$= 6178.57 \times 10^3$ Nm/s $=$ **6178.57 kW** ...**Ans.**

Q.14 The impeller of a centrifugal compressor has the inlet and outlet diameter of 0.3 and 0.6 m, respectively. The intake is from the atmosphere at 100kPa and 300 K, without any whirl component. The outlet blade angle is75°. The speed is 10000 rpm and the velocity of flow is constant at 120m/s. If the blade width at intake is 6 cm, calculate :

(i) Specific work (ii) Exit pressure

(iii) Mass flow rate.

(iv) Power required to drive compressor if the overall efficiency can be assumed at 0.7.

SPPU - May 16, 10 Marks

Ans. : $D_i = 0.3$ m, $D_o = 0.6$ m,

$p_1 = 100$ kPa, $T_1 = 300$ K

$\phi = 75°$, $N = 10000$ rpm,

$C_{fi} = C_{fo} = 120$ m/s $B_i = 6$ cm $= 0.06$ m

Refer Fig. 6.18

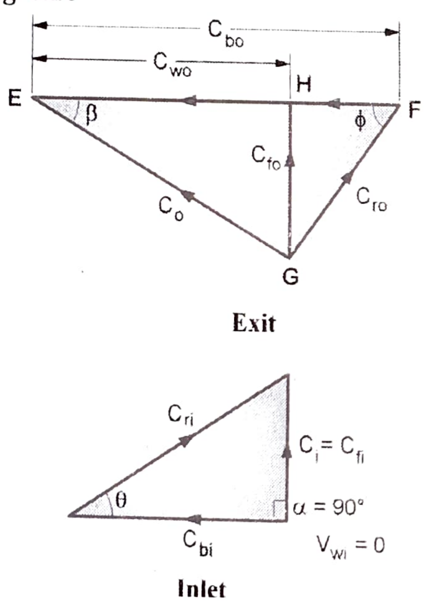

Fig. 6.18

$$C_{bi} = \frac{\pi \times D_i \times N}{60} = \frac{\pi \times 0.3 \times 10000 \text{ rpm}}{60}$$

$$= 157.1 \text{ m/s} = C_i$$

$$C_{bo} = \frac{\pi \times D_o \times N}{60} = \frac{\pi \times 0.6 \times 10000}{60} = 314.2 \text{ m/s}$$

From exit Δ EGF :

$$C_{wo} = C_{bo} - \frac{C_{fo}}{\tan \phi} = 314.2 - \frac{120}{\tan 75} = 282 \text{ m/s}$$

(i) Specific work, W

$$W = C_{bo}\, C_{wo} = \frac{314.2 \times 282}{1000} \text{ kJ/kg}$$

$$= \mathbf{88.60\ kJ/kg}$$ ...**Ans.**

(ii) Exit pressure, p_2

$$W = C_p (T_2 - T_1)$$

$$88.6 = 1.005 (T_2 - 300)$$

$$T_2 = 388.16 \text{ K}$$

Assuming isentropic compression,

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma - 1)}$$

$$p_2 = 100 \left(\frac{388.16}{300}\right)^{1.4/(1.4-1)}$$

$$= \mathbf{246.4\ kPa}$$ ...**Ans.**

(iii) Mass flow rate, $\dot{m}$

Volume flow rate,

$$\dot{V}_1 = \pi\, D_i\, B_i \times C_{f_i}$$

$$= \pi \times 0.3 \times 0.06 \times 120$$

$$= 6.7858 \text{ m}^3/\text{s}$$

$$P_1 \dot{V}_1 = \dot{m} R T_1$$

$$\dot{m} = \frac{100 \times 6.7854}{0.287 \times 300} = \mathbf{7.8813\ kg/s}$$ ...**Ans.**

(iv) Power required, P it overall efficiency, $\eta_o = 0.7$

$$P = \frac{\dot{m} \times w}{\eta_o}$$

$$= \frac{7.8813 \times 88.6}{0.7}$$

$$= \mathbf{997.5\ kW}$$ ...**Ans.**

Q.15 A centrifugal compressor running at a speed of 15000 rpm admits 25m³/sec air at static states 1 bar and 300 K and compresses it adiabatically by the pressure ratio of 2. The air velocity at inlet and the radial velocity at exit is the same as 75 m/sec. The inlet and outlet impeller diameters are 60 cm and 80 cm respectively. Considering the inlet to be axial find :

i) Blade angles at inlet and outlet of impeller.

ii) Angle at which air leaves the impeller.

iii) Impeller breadth at inlet and exit.

