Savitribai Phule Pune University

Semester VII (Mechanical and Automobile Engineering)

DYNAMICS OF MACHINERY (Code : 402042)

Chapter 3 : Single Degree of Freedom Systems : Free Vibrations

Q.1 Define the following terms :

1. Simple Harmonic Motion
2. Time Period
3. Frequency
4. Amplitude
5. Stiffness of spring
6. Degrees of Freedom

Ans. :

1. Simple Harmonic Motion (S.H.M.) :

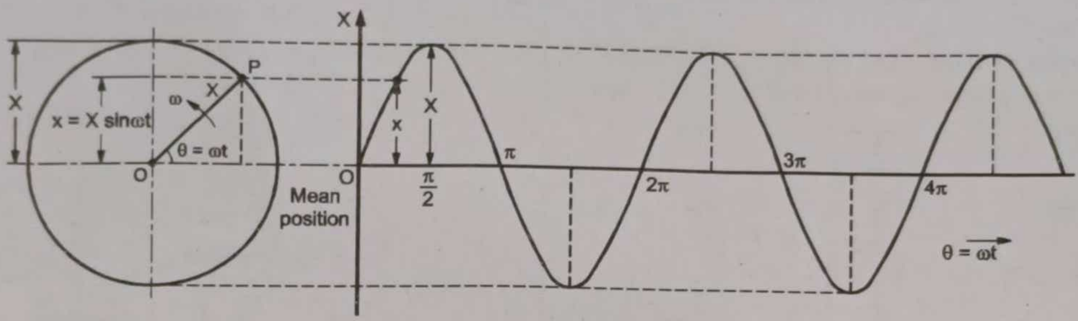


Fig. 3.1 : Simple Harmonic Motion

Let, x = displacement of point from mean position after time 't'.

X = maximum displacement of point from mean position.

From Fig. 3.1;

- **Displacement of point :**

$$x = X \sin \theta$$

or

$$x = X \sin \omega t \quad ...(1)$$

- **Velocity of point :**

$$\dot{x} = \frac{dx}{dt}$$

or

$$\dot{x} = \omega X \cos \omega t$$

- **Acceleration of point :**

$$\ddot{x} = \frac{d^2 x}{dt^2} = -\omega^2 X \sin \omega t$$

or

$$\ddot{x} = -\omega^2 x \quad ... [\because x = X \sin \omega t] \quad ...(2)$$

- Acceleration ∝ Displacement From Mean Position.
- **Simple harmonic motion** : A motion, whose acceleration is proportional to displacement from mean position and is directed towards the mean position, is known as **simple harmonic motion**.
- **Fundamental equation of SHM :**

The Equation (2) can be written as,

$$\ddot{x} + \omega^2 x = 0 \quad ...(3)$$

- The Equation (3) is known as fundamental equation of simple harmonic motion.

2. Time Period (t_p) :

Time period is the time required to complete one cycle (2π).

Mathematically,

$$t_p = \frac{2\pi}{\omega}, \quad \text{sec} \quad ...(4)$$

3. Frequency (f) :

The number of cycles per unit time is known as ***frequency***. It is a reciprocal of time period.

$$f = \frac{1}{t_p} = \frac{1}{\frac{2\pi}{\omega}} = \frac{\omega}{2\pi}, \quad \text{Hz} \quad ..(5)$$

4. Amplitude (X) :

It is the maximum displacement of a vibrating body from its mean position.

5. **Stiffness of spring (K) :**

It is the force required to produce unit displacement in the direction of applied force.

$$K = \frac{F}{\delta}\ ,\ N/m \qquad ...(6)$$

where, K = stiffness of spring, N/m

F = force applied on spring, N

δ = deflection of spring, m.

6. **Degrees of Freedom (D.O.F) :**

- *The minimum number of independent co-ordinates required to specify the motion or configuration of a system at any instant is known as* ***degrees of freedom*** *of that system.*
- In general, degrees of freedom (D.O.F.) is equal to the number of independent displacements that are possible.
- **Examples of D.O.F :** The Fig. 3.2 shows examples of one, two and three degrees of freedom (D.O.F) systems.

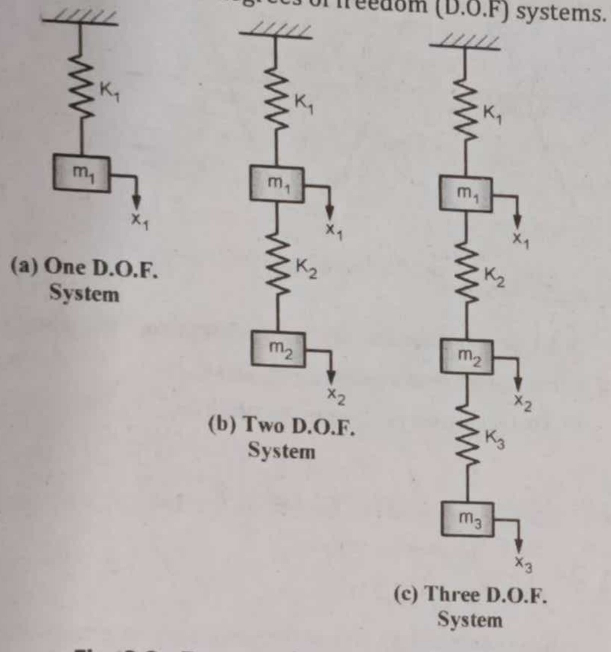


Fig. 3.2 : Degree of Freedom (D.O.F)

Q.2 Define the damping coefficient related to vibrations. **SPPU : Dec. 12, Dec. 13, May 14, Dec. 16, Dec. 17**

Ans. :

Damping Coefficient (c) :

Damping coefficient *is the damping force or resisting force developed per unit velocity.* Mathematically,

$$c = \frac{F}{v},\ \frac{N}{m / sec} \text{ or N-sec/m} \qquad ...(1)$$

where, F = Force applied on damper or damping force in N.

v = Velocity of viscous fluid in m/sec.

Q.3 How to convert multi-springs, into a single spring with linear or rotational coordinate system? **MU : May 19**

Ans. : In many practical applications, more than one springs may be used. To convert such system into equivalent mathematical model, it is necessary to replace springs in system by one equivalent spring. The stiffness of equivalent spring depends upon whether the springs in system are in series or in parallel.

1. **Springs in Series**

- Fig. 3.3(a) shows a system with two springs in series having stiffness $\mathbf{K_1}$ and $\mathbf{K_2}$.
- In case of springs in series :
  i) The total deflection of actual system is equal to sum of the deflections of individual springs (i.e. $\delta = \delta_1 + \delta_2$).
  ii) The force acting on actual system is same as the force acting on each individual spring (i.e. $mg = m_1g = m_2g$).

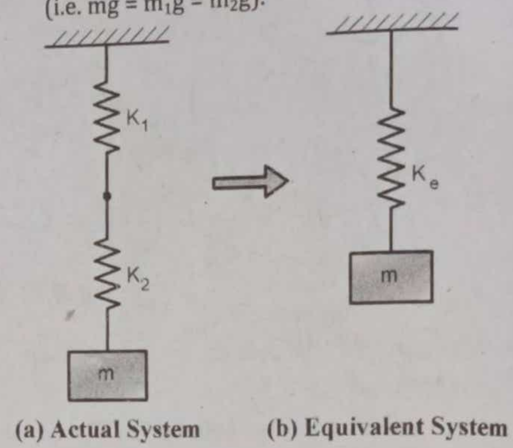


Fig. 3.3 : Spring in series

- The system of two springs in series is to be replaced by an equivalent spring having stiffness K_e, as shown in Fig. 3.3(b). then,

Deflection of equivalent spring = Deflection of actual system

Deflection of equivalent spring

= Deflection of spring 1 + Deflection of spring 2

$$\delta = \delta_1 + \delta_2 \qquad ...(a)$$

$$\text{and,} \quad mg = m_1g = m_2g \qquad ...(b)$$

$$\therefore \frac{mg}{K_e} = \frac{m_1g}{K_1} + \frac{m_2g}{K_2} \qquad ...(c)$$

Substituting Equation (b) in Equation (c), we get,

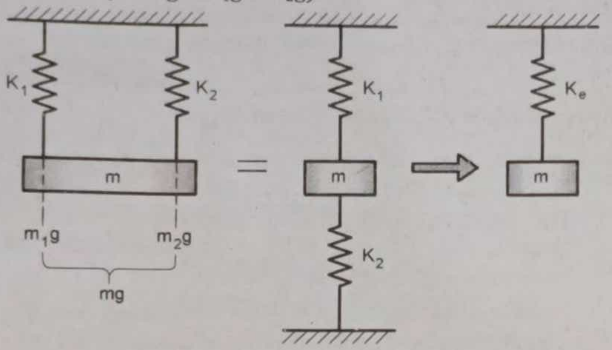
$$\frac{mg}{K_e} = \frac{mg}{K_1} + \frac{mg}{K_2}$$

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} \qquad ...(1)$$

- *Thus, in case of **springs in series**, the reciprocal of the equivalent spring stiffness is equal to the sum of the reciprocal of individual spring stiffnesses.*

2. **Springs in Parallel**

- Fig. 3.4(a) shows a system with two springs in parallel having stiffness K_1 and K_2.
- In case of springs in parallel :
  (i) The deflection of actual system is equal to the deflections of each springs (i.e. $\delta = \delta_1 = \delta_2$).
  (ii) The force acting on actual system is equal to sum of the forces acting on individual springs (i.e. $mg = m_1g + m_2g$).



(a) Actual System (b) Another Form of Actual System (c) Equivalent System

Fig. 3.4 : Springs in Parallel

- The system of two springs in parallel is replaced by an equivalent spring having stiffness K_e , as shown in Fig. 3.4(b). then,

Deflection of equivalent spring = Deflection of spring 1 = Deflection of spring 2

$$\delta = \delta_1 = \delta_2 \quad \text{... (d)}$$

and,

$$mg = m_1g + m_2g \quad \text{...(e)}$$

$$\therefore K_e\delta = K_1\delta_1 + K_2\delta_2 \quad \text{...(f)}$$

Substituting Equation (d) in Equation (f), we get,

$$K_e\delta = K_1\delta + K_2\delta$$

$$\therefore K_e = K_1 + K_2 \quad \text{...(2)}$$

*Thus, in case of **springs in parallel**, the equivalent stiffness is equal to the sum of the individual spring stiffnesses.*

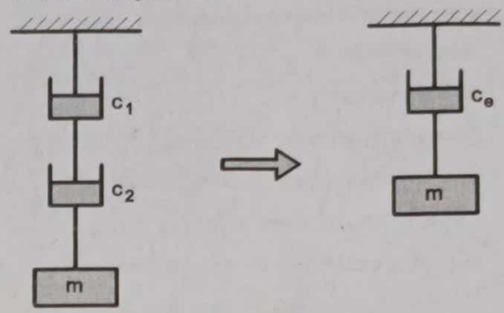
Q.4 How to convert multi dampers into a single damper with linear or rotational coordinate system? **MU : May 19**

Ans. : Equivalent Dampers

In many applications, similar to springs, number of dampers are also used in combinations. To convert such system into equivalent mathematical model, it is necessary to replace dampers in system by one equivalent damper. The damping coefficient of equivalent damper depends upon whether the dampers are in series or parallel.

1. **Dampers in Series**

- Fig 3.5(a) shows a system having two dampers, with damping coefficients c_1 and c_2 in series. These dampers are replaced by an equivalent damper having damping coefficient c_e, as shown in Fig.3.5(b).



(a) Actual System (b) Equivalent System

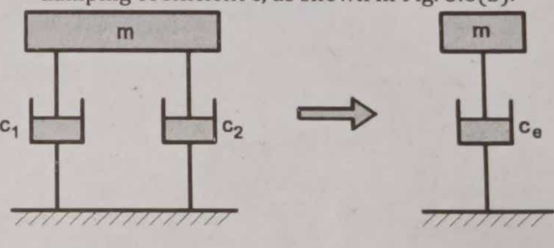
Fig. 3.5 : Dampers in Series

- *The theory of springs in series is also applicable for dampers in series. Thus in case of **dampers in series**, the reciprocal of the equivalent damping coefficient is equal to the sum of the reciprocal of individual damping coefficients.*

$$\frac{1}{c_e} = \frac{1}{c_1} + \frac{1}{c_2} \quad \text{...(1)}$$

2. **Dampers in Parallel**

- Fig. 3.6(a) shows a system, having two dampers with damping coefficients c_1 and c_2, in parallel. These dampers are replaced by an equivalent damper having damping coefficient c, as shown in Fig. 3.6(b).



