frac{V_2^2}{2g}$$

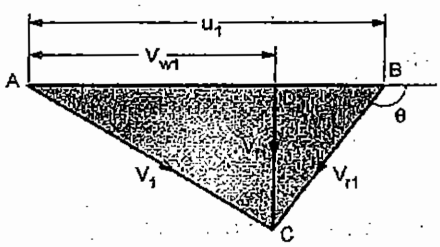


Fig. P. 3.18.23

$$H = \frac{V_{w1} \cdot u_1}{g} + \frac{V_2^2}{2g}$$

$$30 = \frac{V_{w1} \times 20}{9.81} + \frac{(4.244)^2}{2 \times 9.81}$$

$$V_{w1} = 14.265 \text{ m/s}$$

Since $V_{w1} < u_1$, inlet velocity diagram can be drawn as shown in Fig. P. 3.18.23.

(i) Guide vane angle

$$\alpha = \tan^{-1}\left(\frac{V_{f1}}{V_{w1}}\right) = \tan^{-1}\left(\frac{4.244}{14.265}\right)$$

$$= 16.57° \qquad \text{...Ans.}$$

(ii) Power to runner, P

$$P = \rho\, Q\, V_{w1} \times u_1$$

$$= \frac{10^3 \times 0.3 \times 14.265 \times 20}{1000}\ \text{kW}$$

$$= 85.59\ \text{kW} \qquad \text{...Ans.}$$

(iii) Hydraulic efficiency, η_h

$$\eta_h = \frac{V_{w1} \cdot u_1}{g \cdot H} = \frac{14.265 \times 20}{9.81 \times 30}$$

$$= 0.9694 \text{ or } 96.94\% \qquad \text{...Ans.}$$

Ex. 3.18.24 : A Francis turbine working under head of 30 m has a wheel diameter of 1.2 m at the entrance and 0.6 m at the exit. The vane angle at entrance is 90° and the guide blade angle is 15°. The water at exit leaves the vane without any tangential velocity and velocity of flow in the runner is constant. Neglecting the effect of draft tube and losses in the guide and runner passage, determine the speed of the wheel in RPM and vane angle at exit. **SPPU - April 2017, 6 Marks**

Soln. : Refer Fig. P. 3.18.24

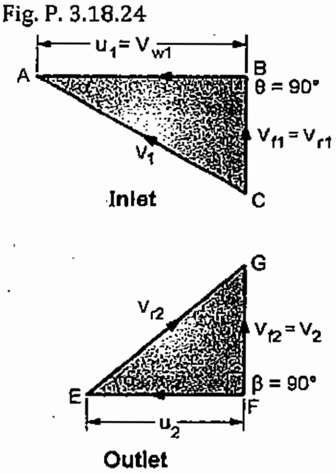


Fig. P. 3.18.24

Given :

$H = 30$ m, $D_1 = 1.2$ m,

$D_2 = 0.6$ m,

Vane angle at inlet, $\theta = 90°$,

$\alpha = 15°$ $\quad \beta = 90°$

i.e. $V_{w_2} = 0$; $\quad V_{f_1} = V_{f_2}$;

$V_{w_1} = u_1$

Since the effect of draft tube and the losses in guide runner passage are neglected, it implies that :

$$H - \frac{V_2^2}{2g} = V_{w_1} \cdot u_1 \qquad \text{...(i)}$$

But, $\quad V_2 = V_{f_2} = V_{f_1}$

From inlet velocity diagram, $V_{f_1} = u_1 \tan \alpha$

i.e. $\quad V_{f_1} = u_1 \tan 15 = 0.268\, u_1 = V_{f_2} = V_2 \qquad$ ...(ii)

On substituting the value of V_2 from Equation (ii) in Equation (i) we get,

$$H - \frac{(0.268\, u_1)^2}{2g} = u_1 \times u_1$$

$$H = u_1^2 + \frac{(0.268\, u_1)^2}{2 \times 9.81}$$

$$30 = u_1^2 + 0.01366\, u_1^2$$

$$u_1 = 5.44\ \text{m/s}$$

(i) Speed of wheel in rpm, N

$$u_1 = \frac{\pi D_1 N}{60}$$

$$5.44 = \frac{\pi \times 1.2 \times N}{60};$$

$$N = 86.58\ \text{rpm} \qquad \text{...Ans.}$$

(ii) Vane angle at exit, ϕ :

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 86.58}{60}$$

$$= 2.72\ \text{m/s}$$

$$V_{f_2} = V_{f_1} = V_2 = 0.268\, u_1$$

$$= 0.268 \times 5.44 = 1.458\ \text{m/s}$$

$$\phi = \tan^{-1}\left(\frac{V_{f_2}}{u_2}\right) = \tan^{-1}\left(\frac{1.458}{2.72}\right)$$

$$= 28.19° \qquad \text{...Ans.}$$

Ex. 3.18.25 : In an inward flow reaction turbine developing 185 kW under a head of 35 m, the inlet diameter is 1.5 times the outlet diameter. The guide blade angle is 20° at inlet and the blade angle at outlet is 25°. The area of flow at outlet is 4/3 times the area of flow at inlet. The water is discharged radially into the atmosphere. Determine the outlet area from guides, pressure head at inlet to wheel and discharge when there is a loss of 10% of velocity head in the guides and 20% of the outlet relative velocity head in the runner blades. **SPPU - May 14,10 Marks**

Soln. : Refer Fig. P. 3.18.25.

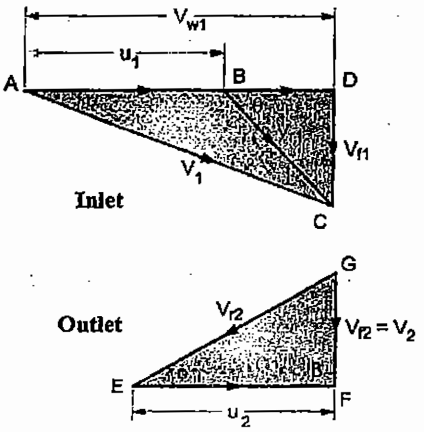


Fig. P. 3.18.25

Given : $P_s = 185$ kW, $H = 35$ m,

$D_1 = 1.5 D_2$, $\alpha = 20°$,

$\phi = 25°$, $A_{f2} = \frac{4}{3} A_{f1}$

$\beta = 90°$ (radial discharge),

$V_{w2} = 0$

Loss of head in guides = 10% of velocity

$$\text{head in guides} = 0.1 \frac{V_2^2}{2g}$$

Loss of head in runner = 20% of relative velocity

$$\text{head in runner} = 0.2 \frac{V_{r2}^2}{2g}$$

(i) Outlet area form guides, A_{f2}

Head available at inlet

H_1 = H [friction loss in guides + friction loss in runner + Exit K.E.]

$$H_1 = H - 0.1 \frac{V_2^2}{2g} - 0.2 \frac{V_{r2}^2}{2g} - \frac{V_2^2}{2g}$$

But $V_{r2} = \frac{V_2}{\cos \phi} = \frac{V_2}{\cos 25} = 1.1034\, V_2$

$$\therefore \quad H_1 = H - 0.1 \frac{V_2^2}{2g} - 0.2 \times \frac{(1.1034\, V_2)^2}{2g} - \frac{V_2^2}{2g}$$

$$= H - \frac{1.3434\, V_2^2}{2g}$$

$$H_1 = 35 - \frac{1.3434 \times V_2^2}{2 \times 9.81} = 35 - 0.06848 V_2^2 \quad \text{...(i)}$$

$$P_s = \rho \cdot g Q H_1 \times 10^{-3} \text{ (kW)}$$

$$185 = 1000 \times 9.81 \times Q \times (35 - 0.06848 V_2^2) \times 10^{-3}$$

$$Q\,(35 - 0.06848\, V_2^2) = 18.8583 \quad \text{...(ii)}$$

Discharge, $Q = A_{f_1} \times V_{f_1} = A_{f_2} \cdot V_{f_1}$

i.e $A_{f_2} \cdot V_{f_1} = \frac{4}{3} A_{f_1} \cdot V_{f_2}$

$$V_{f_1} = \frac{4}{3} V_{f_2} = \frac{4}{3} V_2 \quad \text{...(iii)}$$

From inlet Δ ACD :

$$V_{w1} = \frac{V_{f_1}}{\tan \alpha} = \frac{(4/3)\, V_2}{\tan 20} = 3.6633\, V_2$$

From outlet ΔEFG : $u_2 = \frac{V_{f_2}}{\tan \phi} = \frac{V_2}{\tan 25} = 2.1445\ V_2$

$$\therefore \quad u = \frac{\pi D N}{60},$$

therefore, $\frac{u_1}{u_2} = \frac{D_1}{D_2} = \frac{1.5\, D_2}{D_2} = 1.5$

$$\therefore \quad u_1 = 1.5\, u_2 = 1.5 \times 2.1445\, V_2 = 3.2168\, V_2$$

Now, Net head,

$$H_1 = \frac{V_{w_1} \cdot u_1}{g}$$

$$H - 0.06848\ V_2^2 = \frac{(3.6633\, V_2) \times (3.2168\, V_2)}{9.81}$$

$$35 - 0.06848\, V_2^2 = 1.20123\, V_2^2$$

$$V_2 = 5.25 \text{ m/s}$$

(i) Outlet area, A_{f2} :

From Equation (ii),

$$Q\left(35 - 0.06848\, V_2^2\right) = 18.8583$$

$$Q\,(35 - 0.06848 \times 5.25^2) = 18.8583$$

$$Q = 0.5695 \text{ m}^3/\text{s}$$

$$Q = A_{f2} \times V_2$$

$$0.5695 = A_{f2} \times 5.25$$

$$A_{f2} = \mathbf{0.1085\ m^2} \quad \text{...Ans.}$$

(ii) Pressure head at inlet, $\frac{P_1}{w}$

$$V_{w1} = 3.6633\, V_2$$

$$= 3.6633 \times 5.25 = 19.2323 \text{ m/s}$$

$$V_1 = \frac{V_{w1}}{\cos \alpha} = \frac{19.2323}{\cos 20}$$

$$= 20.467 \text{ m/s}$$

$$H = \frac{p_1}{w} + \frac{V_1^2}{2g}$$

$$\frac{p_1}{w} = H - \frac{V_1^2}{2g}$$

$$= 35 - \frac{(20.467)^2}{2 \times 9.81} = 13.65 \text{ m} \quad ...\text{Ans.}$$

(iii) Discharge, Q :

From above

$$Q = 0.5695 \text{ m}^3/\text{s} \quad ...\text{Ans.}$$

3.19 Axial Flow Reaction Turbines - Propeller and Kaplan Turbines

University Questions

Q. Compare Kaplan and Propeller Turbine. **SPPU : May 14**

Q. Draw construction and details of Kaplan Turbine. **SPPU : Oct. 19(In Sem)**

Q. Compare Kaplan turbine and propeller turbine. Explain which turbine is suitable for part loading condition and why. **SPPU : Dec. 19**

The propeller and kaplan are propeller shaped axial flow reaction turbines in which the water flows parallel to the axis of the shafts. These turbines were developed to meet the requirement of using available large quantity of water at low heads (upto 80 m).

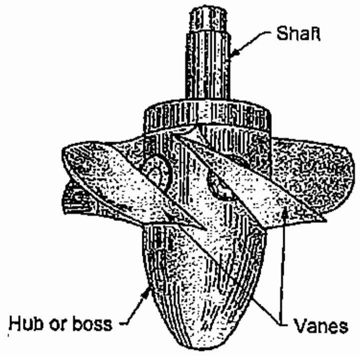


Fig. 3.19.1 : Runner of a Kaplan turbine

In the year 1910, Prof. V. Kaplan developed a **propeller turbine** having fixed runner blades of aerofoil section fixed to the hub of the shaft.

Subsequently, Kaplan modified the design of propeller turbine in the present form of **Kaplan turbine** which has adjustable blades against the non-adjustable blades. The runner of a Kaplan turbine is shown in Fig. 3.19.1.

It has a vertical hollow shaft which is enlarged at the bottom in the shape of propeller called **hub or boss**. The vanes are fixed on the hub which acts as the runner of Kaplan turbine.

The runner vanes are 3 to 6 in numbers (upto 8) made of stainless steel. These vanes are adjustable and can be rotated about pivots fixed to the hub of runner.

These vanes are adjusted automatically by governor mechanism according to the load on the turbine. Due to proper adjustment of runner vanes during the running of turbine, Kaplan turbine gives high efficiency over wide range of load conditions. Other constructional details of a Kaplan turbine are similar to Francis turbine shown in Fig. 3.19.2. Kaplan turbine has the scroll casing, guide mechanism and draft tube similar to Francis turbine except that it has different type of runner. Water from penstock enters the scroll casing into guide vanes. The water from guide vanes turns through 90° and enters the runner vanes axially. In case of Francis turbine, the water enters the runner radially while in Kaplan turbine the water strikes the runner axially.

Kaplan turbine has very few moving vanes compared to Francis turbine. It reduces the friction resistance due to reduced surface contact of water with vanes.

Kaplan turbines with sloping guide vanes taken the place of Francis turbine for certain medium installations.

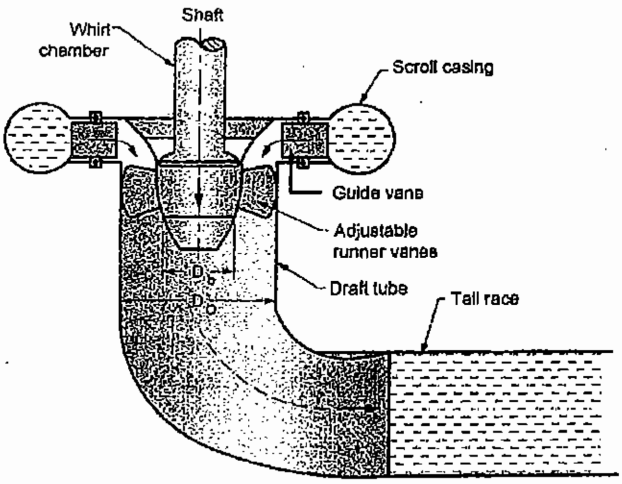


Fig. 3.19.2 : Construction features of a Kaplan turbine

3.19.1 Comparison Between Francis and Kaplan Turbines

University Question

Q. Compare Francis turbine and Kaplan turbine. **SPPU : May 11, May 14, Dec. 15, Feb 16 (In Sem)**

The difference between Francis and Kaplan turbines are explain below :

Sr. No.	Francis turbine	Kaplan turbine
1.	Water enters the runner radially.	Water enters the runner axially.
2.	Number of blades used are generally 16 to 24.	Number of blades used are generally 3 to 8.
3.	Frictional resistance is high.	Friction resistances less due to reduced number of vanes.
4.	Only guide vanes are adjustable.	Guide vanes and moving vanes are both adjustable.
5.	High efficiency is obtained only at high loads.	High efficiency is obtained even at part loads.
6.	It is big in size.	It is compact in size.
7.	Specific speed ranges from 50 - 250.	Specific speed ranges from 250 - 850.

3.19.2 Working Proportions of Propeller and Kaplan turbines

The method of drawing the velocity diagrams and the expressions for work power and efficiency for kaplan turbines are identical as discussed in case of Francis turbine. The working proportions are also obtained in the same manner as for Francis turbine.

However, following important points to be noted for kaplan turbines :

(i) The peripheral velocity at inlet and outlet are equal i.e.

$$u = u_1 = u_2 = \frac{\pi D_o N}{60}$$

where, D_o is the outer diameter of runner.

(ii) The ratio of width to diameter represented by n is taken as

$$n = \frac{D_b}{D_o}, \text{ where } D_b \text{ or } D_h \text{ is the diameter of hub.}$$

(iii) Area of flow at inlet = Area of flow at outlet = $\frac{\pi}{4}\left(D_o^2 - D_b^2\right)$

(iv) Discharge, Q is given as

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) V_f$$

$$= \frac{\pi}{4} D_o^2 (1 - n^2) \times K_f \sqrt{2gH} \qquad \text{...(3.19.1)}$$

where, $n = \frac{D_b}{D_o}$ and its value ranges from 0.35 to 0.6.

(v) Flow ratio, $K_f = \frac{V_{fl}}{\sqrt{2gH}}$ ...(3.19.2)

K_f varies between 0.15 to 0.3.

(vi) Speed ratio, $K_u = \frac{u_1}{\sqrt{2gH}}$ ...(3.19.3)

K_u varies between 0.6 to 0.9.

3.20 Runaway Speed

University Questions

Q. Explain the term Runaway Speed **SPPU : Dec. 15**

Q. Define following terms and explain their importance in selection and design of hydraulic turbines : Run away speed. **SPPU : Dec. 19**

It has been discussed when the turbine is subjected to no load and governor is not operating, the turbine speed will keep on increasing. Therefore, Runaway speed is defined as the maximum speed of turbine at which it would run when the external load on turbine is suddenly reduced to zero and the governor is disangaged while operating under design head and discharge.

The runaway speed is 1.8 to 1.9 times the speed of turbine for pelton turbine, it is 2 to 2.2 times the speed for Francis turbine and 2.5 to 3 times the speed for Kaplan turbine.

Ex. 3.20.1 : Kaplan turbine has a hydraulic efficiency of 90% and a mechanical efficiency of 93% with a runner diameter of 6 m and a boss diameter of 1.8 m. If the discharge of the turbine is 180 m³/s calculate the head on the turbine and the shaft power of the turbine. Assume that there is no whirl at outlet and the discharge is free. Neglect losses in the turbine. **SPPU - May 11, 8 Marks**

Soln. : Hydraulic efficiency, $\eta_h = 90\,\% = 0.9$

$$\eta_m = 93\% = 0.93\,; \qquad D_o = 6\text{m}\,;$$

$$D_b = 1.8\text{ m}\,; \qquad Q = 180\text{ m}^3/\text{s};$$

$$V_{w2} = 0\,; \qquad Q = \frac{\pi}{4}(D_0^2 - D_b^2)\,V_{f1}$$

$$180 = \frac{\pi}{4}(6^2 - 1.8^2)\,V_{f1}\,; \qquad V_{f1} = 7\text{ m/s} = V_{f2}$$

$$V_2 = V_{f2} = 7\text{ m/s (since there is no whirl at outlet)}$$

1. Head on turbine, H :

Hydraulic efficiency,

$$\eta_h = \frac{H - \frac{V_2^2}{2g}}{H}$$

$$0.9 = \frac{H - \frac{(7)^2}{2\times 9.81}}{H}$$

$$\therefore\ H = \mathbf{24.97\ m} \qquad \text{...Ans.}$$

2. Power developed, P_s :

$$P_s = \rho\cdot g\,QH \times \eta_h \times \eta_m$$

$$= \frac{1000\times 9.81\times 180\times 24.97\times 0.9\times 0.93}{1000}\text{ kW}$$

$$= \mathbf{36905.0kW} \qquad \text{...Ans.}$$

Ex. 3.20.2 : A Kaplan turbine has a runner diameter of 4 m and hub diameter of 1.2 m. Discharge through the turbine = 7000 LPS. The hydraulic and mechanical efficiencies are 90% and 93% respectively. Assume no whirl at outlet. Find the net head and power developed by the turbine. **SPPU - May 15, 6 Marks**

Soln. :

Runner diameter, $D_o = 4\text{ m}\,;$

Hub diameter, $D_h = 1.2\text{ m}$

$$Q = 7000\text{ LPS} = \frac{7000}{1000} = 7\text{ m}^3/\text{s}$$

$$\eta_h = 0.9,\ \eta_m = 0.93$$

1. Net head, H

$$Q = \frac{\pi}{4}\left(D_o^2 - D_h^2\right)V_{f_1}\,;$$

$$7 = \frac{\pi}{4}\left(4^2 - 1.2^2\right)V_{f_1}$$

$$V_{f_1} = 0.612\text{ m/s} = V_{f_2}$$

Since no whirl at outlet :

$$V_{f2} = V_2 = 0.612\text{ m/s}$$

Hydraulic efficiency,

$$\eta_h = \frac{H - \frac{(V_2)^2}{2g}}{H}$$

$$0.9 = \frac{H - \left[\frac{(0.612)^2}{2\times 9.81}\right]}{H}\,;$$

$$H = \mathbf{0.191\ m} \qquad \text{...Ans.}$$

2. Power developed, P_s

$$P_s = \rho\cdot g\cdot Q\,H \times \eta_h \times \eta_m \times 10^{-3}\text{ (kW)}$$

$$= 1000\times 9.81\times 7\times 0.191\times 0.9\times 0.93\times 10^{-3}$$

$$= \mathbf{10.978\ kW} \qquad \text{...Ans.}$$

Ex. 3.20.3 : A Kaplan turbine develops 12 MW power under the net head of 24 m, the water leaves the vane at an angle of 30° and enters the runner of the turbine. The outer diameter of the runner is 3 m and hub diameter is 1.3 m the hydraulic and overall efficiency of the turbine are 90% and 85% respectively. The discharge is axial.

Determine :

(i) Discharge through the turbine

(ii) Velocity of flow

(iii) Speed of the turbine

(iv) Runner vane angle at inlet

SPPU - Feb.16 (In Sem), 6 Marks

Soln. :

Refer Fig. P. 3.20.3

$$P_s = 12\text{ MW} = 12000\text{ kW},\ H = 24\text{ m},$$

$$\alpha = 30°,\ \text{outer diameter, } D_0 = 3\text{m},$$

Hub diameter, $D_h = 1.3\text{ m}$

$$\eta_h = 0.9, \qquad \eta_0 = 8.5$$

Discharge is axial i.e. $\beta = 90°$ and $V_{f2} = V_2$, $V_{w2} = 0$

(i) Discharge through the turbine, Q

$$P_s = \rho.g.\,Q\,H \times \eta_o \times 10^{-3}\text{ kW}$$

$$12000 = 1000\times 9.81\times Q\times 24\times 0.85\times 10^{-3}$$

$$Q = \mathbf{59.963\ m^3/s} \qquad \text{...Ans.}$$

(ii) Velocity of flow, $V_{f1} = V_{f2}$

$$Q = \frac{\pi}{4}\,(D_0^2 - D_h)^2\,V_{f1}$$

$$59.963 = \frac{\pi}{4}\,[(3)^2 - (1.3)^2]\,V_{f1}$$

$$V_{f1} = 10.45\text{ m/s} = V_{f2}$$

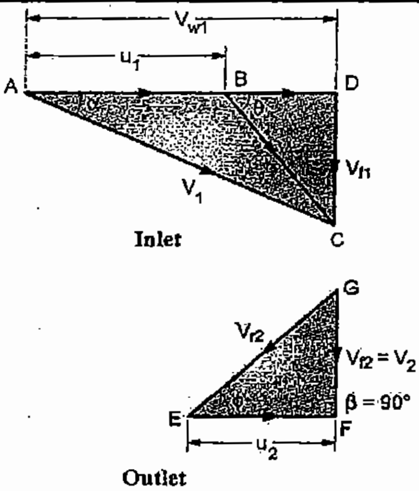


Fig. P. 3.20.3

(iii) Speed of turbine, N

$$\eta_h = \frac{V_{w1} \cdot u_1}{gH} \quad ;$$

$$0.9 = \frac{V_{w1} \times u_1}{9.81 \times 24}$$

$$V_{w1} \cdot u_1 = 211.9 \qquad \text{...(i)}$$

From inlet Δ ACD :

$$\frac{V_{f1}}{V_{w1}} = \tan\alpha \; ;$$

$$\frac{10.45}{V_{w1}} = \tan 30$$

$$\therefore \quad V_{w1} = 18.1 \text{ m/s}$$

$$\because \quad V_{w1} \cdot u_1 = 211.9 \; ;$$

$$18.1 \times u_1 = 211.9$$

$$\therefore \quad u_1 = 11.71 \text{ m/s}$$

But,

$$u_1 = \frac{\pi D_o N}{60} \; ;$$

$$11.71 = \frac{\pi \times 3 \times N}{60}$$

$$N = \mathbf{74.55 \text{ rpm.}} \qquad \text{...Ans.}$$

(iv) Runner vane angle at inlet, θ

$$BD = V_{w1} - u_1$$

$$= 18.1 - 11.71 = 6.39 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{BD}\right)$$

$$= \tan^{-1}\left(\frac{10.45}{6.39}\right)$$

$$\therefore \quad \theta = 58.56° \qquad \text{...Ans.}$$

Ex. 3.20.4 : A Kaplan turbine runner has outer diameter of 4.5 m and the diameter of the hub is 2 m. It is required to develop 20.6 MW when running at 150 rpm under a head of 21 m. Assuming hydraulic efficiency of 94% and overall efficiency of 88%, determine the runner vane angle at inlet and exit at the mean diameter of the vane.

SPPU - April 15 (In Sem), 4 Marks

Soln. : Given :

Outer diameter, $D_o = 4.5$ m

Hub diameter, $D_b = 2$ m

$P_s = 20.6 \text{ MW} = 20600 \text{ kW}$;

$N = 150$ rpm $\qquad H = 21$ m ;

$\eta_h = 0.94$; $\qquad \eta_o = 0.88$

Discharge Q :

$$P_s = \rho \cdot g\, Q\, H \times \eta_o \; ;$$

$$Q = \frac{P_s}{\rho \cdot g\, H\, \eta_o \times 10^{-3}}$$

$$Q = \frac{20600}{1000 \times 9.81 \times 21 \times 0.88 \times 10^{-3}}$$

$$= 113.63 \text{ m}^3/\text{s}$$

Velocity of flow, V_{f1}

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) V_{f1}$$

$$113.63 = \frac{\pi}{4}\,(4.5^2 - 2^2)\, V_{f1}$$

$$V_{f1} = 8.9 \text{ m/s} = V_{f2}$$

Blade Velocity,

$$u_1 = \frac{\pi D_o N}{60} = \frac{\pi \times 4.5 \times 150}{60}$$

$$= 35.34 \text{ m/s} = u_2 = u$$

Assuming discharge is axial i.e. $V_{w2} = 0$. The velocity diagram is shown in Fig. P. 3.20.4.

$$\eta_h = \frac{V_{w1} \cdot u_1}{gH} \quad ;$$

$$0.94 = \frac{V_{w1} \cdot 35.34}{9.81 \times 21}$$

$$V_{w1} = 5.48 \text{ m/s}$$

1. Runner vane angle at inlet, θ

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{u_1 - V_{w1}}\right)$$

$$= \tan^{-1}\left(\frac{8.9}{35.34 - 5.48}\right)$$

$$= 16.6° \qquad \text{...Ans.}$$

2. **Runner vane angle at outlet, ϕ**

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right)$$

$$= \tan^{-1}\left(\frac{8.9}{35.34}\right) = \mathbf{14.14^\circ} \quad \text{...Ans.}$$

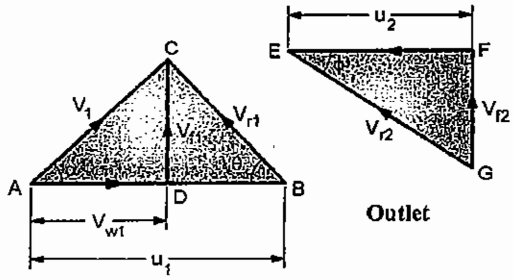


Fig. P. 3.20.4

3. **Mean diameter of the vane, D**

$$u = \frac{\pi D N}{60}$$

$$D = \frac{60 \cdot u}{\pi N} = \frac{60 \times 35.34}{\pi \times 150}$$

$$= \mathbf{4.5\ m} \quad \text{...Ans.}$$

Ex. 3.20.5 : A Kaplan turbine develops 24647.6 kW power at an average head of 39 m. Assuming the speed ratio of 2, flow ratio 0.6, the diameter of boss equal to 0.35 times the diameter of the runner and an overall efficiency 90%. Calculate the diameter, speed and specific speed of the runner. **SPPU : May 18, 6 Marks**

Soln. :

P_s = 24647.6 kW, H = 39 m,

Speed ratio, $K_u = 2 = \frac{u}{\sqrt{2gh}}$

Flow ratio, $K_f = 0.6 = \frac{V_{f1}}{\sqrt{2gH}}$

Diameter of boss, D_b = 0.35 × Diameter of runner, D_o

$$\eta_o = 0.9$$

$$K_u = \frac{u}{\sqrt{2gH}}\ ;$$

$$2 = \frac{u}{\sqrt{2 \times 9.81 \times 39}}\ ;$$

$$u = 55.324\ m/s$$

$$K_f = \frac{V_{f1}}{\sqrt{2gH}}\ ;$$

$$0.6 = \frac{V_{f1}}{\sqrt{2 \times 9.81 \times 39}}\ ;$$

$$V_{f1} = 16.597\ m/s$$

(i) Diameter of runner, D_o

$$P_s = \rho . g. Q. H \times \eta_0 \times 10^{-3}\ (kW)$$

$$24647.6 = 1000 \times 9.81 \times Q \times 39 \times 0.9 \times 10^{-3}$$

$$Q = 71.581\ m^3/s$$

But, $$Q = \frac{\pi}{4}(D_0^2 - D_b^2) \times V_{f_1}$$

$$71.581 = \frac{\pi}{4}\left[D_o^2 - (0.35 D_o)^2\right] \times 16.597$$

$$D_o = 2.5m$$

and $$D_b = 0.35 \times D_o = \mathbf{0.876m} \quad \text{...Ans.}$$

(ii) Speed, N

$$u = \frac{\pi D_o N}{60}\ ;$$

$$55.324 = \frac{\pi \times 2.5 \times N}{60}\ ;$$

$$N = \mathbf{422.64rpm} \quad \text{...Ans.}$$

(iii) Specific speed, N_s

$$N_s = \frac{N \times \sqrt{P}}{H^{5/4}} = \frac{422.64 \times \sqrt{24647.6}}{(39)^{5/4}}$$

$$= \mathbf{680.82} \quad \text{...Ans.}$$

Ex. 3.20.6 : A propeller turbine runner has an external diameter of 5 m and the diameter at hub is 2 m. The turbine has to develop a shaft power of 29430 kW under a head of 25 m at a speed of 160 rpm. If the hydraulic efficiency is 95% and the overall efficiency is 85%, determine the runner vane angles at inlet and outlet at mean diameter and at extreme edge of the runner. Assume that the turbine discharges without whirl at outlet. **SPPU - Dec. 18, 6 Marks**

Soln. :

Velocity diagram is shown Fig P. 3.20.6

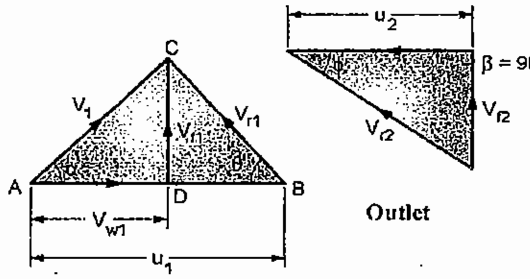


Fig. P. 3.20.6

H = 25m; P_h = 150MW= 150 × 10^3 kW

Outer diameter tip circle, D_o = 5m

Hub diameter, $D_h = 2\text{m}$,

Blockage factor at inlet, $k_{b1} = 0.9$

Blockage factor at outlet, $k_{b2} = 0.93$

Discharge is axial i.e. $\beta = 90°$

$$V_{w1} \times u_1 = \text{Const, C} \;;$$

$$N = 150 \text{ rpm},$$

$$\rho = 1000 \text{ kg/m}^3 \;;$$

$$P_h = \rho . g . Q H$$

$$150 \times 10^3 = 1000 \times 9.81 \times Q \times 25 \times 10^{-3} \text{ (kW)}$$

$$Q = 611.62 \text{ m}^3/\text{s}$$

$$u_1 = \frac{\pi D_0 N}{60} = \frac{\pi \times 5 \times 150}{60} = 39.27 \text{ m/s}$$

$$u_2 = \frac{\pi (D_h) N}{60} = \frac{\pi \times 2 \times 150}{60} = 15.7 \text{ m/s}$$

$$Q = (A_1 . k_{b1} - A_2 . k_{b2}) V_{f1}$$

$$= \left(\frac{\pi}{4} D_0^2 \, k_{b1} - \frac{\pi}{4} . D_h^2 . k_{b2}\right) V_{f1}$$

$$611.62 = \left(\frac{\pi}{4} (5)^2 \times 0.9 - \frac{\pi}{4} \times (2)^2 \times 0.93\right) V_{f1}$$

$$V_{f1} = 41.47 \text{ m/s}$$

(i) Inlet blade angle at blade tip and blade root

$$V_{w1} \cdot u_1 = g \cdot H = 9.81 \times 25 = 245.25 \quad \text{...(i)}$$

Inlet angles at tip

$$\therefore \; V_{w1} \cdot u_1 = \text{Const.}$$

$$\therefore \; V_{w1} = \frac{245.25}{39.27} = 6.25 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{u_1 - V_{w1}}\right)$$

$$= \tan^{-1}\left(\frac{41.47}{39.27 - 6.25}\right)$$

$$= 51.47° \quad \text{...Ans.}$$

Inlet angle at blade root, θ′

$$V'_{w1} = \frac{245.25}{u_2} = \frac{245.25}{15.7} = 15.62 \text{ m/s}$$

$$\theta' = \tan^{-1}\left(\frac{V_{f1}}{u_2 - V_{w2}}\right)$$

$$= \tan^{-1}\left(\frac{41.47}{15.7 - 15.62}\right) = 90° \text{...Ans.}$$

(ii) Outlet blade angle at the blade tip and blade root, ϕ_2

$$V_{f2} = V_{f1} = 41.47 \text{ m/s}$$

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right)$$

$$= \tan^{-1}\left(\frac{41.47}{15.7}\right) = 69.36° \quad \text{...Ans.}$$

Ex. 3.20.7 : A propeller turbine has an outer diameter of 4.5 meter and inner diameter of 2.5 meters. It is developing 20 MW when working under a head of 20 meters. The turbine is directly coupled to an alternator having 22 pairs of poles. The hydraulic efficiency is 94 % and the overall efficiency is 88 %. Find the discharge through turbine. Find the runner vane angles at inlet and outlet at the hub and at the edge of the blade. **SPPU - May 12, 10 Marks**

Soln. : Refer Fig. P. 3.20.7

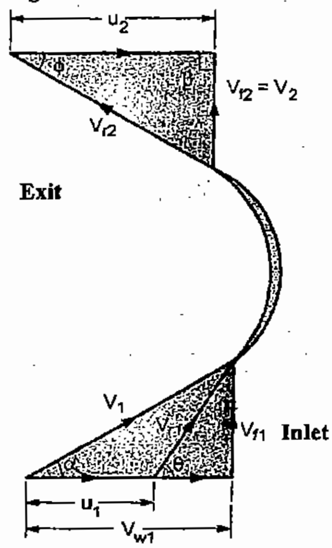


Fig. P. 3.20.7

$$D_0 = 4.5 \text{ m}, \qquad D_b = 2.5 \text{ m},$$

$$P_s = 20 \text{ MW} = 20000 \text{ kW};$$

$$H = 20 \text{ m}$$

Number of poles,

$$p = 22 \text{ pairs},$$

$$\eta_h = 0.94,$$

$$\eta_o = 0.88$$

Speed, $$N = \frac{60 f}{p} = \frac{60 \times 50}{22} = 136.4 \text{ rpm}$$

Assume that at exit discharge is radial i.e. $\beta = 90°$ and $V_{w2} = 0$

$$\eta_h = \frac{V_{w1} . u_1}{g H};$$

$$0.94 = \frac{V_{w1} . u_1}{9.81 \times 20}$$

$$\therefore \; V_{w1} . u_1 = 184.43 \quad \text{...(i)}$$

Area of flow, $A_f = \frac{\pi}{4}\left(D_o^2 - D_b^2\right)$

$$= \frac{\pi}{4}(4.5^2 - 2.5^2) = 10.996 \text{ m}^2$$

(i) Discharge of turbine, Q :

Shaft power, $P_s = \rho . g . Q . H \times \eta_0$

$$20000 \times 10^3 = 1000 \times 9.81 \times Q \times 20 \times 0.88$$

$$Q = \mathbf{115.837 \text{ m}^3/\text{s}} \quad \text{...Ans.}$$

(ii) Runner blade angles at inlet and outlet at Hub :

$$u_1 = \frac{\pi D_b N}{60} = \frac{\pi \times 2.5 \times 136.4}{60}$$

$$= 17.855 \text{ m/s} = u_2$$

From Equation (i) :

$$V_{w1} = \frac{184.43}{17.855} = 10.32 \text{ m/s}$$

$$Q = A_f . V_{f1} ;$$

$$115.837 = 10.996 \times V_{f1}$$

$$V_{f1} = V_{f2} = 10.535 \text{ m/s}$$

Exit vane angle,

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right)$$

$$= \tan^{-1}\frac{10.535}{17.855} = \mathbf{30.54°} . \quad \text{...Ans.}$$

Inlet vane angle,

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{V_{w1} - u_1}\right)$$

$$= \tan^{-1}\left(\frac{10.535}{10.32 - 17.855}\right) = -53.9°$$

$$= \mathbf{180 - 53.9 = 126.1°} \quad \text{...Ans.}$$

(iii) Runner blade angles at inlet and outlet at the edge of the blade

$$u_1 = u_2 = \frac{\pi D_o N}{60} = \frac{\pi \times 4.5 \times 136.4}{60}$$

$$= 32.14 \text{ m/s}$$

$$V_{f1} = V_{f2} = 10.535 \text{ m/s [as in case (ii)]}$$

$$V_{w1} . u_1 = 184.43$$

$$V_{w1} = \frac{184.43}{32.14} = 5.738 \text{ m/s}$$

Inlet vane angle, $\theta = \tan^{-1}\left(\frac{V_{f1}}{V_{w1} - u_1}\right)$

$$= \tan^{-1}\left(\frac{10.535}{5.738 - 32.14}\right) = -21.75°$$

$$= 180 - 21.75 = \mathbf{158.25°} \quad \text{...Ans.}$$

Exit vane angle, $\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right)$

$$= \tan^{-1}\left(\frac{10.535}{5.738}\right) = \mathbf{61.42°} \quad \text{...Ans.}$$

Ex. 3.20.8 : A Propeller reaction turbine of runner diameter 4.5 m is running at 40 r.p.m. The guide blade angle at inlet is 145° and runner blade angle at outlet is 25° to the direction of vane. The axial flow area of water through runner is 25 m². If the runner blade angle at inlet is radial, determine :

(i) Hydraulic efficiency of the turbine

(ii) Discharge through turbine

(iii) Power developed by the turbine

(iv) Specific speed of the turbine **SPPU - May 13,10 Marks**

Soln. : Refer Fig. P. 3.20.8

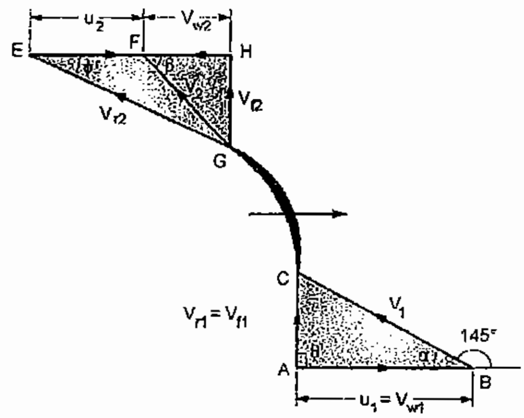


Fig. P. 3.20.8

Runner diameter, $D = D_1 = D_2 = 4.5$ m

$$N = 40 \text{ rpm}$$

$$\alpha = 180 - 145 = 35°$$

$$\phi = 25°$$

Axial Area of flow, $A_f = 25 \text{ m}^2$

$$\theta = 90°$$

($\because$ runner blade angle at inlet is radial)

$$u_1 = u_2 = \frac{\pi D N}{60}$$

$$= \frac{\pi \times 4.5 \times 40}{60} = 9.425 \text{ m/s}$$

From inlet velocity, Δ ABC we have,

$$\frac{u_1}{V_1} = \cos \alpha$$

$$V_1 = \frac{u_1}{\cos \alpha} = \frac{9.425}{\cos 35} = 11.506 \text{ m/s}$$

$$V_{f_1} = u_1 \cdot \tan \alpha = 9.425 \tan 35 = 6.599 \text{ m/s}$$

Since $Q = A_f V_f$ and area of flow is constant, it implies that :

$$V_{f_1} = V_{f_2} = 6.599 \text{ m/s}$$

Consider outlet velocity Δ EFG,

$$EH = \frac{V_{f_2}}{\tan\phi} = \frac{6.599}{\tan 25} = 14.152 \text{ m/s}$$

$$V_{w_2} = EH - u_2 = 14.152 - 9.425 = 4.727 \text{ m/s}$$

$$V_2 = \sqrt{V_{w2}^2 + V_{f2}^2} = \sqrt{(4.727)^2 + (6.599)^2}$$

$$= 8.117 \text{ m/s}$$

(i) Hydraulic efficiency of turbine, η_h

Assuming no loss of energy while the water flows over the vanes then,

$$H - \frac{V_2^2}{2g} = \frac{1}{g}[V_{w_1} \cdot u_1 + V_{w_2} \cdot u_2]$$

$$H - \frac{(8.117)^2}{2 \times 9.81} = \frac{1}{9.81}[9.425 \times 9.425 + 4.727 \times 9.425]$$

$$H - 3.358 = 13.597$$

$$H = 16.955 \text{ m}$$

$$\eta_h = \frac{V_{w1} \cdot u_1 + V_{w2} \cdot u_2}{gH}$$

$$= \frac{9.425 \times 9.425 + 4.727 \times 9.425}{9.81 \times 16.955}$$

$$= \mathbf{0.8019 \text{ or } 80.19\ \%} \qquad \text{...Ans.}$$

(ii) Discharge through turbine, Q

$$Q = A_f V_f = 25 \times 6.599$$

$$= \mathbf{164.975 \text{ m}^3/\text{s}} \qquad \text{...Ans.}$$

(iii) Power developed by the turbine, P or $\dot{W}$

$$P = \frac{(V_{w1} \cdot u_1 + V_{w2} \cdot u_2)\ \rho \times Q}{1000}$$

$$= \frac{(9.425 \times 9.425 + 4.727 \times 9.425)}{1000} \times 1000 \times 164.975$$

$$= \mathbf{2243.17 \text{ kW}} \qquad \text{...Ans.}$$

(iv) Specific speed of turbine,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{40 \times \sqrt{2243.17}}{(16.955)^{5/4}} = \mathbf{55.064} \qquad \text{...Ans.}$$

Ex. 3.20.9 : A Kaplan turbine operates at a discharge of 77 m³/s. The runner diameter and hub diameter are 4.2 m and 1.5 m respectively. Taking the speed ratio of 2.1. Determine :

(i) the net head

(ii) the power developed and

(iii) the specific speed.

Assume the mechanical and hydraulic efficiency of 88% and 92% respectively and no whirl at outlet.

SPPU- Dec. 13, 12 Marks, May 19, Oct. 19 (In sem), 6 Marks

Soln. :

Given : $D_o = 4.2$ m, $D_b = 1.5$ m;

$Q = 77 \text{ m}^3/\text{s}$; $\eta_h = 92\% = 0.92$;

Mechanical efficiency, $\eta_m = 88\% = 0.88$;

$V_{w2} = 0$;

Speed ratio, $K_u = 2.1$;

$u = u_1 = u_2$.

(i) Velocity of flow, $V_{f1} = V_{f2}$

$$Q = \frac{\pi}{4}(D_o^2 - D_b^2)\, V_{f1}$$

$$77 = \frac{\pi}{4}(4.2^2 - 1.5^2) V_{f1}$$

$$\therefore V_{f1} = 6.05 \text{ m/s}$$

$$\therefore V_{f1} = V_{f2} = V_2$$

$$= 6.05 \text{ m/s (Discharge is axial)}$$

(ii) Net head, H

Energy extracted by turbine $= H - \dfrac{V_2^2}{2g}$

Hydraulic efficiency,

$$\eta_h = \frac{\text{Runner power}}{\text{Input power}}$$

$$\therefore \eta_h = \frac{\rho Q g \left(H - \frac{V_2^2}{2g}\right)}{\rho Q g H} = \frac{H - \frac{V_2^2}{2g}}{H}$$

$$0.88 = \frac{H - [(6.05)^2 / 2 \times 9.81]}{H}$$

$$\therefore \mathbf{H = 15.5 \text{ m}} \qquad \text{...Ans.}$$

(iii) Power developed, P

Speed ratio, $K_u = \dfrac{u}{\sqrt{2gH}}$ i.e. $u = K_u\sqrt{2gH}$

$$\therefore u = 2 \times \sqrt{2 \times 9.81 \times 15.5} = 34.87 \text{ m/s}$$

$$P = \rho g Q H \times \eta_n \times \eta_m$$

$$= 10^3 \times 9.81 \times 77 \times 15.5 \times 0.92 \times 0.88$$

$$= 9478.94 \times 10^3 \text{ W} = \mathbf{9478.98 \text{ kW}} \qquad \text{...Ans.}$$

(iv) Specific speed, N_s

$$\text{Speed, } N = \frac{60\,u}{\pi D_o} = \frac{60 \times 34.71}{\pi \times 4} = 165.72 \text{ rpm}$$

$$\therefore \text{ Specific speed, } N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$= \frac{165.72\sqrt{9478.98}}{(15.5)^{5/4}}$$

$$= \mathbf{524.62} \qquad \text{...Ans.}$$

Ex. 3.20.10 : A vertical-shaft Kaplan turbine operates under a net head of 25 m of water and develops hydraulic power of 150 MW. Outer diameter of the tip circle of Kaplan blades is 5 m and the boss (or the hub) diameter is 2 m. Blockage factors, accounting for the thickness of aerofoil-shaped Kaplan blades, are 0.9 and 0.93 at inlet and outlet respectively. The discharge from the turbine is purely axial and hence whirl-free. On inlet side of the runner, the product of the whirl velocity and the blade velocity is constant with respect to the radius (i.e. free vortex type whirl distribution at inlet to the runner). Rotational speed of the turbine is 150 rpm and density of water is 1000 kg/m. Draw inlet and outlet velocity triangles at the tip and root (i.e. at the hub) of a Kaplan blade and determine

(i) Inlet blade angles at the blade tip and the blade root and

(ii) Outlet blade angles at the blade tip and the blade root.

SPPU - Dec. 11, 14 Marks

Soln. :

Velocity diagram is shown Fig P. 3.20.10

$$H = 25\text{m};$$

$$P_h = 150\text{MW} = 150 \times 10^3 \text{ kW}$$

Outer diameter tip circle, $D_o = 5\text{m}$

Hub diameter, $D_h = 2\text{m},$

Blockage factor at inlet, $k_{b1} = 0.9$

Blockage factor at outlet, $k_{b2} = 0.93$

Discharge is axial i.e. $\beta = 90°$

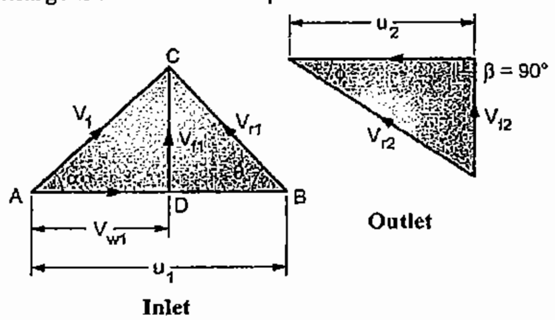


Fig. P. 3.20.10

$$V_{w1} \times u_1 = \text{Const, } C \;;$$

$$N = 150 \text{ rpm},$$

$$\rho = 1000 \text{ kg/m}^3 \;;$$

$$P_h = \rho \cdot g \cdot Q\,H$$

$$150 \times 10^3 = 1000 \times 9.81 \times Q \times 25 \times 10^{-3} \text{ (kW)}$$

$$Q = 611.62 \text{ m}^3\text{/s}$$

$$u_1 = \frac{\pi D_o N}{60} = \frac{\pi \times 5 \times 150}{60} = 39.27 \text{ m/s}$$

$$u_2 = \frac{\pi (D_h) N}{60} = \frac{\pi \times 2 \times 150}{60} = 15.7 \text{ m/s}$$

$$Q = (A_1 . k_{b1} - A_2 . k_{b2})\, V_{f1}$$

$$= \left(\frac{\pi}{4} D_o^2 \, k_{b1} - \frac{\pi}{4} . \, D_h^2 \, . \, k_{b2}\right) V_{f1}$$

$$611.62 = \left(\frac{\pi}{4} (5)^2 \times 0.9 - \frac{\pi}{4} \times (2)^2 \times 0.93\right) V_{f1}$$

$$V_{f1} = 41.47 \text{ m/s}$$

(i) Inlet blade angle at blade tip and blade root

$$V_{w1} \cdot u_1 = g \cdot H$$

$$= 9.81 \times 25 = 245.25 \qquad \text{...(i)}$$

Inlet angles at tip

$$\therefore V_{w1} \cdot u_1 = \text{Const.}$$

$$\therefore V_{w1} = \frac{245.25}{39.27} = 6.25 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{u_1 - V_{w1}}\right)$$

$$= \tan^{-1}\left(\frac{41.47}{39.27 - 6.25}\right)$$

$$= \mathbf{51.47°} \qquad \text{...Ans.}$$

Inlet angle at blade root, θ′

$$V'_{w1} = \frac{245.25}{u_2} = \frac{245.25}{15.7} = 15.62 \text{ m/s}$$

$$\theta' = \tan^{-1}\left(\frac{V_{f1}}{u_2 - V_{w2}}\right)$$

$$= \tan^{-1}\left(\frac{41.47}{15.7 - 15.62}\right)$$

$$= \mathbf{90°} \qquad \text{...Ans.}$$

(ii) Outlet blade angle at the blade tip and blade root, ϕ_2

$$V_{f2} = V_{f1} = 41.47 \text{ m/s}$$

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right) = \tan^{-1}\left(\frac{41.47}{15.7}\right)$$

$$= \mathbf{69.36°} \qquad \text{...Ans.}$$

3.21 Draft Tube

University Questions

Q. Why is the draft tube necessary in the case of hydraulic reaction turbines ? **SPPU : Dec. 11**

Q. State the advantages of using draft tube and justify not using it in Pelton wheel. **SPPU : May 16**

Q. What is function of draft tube ? **SPPU : April 15(In Sem)**

Q. What is draft tube ? What are its advantages ? **SPPU : April 17(In Sem)**

Q. A turbine was originally installed at the tail water level. If it is now proposed to place it above the tail water level without any decrease in head on the turbine, how this can be achieved ? **SPPU : Dec. 18**

Q. Explain the necessity of draft tube in reaction turbines. **SPPU : May 19**

A **draft tube** is a pipe of gradually increasing diameter which connects the exit of runner of a turbine to the tail race. Therefore, it discharges the water from the runner to tail race.

In case of Pelton wheel the available head is converted into kinetic energy before entry to runner buckets and the turbine operate under atmospheric pressure conditions. The velocity of water leaving at turbine exit is small, therefore, the exit of runner is kept above the tail race level.

Whereas in case of reaction turbines, only the part of available head is converted into kinetic energy before entry to runner and rest of energy is in the form of pressure energy. This pressure energy is gradually converted into runner, thus the velocity leaving the runner is at high velocity. In case runner is kept above the tail race, the K.E. corresponding to exit velocity utilise the kinetic energy of water at exit of runner, is wasted the draft tube is made an integral part of Francis, Kaplan and other reaction turbines in which the draft tube exit is kept below the tail race.

The draft tube converts the kinetic energy into pressure head.

Therefore, the function of a draft tube are :

1. It permits the negative head to be established at outlet of runner, hence it increases the net head available to turbine.
2. To convert the kinetic energy of water at exit of runner into pressure energy so that useful head at runner exit is increased.

Utilisation of K.E. at exit of runner increases the power output and efficiency of the turbine. Apart from this, the location of turbine level close to tail race level could cause the flooding of turbine during rainy season.

3.21.1 Draft Tube Theory

Consider a draft tube fitted to a reaction turbine as shown in Fig. 3.21.1.

Section (2 – 2) represents the exit of runner or inlet to draft tube and section (3 – 3) represents the outlet of draft tube.

Let, V_2 = Velocity of water at draft tube or at exit of runner

p_a = Atmospheric pressure at tail race

y = Depth of draft tube below the tail race level

w = Specific weight of water in $N/m^3 = \rho \cdot g$

h_f = Hydraulic energy loss in draft tube due to friction.

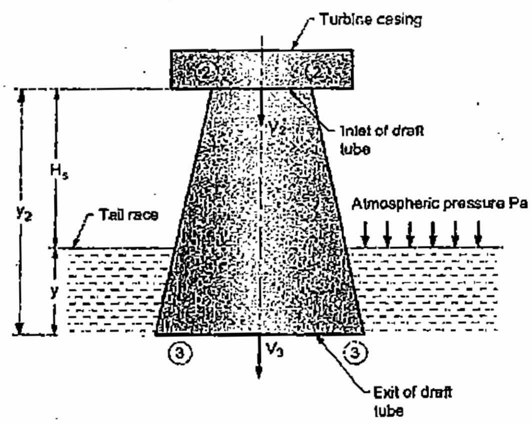


Fig. 3.21.1 : Draft tube theory

Applying Bernoulli's equation between section (2 - 2) and (3 - 3), we have :

$$\frac{p_2}{w}+\frac{V_2^2}{2g}+y_2 = \frac{p_3}{w}+\frac{V_3^2}{2g}+0+h_f$$

$$\frac{p_2}{w} = \frac{p_3}{w}-y_2+\frac{V_3^2-V_2^2}{2g}+h_r \qquad ...(i)$$

$$\text{But,} \quad \frac{p_3}{w} = \frac{p_a}{w}+y \qquad ...(ii)$$

Substituting the value of $\frac{p_3}{w}$ from Equation (ii) in Equation (i),

$$\frac{p_2}{w} = \left(\frac{p_a}{w}+y\right) - y_2 + \frac{V_3^2 - V_2^2}{2g} + h_f$$

The difference $(y_2 - y)$ is called **suction head, H_s**. Therefore, above expression reduces to :

$$\frac{p_2}{w} = \frac{p_a}{w} - H_s - \left(\frac{V_2^2 - V_3^2}{2g} - h_f\right) \qquad \text{...(3.21.1)}$$

$$\frac{p_2}{w} = \frac{p_a}{w} - \left[H_s + \left(\frac{V_2^2 - V_3^2}{2g} - h_f\right)\right] \qquad \text{...(3.21.1(A))}$$

From Equation (3.21.1) it is evident that the pressure at exit of runner drops below the atmospheric pressure due to use of diverging draft tube.

Efficiency of draft tube, η_d

Draft tube efficiency is defined as the ratio of net gain in pressure head to the velocity head at exit of runner or at entry to draft tube. Accordingly,

$$\eta_d = \frac{\text{Net gain in pressure head}}{\text{Velocity head at entry to draft tube}}$$

$$= \frac{\left(\frac{V_2^2 - V_3^2}{2g} - h_f\right)}{\frac{V_2^2}{2g}} \qquad \text{...(3.21.2)}$$

Sometimes the friction loss is expressed as :

$$h_f = k\left(\frac{V_2^2 - V_3^2}{2g}\right)$$

Then Equation (3.21.2) becomes,

$$\eta_d = \frac{(1-k)\left(\frac{V_2^2 - V_3^2}{2g}\right)}{\frac{V_2^2}{2g}} = \frac{(1-k)(V_2^2 - V_3^2)}{V_2^2} \qquad \text{...(3.21.3)}$$

3.21.2 To Show that Net Head Available on Turbine Remains Constant Irrespective of Location of Turbine w.r.t Tail Race Level

Consider a turbine working between the head race level and tail race level of H meters. Three positions of turbine (a), (b) and (c) are shown in Fig. 3.21.2.

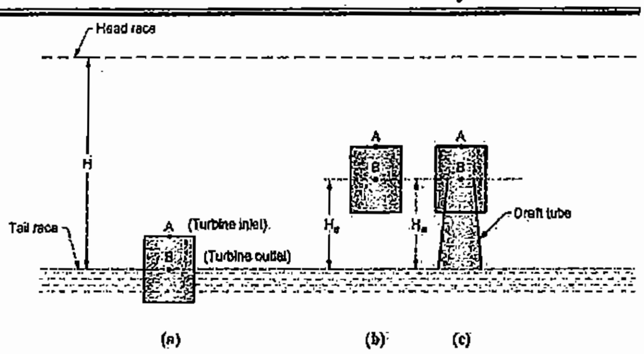


Fig. 3.21.2 : Three locations of turbine with respect to tail race

Point A represents the inlet of water to turbine and point B represents the outlet of water from turbine.

Case (a) : When turbine is installed at tail race level

$$H_A = H \ ; \quad H_B = 0$$

$$\therefore \text{ Net head available} = H_A - H_B = H - 0 = H \qquad \text{...(i)}$$

Case (b) : When turbine exit is at height H_s from tail race level without draft tube

$$H_A = H - H_s \ ; \quad H_B = 0$$

$$\therefore \text{ Net head available} = H_A - H_B = (H - H_s) - 0$$

$$= (H - H_s) \qquad \text{...(ii)}$$

Case (c) : When turbine exit is installed at height H_s from tail race with draft tube

$$H_A = H - H_s;$$

$$H_B = -H_s$$

$$\therefore \text{ Net head available} = H_A - H_B$$

$$= (H - H_s) - (-H_s) = H \qquad \text{...(iii)}$$

This available head is same as in case (a) when turbine exit is installed at the tail race.

It is evident from case (a) and case (c), that net head available remains constant irrespective of location of turbine with respect to tail race level using the draft tube.

3.21.3 Types of Draft Tubes

University Questions

Q. Explain the advantage of elbow type divergent draft tube over a straight divergent draft tube. **SPPU : Dec. 11**

Q. Explain different types of draft tubes used in reaction turbines. **SPPU : May 15**

Q. Mention various types of draft tube. **SPPU : April 15 (In Sem)**

Though there are various types of draft tubes, but the important type of draft tubes are :

1. Conical draft tube
2. Simple elbow draft tube
3. Elbow draft tube with circular inlet and rectangular outlet
4. Moody's spreading draft tube

1. Conical draft tube

Earlier designs of draft tubes were of conical straight divergent type as shown in Fig. 3.21.3(a).

Taper angle of divergent portion of tube is of great importance. If the taper angle, 2θ is large it will cause the separation of flow from the walls of draft tube. In case the taper angle is small, the length of draft tube needed will be long causing higher frictional losses. It may also lead to cavitation problem (discussed in section 3.22 later). Angle θ ranges from 4° to 7° and the proportionate dimensions of the draft tube are shown in given Fig. 3.21.3(a).

It is fabricated out of mild steel plates. It has an efficiency upto 90% and it is employed for vertical shaft turbines.

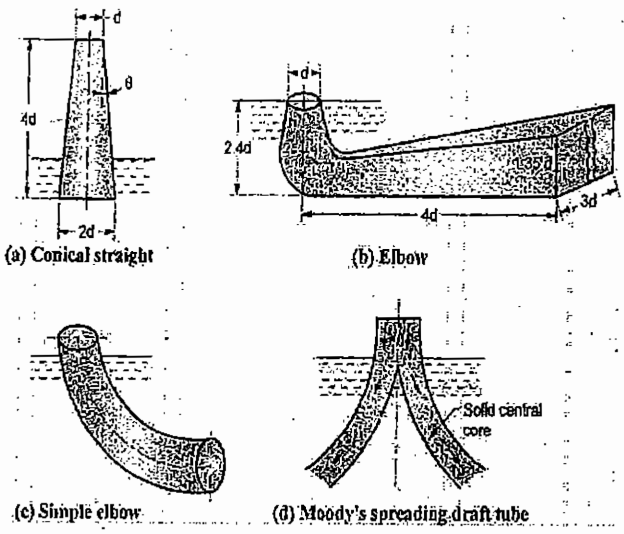


Fig. 3.21.3 : Various types of draft tubes

2. Simple elbow draft tube

It has circular cross-section throughout from inlet to outlet as shown in Fig. 3.21.3(c) which is turned through 90°. Elbow type draft tubes are used for Kaplan turbines. These tubes reduce the depth and cost of excavation. It is made of concrete with steel lining at inlet to resist cavitation. Due to bend, such draft tubes have an efficiency upto 60%.

3. Elbow draft tube

Elbow draft tube is shown in Fig. 3.21.3(b). It is circular in cross-section at inlet in its vertical leg which turns into rectangular cross-section in horizontal portion of tube upto outlet. The horizontal portion of tube is gradually inclined upwards so that the water leaves the tube almost at tail race level.

In this case also the cost and depth of excavation is reduced. The efficiency of these tubes are in the range of 60% to 80%. It is also made of concrete with steel lining near the runner.

4. Moody's spreading draft tube

It is similar to conical draft tube as shown in Fig. 3.21.3(d). It is provided with a solid central core which reduces the whirling action of water. It has an efficiency upto 85% and it is used for vertical shaft turbines having large whirl component at exit of their runner.

Ex. 3.21.1 : A reaction turbine with a straight divergent draft tube operates under a net head of 30 m of water and develops electrical power of 50 MW. Generator and mechanical efficiencies are 98% and 95% respectively. Inlet of the draft tube is located 3 m above the tail race level where the diameter of the draft tube is 5 m. Frictional head loss in the draft tube is 1 m of water and the efficiency of the draft tube is 64.85%. Density of water may be taken to be 1030 kg/m³. Determine : (i) Diameter of the draft tube at its exit (ii) Gauge pressure head at the draft tube inlet (or runner exit). **SPPU- Dec. 11, Aug. 18 (In Sem), 6 Marks**

Soln. :

Refer Fig. P. 3.21.1

Net head, $H = 30\text{m}$; $P_s = 50\text{ MW} = 50000\text{kW}$;

$\eta_{gen} = 0.98$, $\eta_m = 0.95$,

$H_s = 3\text{m}$,

$D_2 = 5\text{m}$, Friction loss in draft tube, $h_f = 1\text{m}$,

$\eta_d = 64.85\,\% = 0.6485$;

$\rho = 1030\text{ kg/m}^3$

(i) Diameter of draft tube at exit, d_3

Hydraulic power developed by turbine,

$$P_h = P_s \times \frac{1}{\eta_m} \times \frac{1}{\eta_{gen}} = 50000 \times \frac{1}{0.95} \times \frac{1}{0.98}$$

$$= 53705.7\text{ kW}$$

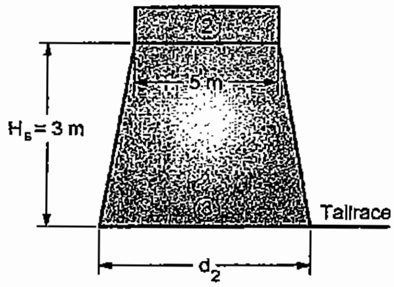


Fig. P. 3.21.1

$$A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(5)^2 = 19.635\ m^2$$

$$P_h = \rho g QH$$

$$53705.7 = 1030 \times 9.81 \times Q \times 30 \times 10^{-3}(kW)$$

$$Q = 177.17\ m^3/s$$

$$V_2 = \frac{Q}{A_2} = \frac{177.17}{19.635} = \mathbf{9.023\ m/s.} \quad \text{...Ans.}$$

Efficiency of draft tube,

$$\eta_d = \frac{\frac{V_2^2 - V_3^2}{2g} - h_f}{\frac{V_2^2}{2g}} = \frac{(V_2^2 - V_3^3) - h_f \times 2g}{V_2^2}$$

$$0.6485 = \frac{(9.023)^2 - V_3^2 - 1 \times 2 \times 9.81}{(9.023)^2} \quad \therefore V_3 = 3\ m/s$$

$$Q = A_3 V_3 = \frac{\pi}{4} D_3^2 \times V_3$$

$$177.17 = \frac{\pi}{4} \times D_3^2 \times 3$$

$$D_3 = \mathbf{8.671\ m} \quad \text{...Ans.}$$

(ii) Gauge pressure head at draft tube inlet, $\frac{p_2}{w}$

Applying Bernoulli's equation at inlet and exit of draft tube,

$$\frac{p_2}{w} + \frac{V_2^2}{2g} + H_s = \frac{V_3^2}{2g} + h_f$$

$$\frac{p_2}{w} + \frac{(9.023)^2}{2 \times 9.81} + 3 = \frac{(3)^2}{2 \times 9.81} + 1$$

$$\frac{p_2}{w} = \mathbf{-5.69 m} \quad \text{...Ans.}$$

Ex. 3.21.2 : A conical draft tube having inlet and outlet diameters 1.2 m and 1.8 m discharges water at outlet with a velocity of 3 m/s. the total length of the draft tube is 7.2 m and 1.44 m of the length of the draft tube is immersed in water. If the atmospheric pressure head is 10.3 m of water and loss of head due to friction in the draft tube is equal to 0.2 times the velocity head at outlet of the tube, determine : (i) pressure head at inlet (ii) Efficiency of the draft tube.

SPPU- Dec. 12, 8 Marks

Soln. : Refer Fig. P. 3.21.2.

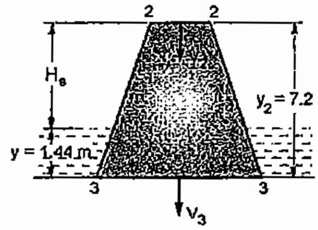


Fig. P. 3.21.2

$d_2 = 1.2$ m, $d_3 = 1.8$ m,

$C_3 = 3$ m/s, $y_2 = 7.2$ m

$H_s = y_2 - y = 7.2 - 1.44 = 5.76$ m,

$p_{atm} = p_a = 10.3$ m, $h_f = 0.2 \times \frac{C_3^2}{2g}$

(i) Pressure head at inlet, $\frac{p_2}{w}$

$$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4} \times 1.2^2 = 1.131\ m^2;$$

$$A_3 = \frac{\pi}{4}d_3^2 = \frac{\pi}{4} \times 1.8^2 = 2.545\ m^2$$

$$Q = A_2 V_2 = A_3 V_3$$

$$V_2 = \frac{A_3}{A_2} \times V_3 = \frac{2.545}{1.131} \times 3 = 6.75\ m/s$$

$$h_f = 0.2 \times \frac{C_3^2}{2g} = 0.2 \times \frac{(3)^2}{2 \times 9.81} = 0.0917\ m$$

Applying Bernoulli's equation between inlet (2) and outlet (3) of tube :

$$\frac{p_2}{w} + \frac{V_2^2}{2g} + H_s = \frac{p_a}{w} + \frac{V_3^2}{2g} + h_f$$

$$\frac{p_2}{w} + \frac{(6.75)^2}{2 \times 9.81} + 5.76 = 10.3 + \frac{(3)^2}{2 \times 9.81} + 0.0917$$

$$\frac{p_2}{w} = \mathbf{2.768\ m}\ \text{(absolute)} \quad \text{...Ans.}$$

(ii) Efficiency of draft tube, η_d

$$\eta_d = \frac{V_2^2 - V_3^2 - h_f}{V_2^2} = \frac{(6.75)^2 - (3)^2 - 0.0917}{(6.75)^2}$$

$$= \mathbf{0.8005\ or\ 80.05\%.} \quad \text{...Ans.}$$

Ex. 3.21.3 : A conical type draft tube, attached to Francis turbine has an inlet diameter of 3 m and its area at outlet is 20 m². The velocity of water in inlet, which is set 5 m above tail race level is 5 m/s. Assuming the loss in draft tube equal to 5% of velocity head at outlet, find :

(i) The pressure head at top

ii) Total head at top taking tail race level as datum.
iii) Power of water at runner outlet.
iv) Power of water at runner outlet.
v) Power lost in draft tube. **SPPU - May 13, 8 Marks**

Soln. : Refer Fig. P. 3.21.3

$$D_1 = 3m \qquad \text{i.e. } A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}\times 3^2 = 7.069\ m^2$$

$$A_2 = 20\ m^2; \qquad H_s = 5m,$$

$$V_1 = 5\ m/s; \quad h_f = 5\% \text{ of velocity head at exit.}$$

$$\therefore \quad h_f = 0.05\times\frac{V_2^2}{2g}$$

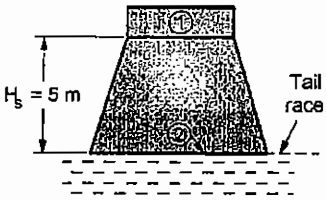


Fig. P. 3.21.3

Discharge rate, $Q = A_1 V_1 = 7.069\times 5 = 35.345\ m^3/s$

But, $Q = A_2 V_2;\ 35.345 = 20\times V_2;$

$$V_2 = 1.7673\ m/s$$

$$h_f = 0.05\times\frac{V_2^2}{2g} = 0.05\times\frac{(1.7673)^2}{2\times 9.81}$$

$$= 0.00796\ m$$

(i) Pressure head at top i.e. at inlet, $H = \frac{p_1}{w}$

Applying Bernoullis equation at inlet and at exit of draft to be :

$$\frac{p_1}{w}+\frac{V_1^2}{2g}+H_s = \frac{p_2}{w}+\frac{V_2^2}{2g}+h_f$$

$$\frac{p_1}{w}+\frac{(5)^2}{2\times 9.81}+5 = 10.3+\frac{(1.7673)^2}{2\times 9.81}+0.00796$$

$$\frac{p_1}{w} = \mathbf{4.193\ m} \text{ (absolute)} \qquad \text{...Ans.}$$

(ii) Total head of top taking tail race as datum,

$$= 10.3+\frac{p_1}{w}$$

$$= 10.3+4.193 = \mathbf{14.493\ m} \qquad \text{...Ans.}$$

(iii) Power of water at runner outlet or at inlet of draft to be,

$$P_1 = \dot{m}\times\frac{V_1^2}{2g} = \frac{\rho Q}{1000}\times\frac{V_1^2}{2g}$$

$$= \frac{1000\times 35.345}{1000}\times\frac{(5)^2}{2\times 9.81}$$

$$= \mathbf{45.037\ kW} \qquad \text{...Ans.}$$

(iv) Power of water at runner outlet,

$$P_2 = \dot{m}\times\frac{V_2^2}{2g} = \frac{\rho Q}{1000}\times\frac{V_2^2}{2}$$

$$= \frac{1000\times 35.345}{1000}\times\frac{(1.7673)^2}{2\times 9.81}$$

$$= \mathbf{5.627\ kW} \qquad \text{...Ans.}$$

(v) Power lost in draft tube,

$$= P_1 - P_2 = 45.037 - 5.627$$

$$= \mathbf{39.41\ kW} \qquad \text{...Ans.}$$

Ex. 3.21.4 : Determine the efficiency of a Kaplan turbine developing 3000 kW under a net head of 5 m. It is provided with a draft tube with its inlet (3 m diameter) set 1.6 m above the tail race level. A vacuum gage connected to the draft tube indicates a reading of 5 m of water. Draft tube efficiency is 78%. **SPPU - May 14, 8 Marks**

Soln. :

Given : $P = 3000$ kW, $H = 5$ m,

diameter of draft tube, d = 3 m,

$$H_s = 1.6\ m,$$

$$\frac{p_2}{w} = 5\ m \text{ (Vacuum)} = 10.3 - 5 = 5.3\ m \text{ (absolute)};$$

$$\eta_d = 78\% = 0.78$$

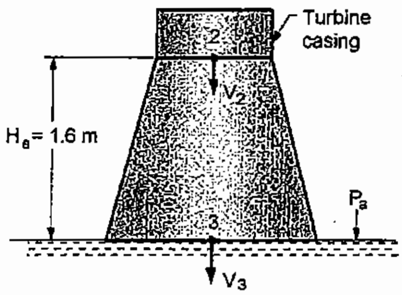


Fig. P. 3.21.4

Let V_2 and V_3 be the velocity of water at turbine exit and draft tube exit respectively.

By applying Bernoulli's equation between point 2 and 3 :

$$\frac{p_2}{w}+\frac{V_2^2}{2g}+H_s = \frac{p_3}{w}+\frac{V_3^2}{2g}$$

$$5.3+\frac{V_2^2}{2\times 9.81}+1.6 = 10.3+\frac{V_3^2}{2g}$$

$$\therefore \quad \frac{V_2^2 - V_3^2}{2 \times 9.81} = 10.3 - 5.3 - 1.6 = 3.4 \qquad ...(i)$$

Draft tube efficiency,

$$\eta_d = \frac{V_2^2 - V_3^2}{V_2^2} \text{ i.e. } 0.78 = 1 - \frac{V_3^2}{V_2^2}$$

$$\therefore \quad V_3^2 = 0.22\, V_2^2 \qquad ...(ii)$$

From Equations (i) and (ii),

$$\frac{V_2^2 - 0.22\, V_2^2}{2 \times 9.81} = 3.4$$

$$\therefore V_2 = 9.248 \text{ m/s}$$

$$\text{Discharge, } Q = \frac{\pi}{4} d^2 \times V_2$$

$$= \frac{\pi}{4} (3)^2 \times 9.248 = 65.37 \text{ m/s}$$

$$\text{Efficiency of turbine, } \eta_o = \frac{\text{Output, P}}{\text{Input power }, \rho g Q H}$$

$$= \frac{3000 \times 10^3}{1000 \times 9.81 \times 65.37 \times 5}$$

$$= \mathbf{0.9356 \text{ or } 93.56\%} \qquad \text{...Ans.}$$

Ex. 3.21.5 : In the scroll casing of a vertical reaction turbine the pressure head is 1.5 bar and water velocity is 4.5 m/s. The flow through the turbine is 11.25 m^3/s. The level of tail race water is 3 m below the centre of the scroll section. Water enters the draft tube without whirl with a velocity of 4.5 m/s and leaves it with half the velocity at the inlet of draft tube. Assuming an overall efficiency and hydraulic efficiencies are 82% and 85% respectively and loss of head of 0.3 m within the draft tube, determine : (i) Loss of head in turbine (ii) Pressure head at inlet to the draft tube.

Soln. :

Refer Fig. P. 3.21.5.

Given : Pressure head p_1 = 1.5 bar,

V_1 = 4.5 m/s, Q = 11.25 m^3/s,

H_s = 3 m ; V_2 = 4.5 m/s,

$$V_3 = \frac{1}{2} V_2 = \frac{1}{2} \times 4.5 = 2.25 \text{ m/s}$$

Overall efficiency, η_o = 82% = 0.82;

Hydraulic efficiency, η_h = 85% = 0.85 ;

Loss of head, h_f = 0.3 m

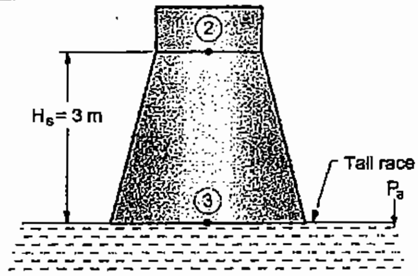


Fig. P. 3.21.5

Let p_2 be the pressure at exit of turbine and p_a = 1.013 bar be the atmospheric pressure = 10.33 m of water.

$$\text{Head on turbine, } H = \frac{p_1}{w} + \frac{V_2^2}{2g} + H_s; \text{ (where, } w = \rho \cdot g)$$

$$= \frac{1.5 \times 10^5}{1000 \times 9.81} + \frac{4.5^2}{2 \times 9.81} + 3$$

$$= 19.32 \text{ m}$$

Input power to turbine,

$$P_i = \rho \cdot g \cdot QH$$

$$= 1000 \times 9.81 \times 11.25 \times 19.32$$

$$= 2132.2 \times 10^3 \text{ W} = 2132.2 \text{ kW}$$

$$\text{Runner power, } P = \eta_h \times P_i = 0.85 \times 2132.2$$

$$= 1812.37 \text{ kW}$$

$$\text{Output power, } P_s = \eta_o \times P_i = 0.82 \times 2132.2$$

$$= 1748.40 \text{ kW}$$

(i) Loss of head in turbine :

Head corresponding to runner power at exit of turbine,

$$H_2 = \frac{P}{\rho \cdot g \cdot Q} = \frac{1812.37 \times 10^3}{1000 \times 9.81 \times 11.25}$$

$$= 16.42 \text{ m}$$

$$\therefore \quad \text{Loss of head} = H - H_2$$

$$= 19.32 - 16.42 = \mathbf{2.9 \text{ m}} \qquad \text{...Ans.}$$

(ii) Pressure head at inlet to draft tube :

By Bernoulli's equation between inlet and exit of draft tube,

$$\frac{p_2}{w} + \frac{V_2^2}{2g} + H_s = \frac{p_a}{w} + \frac{V_3^2}{2g} + h_f$$

$$\frac{p_2}{w} + \frac{4.5^2}{2 \times 9.81} + 3 = 10.3 + \frac{2.25^2}{2 \times 9.81} + 0.3$$

$$\frac{p_2}{w} = 10.33 + \frac{2.25^2}{2 \times 9.81} + 0.3 - \frac{4.5^2}{2 \times 9.81} - 3$$

$$= 10.3 - 3.474$$

$$\therefore \quad \frac{p_2}{w} = \mathbf{6.826 \text{ m (absolute)}} \qquad \text{...Ans.}$$

Note : If $\frac{p_a}{w} = -10.3$ m representing the atmospheric pressure head is taken as zero then it will represent the vacuum pressure head at inlet to draft tube.

When $\frac{p_a}{w} = 0$ Then, $\frac{p_2}{w} = -$ **3.474 m (vacuum)** ...Ans.

Ex. 3.21.6 : A Francis turbine develops 480 kW at an overall efficiency of 80% when working under a head of 6m. It uses a cylindrical draft tube of 3m diameter. In case the cylindrical draft tube is replaced by a conical shape of inlet and outlet diameters of 2m and 3.0 m respectively. The efficiency of conical draft tube is 85%. Assuming that head, discharge and speed remains the same in both cases, find :

(i) Head gained due to installation of conical draft tube

(ii) Increase in efficiency

(iii) Increase in power output

Soln. :

Given : $P_s = 480$ kW ; $\eta_o = 80\% = 0.8$;

$H = 6$ m; $d = 3$m (cylindrical) ;

$d_2 = 2$m $d_3 = 3.0$ m (conical) ;

$\eta_d = 85\% = 0.85$

Shaft power, $P_s = \rho \cdot gQH \times$ Overall efficiency, η_o

$$480 \times 10^3 = 1000 \times 9.81 \times Q \times 6 \times 0.8$$

Discharge, $Q = 10.194$ m³/s

Velocity at inlet using conical draft tube,

$$V_2 = \frac{Q}{4} = \frac{4}{\pi} \cdot \frac{Q}{d_2^2}$$

$$= \frac{4}{\pi} \times \frac{10.194}{(2)^2} = 3.245 \text{ m/s}$$

And velocity at outlet,

$$V_3 = \frac{4}{\pi} \times \frac{Q}{d_3^2}$$

$$= \frac{4}{\pi} \times \frac{10.194}{(3)^2} = 1.442 \text{ m/s}$$

(i) Head gained due to installation of draft tube :

$$\Delta H = \frac{V_2^2 - V_3^2}{2g} \times \eta_d$$

$$= \frac{3.245^2 - 1.442^2}{2 \times 9.81} \times 0.85$$

$$= \mathbf{0.366\ m}$$ ...Ans.

(ii) Increase in efficiency :

$$\Delta\eta = \frac{\text{Head gained}, \Delta H}{\text{Original head, H}}$$

$$= \frac{0.366}{6}$$

$$= \mathbf{0.061 \text{ or } 6.1\%}$$ ...Ans.

(iii) Increase in power :

$$\Delta P_s = \Delta\eta \times \text{Original power, } P_s$$

$$= 0.061 \times 480$$

$$= \mathbf{29.28\ kW}$$ ...Ans.

Ex. 3.21.7 : A Kaplan turbine develops 1500 kW under a head of 6 m. The turbine is set 2.5 m above the tailrace level. A vacuum gauge inserted at the turbine outlet records a suction head of 3.2 m. If the turbine efficiency is 85%, what will be efficiency of the draft tube having inlet diameter of 3m. (Neglect losses in draft tube). **SPPU - Dec. 16, 6 Marks**

Soln. : Refer Fig. P.3.21.7

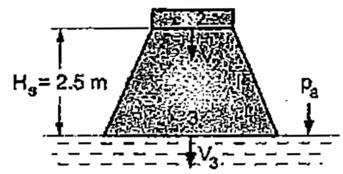


Fig. P. 3.21.7

Power developed $P = 1500$ kW;

Head $H = 6$m

Height of the draft turbine $H_s = 2.5$ m

Pressure at outlet of the turbine $\frac{p_2}{w} = 3.2$ m (vacuum)

$= 10.3 - 3.2$

$= 7.1$ m absolute

Turbine efficiency $\eta_o = 85\%$

Inlet diameter of the draft tube $d_2 = 3$ m

Efficiency of draft tube, η_d

$$P = \rho \cdot g\,Q\,H \times \eta_o \times 10^{-3} \text{ kW}$$

$$1500 = 1000 \times 9.81 \times Q \times 6 \times 0.85 \times 10^{-3}$$

$$\therefore Q = 29.98 \text{ m}^3/\text{s}$$

Now, $Q = \frac{\pi}{4} d_2^2 V_2$; $29.98 = \frac{\pi}{4} \times 3^2 \times V_2$

or $V_2 = 4.24$ m/s

Applying Bernoulli's equation between inlet and outlet of the draft tube.

$$\frac{p_2}{w}+\frac{V_2^2}{2g}+Z_2 = \frac{p_a}{w}+\frac{V_3^3}{2g}+Z_3+h_f \quad ...(i)$$

On neglecting friction losses i.e.

$$h_f = 0 \quad \text{and} \quad Z_2 - Z_3 = H_S = 2.5 \text{ m}$$

$$\frac{p_a}{w} = 10.3 \text{ m (assumed)}$$

From Equation (i) :

$$7.1+\frac{(4.24)^2}{2\times 9.81}+2.5 = 10.3+\frac{V_3^2}{2\times 9.81}$$

$$V_3 = 2.06 \text{ m/s}$$

$$\eta_d = \frac{\left(V_2^2-V_3^2\right)/2g}{V_2^2/2g} = 1-\frac{V_3^2}{V_2^2}$$

$$= 1-\frac{(2.06)^2}{(4.24)^2}$$

$$= \mathbf{0.764 \text{ or } 76.4\%} \quad \text{...Ans.}$$

Ex. 3.21.8 : An axial flow turbine has a vertical conical draft tube. Diameter of the tube at the upper end is 0.5 m and at the outlet is 0.7 m. The tube is running full with water flowing downwards and is 8 m long with 3.5 m of its bottom length in tail water. The frictional losses between the top and the bottom point is 0.2 times the velocity head at the top point where the water has a velocity of 6 m/s. Find the water pressure at the top point of the draft tube.

SPPU : Dec.-19, 5 Marks

Soln. :

Refer Fig. P. 3.21.8

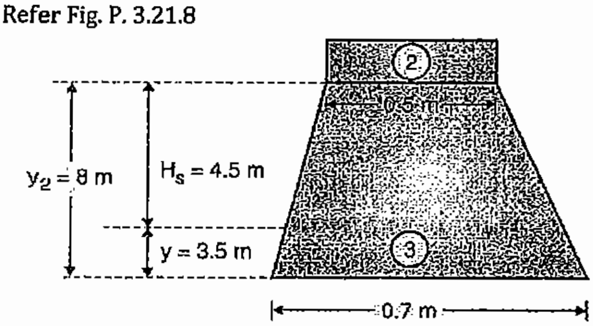


Fig. P. 3.21.8

$$d_2 = 0.5 \text{ m}, \qquad d_3 = 0.7 \text{ m},$$

$$y = 3.5 \text{ m}; \qquad y_2 = 8 \text{ m}$$

$$H_s = \text{Length of tube} - y = 8 - 3.5 = 4.5 \text{ m}$$

$$h_f = 0.2\times\frac{V_2^2}{2g};\; V_2 = 6 \text{ m/s}$$

Pressure at top print, $\frac{p_2}{w}$

$$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(0.5)^2 = 0.19635 \text{ m}^2$$

$$A_3 = \frac{\pi}{4}d_3^3 = \frac{\pi}{4}(0.7)^2 = 0.38485 \text{ m}^2$$

$$Q = A_2 V_2 = A_3 V_3$$

$$\therefore V_3 = \frac{A_2}{A_3}\times V_2 = \frac{0.19635}{0.38485}\times 6 = 3.06 \text{ m/s}$$

Let : $p_{atm} = p_a = 10.3$ m of water pressure

Applying Bernoulli's equation between upper end – 2 and outlet – 3 of the tube :

$$\frac{p_2}{w}+\frac{V_2^2}{2g}+H_s = \frac{p_a}{w}+\frac{V_3^2}{2g}+h_f$$

$$\frac{p_2}{w}+\frac{V_2^2}{2g}+H_s = \frac{p_a}{w}+\frac{V_3^2}{2g}+0.2\frac{V_2^2}{2g}$$

$$\frac{p_2}{w}+\frac{(6)^2}{2\times 9.81}+4.5 = 10.3+\frac{(3.06)^2}{2\times 9.81}+0.2\times\frac{(6)^2}{2\times 9.81}$$

$$\frac{p_2}{w} = \mathbf{4.8094 \text{ m (absolute)}} \quad \text{... Ans.}$$

3.22 Cavitation in Reaction Turbines

University Questions

Q. Write a short note on Cavitations in turbine.

SPPU : Dec. 12

Q. Explain the cavitation phenomenon. Which factors affect the cavitation in water turbines ?

SPPU : Dec. 13

Q. What is cavitation ? **SPPU: May 15**

Q. Explain causes for Cavitation In Reaction Water Turbine. **SPPU : Aug. 18(In Sem)**

Only reaction turbines are subjected to cavitation. The cavitation may occur at inlet of draft tube where the pressure is considerably reduced which may be below the vapour pressure of the liquid flowing through the turbine.

The phenomenon of **cavitation** is defined as the formation of vapour filled bubbles of a flowing fluids in a region where the pressure of liquid falls below its vapour pressure.

Phenomenon of cavitation can be explained as follows :

The vapour pressure of a liquid is the function of temperature and its height from mean sea level. In case the pressure of liquid during flow is reduced below its evaporation pressure at a given temperature, the liquid will boil and small vapour bubbles are formed. These bubbles are carried along by the fluid during flow to high pressure region where the vapours condense and the bubbles suddenly collapse. It results into formation of cavity. The surrounding liquid then rushes from all direction to fill the space thus created.

These streams of liquid a coming from all directions collide at the centre of cavity and a very high pressure in the range of 100 to 1000 times the atmospheric pressure is generated. It generates lot of noise and vibrations and shock waves are formed.

Sudden pressures generated by fluid produces hammering effect and damage to the metallic surface with which it is in contact. It causes the pitting of metallic parts called *erosion*.

3.22.1 Effects of Cavitation

The effects of cavitation are :

1. Flow pattern of fluid is modified with reduced flow rate.
2. Pitting and errosion of metal parts.
3. Collapse of cavities cause noise and vibrations of various parts.
4. Power and efficiency of turbine decreases due to cavitation.
5. Structural failure may take place due fatigue because of high rate of bubble collapse.

3.22.2 Methods of Preventing Cavitation

University Questions

Q. How cavitation can be prevented ? **SPPU : May 15**

Q. Explain Remedies for Cavitation in Reaction water Turbine **SPPU : Aug. 18(In Sem)**

The occurrence and the resulting damage can be prevented by :

1. Installation of turbine near tail race.
2. The pressure of fluid at any point should not fall below its vapour pressure. At any point the absolute pressure head should not fall below 2.5 m of water.
3. The runner blades are either made or coated by special cavitation resistant metals like stainless steel, nickel steel, aluminum bronze.
4. Prestressing the parts likely to be subjected to cavitation.

3.22.3 Recent Development to Prevent Cavitation

Various researches have suggested the following methods to prevent cavitation :

1. **Air injection** at high pressure into flow behind the runner where vertices are formed.
2. **Cathodic protection** of runner blades by electric current of certain voltage to evolve hydrogen from water. Hydrogen produces cushing effect on to runner blades.
3. **By providing** a in the blades. it will cause suction which will prevent separation.
4. By providing small slots in draft tube, the air is sucked from conical draft tube so that the water is not separated from the walls of draft tube.

3.22.4 Thoma's Cavitation Factor for Turbines

University Question

Q. What do you understand by Thomas factor of cavitation and what it signifies for water turbines ? **SPPU : May 12**

Based on experimental results Prof. D Thoma of Germany suggested a dimensionless number called after his name **Thoma's cavitation factor,** σ to find out the region where the cavitation can take place in reaction turbines. It is given as :

Thoma's cavitation factor,

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H} \qquad ...(3.22.1)$$

where, H_b = Barometric pressure head in m f water

H_s = Height of turbine runner from tail race in m of water

H = Net head on turbine in m of water

H_v = Vapour pressure head in m of water

The factor ($H_{atm} - H_v - H_s$) is called the **total suction head above vapour pressure or net positive suction head.** The value of Thoma's cavitation factor 3σ compared with critical cavitation factor, σ_c for a particular type of reaction turbine. Value of σ_c is obtained from experical relations based on experimental results. These are :

(a) For Francis turbine

$$\sigma_c = 0.0431 \left(\frac{N_s}{100}\right) \qquad ...(3.22.2)$$

(b) For Kaplan turbine

$$\sigma_c = 0.28 + 2.415 \times 10^{-3} \cdot N_s^3 \qquad ...(3.22.3)$$

where, N_s is the specific speed.

In order that the cavitation does not occur, the value of σ should be greater than σ_c.

3.23 Introduction to Governing of Water Turbines

In a hydroelectric power plant the turbine is always coupled to a generator in which the mechanical power of turbine is converted into electrical energy by the generator in the form alternating current. The power developed by various power plants is connected to a common grid which supplies power to various consumers.

Various power plants are required to supply power to common grid at certain e.m.f. and frequency. Any mismatch of emf and frequency between the generator and grid may lead to serious trippings.

The voltage and frequency, f of the e.m.f. generated by the alternator depends on the speed of the alternator given by the relation, $f = (p \cdot N/60)$ where p represents the number of pair of poles and N is the speed in r.p.m.

Thus for maintaining the voltage and frequency of the generator, it is necessary to run the generator at constant speed called **synchronous speed.**

Since the generator is coupled to the turbine, it implies that the turbine must be run at constant speed within certain fluctuations of speed permissible.

3.23.1 Governor and it's Functions

The load on the generator keeps on fluctuating while the input power to the turbine is constant. Therefore, the speed of turbine will either increase or decrease with the decrease or increase of load respectively. This leads to fluctuation of speed of the turbine. This in turn will fluctuate the speed of generator which is not desirable as discussed above.

In order to run the generator at constant speed, the turbine speed is required to be maintained under variable load conditions. It is achieved by varying the discharge rate to the runner of the turbine according to the load on the turbine.

The regulation of the speed of the turbine within prescribed limits according to the load on the turbine is called the **governing of turbines.**

The functions of a turbine governor are :

1. It should control the speed of turbine with fluctuating load at synchronous speed of the generator.
2. It helps in starting and shutting down the turbine unit by opening and closing the nozzles of a pelton wheel and wicket gates in case of reaction turbines.

3.24 Governing System for Turbines of a Hydro-Electric Power Plant

The governing system of a turbine is based on a closed loop feedback control system. The block diagram for such a system is shown in Fig. 3.24.1. The basic elements and their functions are as follows :

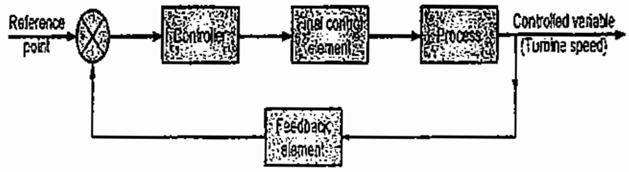


Fig. 3.24.1 : Block diagram of a closed loop control system

The variable which is required to be controlled is called **controlled variable**. In this case the turbine speed is the controlled variable which fluctuates according to the load on the turbine.

The first step in this closed loop control system is done by measuring element called **feedback element**, which is speed in this case. This information is fed back to the controller through a summing point or comparator. This comparator compares the present value of controlled value (i.e. speed) with the *reference point*. (it represents the speed at which turbine is required to run). On comparison between the present value of speed and reference point speed, if any difference between the two, an error signal is produced.

The error signal is transmitted to the **controller**. The function of controller is to produce an output corresponding to the error and initiates an action so as to bring down the error to zero.

The output of the controller is given to a **final control element** which is generally a heavy actuator. This actuator will actuate the mechanism to control the rate of discharge into the turbine.

Thus it could be seen from the above discussion that the closed loop control system would function automatically in a loop till such time it reduces the error to zero and bring back the speed of the turbine to desired constant speed under all varying load conditions.

3.25 Governing Mechanism

Input to a turbine is proportional to the product of head, H and discharge rate, Q. Consider the case when load on turbine increases, the speed of turbine reduces since the input to the turbine, hence its output remains the same.

In order to increase the speed of turbine to its constant value, it is necessary to increase its input according to the increase load which can be achieved either by increasing the head or discharge rate. Since the head on turbine cannot be changed, therefore the input to turbine can only be increased by increasing the discharge rate into the turbine.

Again the discharge rate can be increased by either increasing the velocity of flow or the discharge area. The change in velocity will affect the velocity triangles. Thus the angle of relative velocity at inlet will differ from the inlet tip angle of blades causing shock and hydraulic losses. *Hence, the rate of discharge in turbines is adjusted by changing the flow area in order to meet the varying load conditions with the help of governor mechanism.*

The governor mechanisms are now being discussed for different turbines in succeeding paragraphs.

3.26 Governing of Pelton Turbine

The governing of Pelton turbine is done by **oil pressure governors** as shown in Fig. 3.26.1.

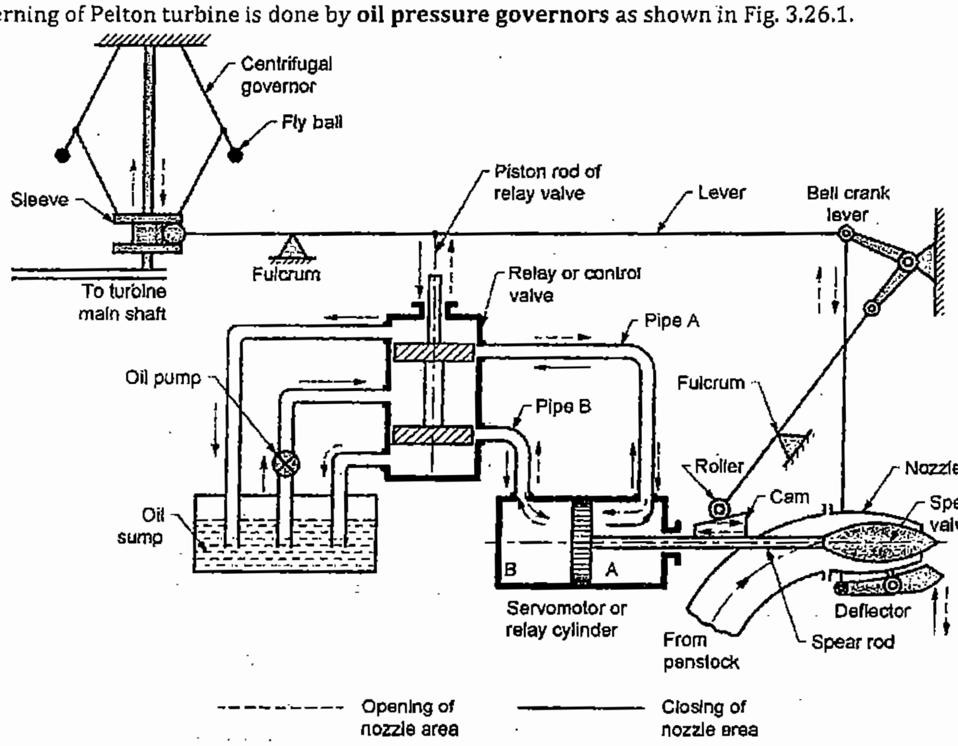


Fig. 3.26.1 : Governing of Pelton turbine

The major components of the governor mechanism are :

1. **Oil pump and oil sump**

System uses oil in servomotor or relay cylinder since the force required to actuate the spear valve would be enormous. For this reason, the system requires an oil sump to store the oil and an oil pump to regulate the oil supply in the mechanism. Oil pump is a positive displacement type of pump like gear pump or axial piston pump. It's function is to pressurise the oil.

2. **Relay or control valve**

Relay valve is a spool valve having 5 ports. It is also called as control valve or distributor valve.

It receives the pressurised oil from the oil pump which is diverted towards the ports connected to pipe A or pipe B. Through these pipes the oil is transferred to corresponding sides of double acting servomotor cylinder. Simultaneously, the oil will be returned from the servomotor from the opposite pipe to the sump.

3. Servomotor or relay cylinder

It is a double acting cylinder which acts as hydraulic actuator. It receives oil from relay valve say through pipe A. The piston of the cylinder will be displaced towards left, thus forcing the oil through the pipe B into the relay valve and finally to oil sump. It will simultaneously more the spear valve to the left and increase the area of flow through the nozzle.

4. Spear valve

Spear valve controls the flow area of the nozzle. It is directly connected to the piston of relay cylinder.

5. Governor and linkages

A centrifugal governor is used as the measuring element of the closed loop control system. It is driven by the turbine shaft through bevel gears.

The sleeve of the governor is connected through linkages to relay valve. The movement of sleeve is transferred through the lever to more the piston rod of relay valve.

Working

Fig. 3.26.1 shows the position when the turbine is running at normal speed.

Consider the case when the load on the generator increases, the speed of the generator and that turbine will decreases. Since the governor is driven by the turbine shaft, its speed will also decrease. As a consequence, the flyballs of the governor will move inwards due to reduced centrifugal force on the balls. As a result the sleeve of the governor will move downwards.

The downward motion of the sleeve will be transferred to the main lever through its fulcrum. It will cause the piston rod of the relay valve to move upwards and simultaneously the bell crank lever also moves upwards. The upward motion of piston rod of control valve causes pressurised oil to flow through the pipe A to the relay cylinder and exerts a force on face A on the piston of servomotor. It moves the piston to the left, thus the spear rod with its valve will also move towards the left. It will increase the nozzle area and the rate of flow of water to the turbine increases. Therefore, the input to turbine and consequently its speed increases.

During this piston movement of servomotor to the left, oil held in the cylinder towards the face B is transferred through pipe B to the oil sump via the relay valve. When the speed of turbine is adjusted to normal speed, the system would return to its original position.

Opposite will be action of the whole mechanism when the load on the generator decreases. The increase in speed of turbine and generator will increase the speed of the governor. The balls flyout and the sleeve moves upwards. It causes the piston rod of relay valve to move downwards, thus opening the valve towards pipe B. Pressurised oil flows into servomotor cylinder towards the face B and causes the spear to move towards right thus closing the nozzle area. The reduced discharge to turbine runner reduces the input, hence its speed. When the speed attained by the turbine is its normal speed, the governing mechanism will return to its original position.

3.26.1 Double Regulation Speed in Pelton Turbine

In above discussion of governing we have not considered the pace at which the turbine is brought back to its synchronous or normal speed with variation in load. Consider the case when governor is too sensitive in that case the spear valve will be opened or closed with small change in speed of the turbine.

When the load decreases, the speed of the turbine and its governor increases. Since governor is too sensitive, it will immediately closed the water supply from the pipe to turbine runner. It would give rise to pressure waves in the pipe line and these pressure waves will travel at sonic velocity along the pipe line. These pressure waves produce many harmful effects even it can lead to bursting of pipe line. This phenomenon is known as **water hammer.**

Therefore, another regulation mechanism is incorporated in the system to control the discharge rate through the nozzle to avoid water hammer. The mechanism is called **jet deflector** as shown in Fig. 3.26.2.

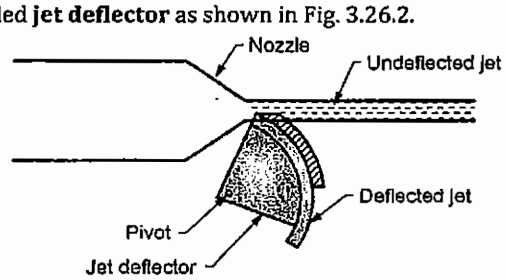


Fig. 3.26.2 : Jet deflector mechanism for pelton turbine (spear is not shown)

It is a curved plate mounted on a second which can be swivelled about a pivot.

The governing mechanism is connected to jet deflector and it actuates about its pivot. When deflector moves upwards, it partially covers the jet area and a portion of jet is deflected which flows out of the main jet. The undeflected jet only strikes the runner buckets of the impulse turbine and the quantity of discharge to runner is thus reduced. Now the spear valve is adjusted so as bring down the discharge to the required value and the jet deflector moved away from the main jet.

3.27 Governing Mechanism of a Francis Turbine

University Questions

Q. Write a short note on Governing of reaction turbine. **SPPU : Dec. 12**

Q. Explain with neat sketch governing mechanism of any one reaction turbine. **SPPU : April 15(In Sem)**

The basic mechanism for governing of various turbines remains the same as discussed in section 3.25, the only difference being the method the control of discharge which varies from turbine to turbine.

In case of pelton turbine, the discharge was controlled using the speed and spear nozzle in Francis turbine a different mechanism is used with modification in relay cylinder to control the discharge as shown in Fig. 3.27.1.

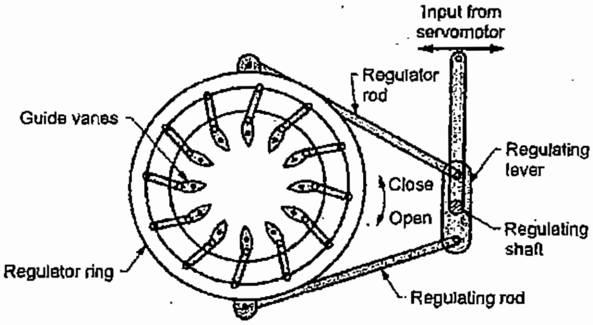


Fig. 3.27.1 : Governing mechanism for Francis turbine

Mechanism has the following components to control the discharge.

1. Regulating ring

– It is a circular ring having guide vanes pivoted at a point through the levers and links. Therefore, when the regulator ring is rotated about its axis, all the guide vane would turn about their pivots. Due to turning of guide vanes the space between two consequetive guide vanes would change.

– Space between guide vanes will increase in one direction of rotation of regulating ring and space will decrease in its opposite direction of rotation.

2. Regulating rod

– It connects the regulating ring to relay cylinder (servomotor) through the various linkages as shown in Fig. 3.27.1. It can be seen by this mechanism, the linear motion of servomotor piston is converted into rotary motion of regulating ring.

– An alternate arrangement providing the rotary motion to regulating ring by using two servomotors is shown in Fig. 3.27.2.

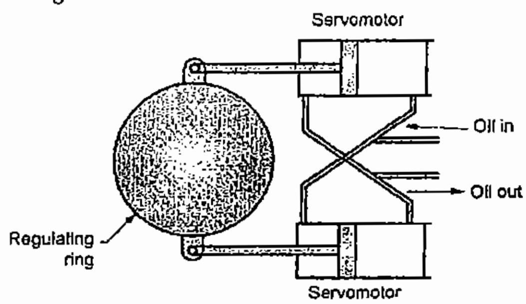


Fig. 3.27.2 : Regulating mechanism for Francis turbine using two servomotors

– This mechanism gives better performance since the force required for operation of Francis turbine guide vanes is much more compared to Pelton turbine.

3.28 Governing Mechanism for Kaplan Turbine

Speed control in Kaplan turbine can also be achieved by varying the discharge by changing the guide vane angle as in case of Francis turbine with an additional feature.

However, *the governing achieved by changing the guide vane angles has a serious drawback.* With the change in guide vane angle, the inlet velocity triangle is changes. It results into the change in direction of absolute velocity at inlet. Hence, the direction of relative velocity at inlet is also changed, thus the water will not enter the moving blades tangential to it.

In other words, the tangential entry to runner blades can be achieved only at a particular speed of the turbine.

In order to overcome this problem, the governing of Kaplan turbine is achieved by double regulation system which controls the movement of guide vanes as well as the moving vane.

Therefore, a separate servomotor is provided to operate the runner blades which are few in numbers. The arrangement is shown in Fig. 3.28.1.

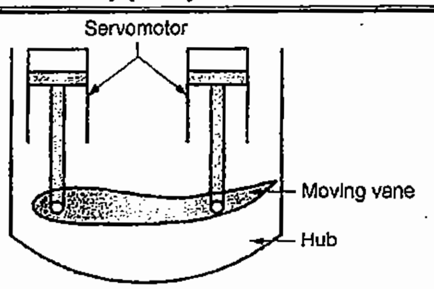


Fig. 3.28.1 : Arrangement for turning the moving vane of a Kaplan turbine

In this methods, the moving blades are pivoted on the hub. It carries a pair servomotors for controlling the movement of each moving blades as shown in Fig. 3.28.1.

During the governing, the discharge is changed by changing the guide vane angles as discussed in case of Francis turbine, simultaneously the moving blade angles are also adjusted in such a way that for new position of guide vanes, the entry of water remains tangential to moving vanes over wide range of guide vane positions.

Due to control of both guide and moving vanes, the Kaplan turbine gives better efficiency over wide range of loads as compared to Francis turbine.

3.29 Main Characteristic Curves of Reaction Turbines

University Questions

Q. What do you understand by the characteristics curves of a turbine? Name it and explain any two characteristic curves. **SPPU : Dec. 12**

Q. Discuss main characteristics curves for hydraulic turbine. **SPPU : May 16**

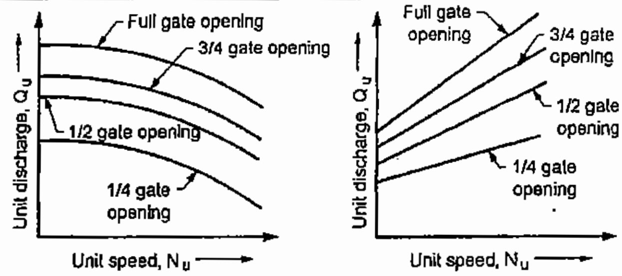
Q. Write a short note on factors influencing performance of turbine. **SPPU : May 18**

These characteristic curves are drawn by testing the turbines under constant head by using unit parameters. These are as follows (Refer Fig. 3.29.1).

(a) Unit speed N_u Vs unit discharge, Q_u curves

Francis turbine

The Q_u Vs N_u curves are shown in Fig. 3.29.1(a). It could be seen that a given gate opening the discharge reduces with the increase in speed it is due to fact the water flowing inwards is retarded by the outward centrifugal forces.



(a) Francis turbine **(b) Kaplan turbine**

Fig. 3.29.1 : Unit discharge Vs unit speed characteristic curves at constant head for Francis and Kaplan turbines

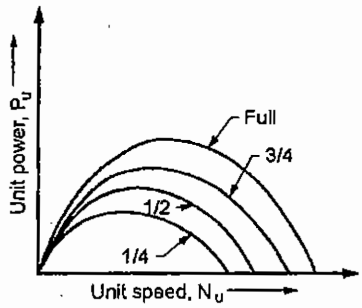
Kaplan turbine

Q_u Vs N_u curve is shown in Fig. 3.29.1(b). It could be noted that at given gate opening, the discharge increases with the increase in speed since the increased speed accelerates the fluid. Also, there is no decelerating force as in case of Francis turbine since the discharge in Kaplan turbine is axial.

(b) Speed Vs power characteristic curves

These characteristics have the similar shape in all the turbines as shown in Fig. 3.29.2 for reaction (Francis and Kaplan) turbines.

It could be noted from the characteristics that the power first increases to a maximum with increase in speed and thereafter the power decreases with further increase in speed. Thus, there are two speeds for each value of power and power developed is maximum at a particular speed.



Francis and Kaplan turbines

Fig. 3.29.2 : P_u Vs N_u characteristic curves under constant head

(c) Unit speed (N_u) Vs efficiency curves

These curves are shown in Fig. 3.29.3 reaction turbines.

The patterns of speed Vs efficiency curves is similar to power Vs speed curves. The efficiency is zero at zero power and speed. Maximum efficiency corresponds to design speed of the turbine. Important point to be noted from the curves is that maximum efficiency occurs at full gate opening in both types of turbines at their rated speeds.

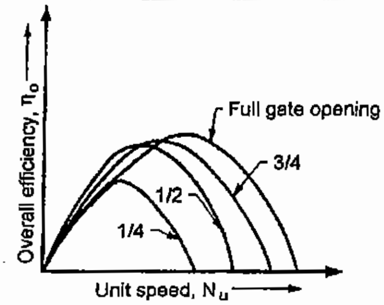


Fig. 3.29.3 : Reaction turbines

3.30 Operating Characteristics (Constant Speed Characteristics)

University Question

Q. Discuss operating characteristics curves for hydraulic turbine. **SPPU : Dec. 12, May 16**

Operating characteristics of the turbine are obtained by testing of turbines under constant speed. Since the head cannot be changed under actual working conditions, therefore, the tests on turbine are conducted under a constant head.

In order to maintain constant speed, the gate openings are adjusted when the load on turbine changes, which is achieved by governing mechanism. The corresponding power input and output are measured and the efficiency is calculated.

From the data obtained, following curves are plotted.

(a) Load Vs efficiency
(b) Discharge Vs power.
(c) Discharge Vs efficiency.

(a) Load Vs efficiency curves

Fig. 3.30.1 shows the load Vs efficiency curve for various reaction type of turbines. Load is represented as the percentage of full load for which a turbine is designed.

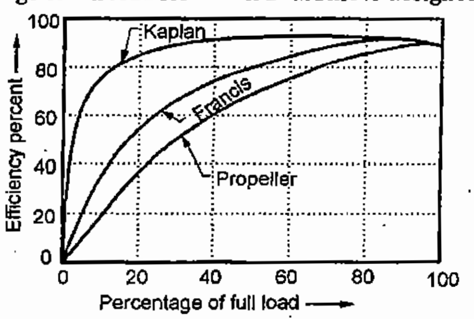


Fig. 3.30.1 : Efficiency Vs load curves for various turbines at constant speed and head

In case of Kaplan turbine, the maximum efficiency is obtained at comparatively low loads compared to Francis and propeller turbines and it remains almost constant with further increase in load.

Thus, these turbines can be operated at high efficiency for wide range of load conditions. Whereas, the Francis and propeller only give high efficiency at 70% or higher loads of full loads.

(b) Discharge (Q) Vs power (P) and Discharge Vs Efficiency (η) curves

Power Vs discharge and efficiency Vs discharge curves are shown in Fig. 3.30.2 at constant head and speed of turbine.

It could be seen that the efficiency of turbine increases with the increase in discharge parabolically. The power output varies linearly with discharge. It could be noted that power output is zero for certain value of discharge, Q_o. It is due to the fact that initially the power developed by the turbine is lost in overcoming the friction losses and the inertia load of rotating parts of a turbine.

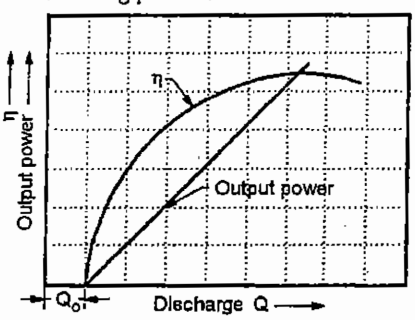


Fig. 3.30.2 : η_o and output power Vs discharge curves

Summary

SECTION - I : Impulse Water Turbine

- A **hydraulic turbine** is a machine which converts hydraulic energy into mechanical energy.
- A **hydro-electric power plant** converts the hydraulic energy into electrical energy by directly coupling the generator with turbine.
- An **impulse turbine** works on the principle of impulse and a **reaction turbine** works on the principle of impulse and reaction.
- A **hydro-electric power plant** has the following main components :

(a) A **dam** to store water under head.

(b) The **penstock** to carry water from dam to turbine.

(c) A water turbine.

(d) **Tailrace** is a canal into which turbine water is discharged.

– Difference in level of head race and tail race is called **gross head, H_g**.

– Head available at inlet of turbine is called **net head, H.**

– Net head is difference of gross head and frictional head, h_f.

$$H = H_g - H_f$$
$$= H_g - \frac{4 f L V^2}{d \cdot 2g};$$

where, d is diameter of penstock.

– **Main parts of Pelton turbine are :**

1. Nozzle and spear assembly.
2. Runner and bucket.
3. Casing
4. Braking jet
5. Deflector

– **Workdone and Efficiencies of Pelton Wheel :**

(i) For Pelton wheel : $\alpha = 0$; $\theta = 0$; $(180 - \phi)$ is called angle of deflection.

(ii) Velocity of jet of inlet, $V_1 = C_v \sqrt{2gH}$;

Blade velocity, $u = \frac{\pi DN}{60}$.

(iii) $V_{w1} = V_1$; $V_{r2} = K V_{r1}$; $V_{w2} = V_{r2} \cos \phi - u$;

$\dot{m} = \rho Q V_1$

(iv) **Workdone,** $W = \dot{m} (V_{w1} \pm V_{w2}) u$ J/s

(v) **Power developed,** $P = W/1000$ kW.

(vi) **Hydraulic efficiency of jet,**

$$\eta_h = \frac{\text{Power developed, P}}{\text{K.E. supplied jet, } \frac{1}{2} \dot{m} V_1^2} = \frac{2 (V_{w1} + V_{w2}) u}{V_1^2}$$

(vii) **Condition for maximum efficiency** is $u = \frac{V_1}{2}$ and

$$(\eta_h)_{max.} = \frac{1 + K \cos \phi}{2}$$

(viii) **Mechanical efficiency,**

$$\eta_m = \frac{\text{Shaft power, } P_S}{\text{Power developed by turbine, P}}$$

(ix) **Volumetric efficiency,**

$$\eta_v = \frac{\text{Actual volume of water strike the runner, } Q_a}{\text{Volume of water issued by nozzle, Q}}$$

(x) **Overall efficiency,**

$$\eta_o = \frac{\text{Shaft power, } P_S}{\text{Power supplied by jet, } P_i \ (= \rho Q H)}$$

$$\eta_o = \eta_v \times \eta_h \times \eta_m$$

(xi) **Plant efficiency,**

$$\eta_p = \frac{\text{Power output generator, } P_g}{\text{Power supplied by jet, } P_i} = \eta_g \cdot \eta_o$$

– **Design aspects of impulse turbine :**

(i) **Jet ratio,** $m = \frac{\text{Pitch diameter of wheel}}{\text{Jet diameter}} = \frac{D}{d}$

(m varies between 11 to 15 ; m = 12 is adopted in most cases)

(ii) **Number of jets,** $n = \frac{\text{Total flow rate}}{\text{Flow rate/nozzle}}$

(iii) **Speed ratio,** $K_u = \frac{\text{Runner speed}}{\text{Velocity of jet}} = \frac{u}{V_1}$

(Value of K_u ranges from 0.43 to 0.47.)

(iv) **Angle of deflection of jet,** $(180 - \phi)$ through buckets varies between 160° to170°. **It is taken as 165° if not specified.**

(v) **Number of buckets, Z**

Theoretical number of buckets,

$$Z = \frac{360°}{\psi°} = \frac{(m + 1)}{(m + 1.2)}$$

where, $\psi = \cos^{-1}\left(\frac{R + 0.5d}{R + 0.6d}\right)$;

$$R = \frac{D}{2}, \quad m = \frac{D}{d}$$

Practical value of buckets,

$$Z = 15 + 0.5\, m = 15 + 0.5 \times \frac{D}{d}.$$

– **Other impulse turbines are :**

1. Turgo 2. Banki 3. Girard

– **Specific speed, N_s** is the speed of a geometrically similar model which would produce unit power under a unit head.

$$N_s = \frac{N \cdot \sqrt{P}}{H^{5/4}}, \text{ where 'N' is the speed of runner.}$$

- **Unit Quantities**
  1. **Unit speed, N_n** of a turbine is defined as its speed while operating under unit head of 1 m $\left(Nu = \frac{N}{\sqrt{H}}\right)$.
  2. **Unit charge, Q_n** is the discharge of a turbine working under a head of 1 m. $\left(Q_u = \frac{Q}{\sqrt{H}}\right)$
  3. **Unit power, P_n** is the power developed under a head 1 m by the turbine. $\left(P_u = \frac{P}{H^{3/2}}\right)$
- Main characterstics of turbine are obtained by keeping the supply head and gate opening constant and varying the discharge and speed by changing load on turbine. The curves are : (N_u V_s N_Q, N_u V_s N_p and N_u V_s N_η)
- Operating characterstics are drawn at constant speed.

SECTION - II : Reaction Water Turbines

- In case of *reaction turbines* only a part of available head is converted into velocity head while passing over the guide vanes before entering the runner. Remainder pressure energy is gradually converted into velocity head until pressure is reduced to atmosphere while producing mechanical work.
- *Reaction turbines are suitable* for low and medium heads between 30 m to 250 m.
- *Reaction turbines are basically* **classified** *as* radial flow, axial flow and mixed flow turbines.
- **Main components of a reaction turbine are :**
  1. **Scroll casing** is of spiral shape of decreasing area. It's function is to distribute water evenly around the periphery of runner at constant velocity and pressure.
  2. **Guide mechanism** has guide vanes to direct water from casing to runner blades and to regulate discharge according to load on turbine.
  3. **Runner and shaft** the function of runner is to convert the available energy into mechanical work.
  4. *Draft tube* to convert the K.E. of water at exit of runner into pressure head.

     Net head, H = Cross head, H_g – Friction head in penstock h_f

     Input power, P_i = ρQgH (Also, called water power)

- **Radial reaction turbines**

$$u_1 = \frac{\pi D_1 N}{60}; \quad u_2 = \frac{\pi D_2 N}{60}$$

(a) *Work, or Runner power,*

$$P = (V_{w1} \cdot u_1 \pm V_{w2} \cdot u_2)\,\rho Q$$

(b) *Condition for maximum work is* $\beta = 90^\circ$ (axial discharge)

$$\therefore W = \rho Q V_{w1} \cdot u_1$$

(c) *Hydraulic efficiency,* $\eta_h = \frac{V_{w1} \cdot u_1 + u_2}{gH}$

(d) *Mechanical efficiency,* $\eta_m = \frac{\text{shaft power}, P_s}{\text{Runner power}, P}$

(e) *Overall efficiency,*

$$\eta_o = \frac{\textbf{shaft power}, P_s}{\textbf{Input power}, \rho gQH} = \eta_h \cdot \eta_m$$

(f) $n = \frac{\text{width of runner}, B_1}{\text{Diameter of runner}, D_1}$; (n = 0.1 to 0.45)

(g) **Speed ratio,** $K_u = \frac{u_1}{\sqrt{2gH}}$; ($K_u$ = 0.6 to 0.9)

(h) **Flow ratio,** $K_f = \frac{V_{f1}}{\sqrt{2gH}}$; (K_f = 0.15 to 0.30)

(i) **Discharge,** $Q = (\pi D_1 - n_1 \cdot t)\, B_1 \cdot V_{f1}$
$= (\pi D_2 - n_2 \cdot t)\, B_2 \cdot V_{f2}$

- *Degree of reaction,* R of runner is defined as the ratio of pressure energy change inside the runner, H_p to the total energy change inside the runner, H_t

$$R = \frac{H_p}{H_t} = \frac{(u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}{(V_1^2 - V_2^2) + (u_1 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}$$

$$= 1 - \frac{V_2^2 - V_1^2}{2g \cdot H_t}$$

For pelton wheel : R = 0, since $H_p = 0$

- **Francis Reaction turbines** in an inward flow reaction turbine with radial discharge. However, **modern Francis turbine** is a mixed flow turbine in which discharge is axial. β = 0. Therefore, workdone /N/S of water becomes,

$$W = \frac{V_{w1} \times u_1}{g}$$

- **Propeller and Kaplan turbines** are axial flow turbines which are used to utilise large discharge at low heads.

- The guide and runner vanes can be adjusted automatically according to load on turbine.

  In this water enters from scroll casing into guide. Vanes from where it turns 90°to enter runner vanes axially.

  (a) $u = u_1 = u_2 = \frac{\pi D_0 N}{60}$ (D_0 = outer diameter of runner)

  (b) $n = \frac{D_b}{D_0}$; (D_b = diameter of hub or boss); value of n ranges from 0.35 to 0.6

  (c) $Q = \frac{\pi}{4}(D_0^2 - D_b^2) V_f$

  (d) Flow ratio, $K_f = \frac{V_f}{\sqrt{2gH}}$; ($K_f$ = 0.6 to 0.7)

- **Other reaction turbines are :**

  (a) *Deriaz reaction turbine* which can works as a reversible pump turbine. It is a mixed flow turbine.

  (b) **Axial flow bulb and tubular turbine.** In these turbine and generator are enclosed in a casing submerged in water. It has adjustable impeller blades. These are used for generation of power at very low heads (upto 12 m).

- **Specific speed,** $N_s = \frac{N\sqrt{P}}{H^{5/4}}$

- **Draft tube** is a pipe of gradually increasing area used for discharging water from the exit of runner of a reaction turbine into tail race.

- Types of draft tubes are :

  (i) Conical

  (ii) Simple elbow

  (iii) Elbow draft tube with circular section at inlet and rectangular section at outlet

  (iv) Moody's spreading draft tube.