SPPU : May 18, 10 Marks

Ans. :

$N = 15000$ rpm, $\dot{V}_1 = 25$ m³/s,

$p_1 = 1$ bar, $T_1 = 300$ K,

$p_2 = 2$ bar; $C_i = C_{fi} = C_{f_0} = 75$ m/s

$d_i = 60$ cm $= 0.6$ m; $d_0 = 80$ cm $= 0.8$ m

Q.20 A centrifugal pump running at 900 rpm is working against a head 20 m. The external diameter of the impeller is 460 mm and outlet width is 50 mm. If the vanes angles at outlet is 40° and manometric efficiency is 70 % determine.

i) Flow velocity at outlet

ii) Absolute velocity of water leaving the vane

iii) Angle made by the absolute at outlet with the direction of motion at outlet

iv) Rate of flow through the pump

SPPU - Dec. 15, 10 Marks

Ans. :

$$N = 900 \text{ rpm}, \quad H_m = 20 \text{ m}$$

$$D_2 = 460 \text{ mm} = 0.46 \text{ m},$$

$$B_2 = 50 \text{ mm} = 0.05 \text{ m},$$

$$\phi = 40°, \quad \eta_m = 70\,\% = 0.7$$

(i) Flow velocity at outlet, V_{f2}

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.46 \times 900}{60} = 21.68 \text{ m/s}$$

Exit velocity diagram is shown in Fig. 5.18.

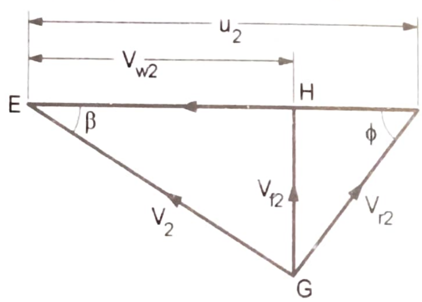

Fig. 5.18

$$\eta_m = \frac{gH_m}{V_{w_2} \cdot u_2}$$

$$0.7 = \frac{9.81 \times 20}{V_{w2} \times 21.68}$$

$$V_{w2} = 12.93 \text{ m/s}$$

$$H_F = u_2 - V_{w2} = 21.68 - 12.93 = 8.75 \text{ m/s}$$

From ΔGHF :

$$\frac{V_{f2}}{H_F} = \tan\phi$$

$$V_{f2} = 8.75 \tan 40 = \mathbf{7.34 \text{ m/s}}$$ ...Ans.

ii) Absolute velocity of water leaving the vane, V_2

$$V_2 = \sqrt{V_{w2}^2 + V_{f2}^2} = \sqrt{(12.93)^2 + (7.34)^2}$$

$$= \mathbf{14.87 \text{ m/s}}$$ ...Ans.

iii) Angle made by absolute velocity at outlet, β

$$\sin\beta = \frac{V_{f2}}{V_2} = \frac{7.34}{14.87} = 0.4936$$

$$\beta = \mathbf{29.58°}$$ ...Ans.

iv) Rate of flow through the pump, Q

$$Q = \pi D_2 B_2 V_{f2} = \pi \times 0.46 \times 0.05 \times 7.34$$

$$= \mathbf{0.5304 \text{ m}^3/\text{s}}$$ ...Ans.

Q.21 Explain multi staging in a centrifugal pump and explain the methods used for multi staging.

Ans. : Multistage Centrifugal Pumps

The centrifugal pumps with two or more number o identical impellers are called multistage centrifugal pumps.

The impellers of these pumps may be attached on th same or on different shafts. The multistage pumps ar needed either to increase the head or the discharg compared to a single stage pump, accordingly the impelle are connected in series or parallel as follows :

Multistage Centrifugal Pumps for High Heads

Two or more identical impellers are connected **series to generate high heads with constant discharg** The arrangement of multistage pump with two impellers series is shown in Fig. 5.19.

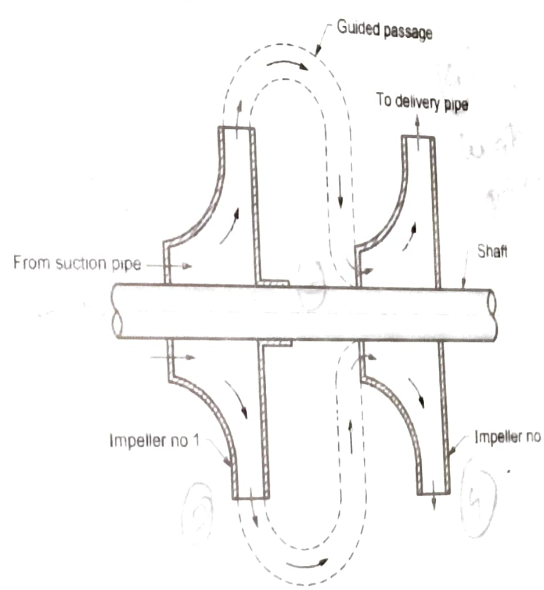

Fig. 5.19 : Impellers in series for heads

The water from suction pipe is supplied to impel where its head is raised. The discharge from impelle guided through the guided passages into the i impeller- 2 where its pressure is further increased. I are more number of impellers in series the operations are conducted and finally the last impe series discharges the water.