(a) Actual System (b) Equivalent System

Fig. 3.6 : Dampers in Parallel

- The theory of springs in parallel is also applicable for dampers in parallel. Thus in case of **dampers in parallel**, the equivalent damping coefficient is equal to the sum of the individual damping coefficients.

$$c_e = c_1 + c_2 \quad \text{...(2)}$$

Q.5 Explain with neat diagram mathematical model of a motorbike.

SPPU : May 16, May 17, Oct. 18(In sem), May 19

Ans. :

Fig. 3.7(a) shows a physical system consisting of motor bike with rider.

- **Mathematical model of motor bike and rider :** In order to develop a mathematical model of a physical system consisting of a motor bike and a rider, following parameters are considered :
  1. Mass of rider, m_r
  2. Stiffness of rider, K_r
  3. Damping coefficient of rider, C_r
  4. Mass of vehicle body (except wheels), m_v
  5. Stiffness of rear suspension, K_{s1}
  6. Damping coefficient of rear suspension, C_{s1}
  7. Stiffness of front suspension, K_{s2}
  8. Damping coefficient of front suspension, C_{s2}
  9. Mass of each wheel, m_w
  10. Stiffness of each tyre K_t
- Fig. 3.7(b) shows a mathematical model of a physical system consisting of a motor bike and a rider.
- **Simplified mathematical model of motor bike :**
  i) If the stiffness and damping coefficient of rider are neglected, the mathematical model can be simplified as shown in Fig. 3.7(c).
  ii) Assuming that front and rear suspensions are identical, the mathematical model can be further simplified, as shown in Fig. 3.7(d).

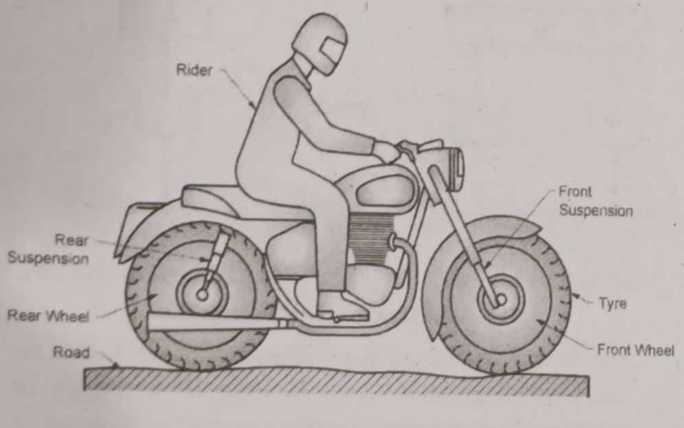
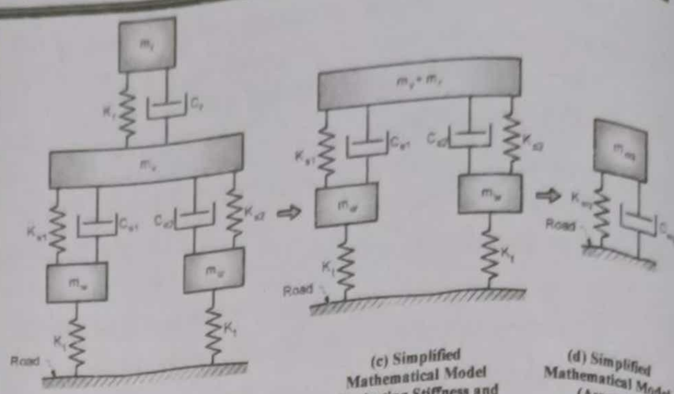


Fig. 3.7(a) : Physical System-Motor Bike and Rider



(b) Real Mathematical Model

(c) Simplified Mathematical Model (Neglecting Stiffness and Damping Coefficient of Rider)

(d) Simplified Mathematical Model (Assuming $K_{s1} = K_{s2} = K_s$ and $C_{s1} = C_{s2} = C_s$)

Fig. 3.7 : Mathematical Model of Bike

Q.6 Explain mathematical model of a bicycle with a rider.

SPPU : Dec. 19

Ans. : Mathematical Modeling of Bicycle :

- The mathematical modeling of a bicycle is similar to that of the motor bike. The only difference is, in bicycle the damping coefficient of suspension system is neglected i.e. $C_{s1} = C_{s2} = 0$. Fig. 3.8(a) shows a physical system consisting of motor bike with rider.
- **Mathematical model of bicycle :** In order to develop a mathematical model of a physical system consisting of a bicycle and a rider, following parameters are considered :
  1. Mass of rider, m_r
  2. Stiffness of rider, K_r
  3. Damping coefficient of rider, C_r
  4. Mass of bicycle (except wheels), m_v
  5. Stiffness of rear suspension, K_{s1}
  6. Stiffness of front suspension, K_{s2}
  7. Mass of each wheel, m_w
  8. Stiffness of each tyre K_t

  Fig. 3.8(a) shows the physical model of bicycle with rider while Fig. 3.8(b) shows the mathematical model of bicycle.

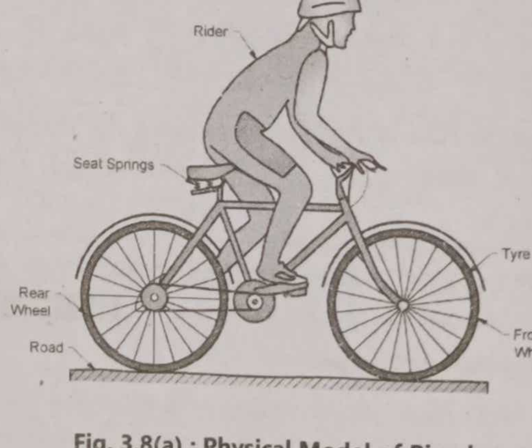
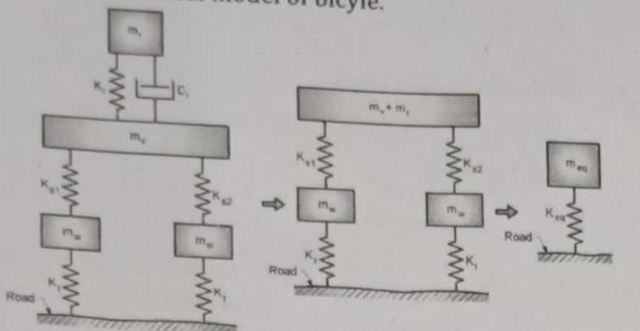


Fig. 3.8(a) : Physical Model of Bicycle

- **Simplified mathematical model of bicycle :**

  Fig. 3.8(c) and Fig. 3.8(d) show the simplified mathematical model of bicyle.



(b) Real Mathematical Model

(c) Simplified Mathematical Model (Neglecting Stiffness and Damping Coefficient of Rider)

(d) Simplified Mathematical Model (Assuming $K_{s1} = K_{s2} = K_s$)

Fig. 3.8 : Mathematical Model of Bicycle

Q.7 Explain with neat diagram mathematical model of a quarter car. **SPPU : Oct. 19(In Sem.)**

Ans. : Mathematical Model of Car :

Fig. 3.9(a) shows a physical system of a car.

- **Mathematical model of car :** In order to develop a mathematical model of a physical system of car, following parameters are considered :
  1. Mass of driver and driver seat, m_d
  2. Stiffness of driver seat spring, K_d
  3. Damping coefficient of driver seat, C_d
  4. Stiffness of back rest, K_b
  5. Damping coefficient of back rest, C_b
  6. Mass of vehicle body, m_v
  7. Stiffness of rear suspension, K_{S1}
  8. Damping coefficient of rear suspension, C_{s1}
  9. Stiffness of front suspension, K_{S2}
  10. Damping coefficient of front suspension, C_{s2}
  11. Mass of rear wheel and axle, mW_1
  12. Mass of front wheel and axle, mw_2
  13. Stiffness of rear tyre, K_{t1}
  14. Stiffness of front tyre, K_{t2}

  Fig. 3.9(b) shows a mathematical model of a car.

Simplified mathematical model of car :

(i) If the stiffness and damping coefficient of driver seat is neglected, the mathematical model can be simplified as shown in Fig. 3.9(c).

(ii) Assuming that the front and rear suspensions are identical, the mathematical model can be simplified as shown in Fig. 3.9(d).

- The chassis is a largest mass supported on the front and rear axles through the suspension systems. The suspension systems between the chassis and the front and rear axles are represented by springs and dampers.
- The wheels and axles are equivalent to the masses supported by the tyres. The tyres have elasticity, and hence, are represented by the springs between the wheels and the road.
- The driver seat is cushioned from the chassis motion. Therefore, a driver seat is equivalent to a mass supported by a spring and a damper on the chassis.
- There is a friction between the driver seat back and the support.
- This modeling is done with the assumption that the vehicle is traveling at a constant speed and there is no horizontal inertia effect.

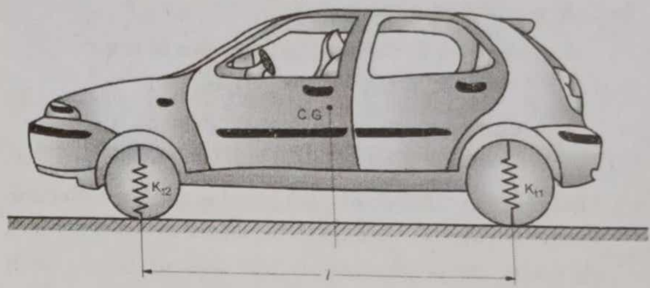
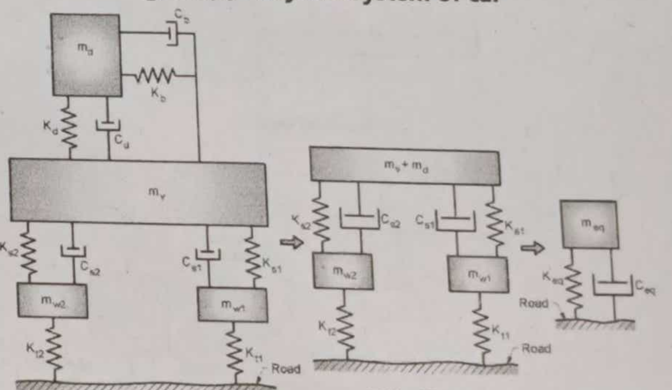


Fig. 3.9(a) : Physical system of car



(b) Real Mathematical Model

(c) Simplified Mathematical Model (Neglecting Stiffness and Damping Coefficient of Driver Seat)

(d) Simplified Mathematical Model (Assuming $K_{s1} = K_{s2} = K_s$ and $C_{s1} = C_{s2} = C_s$)

Fig. 3.9 : Mathematical Model of Car

Q.8 Explain : Free vibrations.

OR What is natural frequency? **SPPU : May 19**

Ans. : Free Vibrations :

- *If the external force is removed after giving an initial displacement to the system, then the system vibrates on its own due to internal elastic forces. Such type of vibrations are knows as* ***free vibrations****. The frequency of free vibrations is known as* ***free*** *or* ***natural frequency*** *(f_n).*

Example : Oscillation of simple pendulum .

Q.9 What are the methods to determine the equation of motion for the vibratory system ?

Ans. : Following three methods are used to determine the natural frequency of the given body or system :

1. Equilibrium Method (D'Alembert's Principle)
2. Energy Method
3. Rayleigh's Method

Q.10 A light cantilever of length l, has a mass M, fixed at its free end. Find the frequency of its lateral vibrations in the vertical plane.

SPPU : Dec. 19

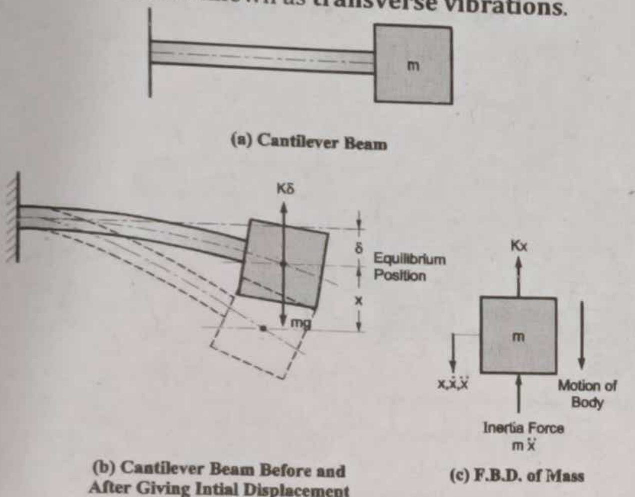
Ans. : Consider a cantilever beam of negligible mass carrying a concentrated mass 'm' at free end, as shown in Fig. 3.10(a).

