34	The impeller of the centrifugal compressor has the inlet and outlet diameter of 0.3 and 0.6 m respectively. The intake is from the atmosphere at 100 kPa and 300 K, without any whirl component. The outlet blade speed is 10000 rpm and velocity of flow is constant at 120 m/s. If the blade width at inlet is 6 cm, calculate (i) Specific work (ii) Exit pressure

Example 6.18 *The impeller of a centrifugal compressor has the inlet and outlet diameter of 0.3 and 0.6 m respectively. The intake is from the atmosphere at 100 kPa and 300 K, without any whirl component. The outlet blade angle is 75°. The speed is 10000 rpm and the velocity of flow is constant at 120 m/s. If the blade width at intake is 6 cm, calculate :*

i) Specific work. *ii) Exit pressure.* *iii) Mass flow rate.*

iv) Power required to drive compressor if the overall efficiency can be assumed at 0.7. **SPPU : May 2016, 10 Marks**

Solution : $D_1 = 0.3$ m, $D_2 = 0.6$ m, $p_1 = 100$ kPa, $T_1 = 300$ K, $V_{w1} = 0$, $\phi = 75°$, N = 10,000 rpm, $V_{f1} = V_{f2} = 120$ m/s,

$B_1 = 0.06$ m, $\eta_0 = 0.7$

To find : i) Specific work ii) Exit pressure iii) $\dot{m}$ iv) power

Step - 1 : Calculate specific work

Refer Fig. 6.34.

We know that,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 10{,}000}{60} = 157.79 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 10{,}000}{60} = 314.16 \text{ m/s}$$

From velocity triangle; (Refer Fig. 6.34)

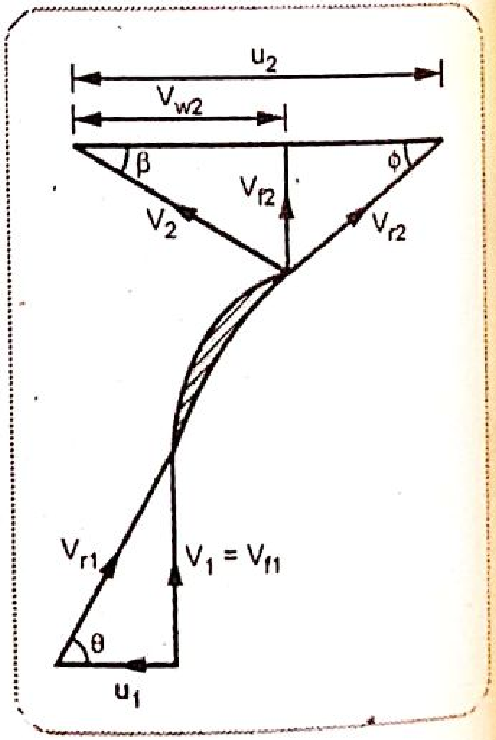


Fig. 6.34

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi} = 314.16 - \frac{120}{\tan 75}$$

$$V_{w2} = 282 \text{ m/s}$$

Specific work is given by,

$$\text{Specific work} = V_{w2}u_2 = 282 \times 314.16 = \mathbf{88.593 \times 10^3\ J/kg}$$ **... Ans.**

Step - 2 : Calculate exit pressure

We know that,

$$W = C_p(\Delta T)_{th} = C_p(T_2' - T_1)$$

$$\therefore \quad 88.593 \times 10^3 = 1.005 \times 10^3\,(T_2' - 300)$$

$$\therefore \quad T_2' = 388.153 \text{ K}$$

But,

$$\frac{p_2'}{p_1} = \left(\frac{T_2'}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{388.153}{300}\right)^{\frac{1.4}{1.4-1}}$$

$$\therefore \quad \frac{p_2'}{p_1} = 2.463 \quad \therefore\ p_2 = 2.463\ p_1$$

$$\therefore \quad p_2 = 2.463 \times 100 = \mathbf{2.463\ kPa}$$

Step ...

10 A Kaplan turbine runner has outer diameter of 4.5 m and the diameter of hub is 2 m. It is required to develop 20.6 MW when running at 150 rpm, under a head of 21 m. Assuming hydraulic efficiency of 94% and overall efficiency of 88% , determine the runner vane angle at inlet and exit at the mean diameter of the vane.