Let, n = number of impellers

H_m = head developed by each impeller

Total head developed,

$$H = n \cdot H_m$$

$$\frac{u_2^2 - u_1^2}{2g} \geq H_m \quad ...(1)$$

$$\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2 \geq 2g H_m$$

∴ For minimum speed,

$$\therefore N = \frac{60}{\pi} \times \frac{\sqrt{2g H_m}}{\sqrt{(D_2^2 - D_1^2)}} \quad ...(2)$$

Equation (2) gives the minimum speed required for pump to start discharging the liquid.

Minimum Diameter of Impeller, D_1

Usually external diameter, $D_2 = 2 \times$ internal diameter, D_1 i.e. $D_2 = 2D_1$. Using the concept of minimum starting speed of pump, N to deliver the given head, H_m, we can determine the minimum diameter of impeller D_1 from Equation (2) as follows :

$$N = \frac{60}{\pi} \times \frac{\sqrt{2g H_m}}{\sqrt{(2D_1^2 - D_1^2)}} = \frac{60}{\pi} \times \frac{\sqrt{2g H_m}}{\sqrt{3} \cdot D_1}$$

$$\therefore D_1 = \frac{48.84}{N} \times \sqrt{H_m} \quad ...(3)$$

$$D_2 = 2 D_1 = \frac{97.68}{N} \times \sqrt{H_m} \quad ...(4)$$

Q.9 A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 rpm work against total head of 40 m. The velocity of flow through the runner is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 50 cm and width at outlet is 5 cm.

Determine :

i) Vane angle at inlet

ii) Workdone by impeller on water per second

iii) Manometric efficiency **SPPU - Dec. 15, 10 Marks**

Soln. :

$D_2 = 2D_1$, $N = 1000$ rpm,

total head, $H_m = 40$ m, $V_{f1} = V_{f2} = 2.5$ m/s,

$\phi = 40°$, $D_2 = 50$ cm $= 0.5$ m,

$B_2 = 5$ cm $= 0.05$ m

Refer Fig. 5.7

$$D_1 = \frac{D_2}{2} = \frac{0.5}{2} = 0.25 \text{ m}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s}$$

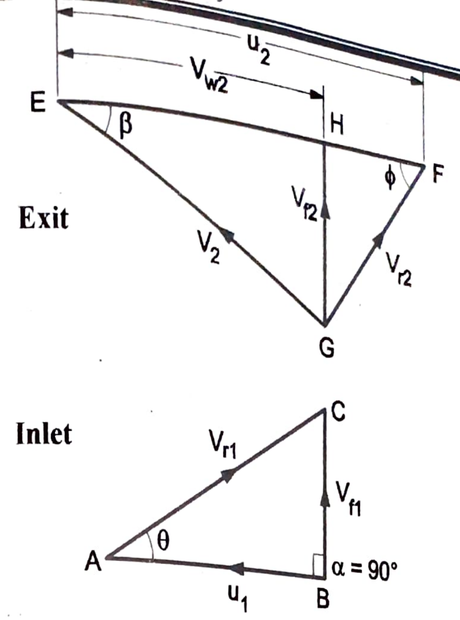

Fig. 5.7

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1000}{60} = 26.18 \text{ m/s}$$

$$Q = \pi D_2 B_2 V_{f2} = \pi \times 0.5 \times 0.05 \times 2.5$$

$$= 0.19635 \text{ m}^3/\text{s}$$

$$\dot{m} = \rho Q = 1000 \times 0.19635 = 196.35 \text{ kg/s}$$

(i) Vane angle at inlet, θ :

Refer inlet ΔABC

$$\theta = \tan^{-1}\left(\frac{V_{f1}}{u_1}\right) = \tan^{-1}\left(\frac{2.5}{13.09}\right)$$

$$= \mathbf{10.81°} \quad \text{...Ans.}$$

(ii) Wordone by impeller on water per second, $\dot{W}$

$$V_{w2} = u_2 - H_f = u_2 - \frac{V_{f2}}{\tan \phi}$$

$$= 26.18 - \frac{2.5}{\tan 40} = 23.2 \text{ m/s}$$

$$\dot{W} = \dot{m} \frac{V_{w_2} \times u_2}{1000} (\text{kW}) = \frac{196.35 \times 23.2 \times 26.18}{1000}$$

$$= \mathbf{119.26\ kW} \quad \text{...Ans.}$$

(iii) Manometric efficiency, η_m

$$\eta_m = \frac{gH_m}{V_{w2} - u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = \mathbf{0.646 \text{ or } 64.6\ \%} \quad \text{...Ans.}$$