Let, K = stiffness of mass, N/m

δ = static deflection of the beam due to mass attached at the end, m

x = displacement of the mass from mean position after applying initial external force, m

- **Transverse vibrations :** If a beam is given a deflection perpendicular to its axis, it oscillates or vibrates in a direction perpendicular to the axis of beam. Such vibrations are known as **transverse vibrations**.



(a) Cantilever Beam

(b) Cantilever Beam Before and After Giving Intial Displacement

(c) F.B.D. of Mass

Fig. 3.10 : Undamped Free Transverse Vibrations

- **At equilibrium (mean) position :** Due to gravitational force **'mg'**, the cantilever beam is deflected by **'δ'** as shown in Fig. 3.10(b).

$$mg = K\delta \quad ...(a)$$

- **Forces acting on beam :** Let, the system is subjected to one time initial external force due to which it will displaced by 'x' from equilibrium (mean) position as shown in Fig. 3.10(b) by dotted line.

Forces acting on the mass beyond equilibrium (mean) position, on the mass are :

(i) Inertia force, $m\ddot{x}$ (upwards)

(ii) Resisting or restoring force, Kx (upwards)

- **D'Alembert's principle :**

$$\sum [\text{Inertia Force + External Forces}] = 0$$

From Fig. 3.10,

$$\therefore \quad m\ddot{x} + Kx = 0$$

[Taking upward force as + ve downward force as – ve]

$$\therefore \quad \ddot{x} + \frac{K}{m}x = 0 \quad ...(b)$$

- Comparing Equation (b) with fundamental equation of simple harmonic motion, we get,

$$\omega_n^2 = \frac{K}{m}$$

or

$$\omega_n = \sqrt{\frac{K}{m}}, \text{ rad/s} \quad ...(c)$$

- **Natural frequency of vibrations (f_n) :**

$$f_n = \frac{\omega_n}{2\pi}$$

or

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}, \text{ Hz} \quad ...(d)$$

- From Equation (a),

$$\frac{K}{m} = \frac{g}{\delta}, \text{ Hz} \quad ...(e)$$

- Substituting the Equation (e) in Equation (d), we get,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}, \text{ Hz} \quad ...(f)$$

- **Static defection for different types of beams :**

Let W = load at the free end, N.

l = length of shaft beam, m.

E = Young's modulus for the material of beam, N/m^2.

I = moment of inertia of beam about horizontal axis, m^4.

Q.11 For the system shown in Fig. 3.11(a), if K_1 = 2400 N/m, K_2 = 1600 N/m, K_3 = 3600 N/m and K_4 = K_5 = 500 N/m; find the mass m such that the system will have a natural frequency of 10 Hz.

SPPU - Oct. 19 (In Sem), 6 Marks

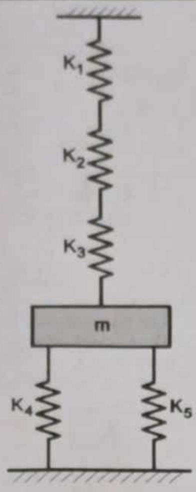
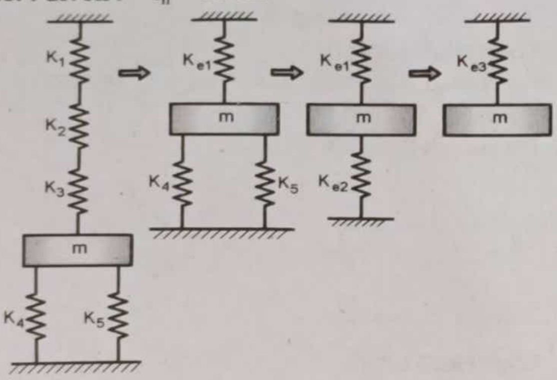


Fig. 3.11(a)

Ans. : Given : $f_n = 10$ Hz.



(b) (c) (d) (e)

Fig. 3.11

From Fig. 3.11(b) :

The springs K_1, K_2, K_3 are in series. Therefore, their equivalent stiffness K_{e1} is given by,

$$\frac{1}{K_{el}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

$$\therefore \quad \frac{1}{K_{el}} = \frac{1}{2000} + \frac{1}{1500} + \frac{1}{3000}$$

$$K_{el} = 666.66 \text{ N/m}$$

- **From Fig. 3.11(c) :**

  The springs K_4 and K_5 are in parallel. Therefore, their equivalent stiffness K_{e2} is given by,

  $$K_{e2} = K_4 + K_5 = 500 + 500 = 1000 \text{ N/m}$$

- **From Fig. 3.11(d) :**

  The spring K_{el} and K_{e2} are in parallel. Therefore, their equivalent stiffness K_{e3} is given by,

  $$K_{e3} = K_{el} + K_{e2} = 666.66 + 1000$$

  $$= 1666.66 \text{ N/m}$$

- **From Fig. 3.11(e) :**

  The natural frequency for spring-mass system is given by,

  $$f_n = \frac{1}{2\pi}\sqrt{\frac{K_{e3}}{m}}$$

  $$\therefore \quad 10 = \frac{1}{2\pi}\sqrt{\frac{1666.66}{m}}$$

  $$\therefore \quad \mathbf{m = 0.4221 \text{ kg}}$$ ...Ans.

Q.12 For the system shown in following Fig. 3.12(A) : $k_1 = 2000$ N/m, $k_2 = 1500$ N/m, $k_3 = 3000$ N/m and $k_4 = k_5 = 500$ N/m. Find mass (M), such that the system has a natural frequency of 10 Hz.

SPPU : Oct. 19(In Sem.), 6 Marks

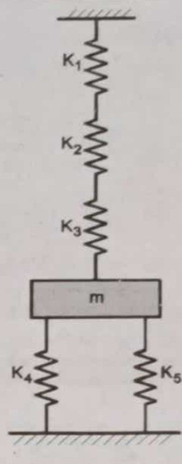
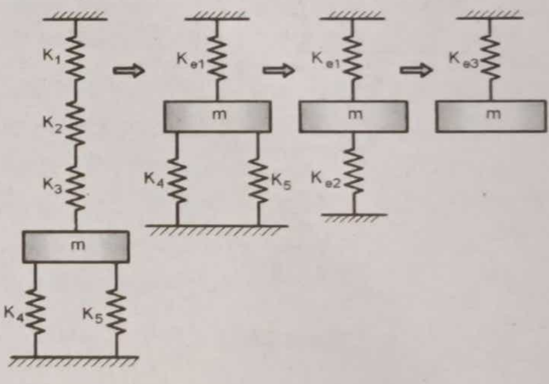


Fig. 3.12(A)

Ans. :

Given : $f_n = 10$ Hz.



(b) (c) (d) (e)

Fig. 3.12(B)

- **From Fig. 3.12(B)(b) :**

The springs K_1, K_2, K_3 are in series. Therefore, their equivalent stiffness K_{e1} is given by,

$$\frac{1}{K_{el}} = \frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}$$

$$\therefore \quad \frac{1}{K_{el}} = \frac{1}{2000}+\frac{1}{1500}+\frac{1}{3000}$$

$$K_{el} = 666.66 \text{ N/m}$$

- **From Fig. 3.12(B)(c):**

The springs K_4 and K_5 are in parallel. Therefore, their equivalent stiffness K_{e2} is given by,

$$K_{e2} = K_4 + K_5 = 500 + 500 = 1000 \text{ N/m}$$

- **From Fig. 3.12(B)(d) :**

The spring K_{el} and K_{e2} are in parallel. Therefore, their equivalent stiffness K_{e3} is given by,

$$K_{e3} = K_{el} + K_{e2} = 666.66 + 1000 = 1666.66 \text{ N/m}$$

- **From Fig. 3.12(B)(e) :**

The natural frequency for spring-mass system is given by,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_{e3}}{m}}$$

$$\therefore \quad 10 = \frac{1}{2\pi}\sqrt{\frac{1666.66}{m}}$$

$$\therefore \quad \mathbf{m = 0.4221 \text{ kg}} \qquad \text{...Ans.}$$

Q.13 For the mathematical model shown in Fig. 3.13(a), determine the equivalent stiffness.

SPPU - Dec. 14, 8 Marks, Dec. 18(In sem), 5 Marks

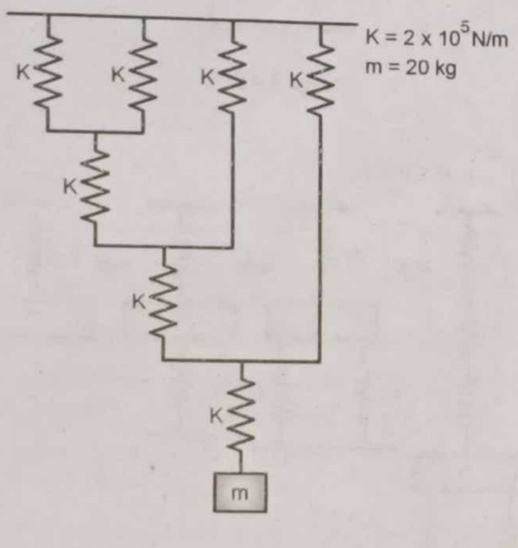


Fig. 3.13(a)

Ans. :

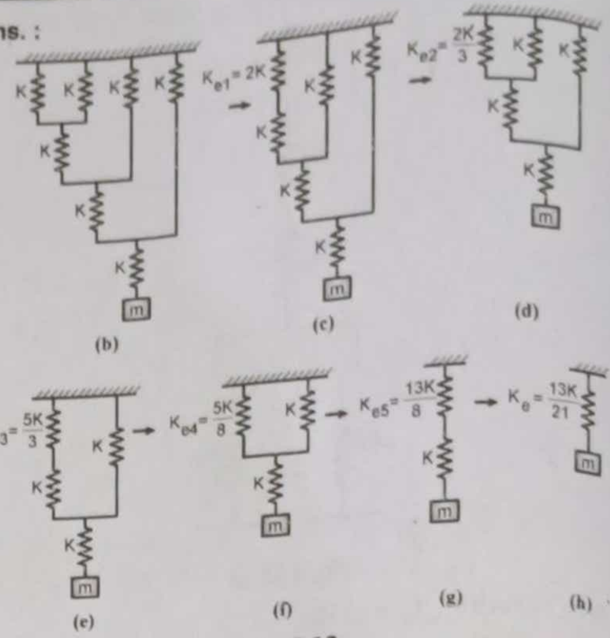


Fig. 3.13

- **From Fig. 3.13(b) :**

For springs K and K in parallel, $K_{e1} = K + K = 2K$

- **From Fig. 3.13(c) :**

For springs K_{e1} and K in series,

$$\frac{1}{K_{e2}} = \frac{1}{K_{e1}}+\frac{1}{K}$$

$$\frac{1}{K_{e2}} = \frac{1}{2K}+\frac{1}{K} = \frac{3}{2K}$$

$$\therefore \quad K_{e2} = \frac{2K}{3}$$

- **From Fig. 3.13(d) :**

For springs K_{e2} and K in parallel,

$$K_{e3} = K_{e2} + K = \frac{2K}{3} + K = \frac{5K}{3}$$

- **From Fig. 3.13(e) :**

For springs K_{e3} and K in series,

$$\frac{1}{K_{e4}} = \frac{1}{K_{e3}}+\frac{1}{K} = \frac{1}{5K/3}+\frac{1}{K} = \frac{3}{5K}+\frac{1}{K} = \frac{8}{5K}$$

$$\therefore \quad K_{e4} = \frac{5K}{8}$$

- **From Fig. 3.13(f) :** For springs K_{e4} and K in parallel

$$K_{e5} = K_{e4} + K = \frac{5K}{8} + K = \frac{13\,K}{8}$$

- **From Fig. 3.13(g) :**

For springs K_{e5} and K is series,

$$\frac{1}{K_e} = \frac{1}{K_{e5}}+\frac{1}{K} = \frac{1}{13\,K/8}+\frac{1}{K} = \frac{8}{13K}+\frac{1}{K} = \frac{21}{13\,K}$$

$$\therefore \quad K_e = \frac{13\,K}{21}$$

- **From Fig. 3.13(h) :**

Natural frequency of the system is,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_e}{m}} = \frac{1}{2\pi}\sqrt{\frac{\frac{13K}{21}}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{13 \times 2 \times 10^5}{21 \times 20}}$$

$$f_n = \mathbf{12.52\ Hz} \qquad \text{...Ans.}$$

Q.14 Find the natural frequency of oscillation for the roller rolling on horizontal surface without slipping, as shown in Fig. 3.14(a). The mass of roller is 5 kg, radius of roller is 50 mm and stiffness of spring is 2000 N/m. What would be the new frequency of oscillation, if radius of roller is made 100 mm without changing the mass ? **SPPU : Oct. 18(In sem), 6 Marks**

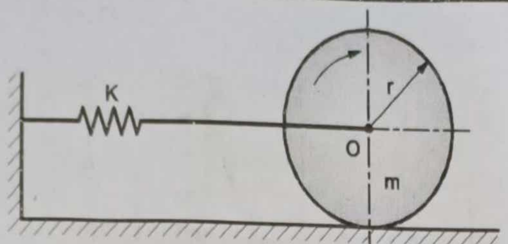


Fig. 3.14(a)

Ans. : If the roller is displaced through a small linear distance 'x', it will also rotate through an angle 'θ', as shown in Fig. 3.14(b).