Example 3.25 *A Kaplan turbine runner has outer diameter of 4.5 m and the diameter of the hub is 2 m. It is required to develop 20.6 MW when running at 150 rpm, under a head of 21 m. Assuming hydraulic efficiency of 94 % and overall efficiency of 88 %. Determine the runner vane angle at inlet and exit at the mean diameter of the vane.*

SPPU : April 2015, 4 Marks, Dec. 2017, 6 Marks

Solution : Given data :

$D_o = 4.5$ m, $D_h = 2$ m, SP = 20.6 MW = 20.6×10^6 W, N = 150 rpm, H = 21 m,

$\eta_h = 94\ \% = 0.94$, $\eta_o = 88\ \% = 0.88$,

To find : θ and ϕ at mean diameter of the vane

Step 1 : Calculate the runner vane angles at inlet and outlet at the mean diameter of runner

The overall efficiency of turbine is given by,

$$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} = \frac{SP}{\rho \times g \times Q \times H}$$

$$\therefore \quad 0.88 = \frac{20.6 \times 10^6}{1000 \times 9.81 \times Q \times 21}$$

$$\therefore \quad Q = 113.6308 \text{ m}^3/\text{sec}$$

But
$$Q = \frac{\pi}{4}(D_o^2 - D_h^2) \times V_{f1}$$

$$\therefore \quad 113.6308 = \frac{\pi}{4}(4.5^2 - 2^2) \times V_{f1}$$

$$\therefore \quad V_{f1} = 8.9033 \text{ m/sec}$$

The vane angles are to be calculated at the mean diameter of runner. The mean diameter is given by,

$$\therefore \quad D_m = \frac{D_o + D_h}{2} = \frac{4.5+2}{2} = 3.25 \text{ m}$$

Now inlet vane velocity at the mean diameter is,

$$u_1 = u_2 = \frac{\pi D_m N}{60} = \frac{\pi \times 3.25 \times 150}{60} = 25.5254 \text{ m/sec}$$

and
$$V_{f1} = V_{f2}$$

Assume no whirl at the outlet hence $V_{w2} = 0$

The hydraulic efficiency is given by,

$$\eta_h = \frac{V_{w1}\, u_1}{gH} \qquad \therefore 0.94 = \frac{V_{w1} \times 25.5254}{9.81 \times 21}$$

$$\therefore \quad V_{w1} = 7.5865 \text{ m/sec}$$

As, $V_{w1} < u_1$, the velocity triangle becomes as shown in Fig. 3.33.

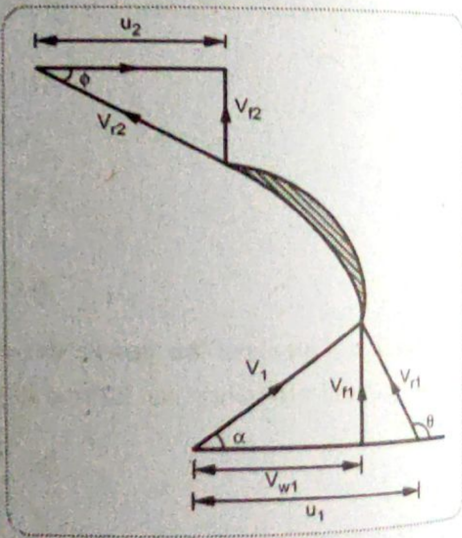


Fig. 3.33

$$V_{f1} = V_{f2} = 8.9033 \text{ m/sec}$$

From inlet velocity triangle,

$$\tan(180-\theta) = \frac{V_{f1}}{u_1 - V_{w1}} = \frac{8.9033}{25.5254 - 7.5865}$$

$$\therefore \quad \tan(180-\theta) = 0.4963$$

$$\therefore \quad 180 - \theta = \tan^{-1}(0.4963) = 26.3957°$$

$$\therefore \quad \theta = 180 - 26.3057° = \mathbf{153.6043°}$$ ...Ans.

From outlet velocity triangle,

$$\tan\phi = \frac{V_{f2}}{u_2} = \frac{8.9033}{25.5254} = 0.3488$$

$$\therefore \quad \phi = \tan^{-1}(0.3488) = \mathbf{19.2288°}$$ ...Ans.