Q.10 A centrifugal pump at 900 rpm has an impeller diameter of 500mm and eye diameter of 300 mm. The blade angle at outlet is 35° with tangent. Determine assuming zero whirl at inlet, the inlet blade angle, absolute velocity at outlet and its direction and the manometric head. The velocity of flow is constant throughout and is 3 m/sec. **SPPU - May 19,10 Marks**

Ans. : Refer Fig. 5.8

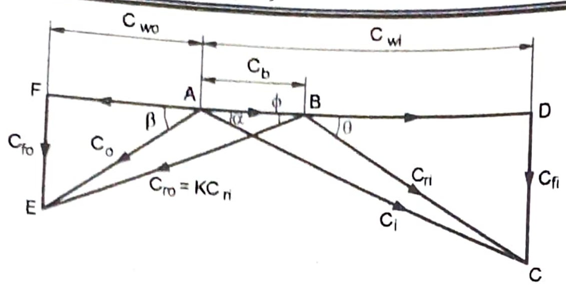

Fig. 4.13

From above :

$$\dot{m} = 0.466 \text{ kg/s}, \quad C_i = 809.7 \text{ m/s},$$

$$\alpha = 8.88°, \quad C_b = 400 \text{ m/s},$$

$$\theta = \phi, \quad K = 0.87$$

Consider ΔACD,

$$C_{wi} = C_i \cos\alpha = 809.7 \cos 8.88 = 800 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 809.7 \sin 8.88 = 125 \text{ m/s}$$

$$BD = C_{wi} - C_b = 800 - 400 = 400 \text{ m/s}$$

$$C_{ri} = \sqrt{BD^2 + C_{fi}^2}$$

$$= \sqrt{(400)^2 + (125)^2} = 419 \text{ m/s}$$

$$K = 0.87 = \frac{C_{ro}}{C_{ri}}; \; 0.87 = \frac{C_{ro}}{419};$$

$$C_{ro} = 364.5 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{125}{400}\right)$$

$$= \mathbf{17.354° = \phi}$$ ...Ans.

$$BF = C_{ro} \cos\phi = 364.5 \cos 17.354 = 347.9 \text{ m/s}$$

$$C_{wo} = BF - AB = 347.9 - 400 = -52.1 \text{ m/s}$$

(ii) Maximum power developed, P:

$$P = \frac{\dot{m}\,(C_{wi} + C_{wo})\,C_b}{1000} \text{ kW}$$

$$= \frac{0.466\,(809.7 - 52.1)\,400}{1000}$$

$$= \mathbf{141.22 \text{ kW}}$$ ...Ans.

Q.12 In a single stage impulse turbine the mean diameter of the blade ring is 1m and the rotational speed is 3000 rpm. The steam is issued from the nozzle at 300 m/sec. and nozzle angle is 20°. The blades are equiangular. If the friction loss in the blade channel is 19% of the Kinetic energy corresponds to relative velocity at the inlet to the blades. What is the power developed in the blading when the axial thrust on the blades is 98 N. Solve the problem graphically.

SPPU: May 18, 6 Marks

Ans. : Refer Fig. 4.14

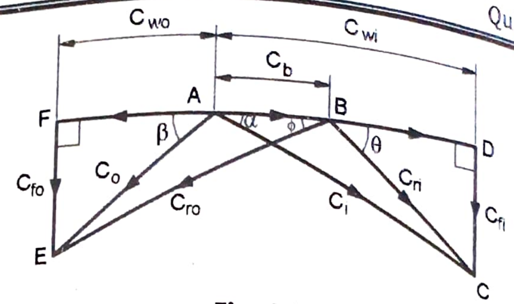

Fig. 4.14

$$d = 1 \text{ m}, \quad N = 3000 \text{ rpm},$$

$$C_i = 300 \text{ m/s}, \quad \alpha = 20^0$$

$$\theta = \phi \text{ (equiangular blades)}$$

Friction loss = 19% of K.E of relative velocity at inlet

$$= 0.19 \times \frac{C_{ri}^2}{2}$$

Axial thrust, F_a = 98 N

$$C_b = \frac{\pi d N}{60} = \frac{\pi \times 1 \times 3000}{60} = 157.1 \text{ m/s}$$