- Let 'F_r' be the frictional force acting at the point of contact between roller and surface.

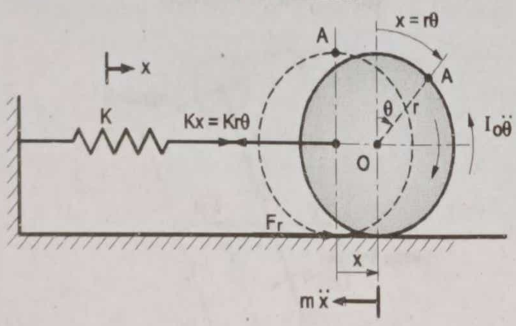


Fig. 3.14(b) : Displaced Position

- **From Fig. 3.14(b) :**

Angular displacement of roller	$= \theta$
Linear displacement of roller	$= x = r\theta$
Linear velocity of roller	$= \dot{x} = r\dot{\theta}$
Linear acceleration of roller	$= \ddot{x} = r\ddot{\theta}$

[I] Equilibrium Method

From Fig. 3.14(b) ;

- **Linear motion of roller :**

$$\sum[\text{Inertia force + External forces}] = 0$$

$$m\ddot{x} + Kr\theta - Fr = 0$$

$$\therefore \ Fr = (m\ddot{x} + Kr\theta) \qquad \text{...(a)}$$

- **Rotary motion of roller :**

$$\sum[\text{Inertia torque + External torques}] = 0$$

$$\therefore \ I_o\ddot{\theta} + F_r \cdot r = 0 \qquad \text{...(b)}$$

Substitute Equation (a) in Equation (b),

$$\frac{1}{2}mr^2\ddot{\theta} + (m\ddot{x} + Kr\theta)\,r = 0$$

$$\frac{1}{2}mr^2\ddot{\theta} + mr^2\ddot{\theta} + Kr^2\theta = 0$$

$$\left(\frac{1}{2}mr^2 + mr^2\right)\ddot{\theta} + Kr^2\theta = 0$$

$$\left(\frac{3}{2}mr^2\right)\ddot{\theta} + Kr^2\theta = 0$$

$$\ddot{\theta} + \left(\frac{Kr^2}{\frac{3}{2}mr^2}\right)\theta = 0$$

$$\therefore \ \ddot{\theta} + \left(\frac{2K}{3m}\right)\theta = 0 \qquad \text{...(c)}$$

- **Natural circular frequency :**

This Equation (c) is the differential equation of motion for a given system. Comparing this equation with fundamental equation of simple harmonic motion, we get,

$$\omega_n^2 = \frac{2K}{3m}$$

$$\omega_n = \sqrt{\frac{2K}{3m}}\ \text{, rad/sec} \qquad \text{...(d)}$$

- **Natural frequency :**

$$\text{New,}\quad f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2K}{3m}},\ \text{Hz} = \frac{1}{2\pi}\sqrt{\frac{2 \times 2000}{3 \times 5}}$$

or $\mathbf{f_n = 2.59\ Hz}$ ...**Ans.**

From Equation (d), it is seen that, the natural frequency of oscillation of roller is independent of its radius. Hence , even, if the radius is increased to 100 mm (by keeping the mass same), the natural frequency remains unchanged.

[II] Energy Method

From Fig. 3.14(a);

- **K. E. of system :**

$$\text{Linear of K.E. roller} = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}mr^2\dot{\theta}^2$$

$$\text{Rotary K. E. of roller} = \frac{1}{2}I_o\omega^2 = \frac{1}{2}I_o\dot{\theta}^2 = \frac{1}{4}mr^2\dot{\theta}^2$$

Total kinetic energy is,

$$KE = \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{4}mr^2\dot{\theta}^2$$

- **P. E. of system :**

Potential energy of the spring $= \frac{1}{2} Kx^2 = \frac{1}{2} Kr^2 \theta^2$

$$PE = \frac{1}{2} K r^2 \theta^2$$

- **Energy method :**

$$\frac{d}{dt}(KE + PE) = 0$$

$$\therefore \frac{d}{dt}\left(\frac{1}{2} mr^2 \dot{\theta}^2 + \frac{1}{4} mr^2 \dot{\theta}^2 + \frac{1}{2} Kr^2 \theta^2\right) = 0$$

$$\frac{1}{2} mr^2\, 2\, \dot{\theta}\, \ddot{\theta} + \frac{1}{4} mr^2\, 2\, \dot{\theta}\, \ddot{\theta} + \frac{1}{2} Kr^2\, 2\, \theta\, \dot{\theta} = 0$$

$$mr^2 \ddot{\theta} + \frac{1}{2} mr^2 \ddot{\theta} + Kr^2 \theta = 0$$

$$\left(mr^2 + \frac{1}{2} mr^2\right)\ddot{\theta} + Kr^2 \theta = 0$$

$$\left(\frac{3\, mr^2}{2}\right)\ddot{\theta} + Kr^2 \theta = 0$$

$$\ddot{\theta} + \left[\frac{Kr^2}{\left(\frac{3}{2} mr^2\right)}\right]\theta = 0$$

$$\ddot{\theta} + \left(\frac{2K}{3m}\right)\theta = 0 \qquad \text{...(e)}$$

- **Natural circular frequency :** This Equation (d) is the differential equation of motion for a given system. Comparing this equation with fundamental equation of simple harmonic motion, we get,

$$\omega_n^2 = \frac{2K}{3m}$$

$$\therefore \omega_n = \sqrt{\frac{2K}{3m}}, \text{ rad/s}$$

- **Natural frequency :**

$$f_n = \frac{\omega_n}{2\pi}$$

$$\text{or} \quad f_n = \frac{1}{2\pi}\sqrt{\frac{2K}{3m}}, \text{ Hz} \qquad \text{...(f)}$$

$$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{2 \times 2000}{3 \times 5}}$$

$$\mathbf{f_n = 2.59\ Hz} \qquad \text{...Ans.}$$

- From Equation (f) it is seen that, the natural frequency of oscillation of roller is independent of its radius. Hence , even, if the radius is increased to 100 mm (by keeping the mass same), the natural frequency remains unchanged.

Q.15 Find the natural frequency of vibration for the system shown in Fig. 3.15(a). Establish the condition for system to be non vibratory, in terms of '**b**'.

SPPU- Dec. 16, Dec, 17, 4 Marks

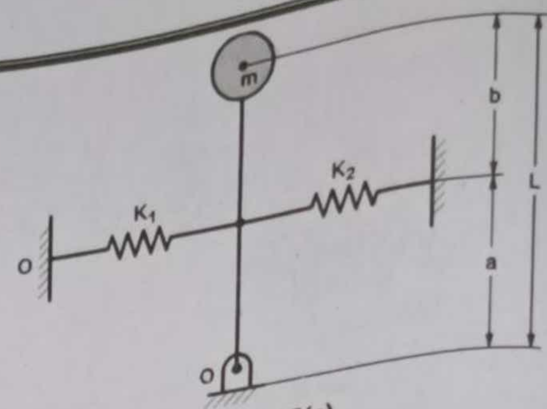
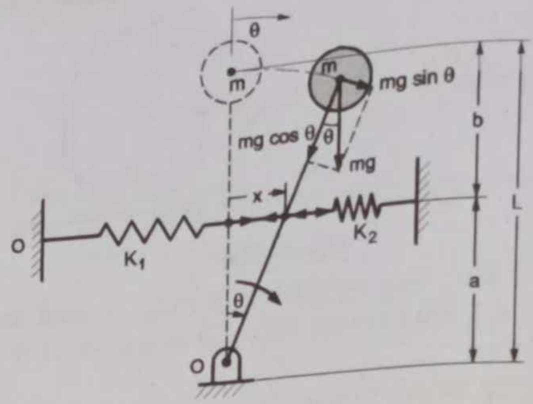


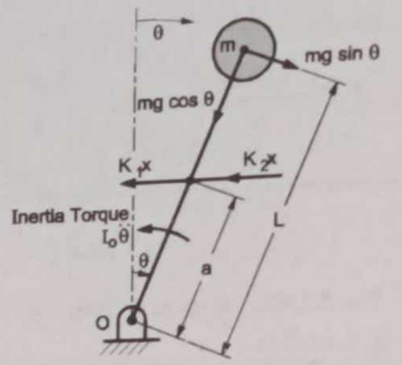
Fig. 3.15(a)

Ans. :

Fig. 3.15 shows the system when mass 'm' is displaced through an angle '**θ**' due to which the spring **K_1** is stretched by distance 'x' and the spring '**K_2**' is compressed by distance 'x'.



(b) Displaced Position



(c) F.B.D. of System

Fig. 3.15

- **From Fig. 3.15(b) :**

Deflection in both springs = $x \simeq a\,\theta$
Spring force or restoring force by spring, $K_1 = K_1 x = K_1 a \theta$
Spring force or restoring force by spring $K_2 = K_2 x = K_2 a \theta$
Mass moment of moment of inertia about O, $I_o = m L^2$

[I] Equilibrium Method : From Fig. 3.15(b) ;

- **Angular motion of body about point 'O' :**

$$\Sigma\,[\text{Inertia torque + External torques}] = 0$$

$$\therefore\ \text{Io}\ \theta + (\text{K1 x a} + \text{K2 x a}) - \text{mg L} \sin\theta = 0$$

$$\therefore\ \text{Io}\ \theta + (\text{K1 a}\ \theta\ \text{a} + \text{K2 a}\ \theta\ \text{a}) - \text{mg L} \sin\theta = 0$$

$$\text{Io}\ \theta + (\text{K1} + \text{K2})\ \text{a2}\ \theta - \text{mgL}\theta = 0$$

$$\ldots[\because \sin\theta \simeq \theta]$$

$$\therefore\ \ddot{\theta} + \left[\frac{(K_1 + K_2)\,a^2 - mgL}{I_o}\right]\theta = 0 \qquad \ldots(a)$$

- **Natural circular frequency :** This Equation (a) is the differential equation of motion for a given system. Comparing this equation with the fundamental equation of simple harmonic motion, we get,

$$\omega_n^2 = \frac{(K_1 + K_2)\,a^2 - mgL}{I_o}$$

$$\therefore\ \omega_n = \sqrt{\frac{(K_1 + K_2)\,a^2 - mgL}{I_o}}$$

$$\text{or}\quad \omega_n = \sqrt{\frac{(K_1 + K_2)\,a^2 - mgL}{mL^2}}\ \text{, rad/sec}$$

- **Natural frequency :**

$$f_n = \frac{\omega_n}{2\pi}$$

$$\text{or}\quad f_n = \frac{1}{2\pi}\sqrt{\frac{(K_1 + K_2)\,a^2 - mgL}{mL^2}}\ \text{, Hz} \qquad \ldots\text{Ans.}$$

- **Condition for system to be non-vibratory :**

for non vibration motion, $f_n \leq 0$.