- Energy balance for draft tube is :

$$\frac{p_2}{w} = \frac{p_a}{w} - H_s - \left(\frac{V_2^2 - V_2^2}{2g} - h_f\right)$$

- **Efficiency of draft tube,** η_d

$$\eta_d = \frac{\frac{V_2^2 - V_3^2}{2g} - h_f}{\frac{V_2^2}{2g}}$$

- The phenomenon of cavitation **in reaction turbines** is defined as the formation of vapour filled bubbles of a flowing fluid in a region where pressure of liquid falls below its vapour pressure.

- Cavitation causes pitting of metal parts, noise, vibration with reduction in power and efficiency of turbine.

- Thoma's cavitation factor, σ for turbine are :

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H}$$

- where, H_v represents the vapour pressure at which cavitation occurs.

- **Critical cavitation factor,** σ_c

  For Francis turbine : $\sigma_c = 0.0431\left(\frac{N_s}{100}\right)^2$

  For Kaplan turbine : $\sigma_c = 0.28 + 2.415 \times 10^{-3} \cdot N_s^3$

- **Function of a governor** are to control the speed fluctuations with fluctuating loads within prescribed limits of turbine speed. It also helps in starting and shutting down the turbine unit by opening and closing the nozzles of a pelton wheel.

- A **good governor** should posses the properties of sensitiveness, stability, quick response to load changes and damping of oscillations.

- Governing mechanism of pelton wheel. It's main components are :

  1. Oil pump and oil sump.
  2. Relay or control valve
  3. Servomotor or relay cylinder
  4. Spear valve
  5. Governor and linkages.

- **Incase of double regulation speed in pelton wheel,** a jet deflector is used in addition to avoid water hammer.

Exercise

Note : For answers please refer the section numbers indicated in bracket.

Q. 1 How do you classify water turbines? What is the difference between the impulse and reaction turbines? **[Section 3.1.1]**

Q. 2 Differentiate between hydraulic turbine and hydro-electric power plant. **[Section 3.2]**

Q. 3 Explain the general layout of hydro-electric plant with the help of a neat sketch. Explain the terms gross head and net head and the relation between these terms. **[Section 3.2]**

Q. 4 What do you mean by gross head, net head and efficiency of turbine? Explain the different efficiency of a turbine. **[Section 3.2]**

Q. 5 What is a Pelton wheel ? Explain its construction and working with a neat sketch. **[Sections 3.3 and 3.3.1]**

Q. 6 State the characteristics features of impulse turbine. **[Section 3.3]**

Q. 7 Draw neat sketch of a Pelton wheel. Show all its dimensions in terms of jet diameter. **[Section 3.3]**

Q. 8 Sketch Pelton wheel bucket and explain the effect of its size, shape and number on its function. **[Section 3.3]**

Q. 9 Sketch a pelton wheel bucket giving its approximate dimensions and answer following questions in brief.

(i) The ideal jet deflection angle is 180°, however buckets deflects the jet through 160°to 165°, why ?

(ii) Why two cups are provided to form a bucket ?

(iii) Why undercut is provided to bucket ?

[Sections 3.3 and 3.3.1]

Q. 10 Draw the inlet and outlet velocity diagram for a Pelton wheel. Obtain an expression for workdone and hydraulic efficiency. Hence, drive the expression for maximum hydraulic efficiency. **[Section 3.4]**

Q. 11 Define and explain as applied to impulse turbine :

(i) Hydraulic efficiency

(ii) Mechanical efficiency

(iii) Volumetric efficiency

(iv) Overall efficiency

(v) Plant efficiency **[Section 3.4]**

Q. 12 Show that the overall efficiency of a hydraulic turbine is the product of volumetric, hydraulic and mechanical efficiencies. **[Section 3.4]**

Q. 13 Show that, the maximum efficiency of the Pelton wheel is given by $\frac{1 + k \cos \beta}{2}$

Where

k = Bucket friction factor

β = Bucket outlet angle. **[Section 3.4]**

Q. 14 Define the terms : 1. Speed ratio, K_u 2. Jet ratio, m **[Section 3.5]**

Q. 15 Prove that the theoretical number of buckets required on a runner of impulse turbine is given as : $Z = \frac{360°}{\psi°}$, where, $\psi = \cos^{-1}\left[\frac{R + 0.5d}{R + 0.6\,d}\right]$, in which R represents the pitch radius of runner and d is the diameter of Jet. **[Section 3.5.1]**

Q. 16 What are the factors on which number of jets depend in case of Pelton wheel ? **[Section 3.5.1]**

Q. 17 Explain the basic principles of working of following impulse turbines

1. Turgo turbine
2. Banki turbine
3. Girard turbine **[Section 3.6.1, 3.6.2 and 3.6.3]**

Q. 18 Discuss the main and operating characteristics of a pelton wheel. **[Section 3.7]**

Q. 19 What is specific speed of a turbine? State its significance and derive an expression for the same. **[Section 3.8]**

Q. 20 Explain the factor which decides the speed of Pelton turbine used for Electrical power generation and discuss the relation between the specific speed and jet ratio. **[Section 3.8]**

Q. 21 Explain the method of selection of hydraulic turbines based on :

(a) According to head and discharge **[Section 3.9(1)]**

(b) According to specific speed **[Section 3.9(2)]**

Q. 22 How the reaction turbines are classified ? **[Section 3.11]**

Q. 23 Why spiral casing of varying area is employed in reaction turbines ? **[Section 3.12]**

Q. 24 Describe with help of neat sketch the main components of Francis turbine. **[Section 3.12]**

Q. 4 Differentiate between the impulse turbine and reaction water turbine. **[Section 3.13]**

Q. 5 Draw general layout of a reaction turbine plant and explain various heads on turbine. **[Section 3.14]**

Q. 6 What is the effect of centrifugal force in an inward flow radial turbines?**[Sections 3.15 and 3.15.1]**

Q. 7 What is degree of reaction ? Explain its significance. Why a pure reaction turbine is not possible ? **[Section 3.15.2]**

Q. 8 What is the effect of centrifugal force in an outward flow radial turbines? **[Section 3.16]**

Q. 9 Explain working of an outward flow reaction turbine. **[Section 3.16]**

Q. 10 Give comparison between inward flow and outward flow reaction turbines. **[Section 3.17]**

Q. 11 With a neat sketch explain the construction and working of a Francis turbine. **[Section 3.18]**

Q. 12 Draw a neat sketch of Kaplan turbine and name the parts. **[Section 3.19]**

Q. 13 Compare Francis, Kaplan and Propeller turbine. Explain why Kaplan turbine is preferred where load on the turbine is fluctuating. **[Section 3.19.1]**

Q. 14 Draw a neat sketch of Kaplan turbine and name the parts. **[Section 3.19]**

Q. 15 What is draft tube ? Why it is used in a reaction turbine and not used in impulse turbine ? Describe with neat sketch two different types of draft tubes. **[Section 3.21]**

Q. 16 Why is the draft tube necessary in the case of hydraulic reaction turbines ? Explain with neat sketches. Explain the advantage of elbow-type divergent draft tube over a straight divergent draft tube. **[Sections 3.21 and 3.21.3]**

Q. 17 Explain the statement when the draft tube is provided in reaction turbine, net head available remains the same even when the turbine is installed above the tail race. **[Section 3.21.2]**

Q. 18 What is the role of a draft tube ? Sketch various types of draft tubes. Define the efficiency of draft tube. **[Sections 3.21 and 3.21.1]**

Q. 19 What do you understand by Thomas factor of cavitation and what it signifies for water turbines ? **[Section 3.22.4]**

Q. 20 Explain the cavitation phenomenon. Which factors affect the cavitation in water turbines ?

[Section 3.22]

Q. 21 Explain with a neat sketch the governing of pelton turbines. **[Section 3.26]**

Q. 22 Write a short note on Governing of reaction turbine. **[Section 3.27]**

Q. 23 Discuss the main characteristic curves of Francis and Kaplan turbines. **[Section 3.29]**

Q. 24 Name characteristics and explain any two characteristic curves. **[Section 3.29]**

❑❑❑

4 Steam Turbines

UNIT - III

Syllabus

Steam Nozzle : Equations for velocity and mass flow rate (No derivation, no numerical)

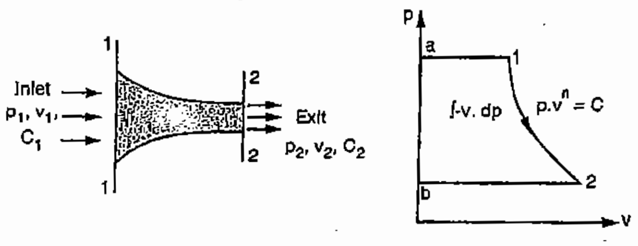
Steam Turbines : Construction and working of Impulse and Reaction steam turbine, velocity diagram, work done efficiencies, Multi-staging, compounding, Degree of reaction, losses in steam turbine, governing of steam turbines

4.1 Steam Nozzles

- In a passage of varying cross-section either the velocity increases or decreases along the section depending upon the inlet conditions of flow.
- A **nozzle** is defined as a passage of uniformly varying cross-section in which the velocity, therefore, the kinetic energy of fluid increases at the expense of its pressure energy.
- In case of nozzles, the flow of fluid takes place at high speeds and the only disturbing influence being the area change, *the process is essentially adiabatic and isentropic in the absence of friction losses without any external work transfers. The changes in potential energy are also negligible.*
- Nozzles are generally used in turbines, jet engines, rockets, injectors etc.

4.2 Velocity and Mass Flow Rate of Steam in a Nozzle

Consider the expansion of steam in a nozzle under steady state conditions between the inlet section (1-1) to any other section (2-2) as shown in Fig. 4.2.1(a) and process is shown in Fig. 4.2.1(b).



(a) Nozzle **(b) (p-v diagram)**

Fig. 4.2.1 : Flow of steam in nozzles

Consider 1 kg mass flow of steam across the nozzle.

Let the inlet conditions be pressure p_1, specific volume v_1 and velocity C_1 and its corresponding exit conditions are (p_2, v_2, C_2).

Steady flow energy equation (S.F.E.E.) per unit mass of steam can be written as :

$$q - w_{sf} = (h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} + g(Z_2 - Z_1) \qquad ...(i)$$

For a nozzle,

(a) Heat transfer, $q = 0$ since the velocity is so high that there is no time available for heat exchange with the surroundings. Therefore, process is adiabatic.

(b) External work transfer, $w_{sf} = 0$.

(c) Change in P.E. is zero since $Z_2 = Z_1$

Substituting the above conditions of $q = 0$, $w_{sf} = 0$ and $Z_2 = Z_1$, in Equation (i), we get,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \qquad ...(ii)$$

Inlet velocity C_1 is called the **velocity of approach**

$$\text{or,} \quad C_2 = \sqrt{2(h_1 - h_2) + C_1^2} \qquad ...(4.2.1)$$

where, **h is in J/kg and velocity C is in m/s.**

In case velocity of approach is negligible, then, $C_2 \simeq 0$. On substituting in Equation (4.2.1),

$$C_2 = \sqrt{2(h_1 - h_2)} \qquad ...(4.2.2)$$

Since the steam expansion is reversible adiabatic or isentropic.

It follows the law $p \cdot v^n = C$ where,

(i) $n = 1.3$, if the steam is initially superheated.

(ii) $n = 1.13$, if the steam is initially wet or dry-saturated.

Note : It should be noted that :

(i) The values of 'n' are approximate and it varies during the expansion process.

(ii) Gas laws are not applicable to steam.

4.2.1 Velocity and Mass Flow Rate of Steam through Nozzle

Velocity of exit,

$$C_2 = \left[2\left(\frac{n}{n-1}\right)p_1 \cdot v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{(n-1)/n}\right\}\right]^{1/2} \quad ...(4.2.3)$$

Mass flow rate from continuity equation, $\dot{m} = \frac{AC}{v}$

Can be written at any section as :

$$\text{Mass flow rate, } \dot{m} = \frac{A_2 \cdot C_2}{v_2} \quad ...(4.2.4)$$

Where, A_2 is the cross-sectional area and v_2 is the specific volume of steam.

Condition for maximum discharge is given by the equation,

Critical pressure ratio,

$$\frac{p_c}{p_1} = \frac{p_t}{p_1} = \left(\frac{2}{n+1}\right)^{n/(n-1)} \quad ...(4.2.5)$$

Maximum mass flow rate,

$$\dot{m}_{max} = A_2 \quad ...(4.2.6)$$

4.2.2 The Value of Adiabatic Expansion Index 'n' and Critical Pressure Ratio

The value of 'n' and $\frac{p_c}{p_1}$ are represented in Table 4.2.1

Table 4.2.1 : The value of 'n' and critical pressure ratio

Condition of steam at inlet	Adiabatic expansion index, n	Critical pressure ratio, $\frac{p_c}{p_1} = \left(\frac{2}{n+1}\right)^{n/(n-1)}$
Wet steam	n = 1.135, value of 'n' varies as n = 1.035 + 0.1 x, where, x is dryness fraction. (Zeuner's equation)	0.5774 ≃ 0.58
Superheated steam	n = 1.3	0.5457 ≃ 0.546

4.3 Types of Nozzles

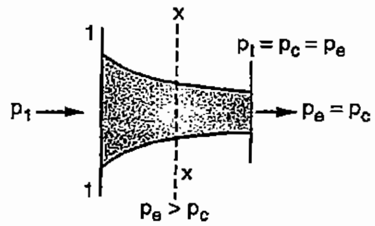
University Question

Q. Explain why subsonic nozzle is convergent while supersonic nozzle is divergent. **SPPU : May 18**

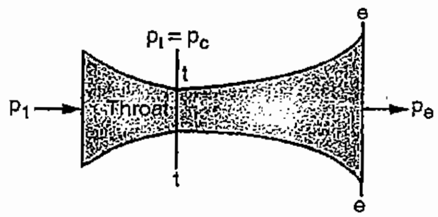
1. Convergent nozzle
2. Convergent – divergent nozzle.

1. Convergent nozzle

- In this type of nozzle, the cross-sectional area of the nozzle decreases from inlet to exit as shown in Fig. 4.3.1(a).
- The minimum exit pressure, p_e can only be maintained equal to critical pressure p_c as defined by Equation (4.2.5) and the nozzle discharges maximum mass flow rate. The velocity at exit equals to sonic velocity.



(a) Convergent nozzle



(b) Convergent-divergent nozzle

Fig. 4.3.1 : Types of nozzles

- If exit pressure $p_e > p_c$, nozzle required will be upto section X-X as shown in Fig. 4.3.1 and the actual mass flow rate is less than maximum mass flow rate with velocity less than sonic velocity.

2. Convergent-divergent nozzle

- In order to accelerate the steam velocity beyond sonic velocity, a convergent-divergent nozzle is needed as shown in Fig. 4.3.1(b).
- The section of minimum area is called throat, where, the throat pressure pt equals to critical pressure pc.
- Exit pressure maintained is less than critical pressure and velocity at exit is more than sonic velocity. Variation of pressure and velocity along the length of nozzle is shown in Fig. 4.3.2.

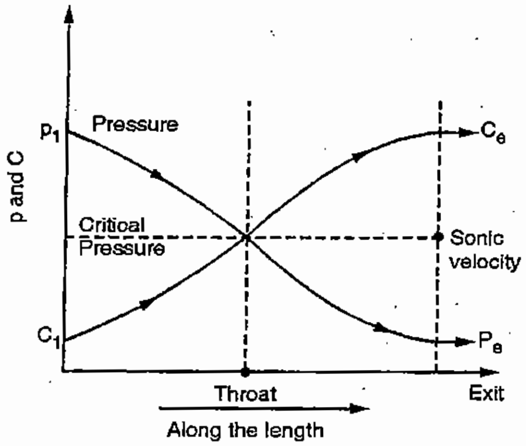


Fig. 4.3.2 : Pressure and velocity variation in a nozzle

4.4 Effect of Friction and Nozzle Efficiency

University Questions

Q. Define the term Nozzle efficiency.

SPPU : April 15 (In Sem), Dec. 16, May 19

Q. Define Nozzle and discuss the effect of friction through convergent – divergent nozzle. With the help of T-S diagram.

SPPU : April 15(In Sem)

- The reversible adiabatic or isentropic expansion without friction is represented by the process (1-2′) on (T-S) diagram in Fig. 4.4.1. However, in an actual expansion process a part of the available isentropic enthalpy drop, (h_1-h_2') of an ideal case is lost in overcoming the frictional losses. The friction in a nozzles are :

  (a) Due to wall friction

  (b) Due to fluid friction

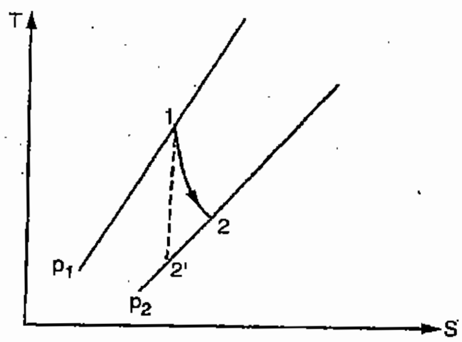


Fig. 4.4.1 : Effect of friction in nozzles

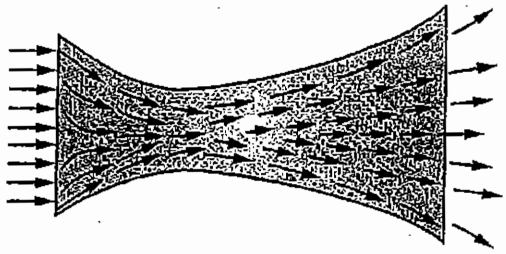


Fig. 4.4.2 : Flow pattern in a nozzle

- The effect of the friction is to increase the entropy of fluid though the process remains adiabatic. Hence, process is not isentropic. The energy lost in overcoming the friction is used to reheat the steam and the actual process is represented by process (1-2) shown in Fig. 4.4.1.
- The flow pattern of fluid in a nozzle is shown in Fig. 4.4.2. It can be observed that the wall friction is small in the convergent portion of the nozzle compared to divergent portion.
- Also, the fluid friction is less in convergent portion of the nozzle compared to divergent portion since the velocity of flow in convergent portion of the nozzle is small.
- If follows that most of the friction occurs only in the divergent portion of the nozzle and the frictional losses in the convergent portion are small.
- The *friction losses in a nozzle depends upon the material, size, shape workmanship of the nozzle, properties of the fluid and flow conditions in the nozzle.*
- We define the **nozzle efficiency**, η_n as the *ratio of actual enthalpy drop to ideal (isentropic) enthalpy drop.* Referring to Fig. 4.4.1.

  Nozzle efficiency,

$$\eta_n = \frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}}$$

or

$$\eta_n = \frac{(h_1 - h_2)}{(h_1 - h_2')} \quad ...(4.4.1)$$

4.4.1 Effect of Nozzle Friction

The effect of friction in a nozzle are :

(i) Reduction in enthalpy drop.

(ii) Reheating of steam i.e. improving the quality of vapour at the exit.

(iii) Reduction in exit velocity.

(iv) Increase in specific volume.

(v) Decrease in mass flow rate.

- If we consider a convergent-divergent nozzle, the (h-S) diagram with friction is as shown in Fig. 4.4.3. It is assumed that the friction losses in convergent portion are negligible and the whole of the friction loss occurs in the divergent portion.
- p_1 represents the pressure at inlet to the nozzle, p_t is the throat pressure which equal to critical pressure defined by the Equation (4.2.5) and p_e is the exit pressure. The nozzle efficiency equation can be written as,

$$\text{Nozzle efficiency, } \eta = \frac{h_1 - h_e}{h_1 - h_e'} \quad ...(4.4.2)$$

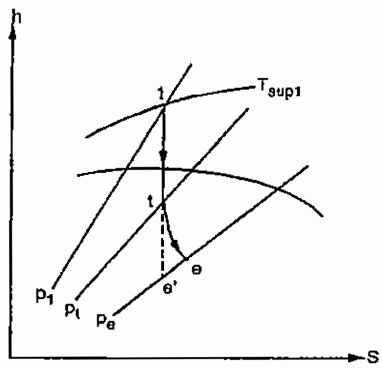


Fig. 4.4.3 : (h-S) diagram with friction

4.4.2 Nozzle or Velocity Coefficient, C_c

It is defined as the ratio of actual exit velocity to the ideal exit velocity. Accordingly,

$$C_c = \frac{\text{Actual velocity}}{\text{Ideal velocity}} \quad ...(4.4.3)$$

$$\text{i.e. } C_c = \sqrt{\frac{\text{Actual enthalpy drop}}{\text{Isentropic enthalpy drop}}} = \sqrt{\frac{h_1 - h_2}{h_1 - h_2'}} \quad ...(4.4.4)$$

$$\therefore \quad C_c = \sqrt{\text{Nozzle efficiency } \eta_n} \quad ...(4.4.5)$$

4.4.3 Coefficient of Discharge, C_d

It is defined as the ratio of actual mass flow rate, $\dot{m}_a$ to the ideal mass flow rate, $\dot{m}_i$.

$$\text{Therefore,} \quad C_d = \frac{\dot{m}_a}{\dot{m}_i} \quad ...(4.4.6)$$

4.5 Method of Solving Problems on Nozzles

For the known inlet conditions of steam p_1 and T_{sup_1} or p_1 and x_1 and given exit pressure p_e, following procedure is to be followed :

1. Type of nozzle required

Determine critical pressure p_c with the help of Equation (4.2.5) (Refer Table 4.2.1).

$ap_c = 0.546\, p_1$ (if the steam is initially superheated)

$p_c = 0.58\, p_1$ (if steam is initially wet)

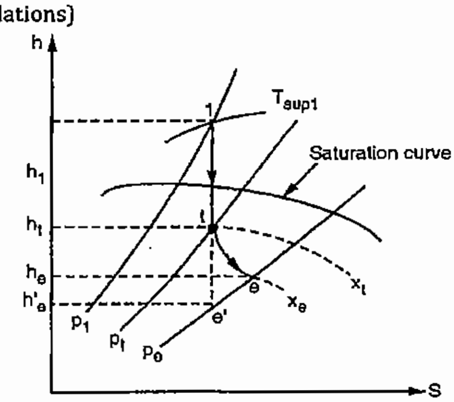
(a) In case $p_e \geq p_c$, a convergent nozzle is needed.

(b) In case $p_e < p_c$, a convergent - divergent nozzle is needed with throat pressure

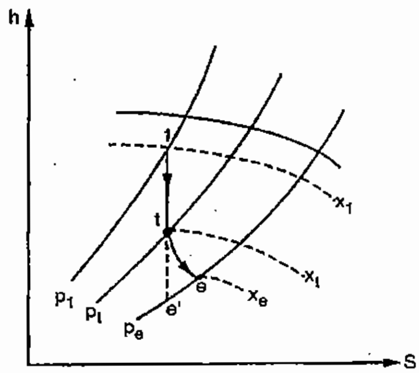
p_t = critical pressure p_c.

2. Determination of velocity at throat and exit

With the help of Mollier's diagram, draw isentropic processes (1-t-e') shown in Fig. 4.5.1. (Problem can also be solved by calculations)



(a) Initially with superheated steam



(b) Initially with wet steam

Fig. 4.5.1

(a) Read h_1, h_t and h_e'.

(b) (i) Using, $\eta_n = \dfrac{h_1 - h_e}{h_1 - h_e'}$ find h_e.

(ii) Plot of h_e and $p'_e = p_e$, fix the point e on Mollier's diagram and read x_e.

(c) Read v_{gt} against p_t and v_{ge} against p_e from steam tables. Then specific volume at throat and exit will be : $v_t = x_t \cdot v_{gt}$ and $v_e = x_e \cdot v_{ge}$.

(d) Since, $C_2 = \sqrt{2\,(h_1 - h_2)}$

[According to Equation (4.2.2)]

If inlet velocity C_1 is negligible.

$$C_2 = \sqrt{2 \times 1000\,(h_1 - h_2)} = 44.7\sqrt{(h_1 - h_2)} \quad ...(4.5.1)$$

Where, h is measured in kJ/kg.

Hence,

Velocity of steam at throat,

$$C_t = 44.7\sqrt{(h_1 - h_t)}$$

Velocity of steam at exit,

$$C_e = 44.7\sqrt{(h_1 - h_e)}$$

3. Dimension of nozzles

- Throat and exit diameters can be calculated by using continuity equation as follows :
- Mass flow rate,

$$\dot{m} = \frac{A_t \times C_t}{v_t} = \frac{A_e \cdot C_e}{v_e}$$

Where, $v_t = x_t \cdot v_{gt}$

and $v_e = x_e \cdot v_{ge}$ ...(4.5.2)

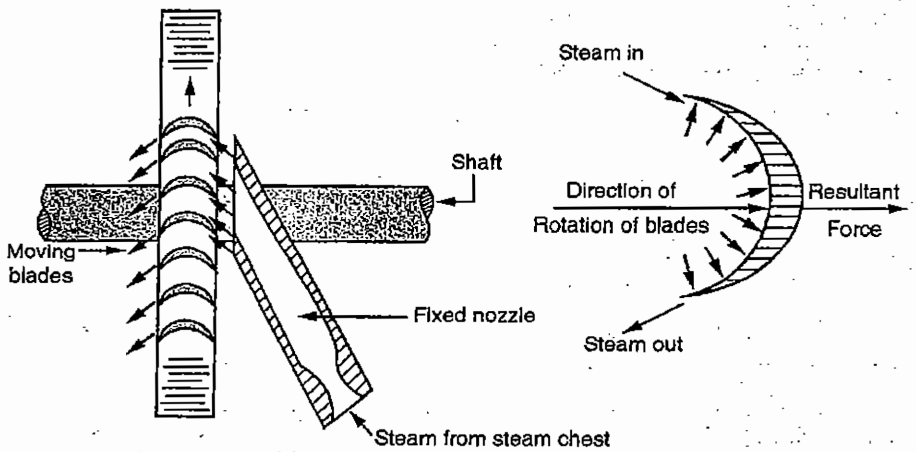
4.6 Introduction to Steam Turbines

University Question

Q. Explain the essential differences in the manner of expansion of steam in impulse and reaction turbines. Illustrate your answer by sketches of pressure, velocity and specific volume changes, which occur as the steam passes over successive blades.

SPPU : May 12, Dec. 12

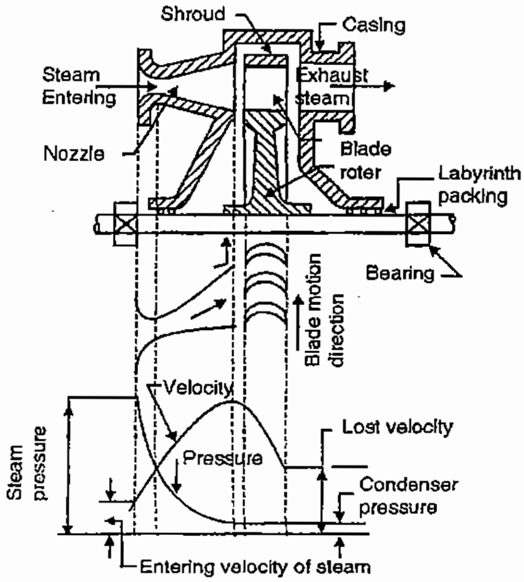
- A ***steam turbine*** is rotary machine which is designed to convert the energy of high pressure and high temperature steam into mechanical power. The operation of steam turbine wholly depends upon the dynamic action of the steam.
- In this the steam is first expanded in a set of nozzles or passages upto exit pressure where in the pressure energy of the steam is converted into kinetic energy.
- The nozzles are fixed to the casing. If the resultant high velocity steam is passed over the curved vanes, or **blades**, the steam changes its direction and it would leave in the direction shown in Fig. 4.6.1(a).
- Due to this there is a change in momentum and it will exert a resultant force on the blades as shown. If these blades are attached to a disc on a **rotor or shaft** which is free to rotate, the resultant force would cause the rotor to rotate. Thus the motive power is developed. The principle of operation of steam turbine is shown in Fig. 4.6.1(b).



(a) (b)

(a)

Fig. 4.6.1 : Conted...



(c)

Fig. 4.6.1 : Principle of operation of steam turbine

- A pair of **ring of nozzles (fixed blades)** fixed to the casing and a **ring of moving blades** fixed to the turbine rotor is called a **stage or a turbine pair.** Both the fixed and moving blades are so designed that the steam jet shall not strike the blades but it should glide over in the direction of blade surfaces.

4.7 Types of Steam Turbines

University Question

Q. Explain the classification of steam turbine.

SPPU : May 11

These are mainly of two types :

(a) Impulse turbine

- These turbines use the principle of impulse in which the kinetic energy of steam obtained after passing over a ring of fixed nozzles is used to exert a force on a ring of moving blades. [Refer Fig. 4.6.1(c)]
- The pressure of steam while passing over the moving blades remain constant (neglecting losses) and its kinetic energy is converted into mechanical work. The examples of impulse turbines, are De-Laval, Curtis and Rateau etc.

(b) Reaction turbines

- In case of a reaction turbines, there is a continuous pressure drop of steam while passing over the rings of fixed and moving blades.
- Accordingly, the moving blade passages are suitably designed for steam to expand, therefore, these blades also act as nozzles.
- The reactive force along with that due to change in momentum of the steam provides the motive force for the turbine to develop power. It follows that these turbines are basically the impulse-reaction turbine but in practice these are called reaction turbines. Example of such a turbine is Parson's reaction turbine.
- Mostly the steam turbines are axial flow type in which the steam flows over the blades in direction parallel to the axis of turbine rotor.
- The only important radial flow turbine is Lungstorm reaction turbine in which the steam enters at the blades tip nearest to the axis of the wheel and flows towards the circumference.

4.8 Advantages of Steam Turbines

The main advantages of steam turbines are :

1. It is a rotary high speed machine.
2. It is compact and it has low weight to power ratio.
3. It has perfect balance and runs vibration free.
4. It needs less floor area.
5. It has low initial and maintenance cost.
6. It needs no internal lubrications, therefore, its condensate is not contaminated and it is suitable as feed water.
7. It is suitable for electrical generators.

4.8.1 Application of Steam Turbines

University Question

Q. Give a list of types of steam turbines used in various applications.

SPPU : May 13

Steam turbines are used for generation of power from small to large scale both for industry and for power grid. In addition the steam turbines are used in process industry both for capative power generation and process heating.

Velocity compounded impulse turbines are suitable for low to medium power generation with process heating because its cost is low, requires less space having ease of operation and low maintenance.

The reaction turbines are efficient, thus these are found suitable for large power generation.

4.8.2 Methods of Improving Efficiency of Turbines

The efficiency of turbines can be increased in the following ways :

(i) Reheating of steam in between the stages.

(ii) Bleeding of steam for feed water heating in between the stages.

(iii) Utilising the low pressure steam for process industry.

(iv) Use of nozzle governing to improve part load efficiency in case of impulse turbines.

(v) Use of by pass or both throttle and by pass governing to improve efficiency at part loads in case of impulse turbines.

4.9 Classification of Steam Turbines

University Question

Q. Explain the classification of steam turbine. **SPPU : May 11, Dec. 13**

Q. How are steam turbines classified ? **SPPU : May 13**

Steam turbines can be classified as follows :

1. According to principle of action of steam :
   (a) Impulse
   (b) Reaction
2. According to direction of steam flow :
   (a) Axial
   (b) Radial
   (c) Tangential
3. According to number of pressure stages :
   (a) Single stage with one or more velocity stages.
   (b) Multistage
4. According to method of governing :
   (a) Throttle
   (b) Nozzle
   (c) By-pass
   (d) Combination of Throttle-bypass or Nozzle-bypass
5. According to heat drop process :
   (a) Non-condensing
   (b) Condensing
   (c) Regeneration
6. According to steam conditions at inlet :
   (a) Low pressure upto 2 bar
   (c) High pressures above 50 bar
   (b) Medium pressure upto 50 bar
   (d) Supercritical pressures above 225 bar.
7. According to their usage :
   (a) Stationary with constant speed
   (b) Stationary with variable speed
   (c) Non-stationary used in steamers, ships, railway locomotives etc.

4.10 Compounding of Steam Turbines

University Questions

Q. What is compounding in steam turbine ? **SPPU : May 11, Dec. 13, April 17 (In Sem)**

Q. Write short notes on necessity and methods of compounding of steam turbines. **SPPU : May 14**

Q. What is compounding ? Explain need of compounding. State the methods of compounding. **SPPU : April 15 (In Sem)**

Q. Explain the need of compounding in steam turbines. **SPPU : Feb 16(In Sem), May 19**

Q. State the different methods of compounding of steam turbine. **SPPU : Dec. 19**

- With the advent of high pressure steam boilers, the total pressure drop from boiler pressure to condenser pressure in a ring of fixed blades (nozzles) due to expansion of steam will cause a high velocity steam at its exit.
- If this high velocity steam is expanded over the single ring of moving blades to develop maximum power, the speed of the rotor may be high as 30000 r.p.m. which is too high for practical purposes.
- Therefore, the various methods are adopted to absorb either the steam pressure or the jet velocity in stages so as to reduce the rotor speed.
- *The method of reducing the rotor speed is called the **compounding of steam turbine.*** It should be noted that each method of compounding consists of multiple rotor system keyed to the shaft with number of stages arranged in series.

- Following are the methods of compounding of impulse turbines.

4.10.1 Velocity Compound Impulse Turbine (Curtis Turbine)

University Question

Q. Discuss any one method of compounding of steam turbines. **SPPU : April 17(In Sem)**

- In this type of turbine the steam expands in a set of nozzles, from the boiler pressure upto the condenser pressure, which converts its pressure energy into kinetic energy.
- This high velocity steam is passed over the rings of moving blades, each ring of moving blades being separated by a ring of fixed blades as shown in Fig. 4.10.1.

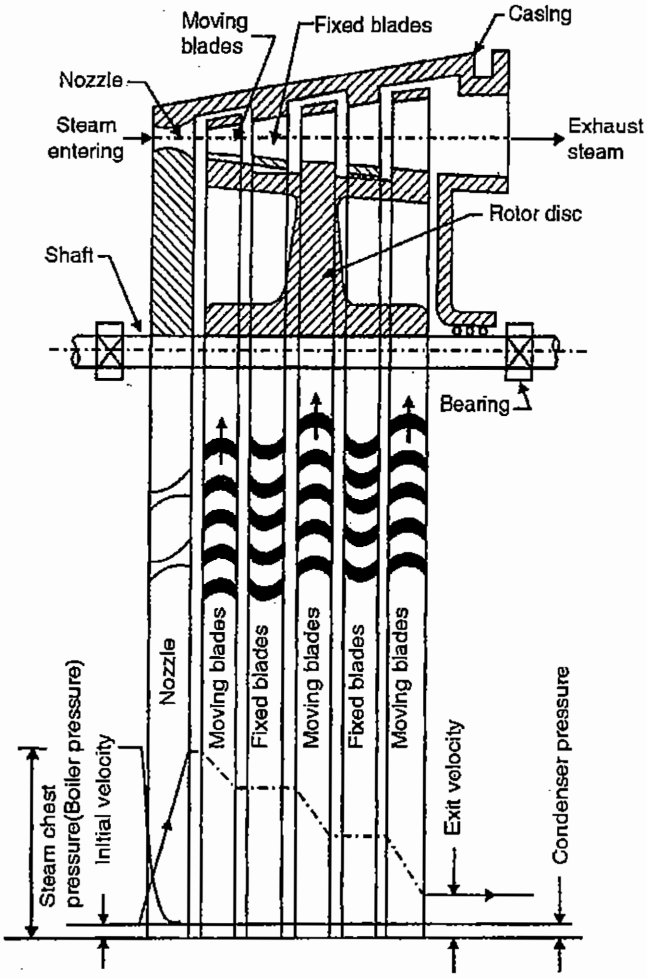


Fig. 4.10.1 : Diagrammatic arrangement of velocity compounded impulse turbine

- A part of high velocity steam is absorbed in the first ring of moving blades and the remainder is passed to the next ring of fixed blades.
- The function of fixed blades is to change the direction of flow of steam so that it can glide over the second ring of moving blades.
- The velocity of steam while passing over the fixed blades is constant except for the energy lost for overcoming the friction losses.
- Again a part of the steam velocity is absorbed in the second ring of moving blades and the process of absorbing the steam velocity continues till it is finally wasted to exhaust.
- Fig. 4.10.1 represents the variation of pressure and velocity of steam passing over the sets of fixed and moving blades along the axis of the turbine.

Advantages

1. Due to relatively large heat drop, a velocity compounded impulse turbine requires comparatively small number of stages and less space.
2. Optimum blade speed ratio decreases with increase in number of stages.
3. Cost of turbine is low.
4. The pressure drop takes place in nozzles, hence turbine casing need not be designed to withstand high pressures.

Disadvantages

1. Friction losses are large since the velocity of steam is high.
2. Its efficiency is low and keeps on decreasing with number of stages.
3. Power developed in later stages keeps on decreasing while these stages require same material space and cost of fabrication.

4.10.2 Pressure Compounded Impulse Turbine (Rateau Turbine)

University Questions

Q. Explain the pressure compounded impulse turbine showing pressure and velocity variations along the axis of the turbine. **SPPU : Dec. 18**

Q. Discuss any one method of compounding in steam turbines. **SPPU : May 19, Dec. 19**

- In this type of turbine the total pressure drop does not take place in a single ring of nozzles but it is divided up in between the set of nozzles rings.

- Steam from boiler is partially expanded in the first ring of nozzles and then it is passed over the ring of moving blades till its velocity is absorbed.
- Exhaust from the moving blade ring is passed over the second ring of nozzles for further expansion partially and its increased velocity is absorbed in the second ring of moving tables.
- Similar process is repeated till the steam is expanded upto the condenser pressure.
- Since the steam is partially expanded in each ring of nozzles, the velocity of steam would not be very high, hence, the turbine velocity would be low. The pressure and velocity variations of steam along the axis of the turbine is shown in Fig. 4.10.2.

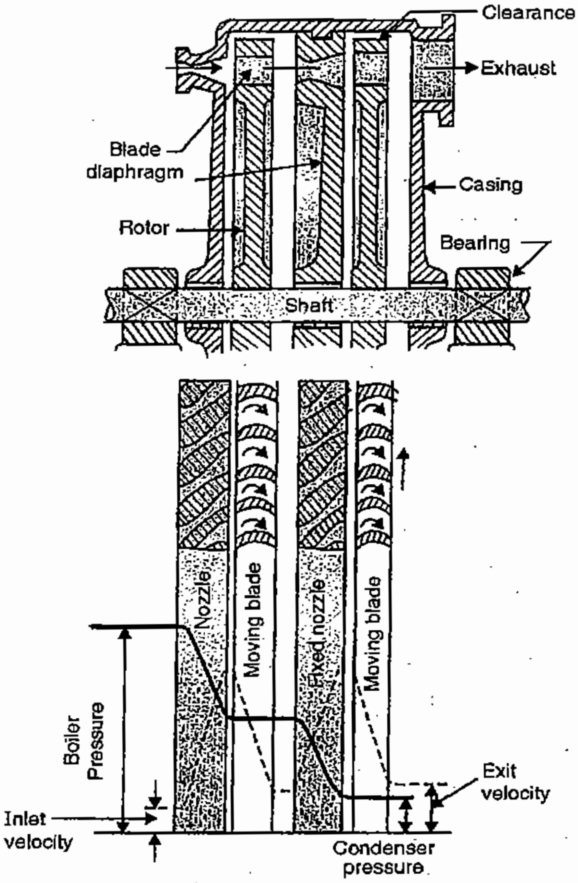


Fig. 4.10.2 : Pressure compounded impulse turbine

4.10.3 Pressure - Velocity Compounded Impulse Turbine

- In this method, both the previous methods of velocity and pressure compounding are utilized.
- The total pressure drop of steam is divided into stages and velocity of steam obtained due to expansion in each stage is also compounded.
- Since the number of stages are less we get a more compact turbine than a pressure compounded turbine.
- This method is well illustrated in Fig. 4.10.3 with pressure-velocity variations along the axis of the turbine.

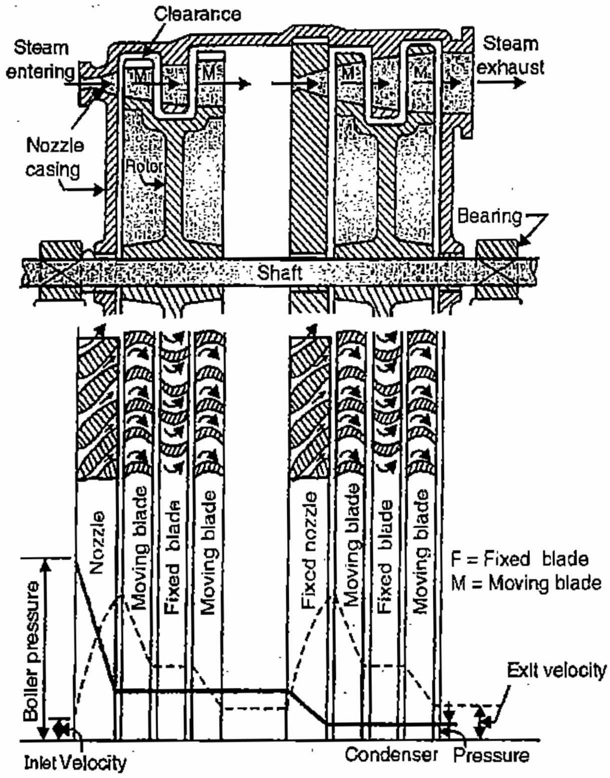


Fig. 4.10.3 : Pressure-velocity compounded impulse turbine

4.10.4 Reaction Turbine

- Unlike impulse turbines nozzles are not provided in this turbine and, also there is a continuous pressure drop in the rings of fixed and moving blades.
- The function of fixed blades, which also act nozzles, is to change the direction of steam so that it can enter into the next ring of moving blades without shock.
- The term reaction is used because the steam expands over the ring of moving blades giving a reaction on moving blades also.
- The variations of pressure and velocity for this type of turbine are shown in Fig. 4.10.4.

– The steam velocity and the turbine velocity is not high because the steam is continuously expanding.

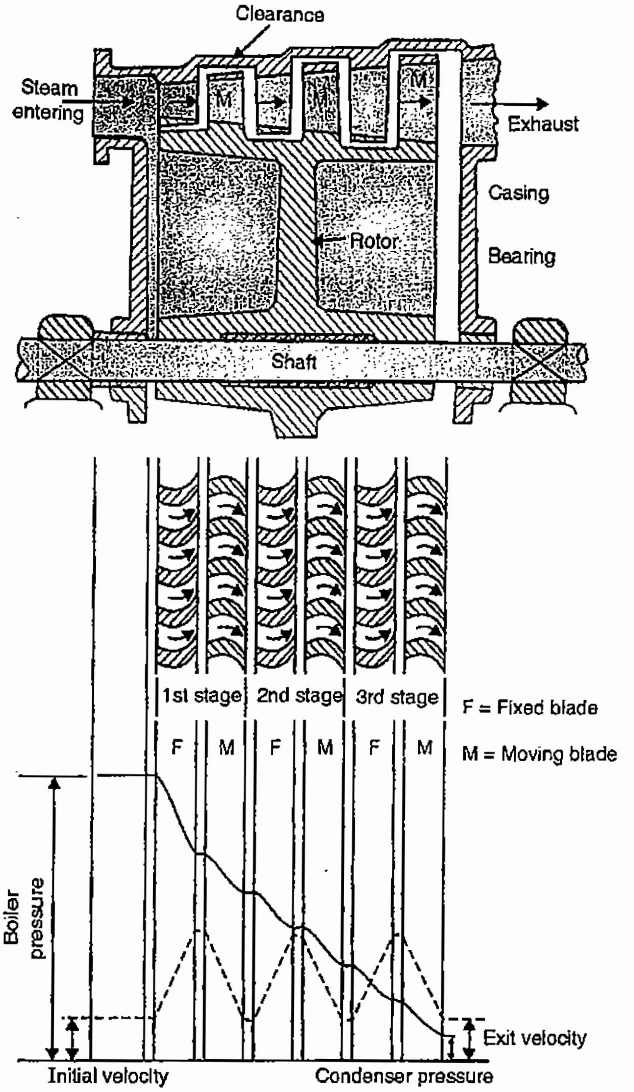


Fig. 4.10.4 : Reaction turbine

– The diameter of the turbine must increase after each set of blade rings in order to cater for the increased specific volume of steam at lower pressure in each of the successive stages.

4.11 Impulse Turbines

4.11.1 Velocity Diagram for Moving Blades

In order to determine the power developed by the turbine it is necessary to determine the rate of change of momentum of steam across the moving blades.

This is done by drawing the velocity diagram at inlet and exit of the moving blades by which the relative velocities can be evaluated correspondingly.

Following are the notations used to deal with turbine theory.

C_b = Linear velocity of blade, m/s

C_i = Absolute velocity of steam at inlet to moving blades i.e. exit velocity of steam from nozzles or fixed blades, m/s

C_o = Absolute velocity of steam at exit to moving blades i.e. the inlet velocity of steam to second ring of nozzles or fixed blades, m/s

C_{wi} = Tangential component of C_i called as velocity of whirl at inlet to moving blades, m/s

C_{wo} = Velocity of whirl at exit of moving blades, m/s

C_{ri} = Relative velocity of steam to moving blades at inlet, m/s

C_{ro} = Relative velocity of steam to moving blades at outlet, m/s

C_{fi} = Axial component of C_i called as velocity of flow at inlet, m/s

C_{fo} = Velocity of flow at exit, m/s

α = Exit angle of nozzles or the angle to the tangent of wheel with which the steam at C_i enters the moving blades, degrees

β = Inlet angle of fixed blades or the angle to the angent of wheel at which the steam at C_o leaves the moving blades, degrees

θ = Inlet angle of moving blades, degrees

ϕ = Exit angle of moving blades, degrees

$\dot{m}$ = Mass flow rate of steam, kg/s

K = Blade velocity coefficient or friction factor

$= \dfrac{C_{ro}}{C_{ri}}$

K = 1, if friction is neglected

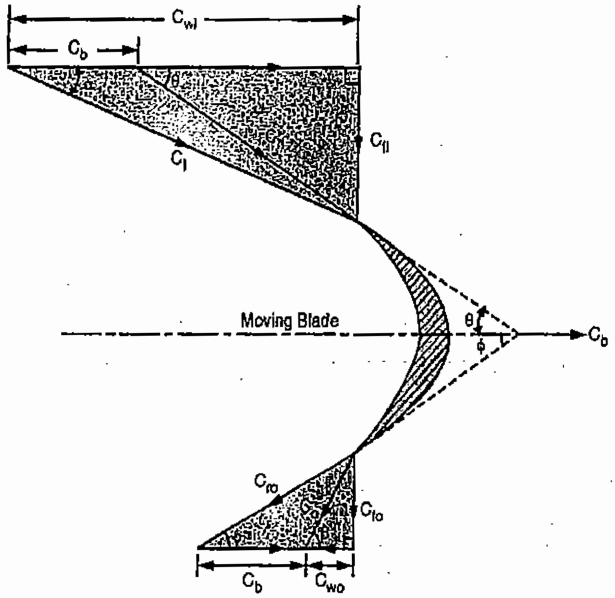
d = Drum diameter, m

N = Speed of turbine, r.p.m.

$$\text{Blade velocity, } C_b = \frac{\pi \cdot d \cdot N}{60} \qquad \text{...(4.11.1)}$$

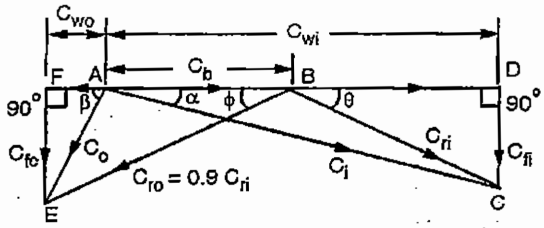
– Fig. 4.11.1(a) represents the velocity diagram for the moving blade of an impulse turbine.

- The jet of steam from nozzle impinges at entrance to the moving blade at an angle α to the plane of rotation. The blade velocity is 'C_b'.
- The tangential component of steam velocity C_i contributes towards the work since it is in the direction of blade velocity vector, C_b.
- The relative velocity of steam to the blade at inlet is C_{ri} which makes an angle θ with the plane of rotation of the wheel. The inlet tip angle of the moving blade must be θ so that the steam enters the moving blades without shock. *The axial component C_{fi} of C_i is responsible for the flow of steam in axial direction.*
- If there is no friction between the moving blade surface and the steam, relative velocity of steam while gliding over its surface will remain constant i.e. $C_{ro} = C_{ri}$.
- The absolute velocity of steam at exit C_o can be determined by taking the vector sum of blade velocity and the relative velocity at exit. It is inclined at an angle β to the plane of rotation of wheel with which it enters into the second ring of fixed blades.
- The components of C_o in tangential and axial directions are represented by C_{wo} and C_{fo} respectively.
- For convenience the combined velocity diagram for inlet and exit of moving blades can be drawn as represented in Fig. 4.11.1(b) to the common base 'C_b'.



(a) Inlet and exit velocity diagram

Fig. 4.11.1(Contd...)



(b) Combined velocity diagram

Fig. 4.11.1

- The procedure to be followed for drawing the velocity diagram in case of more than one stage is identical.

4.11.2 Calculations for Work, Power and Efficiencies

(i) Work and power

From second Newton's Law, the force, F in the direction of moving blades is given by,

$$F = \text{Rate of change of momentum}$$
$$= \text{Mass flow rate of steam} \times (\text{Change in velocity of whirl})$$
$$= \dot{m}\left(\vec{C}_{wi} - \vec{C}_{wo}\right)$$

It could be noticed from Fig. 4.11.1(b) that the velocity of whirl at exit, C_{wo} is in the opposite direction to that of C_{wi}. Hence, C_{wo} is actually negative corresponding to the direction of blade velocity, C_b. Therefore, the total velocity of whirl, C_w becomes the algebraic sum of the two velocities of whirl.

i.e. $C_w = C_{wi} + C_{wo}$

Hence, $$F = \dot{m}\,(C_{wi} + C_{wo}) = \dot{m} \cdot C_w \quad ...(i)$$

$\therefore$ Work done on the moving blades

= Force × Distance moved per second

or, $W = (\dot{m} \cdot C_w)\, C_b$

Power developed,

$$P = \frac{\dot{m}\, C_w C_b}{1000}\ \text{kW} \quad ...(4.11.2)$$

(ii) Diagram or blade efficiency

University Questions

Q. Define the term Blade efficiency. **SPPU : April 15 (In Sem), Dec. 16**

Q. Define the term Diagram efficiency. **SPPU : April 17(In Sem), May 19**

The energy supplied per stage of an impulse turbine is equal to kinetic energy given by $\left(\frac{\dot{m}\,C_i^2}{2}\right)$ and assuming that the kinetic energy leaving the stage is wasted. We can define the **blade efficiency** as,

$$\eta_b = \frac{\text{Workdone on the blades}}{\text{K.E. supplied to the blade}}$$

$$= \frac{(\dot{m} \cdot C_w \cdot C_b)}{\left(\dot{m} \cdot C_i^2/2\right)} = \frac{2C_w \cdot C_b}{C_i^2} \quad ...(4.11.3)$$

– Blade efficiency can also be defined as,

$$\eta_b = \frac{\text{(Change in K.E. of steam)}}{\text{(K.E. supplied)}}$$

$$= \frac{\frac{C_i^2}{2} - \frac{C_o^2}{2}}{\frac{C_i^2}{2}} = \frac{C_i^2 - C_o^2}{C_i^2} \quad ...(4.11.4)$$

Blade efficiency will be **maximum** when C_o is minimum i.e. when **β = 90°** or the **discharge is axial.**

(iii) Stage efficiency

University Question

Q. Define the term Stage efficiency.

SPPU : April 15 (In Sem), April 17(In Sem)

A stage is defined as the combination of a ring of nozzles (fixed blades) and a ring of moving blades. The energy supplied corresponds to the isentropic heat drop, Δ H in the nozzles. The stage efficiency, η_s is given by,

$$\eta_s = \frac{\text{Workdone}}{\text{Energy supplied per stage}}$$

$$= \frac{(\dot{m} \cdot C_w \cdot C_b)}{(\Delta H \times 1000)} = \frac{C_w \cdot C_b}{\Delta h \times 1000} \quad ...(4.11.5)$$

where Δ H is in kJ and it equal to $\dot{m}$ (Δ h) where (Δ h) is the isentropic enthalpy drop per kg of steam in kJ/kg.

The stage efficiency becomes equal to the blade efficiency if there are no friction losses in the nozzles. The expression (4.11.5) can therefore be written as,

$$\eta_s = \eta_n \times \eta_b \quad ...(4.11.6)$$

(iv) Axial thrust

The axial thrust on the wheel is due to the force exerted as a result of the rate of change of momentum in axial direction.

$\therefore$ Axial thrust, F_a = Mass of flow rate × Change in axial velocity

$$= \dot{m}\left(\vec{C}_{fi} - \vec{C}_{fo}\right) \quad ...(4.11.7)$$

In order to take this on thrust on the wheel, the thrust bearings are provided in the turbines.

4.11.3 Definitions : Blade Velocity Coefficient and Blade Speed Ratio

1. Blade velocity coefficient (K)

University Question

Q. Define the term Blade velocity coefficient.

SPPU : April 17(In Sem)

In an impulse turbine, if the friction is neglected, the relative velocity of steam C_{ro} at outlet of moving blades equals to relative velocity of steam C_{ri} at inlet to moving blades i.e. $C_{ro} = C_{ri}$.

Effect of friction is to reduce relative velocity of steam as it passes over the ring of moving blades.

The friction loss is 10 to 15%. Therefore, C_{ro} is less than C_{ri}. Therefore,

Blade velocity coefficient,

$$K = \frac{\text{Relative velocity at exit of moving blade, } C_{ro}}{\text{Relative velocity at inlet to moving blade, } C_{ri}} \quad ...(4.11.8)$$

$$\therefore \quad \mathbf{C_{ro} = K \cdot C_{ri}} \quad ...(4.11.9)$$

2. Blade speed ratio (s)

University Question

Q. Define the term Speed ratio.

SPPU : April 17(In Sem)

Blade speed ratio,

$$\boxed{s = \frac{\text{Blade velocity}, C_b}{\text{Steam velocity at inlet}, C_i}} \quad ...(4.11.10)$$

3. Carry over coefficient (K_c)

A part of K.E. is only converted due to steam velocity C_i at inlet in a stage and the K.E. leaving the stage corresponds to C_o at exit of stage. This K.E. is utilised in subsequent stages for conversion into mechanical work. However, a part of energy is lost in passages before entry into nozzles.

This fraction of K.E. corresponding to C_o which will be available for utilization in nozzles of the succeeding stages is called **Carryover Coefficient**. It's value ranges from 0.9 to 0.95.

$$K_c = \frac{\text{Available K.E. } (C_o')^2/2}{\text{K.E. at exit } (C_o)^2/2} \quad ...(4.11.11)$$

4.12 Condition for Maximum Diagram Efficiency for Impulse Turbines

University Question

Q. Derive an expression for maximum blade efficiency in a single stage impulse turbine. **SPPU : Dec. 12**

Refer Fig. 4.11.1(b).

Workdone per kg of steam,

$$W = (C_{wi} + C_{wo})\, C_b$$

$$= C_b\left[(C_{ri}\cos\theta + C_b) + (C_{ro}\cdot\cos\phi - C_b)\right]$$

$$= C_b\cdot C_{ri}\cos\theta\left(1 + \frac{C_{ro}\cos\phi}{C_{ri}\cos\theta}\right)$$

$$= C_b\,(C_i\cos\alpha - C_b)\,(1 + KC) \quad ...(i)$$

where, K = Friction factor $= \dfrac{C_{ro}}{C_{ri}}$ ...(ii)

$$C = \frac{\cos\phi}{\cos\theta};\ \left(\begin{array}{l}\text{a constant for}\\ \text{given moving blades}\end{array}\right) \quad ...(iii)$$

Blade speed ratio, $s = \dfrac{C_b}{C_i}$ ...(iv)

Blade efficiency, $\eta_b = \dfrac{\text{workdone per kg of steam}}{\text{Energy supplied to blade per kg of steam}}$

$$= \frac{C_b\,(C_i\cos\alpha - C_b)\,(1 + KC)}{\frac{C_i^2}{2}}$$

$$= 2\left(\frac{C_b}{C_i}\cos\alpha - \frac{C_b^2}{C_i^2}\right)(1 + KC)$$

$$= 2\,(s\cos\alpha - s^2)\,(1 + KC) \quad ...(v)$$

Effect of nozzle angle

From Equation (i) above, it is evident that the work would be maximum if $\cos\alpha = 1$ i.e. $\alpha = 0°$ if other parameters like C_b, C_i, K, θ and ϕ are fixed.

In such a case, the axial component of velocity C_{fi} will be zero, therefore, it will not be possible to carry the steam to next ring of blades.

For a given mass flow rate of steam, the annulus area will decrease with the increase in nozzle angle α since the axial component of velocity $C_{fi} = C_i\cdot\sin\alpha$ increases and the work developed by the turbine reduces.

Reduction in area will also increase the friction losses in the blade passages.

In practice to meet these conflicting requirements of the turbine, the nozzle angles are usually kept in the range of 15° to 25°.

From Equation (v), it is clear that the diagram efficiency depends upon the nozzle exit angle α, the blade speed ratio s, the friction factor K and the moving blade inlet and exit angels θ and ϕ respectively.

If α, K and C are kept constant, the diagram efficiency will depend upon the blade speed ratio.

For maximum efficiency, differentiating the expression for efficiency given by Equation (v) with respect 's' and equating to zero, we get,

$$\frac{d(\eta_b)}{d(s)} = 0$$

i.e. $2\,(1 + KC)\,(\cos\alpha - 2s) = 0$

or, Optimum blade speed ratio,

$$(s)_{optimum} = \frac{\cos\alpha}{2} = \frac{C_b}{C_i} \quad ...(4.12.1)$$

Substituting $(s)_{opt}$ from Equation (4.12.1) in Equation (v), we get,

$$(\eta_b)_{max} = 2\left[\frac{\cos\alpha}{2}\cdot\cos\alpha - \left(\frac{\cos\alpha}{2}\right)^2\right](1 + KC)$$

$$= \frac{\cos^2\alpha}{2}\,(1 + KC) \quad ...(4.12.2)$$

For a De-Laval turbine neglecting friction i.e. K = 1 and for symmetrical blades i.e. $\theta = \phi$ hence, C = 1

$$(\eta_b)_{max} = \cos^2\alpha \quad ...(4.12.3)$$

– Expression for maximum work becomes,

$$W_{max} = \frac{C_i\cos\alpha}{2}\left(C_i\cos\alpha - \frac{C_i\cos\alpha}{2}\right)(1 + KC)$$

$$= \frac{C_i^2\cos^2\alpha}{4}\,(1 + KC) \quad ...(4.12.4)$$

In case K = 1, C = 1 (blades symmetrical) and $C_i = 2\,C_b/\cos\alpha$. Then,

$$W_{max} = 2\cdot C_b^2 \quad ...(4.12.5)$$

Ex. 4.12.1 : Steam enters an impulse wheel having a nozzle of 20° at a velocity of 450 m/s. The exit angle of the moving blade is 20° and relative velocity of steam may be assumed to remain constant over the moving blades. If the blade speed is 180 m/s and mass flow rate of steam is 2.5 kg/s, determine :

(i) Blade angle at inlet

(ii) Work done per kg of steam

(iii) Total power developed by the turbine

(iv) Diagram efficiency **SPPU - Dec. 19, 10 Marks**

Soln. :

Refer Fig. P. 4.12.1

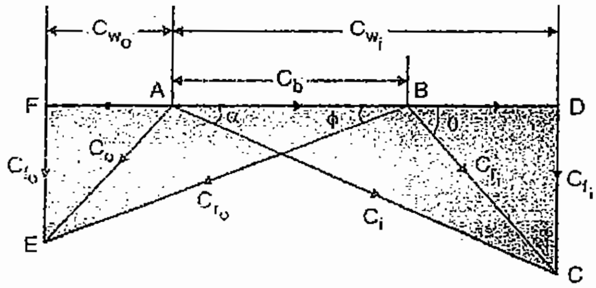


Fig. P. 4.12.1

$\alpha = 20°$, $C_i = 420$ m/s,

$\phi = 20°$, $C_{ro} = C_{ri}$

$C_b = 180$ m/s, $\dot{m} = 2.5$ kg/s

(i) Blade angle at inlet, θ

Consider inlet Δ ACD :

$$C_{wi} = C_i \cos\alpha = 450 \cos 20 = 422.86 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 450 \sin 20 = 153.91 \text{ m/s}$$

$$BD = C_{wi} - C_b = 422.86 - 180$$

$$= 242.86 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{153.91}{242.86}\right)$$

$$= \mathbf{32.36°} \quad \text{... Ans.}$$

Also, $$C_{ri} = \sqrt{(C_{fi})^2 + (BD)^2}$$

$$= \sqrt{(153.91)^2 + (242.86)^2}$$

$$= 287.52 \text{ m/s}$$

$$\therefore \; C_{ro} = C_{ri} = 287.52 \text{ m/s}$$

(ii) Workdone/kg of steam, W

Consider outlet Δ BEF :

$$C_{wo} = AF = FB - C_b = C_{ro}\cos\phi - C_b$$

$$= 287.52 \cos 20 - 180 = 90.18 \text{ m/s}$$

$$W = \frac{(C_{wi} + C_{wo})\, C_b}{1000} \text{ kJ/kg}$$

$$= \frac{(422.86 + 90.18)180}{1000}$$

$$= \mathbf{92.3472 \text{ kJ/kg}} \quad \text{... Ans.}$$

(iii) Total power developed, $\dot{W}$

$$\dot{W} = \dot{m}\, W = 2.5 \times 92.3472$$

$$= \mathbf{230.868 \text{ kW}} \quad \text{... Ans.}$$

(iv) Diagram efficiency, η_b

$$\eta_b = \frac{W}{C_i^2/2} = \frac{2W}{C_i^2} = \frac{2 \times (92.3472 \times 1000)}{(450)^2}$$

$$= \mathbf{0.9121 \text{ or } 91.21\%} \quad \text{... Ans.}$$

Ex. 4.12.2 : Steam issues from the nozzles at an angle of 20° at a velocity of 440 m/s, the friction factor is 0.9 for a single stage turbine designed for a maximum efficiency determine (i) Blade velocity (ii) moving blade angles for equiangular blades (iii) Blade efficiency (iv) stage efficiency if the nozzle efficiency is 93% and power developed for mass flow rate of 3 kg/s. **SPPU : Dec. 17, May 19, 8 Marks**

Soln. :

Refer Fig. P. 4.12.2

$\alpha = 20°$, $C_i = 440$ m/s,

Friction factor, $K = \frac{C_{ro}}{C_{ri}} = 0.9$

(i) Blade velocity, C_b

Since the turbine is designed for maximum efficiency, the condition is,

$$s = \frac{C_b}{C_i} = \frac{\cos\alpha}{2};$$

$$\therefore \; C_b = \frac{C_i}{2}\cos\alpha = \frac{440}{2} \times \cos 20$$

$$= \mathbf{206.7 \text{ m/s}} \quad \text{...Ans.}$$

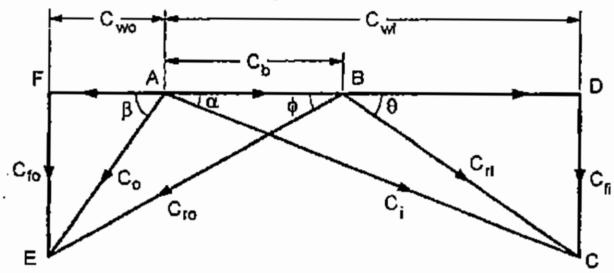


Fig. P. 4.12.2

(ii) Moving blade angles for equiangular blades i.e. $\theta = \phi$

Consider inlet velocity Δ ACD,

$$C_{wi} = C_i \cos\alpha = 440 \cos 20 = 413.5 \text{ m/s}$$

$$BD = C_{wi} - C_b = 413.5 - 206.7 = 206.8 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 440 \sin 20 = 150.5 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{150.5}{206.8}\right)$$

$$= \mathbf{36.04^\circ} \qquad \text{...Ans.}$$

$$\phi = \theta = \mathbf{36.04^\circ} \quad \text{(Being equiangular blades)} \qquad \text{...Ans.}$$

(iii) Blade efficiency, η_b

$$C_{ri} = \sqrt{(BD)^2 + C_{fi}^2} = \sqrt{(206.8)^2 + (150.5)^2}$$

$$= 255.8 \text{ m/s}$$

$$C_{ro} = K\,C_{ri} = 0.9 \times 255.8 = 230.2 \text{ m/s}$$

From Δ BEF,

$$C_{wo} = C_{ro} \times \cos\phi - C_b$$

$$= 230.2 \cos 36.04 - 206.7 = -20.6 \text{ m/s}$$

$$\eta_b = \frac{(C_{wi} + C_{wo})\,C_b}{(C_i)^2/2}$$

$$= \frac{(413.5 - 20.6)\,206.7}{(440)^2/2}$$

$$= \mathbf{0.839 \text{ or } 83.9\%} \qquad \text{...Ans.}$$

(iv) Stage efficiency, η_s if nozzle efficiency, $\eta_n = 0.93$

$$\eta_s = \eta_b \times \eta_n = 0.839 \times 0.93$$

$$= \mathbf{0.78 \text{ or } 78\%} \qquad \text{...Ans.}$$

(v) Power developed, P for mass flow rate of $\dot{m} = 3$ kg/s

$$P = \frac{\dot{m}\,(C_{wi} + C_{wo})\,C_b}{1000} \text{ (kW)}$$

$$= \frac{3\,(413.5 - 20.6) \times 206.7}{1000}$$

$$= \mathbf{243.64 \text{ kW}} \qquad \text{...Ans.}$$

Ex. 4.12.3 : In a single stage steam turbine saturated steam at 10 bar (absolute) is supplied through a convergent divergent steam nozzle. The nozzle angle is 20° and the mean blade speed is 400 m/s. The steam pressure leaving the nozzle is 1 bar (absolute). Find : [illegible]

i) The best blade angles if the blades are equiangular.

ii) The maximum power developed by the turbine, if a number of nozzles used are 5 and the area at the throat of each nozzle is 0.6 cm².

Assume nozzle efficiency 88%, blade friction coefficient of 0.87 and index of expansion n = 1.4. Solve using graphical method. **SPPU : Dec.18, 10 Marks**

Soln. :

Given : $p_1 = 10$ bar (abs), $\alpha = 20°$,

$C_b = 400$ m/s, $p_e = 1$ bar (abs),

$\theta = \phi$ (equiangular),

No. of nozzles, $y = 5$, $A_t = 0.6 \text{ cm}^2$

i.e. $A_t = 0.6 \times 10^{-4} \text{ m}^2$ per nozzle,

$\eta_n = 88\% = 0.88$, $K = \dfrac{C_{ro}}{C_{ri}} = 0.87$

Index of expansion, n = 1.4

From steam tables,

At $p_1 = 10$ bar, dry saturated :

$h_1 = 2013.6$ kJ/kg ;

$v_1 = 0.1943 \text{ m}^3\text{/kg}$

Critical or throat pressure,

$$\frac{p_t}{p_1} = \left(\frac{2}{n+1}\right)^{n/(n-1)} = \left(\frac{2}{1.4+1}\right)^{1.4/(1.4-1)}$$

$$= 0.5283 \text{ bar}$$

$$p_t = 0.5283 \times p_1 = 0.5283 \times 10 = 5.283 \text{ bar}$$

$$C_t = \left[2\left(\frac{n}{n-1}\right) p_1 v_1 \left\{1 - \left(\frac{p_t}{p_1}\right)^{(n-1)/n}\right\}\right]^{\frac{1}{2}}$$

$$= \left[2 \times \frac{1.4}{(1.4-1)} \times (10 \times 10^5) \times 0.1943 \left\{1 - (0.5283)^{(1.4-1)/1.4}\right\}\right]^{\frac{1}{2}}$$

$$= 476.1 \text{ m/s}$$

$$\frac{v_t}{v_1} = \left(\frac{p_1}{p_t}\right)^{\frac{1}{n}};$$

$$v_t = 0.1943\left(\frac{1}{0.5283}\right)^{1/1.4}$$

$$= 0.3065 \text{ m}^3\text{/kg}$$

$$\dot{m} = \frac{A_t C_t}{v_t} \times y = \frac{(0.6 \times 10^{-4}) \times 476.1}{0.3065} \times 5$$

$$\dot{m} = 0.466 \text{ kg/s}$$

Exit velocity form nozzle i.e inlet velocity of steam to turbine,

$$C_e = C_i = \left[2\left(\frac{n}{n-1}\right) p_1 v_1 \left\{1 - \left(\frac{p_e}{p_1}\right)^{(n-1)/n}\right\}\right]^{\frac{1}{2}}$$

$$= \left[2 \times \frac{1.4}{0.4} \times (10 \times 10^5) \times 0.1943 \left\{1 - \left(\frac{1}{10}\right)^{0.4/1.4}\right\}\right]^{\frac{1}{2}}$$

$$= 809.7 \text{ m/s}$$

(i) Best blade angles, (θ and ϕ) if blades are equiangular :

Condition for maximum efficiency is :

$$\frac{C_b}{C_i} = \frac{\cos\alpha}{2};$$

$$\frac{400}{809.7} = \frac{\cos\alpha}{2};$$

$$\alpha = 8.88° \quad \text{...Ans.}$$

Refer Fig. P. 4.12.3 which shows the velocity diagram:

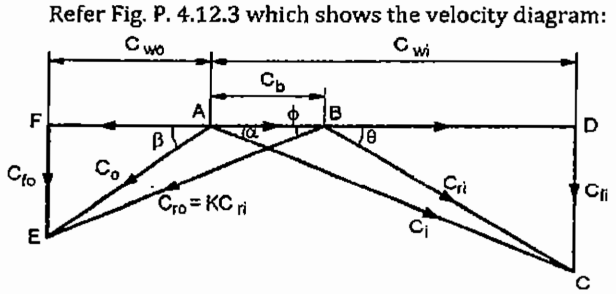


Fig. P.4.12.3

From above :

$\dot{m}$	=	0.466 kg/s,	C_i	=	809.7 m/s,
α	=	8.88°	C_b	=	400 m/s,
θ	=	ϕ,	K	=	0.87

Consider ΔACD,

$$C_{wi} = C_i \cos\alpha = 809.7 \cos 8.88 = 800 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 809.7 \sin 8.88 = 125 \text{ m/s}$$

$$BD = C_{wi} - C_b = 800 - 400 = 400 \text{ m/s}$$

$$C_{ri} = \sqrt{BD^2 + C_{fi}^2}$$

$$= \sqrt{(400)^2 + (125)^2} = 419 \text{ m/s}$$

$$K = 0.87 = \frac{C_{ro}}{C_{ri}};\ 0.87 = \frac{C_{ro}}{419};$$

$$C_{ro} = 364.5 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{125}{400}\right)$$

$$= 17.354° = \phi \quad \text{...Ans.}$$

$$BF = C_{ro}\cos\phi = 364.5 \cos 17.354 = 347.9 \text{ m/s}$$

$$C_{wo} = BF - AB = 347.9 - 400 = -52.1 \text{ m/s}$$

(ii) Maximum power developed, P:

$$P = \frac{\dot{m}\,(C_{wi} + C_{wo})\,C_b}{1000} \text{ kW}$$

$$= \frac{0.466\,(809.7 - 52.1)\,400}{1000}$$

$$= 141.22 \text{ kW} \quad \text{...Ans.}$$

Ex. 4.12.4 : In a single stage impulse turbine the mean diameter of the blade ring is 1m and the rotational speed is 3000 rpm. The steam is issued from the nozzle at 300 m/sec. and nozzle angle is 20°. The blades are equiangular. If the friction loss in the blade channel is 19% of the Kinetic energy corresponds to relative velocity at the inlet to the blades. What is the power developed in the blading when the axial thrust on the blades is 98 N. Solve the problem graphically.

SPPU: May 18, 6 Marks

Soln. :

Refer Fig. P. 4.12.4

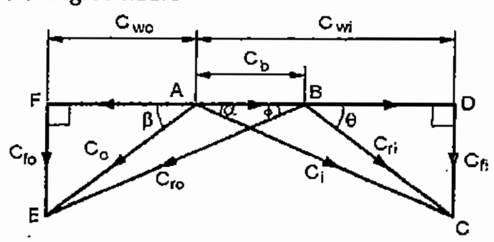


Fig. P. 4.12.4

d	=	1 m,	N	=	3000 rpm,
C_i	=	300 m/s,	α	=	20^0
θ	=	ϕ (equiangular blades)			

Friction loss = 19% of K.E of relative velocity at inlet

$$= 0.19 \times \frac{C_{ri}^2}{2}$$

Axial thrust, F_a = 98 N

$$C_b = \frac{\pi d N}{60} = \frac{\pi \times 1 \times 3000}{60}$$

$$= 157.1 \text{ m/s}$$

From Inlet Δ ACD ,

$$C_{wi} = C_i \cos\alpha = 300 \cos 20 = 281.9 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 300 \sin 20 = 102.6 \text{ m/s}$$

$$BD = C_{wi} - C_b = 281.9 - 157.1 = 124.8 \text{ m/s}$$

$$C_{ri} = \sqrt{BD^2 + C_{fi}^2} = \sqrt{(124.8)^2 + (102.6)^2}$$

$$= 161.6 \text{ m/s}$$

$$\frac{C_{ro}^2}{2} = \frac{C_{ri}^2}{2} - 0.19\frac{C_{ri}^2}{2} = 0.81\frac{C_{ri}^2}{2}$$

$$C_{ro} = \sqrt{0.81} \times C_{ri} = 0.9 \times C_{ri} = 0.9 \times 161.6$$

$$= 145.4 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{102.6}{124.8}\right)$$

$$= 39.424^0 = \phi$$

From exit Δ BEF,

$$C_{fo} = C_{ro} \sin\phi = 145.4 \sin 39.424 = 92.3 \text{ m/s}$$

$$C_{wo} = BF - AB = C_{ro}\cos\phi - C_b$$

$$= 145.4 \cos 39.424 - 157.1 = \mathbf{-44.8 \text{ m/s}}$$

Axial thrust,

$$F_a = \dot{m}\,(\vec{C_{fi}} - \vec{C_{fo}})$$

$$98 = \dot{m}\,(102.6 - 92.3);$$

$$\dot{m} = 9.5146 \text{ kg/s}$$

(i) Power developed, P

$$P = \frac{\dot{m}(C_{wi} + C_{wo})\,C_b}{1000} \text{ (kW)}$$

$$= \frac{9.5146(281.9 - 44.8)\,157.1}{1000}$$

$$= \mathbf{354.4 \text{ kW}}$$ ...Ans.

Ex. 4.12.5 : An impulse turbine has 3 similar stages of the same mean diameter and geometry, each stage develops 500 kW. The peripheral speed of the rotor blades at the mean diameter is 100 m/s, the whirl components of the absolute velocities at entry and exit of the rotor are $c_2 = 200$ m/s and $c_3 = 0$ respectively. The nozzle angles at exit are equal to $\alpha_2 = 65°$. The steam at the exit of the first stage has $P_2 = 8.0$ bar, $t_2 = 200°C$. Determine for the first stage :

(i) mean diameter of the stage for a speed of 3000 r.p.m.

(ii) mass flow rate of steam

(iii) isentropic enthalpy drop for an efficiency of 69%

(iv) rotor blade angles

(v) the blade height of the nozzle and rotor blade at exit.

SPPU - May 13, 10 Marks

Soln. : Refer Fig. P. 4.12.5(a)

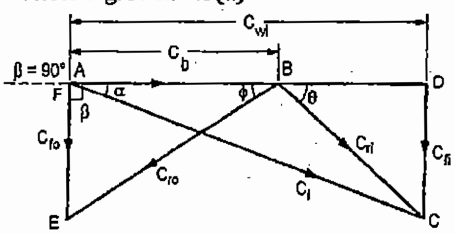


Fig. P. 4.12.5(a)

$$P = 500 \text{ kW}, \qquad C_b = 100 \text{ m/s},$$

$$C_{wi} = C_{y2} = 200 \text{ m/s}, \qquad C_{wo} = C_{y3} = 0$$

$$\alpha_2 = \alpha = 65°, \qquad N = 3000 \text{ rpm},$$

$$p_2 = 8 \text{ bar}, \qquad T_2 = 200°C$$

(i) Mean diameter of stage, d_m

$$C_b = \frac{\pi\, d_m \cdot N}{60};$$

$$d_m = \frac{60\, C_b}{\pi N} = \frac{60 \times 100}{\pi \times 3000}$$

$$= \mathbf{0.6366 \text{ m}}$$ ...Ans.

(ii) Mass flow rate of steam, $\dot{m}$

$$P = \frac{\dot{m}\,(C_{wi} + C_{wo})\,C_b}{1000} \text{ (kW)}$$

$$500 = \frac{\dot{m}\,(200 + 0)\,100}{1000};$$

$$\dot{m} = \mathbf{25 \text{ kg/s}}$$ ...Ans.

(iii) Isentropic enthalpy drop for an efficiency of $\eta_n = 69\% = 0.69$, Δh_i

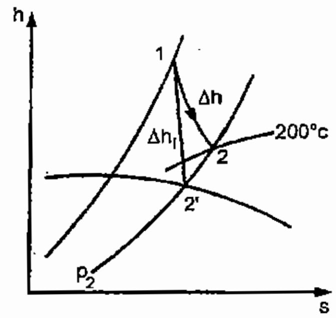


Fig. P. 4.12.5(b)

From Δ ACD ; $\cos\alpha = \dfrac{C_{wi}}{C_i}$

$$C_i = \frac{C_{wi}}{\cos\alpha} = \frac{200}{\cos 65} = 473.24 \text{ m/s}$$

Refer Fig. P. 4.12.5(b). Nozzle efficiency,

$$\eta_n = \frac{\Delta h}{\Delta h_i} = \frac{\frac{1}{2}\frac{C_i^2}{1000} \text{ kJ/kg}}{\Delta h_i}$$

$$\Delta h_i = \frac{1}{2} \times C_i^2 \times \frac{1}{1000} \times \frac{1}{\eta_n}$$

$$= \frac{1}{2} \times (473.24)^2 \times \frac{1}{1000} \times \frac{1}{0.69}$$

$$= \mathbf{162.3 \text{ kJ/kg.}}$$ ...Ans.

(iv) Rotor blade angles at inlet, θ and at exit, ϕ

$$C_{fi} = C_i \sin \alpha = 473.24 \sin 65 = 428.9 \text{ m/s}$$

$$BD = C_{wi} - C_b = 200 - 100 = 100 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{428.9}{100}\right)$$

$$= \mathbf{76.88^\circ} \qquad \text{...Ans.}$$

Assuming negligible friction loss in rotor blades then,

$$C_{ro} = C_{ri} = \sqrt{C_{fi}^2 + BD^2}$$

$$= \sqrt{(428.9)^2 + (100)^2} = 440.4 \text{ m/s}$$

$$\phi = \cos^{-1}\left(\frac{C_b}{C_{ro}}\right) = \cos^{-1}\left(\frac{100}{440.4}\right)$$

$$= \mathbf{76.88^\circ} \qquad \text{...Ans.}$$

(v) Blade height of nozzle and rotor blade at exit (Fig. P. 4.12.5(c))

$$\dot{m} = \frac{\pi (d+h) h C_{fo}}{(v_{sup})_o} = \frac{\pi d_m h C_{fi}}{(v_{sup})_o}$$

From steam tables, specific volume at 8 bar, 200°C we get, $(v_{sup})_o = 0.261 \text{ m}^3/\text{kg}$

$$\therefore \quad 25 = \frac{\pi \times 0.6366 \times h \times 428.9}{0.261}$$

$$h = 7.61 \times 10^{-3} \text{ m} = \mathbf{0.761 \text{ cm}} \qquad \text{...Ans.}$$

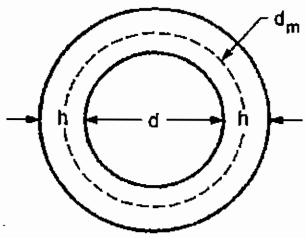


Fig. P. 4.12.5(c)

Ex. 4.12.6 : In a simple impulse turbine the nozzles are inclined at 20° to the direction of motion of moving blades. The steam leaves the nozzle at 375 m/s. The blade speed is 165 m/s. Find suitable inlet and outlet angles for the blades in order that axial thrust is zero. The relative velocity of steam as it flows over the blade is reduced by 15% by friction. Determine also the power developed for steam flow rate of 10 kg/s and diagram efficiency.

SPPU - May 12, 12 Marks

Soln. : Refer Fig. P. 4.12.6(a).

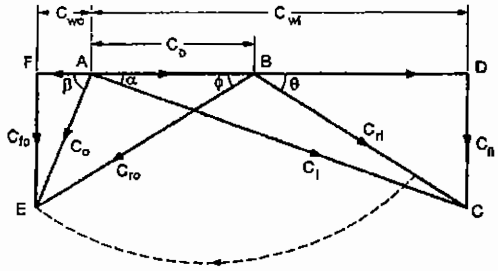


Fig. P. 4.12.6(a)

Given :

$$\alpha = 20^\circ, \qquad C_i = 375 \text{ m/s},$$

$$C_b = 165 \text{ m/s};$$

Axial thrust is zero i.e.

$$F_a = 0 = \dot{m}\,(C_{fi} - C_{fo});$$

Hence,
$$C_{fi} = C_{fo} \qquad \text{...(i)}$$

$$C_{ro} = (1 - 0.15)\, C_{ri} = 0.85\, C_{ri};$$

$$\dot{m} = 10 \text{ kg/s}$$

(i) Inlet blade angle, θ and outlet blade angle, ϕ

From inlet velocity Δ ACD :

$$C_{wi} = C_i \cos \alpha = 375 \cos 20 = 352.4 \text{ m/s}$$

$$C_{fi} = C_i \sin \alpha = 375 \sin 20$$

$$= 128.3 \text{ m/s} = C_{fo} \qquad \text{...(given)}$$

$$BD = C_{wi} - C_b = 352.4 - 165 = 187.4 \text{ m/s}$$

$$C_{ri} = \sqrt{C_{fi}^2 + BD^2}$$

$$= \sqrt{(128.3)^2 + (187.4)^2} = 227.1 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{128.3}{187.4}\right)$$

$$= \mathbf{34.4^\circ} \qquad \text{...Ans.}$$

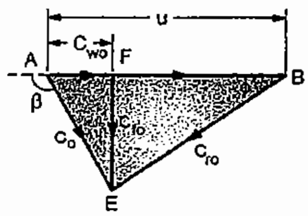


Fig. P. 4.12.6(b) : Modified exit velocity diagram

Consider exit velocity Δ BEF

$$C_{ro} = 0.85\, C_{ri} = 0.85 \times 227.1 = 193 \text{ m/s}$$

$$\phi = \sin^{-1}\left(\frac{C_{fo}}{C_{ro}}\right) = \sin^{-1}\left(\frac{128.3}{193}\right)$$

$$= \mathbf{41.67^\circ} \qquad \text{...Ans.}$$

$$BF = C_{ro} \cos \phi$$

$$= 193 \cos 41.67 = 144.2 \text{ m/s}$$

$$\therefore C_{wo} = BF - C_b = 144.2 - 165 = -20.8 \text{ m/s}$$

C_{wo} being negative, modified exit velocity diagram is also shown.

(ii) Power developed, P

$$P = \dot{m}\,(C_{wi} + C_{wo})\,C_b$$

$$= 10\,(352.4 - 20.8)\,165 \times 10^{-3} \text{ kW}$$

$$= \mathbf{547.14 \text{ kW}} \quad \text{...Ans.}$$

(iii) Diagram efficiency, η_b

$$\eta_b = \frac{2\,(C_{wi} + C_{wo})\,C_b}{C_i^2}$$

$$= \frac{2\,(352.4 - 20.8)\,165}{(375)^2}$$

$$= \mathbf{0.7782 \text{ or } 77.82\%} \quad \text{...Ans.}$$

Ex. 4.12.7 : A single stage turbine is supplied with steam at 4 bar and 160° C and it is exhausted at condenser pressure of 0.15 bar at the rate of 60 kg/min. The steam expands in a nozzle with an efficiency of 90%. The blade speed is 250 m/s and the nozzles are inclined at 20° to the plane of wheel. The blade angle at the exit of the moving blade is 30°. Neglecting friction losses in the moving blades, determine :

(a) Steam jet velocity
(b) Power developed
(c) Blade efficiency, and
(d) Stage efficiency.

Soln. :

Given :

$p_1 = 4$ bar, $160°C$; $\quad p_b = 0.15$ bar,

$\dot{m} = 60$ kg/min $= 1$ kg/s,

$\eta_n = 90\%$, $\quad C_b = 250$ m/s,

$\alpha = 20°$, $\quad \phi = 30°$,

$$K = 1 = \frac{C_{ro}}{C_{ri}}$$

From Mollier's diagram the isentropic heat drop per kg of steam is calculated as,

$$\Delta h' = 513 \text{ kJ/kg}$$

Actual heat drop,

$$\Delta h = \text{Nozzle efficiency} \times \Delta h'$$

$$= 0.9 \times 513 = 461.7 \text{ kJ/kg}$$

(a) Steam jet velocity

$$C_i = 44.7\sqrt{\Delta h}$$

$$= 44.7\sqrt{461.7}$$

$$= \mathbf{960.5 \text{ m/s}} \quad \text{...Ans.}$$

The velocity diagram can be drawn as shown in Fig. P. 4.12.7. By measurement we get,

$$C_w = C_{wi} + C_{wo} = FD = 1280 \text{ m/s}$$

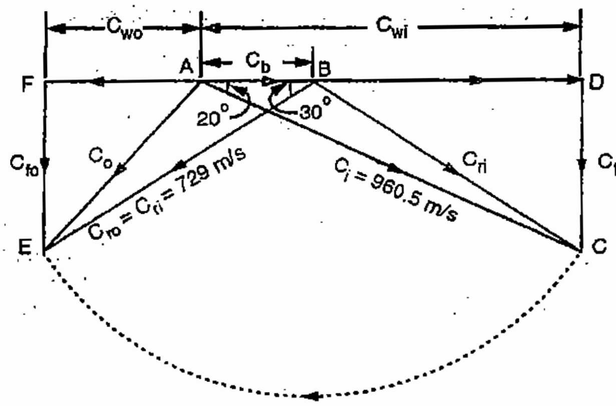


Fig. P. 4.12.7

(b) Power developed

$$P = \frac{\dot{m} \cdot C_w \cdot C_b}{1000} = \frac{1 \times 1280 \times 250}{1000}$$

$$= \mathbf{320 \text{ kW}} \quad \text{...Ans.}$$

(c) Blade efficiency

$$\eta_b = \frac{2\,C_w \cdot C_b}{C_i^2} = \frac{2 \times 1280 \times 250}{(960.5)^2}$$

$$= \mathbf{0.6937 \text{ or } 69.37\ \%} \quad \text{...Ans.}$$

(d) Stage efficiency

$$\eta_s = \frac{C_w \cdot C_b}{\Delta h' \times 1000} = \frac{1280 \times 250}{513 \times 1000}$$

$$= \mathbf{0.6237 \text{ or } 62.37\ \%} \quad \text{...Ans.}$$

Ex. 4.12.8 : Steam issues from the nozzle at an angle of 22° with a velocity of 430 m/s. The friction factor is 0.9. For a stage turbine designed for maximum efficiency, find (i) The blade velocity (ii) Moving blade angles for equiangular blades (iii) Power developed. **SPPU : May 15, 4 Marks**

Soln. :

Refer Fig. P. 4.12.8.

$\alpha = 22°$, $\quad C_i = 430$ m/s ;

$$K = \frac{C_{ro}}{C_{ri}} = 0.9$$

1. Blade velocity, C_b for maximum efficiency :

Condition for maximum efficiency is given as :

$$\text{Blade speed ratio, } S = \frac{C_b}{C_i} = \frac{\cos\alpha}{2}$$

$$\therefore C_b = C_i \times \frac{\cos\alpha}{2} = 430 \times \frac{\cos 22}{2}$$

$$= \mathbf{199.35\ m/s} \qquad \text{...Ans.}$$

2. Moving blade angles for equiangular blades

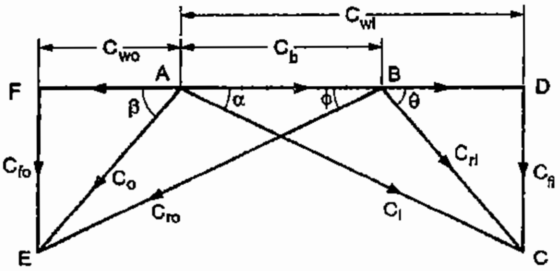


Fig. P. 4.12.8

$$C_{wi} = C_i \cos\alpha = 430 \cos 22 = 398.69 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 430 \sin 22 = 161.08 \text{ m/s}$$

$$BD = C_{wi} - C_b = 398.69 - 199.35 = 199.34 \text{ m/s}$$

Inlet angle of moving blade,

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{161.08}{199.34}\right)$$

$$= \mathbf{38.94^\circ} \qquad \text{...Ans.}$$

Exit angle of moving blade,

$$\phi = \theta = \mathbf{38.94^\circ} \qquad \text{...Ans.}$$

3. Power developed, P

$$C_{ri} = \sqrt{(BD)^2 + (C_{fi})^2}$$

$$= \sqrt{(199.34)^2 + (161.08)^2} = 256.29 \text{ m/s}$$

$$C_{ro} = K \times C_{ri} = 0.9 \times 256.29 = 230.66 \text{ m/s}$$

$$C_{wo} = C_{ro}\cos\phi - C_b$$

$$= 130.66 \cos 38.94 - 199.35 = -19.94 \text{ m/s}$$

Assuming mass flow rate, $\dot{m} = 1$ kg/s

$$P = \frac{\dot{m}\,(C_{wi} + C_{wo})\,C_b}{1000} \text{ kW}$$

$$= \frac{1(398.69 - 19.94)\,199.35}{1000}$$

$$= 75.5 \frac{\text{kW}}{\text{kg/s}} \text{ of mass flow rate} \qquad \text{...Ans.}$$

Ex. 4.12.9 : The nozzles of a De-Laval turbine deliver 1.5 kg / s of steam at a speed of 800 m/s to ring of moving blades having a speed of 200 m/s. The exit angle of the nozzles is 18°. If the blade velocity coefficient is 0.75 and the exit angle of the moving blades is 25°, calculate :

(a) Inlet angle of moving and fixed blades,

(b) Diagram efficiency,

(c) Energy lost in blades per second,

(d) Power developed and

(e) Axial thrust on the turbine rotor.

Soln. :

Given :

$\dot{m} = 1.5$ kg/s; $\alpha = 18^\circ$; $\phi = 25^\circ$; $C_b = 200$ m/s;

$C_i = 800$ m/s and $K = \frac{C_{ro}}{C_{ri}} = 0.75$

The velocity diagram can be drawn as shown in Fig. P. 4.12.9. By measurements from Fig. P. 4.12.9.

we get,

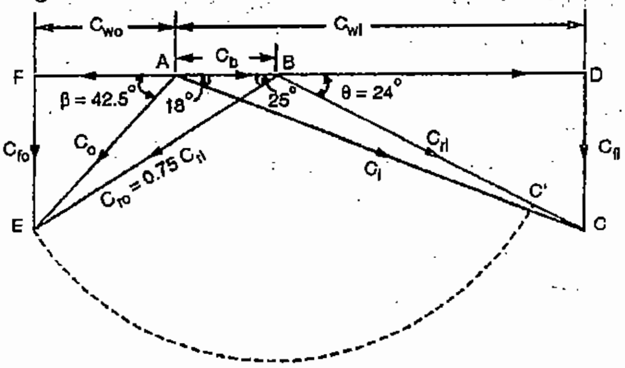


Fig. P. 4.12.9

(a) Inlet angle of moving blades : θ = 24°

Inlet angle of fixed blades, β = 42.5° ...Ans.

To find :

(b) Diagram efficiency

$$\eta = \frac{2C_b \cdot C_w}{C_i^2} = \frac{2\,(AB)\,(FD)}{(AC)^2}$$

$$= \frac{2 \times 200 \times 941}{(800)^2} = \mathbf{0.5881} \qquad \text{...Ans.}$$

(c) Energy lost in blades

$$= \left(\frac{C_{ri}^2 - C_{ro}^2}{2}\right)\dot{m}$$

$$= \left[\frac{(561)^2 - (421)^2}{2}\right] 1.5$$

$$= \mathbf{103110\ Nm\ or\ 103.11\ kJ} \qquad \text{...Ans.}$$

(d) Power developed

$$= \dot{m} \times \frac{C_w \cdot C_b}{1000}$$

$$= 1.5 \times \frac{941 \times 200}{1000}$$

$$= \mathbf{282.3\ kW} \qquad \text{...Ans.}$$

(e) Axial thrust on rotor

$$= \dot{m} \times (C_{fi} - C_{fo}) = 1.5\ (247 - 130)$$

$$= \mathbf{175.5\ N} \qquad \text{...Ans.}$$

Ex. 4.12.10 : A single stage steam turbine is supplied with steam at 5 bar and 200°C at the rate of 50 kg/min. It expands in to condenser at a pressure of 0.2 bar. The blade speed is 400 m/s. the nozzle are inclined at an angle is 20° to the plane of the wheel and the outlet blade angle is 30°. Neglecting frictional losses, determine the power developed, blade efficiency and stage efficiency.

SPPU - April 2017 (In sem), 6 Marks

Soln. :

Refer Fig. P. 4.12.10(a)and (b)

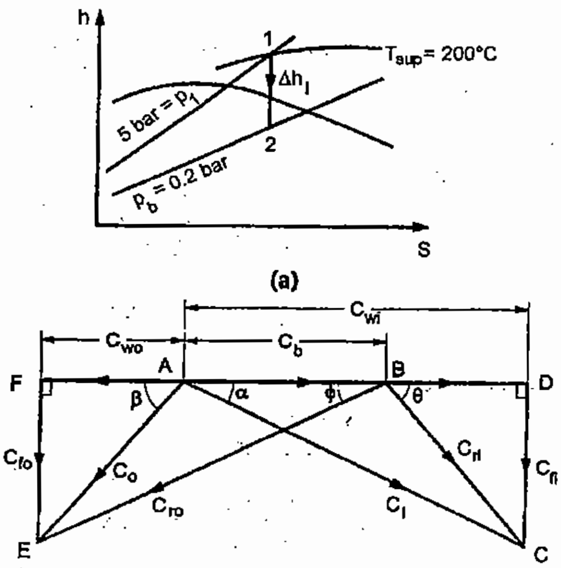
$p_1 = 5$ bar, $T_{sup1} = 200°C$,

$\dot{m} = 50$ kg/s, $p_b = 0.2$ bar,

$C_b = 400$ m/s, $\alpha = 20°$

$\phi = 30°$;

$C_{ro} = C_{ri}$ ($\because$ friction is neglected)



(a)

(b)

Fig. P. 4.12.10

From steam tables,

At $p_1 = 5$ bar, $T_{sup1} = 200°C$

$h_1 = 2855.1$ kJ/kg; $S_1 = 7.059$ kJ/kgK

At $p_b = 0.2$ bar

$h_{f2} = 251.4$ kJ/kg, $h_{fg_2} = 2358.3$ kJ/kg

$S_{f2} = 0.832$ kJ/kgK, $S_{fg_2} = 7.077$ kJ/kgK

For isentropic expansion of steam in nozzles,

$$S_1 = S_2 = S_{f2} + x_2 \times S_{fg_2}$$

$$7.059 = 0.832 + x_2 \times 7.077$$

$$x_2 = 0.8789$$

$$h_2 = h_{f2} + x_2 \times h_{fg_2} = 251.4 + 0.8789 \times 2358.3$$

$$= 2326.4 \text{ kJ/kg}$$

Isentropic enthalpy drop in nozzles,

$$\Delta h_i = h_1 - h_2 = 2855.1 - 2326.4$$

$$= 528.7 \text{ kJ/kg}$$

Note : Δh_i can be determined directly with the help of Mollier's diagram as shown in Fig. P. 4.12.10(a).

$$C_i = 44.7\sqrt{\Delta h_i \times \eta_n}$$

$$= 44.7 \times \sqrt{528.7 \times 1} = 1027.8 \text{ m/s}$$

Refer Fig. P. 4.12.10(b);

$$C_{wi} = C_i \cos\alpha = 1027.8 \cos 20 = 965.8 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 1027.8 \sin 20 = 351.5 \text{ m/s}$$

$$BC = C_{wi} - C_b = 965.8 - 400 = 565.8 \text{ m/s}$$

$$C_{ri} = \sqrt{(BC)^2 + (C_{fi})^2}$$

$$= \sqrt{(565.8)^2 + (351.5)^2} = 666.1 \text{ m/s}$$

$$C_{ro} = C_{ri} = 666.1 \text{ m/s}$$

$$C_{wo} = C_{ro}\cos\phi - C_b$$

$$= 666.1 \cos 30 - 400 = 176.9 \text{ m/s}$$

(i) Power developed, P

$$P = \frac{\dot{m}\ (C_{wi} + C_{wo})\ C_b}{1000}\ (\text{kW})$$

$$= \frac{50\ (965.8 + 176.9)\ 400}{1000}$$

$$= \mathbf{22854\ kW} \qquad \text{...Ans.}$$

(ii) Blade efficiency, η_b

$$\eta_b = \frac{(C_{wi} + C_{wo})\ C_b}{\frac{1}{2} C_i^2} = \frac{2\ (965.8 + 176.9)\ 400}{(1027.8)^2}$$

$$= \mathbf{08654\ or\ 86.54\%} \qquad \text{...Ans.}$$

(iii) Stage efficiency, η_s

Since nozzle efficiency is not given, it was assumed to be 100%. Therefore,

$$\eta_s = \eta_b \times \eta_n = 0.8654 \times 1$$

$$= \mathbf{0.8654 \text{ or } 86.54\%} \quad \text{...Ans.}$$

Ex. 4.12.11 : In a De Laval turbine, steam is issued from the nozzle with a velocity of 1500 m/s whereas the mean blade velocity is 500 m/s. The nozzle angle is 20° and the inlet and outlet angles of blades are equal. The mass of the steam flowing through the turbine is at the rate of 1200 kg/hr. Assuming blade velocity coefficient K = 0.8, draw the velocity diagram and

Determine :

(i) The blade angles,

(ii) The power developed by turbine,

(iii) The blade efficiency. **SPPU - May 16, 6 Marks**

Soln. :

$C_i = 1500$ m/s, $C_b = 500$ m/s

$\alpha = 20°$, $\theta = \phi$

$$\dot{m} = 1200 \text{ kg/hr} = \frac{1200}{3600} \text{ kg/s} = \frac{1}{3} \text{ kg/s},$$

Blade velocity coefficient, $k = \dfrac{C_{ro}}{C_{ri}} = 0.8$

Velocity diagram is shown in Fig. P. 4.12.11.

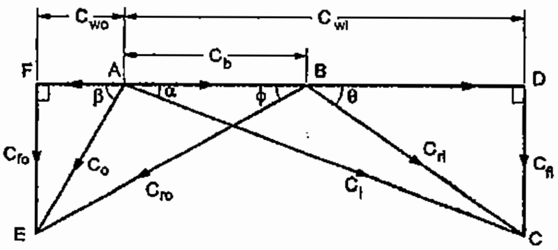


Fig. P. 4.12.11

(1) Blade angles, θ and φ

Consider inlet velocity ΔACD,

$$C_{wi} = C_i \cos\alpha = 1500 \cos 20 = 1409.5 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 1500 \sin 20 = 513 \text{ m/s}$$

$$BD = C_{wi} - C_b = 1409.5 - 500 = 909.5 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right)$$

$$= \tan^{-1}\left(\frac{513}{909.5}\right) = \mathbf{29.43°} \quad \text{...Ans.}$$

(2) Power developed by turbine, P

$$C_{ri} = \sqrt{BD^2 + C_{fi}^2}$$

$$= \sqrt{(909.5)^2 + (513)^2} = 1044.2 \text{ m/s}$$

$$C_{ro} = k\,C_{ri} = 0.8 \times 1044.2 = 835.4 \text{ m/s}$$

$$BF = C_{ro}\cos\phi = 835.4 \cos 29.43 = 727.6 \text{ m/s}$$

$$\therefore C_{wo} = BF - C_b = 727.6 - 500 = 227.6 \text{ m/s}$$

$$P = \frac{\dot{m}\,(C_{wi} + C_{wo})\,C_b}{1000} \text{ (kW)}$$

$$= \frac{1}{3} \times \frac{(1409.5 + 227.6)\,500}{1000}$$

$$= \mathbf{272.85 \text{ kW}} \quad \text{...Ans.}$$

(3) Blade efficiency, η_b

$$\eta_b = \frac{2\,(C_{wi} + C_{wo})\,C_b}{C_i^2} = \frac{2\,(1409.5 + 227.6)\,500}{(1500)^2}$$

$$= \mathbf{0.7276 \text{ or } 72.76\%} \quad \text{...Ans.}$$

Ex. 4.12.12 : A simple impulse turbine has a mean blade ring diameter of 70 cm and runs at 3000 r.p.m. The blade speed ratio is 0.46 and discharge is axial. The nozzle angle is 21° and blade friction factor is 0.95.

Determine,

(i) Blade angles, and

(ii) Theoretical specific power output

Soln. : Given :

Diameter, $d = 70 \text{ cm} = 0.7 \text{ m}$;

Speed, $N = 3000$ r.p.m;

Blade speed ratio $= 0.46 = \dfrac{C_b}{C_i}$

Discharge is axial, i.e. $\beta = 90°$; $\alpha = 21°$

Friction factor, $K = 0.95 = \dfrac{C_{ro}}{C_{ri}}$

(I) Blade angles

Blade velocity $C_b = \dfrac{\pi dN}{60}$

$$= \frac{\pi \times 0.7 \times 3000}{60}$$

$$= 109.95 \text{ m/s}$$

$$\therefore C_i = \frac{C_b}{0.46}$$

$$= \frac{109.95}{0.46}$$

$$= 239 \text{ m/s}$$

Velocity diagram is shown in Fig. P. 4.12.12.

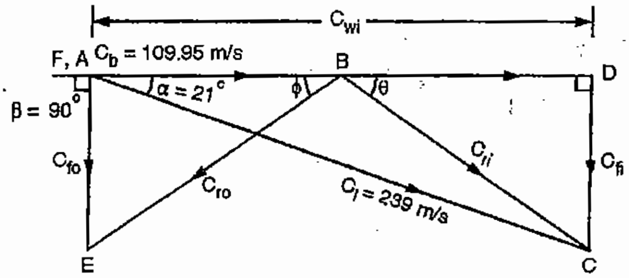


Fig. P. 4.12.12

$$C_{wi} = C_i \cos\alpha = 239 \cos 21 = 223.12 \text{ m/s}$$

$$\therefore BD = C_{wi} - C_b = 223.12 - 109.95 = 113.17 \text{ m/s}$$

$$CD = C_{fi} = C_i \cdot \sin\alpha = 239 \sin 21 = 85.65 \text{ m/s}$$

$$\therefore C_{ri} = \sqrt{(BD)^2 + (CD)^2}$$

$$= \sqrt{(113.17)^2 + (85.65)^2} = 141.93 \text{ m/s}$$

$$\tan\theta = \frac{CD}{BD}$$

$$\therefore \theta = \tan^{-1}\left(\frac{85.65}{113.17}\right)$$

$$= 37.12^\circ \text{ (Inlet angle of moving blade)}$$

...**Ans.**

$$C_{ro} = K\,C_{ri} = 0.95 \times 141.93 = 134.83 \text{ m/s}$$

$$\cos\phi = \frac{C_b}{C_{ro}} = \frac{109.95}{134.83}$$

or, Exit angle of moving blade,

$$\phi = 35.37^\circ$$ ...**Ans.**

$$C_{wo} = 0$$

(ii) Specific power output

$$P = \frac{m\,(C_{wi} + C_{wo})\,C_b}{1000}\text{ kW} = \frac{1\,(223.12 + 0)\,109.95}{1000}$$

$$= 24.532 \text{ kW/kg/s of steam flow rate}$$ ...**Ans.**

Ex. 4.12.13 : The mean diameter of the blades of an impulse turbine with a single row turbine is 1.2 m and the speed is 3000 rpm. The nozzle angle is 18°, the ratio of blade velocity to steam velocity is 0.42 and ratio of relative velocity at outlet from the blades to that at inlet is 0.84. The outlet angle of the blade is to be made 3° less than the inlet blade angle. Steam flow is 8 kg/s. Draw velocity diagram and find resultant thrust on blades, tangential thrust, axial thrust, power developed and blade efficiency. **SPPU - May 11, Dec. 12, 10 Marks**

Soln. :

d = 1.2 m; N = 3000 rpm;

α = 18°; $S = \frac{C_b}{C_i} = 0.42$;

$C_{ro} = 0.84\,C_{ri}$; $\phi = (\theta - 3)°$;

$\dot{m}$ = 8 kg/s

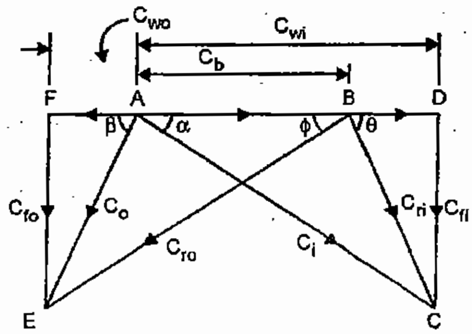


Fig. P. 4.12.13

Blade velocity, $$C_b = \frac{\pi d N}{60} = \frac{\pi \times 1.2 \times 3000}{60}$$

$$= 188.5 \text{ m/s}$$

$$\therefore C_i = \frac{C_b}{S} = \frac{188.5}{0.42} = 448.5 \text{ m/s}$$

Velocity diagram is shown in Fig. P. 4.12.13

$$C_{wi} = C_i \cos\alpha = 448.5 \cos 18 = 426.5 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 448.5 \sin 18 = 138.6 \text{ m/s}$$

$$BD = C_{wi} - C_b = 426.5 - 188.5 = 238 \text{ m/s}$$

$$C_{ri} = \sqrt{C_{fi}^2 + BD^2} = \sqrt{(138.6)^2 + (238)^2}$$

$$= 275.4 \text{ m/s}$$

$$\tan\theta = \frac{C_{fi}}{BD} = \frac{138.6}{238} = 0.58235$$

$$\therefore \theta = 30.2^\circ$$

$$\therefore \phi = \theta - 3 = 30.2 - 3 = 27.2^\circ$$

$$C_{ro} = 0.84\,C_{ri} = 0.84 \times 275.4 = 231.3 \text{ m/s}$$

$$BF = C_{r_o} \cos\phi = 231.3 \cos 27.2 = 205.8 \text{ m/s}$$

$$\therefore C_{wo} = BF - C_b = 205.8 - 188.5 = 17.3 \text{ m/s}$$

$$C_{fo} = C_{ro} \sin\phi = 231.3 \sin 27.2 = 105.7 \text{ m/s}$$

(i) Tangential thrust on blades, F

$$F = \dot{m}\,(C_{wi} + C_{wo}) = 8 \times (426.5 + 17.3)$$

$$= 3550.4 \text{ N}$$ ...**Ans.**

(ii) Axial thrust, F_a

$$F_a = \dot{m}\,(\vec{C}_{fi} - \vec{C}_{fo}) = 8\,(138.6 - 105.7)$$

$$= 263.2 \text{ N}$$ ...**Ans.**

(iii) Resultant thrust, F_r

$$F_r = \sqrt{F^2 + F_a^2} = \sqrt{(3550.4)^2 + (263.2)^2}$$

$$= \mathbf{3560.1\ N} \qquad \text{...Ans.}$$

(iv) Power developed, P

$$P = \frac{F \times C_b}{1000}(\text{kW}) = \frac{3550.4 \times 188.5}{1000}$$

$$= \mathbf{669.25\ kW} \qquad \text{...Ans.}$$

(v) Blade efficiency, η_b

$$\eta_b = \frac{F \cdot C_a}{\dot{m}\frac{C_i^2}{2}} = \frac{3550.4 \times 188.5}{8 \times \frac{(448.5)^2}{2}}$$

$$= \mathbf{0.8318\ or\ 83.18\ \%} \qquad \text{...Ans.}$$

Ex. 4.12.14 : Following data refers to the working of a single stage impulse turbine :

(i) Enthalpy drop in the nozzle = 500 kJ/kg.

(ii) Blade speed = 300 m/s

(iii) Nozzle angle = 25°

(iv) Outlet blade angle = 35°

Calculate :

(1) Power developed by the turbine.

(2) Diagram efficiency and

(3) Stage efficiency.

Neglect the frictional losses.

Assume steam flow rate = 1 kg/s.

Soln. :

Refer Fig. P. 4.12.14.

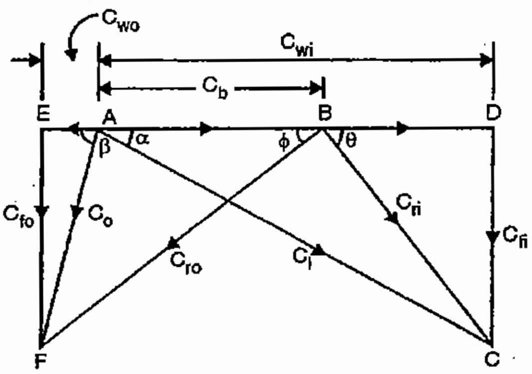


Fig. P. 4.12.14

Given : $\Delta h = 500$ kJ/kg ; $C_b = 300$ m/s

$\alpha = 25°$; $\phi = 35°$;

$\dot{m} = 1$ kg/s

Steam velocity at inlet,

$$C_i = 44.7\sqrt{\Delta h} = 44.7\sqrt{500} = 999.5\ \text{m/s}$$

$$C_{wi} = C_i \cos\alpha = 999.5 \cos 25 = 905.9\ \text{m/s}$$

$$C_{fi} = C_i \sin\alpha = 999.5 \sin 25 = 422.4\ \text{m/s}$$

$$C_{ri} = \sqrt{C_{fi}^2 + (C_{wi} - C_b)^2}$$

$$= \sqrt{(422.4)^2 + (905.9 - 300)^2} = 738.6\ \text{m/s}$$

Assuming no friction losses in blades, then

$$C_{ro} = C_{ri} = 738.6\ \text{m/s}$$

$$C_{wo} = BE - C_b$$

$$= 738.6 \cos 35 - 300 = 305.0\ \text{m/s}$$

(i) Power developed by turbine, P

$$P = \frac{\dot{m}\,(C_{wi} + C_{wo})\,C_b}{1000}\ \text{kW}$$

$$P = \frac{1 \times (905.9 + 305)300}{1000}$$

$$= \mathbf{363.27\ kW} \qquad \text{...Ans.}$$

(ii) Diagram efficiency, η_b

$$\eta_b = \frac{(C_{wi} + C_{wo})\,C_b}{C_i^2/2} = \frac{(905.9 + 305)\,300}{(999.5)^2/2}$$

$$= 0.7273 \text{ or } 72.73\% \qquad \text{...Ans.}$$

(iii) Stage efficiency, η_s

$$\eta_s = \frac{P}{\dot{m} \times (\Delta h)}$$

$$= \frac{363.27}{1 \times 500} = \mathbf{0.7265} \qquad \text{...Ans.}$$

Note : Stage efficiency is equal to diagram efficiency since the nozzle efficiency is not given. Δh represents the isentropic enthalpy drop.

Ex. 4.12.15 : Steam issues from the nozzles of an impulse steam turbine with a velocity of 1200 m/s. The nozzle angle is 20°, the mean blade speed is 400 m/s and the inlet and outlet angles of moving blades are equal. The mass of steam flowing through the turbine is 900 kg/hr. Determine :

(i) The blade angles

(ii) The tangential force on blades

(iii) The power developed

(iv) The blade efficiency

Assume friction factor 0.8.

SPPU - Feb. 16 (In Sem), 8 Marks

Soln. :

Velocity diagram is shown in Fig. P. 4.12.15.

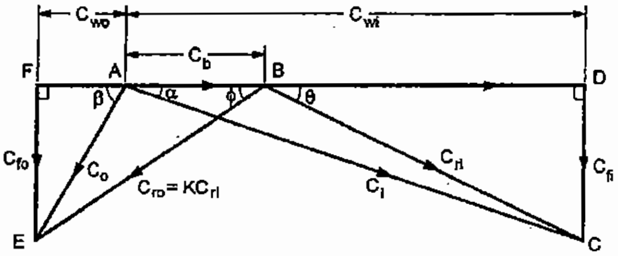


Fig. P. 4.12.15

Given : $C_1 = 1200$ m/s, $\alpha = 20$;

$C_b = 400$ m/s, $\theta = \phi$;

$\dot{m} = 900 \text{ kg/hr} = \frac{900}{3600} = 0.25$ kg/s

Friction factor, k = 0.8

(i) Blade angles

From Δ ACD :

$$C_{wi} = C_i \cos\alpha = 1200 \times \cos 20 = 1127.6 \text{ m/s}$$

$$BD = C_{wi} - C_b$$

$$= 1127.6 - 400 = 727.6 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 1200 \times \sin 20$$

$$= 410.4 \text{ m/s} = CD$$

$$\therefore \tan\theta = \frac{CD}{BD} = \frac{410.4}{727.6} = 0.564$$

$$\theta = 29.425^\circ$$ ...Ans.

∴ Inlet angle of moving blade,

$$\theta = \text{Exit angle of moving blade } \phi$$

$$= 29.425^\circ$$

$$C_{ri} = \sqrt{CD^2 + BD^2}$$

$$= \sqrt{(410.4)^2 + (727.6)^2}$$

$$= 835.4 \text{ m/s}$$ ...Ans.

$$C_{ro} = k \cdot C_{ri} = 0.8 \times 835.4$$

$$= 668.3 \text{ m/s}$$

$$BF = C_{ro}\cos\phi = 668.3 \times \cos 29.425$$

$$= 582 \text{ m/s}$$

$$\therefore C_{wo} = BF - C_b = 582 - 400$$

$$= 182 \text{ m/s} = AF$$

$$FE = C_{fo} = C_{ro}\sin 29.425$$

$$= 668.3 \sin 29.425 = 328.3 \text{ m/s}$$

Inlet angle of fixed blade,

$$\beta = \tan^{-1}\left(\frac{C_{fo}}{AF}\right)$$

$$= \tan^{-1}\left(\frac{328.3}{182}\right) = 61^\circ$$ ...Ans.

(ii) Tangential force on blades, F

$$F = \dot{m}(C_{wi} + C_{wo})$$

$$= 0.25\,(1127.6 + 182) = 327.4 \text{ N}$$ ...Ans.

(iii) Power developed, P

$$P = F \cdot C_b$$

$$= \frac{327.4 \times 400}{1000} \text{ kW} = 130.96 \text{ kW}$$ ...Ans.

(iv) The blade efficiency, η_b

$$\eta_b = \frac{P}{\dot{m} \cdot C_i^2/2}$$

$$= \frac{(130.96 \times 1000)}{0.25 \times (1200)^2} \times 2$$

$$= 0.7276 \text{ or } 72.76\%$$ ...Ans.

4.13 Velocity Diagram for a Velocity Compounded Impulse Turbine

- It has been discussed in section 4.10 that the velocity compounding is done by absorbing the kinetic energy of steam raised in the nozzles by two or more successive rings of moving blades each separated by ring of fixed blade.
- The function of fixed blades is to change the direction of motion of the steam received from a ring of moving blades and to redirect the steam on to the next ring of moving blades as shown in Fig. 4.13.1(a).
- The velocity diagram for each ring of moving blades can be drawn in the similar way as explained in Fig. 4.11.1. All the velocity diagram from different rings of moving blades can be superimposed on the base representing the blade velocity, C_b.
- Fig. 4.13.1(b) represents the velocity diagram for two rings of moving tables separated by a single ring of fixed blade.

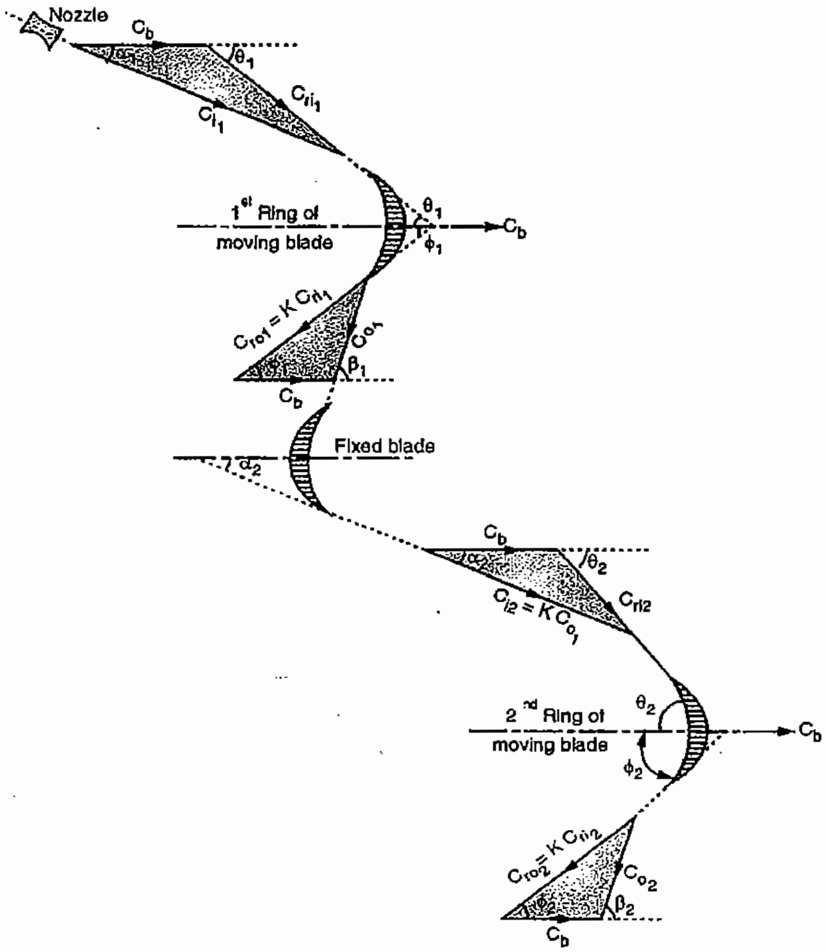


Fig. 4.13.1(a) : Steam flow diagram

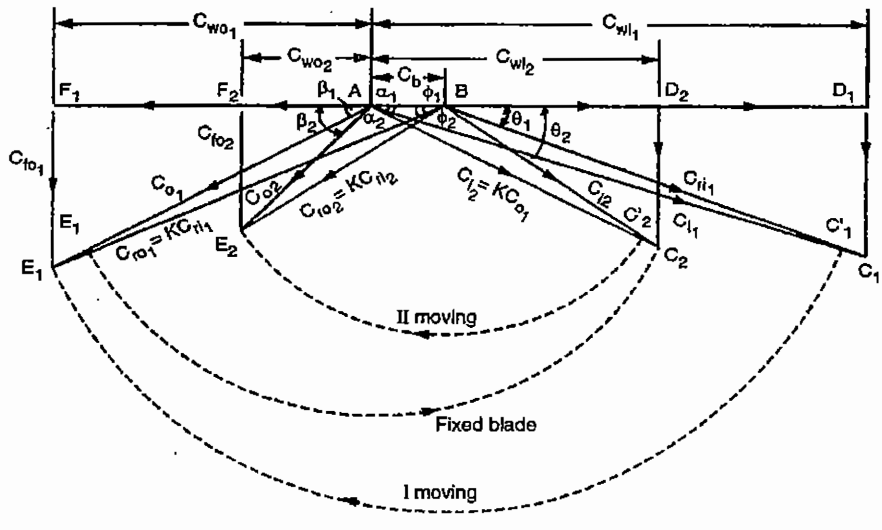


Fig. 4.13.1(b) : Velocity diagram for velocity compounded impulse turbine

Let α_1 = Exit angle of nozzle.

θ_1 = Inlet angle of first ring of moving blades

ϕ_1 = Exit angle of first ring of moving blades

β_1 = Inlet angle of fixed blades

α_2 = Exit angle of fixed blades

θ_2 = Inlet angle second ring of moving blades

ϕ_2 = Exit angle of second ring of moving blades

β_2 = Inlet angle of second ring of fixed blades

K = Friction factor for each ring of moving and fixed blades

$= K_1 = K_2 = K_3$.

The velocity diagram as shown in Fig. 4.13.1(b) can be drawn as follows :

(a) On a convenient scale draw AB to represent the blades velocity, C_b

(b) Draw $AC_1 = C_{i_1}$. The steam velocity at nozzle angle α_1, Join BC_1 which represents the relative velocity at inlet to first ring of moving blades.

Make the point C_1' such that $BC_1' = K \cdot BC_1$ where, K is the friction factor or blade velocity coefficients for the first ring of moving blade.

i.e. $K = \dfrac{C_{ro_1}}{C_{ri_1}}$

(c) Draw $BE_1 = BC_1'$ at an exit angle ϕ of the first ring of moving blades. Join AE_1 which represent the absolute velocity at exit, C_{o1} from the first ring of moving blades or the entrance velocity to the first ring of fixed blades at an angle β_1.