9 The external and internal diameters of an inward flow reaction turbine are 2m and 1m respectively. The head on the turbine is 60 m. the width of the vane at inlet and outlet are same and equal to 0.25 m. The runner vanes are radial at inlet and discharge is radial at outlet. The speed is 200 rpm and the discharge is 6 m^3/sec. Determine (i) The vane angle at outlet of the runner and guide blade angle at inlet. (ii) The hydraulic efficiency.

Example 3.20 *The external and internal diameters of an inward flow reaction turbine are 2 m and 1 m respectively. The head on the turbine is 60 m. The width of the vane at inlet and outlet are same and equal to 0.25 m. The runner vanes are radial at inlet and discharge is radial at outlet. The speed is 200 rpm and the discharge is 6 m^3/s. Determine :*

i) The vane angle at outlet of the runner and guide blade angle at inlet.

ii) The hydraulic efficiency.

SPPU : Dec. 2016, 6 Marks

Solution : Given data :

$D_1 = 2$ m, $D_2 = 1$ m, $H = 60$ m, $B_1 = B_2 = 0.25$ m, $\beta = \theta = 90°$,

$N = 200$ rpm, $Q = 6 m^3/s$

To Find : i) α and ϕ ii) η_h

Step 1 : Calculate the flow velocity at inlet and outlet

The discharge through the turbine is,

$$Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$$

$$\therefore \quad V_{f1} = \frac{Q}{\pi D_1 B_1} = \frac{6}{\pi \times 2 \times 0.25} = 3.8197 \text{ m/s}$$

and

$$V_{f2} = \frac{Q}{\pi D_2 B_2} = \frac{6}{\pi \times 1 \times 0.25} = 7.6394 \text{ m/s}$$

The tangential velocity at inlet and outlet is,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 2 \times 200}{60} = 20.9439 \text{ m/s}$$

and

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1 \times 100}{60} = 10.4719 \text{ m/s}$$

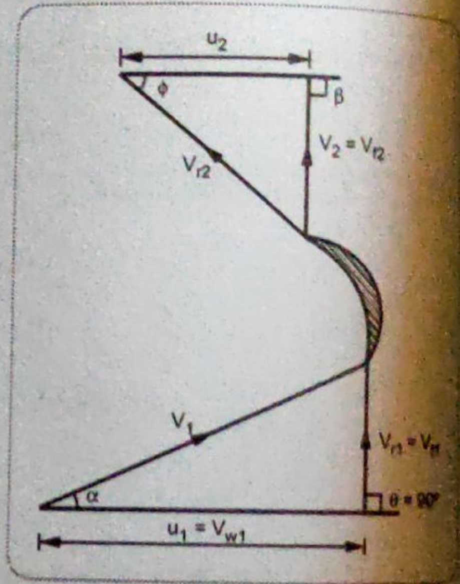


Fig. 3.27

Step 2 : Calculate blade angles and hydraulic efficiency

From inlet velocity triangle ($V_{w1} = u_1$)

$$\tan \alpha = \frac{V_{f1}}{u_1} = \frac{3.8197}{20.9439}$$

$$\therefore \quad \alpha = \mathbf{10.3358°} \quad \text{...Ans.}$$

From outlet velocity triangle,

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{7.6394}{10.4719}$$

$$\therefore \quad \phi = \mathbf{36.1112°} \quad \text{...Ans.}$$

Hydraulic efficiency of turbine is,

$$\eta_h = \frac{V_{w1} u_1}{gH} = \frac{20.9439 \times 20.9439}{9.81 \times 60} \quad \text{...}(V_{w1} = u_1)$$

$$\therefore \quad \eta_h = 0.7452 = \mathbf{74.52\ \%} \quad \text{...Ans.}$$

6 The following data relates to a Pelton wheel.
Head at the base of nozzle = 82 m, Diameter of jet = 105 mm, Discharge of the nozzle = 0.32 m^3/sec., Shaft power = 210 kW, Power absorbed in mechanical resistance = 5 kW. Determine (i) Power lost in the nozzle (ii) Power lost due to hydraulic resistance.