$$\frac{1}{2\pi}\sqrt{\frac{(K_1 + K_2)\,a^2 - mgL}{mL^2}} \leq 0$$

$$\therefore\ \frac{(K_1 + K_2)\,a^2}{mL^2} - \frac{mgL}{mL^2} \leq 0$$

$$\therefore\ \frac{(K_1 + K_2)\,a^2}{mL^2} \leq \frac{g}{L}$$

$$\therefore\ a^2 \leq \frac{gmL}{(K_1 + K_2)}$$

$$\therefore\ a \leq \sqrt{\frac{mgL}{K_1 + K_2}} \qquad \ldots(b)$$

but $\quad a = L - b$

Hence, Equation (b) can be written as,

$$L - b \leq \sqrt{\frac{mgL}{K_1 + K_2}}$$

$$\therefore\ L - \sqrt{\frac{mgL}{K_1 + K_2}} \leq b$$

$$\therefore\ \mathbf{b \geq L - \sqrt{\frac{mgL}{K_1 + K_2}}} \ldots(c) \qquad \ldots\text{Ans.}$$

[II] Energy Method

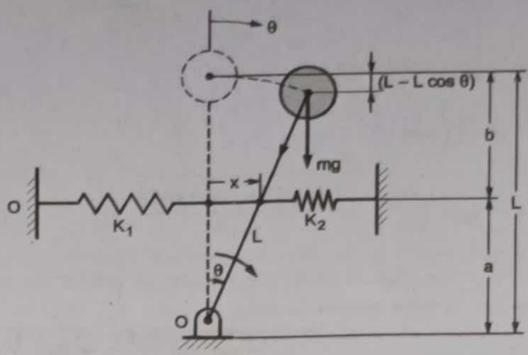


Fig. 3.15(d)

From Fig. 3.15(d);

- **K. E. of system :**

Kinetic energy of mass, $KE = \frac{1}{2} I_o \dot{\theta}^2$

- **P. E. of system :**

$$\text{Potential energy of mass} = -mg\,(L - L\cos\theta)$$

$$\text{Potential energy of spring 1} = \frac{1}{2}K_1 x^2 = \frac{1}{2}K_1 a^2\theta^2$$

$$\text{Potential energy of spring 2} = \frac{1}{2}K_2 x^2 = \frac{1}{2}K_2 a^2\theta^2$$

Total potential energy is,

$$PE = -mg\,(L - L\cos\theta) + \frac{1}{2}K_1 a^2\theta^2 + \frac{1}{2}K_2 a^2\theta^2$$

Energy Method :

$$\frac{d}{dt}(KE + PE) = 0$$

$$\therefore\ \frac{d}{dt}\left(\frac{1}{2}I_o\dot{\theta}^2 - mg\,(L - L\cos\theta) + \frac{1}{2}K_1 a^2\theta^2 + \frac{1}{2}K_2 a^2\theta^2\right) = 0$$

$$\frac{1}{2}I_o\,2\dot{\theta}\,\ddot{\theta} - mgL\sin\theta\,\dot{\theta} + \frac{1}{2}K_1 a^2\,2\theta\,\dot{\theta} + \frac{1}{2}K_2 a^2\,2\theta\,\dot{\theta} = 0$$

$$I_o\ddot{\theta} - mgL\theta + (K_1 + K_2)\,a^2\theta = 0$$

$$\ldots[\because \theta \text{ is small}, \sin\theta \simeq \theta]$$

$$I_o\ddot{\theta} + [(K_1 + K_2)\,a^2 - mgL]\,\theta = 0$$

$$\therefore\ \ddot{\theta} + \left[\frac{(K_1 + K_2)^2 - mgL}{I_o}\right] = 0 \qquad \ldots(d)$$

- **Natural circular frequency :** This Equation (d) is the differential equation of motion for a given system, comparing this equation with the fundamental equation of simple harmonic motion, we get,

$$\omega_n^2 = \frac{(K_1 + K_2)\,a^2 - mgL}{I_o}$$

$$\therefore\ \omega_n = \sqrt{\frac{(K_1 + K_2)\,a^2 - mgL}{I_o}}$$

or $\omega_n = \sqrt{\dfrac{(K_1 + K_2)\,a^2 - mgL}{mL^2}}$, rad/sec

- **Natural frequency :**

$$f_n = \frac{\omega_n}{2\pi}$$

or $f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{(K_1 + K_2)\,a^2 - mgL}{mL^2}}$, Hz ...Ans.

Q.16 Determine the natural frequency of vibration of a system shown in Fig. 3.16(a).

SPPU- Aug. 17(In Sem), 6 Marks

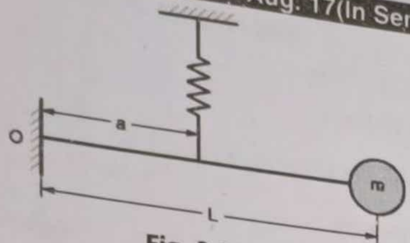


Fig. 3.16(a)

Ans. : Mass moment of inertia of system, $I_o = mL^2$

- **Static deflection of system :** Due to weight **'mg'** the system is initially deflected by small angle **'ϕ'** and spring **'K'** is deflected by distance **'δ'**, as shown in Fig. 3.16(b).

For equilibrium,

$$mg \times L \simeq K\delta \times a \quad \text{...(a)}$$

When system is deflected through an angle **'θ'**, the spring is stretched by distance **'x'** and total deflection of spring is $(x + \delta)$, as shown in Fig. 3.16(b).

[I] Equilibrium Method

From Fig. 3.16(d) ;

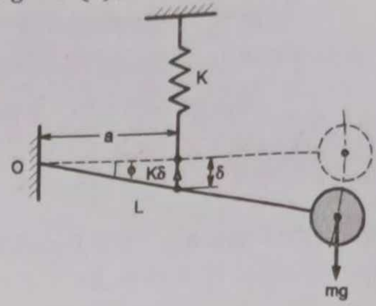


Fig. 3.16(b) : Initial Deflection of System

- **Angular motion about print 'O' :**

$\sum$ [Inertia torque + External torques]=0

$$\therefore\ I_o\ddot{\theta} + K(x+\delta)\,a - mg\cos(\phi+\theta)\,\theta \times L = 0$$

$$I_o\ddot{\theta} + Kxa + K\delta a - mgL = 0$$

...[∵ $\cos(\phi+\theta) \approx 1 \simeq 1$]

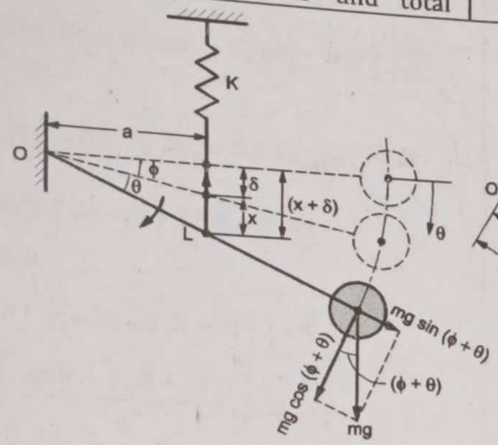
$$I_o\ddot{\theta} + Kxa + mgL - mgL = 0$$

...[∵ from Equation (a) $K\delta\,a \simeq mgL$]

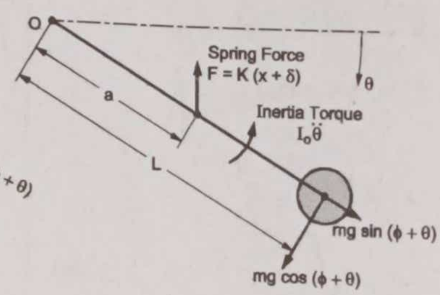
$$I_o\ddot{\theta} + Kxa = 0$$

$$I_o\ddot{\theta} + Ka^2\theta = 0 \quad \text{...[∵ } x \simeq a\theta]$$

$$\therefore\ \ddot{\theta} + \left(\frac{Ka^2}{I_o}\right)\theta = 0 \quad \text{...(b)}$$



(c) Displaced Position



(d) F.B.D. of System

Fig. 3.16

- **Natural circular frequency :** This Equation (b) is the differential equation of motion for a given system. Comparing this equation with the fundamental equation of simple harmonic motion, we get,

$$\omega_n^2 = \frac{Ka^2}{I_o}$$

$$\therefore\ \omega_n = \sqrt{\frac{Ka^2}{I_o}} \quad \text{Or} \quad \omega_n = \sqrt{\frac{Ka^2}{mL^2}}\ \text{, rad/sec}$$

- **Natural frequency :**

$$f_n = \frac{\omega_n}{2\pi}$$

or
$$f_n = \frac{1}{2\pi}\sqrt{\frac{Ka^2}{mL^2}}, \text{ Hz} \quad \text{...Ans.}$$

[II] Energy Method

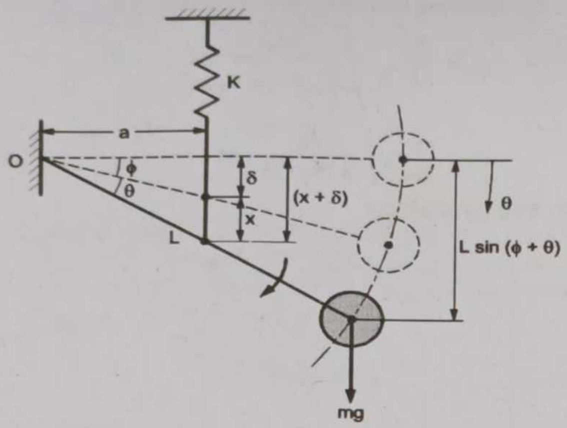


Fig. 3.16(e)

From Fig. 3.16(e) ;

K. E. of system :

Kinetic energy of mass ,

$$KE = \frac{1}{2} I_o \dot{\theta}^2$$

P. E. of system :

Potential energy of mass $= -\text{mgL}\sin(\phi + \theta)$

$= -\text{mgL}\sin\theta$

[as ϕ is small, it is neglected]

Potential energy of spring $= \frac{1}{2}K(x+\delta)^2$

Total potential energy is,

$$PE = -\text{mgL}\sin\theta + \frac{1}{2}K(x+\delta)^2$$

Energy method :

$$\frac{d}{dt}(KE + PE) = 0$$

$$\therefore \frac{d}{dt}\left[\frac{1}{2}I_o\dot{\theta}^2 - \text{mgL}\sin\theta + \frac{1}{2}K(x+\delta)^2\right] = 0$$

$$\frac{1}{2}I_o 2\dot{\theta}\ddot{\theta} - \text{mgL}\cos\theta\,\dot{\theta} + \frac{1}{2}K\,2(x+\delta)\dot{x} = 0$$

$$I_o\dot{\theta}\ddot{\theta} - \text{mgL}\dot{\theta} + K(x+\delta)\dot{x} = 0$$

...[$\because \cos\theta \simeq 1$]

$$I_o\dot{\theta}\ddot{\theta} - \text{mgL}\dot{\theta} + K(a\theta+\delta)a\dot{\theta} = 0$$

...[$\because x = a\theta$ and $\dot{x} = a\dot{\theta}$]

$$I_o\dot{\theta}\ddot{\theta} - \text{mgL}\dot{\theta} + Ka^2\theta\dot{\theta} + K\delta a\dot{\theta} = 0$$

$$\therefore I_o\ddot{\theta} - \text{mgL} + Ka^2\theta + K\delta a = 0$$

$$I_o\ddot{\theta} + Ka^2\theta = 0$$

...[$\because$ from Equation (i) $K\delta a \simeq \text{mgL}$]

$$\therefore \ddot{\theta} + \left(\frac{Ka^2}{I_o}\right)\theta = 0 \quad \text{...(c)}$$

- **Natural circular frequency :** This Equation (c) is the differential equation of motion for a given system. Comparing this equation with the fundamental equation of simple harmonic motion, we get,

$$\omega_n^2 = \frac{Ka^2}{I_o}$$

$$\therefore \omega_n = \sqrt{\frac{Ka^2}{I_o}}$$

$$\therefore \omega_n = \sqrt{\frac{Ka^2}{mL^2}}, \text{ rad/s}$$

- **Natural frequency :**

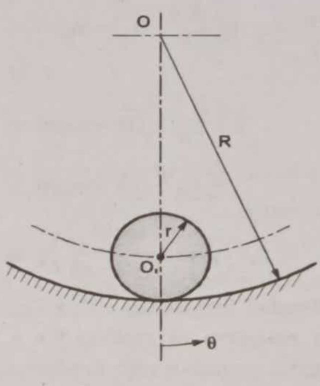
$$f_n = \frac{\omega_n}{2\pi}$$

or
$$f_n = \frac{1}{2\pi}\sqrt{\frac{Ka^2}{mL^2}}, \text{ Hz} \quad \text{...Ans.}$$

Q.17 A cylinder of mass '**m**' and radius '**r**' rolls without slipping on a concave cylindrical surface of radius '**R**'. Find the natural frequency of oscillations,