From Inlet Δ ACD,

$$C_{wi} = C_i \cos\alpha = 300 \cos 20 = 281.9 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 300 \sin 20 = 102.6 \text{ m/s}$$

$$BD = C_{wi} - C_b = 281.9 - 157.1 = 124.8 \text{ m/s}$$

$$C_{ri} = \sqrt{BD^2 + C_{fi}^2} = \sqrt{(124.8)^2 + (102.6)^2}$$

$$= 161.6 \text{ m/s}$$

$$\frac{C_{ro}^2}{2} = \frac{C_{ri}^2}{2} - 0.19\frac{C_{ri}^2}{2} = 0.81\frac{C_{ri}^2}{2}$$

$$C_{ro} = \sqrt{0.81} \times C_{ri} = 0.9 \times C_{ri} = 0.9 \times 161.6$$

$$= 145.4 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{102.6}{124.8}\right) = 39.424^0 = \phi$$

From exit Δ BEF,

$$C_{fo} = C_{ro} \sin\phi = 145.4 \sin 39.424 = 92.3 \text{ m/s}$$

$$C_{wo} = BF - AB = C_{ro} \cos\phi - C_b$$

$$= 145.4 \cos 39.424 - 157.1 = \mathbf{-44.8 \text{ m\textbackslash s}}$$

Axial thrust,

$$F_a = \dot{m}\,(\vec{C_{fi}} - \vec{C_{fo}})$$

$$98 = \dot{m}\,(102.6 - 92.3);$$

$$\dot{m} = 9.5146 \text{ kg/s}$$

(i) Power developed, P

$$P = \frac{\dot{m}(C_{wi} + C_{wo})\,C_b}{1000} \text{ (kW)}$$

$$= \frac{9.5146(281.9 - 44.8)\,157.1}{1000}$$

$$= \mathbf{354.4 \text{ kW}}$$ ...Ans.

$$K = \frac{\text{Relative velocity at exit of moving blade, } C_{ro}}{\text{Relative velocity at inlet to moving blade, } C_{ri}} \quad ...(1)$$

$$\therefore \quad C_{ro} = K \cdot C_{ri}$$

Q.8 Define the term Speed ratio **SPPU : April 17(In Sem)**

Ans. : Blade speed ratio (s)

Blade speed ratio,

$$s = \frac{\text{Blade velocity, } C_b}{\text{Steam velocity at inlet, } C_i}$$

Q.9 Steam enters an impulse wheel having a nozzle of 20° at a velocity of 450 m/s. The exit angle of the moving blade is 20° and relative velocity of steam may be assumed to remain constant over the moving blades. If the blade speed is 180 m/s and mass flow rate of steam is 2.5 kg/s determine :

(i) Blade angle at inlet

(ii) Work done per kg of steam

(iii) Total power developed by the turbine

(iv) Diagram efficiency **SPPU - Dec. 19,10 Marks**

Ans. : Refer Fig. 4.11

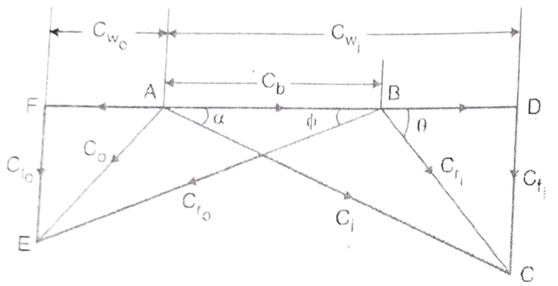

Fig. 4.11

$\alpha = 20°$, $\quad C_i = 420$ m/s,

$\phi = 20°$, $\quad C_{ro} = C_{ri}$

$C_b = 180$ m/s, $\quad \dot{m} = 2.5$ kg/s

(i) Blade angle at inlet, θ

Consider inlet Δ ACD :

$$C_{wi} = C_i \cos\alpha = 450 \cos 20 = 422.86 \text{ m/s}$$

$$C_{fi} = C_i \sin\alpha = 450 \sin 20 = 153.91 \text{ m/s}$$

$$BD = C_{wi} - C_b = 422.86 - 180 = 242.86 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{C_{fi}}{BD}\right) = \tan^{-1}\left(\frac{153.91}{242.86}\right)$$

$$= \mathbf{32.36°} \quad \text{... Ans.}$$

Also, $\quad C_{ri} = \sqrt{(C_{fi})^2 + (BD)^2}$

$$= \sqrt{(153.91)^2 + (242.86)^2} = 287.52 \text{ m/s}$$

$$\therefore \; C_{ro} = C_{ri} = 287.52 \text{ m/s}$$

ii) Workdone/kg of steam, W

Consider outlet Δ BEF :

$$C_{wo} = AF = FB - C_b = C_{ro}\cos\phi - C_b$$

$$= 287.52 \cos 20 - 180 = 90.18 \text{ m/s}$$

$$W = \frac{(C_{wi} + C_{wo})\,C_b}{1000} \text{ kJ/kg} = \frac{(422.86 + 90.18)180}{1000}$$

$$= \mathbf{92.3472 \text{ kJ/kg}} \quad \text{... Ans.}$$

(iii) Total power developed, $\dot{W}$

$$\dot{W} = \dot{m}\,W = 2.5 \times 92.3472 = \mathbf{230.868 \text{ kW}} \quad \text{... Ans.}$$