(d) Mark E_1' such that $AE_1' = K_1 \cdot AE_1$, where K_1 is the friction factor for the fixed blade ring $\left[K_1 = \dfrac{C_{i_2}}{C_{o_1}}\right]$. The AE_1' will represent the velocity of jet at inlet to the second ring of moving blades. Therefore, mark-off $AC_2 = AE_1'$ at an angle α_2. Join BC_2 which represent the relative velocity at inlet to the second ring of moving blades.

(e) Set off $BC_2' = K_2$, BC_2, K_2 being the friction for the second ring of moving blades ($K_2 = C_{ro_2} / C_{ri_2}$). Draw $BE_2 = BC_2'$ at an exit angle of ϕ_2 of the second ring of moving tables.

(f) Join AE_2 which represents the inlet velocity to the second ring of fixed blades.

The velocity diagram can be drawn further if there are more number of rings of moving blades.

4.13.1 Workdone and Efficiencies

Workdone per kg of steam by the first ring of moving blades

$$= C_b (C_{wi_1} + C_{wo_1}) = AB (AD_1 + AF_1) = AB (D_1 F_1)$$

Workdone per kg of steam by second ring moving blades

$$= C_b (C_{wi_2} + C_{wo_2})$$

$$= AB (AD_2 + AF_2) = AB (D_2 F_2)$$

$\therefore$ Total workdone / kg of steam,

$$= C_b \cdot \Sigma (C_{wi} + C_{wo}) \quad ...(4.13.1)$$

$$\therefore \text{Power developed} = m \cdot \frac{C_b \cdot \Sigma (C_{wi} + C_{wo})}{1000} \quad ...(4.13.2)$$

$$\eta_b = \frac{C_b \cdot \Sigma (C_{wi} + C_{wo})}{(C_{i_1})^2 / 2}$$

$$= \frac{2 C_b \cdot \Sigma (C_{wi} + C_{wo})}{C_{i_1}^2} \quad ...(4.13.3)$$

$$\text{Stage efficiency, } \eta_s = \frac{\dot{m} (C_b) \Sigma (C_{wi} + C_{wo})}{\Delta H \times 1000} \quad ...(4.13.4)$$

Where, ΔH is the isentropic enthalpy drop in the nozzle for a given flow rate.

$$\text{Total axial thrust} = \dot{m} \Sigma (C_{fi} - C_{fo}) \quad ...(4.13.5)$$

4.13.2 Condition for Maximum Efficiency

Refer Fig. 4.13.1(b).

Work done per kg of steam in the first row of moving blades is given by,

$$W_1 = (C_{wi_1} + C_{wo_1}) C_b$$

$$= (C_{ri_1} \cos \theta_1 + C_{ro_1} \cos \theta_1) C_b$$

If there is no friction and blading is symmetrical, then,

$$C_{ri_1} = C_{ro_1} \text{ and } \theta_1 = \phi_1$$

$$\therefore W_1 = C_b (2 C_{ri_1} \cos \theta_1) = 2C_b (C_{i_1} \cos \alpha_1 - C_b) \quad ...(i)$$

Magnitude of absolute velocity of steam leaving the first stage and entering into the second stage of moving blades is same if the friction in the fixed blade is neglected and its direction is only changed.

$$\therefore C_{i_2} = C_{o_1}$$

Workdone per kg of steam in the second row of moving blades is given by

$W_2 = C_b \cdot C_{wi_2}$ if the discharge is axial i.e. $\beta_2 = 90°$

$$\text{Alternately, } W_2 = C_b \left[C_{ri_2} \cos \theta_2 + C_{ro_2} \cos \phi_2 \right] \quad ...(ii)$$

For symmetrical blades and without friction,

$$\theta_2 = \phi_2; \qquad C_{ro_2} = C_{ri_2}$$

$$\therefore \quad W_2 = 2C_b\, C_{ri_2} \cos\theta_2 = 2C_b\,(C_{i_2}\cos\alpha_2 - C_b) \quad ...(iii)$$

If, then $\alpha_2 = \beta_1$,

$$C_{i_2}\cos\alpha_2 = C_{o_1}\cos\beta_1 \qquad (\text{since } \alpha_2 = \beta_1 \text{ and } C_{i_2} = C_{o_1})$$

$$= (C_{ro_1}\cos\phi_1 - C_b) = (C_{ri_1}\cos\theta_1) - C_b$$

$$= (C_{i_1}\cos\alpha_1 - C_b) - C_b = C_{i_1}\cos\alpha_1 - 2\,C_b \quad ...(iv)$$

Substituting the value of $C_{i_2}\cos\alpha_2$ from Equation (iv) in Equation (iii) we get,

$$W_2 = 2\,C_b\,(C_{i_1}\cos\alpha_1 - 3\,C_b) \quad ...(v)$$

$\therefore$ Total workdone per kg of steam for both the stages becomes,

$$W = W_1 + W_2$$

$$= C_b\,(C_{i_1}\cos\alpha_1 - C_b) + 2\,C_b\,(C_{i_1}\cos\alpha_1 - 3\,C_b)$$

$$= 4\,C_b\,(C_{i_1}\cos\alpha_1 - 2\,C_b) \quad ...(vi)$$

The blade efficiency for two stage impulse turbine is,

$$\eta_b = \frac{W}{C_{i_1}^2/2} = \frac{4\,C_b\,(C_{i_1}\cos\alpha_1 - 2\,C_b)}{C_{i_1}^2/2}$$

$$= 8\cdot\frac{C_b}{C_{i_1}}\left[\cos\alpha_1 - 2\cdot\frac{C_b}{C_{i_1}}\right]$$

Let, $\quad s = \text{Blade speed to steam velocity ratio} = \dfrac{C_b}{C_{i_1}}$

$$\eta_b = 8s\,(\cos\alpha_1 - 2s) \quad ...(vii)$$

The blade efficiency for two stage will be maximum when, $\dfrac{d\,(\eta_b)}{ds} = 0$

$$\therefore \quad \frac{d}{ds}\,[8s\,(\cos\alpha_1 - 2\,s)] = 0$$

$$\therefore \quad 8\cos\alpha_1 - 32\,s = 0$$

$\therefore$ Condition for maximum efficiency, becomes,

$$s = \frac{\cos\alpha_1}{4} \quad ...(4.13.6)$$

Substituting this value in Equation (vii) we get,

$$(\eta_b)_{max} = 8\cdot\frac{\cos\alpha_1}{4}\left(\cos\alpha_1 - 2\times\frac{\cos\alpha_1}{4}\right)$$

$$= \cos^2\cdot\alpha_1 \quad ...(4.13.7)$$

– The maximum workdone can be calculated with the help of Equation (vi) by using the condition of maximum efficiency i.e.

$$s = \frac{C_b}{C_{i_1}} = \frac{\cos\alpha_1}{4} \quad \text{or,}$$

$$C_b = \frac{C_{i_1}\cos\alpha_1}{4}$$

$$\therefore \quad (W)_{max} = 4\,C_b\,[4\,C_b - 2\,C_b] = 8\,C_b^2 \quad ...(4.13.8)$$

– Above analysis is for two stage impulse turbine. The problem can be extended in a similar way for more stages. If there are 'n' stages, the optimum, blade speed ratio is given by,

$$\therefore \quad s = \frac{\cos\alpha_1}{2\cdot n} \quad ...(4.13.9)$$

The work done in the last row $= \dfrac{1}{2^n} \times$ total W.D. ...(4.13.10)

Where, $\quad n$ = number of rows of moving blades in series

– It can be seen from Equation (4.13.10) that as the number of rows increase, the work output in the subsequent rows keeps on decreasing i.e. the utility of the last rows decreases.

– For this reason, in practice, *more than two or three rows* are hardly preferred for velocity compounded impulse turbines.

4.14 Advantages/Disadvantages of Velocity Compounded impulse Turbine

Advantages

1. Due to relatively large heat drop, a velocity compounded impulse turbine requires comparatively small number of stages and less space.
2. With the increase in number of stages, the optimum blade speed ratio decreases.
3. Due to less number of stages, the cost of turbine is low.
4. The pressure drop in nozzles is considerable, therefore, the turbine casing need not be designed withstand high pressures.

Disadvantages

1. In case of velocity compounded impulse turbines (e.g. Curtis turbine), since the velocity of steam is very high due to total expansion of steam in nozzles, the friction losses are large, its efficiency is low.
2. The efficiency of the turbine keeps on decreasing with the increase in number of stages.
3. The power developed in later stages keeps on decreasing while all the stages require the same material space and the cost of fabrication.

Ex. 4.14.1 : Steam leaves the ring of nozzles of an impulse turbine at 450 m/s. The velocity is compounded in two rings of moving blade separated by a ring of fixed blades. The moving blades are symmetrical and their tip angle are 30° the blade velocity is 75 m/s. The friction for each ring of fixed and moving blades is 0.9. Determine the power developed and blade efficiency if the steam flow rate is 5 kg/s.

Soln. :

Given : $C_b = 75$ m/s; $C_{i_1} = 450$ m/s;

$\theta_1 = \theta_2 = \phi_1 = \phi_2 = 30°$; $K = 0.9$.

The velocity diagram can be drawn from the data given as shown in Fig. P. 4.14.1 on some suitable scale.

By measurement,

Power developed

$$= \dot{m} \times \frac{C_b \Sigma (C_{wi} + C_{wo})}{1000} = \dot{m} \times \frac{AB (F_1 D_1 + F_2 D_2)}{1000}$$

$$= 5 \times \frac{75 (626 + 304)}{1000} = 348.75 \text{ kW} \quad \text{...Ans.}$$

Blade efficiency, $$\eta_b = \frac{2 C_b \Sigma (C_{wi} + C_{wo})}{(C_{i1})^2}$$

$$= \frac{2 \times 74 (626 + 304)}{(450)^2}$$

$$= 0.689 \text{ or } 68.9\% \quad \text{...Ans.}$$

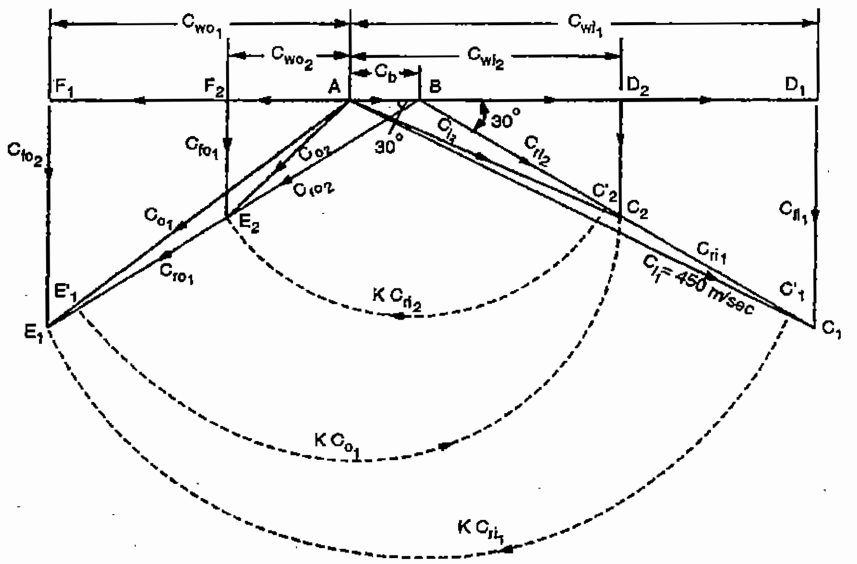


Fig. P. 4.14.1

Ex. 4.14.2 : Draw the velocity diagram for a velocity compounded impulse turbine having two rows of moving blades separated by a ring of fixed blades with the following particulars :

Nozzle angle, $\alpha_1 = 10°$; Moving blade tip discharge angles, $\phi_1 = \phi_2 = 30°$;

fixed blade discharge angle, $\alpha_2 = 13°$.

Assume a loss of 10% in velocity due to friction when the steam passes over a blade ring. Determine the blade velocity for a steam velocity of 500 m/s form the nozzles so that final discharge of steam is axial. Also, determine the power developed for 1 kg/s of steam flow and the stage efficiency.

Soln. :

Given : $K = K_1 = K_2 = (1 - 0.1) = 0.9$ and

$\beta_2 = 90°$ (discharge is axial) ;

$\alpha_1 = 10°$, $\alpha_2 = 13°$,

$\phi_1 = \phi_2 = 30°$

It should be noted that the efficiency of the turbine is maximum when the discharge of steam is axial.

The velocity diagram in such cases is drawn in the reverse direction as follows :

Referring to Fig. P. 4.14.2.

(i) Draw a line AB of certain length to represent blades velocity, C_b. Draw BE_2 at 30° to represent the discharge angle of moving blade and AE_2 at $\beta_2 = 90°$, since the discharge is axial. Then AE_2 represent the axial discharge on an unknown scale. Also the triangle ABE_2 is the final outlet velocity diagram of the stage.

(ii) Working backwards, the steam has just passed over the second ring of moving blades with 10% friction loss in its relative velocity, therefore, the line BE_2 should be extended such that $K = \frac{BE_2}{BE'_2} = 0.9$.

The line BE_2' will represent the relative velocity at inlet of the second ring of moving blades.

(iii) Draw $BC_2 = BE_2'$ and a line AC_2 at a discharge angle α_2. Then the triangle AC_2B is the inlet diagram for the second ring of moving tables.

(iv) Again working backwards, the steam has just passed over the ring of fixed blades with a 10% friction loss in its velocity.

Hence, increase AC_2 by 10% to AC_2'

$$\text{i.e. } \frac{AC_2}{AC_2'} = K = 0.9$$

(v) Cut $AC_2' = AE_1$ on the line BE_1 at discharge angle $\phi_2 = 30°$ of moving blade. Again extend BE_1' such that $\frac{BE_1}{BE_1'} = 0.9$ to account for the friction losses for the relative velocity in the first ring of moving blades

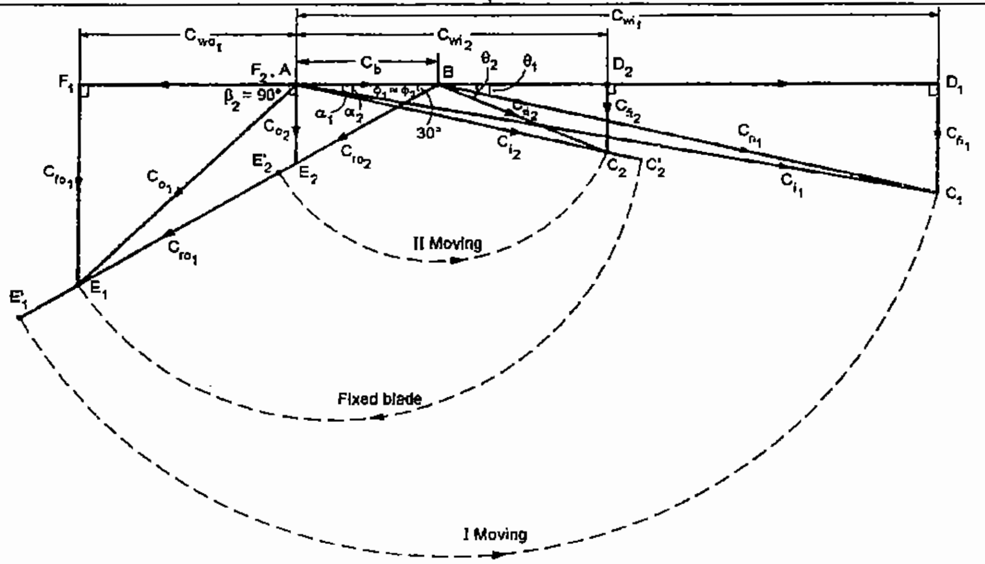


Fig. P. 4.14.2

(vi) Finally, describe an arc with B as center of radius BE_1'. Cut AC_1 at nozzle angle $\alpha_1 = 10°$. The triangle ABC_1 would represent the inlet velocity diagram for the first ring of moving blades where AC_1 represents the steam of jet velocity diagram for the first ring of moving blades where AC_1 represents the steam of jet velocity of 500 m/s.

Therefore, the scale of velocity diagram becomes,

$$1 \text{ cm} = \frac{C_{i_1}}{AC_1}$$

On this basis of the blade velocity,

$$\therefore \; C_b = AB = 107.5 \text{ m/s} \quad \text{...Ans.}$$

$$\text{Power developed} = \frac{\dot{m} C_b \Sigma (C_{wi} + C_{wo})}{1000}$$

$$= \frac{\dot{m} \times AB\,(F_1D_1 + F_2D_2)}{1000}$$

$$= \frac{1 \times 107.5\,(685 + 232)}{1000}$$

$$= \mathbf{98.58\ kW} \quad \text{...Ans.}$$

$$\text{Stage efficiency} = \frac{2\,C_b \cdot \Sigma (C_{wi} + C_{wo})}{(C_{i1})^2}$$

$$= \frac{2 \times 107.5\,(685 + 232)}{(500)^2}$$

$$= \mathbf{0.788 \text{ or } 78.8\%} \quad \text{....Ans.}$$

Ex. 4.14.3 : Steam in two row velocity compounded impulse turbine leaves the nozzles at 600 m/s. The blade speed is 120 m/s. The stage consists of two rows of moving blades with one ring of fixed blades between the rows. The nozzles angle is 16° while the discharge angles are 18° for the first ring of moving blade ring, 21° for the fixed blade ring and 35° for the second blade ring. Assuming 10% loss of velocity during the passage of steam through each ring of blades,

Determine :

(i) Blade inlet angle for each row.

(ii) Driving force and axial thrust for the stage in N.

(iii) Power developed in kW.

(iv) Diagram efficiency.

What would be maximum diagram efficiency for the given steam inlet velocity and nozzle angle ?

Soln. :

Given : $C_{i1} = 600$ m/s; $C_b = 120$ m/s;

$\alpha_1 = 16°$; $\phi_1 = 18°$;

$\alpha_2 = 21°$, $\phi_2 = 35°$;

Friction factor K = 1 - 0.1 = 0.9 (throughout)

$$= \frac{C_{ro_1}}{C_{ri_1}} = \frac{C_{i_2}}{C_{o_1}} = \frac{C_{ro_2}}{C_{ri_2}}$$

Draw velocity diagram as explained in section 4.13 on a convenient scale as shown in Fig. P. 4.14.3.

(i) Blade Inlet angle for each row

By measurement

Inlet angle of first ring of moving blade,

$\theta_1 = 20°$ ...**Ans.**

Inlet angle of fixed blade,

$\beta_1 = 25°$ ...**Ans.**

Inlet angle of second ring of moving blade,

$\theta_2 = 36°$ ...**Ans.**

Inlet angle of second ring of fixed blade,

$\beta_2 = 84°$ ...**Ans.**

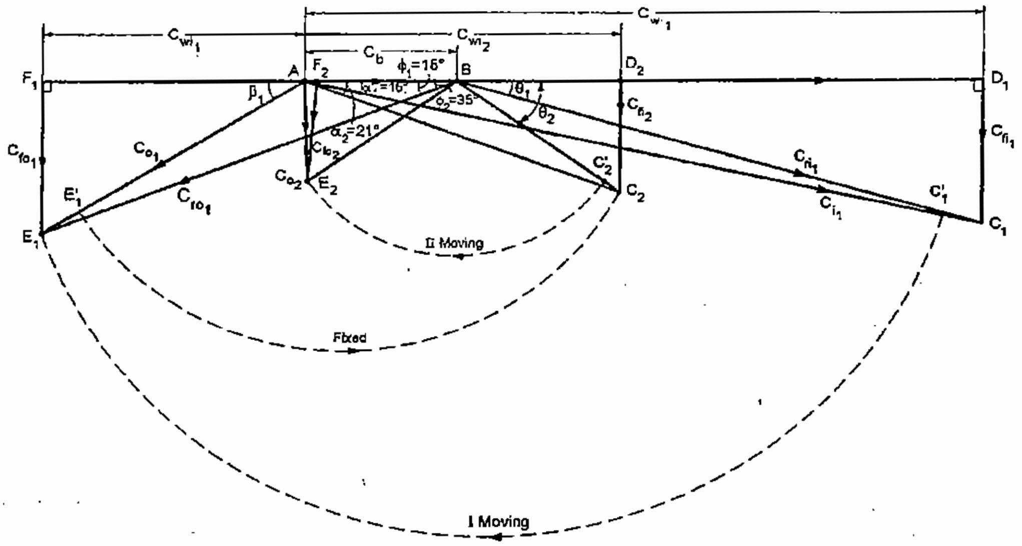


Fig. P. 4.14.3

(ii) Driving force and axial Thrust :

$C_{wi_1} = AD_1 = 540$ m/s

$C_{wo_1} = AF_1 = 266$ m/s

$C_{wi_2} = AD_2 = 250$ m/s

$C_{wo_2} = AF_2 = -10$ m/s

Driving force/unit mass of flow rate

$= \dot{m}\,(C_{wi_1} + C_{wo_1} + C_{wi_2} + C_{wo_2})$

$= 1(540 + 266 + 250 - 10)$

$=$ **1046 N/kg/s of steam flow** ...**Ans.**

Axial Thrust-unit mass of flow rate

$= \dot{m}\,[(C_{fi_1} - C_{fo_1}) + (C_{fi_2} - C_{fo_2})]$

$= 1\,[(152 - 128) + (96 - 84)]$

$=$ **36 N/kg/s of steam flow** ...**Ans.**

(iii) Power developed

Power developed $=$ Driving force $\times C_b$

$= 1046 \times 120$ Nm/s or W

$=$ **125520 W or 125.52 kW** ...**Ans.**

(iv) Diagram efficiency

Diagram efficiency,

$$\eta_b = \frac{2\,(\Sigma C_w)\,C_b}{(C_{i_1})^2} = \frac{2 \times 1046 \times 120}{(600)^2}$$

$=$ **0.6973 or 69.73%** ...**Ans.**

Maximum diagram efficiency,

$$(\eta_b)_{max} = \frac{\cos^2\alpha_1}{2}\left(1 + \frac{C_{ro}\cos\phi_1}{C_{ri_1}\cos\theta_1}\right)$$

$$= \frac{\cos^2 16}{2}\left(1 + 0.9 \times \frac{\cos 18}{\cos 20}\right)$$

$=$ **0.8828 or 88.28%** ...**Ans.**

Ex. 4.14.4 : The following particulars refer to a two row velocity compounded impulse steam turbine. Nozzle angle = 17° and blade speed = 125 m/s. Exit angle of the first row moving blade = 22°. Exit angle of fixed blade = 26°. Exit angle of second row moving blade = 30°. Blade velocity coefficient is 0.9 throughout for moving blades as well as fixed blades. Assuming that absolute velocity of steam leaving the stage is in the axial direction, find:

(i) Absolute velocity of steam leaving the stage

(ii) Diagram efficiency

(iii) Axial thrust produced per kg of steam flow

(iv) Power output per kg of steam.

Soln. :

Given : $\alpha_1 = 17°$; $C_b = 25$ m/s; $\phi_1 = 22°$; $\alpha_2 = 26°$; $\phi_2 = 30°$; $K = 0.9$

Final discharge is axial i.e. $\beta_2 = 90°$.

The velocity diagram can be drawn as per the method explained in Ex. 4.14.2 and it is shown in Fig. P. 4.14.4

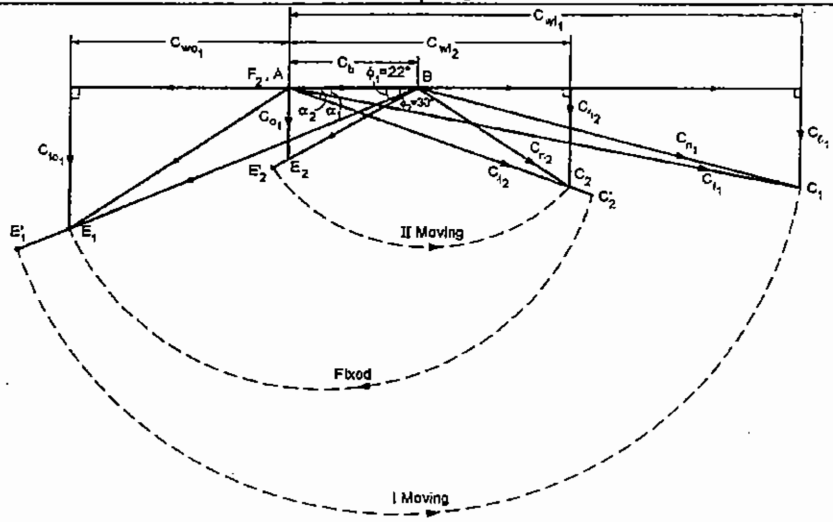


Fig. P. 4.14.4

(i) Absolute velocity of steam

By measurement we get,

$$C_{i_1} = AC_1 = 552.5 \text{ m/s} \quad \text{...Ans.}$$

(ii) Diagram efficiency

$C_{wi_1} = AD_1 = 526$ m/s ; $C_{wo_1} = AF_1 = 244$ m/s

$C_{wi_2} = AD_2 = 235$ m/s ; $C_{wo_2} = AF_2 = 0$

$C_{fi_1} = C_1D_1 = 160$ m/s ; $C_{fo_1} = E_1F_1 = 150$ m/s

$C_{fi_2} = C_2D_2 = 115$ m/s ; $C_{fo_2} = E_2F_2 = 72.5$ m/s

Diagram Efficiency,

$$\eta_b = \frac{2(C_{wi_1} + C_{wi_2} + C_{wo_1} + C_{wo_2})C_b}{C_{i_1}^2}$$

$$= \frac{2(526 + 235 + 244 + 0)125}{(552.5)^2}$$

$$= \frac{2 \times 1005 \times 125}{(552.5)^2}$$

or, η_b = **0.8231 or 82.31%** ...Ans.

(iii) Axial Thrust

$$\text{Axial thrust/kg of steam} = [C_{fi_1} - C_{fo_1} - C_{fi_2} - C_{fo_2}]$$

$$= (160 - 150) + (115 - 72.5)$$

$$= \mathbf{52.5\ N} \quad \text{...Ans.}$$

(iv) Power output

$$\text{Power output per kg of steam} = \frac{(\Sigma C_w)C_b}{100} \text{kNm/kg}$$

$$= \frac{1005 \times 125}{1000}$$

$$= \mathbf{125.625 kNm/kg} \quad \text{...Ans.}$$

Ex. 4.14.5 : Steam at 820 m/s enters the first stage of a two stage impulse turbine. The nozzle angle and exit angle of moving blades of the first stage is 30°. Also the exit angle of fixed blade and exit angle of the moving blades of the second stage is 30°. If the steam flow rate is 5 kg/s determine :

(i) Blade speed so that steam is finally discharged axially. Neglect friction over the moving and fixed blades.

(ii) Power developed by the turbine.

(iii) Diagram efficiency.

Soln. :

Given :

Steam velocity, C_i = 820 m/s

Nozzle exit angles, α_1 = Moving blade exit angle,

ϕ_1 = 30°

Exit angle of fixed blade

α_2 = Exit angle of second stage moving blade, ϕ_2

i.e. $\alpha_2 = \phi_2 = 30°$

Mass flow rate $\dot{m}$ = 5 kg/s

Since friction is neglected over moving and fixed blades, it implies K = 1

Final discharge is axial, therefore $\beta_2 = 90°$

Refer Fig. P. 4.14.5(a) and (b).

Velocity diagram for second stage

(i) Let AB represents the blade velocity C_b on some unknown scale.

(ii) Draw a line AE_2 at angle $\beta_2 = 90°$ and a line BE_2 at moving blades exit angle $\phi_2 = 30°$.
Then $AE_2 = C_{fo_2} = C_{o_2}$ and $BE_2 = C_{ro_2}$.

(iii) Since the steam has passed over the second ring of moving blades. Draw an arc B as center and BE_2 as radius and a line from point A at an angle $\alpha_2 = 30°$. Intersection of arc and line represents the point C_2. Join BC_2. Then $AC_2 = C_{i_2}$ and $BC_2 = C_{ri_2}$.

Velocity diagram for first stage

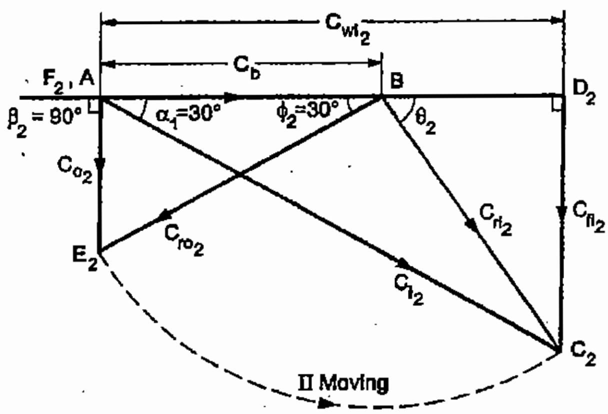
Refer Fig. P. 4.14.5(b).

(iv) Since the steam has earlier passed over the ring of fixed blades, it implies that $C_{o_1} = C_{i_2}$ i.e. $AE_2 = AC_2$. Therefore on blade velocity, C_b = AB the exit velocity diagram can be constructed by drawing a line BE_1 at an angle $\phi_1 = 30°$ and by cutting length $AE_1 = AC_2$. In this BE_1 represents C_{ro_1}.

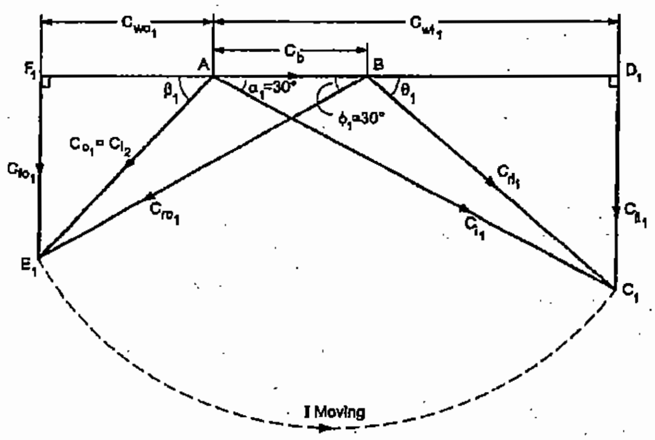
(v) Since the steam has earlier passed over the moving blades, the exit relative velocity C_{ro_1} is equal to inlet relative velocity C_{ri_1}. B as center and BE_1 as radius, describe an arc and draw a line from point A at an angle α_1. Intersection of line AC_1 and arc $E_1 C_1$ fixes the point C_1. Join AC_1 which represents C_{i_1}.

By measurements,

$$\text{Scale of diagram 1 cm} = \frac{C_{i_1} = 820 \text{ m/s}}{\text{Length } AC_1} = 94.2 \text{ m/s}$$



(a) Velocity diagram for second stage



(b) Velocity diagram for first stage

Fig. P. 4.14.5

(i) Blade velocity

$$C_b = AB = 235.6 \text{ m/s} \qquad \text{...Ans.}$$

$$C_{wi_1} + C_{wo_1} = F_1D_1 = 1007.9 \text{ m/s}$$

$$C_{wi_2} + C_{wo_2} = AD_2 = 376.8 \text{ m/s}$$

(ii) Power developed by turbine

$$P = \frac{\dot{m}\,(C_{wi_1} + C_{wo_1} + C_{wi_2} + C_{wo_2})\,C_b}{1000}$$

$$= \frac{5\,(1007.9 + 376.8)\,235.6}{1000}$$

$$= \mathbf{1631.2\ kW} \qquad \text{...Ans.}$$

(iii) Diagram efficiency

$$\eta_b = \frac{P}{\dot{m} \times \frac{C_{i_1}^2}{2}} = \frac{1631.2 \times 10^3}{5 \times \frac{(820)^2}{2}}$$

$$= \mathbf{0.9704 \text{ or } 97.04\%} \qquad \text{...Ans.}$$

4.15 Reheat Factor (R.F)

University Questions

Q. Explain the term Reheat factor related to steam turbine

SPPU : May 11, May 18

The expansion of steam in a turbine is essentially adiabatic. In case the friction is neglected, the process becomes **isentropic in an ideal case.**

However, due to fluid and blade friction leakages, shock etc., the effective enthalpy drop is reduced in the turbine blades.

In order to consider the effect of friction in case of turbines, let us consider the expansion of steam in a four stage turbine.

Let p_1, T_{sup1} be the inlet pressure and temperature of steam to first stage turbine and p_b represents the exit or back pressure from the last stage. Let p_2, p_3, p_4 be the intermediate stage pressures as shown in Fig. 4.15.1.

A_1 represent the state of steam at inlet to first stage turbine at p_1, T_{sup1} (A_1 D) represents the **ideal Rankine adiabatic heat drop** from the initial stage to the exit pressure.

A_1 B_1 represents the isentropic enthalpy drop in the first stage turbine. However, due to friction the actual enthalpy drop is A_1 C_1. Point A_2 can be located at the exit pressure of first stage turbine (at pressure p_2) and on the intersection of horizontal line corresponding to the total exit enthalpy at point C_1.

Point A_2 also represents the actual condition of steam at exit of first stage turbine or at inlet to the second stage turbine.

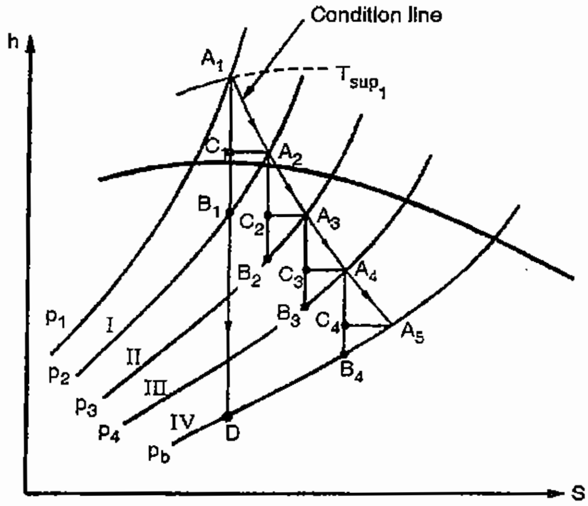


Fig. 4.15.1 : Reheat factor

Similarly, the isentropic and actual enthalpy drops in the successive stages are represented by A_2B_2 and A_2C_2, A_3C_3 etc. respectively. The condition of steam at the exit of different stages is represented by the points A_2, A_3, A_4 and A_5 and the curve joining these points is **called the condition line** representing the process path of expansion of steam approximately.

The algebraic sum of actual heat drops in stage i.e. $(A_1C_1 + A_2C_2 + A_3C_3) + ... = \Sigma AC$ is called the ***total useful heat drop.***

The algebraic sum of isentropic heat drops in stages.

i.e. $(A_1B_1 + A_2B_2 + A_3B_3 + ... = \Sigma AB)$ is called the ***cummulative isentropic heat drop.***

Since the constant pressure lines on (h-s) diagram converge at the origin, the cummulative isentropic heat drop (Σ AB) is always greater than the Rankine heat drop (A_1D)

The ratio of cummulative isentropic heat drop to the Rankine heat drop is defined as the **reheat factor (R.F.)** Hence.

$$\text{Reheat factor, (R.F.)} = \frac{\text{Cummulative isentropic heat drop}}{\text{Rankine heat drop}}$$

$$= \frac{\Sigma AB}{A_1D} \qquad \text{...(4.15.1)}$$

Internal efficiency of the turbine is defined as the ratio of total useful heat drop to the Rankine heat drop.

Accordingly,

Internal efficiency of the turbine,

$$\eta_t = \frac{\text{Total useful heat drop } \Sigma AC}{\text{Rankine heat drop } A_1 D} \quad ...(4.15.2)$$

Stage efficiency for each stage is the ratio of actual heat drop to the isentropic heat drop. Therefore, the expression for stage efficiency for various stages is as follows :

Stage efficiency for the first stage,

$$\eta_{(stage)\,1} = \frac{A_1 C_1}{A_1 B_1}$$

Similarly, $\eta_{(stage)\,2} = \frac{A_2 C_2}{A_2 B_2}$

$$\eta_{(stage)\,3} = \frac{A_3 C_3}{A_3 B_3}; \quad \text{and so on.}$$

If the stage efficiency is same for all the stages, then

$$\eta_{stage} = \frac{\Sigma AC}{\Sigma AB} \quad ...(4.15.3)$$

Equation (4.15.2) can be rewritten as,

$$\eta_t = \frac{\Sigma AC}{A_1 D} \times \frac{\Sigma AB}{\Sigma AB} \text{ or, } \eta_t = \frac{\Sigma AB}{A_1 D} \times \frac{\Sigma AC}{\Sigma AB}$$

Using Equations (4.15.1) and (4.15.3), above expression can be written as,

$$\text{or,} \quad \eta_t = (R.F.) \times \eta_{stage} \quad ...(4.15.4)$$

The reheat factor depends on the turbine stage efficiency, the condition of steam at entry and the exit pressure. **An important point to note is that the reheat factor depends upon the friction due to steam flow and it is no way concerned with the reheating of steam between the stages from the external heat source.**

The value of R.F. usually varies between 1.02 to 1.06

Overall efficiency of plant, η_o

Overall efficiency of the plant is defined as the ratio of useful work to the heat supplied.

$$\therefore \quad \eta_o = \frac{\text{Useful work}}{\text{Heat supplied}} = \frac{h_{A1} - h_{A5}}{h_A - h_{fD}} \quad ...(4.15.5)$$

Ex. 4.15.1 : A three stage turbine is supplied with steam at 30 bar and 350°C. The condenser pressure is 0.04 bar. The intermediate pressures are 5 bar and 1 bar. Assuming efficiency of each stage to be 80 %, determine :

(i) Adiabatic heat drop

(ii) Reheat factor

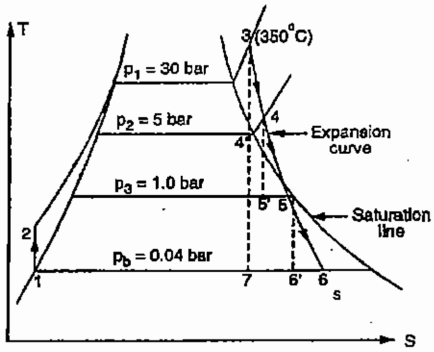
(iii) Internal efficiency of the turbine

Represent the processes on (T – S) and (h – S) diagrams

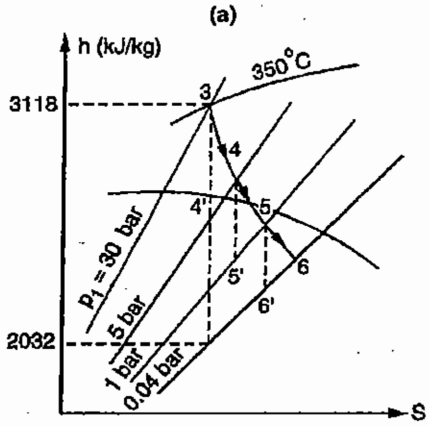
Soln. :

Processes on (T-S) and (h-S) diagram are represented in Fig. P. 4.15.1(a) and Fig. P. 4.15.1(b) respectively.

With the help of Mollier's (h-S) diagram we get, $h_3 = 3118$ kJ/kg, $h_7 = 2032$ kJ/kg



(a)



(b)

Fig. P. 4.15.1

(i) Rankine or adiabatic heat drop :

$$= h_3 - h_7 = 3118 - 2032 = \mathbf{1086\ kJ/kg} \quad ...\text{Ans.}$$

(ii) Reheat factor :

Again considering isentropic process from 30 bar to 5 bar pressure, from (h-S) diagram we get,

$$h_4' = 2718 \text{ kJ/kg}$$

$$\text{Stage efficiency} = \frac{h_3 - h_4}{h_3 - h_4'}$$

$$0.8 = \frac{3118 - h_4}{3118 - 2718}$$

$\therefore\ h_4 = 2798$ kJ/kg

Plot the point 4 at intersection of $h_4 = 2798$ kJ/kg and at $p_4 = 5$ bar pressure line. Again from isentropic process (4 – 5) we get

$$h_5' = 2515 \text{ kJ/kg}$$

$$\text{Stage efficiency} = \frac{h_4 - h_5}{h_4 - h_5'}$$

$$0.8 = \frac{2798 - h_5}{2798 - 2515}$$

$$\therefore\ h_5 = 2571.6 \text{ kJ/kg}$$

Plot point – 5 with known values of h_5 and $p_5 = 1$ bar.

For isentropic process (5 – 6′) we get,

$$h_6' = 2134.0 \text{ kJ/kg}$$

$$\text{Stage efficiency} = 0.8 = \frac{2571.6 - h_6}{2571.6 - 2134}$$

$$= h_6 = 2221.5 \text{ kJ/kg}$$

Hence, $h_3 = 3118$ kJ/kg ; $h_7 = 2032$ kJ/kg

$h_4' = 2718$ kJ/kg ; $h_4 = 2798$ kJ/kg

$h_5' = 2515$ kJ/kg ; $h_5 = 2571.6$ kJ/kg

$h_6' = 2134$ kJ/kg ; $h_6 = 2221.5$ kJ/kg

$$\text{Reheat factor R.F.} = \frac{\text{Cummulative isentropic heat drop}}{\text{Rankine heat drop}}$$

But, Cummulative adiabatic heat drop,

$$= (h_3 - h_4') + (h_4 - h_5') + (h_5 - h_6')$$

$$= (3118 - 2718) + (2798 - 2515) + (2571.6 - 2134)$$

$$= 1120.6 \text{ kJ/kg}$$

$$\therefore\ \textbf{R.F.} = \frac{\mathbf{1120.6}}{\mathbf{1086}} = \mathbf{1.0319} \qquad \textbf{...Ans.}$$

(iii) Internal efficiency of the turbine :

$$\eta_t = \text{R.F.} \times \text{Stage efficiency}$$

$$= 1.0319 \times 0.8$$

$$= \mathbf{0.8255 \text{ or } 82.55\ \%} \qquad \textbf{...Ans.}$$

Ex. 4.15.2 : In a three stage turbine the steam is supplied a 20 bar, 350°C and the exhaust pressure is 0.1 bar. Assuming that the work is shared equally in all the stages, the overall internal efficiency of the turbine is 75.96% and the condition line is straight line. Find : (a) The condition of steam leaving each stage, (b) The R.F. and (c) Efficiency of each stage.

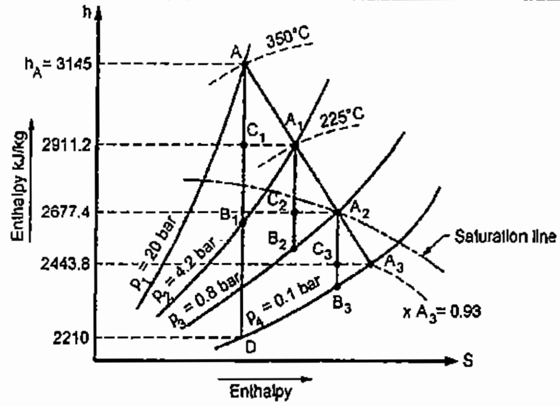


Fig. P. 4.15.2

Soln. :

It is convenient to solve this problem with the help of Mollier's (h-S) diagram as represented in Fig. P. 4.15.2.

Point - A represents the state of steam at inlet to the 1st stage of turbine at a given pressure of 20 bar and temperature of 350°C. Draw isentropic line AD vertically upto the exit pressure of 0.1 bar. From Mollier's diagram, we get $h_A = 3145$ kJ/kg, $h_D = 2210$ kJ/kg.

Given Internal efficiency of turbine, $\eta_t = 0.75$

$$\text{But,}\quad \eta_t = \frac{h_A - h_{A3}}{h_A - h_D}$$

$$0.7596 = \frac{3145 - h_{A3}}{3145 - 2210}$$

$$h_{A3} = 2434.7 \text{ kJ/kg}$$

Draw a horizontal line at $h_{A3} = 2434.7$ kJ/kg till it cuts 0.1 bar line at points A_3.

$(h_A - h_{A3})$ represents the turbine work, W_T. Join points A and A_3 by a straight line since the given condition line is a straight line.

$$W_T = h_A - h_{A3} = 3145 - 2434.7 = 710.3 \text{ kJ/kg}$$

$$\text{Work done per stage} = \frac{W_T}{\text{Number of stages}}$$

$$= \frac{710.3}{3} = 236.8 \text{ kJ/kg}$$

But, Workdone/stage $= (h_A - h_{A1}) = (h_{A1} - h_{A2})$

$= (h_{A2} - h_{A3})$

$236.8 = 3145 - h_{A1}$

or, $h_{A1} = 2908.2$ kJ/kg

Also, $236.8 = h_{A1} - h_{A2}$

$236.8 = 2908.2 - h_{A2}$

or, $h_{A2} = 2671.4$ kJ/kg

At enthalpies $h_{A1} = 2908.2$ and $h_{A2} = 2671.4$ kJ/kg draw horizontal lines which will cut the condition line at points A_1 and A_2 respectively on Mollier's diagram.

(a) The condition of steam leaving each stage :

The pressure line passing through points A_1 and A_2 will represent the pressures of intermediate stages and the condition of steam. Reading the values from (h-S) diagram, we get,

Condition of steam at exit of 1st stage

= **4.2 bar 225°C** ...**Ans.**

Condition of steam at exit of second stage

= **0.8 bar, dry- saturated** ...**Ans.**

Condition of steam at exit of third stage

= **0.1 bar, 0.93 dry** ...**Ans.**

(b) The R.F. :

From points A, A_1, A_2 draw isentropic lines and read,

$$h_{B1} = 2800 \text{ kJ/kg}$$

$$h_{B2} = 2570 \text{ kJ/kg} \quad \text{and} \quad h_{B3} = 2355 \text{ kJ/kg}$$

$$\text{R.F.} = \frac{\text{Cummulative isentropic heat drop}}{\text{Rankine heat drop}}$$

$$= \frac{(h_A - h_{B1}) + (h_{A1} - h_{B2}) - (h_{A2} - h_{B3})}{(h_A - h_D)}$$

$$= \frac{(3145 - 2800) + (2908.2 - 2570) - (2671.4 - 2355)}{(3145 - 2210)}$$

$$= \frac{999.6}{935} = \mathbf{1.069}$$ ...**Ans.**

(c) Efficiency of each stage

Efficiency of stage - 1

$$\eta_{s1} = \frac{h_A - h_{A1}}{h_A - h_{B1}} = \frac{3145 - 2908.2}{3145 - 2800}$$

$$= \mathbf{0.6864 \text{ or } 68.64\ \%}$$ ...**Ans.**

$$\eta_{s2} = \frac{h_{A1} - h_{A2}}{h_{A1} - h_{B2}} = \frac{2908.2 - 2671.4}{2908.2 - 2570}$$

$$= \mathbf{0.7 \text{ or } 70\ \%}$$ ...**Ans.**

$$\eta_{s3} = \frac{h_{A2} - h_{A3}}{h_{A2} - h_{B3}} = \frac{2671.4 - 2908.2}{2671.4 - 2355}$$

$$= \mathbf{0.7484 \text{ or } 74.84\ \%}$$ ...**Ans.**

Ex. 4.15.3 : A three stage turbine is fed at 26 bar and 370°C. Steam is exhausted at 0.05 bar. The interstage pressure are 5 bar and 1 bar. Stage efficiency for all stages is 80%. Assuming condition line to be straight, determine :

(i) Rankine efficiency

(ii) Quality of steam leaving each stage

(iii) Reheat factor

(iv) Workdone/kg of steam in each case

(v) Overall efficiency

Soln. : Refer Fig. P. 4.15.3.

Given : $p_1 = 26$ bar, $T_{sup\,1} = 370°C$; $p_4 = 0.05$ bar; $p_2 = 5$ bar, $p_3 = 1$ bar, $\eta_s = \eta_{s1} = \eta_{s2} = \eta_{s3} = 80\% = 0.8$; condition line is straight line.

(i) Rankine efficiency, η_R

From Mollier's diagram

$$h_{A1} = 3172 \text{ kJ/kg}; \qquad h_D = 2110 \text{ kJ/kg}$$

$$\eta_{stage} = \eta_s = 0.8 = \frac{h_{A1} - h_{A4}}{h_{A1} - h_D} = \frac{3172 - h_{A4}}{3172 - 2110}$$

$$\therefore \quad h_{A4} = 2322 \text{ kJ/kg}$$

Fix point A_4 on Mollier's diagram by drawing horizontal line at $h_{A4} = 2322$ kJ/kg which cuts $p_4 = 0.05$ line at A_4.

Join A_1 A_4 by a straight line since the condition line is a straight line. It cuts p_2 and p_3 lines at A_2 and A_3.

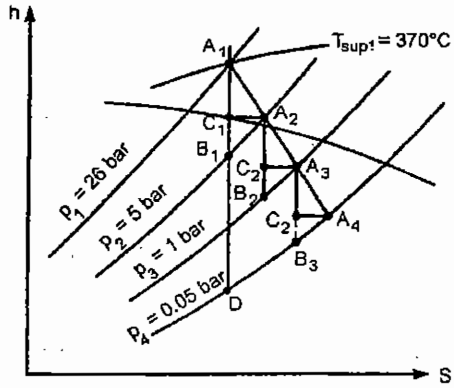


Fig. P. 4.15.3

Draw isentropic lines A_1 B_1, A_2 B_2 and A_3 B_3 and read :

$$h_{A2} = 2890 \text{ kJ/kg} \; ; \quad h_{B1} = 2790 \text{ kJ/kg}$$

$$h_{A3} = 2660 \text{ kJ/kg} \; ; \quad h_{B2} = 2590 \text{ kJ/kg}$$

$$h_{B3} = 2230 \text{ kJ/kg} \; ;$$

Read h_{fD} at 0.05 from steam tables. $h_{fD} = 137.8$ kJ/kg

Neglecting pump work,

∴ Rankine efficiency,

$$\eta_R = \frac{h_{A1} - h_D}{h_{A1} - h_{fD}} = \frac{3172 - 2110}{3172 - 137.8}$$

$$= \mathbf{0.35 \text{ or } 35\%}$$ ...**Ans.**

(ii) Quality of steam leaving each stage

$$T_{supA2} = C;$$

$$x_{A3} = 0.99;$$

$$x_{A4} = \mathbf{0.89}$$ ...**Ans.**

(iii) Reheat factor (R.F.)

$$\text{R.F.} = \frac{\Sigma AB}{A_1D} = \frac{A_1 B_1 + A_2 B_2 + A_3 B_3}{A_1 D}$$

$$= \frac{(3172 - 2790) + (2890 - 2590) + (2660 - 2230)}{(3172 - 2110)}$$

$$= 1.047 \qquad \text{...Ans.}$$

(iv) Workdone in each stage

First stage : $W_1 = h_{A1} - h_{A2} = 3172 - 2890$

$= \mathbf{282\ kJ/kg}$ **...Ans.**

Second stage : $W_2 = h_{A2} - h_{A3} = 2890 - 2660$

$= \mathbf{230\ kJ/kg}$ **...Ans.**

Third stage : $W_3 = h_{A3} - h_{A4} = 2660 - 2322$

$= \mathbf{338\ kJ/kg}$ **...Ans.**

(v) Overall efficiency

$$\eta_o = \frac{h_{A1} - h_{A4}}{h_{A1} - h_{fD}} = \frac{3172 - 2322}{3172 - 137.8}$$

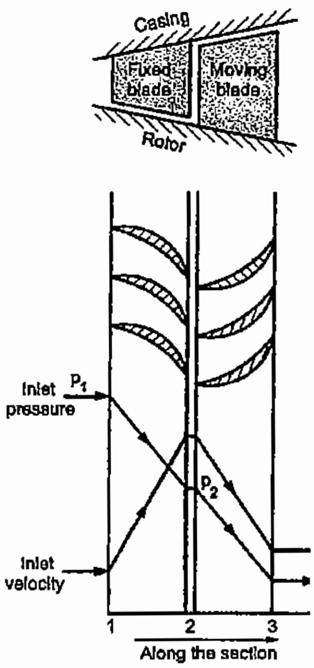
$$= 0.2801 \text{ or } 28.01\% \qquad \text{...Ans.}$$

4.16 Reaction Turbines

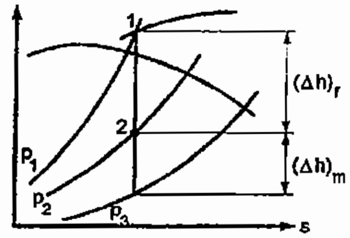
An impulse - reaction turbine, usually called as reaction turbine, uses the principle of both the impulse and reaction since the driving force for the turbine is partly an impulsive force and partly a reactive force.

It has already been discussed in earlier chapter that in case of reaction turbine, the steam continuously expands while it passes over the ring of fixed and moving blades.

Due to this the blade passages between consecutive blades are of converging type like a convergent nozzle as shown in Fig. 4.16.1.



(a)



(a)

Fig. 4.16.1 : A stage of reaction turbine

Fig. 4.16.1 shows the schematic diagram for one stage of a reaction turbine in which the fixed blades are fixed to the casing and the moving blades are fixed to the rotor of the turbine.

Steam enters the ring of fixed blade at pressure p_1 where it expands upto pressure p_2 before entry to ring of moving blades. Due to expansion of steam in fixed blades, the steam velocity increases.

Steam from fixed blades impinges on moving blades having the converging passages due to which steam expands giving rise to reactive force. The change in direction of velocity vector while passing over the moving blades is accompanied by change in momentum, thus exerts an impulsive force.

The effect of expansion of steam over the moving blades is to increase the relative velocity of steam at the exit of moving blade, C_{ro} compared to relative velocity of steam at entry to moving blades C_{ri}.

The variation of pressure and velocity along the axis of turbine is shown in Fig. 4.16.1(a) and the expansion of steam on (h – s) diagram in Fig. 4.16.1(b).

4.17 Degree of Reaction, R_D

University Question

Q. Prove that for Parson's reaction turbine moving and fixed blades are symmetrical in shape. SPPU : May 15

Q. Define the term Degree of reaction. SPPU : Dec. 16

The term **degree of reaction** as applied in case of reaction turbines is a measure of the proportion of the work done by reaction effect (due to pressure drop in moving blades) and it may be defined as :

Degree of reaction,

$$R_D = \frac{[\text{Enthalpy drop in moving blades } (\Delta h)_m]}{[\text{Enthalpy drop in moving blades } (\Delta h)_m + \text{Enthalpy drop in fixed blades } (\Delta h)_f]} \qquad ...(4.17.1)$$

Enthalpy drop in moving blades, $(\Delta h)_m$ is equal to increase in K.E. of the steam corresponding to relative velocity while the steam passes over the ring of moving blades, Therefore,

$$(\Delta h)_m = \frac{C_{ro}^2 - C_{ri}^2}{2}$$

Enthalpy drop in fixed blades, $(\Delta h)_f$ is given by,

$$(\Delta h)_f = \frac{C_i^2 - C_o^2}{2}$$

Therefore, total heat drop for the stage $(\Delta h_m + \Delta h_f)$ is equal to work done by the steam and it equals to,

$$(\Delta h)_m + (\Delta h)_f = C_b (C_{wi} + C_{wo})$$

Hence, Degree of reaction,

$$R_D = \frac{C_{ro}^2 - C_{ri}^2}{2 \cdot C_b (C_{wi} + C_{wo})} \qquad ...(4.17.2)$$

4.17.1 Velocity Diagram

University Question

Q. Prove that for Parson's reaction turbine moving and fixed blades are symmetrical in shape. **SPPU : May 15**

Referring to Fig. 4.17.1

$$C_{ro} = C_{fo} \operatorname{cosec} \phi$$

and $$C_{ri} = C_{fi} \operatorname{cosec} \theta$$

$$(C_{wi} + C_{wo}) = C_{fi} \cot \theta + C_{fo} \cot \phi$$

The *velocity of flow is generally constant* while the steam passes over the blade ring,

i.e. $$C_{fi} = C_{fo} = C_f \text{ (say)}$$

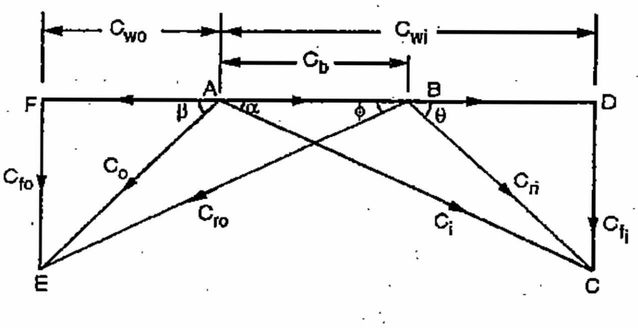


Fig. 4.17.1 : Velocity diagram

Substituting the values of C_{ro}, C_{ri} and $(C_{wi} + C_{wo})$ in Equation (4.17.2), we get,

Degree of reaction,

$$R_D = \frac{(C_f \operatorname{cosec} \phi)^2 - (C_f \operatorname{cosec} \theta)^2}{2 \cdot C_b \cdot C_f (\cot \theta + \cot \phi)}$$

$$= \frac{C_f (\operatorname{cosec}^2 \phi - \operatorname{cosec}^2 \theta)}{2 C_b (\cot \phi + \cot \theta)}$$

$$\therefore R_D = \frac{C_f}{2 C_b} \left[\frac{(\cot^2 \phi + 1) - (\cot^2 \theta - 1)}{\cot \phi + \cot \theta} \right]$$

$$= \frac{C_f}{2 C_b} \left[\frac{\cot^2 \phi - \cot^2 \theta}{\cot \phi + \cot \theta} \right]$$

$$= \frac{C_f}{2 C_b} \times (\cot \phi - \cot \theta) \qquad ...(i)$$

For 50% *degree of reaction turbine, called as **Parson's reaction turbine,*** Equation (i) can be rewritten as,

$$\frac{1}{2} = \frac{C_f}{2 C_b} (\cot \varphi - \cot \theta)$$

$$\therefore C_b = C_f (\cot \phi - \cot \theta) \qquad ...(ii)$$

From Fig. 4.17.1, the value of 'C_b' can also be written as,

$$C_b = C_f (\cot \alpha - \cot \theta) \qquad ...(iii)$$

and, $$C_b = C_f (\cot \phi - \cot \beta) \qquad ...(iv)$$

Comparing the Equations (ii), (iii) and (iv), we get,

$$\alpha = \phi \quad \text{and} \quad \theta = \beta \qquad ...(4.17.3)$$

It follows that for a 50% reaction turbine the moving and fixed blades must be made symmetrical in shape.

The velocity diagram for such a turbine will be symmetrical about a vertical center line as shown in Fig. 4.17.1.

4.17.2 Work, Power and Efficiency

$$\text{W.D. per kg of steam} = (C_{wi} + C_{wo}) C_b \qquad ...(4.17.4)$$

$$\text{Power developed per stage} = \frac{\dot{m} (C_{wi} + C_{wo}) C_b}{1000} \text{ kW} \qquad ...(4.17.5)$$

$$\text{Stage efficiency} = \frac{(C_{wi} + C_{wo}) C_b}{(\Delta h) \times 1000} \qquad ...(4.17.6)$$

Where, (Δh) is the heat drop in kJ/kg in a stage which can be determined with the help of Mollier's diagram Also,

$$\Delta h = \frac{C_i^2}{2} + \frac{C_{ro}^2 - C_{ri}^2}{2} \qquad ...(4.17.7)$$

$$\text{Blade efficiency, } \eta_b = \frac{\text{Work done}}{\text{Energy supplied}}$$

$$= \frac{C_b (C_{wi} + C_{wo})}{\left[\frac{C_i^2}{2} + \frac{C_{ro}^2 - C_{ri}^2}{2} \right]}$$

$$= \frac{2 C_b (C_{wi} + C_{wo})}{C_i^2 + (C_{ro}^2 - C_{ri}^2)} \qquad ...(4.17.8)$$

4.17.3 Blade Height

Now it becomes necessary to determine the height of blades required at a particular stage, let,

N = Speed of turbine, rpm

d = Drum diameter, m

h = Blade height, m

Velocity of flow, $C_f = C_{fi} = C_{fo}$ (m/s)

Referring to Fig. 4.17.2.

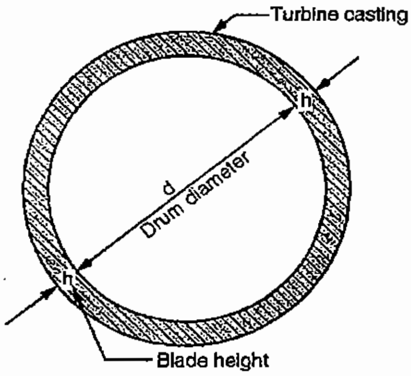


Fig. 4.17.2

Mean drum diameter, $d_m = (d + h)$

Area of flow (neglecting the area of blades occupied),

$$A_f = (\text{Mean circumference}) \times \text{Blade height}$$

$$= [\pi (d + h)]\, h$$

Volume flow rate of steam,

$$\dot{Q} = \text{Area of flow} \times \text{velocity of flow}$$

$$\dot{Q} = [\pi (d + h)\, h]\, C_f \qquad ...(4.17.9)$$

Mass flow rate of steam,

$$\dot{m} = \frac{[\pi (d + h)\, h]\, C_f}{v} \qquad ...(4.17.10)$$

where 'v' is the specific volume of steam in m^3/kg at the given stage.

Also, Blade speed $C_b = \dfrac{\pi (d + h) N}{60}$ (m/s) ...(4.17.11)

4.18 Condition for Maximum Blade Efficiency in Case of 50% Reaction Turbine

University Question

Q. Derive an expression for maximum Utilization factor (diagram efficiency) of Parson's reaction turbine in terms of nozzle angle. **SPPU : May 12, Dec. 15**

Work done per kg of steam from Equation (4.17.4) can be written as,

$$W = (C_{wi} + C_{wo})\, C_b$$

Referring to Fig. 4.17.1

$$C_{wi} = C_i \cos \alpha \quad \text{and,} \quad C_{wo} = C_{ro} \cos \phi - C_b$$

$$= C_i \cos \alpha - C_b \text{ (Since, } \alpha = \phi \text{ and } C_{ro} = C_i)$$

$$\therefore W = C_b (2 C_i \cos \alpha - C_b)$$

$$\text{or,} \quad W = C_i^2 \left[\frac{2\, C_b \cdot C_i \cos \alpha}{C_i^2} - \frac{C_b^2}{C_i^2} \right] \qquad ...(i)$$

Let the ratio of blade velocity to steam velocity called, **Blade velocity ratio,** be represented by 's' i.e.

Blade velocity ratio,

$$s = \frac{\text{Blade velocity, } (C_b)}{\text{Steam velocity, } (C_i)} \qquad ...(4.18.1)$$

Using definition of blade velocity ratio from above Equation, the Equation (i) for work becomes :

$$\text{Then,} \quad W = C_i^2 \,(2 s \cos \alpha - s^2) \qquad ...(ii)$$

Energy supplied / kg of steam

$$= \frac{C_i^2}{2} + \frac{C_{ro}^2 - C_{ri}^2}{2}$$

$$= C_i^2 - \frac{C_{ri}^2}{2} \text{ ...(since } C_1 = C_{ro}) \qquad ...(iii)$$

But, from Δ ABC,

$$C_{ri}^2 = C_i^2 + C_b^2 - 2 C_b \cdot C_i \cos \alpha$$

∴ Energy supplied per kg of steam becomes,

$$= C_i^2 - \frac{C_i^2 + C_b^2 - 2\, C_b \cdot C_i \cos \alpha}{2}$$

$$= \frac{C_i^2 + 2\, C_i \cdot C_b \cos \alpha - C_b^2}{2}$$

$$= \frac{C_i^2}{2} [1 + 2 s \cos \alpha - s^2] \qquad ...(iv)$$

The blade efficiency for the stage,

$$\eta_b = \frac{\text{W.D}}{\text{Energy supplied}}$$

$$= \frac{C_i^2 (2s \cos \alpha - s^2)}{\frac{C_i^2}{2} (1 + 2s \cos \alpha - s^2)}$$

$$= \frac{2\,(2 s \cos \alpha - s^2)}{(1 + 2 s \cos \alpha - s^2)}$$

$$= \frac{2\,(1 + 2 s \cos \alpha - s^2) - 2}{(1 + 2s \cos \alpha - s^2)}$$

$$= 2 - \frac{2}{(1 + 2 s \cos \alpha - s^2)} \qquad ...(4.18.2)$$

Therefore, η_b becomes maximum when factor $(1 + 2s\cos\alpha - s^2)$ becomes maximum

The required condition is,

$$\frac{d}{d(s)}(1 + 2s\cos\alpha - s^2) = 0$$

$$\text{or,} \quad 2\cos\alpha - 2s = 0 \quad \text{or,} \quad s = \cos\alpha$$

i.e. Condition for maximum efficiency,

$$s = \frac{C_b}{C_i} = \cos\alpha \qquad \text{...(4.18.3)}$$

Substituting the value of 's' from Equation (4.18.3) in Equation (4.18.2), we get,

$$(\eta_b)_{max} = 2 - \frac{2}{1 + 2\cos\alpha\cos\alpha - \cos^2\alpha}$$

$$\therefore \text{ Maximum efficiency, } (\eta_b)_{max} = \frac{2\cos^2\alpha}{1 + \cos^2\alpha} \qquad \text{...(4.18.4)}$$

Ex. 4.18.1 : For a certain stage of 50% reaction turbine mean rotor diameter is 1.35 m and speed ratio is 0.69. The rotor speed is 3000 rpm and outlet blade angle is 55°. Find:

(i) Inlet blade angle.

(ii) Blade efficiency and maximum blade efficiency.

SPPU : Dec.19, 10 Marks

Soln. : Refer Fig. P. 4.18.1.

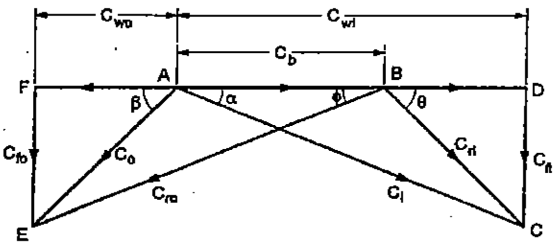


Fig. P. 4.18.1

$$R_D = 50\%, \qquad d_m = 1.35 \text{ m}$$

$$\text{Speed ratio, } S = \frac{C_b}{C_i} = 0.69 \; ; \qquad N = 3000 \text{ rpm}$$

$$\alpha = 55° \text{ from axial direction, therefore,}$$

$$\alpha = 90 - 55 = 35° \text{ from tangential direction}$$

$$\text{Outlet angle, } \phi = 35° = \alpha \qquad (\because R_D = 0.5)$$

$$C_b = \frac{\pi d_m N}{60} = \frac{\pi \times 1.35 \times 3000}{60}$$

$$= 212.1 \text{ m/s}$$

$$C_i = \frac{C_b}{S} = \frac{212.1}{0.69} = 307.4 \text{ m/s}$$

(i) Inlet blade angle, θ :

Consider inlet ΔACD,

$$C_{wi} = C_i\cos\alpha = 307.4\cos 35 = 251.8 \text{ m/s}$$

$$C_{fi} = C_i\sin\alpha = 307.4\sin 35 = 176.3 \text{ m/s}$$

$$BD = C_{wi} - C_{wi} = 251.8 - 212.1$$

$$= 39.7 \text{ m/s} = C_{wo}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right)$$

$$= \tan^{-1}\left(\frac{176.3}{39.7}\right) = 77.3° \qquad \text{...Ans.}$$

$$C_{ri} = \sqrt{(C_{fi})^2 + (BD)^2}$$

$$= \sqrt{(176.3)^2 + (39.7)^2}$$

$$= 180.7 \text{ m/s}$$

(ii) Blade efficiency, η_b : ($C_{ro} = C_i$)

$$\eta_b = \frac{(C_{wi} + C_{wo})\,C_b}{\frac{C_i^2}{2} + \frac{C_{ro}^2 - C_{ri}^2}{2}} = \frac{(C_{wi} + C_{wo})\,C_b}{C_i^2 - \frac{C_{ri}^2}{2}}$$

$$= \frac{(251.8 + 39.7)\,212.1}{(307.4)^2 - \frac{(180.7)^2}{2}}$$

$$= \mathbf{0.7909 \text{ or } 79.07\%} \qquad \text{...Ans.}$$

Ex. 4.18.2 : A 50% reaction turbine (with symmetrical velocity triangles) is running at 400 rpm has the exit angle of blades as 20° and the velocity of the steam relative to the blades at the exit is 1.35 times the mean blade speed. The steam flow rate is 8.33 kg/s and at a particular stage the specific volume is 1.381 m³/kg. Calculate for the stage,

(i) A suitable blade height, assuming the rotor mean diameter 12 times the blade height.

(ii) The diagram work.

SPPU : Dec. 18, 10 Marks

Soln. :

50% reaction turbine i.e. $\alpha = \phi$ and $\theta = \beta$

$$N = 400 \text{ rpm}; \qquad \alpha = \phi = 20°;$$

$$C_{ro} = 1.35, \qquad C_b = C_i,$$

$$\dot{m} = 8.33 \text{ kg/s}, \qquad v = 1.381 \text{ m}^3\text{/kg};$$

Mean rotor diameter, $d_m = (d + h) = 12\,h$

1. Suitable blade height, h

Refer Fig. P. 4.18.2

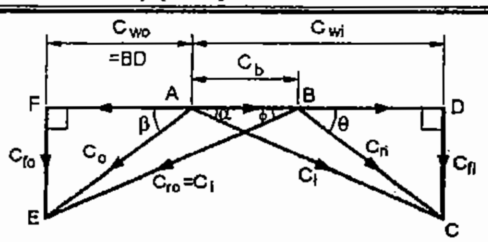


Fig. P. 4.18.2

$$C_b = \frac{\pi \times d_m \times N}{60} = \frac{\pi \times d_m \times 400}{60}$$

$$C_b = 20.944\, d_m \quad \text{...(i)}$$

$$= 20.944 \times 12\, h \quad \text{...(ii)}$$

$$C_{fi} = C_i \sin \alpha = 1.35\, C_b \sin 20$$

$$C_{fi} = 0.462\, C_b$$

$$= 0.462 \times 20.944 \times 12\, h = 251.28h$$

$$\dot{m} = \frac{\pi\, d_m\, h\, C_{fi}}{v};$$

$$8.33 = \frac{\pi \times 12\, h \times h \times 251.28\, h}{1.381};$$

$$h = \mathbf{0.1067\ m} \quad \text{...Ans.}$$

$$d_m = 12\, h = 12 \times 0.1067 = \mathbf{1.2803\ m} \quad \text{...Ans.}$$

2. Diagram work , $\dot{W}$:

$$C_b = 20.944 \times d_m$$

$$= 20.944 \times 1.2803 = 26.81\ m/s$$

$$C_i = 1.35\, C_b = 1.35 \times 26.81 = 36.2\ m/s$$

$$C_{wi} = C_i \cos \alpha = 36.2 \cos 20 = 34.01$$

$$C_{wo} = BD = C_{wi} - C_b = 34.01 - 26.81 = 7.2\ m/s$$

$$\dot{W} = \frac{\dot{m}\ (C_{wi} + C_{wo})\, C_b}{1000}\ kJ/s \text{ or } kW$$

$$= \frac{8.33\ (34.01 + 7.2)\ 26.81}{1000}$$

$$= \mathbf{9.2033\ kW} \quad \text{...Ans.}$$

Ex. 4.18.3 : In a stage of a Turbine with Parsons blading delivers dry saturated steam at 2.7 bar from the fixed blades at 90 m/sec. The mean Blade height is 40 mm and the moving blade exit angle is 20°. The axial velocity of steam is 3/4 of the blade velocity at the mean radius. Steam is supplied to the stage at the rate of 9000 kg/hr the effect of the blade tip thickness on the annulus area can be neglected. Calculate :

i) Wheel speed in RPM

ii) The diagram power

iii) The diagram efficiency

iv) The enthalpy drop of steam in the stage.

SPPU : May 18, 10 Marks

Soln. :

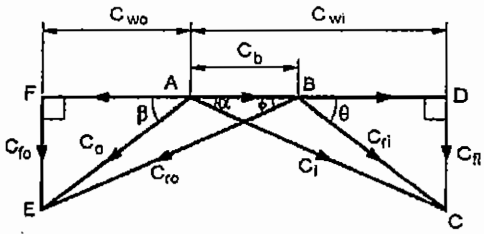


Fig. P. 4.18.3

$$p = 2.7 \text{ bar, dry - saturated}$$

$$C_i = 90\ m/s = C_{ro},$$

$$\text{Blade height, } h = 40\ mm = 0.04\ m$$

$$\phi = 20° = \alpha \qquad (\because \text{ Parson's turbine})$$

$$C_{fi} = C_{fo} = \frac{3}{4} \times C_b = C_f \quad \text{... (i)}$$

$$\dot{m} = 9000\ kg/hr = \frac{9000}{3600} = 2.5\ kg/s$$

(i) Wheel speed in rpm, N

from inlet Δ ACD ,

$$C_{fi} = C_i \sin\alpha = 90 \sin 20 = 30.78\ m/s = C_{fo} = C_f$$

$$C_{wi} = C_i \cos\alpha = 90 \cos 20 = 84.57\ m/s$$

$$\therefore C_{fi} = \frac{3}{4} C_b;$$

$$C_b = \frac{4}{3}\, C_{fi} = \frac{4}{3} \times 30.78 = 41.04\ m/s$$

$$AF = BD = C_{wo} = C_{wi} - C_b$$

$$= 84.57 - 41.04 = 43.53 m/s$$

$$C_{ri} = \sqrt{C_{fi}^2 + (C_{wi} - C_b)^2}$$

$$= \sqrt{(30.78)^2 + (84.57 - 41.04)^2}$$

$$= 53.31 m/s$$

From steam tables at p = 2.7 bar and dry-saturated we get,

Specific volume of steam, $v_g = 0.6695\ m^3/s$

$$\dot{m} = \frac{\pi\, (d + h) \times h \times C_f}{v_g}$$

$$2.5 = \frac{\pi \times (d + 0.04) \times 0.04 \times 30.78}{0.6695};$$

$$d = \mathbf{0.393\ m} \quad \text{...Ans.}$$

$$C_b = \frac{\pi\, d\, N}{60};$$

$$41.04 = \frac{\pi \times 0.393 \times N}{60};$$

$$N = \mathbf{1994.4 rpm} \quad \text{...Ans.}$$

(ii) Diagram Power, P

$$P = \frac{\dot{m}\ (C_{wi} + C_{wo})\, C_b}{1000}\ (kW) = \frac{2.5 \times (84.57 + 43.53)\ 41.04}{1000}$$

$= 13.143$ **kW** ...**Ans.**

(iii) Diagram efficiency, η_b

$$\eta_b = \frac{(C_{wi}+C_{wo})C_b}{\frac{1}{2}[C_i^2+(C_{ro}^2-C_{ri}^2)]} = \frac{2\times(84.57+43.53)\times 41.04}{90^2+(90^2-53.31^2)}$$

$= 0.7871$ **or 78.71%** ...**Ans.**

(iv) Enthalpy drop / stage, Δh

$$\Delta h = \frac{\dot{m}\, C_b\,(C_{wi}+C_{wo})}{1000} = \frac{2.5\times 41.04(84.57+43.53)}{1000}$$

$= 13.143$ **kJ** ...**Ans.**

Ex. 4.18.4 : In Parson's reaction turbine running at 500 rpm with 50% reaction develops 75 kW per kg per second of steam. The exit angle of blades is 20% and steam velocity is 1.5 times the blade velocity. Determine

(i) Blade velocity

(ii) Inlet angle of moving blades

SPPU - Feb. 16(In Sem), 6 Marks

Soln. : Refer Fig. P. 4.18.4.

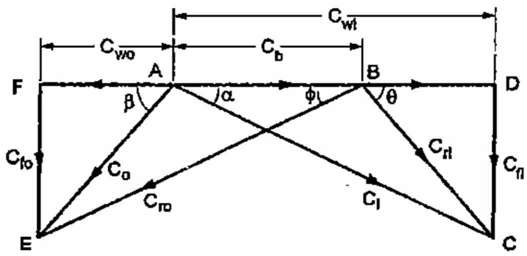


Fig. P. 4.18.4

$N = 500$ rpm ;

Power developed, $P = 75$ kW

When $\dot{m} = 1$ kg/s

Exit angle of blades, $\alpha = \phi = 20°$ (50% reaction) and $\theta = \beta$

Steam velocity, $C_i = 1.5\times C_b$; (C_b = Blade velocity)

(i) Blade velocity, C_b

Consider inlet Δ ACD,

$$C_{wi} = C_i\cos\alpha = 1.5\,C_b\times\cos 20 = 1.40954\,C_b$$

$$BD = C_{wi} - C_b = 1.40954\,C_b - C_b = 0.40954\,C_b$$

But $AF = C_{wo} = BD$ for 50% reaction steam turbine

$$C_{wo} = 0.40954\,C_b$$

$$P = \frac{\dot{m}\,(C_{wi}+C_{wo})\,C_b}{1000}\ (\text{kW})$$

$$75 = \frac{1\,(1.40954\,C_b + 0.40954\,C_b)\,C_b}{1000}$$

$C_b = 203.1$ **m/s** ...**Ans.**

(ii) Inlet angle of moving blades, θ

$$C_{wi} = 1.40954\,C_b = 1.40954\times 203.1 = 286.3\ \text{m/s}$$

$$C_i = 1.5\,C_b = 1.5\times 203.1 = 304.7\ \text{m/s}$$

$$C_{fi} = C_i\sin\alpha = 304.7\sin 20 = 104.2\ \text{m/s}$$

$$BD = C_{wi} - C_b = 286.3 - 203.1 = 83.2\ \text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{83.2}{104.2}\right) = 38.6°$$ ...**Ans.**

Ex. 4.18.5 : A Parsons turbine runs at 400 rpm with 50% reaction and it develops 75kW of power per unit mass of steam flow per second. The exit angle of the blades is 20° and the steam velocity is 1.4 times the blade velocity. Find :

(i) blade velocity and

(ii) inlet angle of the blades

SPPU - May 16, 6 Marks

Soln. : Refer Fig. P. 4.18.5.

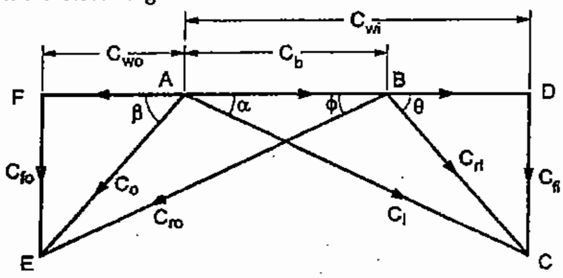


Fig. P. 4.18.5

$N = 400$ rpm, $p = 75$ kW/kg s,

$\phi = 20°$ $C_i = 1.4\times C_b$ ...(i)

$\alpha = \phi = 20°$ and $\theta = \beta = (\because$ 50% Reaction)

(1) Blade velocity, C_b

From Δ ACD :

$$C_{wi} = C_i\cos\alpha = C_i\cos 20 = 0.9397\,C_i = 0.9397\times 1.4\,C_b = 1.3156\,C_b$$

$$BD = AF = C_{wo} = C_{wi} - C_b = 1.3156\,C_b - C_b = 0.3156\,C_b$$

$$P = \frac{\dot{m}\,(C_{wi}+C_{wo})\,C_b}{1000}\ (\text{kW})$$

$$75 = \frac{1\,(1.3156\,C_b + 0.3156\,C_b)\,C_b}{1000}$$

$$C_b = 214.4 \text{ m/s} \qquad \text{...Ans.}$$

(ii) Inlet angle of the blades, $\theta = \beta$

$$BD = 0.3156\, C_b = 0.3156 \times 214.4 = 67.67 \text{ m/s}$$

$$C_i = 1.3156\, C_b = 1.3156 \times 214.4 = 282.06 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 280.06 \sin 20 = 96.47 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{96.47}{67.67}\right)$$

$$= 54.95° = \beta \qquad \text{...Ans.}$$

Ex. 4.18.6 : The following particulars refer to a stage of a an impulse reaction turbine : Mean diameter = 96 cm, speed = 3000 rpm, outlet angle of fixed blades = 20°, height of blades = 12 cm, specific volume of steam at exit of fixed blades = 4.4 m^3 / kg and that at exit of moving blades = 4.8 m^3 / kg. Steam velocity at exit of fixed blades = 275 m/s. Power developed = 260 kW. Calculate the enthalpy drop in the stage, degree of reaction, moving blade outlet angle and gross stage efficiency. Assume carry over coefficient as 0.81, expansion efficiency as 0.94. Thickness of blades may be neglected.

Soln. : Let d be rotor diameter and h height of blades.

Given :

$(d + h) = d_m = 96 \text{ cm} = 0.96 \text{ m}$,

$N = 3000$ rpm, $\alpha = 20°$,

$h = 12 \text{ cm} = 0.12 \text{ m}$, $v_i = 4.4 \text{ m}^3/\text{kg}$,

$v_e = 4.8 \text{ m}^3/\text{kg}$, $C_i = 275 \text{ m/s}$,

$P = 260$ kW

Carryover coefficient,

$$\delta = 0.81, \eta_{exp.} = 0.94$$

Blade speed,

$$C_b = \frac{\pi d_m N}{60} = \frac{\pi \times 0.96 \times 3000}{60}$$

$$= 150.8 \text{ m/s}$$

Fig. P. 4.18.6 shows the velocity diagram for reaction turbine.

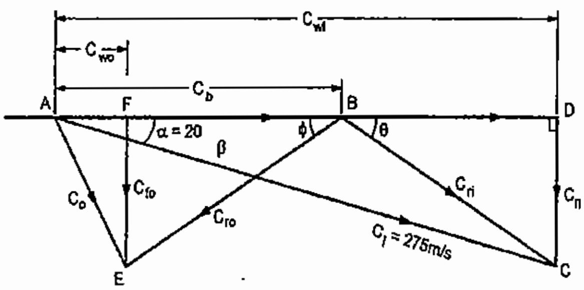


Fig. P. 4.18.6 : Velocity diagram

From Δ ACD :

$$C_{wi} = C_i \cos\alpha = 275 \cos 20 = 258.4 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 275 \sin 20 = 94.1 \text{ m/s}$$

$$BD = C_{wi} - C_b = 258.4 - 150.4 = 107.6 \text{ m/s}$$

$$C_{ri}^2 = \sqrt{BD^2 + C_{fi}^2} = \sqrt{107.6^2 + 94.1^2}$$

$$= 142.9 \text{ m/s}$$

Mass flow rate,

$$\dot{m} = \frac{\pi (d+h)\, h \cdot C_{fi}}{v_i}$$

$$= \frac{\pi \times 0.96 \times 0.12 \times 94.1}{4.4} = 7.74 \text{ kg/s}$$

Power developed,

$$P = \frac{\dot{m}\,(C_{wi} + C_{wo})\, C_b}{1000} \text{kW}$$

$$260 = \frac{7.74\,(258.4 + C_{wo})\, 150.8}{1000}$$

$$\therefore\ C_{wo} = -35.6 \text{ m/s}$$

Also,
$$\dot{m} = \frac{\pi (d+h)\, h\, C_{fo}}{v_o}$$

$$\text{i.e. } 7.74 = \frac{\pi \times 0.96 \times 0.12 \times C_{fo}}{4.8}$$

$$C_{fo} = 102.7 \text{ m/s}$$

Therefore, the exit velocity diagram ABE can be completed a shown in Fig. P. 4.18.6

From Δ FBE

$$C_{ro} = \sqrt{FB^2 + FE^2} = \sqrt{(C_b - C_{wo})^2 + C_{fo}^2}$$

$$= \sqrt{(150.8 - 35.6)^2 + 102.7^2} = 154.3 \text{ m/s}$$

From Δ AEF,

$$C_o = \sqrt{AF^2 + FE^2} = \sqrt{C_{wo}^2 + C_{fo}^2}$$

$$= \sqrt{(35.6)^2 + (102.7)^2} = 108.7 \text{ m/s}$$

Enthalpy drop in stage, ΔH

$$(\Delta h)_{fixed} = \frac{C_i^2 - \delta \cdot C_o^2}{2 \times 1000 \times \eta_{exp.}}$$

where δ is the carryover coefficient and η_{exp} is the expansion efficiency. C_o represent the absolute of steam going to next stage.

$$\therefore\ (\Delta h)_{fixed} = \frac{275^2 - 0.81 \times 108.7^2}{2 \times 1000 \times 0.94} = 35.14 \text{ kJ/kg}$$

$$(\Delta h)_{moving} = \frac{C_{ro}^2 - \delta \cdot C_{ri}^2}{2 \times 1000 \times \eta_{exp.}}$$

$$= \frac{154.3^2 - 0.81 \times 142.9^2}{2 \times 1000 \times 0.94} = 3.87 \text{ kJ/kg}$$

Total enthalpy drop in stage,

$$\Delta H = \dot{m}\ [(\Delta h)_{fixed} + (\Delta h)_{moving}]$$

$$= 7.74\ (35.14 + 3.87)$$

$$= \mathbf{301.9\ kJ/s} \quad \text{...Ans.}$$

Degree of reaction, R

$$R = \frac{(\Delta h)_{moving}}{(\Delta h)_{fixed} + (\Delta h)_{moving}} = \frac{3.87}{35.14 + 3.87}$$

$$= 0.0992$$

Moving blade outlet angle, ϕ

$$\sin \phi = \frac{C_{fo}}{C_{ro}} = \frac{102.7}{154.3} = 0.6655$$

$$\therefore \quad \phi = \mathbf{41.72^\circ} \quad \text{...Ans.}$$

Gross stage efficiency, η_s

$$\eta_s = \frac{P}{\Delta H} = \frac{260}{301.9} = \mathbf{0.8612 \text{ or } 86.12\%} \quad \text{...Ans.}$$

Ex. 4.18.7 : In a stage of 50% Parson's reaction turbine, the steam consumption is 18000 kg/hr and it runs at 300 r.p.m. The discharge blade tip angles are 20° both for fixed and moving blades. The axial velocity of flow is 0.7 times the blade velocity. Determine the drum diameter and blade height of a particular turbine pair where pressure of steam is 2.0 bar of dryness 0.95. The power developed by the turbine amounts to 3.75 kW.

Soln. :

Given : $\alpha = \phi = 20^\circ$; $N = 300$ r.p.m.

$R_D = 50\%$ $C_{fi} = C_{fo} = C_f = 0.7\ C_b$;

$\dot{m} = 18000$ kg/hr $= 5$ kg/s,

$p = 2$ bar, $x = 0.95$

Also $v = x \cdot v_g$

$= 0.95 \times 0.886$ (with the help of steam tables)

$= 0.8417\ m^3/kg$

$$C_b = \frac{\pi (d + h) N}{60} = \frac{\pi (d + h) 300}{60}$$

$$\text{or, } C_b = 5\pi (d + h) \quad \text{...(i)}$$

Referring to Fig. P. 4.18.7

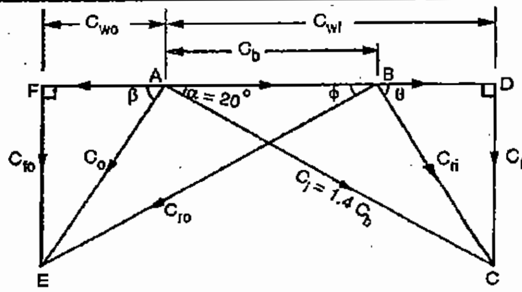


Fig. P. 4.18.7

$$C_{wi} = \frac{C_{fi}}{\tan \alpha} = \frac{0.7\ C_b}{\tan 20} = 1.923\ C_b$$

and $C_{wo} = C_{wi} - C_b = 1.923\ C_b - C_b = 0.923\ C_b$

Power developed,

$$P = \frac{\dot{m}\ (C_{wi} + C_{wo})\ C_b}{1000} \text{kW}$$

$$3.75 = \frac{5\ (1.923\ C_b + 0.923\ C_b)\ C_b}{1000}$$

$$\text{or, } C_b = 16.23 \text{ m/s} \quad \text{...(ii)}$$

$$C_{fi} = 0.7\ C_b = 11.36 \text{ m/s} \quad \text{...(iii)}$$

From Equations (i) and (ii) we get,

$$(d + h) = 1.033 \text{ m}$$

$$\text{Also} \quad \dot{m} = \frac{\pi (d + h)\ h\ C_{fi}}{C}$$

$$5 = \frac{\pi\ (1.033)\ h \times 11.36}{0.8417}$$

$$\text{or, } h = 0.1141 \text{ m} = 11.41 \text{ cm}$$

Since $d + h = 1.033$ m

$\therefore$ $d + 0.1141 = 1.033$

$$\text{or, } d = \mathbf{0.9189\ m \text{ or } 91.89\ cm} \quad \text{...Ans.}$$

Ex. 4.18.8 : A reaction turbine runs at 3000 r.p.m. and the steam consumption is 20000 kg/hr. The pressure of the steam at a certain pair is 2 bar, its dryness fraction is 0.93 and the power developed by the pair is 50 kW. The discharging blade angle is 20° for both the fixed and moving blades and the axial velocity of flow 0.72 times the blade velocity. Find the drum diameter and the blade height. Take the tip leakage steam as 8%. Neglect blade thickness.

Soln. : Refer Fig. P. 4.18.8.

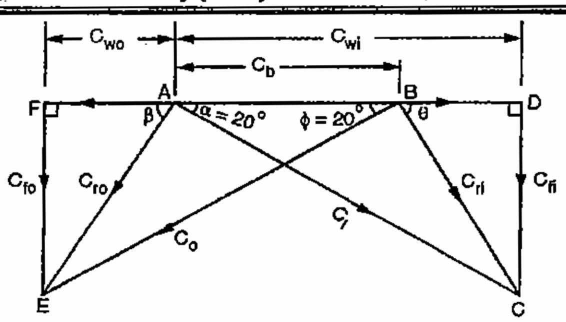


Fig. P. 4.18.8

Given : $N = 3000$ r.p.m.; $P = 50$ kW;

$$\dot{m}_t = 20000 \text{ kg/hr} = \frac{50}{9} \text{ kg/s};$$

$\alpha = \phi = 20°$; $p = 2$ bar

$x = 0.93$; $C_{fi} = C_{fo} = 0.72\, C_b$;

Leakage of steam = 8% = 0.08

Actual mass flow rate of steam,

$$\dot{m}_a = \text{theoretical mass flow rate } (\dot{m}_t) \times (1 - \text{tip leakage})$$

or, $$\dot{m}_a = \frac{50}{9}(1 - 0.08) = 5.111 \text{ kg/s}$$

Specific volume of steam,

$$v = x \cdot v_g = 0.93 \times 0.8857 = 0.8237 \text{ m}^3/\text{kg}$$

Blade speed, C_b

$$C_b = \frac{\pi (d+h) N}{60} = \frac{\pi (d+h)\, 3000}{60}$$

$$= 157.08\,(d+h) \quad ...(i)$$

$$C_{fi} = 0.72\, C_b = 0.72 \times 157.08\,(d+h)$$

$$= 113.09\,(d+h) \quad ...(ii)$$

From velocity triangle ACD,

$$\frac{C_{fi}}{C_{wi}} = \tan 20$$

$$C_{wi} = \frac{113.09\,(d+h)}{\tan 20} = 310.71\,(d+h) \quad ...(iii)$$

$$C_{wo} = C_{wi} - C_b$$

$$= 310.71\,(d+h) - 157.08\,(d+h)$$

$$= 153.63\,(d+h) \quad ...(iv)$$

Power developed,

$$P = \frac{\dot{m}_a (C_{wi} - C_{wo})\, C_b}{1000}$$

$$50 = \frac{5.111\,[310.71\,(d+h) + 153.63\,(d+h)]\, 157.08\,(d+h)}{1000}$$

$$\therefore \quad (d+h)^2 = 0.134124 \quad ...(v)$$

$$\therefore \quad (d+h) = 0.36623 \quad ...(vi)$$

Also, mass flow rate,

$$\dot{m}_t = \frac{\pi (d+h)\, h\, C_{fi}}{x\, v_g}$$

$$\frac{50}{9} = \frac{\pi (d+h)\, h \times 113.09\,(d+h)}{0.8237}$$

Substituting the value of $(d+h)^2$ from Equation (v),

$$\frac{50}{9} = \frac{\pi \times 0.134124\, h \times 113.09}{0.8237}$$

$$h = \mathbf{0.09603\ m} \quad ...\text{Ans.}$$

$$d + h = 0.36623$$

$$d = 0.36623 - 0.09603 = \mathbf{0.2702\ m.} \quad ...\text{Ans.}$$

Note : In the equation of power developed the value of actual mass flow rate must be used while in the equation of mass flow rate the value of theoretical mass flow rate should be used.

Ex. 4.18.9 : A 50% impulse-reaction turbine runs at 3000 rpm. The angles at exit of fixed bladings and inlet of moving bladings are 20° and 30° respectively. The mean ring diameter is 0.7 m and steam condition is 1.5 bar and 0.96 dry. Calculate :

(i) Required height of blades to pass 50 kg/s of steam and

(ii) Power developed by the stage. Solve the problem analytically.

Soln. :

Speed, $N = 3000$ rpm; $\alpha = \phi = 20°$;

$\theta = \beta = 30°$; $d_m = 0.7$ m

$p = 1.5$ bar; $x = 0.96$;

$\dot{m} = 50$ kg/s;

Refer Fig. P. 4.18.9.

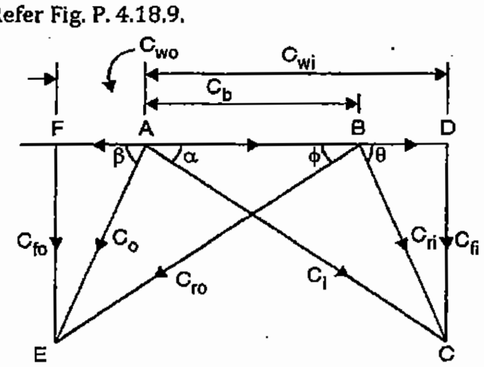


Fig. P. 4.18.9

Blade speed,

$$C_b = \frac{\pi\, d_m \cdot N}{60}$$

$$= \frac{\pi \times 0.7 \times 3000}{60} = 109.96 \text{ m/s}$$

$$C_{fi} = C_{ri} \sin\theta = C_i \sin\alpha$$

$$\therefore C_{ri} \sin 30 = C_i \sin 20$$

$$C_i = 1.462\, C_{ri} \quad \text{...(i)}$$

Also,

$$BD = C_i \cos\alpha - C_b = C_{ri} \cdot \cos\theta$$

$$\therefore C_i \cos 20 - 109.96 = C_{ri} \cos 30$$

$$1.462\, C_{ri} \cos 20 - 109.96 = C_{ri} \cos 30$$

$$\therefore C_{ri} = 216.5 \text{ m/s}$$

$$\text{and} \quad C_i = 1.462$$

$$C_{r_i} = 1.462 \times 216.5 = 316.6 \text{ m/s}$$

$$C_{wi} = C_i \cos\alpha = 316.6 \cos 20 = 297.49 \text{ m/s}$$

$$C_{wo} = C_{wi} - C_b = 297.49 - 109.96 = 187.53 \text{ m/s}$$

$$C_{fi} = C_{fo} = C_i \sin\alpha = 316.6 \sin 20 = 108.3 \text{ m/s}$$

(i) Height of blades, h

From steam tables at 1.5 bar we get,

$$v_g = 1.159 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{A_f \cdot C_{f_i}}{v} = \frac{(\pi\, d_m)\, h \times C_{f_i}}{x \cdot v_g}$$

$$50 = \frac{\pi \times 0.7 \times h \times 108.3}{0.96 \times 1.159}$$

$$h = \mathbf{0.2336 \text{ m}} \quad \text{...Ans.}$$

(ii) Power developed, P

$$P = \frac{\dot{m}\,(C_{wi} + C_{wo})\, C_b}{1000} \text{ (kW)}$$

$$= \frac{50\,(297.49 + 187.53)\, 109.96}{1000}$$

$$= \mathbf{2666.64 \text{ kW}} \quad \text{...Ans.}$$

Ex. 4.18.10 : The total tangential force on one ring of Parsons turbine is 1200 N when the blade speed is 100 m/s. The mass flow rate is 8 kg/s. The blade outlet angle is 20°. Determine blade velocity at outlet from the blade. If friction losses which would occur with pure impulse are 25% of the kinetic energy corresponding to the relative velocity at entry to each ring of blades and expansion losses are 10% of the heat drop in blade. Determine the heat drop per stage, Stage efficiency, Blade efficiency and Maximum utilization factor. **SPPU - May 12, 12 Marks**

Soln. : Refer Fig. P. 4.18.10.

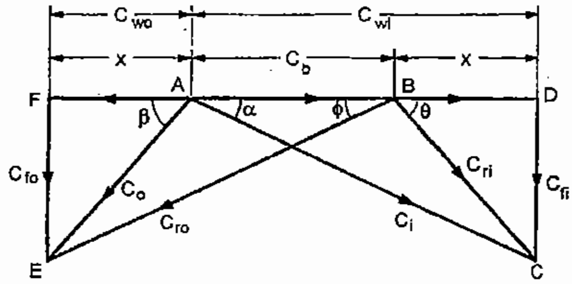


Fig. P. 4.18.10

$$F = 1200 \text{ N}, \qquad C_b = 100 \text{ m/s},$$

$$\dot{m} = 8 \text{ kg/s}, \qquad \phi = 20°$$

Assuming 50% reaction, $\alpha = \phi = 20°$

$$\text{Friction losses} = 0.25 \times \frac{C_{ri}^2}{2} = 0.25 \times \frac{C_o^2}{2}$$

$$\text{Expansion losses} = 10\% \text{ of heat drop in blades}$$

i.e. isentropic efficiency, $\eta_i = 0.9$

(i) Heat drop / stage, Δ h

Power developed,

$$P = F \times C_b = 1200 \times 100 \text{ W} = 120 \times 10^3 \text{ W} = 120 \text{ kW}$$

$$F = \dot{m}\,(C_{wi} + C_{wo}) = \dot{m}\,[C_b + x + x]\,;$$

$$1200 = 8(100 + 2x)$$

$$x = C_{wo} = 25 \text{ m/s}$$

$$C_{wi} = C_b + x = 100 + 25 = 125 \text{ m/s}$$

Consider Δ FBF :

$$C_{fo} = FB \tan\phi = (C_b + x) \tan\phi$$

$$C_{fo} = C_{fi} = (100 + 25) \tan 20° = 45.5 \text{ m/s}$$

$$C_i = C_{ro} = \sqrt{(125)^2 + (45.5)^2} = 133 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{x}\right) = \tan^{-1}\left(\frac{45.5}{25}\right) = 61.21°$$

$$C_{ri} = C_o = \sqrt{C_{fi}^2 + x^2} = \sqrt{(45.5)^2 + (25)^2} = 51.9 \text{ m/s}$$

The turbine is a 50% reaction turbine therefore isentropic enthalpy drop in fixed blade is equal to enthalpy drop in moving blades i.e.

$$(\Delta h_i)_f = (\Delta h_i)_m = \frac{C_i^2 - C_o^2}{2} = \frac{C_{ro}^2 - C_{ri}^2}{2}\,;\ (C_o = C_{ri})$$

Given that energy $\frac{0.25\, C_o^2}{2}$ is lost in blade friction. Then the available energy at entry to fixed blade will be $\left(\frac{C_o^2}{2} - \frac{0.25\,C_o^2}{2} = 0.75\frac{C_o^2}{2}\right)$. Further, the fixed blade acting as nozzle will now convert this energy $\left(0.75\,\frac{C_o^2}{2}\right)$ with isentropic efficiency, $\eta_f = \eta_n = 0.9$

$$\therefore\ (\Delta h_i)_f = \frac{C_i^2 - 0.75\,C_o^2}{2\times\eta_i} = \frac{(133)^2 - 0.75\times(51.9)^2}{2\times 0.9\times 1000}\ \text{kJ/kg}$$

$$= 8.705\ \text{kJ/kg}$$

(i) Enthalpy drop / Stage, $(\Delta h_i)_{stage}$

$$(\Delta h_i)_{stage} = (\Delta h_i)_f + (\Delta h_i)_m$$

$$= 8.705 + 8.705$$

$$= \mathbf{17.41\ kJ/kg} \quad \text{...Ans.}$$

(ii) Blade efficiency, η_b

$$\eta_b = \frac{F\,.\,C_b}{\dot{m}\left(C_i^2 - \frac{C_o^2}{2}\right)} = \frac{1200\times 100}{8\times\left[(133)^2 - \frac{(51.9)^2}{2}\right]}$$

$$= \mathbf{0.9179\ or\ 91.79\%} \quad \text{...Ans.}$$

(iii) Stage efficiency, η_s

$$\eta_b = \frac{W.D.\ \text{i.e.}\ (F\times C_b)}{\dot{m}\,.\,(\Delta h_i)_{stage}}$$

$$= \frac{1200\times 100}{8\times(17.41\times 10^3)}$$

$$= \mathbf{0.8616\ or\ 86.16\%} \quad \text{...Ans.}$$

(iv) Maximum utilization factor, $(\eta_b)_{max}$

$$(\eta_b)_{max} = \frac{2\cos^2\alpha}{1+\cos^2\alpha}$$

$$= \frac{2\times(\cos 20)^2}{1+(\cos 20)^2}$$

$$= \mathbf{0.9379\ or\ 93.79\%} \quad \text{...Ans.}$$

Ex. 4.18.11 : For a certain stage of a reaction turbine, the steam leaves the fixed blade at a pressure of 3 bar, 0.99 dry and a velocity of 120 m/s. The blades are 20 mm high and the discharge angle of both the blades is 20°. The ratio of the axial velocity of flow to the blade velocity is 0.7 at inlet and 0.75 at exit. If the turbine uses 4.8 kg/s of steam, determine the mean blade diameter and Power developed in the ring.

SPPU - Dec. 13, 10 Marks, May 14, 8 Marks

Soln. : Refer Fig. P. 4.18.11.

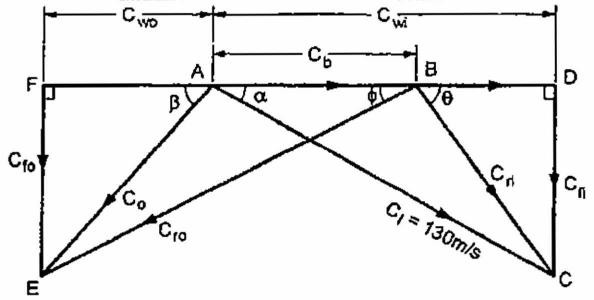


Fig. P. 4.18.11

Given :

p = 3 bar, x = 0.99;

C_i = 120 m/s, h = 20 mm = 0.02 m ;

$\alpha = \phi = 20°$; $\frac{C_{fi}}{C_b} = 0.7$

$\frac{C_{fo}}{C_b} = 0.75$; $\dot{m}$ = 4.8 kg/s,

tip leakage = 5% = 0.05

$\therefore$ Actual mass flow rate,

$$\dot{m}_a = 4.8\times(1-0.05) = 4.56\ \text{kg/s}$$

From steam tables,

$$v_g = 0.606\ \text{m}^3/\text{kg at } p = 3\ \text{bar}$$

From Δ ACD

$$C_{fi} = 0.7\,C_b = C_i \sin\alpha$$

$$0.7\,C_b = 120 \sin 20$$

$$\therefore\ C_b = 58.63\ \text{m/s}$$

$$C_{wi} = C_i\cos\alpha = 120\cos 20 = 112.76\ \text{m/s};$$

$$C_{fi} = 0.7\times C_b = 0.7\times 58.63 = 41.04\ \text{m/s}$$

From Δ BEF

$$EF = C_{fo} = 0.76\,C_b = 0.75\times 58.63 = 43.97\ \text{m/s}$$

$$FB = \frac{EF}{\tan\phi} = \frac{C_{fo}}{\tan\phi} = \frac{43.97}{\tan 20} = 120.81\ \text{m/s}$$

$$\therefore\ C_{wo} = FA = FB - AB = FB - C_b$$

$$= 120.81 - 58.63 = 62.18\ \text{m/s}$$

Average flow velocity,

$$C_f = \frac{C_{fi} + C_{fo}}{2} = \frac{41.04 + 43.97}{2} = 42.50\ \text{m/s}$$

$$\dot{m} = \frac{\pi(d+h)\,h\cdot C_f}{x\cdot v_g}$$

$$4.8 = \frac{\pi(d+0.02)\,0.02\times 42.50}{0.99\times 0.606}$$

$\therefore$ Drum diameter, d = 1.05 m

(i) ∴ Mean drum diameter,

$$d_m = d + h = 1.05 + 0.02 = \mathbf{1.07\ m} \quad \text{...Ans.}$$

(ii) Power developed

$$= \frac{\dot{m}_a \times (C_{wi} + C_{wo})\, C_b}{1000} \text{ kW}$$

$$= \frac{4.56\,(112.76 + 62.18)\, 58.63}{1000}$$

$$= \mathbf{46.77\ kW} \quad \text{...Ans.}$$

Ex. 4.18.12 : A Parsons reaction turbines at 400 rpm develops 5 MW using 6 kg/kWh of steam. The exit angle of the blades are 20° and the velocity of steam is 1.35 times the blade velocity and pressure at exit is 1.2 bar and dryness fractions is 0.95. Calculate for this

(i) a suitable blade height, assuming $\frac{D_m}{h_b} = 12$ and

(ii) Diagram power. **SPPU - May 14, 8 Marks**

Soln. : Fig. P. 4.18.12 shows the velocity diagram.

Given : $N = 400$ rpm, $P = 5$ MW;

Steam consumption = 6 kg/kWh

$\alpha = \phi = 20°$, $\frac{C_i}{C_b} = 1.35$;

$p = 1.2$ bar, $x = 0.95$;

$\frac{D_m}{h_b} = 12$; $\theta = \beta$

Mass flow rate,

$$\dot{m} = \text{steam consumption} \times \text{Power, P}$$

$$= 6 \times (5 \times 10^3) = 30 \times 10^3 \text{ kg/hr}$$

$$= \frac{30 \times 10^3}{3600} = 8.333 \text{ kg/s}$$

$$C_{wi} = C_i \cos\alpha = 1.35\, C_b \times \cos 20 = 1.2686\, C_b$$

$$C_{wo} = C_{wi} - C_{wo} = 1.2686\, C_b - C_b = 0.2686\, C_b$$

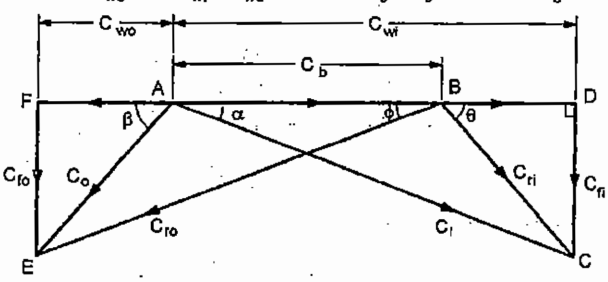


Fig. P. 4.18.12

(i) Blade height, h_b

$$P = \frac{\dot{m}\,(C_{wi} + C_{wo})\, C_b}{1000}$$

$$5 \times 10^3 = \frac{8.333\,(1.2686\, C_b + 0.2686\, C_b)\, C_b}{1000}$$

$$C_b = 624.8 \text{ m/s}$$

$$\therefore\ C_i = 1.35\, C_b = 1.35 \times 624.8$$

$$= 843.4 \text{ m/s} = C_{ro}$$

From steam tables at 1.2 bar, $v_g = 1.428$ m³/kg

∴ Specific volume of steam at exit,

$$v = x \cdot v_g = 0.95 \times 1.428 = 1.3566 \text{ m}^3/\text{kg}$$

$$C_{fi} = C_{fo} = C_f = C_i \sin\alpha = 843.4 \sin 20 = 288.5 \text{ m/s}$$

$$\dot{m} = \frac{\pi D_m \cdot h_b \cdot C_f}{v}$$

$$8.333 = \frac{\pi\, 12\, h_b \times 288.5}{1.3566}$$

$$h_b = 0.0322 \text{ m} = \mathbf{3.22\ cm} \quad \text{...Ans.}$$

(ii) Diagram power, P_1

It represents the power input to turbine.

$$BD = C_{wo} = 0.2686 \times 624.8 = 167.8 \text{ m/s}$$

$$C_{ri} = \sqrt{BD^2 + C_{fi}^2} = \sqrt{167.8^2 + 288.5^2} = 333.8 \text{ m/s.}$$

$$P_1 = m \times \frac{1}{2}\left[C_i^2 + C_{ro}^2 - C_{ri}^2\right] = \frac{\dot{m}}{2}\left[2\, C_i^2 - C_{ri}^2\right]$$

$$= \frac{8.333}{2}\,(2 \times 843.4^2 - 333.8^2) \times \frac{1}{10^6} \text{ MW}$$

$$= \mathbf{5.463\ MW} \quad \text{...Ans.}$$

Ex. 4.18.13 : A Parson's turbine develops 1140 kW at 400 rpm and consumes 7.89 kg of steam per kWh. Steam is supplied at 11 bar and 260°C and the isentropic efficiency of expansion is 85%. The blade angles are 35° and 20° at inlet and outlet respectively. Find the drum diameter and the blade height at a stage where the pressure is 1.4 bar. Height to drum diameter ratio of $\frac{1}{12}$ is recommended. Find also the power developed at that stage. **SPPU - May 19, 8 Marks**

Soln. : Refer Fig. P. 4.18.13(a) and (b).

Given : Power, $P = 1140$ kW

$N = 400$ rpm ; $\dot{m}_s = 7.89$ kg/kWh

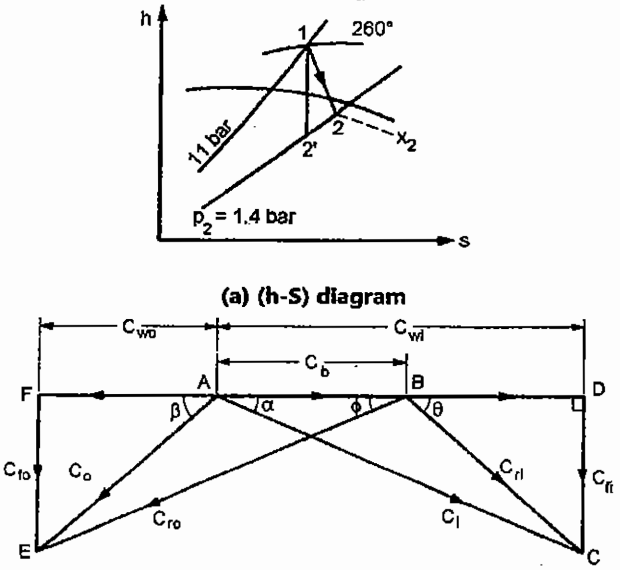
$p_1 = 11$ bar and $T_{sup1} = 260°$ C

Isentropic efficiency,

$\eta_i = 85\% = 0.85$; $\theta = \beta = 35°$

$\alpha = \phi = 20°$, $p_2 = 1.4$ bar

$$\frac{\text{Height, h}}{\text{Drum diameter, d}} = \frac{1}{12} \text{ i.e. } d = 12\,h$$



(a) (h-S) diagram

(b) Velocity diagram

Fig. P. 4.18.13

From Mollier's diagram for isentropic process $(1 - 2')$ we get :

$$h_1 = 2965 \text{ kJ/kg}; \quad h_2' = 2565 \text{ kJ/kg}$$

$$\text{But } \eta_i = 0.85 = \frac{h_1 - h_2}{h_1 - h_2'} = \frac{2965 - h_2}{2965 - 2565}$$

$$h_2 = 2625 \text{ kJ/kg}$$

Fix state 2 on Mollier's diagram by drawing horizontal line at h_2 = 2625 kJ/kg till it cuts the pressure line at p_2 = 1.4 bar.

$$\text{On reading, } x_2 = 0.97$$

$$\text{From steam table 2 at } p_2 = 1.4 \text{ bar,}$$

$$v_{g2} = 1.236 \text{ m}^3/\text{kg}$$

∴ Specific volume of steam,

$$v = x_2 \cdot v_{g2} = 0.97 \times 1.236 = 1.199 \text{ m}^3/\text{kg}$$

Inlet velocity of steam to turbine,

$$C_i = 44.7\sqrt{h_1 - h_2} = 44.7\sqrt{2965 - 2625} = 824.2 \text{ m/s}$$

Mass flow rate of steam,

$$\dot{m} = \dot{m}_s \times \text{Power} = 7.89 \times 1140 \text{ kg/hr}$$

$$= \frac{7.89 \times 1140}{3600} \text{ kg/s} = 2.499 \text{ kg/s}$$

1. Drum diameter, and blade height, h

Let C_b be the blade velocity.

$$C_{fi} = C_i \cdot \sin\alpha = 824.2 \sin 20 = 281.9 \text{ m/s}$$

$$\dot{m} = \frac{\pi(d+h)\,h \cdot C_{fi}}{v}$$

$$2.499 = \frac{\pi(2h+h)\,h \times 281.9}{1.199}$$

$$h = \mathbf{0.0336 \text{ m}} \quad \text{...Ans.}$$

$$\text{Drum diameter, } d = 12h = 12 \times 0.0336$$

$$= \mathbf{0.403 \text{ m}} \quad \text{...Ans.}$$

2. Power developed at the stage, P_1

$$C_b = \frac{\pi(d+h)N}{60}$$

$$= \frac{\pi(0.403 + 0.0336)\,400}{60} = 9.15 \text{ m/s}$$

$$C_{wi} = C_i \cos\alpha = 824.2 \cos 20 = 774.5 \text{ m/s}$$

$$C_{wo} = C_{wi} - C_b = 774.5 - 9.15 = 765.35 \text{ m/s}$$

$$P_1 = \frac{\dot{m}(C_{wi} + C_{wo})\,C_b}{1000} \text{ kW}$$

$$= \frac{2.499\,(774.5 + 765.35)\,9.15}{1000}$$

$$= \mathbf{35.21 \text{ kW}} \quad \text{...Ans.}$$

Ex. 4.18.14 : A 12 stage, 50% reaction turbine has its inlet and exit blades at an angles of 20° and 75° respectively. The mean drum diameter is 1.2 m and the turbine runs at 3500 r.p.m. Determine the enthalpy drop per stage assuming that the velocity of flow remains constant throughout. If the steam enters the turbine at 20 bar, 250°C and the back pressure is maintained at 0.1 bar. Determine the stage efficiency if the reheat factor is 1.05 and the stage efficiency is same for all stages. Also, find the blade height at the exit of eight stage when the turbine develops 10000 kW of power and the condition line is a straight line.

Soln. :

Given : Number of stages = 12,

$$R_D = 50\%, \quad \alpha = \phi = 20°,$$

$$\theta = \beta = 75°$$

$$d_m = (d+h) = 1.2 \text{ m}, \quad N = 3500 \text{ r.p.m.},$$

$$C_{fi} = C_{fo} = C_f$$

Blade velocity,

$$C_b = \frac{\pi d_m N}{60} = \frac{\pi \times 1.2 \times 3500}{60}$$

$$= 219.9 \text{ m/s}$$

Refer velocity diagram given in Fig. P. 4.18.14(a).

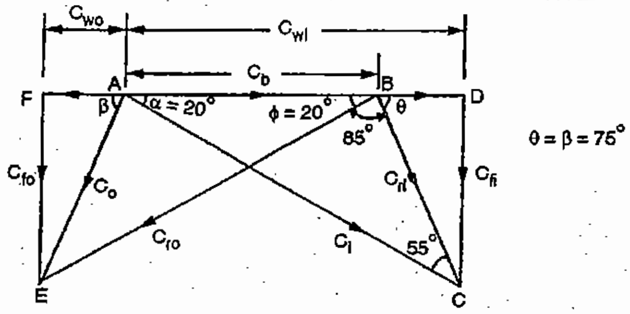


Fig. P. 4.18.14(a)

From Δ ABC, $\dfrac{AB}{\sin BCA} = \dfrac{BC}{\sin CAB}$

i.e. $\dfrac{219.9}{\sin 55} = \dfrac{C_{ri}}{\sin 20}$

or $C_{ri} = 91.8\ \text{m/s}$

$C_{fi} = C_{ri} \sin \theta$

$C_{fi} = 91.8 \times \sin 75 = 88.7\ \text{m/s}$

$AD = C_{wi} = \dfrac{C_{fi}}{\tan \alpha} = \dfrac{88.7}{\tan 20} = 243.6\ \text{m/s}$

$C_{wo} = C_{wi} - C_b = 243.6 - 219.9 = 23.7\ \text{m/s}$

W.D./ kg $= (C_{wi} + C_{wo})\, C_b = (243.6 + 23.7)\ 219.9$

$= 58.8 \times 10^3\ \text{Nm/ kg}$

$= \mathbf{58.8\ kJ/kg\ per\ stage}$ ...Ans.

W.D./kg also represents the enthalpy drop per stage since there are no changes in K.E. at inlet and exit.

Actual or useful enthalpy drop for 12 stages,

$\Delta h = 12 \times 58.8 = 705.6\ \text{kJ/kg}$

Refer Fig. P. 4.18.14(b).

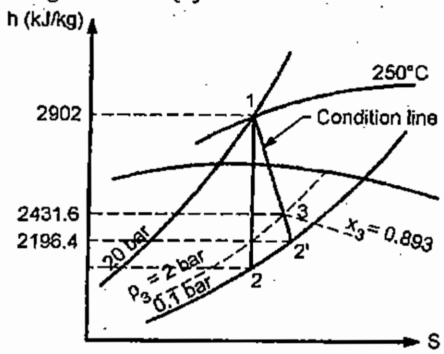


Fig. P. 4.18.14(b)

From Mollier's diagram we get,

$h_1 = 2902\ \text{kJ/kg}, \quad h_2 = 2077\ \text{kJ/kg}$

Isenthalpic or Rankine heat drop,

$h_1 - h_2 = 2902 - 2077 = 825\ \text{kJ/kg}$

∴ Internal efficiency of the turbine,

$$\eta_i = \frac{\text{Total useful heat drop}}{\text{Rankine heat drop}} = \frac{705.6}{825} = 0.8553$$

But, $\eta_i = (\text{R.F}) \times \eta_{Stage}$

$\eta_{Stage} = \dfrac{0.8553}{1.05} = \mathbf{0.8146}$ ...Ans.

Location of condition line on (h-S) diagram

Final enthalpy, $h_2 = h_1 - (\text{total useful heat drop})$

$= 2902 - 705.6 = 2196.4\ \text{kJ/kg}$

Join points (1 and 2) by a straight line since the condition line is a straight line.

Enthalpy drop after eight stage $= 8 \times$ enthalpy drop per stage

$= 8 \times 58.8 = 470.4\ \text{kJ/kg}$

Point - 3, Representing the end of eight stage can be located by drawing a horizontal line at

$h_3 = h_1 - 470.4$

i.e. $h_3 = 2902 - 470.4 = 2431.6$ kJ/kg as shown.

At this stage from Molliers diagram we get, $p_3 = 2$ bar and $x_3 = 0.893$ and from steam tables at $p_3 = 2$ bar, $v_{g3} = 0.885\ \text{m}^3/\text{kg}$.

Total power output = 10000 kW

Workdone /kg of steam = 705.6 kJ/kg

∴ Mass flow rate of steam,

$$\dot{m} = \frac{10000}{705.6} = 14.172\ \text{kg/s}$$

Velocity of flow, $C_{fi} = 88.7$ m/s.

∴ Mass flow rate, $\dot{m} = \dfrac{\text{Area of flow} \times \text{velocity of flow}}{\text{Specific volume}}$

$$\dot{m} = \frac{\pi d_m h}{x_3 \cdot v_{g3}} \times C_{fi}$$

$$14.172 = \frac{\pi \times 1.2 \times h}{0.893 \times 0.885} \times 88.7$$

∴ Height of blade, h $= \mathbf{0.0335\ m\ or\ 3.35\ cm.}$ ...Ans.

Ex. 4.18.15 : In a Parsons turbine running at 1500 rpm, the available enthalpy drop for an expansion is 63 kJ/kg. If the mean diameter of the rotor is 100 cm, find the number of moving rows required. Assume that efficiency of a stage is 0.8, blade outlet angle 20° and speed ratio 0.7.

SPPU - Dec. 16, 6 Marks

Soln. : Refer Fig. P.4.18.15.