Example 2.15 *The following data relate to a Pelton wheel :*

Head at the base of nozzle = 82 m, *Diameter of jet = 105 mm,*

Discharge of the nozzle = 0.32 m^3/s, *Shaft power = 210 kW,*

Power absorbed in mechanical resistance = 5 kW.

Determine :

i) Power lost in nozzle ii) Power lost due to hydraulic resistance.

Solution : Given data :

$$H = 82 \text{ m}, \ d = 105 \text{ mm} = 0.105 \text{ m} \ \therefore \ A = \frac{\pi}{4} \times 0.105^2 = 8.6590 \times 10^{-3} \text{ m}^2,$$

$$Q = 0.32 \text{ m}^3/\text{sec}, \ P = 210 \text{ kW}, \ P_m = 5 \text{ kW}$$

To find : i) P_n ii) P_h

Step 1 : Calculate the power lost in nozzle

Discharge through nozzle is,

$$Q = A \times V_1 \quad \therefore 0.32 = 8.6590 \times 10^{-3} \times V_1$$

$$\therefore \quad V_1 = 36.9557 \text{ m/sec}$$

Power available at the nozzle i.e. water power is,

$$WP = \rho \times g \times Q \times H = 1000 \times 9.81 \times 0.32 \times 82$$

$$\therefore \quad WP = 257.4144 \times 10^3 \text{ Watts} = 257.4144 \text{ kW}$$

Power supplied to the jet in terms of K.E.

$$= \frac{1}{2}\dot{m}V_1^2 = \frac{1}{2} \times \rho \times A \times V_1 \times V_1^2 = \frac{1}{2}\rho\, A V_1^3$$

$$= \frac{1}{2} \times 1000 \times 8.6590 \times 10^{-3} \times 36.9577^3$$

$$= 218.5153 \times 10^3 \text{ Watts} = 218.5153 \text{ kW}$$

The power lost in nozzle is given by,

$$P_n = \text{Power available at nozzle} - \text{Power supplied to jet}$$

$$\therefore \quad P_n = 257.4148 - 218.5153$$

$$\therefore \quad \mathbf{P_n = 38.8994 \text{ kW}} \qquad \textbf{... Ans.}$$

Step 2 : Calculate the power lost due to hydraulic resistance

The power avaiable at the base of nozzle or water power is,

$$WP = \text{Shaft power} + \text{Power lost in runner} + \text{Power lost in nozzle}$$

$$\therefore \quad WP = SP + P_h + P_n$$

$$\therefore \quad 257.4148 = 210 + P_h + 38.8994$$

$$\therefore \quad \mathbf{P_h = 8.5153 \text{ kW}} \qquad \textbf{... Ans.}$$

4	A Pelton wheel of 2.5 m diameter operates under the following conditions. Net available head = 300 m, Jet diameter = 20 cm, Speed = 300 rpm, Blade angle at outlet = 165^0, C_v of the jet = 0.98, Blade friction coefficient = 0.95, Mechanical efficiency = 95% Determine (i) The power developed (ii) Specific speed (iii) Hydraulic efficiency.