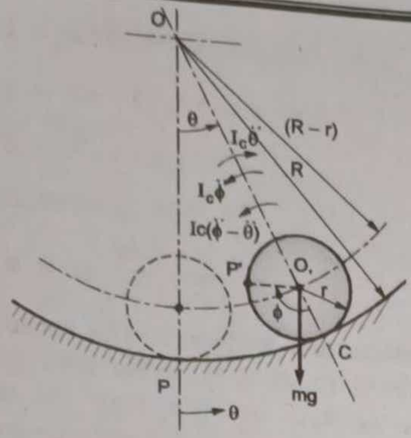
SPPU - Dec. 15, 4 Marks

Ans. :



(a) Equilibrium Position

Fig. 3.17



(b) Displaced Position

Fig. 3.17

- **From Fig. 3.17(b) :**

$$\text{Arc } CP = \text{Arc } CP'$$
$$\therefore\ R\theta = r\phi$$
$$\therefore\ \phi = \frac{R\theta}{r}$$

Translatory displacement of center of cylinder = $(R - r)\,\theta$

Total rotational displacement of cylinder = $\phi - \theta$

[I] Equilibrium Method

From Fig. 3.17(b);

Rotary motion of cylinder :

$$\sum [\text{Inertia torque + External torques}] = 0$$

$$\therefore\ I_c(\ddot{\phi}-\ddot{\theta}) + mgr\sin\theta = 0$$

$$I_c\left(\frac{R\ddot{\theta}}{r}-\ddot{\theta}\right) + mgr\theta = 0 \qquad \text{...}[\because \sin\theta \simeq \theta]$$

$$I_c\left(\frac{R}{r}-1\right)\ddot{\theta} + mgr\theta = 0$$

$$\frac{I_c}{r}(R-r)\,\ddot{\theta} + mgr\theta = 0$$

$$\therefore\ \ddot{\theta} + \left(\frac{mgr^2}{I_c(R-r)}\right)\theta = 0 \qquad \text{...(a)}$$

- **Natural circular frequency :** This Equation (a) is the differential equation of motion for a given system. Comparing this equation with fundamental equation of simple harmonic motion, we get,

$$\omega_n^2 = \frac{mgr^2}{I_c(R-r)}$$

$$\therefore\ \omega_n = \sqrt{\frac{mgr^2}{I_c(R-r)}}\ \text{, rad/s}$$

- **Natural frequency :**

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{mgr^2}{I_c(R-r)}},\ \text{Hz} \qquad \text{...(b)}$$

Mass M.I. of cylinder about point 'C' is,

$$I_c = I_G + mr^2 = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2 \qquad \text{...(c)}$$

Substituting Equation (c) in Equation (b),

$$f_n = \frac{1}{2\pi}\sqrt{\frac{mgr^2}{\frac{3}{2}mr^2(R-r)}}$$

$$\therefore\ f_n = \frac{1}{2\pi}\sqrt{\frac{2g}{3(R-r)}}\ \text{, Hz} \qquad \text{..Ans.}$$

[II] Energy Method

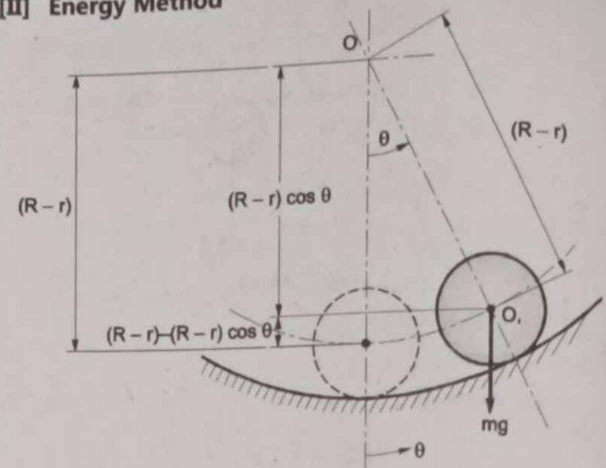


Fig. 3.17(c)

From Fig. 3.17(c) ;

K. E. of system :

$$\text{Translatory K. E. of cylinder} = \frac{1}{2}m\,[(R-r)\,\dot{\theta}]^2$$

$$\text{Rotary K. E. of cylinder} = \frac{1}{2}I_o\,(\dot{\phi}-\dot{\theta})^2$$

Total kinetic energy is,

$$KE = \frac{1}{2}m\,[(R-r)\,\dot{\theta}]^2 + \frac{1}{2}I_o\,(\dot{\phi}-\dot{\theta})^2$$

- **P. E. of system :**

$$\text{P. E. of cylinder} = mg\,[(R-r) - (R-r)\cos\theta]$$

$$\text{or}\quad \text{P. E.} = mg\,(R-r)\,[1-\cos\theta]$$

- **Energy Method :**

$$\frac{d}{dt}(KE + PE) = 0$$

$$\therefore\ \frac{d}{dt}\left[\frac{1}{2}m\left((R-r)\,\dot{\theta}\right)^2 + \frac{1}{2}I_o(\dot{\phi}-\dot{\theta})^2 + mg\,(R-r)\,(1-\cos\theta)\right] = 0$$

$$\frac{d}{dt}\left[\frac{1}{2}m\,(R-r)^2\,\dot{\theta}^2 + \frac{1}{2}I_o\left(\frac{R\dot{\theta}}{r}-\dot{\theta}\right)^2 + mg\,(R-r)\,(1-\cos\theta)\right] = 0$$

$$\frac{d}{dt}\left[\frac{1}{2}m\,(R-r)^2\,\dot{\theta}^2+\frac{1}{2}\frac{1}{2}mr^2\frac{1}{r^2}(R-r)^2\,\dot{\theta}^2+mg\,(R-r)\,(1-\cos\theta)\right] = 0$$

$$\frac{d}{dt}\left[\frac{1}{2}m\,(R-r)^2\,\dot{\theta}^2+\frac{1}{4}m\,(R-r)^2\,\dot{\theta}^2+mg\,(R-r)\,(1-\cos\theta)\right] = 0$$

$$\frac{d}{dt}\left[\frac{3}{4}m\,(R-r)^2\,\dot{\theta}^2+mg\,(R-r)\,(1-\cos\theta)\right] = 0$$

$$\frac{3}{4}m\,(R-r)^2\,2\,\dot{\theta}\,\ddot{\theta}+mg\,(R-r)\sin\theta\cdot\dot{\theta} = 0$$

$$\frac{3}{2}m\,(R-r)^2\,\ddot{\theta}+mg\,(R-r)\,\theta = 0 \qquad \text{...}[\because \sin\theta \simeq \theta]$$

$$\ddot{\theta}+\left(\frac{mg\,(R-r)}{\frac{3}{2}(R-r)^2}\right)\theta = 0$$

$$\therefore\ \ddot{\theta}+\left(\frac{2g}{3\,(R-r)}\right)\theta = 0 \qquad \text{...(d)}$$

- **Natural circular frequency :** This Equation (d) is the differential equation of motion for a given system. Comparing this equation with fundamental equation of simple harmonic motion, we get,

$$\omega_n^2 = \frac{2g}{3\,(R-r)}$$

$$\therefore\ \omega_n = \sqrt{\frac{2g}{3\,(R-r)}}\text{, rad/s}$$

- **Natural frequency :**

$$f_n = \frac{\omega_n}{2\pi}$$

$$\text{or}\quad f_n = \frac{1}{2\pi}\sqrt{\frac{2g}{3\,(R-r)}}\text{, Hz} \qquad \text{...Ans.}$$

Q.18 Classify Dampings in detail.

Ans. : Types of Dampings

Based on the method of providing the resistance to the vibrations, the dampings are classified in to three types [Fig. 3.18] :

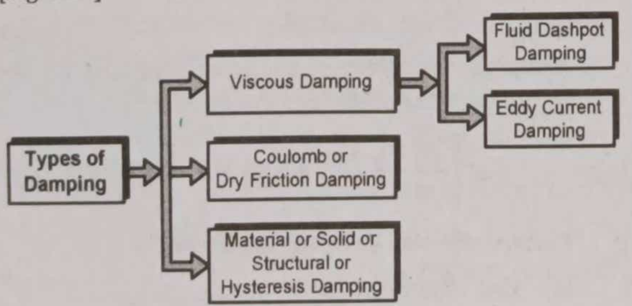


Fig 3.18 : Types of Dampings

1. Viscous Damping :

Principle of Viscous Damping :

- This is the most commonly used damping method to reduce the amplitude of vibrations. When system vibrates in a fluid medium like: air, gas, water or oil, the resistance is offered by the fluid to the vibrating body.
- In viscous, damping the damping resistance depends upon :
  (a) Relative velocity of vibrating body and
  (b) Parameters of the damper (like: viscosity of the fluid used in damper and dimensions of damper).

Viscous Damping :

In viscous damping, the two surfaces having relative motion are separated by a viscous fluid film. This fluid-film offers resistance to motion of one surface with respect to another surface. This resistance is known as **damping resistance** or **damping force.**

Damping Resistance :

- According to Newton's law of viscosity, the damping resistance is given by ,

$$F = \frac{\mu A V}{y} = \left(\frac{\mu A}{y}\right)\dot{x}$$

Or $$F = c\,\dot{x} \qquad \text{...(1)}$$

i.e. $$F \propto \dot{x} \qquad \text{...(2)}$$

Where, F = damping resistance or damping force, N

v = $\dot{x}$ = relative velocity between two surfaces, m/s

c = damping coefficient or damping force per unit velocity, N-s/m

- From Equation (1) it is seen that, in **viscous damping**, the damping resistance or damping force is directly proportional to the relative velocity. Therefore, the viscous damping is a linear type damping.

Damping Coefficient :

- The constant of proportional **'c'** in Equation (1) is known as **damping coefficient** and is defined as the damping force per unit velocity.

- The value of damping coefficient 'c' depends upon :
  (a) The viscosity of fluid
  (b) The dimensions of damper.

Examples of Viscous Damping :

(i) Fluid dashpot damping
(ii) Eddy current damping

(i) Fluid dashpot damping :

- The fluid dashpot damper consist of a piston moving inside a cylinder filled with viscous fluid, as shown in Fig. 3.19.
- The expression for a damping coefficient '**c**' of a fluid dashpot damper can be obtained by using a theory of fluid flow through a rectangular slot [hydrostatic lubrication]
- **Flow rate of fluid through a finite slot :**

$$Q = \frac{\Delta P\, b\, h^3}{12\,\mu\, l} \qquad ...(a)$$

Where, ΔP = pressure difference, N/m^2
b = width of the slot across the flow, m
l = length of the slot in the direction of flow,m
h = thickness of the slot or fluid-film, m
μ = absolute viscosity of the fluid, $N\text{-}s/m^2$

- **Pressure difference across finite slot :**

$$\Delta P = \frac{12\mu\, l\, Q}{b\, h^3} \qquad ...(b)$$

- **Parameters of fluid dashpot damper (Fig. 3.19)**

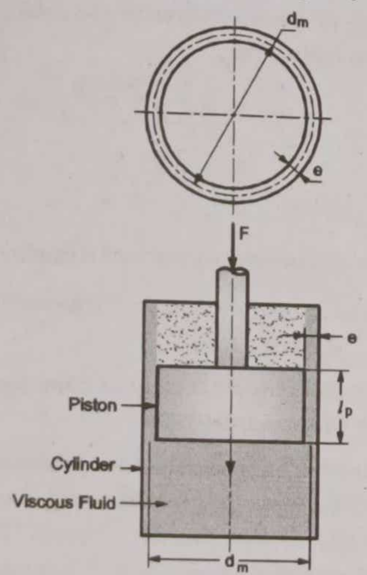


Fig. 3.19 : Fluid Dashpot Damper

Let, A_p = cross-sectional area of the piston, m^2
l_p = length of piston m
e = radial clearance between the piston and cylinder, m
d_m = mean diameter of an annular area between the piston and cylinder, m
μ = absolute viscosity of the fluid, $N\text{-}s/m^2$
ΔP = pressure difference across the two ends of the piston, N/m^2
F = damping resistance or damping force or force acting on the piston, N
Q = flow rate of fluid through, an annular area between the piston and cylinder, m^3/s
v = $\dot{x}$ = relative velocity or velocity of piston, m/s

- Equation (b) can be applied for the flow of fluid through an annular area between the piston and cylinder in fluid dashpot damper by substituting following parameters :

$$\left.\begin{aligned} b &= \pi d_m \\ l &= l_p \\ h &= e \\ \text{and} \quad Q &= V \cdot A_p = \dot{x} \cdot A_p \end{aligned}\right\} \qquad ...(c)$$

- **Pressure difference in fluid dashpot - damper :** Substituting Equation (c) in Equation (b), the pressure difference across an annular area between the piston and cylinder in fluid dashpot damper is given by,

$$\Delta P = \frac{12\,\mu l_p\, \dot{x}\, A_p}{\pi\, d_m\, e^3} \qquad ...(d)$$

- **Damping resistance or force acting on piston :**

$$F = \Delta P \,.\, A_p \qquad ...(e)$$

$$\therefore\ F = \left[\frac{12\,\mu\, l_p\, A_p^2}{\pi\, d_m\, e^3}\right]\dot{x} \qquad ...(f)$$

Again, $$F = c\,\dot{x} \qquad ...(g)$$

- **Damping coefficient :** Comparing Equations (f) and (g), the damping coefficient '**c**' is given by,

$$c = \left[\frac{12\,\mu\, l_p\, A_p^2}{\pi\, d_m\, e^3}\right] \text{N-s/m} \qquad ...(3)$$

- **Factors affecting damping coefficient 'c' :**
  (a) Viscosity of fluid (μ);
  (b) Clearance between piston and cylinder (e);
  (c) Length of piston (lp) ; and
  (d) Diameter of piston (dp) .