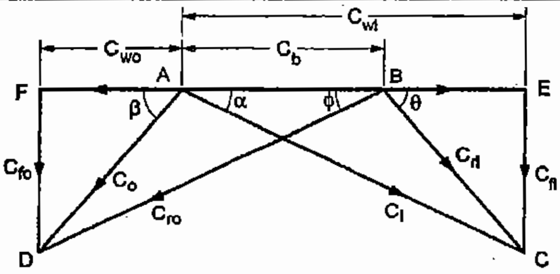


Fig. P. 4.18.15

Given : Turbine is Parson's reaction turbine having degree of reaction,

$$R = 0.5; \qquad \text{Speed } N = 1500 \text{ rpm.}$$

Available enthalpy drop (total available theoretical enthalpy drop) = 63 kJ/kg

Mean diameter, d = 100 cm = 1 m

Stage efficiency η_s = 0.8

Blade outlet angle, ϕ = 20° = α;

Speed ratio, $s = \frac{C_b}{C_i} = 0.7$

In Parson's reaction turbine, i.e. for R = 50%

$$\alpha = \phi, \qquad \theta = \beta,$$

$$C_i = C_{ro}$$

$$C_{ri} = C_o \qquad \text{and} \qquad C_{fi} = C_{fo}$$

$$C_b = \frac{\pi dN}{60} = \frac{\pi \times 1 \times 1500}{60} = 78.5 \text{ m/s}$$

$$s = \frac{C_b}{C_i}; \quad 0.7 = \frac{78.5}{C_i}; \quad C_i = 112 \text{ m/s}$$

$$C_{wi} = C_i \cos \alpha = 112 \times \cos 20° = 105.2 \text{ m/s}$$

$$C_{fi} = C_i \sin \alpha = 112 \sin 20 = 38.3 \text{ m/s}$$

$$BE = C_{wi} - C_b = 105.2 - 78.5 = 26.7 \text{ m/s}$$

$$BE = AF = C_{wo} = 26.7 \text{ m/s}$$

(a) Number of moving rows (stages) required (N_s)

$$\eta_s = \frac{W \cdot D/\text{stage}}{\Delta h}$$

$$= \frac{(C_{wi} + C_{wo})\, C_b}{1000} \times \frac{1}{\Delta h}$$

$$\Delta h = \frac{(105.2 + 26.7) \times 78.5}{1000} \times \frac{1}{0.8}$$

$$= 12.943 \text{ kJ/kg/stage}$$

$$N_s = \frac{\text{Total enthalpy drop}}{\text{Enthalpy drop/stage}} = \frac{63}{12.943}$$

$$= \textbf{4.87 says 5 stages}$$ ...**Ans.**

4.19 Governing of Steam Turbines

University Questions

Q. State various methods employed in practice for governing of steam turbines. **SPPU : Dec. 11, Dec.13**

Q. Why governing of steam turbines is necessary ? **SPPU : May 14**

Q. What is the need of governing system used in steam turbine? **SPPU : Dec.19**

The **function** of a **governor** is to control the fluctuation of speed of prime mover within prescribed limits with the variation of loads on it. In case of steam turbines when it is connected to derive an alternator for converting its mechanical energy into electrical energy, a derive is used to vary the turbine output according to the load on the alternator with very small fluctuations in speed, called **governor**.

The methods used for governing of steam turbines are:

(a) Throttle governing

(b) Nozzles governing

(c) By-pass governing

(d) Combined throttle and nozzle governing

(e) Combined throttle and by-pass governing.

4.20 Throttle Governing

University Questions

Q. Explain throttle governing related to steam turbine. **SPPU : May 11**

Q. Discuss Throttle Governing method in detail along with a neat schematic. **SPPU : Dec. 11, Dec. 13, May 14**

Q. Explain with neat sketch throttle governing of steam turbines. **SPPU : Dec. 15, May 19, Dec. 19**

In this type of governing the steam is throttled down from higher pressure p_1 to a lower pressure p_2 according to the load on the turbine before it is supplied to the turbine, as shown in Fig. 4.20.1.

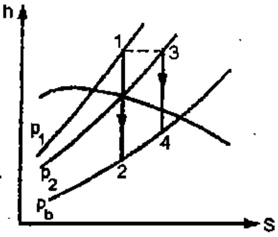


Fig. 4.20.1 : Throttling of steam from p_1 to p_2

It reduces the enthalpy drop i.e. the specific work output is reduced from $(h_1 - h_2)$ to $(h_3 - h_4)$. *Such a method is useful for small capacity power plants since the mechanism is simple with initial low cost.*

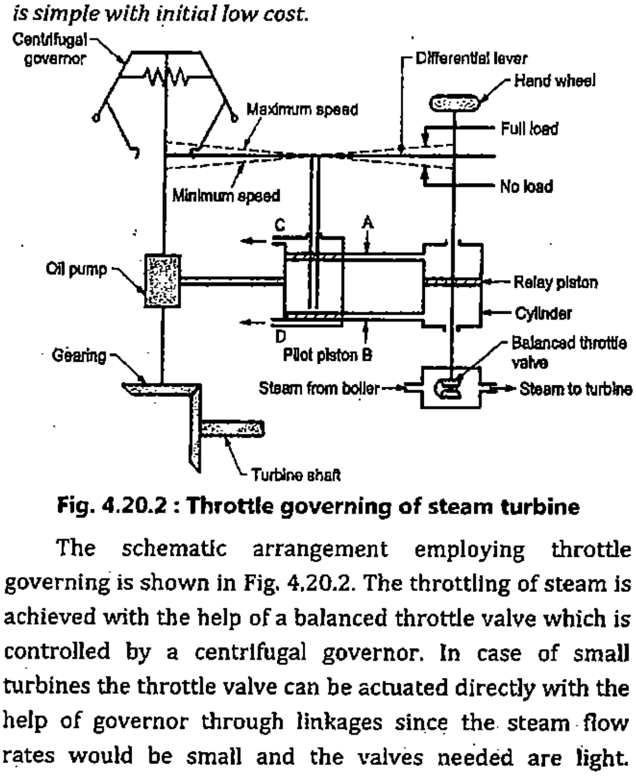


Fig. 4.20.2 : Throttle governing of steam turbine

The schematic arrangement employing throttle governing is shown in Fig. 4.20.2. The throttling of steam is achieved with the help of a balanced throttle valve which is controlled by a centrifugal governor. In case of small turbines the throttle valve can be actuated directly with the help of governor through linkages since the steam flow rates would be small and the valves needed are light. However, in case of medium and large power plants the effort of the governor may not be sufficient to actuate the throttle valve, due to this an oil operated relay is incorporated in the circuit as shown in Fig. 4.20.2. This relay magnifies the small force produced by the governor for a small change of speed to produce a large force to move the throttle valve.

The working of the mechanism is as follows

Consider the case when the load on turbine shaft is equal to the power developed by the turbine, the speed is constant and the system is in equilibrium.

Let us assume that the load on the turbine is reduced suddenly. At this stage since the power developed is more than the load, the turbine and governor speeds will increase due to the excess energy developed by the turbine.

The governor balls will fly out and it will raise the sleave of the governor, consequently, the differential level will cause the pilot piston to be raised.

The oil which is supplied under a pressure of 3-4 bar will flow through the pipe A to the cylinder of relay piston and it would force the relay piston to move downwards, while the oil from relay piston cylinder is drained out through the pipe B.

The downward movement of the relay piston operates the throttle valve which in turn closes the steam ports partially. It throttles the steam and the steam pressure at inlet to the turbine is reduced. When the power developed by the turbine equals to the load on the turbine, the oil ports C - D are covered and the relay piston is locked.

It should be noted that this methods of governing is though simple but it ***reduces the efficiency of power plants at parts loads because a part of the available energy is lost in the irreversible throttling process.*** The Willan's straight line relationship between load and steam consumption is also followed, which is given as follows (Refer Fig. 4.20.3) ;

$$\dot{m}_s = a + b \cdot L \qquad ...(4.20.1)$$

where, $\dot{m}_s$ = Steam consumption in kg/s

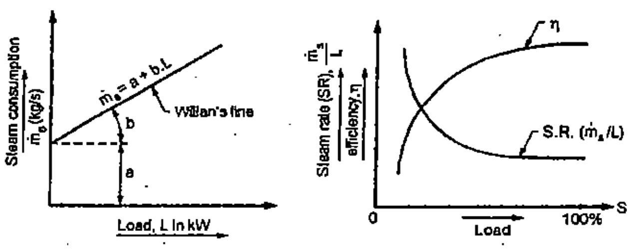
a = Steam consumption at no load in kg/s

b = Steam rate (S.R.) in kg/s kW

L = Load on plant, kW

The specific steam consumption (SSC) or steam rate (S.R.) in kg / kWh can be written as,

$$\text{S.R.} = \frac{\dot{m}_s}{L} = \frac{a}{L} + b \qquad ...(4.20.2)$$



(a) Willan's line (b) Performance of turbine

Fig. 4.20.3 : Throttle governing

The side effects of throttling is to improve the quality of steam at inlet and reduction of stage efficiency at part loads.

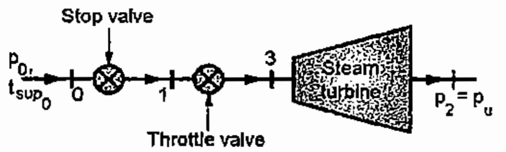
The performance of turbine in throttle governing is shown in Fig. 4.20.3(b).

Representation of throttle governing on (h – s) diagram

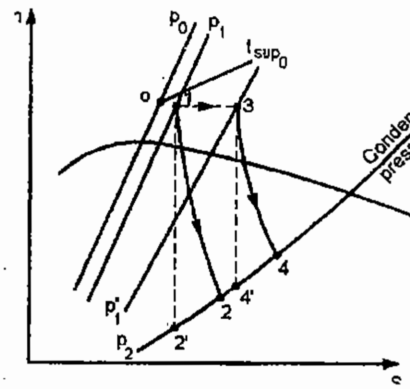
It is assumed that :

(i) There is a negligible loss of heat when steam passes into the throttle valve.

(ii) The pressure in the condenser remains constant.



(a) Schematic diagram



(b) (h-s) diagram

Fig. 4.20.4 : Representation of throttling governing

Let : p_o = Pressure of steam before the stop valve at temperature t_{sup_0} (°C)

p_1 = pressure of steam after stop valve of temperature T_{sup1}

p_2 = condenser pressure

η_i = internal efficiency at full load

$\dot{m}$ = mass flow rate of steam at full load, L (kg/hr)

p_1' = pressure of steam after throttling at part load, L′

$\dot{m}'$ = mass flow rate of steam at fractional load.

η_i' = internal efficiency of turbine at part load

Point 1 represents the condition of steam before throttle valve at full load, L · (1 – 2′) represents the adiabatic expansion to the exhaust pressure p_2 and (1 - 2) is the locus of actual state points.

At some part load where the steam flow is $\dot{m}'$, the first stage nozzle box pressure is p_1' such that :

$$p_1' = \frac{\dot{m}}{\dot{m}'} \times p_1$$

Throttling process is represented as (1 – 3) at constant enthalpy. As before, at part load L′, (3 – 4′) is the adiabatic expansion and (3 – 4) is the locus of actual state points. Then,

$$L = \frac{\eta_i \times \dot{m} \times \Delta h_i}{3600} = \frac{\eta_i \times \dot{m} \times (h_1 - h_2')}{3600} \quad ...(i)$$

$$L' = \frac{\eta_i' \times \dot{m}' \times \Delta h_i'}{3600} = \frac{\eta_i' \times \dot{m}' \times (h_3 - h_4')}{3600} \quad ...(ii)$$

where, $(h_1 - h_2')$ is the isentropic heat drop at full load and $(h3 - h4)$ is the isentropic heat drop at part load.

$$\therefore \frac{L}{L'} = \frac{\eta_i}{\eta_i'} \times \frac{\dot{m}}{\dot{m}'} \times \frac{(h_1 - h_2')}{(h_3 - h_4')} \quad ...(4.20.3)$$

Therefore, we find that the effect of throttling is to reduce the adiabatic heat drop and to reduce the steam flow rate. Usually, the internal efficiency is not much affected.

4.21 Nozzles Governing

University Questions

Q. Discuss Nozzles Governing method in detail along with a neat schematic. **SPPU : Dec. '11, May 14**

Q. Explain the method of nozzle control governing. **SPPU : Feb. 16 (In Sem).**

This method of governing is more efficient than the throttle governing since in this method the mass flow rate of steam supplied to the turbine is only controlled. The method is illustrated in Fig. 4.21.1.

Total number of nozzles required are divided into number of sets like N_1, N_2, N_3. Each set consists of the number of nozzles required to affect the required control on the mass flow rate corresponding to the required power control. Due to this, different sets of nozzles will have different number of nozzles.

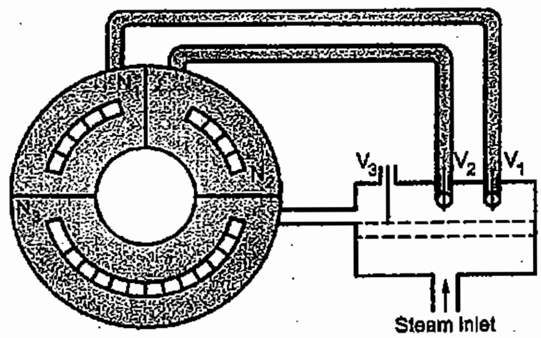


Fig. 4.21.1: Nozzle governing

The supply of steam to the nozzles is controlled by opening and closing of valves V_1, V_2, V_3. At full load all the valves will supply the steam to all the nozzle and on part loads, one or more number of valves are closed

There are several possible mechanical arrangements of nozzles and control valves but in each method of nozzle control is only applied for the first stage of the turbine and the nozzle area for the subsequent stages of the turbine remains constant. It can be seen that in case of nozzle governing the total enthalpy drop is available unlike in the case of throttle governing in which the enthalpy drop is reduced at parts loads. Due to this, the efficiency at part loads remains unaffected in case of nozzle governing.

Nozzle governing cannot be used in reaction turbines since full admission of steam is must over moving blades.

4.22 By-pass Governing

University Question

Q. Discuss By-pass Governing method in detail along with a neat schematic. **SPPU : Dec. 11**

Modern steam power plants employing impulse turbine using steam at very high pressures are usually designed to operate at an economical load which is about 75-80 % of the maximum load.

In such cases it is desirable to have full admission of steam in first few stages of the turbines operating at very high pressure since the enthalpy drop is very small in the initial stages and the nozzle governing cannot be used effectively. It also eliminates the admission losses.

Therefore the load regulation is achieved by throttle governing upto the stage of economical loads.

However at the maximum loads the additional amount of steam required cannot be passed through the first-stages since the required additional number of nozzles are not available. The difficulty of steam regulation is overcome by employing by-pass governing as shown in Fig. 4.22.1.

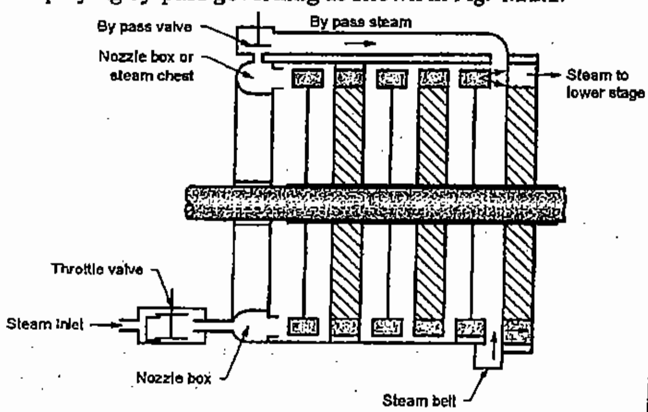


Fig. 4.22.1 : By-pass Governing

Steam required upto economical load is passed through the inlet valve and it is collected in the nozzle-chest. The governing is affected by throttle valve.

But at loads more than economical load, the by-pass valve lifts and a part of the steam is by-passed into the steam belt and this steam mixes with the steam of high pressure turbine stages and it is supplied to lower stages of the turbine. The movement of the by-pass valve according to the load is controlled by the turbine governor.

4.22.1 Comparison between Throttle and Nozzle Governing

	Throttle Governing	Nozzle Governing
1.	Loss of energy due to throttling.	No throttling losses except small loss is nozzle valves due throttling effect.
2.	Can be employed both in impulse and reaction turbine.	Employed in impulse turbines.
3.	Heat drop reduces.	Heat drop per kg of steam remains constant.
4.	Partial admission losses are reduced.	Partial admission losses are increased.
5.	Efficiency is poor thus suitable for small turbines.	Suitable for all sizes of the turbines.

4.23 Combined Governing

University Questions

Q. Discuss Combined Governing method in detail along with a neat schematic. **SPPU : Dec. 11**

It reaction turbines the steam expands continuously while passing over the ring of fixed and moving blades, therefore the full admission of steam in necessary over the moving blades. Therefore, nozzle governing cannot be employed in reaction turbines.

For this reason, a simple method of governing of reaction turbine is to use **combined throttle and by-pass governing.**

In case of parson's reaction turbine a single by-pass valve is used.

Similarly, the combined throttle and nozzle governing is employed in velocity and pressure compounded Impulse turbines.

4.24 Emergency Governors

Steam turbines are generally provided with emergency governors to protect it against over speeding. These governors will come in operation and cut-off the steam supply in case the turbine speed exceeds 10 to 12% of the rated speed.

Emergency governors are of two types :

1. Eccentric bolt type.
2. Eccentric ring type.

Operation of these governors may be either direct or through oil relay (as explained in section Throttle governing.) Fig. 4.24.1 shows the **eccentric bolt type** emergency governor. Eccentric bolt mechanism is screwed into high pressure end of the turbine spindle as shown. When the speed of turbine exceeds, the out of balance force caused by the eccentric ring opposes the spring force.

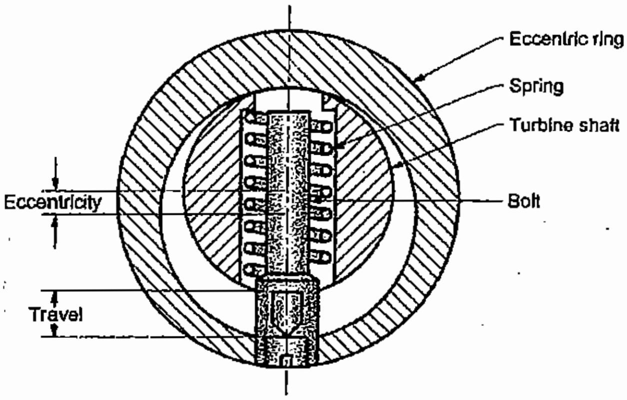


Fig. 4.24.1 : Eccentric bolt type emergency governor

It results in governor instantaneously moving through its full travel and shut-off the steam supply.

Fig. 4.24.2 shows the eccentric ring type emergency governor which is screwed on the end of turbine spindle. The working is similar to eccentric bolt type emergency governor.

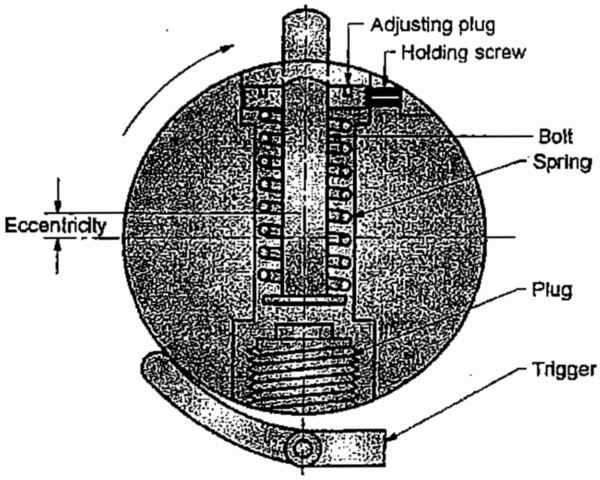


Fig. 4.24.2 : Eccentric ring type emergency governor

Ex. 4.24.1 : A 120 MW power plant is supplied with steam at 100 bar and 550 °C and the condenser pressure is maintained at 0.1 bar. At full load the steam flow rate is 4,80,000 kg/hr while at no load the steam flow rate is 24000 kg/hr. Find the specific steam consumption in kg/kWh at 25%, 50% and 75% loads. Assume that the plant uses the throttle governing. **SPPU - May 11, 8 Marks**

Soln. : Since plant uses the throttle governing, it follows that it obey's the Willan's.

Straight line law, $\dot{m} = a + b \cdot L$ ...(i)

Given : At full load capacity of plant : $L_1 = 120$ MW

$= 120 \times 10^3$ kW

and $\dot{m}_1 = 480000$ kg /hr

At no load : $L_2 = 0$

and $\dot{m}_2 = 24000$ kg/hr

Using the above conditions in equation (i) we get :

$4{,}80{,}000 = a + b \times 120 \times 10^3$ ...(ii)

$24000 = a + b \times 0$

$\therefore \quad a = 24000$ kg/hr ...(iii)

From Equation (ii) ;

$480000 = 24000 + b \times 120 \times 10^3$

$\therefore \quad b = 3.8$ kg/kWh ...(iv)

On substituting the values of 'a' and 'b' in equation (i) we get,

$\dot{m} = 24000 + 3.8\,L$ ...(v)

Specific steam consumption,

$$\frac{\dot{m}}{L} = \frac{24000}{L} + 3.8 \qquad ...(vi)$$

Specific Steam Consumptions

1. **At 25% Load i.e.**

$$L = \frac{25}{100} \times 120 \times 10^3 = 30000 \text{ kW}$$

$$\therefore \frac{\dot{m}}{L} = \frac{24000}{30000} + 3.8$$

$$= \mathbf{4.6 \text{ kg / kWh}} \qquad \text{...Ans.}$$

2. **At 50% Load i.e.**

$$L = \frac{50}{100} \times 120 \times 10^3 = 60000 \text{ kW}$$

$$\therefore \frac{\dot{m}}{L} = \frac{24000}{60000} + 3.8 = \mathbf{4.2 \text{ kg / kWh}} \qquad \text{...Ans.}$$

3. **At 75% Load i.e.**

$$L = \frac{75}{100} \times 120 \times 10^3 = 90000 \text{ kW}$$

$$\therefore \frac{\dot{m}}{L} = \frac{24000}{90000} + 3.8 = \mathbf{4.0667 \text{ kg/Wh}} \qquad \text{...Ans.}$$

Ex. 4.24.2 : A steam turbine operates at 60% load. During test following data were noted :

Steam pressure and temperature before governor valve = 30 bar, 400°C

Steam pressure after governor valve = 10 bar

Pressure at exhaust = 0.1 bar

Steam temperature at exhaust = 55°C

Steam flow rate through turbine = 2000 kg/hr

Find :

(i) Internal efficiency of the turbine.

(ii) Shaft power if the friction losses in bearings and the power required to run the governor amounts to 8.5 kW.

Soln. : Refer Fig. P. 4.24.2

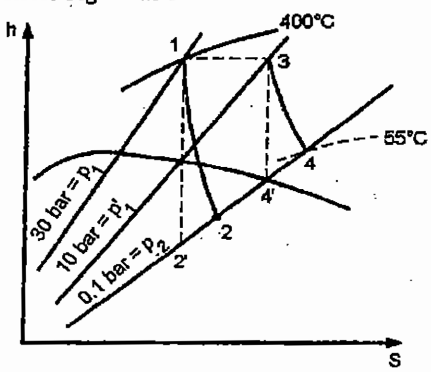


Fig. P. 4.24.2

Given : $p_1 = 30$ bar, $t_{sup_1} = 400°C$,

$p_1' = 10$ bar ; $p_2 = 0.1$ bar ;

Losses, $p_f = 8.5$ kW, $\dot{m}' = 2000$ kg/hr ;

$t_4 = 55°$ C

From Mollier's diagram we get,

$$h_1 = h_3 = 3230 \text{ kJ/kg},$$

$$h_4 = 2610 \text{ kJ/kg},$$

$$h_4' = 2350 \text{ kJ/kg}$$

Actual heat drop,

$$h_3 - h_4 = 3230 - 2610 = 620 \text{ kJ/kg}$$

Isentropic heat drop,

$$\Delta h_i' = h_3 - h_4' = 3230 - 2350$$

$$= 880 \text{ kJ/kg}$$

Internal efficiency of turbine,

$$\eta_i' = \frac{h_3 - h_4}{h_3 - h_4'} = \frac{620}{880} = 0.7045$$

Internal power developed at part load,

$$L' = \frac{\eta_i' \times m \times \Delta h_i'}{3600}$$

$$= \frac{0.7045 \times 2000 \times 880}{3600}$$

$$= 344.45 \text{ kW}$$

Shaft power, $P_s = L' - p_f = 344.45 - 8.5$

$$= \mathbf{335.95 \text{ kW}} \qquad \text{...Ans.}$$

4.25 Losses in Steam Turbines

University Question

Q. Enumerate the energy losses in steam turbine.

SPPU : Dec. 18

There are various losses which occur during the expansion of steam within the turbine. These losses can be broadly divided as :

1. Internal losses
2. External losses

Internal losses in the steam turbine are connected with the flow of steam while the **external losses** are those which occur outside the turbine casing.

4.25.1 Internal Losses in Steam Turbines

The various types of internal losses in steam turbines are as follows :

1. **Nozzle losses :** The expansion of steam in nozzles is ideally isentropic but the loss of energy occurs due to frictional resistance offered by the roughness of nozzle surfaces and viscous friction of fluid. Due to this, the actual kinetic energy available at exit of nozzles is less. These frictional losses are called nozzle losses.
2. **Blade friction losses :** Various factors are responsible for losses in moving blades. These are :
   (i) **Impingement losses** are due to formation of eddies at the leading edge of the moving blades.
   (ii) **Friction losses** in blade passages due to fluid and wall friction.
   (iii) **Turning losses** occur as the steam turns in the blade passages.
   (iv) **Wake losses or trailing edge losses** are caused due to formation of eddies due to mixing of steam jet of nozzles before entering the moving blades.

   Total blade friction losses are taken into account by considering friction factor, K($K = C_{ro}/C_{ri}$) which represents the reduction of relative velocity from C_{ri} to C_{ro}.
3. **Disc friction losses** are due to friction losses due to relative motion between the disc and steam particles.
4 **Partial admission losses or blade windage losses** are occur in impulse turbines since these are partial admission turbines since nozzles are provided only over a part of blade periphery.

   Due to this, turbulence occurs in between the partially filled blade passages which doesnot receive full steam causing the loss of energy called partial admission losses. **These losses do not occur in reaction turbines since these turbines are designed for full admission of steam.**
5 **Gland leakage losses or clearance losses** are due to loss of energy due steam leakage in clearance space between stationary nozzles diaphragm and the shaft in case of impulse turbines.

   Tip leakage occurs in reaction turbines due to clearance between fixed blades and rotor and also the clearance between moving blades and the casing.

   Shaft leakage occurs due to radial clearance between the shaft and the glands provided at the ends of the shaft.
6. **Residual velocity losses** are due to kinetic energy of steam leaving the last stage of turbine which cannot be effectively utilized.
7. **Carry - over losses** are the loss of K.E. in passages when the steam passes from one stage to another stage.
8. **Radiation losses** are the losses of heat energy due to heat transfer by radiation. These losses are negligible in steam turbines.

4.25.2 External Losses

The losses which do not affect the condition of steam while it flows through the turbine are called external losses.

These losses include the **mechanical losses** caused by friction resistance in bearings, governing mechanism, power required to run the oil pump etc. Also, it includes the **loss of energy in steam leakages** through the **end seals** due to pressure difference.

4.26 Comparison Between Impulse and Reaction Turbines

University Question

Q. Compare impulse turbine & reaction steam turbine.

SPPU : May 19

1. In an impulse turbine the steam is first expanded in the nozzles upto the exit pressure and then the pressure remains constant throughout the passage of the turbine. In case of reaction turbine the steam expands continuously while passing over the fixed and moving blades.
2. The relative velocity of steam remains almost constant while it passes over the moving blades in case of impulse turbines. Any reduction in velocity is due to friction losses. The relative velocity of steam increases after passing over the moving blade due to expansion of steam in case of reaction turbine.
3. The number of stages required for a reaction turbine is large compared to impulse turbine because of smaller pressure drop per stage.
4. The blades of an impulse turbine are of profile type. While the blades of a reaction turbine are asymmetrical and use aerofoil section.
5. The steam velocity and blade velocity of an impulse turbine are high compared to reaction turbines.
6. Impulse turbine are compact compared to reaction turbines.
7. The efficiency curve for reaction turbine at the optimum blades speed ratio is flat compared to efficiency curve of an impulse turbine.

4.27 Dimensional Analysis of Steam Turbines

The method of dimensional analysis as discussed in chapter 1 for turbomachines is applied and used to ensure that the experiments on reduced scale models retain the dynamic and geometrical similarity. The turbine performance can be described by the following non-dimensional numbers.

(i) Flow coefficient, $K_f = \frac{V_f}{V}$

(ii) Stage loading coefficient, $\Psi = \frac{\Delta h_o}{V}$

(iii) Reynolds number, $R_e = \frac{\rho V D}{\mu}$

Where V is the steam velocity.

In the above non-dimensional numbers, the stage loading coefficient represents the heat drop per unit velocity and if is usually 1 to 3 in case of reaction turbines.

The value of flow coefficient the non-dimensional axial velocity which is usually in the range of 0.2 to 0.8.

The Reynolds number represents the ratio of fluid inertia force to viscous force. The value of Reynolds number for steam turbines is about 10^7.

Model testing was carried out on large number of steam turbines and the contours of their stage efficiency were plotted by taking the stage loading on the ordinate and the flow coefficient on the abscissa. It was found that the stage efficiency of reaction turbines was within ± 2% of theoretical value.

In dimensional analysis of axial flow steam turbines the fluid velocity triangles are plotted for each stage using the steam tables. Calculations are performed along the mean diameter of the turbine blading. In this analysis, the flow velocity is assumed to be constant in both the axial and radial directions and then the tangential velocities are calculated. It helps in designing and optimizing the blading of steam turbines and evaluate it performance.

4.28 Selection of Steam Turbines

Steam turbines are selected on the basis of the following :

1. Power required to be developed.
2. End use of turbine i.e. for power generation or for process heating or both.
3. Exhaust header pressure and the pressure range.
4. Quantity of steam required.
5. End user must match their requirement with the performance and specification of the manufacturers.
6. Reliability
7. Lower steam rates (S. R.) or specific steam consumption for high performance.

Usually the **reaction turbines are selected** for large power generation since these are more efficient compared to impulse turbine.

The selection of steam turbines for industrial applications is based on their end use. Usually the power required varies from 1000 kW to 10000 kW.

Back pressure turbines are selected if it is required more for process heating. These have high back pressures and their efficiency is high due to elimination of heat losses through the condenser.

Extraction back pressure turbines are selected when the process steam is required at two different pressures. Only problem is the difficulty in controlling the bypass valves to maintain the extraction pressures by steam as per process requirement.

Condensing type of steam turbine is selected when there is the requirement of process steam and it is only to be used for captive power generation.

Extraction-condensing type of turbine is selected when the power requirement is more than the process heat requirements.

SUMMARY

- A **steam turbine** is a rotary machine which is designed to convert the pressure energy of steam into mechanical power.
- A pair of **ring of nozzles** (fixed blades) fixed to the turbine casing and a **ring of moving blades** fixed to turbine rotor is called a **stage** or **turbine pair.**
- Turbines are **classified** as :
  - **(a) Impulse turbine :** In these, the pressure of steam is first converted into K.E. in a ring of nozzles and this K.E. is converted into mechanical work while the steam passes over the ring of moving blades without changes in pressure.
  - **(b) Reaction turbine :** It works on the principle of reaction and there is a continuous pressure drop of steam while passing over the ring of fixed and moving blades.

- **Advantages of steam turbine** are it has smaller size, lesser weight to power ratio, occupies less space, workdone is more, more efficient, perfect balance and vibration free.
- **Compounding of impulse turbine** is done to reduce rotor speeds within practical limits.
- Methods used are :
  (a) Velocity compounding
  (b) Pressure compounding
  (c) Pressure velocity compounding

  Workdone per kg of steam,
  $$W = (C_{wi} + C_{wo})\, C_b$$
  Power developed,
  $$P = \dot{m} \times W$$
  Blade efficiency,
  $$\eta_b = \frac{W.D.}{K.E.\ supplied} = \frac{(C_{wi} + C_{wo})\, C_b}{\frac{C_i^2}{2}}$$
  Stage efficiency,
  $$\eta_b = \frac{W.D.\ /kg\ of\ steam}{Isentropic\ Enthalpy\ drop/kg\ of\ steam,\ \Delta h}$$

Axial thrust $= \dot{m}\,(\vec{C}_{fi} - \vec{C}_{fo})$

Energy lost in friction in blades $= \dot{m}\,(C_{ri}^2 - C_{ro}^2)$

- **Condition for maximum efficiency (Impulse turbines)**
  (a) Optimum blade speed ratio,
  $$s = \frac{C_b}{C_i} = \frac{\cos\alpha}{2} \text{ (for single stage)}$$
  $$(\eta)_{max} = \cos^2\alpha$$
  (b) Optimum blade speed ratio for 'n' stages,
  $$(s) = \frac{C_b}{C_{i_1}} = \frac{\cos\alpha}{2n}$$
  $$\text{W.D. in last row} = \frac{1}{2^n} \times \text{total W.D.}$$
- **Velocity compounded impulse turbines :**

Power developed, $P = \dot{m}\, C_b \cdot \Sigma\,(C_{wi} + C_{wo})$

Blade efficiency, $\eta_b = \dfrac{C_b \cdot \Sigma\,(C_{wi} + C_{wo})}{\frac{C_{i_1}^2}{2}}$

Total axial thrust, $F = \dot{m}\,\Sigma\,(C_{fi} + C_{fo})$

Stage efficiency $= \dfrac{\text{Actual heat drop}}{\text{Isentropic heat drop}}$

Reheat factor (R.F.) $= \dfrac{\text{Cummulative isentropic heat drop}}{\text{Rankine heat drop}}$

- Internal efficiency of turbine,
  $$\eta_t = \frac{\text{Total useful heat drop}}{\text{Rankine heat drop}}$$
  $$\eta_t = (R.F.) \times \eta_{stage}$$
- Overall efficiency of plant,
  $$\eta_o = \frac{\text{Total useful heat drop}}{\text{Heat supplied to plant}}$$
- **Function for governor** is to control the fluctuation of speed of the turbine within prescribed limits according to variation of load on the prime mover.
- **Methods of governing of steam turbines are :**
  1. Throttle governing
  2. Nozzle governing
  3. By-pass governing
  4. Combination nozzle and throttle governing
  5. Combination of throttle and by-pass governing
- In **throttle governing**, the steam is throttled down to a lower pressure according to the load on the turbine. It reduces the efficiency of plant-but the system is simple and less costly.
- It follows the **Willan's line** as : $\dot{m}_s = a + b\,L$ and $(\dot{m}_s / L)$ represents the steam rate :
- In **nozzle governing**, the mass flow rate of steam supplied to the turbine is controlled according to the load by controlling the set of nozzles to supply the steam.
- **By-pass governing** is used by supplying the steam to the turbine at economical loads but at higher loads the steam is by-passed at convenient stage to mix with incoming high pressure steam from nozzle chest.
- **Emergency governors** are used to shut-off the steam supply in case the turbine speed exceeds 10 to 12% more than the rated speed.
- **Internal losses in steam turbines are :**
  1. Nozzle losses due to friction
  2. Blade friction losses
  3. Disc friction losses
  4. Partial admission losses
  5. Clearance losses or gland leakage losses
  6. Residual losses
  7. Carry-over losses
  8. Radiation losses

- **External losses in turbines are due to mechanical and loss of energy in steam at end seals.**

Exercise

Note : For answers please refer the section numbers indicated in bracket.

Q. 1 Define a nozzle. **[Section 4.1]**

Q. 2 Derive an expression for velocity and mass flow rate of steam through nozzles. **[Section 4.2]**

Q. 3 State the condition of maximum of discharge and the critical pressure ratio.**[Sections 4.2, 4.2.1 and 4.2.2]**

Q. 4 State the types of nozzles. What is throat ? **[Section 4.3]**

Q. 5 Define nozzle efficiency, coefficient of velocity and coefficient of discharge.
[Sections 4.4, 4.4.1, 4.4.2 and 4.4.3]

Q. 6 Describe the method of solving the problems on nozzles. **[Section 4.5]**

Q. 7 Define the ring of fixed blades, ring of moving blades and a stage of a steam turbine. **[Section 4.6]**

Q. 8 Explain the working of simple impulse turbine with suitable sketches slowing detailed construction. **[Section 4.6]**

Q. 9 What is the fundamental difference between the operation of impulse and reaction steam turbines ? **[Section 4.7]**

Q. 10 State the advantages of steam turbines. **[Section 4.8]**

Q. 11 Explain the need of compounding in steam turbines. **[Section 4.10]**

Q. 12 Explain the pressure-velocity compounding with a neat sketch. **[Section 4.10.3]**

Q. 13 Explain pressure compounding of impulse turbine. **[Section 4.10.2]**

Q. 14 Draw the variation of pressure and velocity along the length of a reaction turbine. **[Section 4.10.4]**

Q. 15 Draw velocity diagram of an impulse turbine. Write expressions for workdone and diagram or blade efficiency ? **[Sections 4.11.1 and 4.11.2]**

Q. 16 Define the following terms as applied to steam turbines.

(1) Blade velocity coefficient

(2) Blade speed ratio

(3) Carryover coefficient

(4) Stage efficiency **[Section 4.11.2 and 4.11.3]**

Q. 17 What is the condition of maximum blade efficiency in a single stage impulse turbine ? What is its value ? Sketch how efficiency varies with blade steam velocity ratio ? **[Section 4.12]**

Q. 18 Find the optimum ratio of blade speed to steam for a two stage velocity compounded impulse turbine. How does diagram efficiency vary with blade steam velocity with increase in number of stages ? Comment on the result.
[Sections 4.13, 4.13.1 and 4.13.2]

Q. 19 State the condition for maximum efficiency for impulse turbine. **[Section 4.13.2]**

Q. 20 Define the following terms :

(a) Internal efficiency of turbine

(b) Stage efficiency

(c) Overall efficiency of plant. **[Section 4.15]**

Q. 21 What is a reaction turbine ? Why the blade passages are made converging type ? Explain the variation of pressure and velocity along the length of nozzle. **[Section 4.16]**

Q. 22 Define the term stage efficiency in case of reaction turbines. **[Section 4.17.2]**

Q. 23 Define and explain degree of reaction as applied to reaction turbines. **[Section 4.17]**

Q. 24 Prove that for a 50 % reaction turbine, the inlet and exit angles of moving and fixed blades are equal. **[Section 4.17.1]**

Q. 25 Explain the method of determining the blade height at a particular stage of turbine. **[Section 4.17.3]**

Q. 26 Show that the condition for maximum blade efficiency is : Blade speed ratio = cos α. **[Section 4.18]**

Q. 27 What is governing? State methods of governing of steam turbines. **[Section 4.19]**

Q. 28 Explain throttle governing, state its disadvantages. **[Section 4.20]**

Q. 29 Explain by pass governing. **[Section 4.22]**

Q. 30 Compare throttle and nozzle governing. **[Section 4.22.1]**

Q. 31 State and explain various types of losses in steam turbines. **[Sections 4.25, 4.25.1 and 4.25.2]**

Q. 32 Compare the impulse and reaction turbines. **[Section 4.26]**

□□□

5 Centrifugal Pumps

Syllabus

Introduction & classification of rotodynamic Pumps, Main Components of Centrifugal Pump, Construction and Working of Centrifugal Pump, Types of heads, Velocity triangles and their analysis, Effect of outlet blade angle, Work done and Efficiency, Series and parallel operation of pumps, Priming of pumps, specific speed

5.1 Introduction

– The normal duty of a pump is to lift a quantity of liquid from a low level to high level and/or to transfer it from one place to another place. In order to carry out these duties by a pump, it has to provide energy :

 (i) To lift the liquid to required height against the force of gravity.

 (ii) To overcome the fluid resistance to flow of the liquid through the pipe and the pump itself.

 Therefore, a **pump** *is defined as a device which transfers the input mechanical energy of a motor or of an engine into pressure energy or kinetic energy or both of a fluid.*

– Simply we may say that a pump is a device which converts the mechanical energy into hydraulic energy of a fluid.

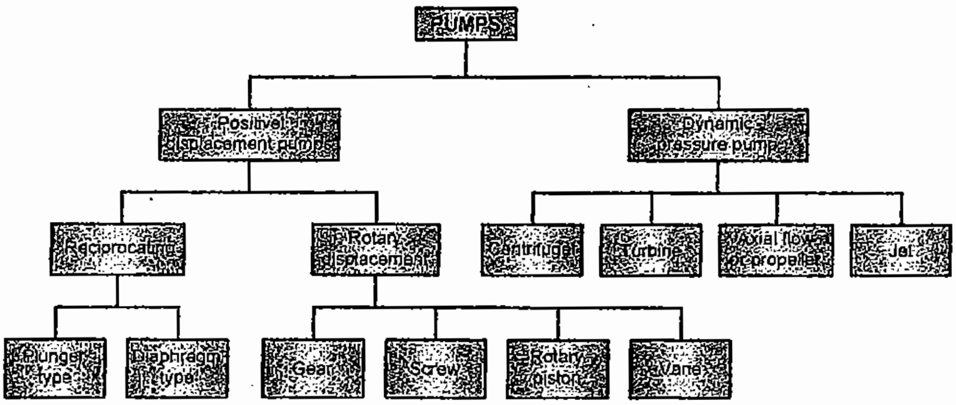
5.2 Classification of Pumps

Pumps can be classified according to the mechanical principle involved in transfer of energy.

These are classified as :

1. Positive displacement pumps
2. Rotodynamic pumps

Above types of pumps are further classified as shown below :



In case of **reciprocating pumps** the pressure energy of a fluid increased due to positive displacement of its piston or plunger. These are used to handle low discharge rates at high pressures.

The **rotary displacement pumps** combine the advantages of reciprocating and centrifugal pumps. They are positive in action, compact, produce an even flow, have no valves and run at high speeds. *These pumps are suitable for handling oils but they are unsuitable for gritty liquids.*

The **centrifugal pump** is far the commonest type of dynamic pump also called as **velocity pump.** These are classified as rotodynamic pumps since the rotating impeller of pump impresses a centrifugal head or pressure on the liquid which leaves the impeller at a high velocity. This pressure enables the liquid to rise to a higher level.

These pumps are suitable for large flow rates.

5.2.1 Difference between Centrifugal Pump and Inward Flow Hydraulic Turbine

Centrifugal pumps are similar to inward flow reaction turbine but it is reverse in action. In case water turbine the water flows from outer periphery radially inwards while the water in centrifugal pump flows radially outwards towards the periphery. In turbine the flow takes from higher pressure to lower pressure and produces mechanical power at the shaft. While in pump the water flows from lower pressure towards higher pressure side on the expense of mechanical energy. In turbine the flow is accelerated due to centrifugal action while the flow is deaccelerated in pumps.

5.2.2 Pump Applications

Some of the important application of pumps are :

1. To pump water from source to fields for agricultural and irrigation purposes.
2. In petroleum installations to pump oil.
3. In steam and diesel power plants to circulate feed water and cooling water respectively.
4. Hydraulic control systems.
5. Transfer of raw materials.
6. Pumping of water in buildings.
7. Fire fighting.

5.2.3 Advantages of Centrifugal Pumps over Reciprocating Pumps

1. Its discharge capacity is much higher.
2. It can also be used for highly viscous fluids like oils, muddy and sewage water, chemicals, paper pulpless.
3. Being high speed machine it can be directly coupled to prime mover.
4. It is compact, smaller in size and has low weight for the same discharge capacity.
5. It can be operated at high speed without any danger of separation and cavitation.
6. Maintenance cost is comparatively very low.
7. These are highly efficient.

However, reciprocating pumps are superior to centrifugal pumps for applications where very high pressures are needed to be developed at moderate discharges e.g. for oil pumping from deep oil wells and certain hydraulic devices like hydraulic jack.

5.3 Components of a Centrifugal Pump (C.F. Pump)

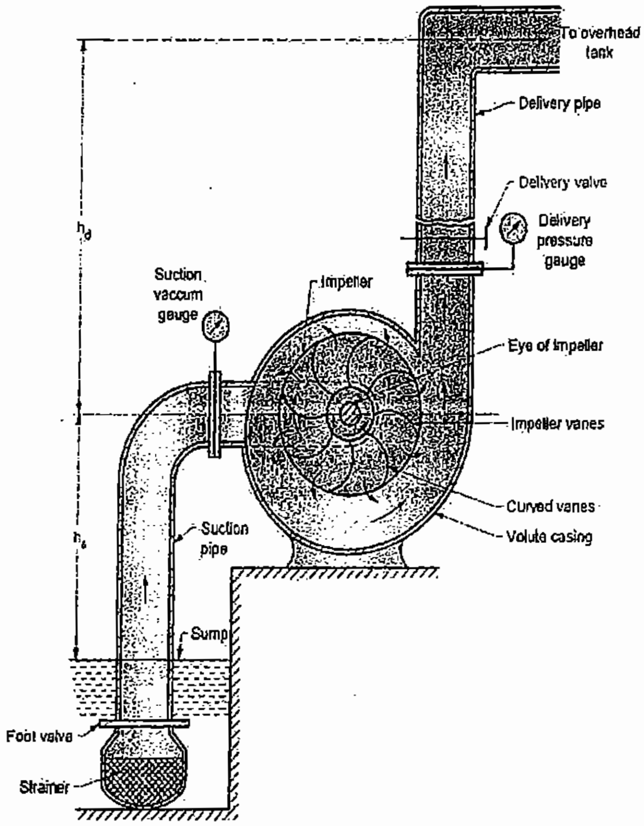


Fig. 5.3.1 : Components of a centrifugal pump

A centrifugal pump has the following main components as shown in Fig. 5.3.1.

1.	Impeller
2.	Casing
3.	Suction pipe with strainer and foot valve
4.	Delivery pipe.

1. Impeller

An impeller is a wheel or rotor having a series of backward curved vanes or blades. The impeller is mounted on a shaft which is usually coupled to a motor. The motor provides the required input energy to rotate the impeller.

2. Casing

The impeller is enclosed in a watertight casing with delivery pipe on one side and with an arrangement on suction side called eye of impeller as shown in Fig. 5.3.1.

Casing has to perform two functions. Firstly, it guides the water from entry to exit of impeller. Secondly, the casing is so designed that it helps in partly converting the kinetic energy of the liquid into pressure energy.

3. Suction pipe with strainer and foot valve

The pipe which connects the sump to the eye of impeller is called *suction pipe*. The sump carries the liquid to be lifted by the pump. The suction pipe at its inlet is provided with a *strainer* and a *foot valve*. The function of strainer is to prevent the entry of any debris into the pump. The foot valve is a non-return valve which allows the flow of water only in upward direction. Therefore, this valve does not allow the liquid to drain out from suction pipe.

4. Delivery pipe

The pipe which connects the outlet of pump upto point it delivers the liquid to required height is called *delivery pipe*.

A valve is provided in the delivery pipe near the outlet of the pump called *delivery valve*. It is a *sluice type valve*. It's function is to regulate the supply of liquid from the pump to delivery pipe.

5.3.1 Working of Centrifugal Pump

Centrifugal pump works on the principle that when a certain mass of liquid is made to rotate along the impeller from the central axis of rotation, it impresses a centrifugal head. It causes the water to move radially outwards at higher velocity and causes the water to rise to a higher level. The motion of water is restricted by casing of pump, it results into pressure build up. In addition, the change in angular momentum of liquid during its flow results into increase in pressure head.

The steps involved in operation of centrifugal pump are as follows :

1. The delivery valve is closed.
2. The *priming* of the pump is carried out. Priming involves the filling the liquid in suction pipe and casing upto the level of delivery valve so that no air pockets are left in the system. If any air or gas pockets are left in this portion of pump, it may result into no delivery of liquid by the pump.
3. The pump shaft and impeller is now rotated with the help of an external source of power like a motor or any other prime mover. The rotation of impeller inside a casing full of liquid produces a forced vortex which is responsible in imparting the centrifugal head to the liquid. It creates a vacuum at the eye of impeller and causes liquid to rise into suction pipe from the sump.
4. The speed of impeller should be sufficient to produce the centrifugal head such that it can initiate discharge from delivery pipe.
5. Now the delivery valve is opened and the liquid is lifted and discharged through the delivery pipe due to its high pressure. Thus the liquid is continuously sucked from the sump to impeller eye and it is delivered from the casing of pump through the delivery pipe.
6. Before stopping the pump, it is necessary to close the delivery pipe otherwise the back flow of liquid may take place from the high head reservoir.

5.4 Classification of Centrifugal Pumps

Based on the design, constructional features and their application, the centrifugal pumps are classified as follows :

1. Based on working head

Based on the range of working head the pumps are classified as :

(a) Low head pumps (upto a head of 15 m). Usually these pumps do not have guide vanes.

(b) Medium head pumps (15 m to 40 m head). These are usually provided with guide vane.

(c) High head pumps (> 40 m head). These are multistage pumps since a single impeller pump cannot build a pressure more than 40 m head.

2. Based on type of casing

University Question

Q. Explain different types of casing used in centrifugal pump. **SPPU : Dec. 16, May 19**

The shape of casing is designed so as to reduce the loss of kinetic head to minimum. Based on the shape of casing used, the pumps are classified as :

(a) Volute pump or constant velocity pump

Fig. 5.4.1 shows a centrifugal pump with a volute or collecting passage round the impeller of gradually increasing area from cut water at A to delivery pipe at B.

The cross-section is so designed to give a constant velocity in the volute of spiral shape. For this reason, it is also called as constant velocity volute.

In such a volute casing the loss of energy is considerably reduce compared to a circular casing if employed. However, the conversion of kinetic energy into pressure energy is not possible. Due to this the efficiency of pump only increases slightly.

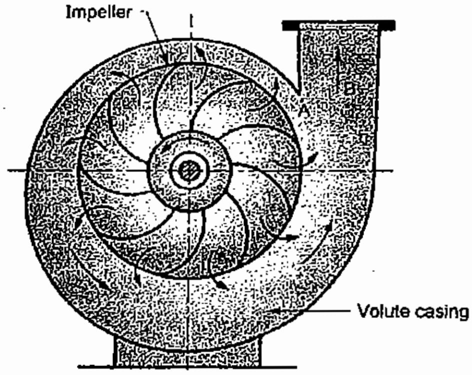


Fig. 5.4.1 : Volute pump

(b) Vortex or variable velocity volute pump

An improvement of the design of volute pump is shown in Fig. 5.4.2. This pump has relatively larger overall diameter compared to pump shown in Fig. 5.4.1 in order to provide an annular space between the impeller and volute passage.

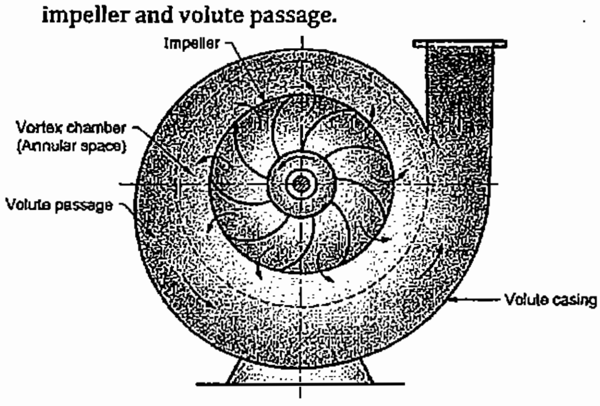


Fig. 5.4.2 : Vortex or variable velocity volute pump

In this annular space called vortex chamber, there is a free vortex in which the velocity of flow of liquid falls as it passes into this chamber from impeller outlet to entry of volute passage. (Since in a free vortex, the velocity of whirl is proportional to radial distance). Due to decrease in velocity the pressure increases radially from centre outwards.

The drawback of this arrangement is that to get an efficient chamber the dimensions become excessive and the pump becomes bulky and expensive.

The volute pumps and vortex volute pumps are single stage pumps with horizontal shaft.

(c) Diffuser or turbine pump

Fig. 5.4.3 shows a diffuser or turbine pump which is similar to vortex volute pump, but a diffuser ring with guide vanes is fixed in annular space.

Function of guide vanes is to guide the liquid leaving the impeller in streamlined diverging passages into the volute chamber from where it flows to the delivery pipe. In case of multistage pumps, the liquid from volute chamber flows into the eye of impeller of the next stage pump and the final stage volute discharges into delivery pipe.

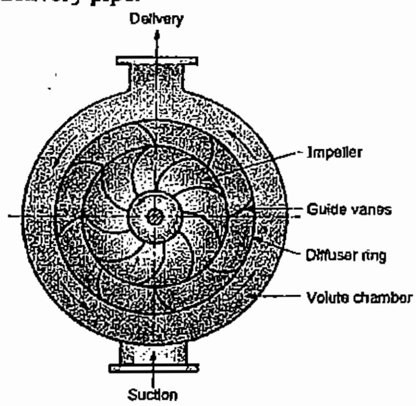


Fig. 5.4.3 : Diffuser or turbine pump

This pump with diffuser ring becomes in fact a reversed reaction turbine and is therefore commonly known as a *turbine pump*.

The guide vane passages so formed have an increasing cross-sectional area which reduces the velocity of flow, hence, the partial kinetic energy of the liquid is converted into pressure energy. Further conversion of kinetic energy into pressure energy takes place in the volute chamber of increasing cross-sectional area.

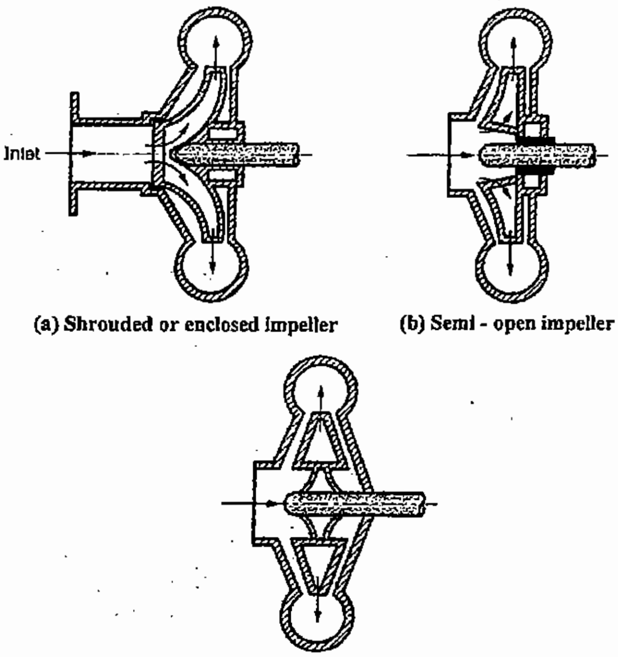
3. Based on liquid handled (Types of impellers)

Depending on the type and viscosity of liquid to be handled, a pump uses three types of impellers, accordingly the pumps are classified as follows :

(a) Shrouded or enclosed impellers

In this type of impellers, the vanes of impeller are cast between two circular discs or plates (shroude) as shown in Fig. 5.4.4(a). The plates on entry side is called *crown plate* and the plate on back or shaft side is called *base plate.*

This arrangement provides better guidance for liquid to flow and prevents leaking of liquid from blade tips from one passage to another passage with high efficiency. These types of impeller pumps are mostly used for clear water or for other liquids of low viscosity free from dirt.



(a) Shrouded or enclosed impeller (b) Semi - open impeller

(c) Open impeller

Fig. 5.4.4 : Types of impellers

(b) Semi-open impellers

These impellers have a plate only on back side called based plate as shown in Fig. 5.4.4(b).

Such an arrangement helps in dealing liquids mixed with fibrous materials. Therefore, these type of impellers can be used in sewage installation, sugar and pulp industry etc. with small amount of debris.

(c) Open impellers

These types of impeller do not have any cover plate on either side of the vanes. Therefore, the vanes of an open impeller are open from both sides as shown in Fig. 5.4.4(c).

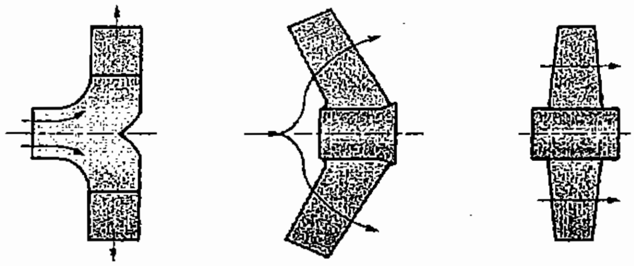
Open impellers are not so efficient but they are useful to deal with liquids which may contain suspended solids such as sand, grit, clay etc. since these pumps do not clog.

4. Based on relative direction of flow through impeller

Based on the direction of flow of liquid through the impeller the pumps are classified as :

(a) Radial flow pumps

Most of the centrifugal pumps are radial flow type in which the liquid flows in the impeller in radial direction only as shown in Fig. 5.4.5(a).



(a) Radial flow (b) Mixed flow (c) Axial flow

Fig. 5.4.5 : Classification of pumps based on relative direction of flow of liquid through impeller

In radial type of impellers, the liquids enter at the centre of impeller axially and then it flows radially over impeller blades upto outer periphery. In this the pressure head is developed due to centrifugal force impressed upon the liquid.

(b) Mixed flow pumps

Refer Fig. 5.4.5(b). It is the modification of radial flow impeller in which the flow is the combination of axial and radial flow and the impeller resembles the propeller of a ship. These are also called as *screw pumps* due to their resemblance to shape of a screw.

The mixed flow impellers have large discharge rates of liquid compared to radial flow impellers at low heads. Therefore, these type of pumps are suitable for irrigation applications.

(c) Axial flow pumps

In axial flow pumps the direction of flow of liquid through its impeller is in the axial direction only from inlet to exit as shown in Fig. 5.4.5(c).

These pumps are designed for very large discharge rates at low heads, hence these are ideally suited for irrigation purposes.

The pressure head developed in axial flow pump is not due to centrifugal action, rather it is due to flow of liquid on blades of aerofoil section similar to generation of lift by the wings of an aeroplane.

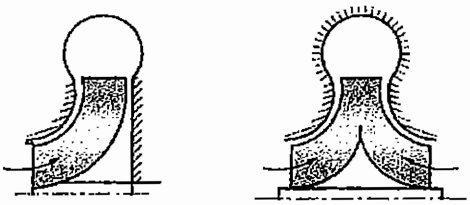
These pumps have adjustable blades similar to runner blades of a Kaplan turbine.

5. Based on number of entrances to impeller

The centrifugal pumps based on number of entrances to the impeller can be classified as follows :

(a) Single entry pump

In this pump the liquid enters only from one side into the impeller from suction pipe as shown in Fig. 5.4.6(a). These are also called as *single suction pump.*



(a) Single entry (b) Double entry

Fig. 5.4.6 : Single and double entry impellers

(b) Double entry or double suction pump

Refer Fig. 5.4.6(b). In these pumps entry to impeller is from both sides of Impeller. In such pumps the axial thrust is negligible.

It is suitable for large discharge rates.

6. Based on number of impellers per shaft

Based on number of impellers used per shaft the pumps are classified as :

(a) Single stage pump : It has one impeller and it is suitable for heads upto 40 m.

(b) Multistage pump : These pumps use two or more number of impellers in series depending upon the head requirements. In these pumps the discharge of one pump from casing enters into the eye of impeller of the next pump in series.

Total head developed by multistage pump is equal to algebraic sum of heads developed by each pump.

7. Based on specific speed, N_S

Specific speed, N_S is defined as the speed of a geometrically similar pump in all respect of actual pump which delivers 1 m^3/s of discharge under a head of 1 m. It is given by the equation,

$$N_S = \frac{N\sqrt{Q}}{H^{3/4}} \quad \text{...(5.4.1)}$$

where, N = speed, rpm

Q = discharge, m^3/s

H = head, m

In above equation H is for single stage pump and single suction pump. In case of multistage pumps, head must be calculated by dividing the total head by number of stage. Similarly, discharge Q for double section pump be taken as half the total discharge.

Based on specific speed, various types of pump are classified as follows :

Pump	Speed, N	Specific speed, N_S
Radial flow	Slow	10 – 30
	Medium	30 – 50
	High	50 – 80
Mixed flow		80 – 160
Axial flow		100 – 450

8. Based on shaft position

Based on shaft position, the pumps are classified as :

(a) Horizontal shaft pump : Usually the centrifugal pumps are with horizontal shaft.

(b) Vertical shaft pump : These pumps are designed for specific applications and to save space. e.g. the deep well and marine pumps.

5.5 Work done by Impeller on Water

University Question

Q. Derive an expression for rise in pressure through impeller of a centrifugal pump. **SPPU : May 16**

A pump is a power absorbing device in which the work is done by impeller on water. The inlet and outlet velocity diagrams as shown in Fig. 5.5.1 can be drawn in similar way as drawn for an inward flow reaction turbine. Notations used are same.

In pumps, the water enter the impeller at its centre and leaves at its outer periphery.

Following *assumptions* are made :

(i) Liquid enters the impeller vane radially for best efficiency i.e. $\alpha = 90$,

therefore, $V_1 = V_{f1}$ and velocity of whirl, $V_{w1} = 0$.

(ii) Liquid enters and leaves the vane without shock.

(iii) The velocity distribution in passages between vanes is uniform.

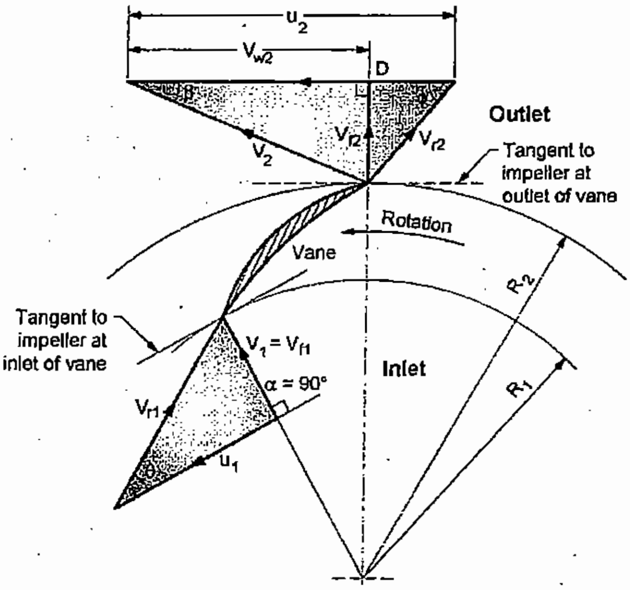


Fig. 5.5.1 : Velocity diagrams at inlet and outlet of impeller vane

Let D_1 = Diameter of impeller at inlet = 2 × Radius, R_1

D_2 = Diameter of impeller at outlet = 2 × Radius, R_2

N = Speed of impeller in rpm.

The tangential blade velocity at inlet, u_1 and at outlet, u_2 can be written as :

$$u_1 = \omega \cdot R_1 = \left(\frac{2\pi N}{60}\right) R_1$$

and

$$u_2 = \omega \cdot R_2 = \left(\frac{2\pi N}{60}\right) R_2$$

V = Absolute velocity

V_r = Relative velocity

V_f = Velocity of flow

V_w = Velocity of whirl

Suffix 1 and suffix 2 represent the velocities at inlet and outlet respectively.

α = Angle made by absolute velocity V_1 at inlet

θ = Inlet angle of vane

ϕ = Outlet angle of vane

β = Discharge angle of absolute velocity at outlet

Workdone (W.D.) by impeller on water per N weight of liquid per second is given as,

$$\text{W.D.} = \frac{1}{g}(V_{w2} \cdot u_2 - V_{w1} \cdot u_1) \qquad ...(5.5.1)$$

Above Equation (5.5.1) is known as Euler's momentum equation for centrifugal pump.

But, $V_{w1} = 0$, since entry is radial.

$$\therefore \quad \text{W.D.} = \frac{V_{w2} \cdot u_2}{g} \text{ (Nm/N/s)} \qquad ...(5.5.2)$$

In case working liquid is water and the weight of water is w, then the work done on water per second becomes,

$$\text{W} \cdot \text{D/s} = w \cdot \frac{V_{w2} \cdot u_2}{g}$$

$$= \rho \cdot Q \cdot V_{w2} \cdot u_2 \text{ (Nm/s)} \qquad ...(5.5.3)$$

where, Weight of water/s,

$$w = \rho \cdot g \cdot Q \qquad ...(5.5.4)$$

Discharge rate,

$$Q = \text{Area} \times \text{Velocity of flow}$$

$$\therefore \quad Q = \pi \cdot D_1 \cdot B_1 \cdot V_{f1}$$

$$= \pi D_2 \cdot B_2 \cdot V_{f2} \qquad ...(5.5.5)$$

Where, B_1 and B_2 is width of impeller at inlet and outlet respectively.

5.6 Various Heads and Efficiencies of a Centrifugal Pump

University Question

Q. What do you mean by manometric head ?

SPPU : May 13

Various heads connected with centrifugal pump installation are shown in Fig. 5.6.1.

Pressure head, H in meter of fluid column is given by the equation,

$$p = \rho \cdot g \cdot H \text{ (N/m}^2\text{)} \qquad ...(i)$$

Let, V_s = Velocity of liquid in suction pipe, m/s

V_d = Velocity of liquid in delivery pipe, m/s

h_{fs} = Loss of head due to friction in suction pipe, m

h_{fd} = Loss of head due to friction in delivery pipe, m

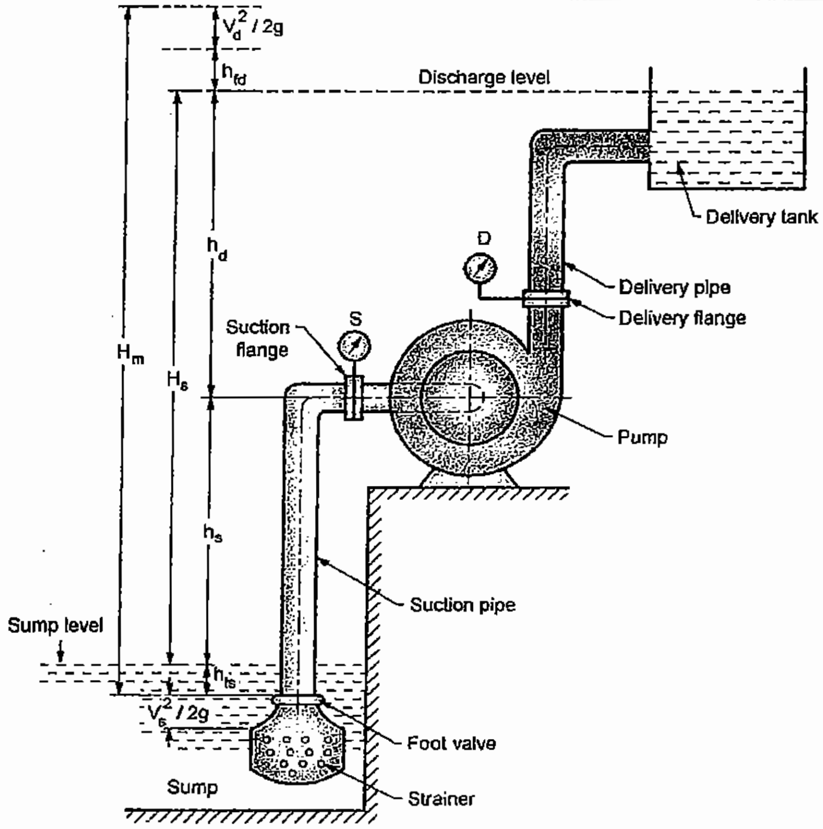


Fig. 5.6.1 : Various heads in a pumping installation

Definitions connected with various types of head are as follows :

(i) Suction lift, h_s

It represents the vertical distance between the top surface level of sump and the centre of impeller.

(ii) Delivery lift, h_d

It represents the vertical distance between the centre of impeller and the discharge level in delivery tank.

(iii) Static head, H_s

It is the sum of suction lift, h_s and delivery lift, h_d. i.e. it represents the vertical distance between the top surface level of sump to discharge level in delivery tank. Hence static head,

$$H_s = \text{Suction lift, } h_s + \text{Delivery lift, } h_d \quad ...(5.6.1)$$

(iv) Gross head, H_g

The pump is required to work against the static head and the other losses like friction losses in pipes and head corresponding to kinetic energy due to suction and delivery velocity of liquid. The total head against which the pump has to work is called the gross head, H_g.

Gross head, H_g = Static head, H_s

+ Friction losses in suction and delivery pipe $(h_{fs} + h_{fd})$

$$+ \text{Velocity heads}\left[\frac{V_s^2}{2g} + \frac{V_d^2}{2g}\right] \quad ...(5.6.2)$$

(v) Manometric head, H_m

The **manometric head** is defined as the minimum amount of head against which the pump has to work to deliver the required discharge. It is required to operate against the following heads :

(a) To develop the static head, H_s.

(b) To overcome the friction losses in suction and delivery pipes and the friction losses in pipes fittings valves, bends etc.

(c) The velocity head at discharge to maintain the delivery of liquid.

∴ Manometric head,

$$H_m = \text{Static head, } H_s + \text{Friction losses, } (h_{fs} + h_{fd}) + \text{Velocity head at discharge, } \frac{V_d^2}{2g} \quad ...(5.6.3)$$

Note that the manometric head, H_m does not include the friction loss head in impeller and casing of the pump. Usually the discharge velocity head $\frac{V_d^2}{2g}$ is small compared to static head and friction head, therefore, this head is mostly neglected.

5.6.1 Relation between Manometer Head and Workdone by impeller on Liquid

University Question

Q. Derive an expression for rise in pressure through impeller of a centrifugal pump. **SPPU : May 16**

The manometric head H_m is also equal to the difference of head imparted by impeller to liquid and the loss of head in impeller and casing. Hence,

$$\text{Manometric head, } H_m = \frac{V_{w2} \cdot u_2}{g} - (h_{fi} + h_{fc}) \quad ...(5.6.4)$$

where, h_{fi} = head loss in impeller

h_{fc} = head loss in casing

5.6.2 Virtual Head, $H_{virtual}$

The virtual head represents the total head through which the liquid can be lifted when all friction losses in pipes, impeller and casing are neglected. Therefore,

$$H_{virtual} = \frac{V_{w2} \cdot u_2}{g} \quad ...(5.6.5)$$

5.6.3 Manometric Head in terms of Suction and Delivery Pressure Measured by Pressure Gauge

Consider that the suction gauge (S) and delivery pressure gauge (D) are installed at suction flange and delivery flange as shown in Fig. 5.6.1.

Let, Z = difference in height between delivery gauge and suction gauge. Using Bernoulli's equation we can write,

$$H_m = \left(\frac{p_d}{w} + \frac{V_d^2}{2g} + Z\right) - \left(\frac{p_s}{w} + \frac{V_s^2}{2g}\right) \quad ...(5.6.6)$$

where, w = Density of liquid in N/m³ = $\rho \cdot g$

In case the suction and delivery pipes are of equal diameters,

Then, $V_s = V_d$ $\quad (\because Q = A \cdot V)$.

Equation (5.6.6) reduces to :

$$H_m = \left(\frac{p_d}{w} - \frac{p_s}{w}\right) + Z \quad ...(5.6.7)$$

5.7 Losses and Efficiencies

5.7.1 Losses in Pumps

Various losses which occur during the operation of a centrifugal pump are as follows :

1. Hydraulic losses

These losses represent the loss of head in pumping installation which are :

(i) Losses in pump

(a) Loss of head due to friction in impeller.

(b) Loss of head due to shock and eddy's from inlet to exit of impeller.

(c) Loss of head in guide vanes due to diffusion and in casing.

(ii) Other hydraulic losses

(a) Friction loss in suction and delivery pipes.

(b) Loss of head in bends, fittings, valves etc.

2. Mechanical losses

Due to disc friction in impeller, friction in bearings and other mechanical parts of the pump.

3. Leakage loss

The difference between theoretical and actual discharge is called as the **slip** of the pump. It affects the manometric efficiency and power input to the pump. It reduces the volumetric efficiency of pump.

A certain amount of energy is lost due to liquid which leaks from high pressure side of pump to low pressure side of pump which is finally lost in eddies. The liquid is leaked through the glands, stuffing box etc. These losses are represented schematically in Fig. 5.7.1.

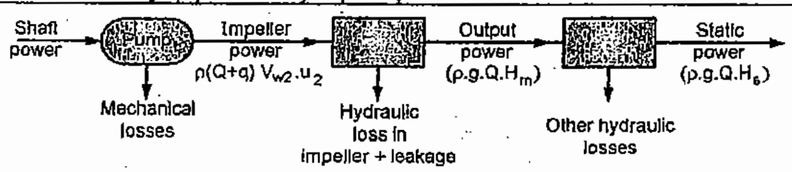


Fig. 5.7.1 : Losses in centrifugal pump

5.7.2 Efficiencies of a Centrifugal Pump

University Question

Q. What do you mean by manometric efficiency, mechanical efficiency and overall efficiency of a centrifugal pump? **SPPU : May 13**

Various efficiencies related to centrifugal pump are :

1. Mechanical efficiency, η_m

The ratio of power available at the impeller i.e. power delivered by impeller to liquid to the power input at the shaft (motor power or shaft power) is known as mechanical efficiency.

Therefore,

$$\eta_m = \frac{\text{Power avilable at impeller,P(kW)}}{\text{Shaft power, } P_s\text{(kW)}} \quad ...(5.7.1)$$

But power available at impeller is given as,

$$P = \rho\,(Q+q)\,V_{w2}\cdot u_2$$

$$\therefore \quad \eta_m = \frac{\rho\cdot(Q+q)\,V_{w2}\cdot u_2}{P_s}$$

$$= \frac{P_s - P_{mech.loss}}{P_s} \quad ...(5.7.2)$$

where, q = leakage loss

Value of mechanical efficiency ranges between 95% to 98%.

2. Manometric efficiency, η_{mano}

It is defined as the ratio of manometric head, H_m developed by the pump to the head imparted by the impeller to liquid. Mathematically,

$$\eta_{mano} = \frac{\text{Manometric head, } H_m}{\text{Head imparted by impeller}\left(\frac{V_{w2}\cdot u_2}{g}\right)}$$

$$= \frac{g\cdot H_m}{V_{w2}\cdot u_2} \quad ...(5.7.3)$$

3. Volumetric efficiency, η_v

It is ratio of actual liquid discharged from the pump in m^3/s to the theoretical liquid passing through the impeller in m^3/s. Mathematically,

$$\eta_v = \frac{\text{Actual discharge, Q}}{\text{Theoretical flow through impeller, } (Q+q)} \quad ...(5.7.4)$$

where, q represents the loss of discharge due to leakage loss.

4. Overall efficiency, η_o

It is defined as the ratio of power output of the pump called **water power** to the shaft power. Thus,

$$\eta_o = \frac{\text{Power output}}{\text{Shaft power, } P_s} = \frac{\rho\cdot g\cdot Q\cdot H_m}{P_s} \quad ...(5.7.5)$$

Also,

$$\eta_o = \eta_{mano}\times\eta_v\times\eta_m \quad ...(5.7.6)$$

$$= \frac{g\cdot H_m}{V_{w2}\cdot u_2}\times\frac{Q}{(Q+q)}\times\frac{\rho(Q+q)\cdot V_{w2}\cdot u_2}{P_s}$$

$$= \frac{\rho\cdot g\cdot Q\cdot H_m}{P_s}$$

This is the same as in Equation (5.7.5).

5.7.3 Specific Speed of Centrifugal Pump, N_s

University Questions

Q. Define specific speed of a hydrodynamic pump. Derive expression for the same. **SPPU : May 15**

Q. Derive an expression of specific speed of centrifugal pump. **SPPU : Dec. 16**

Q. What is the significance of specific speed? Derive the relation for the same for centrifugal pump. **SPPU : May 19**

The **specific speed** of a centrifugal pump is defined as the speed of a geometrically similar pump will deliver a discharge of 1 m^3/s of a liquid under a head of 1 m. It is denoted by N_s.

Let
- Q = Discharge (m^3/s),
- V_f = Velocity of flow (m/s)
- D = Diameter of impeller (m),
- B = Width of impeller (m)
- H_m = Manometric head (m)

We know that : $B \propto D$

Discharge, Q = Area × Velocity of flow = $(\pi DB)\,V_f$

$$Q \propto \pi D^2\cdot V_f \propto D^2\cdot V_f \quad (\because B \propto D) \quad ...(i)$$

Tangential velocity of impeller,

$$u = \frac{\pi D N}{60} \propto DN \quad ...(ii)$$

Also,

$$u \propto V_f \propto \sqrt{H_m} \quad ...(iii)$$

Equating Equations (ii) and (iii),

$$DN \propto \sqrt{H_m} \quad \text{i.e. } D = \frac{\sqrt{H_m}}{N} \quad ...(iv)$$

On substituting the values of V_f and D from equations (iii) and (iv) respectively in Equation (i) we get,

$$Q \propto \left(\frac{\sqrt{H_m}}{N}\right)^2 \times \sqrt{H_m} \propto \frac{H_m^{3/2}}{N^2}$$

$$Q = K \cdot \frac{H_m^{3/2}}{N^2}$$

Where, K is a constant of proportionality. ...(v)

By definition

If $H_m = 1$ m, $Q = 1 m^3/s$, N becomes equal to N_s.

$$\therefore \quad 1 = K \cdot \frac{H_m^{3/2}}{N_s^2} = \frac{K}{N_s^2} \text{ i.e. } K = N_s^2 \quad ...(vi)$$

Substituting the value of K from Equation (vi) in (v),

$$Q = \frac{N_s^2 \cdot H_m^{3/2}}{N^2}; \text{ or, } N_s^2 = \frac{N^2 \cdot Q}{H_m^{3/2}}$$

$$\therefore \quad N_s = \frac{N \cdot \sqrt{Q}}{H_m^{3/4}} \quad ...(5.7.7)$$

5.8 Effect of Vane Discharge Angle ϕ on Manometric Efficiency

University Question

Q. Explain effect of blade angle (outlet) on discharge in centrifugal pump. SPPU : May 15, May 19

At exit of impeller the energy available in liquid has the pressure energy equal to manometric head, H_m and kinetic energy head $\left(\frac{V_2^2}{2g}\right)$.

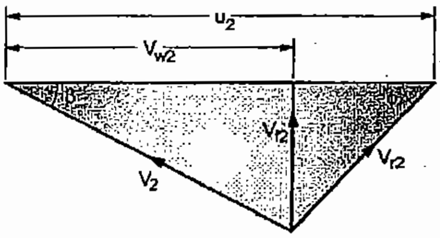


Fig. 5.8.1 : Exit velocity diagram

The energy supplied to impeller per unit weight of liquid is $\frac{V_{w2} \cdot u_2}{g}$. In case the losses in pump are neglected, the input energy to impeller must be equal to energy at exit of impeller i.e.

$$\text{Input energy, } \frac{V_{w2} \cdot u_2}{g} = \left(H_m + \frac{V_2^2}{2g}\right)$$

$$\therefore \quad H_m = \frac{V_{w2} \cdot u_2}{g} - \frac{V_2^2}{2g} \quad ...(5.8.1)$$

From exit velocity diagram shown in Fig. 5.8.1, we have,

$$V_2^2 = V_{w2}^2 + V_{f2}^2$$

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = u_2 - V_{f2} \cdot \cot\phi$$

On substituting above values in Equation (5.8.1) we have,

$$H_m = \frac{(u_2 - V_{f2} \cdot \cot\phi)\, u_2}{g} - \frac{(u_2 - V_{f2} \cdot \cot\phi)^2 + V_{f2}^2}{2g}$$

$$H_m = \frac{2(u_2^2 - V_{f2} \cdot u_2 \cot\phi) - (u_2^2 + V_{f2}^2 \cot^2\phi - 2u_2 \cdot V_{f2} \cot\phi) - V_{f2}^2}{2g}$$

$$H_m = \frac{u_2^2 - V_{f2}^2 \cot^2\phi - V_{f2}^2}{2g} = \frac{u_2^2 - V_{f2}^2 (1 + \cot^2\phi)}{2g}$$

$$= \frac{u_2^2 - V_{f2}^2 \cdot \text{cosec}^2\phi}{2g} \quad ...(5.8.2)$$

Under ideal conditions, manometric efficiency of pump will be,

$$\eta_{mano} = \frac{g \cdot H_m}{(V_{w2} \cdot u_2)} = \frac{g\,(u_2^2 - V_{f2}^2 \,\text{cosec}^2\phi)/2g}{(u_2 - V_{f2} \cot\phi)\, u_2}$$

$$= \frac{u_2^2 - V_{f2}^2 \cdot \text{cosec}^2\phi}{2\, u_2 (u_2 - V_{f2} \cdot \cot\phi)} \quad ...(5.8.3)$$

$$\text{Let Flow ratio, } K_f = \frac{V_{f2}}{\sqrt{2g\, H_m}} = 0.25$$

Using this equation and Equation (5.8.2), the value of u_2 can be calculated in terms of H_m by varying the value ϕ from 20° to 90°. On substituting the various values of u_2, ϕ_2 and V_{f2} in Equation (5.8.3) we find value of η_{mano} decreases from 0.73 to 0.47.

If we further decrease the value of ϕ_2 below 20°, though it would increase the manometric efficiency but it would result into long blades and narrow passages. Such blades will cause increased friction losses.

For above reasons, the vane discharge angle ϕ is kept more than 20° for centrifugal pumps.

5.9 Minimum Starting Speed of Pump

University Question

Q. Derive an expression of minimum starting speed of centrifugal pump. SPPU : Dec. 15, Dec.16, Dec. 19

When pump is started, it will not deliver any liquid until the pressure difference in impeller is large enough to overcome the manometric head.

The pressure head developed is due to centrifugal head caused by the centrifugal force impressed on rotating liquid.

But, Centrifugal head = $(u_2^2 - u_1^2)/2g$

The flow will only commence when the centrifugal head exceeds the manometric head, H_m, therefore,

$$\frac{u_2^2 - u_1^2}{2g} \geq H_m \quad \text{...(5.9.1)}$$

$$\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2 \geq 2g H_m$$

$\therefore$ For minimum speed,

$$\therefore N = \frac{60}{\pi} \times \frac{\sqrt{2g H_m}}{\sqrt{(D_2^2 - D_1^2)}} \quad \text{...(5.9.2)}$$

Equation (5.9.2) gives the minimum speed required for pump to start discharging the liquid.

5.9.1 Minimum Diameter of Impeller, D_1

Usually external diameter, $D_2 = 2 \times$ internal diameter, D_1 i.e. $D_2 = 2D_1$. Using the concept of minimum starting speed of pump, N to deliver the given head, H_m, we can determine the minimum diameter of impeller D_1 from Equation (5.9.2) as follows :

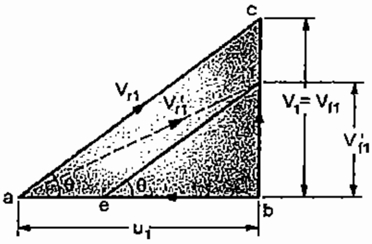
$$N = \frac{60}{\pi} \times \frac{\sqrt{2g H_m}}{\sqrt{(2D_1^2 - D_1^2)}} = \frac{60}{\pi} \times \frac{\sqrt{2g H_m}}{\sqrt{3} \cdot D_1}$$

$$\therefore D_1 = \frac{48.84}{N} \times \sqrt{H_m} \quad \text{...(5.9.3)}$$

$$D_2 = 2 D_1 = \frac{97.68}{N} \times \sqrt{H_m} \quad \text{...(5.9.4)}$$

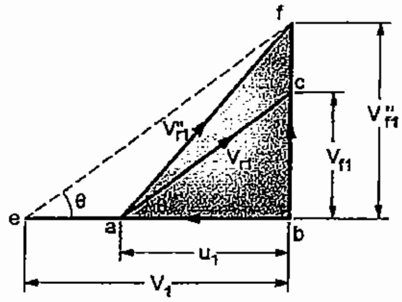
5.10 Effect of Variation of Discharge on Head Loss and Efficiency

- The pump works at maximum efficiency only at designed speed at a specific discharge. In case the discharge from the pump is either increased or decreased, the loss of head at entry to the pump are affected due to shock, as a result the efficiency of the pump is also affected.



(a) Reduced discharge

Fig. 5.10.1 contd...



(b) Increased discharge

Fig. 5.10.1 : Inlet velocity triangle with reduced and increased discharge

- In Fig. 5.10.1 the Δ abc represents the inlet velocity diagram. Under these conditions the relative velocity of liquid is at θ which is equal to vane angle at inlet.
- In case the discharge from pump is reduced by closing the delivery valve, the velocity of flow is reduced from V_{f1} to V'_{f1} as shown in Fig. 5.10.1(a).While the velocity of flow is increased from V_{f1} to V''_{f1} due to increased discharge as shown in Fig. 5.10.1(b).
- With the pump running at the same speed, the vane velocity u_1 at inlet remains the same and the inlet velocity diagram is changed from Δ abc to modified Δ afb.
- As can be seen from the diagrams the relative velocity V'_{r1} is no more tangential to vane at entry. It causes shock at entry and as a consequence the loss of head.
- Now the new velocity of flow is V'_{f1} represented by bf is fixed and the water must pass through the vane at an angle θ, it implies that velocity triangle abf is modified to Δ efb in which ef is parallel to ac.
- It causes vane velocity to change from ab to ef suddenly which results into shock and head loss.

The loss of head at entrance due to sudden change in vane velocity is given as :

(a) When discharge is reduced [Fig. 5.10.1(a)]

Loss of head,

$$h_L = \frac{(ae)^2}{2g} = \frac{(u_1 - V'_{f1} \cot \theta)^2}{2g} \quad \text{...(5.10.1)}$$

(b) When discharged is increased [Refer Fig. 5.10.1(b)]

Loss of head, $h_L = \frac{(ae)^2}{2g} = \frac{(eb - ab)^2}{2g}$

$$= \frac{(V''_{f1} \cot \phi - u_1)^2}{2g} \quad \text{...(5.10.2)}$$

5.11 Effect of Finite Number of Vanes of Impeller on Head and Efficiency

The velocity diagram shown in Fig. 5.5.1and explained in section 5.5 represents the **Euler's velocity diagram** and such a diagram can only be obtained *if the impeller has the infinite number of vanes and equally spaced.*

The head imparted by vanes in such a case is called **Euler or ideal head, H_e**

where, $$H_e = \frac{V_{w2} \cdot u_2}{g}.$$

Practically, it is not possible to infinite number of vanes for the following reasons :

(i) Restricted area of flow between vanes will increase friction head losses.

(ii) Fabrication of pump is difficult.

(iii) Choking of pump may take place due to non-clear water having debris.

Although the design of vanes is based on the Euler's velocity triangle with finite number of vanes, however, the actual velocity diagram developed are different than Euler's diagram. The actual velocity of whirl, V_{w2} at outlet is less than Euler's velocity of whirl due to *secondary or circulatory flows.* Consequently, the actual head developed by the impeller with finite number of vanes is less than the Euler's head with infinite number of vanes i.e. $H_{actual} < H_e$.

The ratio of (H_{actual}/H_e) is called **the vane effectiveness,** ϵ or vane efficiency. The value of ϵ depends on the number of vanes in the impeller. It is found that ϵ increases from 0.6 to 0.8 if the vanes are increased from 4 to 12.

The value of vane effectiveness for impellers having vanes more than 24 is taken as unity.

5.12 Working Proportions of Centrifugal Pump

Following working proportions are generally adopted in case of centrifugal pumps.

1. **Speed ratio, K_u**

   It is defined as the tangential velocity of vane at outlet, u_2 to the theoretical velocity of jet corresponding to manometric head, H_m.

   Hence, $$K_u = \frac{u_2}{\sqrt{2g \cdot H_m}}$$

   Speed ratio, K_u varies from 0.95 to 1.25.

2. **Flow ratio, K_f**

   It is defined as the ratio of velocity of flow at exit, V_{f2} to the theoretical velocity of jet corresponding to manometric head, H_m. Hence,

   $$K_f = \frac{V_{f2}}{\sqrt{2g\,H_m}}$$

   Value of K_f varies from 0.1 to 0.25.

3. **Diameters of impeller**

   Internal diameter of impeller D_1 is kept $\frac{1}{3} D_2$ to $\frac{2}{3} D_2$, where D_2 represents the external diameter of impeller.

   Since,

$$u_2 = \frac{\pi D_2 N}{60} \text{ and } K_u = \frac{u_2}{\sqrt{2g\,H_m}}$$

$$\therefore \quad K_u \cdot \sqrt{2gH_m} = \frac{\pi}{60} \times D_2 \times N$$

$$\therefore \quad D_2 = K_u \cdot \frac{60}{\pi} \times \sqrt{2g} \times \frac{\sqrt{H_m}}{N}$$

$$\therefore \quad D_2 = \frac{84.6\,K_u \sqrt{H_m}}{N} \qquad \text{...(5.12.1)}$$

and $$D_1 = \frac{1}{3} D_2 \text{ to } \frac{2}{3} D_2 \qquad \text{...(5.12.2)}$$

5.13 Effect of Discharge Vane Angle, ϕ (Vane Shapes)

University Question

Q. Discuss the influence of blade angles on performance of the centrifugal pump. **SPPU : May 12**

Rate of energy transfer per unit mass by the impeller is called Euler's head, H_e.

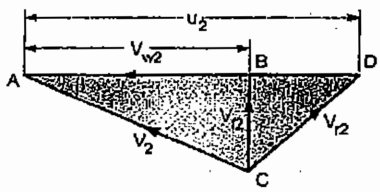


Fig. 5.13.1 : Outlet velocity diagram

$$H_e = \frac{V_{w2} \cdot u_2}{g} \qquad \text{...(i)}$$

From outlet velocity diagram shown in Fig. 5.13.1.

$$V_{w2} = u_2 - V_{f2} \cdot \cot \phi$$

But, $$V_{f2} = \frac{Q}{A_2}$$

$$\therefore \quad V_{w2} = u_2 - \frac{Q}{A_2} \cot \phi \text{ (where, A is area of flow)}$$

On substituting the value of V_{w2} in Equation (i) we get,

Euler's head, $$H_e = \frac{u_2}{g}\left(u_2 - \frac{Q}{A_2}\cot\phi\right) \quad ...(5.13.1)$$

At a *given speed*, u_2 is constant and so is the exit area A_2 of impeller and discharge angle ϕ. Therefore, Equation (5.13.1) can be written in the form,

$$H_e = K - K_1 Q \quad ...(5.13.2)$$

where, K and K_1 are constants as

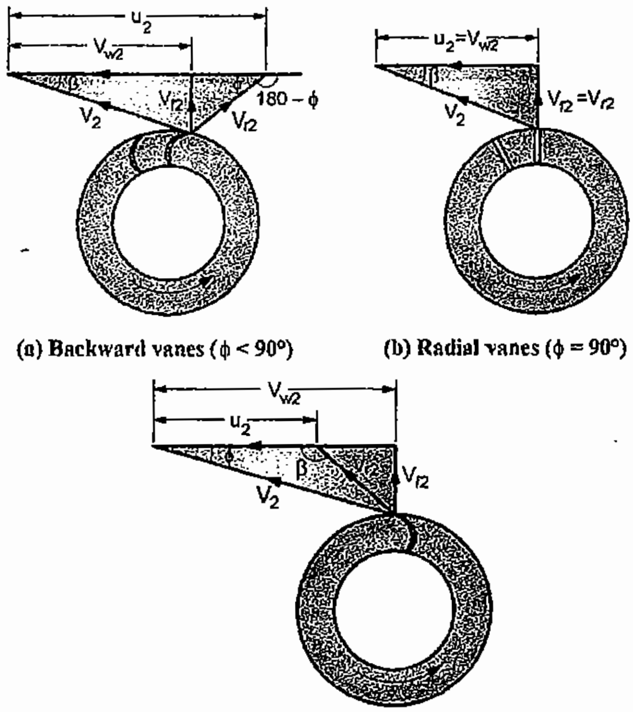
$$K = \frac{u_2^2}{g} \quad \text{and} \quad K_1 = \frac{u_2 \cot\phi}{g \cdot A_2}$$

Various vane shapes in centrifugal pumps

Vane shapes are of three types as follows :

(a) Backward vanes ($\phi < 90°$)

In these vanes, the outlet tip of vane is curved in the direction opposite to motion of impeller as shown in Fig. 5.13.2(a). Therefore, angle ϕ becomes less than 90°.



(a) Backward vanes ($\phi < 90°$) (b) Radial vanes ($\phi = 90°$)

(c) Forward vanes ($\phi > 90°$)

Fig. 5.13.2 : Vane shapes with outlet velocity diagram

(b) Radial vanes ($\phi = 90°$)

(c) Forward vanes ($\phi > 90°$)

The outlet tip angle of vane is in the direction of motion of impeller and it makes an obtuse angle with the tangent to the rotor.

These three types of vanes with outlet velocity diagram are shown in Fig. 5.13.2.

From Equation (5.13.1) it is evident that

1. For backward vanes, cot ϕ is positive (since $\phi < 90°$), the Euler's head, H_e will keep on decreasing with the increase in discharge rate Q. Therefore, the energy transfer with backward vanes is reduced.
2. For radial vanes ($\phi = 90°$), cot ϕ = 0, therefore the Euler's head remains constant irrespective of discharge rates.
3. In case of forward vanes ($\phi > 90°$) the Euler's head keeps on increasing with increase in discharge rates. However, the absolute velocity V_2 at exit of impeller is also increased. The conversion of this kinetic energy $\left(\frac{V_2^2}{2g}\right)$ into pressure energy cannot be carried out very effectively in diffuser section of pump since the liquid has a tendency to break away from the walls of the diverging passages.

   However, in case the diffusion process is too rapid in a small diffuser section, the liquid may reverse its direction of flow due to resultant high pressure gradient. It causes formation of eddies and loss of energy.
4. For the reasons discussed above, the pumps are usually designed with backward vanes in the range of ϕ = 20° to 35° except in cases where the impeller diameter and high head is the major consideration.

The theoretical head-discharge characteristic curves based on above discussion is shown in Fig. 5.13.3.

These curves are straight lines since,

$$H_e = K - K_1 \cdot Q \quad \text{[From Equation (5.13.2)]}$$

Since power required by impeller,

$P = \rho \cdot g \cdot Q\, H_e$, it follows from Equation (5.13.2) that,

$$P = \rho g \cdot Q\,(K - K_1 Q) = K_2 Q - K_3 \cdot Q^2 \quad ...(5.13.3)$$

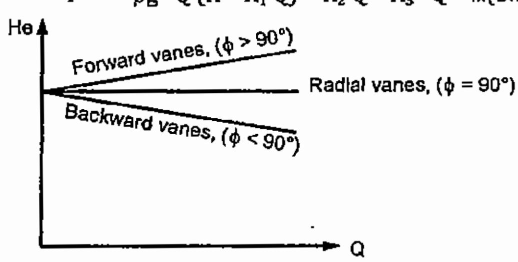


Fig. 5.13.3 : Theoretical head-discharge characteristics for various vane shapes

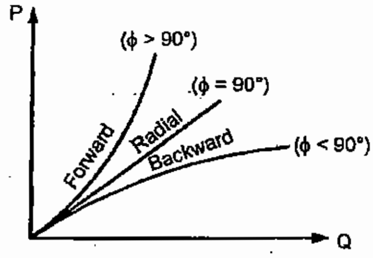


Fig. 5.13.4 : Theoretical power-discharge characteristics for various vane shapes

The power variation with variation in discharge Q for forward, radial and backward vanes is shown in Fig. 5.13.4.

It could be seen that with forward vanes, the input power required increases very rapidly with increase in discharge rates.

Ex. 5.13.1 : A centrifugal pump is required to lift water against total head of 40 m at the rate of 50 litres per second find the power of pump if the overall efficiency is 62%.

Soln. :

$$H = 40 \text{ m},$$

$$Q = 50 \text{ litres/s} = \frac{50}{1000} = 0.05 \text{ m}^3/\text{s},$$

Overall efficiency, $\eta_0 = 62\% = 0.62$

Power required to lift the water by the pump,

$$P = \frac{\rho \cdot g \cdot Q H \times 10^{-3}}{\eta_0} \text{ (kW)}$$

$$= \frac{1000 \times 9.81 \times 0.05 \times 40 \times 10^{-3}}{0.62}$$

$$= \mathbf{31.645 \text{ kW}}$$ **...Ans.**

Ex. 5.13.2 : A centrifugal pump delivers 0.1 m³/s of water to a height of 25 m through a pipe of 80 m long and 12 cm diameter pipe. Suction losses of the pump amounts to 0.4 m head. Find the shaft power to drive the pump if its overall efficiency is 73%. Assume, friction factor for delivery pipe = 0.013.

Soln. :

Given : Discharge, $Q = 0.1 \text{ m}^3/\text{s}$,

Diameter of pipe, $D = 12 \text{ cm} = 0.12 \text{ m}$;

$l = 80 \text{ m}$, $f = 0.001$,

$\eta_o = 73\% = 0.73$, $H_d = 30 \text{ m}$,

$h_{fs} = 0.4 \text{ m}$.

Velocity of discharge through pipe,

$$V_2 = \frac{Q}{A_{pipe}} = \frac{Q}{\frac{\pi}{4} \cdot D^2} = \frac{0.1 \times 4}{\pi \times (0.12)^2} = 8.84 \text{ m/s}$$

Friction head loss in pipe,

$$h_{fd} = \frac{4 f l V_2^2}{D \times 2g}$$

$$= \frac{4 \times 0.001 \times 80 \times (8.84)^2}{0.12 \times 2 \times 9.81} = 10.62 \text{ m}$$

Total head, $H = H_d + h_{fs} + h_{fd}$

$$= 30 + 0.4 + 10.62 = 41.02 \text{ m}$$

Shaft power required to drive the pump,

$$P_s = \frac{\rho \cdot g\, Q\, H}{\text{Overall efficiency, } \eta_o}$$

$$= \frac{1000 \times 9.81 \times 0.1 \times 41.02}{0.73} \times \frac{1}{10^3} \text{ kW}$$

$$= \mathbf{55.124 \text{ kW}}$$ **...Ans.**

Ex. 5.13.3 : The diameter of a centrifugal pump at inlet and outlet are 30 cm and 60 cm respectively. Find the minimum starting speed if it works against a head of 30 m.

Soln. : Given :

$D_1 = 30 \text{ cm} = 0.3 \text{ m}$, $D_2 = 60 \text{ cm} = 0.6 \text{ m}$

$H_m = 30 \text{ m}$

The flow will only commence when the centrifugal head exceeds the manometric head, H_m

$$\text{i.e.} \quad \frac{u_2^2 - u_1^2}{2g} \geq H_m$$

For minimum speed,

$$\frac{u_2^2 - u_1^2}{2g} = H_m$$

$$\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2 = 2g \times H_m$$

$$\left(\frac{\pi \times 0.6 \times N}{60}\right)^2 - \left(\frac{\pi \times 0.3 \times N}{60}\right)^2 = 2 \times 9.81 \times 30$$

$$N = \mathbf{204.05 \text{ rpm}}$$ **...Ans.**

Ex. 5.13.4 : A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 rpm work against total head of 40 m. The velocity of flow through the runner is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 50 cm and width at outlet is 5 cm.

Determine :

i) Vane angle at inlet

ii) Workdone by impeller on water per second

iii) Manometric efficiency **SPPU - Dec. 15, 10 Marks**

Soln. :

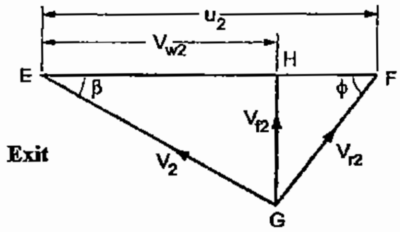
$$D_2 = 2D_1, \quad N = 1000 \text{ rpm},$$

$$\text{total head, } H_m = 40 \text{ m} \quad V_{f1} = V_{f2} = 2.5 \text{ m/s},$$

$$\phi = 40° \quad D_2 = 50 \text{ cm} = 0.5 \text{ m},$$

$$B_2 = 5 \text{ cm} = 0.05 \text{ m}$$

Refer Fig. P. 5.13.4



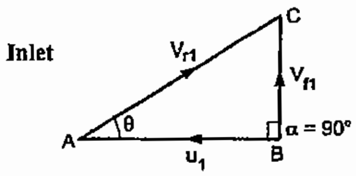


Fig. P. 5.13.4

$$D_1 = \frac{D_2}{2} = \frac{0.5}{2} = 0.25 \text{ m}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1000}{60} = 26.18 \text{ m/s}$$

$$Q = \pi D_2 B_2 V_{f2} = \pi \times 0.5 \times 0.05 \times 2.5$$

$$= 0.19635 \text{ m}^3/\text{s}$$

$$\dot{m} = \rho Q = 1000 \times 0.19635 = 196.35 \text{ kg/s}$$

(i) Vane angle at inlet, θ :

Refer inlet ΔABC

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{u_1}\right) = \tan^{-1}\left(\frac{2.5}{13.09}\right)$$

$$= \mathbf{10.81°} \qquad \text{...Ans.}$$

(ii) Wordone by impeller on water per second, $\dot{W}$

$$V_{w2} = u_2 - H_f = u_2 = \frac{V_{f2}}{\tan\phi}$$

$$= 26.18 - \frac{2.5}{\tan 40} = 23.2 \text{ m/s}$$

$$\dot{W} = \dot{m}\,\frac{V_{w2} \times u_2}{1000} \text{ (kW)}$$

$$= \frac{196.35 \times 23.2 \times 26.18}{1000} = \mathbf{119.26 \text{ kW}} \text{ ...Ans.}$$

(iii) Manometric efficiency, η_m

$$\eta_m = \frac{gH_m}{V_{w2} - u_2} = \frac{9.81 \times 40}{23.2 \times 26.18}$$

$$= \mathbf{0.646 \text{ or } 64.6\ \%} \qquad \text{...Ans.}$$

Ex. 5.13.5 : A centrifugal pump running at 800 rpm is working against a total head of 20.2 m. The external diameter of the impeller is 480 mm and outlet width 60 mm. If the vanes angle at outlet is 40° and manometric efficiency is 70%, determine :

(i) Flow velocity at outlet

(ii) Absolute velocity of water leaving the vane.

(iii) Angle made by the absolute velocity at outlet with the direction of motion at outlet, and

(iv) Rate of flow through the pump.

(v) Specific speed.

SPPU - Dec. 12, May 14, May 15,10 Marks

Soln. :

$$\text{Speed,} \quad N = 800 \text{ rpm}; \quad \text{Head, } H_m = 20.2 \text{ m},$$

$$D_2 = 480 \text{ mm} = 0.48 \text{ m};$$

$$B_2 = 60 \text{ mm} = 0.06 \text{ m};$$

$$\text{Vane outlet angle } \phi = 40°$$

$$\text{Manometric efficiency, } \eta_{mano} = 70\% = 0.7$$

(i) Flow velocity at outlet, V_{f2}

Vane velocity at outlet,

$$u_2 = \frac{\pi D_o N}{60} = \frac{\pi \times 0.48 \times 800}{60} = 20.11 \text{ m/s}$$

Exit velocity diagram is shown in Fig. P.5.13.5.

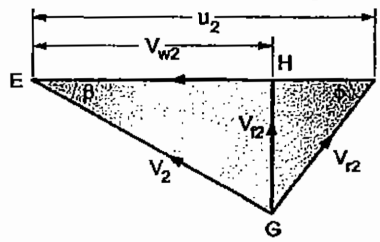


Fig. P. 5.13.5

$$\eta_{mano} = \frac{g \cdot H_m}{V_{w2} \cdot u_2}; \quad 0.7 = \frac{9.81 \times 20.2}{V_{w2} \times 20.11}$$

$$V_{w2} = 14.08 \text{ m/s}$$

$$\therefore \quad HF = u_2 - V_{w2} = 20.11 - 14.08 = 6.03 \text{ m/s}$$

$$\frac{V_{f2}}{HF} = \tan\phi$$

$$V_{f2} = HF \times \tan\phi = 6.03 \times \tan 40$$
$$= \mathbf{5.059\ m/s} \quad \text{...Ans.}$$

(ii) Absolute velocity of water at exit, V_2

$$V_2 = \sqrt{V_{w2}^2 + V_{f2}^2} = \sqrt{(14.08)^2 + (5.05)^2}$$
$$= \mathbf{14.95\ m/s} \quad \text{...Ans.}$$

(iii) Angle made by absolute velocity at outlet, β

$$\sin\beta = \frac{V_{f2}}{V_2} = \frac{5.05}{14.08} = 0.358$$
$$\beta = \mathbf{20.97^\circ} \quad \text{...Ans.}$$

(iv) Rate of flow through the pump, Q

$$Q = \pi D_2 B_2 V_{f2} = \pi \times 0.48 \times 0.06 \times 5.05$$
$$= \mathbf{0.4566\ m^3/s} \quad \text{...Ans.}$$

(v) Specific speed, N_s

$$Ns = \frac{N\sqrt{Q}}{Hm^{3/4}} = \frac{800 \times \sqrt{0.4566}}{20.2^{3/4}}$$
$$= \mathbf{56.734} \quad \text{...Ans.}$$

Ex. 5.13.6 : A centrifugal pump at 900 rpm has an impeller diameter of 500mm and eye diameter of 300 mm. The blade angle at outlet is 35° with tangent. Determine assuming zero whirl at inlet, the inlet blade angle, absolute velocity at outlet and its direction and the manometric head. The velocity of flow is constant throughout and is 3 m/sec.

SPPU - May 19,10 Marks

Soln. :

Refer Fig.P.5.13.6

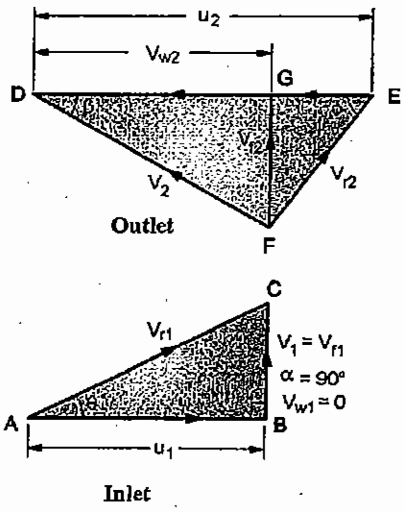


Fig. P.5.13.6

$$N = 900\text{ rpm}; \quad D_1 = 500\text{ mm} = 0.5\text{ m}$$
$$D_2 = 300\text{ mm} = 0.3\text{ m}, \quad V_{w_1} = 0,$$

(∵ No write at inlet)

$$\phi = 35^\circ, \quad V_{f_1} = V_{f_2} = 3\text{ m/s}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 900}{60} = 14.14\text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 900}{60} = 23.56\text{ m/s}$$

(i) Inlet blade angle, θ.

Consider Δ ABC,

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{u_1}\right) = \tan^{-1}\left(\frac{3}{14.14}\right)$$
$$= \mathbf{11.98^\circ} \quad \text{...Ans.}$$

(ii) Absolute velocity at outlet, V_2 and its direction, β

Consider outlet Δ DFE

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = 23.56 - \frac{3}{\tan 35} = 19.28\text{ m/s}$$

$$V_2 = \sqrt{(V_{w2})^2 + (V_{f2})^2} = \sqrt{(19.28)^2 + (3)^2}$$
$$= \mathbf{19.51\ m/s} \quad \text{...Ans.}$$

$$\beta = \text{Sin}^{-1}\left(\frac{V_{f1}}{V_2}\right) = \text{Sin}^{-1}\left(\frac{3}{19.51}\right) = \mathbf{8.85^\circ} \quad \text{...Ans.}$$

(iii) Manometric head, H_m,

On neglecting loss of head in impeller and casing,

$$H_m = \frac{V_{w2} \cdot u_2}{g}$$
$$= \frac{19.28 \times 23.56}{9.81} = \mathbf{46.3\ m} \quad \text{...Ans.}$$

Ex. 5.13.7 : A centrifugal pump impeller has an external diameter of 500 mm and a discharge area of 0.15 m². The vanes are set back at an angle of 30° to the tangent at exit. The diameters of suction and delivery pipe are 300 mm and 250 mm respectively. Pressure gauge at points on suction and delivery pipe fitted close to pump and at a height of 1.75 meters above the level in supply sump showed gauge pressure heads of 3.5 meter below and 20 meters above the atmospheric pressure head respectively when the pump is delivering 225 liters of water per second at 820 rpm. If it requires 73.5 kW to drive the pump. Find :

(i) The loss of head in suction pipe

(ii) Manometric efficiency

(iii) Overall efficiency

SPPU - May 14,10 Marks

Soln. : Refer Fig. P. 5.13.7

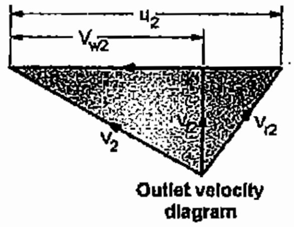


Fig. P. 5.13.7

$D_2 = 500 \text{ mm} = 0.5 \text{ m}, \quad A_{f_2} = 0.15 \text{ m}^2; \phi = 30°;$

$d_s = 300 \text{ mm} = 0.3 \text{ m}; \quad d_d = 250 \text{ mm} = 0.25 \text{ m}$

Gauge pressure head,

$$\frac{p_s}{\rho \cdot g} = -3.6 \text{ m} = 10.33 - 3.6 = 6.73 \text{ m (absolute)}$$

and

$$\frac{p_d}{\rho \cdot g} = 20 \text{ m (gauge)} = 20 + 10.33 = 30.33 \text{ (abs)}$$

Height between pressure gauges,

$$h' = 1.75 \text{ m} = Z_d - Z_s;$$

$$Q = 225 \text{ litre/s} = 0.225 \text{ m}^3/\text{s}$$

$$N = 820 \text{ rpm}; \quad P_s = 93.58 \text{ kW}$$

(i) Loss of head in suction pipe

Velocity in suction pipe,

$$V_s = \frac{\text{Discharge, } Q}{\text{Area of pipe } A_s} = \frac{Q}{(\pi/4)\, d_s^2}$$

$$= \frac{4}{\pi} \times \frac{0.225}{(0.3)^2} = 3.183 \text{ m/s}$$

Velocity in delivery pipe,

$$V_d = \frac{4}{\pi} \times \frac{Q}{d_d^2} = \frac{4}{\pi} \times \frac{0.225}{(0.25)^2} = 4.584 \text{ m/s}$$

$$V_{f_2} = \frac{Q}{A_{f_2}} = \frac{0.225}{0.15} = 1.5 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 820}{60} = 21.468 \text{ m/s}$$

$$V_{w_2} = u_2 - \frac{V_{f_2}}{\tan \phi} = 21.468 - \frac{1.5}{\tan 30} = 18.870 \text{ m/s}$$

Assuming suction gauge at centre of pump, $Z_s = 0$

Loss of head in suction pipe,

$$= p_{atm} - \left(\frac{p_s}{\rho \cdot g} + \frac{V_s^2}{2g} + Z_s\right)$$

$$= 10.33 - \left[6.73 + \frac{(3.183)^2}{2 \times 9.81} + 0\right]$$

$$= \mathbf{3.0836 \text{ m}}$$ ...**Ans.**

(ii) Manometric efficiency, η_{mano} :

Manometric head,

$$H_m = \left[\frac{p_d}{\rho \cdot g} + \frac{V_d^2}{2g} + Z_d\right] - \left[\frac{p_s}{\rho \cdot g} + \frac{V_s^2}{2g} + Z_s\right]$$

$$= \left[30.33 + \frac{4.584^2}{2 \times 9.81} + 1.75\right] - \left[6.73 + \frac{3.183^2}{2 \times 9.81} + 0\right]$$

$$= 25.905 \text{ m}$$

$$\eta_{mano} = \frac{gH_m}{V_{w_2} \cdot u_2} = \frac{9.81 \times 25.905}{18.87 \times 21.468} = \mathbf{0.6273 \text{ or } 62.73\%}$$

(iii) Overall efficiency, η_o

$$\eta_o = \frac{\rho \cdot g\, Q\, H_m \times 10^{-3}}{P_s}$$

$$= \frac{1000 \times 9.81 \times 0.225 \times 25.905 \times 10^{-3}}{93.58}$$

$$= \mathbf{0.6110 \text{ or } 61.10\%}$$ ...**Ans.**

Ex. 5.13.8 : A centrifugal pump having outer diameter equal to two times inner diameter and running at 1200 r.p.m. works against a total head of 75 m. The velocity of flow through the impeller is constant and equal to 3 m/s. The vanes are set back at width at an angle of 30° at outlet. If the outer diameter of the impeller is 600 mm and width at outlet is 50 mm. Determine :

(i) Vane angle at inlet

(ii) Work done per second by impeller

(iii) Manometric efficiency. **SPPU - May 13,10 Marks**

Soln. : Refer Fig. P.5.13.8

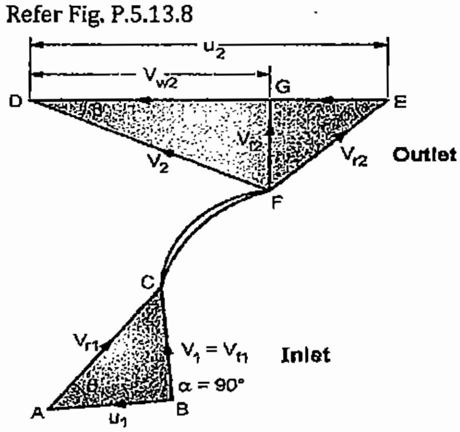


Fig. P. 5.13.8

Outer diameter,

$$D_2 = 2 \times \text{Inner diameter, } D_1$$

$$N = 1200 \text{ rpm}, \quad H_m = 75$$

$$V_{f1} = V_{f2} = 3 \text{ m/s}, \quad \phi = 30°$$

$$D_2 = 600\text{ mm} = 0.6\text{ m},$$
$$B_2 = 50\text{ mm} = 0.05\text{ m}$$
$$\therefore D_1 = (D_2/2) = (0.6/2) = 0.3\text{ m}$$

Vane velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 1200}{60} = 18.85\text{ m/s}$$
$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 1200}{60} = 37.7\text{ m/s}$$

Discharge, $Q = \pi D_2 B_2 V_{f2}$

$$= \pi \times 0.6 \times 0.05 \times 3 = 0.28274\text{ m}^3/\text{s}$$

(i) Vane angle at inlet, θ

$$\theta = \tan^{-1}\left[\frac{V_{f1}}{u_1}\right] = \tan^{-1}\left[\frac{3}{18.85}\right]$$
$$= 9.043^\circ \qquad \text{...Ans.}$$

(ii) Workdone by impeller/s, $\dot{W}$

From outlet velocity diagram,

$$V_{w2} = u_2 - GE = u_2 - \frac{V_{f2}}{\tan \phi}$$
$$= 37.7 - \frac{3}{\tan 30} = 32.5\text{ m/s}$$
$$\dot{W} = \frac{\dot{m}\, V_{w2} \cdot u_2}{g} = \frac{\rho \cdot gQ}{g} \times V_{w2} \cdot u_2$$
$$= \rho \cdot Q\, V_{w2} \cdot u_2\ (\text{kW})$$
$$= 1000 \times 0.28274 \times 32.5 \times 37.7$$
$$= \mathbf{346.43 \times 10^3\ Nm/s\ or\ W.} \qquad \text{...Ans.}$$

(iii) Manometric efficiency, η_{mano}

$$\eta_{mano} = \frac{g H_m}{V_{w2} \cdot u_2} = \frac{9.81 \times 75}{32.5 \times 37.7}$$
$$= \mathbf{0.6\ or\ 60\%} \qquad \text{...Ans.}$$

Ex. 5.13.9 : The outer diameter of an impeller of a centrifugal pump is 400 mm and outlet width is 50 mm. The pump is running at 800 r.p.m. and is working against a total head of 15 m. The vane angle at outlet is 40° and manometric efficiency is 75%.

Determine :

(i) Velocity of the flow at outlet

(ii) Velocity of water leaving the vane

(iii) Angle made by the absolute velocity at outlet with the direction of motion

(iv) Discharge. **SPPU - May 13, May 18, 8 Marks**

Soln. : Refer Fig. P. 5.13.9

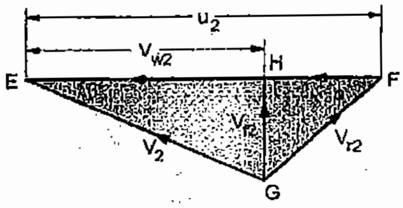


Fig. P. 5.13.9

Diameter, $D_2 = 400\text{ mm} = 0.4\text{m},$

Width, $B_2 = 50\text{ mm} = 0.05\text{m}$

$$N = 800\text{ rpm}; \quad H_m = 15\text{m},$$
$$\phi = 40^\circ,$$
$$\eta_{mano} = 0.75$$
$$u_2 = \frac{\pi D_2 N}{60}$$
$$= \frac{\pi \times 0.4 \times 800}{60} = 16.755\text{ m/s}$$
$$\eta_{mano} = \frac{g H_m}{V_{w2} \cdot u_2}$$
$$0.75 = \frac{9.81 \times 15}{16.755 \times V_{w2}}$$
$$V_{w2} = 11.71\text{ m/s}$$

(i) Velocity of flow at outlet, V_{f2}

From velocity diagram,

$$HF = EF - EH = u_2 - V_{w2}$$
$$= 16.755 - 11.71 = 5.045\text{ m/s}$$

But, $$HF = \frac{V_{f2}}{\tan \phi}$$
$$V_{f2} = 5.045 \tan 40$$
$$= \mathbf{4.233\ m/s} \qquad \text{...Ans.}$$

(ii) Velocity of water leaving the vane, V_2

$$V_2 = \sqrt{V_{w2}^2 + V_f^2}$$
$$= \sqrt{(11.71)^2 + (4.233)^2}$$
$$= \mathbf{12.452\ m/s} \qquad \text{...Ans.}$$

(iii) Angle made by absolute velocity at outlet with the direction of motion, β

$$\beta = \tan^{-1}\left(\frac{V_{f2}}{V_{w2}}\right)$$
$$= \tan^{-1}\left(\frac{4.233}{11.71}\right)$$
$$= \mathbf{19.874^\circ} \qquad \text{...Ans.}$$

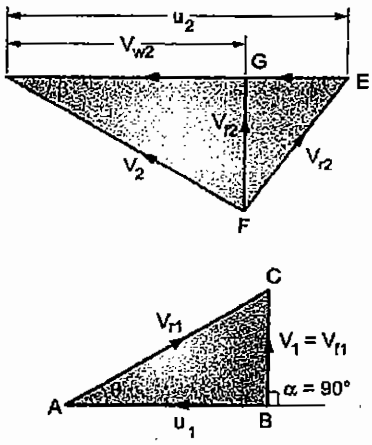
(iv) Discharge, Q

$$Q = \pi D_2 B_2 V_{f2}$$
$$= \pi \times 0.4 \times 0.05 \times 4.233$$
$$= \mathbf{0.266\ m^3/s} \quad \text{...Ans.}$$

Ex. 5.13.10 : A centrifugal pump discharges 3000 lpm of water and the outer diameter of the impeller is 35 cm. The vanes are curved backwards at an angle of 30 degrees to the wheel tangent at the impeller tip. The thickness of the vanes occupies 10% of the peripheral area. The passage is 50 mm wide and the impeller rotates at a speed of 1500 rpm. If the manometric efficiency is 82%, find the pressure head developed across the impeller in metres of water column. Assume that there is no loss of energy in the impeller and that the flow velocity is constant throughout.

SPPU - Dec. 11, 10 Marks

Soln. : Refer Fig. P. 5.13.10.



Inlet

Fig. P. 5.13.10.

Discharge, $Q = 3000\ lpm = 3\ m^3/min$

$$= \frac{3}{60}\ m^3/s = 0.05\ m^3/s$$

$D_2 = 35\ cm = 0.35\ m,$

$\phi = 30°$

Area occupied by vane, $a_f = 10\ \%$ of peripheral area, A_f

$B_2 = 50\ mm = 0.05\ m$

$N = 1500\ rpm$

$\eta_{mano} = 0.82,\ V_{f_1} = V_{f_2}$

(i) Pressure head developed across impeller, $\left(\frac{p_2 - p_1}{w}\right)$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.35 \times 1500}{60}$$
$$= 27.49\ m/s$$
$$Q = A_{f2} \cdot V_{f2} = \pi D_2 B_2 (1 - a_f) V_{f2}$$
$$0.05 = \pi \times 0.35 \times 0.05 (1 - 0.1) V_{f2}$$
$$V_{f2} = 1.01\ m/s$$
$$V_1 = V_{f1} = V_{f2} = 1.01\ m/s$$

From outlet velocity Δ DFE

$$V_{\omega 2} = u_2 - GE = u_2 - \frac{V_{f2}}{\tan\phi}$$
$$= 27.49 - \frac{1.01}{\tan 30} = 25.74\ m/s$$
$$\therefore V_2 = \sqrt{V_{\omega 2}^2 + V_{f2}^2}$$
$$= \sqrt{(25.74)^2 + (1.01)^2} = 25.776\ m/s$$
$$\eta_{mano} = \frac{g H_m}{V\omega_2 \cdot u_2}$$
$$0.82 = \frac{9.81 \times H_m}{25.74 \times 27.49}\ ;\ H_m = 60.07\ m$$

Applying Bernoulli's theorem at inlet and exit of impeller on neglecting losses we get,

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + \frac{V_{w2} \cdot u_2}{g} = \frac{p_2}{w} + \frac{V_2^2}{2g}$$
$$\frac{p_1}{w} + \frac{(1.01)^2}{2 \times 9.81} + \frac{25.74 \times 27.49}{9.81} = \frac{p_2}{w} + \frac{(25.74)^2}{2 \times 9.81}$$
$$\frac{p_2 - p_1}{w} = \mathbf{38.41 m} \quad \text{...Ans.}$$

Ex. 5.13.11 : Impeller of a centrifugal pump is 185 mm in diameter and width at the outlet is 60 mm. The blades are curved backwards at an angle of 20 degrees. The H-Q characteristic of the pump is given by

$H = 45 + 220\ Q - 1660\ Q^2$

The pump delivers water through a 150 mm diameter and 80 m long pipe to a static lift of 32 m. Calculate the head developed, discharge and the manometric efficiency if the pump speed is 3200 rpm. Friction factor for the outlet pipe material as 0.028 and the frictional head loss in the suction pipe is negligible.