Q

→

Given Data :-

$D = 2.5\ m$

$H = 300m$

$N = 300\ rpm$

Angle of deflection = $165°$

$\therefore\ \phi = 180 - 165 = 15°$

$C_v = 0.98$

$K = 0.95$

$d = 20\ cm = 0.2m$

$\eta_m = 95\%$

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4} \times 0.2^2$$

$$= 0.0314\ m^2$$

To find :- ① P

② N_s

③ η_h

① P

Velocity of jet,

$V_1 = C_v \sqrt{2gH}$

$= 0.98 \sqrt{2 \times 9.81 \times 300}$

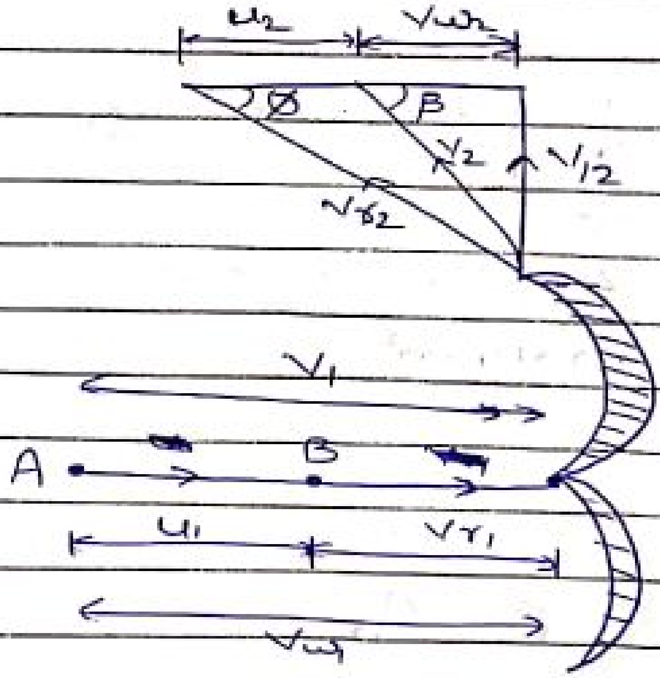
$= 75.1858$ m/sec

Bucket velocity,

$u = \frac{\pi D N}{60}$

$= \frac{\pi \times 2.5 \times 300}{60}$

$u = 39.2699$ m/sec.



$$V_{w1} = V_1 = 75.1858 \ m/sec$$

$$V_{r1} = V_1 - u = 75.1858 - 39.2699$$

$$\therefore \quad V_{r1} = 35.9158 \ m/sec$$

$$V_{r2} = K V_{r1} = 0.95 \times 35.9158$$

$$\therefore \quad V_{r2} = 34.12 \ m/sec$$

$$\therefore \quad V_{w2} = V_{r2} \cos\phi - u_2$$

$$= 34.12 \times \cos(15) - 39.2699$$

$$V_{w2} = -6.3125 \ m/sec.$$

$$W.D = \rho A V_1 (V_{w1} + V_{w2}) \times u$$

$$= 1000 \times 0.0314 \times 75.1858 \times (75.1858 - 6.3125) \times 39.2699$$

$$\therefore \quad W.D = 6.3852 \times 10^6 \ W$$

$$P = \frac{WD}{1000}$$

$$= \frac{6.3852 \times 10^6}{1000}$$

$$\boxed{P = 6.3852 \times 10^3 \ KW}$$

② $N_s = ?$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$= \frac{300\sqrt{6.3852 \times 10^3}}{300^{5/4}}$$

$$\boxed{N_s = 19.2002}$$

③ $\eta_h = ?$

$$\eta_h = \frac{2[V_{w_1} + V_{w_2}] \times u}{V_1^2}$$

$$= \frac{2[75.1858 + (-6.3125)] \times 39.2699}{75.1858^2}$$

$$\therefore \eta_h = 0.9569$$

$$\boxed{\therefore \eta_h = 95.69\%}$$

1	A jet of water moving with V m/s strikes at the centre of a curved vane which is moving with u m/s. If the outgoing jet makes an angle Θ with the incoming jet, prove that 1. Maximum efficiency, $\eta_{max} = \frac{8}{27}(1 + cos\Theta)$ 2. Blade speed, $u = \frac{V}{3}$

1.15 Force on Curved Plate (Vane) Moving in the Direction of Jet

SPPU : Dec. 2011, May 2015

- Consider a jet of water which strikes a curved plate at the center and the curved plate moves with uniform velocity in the direction of jet as shown in Fig. 1.15.
- In this case, the relative velocity of jet or the velocity with which jet strikes the curved plate is **(V – u).**
- As the plate is smooth, there is no loss of energy due to impact of jet. Hence, the velocity with which the jet will leave the curved plate is (V – u).
- In this case, the jet leaves the vane in tangential direction at an angle θ with horizontal at relative velocity (V – u).
- The velocity at the outlet of vane can be resolved into two components i.e. in the direction of jet and in the direction perpendicular to the jet.
- In this case,