(iv) Diagram efficiency, η_b

$$\eta_b = \frac{W}{C_i^2/2} = \frac{2W}{C_i^2} = \frac{2 \times (92.3472 \times 1000)}{(450)^2}$$

$$= \mathbf{0.9121 \text{ or } 91.21\%} \quad \text{... Ans.}$$

Q.10 Steam issues from the nozzles of an angle of 20 ° at a velocity of 440 m/s, the friction factor is 0.9, for a single stage turbine designed for a maximum efficiency determine (i) Blade velocity (ii) moving blade angles for equiangular blades (iii) Blade efficiency (iv) stage efficiency if the nozzle efficiency is 93% and power developed for mass flow rate of 3 kg/s. **SPPU : Dec. 17, May 19, 8 Marks**

Ans. : Refer Fig. 4.12

$\alpha = 20°$, $\quad C_i = 440$ m/s,

Friction factor, $\quad K = \dfrac{C_{ro}}{C_{ri}} = 0.9$

(i) Blade velocity, C_b

Since the turbine is designed for maximum efficiency, the condition is,

$$s = \frac{C_b}{C_i} = \frac{\cos\alpha}{2};$$

$$\therefore \; C_b = \frac{C_i}{2}\cos\alpha = \frac{440}{2} \times \cos 20$$

$$= \mathbf{206.7 \text{ m/s}} \quad \text{...Ans.}$$

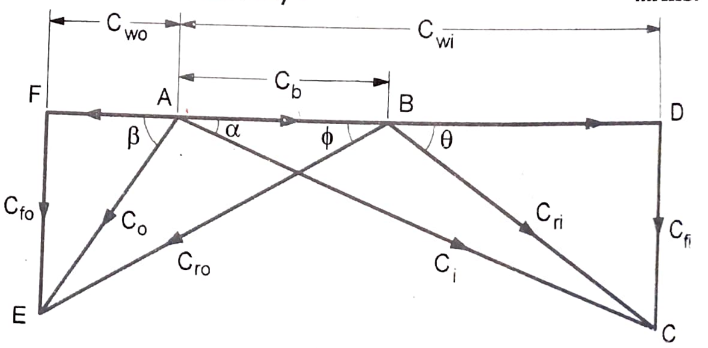

Fig. 4.12

(ii) Moving blade angles for equiangular blades i.e. θ = ϕ

Consider inlet velocity Δ ACD,

$$C_{wi} = C_i \cos\alpha = 440 \cos 20 = 413.5 \text{ m/s}$$

$$BD = C_{wi} - C_b = 413.5 - 206.7 = 206.8 \text{ m/s}$$

$H = 22$ m, $\quad V_{f1} = V_{f2} = 2.5$ m/s

$\alpha = 10°$, $\quad \theta = 90°$ (radial vanes)

$\beta = 90°$ (radial discharge)

From inlet velocity Δ ABC,

$$u_1 = \frac{V_{f1}}{\tan\alpha} = \frac{2.5}{\tan 10} = 14.18 \text{ m/s} = V_{w_1}$$

$$V_1 = \frac{V_{f1}}{\sin\alpha} = \frac{2.5}{\sin 10} = 14.4 \text{ m/s}$$

(i) Speed of the turbine, N

$$u_1 = \frac{\pi D_1 N}{60};$$

$$14.18 = \frac{\pi \times 1.2 \times N}{60}$$

$$\mathbf{N = 225.68 \text{ rpm}} \quad \text{...Ans.}$$

(ii) Vane angle of outlet, ϕ

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 225.64}{60} = 7.09 \text{ m/s}$$

From outlet Δ EFG,

$$\phi = \tan^{-1}\left(\frac{V_{f2}}{u_2}\right) = \tan^{-1}\left(\frac{2.5}{7.09}\right)$$

$$\mathbf{\phi = 15.945°} \quad \text{...Ans.}$$

iii) Hydraulic efficiency, η_h

$$\eta_h = \frac{V_{w1} \cdot u_1}{g H} = \frac{14.18 \times 14.18}{9.81 \times 22} = \mathbf{0.9317 \text{ or } 93.17\%} \text{...Ans.}$$