SPPU - Dec. 11, 10 Marks

Soln. :

Impeller diameter, $D_2 = 185\ mm = 0.185\ m$

Width, $B_2 = 80\ mm = 0.08\ m;\ \phi = 20°$

Characteristic curve of pump is

$$H = 44 + 260\,Q - 3850\,Q^2\,(m) \quad ...(i)$$

$$d_d = 150\text{ mm} = 0.15\text{m}, \quad l_d = 80\text{ m},$$

$$h_s = 32\text{m}, \quad N = 3200\text{ rpm}, \quad f' = 0.028$$

1. Head, H and discharge, Q delivered by pump

$$h_{fd} = \frac{f' \cdot l_d \cdot V_d^2}{d_d \cdot 2g}$$

and $$V_d = \frac{Q}{A_d} = \frac{4Q}{\pi d_d^2}$$

$$h_{fd} = f' \times L_d \times \left(\frac{4Q}{\pi d_d}\right)^2 \times \frac{1}{d_d \cdot 2g}$$

$$= \frac{f' \times l_d \times 16\,Q^2}{2\pi \cdot d_d^5 \cdot 2g} = \frac{0.028 \times 80 \times 16 \times Q^2}{\pi^2 \times (0.15)^5 \times 2 \times 9.81}$$

$$= 2437.3\,Q^2$$

$$V_d = \frac{4Q}{\pi \cdot d_d^2} = \frac{4Q}{\pi\,(0.15)^2} = 56.59\,Q$$

$$\text{Total head,}H = h_s + h_{fd} + \frac{V_d^2}{2g}$$

$$= 32 + 2437.3\,Q^2 + \frac{(56.59\,Q)^2}{2 \times 9.81}$$

$$= 32 + 2600.5\,Q^2 \quad ...(ii)$$

On equating Equations (i) and (ii)

$$44 + 260\,Q - 3850\,Q^2 = 32 + 2600.5\,Q^2$$

$$6450.5\,Q^2 - 260\,Q - 12 = 0$$

$$Q = \frac{260 \pm \sqrt{(260)^2 - 4\,(-12)\,(6450.5)}}{2 \times 6450.5}$$

$$= \frac{260 \pm 614.2}{12901}$$

$$= \mathbf{0.06776\ m^3/s}$$

(on neglecting negative value) ...Ans.

$$H = 32 + 2600.5\,Q^2$$

$$= 32 + 2600.5 \times (0.06776)^2$$

$$= \mathbf{43.94m} \quad ...\text{Ans.}$$

2. Manometric efficiency, η_{mano}

Fig. P. 5.13.11 Shows the outlet diagram

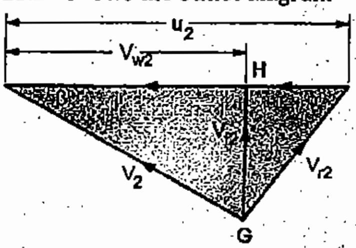


Fig. P.5.13.11

$$Q = \pi\,D_2\,B_2\,V_{f2}$$

$$0.06776 = \pi \times 0.185 \times 0.08\,V_{f2}$$

$$V_{f2} = 1.457\text{ m/s}$$

$$u_2 = \frac{\pi\,D_2\,N}{60} = \frac{\pi \times 0.185 \times 3200}{60} = 31\text{ m/s}$$

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = 31 - \frac{1.457}{\tan 20} = 27\text{ m/s}$$

$$\eta_{mano} = \frac{gH}{V_{w2} \cdot u_2} = \frac{9.81 \times 43.94}{27 \times 31}$$

$$= \mathbf{0.515\ or\ 51.5\%} \quad ...\text{Ans.}$$

Ex. 5.13.12: The diameter of an impeller of a centrifugal pump at inlet and outlet are 30 cm and 60 cm respectively. The velocity of flow at outlet is 2 m/s and the vanes are set back at an angle of 45° at the outlet. Determine the minimum starting speed of the pump if its manometric efficiency is 70%. **SPPU - Dec. 13, 12 Marks**

Soln. :

Given : $D_1 = 30\text{ cm} = 0.3\text{ m}$, $D_2 = 60\text{ cm} = 0.6\text{ m}$,

$V_{f2} = 2\text{m/s}$, $\phi = 45°$;

$\eta_{mano} = 70\% = 0.7$

From exit velocity diagram shown in Fig. P.5.13.12.

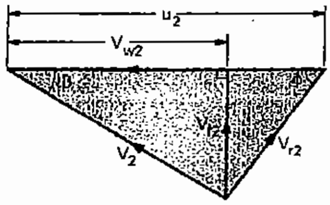


Fig. P.5.13.12

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$\text{i.e.}\quad \tan 45° = \frac{2}{u_2 - V_{w2}}$$

$$\therefore\ V_{w2} = u_2 - 2 \quad ...(i)$$

$$\eta_{mano} = \frac{g \cdot H_m}{V_{w2} \times u_2}$$

$$0.7 = \frac{9.81 \times H_m}{(u_2 - 2)\,u_2}$$

$$\therefore\ H_m = 0.0714\,(u_2 - 2)\,u_2$$

Let N be the minimum speed of the pump required for starting.

Vane velocity at exit,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times N}{60} = 0.03142\,N$$

Vane velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times N}{60} = 0.01571\ N$$

Condition for minimum starting speed is,

$$\frac{u_2^2 - u_1^2}{2g} = H_m = 0.0714\,(u_2 - 2)\,u_2$$

$$\frac{(0.03142\ N)^2 - (0.01571\ N)^2}{2 \times 9.81} = 0.0714\,(0.03142\ N - 2)\ 0.03142\ N$$

$$0.01682\ N = (0.03142\ N - 2)$$

$$N = \mathbf{137.0\ rpm} \qquad \text{...Ans.}$$

Ex. 5.13.13 : Following data relates to centrifugal pump. Eye and rim diameter = 10 cm and 20 cm respectively, outer width = 1.25 cm, vane angle at outer rim = 25°, speed = 3000 rpm, constant flow velocity = 3 m/s, manometric efficiency = 78% and overall efficiency = 72%. Determine :
(i) Inlet vane angle, (ii) Discharge,
(iii) Manometric head and (iv) Shaft power
(v) Mechanical efficiency. **SPPU - May 16, 12 Marks**

Soln. : Refer Fig. P. 5.13.13

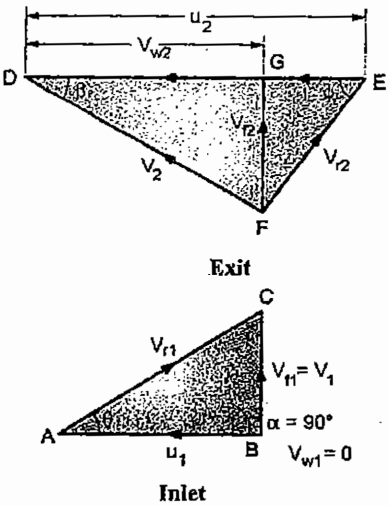


Fig. P. 5.13.13

$D_1 = 10\ cm = 0.1 m,\quad D_2 = 20\ cm = 0.2 m$

$B_2 = 1.25\ cm = 0.0125\ m,$

$\phi = 25°,\quad N = 3000\ rpm$

$V_{f1} = V_{f2} = 3\ m/s\quad \eta_{mano} = 0.78,$

$\eta_o = 0.72$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.2 \times 3000}{60} = 31.42\ m/s$$

$$Q = \pi\ D_2 B_2 V_{f_2} = \pi \times 0.2 \times 0.0125 \times 3$$

$$= 0.023562\ m^3/s$$

$$V_{w_2} = u_2 - \frac{V_{f_2}}{\tan\phi} = 31.42 - \frac{3}{\tan 25} = 24.99\ m/s$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.1 \times 3000}{60} = 15.71\ m/s$$

(i) Inlet vane angle, θ

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{u_1}\right)$$

$$= \tan^{-1}\left(\frac{3}{15.71}\right)$$

$$= \mathbf{10.81°} \qquad \text{...Ans.}$$

(ii) Discharge, Q

From above $\quad Q = 0.023562\ m^3/s$

$$= \mathbf{23.562\ l/s} \qquad \text{...Ans.}$$

(iii) Manometric head, H_m

$$\eta_{mano} = \frac{g\ H_m}{V_{w_2} \cdot u_2}$$

$$0.78 = \frac{9.81 \times H_m}{24.99 \times 31.42}$$

$$H_m = \mathbf{62.43\ m} \qquad \text{...Ans.}$$

(iv) Shaft power, P_s

$$\eta_o = \frac{\rho.g.Q.H_m \times 10^{-3}}{P_s}$$

$$0.72 = \frac{1000 \times 9.81 \times 0.023562 \times 62.43 \times 10^{-3}}{P_s}$$

$$P_s = \mathbf{20.04\ kW} \qquad \text{...Ans.}$$

(v) Mechanical efficiency, η_m

$$\eta_m = \frac{\rho\ Q\ V_{w2} \cdot u_2}{P_s}$$

$$= \frac{1000 \times 0.023562 \times 24.99 \times 31.42}{20.04 \times 1000}$$

$$= \mathbf{0.9232\ or\ 92.32\ \%} \qquad \text{...Ans.}$$

Ex. 5.13.14 : A three stage centrifugal pump has impellers 50 cm diameter and 3 cm width at outlet. The thickness of the blades has reduced the circumferential area by 10%. The manometric efficiency is 90%, overall efficiency is 80%. While velocity at outlet is 20 m/s, velocity of flow at outlet is 2.25 m/s and speed is 1000 rpm. Calculate :
(i) Head generated (ii) Discharge
(iii) Exit vane angle (iv) Shaft power. **SPPU - Dec. 16, 8 Marks**

Soln. : Refer Fig. P. 5.13.14

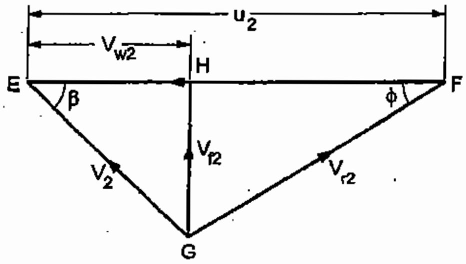


Fig. P. 5.13.14

Number of stages, $n = 3$

Impeller outer diameter, $D_2 = 50\text{ cm} = 0.5\text{ m}$

Width at outlet $B_2 = 3\text{cm} = 0.03\text{ m}$

Area reduced due to thickness, $k = 10\% = 0.1$

Manometric efficiency $\eta_{mano} = 0.9$

Overall efficiency $\eta_o = 0.8$

Whirl velocity at exit, $V_{w2} = 20\text{ m/s}$

Radial velocity at exit, $V_{f2} = 2.25\text{ m/s}$

Speed, $N = 1000\text{ rpm}$

$$\text{Area of flow} = \pi \times D_2 \times B_2 \times (1-k)$$

$$= \pi \times 0.5 \times 0.03 \times (1-0.1)$$

$$= 0.04241\text{ m}^2$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1000}{60}$$

$$= 26.18\text{ m/s}$$

(a) Discharge (Q)

$$Q = \text{area of flow} \times V_{f2}$$

$$= 0.04241 \times 2.25$$

$$= \mathbf{0.09542\ m^3/s} \quad \text{...Ans.}$$

(b) Exit vane angle, ϕ :

From exit velocity triangle,

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$= \frac{2.25}{26.18 - 20.0} = 0.36408$$

$$\phi = \mathbf{20.0^\circ} \quad \text{...Ans.}$$

(c) Total Head, H_{mt},

$$\eta_{mano} = \frac{g H_m}{V_{w2} u_2}$$

$$\text{Head/stage,} \quad H_m = \frac{\eta_{mano} V_{w2} u_2}{g}$$

$$= \frac{0.9 \times 20 \times 26.18}{9.81} = 48.04\text{ m}$$

$$H_{mt} = n \times H_m = 3 \times 48.04$$

$$= \mathbf{144.11\ m} \quad \text{...Ans.}$$

$$H_{mt} = \mathbf{144.11\ m} \quad \text{...Ans.}$$

(d)Shaft power (P_s)

$$\eta_o = \frac{\rho \cdot g \cdot Q\, H_{mt} \times 10^{-3}}{P_s}\text{kW}$$

$$0.8 = \frac{1000 \times 9.81 \times 0.09542 \times 144.12 \times 10^{-3}}{P_s}$$

$$P_s = \mathbf{168.633\ kW} \quad \text{...Ans.}$$

Ex. 5.13.15 : A centrifugal pump delivers 1800 lit/min against a total head of 20 m. Its speed is 1450 rpm. Inner and outer diameters of impeller are 120 mm and 240 mm respectively and the diameter of suction and delivery pipes are both 100 mm. Determine the blade angles of the impeller vane at inlet and outlet respectively if the water enters radially. Assume manometric efficiency is 0.90. **SPPU - Dec. 16, 8 Marks**

Soln. :

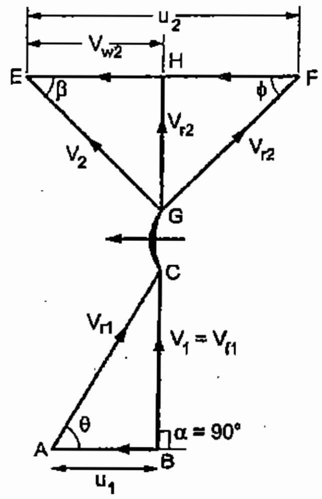


Fig. P.5.13.15

Given data :

$$\text{Discharge, } Q = 1800\text{ lit/min} = \frac{1800}{1000} \times \frac{1}{60}$$

$$= 0.03\text{ m}^3/\text{s}$$

Head, $H_m = 20\text{ m}$

Speed, $N = 1450\text{ rpm}$

$\eta_{mano} = 0.9$

Inner diameter of impeller	d_1	$= 0.12$ m
Outer diameter of impeller	d_2	$= 0.24$ m
Diameter of suction pipe	d_s	$= 0.1$ m
Diameter of delivery pipe	d_d	$= 0.1$ m
Radial water entry	α	$= 90°$,
	i.e. V_{w1}	$= 0,\ V_{f1} = V_1$

(a) Inlet blade angle θ and exist blade angle, ϕ

$$u_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.12 \times 1450}{60} = 9.11 \text{ m/s}$$

$$u_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.24 \times 1450}{60} = 18.22 \text{ m/s}$$

$$\eta_{mano} = \frac{g H_m}{V_{w2} \cdot u_2}$$

$$0.9 = \frac{9.81 \times 20}{V_{w2} \times 18.22}$$

$$V_{w2} = 11.97 \text{ m/s}$$

$$V_{f1} = V_s = \frac{Q}{\frac{\pi}{4} d_s^2} = \frac{0.03 \times 4}{\pi \times 0.1^2} = 3.82 \text{ m/s}$$

$$V_{f2} = V_d = \frac{Q}{\frac{\pi}{4} d_d^2} = \frac{0.03 \times 4}{\pi \times 0.1^2} = 3.82 \text{ m/s}$$

$$\tan\theta = \frac{V_{f1}}{u_1} = \frac{3.82}{9.11}$$

$$\theta = 22.75° \qquad \text{...Ans.}$$

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}} = \frac{3.82}{18.22 - 11.97} = 0.6112$$

$$\phi = 31.43° \qquad \text{...Ans.}$$

Ex. 5.13.16 : Power input to centrifugal pump is 50kW at the shaft while running the pump at 1440 rpm. The impeller tip diameter is 30 cm and the blade width at the tip is 1.5cm. The water flowe rate is 110 lit/s. The vacuum gauge reading at the suction flange is – 20cm of mercury and at the delivery flange the pressure gauge reading is 370 kPa. The blade outlet angle is 65°. Calculate :

(i) Theoretical head, (ii) Ideal head

(iii) Hydraulic efficiency (iv) Mechanical efficiency

(v) Overall efficiency (vi) Specific speed of the pump.

Assume radial entry and constant flow velocity.

SPPU - May 16, 8 Marks

Soln. : Refer Fig. P. 5.13.16

$P_s = 50$ kW, $\quad N = 1440$ rpm,

$D_2 = 30$ cm $= 0.3$ m $\quad B_2 = 1.5$ cm $= 0.015$ m,

$$Q = 110 \text{ lit/s} = 0.11 \text{ m}^3\text{/s}$$

$$\frac{p_s}{\rho . g} = \text{Vacuum} = -20 \text{ cm of Hg}$$

$$= \frac{-20}{76} \times 10.33 = -2.72 \text{ m}$$

$$p_d = 370 \text{ kPa (gauge)}$$

$$\phi = 65°$$

$$Q = \pi D_2 \cdot B_2 \cdot V_{f_2}$$

$$0.11 = \pi \times 0.3 \times 0.015 \times V_{f_2}$$

$$V_{f_2} = 7.78 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 1440}{60}$$

$$= 22.62 \text{ m/s}$$

From exit velocity ΔEGF we have,

$$HF = \frac{V_{f_2}}{\tan\phi} = \frac{7.78}{\tan 65} = 3.63 \text{ m/s}$$

$$\therefore V_{w_2} = u_2 - HF$$

$$= 22.62 - 3.63 = 18.99 \text{ m/s}$$

Absolute suction head ,

$$\frac{p_s}{\rho g} = 10.33 - 2.72 = 7.61 \text{ m}$$

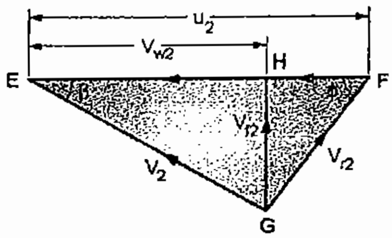


Fig. P.5.13.16

$$\frac{p_d}{\rho g} = 370 \text{ kPa (Gauge)} = \frac{370}{100} \text{ bar}$$

$$= 3.7 \times \frac{10.33}{1.013} = 37.33 \text{ m (gauge)}$$

$$\therefore \frac{p_d}{\rho g} = 37.73 + 10.33 = 48.06 \text{ m (absolute)}$$

(i) Theoretical head,

$$H_m = \frac{p_d}{\rho g} - \frac{p_s}{\rho g}$$

$$= 48.06 - 7.61$$

$$= 40.45 \text{ m} \qquad \text{...Ans.}$$

(ii) Ideal head,

$$H_m = \frac{V_{w_2} \cdot u_2}{g}$$

$$= \frac{18.99 \times 22.62}{9.81}$$

$$= 43.79 \text{ m} \qquad \text{...Ans.}$$

(iii) Hydraulic efficiency, η_h

$$\eta_h = \frac{\text{Theoretical head}}{\text{Ideal head}} = \frac{40.45}{43.79}$$

$$= 0.9238 \text{ or } 92.38\ \% \qquad \text{...Ans.}$$

(iv) Mechanical efficiency, η_m

Hydraulic power,

$$P_h = \frac{\rho.Q\,(V_{w_2} \cdot u_2)}{1000} \text{ kW}$$

$$= \frac{1000 \times 0.11 \times 18.99 \times 22.62}{1000} = 47.25 \text{ kW}$$

$$\eta_m = \frac{P_h}{P_s} = \frac{47.25}{50}$$

$$= 0.945 \text{ or } 94.5\ \% \qquad \text{...Ans.}$$

(v) Overall efficiency, η_o

$$\eta_0 = \frac{\rho.g.Q\ H_m \times 10^{-3}}{P_s}$$

$$= \frac{1000 \times 9.81 \times 0.11 \times 40.45 \times 10^{-3}}{50}$$

$$= 0.873 \text{ or } 87.3\% \qquad \text{...Ans.}$$

(vi) Specific speed of the pump, N_s

$$N_s = \frac{N\sqrt{P}}{H_m^{3/4}} = \frac{1440\sqrt{47.25}}{(40.45)^{3/4}}$$

$$= 617.13 \qquad \text{...Ans.}$$

Ex. 5.13.17 : A three stage centrifugal pump has impeller diameter 400 mm and 20 mm wide. The vane angle at outlet is 45° and the area occupied by the thickness of vane is 8% of total area. Inner diameter of impeller is half of outer diameter and inlet width is twice that of outlet. The pump discharge is 3.6 m³ per minute and runs at 920 rpm. Flow velocity is constant from inlet to outlet. Find :

(i) Power pump in kW

(ii) Total manometric head

(iii) Specific speed

(iv) Shaft power

(v) Vane angle at inlet.

Take mechanical efficiency = 88%.

Manometric efficiency = 77%.

SPPU - May 15, May 19, 12 Marks

Soln. : Refer Fig. P. 5.13.17.

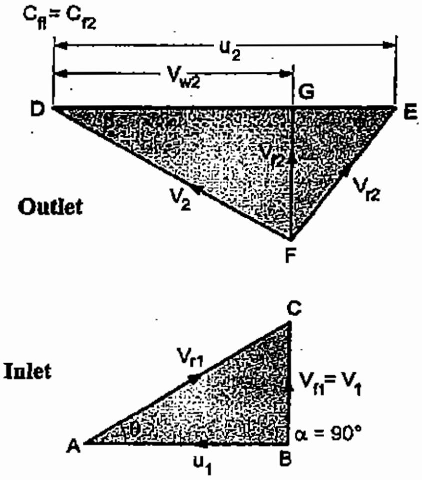


Fig. P. 5.13.17

Number and stages, $n = 3$, $D_2 = 400$ mm $= 0.4$ m,

$$B_2 = 20 \text{ mm} = 0.02 \text{ m}$$

$$\phi = 45°,$$

Vane area occupied by blades, $k_b = 8\% = 0.08$

$$D_1 = \frac{D_2}{2} = \frac{0.4}{2} = 0.2 \text{ m};$$

$$B_1 = 2B_2 = 2 \times 0.02 = 0.04 \text{ m}$$

$$Q = 3.6 \text{ m}^3/\text{min}, \qquad N = 920 \text{ rpm},$$

$$\eta_m = 0.88, \qquad \eta_{mano} = 0.77$$

Assuming pumps in series then discharge through each pump remains the same while the total head,

$$H = n \times H_m$$

(H_m = manometric head by each pump)

$$Q = \pi D_2 B_2 (1 - k_b) V_{f2}$$

$$= \pi D_1 B_1 (1 - k_b) V_{f1}$$

$$\frac{3.6}{60} = \pi \times 0.4 \times 0.02\,(1 - 0.08)\, V_{f2}$$

$$V_{f2} = 2.595 \text{ m/s}$$

Also, $V_{f1} = V_{f2} = 2.595$ m/s

($\because D_2 B_2 = D_1 B_1$ i.e. $0.4 \times 0.02 = 0.2 \times 0.04 = 0.2$)

(i) Power output of pump or power required to drive the pump, P

$$u_1 = \frac{(\pi D_1 N)}{60} = \frac{\pi \times 0.2 \times 920}{60} = 9.634 \text{ m/s}$$

$$u_2 = \frac{(\pi D_2 N)}{60} = \frac{\pi \times 0.4 \times 920}{60} = 19.268 \text{ m/s}$$

From exit Δ DFE,

$$V_{w_2} = u_2 - GE = u_2 - \frac{V_{f_2}}{\tan\phi}$$

$$= 19.268 - \frac{2.595}{\tan 45} = 16.673 \text{ m/s}$$

$$\eta_{mano} = \frac{g H_m}{V_{w_2} - u_2}$$

$$0.77 = \frac{9.81 \times H_m}{16.673 \times 19.268}$$

$$H_m = 25.216$$

Actual head,

$$H/\text{pump} = \frac{H_m}{\eta_{mano}} = \frac{25.216}{0.77} = 32.748 \text{ m}$$

$$P = \text{No. of stages} \times \rho \cdot g \cdot Q \cdot H \times 10^{-3} \text{ (kW)}$$

$$= 3 \times 1000 \times 9.81 \times \frac{3.6}{60} \times 32.748 \times 10^{-3}$$

$$= 57.826 \text{ kW}$$

Power/pump, $P_1 = \frac{P}{n} = \frac{57.826}{3} = \mathbf{19.275\ kW}$ **...Ans.**

(ii) Total manometric head, H_{mt} :

$H_{mt} = n \times H_m = 3 \times 25.216 = \mathbf{75.648\ m}$ **...Ans.**

(iii) Specific speed, N_s :

$$N_s = \frac{N \times \sqrt{P_1}}{H_m^{3/4}} = \frac{920 \times \sqrt{19.275}}{(32.748)^{3/4}} = \mathbf{295.05} \quad \text{...Ans.}$$

(iv) Shaft power, P_s :

$$P_s = \frac{P}{\eta_m} = \frac{57.826}{0.88} = \mathbf{65.71\ kW} \quad \text{...Ans.}$$

(v) Vane angle at inlet, θ :

$$\theta = \tan^{-1}\left(\frac{V_{f_1}}{u_1}\right) = \tan^{-1}\left(\frac{2.595}{9.634}\right)$$

$$= \mathbf{15.07°} \quad \text{...Ans.}$$

Ex. 5.13.18 : Show that the pressure rise across the impeller of a centrifugal pump is given by $\frac{1}{2g}[V_{f1}^2 + u_2^2 - V_{f2}^2 \text{cosec}^2 \phi]$

Where, V_{f1} and V_{f2} are velocities of flow at inlet and outlet

u_2 = tangential velocity of impeller at outlet

ϕ = Vane angle at outlet

SPPU - May 12, May 18, Dec.18, 6 Marks

Soln. :

Let the conditions at inlet and outlet of impeller be represented by suffix 1 and suffix 2 respectively.

Neglecting losses in impeller and applying Bernoulli's equation,

$(\text{Total energy})_{inlet} = (\text{Total energy})_{outlet}$ – Workdone by impeller on water

$$\text{i.e. } \frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 = \left(\frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2\right) - \frac{V_{w2} \cdot u_2}{g}$$

Assuming, $Z_1 = Z_2$ = i.e. inlet and outlet are at the same height.

$$\therefore \text{Pressure rise, } \left(\frac{p_2}{w} - \frac{p_1}{w}\right) = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + \frac{V_{w2} \cdot u_2}{g} \quad \text{...(i)}$$

From outlet velocity diagram shown in Fig. P. 5.13.18, we have

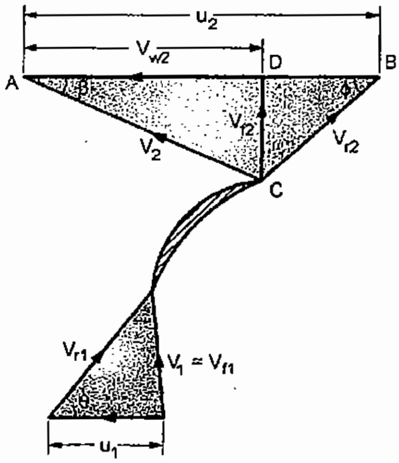


Fig. P. 5.13.18

Also, from ΔACD,

$$V_2^2 = V_{w2}^2 + V_{f2}^2 = (u_2 - V_{f2} \cot\phi)^2 + V_{f2}^2$$

$$= u_2^2 - 2u_2 \cdot V_{f2} \cot\phi + V_{f2}^2 \cot^2\phi + V_{f2}^2$$

$$= u_2^2 - 2u_2 \cdot V_{f2} \cot\phi + V_{f2}^2 (1 + \cot^2\phi)$$

$$= u_2^2 - 2u_2 \cdot V_{f2} \cot\phi + V_{f2}^2 \text{ cosec}^2\phi$$

$$(\because 1 + \cot^2\phi = \text{cosec}^2\phi) \quad \text{..(ii)}$$

On substituting the values of V_1, V_2 and V_{w2} in Equation (i), we get,

$$\left(\frac{p_2}{w} - \frac{p_1}{w}\right) = \frac{V_{f1}^2}{2g} - \frac{(u_2^2 - 2u_2 \cdot V_{f2} \cot\phi + V_{f2}^2 \text{cosec}^2\phi)}{2g} + \frac{(u_2 - V_{f2}\cot\phi)\,u_2}{g}$$

$$= \frac{1}{2g}[V_{f1}^2 - u_2^2 + 2u_2 \cdot V_{f2}\cot\phi - V_{f2}^2 \text{cosec}^2\phi + 2u_2^2 - 2u_2 \cdot V_{f2}\cot\phi]$$

$$= \frac{1}{2g}[V_{f1}^2 + u_2^2 - V_{f2}^2 \text{ cosec}^2\phi] \quad \text{.... proved}$$

Ex. 5.13.19 : A centrifugal pump has an impeller it internal diameter 125 mm and exit diameter 250 mm and rotates at 1800 rpm. The absolute velocity of water at inlet is radial and the vanes are bent back at an angle of 30° to the tangent at discharge. The breadth of the impeller at inlet and outlet are 12.5 mm and 6.25 mm respectively. Determine the rise in pressure head as water passes through the impeller neglecting losses. The discharge of the pump is 8.5 lit/sec.

SPPU : Dec. 18, 12 Marks

Soln. : Refer Fig. P.5.13.19

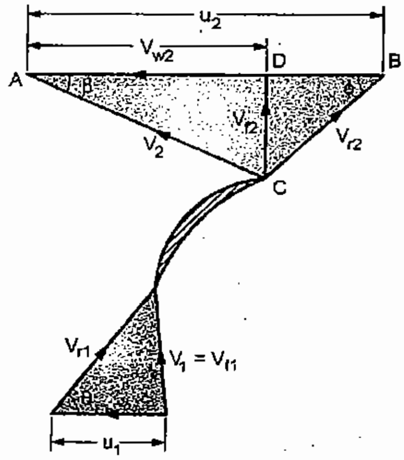


Fig. P.5.13.19

$$D_1 = 125 \text{ mm} = 0.125 \text{ m}$$

$$D_2 = 250 \text{ mm} = 0.25 \text{ m}$$

$$N = 1800 \text{ rpm}; \quad \alpha = 90^\circ \text{ i.e. } V_1 = V_{f1}$$

$$\phi = 30^\circ; \quad B_1 = 12.5 \text{ mm} = 0.0125 \text{ m}$$

$$B_2 = 6.25 \text{ mm} = 0.00625 \text{ m}$$

$$Q = 8.5 \text{ lit/s} = \frac{8.5}{1000} = 0.0085 \text{ m}^3/\text{s}$$

$$Q = \pi D_1 B_1 v_{f1};$$

$$0.0085 = \pi \times 0.125 \times 0.0125 \times V_{f1};$$

$$V_{f1} = 1.732 \text{ m/s}$$

$$Q = \pi D_2 B_2 V_{f2};$$

$$0.0085 = \pi \times 0.25 \times 0.00625 \times V_{f2};$$

$$V_{f2} = 1.732 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1800}{60} = 23.562 \text{ m/s}$$

$\therefore$ Pressure rise in impeller,

$$\frac{p_2}{w} - \frac{p_1}{w} = \frac{1}{2g}(V_{f1}^2 + u_2^2 - V_{f2}^2 \operatorname{cosec}^2\phi)$$

$$= \frac{1}{2 \times 9.81}[(1.732)^2 + (23.562)^2 - (1.732)^2 \times (\operatorname{cosec} 30)^2]$$

$$= \frac{1}{2 \times 9.81}[2.999 + 555.168 - 11.996]$$

$$= \mathbf{27.8375 \text{ m}} \qquad \text{...Ans.}$$

Ex. 5.13.20 : The effective inlet and outlet radial area of flow of a centrifugal pump are respectively 645 cm² and 580 cm², the water entering with the radial velocity of 5.50 m/s, the impeller vanes are set back at an angle of 45° to the tangent at outlet. The outer peripheral velocity is 27.50 m/s and manometric efficiency is 80%. Assuming the losses of head due to friction are i) between suction flange and impeller inlet = 3.05 m, ii) through impeller = 4.80 m, iii) between guide vanes and delivery flange = 1.52 m, and that at the outlet velocity from the guides is two fifths of the inlet velocity to them. Find 1) the loss of head due to friction in guides vanes 2) The guide vane efficiency.

SPPU : Dec. 18, 12 Marks

Soln. :

$$A_{f1} = 645 \text{ cm}^2 = 645 \times 10^{-4} \text{ m}^2,$$

$$A_{f2} = 580 \text{ cm}^2 = 580 \times 10^{-4} \text{ m}^2$$

$$V_{f1} = 5.5 \text{ m/s}, \phi = 45^\circ, u_2 = 27.5 \text{ m/s}, \eta_{mano} = 0.8$$

Losses in pump are :

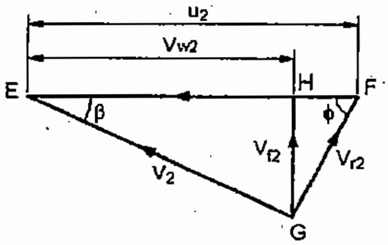
(i) Between suction flange and impeller inlet = 3.05 m

(ii) Through impeller = 4.8 m

(iii) Between guide vanes and delivery flange = 1.52 m

(iv) Outlet velocity from guides = $\frac{2}{5}$ × inlet velocity

Refer Fig. P.5.13.20



Outlet

Fig. P.5.13.20

$$Q = A_{f1} V_{f1} = A_{f2} V_{f2}$$

$$\therefore \quad 645 \times 10^{-4} \times 5.5 = 5.80 \times 10^{-4} \times V_{f2}$$

$$V_{f2} = 6.12 \text{ m/s}$$

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = 27.5 - \frac{6.12}{\tan 45}$$

$$= 21.38 \text{ m/s}$$

$$\eta_{mano} = \frac{g \cdot H_m}{V_{w_2} \cdot u_2};$$

$$0.8 = \frac{9.81 \times H_m}{21.38 \times 27.5};$$

$$H_m = 47.95 \text{ m}$$

$$V_2 = \sqrt{V_{w2}^2 + V_{f2}^2} = \sqrt{(21.38)^2 + (6.12)^2}$$

$$= 22.24 \text{ m/s}$$

Work supplied per kg of water $= V_{w2} \cdot u_2$

$$= 21.38 \times 27.5 = 587.95 \text{ Nm/kg}$$

$$\text{Work supplied / kg} = g\left[H_{mano} + \begin{matrix}\text{Losses in impeller} \\ \text{and diffuser} \\ \text{(guide vanes)}\end{matrix}\right]$$

$$\frac{587.75}{g} = 47.95 + (3.05 + 4.8 + 1.52 + \text{diffuser losses})$$

$$59.91 = 47.95 + 3.05 + 4.8 + 1.52 + \text{diffuser losses}$$

Diffuser losses = **2.593 m**

Velocity head, recovered in guides i.e. guide vane, efficiency

$$= \frac{\frac{V_2^2}{2g} - \text{Losses in diffuser}}{\frac{V_2^2}{2g}} = \frac{(22.24)^2 - 2g \times 2.593}{(22.24)^2}$$

$$= \mathbf{0.8971 \text{ or } 89.71\ \%} \qquad \text{... Ans.}$$

Ex. 5.13.21 : A centrifugal pump impeller has an external diameter of 450 mm and discharge area of 0.11 m². The vanes are bent backwards at an angle of 35° at outlet. The diameter of the suction and delivery pipes is 300 mm and 230 mm respectively. Pressure gauge at points on suction and delivery pipes close to the pump and each gauge 1.50 m above the level of supply sump showed gauge pressure head of 3.70 m below and 19 m above atmospheric head respectively. When the pump was delivering 200 lit/sec of water at 800 rpm. It requires 70 kW to drive the pump. Find the loss of head in the suction pipe, manometric efficiency and overall efficiency of the pump.

SPPU : May 18, 12 Marks

Soln. :

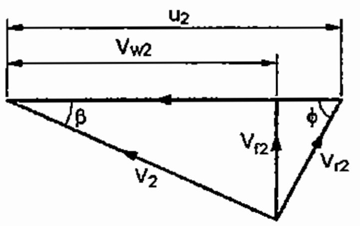


Fig. P.5.13.21

Refer Fig. P.5.13.21

$D_2 = 450 \text{ mm} = 0.45 \text{ m}$, $A_{f2} = 0.11 \text{ m}^2$

$\phi = 35°$, $d_s = 300 \text{ mm} = 0.3 \text{ m}$,

$d_d = 230 \text{ mm} = 0.23 \text{ m}$,

Gauge pressure head,

$$\frac{p_s}{\rho . g} = -3.7 \text{m} = 10.33 - 3.7$$

$$= 6.63 \text{ m (absolute)}$$

$$\text{and, } \frac{p_d}{\rho g} = 19 \text{ m};$$

Height between pressure gauges, $h' = 1.5 \text{ m} = Z_d - Z_s$

$$Q = 200 \text{ lit/s} = \frac{200}{1000} \text{ m}^3/\text{s} = 0.2 \text{ m}^3/\text{s};$$

$$N = 800 \text{ rpm};$$

$$P_s = 70 \text{ kW}$$

(i) Loss of head in suction pipe

Velocity in suction pipe,

$$V_s = \frac{Q}{A_s} = \frac{Q}{\frac{\pi}{4} d_s^2} = \frac{0.2}{\frac{\pi}{4}(0.3)^2} = 2.829 \text{ m/s}$$

Velocity in delivery pipe,

$$V_d = \frac{Q}{A_d} = \frac{Q}{\frac{\pi}{4}(d_d)^2} = \frac{0.2}{\frac{\pi}{4}(0.23)^2} = 4.814 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.45 \times 800}{60} = 18.85 \text{ m/s}$$

$$V_{f_2} = \frac{Q}{A_{f_2}} = \frac{0.2}{0.11} = 1.818 \text{ m/s}$$

$$V_{w_2} = u_2 - \frac{V_{f_2}}{\tan\phi} = 18.85 - \frac{1.818}{\tan 35} = 16.253 \text{ m/s}$$

Assuming suction gauge at center of pump i.e. $Z_s = 0$

Loss of head in suction pipe,

$$= p_{atm} - \left(\frac{\rho_s}{\rho \cdot g} + \frac{V_s^2}{2g} + Z_s\right)$$

$$= 10.33 - \left(6.63 + \frac{(2.829)^2}{2 \times 9.81} + 0\right)$$

$$= \mathbf{3.292 \text{ m}} \qquad \text{... Ans.}$$

(ii) Manometric efficiency, η_{mano}

Manometric head,

$$H_m = \left(\frac{\rho_d}{\rho . g} + \frac{V_\phi^2}{2g} + Z_d\right) - \left(\frac{\rho_s}{\rho \cdot g} + \frac{V_s^2}{2g} + Z_s\right)$$

$$= \left(19 + \frac{4.814^2}{2 \times 9.81} + 1.5\right) - \left(6.63 + \frac{2.829^2}{2 \times 9.81} + 0\right)$$

$$= 14.643 \text{ m}$$

$$\eta_{mano} = \frac{g H_m}{V_{w_2} \cdot u_2} = \frac{9.81 \times 14.643}{16.253 \times 18.85}$$

$$= 0.4689 \text{ or } 46.89\% \quad \text{... Ans.}$$

(iii) Overall efficiency, η_0

$$\eta_0 = \frac{\rho . g\, Q\, H_m \times 10^{-3}}{P_s}$$

$$= \frac{1000 \times 9.81 \times 0.2 \times 14.643 \times 10^{-3}}{70}$$

$$= 0.4104 \text{ or } 41.04\% \quad \text{... Ans.}$$

Ex. 5.13.22 : Centrifugal pump delivers water at a rate of 0.6 m³/s against a head of 20 m. It runs at 1000 rpm. Water enters the impeller radially and the velocity of flow remains constant throughout at 3 m/s. The manometric efficiency of pump is 80% and the loss of head due to friction over the impeller is 0.025 V_2^2 m of water. Assume inner diameter as half of the outer diameter.

Determine :

(i) Vane angle at inlet and outlet

(ii) Diameter of impeller

(iii) Area of flow at outlet **SPPU : Dec. 19, 9 Marks**

Soln. :

$$Q = 0.6\ m^3/s, \quad H_m = 20\ m,$$

$$N = 1000 \text{ rpm} \quad \alpha = 90° \text{ (radial entry)},$$

$$V_{f_1} = V_{f_2} = 3\ m/s, \quad \eta_{mano} = 0.8$$

Loss of head over impeller, $h_f = 0.025\ V_2^2$ (m of water)

$$D_1 = \frac{1}{2} D_2;$$

$$V_{f_1} = V_1 = 3\ m/s$$

Refer Fig. P.5.13.22

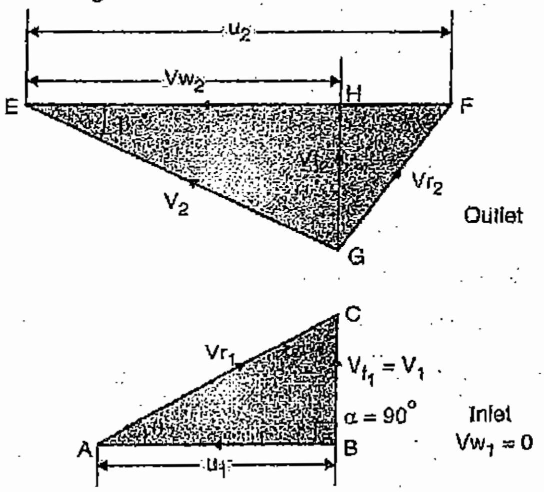


Fig. P.5.13.22

(i) Vane angle at inlet, θ and at outlet, ϕ

$$\eta_{mano} = \frac{g H_m}{V_{w2} . u_2}$$

$$0.8 = \frac{9.81 \times 20}{V_{w2} . u_2}$$

$$\therefore \quad V_{w2} . u_2 = 245.25 \quad \text{...(i)}$$

$$\text{But,} \quad \frac{V_{w2} . u_2}{g} = H_m + \text{Losses } (h_f)$$

$$\frac{245.75}{9.81} = 20 + 0.025\, V_2^2$$

$$V_2 = 14.142\ m/s$$

From Δ EGH :

$$V_{w2} = \sqrt{V_2^2 - V_{f2}^2} = \sqrt{(14.142)^2 - (3)^2}$$

$$= 13.82\ m/s$$

From eq (i) :

$$V_{w2} . u_2 = 245.25 ;$$

$$u_2 = \frac{245.25}{V_{w2}} = \frac{245.25}{13.82} = 17.746\ m/s$$

$$u_2 = \frac{\pi D_2 N}{60}$$

$$17.746 = \frac{\pi D_2 \times 1000}{60}$$

$$D_2 = 0.3389\ m ;$$

$$\text{and} \quad D_1 = \frac{D_2}{2} = \frac{0.3389}{2} = 0.1695\ m$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.1695 \times 1000}{60}$$

$$= 8.873\ m/s$$

From inlet Δ ABC,

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{u_1}\right) = \tan^{-1}\left(\frac{3}{8.873}\right)$$

$$= 18.68° \quad \text{... Ans.}$$

From Δ EGF :

$$HF = u_2 - V_{w2} = 17.746 - 13.82$$

$$= 3.926\ m/s$$

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{HF}\right) = \tan^{-1}\left(\frac{3}{3.926}\right)$$

$$= 37.385° \quad \text{... Ans.}$$

(ii) Diameter of impeller, D_2

From above : $D_2 = 0.3389$ m ... **Ans.**

(iii) Area of flow at outlet, A_{f_2}

$$Q = A_{f_2} \times V_2 ; \ 0.6 = A_{f_2} \times 14.142$$

$$A_{f_2} = 0.04243 \text{ m}^2 \quad \text{... Ans.}$$

Ex. 5.13.23 : Prove that in general, for a centrifugal pump having a discharge Q at speed N, the manometric head is expressible in the form :

$H_m = A \cdot N^2 + B \cdot NQ + CQ^2$, where A, B and C are constants. SPPU - May 11

Soln. : As derived in Ex. 5.13.18.

Pressure rise,

$$\left(\frac{p_2}{w} - \frac{p_1}{w}\right) = \frac{1}{2g}\left(V_{f1}^2 + u_2^2 - V_{f2}^2 \operatorname{cosec}^2 \phi\right) \quad ...(i)$$

The manometric head, H_m is equal to pressure rise in the impeller and the fraction of kinetic head equivalent to exit velocity which is recovered in volute casing if other losses in the pump are neglected. Hence,

$$H_m = \frac{p_2 - p_1}{w} + k \cdot \frac{V_2^2}{2g} \quad ...(ii)$$

From exit velocity diagram,

$$V_2^2 = V_{f2}^2 + V_{w2}^2 = V_{f2}^2 + (u_2 - V_{f2} \cot \phi)^2$$

$$V_2^2 = V_{f2}^2 + u_2^2 - 2\, u_2 \cdot V_{f2} \cot \phi + V_{f2}^2 \cdot \cot^2 \phi \quad ...(iii)$$

On substituting the values from Equations (i) and (iii) in Equation (ii)

$$H_m = \frac{1}{2g}\left(V_{f1}^2 + u_2^2 - V_{f2}^2 \operatorname{cosec}^2 \phi\right)$$

$$+ \frac{k}{2g}\left(V_{f2}^2 + u_2^2 - 2\, u_2 \cdot V_{f2} \cot \phi + V_{f2}^2 \cdot \cot^2 \phi\right)$$

$$H_m = \frac{1}{2g}\left[u_2^2(1+k) - 2k \cot \phi \cdot u_2 \cdot V_{f2} + V_{f2}^2(k \cot^2 \phi - \operatorname{cosec}^2 \phi) + V_{f1}^2\right]$$

But, $V_{f1} = V_{f2}$

$$H_m = \frac{1}{2g}\left[u_2^2(1+k) - 2k \cdot \cot \phi\, u_2 \cdot V_{f2} + V_{f2}^2(1 + k \cot^2 \phi - \operatorname{cosec}^2 \phi)\right]$$

Let, $\frac{(1+k)}{2g} = a$, $\frac{-2k \cot \phi}{2g} = b$

and $\frac{(1 + k \cot^2 \phi - \operatorname{cosec}^2 \phi)}{2g} = c$

$$\therefore H_m = (a \cdot u_2^2 + b \cdot u_2 \cdot V_{f2} + c \cdot V_{f2}^2) \quad ...(iv)$$

Since, $u_2 = \frac{\pi D_2 N}{60}$ i.e. $u_2 \propto N$

and $V_{f2} = \frac{Q}{A}$ i.e. $V_{f2} \propto Q$

Equation (iv) can be reduced to :

$$H_m = A \cdot N^2 + B \cdot N \cdot Q + C \cdot Q^2 \quad \text{.... proved}$$

where, A, B and C are constants.

Ex. 5.13.24 : A centrifugal pump lifts water against a static head of 40 m, of which 4 m is suction lift. The suction and delivery pipes are both 150 mm diameter, the head loss in the suction pipe is 2.3 m and in the delivery pipe, 7.4 m. The impeller is 420 mm diameter and 25 mm wide at the exit, it revolves at 1200 rpm and its effective vane angle at exit is 35°. If manometric efficiency is 82% and overall efficiency is 72%, determine the discharge delivered by the pump and power required to drive the pump. Also find the pressure head indicated at the suction and delivery branches of the pipe.

Soln. :

Given : Static head, H_s = 40 m, Suction lift,

h_s = 4 m, $d_s = d_d$ = 150 mm = 0.15 m,

h_{fs} = 2.3, h_{fd} = 7.4 m,

D_2 = 420 mm = 0.42 m,

B_2 = 25 mm = 0.025 m;

N = 1200 rpm, ϕ = 35°

η_{mano} = 82% = 0.82; η_o = 72% = 0.72

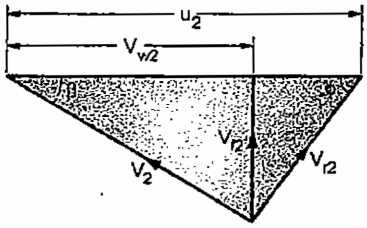


Fig. P. 5.13.24 : Outlet velocity diagram

(i) Discharge, Q

Vane velocity at exit,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.42 \times 1200}{60} = 26.39 \text{ m/s}$$

Discharge, $Q = \frac{\pi}{4} \cdot d_d^2 \cdot V_d$

$$Q = \frac{\pi}{4} \times 0.15^2 \times V_d$$

$$= 0.01767\, V_d \quad ...(i)$$

Also, $Q = \pi D_2 B_2 \cdot V_{f2}$

$$Q = \pi \times 0.42 \times 0.025 \times V_{f2} \quad ...(ii)$$

From Equations (i) and (ii),

$$0.01767\, V_d = 0.03299\, V_{f2}$$

$$\therefore V_{f2} = 0.5357\, V_d \quad ...(iii)$$

From outlet velocity diagram shown in Fig. P. 5.13.24

$$V_{w2} = u_2 - V_{f2} \cot \phi = 26.39 - V_{f2} \cot 35°$$

$V_{w2} = 26.39 - 1.428\, V_{f2}$; On substituting the value of V_{f2}

$$V_{w2} = 26.39 - 1.428 \times 0.5357\, V_d$$

$$= 26.39 - 0.765\, V_d \quad \text{...(iv)}$$

Manometric head,

$$H_m = H_s + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$$

$$= 40 + 2.3 + 7.4 + \frac{V_d^2}{2 \times 9.81}$$

$$= 49.7 + \frac{V_d^2}{2 \times 9.81} \quad \text{...(v)}$$

$$\eta_{mano} = \frac{g \cdot H_m}{V_{w2} \cdot u_2}$$

$$0.82 = \frac{9.81 \times \left(49.7 + \frac{V_d^2}{2 \times 9.81}\right)}{(26.39 - 0.765\, V_d)\, 26.39}$$

$$26.39 \times 0.82\, (26.39 - 0.765\, V_d) = 9.81 \times 49.7 + 0.5\, V_d^2$$

$$21.64\, (26.39 - 0.765\, V_d) = 487.6 + 0.5 d^2$$

$$571.1 - 16.555\, V_d = 487.6 + 0.5\, V_d^2$$

$$V_d^2 + 33.11\, V_d - 167 = 0$$

$$\therefore \quad V_d = \frac{-33.11 \pm \sqrt{33.11^2 + 4 \times 167}}{2}$$

$$= \frac{-33.11 \pm 42}{2} = 4.445 \text{ m/s}$$

(By taking positive value)

On substituting the value of V_d in Equation (i),

$$Q = 0.01767\, V_d = 0.01767 \times 4.445$$

$$= \mathbf{0.0785\ m^3/s} \quad \text{...Ans.}$$

(ii) Power required, P

From Equation (v),

$$H_m = 49.7 + \frac{V_d^2}{2 \times 9.81} = 49.7 + \frac{(4.445)^2}{2 \times 9.81}$$

$$= 49.7 + 1 = 50.7 \text{ m}$$

$$\therefore \quad P = \frac{\rho Q g H_m}{\eta_o} = \frac{1000 \times 0.078 \times 9.81 \times 50.7}{0.72}$$

$$= 53.88 \times 10^3 \text{ W} = \mathbf{53.88\ kW} \quad \text{...Ans.}$$

(iii) Pressure head at suction and delivery pipes :

$$\text{Velocity head} = \frac{V_s^2}{2g} = \frac{V_d^2}{2g}$$

$$= \frac{(4.445)^2}{2 \times 9.81} = 1 \text{ m}$$

Pump suction pressure head,

$$\frac{p_s}{w} = -\left(h_s + h_{fs} + \frac{V_s^2}{2g}\right)$$

$$= -(4 + 2.3 + 1) = \mathbf{-7.3\ m} \quad \text{...Ans.}$$

Pressure head at delivery,

$$\frac{p_d}{w} = H_m + \frac{p_s}{w} = 50.7 - 7.3 = \mathbf{43.4 m} \quad \text{...Ans.}$$

Ex. 5.13.25 : A three stage centrifugal pump has impeller 400 mm in diameter and 20 mm wide. The vane angle at outlet is 45° and the area occupied by the thickness of vanes may be assumed 8 percent of the total area. If the pump delivers 3.6 m^3 of water per minute when running at 920 r.p.m. determine.

(i) Power of the pump

(ii) Manometric head

(iii) Specific speed

Assume mechanical efficiency of 88 percent and manometric efficiency of 77 percent.

Soln. :

Given : No. of stage, $n_1 = 3$; $D_2 = 400$ mm $= 0.4$ m

$B_2 = 20$ mm $= 0.02$m. $\phi = 45°$

Reduction in area = 8% = 0.08

therefore, $K_b = (1 - 0.08) = 0.92$

$Q = 3.6\ m^3/\text{min}$; $N = 920$ rpm ;

$\eta_m = 88\% = 0.88$; $\eta_{mano} = 77\% = 0.77$

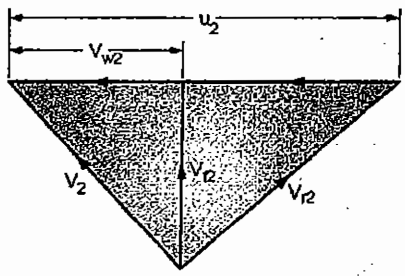


Fig. P. 5.13.25

(i) Total head generated, H_m

$$V_{f2} = \frac{Q}{\pi D_2 B_2 \cdot K_b} = \frac{3.6}{60} \times \frac{1}{\pi \times 0.4 \times 0.02 \times 0.92}$$

$$= 2.595 \text{ m/s}$$

Blade velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 920}{60} = 19.27 \text{ m/s}$$

From outlet triangle shown in Fig. P. 5.13.25,

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = 19.27 - \frac{2.595}{\tan 45}$$

$$= 16.675 \text{ m/s}$$

$$\eta_{mano} = \frac{g \cdot H_m}{V_{W2} \cdot u_2}$$

$$0.77 = \frac{9.81 \times H_m}{16.675 \times 19.27}$$

$$H_m = 25.22 \text{ m/stage}$$

Since pump is a multistage pump in series, total head generated,

$$H = 3 \times H_m = 3 \times 25.22$$

$$= 75.66 \text{ m} \quad \text{...Ans.}$$

(ii) Power of pump, P_s

$$\eta_m = \frac{\rho g Q H}{P_s}$$

$$P_s = \frac{\rho \cdot g \cdot Q \cdot H}{\eta_m}$$

$$= \frac{10^3 \times 9.81}{0.88} \times \frac{3.6}{60} \times 75.66 \times \frac{1}{1000} \text{ kW}$$

$$= 50.6 \text{ kW} \quad \text{...Ans.}$$

(iii) Specific speed, N_s

$$N_s = \frac{N\sqrt{P}}{H_m^{5/4}} = \frac{920\sqrt{50.6}}{(25.22)^{5/4}}$$

$$= 115.79 \quad \text{...Ans.}$$

Ex. 5.13.26 : A centrifugal pump delivers 1565 LPS against a manometric head of 6.1 m. when the impeller rotates at 200 rpm. The impeller diameter is 1.22 m and the area at outer periphery is 6450 cm^2. If the vanes are set back at an angle of 26° at the outlet determine.

(1) Manometric efficiency

(2) Power required to drive the pump

(3) Minimum starting speed if ratio of external to internal diameter is 2.

Soln. :

Given : $Q = 1565 \text{ LPS} = \frac{1565}{1000} \text{ m}^3/\text{s} = 1.565 \text{ m}^3/\text{s}$;

$H_m = 6.1$ m;

$N = 200$ rpm; $D_2 = 1.22$ m; $A_2 = 6450 \text{ cm}^2$

$$= \frac{6450}{(100)^2} = 0.645 \text{ m}^2 \; ; \; \phi = 26°$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.22 \times 200}{60} = 12.78 \text{ m/s}$$

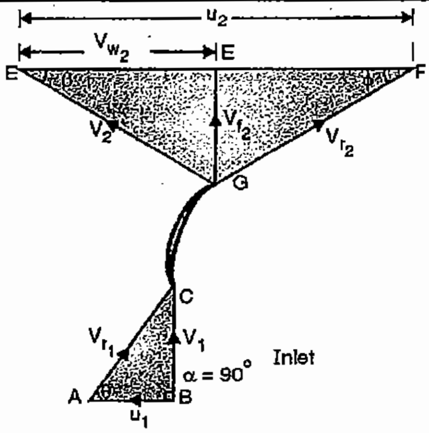


Fig. P. 5.13.26

1. Manometric efficiency, η_{mano}

$$Q = A_2 \times V_{f_2}$$

$$1.565 = 0.645 \times V_{f_2}$$

$$V_{f2} = 2.4264 \text{ m/s}$$

$$EF = \frac{V_{f2}}{\tan\phi} = \frac{2.4264}{\tan 26} = 4.975 \text{ m/s}$$

$$V_{w_2} = u_2 - EF = 12.78 - 4.975 = 7.805 \text{ m/s}$$

$$\eta_{mano} = \frac{\text{Manometric head, } H_m}{\text{Head imparted by impeller}\left(\frac{V_{w_2} \cdot u_2}{g}\right)}$$

$$= \frac{g \cdot H_m}{V_{w_2} \cdot u_2} = \frac{9.81 \times 6.1}{7.805 \times 12.78}$$

$$= 0.5999 \text{ or } 59.99\% \quad \text{...Ans.}$$

2. Power required to drive the pump, P

$$\dot{m} = \rho Q = 1000 \times 1.565 = 1565 \text{ kg/s}$$

$$P = \frac{\dot{m}\, V_{w_2} \cdot u_2}{1000} \text{ (kW)}$$

$$= \frac{1565 \times 7.805 \times 12.75}{1000}$$

$$= 156.2 \text{ kW} \quad \text{...Ans.}$$

3. Minimum starting speed if $\frac{D_2}{D_1} = 2$

Condition for minimum starting speed, N is,

$$\frac{u_2^2 - u_1^2}{2g} \geq H_m$$

$$\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2 = 2g \times H_m$$

$$\left(\frac{\pi \times 1.22 \times N}{60}\right)^2 - \left(\frac{\pi \times 0.61 \times N}{60}\right)^2 = 2 \times 9.81 \times 6.1$$

$$N = 197.754 \text{ rpm} \quad \text{...Ans.}$$

Ex. 5.13.27 : A centrifugal pump discharges 57 litres per second water when operated by a motor running at 1760 rpm. The flow is radial at inlet and the relative velocity of 15 m/s makes an angle of 25° with the tangent to the periphery. If the pump efficiency is 70% and motor efficiency is 80%. Find the input power to the motor. Impeller diameter of the pump is 37.5 cm.

Soln. : Given :

$$Q = 57 \text{ lps} = 5.7 \times 10^{-3} = 0.057 \text{ m/s};$$

$$N = 1760 \text{ rpm}, \qquad \alpha = 90 \text{ (radial entry)},$$

$$V_{r1} = 15 \text{ m/s}, \qquad \theta = 25°,$$

$$\eta_{mano} = 70\% = 0.7; \qquad \eta_m = 80\% = 0.8;$$

$$D_2 = 37.5 \text{ cm} = 0.375 \text{ m}.$$

Vane velocity at exit,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.375 \times 1760}{60} = 34.56 \text{ m/s}$$

From inlet velocity diagram shown in Fig. P. 5.13.27.

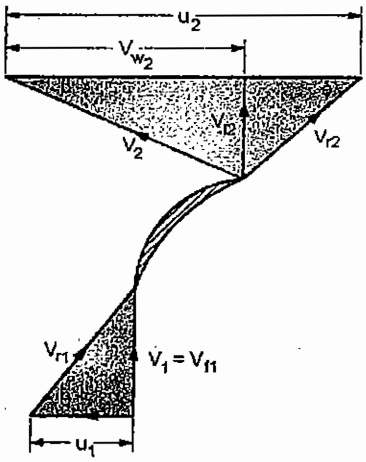


Fig. P. 5.13.27

$$V_{f1} = V_{r1} \cdot \sin\theta = 15 \sin 25 = 6.34 \text{ m/s}$$

$$\text{But, } V_{f2} = V_{f1} = 6.34 \text{ m/s}$$

$$\text{Assuming } \theta = \phi, \qquad u_2 - V_{w2} = V_{f2} \cdot \cot\phi$$

$$34.56 - V_{w2} = 6.34 \cot 25° = 13.60$$

$$\therefore \quad V_{w2} = 20.96 \text{ m/s}$$

$$H_m = \frac{V_{w2} \cdot u_2 \times \eta_{mano}}{g}$$

$$= \frac{20.96 \times 34.56}{9.81} \times 0.7 = 51.69 \text{ m}$$

Power input, P,

$$P_S = \frac{\rho \cdot g \cdot Q \cdot H_m}{\eta_m \times \eta_{mano}} \times 10^{-3} \text{ kW}$$

$$= \frac{1000 \times 9.81 \times 0.057 \times 51.69 \times 10^{-3}}{0.8 \times 0.7}$$

$$= 51.61 \text{ kW} \qquad \text{...Ans.}$$

Ex. 5.13.28 : A centrifugal pump running at 900 rpm is working against a head 20 m. The external diameter of the impeller is 460 mm and outlet width is 50 mm. If the vanes angles at outlet is 40° and manometric efficiency is 70%, determine :

i) Flow velocity at outlet
ii) Absolute velocity of water leaving the vane
iii) Angle made by the absolute at outlet with the direction of motion at outlet
iv) Rate of flow through the pump

SPPU - Dec. 15, 10 Marks

Soln. :

$$N = 900 \text{ rpm}, \qquad H_m = 20 \text{ m}$$

$$D_2 = 460 \text{ mm} = 0.46 \text{ m},$$

$$B_2 = 50 \text{ mm} = 0.05 \text{ m},$$

$$\phi = 40°, \qquad \eta_m = 70\% = 0.7$$

(i) Flow velocity at outlet, V_{f2}

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.46 \times 900}{60} = 21.68 \text{ m/s}$$

Exit velocity diagram is shown in Fig. P. 5.13.28.

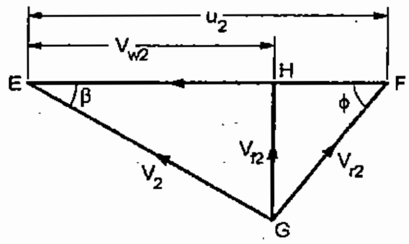


Fig. P.5.13.28

$$\eta_m = \frac{gH_m}{V_{w2} \cdot u_2}$$

$$0.7 = \frac{9.81 \times 20}{V_{w2} \times 21.68}$$

$$V_{w2} = 12.93 \text{ m/s}$$

$$H_F = u_2 - V_{w2}$$

$$= 21.68 - 12.93 = 8.75 \text{ m/s}$$

$$\text{From } \Delta GHF: \quad \frac{V_{f2}}{H_F} = \tan\phi$$

$$V_{f2} = 8.75 \tan 40 = \mathbf{7.34 \text{ m/s}} \qquad \text{...Ans.}$$

(ii) Absolute velocity of water leaving the vane, V_2

$$V_2 = \sqrt{V_{w2}^2 + V_{f2}^2} = \sqrt{(12.93)^2 + (7.34)^2}$$

$$= \mathbf{14.87 \text{ m/s}} \qquad \text{...Ans.}$$

(iii) Angle made by absolute velocity at outlet, β

$$\sin\beta = \frac{V_{f2}}{V_2} = \frac{7.34}{14.87} = 0.4936$$

$$\beta = 29.58° \qquad \text{...Ans.}$$

(iv) Rate of flow through the pump, Q

$$Q = \pi D_2 B_2 V_{f2} = \pi \times 0.46 \times 0.05 \times 7.34$$

$$= 0.5304\ m^3/s \qquad \text{...Ans.}$$

Ex. 5.13.29 : A centrifugal pump impeller has a outer diameter of 360 mm and width of 60 mm. The vanes are curved backwards at 35° to the tangent at outer periphery and thickness of vanes occupy 20% of the peripheral area. Velocity of flow is constant from inlet to outlet. The impeller rotates at 800 rpm. If the rate of flow through the pump is 0.13 m^3/s,

determine :

(i) The pressure rise in the impeller and

(ii) The percentage of total work converted to kinetic energy.

Soln. :

Given : $D_2 = 360\ mm = 0.36\ m$;

Width, $B_2 = 60\ mm = 0.06\ m$, $\phi = 35°$

thickness area = 20% of peripheral area

i.e. $A = (1 - 0.2)\,\pi D_2 B_2$; $N = 800$ rpm

$Q = 0.13\ m^3/s$; $V_{f1} = V_{f2}$.

(i) Pressure rise in impeller

Vane velocity at exit,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.36 \times 800}{60} = 15.08\ m/s$$

$$V_1 = V_{f1} = V_{f2}$$

$$= \frac{Q}{A} = \frac{Q}{(1-0.2)\,\pi D_2 B_2}$$

$$= \frac{0.13}{0.8 \times \pi \times 0.36 \times 0.06} = 2.39\ m/s$$

From outlet velocity diagram shown in Fig. P. 5.13.29.

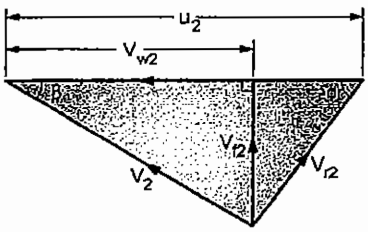


Fig. P. 5.13.29

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = 15.08 - \frac{2.39}{\tan 35} = 11.67\ m/s$$

$$\therefore V_2 = \sqrt{V_{\omega 2}^2 + V_{f2}^2} = \sqrt{11.67^2 + 2.39^2}$$

$$= 11.91\ m/s$$

From Bernoulli's theorem at inlet and exit of impeller (on neglecting losses) can be written as,

$$\frac{p_1}{W} + \frac{V_1^2}{2g} + \frac{V_{w2} \cdot u_2}{g} = \frac{p_2}{W} + \frac{V_2^2}{2g}$$

∴ Pressure rise in impeller,

$$\frac{(p_2 - p_1)}{W} = \frac{V_1^2}{2g} + \frac{V_{w2} \cdot u_2}{g} - \frac{V_2^2}{2g}$$

$$= \frac{2.34^2}{2 \times 9.81} + \frac{11.67 \times 15.08}{9.81} - \frac{11.91^2}{2 \times 9.81}$$

$$= 10.99\ m \qquad \text{...Ans.}$$

(ii) Percentage of total work converted to kinetic energy

$$= \frac{(V_2^2/2g)}{(V_{w2} \cdot u_2/g)} \times 100\% = \frac{V_2^2}{2 \cdot V_{w2} \cdot u_2} \times 100$$

$$= \frac{11.91^2}{2 \times 11.67 \times 15.08} \times 100$$

$$= 40.3\% \qquad \text{...Ans.}$$

Ex. 5.13.30 : A centrifugal pump, in which water enters radially delivers water to a head of 165 m. The impeller has a diameter of 360 mm and width 180 mm at inlet and the corresponding dimensions at the outlet are 720 mm and 90 mm respectively. Its rotational speed is 1200 rpm. The blades are curved backward at 30° to the tangent at exit and the discharge is 0.389 m^3/s.

Determine

(i) Theoretical head developed

(ii) Manometric efficiency

(iii) Pressure rise across the impeller assuming losses equal to 12% of velocity head at exit

(iv) Pressure rise and the loss of head in the volute casing

(v) The vane angle at inlet and

(vi) Power required to drive the pump assuming an overall efficiency of 70%. What would be corresponding mechanical efficiency ?

SPPU - May 12, 12 Marks, May 14, 10 Marks

Soln. :

Given : $\alpha = 90$ (radial entry), $H_m = 165$ m

$D_1 = 360 \text{ mm} = 0.36 \text{ m}$

$B_1 = 180 \text{ mm} = 0.18 \text{ m};$

$D_2 = 720 \text{ mm} = 0.72 \text{ m}$

$B_2 = 90 \text{ mm} = 0.09 \text{ m};$

$N = 1200 \text{ rpm}, \phi = 30°$

$Q = 0.389 \text{ m}^3/\text{s}$

(i) Theoretical head required, H_e

$$\text{Discharge, } Q = \pi D_1 B_1 V_{f1}$$

$$0.389 = \pi \times 0.36 \times 0.18\, V_{f1}$$

$$\therefore V_{f1} = 1.91 \text{ m/s}$$

$$\text{Similarly, } V_{f2} = \frac{Q}{\pi D_2 B_2} = \frac{0.389}{\pi \times 0.72 \times 0.09} = 1.91 \text{ m/s}$$

Vane velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.36 \times 1200}{60} = 22.62 \text{ m/s}$$

$$\text{Vane velocity at exit, } u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.72 \times 1200}{60} = 45.24 \text{ m/s}$$

From exit velocity diagram shown in Fig. P. 5.13.30.

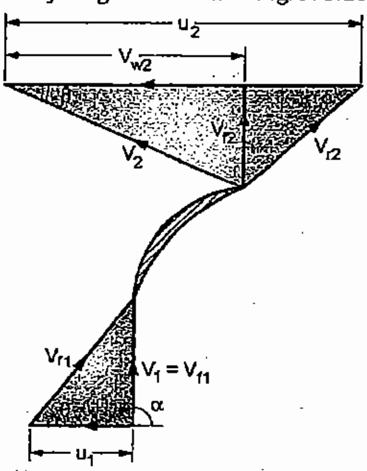


Fig. P. 5.13.30

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = 45.24 - \frac{1.91}{\tan 30}$$

$= 41.93$ m/s i.e. Euler's head

Theoretical head developed,

$$H_e = \frac{V_{w2} \cdot u_2}{g} = \frac{41.93 \times 45.24}{9.81}$$

$= \mathbf{193.37m}$ ...**Ans.**

(ii) Manometric efficiency, η_{mano}

$$\eta_{mano} = \frac{\text{Manometric head}, H_m}{\text{Theoretical head}, H} = \frac{165}{193.37}$$

$= \mathbf{0.8533 \text{ or } 85.33\%}$ ...**Ans.**

(iii) Pressure rise in impeller, $\left(\frac{p_2 - p_1}{w}\right)$

Given :

Losses, h_e = 12% of velocity head at exit

$$h_l = 0.12 \times \frac{V_2^2}{2g}; \quad V_1 = V_{f1} = 1.91 \text{ m/s}$$

From outlet velocity diagram,

$$V_2 = \sqrt{V_{\omega 2}^2 + V_{f2}^2} = \sqrt{41.93^2 + 1.91^2} = 41.97 \text{ m/s}$$

On applying Bernoulli's equation at inlet and exit,

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 + H_e = \frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2 + h_l$$

$$\therefore \frac{p_2 - p_1}{w} = H_e + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_l$$

$$= H_e + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - 0.12 \times \frac{V_2^2}{2g}$$

$$= 193.37 + \frac{1.91^2}{2 \times 9.81} - \frac{41.97^2}{2 \times 9.81} - 0.12 \times \frac{41.97^2}{2 \times 9.81}$$

$= \mathbf{93.0 \text{ m}}$ ...**Ans.**

(iv) Pressure rise and loss of head in casing

Pressure rise in casing = H_{mano} – Pressure rise in impeller

$= \mathbf{165 - 93 = 72 \text{ m}}$ ...**Ans.**

Loss of head in casing $= H_e - H_{mano} - h_l$

$$= 193.37 - 165 - 0.12 \times \frac{41.97^2}{2 \times 9.81}$$

$= \mathbf{17.6 \text{ m}}$ ...**Ans.**

(v) Inlet vane angle, θ

From velocity diagram at inlet,

$$\tan\theta = \frac{V_{f1}}{u_1} = \frac{1.91}{22.62} = 0.08444$$

$\therefore \theta = \mathbf{4.827°}$ ...**Ans.**

(vi) Power required to drive the pump, P_s

Given : Overall efficiency,

$$\eta_o = 70\% = 0.7$$

$$\eta_o = \frac{\rho \cdot g \cdot Q \cdot H_m}{P_s}$$

$$\text{i.e. } P_s = \frac{\rho \cdot g \cdot Q \cdot H_m \times 10^{-3}}{\eta_o} \text{ (kW)}$$

$$= \frac{1000 \times 9.81 \times 0.389 \times 165 \times 10^{-3}}{0.7}$$

$$= \mathbf{899.5\ kW} \qquad \text{...Ans.}$$

(vii) Mechanical efficiency, η_m

$$\text{Since, } \eta_o = \eta_{mano} \times \eta_v \times \eta_m;$$

(Assuming volumetric efficiency, $\eta_v = 1$)

$$\therefore \eta_m = \frac{\eta_o}{\eta_{mano}} = \frac{0.7}{0.8533}$$

$$= \mathbf{0.8203 \text{ or } 82.03\%} \qquad \text{...Ans.}$$

Ex. 5.13.31 : A centrifugal pump lifts water under a static lift of 40 m of which 3 m is suction lift. The suction and delivery pipes are both 350 mm in diameter. The friction loss in suction pipe is 2 m and in delivery pipe it is 6 m. The pump impeller is 0.5 m in diameter and 30 mm wide at outlet and runs at 1200 rpm. The exit blade angle is 20°. If manometric efficiency is 85% determine the pressure at suction and delivery ends of pump and the discharge.

Soln. :

Static head, $H_s = h_s + h_d = 40$ m

Suction lift $h_s = 3$m

$d_s = d_d = 350$ mm $= 0.35$ m;

$h_{fs} = 2$m ; $h_{fd} = 6$ m

$D_2 = 0.5$ m ; $B_2 = 30$ mm $= 0.030$ m

$N = 1200$ rpm ; $\phi = 20°$;

$\eta_m = 85\% = 0.85$

$$u_2 = \frac{\pi \cdot D_2 \cdot N}{60} = \frac{\pi \times 0.5 \times 1200}{60} = 31.416 \text{ m/s}$$

Refer Fig. P. 5.13.31

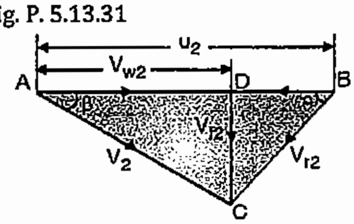


Fig. P. 5.13.31 : Velocity diagram at outlet

Manometric head,

$$H_m = H_s + H_{fs} + h_{fd} + \frac{V_d^2}{2g}$$

$$\text{On neglecting } \frac{V_d^2}{2g},\ H_m = 40 + 2 + 6 = 48 \text{ m}$$

$$\eta_m = \frac{g \cdot H_m}{V_{w2} \times u_2};$$

$$0.85 = \frac{9.81 \times 48}{V_{w2} \times 31.416}$$

$$\therefore V_{w2} = 17.634 \text{ m}$$

$$\therefore DB = u_2 - V_{w2} = 31.416 - 17.634$$

$$= 13.782 \text{ m/s}$$

$$V_{f2} = DB \tan \phi = 13.782 \tan 30$$

$$= 7.957 \text{ m/s}$$

$$\text{From } \Delta \text{ ACD} : V_2 = \sqrt{V_{w2}^2 + V_{f2}^2}$$

$$\therefore V_2 = \sqrt{(17.634)^2 + (7.957)^2}$$

$$= 19.346 \text{ m/s}$$

1. Discharge, Q

$$Q = \pi D_2 B_2 \cdot V_{f2} = \pi \times 0.5 \times 0.030 \times 7.957$$

$$= \mathbf{0.375\ m^3/s} \qquad \text{...Ans.}$$

Let V_d = Velocity in discharge pipe and V_s in suction pipe

$$Q = \frac{\pi}{4} \cdot d_d^2 \times V_d$$

$$0.375 = \frac{\pi}{4} \times (0.35)^2 \times V_d$$

$$\therefore V_d = 3.9 \text{ m/s} = V_s$$

(since diameter of suction and delivery pipes is same)

2. Pressure head at suction and delivery ends of pump

$$\text{Velocity head} = \frac{V_s^2}{2g} = \frac{V_d^2}{2g} = \frac{(3.9)^2}{2 \times 9.81} = 0.775 \text{ m}$$

Pump suction pressure head,

$$\frac{p_s}{w} = -\left(h_s + h_{fs} + \frac{V_s^2}{2}\right)$$

$$= -(3 + 2 + 0.775) = \mathbf{-5.775\ m} \qquad \text{...Ans.}$$

Pressure at delivery head,

$$\frac{p_d}{w} = H_m + \frac{P_s}{w}$$

$$= 48 - 5.775$$

$$= \mathbf{42.225\ m} \qquad \text{...Ans.}$$

Ex. 5.13.32 : A centrifugal pump delivers 250 lps of water against a head of 50 m at a designed speed of 900 rpm. The internal and external diameter of impeller are 350 mm and 700 mm respectively. The vane angle at exit is 35°. The width of impeller at inlet is 70 mm and at outlet is 35 mm. Determine the following :

(i) Manometric efficiency

(ii) Inlet vane angle

(iii) Loss of head at inlet if discharge is reduced by 40%. Assume, radial entry.

Soln. : Refer Fig. P. 5.13.32.

Given : $Q = 250$ lps $= 0.25\ m^3/s$, $H_m = 50$ m;

$N = 900$ rpm, $D_1 = 350$ mm $= 0.35$ m,

$D_2 = 700$ mm $= 0.7$ m, $\phi = 35°$,

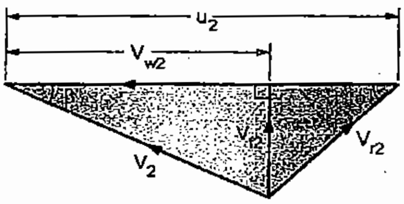
$B_1 = 70$ mm $= 0.07$ m;

$B_2 = 35$ mm $= 0.035$ m.

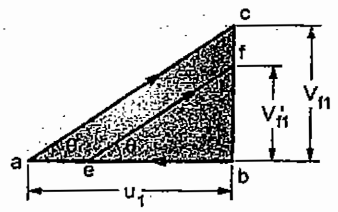
(i) Manometric efficiency, η_{mano}

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.35 \times 900}{60} = 16.49 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.7 \times 900}{60} = 32.98 \text{ m/s}$$



(a) Outlet diagram



(b) Inlet velocity diagram

Fig. P. 5.13.32

$$V_{f1} = \frac{Q}{\pi D_1 B_1} = \frac{0.25}{\pi \times 0.35 \times 0.07} = 3.248 \text{ m/s}$$

$$V_{f2} = \frac{Q}{\pi D_2 B_2} = \frac{0.25}{\pi \times 0.7 \times 0.035} = 3.248 \text{ m/s}$$

$$\therefore\ V_{f1} = V_{f2} = 3.248 \text{ m/s}$$

From outlet velocity diagram,

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan \phi} = 32.98 - \frac{3.248}{\tan 35} = 28.34 \text{ m/s}$$

$$\eta_{mano} = \frac{g \cdot H_m}{V_{w2} \cdot u_2} = \frac{9.81 \times 50}{28.34 \times 32.98}$$

$$= \mathbf{0.5248 \text{ or } 52.48\%}$$ ...**Ans.**

(ii) Inlet vane angle, θ

From inlet velocity triangle,

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{u_1}\right)$$

$$= \tan^{-1}\left(\frac{3.248}{16.49}\right)$$

$$= \mathbf{11.143°}$$ ...**Ans.**

(iii) Loss of head at inlet if discharge is reduced by 40%

New discharge,

$$Q' = \left(1 - \frac{40}{100}\right) Q = \frac{60}{100} \times 0.25$$

$$= 0.15\ m^3/s$$

$$V'_{f1} = \frac{Q'}{\pi D_1 B_1} = \frac{0.15}{\pi \times 0.35 \times 0.07}$$

$$= 1.949 \text{ m/s}$$

Loss of head at inlet,

$$h_L = \frac{(ae)^2}{2g} = \frac{(u_1 - V'_{f1} \cot \theta)^2}{2g}$$

$$= \frac{(16.49 - 1.949 \cot 11.143)^2}{2 \times 9.81}$$

$$= \mathbf{2.217 \text{ m of water}}$$ ...**Ans.**

5.14 Multistage Centrifugal Pumps

University Questions

Q. Why is multistaging used for a centrifugal pump ? Describe the methods use for multistaging. **SPPU : Dec. 11, Dec. 12, May 14**

Q. Explain multi staging in a centrifugal pump and explain the methods used for multi staging. **SPPU : Dec. 13**

The centrifugal pumps with two or more number of identical impellers are called multistage centrifugal pumps.

The impellers of these pumps may be attached on the same or on different shafts. The multistage pumps are needed either to increase the head or the discharge compared to a single stage pump, accordingly the impellers are connected in series or parallel as follows :

5.14.1 Multistage Centrifugal Pumps for High Heads

Two or more identical impellers are connected in **series to generate high heads with constant discharge.** The arrangement of multistage pump with two impellers in series is shown in Fig. 5.14.1.

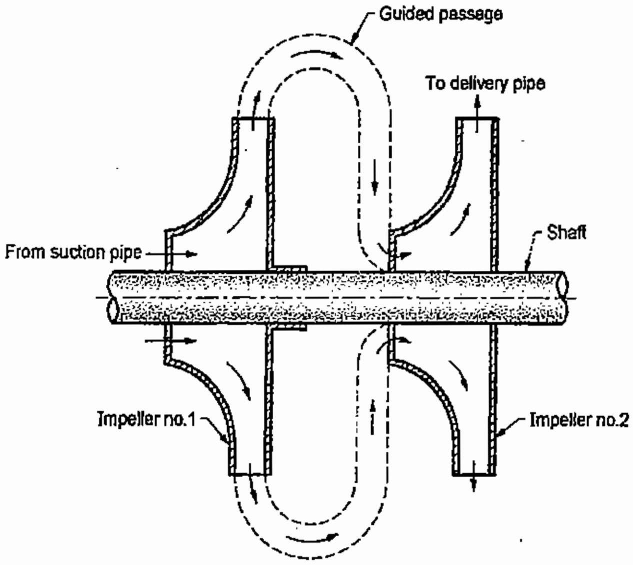


Fig. 5.14.1 : Impellers in series for heads

The water from suction pipe is supplied to impeller - 1 where its head is raised. The discharge from impeller - 1 is guided through the guided passages into the inlet of impeller- 2 where its pressure is further increased. If there are more number of impellers in series the similar operations are conducted and finally the last impeller in series discharges the water.

Let, n = number of impellers

H_m = head developed by each impeller

Total head developed,

$$H = n \cdot H_m \quad ...(i)$$

Total discharge passing through each impeller remains the same.

Therefore, $$Q_{total} = Q \quad ...(ii)$$

These pumps are used where high heads are needed at relatively low discharge as in boiler practice.

Advantages of multistage pumps

Advantages of multistage pumps in series compared to single stage pump are :

1. Friction losses are reduced due to reduced head on each impeller requiring lower speeds.
2. Stresses are reduced.
3. Impeller diameter is reduced.
4. Axial thrust can be eliminated by suitable arrangement.
5. Due to lower specific speed, higher suction lift is possible.

5.14.2 Multistage Centrifugal Pumps for High Discharge

Two or more identical impellers are connected in **parallel to generate high discharge at constant heads.** The arrangement of multistage pumps in parallel are shown in Fig. 5.14.2.

Each pump works as a separate unit and lifts the water from the sump by their respective suction pipes and their delivery pipes are connected to common delivery pipe as shown in Fig. 5.14.2. Each pump develops the same head and discharge of pumps are added.

Therefore,

Total head, H_{total} = Head developed by each impeller, H_m

Total discharge, Q_{total} = Number of pumps × Q

Where, Q is the discharge of each centrifugal pump.

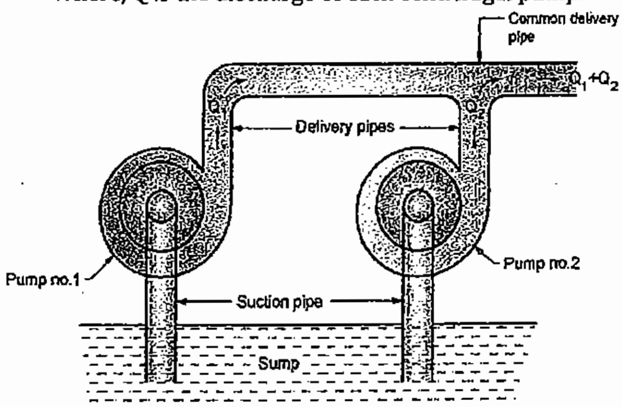


Fig. 5.14.2 : Pumps in parallel

5.15 Performance Characteristics of a Centrifugal Pump

University Question

Q. Discuss the performance characteristics of the centrifugal pump. **SPPU : May 11**

A centrifugal pump is designed to develop certain manometric head and discharge at constant speed since the pumps are usually driven by A.C. motors.

In certain cases the pumps may be driven by an I.C. engine at variable speed or the pump in actual practice may needed to develop a certain head or discharge. Under these actual conditions, the behaviour of the pump will be different than expected. Therefore, various tests on the pump under variable conditions are conducted in order to predict the behaviour and performance of the pump. The test results are then plotted on a graph under different flow rates, head and speed.

The curves thus obtained are known as **characteristic curves for the pump.**

5.15.1 Test Set up for Conduct of Tests on Centrifugal Pump in a Laboratory

Fig. 5.15.1 shows a schematic layout of a test set up for testing centrifugal pumps.

It consists of a water storage tank of suitable capacity on which the pump along with motor and variable speed arrangement is fixed. The suction of the pump is connected through the suction line and the strainer cum foot valve arrangement to the storage tank.

The pump delivery is connected through a flow control valve to the measuring tank.

On the suction line, a vacuum gauge is fitted and a pressure gauge is fixed on the delivery line. Arrangement for priming the pump is provided using a priming cock and funnel on to the delivery line.

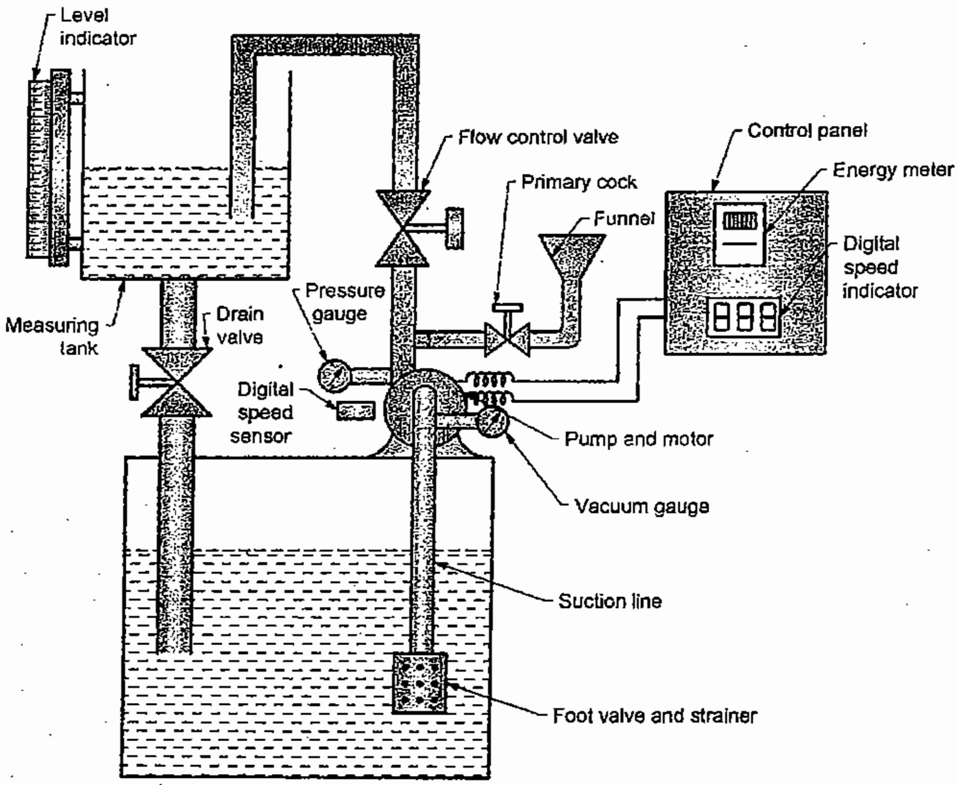


Fig. 5.15.1 : Test set up for centrifugal pump

Measuring tank consists of level indicator glasstube and the scale to measure discharge. Below the measuring tank, the drain value is fitted which allows to drain the water from measuring tank to storage tank.

The motor is connected through an energy motor which is fixed on the control panel. The motor speed is measured by the digital pickup indicator located on the control panel. The delivery head is measured by pressure gauge and the vacuum at pump suction is measured by vacuum gauge.

The pump is tested at constant speed by varying the discharge rates controlled by the flow control valve. The power input is measured by energy meter. The pump efficiency are calculated.

Various graphs are plotted for power, head and efficiency Vs the discharge.

Experiments are repeated at variable speed by keeping the constant valve opening. Head, discharge and power input are measured. Various performance curves are plotted.

5.15.2 Types of Performance Characteristic Curves

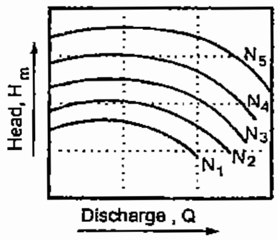
The performance characteristic curves are broadly divided into following four categories :

1. Main characteristic curves.
2. Operating characteristic curves.
3. Iso-efficiency or Muschel curves.
4. Constant head and constant discharge curves.

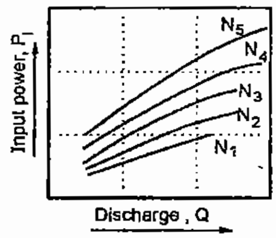
5.15.3 Main Characteristic Curves

Main characteristic curves are obtained by test run at constant speed and the discharge is varied by means of delivery valve.

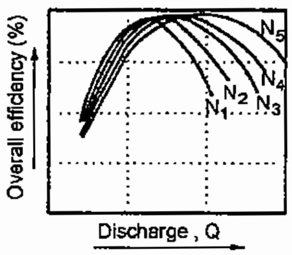
At each discharge, the manometric head H_m and input power P are measured and the overall efficiency η_o is calculated.



(a) H Vs Q



(b) P_i Vs Q



(c) η_o Vs Q

Fig. 5.15.2 : Main characteristic curves of a centrifugal pump

Test curves are plotted between H_m Vs Q, P Vs Q and η_o Vs Q as shown in Fig.5.15.2 for that constant speed.

The test run is repeated by running the pump at another constant speed. A family of curves will be obtained at various constant speeds N_1, N_2.... as shown in Fig. 5.15.2.

5.15.4 Operating Characteristic Curves

University Question

Q. Draw and discuss the operating characteristics of a centrifugal pump. SPPU : May 13

The pumps are designed for maximum, efficiency at a given speed called designed speed.

Therefore the pumps are test run at designed speed as provided by the manufacturer of the pump.

The discharge is varied as discussed in case of main characteristic curve and the head and power input are measured. The overall efficiency of the pump is calculated.

The performance curve thus obtained at design speed are called operating characteristic curve at shown in Fig. 5.15.3.

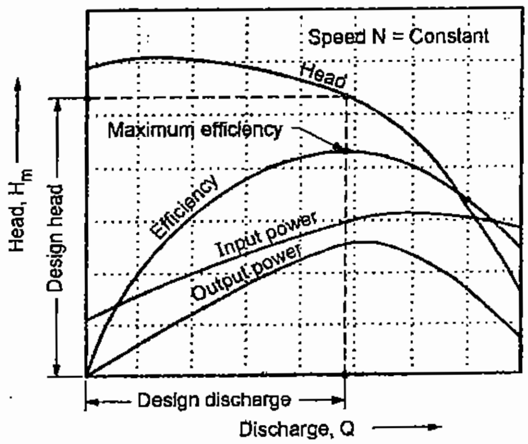


Fig. 5.15.3 : Operating characteristic curves of a centrifugal pump

5.15.5 Iso-efficiency or Maschel Curves

Iso-efficiency i.e. constant efficiency curves are useful in predicting the performance on entire operations and its best performance.

These characteristic curves can be drawn with the help of η_o Vs Q and H_m Vs Q curves shown in Fig. 5.15.2(c) and Fig. 5.15.2(a) respectively. The method is as follows :

(i) Draw a horizontal line on η_o Vs Q curve. It represent the constant efficiency line.

(ii) The points at which the constant efficiency line cuts the constant speed lines, the discharges are noted.

(iii) At a given discharge and speed, the H_m is noted from H Vs Q graph.

(iv) These values of H_m and Q at constant efficiency and speed are projected on a graph of H_m Vs Q shown in Fig. 5.15.4.

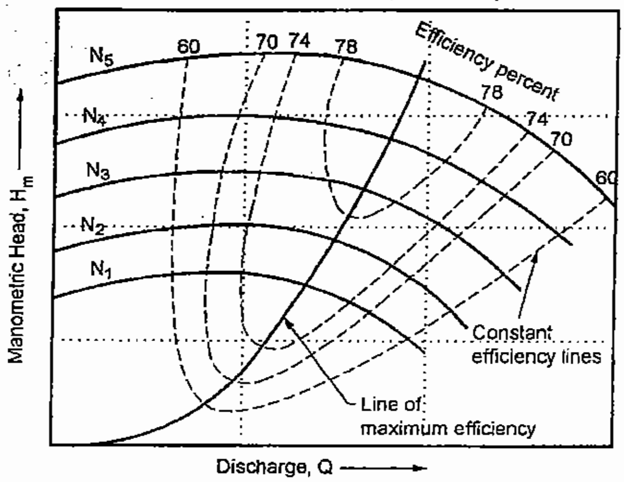


Fig. 5.15.4 : Iso-efficiency curves of a centrifugal pump

(v) The points corresponding to same overall efficiency are then joined with a smooth curve as shown in Fig. 5.15.4. These curves represent the iso-efficiency curves.

These iso-efficiency curves help to locate the regions where the pump would operate at maximum efficiency.

5.15.6 Constant Head and Constant Discharge Curves

Often a centrifugal pump is draw required to operate variable speed than the designed speed. Therefore it is necessary to the performance curves of a pump at variable speed so that these curves can be used to predict the performance.

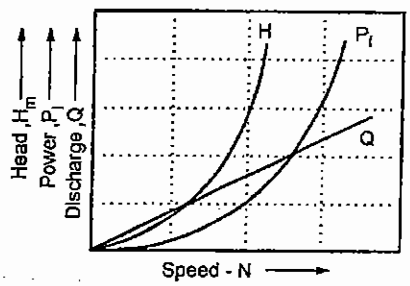


Fig. 5.15.5 : Head, Discharge and power characteristic curves at variable speeds

The procedure is as follows : The delivery valve opening is fixed and kept constant during the test on pump. Then it is operated at variable speed. For each speed the manometric head H_m, discharge Q and power input P_i are measured. The graphs H Vs N, P_i Vs N and Q Vs N can be drawn as shown in Fig. 5.15.5.

5.16 Selection of Pump Based on System Resistance Curve

University Question

Q. State the criteria for selection of a centrifugal pump for a given application. **SPPU : Dec. 11, Dec. 12, May 14**

Manufacturer supplies the head-discharge (H–Q) characteristic curve for their designed pump and operated under test conditions.

However, this pump is required to operate under different conditions with regard to suction and discharge pipe lines, bends, number of valves, elbows, tees etc.

Therefore, the pump to be employed under actual conditions is required to overcome the static head and the friction head loss in suction and delivery pipes and the connected fittings. Usually the static head is constant but the friction losses are proportional to the square of discharge. Hence, the user of the pump evaluates his system requirement and a curve is drawn as head-discharge curve called system resistance curve or system characteristic curve as shown in Fig. 5.16.1.

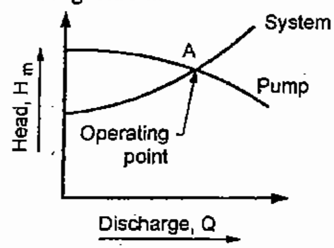


Fig. 5.16.1 : System resistance curve with pump characteristic curve

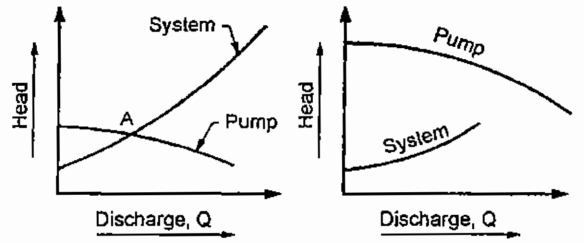
The pump characteristic curves as supplied by the manufacturer are superimposed on system characteristic curve as evaluated. The intersection of the two curves (point A) represents the operating point as shown in Fig. 5.16.1. This operation point on the pump characteristic curve should be in the vicinity of the maximum efficiency point of the pump. It would represent the economical running of the pump.

Also, it is necessary that the pump discharge should meet the limiting discharge of the system requirement.

Based on the above concept and other considerations the best pump is selected from various manufacturer's designed characteristic curves according to particular application.

5.16.1 Undersized and Oversized Pumps

As discussed above in section 5.15, the intersection of pump system characteristic curve represents the point of operation of pump which should be near the maximum efficiency point.



(a) Undersized pump **(b) Oversized pump**

Fig. 5.16.2

In case, the interaction between system and pump is not proper then pump may have either poor efficiency at part loads or it may not be able to meet the head requirements of the system, accordingly the pumps are called undersized and oversized pumps.

A pump is said an **undersized pump** or **under capacity** if it cannot meet the head and discharge requirements of the pump as shown in Fig. 5.16.2(a).

A pump is said to be **oversized pump** which can deliver much higher head and discharge.

Than the system requirements as shown in Fig. 5.16.2(b). The over capacity of the pump leads to poor part load efficiency and uneconomical running of the pump.

5.17 Characteristic Curves of Multistage Pumps

Characteristic curves for multistage pumps in series and parallel are discussed below.

5.17.1 Multistage Pumps in Series

When two identical pumps in series are connected in series, the total head developed by the multistage pump is the sum of head developed by each pump. while the total discharge remain the same equal to discharge of each pump.

Therefore the *Combined characteristic curve* can be plotted by adding the head of each pump at constant different discharges as shown in Fig. 5.17.1, e.g. $H_C = H_A + H_B$ at discharge Q_1. Locus of such points will represent the combined characteristic curve.

The system characteristic curve is also shown in Fig. 5.17.1. The intersection of system and combined pump characteristic curve at point G represents the point of operation.

Total head developed = Head developed by pump 1 + Head developed by pump 2

$$\text{i.e. } H_{Total}, H_G = H_E + H_F$$

$$Q_{Total} = Q$$

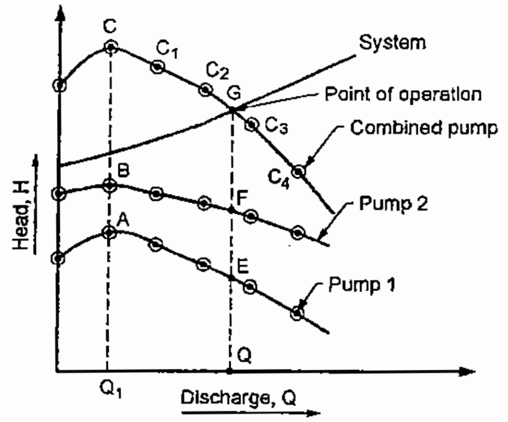


Fig. 5.17.1 : Characteristic curve of centrifugal pumps in series

5.17.2 Multistage Pumps in Parallel

When two identical pumps are connected in parallel, the total discharge of the pump is equal to the sum of discharges of individual pumps at the same head.

Therefore, the *combined characteristic curve* can be plotted by taking the locus of points of combined discharges at various constant heads as shown in Fig. 5.17.2 e.g. at constant head H, point G is plotted as $Q_A + Q_B = Q_G$.

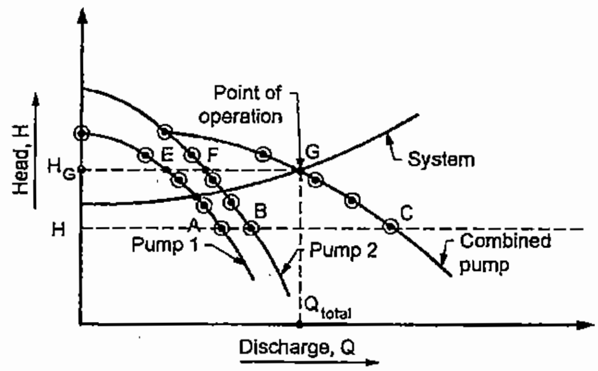


Fig. 5.17.2 : Characteristic curves of centrifugal pump in parallel

The system characteristic curve is also shown in Fig. 5.17.2 which intersects the combine characteristic curve at G. Point G represents the point of operation of multistage pump in parallel.

Total discharged at point G = Sum of discharges of pumps 1 and 2 at head H_G

i.e.

$$Q_{total} = Q_E + Q_F$$

$$H_{total} = H_G = H_E = H_F$$

Ex. 5.17.1 : A three stage centrifugal pump in series is designed for a total head of 90 m when running at 1440 rpm. Each pump has a discharge of 0.25 m^3/s. Entry to pump impeller is radial and its discharge angle is 30°. The velocity of flow is constant throughout and it is equal to 0.28 of vane velocity at outlet. The losses in impeller are equivalent to 30% of kinetic energy of water at exit. Determine the following :

(i) Vane velocity at exit

(ii) Impeller diameter and width of impeller at outlet.

(iii) Manometric efficiency and power to run the unit.

Soln. :

Refer outlet velocity diagram is shown in Fig. P. 5.17.1.

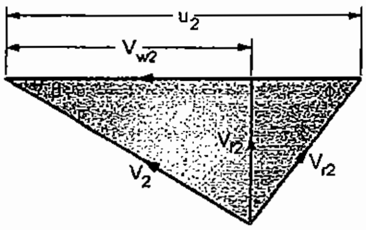


Fig. P. 5.17.1

Given : Number of stages, n = 3,

H_{total} = 90 m; N = 1440 rpm,

Q = 0.25 m^3/s ; α = 90 (radial entry),

hence, V_{w1} = 0 ; φ = 30° ;

$V_{f1} = V_{f2} = 0.28\, u_2$;

Head losses in impeller, $h_l = 0.3 \times \dfrac{V_2^2}{2g}$

(i) Vane velocity at exit, u_2 :

$$H_{total} = n \times H_m$$

$$\therefore \text{ Useful head, } H_m = \frac{90}{3} = 30 \text{ m}$$

$$\text{Actual head needed, } H = H_m + 0.3 \times \frac{V_2^2}{2g}$$

$$= 30 + 0.3 \times \frac{V_2^2}{2g} \quad \text{...(i)}$$

$$\text{But, } \frac{V_{w2} \cdot u_2}{g} = 30 + 0.3 \times \frac{V_2^2}{2g} \quad \text{...(ii)}$$

$$V_{w2} = u_2 - V_{f2} \cot \phi$$

$$= u_2 - 0.28\, u_2 \cot 30 = 0.515\, u_2 \quad \text{...(iii)}$$

$$V_2^2 = V_{f2}^2 + V_{\omega 2}^2 = (0.28\, u_2)^2 - (0.515\, u_2)^2$$

$$= 0.3436\, u_2^2 \quad \text{...(iv)}$$

On substituting the values from Equation (iii) and (iv) in Equation (ii),

$$\frac{0.515 \times u_2 \times u_2}{9.81} = 30 + \frac{0.3 \times 0.3436\, u_2^2}{2 \times 9.81}$$

$$\mathbf{u_2 = 25.2 \text{ m/s}} \quad \text{...Ans.}$$

(ii) Impeller diameter, D_2 and width, B_2 at outlet

$$u_2 = \frac{\pi D_2 N}{60}$$

$$\therefore D_2 = \frac{60\, u_2}{\pi N} = \frac{60 \times 25.2}{\pi \times 1440}$$

$$= \mathbf{0.3342 \text{ m} = 334.2 \text{ mm}} \quad \text{...Ans.}$$

$$V_{f2} = 0.28\, u_2 = 0.28 \times 25.2 = 7.056 \text{ m/s}$$

$$Q = \pi D_2 B_2 V_{f2}$$

$$0.25 = \pi \times 0.3342 \times B_2 \times 7.056$$

$$B_2 = 0.03375 \text{ mm} = \mathbf{33.75 \text{ mm}} \quad \text{...Ans.}$$

(iii) Manometric efficiency, η_{mano} and power to run the unit

$$H = 30 + 0.3 \frac{V_2^2}{2g}$$

$$\text{But, } V_2^2 = 0.3436\, u_2^2$$

$$\therefore H = 30 + 0.3 \times 0.3436 \times \frac{25.2^2}{2 \times 9.81}$$

$$= 33.336 \text{ m}$$

$$\eta_{mano} = \frac{H_m}{H} = \frac{30}{33.336}$$

$$= \mathbf{0.8999 \text{ or } 89.99\%} \quad \text{...Ans.}$$

Power to drive the pump,

$$P = \text{Number of stages} \times \rho \cdot g \cdot Q \cdot H \times 10^{-3} \text{ (kW)}$$

$$= 3 \times 1000 \times 9.81 \times 0.25 \times 33.336 \times 10^{-3}$$

$$= \mathbf{245.27 \text{ kW}} \quad \text{...Ans.}$$

Ex. 5.17.2 : A four stage centrifugal pump in series is required to develop a total manometric head of 640 m when running at 1500 rpm and discharging at 0.2 m^3/s. Vane outlet angle are 35°. Impeller diameter and width at outlet are 600 mm and 50 mm respectively. Find the manometric efficiency.

Soln. : Given : Number of stages, n = 4;

$(H_m)_{total}$ = 640 m; N = 1500 rpm;

Q = 0.2 m^3/s; ϕ = 35°;

D_2 = 600 mm = 0.6 m, B_2 = 50 mm = 0.05 m.

Vane velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 1500}{60} = 47.12 \text{ m/s}$$

$$V_{f2} = \frac{Q}{\pi D_2 B_2} = \frac{0.2}{\pi \times 0.6 \times 0.05} = 2.122 \text{ m/s}$$

$$\therefore V_{w2} = u_2 - V_{f2} \cot \phi$$

$$= 47.12 - 2.122 \cot 35 = 44.09 \text{ m/s}$$

$$H_m \text{ per impeller} = \frac{(H_m)_{total}}{n} = \frac{640}{4} = 160 \text{ m}$$

Manometric efficiency,

$$\eta_{mano} = \frac{g \cdot H_m}{V_{w2} \cdot u_2} = \frac{9.81 \times 160}{44.09 \times 47.12}$$

$$= 0.7555 \text{ or } 75.55\% \qquad \text{...Ans.}$$

Ex. 5.17.3 : A pump is to deliver water from a tank against a static head of 40 m. The suction pipe is 50 m long and 25 cm diameter. The delivery pipe is 20 cm diameter and 1600 m long. The pump characteristic can be defined as H = 100 – 6000 Q^2 where H is the Head in meters and Q is discharge in m^3/s. Calculate the net head and discharge of the pump. 'f' for both the pipes is 0.02. Calculate power required to drive the pump if overall efficiency of the pump is 85%.

Soln. :

Given : H_s = 40 m, l_s = 50 m,

d_s = 25 cm = 0.25 m,

d_d = 20 cm = 0.2 m, l_d = 1600 m

Pump characteristic is

$$H = 100 - 6000 Q^2 \qquad \text{...(i)}$$

$$f = 0.02; \eta_o = 85\% = 0.85$$

(i) Net head, H and discharge, Q

Velocity in suction pipe,

$$V_s = \frac{Q}{\frac{\pi}{4} \cdot d_s^2} = \frac{4 \times Q}{\pi \times 0.25^2} = 20.37 \text{ Q}$$

Velocity in discharge pipe,

$$V_d = \frac{4}{\pi} \cdot \frac{Q}{d_d^2} = \frac{4 \times Q}{\pi \times 0.2^2} = 31.83 \text{ Q}$$

Friction losses,

$$h_{fs} = \frac{f \cdot l_s \cdot V_s^2}{d_s \cdot 2g} = \frac{0.02 \times 50 \times (20.37 \text{ Q})^2}{0.25 \times 2 \times 9.81} = 84.6 \text{ Q}^2$$

$$h_{fd} = \frac{f \cdot l_d \cdot V_d^2}{d_d \cdot 2g} = \frac{0.02 \times 1600 \times (31.83 \text{ Q})^2}{0.2 \times 2 \times 9.81}$$

$$= 8262.2 \text{ Q}^2$$

$\therefore$ Total head,

$$H = H_s + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$$

$$= 40 + 80.6 \text{ Q}^2 + 8262.2 \text{ Q}^2 + \frac{(31.83 \text{ Q})^2}{2 \times 9.81}$$

$$= 40 + 8394.4 \text{ Q}^2 \qquad \text{...(ii)}$$

(ii) Discharge of pump, Q

On equating Equations (i) and (ii) which would represent operation point of the pump,

$$100 - 6000 Q^2 = 40 + 8394.4 Q^2$$

$$\therefore \mathbf{Q = 0.0646 \ m^3/s} \qquad \text{...Ans.}$$

(iii) Head of pump, H :

From Equation (i)

$$H = 100 - 6000 Q^2$$

$$= 100 - 6000 (0.0646)^2 = \mathbf{74.96 \ m}$$

(iv) Power required to drive the pump, P_s

$$P_s = \frac{\rho \cdot g \cdot Q \cdot H \times 10^{-3}}{\eta_o} (kW)$$

$$= \frac{1000 \times 9.81 \times 0.0646 \times 74.96 \times 10^{-3}}{0.85}$$

$$= 55.89 \text{ kW} \qquad \text{...Ans.}$$

Ex. 5.17.4 : A centrifugal pump has a head discharge characteristic given by H = 35 – 2200 Q^2 where H = head developed by the pump in m and Q = corresponding discharge in m^3/s. The pump is to deliver a discharge against a static head of 12 m. The suction pipe is 150 mm in diameter, 20 m long and has friction factor value of 0.018. The delivery pipe is 200 mm in diameter, 400 m long and has a friction factor value of 0.02. Calculate the head and discharge delivered by the pump. If the overall efficiency of the pump is 0.7, calculate the driving power supplied to the pump.

Soln. :

Given : $H_m = H = 35 - 2200\,Q^2$ ...(i)

where H is in m, Q in m^3/s, $H_s = 12$ m;

d_s = 150 mm = 0.15 m, l_s = 20 m,

f_s = 0.018, d_d = 200 mm = 0.2 m,

l_d = 400 m, f_d = 0.02;

η_o = 0.7

(i) Head, H and discharge, Q delivered by pump :

Manometric head,

$$H_m = H_s + h_{fs} + h_{fd} \quad ...(ii)$$

$$\therefore H_m = H_s + \frac{f_s \cdot l_s \cdot V_s^2}{d_s \times 2g} + \frac{f_d \cdot l_d \cdot V_d^2}{d_d \times 2g} \quad \left(\because h_f = \frac{f/V^2}{d \cdot 2g}\right)$$

But,
$$V_s = \frac{Q}{\frac{\pi}{4} \cdot d_s^2} = \frac{4 \times Q}{\pi \times (0.15)^2} = 56.59\,Q \quad ...(iii)$$

and
$$V_d = \frac{4Q}{\pi \cdot d_d^2} = \frac{4}{\pi} \times \frac{Q}{(0.2)^2} = 31.83\,Q \quad ...(iv)$$

From Equations (ii), (iii) and (iv)

$$\therefore H_m = 12 + \frac{0.018 \times 20 \times (56.59\,Q)^2}{0.15 \times 2 \times 9.81} + \frac{0.02 \times 400 \times (31.83\,Q)^2}{0.2 \times 2 \times 9.81}$$

$$H_m = 12 + 391.73\,Q^2 + 2065.54\,Q^2$$

$$= 12 + 2457.27\,Q^2 \quad ...(v)$$

Equating Equations (i) and (iv), we get

$$35 - 2200\,Q^2 = 12 + 2457.27\,Q^2$$

$\therefore$ Discharge, Q = **0.07 m³/s** ...**Ans.**

Head, H = $35 - 2200\,Q^2 = 35 - 2200 \times 0.07^2$

= **24.22 m** ...**Ans.**

Ex. 5.17.5 : A centrifugal pump running at 1000 rev/min gave the following relations between head and discharge :

Discharge (m^3/min)	0.0	4.5	9.0	13.5	18.0	22.5
Head (m)	22.5	22.2	21.6	19.5	14.1	0.0

The pump is delivering water against a static head of 15 m. The total required system head static + friction + velocity head) is given by $H_m = 15 + 19.98 \times 10^{-3}\,Q^2$ where Q is in m^3/min. Determine the discharge in m^3/min and power required to drive the pump if its overall efficiency is 75%

Soln. :

Given : N = 1000 rpm; Static head, h_s = 15 m;

total head (static + friction + velocity head),

$H_m = 15 + 19.98 \times 10^{-3}\,Q^2$ (Q is in m^3/min);

Overall efficiency, η_o = 75% = 0.75.

For pump

Discharge (m^3/min)	0.0	4.5	9.0	13.5	18.0	22.5
Head (m)	22.5	22.2	21.6	19.5	21.5	0.0

For system

Using, $H_m = 15 + 19.98 \times 10^{-3}\,Q^2$

Discharge (m^3/min)	0.0	4.5	9.0	13.5	18.0	22.5
Head (m)	15.0	15.4	16.62	18.84	21.5	25.1

The characteristic curves for pump and system can be calculated as shown in Fig. P. 5.17.5.

Discharge, Q_p

The intersection of two graphs is shown by point Y. It represents the operating point. From graph we get the discharge and head as follows :

Q_p = 14 m³/s and H_p = 19 m ...**Ans.**

Power required to drive the pump, P_s

$$P_s = \frac{\rho \cdot g \cdot Q \cdot H_p \times 10^{-3}}{\eta_o} \text{(kW)}$$

$$= 1000 \times 9.81 \times \frac{14}{60} \times 19 \times 10^{-3} \times \frac{1}{0.75}$$

= **58 kW** ...**Ans.**

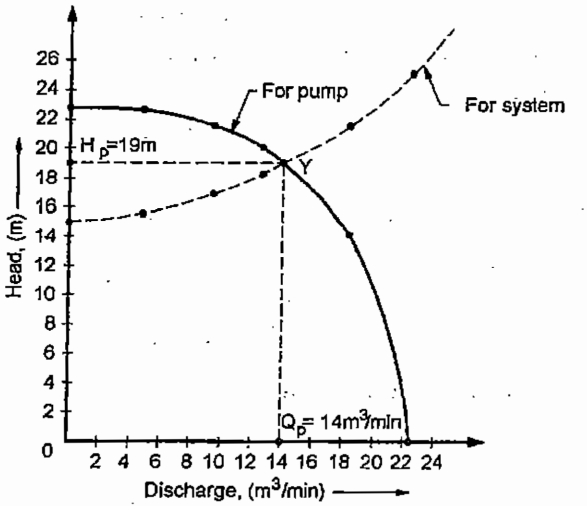


Fig. P. 5.17.5

Ex. 5.17.6 : A certain centrifugal pump has a head discharge relationship is as given in the table below.

Discharge, Q(lit/s)	0	10	20	30	40	50
Head H(m)	20.2	20.6	19.7	17.5	14.2	8.0

The pump delivers water through a 150 mm diameter and 500 m long pipeline. The coefficient of friction for the pipe is 0.025. The pump is to operate against a head of 15 m. Assuming the efficiency of the pump as 70%, determine the discharge and power required. **SPPU : Dec.19, 11 Marks**

Soln. :

Diameter of pipe, $d = 150 \text{ mm} = 0.15 \text{ m}$,

Length, $L = 500 \text{ m}$,

$f = 0.025$, Static head,

$H_S = 15 \text{ m}$

$\eta_{pump} = 70\% = 0.7$

$$h_f = \frac{fLV^2}{d \cdot 2g};$$

$$\text{But } V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$$

$$\therefore \quad h_f = \frac{fL}{d \cdot 2g} \times \left(\frac{4Q}{\pi d^2}\right) = \frac{16\, LQ^2}{2gd^5 \cdot \pi^2}$$

$$h_f = \frac{16 \times 0.025 \times 500\, Q^2}{2 \times 9.81 \times (0.15)^5 \times \pi^2}$$

$$h_f = 13601.1\, Q^2 \text{ (Where, Q is in m}^3\text{/s)}$$

$$h_f = 13601.1 \left(\frac{Q}{1000}\right)^2 \qquad \text{(if Q is in litres/s)}$$

$$h_f = 13601.1 \times 10^{-6}\, Q^2$$

System resistance,

$$H_{syst.} = H_s + h_f$$

$$H_{syst.} = 15 + 13601.1 \times 10^{-6}\, Q^2 \qquad \text{...(i)}$$

$$= 15 + 0.0136011\, Q^2$$

The system resistance characteristic table using equation (i) can be prepared to provide the relation between H and Q as follows :

Discharge Q (lps)	0	10	20	30	40	50
Head H(m)	15	16.36	20.44	32.24	36.76	49.00

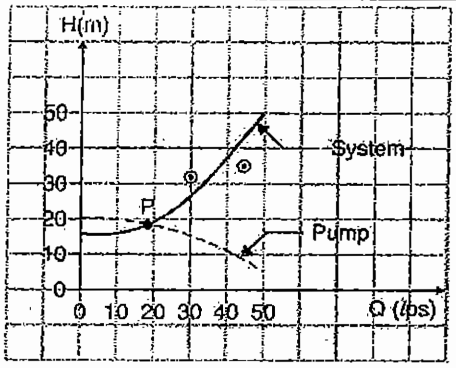


Fig. P.5.17.6 : Head-Discharge Curve

Fig. P.5.17.6 shows the plot of H-Q curves for the pump and system resistance. The intersection of these two curves on graph is point P. This point represents the operating point of the pump,. At point P from graph we get :

$$Q_p = 18.5 \text{ lps and } H_p = 19.4 \text{ m}$$

(i) Discharge,

$$Q_p = \mathbf{18.5 \text{ lps}} \qquad \text{...Ans.}$$

(ii) Power required to operate the pump, P

$$P = \rho g Q_p H_p \times \frac{1}{\eta_{pump}} \times 10^{-3} \text{ (kW)}$$

$$= 1000 \times 9.81 \times \frac{18.5}{1000} \times 19.4 \times \frac{1}{0.7} \times 10^{-3}$$

$$= \mathbf{5.03 \text{ kW}} \qquad \text{... Ans.}$$

Ex. 5.17.7 : Test on a single centrifugal pump running at constant speed gave the following results :

Q (LMP)	0	225	455	680	910	1135
H(m)	12.7	12.6	11.9	10.4	7.8	3.7
η(%)	0	48	68	76	70	50

Two such pumps are installed to run in parallel with common suction and delivery pipes for a static head of 6.4 m. The friction and other losses are given by $2.02\, Q^2 \times 10^{-6}$ m where Q is in LPM. Calculate the discharge and power required when (i) only one pump is used (ii) two pumps are used in parallel. **SPPU - May 11**

Soln. : Given : Static head, $H_s = 6.4$ m;

Friction losses, $h_{fs} = 2.02\, Q^2 \times 10^{-6}$ m (Q is in LPM)

$$\therefore \quad H_{system} = H_s + h_{fs} = 6.4 + 2.02\, Q^2 \times 10^{-6}$$

Discharge (LPM)	0	225	455	680	910	1135
H_{sys}	6.4	6.50	6.82	7.33	8.07	9.00

Head discharge characteristics for pump and system can now be plotted as shown in Fig. P.5.17.7.

The operating point gives,

$$Q = 870 \text{ LPM}, H = 7.9 \text{ m}$$

(i) Discharge of one pump,

$$Q = 870 \text{ LPM} = 0.87 \text{ m}^3/\text{min}$$

(ii) Discharge of two pumps in parallel

H	Q	$h_f = 2.02\times10^{-6}Q^2$	H_m	η
12.7	0	0	6.4	0
12.5	450	0.4091	6.809	48
11.9	910	1.673	8.07	68
10.4	1360	3.736	10.136	76
7.3	1820	6.69	13.09	70
3.7	2270	10.41	16.80	50

For parallel pumps, theoretically discharge is doubled

$\therefore$ New table will be $H_m = H_{st} + h_f$; $H_{st} = 6.4$ m

Now, a graph is plotted, against pump head $\times$ Q and $H_m \times Q$

Intersection gives new output

This gives $Q = 1395$ lpm i.e. $Q = 0.02325 \text{ m}^3/\text{s}$

$H = 10.2$ m ...**Ans.**

From graph, an output,

$$\eta_0 = 76\%$$

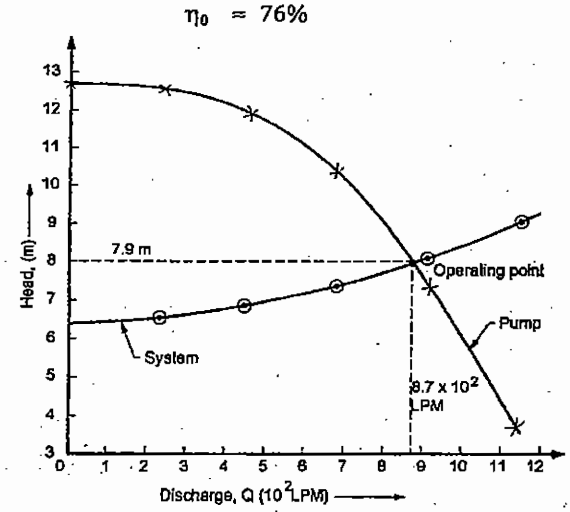


Fig. P. 5.17.7

(iii) Power required for one pump,

$$P = \frac{\rho \cdot g \cdot Q \cdot H}{\eta} \times 10^{-3} \text{ kW}$$

$$= 1000 \times 9.81 \times \left(\frac{0.87}{60}\right) \times \frac{7.9}{0.7} \times 10^{-3} = \mathbf{1.605 \text{ kW}}$$

(iv) Power required for two pumps, P_1

$$H = 10.2$$

Now, Total power $= \dfrac{\rho g Q H}{1000 \times \eta_0}$ kW

$$= \frac{9.81 \times 0.02325 \times 10.2}{0.76}$$

$$= \mathbf{3.061 \text{ kW}}$$ ...**Ans.**

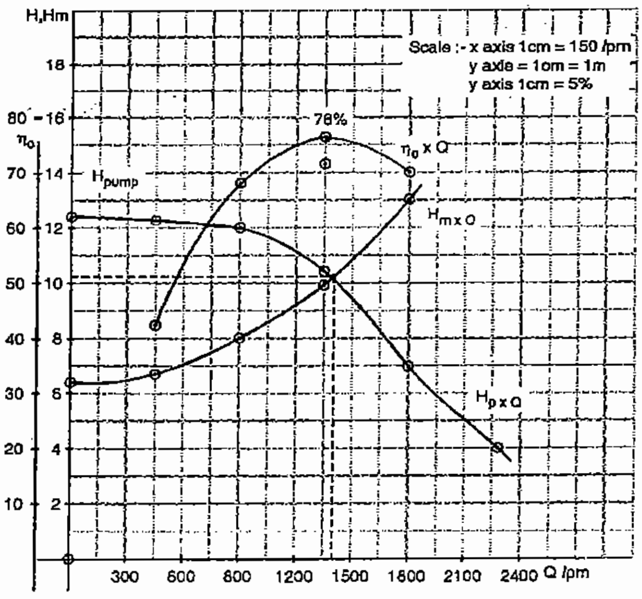


Fig. P. 5.17.7(a)

Ex. 5.17.8 : Two centrifugal pumps have the head discharge characteristics as follows :

Discharge lps	0	4	8	12	16	20	24
Head of pump number 1 in m	50.0	51.8	50.8	48	42.5	32.5	18.3
Head of pump number 2 in m	46.7	45.9	44.2	40.3	34.3	26	17

Both pumps are installed together and are required to pump water through a pipe 150 mm diameter having f = 0.02. Calculate the head under which the pumps are working and discharges in lps if they are connected in parallel, static lift is 15 m and suction and delivery pipes are 360 m long.