  x component of striking velocity = $V_{x1} = (V - u)$

  y component of striking velocity = $V_{y1} = 0$

  x component of leaving velocity = $V_{x2} = -(V - u)\cos\theta$

  y component of leaving velocity = $V_{y2} = (V - u)\sin\theta$

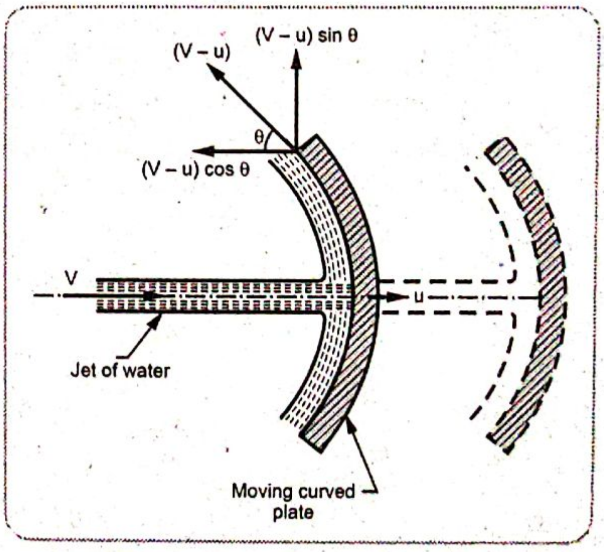


Fig. 1.15 : Jet striking a moving curved plate

- The mass of water striking the plate per second is,

$$\dot{m} = \rho A (V - u)$$

- The force exerted by the jet on the curved plate in the direction of jet is,

$$F_x = \dot{m}(V_{x1} - V_{x2}) = \rho A (V - u)\,[(V - u) - (-(V - u)\cos\theta)]$$

$$\therefore \quad F_x = \rho A (V - u)\,[(V - u) + (V - u)\cos\theta]$$

$$\therefore \quad F_x = \rho A (V - u)^2 (1 + \cos\theta) \qquad \ldots (1.31)$$

- The work done by the jet on the plate per second (Power) is,

$$WD/sec = P = F_x \times u = \rho A (V - u)^2 (1 + \cos\theta) \times u$$

- The efficiency of the system is,

$$\eta = \frac{\text{Work done per second}}{\text{K.E. supplied per second}} = \frac{\rho A (V - u)^2 (1 + \cos\theta) \times u}{\frac{1}{2}\dot{m} V^2}$$

$$\therefore \quad \eta = \frac{\rho A (V - u)^2 (1 + \cos\theta) \times u}{\frac{1}{2}\rho A V \times V^2} = \frac{2 (V - u)^2 (1 + \cos\theta) \times u}{V^3}$$

$$\eta = \frac{2(1+\cos\theta)}{V^3}(V^2u - 2Vu^2 + u^3) \qquad \ldots (i)$$

- For maximum efficiency differentiate the above equation (i) with respect to u and equate to zero.

$$\therefore \quad \frac{d\eta}{du} = 0$$

$$\therefore \quad \frac{d}{du}\left[\frac{2(1+\cos\theta)}{V^3}(V^2u - 2Vu^2 + u^3)\right] = 0$$

$$\therefore \quad \frac{2(1+\cos\theta)}{V^3}[V^2 - 4Vu + 3u^2] = 0$$

$$\therefore \quad V^2 - 4Vu + 3u^2 = 0$$

$$\therefore \quad (V - 3u)(V - u) = 0$$

$$\therefore \quad V = 3u \quad \text{or} \quad V = u$$

- If V = u, then there is no work done on the curved plate.

  Therefore consider V = 3u and substitute this value in equation (i),

$$\eta_{max} = \frac{2(1+\cos\theta)}{(3u)^3}[(3u)^2 \times u - 2(3u)u^2 + u^3]$$

$$\therefore \quad \eta_{max} = \frac{2(1+\cos\theta)}{27u^3}(9u^3 - 6u^3 + u^3)$$

$$\therefore \quad \eta_{max} = \frac{8(1+\cos\theta)}{27} = \frac{8}{27}(1+\cos\theta) \qquad \ldots (1.32)$$