Q.33 The external and internal diameters of an inward flow reaction turbine are 2 m and 1 m respectively. The head on the turbine is 60 m. The width of the vane at inlet and outlet are same and equal to 0.25 m. The runner vanes are radial are inlet and discharge is radial at outlet. The speed is 200 rpm and the discharge is 6 m^3/s. Determine :

(i) The vane angle at outlet and inlet of the runner

(ii) The hydraulic efficiency **SPPU - Dec. 16, 6 Marks**

Ans. : Refer Fig. 3.17

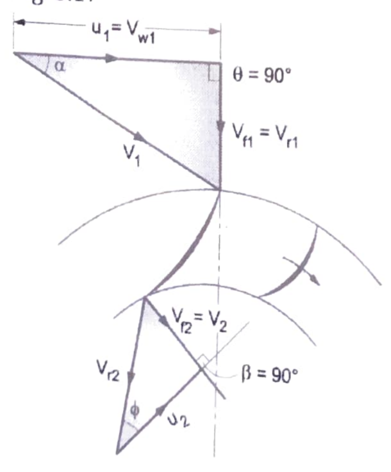

Fig. 3.17

Given :

Speed, N = 200 rpm ;

External diameter, D_1 = 2.0 m ;

Internal diameter D_2 = 1.0 m ;

Head, H = 60m ,

Discharge, $Q = 6\ m^3/s$;

$V_{f1} = V_{r1}$

$B_1 = B_2 = 0.25$ m

Runner vane angle at inlet $\theta_1 = 90°$ (radial vanes),

Radial discharge $\beta_2 = 90°$, $\quad V_{w2} = 0$

From the inlet velocity triangle

$$u_1 = V_{w1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 2.0 \times 200}{60} = 20.95 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.0 \times 200}{60} = 10.47 \text{ m/s}$$

Q = discharge

$= \pi D_1 B_1 V_{f1}$ (neglecting the thickness of vane)

$$6 = \pi \times 2.0 \times 0.25 \times V_{f1}$$

$$V_{f1} = 3.82 \text{ m/s}$$

Also, $Q = \pi D_2 B_2 V_{f2}$

$$6 = \pi \times 1.0 \times 0.25 \times V_{f2}$$

$$V_{f2} = 7.64 \text{ m/s}$$

(i) Vane angle at outlet ϕ and inlet angle, α

$$\tan\phi = \frac{V_{f2}}{u_2} = \frac{7.64}{10.47} = 0.729$$

$$\mathbf{\phi = 36.11°} \quad \text{...Ans.}$$

$$\alpha = \tan^{-1}\left(\frac{V_{f1}}{u_1}\right)$$

$$= \tan^{-1}\left(\frac{3.82}{20.95}\right) = \mathbf{10.33°} \quad \text{...Ans.}$$

(ii) Hydraulic efficiency (η_H)

$$\eta_H = \frac{V_{w1}\, u_1}{gH} = \frac{20.95 \times 20.95}{9.81 \times 60}$$

$$= \mathbf{0.7456 \text{ or } 74.56\%} \quad \text{...Ans.}$$

Q.34 For the Francis turbine following data is available :

Shaft power = 130 kW, Net head = 90, Speed = 120 r.p.m.

Overall efficiency = 75%, Hydraulic efficiency = 90%

Velocity of flow at inlet = $1.15\sqrt{H}$,

Vane speed at inlet = $3.45\sqrt{H}$

Assume discharge radial at exit. Find :

(i) Guide blade and moving blade angle at inlet

(ii) Diameter of runner at inlet.

SPPU - May 19, Oct. 19(In Sem.), 6 Marks

Q.13 A jet of water moving with V m/s strikes at the centre of a curved vane which is moving with 'u' m/s. If the outgoing jet makes an angle θ with the incoming jet, prove that,

(i) Maximum efficiency = $\eta_{max} = \frac{8}{27}(1 + \cos\theta)$

(ii) Blade speed u = V/3 **SPPU : May 15**

Ans. : Consider a jet of water of area **A** at velocity **V** strikes a single symmetrical moving curved vane at velocity **u** as shown in Fig. 2.12.

Jet leaves the vane in tangential direction at angle θ from horizontal at relative velocity (V – u).

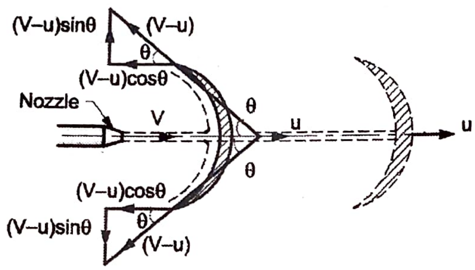

Fig. 2.12 : Impact of jet on single moving curved vane

Mass flow rate of jet of water striking the vane, $\dot{m} = \rho \cdot A\,(V - u)$. Component of velocity in direction of jet after striking the vane is $[-\,(V - u)\cos\theta]$ as shown in Fig. 2.12.