Soln. :

Given : Pipe diameter, d = 150 mm = 0.15 m, f = 0.02;
Pumps are connected in parallel;

$H_s = 15$ m, $\quad l = l_s + l_d = 360$ m;

$h_s = 15 \text{ m} = l_s$

Let, Q be the total discharge in lps and Q_1 and Q_2 be discharges of pump number 1 and 2 respectively.

Total head,

$$H = H_s + \text{friction losses } (h_{fs} + h_{fd})$$

$$= H_s + \frac{f \cdot l \cdot V^2}{d \times 2g}\left(\text{But}, V = \frac{Q \times 10^{-3}}{A} = \frac{Q \times 10^{-3}}{\frac{\pi}{4}d^2}\right)$$

$$= 15 + \frac{0.02 \times 360}{2 \times 9.81 \times 0.15}\left[\frac{4\,Q \times 10^{-3}}{\pi \times 0.15^2}\right]^2$$

$$= 15 + \frac{Q^2}{127.64} = 15 + \frac{(Q_1 + Q_2)^2}{127.64} \quad \text{....(i)}$$

System characteristic can be calculated from Equation (i) as shown in Table P.5.17.8(a) below :

Table P. 5.17.8(a)

Discharge Q (lps)	0	8	16	24	32	40	48	56
Head, H (m)	15	15.5	17.0	19.51	23.02	27.54	33.05	39.57

Refer Fig. P. 5.17.8.

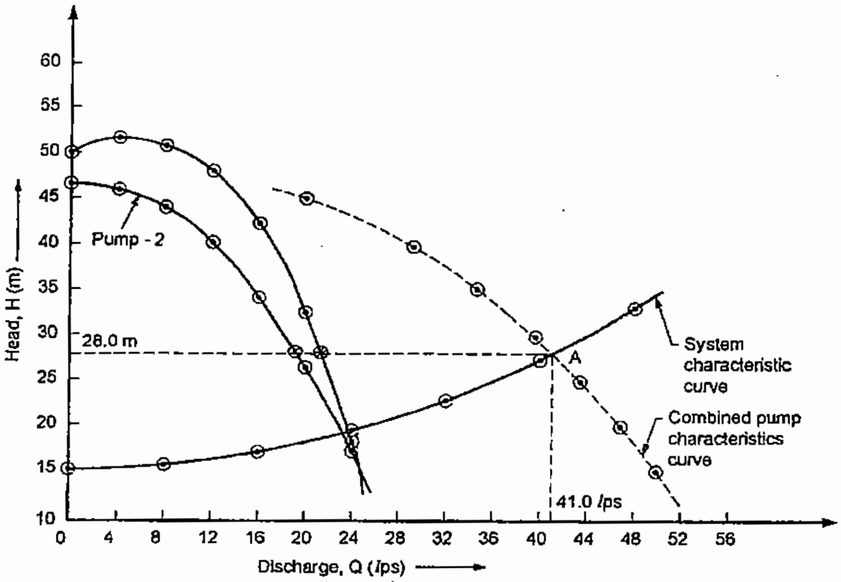


Fig. P. 5.17.8

Head-discharge characteristic curves can be drawn as follows :

1. From given data in the example, draw characteristic curves for individual pump numbers 1 and 2.
2. Since pumps run in parallel, total discharge $Q = Q_1 + Q_2$ at same head H.
3. Assuming H = 15, 20, 25, 30 ... to 45 m determine$(Q_1 + Q_2)$ from graph. These values are given in Table P.5.17.8(b).
4. Draw the combined characteristic curve as shown by dotted lines in the Fig. P. 5.17.8.
5. From Table 1, draw the system characteristic curve.
6. Intersection of combined pump and system characteristic curves will represent the operational point as represented by point A. From Fig. P. 5.17.8 at point A we get,

Table P. 5.17.8(b)

Head (m)	15	20	25	30	35	40	45
Q_1 (lps)	24.7	23.4	22.4	20.8	19.4	17.2	14.5
Q_2 (lps)	24.8	22.7	20.5	18.4	15.6	12.4	6.0
$Q_1 + Q_2$ (lps)	49.5	46.1	42.9	39.2	35.0	29.6	20.5

Q = **40.8 litres/s and H = 28 m** ...Ans.

– Discharge by pump number 1,

Q_1 = **21.6 lps at H = 28 m** ...Ans.

- Discharge by pump number 2,

$$Q_2 = 19.2 \text{ lps at } H = 28 \text{ m} \qquad \text{...Ans.}$$

Ex. 5.17.9 : In Example 5.17.8, if the pump characteristics remain the same as shown with the same data for pipe size, friction factor. Calculate the heads and discharge in lps if the pumps are connected in series having the suction lift of 60 and the suction and delivery pipes are 900 m long.

Soln. :

Given : $H_s = 60$ m, $l_s + l_d = 900$ m; $d = 0.15$ m, $f = 0.02$

$$\text{Total head, } H = H_s + (h_{fs} + h_{fd}) = H_s + \frac{f \cdot l \cdot V^2}{d \cdot 2g} \qquad \text{...(i)}$$

$$\text{But, } V = \frac{Q \times 10^{-3}}{\frac{\pi}{4} \cdot d^2}$$

$$\text{where, Q is in lps } = \frac{4}{\pi} \times \frac{10^{-3} \times Q}{(0.15)^2} = \frac{Q}{17.671}$$

From Equation (i),

$$H = 65 + 0.02 \times 900 \times \left(\frac{Q}{17.671}\right)^2 \times \frac{1}{0.15}$$

$$\times \frac{1}{2 \times 9.81} \quad 65 + \frac{Q^2}{51.06} \qquad \text{...(ii)}$$

Computed value of H at different discharge from Equation (ii) are shown in Table 5.17.9(a).

Table P.5.17.9(a) : System characteristics

Q (lps)	0	4	8	12	16	20	24
H	65	65.31	66.25	67.82	70.01	72.83	76.28

The discharge characteristic curves for the system as per Table P.5.17.9(a) are shown in Fig. P. 5.17.9 *Given as per example.*

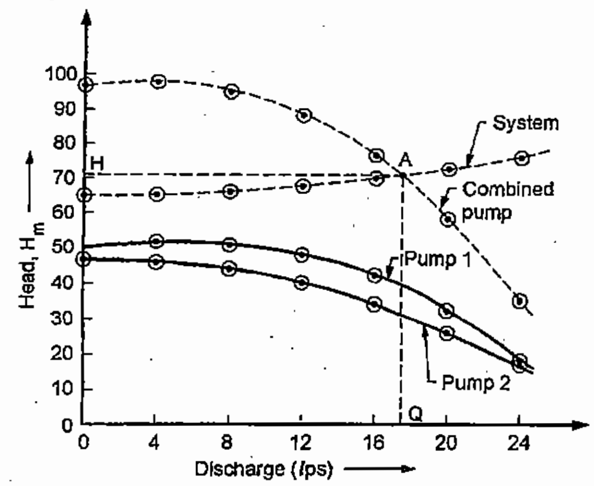


Fig. P. 5.17.9 : Characteristic curves

Table 5.17.9(b) : Individual pump and combined characteristics

Q (lps)	0	4	8	12	16	20	24
Head for pump number 1 (m) H_1	50	51.8	50.8	48	42.5	32.5	18.3
Head for pump number 2 (m) H_2	46.7	45.9	44.2	40.3	34.3	26	17
Combined head	96.7	97.7	95.0	88.3	76.8	58.5	35.3

1. Since the pumps run in series, the total head,
   H = Head of pump 1, H_1 + Head of pump 2, H_2.
2. Therefore, from Table P.5.17.9(b) the pump characteristic curves for pump 1, pump 2 and combined pump in series can be drawn.
3. The intersection of system and combined pump characteristic curves intersect at A. It gives the operating conditions of pump when working in series.

On reading the values from Fig. P. 5.17.9 we have,

Total head, $H = 71$ m;

Total discharge, $Q = Q_1 = Q_2 = 16.6$ lps,

$H_1 = 39$ m, $H_2 = 32$ m. ...**Ans.**

5.18 Cavitation in Centrifugal Pumps

University Questions

Q. Explain cavitation in centrifugal pumps. **SPPU : Dec. 13, Dec.15**

Q. What do you mean by cavitation ? What are its effects ? How we can overcome the cavitation effect in centrifugal pump. **SPPU : May 18**

Q. What is cavitation ? Explain the phenomenon of cavitation. **SPPU : Dec. 18**

Q. Explain NPSH and cavitation with respect to centrifugal pump. **SPPU : May 19**

Q. What did you understand by the term cavitation and NPSH in centrifugal pump ? How cavitation can be avoided. **SPPU : Dec. 19**

Cavitation is defined as the phenomenon of formation of vapour bubbles in the region of flowing liquid where its pressure falls below the vapour pressure of liquid, then the liquid will vapourise and flow will no longer will be continuous.

When these vapour bubbles travel into the region of higher pressure, they suddenly collapse on the metallic surfaces and the surrounding liquid rushes to fill the cavities of vapour bubbles. The severe rush of liquid causes the development of extremely high pressures.

Prolonged cavitation causes erosion and pitting of metals, severe vibrations and noise. Fig. 5.18.1 shows the suction side of pump. Consider two points A and B at inlet to eye of impeller and on the sump level respectively.

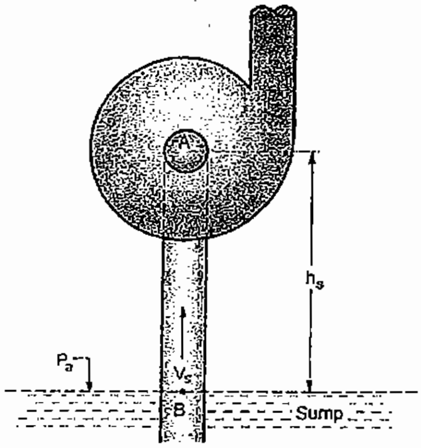


Fig. 5.18.1 : Cavitation in pumps

Let,
- p_a = absolute atmospheric pressure;
- P_s = absolute pressure at eye of impeller
- h_s = suction lift
- V_s = velocity in suction pipe
- h_{fs} = friction losses in suction pipe

By Bernoulli's theorem,

$$\frac{p_a}{w} = \frac{p_s}{w} + h_s + \frac{V_s^2}{2g} + h_{fs}$$

$$\therefore \quad \frac{p_s}{w} = \frac{p_a}{w} - \left(h_s + \frac{V_s^2}{2g} + h_{fs}\right) \qquad ...(5.18.1)$$

Therefore, the absolute pressure at inlet to eye of impeller falls below that of atmospheric pressure.

5.18.1 Estimation of Maximum Permissible Suction Lift (Limited by Cavitation)

University Questions

Q. Define the maximum suction lift. State the expression to calculate it. What factors affect its values ? **SPPU : May 16, Dec. 18**

Q. Derive relation for maximum suction lift of a centrifugal pump. **SPPU : May 18**

Q. State some methods of eliminating or reducing cavitation. **SPPU : Dec. 18**

***Definition of maximum suction lift :** The suction lift of pump at which the pressure of inlet does not fall below the vapour pressure of liquid is called the maximum suction lift.*

As discussed above, the cavitation occurs when the pressure at inlet to impeller falls below the vapour pressure of liquid, p_v. Therefore, to avoid cavitation the condition is that:

$$\frac{p_s}{w} \geq \frac{p_a}{w} \qquad ...(5.18.2)$$

Therefore, the limiting value of absolute suction pressure at inlet to impeller is $p_s = p_v$. On substituting this value in Equation (5.18.1), **the maximum permissible suction lift**, h_s to avoid cavitation in the pump can be estimated as follows :

$$\frac{p_v}{w} = \frac{p_a}{w} - \left(h_s + \frac{V_s^2}{2g} + h_{fs}\right)$$

$\therefore$ **Permissible suction lift,**

$$h_s = \left(\frac{p_a - p_v}{w}\right) - \frac{V_s^2}{2g} - h_{fs} \qquad ...(5.18.3)$$

The factors which affect the permissible suction lift are diameter of suction pipe, inlet and outlet diameters of impeller, shape of impeller vanes and their number, area of flow and location of pump above sump level.

Note : The vapour pressure of liquid increases with the increase in temperature.

5.18.2 Net Positive Suction Head (NPSH)

University Questions

Q. Explain N.P.S.H in centrifugal pumps. **SPPU : Dec. 13**

Q. Explain NPSH and cavitation with respect to centrifugal pump. **SPPU : May 19**

Q. What did you understand by the term cavitation and NPSH in centrifugal pump? **SPPU : Dec. 19**

The **net positive suction head (NPSH)** *is defined as the absolute pressure head at the inlet to the pump to force the liquid into the pump at a given temperature.*

Fig. 5.18.2 shows how the pressure at the eye of impeller falls below the atmospheric pressure.

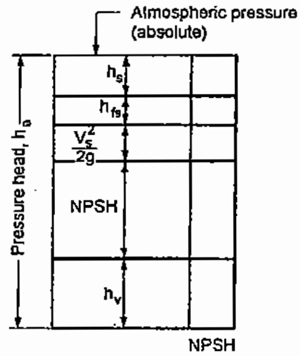


Fig. 5.18.2 : NPSH

It is necessary that NPSH is positive to avoid cavitation and force the liquid into the eye of impeller.

Therefore, $$\text{NPSH} = \frac{p_a}{W} - \left(\frac{p_v}{W} + h_s + \frac{V_s^2}{2g} + h_{fs}\right)$$

$$\text{NPSH} = h_a - \left(h_v + h_s + \frac{V_s^2}{2g} + h_{fs}\right) \quad ...(5.18.4)$$

The value of NPSH is mainly affected by the temperature of liquid to be handled by the pump since h_v and h_{fs} are the function of temperature.

5.18.3 NPSH (Required)

University Question

Q. Explain the term NPSH required. **SPPU : May 11, Dec. 19**

Required NPSH is based on the pump design and its speed and capacity. It is then tested to determine the NPSH at which the pump gives the maximum efficiency. Accordingly, the value of NPSH required is specified by the manufacturer of the pump. It represents the minimum value of NPSH to avoid cavitation at specified discharge and speed.

5.18.4 NPSH (Available)

University Questions

Q. Explain the term NPSH available. **SPPU : May 11, Dec. 19**

Q. State some methods of eliminating or reducing cavitation. **SPPU : Dec. 18**

Available NPSH of the pump is estimated after installation of pump which is based on the suction pipe diameter, liquid to be handled and temperatures of operation (both the place of installation and the liquid temperature), length of suction pipe and its coefficient of friction and flow rates. While estimating this NPSH, the diameter of suction pipe must be same as per manufacturer's design specifications.

In order to have cavitation free operation of pump the available NPSH must be greater than the required NPSH. NPSH available must be higher than NPSH required.

5.18.5 Thoma's Cavitation Factor

We have seen above, the cavitation may occur when the pressure of the liquid on suction side drops below the vapour pressure of liquid.

The cavitation in a pump can be noted by the sudden drop in discharge and it's efficiency alongwith noise and vibrations and the intensity of cavitation increases with the decrease in NPSH.

Therefore, the cavitation imposes a limit on the discharge by the pump and its speed of rotation.

Equation (5.18.3) can be rewritten in the following form :

$$\frac{V_s^2}{2g} = \left(\frac{p_a - p_v}{W}\right) - h_s - h_{fs} = h_a - h_v - h_s - h_{fs}$$

Dividing by manometric head, H_m.

$$\frac{V_s^2}{2g\,H_m} = \frac{h_a - (h_v + h_s + h_{fs})}{H_m}$$

In the above equation $(V_s^2/2g\,H_m)$ is called **Thoma's cavitation factor, σ.** Therefore,

Thomas cavitation factor,

$$\sigma = \frac{h_a - (h_v + h_s + h_{fs})}{H_m} = \frac{\text{NPSH}}{H_m} \quad ...(5.18.5)$$

The cavitation will occur if the value of Thomas cavitation factor, σ is less than the critical value, σ_c at which the cavitation just begins. It implies that :

$$\sigma \geq \sigma_c \text{ i.e. NPSH} \geq \sigma_c\, H_m \quad ...(5.18.6)$$

5.18.6 Suction Specific Speed (N_{SS})

Suction specific speed is another parameter used in pumps to indicate whether the cavitation will occur. It is defined as the speed of geometrically similar model pump which will operate with similar degree of cavitation.

Specific speed of the pump is given as :

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}} \quad ...(5.18.7)$$

Therefore, replacing H_m by NPSH for similar degree of cavitation, the **suction specific speed**, can be written as :

Suction specific speed, $$N_{SS} = \frac{N\sqrt{Q}}{(\text{NPSH})^{3/4}} \quad ...(5.18.8)$$

On dividing the Equation (5.18.7) by Equation (5.18.8) we have,

$$\frac{N_S}{N_{SS}} = \frac{(\text{NPSH})^{3/4}}{(H_m)^{3/4}} = (\sigma)^{3/4} \qquad \left(\because \frac{\text{NPSH}}{H_m} = \sigma\right)$$

$$\therefore \text{ Cavitation factor, } \sigma = \left(\frac{N_S}{N_{SS}}\right)^{4/3} \quad ...(5.18.9)$$

5.19 Priming of a Centrifugal Pump

University Questions

Q. Explain Priming of centrifugal pump. SPPU : Dec. 12,May 14, Dec.15

Q. What is priming of centrifugal pump and why it is necessary ? SPPU : May 13

Q. Write a short note on Priming of a centrifugal pump. SPPU : Dec. 16

The operation of filling the casing, impeller and suction pipe and the portion of delivery pump upto delivery valve is called **priming.**

In case the priming of pump is not done and the pump is not under the operation, the water present in the pump and suction pipe will flow back to the sump. The space occupied by water will be filled by air.

If the pump is now started, the air pockets inside the impeller may give rise to vortices and causes the discontinuity of flow. Under these conditions, the flow of fluid does not commence and the pump runs dry. It causes the rubbing and seizing of the wearing rings, increases noise level and vibrations and finally may cause the serious damage to pump.

5.19.1 Methods of Priming

The pump can be primed in any of the following ways :

1. Priming by hand

It is used for small pumps. A foot valve is essential at the bottom of suction pipe.

A funnel or priming cup is provided to fill the water by hand. An air vent is provided in the casing of pump. When the water is filled, the air escapes through the air vent. It is closed once the priming is completed.

2. Connection with city water mains

The pump may be connected to city water supply mains which can be opened to fill water in the pump for priming.

3. By providing priming pumps

These pumps are small reciprocating pumps connected to the pump casing and operated by hand or by power. **Steam ejectors** are also sometimes used.

4. Self priming devices

These are particularly valuable under the following conditions :

(a) Where automatic starting and stopping of pump is required.

(b) Where absolute reliability of starting of pump is essential at short notice e.g. for fire pumps.

(c) Where pumps are needed to work on leaky suction pipe or with air pockets.

A self priming device generally consists of some form of automatic air exhauster like vacuum pump incorporated in the pump casing. The vacuum pump shaft is coupled and driven by the shaft of centrifugal pump. When the pump is started the air is sucked out and water is pumped.

Another self priming device is shown in Fig. 5.19.1.

This system is suitable even if the foot valve of the pump is leaking.

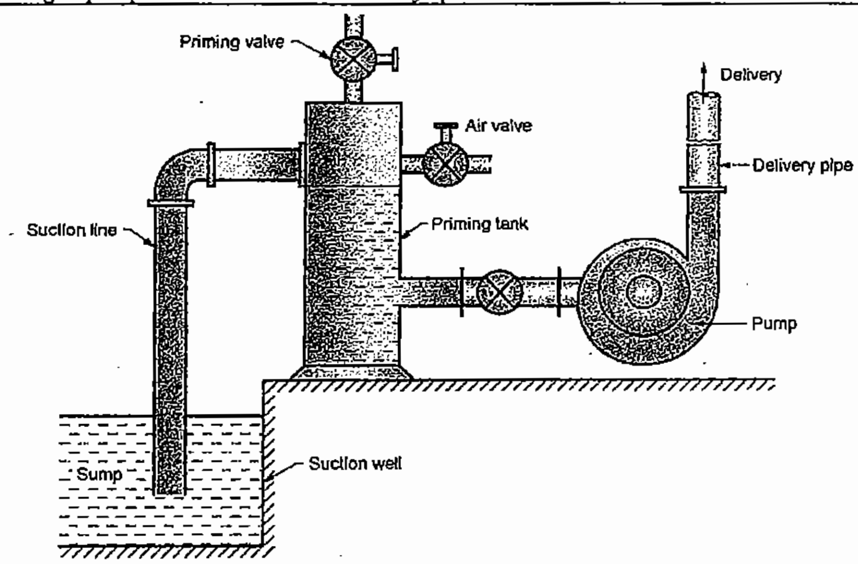


Fig. 5.19.1 : Self priming of centrifugal pump

It consists of a priming tank between the suction line and the pump. It is provided with an air valve and a priming valve at its top. Suction line is connected at the top of priming tank as shown in Fig. 5.19.1.

Initially the priming tank is filled with water through priming valve with an open air valve. Then both valves are closed.

When pump is started, it draws water from priming tank. The water level in it falls. Space created by flow of water to pump is filled by expanding air in the priming tank. It creates a vacuum due to which water rushes into the primary tank through the suction line. The primary tank remains full of water even when pump stops.

5.20 Axial Thrust in Centrifugal Pumps

The open type of impellers do not produce no axial thrust. However, the single entry impeller with enclosed vanes produces an axial thrust towards the suction.

Axial thrust is produced due to following :

(i) The change in momentum of liquid flow from axial to radial direction on the vanes causes a force in direction of flow at inlet.

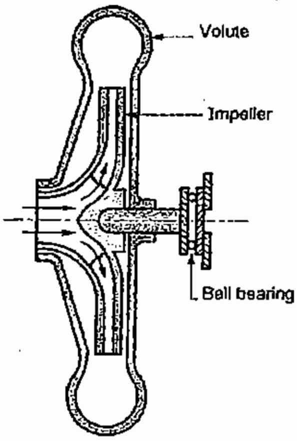
(ii) The pressure difference prevailing across the impeller causes static axial thrust towards the impeller inlet.

5.20.1 Methods of Balancing the Axial Thrust

In order to balance the applied thrust, following methods are adopted.

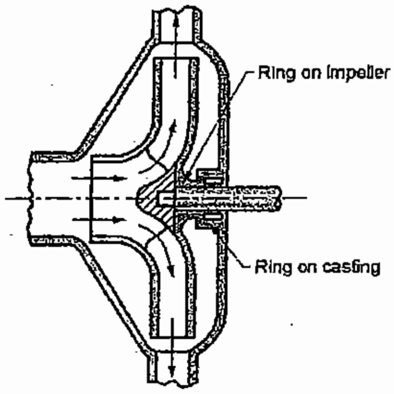
1. For small pumps

(i) A ball thrust bearing is provided in the direction of axial thrust as shown in Fig. 5.20.1(a).



(a) By ball thrust bearing

(ii) A cast iron ring in the casing is inserted which should fit in with similar ring cast integral with the impeller as shown in Fig. 5.20.1(b).



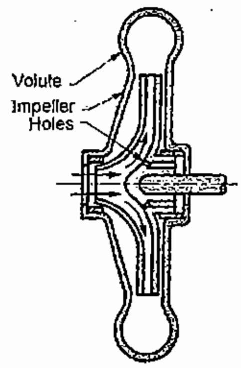
(b) By cast ring fitted in casing

Fig. 5.20.1 : Balancing of axial thrust in small pumps

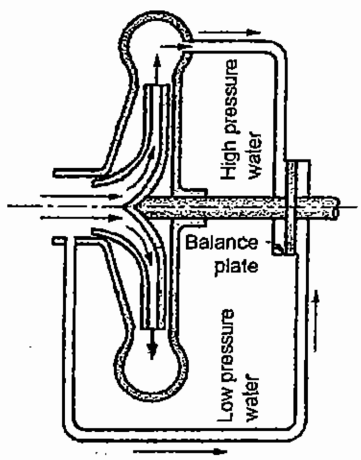
2. For large pumps

(i) *By using double suction impellers* as shown in Fig. 5.20.2(a) may be used to eliminate axial thrust. In this case, there would be equal and opposite axial thrusts acting on each suction side, hence they would balance out each other.

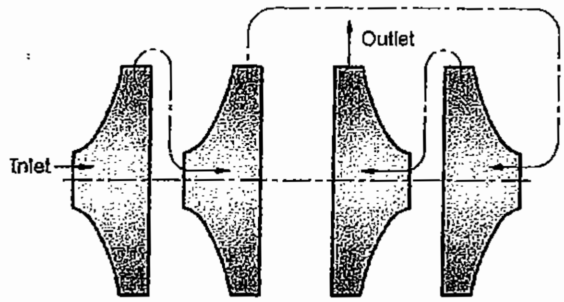
(a) Double suction impeller



(b) Relieving holes in impeller



(c) Balance plate



(d) Even number of impellers of a multistage pump

Fig. 5.20.2 : Balancing of axial thrust in large pumps

(ii) *By providing relieving holes in the impeller* which allows the suction pressure to act equally from both sides as shown in Fig. 5.20.2(b).

(iii) *By provision of balance plate or disc* fitted at the end of pump shaft as shown in Fig. 5.20.2(c). It is so connected to high pressure side and low pressure side that it produces a net force equal and opposite to axial thrust on suction side.

(iv) *The number of impellers of multistage centrifugal pumps are made even in number.* The suction of half of the impellers is kept on one side and the suction of remainder half of the impellers is kept from the other side so that the total axial thrust exerted on each side will neutralise each other. The system is shown in Fig. 5.20.2(d). It is similar to concept of double suction impellers.

5.21 Trouble Shooting of Centrifugal Pumps and their Remedies

Some of the faults commonly experienced during the operation of centrifugal pumps along with their causes and remedies are listed below in Table 5.21.1.

Table 5.21.1 : Faults in working centrifugal pumps

Sr. No.	Fault	Other Symptoms	Cause	Remedies
1.	No output	(i) Suction and delivery gauges read zero.	(i) Pump has lost its water due to air lock in suction pipe, air leaks in suction pipe, air leaks in stuffing boxes, or level of water dropped below strainers.	(i) STOP and Reprime Remake pipe joints Tighten glands or repack Lengthen suction pipe to lower strainer.
		(ii) Suction gauge reads zero but delivery gauge normal.	(ii) Speed too low to overcome the total head, or choke in delivery pipe.	(ii) Increase speed. Clear choke in delivery pipe.
2.	Poor output : It should be noted that unless a flow meter is provided, the pump attendant has very little to inform him that the output of the pump is not normal.	(i) Surging in delivery pressure gauge.	(i) Air leak in suction pipe or stuffing boxes, or air entering strainer.	(i) As above
		(ii) Suction gauge shows high vacuum, vibration and noise.	(ii) Choked strainer, foot valve or suction pipe. Vibration and noise caused by cavitation.	(ii) Clean strainer, foot valve or suction pipe.
		(iii) Suction gauge shows low vacuum.	(iii) Impeller partly choked. (iv) Partial choke in delivery pipe.	(iii) Clean impeller. (iv) Clear choke in delivery pipe.
		(iv) Pump not running upto speed.	(v) Engine or motor defective. (vi) Voltage low. (vii) Belt slipping.	(v) Replace or overhaul. (vi) Check and rectify. (vii) Tighten belt.

Ex. 5.21.1 : Tests on a pump model indicate a Thoma's cavitation factor $\sigma_C = 0.1$. A homologous unit is to be installed at a location where atmospheric pressure is 0.91 bar and vapour pressure as 0.035 (bar) (abs) and is to pump water against a head of 25 m. What is the maximum permissible suction lift/head ?

Soln. :

Given : $\sigma_c = 0.1$, $p_a = 0.91$ bar,

$p_v = 0.035$ bar, $H_m = 25$ m,

$$\sigma_c = \frac{(h_{atm} - h_v) - h_s}{H_m}$$

$$= \frac{\left(\frac{p_a}{\rho g} - \frac{p_v}{\rho g}\right) - h_s}{H_m} \quad \left(\because h = \frac{p}{\rho \cdot g}\right)$$

$$0.1 = \frac{\frac{0.91 \times 10^5}{9.81 \times 1000} - \frac{0.035 \times 10^5}{9.81 \times 1000} - h_s}{25}$$

Maximum permissible suction head,

$$h_s = 6.42 \text{ m} \quad \text{...Ans.}$$

Ex. 5.21.2 : A pump can deliver a discharge of 0.1 m³/s to a head of 30 m. The critical cavitation number for the pump is found to be 0.12. The pump is to be installed at a location where the barometric pressure is 96 kPa (abs) and the vapour pressure is 3.0 kPa (abs). Assuming an intake pipe friction of 0.3 m, determine the minimum value of NPSH. What would be the maximum allowable elevation above the sump water surface at which the pump can be located ?

Soln. :

Given : $Q = 0.1 \text{ m}^3/\text{s}$, $H = 30$ m;

$\sigma_C = 0.12$; $p_a = 96$ kPa (abs),

$p_v = 3.0$ kPa (abs.); $h_{fs} = 0.3$ m

$$h_a = \frac{p_a}{W} = \frac{p_a}{\rho \cdot g}$$

$$h_a = \frac{96 \times 10^3}{1000 \times 9.81} = 9.786 \text{ m}$$

Similarly, $$h_v = \frac{p_a}{\rho \cdot g} = \frac{3 \times 10^3}{1000 \times 9.81} = 0.306 \text{ m}$$

Cavitation factor, $$\sigma_c = \frac{\text{NPSH}, \Delta H}{H} \quad \text{i.e. } 0.12 = \frac{\Delta H}{30}$$

$\therefore$ NPSH, $\Delta H = 3.6$ m

ΔH = 3.6 m represents the NPSH required.

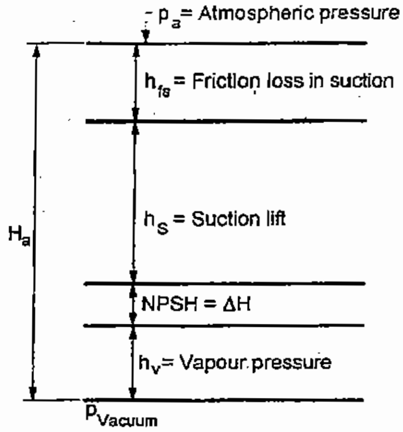


Fig. P. 5.21.2

$\therefore$ **Maximum suction lift possible, h_s**

$$h_s = h_a - (h_v + \text{NPSH} + h_{fs})$$

$$= 9.786 - (0.306 + 3.6 + 0.3) = \mathbf{5.58 \text{ m}} \text{...Ans.}$$

Ex. 5.21.3 : A centrifugal pump delivers 1 m³/min to a total head of 30 m. The critical cavitation number is 0.12. The pump is to be installed at a location where the barometric pressure is 96 kPa (ab) and the vapour pressure is 3.0 kPa (ab). Assume suction pipe friction equivalent of 0.3 m. Determine maximum allowable elevation above the sump water surface at which the pump can be installed. You can use following formula. Critical cavitation number = $\frac{\text{NPSH}}{H}$.

Soln. :

Given : $Q = 1 \text{ m}^3/\text{min} = \frac{1}{60} \text{ m}^3/\text{s}$,

$H_m = 30$ m;

Critical cavitation number,

$\sigma = 0.12$; $p_a = 96$ kPa (abs),

$p_v = 3$ kPa (abs), $h_{fs} = 0.3$ m;

Critical cavitation number,

$$\sigma = \frac{\text{NPSH}}{H_m} \quad \text{i.e. } 0.12 = \frac{\text{NPSH}}{30}$$

$\therefore$ Critical value of NPSH $= 0.12 \times 30$

$= \mathbf{3.6 \text{ m}}$ ...Ans.

Maximum allowable suction lift, h_s :

$$\text{NPSH} = \frac{p_a}{w} - \frac{p_v}{w} - h_s - h_{fs}; \text{ (But, } W = \rho \cdot g)$$

$$3.6 = \frac{96 \times 10^3}{1000 \times 9.81} - \frac{3 \times 10^3}{1000 \times 9.81} - h_s - 0.3$$

$\therefore$ h_s = **5.58 m** ...Ans.

Ex. 5.21.4 : Oil of specific gravity 0.9 is lifted by a centrifugal pump from an oil tank which is open to atmosphere. The loss of head in the suction pipe and foot valve amounts to 1.5 m. The velocity in the suction pipe is 3 m/s. If the oil starts vapourising at a pressure of 0.5 bar (Abs), estimate the maximum permissible suction lift. The barometer reading is 760 mm of mercury.

Soln. : Given : Specific gravity of oil, $S_o = 0.9$,

$h_{fs} = 1.5$ m, $V_s = 3$ m/s,

$p_v = 0.5$ bar (abs.) ;

$p_a = 760$ mm of Hg $= 10.33$ m of water column

$$\text{Velocity head} = \frac{V_s^2}{2g} = \frac{3^2}{2 \times 9.81} = 0.459 \text{ m of oil}$$

Atmospheric pressure in terms of oil column

$$= \frac{10.33 \text{ m of water columm}}{\text{Specific gravity of oil, } S_o}$$

$$= \frac{10.33}{0.9} = 11.48 \text{ m of oil column}$$

Vapour pressure equivalent to head of oil ($\because p = \rho gh$)

$$\therefore h_v = \frac{p_v}{\rho_o \cdot g} = \frac{0.5 \times 10^5}{(0.9 \times 1000) \times 9.81}$$

$$= 5.663 \text{ m}$$

We know that, $$h_a = h_s + \frac{V_s^2}{2g} h_{fs} + h_v$$

$$11.48 = h_s + 0.459 + 1.5 + 5.663$$

$$\therefore h_s = \mathbf{3.858 \text{ m}} \text{ of oil column} \quad \text{...Ans.}$$

Ex. 5.21.5 : A centrifugal pump delivers 1200 litres of water per minute. The diameter of suction pipe is 80 mm and the length of suction pipe is 1.25 times suction lift. The NPSH required as specified by the pump manufacturer is 1.5 m. The pump is to operate at a place having a barometric pressure of 10.1 m of water.

Estimate the maximum permissible suction lift and length of suction pipe to avoid the cavitation. Assume vapour pressure of water = 2.5 m(ab).

Use $h/s = \dfrac{flV^2}{2g \times d}$ where f = 0.02.

Soln. :

Given : $Q = 1200 \text{ litres/min} = \dfrac{1200}{1000} \times \dfrac{1}{60} = 0.02 \text{ m}^3/\text{s}$

$d_s = 80 \text{ mm} = 0.08$ m, $l_s = 1.25 h_s$;

NPSH = 1.5 m $\dfrac{p_a}{w} = h_a = 10.1$ m,

$h_v = 2.5$ m (absolute)

$$h_{ls} = \frac{f \cdot l \cdot V^2}{d \cdot 2g}, \text{ where } f = 0.02$$

$$V_s = \frac{Q}{A_s} = \frac{0.02}{\frac{\pi}{4} \times 0.08^2} = 3.98 \text{ m/s}$$

Friction loss in suction pipe,

$$h_{fs} = \frac{f \cdot l_s \cdot V_s^2}{d_s \cdot 2g} = \frac{0.02 \times 1.25 h_s \times 3.98^2}{0.08 \times 2 \times 9.81}$$

$$= 0.2523 h_s$$

$$\text{NPSH} = h_a - h_v - h_s - h_{fs}$$

$$1.5 = 10.1 - 2.5 - h_s - 0.2523 h_s$$

$\therefore$ Permissible suction lift,

$$h_s = \mathbf{4.871 \text{ m}} \quad \text{...Ans.}$$

Length of suction pipe,

$$l_s = 1.25 h_s = 1.25 \times 4.871 = \mathbf{6.089 \text{ m}} \quad \text{...Ans.}$$

5.22 Similarity Relations for Model Testing

Model (m) and prototype (p) : Similarity relations for identical centrifugal pumps are based on the following assumptions :

(i) All velocities are function of manometric head, H_m.

(ii) Pumps are geometrically similar and all linear dimensions are proportional to the impeller diameter.

(iii) All pumps have similar hydraulic and volumetric efficiencies.

The similarity between the pumps will be satisfied when :

1. **Specific speeds of model and prototypes are equal i.e.**

$$N_{sm} = N_{sp}$$

$$\left[\frac{N\sqrt{Q}}{H_m^{3/4}}\right]_m = \left[\frac{N\sqrt{Q}}{H_m^{3/4}}\right]_p \quad \text{...(5.22.1)}$$

2. **Head coefficient, K_H are equal**

$$u = \frac{\pi D N}{60}$$

$$u \propto DN \quad \text{...(i)}$$

$$u \propto V_f \propto \sqrt{H_m} \quad \text{...(ii)}$$

$$\therefore DN \propto \sqrt{Hm}$$

$$\frac{Hm}{D2\ N2} = \text{Constant, } K_H \quad \text{...(5.22.2)}$$

$$\therefore \left(\frac{H_m}{D^2 N^2}\right)_m = \left(\frac{H_m}{D^2 N^2}\right)_p \quad \text{...(5.22.3)}$$

3. **Discharge coefficient, K_Q are equal**

$$Q = \pi D B V_f$$

$$Q \propto D^2 \cdot V_f \propto D^2 \cdot \sqrt{H_m} \quad ...(i)$$

From Equation (5.22.2) ;

$$\frac{H_m}{D^2 N^2} = \text{Constant i.e.} \sqrt{H_m} \propto DN \quad ...(ii)$$

From Equations (i) and (ii) : $\sqrt{H_m} \propto \frac{Q}{D^2} \propto DN$

$$\text{or} \quad Q \propto D^3 \cdot N$$

$$\frac{Q}{D^3 \cdot N} = \text{Constant}, K_Q \quad ...(5.22.4)$$

$$\therefore \quad \left(\frac{Q}{D^3 \cdot N}\right)_m = \left(\frac{Q}{D^3 \cdot N}\right)_p \quad ...(5.22.5)$$

4. **Power coefficient, K_p are equal**

$$P = \rho \cdot g \cdot Q H_m \times \eta_o$$

$$\text{i.e.} \quad P \propto Q \cdot H_m \quad ...(i)$$

From Equation (5.22.4) ; $Q = D^3 N$ ...(ii)

$\therefore$ From Equation (5.22.2) ; $H_m \propto D^2 \cdot N^2$ ...(iii)

On substituting the values of Q and H_m in equation (i),

$$P \propto (D^3 N) \times D^2 N^2$$

$$P \propto D^5 N^3$$

$$\therefore \quad \frac{P}{D^5 \cdot N^3} = \text{Const}, K_p \quad ...(5.22.6)$$

$$\therefore \quad \left(\frac{P}{D^5 \cdot N^3}\right)_m = \left(\frac{P}{D^5 \cdot N^3}\right)_p \quad ...(5.22.7)$$

5.22.1 Ackeret Relationship for Pumps

Based on experimental results, the ackeret had suggested the emperical relation by considering the scale effect (difference in efficiency of model and prototype) as follows :

$$\frac{1-\eta_p}{1-\eta_m} = 0.5 + 0.5\left(\frac{D_m}{D_p}\right)^{0.2} \times \left(\frac{H_m}{H_p}\right)^{0.1} \quad ...(5.22.8)$$

Ex. 5.22.1 : The diameter of a centrifugal pump, which is discharging 0.03 m^3/s of water against a total head of 20 m is 0.40 m. The pump is running at 1500 r.p.m. Find the head discharge and ratio of powers of a geometrically similar pump of diameter 0.25 m when it is running at a speed at 3000 r.p.m.

Soln. : Given :

Pump 1 : Q_1 = 0.03 m^3/s, H_1 = 20 m, D_1 = 0.4 m, N_1 = 1500 rpm.

Pump 2 : D_2 = 0.25 m, N_2 = 3000 rpm.

(i) Discharge of pump-2, Q_2

For similar pumps, the discharge coefficient, $K_Q = \frac{Q}{D^3 N}$ must be equal. Therefore,

$$\frac{Q_1}{D_1^3 \cdot N_1} = \frac{Q_2}{D_2^3 \cdot N_2}$$

$$\text{i.e.} \quad Q_2 = Q_1 \cdot \left(\frac{D_2}{D_1}\right)^3 \cdot \left(\frac{N_2}{N_1}\right)$$

$$\therefore \quad Q_2 = 0.03\left(\frac{0.25}{0.4}\right)^3 \times \left(\frac{3000}{1500}\right)$$

$$= \mathbf{0.01465\ m^3/s} \quad \text{...Ans.}$$

(ii) Head, H_2

For similar pumps, the head coefficient, $K_H = \frac{H}{D^2 N^2}$ must be equal. Therefore,

$$\frac{H_1}{D_1^2 \cdot N_1^2} = \frac{H_2}{D_2^2 \cdot N_2^2}$$

$$H_2 = H_1\left(\frac{D_2}{D_1}\right)^2 \left(\frac{N_2}{N_1}\right)^2$$

$$= 20\left(\frac{0.25}{0.4}\right)^2 \left(\frac{3000}{1500}\right)^2$$

$$= \mathbf{31.25\ m} \quad \text{...Ans.}$$

(iii) Ratio of powers, $\frac{P_1}{P_2}$

For similar pumps, the power coefficient, $K_p = \frac{P}{D^5 \cdot N^3}$ must be equal.

Therefore,

$$\frac{P}{D_1^5 \cdot N_1^3} = \frac{P}{D_2^5 \cdot N_2^3}$$

$$\therefore \quad \frac{P_1}{P_2} = \left(\frac{D_1}{D_2}\right)^5 \left(\frac{N_1}{N_2}\right)^3 = \left(\frac{0.4}{0.25}\right)^5 \left(\frac{1500}{3000}\right)^3$$

$$= \mathbf{1.31072} \quad \text{...Ans.}$$

Ex. 5.22.2 : Two geometrically similar pumps are running at the same speed of 1500 rpm. One pump has impeller diameter of 350 mm and lifts water at the rate of 26 lit/sec against a head of 18 m. Determine the head and impeller diameter of the other pump to deliver half the discharge.

Soln. :

Given : $N_1 = N_2$ = 1500 rpm

Pump 1 : D_1 = 350 mm = 0.35 m;

$$Q_1 = 26 \text{ lit/s} = \frac{26}{1000} = 0.026 \text{ m}^3/\text{s},\ H_1 = 18 \text{ m.}$$

Pump 2 : $Q_2 = \frac{Q_1}{2} = \frac{0.026}{2} = 0.013 \text{ m}^3/\text{s}$

(i) Head of pump 2

For similar pumps, their head coefficient,

$K_H = \frac{H}{D^2 N^2}$ must be equal. Therefore,

$$\frac{H_1}{D_1^2 \cdot N_1^2} = \frac{H_2}{D_2^2 \cdot N_2^2}$$

But $N_1 = N_2$

$$\therefore \frac{H_2}{H_1} = \left(\frac{D_2}{D_1}\right)^2 \quad \text{...(i)}$$

Their discharge coefficients $K_Q = \frac{Q}{D^3 N}$ are also equal.

$$\therefore \frac{Q_1}{D_1^3 \cdot N_1} = \frac{Q_2}{D_2^3 \cdot N_2}$$

$$\therefore \frac{D_2}{D_1} = \left(\frac{Q_2}{Q_1} \cdot \frac{N_1}{N_2}\right)^{1/3} = \left(\frac{0.013}{0.026}\right)^{1/3}$$

$$= 0.7937 \ (\because N_1 = N_2) \quad \text{...(ii)}$$

Substituting the value of D_2 / D_1 in Equation (i) we get,

$$H_2 = H_1 \left(\frac{D_2}{D_1}\right)^2 = 18\,(0.7937)^2$$

$$= \mathbf{11.339 \text{ m}} \quad \text{...Ans.}$$

(ii) Impeller diameter of pump 2, D_2

From Equation (ii),

$$D_2 = 0.7937\, D_1 = 0.7937 \times 0.35$$

$$= \mathbf{0.2778 \text{ m}} \quad \text{...Ans.}$$

Ex. 5.22.3 : A model of a centrifugal pump is made $\frac{1}{8}^{th}$ of the actual full size pump. The model when tested at 6.25 r.p.m. developed a head of 7.5 m consuming 18 kW power. If the full size pump is to work against a head of 30 m, determine :

i) speed of pump

ii) power required for the pump

iii) discharge of the model and actual pump

Soln. :

Given : Scale ratio, $n = \frac{D_m}{D_p} = \frac{1}{8}$

Model : $N_m = 625$ rpm, $H_m = 7.5$ m,

$P_m = 18$ kW

Prototype : $H_p = 30$ m.

(i) Speed of prototype pump, N_p

Head coefficient, $K_H = \frac{H}{N^2 D^2}$ for model and prototype must be same. Accordingly,

$$\frac{H_m}{N_m^2 \cdot D_m^2} = \frac{H_p}{N_p^2 \cdot D_p^2}$$

$$\therefore N_p = N_m \cdot \left(\frac{D_m}{D_p}\right) \times \sqrt{\frac{H_p}{H_m}}$$

$$= 625 \left(\frac{1}{8}\right) \times \sqrt{\frac{30}{7.5}}$$

$$= \mathbf{156.25 \text{ rpm}} \quad \text{...Ans.}$$

(ii) Power required by prototype pump, P_p

Power coefficient,

$K_p = P/D^5 \cdot N^3$ must be same for model and prototype.

Accordingly,

$$\frac{P_p}{D_p^5 \cdot N_p^3} = \frac{P_m}{D_m^5 \cdot N_m^3};$$

$$\therefore P_p = P_m \times \left(\frac{D_p}{P_m}\right)^5 \times \left(\frac{N_p}{N_m}\right)^3$$

$$= 18 \times (8)^5 \times \left(\frac{156.25}{625}\right)^3 = \mathbf{9216 \text{ kW}} \quad \text{...Ans.}$$

(iii) Discharge of model, Q_m and discharge of actual pump, Q_p

Assuming 90% efficiency in absence of data power developed by model,

$$P_m = \rho \cdot g \times Q_m \times H_m \times (\eta_o)_m$$

$$18 \times 10^3 = 1000 \times 9.81 \times Q_m \times 7.5 \times 0.9$$

$$\therefore Q_m = \mathbf{0.2148 \text{ m}^3/\text{s}} \quad \text{...Ans.}$$

Similarly, $P_p = \rho \cdot g \times Q_p \times H_p \times (\eta_o)_p$

$$9216 \times 10^3 = 1000 \times 9.81 \times Q_p \times 30 \times 0.9$$

$$\therefore Q_p = \mathbf{34.794 \text{ m}^3/\text{s}} \quad \text{...Ans.}$$

Ex. 5.22.4 : A model of a centrifugal pump coupled to 5.5 kW motor is found to deliver 15 litres/s of water, against a head of 28 m at 1450 r.p.m. If the prototype pump is required to develop a head of 60 m, determine its speed, power input and flow capacity. Assume a scale of 1 : 3 between model and prototype.

Soln. :

Given : Model : $P_m = 5.5$ kW, $Q_m = 15$ litres/s,

$H_m = 28$ m, $N_m = 1450$ rpm.

Prototype : $H_p = 60$ m, Scale, $n = \frac{D_m}{D_p} = \frac{1}{3}$

(i) Speed of prototype, N_p

Head coefficient, $K_H = \frac{H}{N^2 \cdot D^2}$ for model and prototype must be same. Accordingly,

$$\frac{H_m}{N_m^2 \cdot D_m^2} = \frac{H_p}{N_p^2 \cdot D_p^2}$$

$$N_p = \sqrt{\frac{H_p}{H_m} \cdot \frac{D_m^2}{D_p^2} \cdot N_m^2}$$

$$= \frac{D_m}{D_p} \times N_m \times \sqrt{\frac{H_p}{H_m}}$$

$$= \frac{1}{3} \times 1450 \times \sqrt{\frac{60}{28}}$$

$$= \mathbf{707.53\ rpm} \quad \text{...Ans.}$$

(ii) Power input to prototype, P_p

Power coefficient, $K_p = \frac{P}{D^5 \cdot N^3}$ must be same for model and prototype. Accordingly,

$$\frac{P_p}{D_p^5 \cdot N_p^3} = \frac{P_m}{D_m^5 \cdot N_m^3}$$

$$P_p = P_m \cdot \left(\frac{D_p}{P_m}\right)^5 \times \left(\frac{N_p}{N_m}\right)^3$$

$$= 5.5\,(3)^5 \left(\frac{707.53}{1500}\right)^3$$

$$= \mathbf{140.26\ kW} \quad \text{...Ans.}$$

(iii) Flow capacity of prototype pump, Q_p

Discharge coefficient, $K_p = \frac{Q}{ND^3}$ must be equal to model and prototype pumps. Therefore,

$$\frac{Q_p}{N_p \cdot D_p^3} = \frac{Q_m}{N_m \cdot D_m^3}$$

$$Q_p = Q_m \times \frac{N_p}{N_m} \times \left(\frac{D_p}{D_m}\right)^3$$

$$= 15 \times \left(\frac{707.53}{1500}\right)(3)^3$$

$$= \mathbf{191.033\ litres/s} \quad \text{...Ans.}$$

Summary

- A **pump** is a device which transfers the input mechanical energy of a motor or of an engine into pressure energy or kinetic energy or both of a liquid.
- **Pumps are classified as :**
  1 Positive displacement pumps e.g. reciprocating and rotary displacement.
  2. Dynamic pressure pumps e.g. centrifugal, axial flow, jet pump etc.
- Some of important **pump applications** are :
  1. Pumping of water for irrigation, agricultural and fire fighting purposes.
  2. Pumping of oil in petroleum installations.
  3. Pumping of water in steam and diesel power plants, in buildings.
  4. Hydraulic control systems.
  5. Transfer of raw materials.
- **Advantages** of C.F. pumps over reciprocating pumps are :
  1. High discharge
  2. Can use highly viscous fluids.
  3. Can be directly coupled to prime mover.
  4. Compact and small in size.
  5. Low weight with high discharge
  6. Can operate at high speeds without danger of separation and cavitation
  7. Low maintenance cost
  8. Highly efficient
- **Main components of a C.F. pump** are impeller, casing, suction pipe with strainer and foot valve and the delivery pipe.
- **Centrifugal pumps are classified** based on the following :
  1. Head
  2. Type of casing (volute, vortex, diffuser)
  3. Type of liquid handled (enclosed, semi-open and open impellers)
  4. On relative direction of flow through impeller (radial, mixed and axial flows)
  5. Number of entrances to impeller (single and double entry).

6. Number of impellers per shaft (single stage, multistage)
7. Specific speed (Radial 10-80, Mixed 80-160 and axial 100-450)
8. Shaft position (horizontal, vertical).

– **Work and discharge of centrifugal pump are**

(1) Work input, $W = \frac{V_{w1} \cdot u_2}{g}$ (Nm/N/s)

(2) $\dot{W} = \rho \cdot Q\, V_{w2} \cdot u_2 \times 10^{-3}$ (kW)

(3) $Q = \pi \cdot D_1 \cdot B_1 \cdot V_{f1} = \pi \cdot D_2 \cdot B_2 \cdot V_{f2}$

– **Various heads of pumps are**

1. Static head, H_s = Suction lift, h_s + delivery lift, h_d
2. Gross head, $H_g = H_s$ + Friction losses $(h_{fs} + h_{fd})$ + Velocity heads $\left(\frac{V_s^2}{2g} + \frac{V_d^2}{2g}\right)$.
3. **Manometric head, H_m** represents the minimum head against which the pump has to work to deliver the required to discharge.

$$H_m = H_s + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$$

$$= \left(\frac{p_d}{W} + \frac{V_d^2}{2g} + Z\right) - \left(\frac{p_d}{W} + \frac{V_d^2}{2g}\right)$$

4. Virtual head, $H_{virt} = V_{w1} \cdot u_2/g$

– **Various losses in pumps are** : (Refer Fig. 5.7.1)

1. Hydraulic losses
2. Mechanical losses
3. Leakage losses

– **Various efficiencies are**

1. **Mechanical efficiency,**

$$\eta_m = \frac{\text{Power available at impeller, P}}{\text{Shaft power, } P_s}$$

$$P = \rho\,(Q + q)\, V_{w1} \cdot u_2 = P_s - P_{mech.\,losses}$$

2. **Manometric efficiency,**

$$\eta_{mano} = \frac{H_m}{(V_{w1} \cdot u_2/g)} = \frac{g \cdot H_m}{V_{w1} \cdot u_2}$$

3. **Volumetric efficiency,**

$$\eta_v = \frac{\text{Actual discharge, Q}}{\text{Theoretical flow through impeller, } (Q + q)}$$

q = leakage loss

4. **Overall efficiency,**

$$\eta_o = \frac{\text{Power output}}{\text{Shaft power}} = \frac{\rho \cdot g \cdot Q \cdot H_m}{P_s}$$

$$= \eta_{mano} \times \eta_v \times \eta_m$$

– **Manometric head,** $H_m = \frac{V_{w1} \cdot u_2}{g} - \frac{V_2^2}{2g}$

– **Specific speed of pump, N_s** is defined as the speed of geometrically similar pump will deliver a discharge of 1 m^3/s of liquid under a head of 1 m.

$$Ns = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

– **Vane discharge angle, ϕ** is kept more than 20°.

– **Minimum starting speed of pump** represents the speed at which the impeller will develop the head more than manometric head to start discharging

i.e. $(u_2^2 - u_1^2)/2g = H_m$

$$\therefore\; N = \frac{60}{\pi} \times \frac{\sqrt{2g\,H_m}}{\sqrt{D_2^2 - D_1^2}}$$

Minimum diameter of impeller

$$D_2 = 2\,D_1 = \frac{97.68}{N} \times \sqrt{H_m}$$

– **Effect of finite number of vanes of impeller** is to develop head, H less than Euler's head, H_e. Therefore, **vane effectiveness** $\in = H/H_e$.

$\in$ = 1 for more than 24 vanes.

– **Working proportions of C.F. pump**

1. Speed ratio, $K_u = u_2/\sqrt{2g \cdot H_m}$ (Varies between 0.95 to 1.25).
2. Flow ratio, $K_f = V_{f2}/\sqrt{2g \cdot H_m}$ (Varies between 0.1 to 0.25)
3. Diameters of impeller :

$$D_2 = (84.6 \cdot K_u \cdot \sqrt{H_m})/N \text{ and } D_1 = \frac{1}{3} D_2 \text{ to } \frac{1}{3} D_2$$

– **Effect of vane shapes :** $H_e = \frac{u_2}{g}\left(u_2 - \frac{Q}{A_2} \cdot \cot\phi\right)$

1. *Backward vanes* ($\phi < 90°$) : Euler's head decreases with increase in discharge.
2. *Radial vanes* ($\phi = 90°$) : Euler's head remains constant.
3. Forward vanes ($\phi > 90°$) : Euler's head increases with increase in discharge.

– Centrifugal pumps with or more number of impellers are called **multistage pumps.** If n = Number of impellers :

1. **In series :** $H_{total} = H_1 + H_2 ...;$

   $Q_{total} = Q = Q_1 = Q_2 = Q_3 ...$

2. **In parallel :** $Q_{total} = Q_1 + Q_2 + ...;$

   $H_{total} = H = H_1 = H_2 = H_3 = ...$

– **Various types of characteristics curves of a pump are :**
  1. *Main characteristic curves* are drawn at constant speed by varying discharge. (H Vs N, P Vs Q and η_o Vs Q).
  2. *Operating characteristic curves* are drawn at designed speed.
  3. *Iso-efficiency curves* are useful in predicting the performance on entire operations and its best performance.
  4. Constant head and constant discharge curves are drawn at variable speed.

– A pump is said to be **undersized pump** if it cannot meet the head and discharge requirements.

– In case the pump delivers much higher head and discharge than required, it is called **oversized pump.**

– **Pump is selected** based on the intersection of characteristic curves of a pump and the system which should be in the vicinity of maximum efficiency point of the pump for its economical running.

– **Characteristic curves of a multistage pump** are obtained by combining the characteristic curves of individual pumps according to their operation in series or parallel as follows :

  In series : $H_{total} = H_1 + H_2$ at the same discharge

  In parallel : $Q_{total} = Q_1 + Q_2$ at the same head

– **Cavitation in C.F. pumps** is defined as the phenomenon of formation of air bubbles in the region of flowing liquid where its pressure falls below the vapour pressure of liquid, then liquid will vapourise with discontinuity of flow.

– Cavitation develops very high pressures causing the erosion and pitting of metals, noise and vibrations.

– Permissible suction lift, h_s is given as

$$h_s = \left(\frac{p_a - p_v}{w}\right) - \frac{V_s^2}{2g} - h_{fs}$$

– **NPSH** is defined as the absolute pressure head at inlet to pump to force the liquid into pump at a given temperature.

– NPSH must be positive to avoid cavitation and force the liquid into eye of impeller.

$$NPSH = h_a - \left(h_v + h_s + \frac{V_s^2}{2g} + h_{fs}\right)$$

– **NPSH required** is the minimum value of NPSH to avoid cavitation at specified discharge and speed as per manufacturer's design data. While, *NPSH available* represents the estimated head available inlet of impeller as per actual conditions.

  NPSH available must be greater than NPSH required.

– Thoma's cavitation factor, σ is given as :

$$\sigma = \frac{h_a - (h_v + h_s + h_{fs})}{H_m} = \frac{NPSH}{H_m}$$

– **Suction specific speed,** N_{SS} is defined as the speed of geometrically similar model pump which will operate with similar degree of cavitation.

$$\text{Specific speed of pump, } N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

$$\text{and, suction specific speed of pump, } N_{ss} = \frac{N\sqrt{Q}}{(NPSH)^{3/4}}$$

$$\therefore \frac{N_s}{N_{ss}} = \frac{(NPSH)^{3/4}}{(H_m)^{3/4}} = (\sigma)^{3/4}$$

$$\text{i.e. Cavitation factor, } \sigma = \left(\frac{N_s}{N_{ss}}\right)^{4/3}$$

– Operation of filling the casing, impeller and suction pipe and portion of delivery pipe upto delivery valve is called **priming.**

– **Methods of priming are**
  1. By hand
  2. By connecting with city water mains.
  3. By provision of priming pumps and steam ejectors.
  4. Self priming devices like vacuum pump in casing of C.F. pump priming tanks etc.

– **Axial thrust** in pumps is produced due to change of direction of liquid flow from axial entry to radial discharge over the vanes and due to pressure difference prevailing across the impeller.

– **Axial thrust can be balanced by following methods**

  1. **Small pumps**
     (a) By provision of ball thrust bearings in direction of axial thrust.
     (b) By cast ring fitted on casing.

2. **Large pumps**
   (a) By using double suction pumps.
   (b) By providing relieving holes in the impeller.
   (c) By provision of balance plate or disc at the end of pump shaft.
   (d) Using even number of impellers in multistage pumps with half of impellers having suction on opposite sides.

Exercise

Q. 1 Write a short note on various applications of a centrifugal pump. **[Section 5.2.2]**

Q. 2 Explain the operation of a centrifugal pump with the help of a neat sketch. **[Sections 5.3 and 5.3.1]**

Q. 3 How does a centrifugal pump develops pressure energy to flowing fluid across it ? **[Section 5.3.1]**

Q. 4 Explain different type of casings used in centrifugal pumps. **[Section 5.4]**

Q. 5 Show by sketch the difference between the volute and diffuser pump. **[Section 5.4(2)]**

Q. 6 Explain different types of impellers used in centrifugal pumps. **[Section 5.4(3)]**

Q. 7 State the difference between closed, semi-closed and open impeller. **[Section 5.4(3)]**

Q. 8 Define specific speed of a centrifugal pump. How the pumps are classified based on specific speed ? **[Section 5.4(7)]**

Q. 9 What is the difference between single suction and double suction centrifugal pump ? **[Section 5.4(5)]**

Q. 10 Explain velocity diagram of an impeller pump and write expressions for work done and discharge through the pump. **[Section 5.5]**

Q. 11 Write a short note on : Various heads. **[Sections 5.6, 5.6.1, 5.6.2 and 5.6.3]**

Q. 12 The difference between the water levels in the pump and overhead tank is H. What additional head the pump should generate ? **[Section 5.6]**

Q. 13 What do you mean manometric efficiency, mechanical efficiency and overall efficiency of a centrifugal pump? **[Section 5.7.2]**

Q. 14 Explain the effect of change of outlet blade angle of impeller on pressure head developed and efficiency of the pump. **[Section 5.8]**

Q. 15 Explain in brief "minimum speed for starting a centrifugal pump". **[Sections 5.9 and 5.9.1]**

Q. 16 Discuss effect of variation in discharge on the efficiency of a centrifugal pump. **[Section 5.10]**

Q. 17 How the vane angle at exit influence the work done/discharge characteristics? **[Section 5.13]**

Q. 18 Discuss the influence of blade angles on performance of the centrifugal pump. **[Section 5.13]**

Q. 19 Why is multistaging used for a centrifugal pump ? Describe the methods use for multistaging. **[Section 5.14, 5.14.1, and 5.14.2]**

Q. 20 How will you show the difference on performance characteristics ? **[Section 5.15]**

Q. 21 Draw and discuss the operating characteristics of a centrifugal pump. **[Sections 5.15.4]**

Q. 22 Explain with the help of a head-discharge curve, the interaction of a centrifugal pump and the system. What happens if a pump is undersized or oversized compared to the system ? **[Sections 5.16.1 and 5.15.6]**

Q. 23 State the criteria for selection of a centrifugal pump for a given application.**[Sections 5.16]**

Q. 24 Explain series and parallel operations of identical centrifugal pumps. Draw the combined head (H) - discharge (Q) curves in both the cases. **[Section 5.17.1 and 5.17.2]**

Q. 25 Write short note on : Cavitations in centrifugal pump. **[Sections 5.18]**

Q. 26 Explain the terms : Available NPSH and required NPSH. **[Sections 5.18.4 and 5.18.3]**

Q. 27 Explain cavitations and N.P.S.H in centrifugal pumps.**[Sections 5.18.1, 5.18.2, 5.18.3 and 5.18.4**

Q. 28 What is meant by "Priming" of a pump ? Describe some priming devices. **[Sections 5.19]**

Q. 29 What is priming of centrifugal pump and why it is necessary? **[Sections 5.19 and 5.19.1]**

Q. 30 How the axial thrust is developed in a centrifugal pump ? Discuss the methods used to reduce this thrust. **[Sections 5.20 and 5.20.1]**

Q. 31 A centrifugal pump is making noise during operations. What are probable causes and suitable remedies for this problem ? **[Section 5.21]**

□□□

6

UNIT - IV

Centrifugal Compressors

Syllabus

Classification of Centrifugal Compressor, construction and working, velocity diagram, flow process on T-S Diagram, Euler's work, actual work input, various losses in Centrifugal Compressor.

6.1 Introduction to Steady Flow Compressors

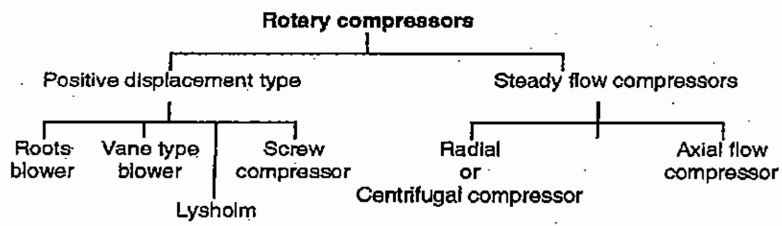
- The true steady flow compressors are also termed as **turbo-compressors** which include both centrifugal and axial flow compressors. These are **power absorbing machines.**
- In these type of compressors the air is not trapped within the specified boundaries but it flows at a steady rate continuously through the compressor.
- Both centrifugal and axial flow compressors operate on the air or gas to produce a change in momentum on the expense of external work supplied which results into increase in pressure of the gas.
- The centrifugal compressors were initially used in gas turbines for aircrafts which were subsequently replaced by axial flow compressors after World War II.
- The axial flow compressors are more efficient (3% to 5% higher) compared to centrifugal compressors. A centrifugal compressor can develop pressure ratio upto 4 per stage while an axial flow compressor can develop a pressure ratio only 1.2 to 1.3 per stage. But the total pressure ratio developed in multistage centrifugal compressors is limited to 10 while axial flow compressors can develop pressure ratio upto 20 in 10 to 14 stage. *Usually the use of multistage centrifugal compressors in series are not preferred due to problem in designing the air passages.*
- The mass flow rate of air compressed in centrifugal compressors is much lower than axial flow compressors.
- Therefore, *centrifugal compressors are suitable* for high pressure ratios compared to blowers but comparatively lower mass flow rate applications.
- The advantages and applications centrifugal compressors are :
  (i) Occupies less length compared to axial flow compressors.
  (ii) It can resist the built up of deposits in channels and blades. Thus it can work well in contaminated atmosphere.
  (iii) It can operate efficiently at wide range of mass flow rates at a given speed.
  (iv) These are suitable for large refrigeration units, small turbojet units, large petrochemical and industrial units, supercharging of I.C. engines etc.

6.1.1 Classification of Rotary Compressors

University Question

Q. Describe the classification of compressors. **SPPU : Dec. 13, May 19**

The rotary compressors can be broadly classified as shown below :



- *Positive displacement type of single stage blowers* can produce pressures upto 2.5 bar with handling capacity ranging from 0.5 m^3/min to 1500 m^3/min.
- *The centrifugal compressors* can produce pressure ratios upto 4:1 per impeller and with number of impellers pressure ratio can reach upto 10:1. These compressors can be adapted for air flows ranging from 15 m^3/min to 1200 m^3/min.
- In case of *axial flow compressors* the maximum compression ratio per stage is 1.2 to 1.3 and the number of stages used are in the range of 8 to 20. The total compression ratio could be achieved upto 20 with the discharge pressure reaching upto 400 bar.
- These compressors are only suitable for large volume flow rates ranging from 1200 m^3/min to 42000 m^3/min.

6.1.2 Difference between Fan and Blower

University Question

Q. Write short note on
(i) Fan (ii) Blower **SPPU : May 15**

- Fan and blower are both pressure raising devices on the expense of external work supplied.
- Fan are used to raise the pressure upto 20 cm of water pressure. Whereas, blowers are used to increase the pressure ratios upto 2.
- Fans are used for comparatively low discharge rates for household applications while blowers are used for large discharge rates for industrial applications.

6.2 Components and Working of Centrifugal Compressor

University Question

Q. Why diffusers are necessary in centrifugal compressors.? **SPPU : Dec. 16**

Fig. 6.2.1 and 6.2.2 shows the main components of a centrifugal compressor.

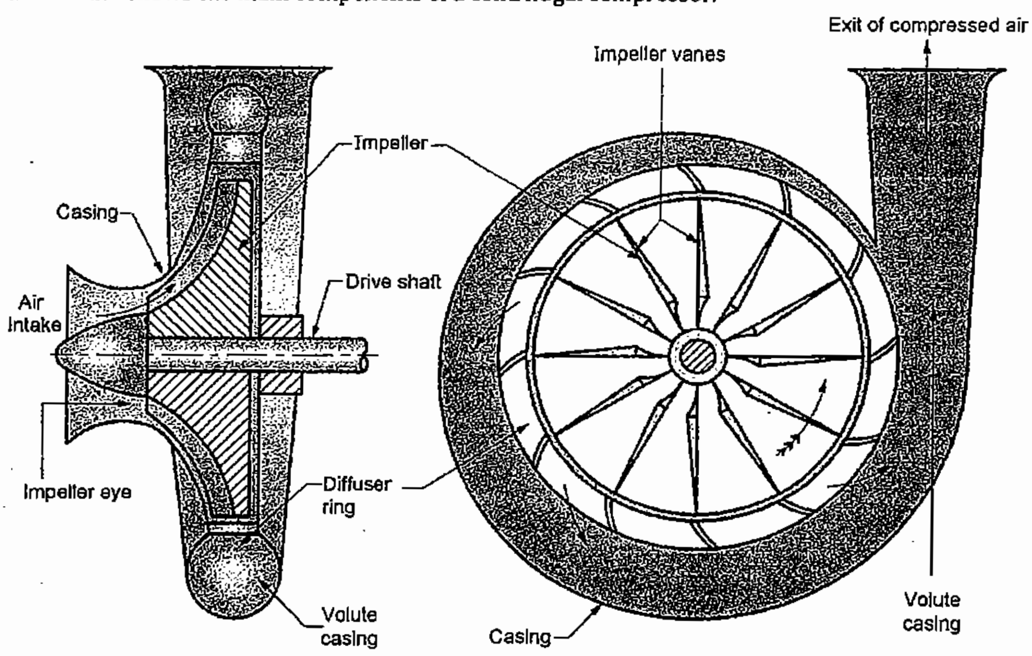


Fig. 6.2.1 : Centrifugal compressors

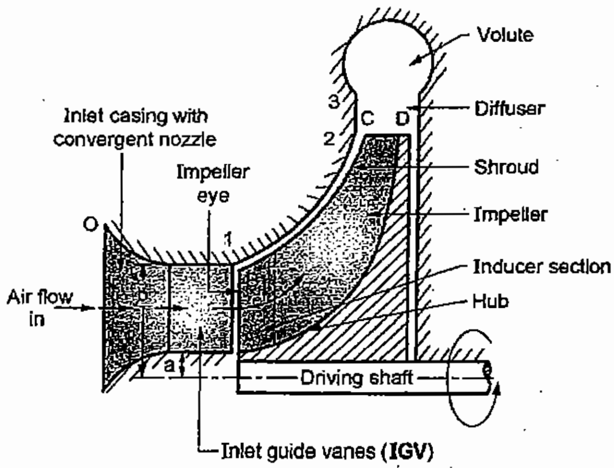
It consists of the following :

(i) Inlet casing with convergent nozzle and inlet guide vanes

The air or gas enters from surroundings at section '0' into a convergent nozzle where the flow is accelerated. The air enters into the eye of the compressor at uniform rate from nozzles through the inlet guide vanes (IGV's). The function of IGV is to direct the flow in the desired direction at entry into the eye of impeller at section 1.

(ii) Impeller

- It can rotate at very high speeds upto 30000 rpm. It produces centrifugal head on the air within and causes the air to flow from surroundings axially into the impeller eye.
- The air then flows radially outwards into the impeller blades. The fresh air enters at the eye of impeller to take the place of displaced air.
- The function of impeller is to impart the energy to gas by the rotating blades.
- The rotation of impeller causes a static pressure rise alongwith increase in kinetic energy of the gas.



$a = \frac{1}{2} \times$ **hub diameter,** d_h $b = \frac{1}{2}$ **tip diameter,** d_t

$$\textbf{Mean radius at inlet} = \frac{d_h + d_t}{2}$$

Fig. 6.2.2 : Elements of a compressor stage

- An impeller has an inducer section, hub, vanes and shroud.
- The inducer receives the flow between the hub and tip diameters of eye.
- The inducer section of impeller increases the angular momentum of the gas without the increase its radii of rotation. The impeller vanes transfers the energy from impeller to the gas.
- The hub corresponds to the surface AD. It's diameter varies from entry to exit.
- The tips of the blades can be shrouded to prevent the leakages. The impellers may be shrouded impellers or open impellers.

(iii) Diffuser

- The function of diffuser is to convert the high kinetic energy into pressure energy between diffuser blade passages.
- The diffuser blades are so shaped that they provide an increased passage of area to the air. It causes to convert the kinetic energy into pressure energy by diffusion.

(iv) Volute casing

- The high pressure air leaving the diffuser is carried through the volute casing to the exit of compressor. The increased cross-sectional area of volute casing also causes small pressure rise with reduction in its kinetic energy.
- Pressure and velocity variation in centrifugal compressor is shown in Fig. 6.2.3.

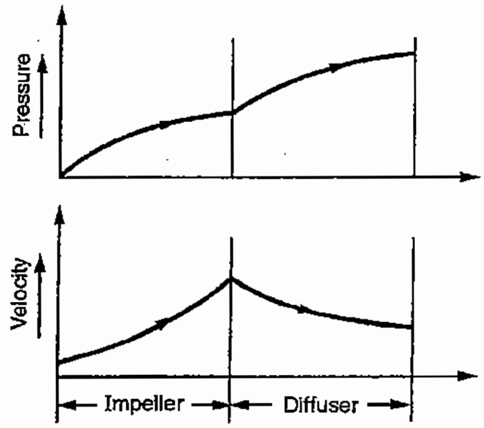
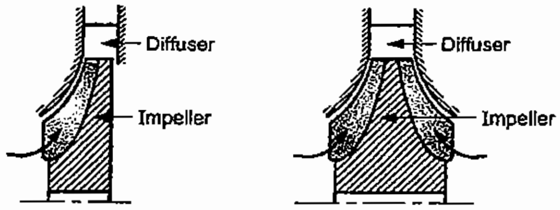


Fig. 6.2.3 : Pressure and velocity variation

- The normal practice is to design the compressor so that about half of pressure rise takes place in impeller and half in the diffuser.
- The impellers of centrifugal compressors may also be **double sided** so that the air may enter from both sides of the impeller. Thus, double sided impellers will have double impeller eye compared to single sided impeller shown in Fig. 6.2.4.

- A pressure ratio upto 4 can be achieved in case of single stage centrifugal compressors and a pressure ratio upto 12 can be achieved in case of multistage centrifugal compressors.



(a) Single sided **(b) Double sided**

Fig. 6.2.4 : Types of impellers

6.2.1 Vaneless and Vaned Diffusers

- The diffuser consists of an annular space surrounding the impeller. If the annular space is without the vanes, it is called **vaneless diffuser** and if the diffuser has the set of guide vanes to direct the gases it is called **vaned diffuser.**
- The function of diffuser is to convert the kinetic energy of gases leaving the impeller into pressure energy before discharge through the volute casing.
- The static pressure rise in vaneless diffuser is simply due to diffusion process of the gases during the flow from smaller diameter to larger diameter. *Such a flow in vaneless space is called* **free vortex** *in which the angular momentum of gas remains constant.*
- It leads to large sized diffuser having low energy conversion efficiency. But these are economical for wide range of industrial applications and they are free from blade stalling and shock waves formation. *Vaneless diffusers are suitable only for low pressure rise and unsuitable for aeronautical applications.*
- In vaned diffusers, the diffusion process is achieved in short radial distance by passing the gas over aerofoil blades to avoid flow separation.
- The use of blades increases the friction losses but provides high energy conversion efficiency. In order to reduce the friction losses and to avoid the problem of blade stalling and shock waves, the number of fixed blades are kept low in number.
- Generally, the diffuser blades are kept $\frac{1}{3}$ rd of the number of impeller blades.

6.3 Representation of Flow Processes on (T-S) Diagram

University Question

Q. Represent and explain the process involved in centrifugal compressor on (T-S) diagram and derive the expression for isentropic efficiency based on total values. **SPPU : May 11, May 14, Dec. 15**

- Air is drawn from surrounding air at stagnation conditions into inlet casing and then it enters at eye of impeller with some velocity where it is compressed. The air leaves the impeller and enters into diffuser when the partial recovery of velocity head is carried out and the air leaves the diffuser in collector called volute or scroll with very small velocity.
- The combined (T – S) diagram for total or stagnation values and static values is shown in Fig. 6.3.1. The corresponding points on rotary compressor are shown in Fig. 6.3.2.

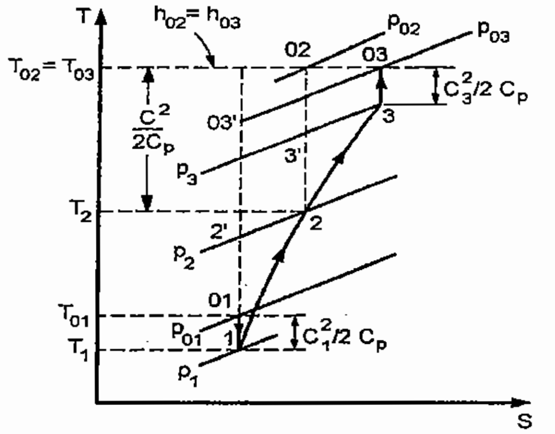


Fig. 6.3.1 : Representation of process in centrifugal compressor

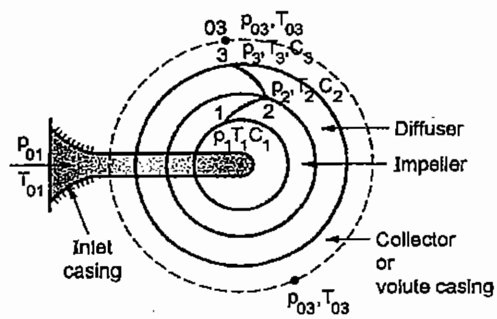


Fig. 6.3.2 : Representation of points on centrifugal compressor

- Point 01 represents the condition of ambient air (static air with velocity, C = 0) at conditions of p_{01}, T_{01}.

- Air is drawn by the impeller and during the isentropic process (01 - 1), the velocity increases to C_1 on the expense of its pressure energy in the convergent nozzle of inlet casing. The losses in inlet casing have been neglected.
- Point 1 represents the static conditions of air at inlet to the impeller as p_1, T_1 and velocity C_1.
- Air is compressed in the impeller and its pressure and temperature rises on the expense of external work. Process (1 - 2′) represents reversible adiabatic compression and process (1 - 2) actual compression process with friction.
- Point 2 represents the state of air just at the exit of impeller given as p_2, T_2 and velocity C_2.
- Pressure p_{02} corresponds to delivery pressure if the total kinetic energy $\left(C_2^2/2\right)$ is converted into pressure energy in the diffuser.
- From impeller the air enters into the fixed diffuser blades where its kinetic energy is converted into pressure energy by diffusion process. Since diffusion process is not isentropic, the final delivery pressure corresponds to pressure p_3 and having K.E. $\left(C_3^2/2\right)$ entering into collector (volute casing) $p_{03} < p_{02}$ since diffusion process is irreversible.
- Point 03 represents the condition of air at discharge at p_{03}, T_{03} with negligible velocity.
- The (T - S) diagram with ideal diffusion process with negligible velocity is shown in Fig. 6.3.3. Process (01 - 1) represents the isentropic process in inlet casing with increase in velocity C_1 and decrease to static pressure p_1.

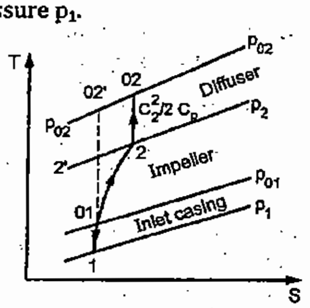


Fig. 6.3.3 : Ideal processes with ideal diffusion process

- Process (1 - 2′) represents the ideal isentropic compression and process (1 - 2) as actual compression process in impeller blade. The condition of air leaving the impeller is at p_2, T_2 and velocity C_2.
- This air enters into diffuser where the K.E. is converted into pressure energy completely in an ideal (isentropic) diffusion process (2 - 02). Thus the state at exit of compressor is at p_{02}, T_{02} with negligible velocity at exit ($C_2 \simeq 0$).

Actual and Isentropic workdone

Neglecting the changes in potential energy and process being adiabatic, the actual workdone per kg of air can be calculated from steady flow energy equation as follows :

$$q - w_a = (h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} + g(Z_2 - Z_1)$$

But, $q = 0$, $Z_2 \simeq Z_1$, work is negative,

$$\text{Therefore, } -(-w) = (h_2 - h_1) + \frac{C_2^2 - C_1^2}{2}$$

But, Stagnation enthalpy,

$$h_o = \text{static enthalpy, } h + \text{K.E}\left(\frac{C^2}{2}\right)$$

$$\therefore \quad w_a = h_{02} - h_{01} = C_p(T_{02} - T_{01}) \qquad \text{...(6.3.1)}$$

$$\text{But,} \quad \frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{(n-1)/n}$$

$$\therefore \quad w_a = C_p\left[\left(\frac{p_{02}}{p_{01}}\right)^{(n-1)/n} - 1\right] \qquad \text{...(6.3.2)}$$

Similarly isentropic workdone can be written as,

$$w_i = C_p\left[\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1\right] \qquad \text{...(6.3.3)}$$

Isentropic efficiency, η_i is defined as the ratio of isentropic or ideal work to the actual work.

$$\text{Therefore, } \eta_i = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{C_p(T'_{02} - T_{01})}{C_p(T_{02} - T_{01})}$$

$$\text{i.e.} \quad \eta_i = \frac{(T'_{02} - T_{01})}{(T_{02} - T_{01})} \qquad \text{...(6.3.4)}$$

Ex. 6.3.1 : For an axial flow compressor the intake air pressure [illegible] bar and [illegible]°C. The pressure head and temperature at delivery pipe are [illegible] bar and [illegible]°C respectively. Static pressure in the delivery pipe is [illegible] bar. Determine : [illegible]
(i) The total head isentropic efficiency
(ii) Air velocity in delivery pipe. **SPPU - Dec. 13,10 Marks**

Soln. : Refer Fig. P. 6.3.1

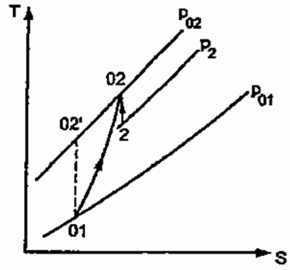


Fig. P. 6.3.1

Given : p_{01} = 1 bar, T_{01} = 18° C = 291 K

p_{02} = 3.5 bar, T_{02} = 185° C = 458 K,

p_2 = 3 bar

(i) Total head isentropic efficiency, η_{ts}

Consider isentropic process (01-02′)

$$T'_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} = 291\left(\frac{3.5}{1}\right)^{0.4/1.4} = 416.2 \text{ K}$$

$$\eta_{ts} = \frac{T'_{02}-T_{01}}{T_{02}-T_{01}} = \frac{416.2-291}{458-291} = \mathbf{0.75 \text{ or } 75\%} \quad \text{...Ans.}$$

(ii) Air velocity in delivery pipe, C_2 :

Consider isentropic process (02 – 2)

$$\frac{T_2}{T_{02}} = \left(\frac{p_2}{p_{02}}\right)^{(\gamma-1)/\gamma}$$

$$T_2 = 458\left(\frac{3}{3.5}\right)^{(0.4)/1.4} = 438.27 \text{ K}$$

$$T_{02} = T_2 + \frac{C_2^2}{2\,C_p} \quad \text{(Assume, } C_p = 1005 \text{ J/kg K for air)}$$

$$458 = 438.27 + \frac{C_2^2}{2\times 1005}$$

$$C_2 = \mathbf{199.14 \text{ m/s}} \quad \text{...Ans.}$$

Ex. 6.3.2 : A centrifugal compressor used as a supercharger for aero engines handles 150 kg/min of air. The suction pressure and temperature are 1 bar and 290 K. The suction velocity is 80 m/s. After compression in the impeller the conditions are 1.5 bar, 345 K and 220 m/s, calculate :

(i) Isentropic efficiency

(ii) Power required to drive the compressor

(iii) The overall efficiency of the unit.

It may be assumed that Kinetic Energy of air gained in the impeller is entirely converted into pressure in the diffuser.

SPPU - May 2011, 8 Marks

Soln. : Refer Fig. P. 6.3.2.

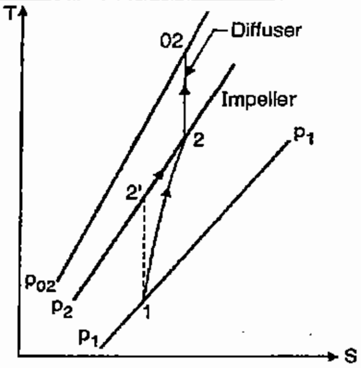


Fig. P.6.3.2

Given : $\dot{m}$ = 150 kg/min = $\frac{150}{60}$ = 2.5 kg/s

p_1 = 1 bar, T_1 = 290 K,

C_1 = 80 m/s, p_2 = 1.5 bar,

T_2 = 345 K, C_2 = 220 m/s

(i) Isentropic efficiency based on static values, η_{ts}

$$T'_2 = T_1\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = 290\left(\frac{1.5}{1}\right)^{0.4/1.4} = 325.6 \text{ K}$$

$$\eta_{ts} = \frac{T'_2-T_1}{T_2-T_1} = \frac{325.6-290}{345-290}$$

$$= \mathbf{0.6476 \text{ or } 64.76\%} \quad \text{...Ans.}$$

(ii) Power required to drive the compressor, W_{sf} or P.

S.F.E.E is :

$$\dot{Q} - \dot{W}_{sf} = \dot{m}\left[C_p(T_2-T_1) + \frac{C_2^2-C_1^2}{2\times 1000} + g(Z_2-Z_1)\right]$$

But $\dot{Q}$ = 0, $Z_2 = Z_1$

$$0 - \dot{W}_{sf} = 2.5\left[1.005\,(345-290) + \frac{(220)^2-(80)^2}{2\times 1000} + 0\right]$$

$$\dot{W}_{sf} = \mathbf{-190.69 \text{ kW}}$$

(Negative sign shows that work is supplied)

$$\therefore \dot{W}_{sf} = \mathbf{P = 190.69 \text{ kW}} \quad \text{...Ans.}$$

(iii) Overall efficiency i.e. isentropic efficiency based on total values, η_t

$$T_{01} = T_1 + \frac{C_1^2}{2C_p} = 290 + \frac{(80)^2}{2\times 1005} = 293.2 \text{ K}$$

$$T_{02} = T_2 + \frac{C_2^2}{2C_p} = 345 + \frac{(220)^2}{2\times 1005} = 369.1 \text{ K}$$

$$\frac{p_{01}}{p_1} = \left(\frac{T_{01}}{T_1}\right)^{\gamma/(\gamma-1)}$$

$$p_{01} = 1\left(\frac{293.2}{290}\right)^{1.4/0.4} = 1.039 \text{ bar}$$

$$p_{02} = p_2\left(\frac{T_{02}}{T_2}\right)^{\gamma/(\gamma-1)} = 1.5\left(\frac{369.1}{345}\right)^{1.4/0.4} = 1.9 \text{ bar}$$

$$T_{02}' = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} = 293.2\left(\frac{1.9}{1.039}\right)^{0.4/1.4}$$

$$= 348.4 \text{ K}$$

$$\eta_i = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}} = \frac{348.4 - 293.2}{369.1 - 293.2}$$

$$= \mathbf{0.727 \text{ or } 72.7\%}$$ ...Ans.

Ex. 6.3.3 : A gas compressor compresses the gas at the rate of 2 kg/s from inlet static pressure of 1 bar to a static pressure of 4 bar. The power input to the compressor is 400 kW. The velocity of air at entry to impeller blades is 100 m/s and at exit of impeller blades is 160 m/s. Determine the stagnation pressures and temperatures at inlet and exit of the compressor, diameter of suction pipe required and adiabatic efficiency based on static and total values. Assume, $\gamma = 1.4$, $C_p = 1.05$ kJ / kg K and $R = 300$ Nm/kg K. Temperature at inlet to impeller blades is 280 K.

Soln. :

Given : $\dot{m} = 2$ kg/s, $p_1 = 1$ bar,

$p_2 = 4$ bar, $W = -400$kW,

$C_1 = 100$ m/s, $C_2 = 160$ m/s,

$\gamma = 1.4$, $C_p = 1.05$ kJ/kg K,

$R = 300$ Nm/kg K, $T_1 = 280$ K.

Refer Fig. P. 6.3.3

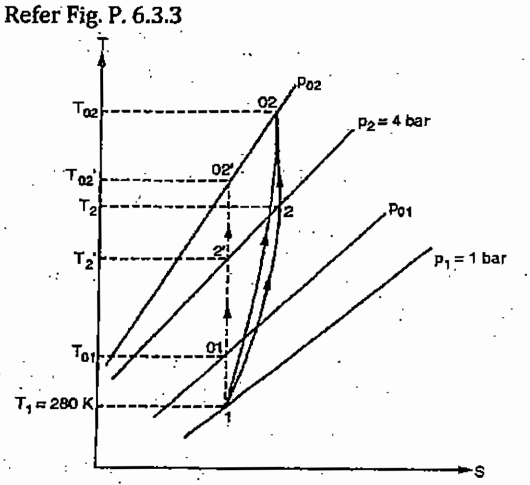


Fig. P. 6.3.3

Applying steady flow energy equation,

$$\dot{Q} - \dot{W} = \dot{m}\left[C_p(T_2 - T_1) + \frac{C_2^2 - C_1^2}{2} + g(Z_2 - Z_1)\right]$$

But $\dot{Q} = 0$ and $(Z_2 - Z_1) = 0$

$$\therefore 0 - (-400) = 2\left[1.05(T_2 - 280) + \frac{(160)^2 - (100)^2}{2 \times 1000} + 0\right]$$

$$T_2 = 466.8 \text{ K}$$

From isentropic process (01 – 1) we have,

$$T_{01} - T_1 = \frac{C_1^2}{2C_p}$$

$$T_{01} - 280 = \frac{(100)^2}{2 \times 1.05 \times 1000}$$

$$\therefore \mathbf{T_{01} = 284.76 \text{ K}}$$ ...Ans.

$$\frac{p_{01}}{p_1} = \left(\frac{T_{01}}{T_1}\right)^{(\gamma-1)/\gamma}$$

$$\frac{p_{01}}{1} = \left(\frac{284.76}{280}\right)^{(1.4-1)/1.4}$$

$$\therefore \mathbf{p_{01} = 1.0094 \text{ bar}}$$ ...Ans.

For isentropic process (2 – 02),

$$T_{02} - T_2 = \frac{C_2^2}{2C_p}$$

or, $$T_{02} = 466.3 + \frac{(160)^2}{2 \times 1.05 \times 1000}$$

$$= \mathbf{479 \text{ K}}$$ ...Ans.

$$\frac{p_{02}}{p_2} = \left(\frac{T_{02}}{T_2}\right)^{(\gamma-1)/\gamma}$$

$$p_{02} = 4\left(\frac{479}{466.8}\right)^{(1.4-1)/1.4}$$

$$= \mathbf{4.38 \text{ bar}}$$ ...Ans.

$$v_1 = \frac{RT_1}{p_1} = \frac{300 \times 280}{1 \times 10^5} = 0.84 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{A_1 C_1}{v_1}$$

$$2 = \frac{A_1 \times 100}{0.84}$$

$$A_1 = 1.68 \times 10^{-2} \text{m}^2$$

Let, 'd_1' be the diameter of inlet pipe,

$$A_1 = \frac{\pi}{4} \times d_1^2 = 1.68 \times 10^{-2}$$

$$\therefore d_1 = 0.1463 \text{ m} = \mathbf{14.63 \text{ cm}}$$ ...Ans.

Ex. 6.3.4 : A centrifugal compressor used for supercharging an aircraft draws air at its inlet conditions of 0.8 bar, 7°C and velocity of 100 m/s. It is compressed adiabatically in impeller upto 1.5 bar and 70°C and velocity leaving the impeller is at 300 m/s.

This air enters into the diffuser where its kinetic energy is completely converted into pressure energy. The mass flow rate of air is 3 kg/s. Find :

(i) Impeller power

(ii) Isentropic efficiency of impeller based on static conditions

(iii) Isentropic efficiency based on stagnation conditions.

Assume, $C_p = 1.005$ kJ/kg K

Soln. :

Processes are shown on T – S diagram in Fig. P. 6.3.4.

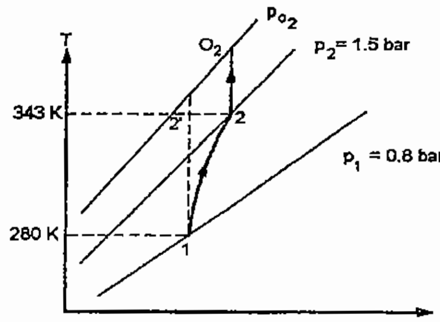


Fig. P. 6.3.4

Given : $p_1 = 0.8$ bar, $C_1 = 100$ m/s

$C_2 = 300$ m/s, $\dot{m} = 3$ kg/s

$T_1 = 7^\circ C = 7 + 273 = 280$ K

$T_2 = 70^\circ C = 70 + 273 = 343$ K

(i) Impeller power, $\dot{W}$

From steady flow energy equation

$$\dot{W} = \dot{m}\left[C_p(T_2 - T_1) + \frac{C_2^2 - C_1^2}{2\times1000}\right] \text{ kW}$$

$$= 3\left[1.005(343-280) + \frac{(300)^2-(100)^2}{2\times1000}\right]$$

$$= 309.95 \text{ kW} \quad \text{...Ans.}$$

(ii) Isentropic efficiency, based on static values, η_{is}

$$T_2' = T_1\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = 280\left(\frac{1.5}{0.8}\right)^{(1.4-1)/1.4}$$

$$= 335.1 \text{ K}$$

$$\eta_{is} = \frac{T_2' - T_1}{T_2 - T_1} = \frac{(335.1-280)}{343-280}$$

$$= 0.8746 \text{ or } 87.46\% \quad \text{...Ans.}$$

(iii) Isentropic efficiency based on stagnation conditions, η_t

$$T_{01} = T_1 + \frac{C_1^2}{2C_p} = 280 + \frac{(100)^2}{2\times1.005\times1000}$$

$$= 284.98 \text{ K}$$

$$T_{02} = T_2 + \frac{C_2^2}{2C_p} = 343 + \frac{(300)^2}{2\times1.005\times1000}$$

$$= 387.78 \text{ K}$$

$$p_{02} = p_2\left(\frac{T_{02}}{T_2}\right)^{\gamma/(\gamma-1)} = 1.5\left(\frac{387.78}{343}\right)^{1.4/(1.4-1)}$$

$$= 2.3046 \text{ bar}$$

$$p_{01} = p_1\left(\frac{T_{01}}{T_1}\right)^{(\gamma)/(\gamma-1)}$$

$$= 0.8\left(\frac{284.98}{280}\right)^{1.4/(1.4-1)} = 0.8509 \text{ bar}$$

$$T_{02}' = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma}$$

$$= 284.98\left(\frac{2.3046}{0.8509}\right)^{(1.4-1)/1.4} = 378.83 \text{ K}$$

$\therefore$ Isentropic efficiency based on total values,

$$\eta_t = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}} = \frac{378.83 - 284.98}{387.78 - 284.98}$$

$$= 0.9129 \text{ or } 91.27\% \quad \text{...Ans.}$$

Ex. 6.3.5 : A Centrifugal compressor used as a supercharger for aero engine handles 180 Kg / min of air. The suction pressure and temperature are 1 bar and 280 K. The suction velocity is 90 m/sec. After isentropic compression in the impeller conditions are 1.5 bar, 335 K and 230 m/sec. Calculate :

i) Isentropic efficiency

ii) Power required to drive compressor

iii) Overall efficiency of the unit.

Assume that kinetic energy of the air gained in impeller is entirely converted into pressure in diffuser. Take $\gamma = 1.4$ for air. **SPU : May 18, 8 Marks**

Soln. :

$$\dot{m} = 180 \text{ kg/min} = \frac{180}{60} = 3 \text{ kg/s}$$

$p_1 = 1$ bar, $T_1 = 280$ K,

$C_1 = 90$ m/s, $p_2 = 1.5$ bar,

$T_2 = 335$ K, $C_2 = 230$ m/s;

$\gamma = 1.4$

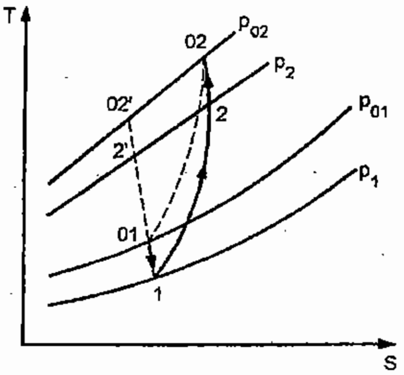


Fig. P. 6.3.5

Refer Fig. P. 6.3.5

(i) Isentropic efficiency based on static Values, η_{is}

$$T_2 = T_1 = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$

$$= 280\left(\frac{1.5}{1}\right)^{(1.4-1)/1.4} = 314.4 \text{ K}$$

$$\eta_{is} = \frac{T_2^1 - T_1}{T_2 - T_1} = \frac{314.4-280}{335-280}$$

$$= 0.6255 \text{ or } 62.55\% \qquad \text{...Ans.}$$

(ii) Power required to drive the compressor, $\dot{W}_{sf}$ or P

S.F.E.E is,

$$Q - \dot{W}_{sf} = \dot{m}\left[(h_2 - h_1) + \frac{C_2^2 - C_1^2}{2 \times 1000} + \frac{g(Z_2 - Z_1)}{1000}\right]$$

Assuming $Z_2 - Z_1 \simeq 0$, $\dot{Q} \doteq 0$ (Process is adiabatic);

$$h_2 - h_1 = C_P(T_2 - T_1)$$

$$\therefore \quad 0 - \dot{W}_{sf} = 3\left[1.005(335-280) + \frac{(230)^2 - (90)^2}{2 \times 1000} + 0\right]$$

$$\dot{W}_{sf} = -3\,[45.225 + 22.4]$$

$$= \mathbf{-202.875 \text{ kW}} \qquad \text{...Ans.}$$

Negative sign shows that power is supplied to run the compressor.

(iii) Overall efficiency i.e isentropic efficiency based on total values, η_t :

$$C_p = 1.005 \text{ kJ/kg K} = 1005 \text{ J/kg K}$$

$$T_{01} = T_1 + \frac{C_1^2}{2C_p} = 280 + \frac{(90)^2}{2 \times 1005} = 284.03 \text{ K}$$

$$T_{02} = T_2 + \frac{C_2^2}{2C_p} = 335 + \frac{(230)^2}{2 \times 1005} = 361.32 \text{ K}$$

$$\frac{p_{01}}{p_1} = \left(\frac{T_{01}}{T_1}\right)^{(\gamma)/(\gamma-1)} ;\ p_{01} = 1\left(\frac{284.03}{280}\right)^{1.4/0.4}$$

$$= 1.0513 \text{ bar}$$

$$p_{02} = p_2\left(\frac{T_{02}}{T_2}\right)^{\gamma/(\gamma-1)} = 1.5 \times \left(\frac{361.32}{335}\right)^{1.4/0.4}$$

$$= 1.9546 \text{ bar}$$

$$T_{02}^1 = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} = 284.03\left(\frac{1.9546}{1.0513}\right)^{0.4/1.4}$$

$$= 339.09 \text{ K}$$

$$\eta_t = \frac{T_{02}^1 - T_{01}}{T_{02} - T_{01}} = \frac{339.09 - 284.03}{361.32 - 284.03}$$

$$= \mathbf{0.7124 \text{ or } 71.24\%} \qquad \text{...Ans.}$$

Ex. 6.3.6 : A centrifugal compressor develops a pressure ratio of 5.0 and an air consumption of 30 kg/s. The inlet temperature and pressure are 15°C and 1 bar respectively. If isentropic efficiency is 0.85, calculate

(i) The workdone

(ii) Exit total temperature

(iii) The power required **SPPU - Dec. 16, 6 Marks**

Soln. :

Refer Fig. P. 6.3.6

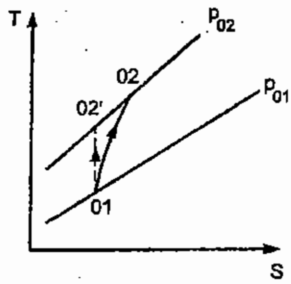


Fig. P. 6.3.6

Pressure ratio, $\quad R_p = \dfrac{p_{02}}{p_{01}} = 5$

Mass flow rate, $\quad \dot{m} = 30$ kg/s

Inlet temperature $\quad T_{01} = 15 + 273 = 288$ K

Inlet pressure $\quad p_{01} = 1$ bar

Isentropic efficiency, $\quad \eta_i = 85\%$

(a) Workdone (W)

$$T_{02}' = T_{01}\,(R_p)^{(\gamma-1)\gamma}$$

$$= 288\,(5)^{0.4/1.4} = 456.1 \text{ K}$$

$$\eta_i = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}}$$

$$0.85 = \frac{456.1 - 288}{T_{02} - 288}$$

$$T_{02} = 485.76 \text{ K}$$

$$W = C_p(T_{02} - T_{01})$$
$$= 1.005\,(485.76 - 288)$$
$$W = \mathbf{198.8\ kJ/kg\ of\ air} \quad \text{...Ans.}$$

(b) Exit total temperature (T_{02}).

From above, $T_{02} = 485.76$ K ...Ans.

(c) Power required ,(P)

$$P = \dot{m} \times W = 30 \times 198.8$$
$$= \mathbf{5964\ kW} \quad \text{...Ans.}$$

6.4 Analysis of Centrifugal Compressors - Velocity Diagrams

Following are the notations used in the analysis of centrifugal compressors.

ω = angular velocity (rad/s)

N = speed (r.p.m.)

C_i = absolute velocity of air at inlet to impeller blades (m/s)

C_{wi} = whirl or tangential component of absolute velocity at inlet (m/s)

C_{fi} = velocity of flow at inlet or radial component of absolute velocity (m/s)

C_{ri} = relative velocity at inlet (m/s)

C_{bi} = mean blade velocity at inlet (m/s)

$= \omega \cdot r_i = \frac{2\pi N}{60} \times r_i$

C_o = absolute of air at exit from impeller blades (m/s)

C_{wo} = whirl component of absolute velocity at exit (m/s)

C_{fo} = velocity of flow at exit in radial direction (m/s)

C_{ro} = relative velocity at exit (m/s)

C_{bo} = blade velocity at the tip of impeller (m/s)

$= \omega \cdot r_o = \frac{2\pi N}{60} \times r_o$

$\dot{m}$ = mass flow rate of air (kg / s)

r_i = inlet radius of impeller

$= \frac{1}{2}$ (hub radius, r_h + tip radius, r_t)

r_o = tip or exit radius of impeller

α = exit angle of fixed blade or guide vanes at inlet to impeller blades

θ = inlet angle of impeller blades

ϕ = exit angle of impeller blades

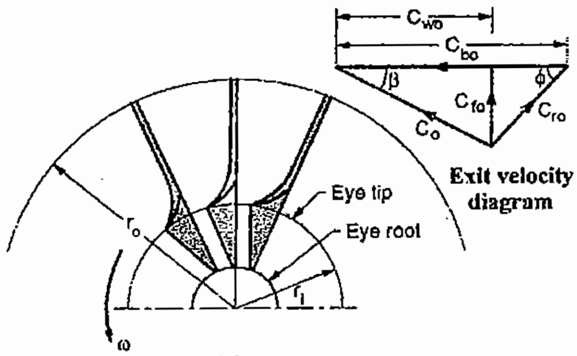
β = inlet angle of the fixed blade i.e. the inlet angle to diffuser blades.

Note that, as per sign convention for radial machines, the angles of blades are measured with reference to blade velocity direction. i.e. from tangential direction.

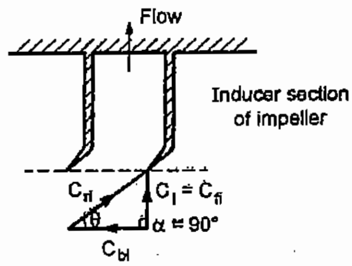
Assumptions

1. The flow of gas is steady.
2. There is no separation of flow.
3. There is no formation of shock wave anywhere during the gas flow.
4. The flow through the impeller is frictionless.

The velocity diagram at inlet and outlet of impeller blades is shown in Fig. 6.4.1(a), (b) and (c).



(a)



(b)

Fig. 6.4.1

Inlet velocity diagram

The air enters the eye of impeller blades with absolute velocity in axial direction and it is turned through 90° during the passage at inlet to impeller. Therefore, $\alpha = 90°$ at inlet without inlet guide vanes as shown in Fig. 6.4.1(b).

The relative velocity, c_{ri} is at an angle θ from radial direction.

It implies that absolute velocity C_i equals to velocity of flow C_{fi} and the velocity of whirl at inlet $C_{wi} = 0$.

From velocity triangle,

$$\tan\theta = \frac{C_i}{C_{bi}} = \frac{C_{fi}}{C_{bi}}$$

The curved impeller and diffuser blades are so shaped that the air enters and leaves their tips tangentially without shock, therefore, the relative velocities C_{ri} and C_{ro} are tangential at the tip of blades at inlet and exit respectively.

Exit velocity diagram

Impeller shown has the backward curved vane having exit angle ϕ as shown in Fig. 6.4.1(a) and (c). The flow leaves at relative velocity C_{ro} at an angle ϕ. The absolute velocity leaving is at C_o at angle β. It also represents the inlet angle of diffuser blades. C_{wo} is the whirl component of C_o. From exit velocity diagram

$$C_{fo} = C_{ro}\sin\phi = C_o \sin\beta$$

$$C_{wo} = C_o\cos\beta$$

$$= C_{bo} - \frac{C_{fo}}{\tan\phi} = C_{bo} - C_{ro}\cos\phi$$

According to Newton's second law of motion, the torque in the direction of motion of blade is equal to rate of change of angular momentum. Therefore,

Torque, $$T = \dot{m}\,(C_{wo}\cdot r_o - C_{wi}\cdot r_i) \quad ...(6.4.1)$$

Since, $$C_{wi} = 0$$

Torque, $$T = \dot{m}\cdot C_{wo}\cdot r_o \quad ...(6.4.2)$$

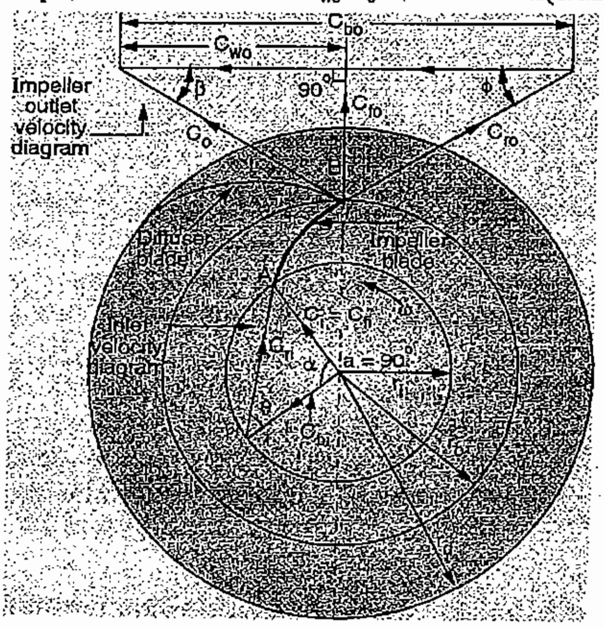


Fig. 6.4.1(c) : Velocity diagram centrifugal compressor

Hence, the rate of energy transfer,

$$\dot{W} = \text{Torque} \times \text{angular speed} = (\dot{m}\,C_{wo}\cdot r_o)\cdot\omega_o$$

$$\boxed{\dot{W} = \dot{m}\cdot C_{wo}.C_{bo}} \quad ...(6.4.3)$$

$$\text{Workdone per kg} = C_{wo}\cdot C_{bo} \quad ...(6.4.4)$$

The rate of energy transfer represents the indicated power required to drive the compressor.

If the exit from the impeller is ***radial (ideal case)***, then $\phi = 90°$. In this case $C_{wo} = C_{bo}$. The power required to drive the compressor for ideal case from Equation (6.4.3) can be written as,

Ideal power, $$\dot{W} = \dot{m}\cdot C_{bo}^2 \quad ...(6.4.5)$$

∴ Workdone per kg of air becomes,

$$W = C_{bo}^2 \text{ (Ideal case)} \quad ...(6.4.6)$$

The ideal work is called **Euler work.**

Theoretical or the Euler's head,

$$H = \frac{C_{bo}^2}{g} \quad ...(6.4.7)$$

Considering the reversible adiabatic conditions (i.e. Q = 0) and applying steady flow energy equation in case of an ideal compressor, (neglecting Δ P. E.), we get,

$$h_1 + \frac{C_1^2}{2} + Q - W = h_2 + \frac{C_2^2}{2}$$

and, $$h_0 = h + \frac{C^2}{2}$$

$$\therefore h_{01} + 0 - (-C_{bo}^2) = h_{02}$$

$$\therefore \quad C_{bo}^2 = h_{02} - h_{01}$$

$$= C_p(T_{02} - T_{01}) \quad ...(6.4.8)$$

$$\therefore \quad C_{bo}^2 = C_p\cdot T_{01}\left[\left(\frac{p_{02}}{p_2}\right)^{(\gamma-1)/\gamma} - 1\right] \quad ...(6.4.9)$$

Where, T_{01}, p_{01} and T_{02}, p_{02} are total head temperatures and pressures respectively at inlet and outlet of the impeller of a centrifugal compressor in an ideal case.

Note that Impeller blade angle of eye root and tip can be calculated by drawing inlet velocity diagram having blade velocity corresponding to radii at eye root and eye tip respectively.

6.4.1 Euler's Work

Euler's work per kg of air from Equation (6.4.3) can be written as :

$$\text{Euler's work, } W = C_{wo}\cdot C_{bo} \quad ...(i)$$

Equation (i) in terms of various energies can be modified as follows :

Refer velocity diagrams shown in Fig. 6.4.1(c) from inlet velocity diagram,

$$C_{ri}^2 = C_i^2 + C_{bi}^2 \quad \text{...(ii)}$$

From outlet velocity diagram,

$$C_{ro}^2 = C_{fo}^2 + (C_{bo} - C_{wo})^2$$

$$= C_{fo}^2 + C_{bo}^2 + C_{wo}^2 - 2C_{bo} \cdot C_{wo}$$

But, $C_{fo}^2 + C_{wo}^2 = C_o^2$. Therefore, above Equation reduces to,

$$C_{ro}^2 = C_o^2 + C_{bo}^2 - 2C_{bo}\, C_{wo} \quad \text{...(iii)}$$

On substracting Equation (iii) from Equation (ii) we can write,

$$C_{ri}^2 - C_{ro}^2 = C_i^2 - C_o^2 + C_{bi}^2 - C_{bo}^2 + 2\, C_{bo} \cdot C_{wo}$$

$$\therefore \quad C_{bo} \cdot C_{wo} = \frac{C_{ri}^2 - C_{ro}^2}{2} + \frac{C_o^2 - C_i^2}{2} + \frac{C_{bo}^2 - C_{bi}^2}{2} \quad \text{...(iv)}$$

On combining Equations (i) and (iv),

Euler's work, $W = C_{bo}\, C_{wo}$

$$= \left(\frac{C_{ri}^2 - C_{ro}^2}{2}\right) + \left(\frac{C_o^2 - C_i^2}{2}\right) + \left(\frac{C_{bo}^2 - C_{bi}^2}{2}\right) \quad \text{...(6.4.10)}$$

In the Equation (6.4.10) we have :

(i) Term $\left(\frac{C_o^2 - C_i^2}{2}\right)$ represents the kinetic energy change which represents the **dynamic head.** It gives rise to static pressure in the rotor. This energy is needed to be converted in the diffuser section of the compressor.

(ii) Term $\left(\frac{C_{bo}^2 - C_{bi}^2}{2}\right)$ is the change in kinetic energy due to movement of impeller from inlet to outlet. It corresponds to centrifugal head imparted to gas which is responsible for static pressure rise.

In axial flow machines like axial flow compressors, since $C_{bo} = C_{bi}$, the centrifugal energy imparted by rotor becomes zero.

(iii) Term $\left(\frac{C_{ri}^2 - C_{ro}^2}{2}\right)$ represents the rise in pressure energy in rotor due to decrease in relative velocity in compressor impeller due to diverging passages.

6.4.2 Width of Impeller Blades

Fig. 6.4.2 shows the section of the single and double sided impellers. The width of impeller blades can be obtained by using the continuity equation since the mass flow rate remains constant throughout. Let,

b_i = width of impeller blades at inlet

b_o = width of impeller blades at outlet

Let, v_i = specific volume of air at inlet

v_o = specific volume of air at outlet

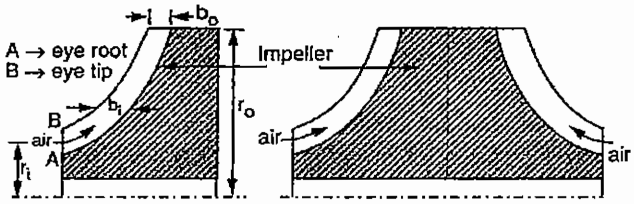
$$\text{Mass flow rate, } \dot{m} = \frac{\text{Area of flow} \times \text{Velocity of flow}}{\text{Specific Volume}}$$

$$\text{i.e.} \quad \dot{m} = \frac{(2\pi r_i b_i)\, C_{fi}}{v_i} = \frac{(2\pi r_o b_o)\, C_{fo}}{v_o} \quad \text{...(6.4.11)}$$

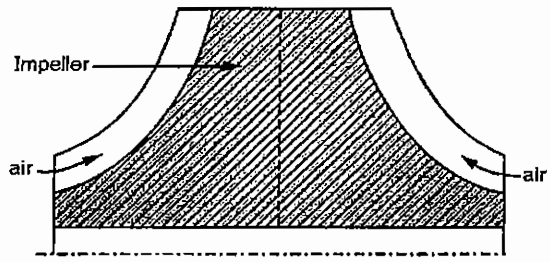
In case the thickness 't' of blades is taken into account having 'n' number of blades, the Equation (6.4.11) reduces to :

$$\dot{m} = \frac{(2\pi r_i - n \cdot t)\, b_i \cdot C_{fi}}{v_i}$$

$$= \frac{(2\pi r_o - n \cdot t)\, b_o\, C_{fo}}{v_o} \quad \text{...(6.4.12)}$$



(a) Single sided



(b) Double sided

Fig. 6.4.2 : Single and double sided impellers

6.5 Vane Shapes and Their Characteristics

The rate of energy transfer per unit mass for a centrifugal compressor can be written as follows :

Work done per unit mass of air,

$$W = C_{bo} \cdot C_{wo}$$

Above work is called **Euler Work** and this work supplied to the compressor is responsible to develop the change in head H of the fluid. Therefore,

$$\text{Euler head,} \quad H = C_{bo} \cdot C_{wo} \quad \text{...(6.5.1)}$$

From Fig. (6.5.1(a)), $C_{wo} = C_{bo} - C_{fo} \cdot \cot\phi$

$\therefore$ Euler head,

$$H = C_{bo}(C_{bo} - C_{fo} \cdot \cot\phi)$$

$$= C_{bo}\left(C_{bo} - \frac{Q}{A} \cdot \cot\phi\right) \qquad ...(6.5.2)$$

Where, Q and A represent the volume flow rate of air and the area of flow at exit respectively.

For a **given compressor running at constant speed**, the tip velocity C_{bo}, area of flow A and the exit angle ϕ are constant, hence, we can write,

$$H = K - K_1 Q \qquad ...(6.5.3)$$

Where K and K_1 are specified constants.

$$K = C_{bo}^2 \text{ and } K_1 = C_{bo} \times \frac{\cot\phi}{A}$$

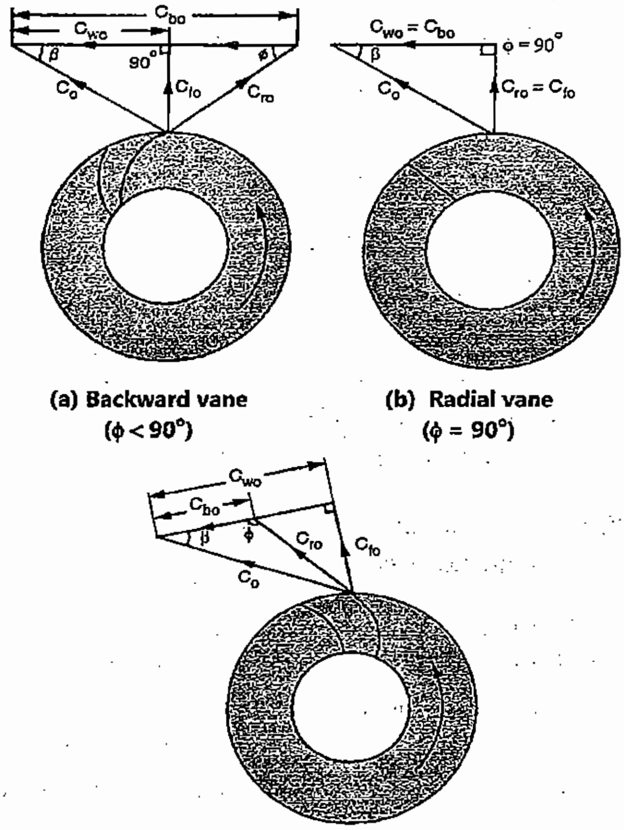
Vane shapes in centrifugal machines are of three types :

(a) Backward curved vanes, with $\phi < 90°$.

(b) Radial vanes, with $\phi = 90°$.

(c) Forward vanes, with $\phi > 90°$

These three types of vanes with their outlet velocity diagrams are shown in Fig. 6.5.1.



(a) Backward vane ($\phi < 90°$)

(b) Radial vane ($\phi = 90°$)

(c) Forward vane ($\phi > 90°$)

Fig. 6.5.1 : Shape of vanes of a centrifugal compressor with outlet velocity diagram

From Equation (6.5.2) it is evident that :

(i) **For backward vanes,** cot ϕ is positive (since $\phi < 90°$), the Euler head H will keep on falling with the increase in volume flow rate, Q.

(ii) **For radial vanes** ($\phi = 90°$), cot $\phi = 0$. Therefore, Euler head, H remains constant irrespective of volume flow rate.

(iii) **For forward vanes** ($\phi > 90°$), cot ϕ is negative. Therefore, Euler head H keeps on increasing with the increase in volume flow rate.

The theoretical Euler head characteristic curve for various types of vanes are shown in Fig. 6.5.2.

When discharge rate Q = 0, the head developed equals to C_{bo}^2, it is called as ***shut off*** **head, H′** as shown in Fig. 6.5.2.

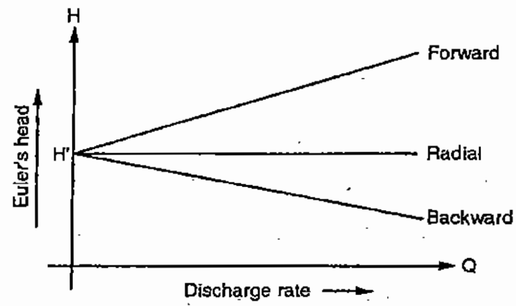


Fig. 6.5.2 : Discharge - Head characteristics of forward, radial and backward vanes

Following conclusions can be drawn with the help Equation (6.5.2) and Fig. 6.5.2 and Fig. 6.5.1 :

(i) Impellers with backward vanes have low whirl velocity C_{wo} due to which the energy transfer is low for a given tip speeds of the impeller.

(ii) Impellers with forward curved vanes have large value of C_{wo} due to high value of ϕ. Therefore such impellers give high energy transfers. However it also gives high absolute velocity C_o at outlet of the impeller and the conversion of this kinetic energy $\left(\frac{C_o^2}{2}\right)$ into pressure energy cannot be carried out efficiently in the diffuser section since the air has a tendency to break away from the walls of the diverging passages.

In case the diffusion process is too rapid, the pressure rise will be too steep and the flow of air may reverse it direction (due to high pressure gradient). This results in formation of eddies, as a result some kinetic energy is converted into internal energy and reduction in pressure rise.

Small angle of divergence angle results into long diffuser and high viscous friction losses.

For above reasons the divergence angle β at inlet to diffuser is kept between 7° to 14° based on experimental results.