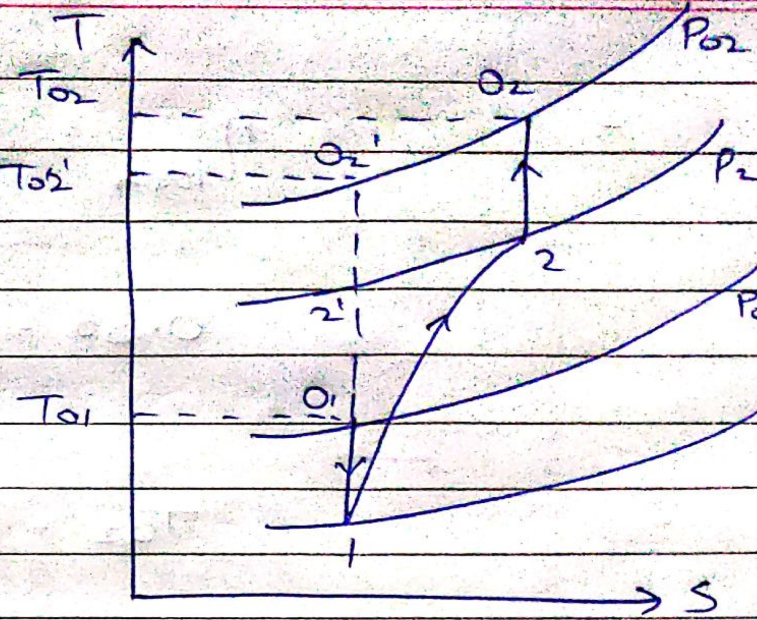
- For semicircular vane $\theta = 0°$

$$\therefore \quad \eta_{max} = \frac{8}{27}(1+\cos 0) = \frac{8}{27}(1+1)$$

$$\eta_{max} = \frac{16}{27} = 0.5926 = 59.26\ \% \qquad \ldots (1.33)$$

28	A centrifugal compressor delivers 16.5 kg/s of air with a total head pressure ratio of 4:1. The speed of the compressor is 15000 rpm. Inlet total head temperature is 20 C , slip factor 0.9, power input factor 1.04 and 80% isentropic efficiency. Calculate (i) Overall diameter of impeller (ii) Power Input

Page No.: Date: / /



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→ Given Data :-

$\dot{m} = 16.5\ kg/s$

$\frac{P_{02}}{P_{01}} = 4$

~~N = 16000 rpm~~

$N = 15000$ rpm

$T_{01} = 20°C = 20 + 273 = 293\ K$

$\phi_s = 0.9$

$\phi_w = 1.04$

$\eta_{isen} = 80\% = 0.80$

To find :- ① D_2

② P

① $D_2 =$

$$\frac{T_{02}'}{T_{01}} = \left(\frac{P_{02}'}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} \cdots$$

$\because (P_{02}' = P_{02})$

$$\therefore \frac{T_{02}'}{293} = \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$\therefore T_{02}' = 435.3963\ K$$

Page No.
Date:

- $$\eta_{isen} = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$0.80 = \frac{435.3963 - 293}{T_{02} - 293}$$

$$\therefore T_{02} = 470.99 \text{ K}$$

Work Done (actual) :-

- $$WD_{act} = C_p(T_{02} - T_{01})$$

$$= 1.005 \times 10^3 (470.99 - 293)$$

$$= 178.8799 \times 10^3 \text{ J/kg}$$

- Slip factor,

$$\phi_s = \frac{V_{w2}}{u_2}$$

$$0.9 = \frac{V_{w2}}{u_2}$$

$$\therefore V_{w2} = 0.9 \times u_2$$

- $$WD_{act} = \phi_w V_{w2} u_2$$

$$\therefore 178.8799 \times 10^3 = 1.04 \times 0.9 u_2 \times u_2$$

$$u_2^2 = 191.11 \times 10^3$$

$$\boxed{u_2 = 437.162 \text{ m/sec}}$$

Page No.
Date: /

- Blade velocity at outlet

$$u_2 = \frac{\pi D_2 N}{60}$$

$$437.162 = \frac{\pi \times D_2 \times 15000}{60}$$

$$\boxed{D_2 = 0.5566\ m}$$

(2) $P = ?$

Assume $\eta_{mech} = 100\%$

$$\eta_{mech} = \frac{P_{cons}}{P_{req}} = 1$$

$$P_{req} = \dot{m}\, W_{Dact}$$

$$= 16.5 \times 178.8799 \times 10^3$$

$$= 2.951\ MW$$

$$\therefore \boxed{P_{req} = 2.951\ MW}$$