(ii) Eddy current damping :

- Eddy current damping is based on the principle of magnetic flux. It consist of a magnet and non - ferrous metal, as shown in Fig. 3.20.

- When the plate moves between north and south poles of a magnet, in a direction perpendicular to magnetic flux, a current is induced in the plate and it is proportional to the velocity of the plate. This current is in the form of eddy current that sets up a magnetic field in a direction opposing the original magnetic field that causes them. Thus, there is resistance to the motion of the plate in a magnetic field which results in damping.

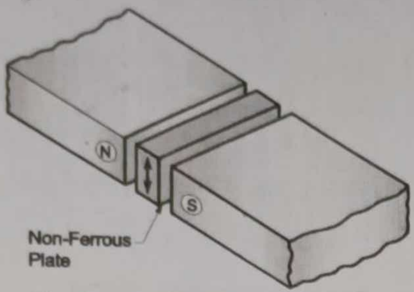


Fig. 3.20 : Eddy Current Damper

- This type of damping is used in vibrometers and in vibration control devices.

2. Coulomb or Dry Friction Damping :

- This type of damping occurs due to friction between two rubbing surfaces which are dry or unlubricated.
- The force of friction acting on each of the two mating surfaces is given by,

$$F_r = \mu R_N$$

where, μ = coefficient of friction

R_N = normal reaction between two mating surfaces

- The variation of 'μ' with respect to sliding velocity '**v**' for different surface conditions is shown in Fig. 3.21. From Fig. 3.21 it is seen that, for ideally smooth surfaces, coefficient of friction 'μ' is independent of velocity. For rough surfaces, coefficient of friction 'μ' decreases some what initially with the increase in velocity, and then is practically constant. For all practical purpose 'μ' is taken as constant throughout the velocity range.

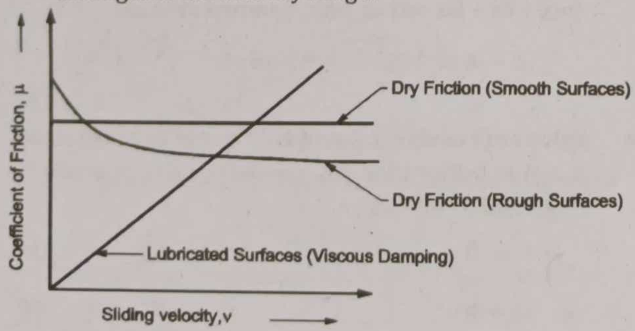


Fig. 3.21 : Coefficient of Friction Versus Sliding Velocity

3. Material or Solid or Structural or Hysteresis Damping :

- This type of damping occurs in all vibrating systems due to elasticity of material. The amount of such damping is very small. When materials are deformed, energy is absorbed and dissipated to surrounding in the form of heat. This effect is due to the internal friction of the molecules of material.
- When a body with material damping is subjected to vibrations, the stress- strain diagram for a vibrating body is not a straight line but forms a hysteresis loop, as shown in Fig. 3.22.

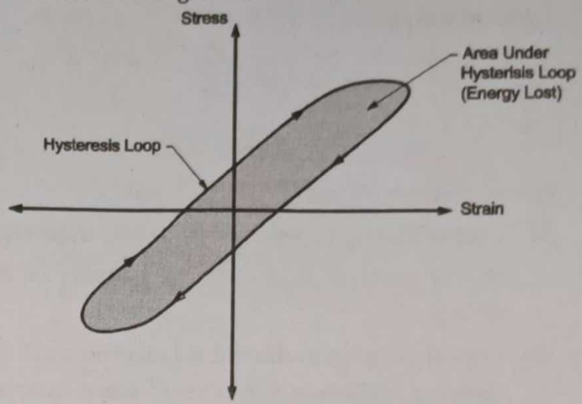


Fig. 3.22 : Hysteresis Loop for Elastic Material Subjected to Vibrations

- The area under hysteresis loop represents the energy dissipated due to molecular friction per cycle per unit volume of the body. The size of hysteresis loop depends upon the type of a material of the vibrating body.

Q.19 Define the term : Critical damping coefficient, related to vibrations **SPPU : May 16, Dec. 16, Dec. 17**

Ans. : Critical Damping Coefficient (c_c) :

- **Critical damping coefficient** : Critical damping coefficient 'c_c' is that value of damping coefficient '**c**' at which the frequency of free damped vibrations is zero and the motion is aperiodic. The critical damping coefficient 'c_c' is also defined as that value of the damping coefficient '**c**' that makes the expression within the radial sign to zero, thereby giving two equal roots of '**S**' (i.e. S_1 and S_2).
- **At c = c_c :**

$$\sqrt{\left(\frac{c}{2m}\right)^2 - \frac{K}{m}} = 0$$

$$\therefore \sqrt{\left(\frac{c_c}{2m}\right)^2 - \frac{K}{m}} = 0$$

$$\left(\frac{c_c}{2m}\right)^2 = \frac{K}{m}$$

$$\frac{c_c}{2m} = \sqrt{\frac{K}{m}}$$

$$\frac{c_c}{2m} = \omega_n$$

or $$\mathbf{c_c = 2m \cdot \omega_n} \quad ...(1)$$

where, ω_n = natural frequency of undamped free vibrations, **rad/s**

$$= \sqrt{\frac{K}{m}}$$

- Again from Equation (3.15.7),

$$c_c = 2m\sqrt{\frac{K}{m}}$$

or $$\mathbf{c_c = 2\sqrt{Km}} \quad ...(2)$$

- From Equations (1) and (2) it is seen that :
  (i) Critical damping coefficient 'c_c' is only dependent on the mass of body and the stiffness of the spring.
  (ii) Critical damping coefficient is independent of the damping coefficient '**c**' and hence actual damping condition.

Q.20 Define the term : Damping factor

SPPU : Dec. 12, Dec. 13, May 16, Dec. 16, Dec. 17, Oct. 18(In sem), May 19, Oct. 19(In Sem.)

Ans. : Damping factor or **damping ratio** 'ξ' is defined as the ratio of the damping coefficient to the critical damping coefficient. Mathematically,

$$\xi = \frac{c}{c_c}$$

Q.21 Explain with displacement-time plot, the overdamped, critically-damped and under-damped vibratory systems. Give suitable examples.

SPPU : May 16, Aug. 17(In Sem)

OR. Draw and explain displacement-time curves for over damped system, critically damped and under damped vibratory system. **SPPU : May 19**

Ans. : General Solution to Differential Equation and Types of Damped Systems :

Two Roots of General Solution :

- From Equation

$$\xi = \frac{c}{c_c}$$

or $$\xi = \frac{c}{2m\,\omega_n} \quad [\text{as } c_c = 2m\,\omega_n]$$

$$\therefore \quad \frac{c}{2m} = \xi\,\omega_n \quad ...(e)$$

and $$\omega_n^2 = \frac{K}{m} \quad ...(f)$$

- Substituting Equations (e) and (f) in Equation,

$$S_{1,2} = \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{K}{m}}$$

we get,

$$S_{1,2} = -\xi\,\omega_n \pm \sqrt{\xi^2\,\omega_n^2 - \omega_n^2}$$

$$\therefore S_{1,2} = \left[-\xi \pm \sqrt{\xi^2-1}\right]\omega_n$$

$$\therefore S_1 = \left[-\xi + \sqrt{\xi^2-1}\right]\omega_n \quad ...(g)$$

$$\text{and } S_2 = \left[-\xi - \sqrt{\xi^2-1}\right]\omega_n \quad ...(h)$$

Types or Cases of Damped Systems :

Depending upon the value of damping factor or damping ratio 'ξ', there are three types or cases of damped systems :

Case 1 : Over-Damped System ($\xi > 1$)

Case 2 : Critically Damped System ($\xi = 1$)

Case 3 : Under-Damped System ($\xi < 1$)

1. **Over-Damped System ($\xi > 1$) :**

- **Over-damped system :** If the damping factor 'ξ' is greater than one or the damping coefficient '**c**' is greater than critical damping coefficient then the system is said to be **over -damped**. For over-damped system ;

$$\xi > 1 \text{ or } c > c_c \quad ...(i)$$

- **Two roots for over-damped system**

$$\left.\begin{aligned} S_1 &= \left[-\xi + \sqrt{\xi^2-1}\right]\omega_n \\ S_2 &= \left[-\xi - \sqrt{\xi^2-1}\right]\omega_n \end{aligned}\right\} \quad ...(j)$$

In this case two roots $\mathbf{S_1}$ and $\mathbf{S_2}$ real and negative.

- **Solution to differential Equation ($m\ddot{x} + c\dot{x} + Kx = 0$) in over-damped system :**

$$x = A\,e^{\left[-\xi+\sqrt{\xi^2-1}\right]\omega_n t} + B\,e^{\left[-\xi-\sqrt{\xi^2-1}\right]\omega_n t} \quad ...(1)$$

- **Values of constants A and B :** The values of constants A and B, in Equation (1), are determined from initial conditions as follows :

at $t = 0$; $\quad x = X_0 \quad ...(k)$

at $t = 0$; $\quad \dot{x} = 0 \quad ...(l)$

From Equation (1),

$$X = A\,e^{[-\xi+\sqrt{\xi^2-1}]\,\omega n\,t} + B\,e^{[-\xi+\sqrt{\xi^2-1}]\,\omega^n\,t} \quad ...(m)$$

Differentiating Equation (m) with respect to 't', we get,

$$\dot{x} = A\left[-\xi+\sqrt{\xi^2-1}\right]\omega_n\,e^{[-\xi+\sqrt{\xi^2-1}]\,\omega_n t} + B\left[-\xi-\sqrt{\xi^2-1}\right]\omega_n\,e^{[-\xi+\sqrt{\xi^2-1}]\,\omega_n t} \quad ...(n)$$

Substituting Equation (k) in Equation (m),

$$X_0 = A + B \quad ...(o)$$

Substituting Equation (l) in Equation (n),

$$0 = A\left[-\xi+\sqrt{\xi^2-1}\right]\omega_n + B\left[-\xi-\sqrt{\xi^2-1}\right]\omega_n \quad ...(p)$$

From Equations (o) and (p), we get,

$$\left.\begin{aligned} A &= \frac{\left[\xi+\sqrt{\xi^2-1}\right]X_0}{2\sqrt{\xi^2-1}} \\ \text{and}\quad B &= \frac{\left[-\xi+\sqrt{\xi^2-1}\right]X_0}{2\sqrt{\xi^2-1}} \end{aligned}\right\} \quad ...(q)$$

- **General solution to differential equation in over damped system :**

Substituting Equation (q) in Equation (1), we get,

$$x = \frac{X_0}{2\sqrt{\xi^2-1}}\left\{\left[\xi+\sqrt{\xi^2-1}\right]e^{[-\xi+\sqrt{\xi^2-1}]\,\omega^n t} + \left[-\xi+\sqrt{\xi^2-1}\right]e^{[-\xi+\sqrt{\xi^2-1}]\,\omega_n t}\right\} \quad ...(2)$$

The motion represented by Equation (2), in over-damped system, is **a periodic motion**. Therefore, the system is non-vibratory. Once the system is displaced from the equilibrium (mean) position, it will take infinite time to come back to the equilibrium position (Fig. 3.23). As most of the systems do not have so much of damping, this type of motion is rarely encountered.