(iii) For the reasons discussed above, the compressors are usually designed with backward vanes having ϕ in the range of 20° to 35° except in the case where the rotor diameter is to be limited and the high head is the major consideration

(iv) Use of radial vanes is a compromise between the backward and forward vanes. Compressors with radial vanes can be run at high speeds with high pressure rise. Moreover, these vanes are easy to design since these vanes are free from complex bending and twisting stresses. These vanes can be cheaply manufactured on mass scale.

6.6 Slip Factor, Power Input Factor, Pressure Coefficient and Prewhirl

6.6.1 Slip Factor (Ψ_s)

University Questions

Q. Define slip. **SPPU : May 14**

Q. Explain slip in connection to centrifugal compressors. **SPPU : Dec. 11, Dec. 12, Dec. 19**

Q. Explain slip and slip factor its importance in centrifugal compressor. **SPPU : May 15**

Q. Explain Slip coefficient. **SPPU : May 14, Dec. 15**

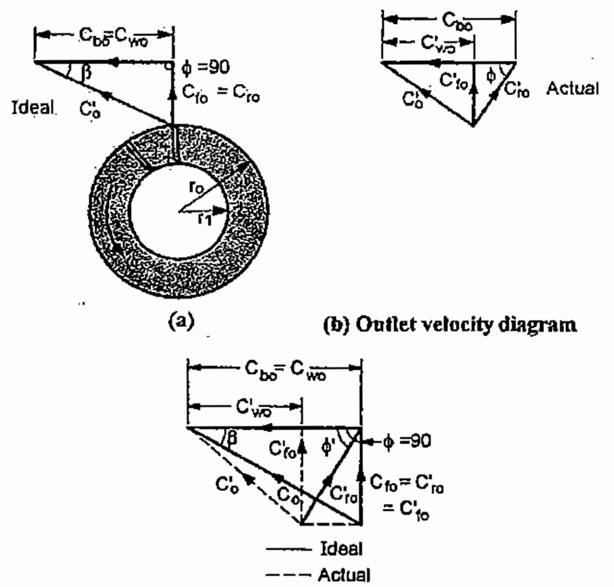
Q. What do you mean by slip and slip factor ? **SPPU : Dec. 16**

Q. Write short note on Slip and Slip Factor in compressors. **SPPU : May 18**

- In case of radial or straight vane centrifugal compressors, the ideal velocity of whirl C_{wo} equals to blade velocity C_{bo} at exit as shown in Fig. 6.6.1(a).
- However, this condition is not satisfied in actual practice because of the inertia of gas trapped between the impeller blades lags behind compared to the movement of impeller.
- The effect of this is to create a pressure difference across the impeller blades with a high pressure on leading face and comparatively low pressure on trailing face as shown in Fig. 6.6.1(a).
- Due to this the gas flows at a higher speed on the low pressure side compared to high pressure side, hence, a velocity gradient would exist across the impeller blades.
- As a result, the gas leaves tangentially only on the high pressure side and nowhere else while at other points the direction of velocity vector differs that from exit blade angle ϕ.
- The gas will thus be discharged at a certain average angle ϕ' which is less than the impeller blade exit angle $\phi = 90^\circ$ for radial vanes is shown by actual velocity diagram in Fig. 6.6.1(b).
- It results into reduction in whirl velocity component from C_{wo} to C'_{wo} This phenomenon is called **slip.**

The difference of ideal and actual whirl velocities $(C_{wo} - C'_{wo})$ *is called* ***slip.***

The ratio of actual whirl velocity C'_{wo} *to ideal whirl velocity* C_{wo} *is called* ***slip factor*** Ψ_s.



(a) **(b) Outlet velocity diagram**

(c) Superimposed ideal and actual diagrams

Fig. 6.6.1 : Effect of slip in case of centrifugal compressor

$$\therefore \text{Slip factor, } \Psi_s = \frac{\text{Actual whirl velocity } (C'_{wo})}{\text{Ideal whirl velocity } (C_{wo})} \quad ...(6.6.1)$$

But, for an impeller with radial vanes, $C_{wo} = C_{bo}$

$$\therefore \quad \text{Slip factor, } \Psi_s = \frac{C'_{wo}}{C_{bo}} \quad ...(6.6.2)$$

The slip factor depends upon the number of impeller vanes. Lesser the number of vanes, higher is the slip. It's usual value varies between 0.9 to 0.96.

Ideal work/kg of air $= C_{bo} C_{wo}$ ...[Refer Equation (6.4.4)]

$\therefore$ Using slip factor, the workdone becomes,

$$\text{Theoretical work, } W = C_{bo} \times \Psi_s \cdot C_{bo}$$

$$= \Psi_s \cdot C_{bo}^2 \qquad \text{...(6.6.3)}$$

6.6.2 Work Factor or Power Input Factor, Ψ_w

University Questions

Q. Define work factor. **SPPU : May 11, May 14**

Q. Explain Work input factor. **SPPU : May 15**

- The actual work required by the compressor is always greater than theoretical work given by the Equation (6.6.3) due to friction and turbulence losses.
- Actual work is obtained by multiplying the theoretical work by an emperically determined factor Ψ_w called **work factor.** It's value usually varies between 1.03 to 1.06.

$$\therefore \text{ Actual workdone, } W = \Psi_s \cdot \Psi_w \cdot C_{bo}^2 \qquad \text{...(6.6.4)}$$

6.6.3 Pressure Coefficient Ψ_p

University Questions

Q. Define pressure coefficient. **SPPU : May 11, May 14**

Q. Explain Pressure coefficient. **SPPU : May 15, Dec. 15**

The ratio of isentropic work to Euler work of compression is defined as **pressure coefficient.**

$$\therefore \Psi_p = \frac{\text{Isentropic work}}{\text{Euler work}} = \frac{C_p (T'_{02} - T_{01})}{C_{bo} C_{wo}} \qquad \text{...(6.6.5)}$$

6.6.4 Prewhirl

University Question

Q. Explain prewhirl in connection to centrifugal compressors. **SPPU: Dec. 11, Dec. 12, May 14, Dec. 19**

Q. What is Pre-whirl in centrifugal compressor? Why it is necessary? **SPPU : Dec. 16**

- If the relative velocity of air at inlet is such that the mach number at entrance exceeds a value beyond which the shock waves are formed, it is desirable to reduce the inlet relative velocity by giving the air an initial pre-rotation to inlet absolute velocity of air since the formation of shock waves increases the loss of energy.

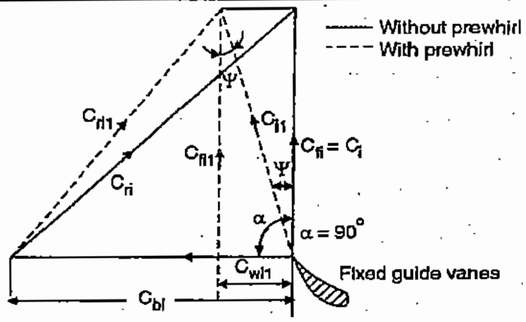


Fig. 6.6.2 : Effect of prewhirl

- This process which causes the air to enter impeller blades at a reduced relative velocity is called **pre-rotation or prewhirl.**
- The prewhirl to the air is given by introducing a set of fixed guide vanes preceding the impeller. The inlet velocity diagram with and without prewhirl is shown in Fig. 6.6.2.
- The angle Ψ is called the **angle of prewhirl** which equals to inlet angle of fixed guide vane. The workdone per kg of air by the impeller with prewhirl is given as :

$$W = C_{wo} \cdot C_{bo} - C_{wi1} \cdot C_{bi} \qquad \text{...(6.6.6)}$$

Therefore, the prewhirl reduces the compressor input work by an amount equal to ($C_{wi1} \times C_{bi}$).

6.7 Calculations for Pressure Ratio and Relationship between Ψ_p, Ψ_s and Ψ_w

University Question

Q. Derive an expression for the overall pressure ratio developed in the Centrifugal Compressor. **SPPU : May 18**

$$\text{Actual workdone, } W = \Psi_s \times \Psi_w \cdot C_{bo}^2$$

... [Refer Equation (6.6.4)]

Fig. 6.7.1 shows the pressure rise in centrifugal compressor in which process (01 – 02') shows the isentropic or ideal compression and process (01 – 02) shows the actual compression with friction.

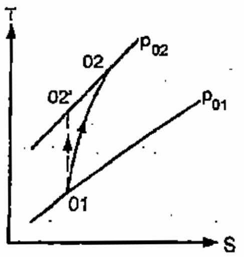


Fig. 6.7.1 : Pressure rise in centrifugal compressor

This work is transferred to the air due to which its pressure and temperature rises. The work required to compress the air with the help of S.F.E.E. can be written as,

$$W = C_p (T_{02} - T_{01}) \quad \text{...(i)}$$

Equating the two works of compression,

$$C_p (T_{02} - T_{01}) = \Psi_s \times \Psi_w \cdot C_{bo}^2 \quad \text{...(ii)}$$

$$\therefore \quad (T_{02} - T_{01}) = \frac{\Psi_s \cdot \Psi_w \cdot C_{bo}^2}{C_p} \quad \text{...(iii)}$$

Isentropic efficiency is given as,

$$\eta_i = \frac{C_p (T'_{02} - T_{01})}{C_p (T_{02} - T_{01})} \quad \text{...(6.7.1)}$$

$$= \frac{C_p (T'_{02} - T_{01})}{\Psi_s \cdot \Psi_w \cdot C_{bo}^2}; \quad \text{[From Equation (ii)]}$$

$\therefore$ Isentropic temperature rise,

$$(T'_{02} - T_{01}) = \frac{\Psi_s \cdot \Psi_w \cdot C_{bo}^2 \cdot \eta_i}{C_p} \quad \text{...(iv)}$$

Pressure ratio is given as,

$$\left(\frac{p_{02}}{p_{01}}\right) = \left(\frac{T'_{02}}{T_{01}}\right)^{\gamma/(\gamma-1)} = \left(1 + \frac{T'_{02}}{T_{01}} - 1\right)^{\gamma/(\gamma-1)}$$

$$\frac{p_{02}}{p_{01}} = \left(1 + \frac{T'_{02} - T_{01}}{T_{01}}\right)^{\gamma/(\gamma-1)}$$

$$\frac{p_{02}}{p_{01}} = \left[1 + \frac{\Psi_s \cdot \Psi_w \cdot C_{bo}^2 \cdot \eta_i}{C_p \cdot T_{01}}\right]^{\gamma/(\gamma-1)} \quad \text{...(6.7.2)}$$

Also, Pressure coefficient,

$$\Psi_p = \frac{\text{Isentropic work}}{\text{Euler work}} \quad \text{...[Refer Equation (6.6.5)]}$$

i.e.

$$\Psi_p = \frac{C_p (T'_{02} - T_{01})}{C_{bo}^2} = \frac{C_p \eta_i (T_{02} - T_{01})}{C_{bo}^2}$$

$$\therefore \quad (T_{02} - T_{01}) = \frac{\Psi_p \cdot C_{bo}^2}{C_p \cdot \eta_i} \quad \text{...(v)}$$

Equating Equations (iii) and (v),

$$\frac{\Psi_s \cdot \Psi_w \cdot C_{bo}^2}{C_p} = \frac{\Psi_p \cdot C_{bo}^2}{C_p \cdot \eta_i}$$

$$\text{i.e.,} \quad \Psi_p = \eta_i \cdot \Psi_s \cdot \Psi_w \quad \text{...(6.7.3)}$$

Ex. 6.7.1 : A centrifugal compressor runs at 2400 r.p.m. and it receives air at 27°C. It's tip diameter is 0.7 m. Find the temperature of air leaving the compressor. Assume, $C_p = 1.005$ kJ / kg K and $\gamma = 1.4$. Neglect slip and changes in kinetic energy.

Soln. :

Given :

Inlet temperature, $T_{01} = 27^\circ C = 300$ K ;

$N = 2400$ rpm ; $\quad D_o = 0.7$ m.

Tip velocity,

$$C_{bo} = \frac{\pi D_o N}{60} = \frac{\pi \times 0.7 \times 2400}{60} = 87.96 \text{ m/s}$$

Let T_{02} be the exit temperature.

Workdone per kg of air,

$$W = m \cdot C_{bo}^2 = 1 \times (87.96)^2$$

$$= 7737 \text{ Nm/kg} = 7.737 \text{ kNm/kg}$$

$$\text{But, } W = C_p (T_{02} - T_{01})$$

$$7.737 = 1.005 (T_{02} - 300)$$

$$\mathbf{T_{02} = 307.7\ K} \quad \text{...Ans.}$$

Ex. 6.7.2 : A centrifugal compressor running at 9000 rpm delivers 600 m³/min of free air. The air is compressed from 1 bar and 20° C to a pressure ratio of 4 with an isentropic efficiency of 82 %. Blades are radial at outlet of impeller and flow velocity of 62 m/s may be assumed throughout constant. The outer radius of impeller is twice the inner and slip factor may be assumed as 0.9. The blade area coefficient of 0.9 may be assumed at inlet.

Calculate :

(i) Final temperature of air

(ii) Theoretical power

(iii) Impeller diameters at inlet and outlet

(iv) Impeller blade angle at inlet

(v) Diffuser blade angle at inlet

(vi) Breadths of impeller at inlet **SPPU - Dec. 15, 12 Marks**

Soln. :

$N = 9000$ rpm

$\dot{Q} = 600 \text{ m}^3/\text{mm} = \frac{600}{60} = 10 \text{ m}^3/\text{s}$

$p_1 = 1$ bar, $\quad T_1 = 20°C = 293$ K

$\frac{p_2}{p_1} = r_p = 4, \quad \eta_i = 0.82$

$\phi = 90°$ (radial blades at outlet),

therefore $C_{wo} = C_{bo}$

$C_{fi} = C_{fo} = C_f = 62$ m/s

$D_0 = 2 D_i, \quad \psi_s = 0.9$,

Blade area coefficient, $k_f = 0.9$

Refer Fig. P. 6.7.2

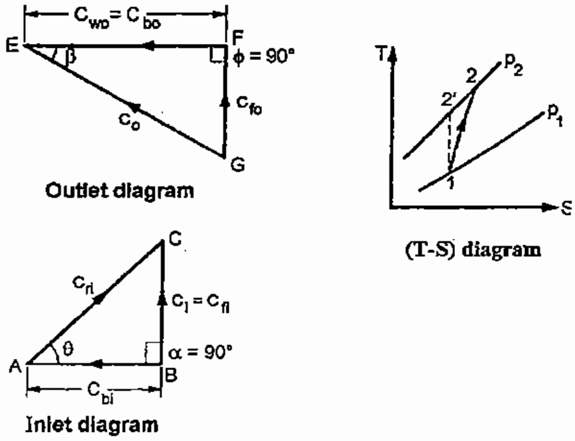


Fig. P. 6.7.2

(i) Final temperature of air, T_2

$$T_2' = T_1 (r_p)^{(\gamma-1)/\gamma} = 293(4)^{0.4/1.4}$$

$$= 435.4 \text{ K}$$

$$\eta_i = \frac{T_2' - T_1}{T_2 - T_1}$$

$$0.82 = \frac{435.4 - 293}{T_2 - 293}$$

$$\mathbf{T_2 = 466.65 \text{ K}} \qquad \text{...Ans.}$$

(ii) Theoretical power, P_t

Density of air inlet,

$$\rho_i = \frac{p_1}{RT_1} = \frac{1 \times 10^5}{287 \times 293} = 1.1892 \text{ m}^3/\text{kg}$$

$$\dot{m} = \rho_i \dot{Q} = 1.1892 \times 10 = 11.892 \text{ kg/s}$$

$$P_t = \dot{m}\ C_p (T_2 - T_1)$$

$$= 11.892 \times 1.005 \times (466.65 - 293)$$

$$= \mathbf{2075.4 \text{ kW}} \qquad \text{...Ans.}$$

(iii) Impeller diameter at inlet, D_i and at outlet, D_0

Work input / kg of air, ,

$$\psi_S C_{bo}^2 = C_p (T_2 - T_1)$$

$$0.9 \times C_{bo}^2 = (1.005 \times 1000) \times (466.65 - 293)$$

$$C_{bo} = 440.35 \text{ m/s}$$

$$C_{bo} = \frac{\pi D_0 N}{60}$$

$$440.35 = \frac{\pi \times D_0 \times 9000}{60}$$

$$\mathbf{D_0 = 0.9345 \text{ m}} \qquad \text{...Ans.}$$

$$D_i = \frac{D_0}{2} = \frac{0.9345}{2} = \mathbf{0.46725 \text{ m}} \qquad \text{...Ans.}$$

(iv) Impeller blade angle at inlet, θ

$$C_{bi} = \frac{\pi D_i N}{60} = \frac{\pi \times 0.46725 \times 9000}{60}$$

$$= 220.18 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_{fi}}{C_{bi}}\right) = \tan^{-1}\left(\frac{62}{220.18}\right)$$

$$= \mathbf{15.727°} \qquad \text{...Ans.}$$

(v) Diffuser blade angle at inlet, B

$$B = \tan^{-1}\left(\frac{C_{fo}}{C_{bo}}\right) = \tan^{-1}\left(\frac{62}{440.35}\right)$$

$$= \mathbf{8.014°} \qquad \text{...Ans.}$$

(vi) Breadth at inlet, B_i and at outlet, B_0

$$\dot{m} = \frac{\pi D_i B_i k_f C_{fi}}{v_i} = \rho_i \cdot \pi D_i B_i k_f C_{fi} \qquad \left(\because \frac{1}{v_i} = \rho_i\right)$$

$$11.892 = 1.1892 \times \pi \times 0.46725 \times B_i \times 0.9 \times 62$$

$$\mathbf{B_i = 0.1221 \text{ m}} \qquad \text{...Ans.}$$

$$p_2 = \rho_0 RT_2 ;\ 4 \times 10^5$$

$$= \rho_0 \times 287 \times 466.65$$

$$\rho_0 = 2.9867 \text{ kg/m}^3$$

$$\dot{m} = \rho_0 \times \pi D_0 B_0 k_f C_{fo}$$

$$11.892 = 2.9867 \times \pi \times 0.9345 \times B_0 \times 0.9 \times 62$$

$$\mathbf{B_0 = 0.0243 \text{ m}} \qquad \text{...Ans.}$$

Ex. 6.7.3 : A centrifugal compressor has an inlet eye 150 mm diameter. The impeller revolves at 20,000 rpm and the inlet air has an axial velocity of 107 m/s, inlet stagnation temperature 294 K and inlet pressure 1.03 bar. Determine :

(a) Theoretical angle of the blade at this points and

(b) Mach number of the flow at the tip of the eye.

Soln. : Refer Fig. P 6.7.3

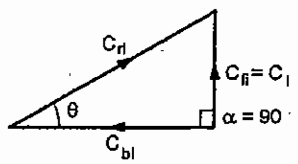


Fig. P. 6.7.3

Given : $d_i = 150 \text{ mm} = 0.15 \text{ m}$,

$N = 20000 \text{ rpm}$ $\quad C_i = C_{fi} = 107 \text{ m/s}$

$T_{01} = 294 \text{ K}$ $\quad p_{01} = 1.03 \text{ bar}$

$$C_{bi} = \frac{\pi \cdot d_i \cdot N}{60} = \frac{\pi \times 0.15 \times 20000}{60} = 157.08 \text{ m/s}$$

(i) Theoretical angle of blade, θ

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{C_{bi}}\right) = \tan^{-1}\left(\frac{107}{157.08}\right)$$

$$= \mathbf{34.26^\circ} \qquad \text{...Ans.}$$

(ii) Mach number at the tip of eye, M

Static temperature,

$$T_1 = T_{01} - \frac{C_i^2}{2\,C_p}$$

$$= 294 - \frac{(107)^2}{2 \times (1.005 \times 10^3)} = 288.3 \text{ K}$$

Sonic velocity,

$$a = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 288.3}$$

$$= 340.35 \text{ m/s}$$

$$C_{ri} = \frac{C_i}{\sin\theta} = \frac{107}{\sin 34.26} = 190.06 \text{ m/s}$$

$$M = \frac{C_{ri}}{a} = \frac{190.06}{340.35} = \mathbf{0.5584} \qquad \text{...Ans.}$$

Ex. 6.7.4 : A centrifugal compressor has an inlet diameter of 180 mm. The impeller rotates at 21000 rpm. The air at inlet has a velocity of 115 m/s. The inlet stagnation temperature is 300 K.

Determine :

(i) Mach number of the flow at the tip of the eye.

(ii) Theoretical blade angle at inlet.

(Take R = 0.287 kJ/kg K). **SPPU - Dec. 13,10 Marks**

Soln. : Refer Fig. P. 6.7.4

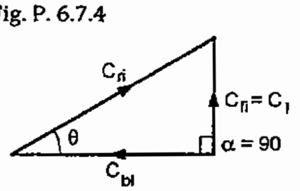


Fig. P. 6.7.4

Given : d_i = 180 mm = 0.18 m, N = 21000 rpm

$C_i = C_{fi}$ = 115 m/s, T_{01} = 300 K

p_{01} = 1.03 bar

$$C_{bi} = \frac{\pi \cdot d_i \cdot N}{60} = \frac{\pi \times 0.18 \times 21000}{60}$$

$$= 197.92 \text{ m/s}$$

(i) Theoretical angle of blade, θ

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{C_{bi}}\right)$$

$$= \tan^{-1}\left(\frac{115}{197.92}\right) = \mathbf{30.15^\circ}$$

(ii) Mach number at the tip of eye, M :

Static temperature,

$$T_1 = T_{01} - \frac{C_i^2}{2\,C_p} = 300 - \frac{(115)^2}{2 \times (1.005 \times 10^3)}$$

$$= 293.42 \text{ K}$$

Sonic velocity,

$$a = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 293.42}$$

$$= 343.36 \text{ m/s}$$

$$C_{ri} = \frac{C_i}{\sin\theta} = \frac{115}{\sin 30.15} = 228.96 \text{ m/s}$$

$$M = \frac{C_{ri}}{a} = \frac{228.96}{343.36} = \mathbf{0.666}$$

Ex. 6.7.5 : A centrifugal compressor running at 10000 r.p.m. delivers 660 m³/min. of free air. The air is compressed from 1 bar and 20°C to a pressure ratio of 4 with an isentropic efficiency of 82%. Blades are radial at outlet of impeller and flow velocity of 62 m/s may be assumed throughout constant. The outer radius of impeller is twice the inner and the slip factor may be assumed as 0.9. The blade area coefficient may be assumed 0.9 at inlet, determine :

(i) Final temperature of air

(ii) Theoretical power

(iii) Impeller diameter at inlet and outlet

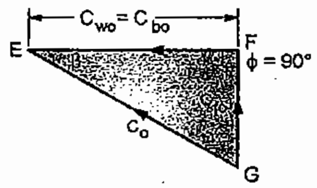
(iv) Breadth of impeller at inlet

(v) Impeller blade angle at inlet

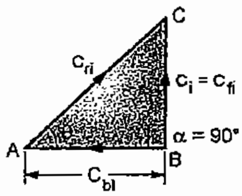
(vi) Diffuser blade angle at inlet

SPPU - Dec. 12,Dec. 18,10 Marks

Soln. : Refer Fig. P. 6.7.5



Exit velocity diagram



Inlet velocity diagram

Fig. P. 6.7.5

$$N = 10000 \text{ rpm}, \quad \dot{Q} = 660 \text{ m}^3/\text{min}$$

i.e. $$\dot{Q} = \frac{660}{60} = 11 \text{ m}^3/\text{s} \quad p_1 = 1 \text{ bar},$$

$$T_1 = 20°C = 293 \text{ K} \quad r_p = \frac{p_2}{p_1} = 4 \;;$$

$$\eta_i = 0.82$$

Blades are radial at outlet i.e.

$$\phi = 90°,$$

therefore, $$C_{wo} = C_{bo}$$

$$C_{fi} = C_{fo} = C_i = 62 \text{ m/s}$$

$$R_o = 2 R_i \quad \text{i.e.} \quad D_o = 2 D_i, \qquad \psi_s = 0.9$$

Blade area constant at inlet, $k_f = 0.9$

(i) Final temperature of air, T_2

$$T_2' = T_1 (r_p)^{(\gamma-1)/\gamma} = 293\,(4)^{0.4/1.4} = 435.4 \text{ K} \quad \text{...Ans.}$$

$$\eta_i = \frac{T_2' - T_1}{T_2 - T_1}$$

$$0.82 = \frac{435.4 - 293}{T_2 - 293}$$

$$T_2 = \mathbf{466.65\ K} \quad \text{...Ans.}$$

(ii) Theoretical power, P_t

Density of air at inlet,

$$\rho_i = \frac{p_1}{RT_1} = \frac{1 \times 10^5}{287 \times 293} = 1.1892 \text{ m}^3/\text{kg}$$

$$\dot{m} = \rho_i \times \dot{Q} = 1.1892 \times 11 = 13.08 \text{ kg/s}$$

$$P_t = \dot{m}\, C_p (T_2 - T_1)$$

$$= 13.081 \times 1.005 \times (466.65 - 293)$$

$$= \mathbf{2282.87\ kW} \quad \text{...Ans.}$$

(iii) Impeller diameter at inlet, D_i and outlet, D_o

Specific volume of air at inlet,

$$v_i = \frac{1}{\rho_i} = \frac{1}{1.1892}$$

$$= 0.8409 \text{ m}^3/\text{kg}$$

$$\text{Work done/ kg of air} = \psi_s \cdot C_{bo}^2$$

i.e. $$C_p (T_{o2} - T_{o1}) = \psi_s C_{bo}^2$$

$$1005\,(466.65 - 293) = 0.9 \times C_{bo}^2$$

$$C_{bo} = 440.35 \text{ m/s}$$

$$C_{bo} = \frac{\pi D_o N}{60}$$

$$440.35 = \frac{\pi \times D_o \times 10000}{60}$$

$$D_o = \mathbf{0.841\ m} \quad \text{...Ans.}$$

$$D_i = \frac{D_o}{2} = \frac{0.841}{2} = \mathbf{0.4205\ m} \quad \text{...Ans.}$$

(iv) Breadth of impeller at inlet, B_i

$$\dot{m} = \frac{\pi D_i B_i K_f C_{fi}}{v_i}$$

$$13.081 = \frac{\pi \times 0.4205 \times B_i \times 0.9 \times 62}{0.8409}$$

$$B_i = \mathbf{0.1492\ m} \quad \text{...Ans.}$$

(v) Impeller blade angle at inlet, θ

$$C_{bi} = \frac{1}{2} C_{bo} = \frac{1}{2} \times 440.35$$

$$\left(\because D_i = \frac{1}{2} D_o\right) = 220.175 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{C_{bi}}\right) = \tan^{-1}\left(\frac{62}{220.175}\right)$$

$$= \mathbf{15.727°} \quad \text{...Ans.}$$

(vi) Diffuser blade angle at inlet i.e. β

$$\beta = \tan^{-1}\left(\frac{C_{fo}}{C_{bo}}\right) = \tan^{-1}\left(\frac{62}{440.35}\right)$$

$$= \mathbf{8.014°} \quad \text{...Ans..}$$

Ex. 6.7.6 : Prove that for the case of a simple centrifugal impeller with axial entry of flow into impeller eye and possessing radial vanes at impeller tip, the pressure coefficient (ψ_p) and adiabatic efficiency (η_c) of the compressor can be related with its blade velocity V_b at impeller tip by the following relation :

[illegible]

Where $(T_2 - T_1)$ is the temperature gained by air during compression process. **SPPU : May 19, 6 Marks**

Soln. : Pressure coefficient : It is defined as the ratio of isentropic work to Euler's work.

Refer Fig. P. 6.7.6.

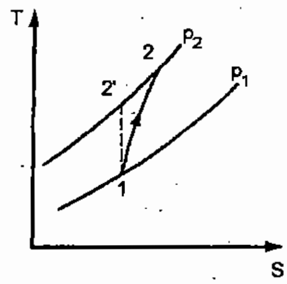


Fig. P. 6.7.6

Isentropic work, $W_i = C_p\left(T_2' - T_1\right)$ ...(i)

Isentropic efficiency, $\eta_i = \dfrac{C_p\left(T_2' - T_1\right)}{C_p\,(T_2 - T_1)}$ ...(ii)

$\therefore \left(T_2' - T_1\right) = \eta_i\,(T_2 - T_1)$ ...(iii)

Pressure coefficient,

$$\varphi_p = \frac{\text{Isentropic work, } W_i}{\text{Euler's work}} = \frac{C_p\left(T_2' - T_1\right)}{C_{bo}^2}$$

$$\therefore \varphi_p = \frac{C_p \times \eta_i\,(T_2 - T_1)}{C_{bo}^2}$$

$$\therefore (T_2 - T_1) = \frac{C_{bo}^2}{C_p} \times \frac{\varphi_p}{\eta_i} = \frac{C_{bo}^2}{1005} \times \frac{\varphi_p}{\eta_i} \qquad \textbf{Proved}$$

$\because$ C_p for air = 1005 J/kg K

Ex. 6.7.7 : A centrifugal compressor impeller admits 20 kg/s air at static state of 1 bar, 300 K and runs at 15000 rpm. Isentropic efficiency is 90% for the compression upto 5 bar total pressure. The air enters the impeller eye without prewhirl with the velocity of 120 m/s. Considering the ratio of whirl velocity to tip speed as 0.9 and the internal diameter of the impeller eye as 20 cm, determine :

(i) Rise in the total temperature in the compressor

(ii) Impeller tip speed

(iii) Impeller tip diameter

(iv) Power required to drive compressor

(v) Outer diameter of the impeller eye.

SPPU - May 16, 10 Marks

Soln. :

$\dot{m} = 20$ kg/s, $p_1 = 1$ bar,

$T_1 = 300$ K, $N = 1500$ rpm,

$\eta_i = 0.9$, $p_{02} = 5$ bar

$C_{fi} = C_i = 120$ m/s, $C_{wo} = 0.9\,C_{bo}$

$d_i = 20$ cm $= 0.2$ m

Refer Fig. P. 6.7.7.

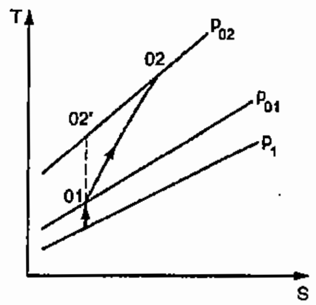


Fig. P. 6.7.7

$$T_{01} = T_1 + \frac{C_1^2}{2C_p}$$

$$= 300 + \frac{(120)^2}{2 \times 1005} = 307.2 \text{ K}$$

$$\frac{p_{01}}{p_1} = \left(\frac{T_{01}}{T_1}\right)^{\gamma/(\gamma-1)}$$

$$p_{01} = 1 \times \left(\frac{307.2}{300}\right)^{1.4/0.4} = 1.086 \text{ bar}$$

$$T_{02}' = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma}$$

$$= 307.2\left(\frac{5}{1.086}\right)^{0.4/1.4} = 475.2 \text{ K}$$

$$\eta_i = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}}$$

$$0.9 = \frac{475.2 - 307.2}{T_{02} - 307.2}$$

$$T_{02} = 493.9 \text{ K}$$

(i) Rise in total temperature, ΔT_o

$$\Delta T_0 = T_{02} - T_{01} = 493.9 - 307.2$$

$$= \mathbf{186.7°\,C} \qquad \textbf{...Ans.}$$

(ii) Impeller tip speed, C_{b0}

$$W = C_P\,(T_{02} - T_{01})$$

$$= 1.005 \times 186.7 = 187.634 \text{ kJ/kg}$$

But, $W = C_{wo} \times C_{bo}$

$$187.634 \times 10^3 = (0.9\,C_{bo}) \times C_{bo}$$

$$C_{bo} = \mathbf{456.6 \text{ m/s}} \qquad \textbf{...Ans.}$$

(iii) Impeller tip diameter, D_o

$$C_{bo} = \frac{\pi D_o N}{60}$$

$$456.6 = \frac{\pi \times D_o \times 15000}{60}$$

$$D_o = \mathbf{0.5814 \text{ m}} \qquad \textbf{...Ans.}$$

(iv) Power required to drive the compressor, W or P

$$P = \dot{m}\,W = 20 \times 187.634$$

$$= \mathbf{3752.68 \text{ kW}} \qquad \textbf{...Ans.}$$

(v) Outer diameter of impeller eye, d_o

Specific volume air at inlet,

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 300}{1 \times 10^5} = 0.861 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{A_1 C_{fi}}{v_1} = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) \times \frac{C_{fi}}{v_1}$$

$$20 = \frac{\pi}{4} \times \left[d_o^2 - (0.2)^2\right] \times \frac{120}{0.861}$$

$$d_o = \mathbf{0.472 \text{ m}} \qquad \textbf{...Ans.}$$

Ex. 6.7.8 : A centrifugal compressor has to deliver 6 kg/s of air with pressure ratio of 4.4 : 1 at 17500 rpm. Initial conditions are static air at 1 bar and 15°C. Assuming an adiabatic efficiency of 78%, ratio of whirl speed to tip speed 0.94 and neglecting all other losses, calculate the tip speed, diameter and rise in total pressure. Also determine the external diameter of eye for which the internal diameter is 12cm and the axial velocity at inlet is 150m/sec. Assume C_p = 1.05 kJ/kgK and γ = 1.4.

Soln. :

Refer Fig. P. 6.7.8(a).

Since the condition of still air at A is at p_{01} = 1 bar and T_{01} = 15°C or 288 K, its pressure and temperature will be reduced at inlet to impeller blades represented by point B since some of the pressure energy is converted into kinetic energy.

Process BD represents the actual compression of air in the impeller blades having some kinetic energy equal to $C_0^2 / 2$ at point D.

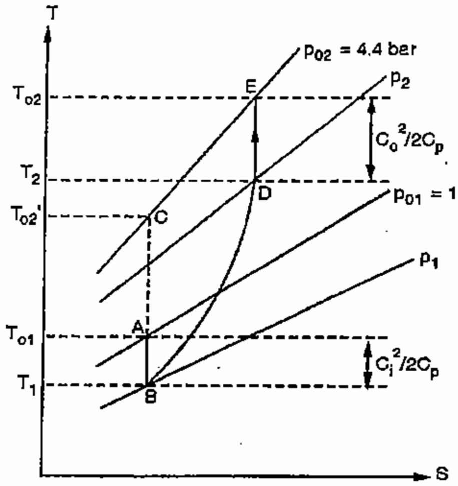


Fig. P. 6.7.8(a)

This kinetic energy will be converted into pressure energy in the diffuser section represented by isentropic process DE.

Given : Inlet velocity of air $C_i = C_{fi}$ = 150 m/s

Temperature drop equivalent to kinetic energy at inlet,

$$T_{01} - T_1 = \frac{C_1^2}{2\,C_p} = \frac{(150)^2}{2\times(1.05\times1000)} = 10.71°\,C$$

$$\therefore\ T_1 = T_{01} - 10.71 = 288 - 10.71 = 277.29\ K$$

$$\frac{T_{02}'}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma}$$

$$\text{or,}\quad T_{02}' = 288\left(\frac{4.4}{1}\right)^{(1.4-1)/1.4} = 439.78\ K$$

Isentropic efficiency based on total values,

$$\eta_i = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}}$$

$$0.78 = \frac{439.78 - 288}{T_{02} - 288}$$

$$T_{02} = 482.59\ K$$

Workdone per kg of air,

$$W = C_p\,(T_{02} - T_{01})$$

$$= 1.05\,(482.59 - 288)$$

$$= 204.3\ kJ/kg$$

$$\text{Also,}\quad W = C_{wo}\times C_{bo} = \psi_s\cdot C_{bo}^2$$

$$204.3\times1000 = 0.94\times C_{bo}^2$$

$$C_{bo} = \mathbf{466.2\ m/s} \quad \text{...Ans.}$$

$$\text{But,}\quad C_{bo} = \frac{\pi\,D_o\,N}{60}$$

... where D_o = tip diameter

$$466.2 = \frac{\pi\times D_o\times17500}{60}$$

$$D_o = \mathbf{0.509\ m} \quad \text{...Ans.}$$

Rise in total temperature,

$$T_{02} - T_{01} = 482.59 - 288 = \mathbf{194.59°C} \quad \text{...Ans.}$$

For isentropic process AB,

$$\left(\frac{T_{01}}{T_1}\right)^{\gamma/(\gamma-1)} = \frac{p_{01}}{p_1}$$

$$p_1 = p_{01}\left(\frac{T_1}{T_{01}}\right)^{\gamma/(\gamma-1)}$$

$$= 1\left(\frac{277.29}{288}\right)^{1.4/(1.4-1)}$$

$$= 0.876\ bar$$

$$R = C_p - C_v = C_p\left(1-\frac{1}{\gamma}\right)$$

$$= 1.05\left(1-\frac{1}{1.4}\right)$$

$$= 0.3\ kJ/kg\,K$$

Specific volume of air at inlet,

$$v_1 = \frac{R\cdot T_1}{p_1} = \frac{300\times277.29}{0.876\times10^5}$$

$$= 0.9496\ m^3/kg$$

Mass flow rate, $\dot{m} = \dfrac{\text{Area of flow at inlet} \times C_{fi}}{v_1}$

$$6 = \frac{A \times 150}{0.9496}$$

$$A = 0.03798 \text{ m}^2$$

Refer Fig. P. 6.7.8(b).

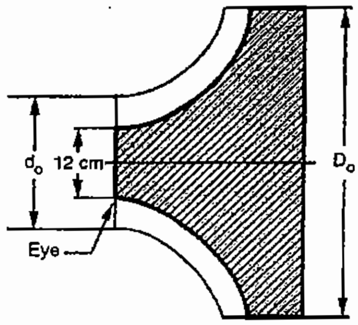


Fig. P. 6.7.8(b)

Internal diameter of eye, d_i = 12 cm.

Let, external diameter of eye be 'd_o'.

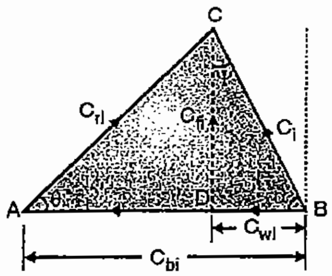
$$\text{Then, } A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)$$

$$0.03798 = \frac{\pi}{4}\left(d_o^2 - 0.12^2\right)$$

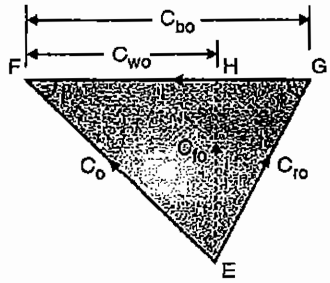
or, d_o = **0.2505 m, or 25.05 cm** ...Ans.

Ex. 6.7.9 : A double sided centrifugal compressor has impeller eye root and tip diameters of 17.5 cm and 31.25 cm and is to deliver 18 kg of air/sec. at 16000 rpm. The ambient conditions are 15°C and 1 bar and the compressor is to be a part of stationary power plant. Find suitable values for the impeller vane angles at root and tip of the eye if the air is given 20° of prewhirl at all radii. The axial component of the inlet velocity is constant over the eye and is about 155 m/s. Find also the maximum Mach number at the eye.

Soln. : Please refer Fig. P. 6.7.9.



Inlet velocity diagram



Exit velocity diagram

Fig. P. 6.7.9

Given :

Internal tip diameter, D_i = 17.5 cm = 0.175 m

Outer tip diameter, D_o = 31.25 cm = 0.3125 m

$\dot{m}$ = 18 kg/s

N = 16000 rpm

Ambient, p_0 = 1 bar, T_o = 15°C = 288 K

Prewhirl angle, ψ = 20°

Axial velocity, $C_{fi} = C_{fo}$ = 155 m/s

Blade velocity, $C_{bi} = \dfrac{\pi D_i N}{60} = \dfrac{\pi \times 0.175 \times 16000}{60}$

= 146.6 m/s

$$C_{bo} = \frac{\pi D_o N}{60}$$

$$= \frac{\pi \times 0.3125 \times 16000}{60}$$

= 261.8 m/s

1. **Vane angle at inlet, θ**

$C_{wi} = C_{fi} \tan \psi$

= 155 tan 20

(where, α = 90 – ψ = 90 – 20 = 70°)

= 56.42 m/s

AD = AB – DB = $C_{bi} - C_{wi}$

= 146.6 – 56.42 = 90.18 m/s

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{AD}\right) = \tan^{-1}\left(\frac{155}{90.18}\right)$$

= 59.8° ...Ans.

$$C_{ri} = \sqrt{C_{fi}^2 + AD^2}$$

$$= \sqrt{(155)^2 + (90.18)^2} = 179.33 \text{ m/s}$$

2. **Vane angle at outlet, φ**

$C_{ro} = C_{ri}$ = 179.33 m/s (On neglecting friction)

$$\sin\phi = \frac{C_{fo}}{C_{ro}} = \frac{155}{179.33} = 0.8643$$

$$\phi = \mathbf{59.8°} \quad \text{...Ans.}$$

3. Mach number at the eye, M_i

Sonic velocity,

$$a_i = \sqrt{\gamma R T_o} = \sqrt{1.4 \times 287 \times 288}$$

$$= 340.2 \text{ m/s}$$

$$M_i = \frac{C_{ri}}{a_i} = \frac{179.33}{340.2} = \mathbf{0.5272} \quad \text{...Ans.}$$

Ex. 6.7.10 : Following data is given for a centrifugal compressor :

(i) R.P.M. = 15000.

(ii) Air flow rate = 30 kg/s.

(iii) Air enters the compressor axially.

(iv) Conditions at exit : Radius = 0.3 m and relative velocity of air at the tip = 100 m/s at an angle of 80°.

Find the torque and power required to drive the compressor.

SPPU - May 19, 8 Marks

Soln. : From Fig. P. 6.7.10,

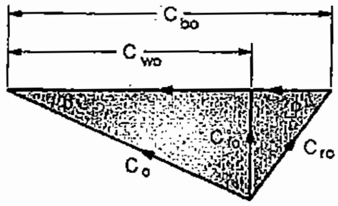


Fig. P. 6.7.10

Given : $N = 15000$ rpm, $\dot{m} = 30$ kg/s

$R_0 = 0.3$ m, $C_{r0} = 100$ m/s

$\phi = 80°$.

Exit velocity diagram is shown in Fig. P. 6.7.10.

Entry to compressor is axial i.e. $C_{wi} = 0$

Blade velocity at exit,

$$C_{b0} = \frac{\pi D_0 N}{60} = \frac{\pi \times 2 R_0 \times N}{60}$$

$$= \frac{\pi \times (2 \times 0.3) \times 15000}{60} = 471.24 \text{ m/s}$$

$$C_{w0} = C_{b0} - C_{r0} \cos\phi = 471.24 - 100 \cos 70$$

$$= 437.04 \text{ m/s}$$

(i) Torque, T :

$$T = \dot{m}\, C_{w0} \cdot R_o = 30 \times 437.04 \times 0.3$$

$$= \mathbf{3933.4 \text{ Nm}} \quad \text{...Ans.}$$

(ii) Power input, P :

$$P = T \cdot \omega = T \cdot \frac{2\pi N}{60}$$

$$= 3933.4 \times \frac{2 \times \pi \times 15000}{60} \text{ Nm/s or W}$$

$$= 6178.57 \times 10^3 \text{ Nm/s}$$

$$= \mathbf{6178.57 \text{ kW}} \quad \text{...Ans.}$$

Ex. 6.7.11 : The impeller of a centrifugal compressor has the inlet and outlet diameter of 0.3 and 0.6 m respectively. The intake is from the atmosphere at 100 kPa and 300 K without any whirl component. The outer blade angle is 75°. The speed is 10000 rpm and the velocity of flow is constant at 120 m/s. If the blade width at intake is 6 cm, calculate :

(i) Specific work

(ii) Exit pressure

(iii) Mass flow rate

(iv) Power required to drive compressor if the overall efficiency can be assumed at 0.7.

SPPU - May 16, 10 Marks

Soln. :

$D_i = 0.3$ m, $D_o = 0.6$ m,

$p_1 = 100$ kPa, $T_1 = 300$ K

$\phi = 75°$, $N = 10000$ rpm,

$C_{fi} = C_{fo} = 120$ m/s $B_i = 6$ cm $= 0.06$ m

Refer Fig. P. 6.7.11

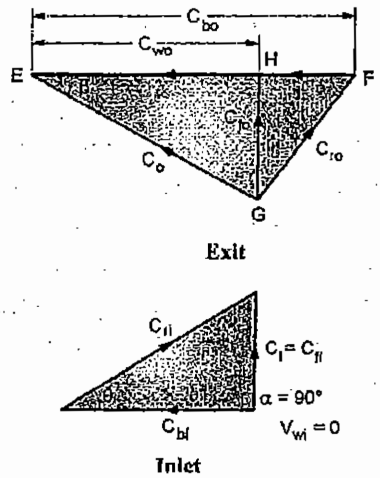


Fig. P. 6.7.11

$$C_{bi} = \frac{\pi \times D_i \times N}{60} = \frac{\pi \times 0.3 \times 10000 \text{ rpm}}{60}$$

$$= 157.1 \text{ m/s} = C_i$$

$$C_{bo} = \frac{\pi \times D_o \times N}{60} = \frac{\pi \times 0.6 \times 10000}{60}$$

$$= 314.2 \text{ m/s}$$

From exit Δ EGF :

$$C_{wo} = C_{bo} - \frac{C_{fO}}{\tan\phi} = 314.2 - \frac{120}{\tan 75} = 282 \text{ m/s}$$

(i) Specific work, W

$$W = C_{bo}\, C_{wo} = \frac{314.2 \times 282}{1000} \text{ kJ/kg}$$

$$= \mathbf{88.60\ kJ/kg} \qquad \text{...Ans.}$$

(ii) Exit pressure, p_2

$$W = C_p (T_2 - T_1)$$

$$88.6 = 1.005 (T_2 - 300)$$

$$T_2 = 388.16 \text{ K}$$

Assuming isentropic compression,

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{(\gamma)/(\gamma-1)}$$

$$p_2 = 100 \left(\frac{388.16}{300}\right)^{1.4/(1.4-1)}$$

$$= \mathbf{246.4\ kPa} \qquad \text{...Ans.}$$

(iii) Mass flow rate, $\dot{m}$

Volume flow rate,

$$\dot{V}_1 = \pi D_i B_i \times C_{f_i} = \pi \times 0.3 \times 0.06 \times 120$$

$$= 6.7858 \text{ m}^3/\text{s}$$

$$P_1 V_1 = \dot{m} R T_1$$

$$\dot{m} = \frac{100 \times 6.7854}{0.287 \times 300} = \mathbf{7.8813\ kg/s} \qquad \text{...Ans.}$$

(iv) Power required, P it overall efficiency, $\eta_o = 0.7$

$$P = \frac{\dot{m} \times w}{\eta_o} = \frac{7.8813 \times 88.6}{0.7}$$

$$= \mathbf{997.5\ kW} \qquad \text{...Ans.}$$

Ex. 6.7.12 : A centrifugal air compressor compresses the air from 1 bar to 3 bar. Internal and outer diameter of the impeller are 0.2 m and 0.4 m respectively. The impeller blade angle at inlet is 30° and at exit is 40°. Air enters the impeller blade radially at a speed of 12 m/s. Determine :

(a) Speed of impeller in r.p.m.

(b) Workdone per kg of air.

(c) Thickness of the impeller blades for a mass flow rate of air as 0.5 kg/s if the impeller has 30 blades and width of each impeller blade is 5.5 cm. Assume the specific volume of air as 0.82 m³/kg and velocity of flow constant throughout.

Assume, $C_p = 1.005$ kJ/kg K and $\gamma = 1.4$.

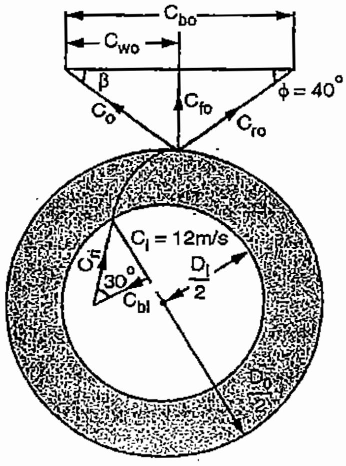


Fig. P. 6.7.12

Soln. :

Given : $D_i = 0.2$ m ; $D_o = 0.4$ m

$\theta = 30°$; $\phi = 40°$.

Velocity of air at inlet, $C_i = C_{fi} = 12$ m/s ;

Entry is radial i.e. $\alpha = 90°$

$\dot{m} = 0.5$ kg/s, no. of blades,

$n = 30$,

specific volume of air, $v = 0.82$ m³/kg

(a) Speed of impeller

Refer Fig. P. 6.7.12.

$$C_{bi} = \frac{C_i}{\tan\theta} = \frac{12}{\tan 30} = 20.78 \text{ m/s}$$

$$C_{bi} = \frac{\pi D_i N}{60}$$

...where N is speed of impeller in r.p.m.

$$\therefore \quad 20.78 = \frac{\pi \times 0.2 \times N}{60}$$

$$N = \mathbf{1984.3\ r.p.m.} \qquad \text{...Ans.}$$

(b) Workdone per kg of air

Tip blade velocity,

$$C_{bo} = \frac{\pi D_o N}{60} = \frac{\pi \times 0.4 \times 1984.3}{60}$$

$$= 41.56 \text{ m/s}$$

Velocity of whirl at outlet,

$$C_{wo} = C_{bo} - \frac{C_{fo}}{\tan 40} = 41.56 - \frac{12}{\tan 40}$$

$$= 27.26 \text{ m/s}$$

Workdone per kg of air, $W = C_{wo} \cdot C_{bo} = 27.26 \times 41.56$

$$=1132.93 \text{ Nm/kg} \quad \text{...Ans.}$$

(c) Thickness of blade

Let the blade thickness be 't'.

$$\dot{m} = \frac{(\pi D_i - n \cdot t)\, b_i\, C_{fi}}{v}$$

$$0.5 = \frac{(\pi \times 0.2 - 30 \times t) \times 0.055 \times 12}{0.82}$$

or, $t = 0.000237 \text{ m} = \mathbf{0.0237 \text{ cm}}$ **...Ans.**

Ex. 6.7.13 : A centrifugal compressor has to deliver 8 kg/sec of air with a pressure ratio of 4.4:1 at 18000 r.p.m. Initial conditions are static air at 1 bar and 15°C. Assuming an adiabatic efficiency (based on total values) of 78%, ratio of whirl speed to tip speed 0.94 and neglecting all other losses, calculate the tip speed, diameter and the rise in total temperature. Also, determine the external diameter of eye for which the internal diameter is 12 cm and the axial velocity at inlet is 150m/sec. Assume, $C_p = 1.05$ kJ/kg K and $\gamma = 1.4$.

Soln. : Refer Fig. P. 6.7.13(a).

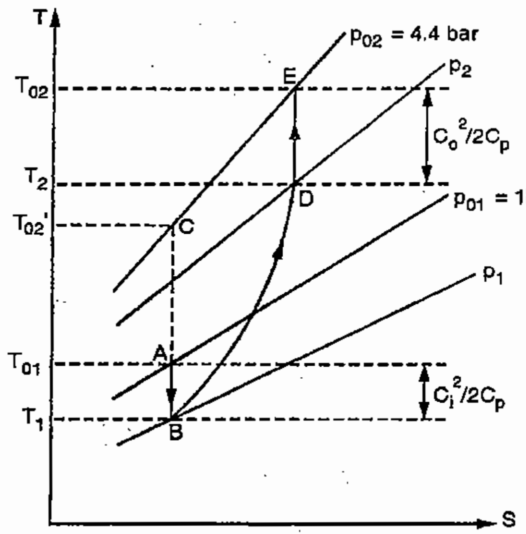


Fig. P. 6.7.13(a)

Since the condition of still air at A is at p_{01} = 1 bar and T_{01} = 15°C or 288 K, its pressure and temperature will be reduced at inlet to impeller blades represented by point B since some of the pressure energy is converted into kinetic energy.

Process BD represents the actual compression of air in the impeller blades having some kinetic energy equal to $C_o^2/2$ at point D.

This kinetic energy will be converted into pressure energy in the diffuser section represented by isentropic process DE.

Given : Inlet velocity of air $C_i = C_{fi} = 150$ m/s

Temperature drop equivalent to kinetic energy at inlet,

$$T_{01} - T_1 = \frac{C_1^2}{2C_p}$$

$$= \frac{(150)^2}{2 \times (1.05 \times 1000)} = 10.71^\circ\text{C}$$

$$\therefore \quad T_1 = T_{01} - 10.71$$

$$= 288 - 10.71 = 277.29 \text{ K}$$

$$\frac{T_{02}'}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma}$$

or, $$T_{02}' = 288\left(\frac{4.4}{1}\right)^{(1.4-1)/1.4}$$

$$= 439.78 \text{ K}$$

Isentropic efficiency based on total values,

$$\eta_t = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}}$$

$$0.78 = \frac{439.78 - 288}{T_{02} - 288}$$

$$T_{02} = 482.59 \text{ K}$$

Workdone per kg of air,

$$W = C_p (T_{02} - T_{01})$$

$$= 1.05\,(482.59 - 288) = 204.3 \text{ kJ/kg}$$

Also, $$W = C_{wo} \times C_{bo} = \Psi_s \cdot C_{bo}^2$$

$$204.3 \times 1000 = 0.94 \times C_{bo}^2$$

$$C_{bo} = \mathbf{466.2 \text{ m/s}} \quad \text{...Ans.}$$

But, $$C_{bo} = \frac{\pi D_o N}{60}$$ ...where, D_o = tip diameter

$$466.2 = \frac{\pi \times D_o \times 18000}{60}$$

$$D_o = \mathbf{0.495 \text{ m}} \quad \text{...Ans.}$$

Rise in total temperature,

$$T_{02} - T_{01} = 482.59 - 288 = \mathbf{194.59^\circ C} \quad \text{...Ans.}$$

For isentropic process AB,

$$\left(\frac{T_{01}}{T_1}\right)^{\gamma/(\gamma-1)} = \frac{p_{01}}{p_1}$$

or, $$p_1 = p_{01}\left(\frac{T_1}{T_{01}}\right)^{\gamma/(\gamma-1)} = 1\left(\frac{277.29}{288}\right)^{1.4/(1.4-1)}$$

$$= 0.876 \text{ bar}$$

$$R = C_p - C_v = C_p\left(1 - \frac{1}{\gamma}\right) = 1.05\left(1 - \frac{1}{1.4}\right)$$

$$= 0.3 \text{ kJ/kg K}$$

Specific volume of air at inlet,

$$v_1 = \frac{R \cdot T_1}{p_1} = \frac{300 \times 277.29}{0.876 \times 10^5} = 0.9496 \text{ m}^3\text{/kg}$$

Mass flow rate,

$$\dot{m} = \frac{\text{Area of flow at inlet} \times C_{fl}}{v_1}$$

$$8 = \frac{A \times 150}{0.9496}$$

$$A = 0.05064 \text{ m}^2$$

Refer Fig. P. 6.7.13(b).

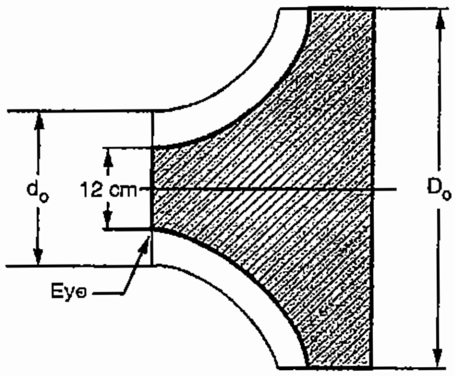


Fig. P. 6.7.13(b)

Internal diameter of eye,

$$d_i = 12 \text{ cm.}$$

Let, external diameter of eye be 'd_o'. Then,

$$A = \frac{\pi}{4}(d_o^2 - d_i^2)$$

$$0.05064 = \frac{\pi}{4}(d_o^2 - 0.12^2)$$

$$\mathbf{d_o = 0.2808 \text{ m, or } 28.08 \text{ cm}} \quad \textbf{...Ans.}$$

Ex. 6.7.14: A centrifugal compressor runs at 15000 rpm with an overall stagnation pressure ratio of 4. Ambient air conditions are 25°C and 1 bar. Vanes are radial. Slip factor is 0.96 and power input factor (work factor) is 1.04. Flow in the inlet section up to the impeller entry is isentropic and that in the impeller and the diffuser is adiabatic. There is no prewhirl at axial entry to the impeller. Mechanical efficiency is 96% and the electric motor driving the compressor is 98% efficient. The loss of stagnation pressure from impeller exit to diffuser exit is 0.1 bar. Isentropic efficiency of the impeller alone is 90%. Taking γ = 1.4 and C_p = 1.005 kJ/kgK for air, determine:

(i) Electrical energy consumed by the electric motor per kg of air.

(ii) Overall isentropic efficiency of the compressor.

(iii) Impeller tip diameter.

Draw neat T-s diagram showing only the stagnation temperatures and pressures involved in the present problem. **SPPU - Dec. 2011, 12 Marks**

Soln. :

Refer Fig. P. 6.7.14 showing (T-S) diagram.

Given : $N = 15000$ rpm, $\dfrac{p_{o2}}{p_{o1}} = 4$;

$p_{o1} = 1$ bar, $T_{o1} = 25°\text{ C} = 298$ K

$\alpha = 90°$ (radial vanes), slip factor, is $\psi_s = 0.96$,

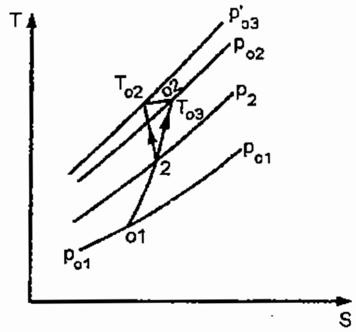


Fig. P. 6.7.14

Power input factor, $\psi_w = 1.04$

Mechanic efficiency, $\eta_m = 0.96$, $\eta_{motor} = 0.98$,

Pressure loss in diffuser = 0.1 bar, $\eta_I = 0.9$ (impeller)

$\gamma = 1.4$, $C_p = 1.005$ kJ/kg K

(i) Electric energy consumed by Electric motor/kg of air, W_{motor} :

$$T'_{o2} = T_{o1}\left(\frac{p_{o2}}{p_{o1}}\right)^{(\gamma - 1)/\gamma}$$

$$= 298\,(4)^{0.4/1.4} = 370.6 \text{ K}$$

$$(\Delta T)_o = T'_{o2} - T_{o1} = 370.6 - 298 = 72.6 \text{ K}$$

Actual temperature rise,

$$\Delta T_o = \frac{\Delta T'_{o2}}{\eta_I} = \frac{72.6}{0.9} = 80.667 \text{ K}$$

Work consumed / kg of air

$$W = C_p(\Delta T_o) = 1.005 \times 80.667$$

$$= 81.07 \text{ kJ/kg}$$

$$W_{motor} = W \times \frac{1}{\eta_m} \times \frac{1}{\eta_{motor}}$$

$$= 81.07 \times \frac{1}{0.96} \times \frac{1}{0.98}$$

$$= \mathbf{86.17\ kJ/kg} \quad \text{...Ans.}$$

(ii) Overall isentropic efficiency of the compressor, η_o :

$$\eta_o = \eta_i \times \eta_m \times \eta_g = 0.9 \times 0.96 \times 0.98$$

$$= \mathbf{0.8467\ or\ 84.67\ \%} \quad \text{...Ans.}$$

(iii) Impeller tip diameter, D_o

$$W = \psi_s \cdot \psi_w\ C_{bo}^2$$

$$81.07 \times 10^3 = 0.96 \times 1.04 \times C_{bo}^2$$

$$C_{bo} = 284.96\ m/s = \frac{\pi D_o N}{60}$$

$$= \frac{\pi \times D_o \times 15000}{60}$$

$$D_o = \mathbf{0.3628\ m} \quad \text{...Ans.}$$

Ex. 6.7.15 : A centrifugal compressor running at a speed of 15000 rpm admits 25 m³/sec air at standard states 1 bar and 300 K and compresses it adiabatically by the pressure ratio of 2. The air velocity at inlet and the radial velocity at exit is the same as 75 m/sec. The inlet and outlet impeller diameters are 60 cm and 80 cm respectively. Considering the inlet to be axial find :

(i) Blade angles at inlet and outlet of impeller.

(ii) Angle at which air leaves the impeller.

(iii) Impeller breadth at inlet and ex

SPPU : May 18, 10 Marks

Soln. :

$N = 15000$ rpm , $\dot{V}_1 = 25\ m^3/s$,

$p_1 = 1$ bar , $T_1 = 300$ K,

$p_2 = 2$ bar; $C_i = C_{fi} = C_{f_0} = 75$ m/s

$d_i = 60$ cm $= 0.6$ m ; $d_0 = 80$ cm $= 0.8$ m

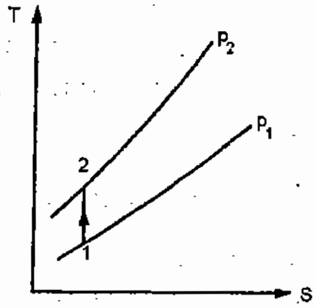


Fig. P.6.7.15

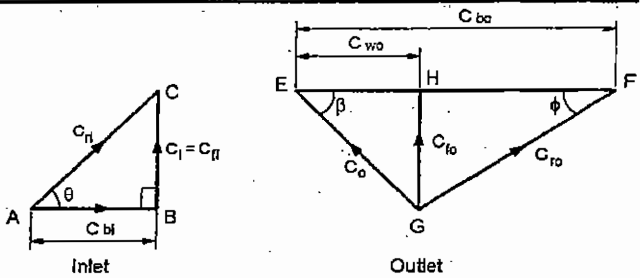


Fig. P.6.7.15

Refer Fig. P.6.7.15

$$p_1 \dot{V}_1 = \dot{m}\ R\ T_1$$

$$1 \times 10^5 \times 25 = \dot{m} \times 287 \times 300$$

$$\dot{m} = 0.03444\ kg/s$$

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(\gamma - 1)/\gamma}$$

$$= 300 \left(\frac{2}{1}\right)^{0.4/1.4} = 365.7\ m/s$$

$$C_{bi} = \frac{\pi\ d_i\ N}{60} = \frac{\pi \times 0.6 \times 15000}{60} = 471.24\ m/s$$

$$C_{bo} = \frac{\pi \cdot d_o\ N}{60} = \frac{\pi \times 0.8 \times 15000}{60}$$

$$= 628.32\ m/s = C_{wo}$$

(i) Blade angle at inlet, θ and at outlet, ϕ

Work done /kg of air,

$$W = C_p (T_2 - T_1) = 1005\ (365.7 - 300)$$

$$= 660285\ Nm/kg$$

$$W = C_{wo} \cdot C_{bo}$$

$$660285 = C_{wo} \times 628.32$$

$$C_{wo} = 105.09\ m/s$$

Consider inlet Δ ABC,

$$\theta = \tan^{-1}\left(\frac{C_i}{C_{bi}}\right) = \tan^{-1}\left(\frac{75}{471.24}\right)$$

$$= \mathbf{9.043^\circ} \quad \text{...Ans.}$$

Consider outlet Δ E F G

$$HF = C_{bo} - C_{wo} = 628.32 - 105.09 = 523.23\ m/s$$

$$\phi = \tan^{-1}\left(\frac{C_{fo}}{HF}\right) = \tan^{-1}\left(\frac{75}{523.23}\right)$$

$$= \mathbf{8.157^\circ} \quad \text{...Ans.}$$

(ii) Angle of which air leaves the impeller, β

$$\beta = \tan^{-1}\left(\frac{C_{fo}}{C_{wo}}\right) = \tan^{-1}\left(\frac{75}{105.09}\right)$$

$$= \mathbf{35.515^\circ} \quad \text{...Ans.}$$

(iii) Impeller width at inlet b_i and at outlet b_o

Specific volume of air at inlet,

$$v_i = \frac{RT_1}{p_1} = \frac{287 \times 300}{1 \times 10^5}$$

$$= 0.861 \text{ m}^3/\text{kg} \qquad \text{...Ans.}$$

Specific volume at outlet,

$$v_o = \frac{RT_2}{p_2} = \frac{287 \times 365.7}{2 \times 10^5} = 0.5248 \text{ m}^3/\text{s}$$

$$\dot{m} = \frac{\pi\, d_i\, b_i\, C_{fi}}{v_i} = \frac{\pi\, d_o\, b_o\, C_{fo}}{v_o}$$

$$0.03444 = \frac{\pi \times 0.6 \times b_i \times 75}{0.861}$$

$$= \frac{\pi \times 0.8 \times b_o \times 75}{0.5248}$$

$$\therefore\ b_i = \mathbf{2.0975 \times 10^{-4} m}$$

$$\text{and } b_o = \mathbf{9.5886 \times 10^{-5} m} \quad \text{...Ans.}$$

Ex. 6.7.16 Following data pertains to a centrifugal compressor.

Diameter of inlet eye of

Compressor impeller = 35 cm

Axial velocity at inlet = 140 m/s

Mass flow = 12 kg/s

The velocity in the delivery duct = 120 m/s

The tip speed of impeller = 460 m/s

Speed of the impeller = 16000 rpm

Total head isentropic efficiency = 80%

Pressure coefficient = 0.73

Ambient conditions = 1.0132 bar and 15 °C

Calculate

i) The static pressure and temperature at inlet and outlet of the compressor.

ii) The static pressure ratio. **SPPU : Dec. 18, 10 Marks**

Soln. :

$$\frac{p_{02}}{p_{01}} = 3.6; \text{ Diameter of impeller eye,}$$

$$d_o = 35 \text{ cm} = 0.35 \text{ m}$$

$$C_{fi} = 140 \text{ m/s} = C_1;\ \dot{m} = 12 \text{ kg/s},\ C_o = 120 \text{ m/s},$$

$$C_{bo} = 460 \text{ m/s}$$

$$N = 16000 \text{ rpm},\ \eta_i = 0.8;\ \psi_p = 0.73,$$

$$P_{01} = 1.0132 \text{ bar}$$

$$T_{01} = 15^\circ\text{c} = 15 + 273 = 288 \text{ K}$$

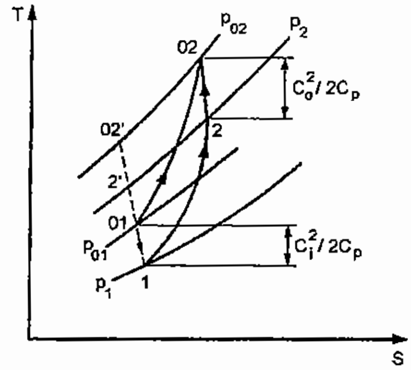
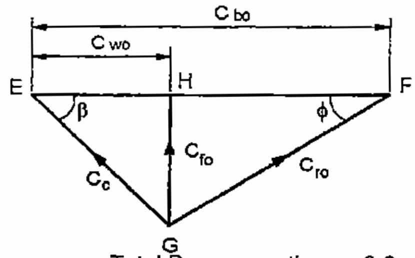
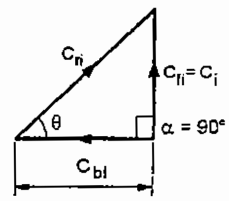


Fig. P.6.7.16



Total Pressure ratio = 3.6 : 1

Outlet



Inlet

Fig. P. 6.7.16(a)

Refer Fig. P.6.7.16and P.6.7.16(a)

(i) Static pressure, p_1 and temperature, T_1 at inlet and at exit p_2, T_2

$$T_1 = T_{01} - \frac{C_i^2}{2C_p}$$

$$= 288 - \frac{(140)^2}{2 \times (1.005 \times 1000)}$$

$$= \mathbf{278.25 K} \qquad \text{...Ans.}$$

$$\frac{p_1}{p_{01}} = \left(\frac{T_1}{T_{01}}\right)^{(\gamma-1)/\gamma}$$

$$p_1 = 1.0132\left(\frac{278.25}{288}\right)^{(1.4-1)/1.4} = 1.0 \text{ bar} \quad \text{...Ans.}$$

$$T_{02}' = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} = 288\,(3.6)^{0.4/1.4}$$

$$= 415.3 \text{ K}$$

$$\eta_i = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}};\quad 0.8 = \frac{415.3 - 288}{T_{02} - 288}$$

$$T_{02} = 447.1\text{ K}$$

$$T_2 = T_{02} - \frac{C_0^2}{2C_p} = 447.1 - \frac{(120)^2}{2 \times (1.005 \times 1000)}$$

$$= \mathbf{439.93K}$$

$$\frac{p_2}{p_{02}} = \left(\frac{T_2}{T_{02}}\right)^{\gamma/(\gamma-1)};$$

$$p_2 = p_{02}\left(\frac{T_2}{T_{02}}\right)^{\gamma/(\gamma-1)} \quad \text{(But } p_{02} = 3.6\,p_{01}\text{)}$$

$$p_2 = (3.6 \times 1.0132)\left(\frac{439.93}{447.1}\right)^{1.4/(1.4-1)}$$

$$= \mathbf{3.447\ bar} \quad \text{...Ans.}$$

(ii) Static pressure ratio, (p_2/p_1)

$$\frac{p_2}{p_1} = \frac{3.447}{1.0} = \mathbf{3.447} \quad \text{... Ans.}$$

Ex. 6.7.17 : A centrifugal compressor is used as a supercharger for an aircraft engine developing 750 kW power having specific fuel consumption of 0.27 kg/kWh. The supercharger supplies air fuel mixture at 1.25 bar. The air fuel ratio is 17 : 1. Air enters the supercharger at pressure of 0.55 bar and temperature of 0°C. Assuming adiabatic efficiency of supercharger as 85%, calculate volume flow rate of mixture to be supplied to the engine and power required to drive the supercharger. Take C_p = 1 kJ/kg K; R = 0.277 kJ/kgK. **SPPU : Dec. 19, 11 Marks**

Soln. :

Engine power, $P_s = 750\text{ kW},\ \dot{m}_{f1} = 0.27\text{ kg/kWh}$

$$p_2 = 1.25\text{ bar},\ \frac{\dot{m}_a}{\dot{m}_f} = \text{A. F. ratio} = 17;$$

$$p_1 = 0.55\text{ bar}$$

$$T_1 = 0\text{ °C} = 273\text{ K};$$

Adiabatic efficiency, $\eta_i = 0.85$

1. Volume flow rate of the mixture supplied to the engine, $\dot{V}_2$

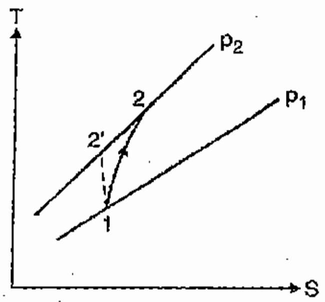


Fig. P.6.7.17

$$C_p\left(1 - \frac{1}{\gamma}\right) = R$$

$$1\left(1 - \frac{1}{\gamma}\right) = 0.277$$

$$\gamma = 1.383$$

Refer Fig. P.6.7.17

$$T_2' = T_1\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$

$$T_2' = 273\left(\frac{1.25}{0.55}\right)^{(1.383-1)/1.383}$$

$$= 342.7\text{ K}$$

$$\eta_i = \frac{T_2' - T_1}{T_2 - T_1};\quad 0.85 = \frac{342.7 - 273}{T_2 - 273};$$

$$T_2 = 355\text{ K}$$

$$\dot{m}_f = \dot{m}_{f_1} \times \text{Power, } P_s$$

$$= 0.27 \times 750\text{ kg/hr}$$

$$= \frac{0.27 \times 750}{3600}\text{ kg/s}$$

$$= 0.05625\text{ kg/s}$$

$$\dot{m}_g = \dot{m}_a + \dot{m}_f = \dot{m}_f\left(\frac{\dot{m}_a}{\dot{m}_f} + 1\right)$$

$$= 0.05625\,(17 + 1) = 1.0125\text{ kg/s}$$

$$p_2\dot{V}_2 = \dot{m}_g \times R \times T_2$$

$$\frac{1.25 \times 10^5}{10^3}\text{ (kPa)} \times \dot{V}_2 = 1.0125 \times 0.277 \times 355$$

$$\dot{V}_2 = \mathbf{0.796514\ m^3/s} \quad \text{... Ans.}$$

2. Power required to drive the supercharger, P

$$P = \dot{m}_g C_p (T_2 - T_1)$$

$$= 1.0125 \times 1 \times (355 - 273)$$

$$= \mathbf{83.025\ kW} \quad \text{... Ans.}$$

Ex. 6.7.18 : A centrifugal compressor inducing air at 20°C is running at 15,000 rpm. The pressure ratio is 4:1 with an isentropic efficiency of 80%. Curved vanes at inlet give the air a prewhirl of 25° to the axial direction at all radii and mean diameter of eye is 25 cm; impeller tip diameter is 60 cm. The absolute velocity of air at inlet is 150 m/s. Find the slip factor. **SPPU - May 15,10 Marks**

Soln. : Refer Fig. P. 6.7.18(a)

$T_1 = 20°C = 293\text{ K} \qquad N = 15000\text{ rpm}$

$\frac{p_2}{p_1} = 4 \qquad \eta_i = 0.8$

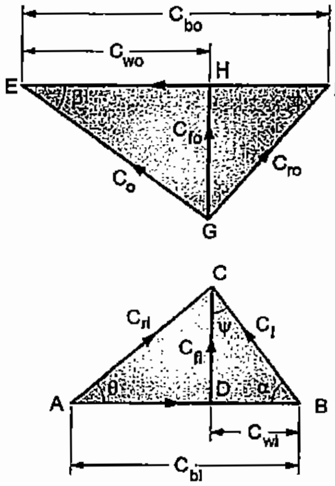


Fig. P. 6.7.18(a)

Prewhirl angle, $\psi = 25°$,

$d_i = 25\text{ cm} = 0.25\text{ m}$

$d_o = 60\text{ cm} = 0.6\text{ m}$,

$C_i = 150\text{ m/s}$

$\phi = \psi = 25°$,

since prewhirl is same at all radii

$$C_{bi} = \frac{\pi d_i N}{60} = \frac{\pi \times 0.25 \times 15000}{60} = 196.35\text{ m/s}$$

$$C_{bo} = \frac{\pi d_o N}{60} = \frac{\pi \times 0.6 \times 15000}{60} = 471.24\text{ m/s}$$

$$T_2' = T_1\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = 293(4)^{0.4/1.4} = 435.4\text{ K}$$

$$\eta_i = \frac{T_2' - T_1}{T_2 - T_1}$$

$$0.8 = \frac{435.4 - 293}{T_2 - 293}$$

$$T_2 = 471\text{ K}$$

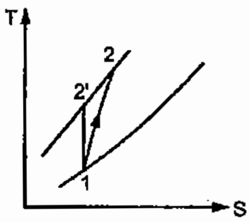


Fig. P. 6.7.18(b)

W.D/kg of air, $W = (h_2 - h_1) = C_p(T_2 - T_1)$

$= 1.005(471 - 293)$

$= 178.89\text{ kJ/kg}$

From inlet velocity Δ ACB :

$$C_{wi} = C_i \sin\psi = 150 \sin 25 = 63.39\text{ m/s}$$

$$C_{fi} = C_i \cos\psi = 150 \cos 25 = 135.95\text{ m/s}$$

$$W = C_{wo} C_{bo} - C_{wi} C_{bi}$$

$$178.89 \times 10^3 = C_{wo} \times 471.24 - 63.39 \times 196.35$$

$$C_{wo} = 406.03\text{ m/s}$$

Slip factor, $\psi_s = \frac{C_{wo}}{C_{bo}} = \frac{406.03}{471.24} = \mathbf{0.8615}$ **...Ans.**

Ex. 6.7.19 : The inlet conditions of a centrifugel compressor are 1 bar, 30°C and running at 10000 rpm. It delivers a free air stream of 1.5 m³/s. The compression ratio is 5. The velocity of flow is 50m/s and is constant. Assume that the blades are radial outlet and the slip factor is 0.92. Calculate :

(i) Temperature of air outlet

(ii) Power required

(iii) Impeller diameter

(iv) Diffuser inlet angle

(v) Blade angle at inlet

Assume that power factor is 1.11 and isentropic efficiency is 0.90. **SPPU - Dec. 16, 10 Marks**

Soln. : Refer Fig. P. 6.7.19(a)

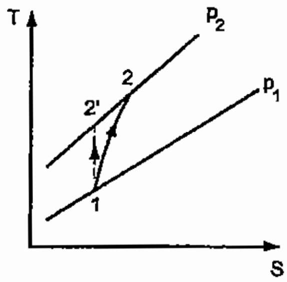


Fig. P. 6.7.19(a)

Given :

Static inlet pressure, $p_1 = 1\text{ bar} = 100\text{ kPa}$

Static inlet temperature, $T_1 = 30 + 273 = 303\text{ K}$

Speed, $N = 10,000\text{ rpm}$

Volume flow rate, $\dot{V}_1 = 1.5\text{ m}^3/\text{s}$

Compression ratio, $r_p = 5 = \frac{p_2}{p_1}$

Flow velocity, $V_{f1} = V_{f2} = 50$ m/s

Radial blade at exit, $\phi_2 = 90°$, $C_{bo} = C_{wo}$

Slip factor, $\Psi_s = 0.92$

Power factor, $\Psi_w = 1.11$

Isentropic efficiency, $\eta_i = 0.9$

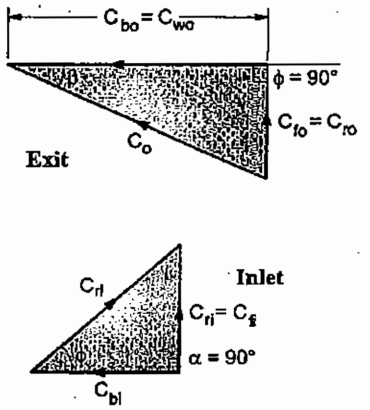


Fig. P. 6.7.19(b)

(a) Temperature at exit, T_2

Isentropic temperature at exit,

$$T_2' = T_1\left[\frac{p_2}{p_1}\right]^{\frac{\gamma-1}{\gamma}} = 303\,(5)^{0.4/1.4}$$

$$T_2' = 479.9 \text{ K}$$

$$\eta_i = \frac{(T_2' - T_1)}{(T_2 - T_1)}$$

$$0.9 = \frac{(479.9 - 303)}{(T_2 - 303)}$$

$$\therefore \quad T_2 = 499.55 \text{ K} \qquad \text{...Ans.}$$

(b) Power input to compressor, P_c

$$\therefore\ p\dot{V} = \dot{m} RT$$

$$\dot{m} = \frac{p_1 \dot{V}}{RT_1} = \frac{100 \times 1.5}{0.287 \times 303} = 1.725 \text{ kg/s}$$

$$P_c = \dot{m}\, C_p (T_2 - T_1)$$

$$\therefore\ P_c = 1.725 \times 1.005\,(499.55 - 303)$$

$$P = 340.74 \text{ kW} \qquad \text{...Ans.}$$

(c) Impeller diameter (D_o)

Work input / kg of air,

$$W = C_p(T_2 - T_1) = \Psi_s \cdot \Psi_w \times C_{bo}^2$$

$$1.005 \times 1000\,(499.55 - 303) = 0.92 \times 1.11 \times C_{bo}^2$$

$$\therefore\ C_{bo} = 439.8 \text{ m/s}$$

$$C_{bo} = \frac{\pi D_o \times 10000}{60}$$

$$\therefore\ 439.8 = \frac{\pi D_o \times 10000}{60}$$

$$\therefore\ D_o = 0.84 \text{ m} \qquad \text{...Ans.}$$

(d) Blade angle at inlet, θ

Let impeller diameter at inlet,

$$D_i = \frac{1}{2} D_o = \frac{1}{2} \times 0.84 = 0.42 \text{ m}$$

$$C_{bi} = \frac{\pi D_i N}{60} = \frac{\pi \times 0.42 \times 10000}{60} = 219.9 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{C_{bi}}\right) = \tan^{-1}\left(\frac{50}{219.9}\right)$$

$$= 12.8° \qquad \text{...Ans.}$$

(e) Diffuser angle at inlet β

From exit velocity triangle

$$\beta = \tan^{-1}\left(\frac{C_{fo}}{C_{bo}}\right) = \tan^{-1}\left(\frac{50}{439.8}\right)$$

$$= 6.49° \qquad \text{...Ans.}$$

6.8 Losses in Centrifugal Compressors and Actual Head – Capacity (H - Q) Curve

The theoretical (Eulers) head - capacity curve has already been discussed in section 6.5, however, actual head obtained is reduced due to the following types of losses in the compressor :

(a) **Channel Losses :** These losses are due to skin friction and turbulence in the flow passages formed by the impeller blades. Frictional losses are proportional to square of velocity.

(b) **Exit losses in the diffuser :** These represent the loss of head equivalent to non-conversion of kinetic energy of fluid between entrance and exit of the diffuser. These losses are also proportional to square of velocity.

(c) **Inlet losses :** These losses are due to the fact that the angle of incidence of fluid on the impeller blades varies with the discharge rates and it doesnot coincide with the angle of entrance (θ) of the blade if the compressor is not operating at the designed conditions. It results into loss of kinetic energy since a component of relative velocity at inlet is lost.

Both the channel and exit losses in the diffuser can be combined as both are proportional to square of velocity. Since velocity is proportional to discharge rates, it implies that these combined losses (H_f) are also proportional to square of discharge rates i.e. $H_f \propto (Q)^2$.

∴ These losses are represented in Fig. 6.8.1 and the actual head discharge characteristic curve can be obtained by substracting these losses from theoretical head.

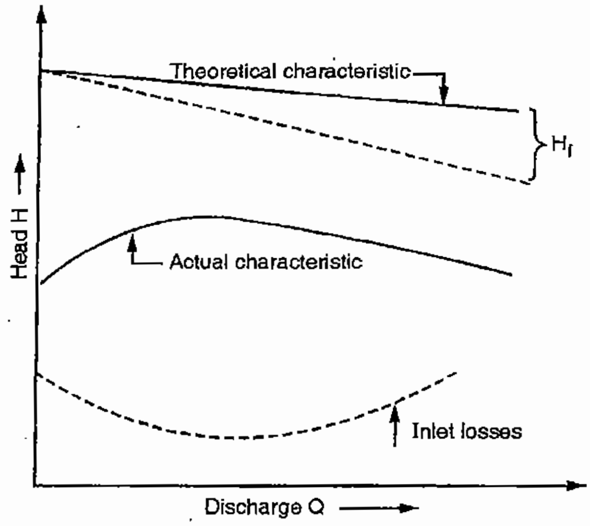


Fig. 6.8.1 : Ideal and actual (H – Q) characteristics curve

6.9 Non-Dimensional Parameters for Plotting Compressor Characteristics

University Question

Q. Discuss the dimensionless parameters used to predict the performance characteristics of centrifugal and axial flow compressor. State the importance of same.

SPPU : May 12

The performance of a compressor may be specified by curves of delivery pressure and temperature against the mass flow rate for various speeds of the compressor. These performance characteristics depend upon a number of variables as follows :

(a) Inlet stagnation pressure, p_{01} , temperature, T_{01} and specific volume v_1

(b) Mass flow rate, $\dot{m}$

(c) Delivery stagnation pressure, p_{02}, temperature, T_{02} and specific volume v_2

(d) Diameter of impeller, D and its speed, N

(e) Physical properties of the working fluid.

In order to evaluate the performance of a compressor by taking into account all the variables enumerated above, it will involve large number of tests to be performed on the compressor. Even after obtaining the test results, it will not be possible to present precisely the test results and the curves thus obtained.

However, many of the complications involved can be eliminated by using the dimensional analysis techniques by which few of the variables involved may be combined and form a convenient number of parameters as dimensionless groups.

The pressure at outlet of a compressor can be written as the function of various variables as follows :

$$p_{02} = f(p_{01}, RT_{01}, D, \dot{m}, N) \quad \text{...(6.9.1)}$$

In the above function, the specific volume of air has been omitted since it is dependent p_{01} and T_{01}. The variables D represents the characteristic constant dimensions of the compressor and specific heats have not been included since they are constants for a gas or air.

The chosen dimensions are :

Mass (kg) = M, Length (m) = L and time (seconds) = t

Dimensions of pressure are :

$$p = \frac{N}{m^2} = \frac{kg \cdot m}{s^2} \cdot \frac{1}{m^2}$$

$$= \frac{M \times L}{t^2} \times \frac{1}{L^2} = \frac{M}{L\,t^2} = ML^{-1} \cdot t^{-2}$$

$$T = \frac{\text{Heat}}{\text{Mass}} = \frac{Nm}{kg}$$

$$= \frac{kg \cdot m}{s^2} \times \frac{m}{kg} = \frac{m^2}{s^2} = L^2 \cdot t^{-2}$$

$$D = L$$

$$\dot{m} = \frac{kg}{s} = \frac{M}{t} = M\,t^{-1}$$

$$N = \frac{\text{radians}}{s} = \frac{1}{s} = t^{-1}$$

According to Buckingham's theorem, referred as π (pi) theorem, three dimensionless groups are possible.

In Equation (6.9.1), there are six variables and three basic dimensions (M, L, t). Therefore, the above function is reducible to (6 – 3) = 3 non-dimensional groups formed by these variables.

The three dimensionless groups are formed by combining p_{01}, T_{01} and D with each of the remaining variables so that,

$$\pi_1 = p_{01}^{a}\, T_{01}^{b}\, D^{c} \cdot p_{02} \quad \text{...(i)}$$

$$\pi_2 = p_{01}^{a_1}\, T_{01}^{b_1}\, D^{c_1} \cdot \dot{m} \quad \text{...(ii)}$$

$$\pi_3 = p_{01}^{a_2}\, T_{01}^{b_2}\, D^{c_2} \cdot N \quad \text{...(iii)}$$

On substituting the basic dimensions for the variables in Equation (i) we have,

$$\pi_1 = (ML^{-1}t^{-2})^a (L^2 t^{-2})^b (L)^c (ML^{-1}t^{-2})^1$$

For M, $0 = a + 1$ ...(iv)

For L, $0 = -a + 2b + c - 1$ ...(v)

For t, $0 = -2a - 2b - 2$ ...(vi)

On solving the Equation (iv) to Equation (vi), we get

$$a = -1, b = 0, c = 0 \quad \text{...(vii)}$$

On substituting the values of a, b and c in Equation (i),

$$\pi_1 = p_{01}^{-1}\, T_{01}^{0}\, D^0\, p_{02} = \frac{p_{02}}{p_{01}} \quad \text{...(6.9.2)}$$

Similarly we can solve for π_2 and π_3. On solving we get,

$$\pi_2 = p_{01}^{-1} \cdot T_{01}^{0.5} \cdot D^{-2}\, \dot{m}^1 \frac{\dot{m}\sqrt{T_{01}}}{p_{01} \cdot D^2} \quad \text{...(6.9.3)}$$

$$\pi_3 = p_{01}^{0} = T_{01}^{0.5} \cdot D^1 N = \frac{DN}{\sqrt{T_{01}}} \quad \text{...(6.9.4)}$$

In case T_{02} is also taken as a function of variable representing the outlet temperature of the compressor. The ratio $\frac{T_{02}}{T_{01}}$ will also be found as dimensionless group in Equation (6.9.2) to (6.9.4).

It means that the performance of a compressor of fixed size with regard to variation of delivery pressure and temperature with mass flow rate, speed and inlet conditions can be expressed in the form of dimensionless groups $\left(\frac{p_{02}}{p_{01}}\right), \left(\frac{T_{02}}{T_{01}}\right), \frac{\dot{m}\sqrt{T_{01}}}{p_{01}}$ **and** $\frac{N}{\sqrt{T_{01}}}$ **by omitting D from the Equations derived above.**

6.10 Surging and Choking

- Actual head-mass flow rate ($\dot{m} \propto Q$) characteristics curve is shown in Fig. 6.10.1.
- ABC part of the curve has the positive slope and CDEF part has the negative slope.
- Point A represents the **shut-off-head** when mass flow rate is zero i.e. when discharge valve is fully closed, while, the point F represents the maximum mass flow rate corresponding to fully open discharge valve but head developed is zero.
- Point C represents the condition of maximum head developed at a certain mass flow rate and at this point it has the maximum compression efficiency.

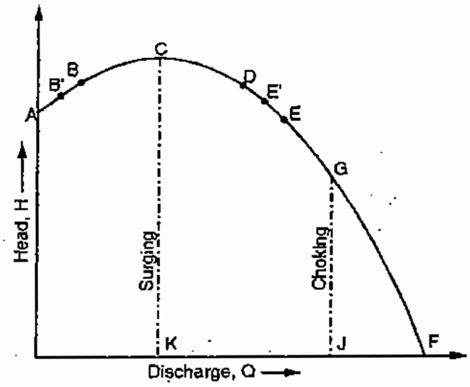


Fig. 6.10.1 : Surging and choking in compressor

- However, it is observed that the operation of the compressor in the entire range of characteristic curve is not stable for the reasons explained below.

6.10.1 Surging

University Questions

Q. Explain the terms surging in a rotary compressor.

SPPU : May 11, Dec. 15, May 14, May 15, Dec. 16)

Q. Explain surging with reference to compressors.

SPPU : Dec. 12, May 19

Q. Describe surging in a centrifugal compressor.

SPPU : May 16

Q. Explain the term surging in a centrifugal compressor. How does it affect the performance of compressor? Suggest method to minimize its effect.

SPPU : Dec. 19)

- Consider the operation of the curve at point E at any instant. Head developed by the compressor (H_E) at the given mass flow rate $\dot{m}_E$ caters for the friction and other losses of the compressor as explained above in section 6.8.
- In case the load on the compressor increases caused by the increased friction losses instantaneously due to small disturbances in operation of the machine, it would tend to decrease the mass flow rate and the operation of the curve will shift to E'. At this condition the operation of the compressor is stable since it can develop the required delivery head to cater for increased losses.
- It would again tend to increase the mass flow rate and the compressor would return back to its operation point E. **Therefore the operation of the compressor is stable between FC portion of the curve.**

- However, operation of compressor between the curve having positive slope (curve AC) is unstable.
- Consider the operation of compressor at some point B. Any increased friction losses due to disturbances in compressor operation will tend to reduce the mass flow rate and the point of operation will shift to point B'. At this point the compressor develops lesser head than the required head. Under these conditions the compressor produces less head while the pressure in the delivery line is more. This would result in momentarily reversal of flow.
- After some time, due to flow of fluid from discharge line, the back pressure will fall and the compressor will again start delivering the fluid.
- Consequently, the compressor operation would again shift to point B where the similar unstable conditions will occur again. *This unsteady to and fro motion of the fluid will cause pulsations in the compressor and this phenomenon is called **surging**.*
- The machine experiences abnormal vibrations, noise and rapid variation in power due to prolonged surging and may result in damaging the compressor.
- Therefore, surging puts a limit on the mass flow rate on the downstream side.

6.10.1.1 Rotating Stall In Centrifugal Compressor

University Question

Q. Explain stalling with reference to compressors. **SPPU : Dec. 12**

Q. Explain the stalling in centrifugal compressor. Also describe its effect on the compressor performance. **SPPU : May 16**

As observed in section 6.10.1, there is a local disruption of air flow within the compressor. Though the compressor continues to provide compressed air but at lower effectiveness. Such a momentary stalling of rotor during the reversal of flow is called rotor

Stall or compressor surge Causing abnormal vibrations and noise and may damage the compressor. It can be prevented by keeping the limit of closing the delivery valve.

6.10.2 Choking

University Questions

Q. Explain the term chocking in a rotary compressor. **SPPU : May 11, May 14, Dec. 15, Dec. 16**

Q. Explain choking with reference to compressors. **SPPU: Dec. 12, May 15, May 19**

Q. Explain choking **SPPU : May 15**

Q. Describe choking in a centrifugal compressor. **SPPU : May 16**

Q. Explain the term chocking in a centrifugal compressor. How does it affect the performance of compressor? Suggest method to minimize its effect. **SPPU : Dec. 19)**

- At point F, though the mass flow rate is maximum but the pressure head developed is zero i.e. pressure ratio (p_2/p_1) is unity, hence, at this point the compressor efficiency is zero.
- The point G on compressor characteristic curve represents the maximum flow rate which is practically possible and this mass flow rate is called **choking mass flow rate.**
- Choking means the maximum mass flow rate at a given speed of rotation and at this point the compressor can no longer assure a fixed discharge head and the energy of compression is dissipated. This implies that at choking the characteristics curve becomes vertical.
- Due to above reasons the rotary compressors are always operated between the limits of surging and choking.

6.11 Performance Characteristic Curves of Rotary Compressors

University Question

Q. How do stalling and surging take place in centrifugal compressor stages? How does it affect the performance of compressor? Suggest methods to minimize. **SPPU : May 13**

- The operation of turbo-compressors is characterised by the plot of pressure ratio $\left(\frac{p_{02}}{p_{01}}\right)$, efficiency ($\eta$) and power (P) against the discharge rates ($\dot{Q}$) as shown in Fig. 6.11.1.

- However, these plots are at a given intake conditions of pressure and temperature, the speed and the outer diameter of the impeller.
- The variables affecting the performance of a compressor are intake pressure p_{01}, intake temperature T_{01}, mass flow rate $\dot{m}$, outer diameter of impeller D and the speed N.
- In order to predict the performance of the compressor to that of a geometrically similar model and in order to take into account the variables outlined above, the performance curves are plotted with dimensionless parameters.
- The pressure ratio $\left(\frac{p_{02}}{p_{01}}\right)$ is the function of the dimensionless parameters as speed parameter $\frac{N}{\sqrt{T_{01}}}$ and mass flow parameter $\left(\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}\right)$.

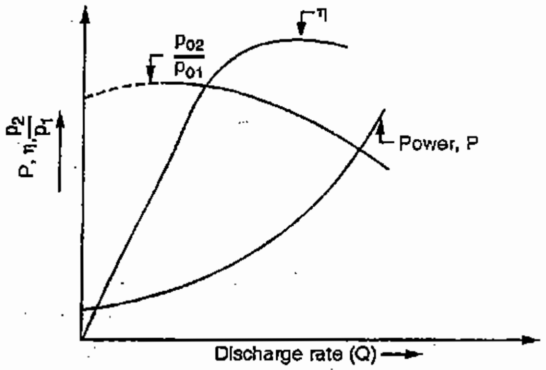


Fig. 6.11.1 : Characteristics curve for rotary compressor

- A plot of family of curves between pressure ratio and mass flow parameter by varying speed parameter is shown in Fig. 6.11.2.

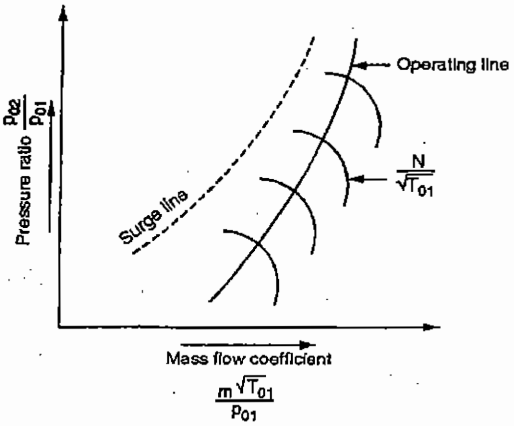


Fig. 6.11.2 : Plot of pressure ratio against mass flow coefficient at various speed parameters

The unstable range of curve is limited by surge line which corresponds to low mass flow rates and by the choking at high mass flow rates.

6.11.1 Performance Characteristic Curve of a Centrifugal Compressor

The performance characteristic curves of a centrifugal compressor are shown in Fig. 6.11.3 between pressure ratio $\left(\frac{p_{02}}{p_{01}}\right)$ and discharge rates (Q) at different speeds.

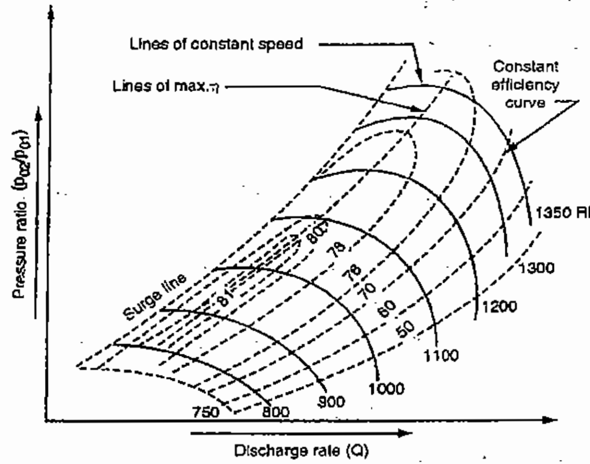


Fig. 6.11.3 : Performance characteristic curves of a centrifugal compressor

Following are the conclusions drawn :

(i) Surge line limits the operation of compressor at low mass flow rates.

(ii) There is a rapid reduction in discharge flow rates at a given speed as the pressure ratio increases.

(iii) Increase in speed at a given pressure ratio increases the discharge rates but decreases the efficiency at a rapid rate.

(iv) At low speeds the curves are fairly flat.

(v) Maximum mass flow rates are limited by choking limit.

Summary

- In **centrifugal compressors** the air is not trapped within the specified boundaries but it flows steadily through the compressor. In this compressor the pressure is developed due to change in momentum of gas on the expense of external work supplied.
- The main components of a centrifugal compressor are :

(i) Impeller (ii) Diffuser

(iii) Volute casing

- Impellers may be single sided or double sided.
- **Theoretical Euler head characteristic curve** i.e. plot of head (H) discharge (Q) for various types of vanes is,
  - (i) For backward vanes, $\phi < 90°$, H decreases with increase in Q.
  - (ii) For radial vanes, $\phi = 90°$, H remains constant.
  - (iii) For forward vanes, $\phi > 90°$, H increases with increase in Q.
- The head developed by compressor when discharge valve is closed i.e. at Q = 0 is called **shut-off head.**
- **Slip factor,** Ψ_s is defined as the ratio of actual whirl velocity to ideal whirl velocity and the difference between actual and ideal whirl velocities is called **slip.**
- **Work factor,** Ψ_w is an emperically determined factor to take into account the friction and turbulence losses. Its value varies between 1.03 to 1.06. Actual work of compression becomes $\Psi_s \cdot \Psi_w \cdot C_{bo}^2$.
- Ratio of isentropic work to Euler work of compression is called **pressure coefficient,** Ψ_p,
- In order to reduce the inlet relative velocity at M < 1 to avoid shock formation, air is given pre-rotation at inlet by providing fixed vanes at inlet to impeller blades. It is called **prewhirl.**

$$\Psi_p = \eta_i \cdot \Psi_s \cdot \Psi_w$$

- Actual head developed by the compressor is reduced due to channel losses, exit losses in the compressor and the inlet losses.
- Actual operation of a compressor is limited due to surging and choking. **Surging** in a compressor is due to unsteady to and fro motion of the fluid between the impeller and discharge line causing pulsations in the compressor.
- **Choking** represents the permissible mass flow rate at a given speed of rotation. At chocking the compressor characteristic curve is vertical.

Exercise

Note : For answers please refer the section numbers indicated in bracket.

Q. 1 Classify compressors . **[Section 6.1.1]**

Q. 2 Draw a neat sketch of various components of the centrifugal compressor and show the variation of pressure and velocity of air being compressed. **[Section 6.2]**

Q. 3 What is centrifugal compressor ?. State its important features. **[Section 6.1]**

Q. 4 Define isentropic efficiency of centrifugal compressor based on total values. **[Section 6.3]**

Q. 5 Represent the rotary compressor cycle on (T – S) diagram and explain the various processes involved. **[Section 6.3]**

Q. 6 Represent and explain the processes involved in a centrifugal compressor on (T-S) diagram and derive the expression for isentropic efficiency based on total values. **[Section 6.3]**

Q. 7 Draw velocity diagram for a centrifugal compressor and write the expression for work of compression. **[Section 6.4]**

Q. 8 Define Euler's work. **[Sections 6.4 and 6.4.1]**

Q. 9 Discuss the effect of vane shapes on head-discharge characteristics of a centrifugal compressor. **[Section 6.5]**

Q. 10 What is Euler's head ? How the actual head-discharge characteristic curve can be obtained from theoretical curve ? **[Sections 6.5 and 6.8]**

Q. 11 The head pressure ratio for forward curved vane centrifugal compressor rises with capacity while that for backward curved vane compressor reduces with capacity. **[Section 6.5]**

Q. 12 Define : Power input factor. **[Section 6.6.2]**

Q. 13 Define slip coefficient, work factor and pressure coefficient. **[Sections 6.61, 6.6.2 and 6.6.3]**

Q. 14 Explain slip and prewhirl in connection to centrifugal compressors. **[Section 6.61 and 6.6.4]**

Q. 15 Define slip coefficient and pressure coefficient. **[Sections 6.6.1and 6.6.3]**

Q. 16 Discuss the various losses of a centrifugal compressor. **[Section 6.8]**

Q. 17 Why non-dimensional parameters are used to predict the performance characteristics of a compressor ? Derive these parameters from the concept of dimensional analysis. **[Section 6.9]**

Q. 18 Explain the terms surging and chocking in a rotary compressor **[Sections 6.10.1 and 6.10.2]**

Q. 19 Discuss the performance characteristic curves of centrifugal compressor. **[Sections 6.11 and 6.11.1]**

□□□

7

UNIT - IV

Axial Flow Compressors

Syllabus

Construction and working, stage velocity triangle and it's analysis, enthalpy entropy diagram, stage losses and various efficiencies of axial flow compressors. (No numerical)

7.1 Introduction to Axial Flow Compressors

For jet aircraft applications, compressors are required to have low weight to power ratio, high volume flow rates of air, high compression ratio with less frontal area.

The centrifugal compressors developed could not meet these requirements since they can only develop pressure ratios upto 5 : 1 per stage having comparatively low volume flow rates. This has led to development of axial flow compressors.

In earlier designs the axial flow compressors could develop a pressure ratio of 5 : 1 in a 10 stage compressor. However, with the advancement technology and aerodynamic theory, present axial flow compressors can develop pressure ratio even upto 30 : 1 with reduction in weight and frontal area. These compressors are capable of dealing with very high discharge rates needed for aircraft applications.

Now a days, axially flow compressors are generally used in gas turbine power plants and aircrafts since it can develop high pressure ratio with high volume flow rates at high efficiency in multistage compressors. These compressors generally run at high speeds in range of 10000 to 30000 rpm. These compressors are suitable for multistaging as against the centrifugal compressors.

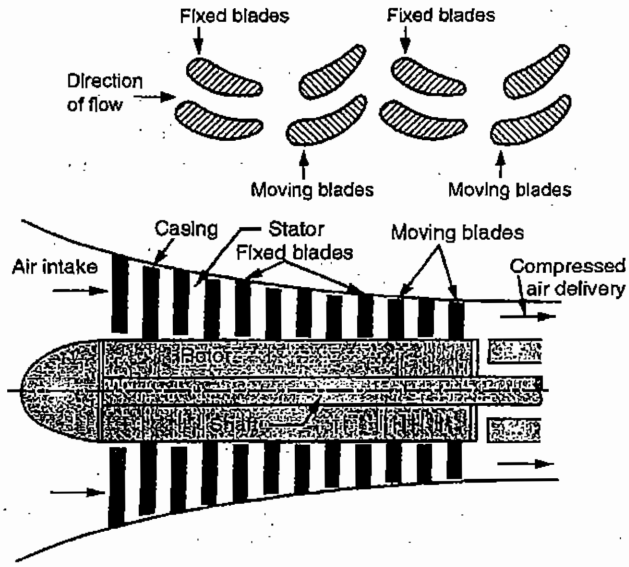
7.2 Components and Working of Axial Flow Compressor

University Question

Q. Explain the construction and working of an axial flow compressor.

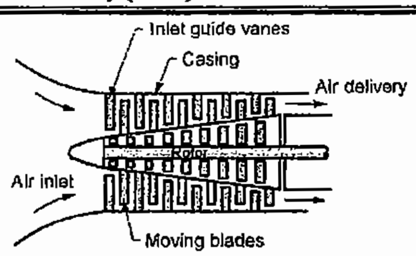
SPPU : May 11, Dec.15, May 16, May 19

- An axial flow compressor is similar in appearance to an axial flow reaction turbine. This type of compressor is shown diagrammatically in Fig. 7.2.1.
- In this compressor the air flows parallel to the axis of compressor. Each row of rotor blade is followed by a companion set of stator blades fixed on the casing. A pair of fixed blade and rotor blade is called a **stage.**
- The function of the fixed blades is to receive the high velocity gas from the preceding rotor blades and to direct the flow to the succeeding row of rotor blades.
- An axial flow compressor may have a *drum type rotor* or *disc type rotor* as shown in Fig. 7.2.1(a) and (b) respectively. For the same weight, the centrifugal stresses are lower in disc type as compared to drum type rotor.
- As shown in Fig. 7.2.1, it consists of fixed and moving blades alternately.



(a) Drum type rotor

Fig. 7.2.1 contd...



(b) Disc type rotor

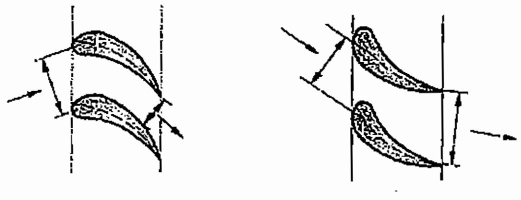
Fig. 7.2.1 : Axial flow compressor

- The fixed blades are attached to the casing called stator and the moving blades are attached to the rotor or spindle.
- The ring of fixed and moving blades must be close for smooth and efficient flow.
- At the inlet of compressor, an extra row of fixed vanes are provided called *inlet guide vanes* (IGV's)
- The gas space is restricted as staging progresses by decreasing the radial distance between the rotor drum and the casing and also by shortening the blades due to reduction in volume due to increasing pressure from stage to stage.
- The basic principle of operation of axial flow compressors is same as that of the centrifugal compressors.
- In this also, the dynamic energy imparted to the gas by the rotor blades is converted into kinetic energy which is then converted into pressure energy in the stator blades by diffusion process carried out in the diverging blade passages.
- The blades are made of aerofoil section to reduce the friction losses.
- The pressure rise per stage in case of axial flow compressor ranges from 1.1 to 1.25 due to which a large number of stages ranging from 5 to 14 are used which can give pressure ratio upto 12 : 1. Pressure ratios can be further increased by increasing the number of stages.
- The multistage axial flow compressors can supply air upto 30000 m^3/min.

Design of blades are based on aerofoil theory and the area of flow is reduced from inlet to delivery end due to increased pressure in order to maintain the velocity of flow reasonably constant along the length of compressor. It is achieved by reducing the height of blades.

7.2.1 Comparison between Turbine and Compressor Blades

Rotor blades for turbine and compressor are shown in Fig. 7.2.2. It can be noted that there is a marked difference in turbine and compressor blading. The section of compressor blades are of aerofoil section and the area of passages through the blades is different.



(a) Turbine blades (b) Compressor blades

Fig. 7.2.2 : Comparison of form of turbine and axial flow compressor rotor blades

The reasons are :

(1) In turbines the blade passages are converging since the flow is accelerating. Whereas the compressor blade passages are diverging since the flow is decelerating or diffusing.

(2) In compressors the blades are made of aerofoil section based on aerofoil theory since the diffusing flow is less stable and rapid diffusion causes the fluids break away from the walls of the diverging passages, reverses its direction and flows back in direction of pressure gradient.

However, such problem does not exist in turbine blades, hence these blades can have profile consisting of circular arcs and straight line.

7.3 Velocity Diagram for Axial Flow Compressors Stage

University Question

Q. Describe axial flow compressor with velocity diagrams.

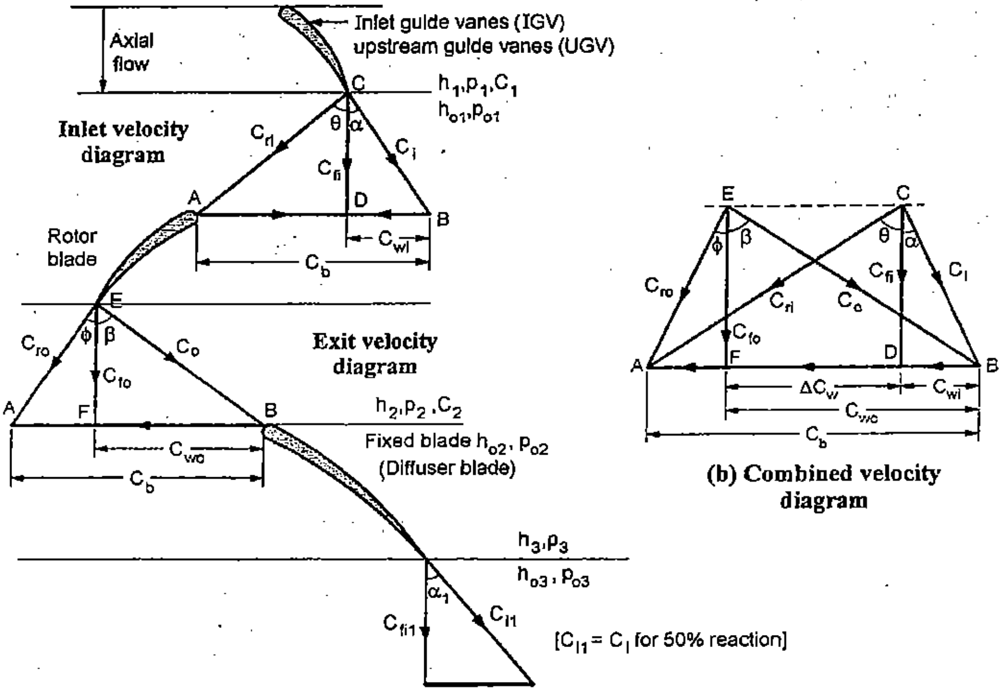
SPPU : Dec.18

Velocity diagram for a general stage of axial flow compressor is shown in Fig. 7.3.1(a) and the combined diagram for 50% reaction compressor is shown in Fig. 7.3.1(b).

Notations used for gas velocities are :

C → absolute velocity,
C_r → relative velocity
C_f → flow velocity
C_w → whirl velocity
C_b → mean blade velocity

The subscript ' i ' denotes at inlet and ' o ' denotes at exit. α and β are exit and inlet angles of fixed blades respectively, θ and ϕ are inlet and exit angles of rotor blades respectively.



(a)

(b) Combined velocity diagram

Fig. 7.3.1 : Velocity diagram for axial compressor stage

Note that, $\vec{C} = \vec{C_r} + \vec{C_b}$ *and* C_w *and* C_f *are tangential and axial component of absolute velocity(C) respectively.*

For any stage, the rotor receives the gas with an absolute velocity C_i and angle α from the previous stage. In case of first stage of compressor, the air is received in desired direction α by providing IGVs or upstream guide vanes (UGV). It results into additional loss of energy from flow through these vanes.

- In an axial flow compressor the air flows over the rotor blades in a direction parallel to the axis of compressor. The inlet and outlet velocity diagrams are shown in Fig. 7.3.1(a) and the superimposed velocity diagram is shown in Fig. 7.3.1(b) for a 50% reaction axial flow compressor.
- The notations used are same as discussed in case of centrifugal compressors. Again these have been mentioned above. In case of axial flow compressors, ***the angles are measured from the axial direction and not from the tangential direction as in case of centrifugal compressors.***
- The blade velocity is same at inlet and outlet of the rotor blade and due to this the inlet and outlet velocity diagrams can be superimposed as shown in Fig. 7.3.1(b).
- Since the rotor blades are diverging, the relative velocity at outlet to the moving blade becomes less than the relative velocity at inlet due to conversion of kinetic energy into pressure energy by diffusion process ($C_{ro} < C_{ri}$).
- The object of the fixed blades is to conduct the air in the correct direction, so that entry to the next ring of moving blades is without shock.
- The passage between the fixed blades acts as a diffuser section, thus the kinetic energy is converted into pressure energy ($C_i < C_o$).
- Due to this the inlet velocity C_o to the fixed blades is again reduced to inlet velocity to rotor blades C_i at the exit of fixed or stator blades.

The axial velocity of flow remains constant in a stage.

Work input

The work done on the compressor per kg of air per stage is given as,

$$W = (C_{wo} - C_{wi})\,C_b \qquad ...(7.3.1)$$

Applying the Steady Flow Energy Equation (S.F.E.E) and assuming process to be adiabatic, the work of compression can be written as,

$$W = C_p(T_{02} - T_{01}) \quad ...(7.3.2)$$

where, T_{02} = Exit stagnation temperature of air

T_{01} = Inlet stagnation temperature of air

Equating the Equations (7.3.1) and (7.3.2),

$$W = (C_{wo} - C_{wi})C_b = C_p(T_{03} - T_{01})$$

$$= C_p(T_{02} - T_{01}) \quad ...(7.3.3)$$

$$\therefore (T_{02} - T_{01}) = \frac{(C_{wo} - C_{wi})C_b}{C_p} \quad ...(7.3.4)$$

7.3.1 Representation of Processes on Enthalpy-Entropy (h – S) Diagram for Single Stage Compression

University Question

Q. Represent and explain the process involved in axial flow compressor on (h-s) diagram and derive an expression for isentropic efficiency and stage pressure ratio. **SPPU : Dec.15, May 16, Dec. 16**

Fig. 7.3.2 shows the stage of a single stage compressor. Air enters the rotor blades with velocity C_i and leaves at higher velocity C_0. This velocity is reduced to C_{i1} ($\simeq C_i$) in stator or diffuser blades.

At inlet to compressor, $$h_{01} = h_i + \frac{1}{2}C_i^2$$

$$p_{01} = p_1 + \frac{1}{2}\rho_i \times C_i^2$$

At outlet to rotor, $$h_{02} = h_2 + \frac{1}{2}C_0^2$$

$$p_{02} = p_2 + \frac{1}{2}\rho_0 \times C_0^2$$

At outlet to stator, $$h_{03} = h_3 + \frac{1}{2}C_{i1}^2$$

$$p_{03} = p_3 + \frac{1}{2}\rho_{i1} \cdot C_{i1}^2$$

But the outlet condition of stator is same as inlet to the first stage, $C_{i1} = C_i$ (for 50% reaction). Therefore, $h_{03} > h_{01}$, there is an increase in enthalpy in the stage. It causes the temperature of air to rise by $\Delta T_0 = (T_{03} - T_{01})$. All the power is absorbed by the rotor which is used to increase the static pressure and in friction heating. The fixed blades in the stator merely transform the K.E. $\left(\frac{C_o^2}{2} - \frac{C_{i1}^2}{2}\right)$ to increase its static pressure. Therefore, T_{03} can be assumed equal to T_{02} and p_{03} is less than p_{02} due to stagnation pressure loss and increase in entropy due to irreversibility's. However, total stagnation enthalpy remains same i.e. $h_{03} = h_{02}$.

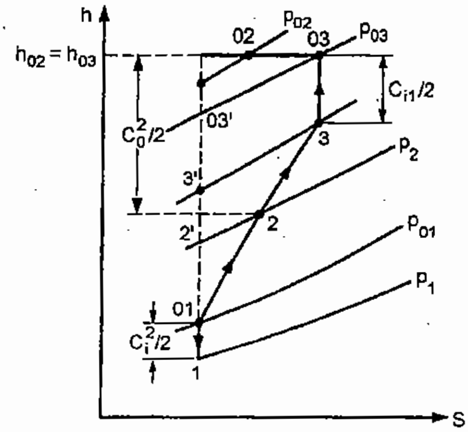


Fig. 7.3.2 : (h – S) diagram for single stage axial flow compressor

Hence the total temperature rise per stage becomes $\Delta T_0 = (T_{03} - T_{01}) = (T_{02} - T_{01})$

If T_{03}' is the isentropic temperature after compression in ideal case, then isentropic efficiency, η_i of compressor is,

Isentropic efficiency,

$$\eta_i = \frac{\text{Isentropic work input}}{\text{Actual work input}} = \frac{h_{03}' - h_{01}}{h_{03} - h_{01}}$$

$$\eta_i = \frac{T_{03}' - T_{01}}{T_{03} - T_{01}} = \frac{T_{03}' - T_{01}}{T_{02} - T_{01}} \quad ...(7.3.5)$$

$$T_{03}' - T_{01} = \eta_i(T_{03} - T_{01})$$

$$T_{01}\left(\frac{T_{03}'}{T_{01}} - 1\right) = \eta_i(T_{03} - T_{01})$$

$$T_{01}\left[\left(\frac{p_{03}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1\right] = \eta_i(T_{03} - T_{01})$$

$$\frac{p_{03}}{p_{01}} = \left[\frac{\eta_i(T_{03} - T_{01})}{T_{01}} + 1\right]^{\gamma/(\gamma-1)} \quad ...(7.3.6)$$

Also $C_p(T_{03} - T_{01}) = (C_{wo} - C_{wi})C_b$

Refer Fig. 7.3.1(a) :

$$C_{wo} - C_{wi} = C_f(\tan\theta - \tan\phi)$$

$$\therefore T_{03} - T_{01} = \frac{(C_{wi} - C_{wi})C_b}{C_p}$$

$$= \frac{C_f \cdot C_b(\tan\theta - \tan\phi)}{C_p}$$

On substituting the value of $(T_{03} - T_{01})$ in Equation (7.3.6) we get,

$$\frac{p_{03}}{p_{01}} = \left[\frac{\eta_i \cdot C_f \cdot C_b \cdot (\tan\theta - \tan\phi)}{C_p \cdot T_{01}} + 1\right]^{\gamma/(\gamma-1)} \quad ...(7.3.7)$$

Since no work is done in diffuser, in ideal case we have $p_{03} = p_{02}$ and $T_{03} = T_{02}$.

7.3.2 Degree of Reaction (R)

The **degree of reaction** as applied to axial flow compressors is defined as the ratio of pressure rise in rotor blades to the pressure rise in the stage i.e.

Degree of reaction,

$$R = \frac{\text{Pressure rise in rotor blades}}{\text{Pressure rise in stage}} \quad ...(7.3.8)$$

But, pressure rise in rotor $= \dfrac{C_{ri}^2 - C_{ro}^2}{2}$ ...(7.3.9)

Pressure rise in stator $= \dfrac{C_o^2 - C_i^2}{2}$ ...(7.3.10)

Total pressure rise in stage, $(\Delta p) = (C_{wo} - C_{wi})\,C_b$ ...(7.3.11)

$\therefore$ Degree of reaction, $R = \dfrac{(C_{ri}^2 - C_{ro}^2\ /\ 2)}{(C_{wo} - C_{wi})\,C_b}$ ...(7.3.12)

7.3.3 To Show that Compressor has Symmetrical Blades for 50% Reaction

From velocity triangles shown in Fig. 7.3.1(b), we get,

$$C_{wi} = C_b - C_{fi} \cdot \tan\theta \quad ...(i)$$

$$C_{wo} = C_b - C_{fo} \tan\phi \quad ...(ii)$$

$$\therefore \quad C_{wo} - C_{wi} = (C_b - C_{fo}\tan\phi) - (C_b - C_{fi}\tan\theta)$$

$$= C_{fi}(\tan\theta - \tan\phi) = C_f(\tan\theta - \tan\phi) \quad ...(iii)$$

Where, $C_{fi} = C_{fo} = C_f$

Therefore, pressure rise in the stage from Equations (7.3.9) and Equation (iii) becomes,

$$(\Delta p_o)\ \text{stage} = C_b \cdot C_f(\tan\theta - \tan\phi) \quad ...(iv)$$

Also from velocity diagrams shown in Fig. 7.3.1(a) and from law of triangles, we can write,

$$C_{ri}^2 = C_f^2 + (C_f \tan\theta)^2 \quad ...(v)$$

$$C_{ro}^2 = C_f^2 + (C_f \tan\phi)^2 \quad ...(vi)$$

Therefore pressure rise in the rotor becomes,

$$(\Delta p)\ \text{rotor} = \frac{C_{ri}^2 - C_{ro}^2}{2}; \ \text{...from Equation (7.3.9)}$$

On substituting the values from Equation (v) and (vi), we get,

$$(\Delta p)\ \text{rotor} = \frac{[C_f^2 + (C_f \tan\theta)^2 - [C_f^2 + (C_f \tan\phi)^2]}{2}$$

$$(\Delta p)\ \text{rotor} = \frac{C_f^2}{2}(\tan^2\theta - \tan^2\phi)$$

$$= \frac{C_f^2}{2}(\tan\theta - \tan\phi)(\tan\theta + \tan\phi) \quad ...(vii)$$

Substituting the values of pressure rise for stage and rotor from Equations (iv) and (vii) in Equation (7.3.12),

Degree of reaction,

$$R = \frac{\frac{C_f^2}{2}(\tan\theta - \tan\phi)(\tan\theta + \tan\phi)}{C_b \cdot C_f(\tan\theta - \tan\phi)}$$

$$R = \frac{C_f}{2\,C_b}(\tan\theta + \tan\phi) \quad ...(7.3.13)$$

Axial flow compressors are usually designed for 50% reaction i.e. R = 0.5, Equation (7.3.13) reduces to,

$$0.5 = \frac{C_f}{2\,C_b}(\tan\theta + \tan\phi)$$

or, $\dfrac{C_b}{C_f} = (\tan\theta + \tan\phi)$ ...(viii)

From the geometry of velocity triangles,

$$\frac{C_b}{C_f} = \tan\alpha + \tan\theta = (\tan\phi + \tan\beta) \quad ...(ix)$$

It follows from Equations (viii) and (ix),

$$\alpha = \phi \ \text{ and } \ \theta = \beta \quad ...(7.3.14)$$

Conditions given in Equation (7.3.14) are only applicable for 50% reaction axial flow compressors. It is obvious that such compressors will have symmetrical blades.

