Formulaes-:

(I) Frequency $F = \frac{\omega_n}{2\pi}$

(II) Natural Frequency $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K_t}{I}}$

stiffness (K) | mass (m) | Inertia (I)

$I = mK^2$

(III) Damped frequency $\omega_d = \frac{2\pi}{t_d} = \omega_n\sqrt{1-\xi^2}$

Time ($t_d$)

(IV) Logarithmic decrement ($\delta$)

$$\delta = \frac{1}{n}\ln\left(\frac{\beta_1}{\beta_{n+1}}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

No. of cycle ($n$) | ratio of amplitude

(V) Damping factor ($\xi$) $= \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{c}{c_c}$

(VI) Damping coeff. ($c$) $= \xi = \frac{c}{2I\omega_n}$

C = damping coefficient | $C_c$ = Critical damping coeff

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Unit :- 3

Q.1 May-Jun 2023

Q.1] a] Given :- $m = 5kg$

$K = 980 N/m$

$\mu = 0.25$

To find :- (i) frequency, f

(ii) number of cycle, n

(iii) time, t

$x_0 = 5cm = 0.05m$

$x_n = 0.5 x_0$

$\boxed{x_n = 0.025m}$

Soln :- (1) Frequency $f = \frac{\omega_n}{2\pi}$

$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{980}{5}} = 14 \ rad/sec$

$\therefore \boxed{f = 2.228 Hz}$

(2) $n\Delta = x_0 - x_n$

$\Delta = \frac{4 F_r}{K} = \frac{4 \times (\mu m g)}{K}$

$= \frac{4 \times (0.25 \times 5 \times 9.81)}{980}$

$\boxed{\Delta = 0.05}$

$\therefore n = \frac{x_0 - x_n}{\Delta}$

$= \cancel{0.025} \ \frac{0.05 - 0.025}{0.05}$

$\boxed{n = 0.5 \text{ cycles}}$

(3) $t = \frac{n \times 2\pi}{\omega_n} = \frac{0.5 \times 2\pi}{14}$

$\boxed{n = 0.2244 \ Sec}$

Q.2

Q.2] b Given :- $m = 10 Kg$ $K = 0.3 m$

$K_t = 5 Nm/rad$ $n = 2$

$\frac{\beta_0}{\beta_2} = 100$

1. Logarithmic decrement

$$\delta = \frac{1}{n} \ln\left(\frac{\beta_0}{\beta_2}\right)$$

$$= \frac{1}{2} \ln(100)$$

$$\boxed{\delta = 2.3025}$$

(2) Mass moment of inertia

$$I = mK^2$$

$$= 10 \times 0.3^2$$

$$\boxed{I = 0.9 \ Kg\text{-}m^2}$$

(3) Natural frequency

$$\omega_n = \sqrt{\frac{K_t}{I}} = \sqrt{\frac{5}{0.9}}$$

$$\boxed{\omega_n = 2.357 \ rad/s}$$

(4) Damping factor

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

$$= \frac{2.3025}{\sqrt{4\pi^2 + 2.3025^2}}$$

$$\boxed{\xi = 0.344}$$

(5) Damping coefficient

$$\xi = \frac{C_t}{2 I \omega_n}$$

$$0.344 = \frac{C_t}{2 \times 0.4 \times 2.357}$$

$\cancel{C_t = 1.4595}$ $\boxed{C_t = 1.4595 \text{ N-m-sec/rad}}$

(c) Periodic time Damped frequency:

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$
$$= 2.357\sqrt{1 - 0.344^2}$$

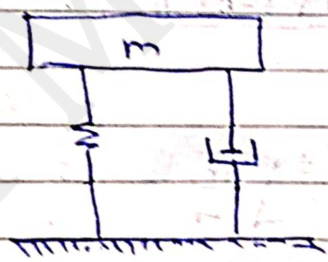
$$\boxed{\omega_d = 2.2131 \text{ rad/s}}$$

(d) Periodic time

$$t = \frac{2\pi}{\omega_d}$$
$$= \frac{2\pi}{2.2131}$$
$$\boxed{t = 2.84 \text{ sec}}$$

Q.3]

Q.1] c]




Given:

$m = 3\,kg$ $K = 3000\,N/m$ $t_d = 2\,sec$

$n = 1$

$$\beta_2 = \frac{\beta_0}{2}$$

$$\boxed{\frac{\beta_0}{\beta_2} = 2}$$

(1) Logarithmic decrement

$$\delta = \frac{1}{n} \ln\left(\frac{\beta_0}{\beta_2}\right)$$
$$= \ln(2)$$
$$\boxed{\delta = 0.6931}$$