∴ Normal force on vane,

$$F_n = \dot{m}\,[\text{Initial velocity} - \text{Final velocity}]$$

$$F_n = \rho \cdot A \cdot (V - u)\,[(V - u) - \{-(V - u)\cos\theta\}]$$

$$F_n = \rho \cdot A \cdot (V - u)^2\,(1 + \cos\theta) \qquad ...(1)$$

Rate of workdone,

$$W = \text{Force} \times \text{distance travelled/second by vane}$$

$$W = \rho \times A \cdot (V - u)^2\,(1 + \cos\theta) \times u \qquad ...(1(A))$$

$$\text{K.E. supplied to jet} = \frac{1}{2}\dot{m}\,V^2 = \frac{1}{2}(\rho\,A\,V)\,V^2 = \frac{1}{2}\rho \cdot A \cdot V^3$$

Efficiency of system, η

$$\eta = \frac{\text{Workdone, W}}{\text{K.E}} = \frac{\rho \cdot A\,(V - u)^2\,(1 + \cos\theta)\,u}{\frac{1}{2}\cdot \rho \cdot A \cdot V^3}$$

$$= \frac{2\,(V - u)^2\,(1 + \cos\theta)\,u}{V^3} \qquad ...(2)$$

Condition for maximum efficiency is, $\frac{d\eta}{du} = 0$

$$\therefore \frac{d}{du}\left(\frac{2\,(V - u)^2\,(1 + \cos\theta)\,u}{V^3}\right) = 0$$

$$\frac{2\,(1 + \cos\theta)}{V^3}\left(\frac{d}{du}\,(V^2 - 2Vu + u^2)\,u\right) = 0$$

$$\frac{2\,(1 + \cos\theta)}{V^3}\,[V^2 - 2V \cdot 2u + 3u^2] = 0$$

But $\frac{2\,(1 + \cos\theta)}{V^3} \neq 0$, it follows that :

$$V^2 - 4Vu + 3u^2 = 0$$

$$(V - 3u)\,(V - u) = 0$$

$$\therefore V = 3u \text{ or } V = u \qquad ...(i)$$

V = u is not possible since jet will never strike the vane.

Hence, the system has the maximum efficiency when,

$$V = 3u \qquad ...(3)$$

On substituting the above value in Equation (2), we have,

$$\eta_{max} = \frac{8}{27}(1 + \cos\theta) \qquad ...(4)$$

For semicircular vane, θ = 0, on substituting the value of θ in Equation (4), we have,

$$\therefore \eta_{max} = \frac{16}{27} = 0.5926 = 59.26\%$$

Q.14 Prove that the work done per second on a series of moving curved vanes by a jet of water striking at one of the tips of the vane tangentially is given by, workdone/sec = $\rho a V_1\,[V_{w1} \pm V_{w2}] \times u$. **SPPU : May 19**

Ans. : Force Exerted on a Series of Moving Radial Curved Vanes

For a radial curved vane, the radius of vane at inlet and outlet is different, hence, the blade velocity u will be different at inlet and outlet.

Consider a series of radial vanes mounted on a wheel as shown in Fig. 2.13.

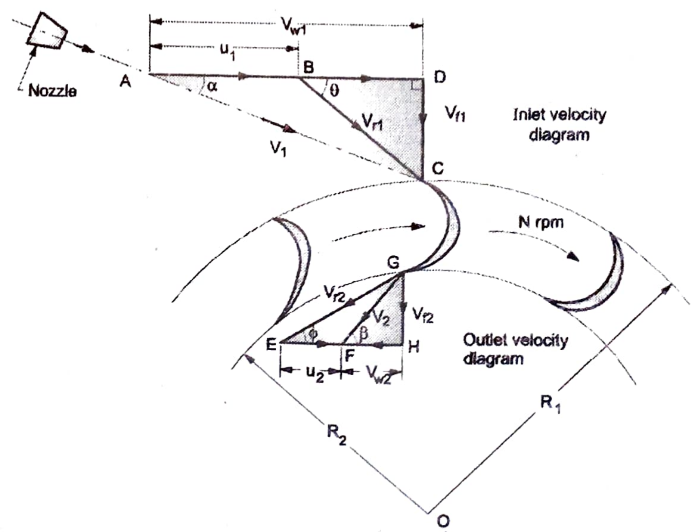

Fig. 2.13 : Series of radial vanes mounted on a wheel