7.3.4 Work Input Factor or Workdone Factor for Axial Flow Compressors (ψ_w)

- Since the work required to compress the air given by Equations (7.3.1) and (7.3.2) are equal, it follows that,

$$C_p(T_{02} - T_{01}) = (C_{wo} - C_{wi})\,C_b \quad ...(7.3.15)$$

- In the analysis of axial flow compressors it is assumed that the axial velocity of flow remains constant throughout.
- However, there exists a negative pressure gradient in the direction of flow (pressure increase in the direction of flow), the boundary layer along at hub and casing increases as the flow progress due to secondary flows. It results into variation in axial velocity along the blade height as shown in Fig. 7.3.3.

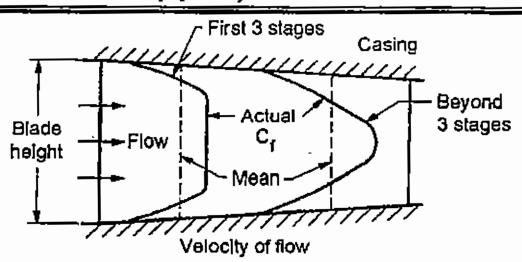


Fig. 7.3.3

As it can be seen that the velocity of flow is much less at hub and casing and it is high in the central region of flow of the mean velocity of flow. In the first few stages the variation of C_f is large and after 3 to 6 stages the axial velocity of flow becomes constant.

- It is due to the fact that in case of axial flow compressors the passages between the blades are diverging and the flow is decelerating.
- Very rapid diffusion in diverging passages of the blades is not possible since there is tendency for the fluid to break away from the walls, reverses its direction and flows back in the direction of pressure gradient. These factors tend to result into increased mean velocity of flow and reduction in whirl component of velocity.
- Effect of axial velocity on the stage velocity triangles and work is shown in Fig. 7.3.4.
- It is evident from Fig. 7.3.4, the work absorbing capacity in the central region decreases with increase in velocity of flow whereas it increases near the hub and casing due to reduction in C_f. But in the region of hub and casing the work absorbing capacity is less due to energy losses. As a result, the overall capacity of work in a stage reduces. The ratio of actual work absorbed to the theoretical work is defined as **work input factor or workdone factor, ψ_w.**

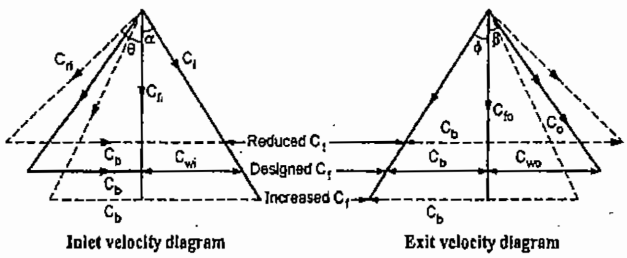


Fig. 7.3.4 : Effect of axial velocity of flow (C_f) on velocity triangles and work

The effect of number of stages on work input factor is shown in Fig. 7.3.4(a).

The actual work transferred to gas or air can be rewritten as :

Actual work input, W $= (C_{wo} - C_{wi})\, C_b \cdot \psi_w$

The value of ψ_w varies from 0.9 to 0.85.

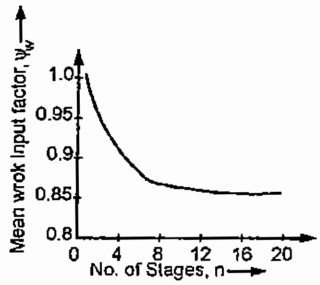


Fig. 7.3.4(a) : Effect of number of stages on work input factor

7.4 Polytropic Efficiency of an Axial Flow Compressor

The (T - S) diagram for four stages of a multistage axial flow compressor is shown in Fig. 7.4.1. $\left(T'_{02} - T_{01}\right)$ represents the overall isentropic temperature rise for a pressure ratio of $\left(\frac{p_{02}}{p_{01}}\right)$. Whereas, the total isentropic temperature rise for all the stages is $\Sigma\,(\Delta T)$ which is greater than the overall isentropic temperature rise due to the fact that in multistage compression the subsequent stages start at a higher level of entropy and the pressure lines are of diverging nature on (T - S) diagram. Hence, we define **polytropic efficiency as the isentropic efficiency of an infinitesimal stage of the compressor which is constant for each stage of compression. The polytropic efficiency for a stage is given as :**

$$\eta_p = \frac{\text{Polytropic work, } W_P}{\text{Actual work, W}}$$

$$= \frac{\left(\frac{n}{n-1}\right) mR\,(T_{02} - T_{01})}{m\,C_p\,(T_{02} - T_{01})}$$

$$= \left(\frac{n}{n-1}\right)\frac{R}{C_p} = \left(\frac{n}{n-1}\right)\frac{R}{\left(\frac{\gamma}{\gamma-1}\right)R}$$

$$\eta_p = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)$$

and $$\left(\frac{n-1}{n}\right) = \left(\frac{\gamma-1}{\gamma}\right)\frac{1}{\eta_p} \qquad ...(7.4.1)$$

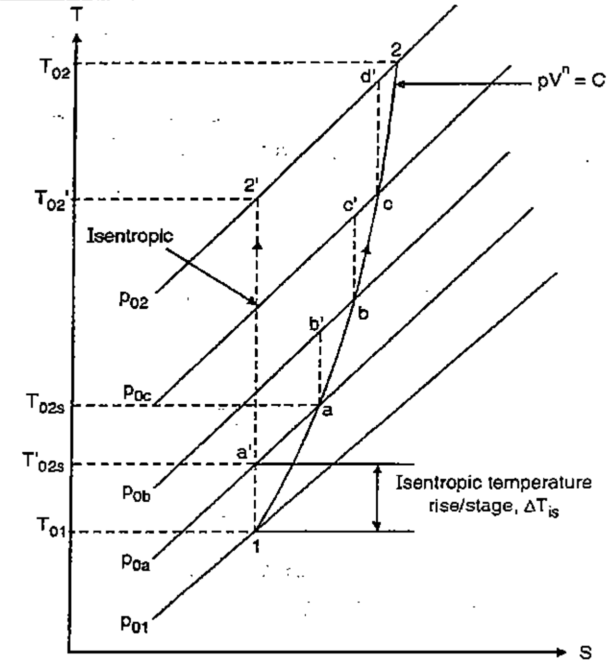


Fig. 7.4.1 : Polytropic efficiency

Generally, the value of index-n is unknown, therefore, it is more convenient to express this value in terms of total pressure and temperature ratios.

Since, $$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{(n-1)/n}$$

– Taking log on both the sides and on simplifying we get,

$$\left(\frac{n}{n-1}\right) = \frac{\log\left(\frac{p_{02}}{p_{01}}\right)}{\log\left(\frac{T_{02}}{T_{01}}\right)}$$

– Therefore the expression for polytropic efficiency can be written as :

$$\eta_p = \left(\frac{\gamma-1}{\gamma}\right) \times \frac{\log\left(\frac{p_{02}}{p_{01}}\right)}{\log\left(\frac{T_{02}}{T_{01}}\right)} \quad \text{...(7.4.2)}$$

7.5 Flow Coefficient ϕ and Rotor Blade Loading Coefficient, ψ

University Question

Q. Explain flow co-efficient. **SPPU : May 15**

Flow Coefficient, ϕ is defined as the ratio of axial velocity of flow C_f to the peripheral velocity of blades C_b, i.e.

$$\phi = \frac{\text{Axial velocity flow}, C_f}{\text{Peripheral velocity of blade}, C_b} \quad \text{...(7.5.1)}$$

Blade loading coefficient, ψ for an axial flow compressor is defined as the ratio of workdone to the square of peripheral velocity of blade i.e.

$$\psi = \frac{\text{Workdone}}{(\text{Blade velocity})^2} = \frac{W}{C_b^2} \quad \text{...(7.5.2)}$$

Both flow coefficient and blade loading coefficients are dimensionless parameters.

Refer Fig. 7.3.1(a),

$$W = (C_{wo} - C_{wi})\, C_b$$

$$W = [(C_b - C_{fo}\tan\phi) - (C_b - C_{fi}\tan\theta)]\, C_b$$

If $C_{fi} = C_{fo} = C_f$, then

$$W = C_f \cdot C_b (\tan\theta - \tan\phi) \quad \text{...(i)}$$

∴ Blade loading coefficient,

$$\psi = \frac{W}{C_b^2} = \frac{C_f \cdot C_b (\tan\theta - \tan\phi)}{C_b^2}$$

$$\psi = \frac{C_f}{C_b} \times (\tan\theta - \tan\phi)$$

$$= \phi(\tan\theta - \tan\phi) \quad \text{...(7.5.3)}$$

∵ $\alpha = \phi$ and $\theta = \beta$ for 50% reaction compressor,

Equation (7.5.1) can be rewritten as :

$$\psi = \phi(\tan\theta - \tan\phi)$$

$$= \phi(\tan\beta - \tan\alpha) \quad \text{...(7.5.3(A))}$$

The performance of 50% reaction axial flow compressor can be predicted by using blade loading coefficient ψ and flow coefficient ϕ as shown in Fig. 7.5.1.

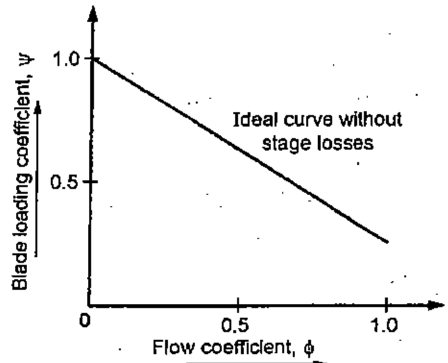


Fig. 7.5.1

7.6 Flow Passages and Vortex Theory

In the earlier sections we have assumed that the flow of gas in annular section (between stator and rotor) is two dimensional neglecting the effect of radial component of velocity. This assumption is reasonable for later stages of compressor where the blade height is small due to hub to tip diameter ratio being high in the range of 0.8 and above.

However, in earlier stages of compressor, the hub to tip ratio of about 0.4 is kept for the following reasons :

(i) To keep the low frontal area

(ii) To allow high mass flow rate.

As the pressure ratio increases along the axis of compressor, the hub to tip ratio increases with number of stages therefore, the blade height considerably reduces. Now the flow stream lines will no more lie on surface of rotation parallel to the axis of rotor.

Thus it becomes necessary to consider the small radial component of velocity along with axial and tangential components of velocity i.e. three dimensional flow must be considered for large mass flow rates.

The variation in blade velocity from root to tip of blades will be large for small hub to tip ratios, hence it would vary the velocity triangles at various radii of blading. It affects considerably the shapes of blades.

The flow in an annular passage in which there is no radial component of velocity, whose stream lines lie in circular, cylindrical surfaces is known as radial equilibrium theory.

7.6.1 Radial Equilibrium Theory (Vortex Theory)

In an axial flow compressor, the fluid has the combination of rotational and axial motion. The fluid is subjected to a centrifugal force and for its equilibrium, it must be balanced by pressure gradient in radial direction.

Fig. 7.6.1 shows section two stream lines AB and CD at radius (r) and (r + dr) of unit width.

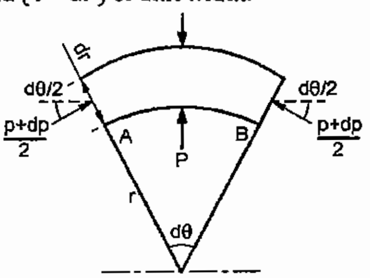


Fig. 7.6.1 : Vortex theory

When the fluid flows between curved stream lines the centrifugal forces are set up and these are counterbalance by pressure forces acting in radial direction as shown in Fig. 7.6.1. Effect of acceleration due to gravity is neglected being too small compared to centripetal acceleration.

$$\text{Volume of fluid in prism} = (r \cdot d\theta)\, dr$$

$$\text{Mass in prism, } dm = \rho \cdot (r\, d\theta)\, dr$$

$$\text{Centrifugal force, } F_c = dm \cdot \omega^2 \cdot r = \rho \cdot r \cdot d\theta \cdot dr \cdot \omega^2 \cdot r$$

$$\text{But,} \quad C_w = \omega \cdot r$$

$$\therefore \quad F_c = \rho \cdot d\theta \cdot dr \cdot C_w^2 \quad \text{...(i)}$$

Radial pressure force,

$$F_p = (p + dp)(r + dr)\, d\theta - p \cdot r \cdot d\theta - 2\left(\frac{p + dp}{2}\right) dr \cdot \frac{d\theta}{2}$$

Third term in above expression results due to resolution of average pressure forces on two sides of element in radial-axial plane. On neglecting second order terms, above equation reduces to,

$$\text{Radial pressure force,} \quad F_p = dp \cdot r\, d\theta \quad \text{...(ii)}$$

$$\text{For equilibrium,} \quad F_c = F_p$$

$$\therefore \rho \cdot d\theta \cdot dr \cdot C_w^2 = dp \cdot r \cdot d\theta$$

$$\therefore \frac{1}{\rho} \cdot \frac{dp}{dr} = \frac{C_w^2}{r} \quad \text{...(7.6.1)}$$

Equation (7.6.1) represents the **radial equilibrium equation.**

7.6.2 Energy Equation for Vortex Flow

Considering the flow of an ideal gas, the total stagnation enthalpy, h_0 at radius r can be written as,

$$h_0 = h + \frac{C^2}{2} = C_p \cdot T + \left(\frac{C_f^2 + C_w^2}{2}\right)$$

$$= \left(\frac{\gamma}{\gamma - 1}\right)\frac{p}{\rho} + \frac{1}{2}\left(C_f^2 + C_w^2\right)$$

On differentiating with respect to 'r',

$$\frac{dh_0}{dr} = \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{1}{\rho} \cdot \frac{dp}{dr} - \frac{p}{\rho^2} \cdot \frac{d\rho}{dr}\right) + C_f \cdot \frac{dC_f}{dr} + C_w \cdot \frac{dC_w}{dr} \quad \text{...(7.6.2)}$$

For small change in pressure across the two stream lines, the process can be assumed isentropic i.e.

$$p / \rho^{\gamma} = \text{constant.}$$

On taking its log-differential,

$$\frac{1}{p} \cdot \frac{dp}{dr} - \gamma \cdot \frac{1}{\rho} \cdot \frac{d\rho}{dr} = 0$$

$$\text{i.e.} \quad \frac{d\rho}{dr} = \frac{\rho}{\gamma \cdot p} \cdot \frac{dp}{dr} \quad \text{...(7.6.3)}$$

On substituting the value of $\frac{d\rho}{dr}$ from Equation (7.6.3) in Equation (7.6.2) we have,

$$\frac{dh_0}{dr} = \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{1}{\rho} \cdot \frac{dp}{dr} - \frac{p}{\rho^2} \cdot \frac{\rho}{\gamma p} \frac{dp}{dr}\right)$$

$$+ C_f \cdot \frac{dC_f}{dr} + C_w \cdot \frac{dC_w}{dr}$$

$$\frac{dh_0}{dr} = \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{1}{\rho} \cdot \frac{dp}{dr} - \frac{1}{\gamma \cdot \rho} \cdot \frac{dp}{dr}\right)$$

$$+ C_f \cdot \frac{dC_f}{dr} + C_w \cdot \frac{dC_w}{dr}$$

On substituting the value of $\left(\frac{1}{\rho} \cdot \frac{dp}{dr}\right)$ from Equation (7.6.1) in the above equation, we get,

$$\frac{dh_0}{dr} = \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{C_w^2}{r} - \frac{1}{\gamma} \cdot \frac{C_w^2}{r}\right) + C_f \cdot \frac{dC_f}{dr} + C_w \cdot \frac{dC_w}{dr}$$

$$\therefore \frac{dh_0}{dr} = \frac{C_w^2}{r} + C_f \cdot \frac{dC_f}{dr} + C_w \cdot \frac{dC_w}{dr} \quad \text{...(7.6.4)}$$

7.6.3 Free Vortex Flow

Following assumptions are made for free vortex flow :

(1) Equal work input is assumed constant at all radii of rotation then stagnation enthalpy at all radii of rotation across the annular space remains constant i.e.

$$\frac{dh_0}{dr} = 0 \quad \text{...(iii)}$$

(2) Axial flow velocity is assumed constant at all radii of rotation then,

$$\frac{dC_f}{dr} = 0 \quad \text{...(iv)}$$

Applying the above conditions for free vortex flow in Equation (7.6.4), we have,

$$0 = \frac{C_w^2}{r} + 0 + C_w \cdot \frac{dC_w}{dr}$$

$$\text{i.e.} \quad \frac{dC_w}{C_w} + \frac{dr}{r} = 0$$

On integration, $\log_e C_w + \log_e r = \log C$

or, $C_w \cdot r$ = constant, C

$$\text{i.e. } C_w \propto \frac{1}{r} \quad \text{...(7.6.5)}$$

Above Equation (7.6.5) shows that the whirl velocity component varies inversely with radius as a free vortex condition satisfying the condition of radial equilibrium.

Following conditions naturally satisfy the radial equilibrium theory given by Equation (7.5.2) and conducive for design are :

(a) Constant specific work input.

(b) Constant axial velocity

(c) Free vortex variation of whirl velocity.

7.6.4 Free Vortex Design

In case of free vortex design, the degree of reaction, R can be expressed as :

$$R = 1 - \frac{\text{constant}}{r^2}$$

It has the following disadvantages :

(1) Degree of reaction increases from root to tip of blade. Due to low root velocity (r is less), more fluid deflection is needed for a given work input i.e. much higher rate of diffusion is needed at root section compared to tip section of blade. It reduces pressure ratio per stage for free vortex design.

(2) Free vortex blades will have more twist with large variation in Mach number causing shocks. In order to avoid formation of shocks, the blade and axial velocities have to be kept low.

It is usually desirable to keep specific work input condition to provide a constant pressure ratio upto blade height. It is found that it gives,

$C_b (C_{wo} - C_{wi})$ = a constant × C_{bm}, which is independent of radius. This condition is met in design of axial flow compressors with 50% degree of reaction.

7.7 Aerofoil Blading

– The blades of an axial flow compressor if designed based on the forces caused by the rate of change of momentum theory, such blades are not efficient on account of boundary layer separation from blade surfaces. It causes the loss of pressure due to formation of eddies with distortion of flow.

– It should be noted that the problem of boundary flow separation does not occur in radial blades since the increase in pressure is due to centrifugal head imparted to it.

– **The problem of boundary layer separation can be avoided by using aerofoil blading.**

7.7.1 Nomenclature Related to Aerofoil Blades

The related nomenclature used in aerofoil blades are as follows (Refer Fig. 7.7.1).

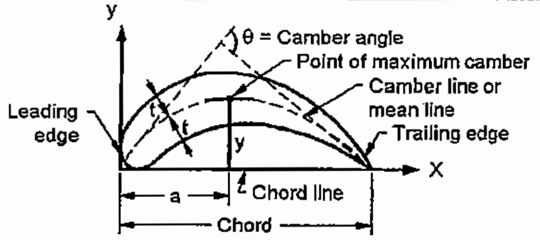


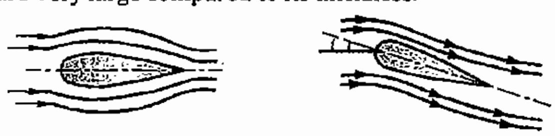
Fig. 7.7.1 : Aerofoil blade profile

1. **Camber line :** It represents the locus of all points midway between the upper and lower mean surfaces of an aerofoil section as measured perpendicular to the mean line.
2. **Blade thickness, t :** It represents the perpendicular distance of the outer or inner surface from camber line.
3. **Camber angle, θ :** The angle made by the tangents to camber line at the leading and trailing edge of the blade profile.
4. **Chord line :** It represents the line joining the ends of the mean camber line.
5. **Camber :** It is the maximum rise of camber line from the chord line.
6. **Distance, a :** It is the distance along the X-axis from leading edge upto the point of maximum camber.
7. **Pitch, s :** It represents the distance between the two consecutive blades.

A blade designated as 11C1/45/C50 according to British standards shows that 11% blade thickness of chord with circular arc having 45° camber angle with a = 50 % of chord length.

7.7.2 Elementary Theory of Aerofoil Blading

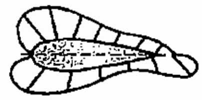
An **aerofoil** may be defined as a stream lined form bounded by two flattened curves whose length and width are very large compared to its thickness.



(a) Symmetrical aerofoil (b) Non - symmetrical aerofoil with angle of incidence '*i*'

Fig. 7.7.2 : Aerofoil blades

Consider a symmetrical aerofoil placed in a stream of gas as shown in Fig. 7.7.2(a). It could be seen that there is no permanent deflection of main stream except for local disturbance as the flow approaches from its leading edge to trailing edge.



(c) Static pressure distribution

Fig. 7.7.2 : Streamline and static pressure distribution in aerofoil blades

Only forces exerted are viscous forces.

Fig. 7.7.2(b) shows a non-symmetrical aerofoil blading which is inclined at an angle 'i' called the **angle of attack or angle of incidence** with the direction of undisturbed approaching flow to the leading edge of the blade. It could be seen that there is pronounced disturbance of stream flow causing into greater local deflection of flow.

This deflection of flow can only be caused if the blade section exerts a force on the gas stream. As a reaction to it, the gas exerts an equal and opposite force on the blade section in addition to shear forces due to wall and fluid friction. Thus the flow around the aerofoil section is both rectilinear and circulatory.

The static pressure developed by these forces can be expressed as force per unit of projected area. If input work per stage is constant, the static pressure rise would be greater than the blades kept in direction of stream flow. The static pressure distribution on aerofoil blade is shown in Fig. 7.7.2(c).

7.7.3 Coefficient of Lift, C_L and Coefficient of Drag, C_D

Fig. 7.7.3 shows the cross-section of two blades with their camber, pitch(s), chord and angle of incidence 'i' used for a typical cascade test.

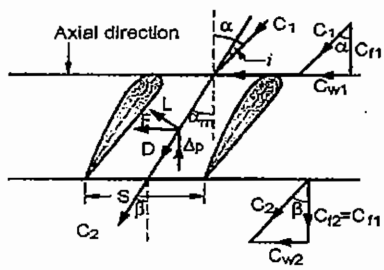


Fig. 7.7.3 : Forces acting on blades

As discussed above, there are two types of forces acting on aerofoil blading due to flow of gas as follows :

(i) Shearing force due to viscous friction of air and the wall friction between air and blade surface.

(ii) Pressure forces on aerofoil section.

- The total or resultant force acting on blade section, called aerodynamic force, is proportional to the kinetic energy of stream and the projected area of blading.
- The resultant force can be resolved into two components as follows :
  (a) Lift, L which is normal to the direction of approach velocity. It is responsible for an aero plane to maintain its lift. It is caused due to unbalanced pressure distribution over aerofoil surface.
  (b) Drag, D which is parallel to the direction of approach velocity. It represents the friction forces.
- The boundary layer from the leading edge upto a short distance is usually laminar and in later part it becomes turbulent. This transition from laminar to turbulent needs to be prevented.
- In case the curvature of aerofoil section is large, it will cause separation of flow due to adverse pressure gradient caused by the loss of pressure head in turbulent motion. In such a case the lift will reduce and drag will increase.
- **Analysis :** Consider unit height of blade and let C_1 and C_2 be the absolute velocity at inlet and outlet of cascade and assume flow to be incompressible.
- The stagnation pressure loss across the cascade shown in Fig. 7.7.3 due to friction is given as,

$$\Delta p_0 = p_{01} - p_{02} \quad \text{...(i)}$$

Total pressure loss coefficient,

$$K_p = \frac{p_{01} - p_{02}}{\frac{1}{2}\rho C_1^2} \quad \text{...(ii)}$$

From velocity diagram,

$$C_1 = \frac{C_f}{\cos\alpha}\text{, using in Equation (ii),}$$

$$\therefore \Delta p_0 = p_{01} - p_{02} = K_p \frac{1}{2}\rho \cdot \frac{C_f^2}{\cos^2\alpha} \quad \text{...(iii)}$$

Actual pressure rise through the cascade,

$$\Delta p = p_2 - p_1 = \frac{\rho}{2}\left(C_1^2 - C_2^2\right) - \Delta p_0$$

$$= \frac{\rho}{2}\left(C_{f1}^2 + C_{w1}^2 - C_{f2}^2 - C_{w2}^2\right) - \Delta p_0$$

$$= \frac{\rho}{2}\left(C_{f1}^2 + C_{f1}^2 \tan^2\alpha - C_{f2}^2 - C_{f2}^2\tan^2\beta\right) - \Delta p_0$$

$$= \frac{\rho}{2}\cdot C_f^2(\tan^2\alpha - \tan^2\beta) - \Delta p_0 \; [\because C_{f1} = C_{f2} = C_f]$$

$$= \frac{\rho}{2}C_f^2(\tan\alpha - \tan\beta)(\tan\alpha + \tan\beta) - \Delta p_0$$

$$= \frac{\rho}{2}C_f^2(\tan\alpha - \tan\beta)\tan\alpha_m - \Delta p_0 \quad \text{...(iv)}$$

Where, $\tan\alpha_m = \frac{\tan\alpha + \tan\beta}{2}$ represents tangent of the angle between the direction of mean velocity C_m from axial direction.

On substituting the value of Δp_0 from Equation (iii) in Equation (iv) we get,

$$\Delta p = \rho C_f^2(\tan\alpha - \tan\beta)$$

$$\tan\alpha_m - K_p \cdot \frac{1}{2}\cdot\rho\frac{C_f^2}{\cos^2\alpha} \quad \text{...(v)}$$

Force acting along the cascade (peripheral component)

$$F_w = S\cdot\rho\cdot C_f \times \text{Change in velocity component}$$

$$= S\cdot\rho\cdot C_f \times C_f(\tan\alpha - \tan\beta)$$

$$= S\cdot\rho\cdot C_f^2(\tan\alpha - \tan\beta) \quad \text{...(7.7.1)}$$

Axial component of force on blade,

$$F_a = S\Delta p$$

On substituting the value of Δp from Equation (v), F_a becomes,

$$F_a = S\rho C_f^2(\tan\alpha - \tan\beta)\tan\alpha_m$$

$$-\frac{1}{2}S\cdot K_p\cdot\rho\cdot\frac{C_f^2}{\cos^2\alpha} \quad \text{...(7.7.2)}$$

Resolving the forces F_w and F_a in the direction of lift, L and drag, D shown in Fig. 7.7.2 we have,

Lift, $L = F_w\cos\alpha_m + F_a\cdot\sin\alpha_m$ ...(vi)

Drag, $D = F_w\sin\alpha_m - F_a\cdot\sin\alpha_m$ ...(vii)

Let 'c' be the length of chord, and C_m is the average velocity.

The coefficient of lift, C_L and coefficient of Drag, C_D can be written as,

$$C_L = \frac{L}{\frac{1}{2}\rho\cdot c\cdot C_m^2}$$

and

$$C_D = \frac{D}{\frac{1}{2}\rho\cdot c\cdot C_m^2} \quad \text{...(7.7.3)}$$

The values of C_L and C_D depend upon the aerofoil shape, degree of curvature, Reynold number, Mach number and angle of incidence or attack, 'i'.

Fig. 7.7.3 shows the variation of C_L and C_D at various angle of attack, i based on experimental results for various cascades.

7.7.4 Stalling

University Questions

Q. Explain the term Stalling in an axial flow compressor. **SPPU : Dec.15**

Q. Explain the phenomena of stalling of the blades. **SPPU : Dec.18**

It could be noted from the Fig. 7.7.4, as the angle of incidence increases, the C_L increases almost linearly upto 'd' and then the rate of increase of C_L decreases. It is for the reason that at higher angle of incidence the air stream tends to break away from the surface with separation of flow over large part of the blade beyond point 'e'. Separation of flow causes the formation of eddies and loss of lift with simultaneous increase in drag as shown in Fig. 7.7.4.

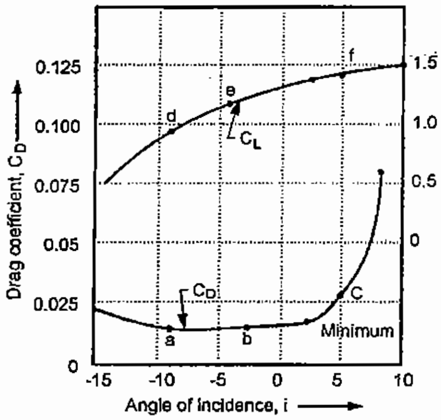


Fig. 7.7.4 : Drag and lift coefficient Vs angle of incidence

- Beyond the point 'f' with further increase in angle of incidence, the flow separation will occupy more surface area and extending away from the blade surface.
- It would result into complete breakdown of flow pattern in the blade passages and the aerofoil blade is said to have **stalled.**
- The incidence corresponding to maximum C_L at which blades are installed is called the **stalling incidence angle.** Therefore, **stalling** of an axial flow compressor can be defined as the breakaway of the flow from the suction side of the aerofoil blade.
- The rotating stall can produce resonant vibration and may cause the blade failure.

7.7.5 Surging and Choking

University Question

Q. Explain choking and surging. **SPPU : May 15**

The phenomenon of surging and choking is similar to as discussed in case of centrifugal compressors. (Refer sections 6.10, 6.10.1 and 6.10.2)

7.8 Losses in Axial Flow Compressors

University Questions

Q. Explain the Losses in axial flow compressor. **SPPU : Dec. 15, Dec. 19**

Q. Compare the effect of different factors affecting the stage pressure ratio in axial flow compressor. **SPPU : May 16**

Q. Write a short note on losses in axial flow compressor. **SPPU : Dec. 16**

Q. What are the various losses in Axial Flow Compressor. **SPPU : May 18**

Various pressure losses which occur in an axial flow compressor are as follows :

1. **Skin friction losses :** The pressure losses are caused due to viscous friction that arise in the boundary layers of vanes and flow passage walls in the annulus. Separation eddies and boundary layer accounts most of the losses which are difficult to assess.
2. **Profile losses :** It represents the pressure losses due to skin friction on the blade surfaces. It can be obtained from experimental cascade testing which is expressed as drag coefficient. It mainly depends on Reynold number and the angle of incidence.
3. **Secondary flow losses :** These pressure losses are associated with secondary flows produced by the combined effect of curvature and boundary layer as a result of finite blade spacing.

   The secondary flow is produced due to viscous effects. It modifies the main flow pattern of stream and the flow gets deflected. The losses are greatly influenced by tip clearance and this should not exceed more than 2% of blade height.

   The additional drag coefficient arising due to secondary losses as estimated by Cartev are given by the equation,

$$C_{DS} = 0.018\, C_L^2$$

7.9 Performance Characteristic Curves of an Axial Flow Compressor

The characteristic curves of an axial flow compressor take a form similar to centrifugal compressors as discussed in section 6.11. These characteristic curves are plotted on the same non-dimensional parameters as follows.

1. The pressure ratio parameter, $\frac{p_{02}}{p_{01}}$
2. Speed parameter, $\frac{N}{\sqrt{T_{01}}}$
3. Mass flow parameter, $\frac{m\sqrt{T_{01}}}{p_{01}}$

Fig. 7.9.1 shows characteristic curve plotted between pressure ratio $\left(\frac{p_{02}}{p_{01}}\right)$ against the mass flow parameter $\left(m\sqrt{T_{01}}/p_{01}\right)$ at various speed parameters $N/\sqrt{T_{01}}$ relative to designed value.

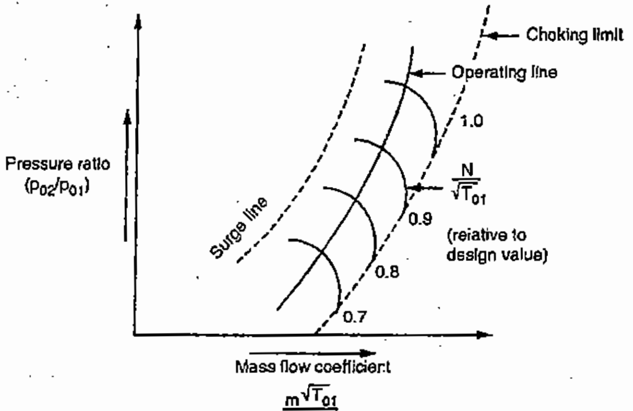


Fig. 7.9.1 : Plot of pressure ratio against mass flow coefficient at various speed parameters

The unstable range of the curve is limited by surge line which corresponds to low mass flow rates and by the choking at high mass flow rates. The operating line is near the peak of the characteristic curve.

Fig. 7.9.2 shows the characteristic curve between the isentropic efficiency and mass flow parameter at various speed parameters. It could be seen that the efficiency varies with mass flow rate at a given speed in a similar manner as pressure ratio. Maximum efficiency is almost same at all speeds. The locus of points of maximum efficiency gives the operating line.

Fig. 7.9.3 shows the characteristic curves for the compressor with constant efficiency curves. Following conclusions can be drawn.

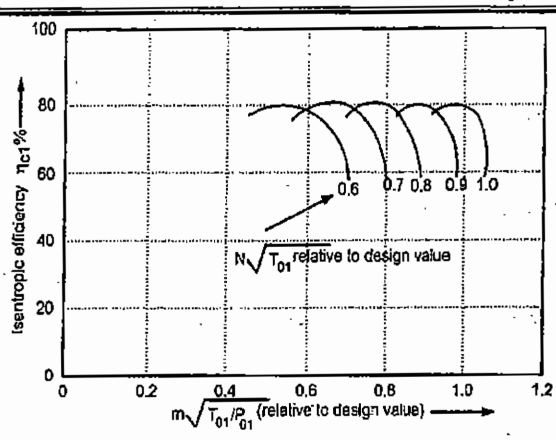


Fig. 7.9.2 : Plot of isentropic efficiency against mass flow coefficient at various speed parameters

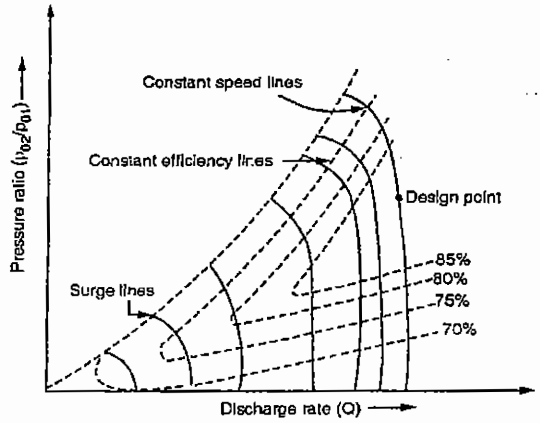


Fig. 7.9.3 : Characteristic curves of an axial flow compressor

The salient points are :

(i) The increase in pressure does not affect appreciably the discharge rates as the constant speed lines are almost vertical except at low speeds.

(ii) The efficiency of the compressor decreases with the decrease in pressure ratio at a given mass flow rate.

(iii) The optimum performance of axial flow compressors is obtained at large discharge rates and at high speeds.

7.10 Comparison between Centrifugal and Axial Flow Compressors

University Questions

Q. Compare centrifugal and axial flow compressor.
SPPU : Dec. 13, Dec. 16

Q. Write advantages and disadvantages of axial flow compressor over centrifugal compressor.
SPPU : Dec. 19

Following are the main features of centrifugal and axial flow compressors. Such comparison helps us in selection of these compressors.

Sr. No	Centrifugal compressor	Axial flow compressor
1.	The flow is radial.	The flow is axial.
2.	It develops high pressure ratio per stage of about 4.5:1.	It develops low pressure ratio per stage of about 1.2:1. Therefore to get high pressure ratios, a large number of stages are required.
3.	It has high efficiency over wide range of speeds.	It has high peak efficiency of 86 to 89% in narrow range of speeds when multistage compression is used with aerofoil blades.
4.	Easy to manufacture.	Difficult to manufacture.
5.	It has low starting power requirements.	It requires high starting powers.
6.	It has low weight and cost.	It has high weight and cost.
7.	It needs large frontal area for given mass flow rates.	It needs less frontal area for given mass flow rates.
8.	It is not suitable for multistaging due to large losses in between the stages.	It is suitable for multistaging.
9.	The part load performance is good.	The part load performance is poor.
10.	These are suitable for supercharging of I.C. engines, for compression of refrigerants and other industrial applications.	These are suitable for jet engines, gas turbine power plants and steel mills.

7.11 Compressor Materials

- Compressor and turbine blades are subjected the high temperature and high forces in all the directions due to gas flow, centrifugal action and driving forces.
- Therefore the materials used for rotor and blades should have high strength, able to withstand high temperatures and the blades should be light in weight.
- The materials used should have properties for easy forging, welding and machining.
- Commonly used materials for compressors are :
  1. Titanium alloys
  2. Steel alloys
  3. Nickel alloys
- Usually chromium and aluminium in various proportions are used to improve the strength and resistance to corrosion in the above alloys. Above listing of materials is in the increasing order of their ability to withstand high temperature and weight.
- **Steel is used as rotor material.** However in aircraft applications, titanium alloy is in the front few stages and nickel alloy is used for remainder stages.
- Titanium, steel and nickel alloys are used for **rotor blades** depending on the particular design and application.
- Blades of Titanium alloy material are light in weight compared to steel and nickel alloys with high oxidation resistance, but their strength decreases with increase in temperature. Thus, these alloys are suitable comparative for low temperature application like for compressor blades but not for turbine blades.
- Nickel alloys is used for gas turbine blades due to their high strength. Various elements added in nickel are titanium, aluminium, chromium, cobalt, molybdenum etc to improve its strength and to withstand high temperatures.

Steel is most commonly used for **stator blades.**

Note : Numericals are not in syllabus, however few examples are given below to understand the application of theory on axial flow compressors.

Ex. 7.11.1 : An axial flow air compressor comprises of similar stages with equal work consumption and 50% degree of reaction for each stage. Inlet and Outlet air angles for compressor blades are 52° and 35°, respectively. Air enters the compressor at 15.5°C and compression efficiency is 80%. Mean blade speed may be taken as 220 m/s and axial velocity of flow may be treated as constant. Plot velocity triangles for the compressor and determine pressure ratio per stage.

Soln. : Refer Fig. P. 7.11.1(a) and (b)

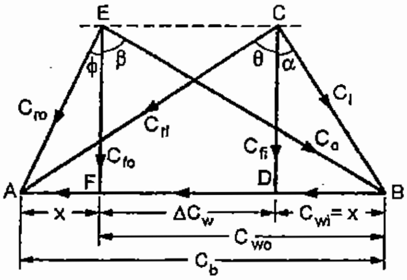


Fig. P. 7.11.1(a)

R_D = 50% i.e. $\alpha = \phi = 35°$ and $\theta = \beta = 52°$

T_1 = 15.5°C = 288.5 K, η_i = 80% = 0.8

C_b = 220 m/s, $C_{fi} = C_{fo} = C_f$

$$\Delta C_w = C_{wo} - C_{wi} = C_{fo} \tan\beta - C_{fi} \tan\alpha$$

$$\Delta C_w = C_f (\tan 52 - \tan 35) = 0.57973\, C_f \qquad ...(1)$$

$$C_b = \Delta C_w + 2x = \Delta C_w + 2 \times C_f \tan\alpha$$

$$= \Delta C_w + 2\, C_f \tan 35$$

$$220 = 0.57973\, C_f$$

$$C_b = AD + DB = C_f \tan\theta + C_f \tan\alpha$$

$$220 = C_f (\tan 52 + \tan 35)$$

$$C_f = 111.1 \text{ m/s}$$

$$\therefore \Delta C_w = 0.57973\, C_f = 0.57973 \times 111.1$$

$$= 64.41 \text{ m/s}$$

$$W = (\Delta C_w)\, C_b = C_p \cdot (T_{02} - T_{01})$$

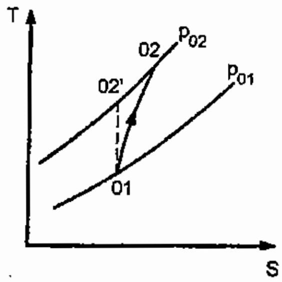


Fig. P. 7.11.1(b)

$$64.41 \times 220 = 1005\,(T_{02} - 288.5)$$

$$T_{02} = 302.6 \text{ K}$$

$$\therefore \eta_i = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$0.8 = \frac{T'_{02} - 288.5}{302.6 - 288.5}$$

$$T'_{02} = 299.78 \text{ K}$$

Pressure ratio per stage, $p_r = \dfrac{p_{02}}{p_{01}}$

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{(\gamma-1)}{\gamma}}$$

$$p_r = \left(\frac{T'_{02}}{T_{01}}\right)^{\frac{\gamma}{(\gamma-1)}}$$

$$p_r = \left(\frac{299.78}{288.5}\right)^{\frac{1.4}{0.4}}$$

$$= 1.1437 \qquad \text{...Ans.}$$

Ex. 7.11.2 : Determine the compressor speed, absolute velocity of the air leaving the stationary inlet guide vane for an axial flow compressor having following specifications. The first stage has a velocity diagram which is symmetric the ratio of change of whirl velocity to axial velocity is 0.6 the first stage pressure ratio is 1.8, inlet pressure and temperature = 1.01 bar and 300 K respectively, flow coefficient is 0.4, compressor efficiency = 85% and the mean radius is 30 cm.

SPPU - May 16, 10 Marks

Soln. : Refer Fig. P.7.11.2 (a) and (b)

Change in whirl velocity,

$$\Delta C_W = C_{Wo} - C_{Wi} = 0.6\, C_f \qquad ...(i)$$

Where, $C_f = C_{fi} = C_{fo}$

$$R_p = \frac{p_{02}}{p_{01}} = 1.8$$

$$p_{01} = 1.01 \text{ bar}$$

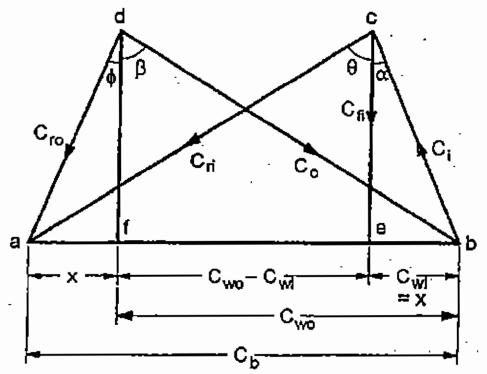
$$T_{01} = 300 \text{ K}$$

Flow coefficient, $\phi_1 = \dfrac{C_f}{C_b} = 0.4$ ...(ii)

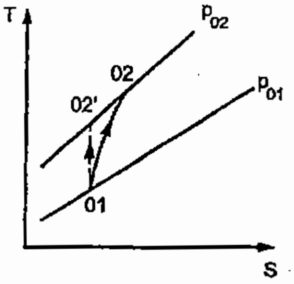
η_i = 0.85, Mean radius

R = 30 cm = 0.3 m

$\therefore D = 2R = 2 \times 0.3 = 0.6$ m



(a)



(b)
Fig. P.7.11.2

1. Compressor speed, N and Absolute velocity of air leaving stationary inlet guide vanes i.e. C_i

$$T'_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{(\gamma-1)/\gamma}$$

$$= 300\,(1.8)^{0.4/1.4} = 354.86 \text{ K}$$

$$\eta_i = \frac{T'_{02}-T_{01}}{T_{02}-T_{01}}$$

$$0.85 = \frac{354.86-300}{T_{02}-300}$$

$$T_{02} = 364.54 \text{ K}$$

Workdone / stage,

$$W = C_p\,(T_{02}-T_{01})$$

$$= 1.005\,(364.54-300)$$

$$= 64.863 \text{ kJ/kg of air}$$

But, $W = (C_{Wo}-C_{Wi})\,C_b$

$$64.863\times10^3 = 0.6\,C_f\times\frac{C_f}{0.4}$$

$$C_f = 207.95 \text{ m/s} = C_{fi} = C_{fo}$$

$$\therefore\ C_b = \frac{C_f}{0.4} \text{ (From equation (ii))}$$

$$= \frac{207.95}{0.4} = 519.87 \text{ m/s}$$

$$C_b = \frac{\pi DN}{60}$$

$$N = \frac{60\,C_b}{\pi D} = \frac{60\times519.87}{\pi\times0.6}$$

$$= \mathbf{16547.9\ rpm}$$ ...**Ans.**

From Fig. P.7.11.2:

$$C_b = (C_{wo}-C_{wi}) + 2x = 0.6\,C_f + 2x$$

$$519.87 = 0.6\times207.95 + 2x$$

$$x = 197.55 \text{ m/s}$$

From Δ ceb : $C_i = \sqrt{(C_{fi})^2+(x)^2}$

$$= \sqrt{(207.95)^2+(197.55)^2}$$

$$= \mathbf{286.83\ m/s}$$ ...**Ans.**

Ex. 7.11.3 : An axial flow compressor is designed for 50% reaction with inlet and outlet air angles for rotor blades as 80° and 45° respectively measured from axial direction. The mean blade speed is 200 m/s and the axial velocity of flow is constant throughout. Assuming a work factor of 0.88, find the number of stages required if the total pressure ratio is 4:1 with an isentropic efficiency of 85%. The stagnation inlet temperature may be taken as 290 K.

Assume, $\gamma = 1.4$, $R = 287$ Nm/kg K, $C_p = 1.005$ kJ/kg K.

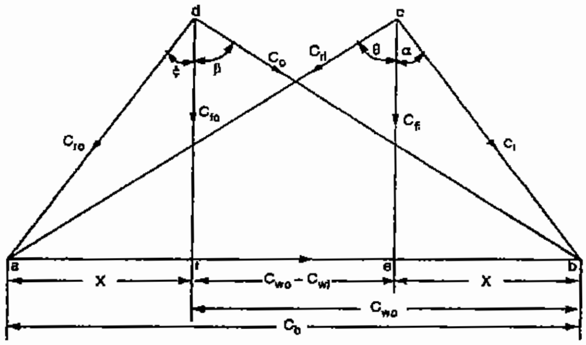


Fig. P. 7.11.3(a)

Soln. : Refer Fig. P. 7.11.3(a)

Given : Degree of reaction, R = 0.5 ;

$$\theta = \beta = 80° \text{ and } \quad \alpha = \phi = 45°;$$

Blade velocity,

$$C_b = 200 \text{ m/s}; \quad \psi_w = 0.88;$$

$$\frac{p_{02}}{p_{01}} = 4; \quad \eta_i = 85\%;$$

$$T_{01} = 290 \text{ K};$$

The velocity diagram can be drawn as shown in Fig. P. 7.11.3(a).

Let, af = eb = x

From triangles cbe and bdf,

$$C_{fi} = x\cot\alpha$$

$$C_{fo} = df = (C_b - x)\cot\beta$$

Since, axial velocity of flow are equal i.e. $C_{fi} = C_{fo}$, it follows :

$$x\cot\alpha = (C_b - x)\cot\beta$$

$$x\cot 45 = (200-x)\cot 80$$

On solving, $x = 29.98$ m/s

$$\Sigma C_w = C_{wo} - C_{wi} = (C_b - x) - x = C_b - 2x$$

$$= 200 - 2\times29.98 = 140.04 \text{ m/s}$$

Theoretical work done per kg of air,

$$W' = C_b \cdot \Sigma C_w = 200 \times 140.04$$

$$= 28008 \text{ Nm/kg}$$

Actual work done = Theoretical work done × Work factor

$$W = 28008 \times 0.88 = 24647 \text{ Nm/kg}$$

Isentropic efficiency,

$$\eta_i = \frac{\text{Isentropic work}}{\text{Actual work}}$$

$$\therefore \quad 0.85 = \frac{\text{Isentropic work}}{24647}$$

Isentropic work W_i = 20950 Nm/kg

Let the pressure ratio per stage be p_r.

$$W_i = \left(\frac{\gamma}{\gamma-1}\right) R \cdot T_{01} \left[(p_r)^{(\gamma-1)/\gamma} - 1\right]$$

$$20950 = \left(\frac{1.4}{1.4-1}\right) 287 \times 290 \left[(p_r)^{(1.4-1)/1.4} - 1\right]$$

$$\therefore \quad p_r = 1.2752 \text{ per stage}$$

Let the number of stages required be 'm', it follows that,

$$(p_r)^m = 4; \quad (1.2752)^m = 4$$

Taking log on both sides and on solving,

Number of stages, m = **5.7 say 6 stages.** ...Ans.

Refer Fig. P. 7.11.3(b),

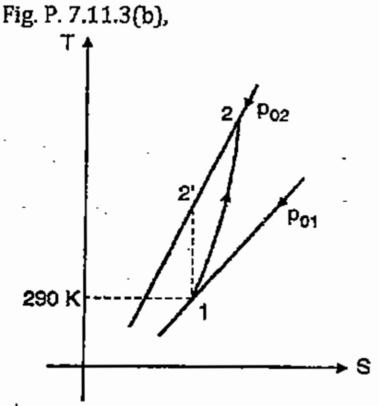


Fig. P. 7.11.3(b)

$$T'_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} = 290\,(4)^{(1.4-1)/1.4}$$

$$= 430.94 \text{ K}$$

Isentropic efficiency,

$$\eta_i = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$0.85 = \frac{430.94 - 290}{T_{02} - 290} \quad \therefore T_{02} = 455.8 \text{ K}$$

Polytropic efficiency,

$$\eta_p = \left(\frac{\gamma-1}{\gamma}\right) \times \frac{\log\left(\frac{p_{02}}{p_{01}}\right)}{\log\left(\frac{T_{02}}{T_{01}}\right)}$$

$$= \frac{(1.4-1)}{1.4} \times \frac{\log(4)}{\log(455.8/290)}$$

$$= \mathbf{0.876 \text{ or } 88.6\%}$$ ...Ans.

Referring to Fig. P. 7.11.3(a), from triangle cae we get,

$$\frac{C_b - x}{C_{ri}} = \sin\theta$$

$$\frac{200 - 29.98}{C_{ri}} = \sin 80$$

$$C_{ri} = 172.264 \text{ m/s}$$

$$C_i = \frac{x}{\sin 45} = \frac{29.98}{\sin 45} = 42.4 \text{ m/s}$$

Static temperature, $T_1 = T_{01} - \frac{C_i^2}{2\,C_p}$

$$= 290 - \frac{(42.4)^2}{2 \times (1.005 \times 1000)}$$

$$= 289.1 \text{ K}$$

Sonic velocity, $a = \sqrt{\gamma R T_1}$

$$= \sqrt{1.4 \times 287 \times 289.1} = 340.82 \text{ m/s}$$

∴ Mach number at inlet,

$$M = \frac{C_{ri}}{a} = \frac{172.64}{340.82} = \mathbf{0.507}$$ ...Ans.

Ex. 7.11.4 : An axial flow compressor has an overall pressure ratio of 4 and mass flow rate of 3 kg/s. If the polytropic efficiency is 88% and the stagnation temperature rise per stage must not exceed 25 K, calculate the number of stages required and pressure ratio of first and last stages. Assume equal temperature rise in all stages. If the absolute velocity approaching the last rotor is 165m/s at an angle of 20° from the axial direction, the work done factor is 0.83, the velocity diagram is symmetrical and the mean diameter of last stage rotor is 18cm calculate the rotational speed and length of the last stage rotor blade inlet to the stage. Ambient conditions are 1.01 bar 288K.

Soln. :

Refer Fig. P. 7.11.4.

Given : $r_p = 4 = \frac{p_{02}}{p_{01}}$; $\dot{m}_a$ = 3 kg/s;

η_p = 0.88$(\Delta T)_0$ per stage = 25 K

C_i = 165 m/s; α = 20°

ψ_w = 0.83; d_m = 18 cm = 0.18 m

p_1 = 1.01 bar; T_{01} = 288 K

Workdone / stage,

$$w = C_p \cdot (\Delta T)_0 = 1.005 \times 25 = 25.125 \text{ kJ/kg}$$

Isentropic efficiency,

$$\eta_i = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}} = \frac{T_{01}\left(\frac{T_{02}'}{T_{01}} - 1\right)}{T_{01}\left(\frac{T_{02}}{T_{01}} - 1\right)}$$

$$\therefore \quad \eta_i = \frac{\left[(p_{02}/p_{01})^{\frac{(\gamma-1)}{\gamma}} - 1\right]}{\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{(\gamma-1)}{\gamma} \times \frac{1}{\eta_p}} - 1\right]}$$

$$= \frac{[(4)^{0.4/1.4} - 1]}{\left[(4)^{\frac{0.4}{1.4 \times 0.88}} - 1\right]} = 0.855$$

Isentropic work / stage,

$$w_i = w \times \eta_i$$

$$= 25.125 \times 0.855 = 21.482 \text{ kJ/kg}$$

$$w_i = C_p (T_2' - T_1)$$

$$21.482 = 1.005 (T_2' - 288)$$

$$T_2' = 309.4 \text{ K.}$$

Pressure ratio / stage,

$$r_{ps} = \frac{p_2}{p_1} = \left(\frac{T_2'}{T_1}\right)^{\gamma/(\gamma-1)} = \left(\frac{309.4}{288}\right)^{1.4/0.4} = 1.285$$

1. Number of stages required, m

$$(r_{ps})^m = r_p; \quad (1.285)^m = 4$$

$$m = \frac{\log 4}{\log 1.285} = 5.53 \text{ (say 6 stages)} \qquad \text{...Ans.}$$

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \frac{p_7}{p_6} = 1.285$$

2. Rotational speed, N

Refer Fig. P. 7.11.4

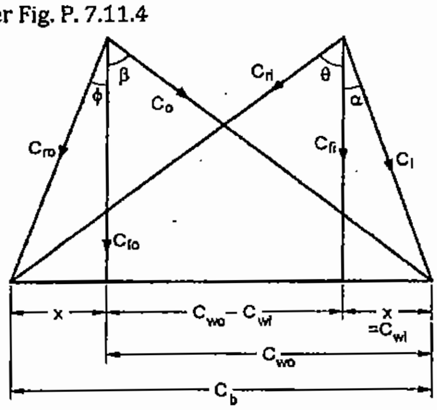


Fig. P. 7.11.4

W.D./stage transferred = work factor x

$$w = 0.83 \times 25.125 = 20.854 \text{ kJ/kg}$$

$$x = C_i \sin \alpha = 165 \sin 20$$

$$= 56.43 \text{ m/s}$$

$$\text{W.D/ stage} = (C_{wo} - C_{wi})\, C_b$$

$$20.854 \times 10^3 = (C_b - 2x)\, C_b$$

$$20854 = (C_b - 2 \times 56.43)\, C_b$$

$$C_b^2 - 112.86\, C_b - 20854 = 0$$

$$C_b = \frac{112.86 \pm \sqrt{(112.86)^2 + 4 \times 20854}}{2}$$

$$= \frac{112.86 \pm 310.84}{2} = 211.50 \text{ m/s}$$

($\because$ C_b cannot have negative value)

$$C_b = \frac{\pi dm\, N}{60}$$

$$211.5 = \frac{\pi \times 0.18 \times N}{60}$$

$$N = \mathbf{22438.9 \text{ rpm}} \qquad \text{...Ans.}$$

3. Length of blade

$$C_{fi} = C_i \cos \alpha = 165 \cos 20 = 155 \text{ m/s}$$

Density of air, $\rho = \frac{p_1}{RT_1} = \frac{1.01 \times 10^5}{287 \times 288} = 1.2219 \text{ kg/m}^3$

Let h be the length of blade,

$$\dot{m} = \rho \cdot A \cdot C_{fi} = \rho\, (\pi \cdot d_m \cdot h)\, C_{fi}$$

$$3 = 1.2219 \times \pi \times 0.18 \times h \times 155$$

$$h = \mathbf{0.028 \text{ m}} \qquad \text{...Ans.}$$

Ex. 7.11.5 : An eight stage axial flow compressor takes in air at a temperature of 30°C at the rate of 3 kg/s. The pressure ratio is 6 and isentropic efficiency is 89%. The compressor is designed for 50% reaction. The blade speed for each stage is constant and is equal to 180 m/s, flow velocity is 100 m/s. Find the power required to run the compressor and the direction of air at entry and exit from the rotor and stator. The total work is equally shared between the stages.

SPPU - May 15,10 Marks

Soln. : Refer Fig. P. 7.11.5(a) and (b).

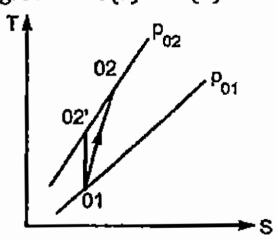


Fig. P. 7.11.5(a)

No. of stages, $y = 8,$

$$T_1 = 30°C = 303\text{ K}, \quad \dot{m} = 3\text{ kg/s}$$

Total pressure ratio,

$$(p_r)_t = 6, \quad \eta_i = 0.89$$

$$\alpha = \phi,$$

$$\theta = \beta \quad (\because \text{50\% reaction compressor})$$

$$C_b = 180\text{ m/s}, \quad C_{fi} = C_{fo} = 100\text{ m/s}$$

Total work is equally shares

(i) Power required, P

Isentropic workdone / kg of air,

$$W_i = \left(\frac{\gamma}{\gamma-1}\right) R\,T_{01}\left[(p_r)_t^{(\gamma-1)/\gamma} - 1\right]$$

$$= \frac{1.4}{0.4} \times 287 \times 303\,[(6)^{0.4/1.4} - 1]$$

$$= 203470.2\text{ Nm/kg}$$

$$\eta_i = \frac{\text{Isentropic work, } W_i}{\text{Actual work, } W_a}$$

$$W_a = \frac{W_i}{\eta_i} = \frac{203470.2}{0.89}$$

$$= 228618.2\text{ J/kg}$$

$$P = \dot{m} \times W_a = 3 \times \frac{228618.2}{1000}\text{ kW}$$

$$= \mathbf{685.85\ kW} \qquad \text{...Ans.}$$

(ii) Blade angles, θ and φ

$$\text{Workdone / stage, } W = \frac{\text{Actual workdone}}{\text{No. of stages}}$$

$$= \frac{228618.2}{8} = 28578.3\text{ Nm/kg}$$

Let $AF = EB = x$

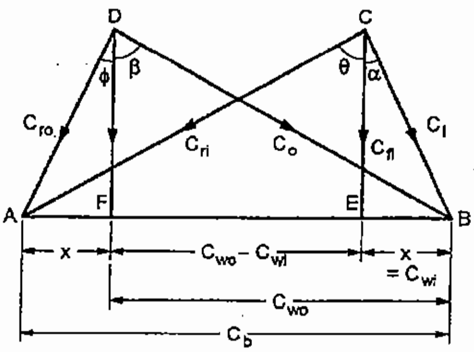


Fig. P. 7.11.5(b)

From ΔCBE and ΔBDF we have,

$$C_{fi} = x \cot\alpha$$

$$100 = x\cot\alpha \qquad \text{...(i)}$$

$$DF = C_{fo} = (C_b - x)\cot\beta$$

$$100 = (180 - x)\cot\beta \qquad \text{...(ii)}$$

$$\text{Also, } W = (C_{wo} - C_{wi})\,C_b = (C_b - 2x)\,C_b$$

$$\therefore\ 28578.3 = (180 - 2x)\,180$$

$$x = 10.62\text{ m/s}$$

On substituting the values of x in Equations (i) and (ii) we get,

$$100 = 10.62\cot\alpha$$

$$\alpha = \mathbf{6.062° = \phi} \qquad \text{...Ans.}$$

$$100 = (180 - x)\cot\beta$$

$$100 = (180 - 10.62)\cot\beta$$

$$\beta = \mathbf{59.44° = \theta} \qquad \text{...Ans.}$$

Ex. 7.11.6 : An axial flow compressor with eight stages and 50% reaction compresses air with a pressure ratio of 4 : 1. The air enters the compressor at 20°C and flows through it with a constant velocity of 90 m/s. The blades of compressor runs with a mean speed of 180 m/s. Take isentropic efficiency = 82%. Find :

(i) Work done by machine

(ii) Blade angles. **SPPU - May 15, 8 Marks**

Soln. : Refer Fig. P. 7.11.5(a) and (b).

No. of stages, $y = 8,$

$$\alpha = \phi,$$

$$\theta = \beta \quad (\because \text{50\% reaction})$$

$$p_r = 4, \quad T_{01} = 20°C = 273 + 20 = 293\text{ K}$$

$$C_{fi} = C_{fo} = 90\text{ m/s},$$

$$C_b = 180\text{ m/s}, \quad \eta_i = 0.82$$

(i) Workdone by machine

Isentropic workdone / kg of air,

$$T'_{02} = T_{01}\,(p_r)^{(\gamma-1)/\gamma}$$

$$= 293(4)^{0.4/1.4} = 435.4\text{ K}$$

$$\eta_i = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$0.82 = \frac{435.4 - 293}{T_{02} - 293}$$

$$T_{02} = 466.7\text{ K}$$

$$\text{Total work input, } W_a = C_p\,(T_{02} - T_{01})$$

$$= 1.005\,(466.7 - 293)$$

$$= \mathbf{174.57\ kJ/kg} \qquad \text{...Ans.}$$

(ii) Blade angles

Work input / stage,

$$W = \frac{W_a}{y}$$

$$= \frac{174.57}{8} = 21.821 \text{ kJ/kg}$$

$$W = (C_{wo} - C_{wi})\, C_b$$

$$21.821 \times 10^3 = (C_b - 2x)\, C_b$$

$$21821 = (180 - 2x)\, 170$$

$$x = 29.39 \text{ m/s}$$

From Δ CBE,

$$C_{fi} = x \cot \alpha$$

$$100 = 29.39 \cot \alpha$$

$$\alpha = \phi = \mathbf{16.38°} \qquad \text{...Ans.}$$

From Δ BDF,

$$C_{fo} = (C_b - x) \cot \beta$$

$$100 = (180 - 29.39) \cot \beta$$

$$\beta = \theta = \mathbf{56.42°} \qquad \text{...Ans.}$$

Ex. 7.11.7 : An axial flow compressor is required to deliver air at the rate of 50 kg/s and provide a total pressure ratio of 5 : 1, the inlet stagnation conditions being 288 K and 1 bar. The isentropic efficiency is 86 %. The compressor shall have 10 stages with equal rise in total temperature in each stage. The axial velocity of flow is 150 m/s and the blade speed is kept at 200 m/s to minimize noise generation. The stage degree of reaction at mean blade height is 50 %. Assuming workdone factor as 0.86, calculate all the fluid angles of the first stage. Also calculate the tip and hub diameter if hub-tip diameter ratio is 0.8. Determine the speed in rpm. (R = 287 J/kg K, C_p = 1.005 kJ/kg K) **SPPU - Dec. 15, 10 Marks**

Soln. :

$$\dot{m} = 50 \text{ kg/s}, \qquad r_p = 5 = \frac{p_{02}}{p_{01}},$$

$$p_{01} = 1 \text{ bar}, \qquad T_{01} = 288 \text{ K}$$

$$\therefore \quad p_{02} = 5\, p_{01} = 5 \times 1 = 5 \text{ bar},$$

$$\eta_i = 0.86, \qquad \text{number of stages, } y = 10$$

$$C_f = C_{fi} = C_{fo} = 150 \text{ m/s},$$

$$C_b = 200 \text{ m/s}, \qquad R_D = 50\ \%$$

$$\text{i.e. } \alpha = \phi \text{ and } \theta = \beta,$$

Wordone factor, $\psi_w = 0.8$

$$\frac{\text{Hub diameter, } D_h}{\text{Tip diameter, } D_t} = 0.8$$

$$R = 287 \text{ J/kg K},$$

$$C_p = 1.005 \text{ kJ/kg K}$$

Refer Fig. P. 7.11.7(a)

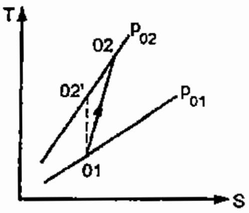


Fig. P. 7.11.7(a)

$$T'_{02} = T_{01} \left(\frac{p_{02}}{p_{01}}\right)^{(\gamma - 1)/\gamma}$$

$$= 288\, (5)^{0.4/1.4} = 456.14 \text{ K}$$

$$\eta_i = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$0.86 = \frac{456.14 - 288}{T_{o2} - 288}$$

$$T_{02} = 483.51 \text{ K}$$

Total theoretical compressor work per kg of air

$$W_T = C_p\, (T_{02} - T_{01})$$

$$= 1.005\, (483.51 - 288)$$

$$= 196.49 \text{ kJ/kg of air}$$

Actual workdone stage,

$$W'_T = \frac{W'_T}{\text{No. of stages}} = \frac{159.19 \times 10^3}{10}$$

$$= 15.719 \text{ Nm/kg}$$

Refer Fig. P. 7.11.7(b) on which combined velocity diagram is shown.

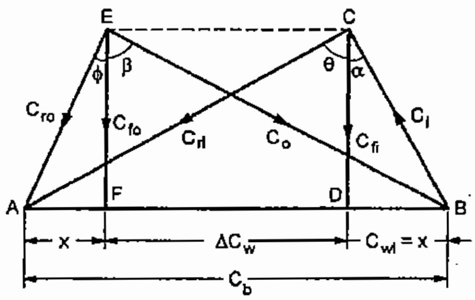


Fig. P. 7.11.7(b)

1. All fluid angles i.e. α, θ, φ, β

$$\dot{W} = \Delta C_W \times C_b$$

$$15719 = \Delta C_W \times 200$$

$$\Delta C_W = 78.6 \text{ m/s}$$

But $\Delta C_W = C_b - 2x$

$$78.6 = 200 - 2x \; ; \; x = 60.7 \text{ m/s}$$

$$\tan\alpha = \frac{x}{C_{fi}} = \frac{60.7}{150} = 0.4057$$

$$\alpha = 22.03° = \phi$$

$$AD = C_b - x = 200 - 60.7 = 139.3 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{AD}{CD}\right) = \tan^{-1}\left(\frac{AD}{C_{fi}}\right)$$

$$= \tan^{-1}\left(\frac{139.3}{150}\right) = \mathbf{42.88° = \beta}$$

$$\therefore \alpha = \phi = \mathbf{22.03°}$$

and $\theta = \beta = 42.88°$ ...Ans.

2. Tip diameter, D_t and hip diameter, D_n

$$p_{01} = \rho R T_{01}$$

$$1 \times 10^5 = \rho \times 287 \times 288$$

$$\rho = 1.20983 \text{ kg/m}^3$$

$$\dot{m} = \rho \times \frac{\pi}{4}\left(D_t^2 - D_h^2\right) C_f \; ;$$

$$50 = 1.20983 \times \frac{\pi}{4}\left[D_t^2 - (0.8D_t)^2\right] 150$$

$$D_t = 0.987 \text{ m}$$

and $D_n = 0.8 \times 0.987 = \mathbf{0.7896 \text{ m}}$ **...Ans.**

3. Speed in rpm, N

Mean diameter of rotor,

$$D = \frac{D_t + D_h}{2} = \frac{0.987 + 0.7896}{2} = 0.8883 \text{ m}$$

$$C_b = \frac{\pi D N}{60} \; ;$$

$$200 = \frac{\pi \times 0.8883 \times N}{60}$$

$$N = \mathbf{4300 \text{ rpm}}$$ **...Ans.**

Ex. 7.11.8 : In an eight stage axial flow compressor, the overall stagnation pressure ratio achieved is 5 : 1 with an overall isentropic efficiency of 92%. The inlet stagnation temperature and pressure are 290 K and 1 bar. The work is divided equally between the stages. The mean blade speed is 160 m/s and 50% reaction design is used. The axial velocity through the compressor is constant and is equal to 90 m/s. Calculate the blade angle and the power required.

SPPU - Dec.12,10 Marks

Soln. : Refer Fig. P. 7.11.8(a) and (b)

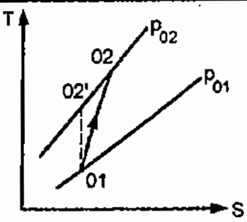


Fig. P. 7.11.8(a)

Number of stages, $m = 8$; $\frac{p_{02}}{p_{01}} = 5$

$\eta_i = 0.92$, $T_{01} = 290$ K,

$p_{01} = 1$ bar

$C_b = 160$ m/s ; 50% degree of reaction

i.e. $\alpha = \phi$, $\theta = \beta$, $C_{fi} = C_{fo} = C_f = 90$ m/s

(i) Power, required, P

$$T'_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} = 290(5)^{0.4/1.4} = 459.3 \text{ K}$$

$$\eta_i = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}} \; ; \quad 0.92 = \frac{459.3 - 290}{T_{02} - 290}$$

$$T_{02} = 474 \text{ K}$$

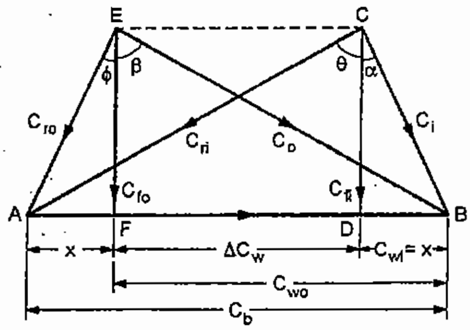


Fig. P. 7.11.8(b)

Total compressor work per kg of air,

$$W_T = C_p(T_{02} - T_{01}) = 1.005(474 - 290)$$

$$= 184.92 \text{ kJ/kg} = 184920 \text{ Nm/kg}$$

$$\therefore P = \dot{m} \times W_T = 1 \times 184.92$$

$$= \mathbf{184.92 \text{ kW/kg/s}}$$ **...Ans.**

$\because$ Since work is shared equally in all stages,

work input / stage,

$$W = \frac{W_T}{\text{No. of stages}} = \frac{184920}{8} = 23115 \text{ Nm/kg}$$

But, $W = \Delta C_W \times C_b$

$$23115 = \Delta C_W \times 160$$

$$\Delta C_W = 144.47 \text{ m/s}$$

From Fig. P 7.11.8(b)

$$\Delta C_W = C_b - 2x$$

$$x = \frac{C_b - \Delta C_W}{2} = \frac{160 - 144.47}{2} = 7.766 \text{ m/s}$$

Blade angles :

$$\alpha = \tan^{-1}\left(\frac{x}{C_{fi}}\right) = \tan^{-1}\left(\frac{7.766}{90}\right) = \mathbf{4.932^\circ} \quad \text{...Ans.}$$

$$\theta = \tan^{-1}\left(\frac{C_b - x}{C_{fi}}\right) = \tan^{-1}\left(\frac{160 - 7.766}{90}\right) = \mathbf{59.41^\circ} \quad \text{...Ans.}$$

Ex. 7.11.9 : An axial compressor stage has following data :

Temperature and pressure at entry 300K, 1 bar

Degree of reaction 50%

Mean blade ring diameter 36 cm

Rotational speed 18000 r.p.m.

Blade height at entry 6 cm

Air angles at rotor and stator exit 25°

Axial velocity 180 m/s

Work done factor 0.88

Stage efficiency 85%

Mechanical efficiency 96.7%

Determine :

i) Air angles at the rotor and stator entry

ii) The mass flow rate of air

iii) The power required to drive the compressor

iv) The loading coefficient

v) The pressure ratio developed the stage

vi) The Mach number at the rotor entry

SPPU- May 13,10 Marks

Soln. : Refer Fig. P. 7.11.9(a).

$$p_1 = 1 \text{ bar}, T_1 = 300 \text{ K},$$

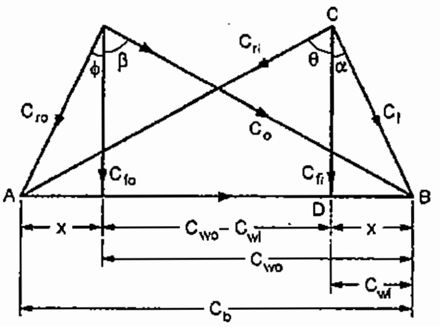


Fig. P. 7.11.9(a)

Degree of reaction,

$$R_D = 0.5, \text{ therefore,}$$

$$\alpha = \phi, \qquad \theta = \beta$$

$$d_m = 36 \text{ cm} = 0.36 \text{ m}, \quad N = 18000 \text{ rpm}$$

$$h_1 = 6 \text{ cm} = 0.06\text{m}; \quad \alpha = \phi = 25^\circ$$

$$C_{fi} = C_{fo} = 180 \text{ m/s}$$

work done factor, $\Psi_w = 0.88$

Stage efficiency, $\eta_s = 0.85$

Mechanical efficiency, $\eta_m = 0.967$

$$C_b = \frac{\pi \, dm \, N}{60} = \frac{\pi \times 0.36 \times 18000}{60} = 339.29 \text{ m/s}$$

(i) Air angles at the rotor entry, ϕ and at stator entry, β

From Δ CBD :

$$x = C_{fi} \tan \alpha = 180 \tan 25 = 83.94 \text{ m/s}$$

$$AD = C_b - x = 339.29 - 83.94 = 255.35 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{AD}{CD}\right) = \tan^{-1}\left(\frac{255.35}{180}\right) = \mathbf{54.819^\circ} \quad \text{...Ans.}$$

$$\beta = \theta = \mathbf{54.819^\circ \text{ (50\% reaction)}} \quad \text{...Ans.}$$

(ii) Mass flow rate of air, $\dot{m}$

Density of inlet, $\rho_1 = \frac{p_1}{RT_1} = \frac{1 \times 10^5}{287 \times 300} = 1.1614 \text{ kg/m}^3$

$$\dot{m} = \rho_1 (\pi \, d_m \cdot h_1) \, C_{fi}$$

$$= 1.1614 \times \pi \times 0.36 \times 0.06 \times 180$$

$$= \mathbf{14.186 \text{ kg/s}} \quad \text{...Ans.}$$

(iii) Power required to drive the compressor, P_s

Workdone / kg of air,

$$w = \Psi_w (C_{wo} - C_{wi}) \, C_b = \Psi_w (C_b - 2x) \, C_b$$

$$= 0.88 \times \frac{(339.29 - 2 \times 83.94) 339.29}{1000}$$

$$= 51.179 \text{ kJ/kg}$$

$$P_s = \frac{\dot{m} \cdot w}{\eta_{mech}} = \frac{14.186 \times 51.179}{0.967}$$

$$= \mathbf{750.8 \text{ kW}} \quad \text{...Ans.}$$

(iv) Loading coefficient, Ψ

$$\Psi = \frac{w}{C_b^2} = \frac{51.179 \times 10^3}{(339.29)^2} = \mathbf{0.4446} \quad \text{...Ans.}$$

(v) Pressure ratio developed the stage, p_r

(Refer Fig. P. 7.11.9(b))

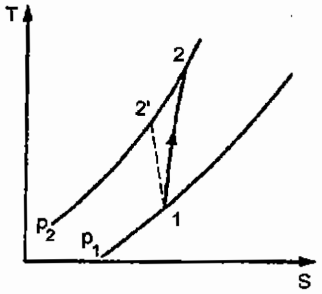


Fig. P. 7.11.9(b)

$$w = C_p (T_2 - T_1);$$

$$(T_2 - T_1) = \frac{51.179}{1.005} = 50.924 \text{ K}$$

$$\eta_s = \frac{T_2' - T_1}{T_2 - T_1}; \quad 0.85 = \frac{T_2' - T_1}{50.924}$$

$$T_2' - T_1 = 43.286 \text{ K}$$

i.e. $$T_2' = 300 + 43.286$$

$$= 343.286 \text{ K}$$

$$p_r = \frac{p_2}{p_1} = \left(\frac{T_2'}{T_1}\right)^{\gamma/(\gamma-1)}$$

$$= \left(\frac{343.286}{300}\right)^{1.4/0.4} = \mathbf{1.6028}$$

(vi) Mach number at rotor entry, M_i From Δ ACD,

$$C_{ri} = \frac{AD}{\sin\theta} = \frac{C_b - x}{\sin\theta}$$

$$= \frac{339.29 - 83.94}{\sin 54.819} = 312.417 \text{ m/s}$$

Sonic velocity,

$$a = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 300} = 348.189 \text{ m/s}$$

$$M_i = \frac{C_{ri}}{a} = \frac{312.417}{347.189}$$

$$= \mathbf{0.8998}$$ **...Ans.**

Ex. 7.11.10 : Derive the following relation for efficiencies and degree of reaction of axial compressor $\eta_{st} = R\,\eta_R + (1 - R)\,\eta_D$. Calculate the value of the stage efficiency of 50% reaction compressor stage with the following efficiencies of the blade rows : $\eta_R = 0.849$ and $\eta_D = 0.849$.

SPPU - May 13, 8 Marks

Soln. : Refer Fig.P.7.11.10.

Stage efficiency,

$$\eta_{st} = \frac{h_3' - h_1}{h_3 - h_1} = \frac{(T_3' - T_1)}{(T_3 - T_1)} \quad \text{...(i)}$$

Degree of reaction,

$$R = \frac{(\Delta h)_R}{(\Delta h)_R + (\Delta h)_D} = \frac{(h_2 - h_1)}{(h_2 - h_1) + (h_3 - h_2)} \quad \text{...(ii)}$$

$$R = \frac{h_2 - h_1}{h_3 - h_1} = \frac{T_2 - T_1}{T_3 - T_1} \quad \text{...(iii)}$$

$$\eta_R = \frac{h_2' - h_1}{h_2 - h_1} = \frac{T_2' - T_1}{T_2 - T_1} \quad \text{...(iv)}$$

$$\eta_D = \frac{h_3'' - h_2}{h_3 - h_2} \equiv \frac{h_3' - h_2'}{h_3 - h_2} = \frac{T_3' - T_2'}{T_3 - T_2}$$

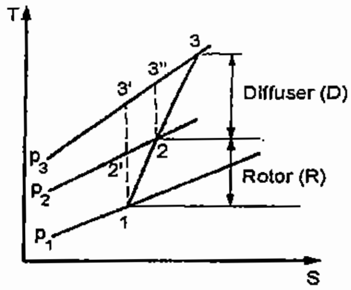


Fig. P. 7.11.10

Rewriting for stage efficiency,

$$\eta_{st} = \frac{T_3' - T_1}{T_3 - T_1} = \frac{T_3' - T_1 + T_2' - T_2'}{(T_3 - T_1)}$$

$$= \frac{(T_3' - T_2') + (T_2' - T_1)}{(T_3 - T_1)} \quad \text{...(v)}$$

$$\eta_{st} = \frac{(T_2' - T_1)}{(T_3 - T_1)} + \frac{(T_3' - T_2')}{(T_3 - T_1)}$$

$$= \frac{(T_2' - T_1)}{(T_2 - T_1)} \times \frac{(T_2 - T_1)}{(T_3 - T_1)} + \frac{T_3' - T_2'}{T_3 - T_1}$$

On substituting the values from Equations (iii) and (iv)

$$\eta_{st} = \eta_R \times R + \frac{T_3' - T_2'}{(T_3 - T_1)} \quad \text{...(vi)}$$

$$\frac{T_3' - T_2'}{T_3 - T_1} = \frac{(T_3' - T_2')}{T_3 - T_1} \times \frac{(T_3 - T_2)}{(T_3 - T_2)}$$

$$= \left(\frac{T_3 - T_2}{T_3 - T_1}\right) \times \frac{(T_3' - T_2')}{(T_3 - T_2)} = \left(\frac{T_3 - T_2}{T_3 - T_1}\right) \times \eta_D$$

$$= \left[\frac{T_3 - T_2}{T_3 - T_1} + 1 - 1\right] \times \eta_D = \left[1 + \frac{T_3 - T_2}{T_3 - T_1} - 1\right] \eta_D$$

$$= \left[1 + \frac{(T_3 - T_2 - T_3 + T_1)}{(T_3 - T_1)}\right] \eta_D$$

$$= \left[1 + \frac{(-T_2 + T_1)}{(T_3 - T_1)}\right]\eta_D$$

$$= \left[1 - \frac{(T_2 - T_1)}{(T_3 - T_1)}\right]\eta_D = [1 - R]\,\eta_D \qquad \text{...(vii)}$$

On substituting the value from Equation (vii) in equation (vi) we get,

$$\eta_{st} = \eta_R \times R + (1 - R)\,\eta_D \qquad \text{...Proved}$$

Ex. 7.11.11 : The ambient conditions at inlet are 20°C and 1 bar. At exit the total head temperature and pressure are 150°C and 3.5 bar and static pressure at exit is 3 bar. Calculate :

(i) Isentropic efficiency (ii) Polytropic efficiency

(iii) Air velocity at exit **SPPU-Dec.16, 8 Marks**

Soln. : Refer Fig. P. 7.11.11

Given : Ambient temperature, $T_{01} = 20°C = 293$ K

Ambient pressure, $p_{01} = 1$ bar

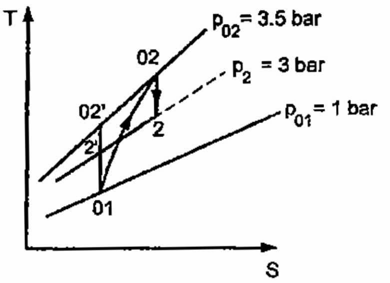


Fig. P. 7.11.11

Stagnation temperature at exit,

$$T_{02} = 150 + 273 = 423 \text{ K}$$

Stagnation pressure at exit, p_{02} = 3.5 bar

Static pressure at exit p_2 = 3 bar

$$T_{02}' = T_{01}\left[\frac{p_{02}}{p_{01}}\right]^{\frac{\gamma-1}{\gamma}} = 293\left[\left(\frac{3.5}{1}\right)\right]^{04/1.4}$$

$$T_{02}' = 419.1 \text{ K}$$

(a) Isentropic efficiency (η_i)

$$\eta_i = \left[\frac{(T_{02}' - T_{01})}{(T_{02} - T_{01})}\right]$$

$$= \frac{419.1 - 293}{423 - 293} = \mathbf{0.97 \text{ or } 97\%} \qquad \text{...Ans.}$$

(b) Polytropic efficiency (η_p)

$$\eta_p = \frac{\frac{(\gamma-1)}{\gamma} \ln\left(\frac{p_{02}}{p_{01}}\right)}{\ln\left(\frac{T_{02}}{T_{01}}\right)} = \frac{\frac{0.4}{1.4}\ln(3.5)}{\ln\left(\frac{423}{293}\right)}$$

$$= \mathbf{0.9748 \text{ or } 97.48\%} \qquad \text{...Ans.}$$

(c) Air velocity at exit, C_2

Apply Isentropic process between (2 – 02),

$$T_2 = T_{02} \times \left[\frac{p_2}{p_{02}}\right]^{(\gamma-1)/\gamma}$$

$$= 423\left[\frac{3}{3.5}\right]^{0.4/1.4}$$

$$= 404.77 \text{ K}$$

$$\therefore T_{02} = T_2 + \frac{C_2^2}{2\,C_p}$$

$$423 = 404.77 + \frac{C_2^2}{2 \times (1.005 \times 1000)}$$

$$C_2 = \mathbf{191.4 \text{ m/s}} \qquad \text{...Ans.}$$

Ex. 7.11.12 : The speed of an axial flow compressor is 15000 rpm. The mean diameter is 0.6 m. The axial velocity is constant and is 225 m/s. The velocity of whirl at inlet is 85 m/s. The work done is 45 kJ/kg of air. The inlet conditions are 1 bar and 300 K. Assume a stage efficiency of 0.89, mechanical efficiency of 0.95 and power developed is 425 kW. Calculate :

(i) Fluid deflection angle (ii) Pressure ratio

(iii) Mass flow rate (iv) Shaft power **SPPU - Dec. 16, 10 Marks**

Soln. : Given :

Speed,	N	= 15,000 rpm ;
Mean diameter,	d_m	= 0.6 m
Axial velocity,	C_{fi}	= C_{fo} = 225 m/s
Whirl velocity at inlet,	C_{wi}	= 85 m/s
Work done,	W	= 45 kJ/kg
Inlet temperature	T_1	= 300 K
Inlet pressure	p_1	= 1 bar
Stage efficiency	η_s	= 0.89

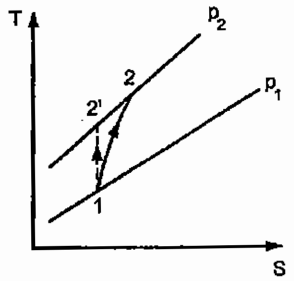


Fig. P. 7.11.12(a)

C_b = mean blade velocity

$$C_b = \frac{\pi\, d_m N}{60} = \frac{\pi \times 0.6 \times 15000}{60}$$

$$C_b = 471.24 \text{ m/s}$$

Workdone/kg, $$W = C_p(T_2 - T_1) \quad \text{...(i)}$$

$$45 = 1.005 \times (T_2 - 300)$$

$$\therefore \; T_2 = \mathbf{344.78\ K}$$

(a) Pressure ratio, $R_p = \frac{p_2}{p_1}$

$$\eta_s = \frac{(T_2' - T_1)}{(T_2 - T_1)}$$

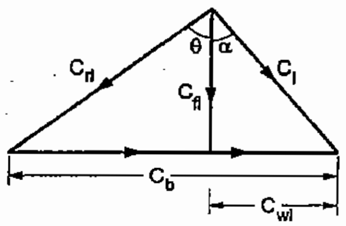
$$\therefore \; 0.89 = \frac{(T_2' - 300)}{(344.78 - 300)}$$

$$\therefore \; T_2' = 339.85 \text{ K}$$

$$R_p = \frac{p_2}{p_1} = \left[\frac{T_2'}{T_1}\right]^{\frac{\gamma}{(\gamma-1)}}$$

$$= \left[\frac{339.85}{300}\right]^{\frac{1.4}{0.4}}$$

$$R_p = 1.547 \quad \text{...Ans.}$$



Inlet triangle

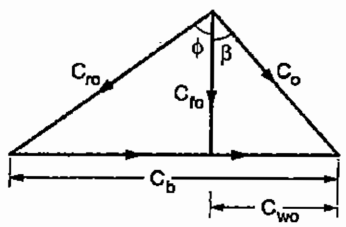


Fig. P. 7.11.12

(b)

(b) Fluid deflection angle $(\beta - \alpha)$

$$W = \frac{(C_{wo} - C_{wl})\, C_b}{1000}$$

$$45 = \frac{(C_{wo} - 85)\, 471.24}{1000}$$

$$C_{wo} = 180.49 \text{ m/s}$$

From inlet velocity triangle,

$$\tan\alpha = \frac{C_{wi}}{C_{fi}} = \frac{85}{225}$$

$$\alpha = \mathbf{20.7^\circ}$$

From exit velocity triangle

$$\beta = \tan^{-1}\left(\frac{C_{wo}}{C_{fo}}\right) = \tan^{-1}\left(\frac{180.49}{225}\right) = 38.74^\circ$$

$$\beta - \alpha = 38.74 - 20.7 = \mathbf{18.04^\circ} \quad \text{...Ans.}$$

(c) Degree of Reaction, R :

$$C_{rl} = \sqrt{(C_{fi})^2 + (C_b - C_{wi})^2}$$

$$= \sqrt{(225)^2 + (471.24 - 85)^2}$$

$$= \mathbf{447\ m/s} \quad \text{...Ans.}$$

$$C_{ro} = \sqrt{(C_{fo})^2 + (C_b - C_{wo})^2}$$

$$= \sqrt{(225)^2 + (471.24 - 180.49)^2}$$

$$= \mathbf{367.6\ m/s} \quad \text{...Ans.}$$

$$R = \frac{\left(C_{rl}^2 - C_{ro}^2\right)}{2} \times \frac{1}{(C_{wo} - C_{wi})\, C_b}$$

$$= \frac{(447)^2 - (367.6)^2}{2} \times \frac{1}{(180.49 - 85)\, 471.24}$$

$$R = \mathbf{0.7187} \quad \text{...Ans.}$$

(d) Mass flow rate $\dot{m}$

$$P = \dot{m} \times \text{W.D.} / \text{kg}$$

$$425 \text{ kW} = \dot{m}\,(\text{kg/s}) \times 45 \text{ kJ/kg}$$

$$\therefore \; \dot{m} = 9.444 \text{ kg/s} \quad \text{...Ans.}$$

(e) Shaft power (P_s)

$$\eta_m = \frac{P}{P_s}$$

$$P_s = \frac{P}{\eta_m}$$

$$= \frac{425}{0.95}$$

$$= \mathbf{447.37\ kW} \quad \text{...Ans.}$$

Summary

- **An axial flow compressor** is similar in appearance to an axial flow turbine.
- **The main components of axial flow compressors are :**
  1. Rotor
  2. Casing
  3. Moving blades mounted on rotor.
  4. Fixed blades mounted on casing.
- Space between rotor and casing is called **stator.**

- In an **axial flow compressor** the dynamic energy imparted to gas by rotor blades is converted into K. E. which is then converted into pressure energy in stator blades by diffusion process carried out in diverging blade passes.

  Pressure rise/stage is 1.1 to 1.25. It uses large number of stages to develop required pressure ratio.

- **In an axial flow compressor**

  (i) **Blades angles** α, θ, ϕ and β are measured from axial direction.

  (ii) **Work done / stage** = $(C_{wo} - C_{wi})\, C_b$

  (iii) **Temperature rise,**

  $$\Delta T_0 = T_{02} - T_{01} = \frac{(C_{wo} - C_{wi})}{C_p} C_b$$

  (iv) **Degree of reaction,**

  $$R = \frac{\text{Pressure rise in rotor blades}}{\text{Pressure rise in stage}}$$

  For 50% degree of reaction : $\alpha = \phi$, $\theta = \beta$

  (v) **Work input factor,** ψ_w is defined as the ratio of actual work done on the air to theoretical work done.

  $\therefore$ Actual work done on air $= (C_{wo} - C_{wi})\, C_b \times \psi_w$

  (vi) **Polytropic efficiency,**

  $$\eta_p = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right) = \frac{\log(p_{02}/p_{01})}{\log(T_{02}/T_{01})} \times \left(\frac{\gamma-1}{\gamma}\right)$$

- **According radial equilibrium (vortex theory)**

  $$\frac{1}{\rho} \cdot \frac{dp}{dr} = \frac{C_w^2}{r}$$

- **Energy equation for vortex flow**

  $$\frac{dh_0}{dr} = \frac{C_w^2}{r} + C_f \cdot \frac{dC_f}{dr} + C_w \cdot \frac{dC_w}{dr}$$

- **For free vortex flow,** the assumptions are :

  1. Equal work input at all radii, $\frac{dh_0}{dr} = 0$
  2. Constant axial velocity of flow, $\frac{dC_f}{C_f} = 0$

  Therefore, the free vortex equation becomes,

  $$C_w \propto \frac{1}{r}$$

- Problem of boundary layer separation in axial flow compressors can be reduced by use of **aerofoil blades.**

- An **aerofoil** is defined as a streamlined form bounded by two flattened curves whose length and width are very large compared to its thickness.

  Coefficient of lift, $C_L = \dfrac{L}{\frac{1}{2}\rho \cdot c \cdot C_m^2}$

  Coefficient of drag, $C_D = \dfrac{D}{\frac{1}{2}\rho \cdot c \cdot C_m^2}$

- **Various losses in axial flow compressor** are skin friction losses, profile losses and secondary flow losses.

- **Performance characteristic curves** of the compressor are drawn with non-dimensional parameters as :

  (a) Pressure ratio, (p_{02}/p_{01})

  (b) Speed parameter, $(N/\sqrt{T_{01}})$

  (c) Mass flow parameter, $(m\sqrt{T_{01}}/p_{01})$

- **Materials used for rotor and blades** are titanium, steel and nickel alloys of chromium and aluminium. Additional elements added are cobalt, molybdenum for strength and to withstand high temperatures.

Exercise

Q. 1 Why axial flow compressors are used for jet aircraft applications ? **[Section 7.1]**

Q. 2 Why centrifugal compressors are not suitable for aircraft applications ? **[Section 7.1]**

Q. 3 State the function of rotor and stator blades in an axial flow compressors. **[Section 7.2]**

Q. 4 State the difference between turbine and compressor blades. **[Section 7.2.1]**

Q. 5 Explain the construction and working of an axial flow compressors. **[Section 7.2]** **(May 11)**

Q. 6 Draw the inlet and exit velocity diagrams for axial flow compressor and write the expression for work input. **[Section 7.3]**

Q. 7 Explain the processes involved in an axial flow compressor with the help of (T – S) diagram. **[Section 7.3.1]**

Q. 8 Define degree of reaction as applied to axial flow compressors and show that the blades are symmetrical for 50% reaction. **[Section 7.3.2 and 7.3.3]**

Q. 9 Define work input factor and explain why the actual work transferred to gas is less than theoretical work input ? How does the axial velocity varies with blade height ? **[Section 7.3.4]**

Q. 10 Define polytropic efficiency of an axial flow compressor. **[Section 7.4]**

Q. 11 Define polytropic efficiency of a compressor with the help of (T – S) diagram and obtain an expression for polytropic efficiency. **[Section 7.4]**

Q. 12 Define : flow coefficient and rotor blade loading coefficient. **[Section 7.5]**

Q. 13 Write the equation in case of free vortex for radial equilibrium condition. **[Sections 7.6.1, 7.6.2, 7.6.3, 7.6.4]**

Q. 14 State the reasons for using aerofoil blading for axial flow compressors. **[Section 7.7]**

Q. 15 Explain why aerofoil blading is needed in axial flow compressors ? Define angle of attack and its effect on static pressure distribution on blades. **[Section 7.8.2]**

Q. 16 Define coefficient of lift and coefficient of drag. **[Section 7.7.3]**

Q. 17 What do you understand by stalling of compressors ? **[Section 7.7.4]**

Q. 18 State the various losses in axial flow compressors. **[Section 7.8]**

Q. 19 Discuss various performance characteristic curves of an axial flow compressors.**[Section 7.9]**

Q. 20 Compare centrifugal and axial flow compressor. **[Section 7.10]**

Q. 21 State any three materials used for compressor blades and the material used for its rotor. **[Section 7.11]**

□□□

APPENDIX – A

Appendix A 1

Table-1 : Properties of Saturated Water and Steam (temperature) Tables

Saturation Temp. t_s (°C)	Absolute Pressure (p) bar	Specific Volume (m^3/kg)		Specific Enthalpy (kJ/kg)			Specific Entropy (kJ/kgK)		
		Water v_f	Steam v_g	Water h_f	Evaporation h_{fg}	Steam h_g	Water S_f	Evaporation S_{fg}	Steam S_g
0.00	0.00611	0.001000	206.31	0.00	2501.0	2501.0	0.000	9.156	9.156
0.01	0.00611	0.001000	206.16	0.04	2501.5	2501.5	0.000	9.157	9.157
5.00	0.00872	0.001000	147.20	21.0	2489.6	2510.6	0.076	8.951	9.027
10.00	0.01227	0.001000	106.42	42.0	2477.8	2519.8	0.151	8.751	8.902
15.00	0.01704	0.001001	77.98	62.9	2466.0	2528.9	0.224	8.559	8.783
20.00	0.02337	0.001002	57.84	83.9	2454.2	2538.1	0.296	8.372	8.668
25.00	0.03166	0.001003	43.40	104.8	2442.4	2547.2	0.367	8.192	8.559
30.00	0.04242	0.001004	32.93	125.7	2430.6	2556.3	0.437	8.018	3.455
35.00	0.05622	0.001006	25.25	146.6	2418.7	2565.3	0.505	7.849	8.354
40.00	0.07375	0.001008	19.55	167.5	2408.8	2574.3	0.572	7.686	8.258
45.00	0.09582	0.001010	15.28	188.4	2394.8	2583.2	0.638	7.528	8.166
50.00	0.12335	0.001012	12.05	209.3	2382.8	2592.1	0.704	7.374	8.078
55.00	0.15740	0.001015	9.58	230.2	2370.7	2600.9	0.768	7.225	7.993
60.00	0.19920	0.001017	7.679	251.1	2358.5	2609.6	0.831	7.080	7.911
65.00	0.25009	0.001020	6.202	272.0	2346.2	2618.2	0.893	6.939	7.832
70.00	0.31162	0.001023	5.046	293.0	2333.9	2626.9	0.955	6.802	7.757
75.00	0.38549	0.001026	4.134	313.9	2321.4	2635.3	1.015	6.568	7.683
80.00	0.47360	0.001029	3.409	334.9	2308.8	2643.7	1.075	6.538	7.613
85.00	0.57803	0.001033	2.829	355.9	2296.0	2651.9	1.134	6.411	7.545
90.00	0.70109	0.001036	2.361	376.9	2283.1	2660.0	1.193	6.287	7.480
95.00	0.84526	0.001040	1.982	398.0	2270.1	2668.1	1.250	6.167	7.417
100.00	1.01330	0.001043	1.673	419.0	2257.0	2676.0	1.307	6.048	7.355
105.00	1.20800	0.001047	1.418	440.0	2244.1	2684.1	1.363	5.934	7.297
110.00	1.43270	0.001052	1.210	461.1	2229.9	2691.0	1.418	5.820	7.238
115.00	1.69050	0.001057	1.036	481.9	2216.2	2698.1	1.473	5.710	7.183
120.00	1.98510	0.001061	0.891	504.1	2201.8	2705.9	1.527	5.602	7.129

Appendix A 2

Table-2 : Properties of Saturated Water and Steam (pressure) Tables

Absolute Pressure P (bar)	Saturation Temp. t (°C)	Specific Volume (m³/kg)		Specific Enthalpy (kJ/kg)			Specific Entropy (kJ/kgK)		
		Water v_f	Steam v_g	Water h_f	Evaporation h_{fg}	Steam h_g	Water S_f	Evaporation S_{fg}	Steam S_g
0.01	6.98	0.001000	129.21	29.3	2484.9	2514.2	0.106	8.871	8.977
0.02	17.03	0.001001	67.00	73.5	2460.0	2533.5	0.261	8.463	8.724
0.03	24.08	0.001003	45.67	101.1	2444.4	2545.6	0.355	8.223	8.578
0.04	28.96	0.001004	34.80	121.5	2432.9	2554.4	0.423	8.052	8.475
0.05	32.88	0.001005	28.19	137.8	2423.7	2561.5	0.476	7.919	8.395
0.06	36.18	0.001006	23.74	151.5	2416.0	2567.5	0.521	7.810	8.331
0.07	39.03	0.001007	22.53	163.4	2409.2	2572.6	0.559	7.719	8.278
0.08	41.53	0.001008	18.11	173.9	2403.2	2577.1	0.593	7.637	8.230
0.09	43.79	0.001009	16.20	183.3	2397.9	2581.2	0.622	7.566	8.188
0.10	45.81	0.001010	14.67	191.8	2392.8	2584.6	0.649	7.501	8.150
0.15	53.97	0.001014	10.02	225.9	2373.1	2599.0	0.755	7.254	8.009
0.20	60.06	0.001017	7.65	251.4	2358.3	2609.7	0.832	7.077	7.909
0.25	64.97	0.001020	6.20	271.9	2346.3	2618.2	0.893	6.938	7.831
0.30	69.10	0.001022	5.23	289.2	2336.1	2625.3	0.944	6.825	7.769
0.40	75.87	0.001027	3.99	315.6	2319.2	2636.8	1.026	6.645	7.671
0.50	81.33	0.001030	3.24	340.5	2305.4	2645.9	1.091	6.503	7.594
0.60	85.95	0.001033	2.73	359.9	2293.6	2653.5	1.145	6.388	7.533
0.70	89.96	0.001036	2.37	376.8	2283.3	2660.1	1.192	6.288	7.480
0.80	93.51	0.001039	2.09	391.6	2274.1	2665.8	1.233	6.202	7.435
0.90	96.71	0.001041	1.87	405.2	2265.6	2670.8	1.269	6.126	7.395
1.0	99.63	0.001043	1.694	417.5	2258.0	2675.5	1.303	6.057	7.360
1.013	100.00	0.001043	1.673	419.0	2257.0	2676.0	1.307	6.048	7.355
1.2	104.81	0.001047	1.428	439.4	2244.1	2683.5	1.361	5.938	7.299
1.4	109.32	0.001051	1.236	458.4	2231.9	2690.3	1.411	5.836	7.247
1.6	113.37	0.001055	1.091	475.4	2220.9	2696.3	1.455	5.747	7.202
1.8	116.94	0.001058	0.977	490.7	2210.7	2701.4	1.494	5.668	7.162
2.0	120.23	0.001061	0.885	504.7	2201.6	2706.3	1.530	5.597	7.127
2.2	123.27	0.001064	0.810	517.6	2193.0	2710.6	1.563	5.532	7.095
2.4	126.10	0.001066	0.747	529.6	2184.9	2714.5	1.593	5.473	7.066
2.6	128.73	0.001069	0.693	540.9	2177.3	2718.2	1.621	5.418	7.039
2.8	131.20	0.001071	0.646	551.4	2170.1	2721.5	1.646	5.367	7.014
3.0	133.55	0.001074	0.606	561.4	2163.2	2724.7	1.672	5.319	6.991
3.2	135.75	0.001076	0.570	570.9	2156.7	2727.6	1.695	5.274	6.969
3.4	137.86	0.001078	0.538	579.9	2150.5	2730.4	1.717	5.232	6.949
3.6	139.84	0.001080	0.510	588.5	2144.4	2732.9	1.738	5.192	6.930
3.8	141.78	0.001082	0.485	596.8	2138.5	2735.3	1.757	5.155	6.912
4.0	143.61	0.001084	0.462	604.7	2132.9	2737.6	1.776	5.118	6.894
4.2	145.39	0.001086	0.442	612.3	2127.5	2739.8	1.795	5.083	6.878
4.4	147.09	0.001088	0.423	619.6	2122.3	2741.9	1.812	5.050	6.862

Absolute Pressure P (bar)	Saturation Temp. t_s (°C)	Specific Volume (m^3/kg)		Specific Enthalpy (kJ/kg)			Specific Entropy (kJ/kgK)		
		Water v_f	Steam v_g	Water h_f	Evaporation h_{fg}	Steam h_g	Water S_f	Evaporation S_{fg}	Steam S_g
4.6	148.74	0.001089	0.405	626.7	2117.1	2743.8	1.829	5.018	6.847
4.8	150.31	0.001091	0.389	633.5	2112.1	2745.6	1.845	4.988	6.833
5.0	151.84	0.001093	0.373	640.1	2107.4	2747.5	1.860	4.959	6.819
5.2	153.33	0.001095	0.361	646.5	2102.7	2749.2	1.876	4.930	6.806
5.4	154.76	0.001096	0.348	652.8	2098.0	2750.8	1.890	4.903	6.763
5.6	156.16	0.001098	0.337	658.8	2093.7	2752.5	1.904	4.877	6.781
5.8	157.52	0.001099	0.326	664.7	2089.7	2754.0	1.918	4.851	6.769
6.0	158.84	0.001100	0.316	670.4	2085.0	2755.4	1.931	4.827	6.758
6.5	161.99	0.001104	0.293	684.0	2074.8	2758.8	1.962	4.768	6.730
7.0	164.96	0.001108	0.273	697.0	2064.9	2761.9	1.992	4.713	6.705
7.5	167.66	0.001112	0.256	709.3	2059.3	2768.6	2.020	4.671	6.691
8.0	170.41	0.001115	0.240	720.9	2046.5	2767.4	2.046	4.614	6.660
8.5	172.95	0.001118	0.227	732.5	2037.9	2770.4	2.061	4.578	6.639
9.0	175.36	0.001121	0.215	742.6	2029.5	2772.1	2.094	4.525	6.619
9.5	177.69	0.001124	0.204	752.8	2021.4	2774.2	2.117	4.484	6.601
10.0	179.88	0.001127	0.1943	762.6	2013.6	2776.2	2.138	4.445	6.583
11.0	184.07	0.001133	0.1774	781.1	1998.5	2779.6	2.179	4.371	6.550
12.0	187.96	0.001139	0.1632	798.4	1984.3	2782.5	2.216	4.403	6.519
13.0	191.61	0.001144	0.1511	814.7	1970.7	2785.4	2.251	4.240	6.491
14.0	195.04	0.001149	0.1407	830.1	1957.7	2787.8	2.284	4.181	6.465
15.0	198.30	0.001154	0.1317	844.7	1945.2	2789.9	2.315	4.126	6.441
16.0	201.37	0.001159	0.1237	858.6	1933.2	2791.8	2.344	4.074	6.418
17.0	204.31	0.001163	0.1166	871.8	1921.5	2793.3	2.371	4.025	6.396
18.0	207.11	0.001168	0.1103	884.6	1910.3	2794.9	2.398	3.977	6.375
19.0	209.80	0.001172	0.1047	896.8	1899.3	2796.1	2.423	3.931	6.354
20.0	212.37	0.001177	0.09954	908.6	1888.6	2797.2	2.447	3.890	6.337
25.0	223.94	0.001197	0.07905	962.0	1839.0	2801.0	2.554	3.700	6.254
30.0	233.84	0.001216	0.06663	1008.4	1793.9	2802.3	2.664	3.538	6.184
35.0	242.54	0.001235	0.05703	1049.8	1752.2	2802.0	2.725	3.398	6.123
40.0	250.33	0.001252	0.04975	1087.4	1712.9	2800.3	2.797	3.272	6.069
45.0	257.41	0.001262	0.04404	1122.1	1675.6	2797.7	2.861	3.158	6.019
50.0	263.91	0.001286	0.03943	1154.5	1639.7	2794.2	2.921	3.053	5.974
60.0	275.55	0.001319	0.03244	1213.7	1571.3	2785.0	3.047	2.864	5.891
70.0	285.79	0.001351	0.02737	1267.4	1506.0	2773.5	3.122	2.694	5.816
80.0	294.97	0.001384	0.02353	1317.1	1442.8	2759.9	3.208	2.539	5.747
90.0	303.31	0.001418	0.02050	1363.7	1380.9	2744.6	3.287	2.395	5.682
100.0	310.96	0.001453	0.01804	1408.0	1319.7	2727.7	3.361	2.259	5.620
120.0	324.65	0.001527	0.01428	1491.8	1197.4	2689.2	3.497	2.003	5.500
140.0	336.64	0.001611	0.01150	1571.6	1070.7	2642.3	3.624	1.756	5.380
160.0	347.33	0.001710	0.009308	1650.5	934.3	2584.8	3.747	1.506	5.253
180.0	356.96	0.001840	0.007498	1734.8	779.1	2513.9	3.877	1.236	5.113
200.0	365.70	0.002037	0.005877	1826.5	591.9	2418.4	4.015	0.926	4.941
220.0	373.69	0.002671	0.003728	2011.1	184.5	2195.6	4.295	0.285	4.580
221.2	374.15	0.003170	0.003170	2107.4	000.0	2107.4	4.443	0.000	4.443

Appendix A 3

Table – 3 : Properties of Superheated Steam

(a) Specific Volume (v_g), m^3 / kg (b) Specific Enthalpy (h), kJ/kg (c) Specific Entropy (s), kJ / kg K

Pressure (p) bar	Sat. Temp. t_s (°C)	Properties	Sat. Steam v_g, h_g, s_g	Temperature (°C)								
				100	150	200	250	300	400	500	600	700
0.02	17.03	v	67.0	86.08	97.65	109.2	120.7	132.2	155.3	178.4	210.5	224.6
		h	2533.5	2688.5	2783.7	2888.0	2977.7	3076.8	3297.7	3489.2	3705.6	3928.8
		s	8.578	9.193	9.433	9.648	9.844	10.164	10.454	10.717	10.959	11.184
0.04	28.96	v	34.8	43.03	48.83	54.48	60.35	66.10	77.70	89.20	100.70	112.300
		h	2554.4	2688.3	2783.6	2879.9	2977.6	3076.8	3297.7	3489.2	3705.6	3928.800
		s	8.475	9.328	9.524	10.031	10.321	10.585	10.827	11.051	11.272	11.480
0.06	36.18	v	23.74	28.68	32.55	36.38	40.250	44.10	51.80	59.50	67.20	74.990
		h	2567.5	2688.0	2783.4	2879.8	2977.600	3076.7	3279.6	3489.2	3705.6	3928.800
		s	8.331	8.685	8.925	9.141	9.337	9.518	9.844	10.134	10.397	10.639
0.08	41.53	v	18.11	21.50	24.42	27.30	30.18	33.10	38.80	44.60	50.40	56.100
		h	2577.1	2687.8	2783.2	2879.7	2977.5	3076.6	3279.6	3489.1	3705.5	3928.700
		s	8.23	8.552	8.792	9.008	9.204	9.385	9.711	10.001	10.265	10.507
0.10	45.81	v	14.67	17.20	19.53	21.82	24.15	26.50	31.10	35.70	40.30	44.900
		h	2584.6	2687.5	2783.1	2879.6	2977.4	3076.5	3279.6	3489.1	3705.5	3928.700
		s	8.150	8.449	8.689	8.905	9.101	9.282	9.608	9.894	10.162	10.404
0.15	53.90	v	10.02	11.51	13.06	14.61	16.17	17.71	20.80	23.89	26.99	30.050
		h	2599.0	2686.9	2782.4	2879.5	2977.3	3076.5	3279.5	3489.0	3705.5	3928.600
		s	8.009	8.261	8.502	8.718	8.915	9.095	9.421	9.711	9.974	10.216
0.20	60.06	v	7.65	8.59	9.75	10.91	12.08	13.22	15.53	17.84	20.15	22.450
		h	2609.7	2686.3	2782.3	2879.2	2977.1	3076.4	3279.4	3489.0	3675.4	3928.700
		s	7.909	8.126	8.368	8.584	8.781	8.962	9.288	9.578	9.842	10.084
0.25	64.97	v	6.20	6.87	7.81	8.74	9.67	10.59	12.44	14.29	16.14	17.990
		h	2618.2	2685.7	2782.0	2879.0	2977.0	3076.3	3279.2	3489.0	3675.4	3928.700
		s	7.831	8.022	8.262	8.481	8.678	8.859	9.186	9.476	9.739	9.981
0.30	69.10	v	5.23	5.71	6.49	7.27	8.04	8.81	10.35	10.89	13.43	14.970
		h	2625.3	2685.1	2781.6	2878.7	2976.8	3076.1	3279.3	3488.9	3705.4	3928.700
		s	7.769	7.936	8.179	8.396	8.593	8.774	9.101	9.391	9.654	9.897
0.40	75.87	v	3.99	4.28	4.87	5.45	6.03	6.60	7.76	8.92	10.07	11.230
		h	2636.8	2683.8	2780.9	2878.2	2976.5	3075.9	3279.1	3488.8	3705.3	3928.600
		s	7.671	7.801	8.045	8.263	8.460	8.641	8.969	9.258	9.522	9.764
0.50	81.33	v	3.24	3.42	3.89	4.36	4.82	5.28	6.21	7.13	8.06	8.980
		h	2645.9	2682.6	2780.1	2877.6	2976.1	3075.7	3279.0	3488.7	3705.2	3928.600
		s	7.594	7.695	7.941	8.159	8.356	8.538	8.865	9.155	9.419	9.661
0.60	85.85	v	2.73	2.84	3.24	3.63	4.02	4.40	5.17	5.94	6.11	7.480
		h	2653.5	2681.3	2779.4	2877.3	2975.8	3075.4	3278.8	3488.6	3705.1	3928.500
		s	7.533	7.609	7.855	8.074	8.272	8.453	8.781	9.071	9.334	9.576
0.70	89.96	v	2.37	2.43	2.77	3.11	3.44	3.77	4.43	5.10	5.76	6.42
		h	2660.1	2680.0	2778.6	2876.8	2975.6	3075.2	3278.6	3488.5	3705.0	3928.4
		s	7.480	7.535	7.783	8.002	8.201	8.382	8.709	9.001	9.263	9.505
0.80	93.51	v	2.090	2.13	2.43	2.72	3.01	3.30	3.88	4.46	5.03	5.16
		h	2665.8	2678.8	2777.8	2876.2	2975.2	3074.0	3278.5	3488.4	3705.0	3928.4
		s	7.435	7.471	7.720	7.940	8.138	8.320	8.648	8.938	9.201	9.444

Pressure (p) bar	Sat. Temp. t_s (°C)	Properties	Steam v_g, h_g, s_g	Temperature t(°C) 100	150	200	250	300	400	500	600	700
1.00	96.71	v	1.694	1.696	1.936	2.172	2.406	2.639	3.103	3.565	4.028	4.490
		h	2670.8	2676.2	2776.3	2875.4	2974.5	3074.5	3278.2	3488.1	3704.8	3928.2
		s	7.360	7.362	7.614	7.835	8.034	8.217	8.544	8.835	9.098	9.341
1.50	111.35	v	1.164	...	1.285	1.444	1.601	1.757	2.067	2.376	2.685	2.993
		h	2693.3	...	2772.5	2872.9	2972.9	3073.3	3277.6	3487.6	3704.4	3927.9
		s	7.225	...	7.419	7.644	7.845	8.028	8.356	8.647	8.911	9.153
2.00	120.23	v	0.885	...	0.960	1.080	1.199	1.316	1.549	1.781	2.013	2.244
		h	2706.3	...	2768.5	2870.5	2971.2	3072.1	3276.7	3487.0	3704.1	3927.6
		s	7.127	...	7.279	7.507	7.710	7.894	8.223	8.514	8.777	9.020
3.00	133.55	V	0.606	...	0.634	0.716	0.796	0.875	1.031	1.187	1.341	1.496
		h	2724.7	...	2760.4	2865.5	2967.9	3069.7	3275.2	3486.0	3703.2	3927.0
		s	6.991	...	7.077	7.312	7.518	7.703	8.034	8.326	8.590	8.832
4.00	143.61	v	0.462	...	0.471	0.534	0.595	0.655	0.773	0.889	1.005	1.121
		h	2737.6	...	2752.0	2860.4	2964.5	3067.2	3273.6	3484.9	3702.3	3926.4
		s	6.894	...	6.929	7.171	7.380	7.568	7.799	8.192	8.456	8.699
5.00	151.84	v	0.375	...	...	0.425	0.474	0.523	0.617	0.711	0.804	0.897
		h	2747.5	...	...	2855.1	2961.1	3064.8	3272.1	3483.8	3701.5	3925.8
		s	6.819	...	...	7.059	7.272	7.461	7.795	8.088	8.353	8.596
6.00	158.84	v	0.316	...	...	0.352	0.394	0.434	0.514	0.592	0.670	0.747
		h	2755.4	...	...	2849.7	2957.6	3962.3	3270.6	3482.7	3700.7	3925.1
		s	6.758	...	...	6.966	7.183	7.374	7.709	8.003	8.268	8.511
7.00	164.96	v	0.273	...	...	0.300	0.336	0.371	0.440	0.507	0.574	0.640
		h	2761.9	...	...	2844.2	2954.0	3059.8	3269.0	3481.6	3699.9	3924.5
		s	6.705	...	...	6.886	7.107	7.300	7.636	7.931	8.196	8.440
8.00	170.41	v	0.240	...	...	0.261	0.293	0.324	0.384	0.443	0.502	0.560
		h	2767.4	...	...	2838.6	2950.4	3057.3	3267.5	3480.5	3699.1	3923.9
		s	6.660	...	...	6.815	7.040	7.235	7.573	7.868	8.134	8.377
9.00	175.36	v	0.2150	...	...	0.230	0.260	0.287	0.341	0.394	0.446	0.498
		h	2772.1	...	...	2838.7	2946.8	3054.7	3266.0	3479.4	3698.2	3923.3
		s	6.619	...	...	6.751	6.980	7.177	7.517	7.812	8.079	8.323
10.00	179.88	v	0.1943	...	...	0.2059	0.2328	0.2580	0.3065	0.3540	0.4010	0.4943
		h	2776.2	...	...	2826.8	2943.000	3052.1	3264.3	3678.4	3697.4	3922.7
		s	6.583	...	...	6.692	6.926	7.125	7.763	8.029	8.029	8.273
15.00	198.30	v	0.1317	...	...	0.1324	0.1520	0.1697	0.2029	0.2350	0.2666	0.2980
		h	2789.9	...	...	2794.7	2923.5	3038.9	3256.6	3472.8	3693.3	3919.6
		s	6.441	...	...	6.451	6.710	6.921	7.271	7.570	7.838	8.084
20.00	212.37	v	0.09954	...	...	...	0.1115	0.1255	0.1511	0.1756	0.1995	0.2232
		h	2797.2	...	...	...	2902.4	3025.0	3248.7	3467.4	3689.2	3916.5
		s	6.337	...	...	...	6.545	6.770	7.130	7.432	7.702	7.947
25.00	223.94	v	0.07905	...	...	...	0.0872	0.991	0.1202	0.1401	0.1594	0.1786
		h	2801.0	...	...	...	2880.5	3010.4	3240.7	3461.6	3685.1	3913.3
		s	6.254	...	...	...	6.408	6.648	7.018	7.324	7.594	7.844
30.00	233.84	v	0.0663	...	...	...	0.07055	0.08116	0.09931	0.11611	0.13232	0.14830
		h	2802.3	...	...	...	2854.8	2995.1	3232.5	3456.2	3681.0	3910.3
		s	6.069	...	...	...	6.286	6.542	6.925	7.235	7.508	7.756

Pressure (p) bar	Sat. Temp. t_s (°C)	Properties	Steam	Temperature (°C)								
			v_g, h_g, s_g	100	150	200	250	300	400	500	600	700
40.00	250.33	v	0.04975	...	...	...	...	0.05883	0.07338	0.08634	0.09876	0.11091
		h	2800.3	...	...	...	...	2962.0	3215.7	3445.0	3672.8	3904.1
		s	6.069	...	...	...	...	6.346	6.773	7.091	7.619	7619
50.00	263.91	v	0.03943	...	...	...	...	0.4530	0.05780	0.06849	0.07862	0.08845
		h	2794.2	...	...	...	...	2925.5	3198.3	3433.7	3644.5	3897.9
		s	5.891	...	...	...	...	6.211	6.651	6.977	7.258	7.511
60.00	275.55	v	0.03244	...	...	...	...	0.03615	0.04738	0.05659	0.06518	0.07348
		h	2745.0	...	...	...	...	2885.0	3180.1	3422.2	3656.2	3891.7
		s	5.891	...	...	...	...	6.069	6.546	6.882	7.166	7.422
70.00	285.79	v	0.02737	...	...	...	...	0.02946	0.03992	0.04809	0.05560	0.06279
		h	2773.5	...	...	...	...	2839.0	3161.2	3410.6	3647.9	3885.40
		s	5.816	...	...	...	...	5.933	6.454	6.799	7.088	7.346
80.0	294.97	v	0.02353	...	...	...	...	0.02426	0.03431	0.04170	0.04840	0.05477
		h	2759.9	...	...	...	...	2786.8	3141.6	3398.8	3639.5	3879.2
		s	5.747	...	...	...	...	5.794	6.369	6.726	7.019	7.279
90.00	303.31	v	0.02050	...	...	...	...	...	0.02993	0.03674	0.0428	0.04853
		h	2744.6	...	...	...	...	...	3121.2	3386.8	3631.1	3873.0
		s	5.628	...	...	...	...	...	6.292	6.660	6.957	7.220
100.00	310.96	v	0.01804	...	...	...	...	...	0.02641	0.3276	0.03831	0.04355
		h	2727.7	...	...	...	...	...	3099.9	3374.6	3622.7	3866.7
		s	5.620	...	...	...	...	...	6.218	6.599	6.901	7.166
120.00	324.65	v	0.01426	...	...	...	...	...	0.02108	0.02679	0.03160	0.03607
		h	2689.2	...	...	...	...	...	3054.8	3349.5	3605.7	3854.3
		s	5.550	...	...	...	...	...	6.081	6.491	6.802	7.072
140.00	336.64	v	0.01150	...	...	...	...	...	0.01723	0.02250	0.02680	0.03072
		h	2642.3	...	...	...	...	...	3005.6	3323.8	3588.5	3841.7
		s	5.380	...	...	...	...	...	5.951	6.394	6.716	6.991
160.00	347.33	v	0.009308	...	...	...	...	...	0.01428	0.01930	0.02320	0.02672
		h	2584.8	...	...	...	...	...	2951.4	3297.1	3571.0	3829.1
		s	5.253	...	...	...	...	...	5.824	6.305	6.639	6.919
180.00	356.96	v	0.007498	...	...	...	...	...	0.01190	0.01679	0.02040	0.02361
		h	2513.9	...	...	...	...	...	2890.3	3269.6	3553.5	3816.5
		s	5.113	...	...	...	...	...	5.695	6.223	6.569	6.854
200.00	365.70	v	0.005877	...	...	...	...	...	0.00995	0.01477	0.01816	0.02111
		h	2418.4	...	...	...	...	...	2820.5	3241.1	3535.5	3830.8
		s	4.941	...	...	...	...	...	5.559	6.146	6.504	6.795
220.00	373.69	v	0.003728	...	...	...	...	...	0.00825	0.01312	0.01633	0.01907
		h	2195.6	...	...	...	...	...	2738.8	3211.7	3517.4	3791.1
		s	4.580	...	...	...	...	...	5.410	6.072	6.445	6.741
221.00	374.15	v	0.003170	...	...	...	...	...	0.00816	0.01303	0.01622	0.01895
		h	2107.4	...	...	...	...	...	2734.6	3210.7	3516.4	3789.1
		s	4.443	...	...	...	...	...	5.399	6.068	6.441	6.738

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