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(II) Damping factor

$$\varepsilon = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$$

$$= \frac{0.6931}{\sqrt{4\pi^2+0.6931}}$$

$$\boxed{\varepsilon = 0.1046}$$

(III) Natural frequency

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3000}{3}}$$

$$\boxed{\omega_n = 31.62 \text{ rad/sec}}$$

(IV) For damping coefficient

$$\varepsilon = \frac{C}{C_c}$$

$$\varepsilon = \frac{C}{2m \times \omega_n}$$

$$0.1046 = \frac{C}{2 \times 3 \times 31.622}$$

$$\boxed{C = 20.745 \text{ N-sec/m}}$$

— x — x —

May-Jun 2024

Q.3]
Q.2 a] Given $m = 5$ kg, $K = 1460$ N/m

$V = 1$ m/s, $C = \frac{1.46}{1} = 1.46$ N-sec/m

$n = 5$

(I) Natural frequency

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1460}{5}}$$

$$\boxed{\omega_n = 19.8 \text{ rad/s}}$$

(ii) Critical dumping coeff.

$$C_c = 2 \times m \times \omega_n$$

$$= 2 \times 5 \times 14.8$$

$$\boxed{C_c = 148 \text{ N-sec/m}}$$

(iii) Dumping Factor

$$\xi = \frac{C}{C_c}$$

$$= \frac{1.46}{148}$$

$$\boxed{\xi = 0.00984}$$

(iv) logarithmic decrement

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$= \frac{2\pi \times 0.00984}{\sqrt{1-0.00984^2}}$$

$$\boxed{\delta = 0.0622}$$

(v) ratio

$$\delta = \frac{1}{n} \ln\left(\frac{\beta_1}{\beta_6}\right)$$

$$0.0622 = \frac{1}{5} \ln\left(\frac{\beta_1}{\beta_6}\right)$$

$$\boxed{\frac{\beta_1}{\beta_6} = 1.3647}$$

$$\boxed{\frac{\beta_6}{\beta_1} = \frac{1}{1.3647} = 0.7327}$$

NOV-DEC 2022

Q.5]

Q.1] a] Given $m = 3\,kg$

$K = 100\,N/m$

$C = 3\,N\text{-}sec/m$

$n = 1$

(i) Natural frequency

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{100}{3}}$$

$$\boxed{\omega_n = 5.7735\ rad/sec}$$

(ii) Dumping factor

$$\xi = \frac{C}{C_c}$$

$$= \frac{C}{2 \times m \times \omega_n}$$

$$\xi = \frac{3}{2 \times 3 \times 5.7735}$$

$$\boxed{\xi = 0.0866}$$

(iii) Logarithmic decrement

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$= \frac{2\pi \times 0.0866}{\sqrt{1 - 0.0866^2}}$$

$$\boxed{\delta = 0.5462}$$

(iv) Ratio

$$\delta = \frac{1}{n} \log\left(\frac{\beta_1}{\beta_2}\right)$$

$$0.5462 = \ln\left(\frac{\beta_1}{\beta_2}\right)$$

$$\frac{\beta_1}{\beta_2} = 1.7266$$

$$\therefore \frac{\beta_2}{\beta_1} = \frac{1}{1.7266} \Rightarrow \boxed{\frac{\beta_2}{\beta_1} = 0.5791}$$

(V) Number of cycle

$$\frac{B_1}{B_2} = \frac{1}{0.2} = 5$$

$$\delta = \frac{1}{n} \ln\left(\frac{B_1}{B_2}\right)$$

$$0.5462 = \frac{1}{n} \ln(5)$$

$$\boxed{n = 2.946 \text{ cycle}}$$

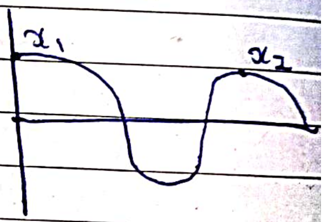
Nov-Dec 2023

Q.6] Q.2] b] Given $\frac{B_1}{B_2} = 16$ $m = 200\,Kg$ $t = 2\,sec$

$n = 1$ $\omega_d = \frac{2\pi}{T_d} = \frac{2\pi}{2} = \pi$

(I) Logarithmic decrement

$$\delta = \frac{1}{n} \ln\left(\frac{B_1}{B_2}\right) \Rightarrow 2.77$$




(II) Damping factor

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$2.77 = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \Rightarrow \boxed{\xi = 0.4034}$$

(III) Natural frequency

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\frac{\pi}{\sqrt{1-0.4034^2}} = \omega_n \Rightarrow \boxed{\cancel{\omega_n = 4.33\ rad/sec} \quad \omega_n = 3.433\ rad/sec}$$

(IV) Damping coeff.

$$\xi = \frac{c}{c_c} \Rightarrow \xi = \frac{c}{2m\omega_n}$$

i.e. $0.4034 = \frac{c}{2 \times 200 \times 3.433}$

$$\boxed{c = 553.94\ Ns/m}$$

(v) Spring Stiffness

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$3.433 = \sqrt{\frac{K}{200}}$$

$$\boxed{K = 2357.0478 \ N/m}$$