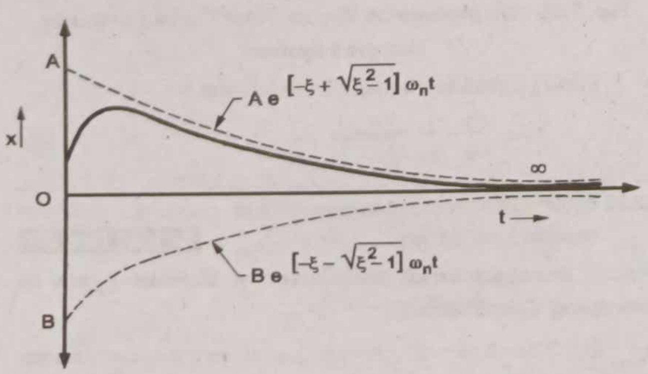


Fig. 3.23: Displacement Versus Time Curve for Over-Damped System

2. Critically Damped System ($\xi = 1$) :

- **Critically damped system :** If the damping factor 'ξ' is equal to one or the damping coefficient '**c**' is equal to the critical damping coefficient 'c_c', the system is said to be **critically damped**.
- **Two roots for critically damped system :**

$$S_1 = -\omega_n \quad ...[\text{Putting } \xi = 1 \text{ in Equation (g)}] \quad ...(r)$$

$$S_2 = -\omega_n \quad ...[\text{Putting } \xi = 1 \text{ in Equation (h)}] \quad ...(s)$$

- The two roots are real and equal. As the roots are equal, in critically damped system, solution to the differential Equation ($m\ddot{x} + c\dot{x} + Kx = 0$)

$$x = A\,e^{S_1 t} + B\,t\,e^{S_2 t}$$

$$\therefore\ x = A\,e^{-\omega_n t} + B\,t\,e^{-\omega_n t}$$

or $$\boxed{x = (A + Bt)\,e^{-\omega_n t}} \quad ...(3)$$

- **Values of constants A and B :** The values of constants A and B, in Equation (3), are determined from initial conditions as follows :

at $t = 0$; $x = X_0$ ...(r)

at $t = 0$; $\dot{x} = 0$ ...(s)

From Equation (3),

$$x = (A + Bt)\,e^{-\omega_n t} \quad ...(t)$$

Differentiating Equation (t) with respect to 't', we get,

$$\dot{x} = B\,e^{-\omega_n t} - (A + Bt)\,\omega_n\,e^{-\omega_n t} \quad ...(u)$$

Substituting Equation (r) in Equation (t),

$$X_0 = A$$

$$\therefore\ A = X_0$$

Substituting Equation (s) in Equation (u),

$$0 = B - A\,\omega_n$$

$$\therefore\ B = A\,\omega_n$$

or $$B = X_0\,\omega_n \quad ...(v)$$

- **General solution to differential equation in critically damped system :**

Substituting Equations (u) and (v) in Equation (3), we get,

$$x = \left[X_0 + X_0\,\omega_n\,t\right]e^{-\omega_n t}$$

or $$x = X_0\left[1 + \omega_n\,t\right]e^{-\omega_n t} \quad ...(4)$$

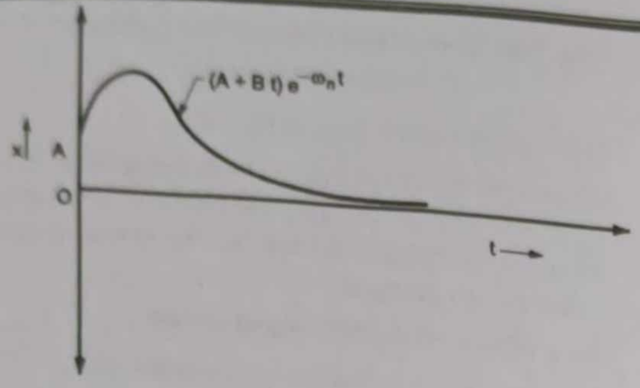


Fig. 3.24 : Displacement Versus Time Curve for Critically Damped System

- From Equation (4) it is seen that, as time '**t**' increases, displacement '**x**' decreases.(As $t \rightarrow \infty$, $x \rightarrow 0$). Fig. 3.24 shows the displacement versus time curve for critically damped system.
- The displacement versus time curve for critically damped system lies below any of the curves for over damped system. The motion represented by Equation (4), in critically damped system, is also a periodic. Therefore, the system is non-vibratory.
- In critically damped system, once the system is disturbed it will move back rapidly close to its equilibrium position in shortest possible time, after that it will take infinite time to come exactly to equilibrium position.
- **Application of critically damped system :** Application of critical damping is in hydraulic door closer, in which it is necessary that the door to return to its original position, after it has been opened, within shortest possible time.

3. Under-Damped System ($\xi < 1$) :

- **Under-damped system :** If the damping factor '**ξ**' is less than one or the damping coefficient '**c**' is less than the critical damping coefficient 'c_c', the system is said to be **under damped**.
- **Two roots for under – damped system :** In such case, the two roots S_1 and S_2 are complex conjugate (imaginary) and are given by,

$$S_1 = \left[-\xi + i\sqrt{1-\xi^2}\right]\omega_n \quad ...(w)$$

and $$S_2 = \left[-\xi - i\sqrt{1-\xi^2}\right]\omega_n \quad ...(x)$$

where, $i = \sqrt{-1}$ is the imaginary unit of the complex root.

- **Solution to differential equation in under-damped system**

$$x = A\, e^{\left[-\xi + i\sqrt{1-\xi^2}\right]\omega_n t} + B\, e^{\left[-\xi - i\sqrt{1-\xi^2}\right]\omega_n t}$$

$$\therefore \quad x = e^{-\xi\omega_n t}\left\{A\, e^{\left[+i\sqrt{1-\xi^2}\right]\omega_n t} + B\, e^{\left[-i\sqrt{1-\xi^2}\right]\omega_n t}\right\} \quad ...(5)$$

putting $\boxed{\sqrt{1-\xi^2}\,.\,\omega_n = \omega_d}$ in Equation (5), we get,

$$\therefore \quad x = e^{-\xi\omega_n t}\left[A\, e^{i\omega_d t} + B\, e^{-i\omega_d t}\right] \quad ...(6)$$

According to Euler's theorem, the above Equation (6) can also be written as,

$$\boxed{x = X\, e^{-\xi\omega_n t}\sin(\omega_d t + \phi)} \quad ...(7)$$

where, X and ϕ are constants to be determined from initial conditions.

- The Equation (7) describe the simple harmonic motion of angular frequency 'ω_d' and amplitude $\mathbf{X\, e^{-\xi\omega_n t}}$, which decreases exponentially with increase in time, as shown in Fig. 3.25. Thus the resultant motion is oscillatory having frequency 'ω_d' and decreasing amplitude $\mathbf{X\, e^{-\xi\omega_n t}}$, which ultimately dies out after some considerable time.
- **Natural frequency of damped vibrations :**

$$\omega_d = \left[\sqrt{1-\xi^2}\right]\omega_n \quad ...(8)$$

- In Equation (8), as $\xi < 1$, the natural frequency of damped vibrations 'ω_d' is always less than the natural frequency of undamped vibrations 'ω_n'

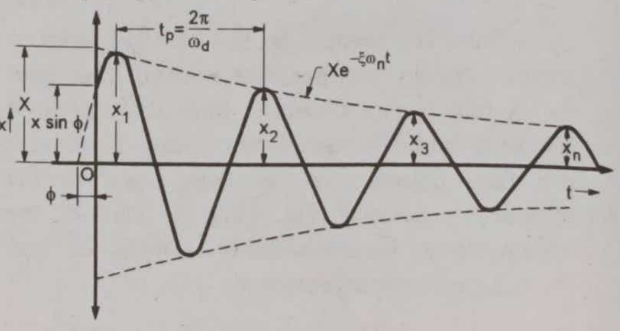


Fig. 3.25 : Displacement Versus Time Curve for Under Damped System

- **Time period for damped vibrations :**

$$t_P = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}\ , \text{ s.} \quad ...(9)$$

Q.22 Draw Displacement versus Time curve for different damping conditions. **SPPU : Dec. 18**

Ans. : Comparison of Responses of Various Types of Damping Conditions :

- Fig. 3.26 shows the comparison of responses of various types of damping conditions.

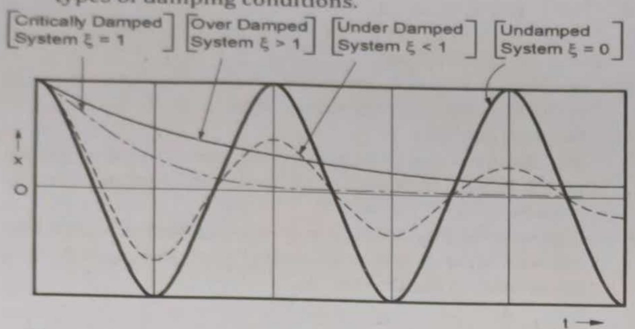


Fig. 3.26 : Displacement Versus Time Plot for Various Types of Damping Conditions

Q.23 Define the term : Logarithmic decrement, related to vibration. **SPPU : May 16, Dec. 16, Dec. 17**

OR What is logarithmic decrement? Derive an expression for the same. **SPPU : Dec. 12, Dec. 14, May 15**

Ans. : Logarithmic Decrement (δ)

- **Two important parameters indicative of damped free vibrations :**
  (i) Natural frequency of damped vibrations; and
  (ii) Rate of decay of amplitude.
- **Measurement of rate of decay :** The rate of decay of amplitude is measured by parameter known as **logarithmic decrement.** The rate of decay of amplitude is proportional to the amount of damping present in a system. The larger the damping, the grater will be the rate of decay.

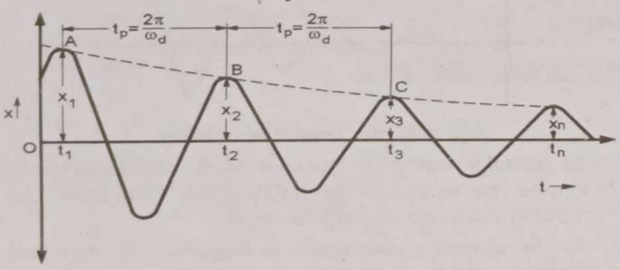


Fig. 3.27 : Displacement Versus Time Curve for Under Damped System

- **Logarithmic decrement (δ) : Logarithmic decrement** is defined as the natural logarithm of, the ratio of any two successive amplitudes on the same side of the mean position.
- Fig. 3.27 shows the free vibrations of an under-damped system. Let A and B are the corresponding points on the two successive cycles where displacement is maximum.
- **Periodic time (t_P) :**

$$t_P = t_2 - t_1 \quad ...(a)$$
$$= \frac{2\pi}{\omega_d}$$

or
$$t_P = \frac{2\pi}{\left(\sqrt{1-\xi^2}\right)\omega_n} \quad ...(1)$$

- **Amplitudes at time 't_1' and 't_2' :**

$$x_1 = Xe^{-\xi\omega_n t_1}\sin[\omega_d t_1 + \phi] \quad ...(b)$$
$$x_2 = Xe^{-\xi\omega_n t_2}\sin[\omega_d t_2 + \phi]$$
$$= Xe^{-\xi\omega_n (t_1+t_p)}\sin[\omega_d(t_1+t_p)+\phi]$$
$$...\left[\because t_2 = t_1 + t_p\right]$$
$$= Xe^{-\xi\omega_n (t_1+t_p)}\sin[\omega_d t_1 + \omega_d t_p + \phi]$$
$$= Xe^{-\xi\omega_n (t_1+t_p)}\sin\left[\omega_d t_1 + \omega_d\frac{2\pi}{\omega_d} + \phi\right]$$
$$= Xe^{-\xi\omega_n (t_1+t_p)}\sin[\omega_d t_1 + 2\pi + \phi]$$

or
$$x_2 = Xe^{-\xi\omega_n (t_1+t_p)}\sin[\omega_d t_1 + \phi] \quad ...(c)$$

- **Logarithmic decrement (δ) :**

From Equations (b) and (c),

$$\frac{x_1}{x_2} = \frac{Xe^{-\xi\omega_n t_1}\sin[\omega_d t_1+\phi]}{Xe^{-\xi\omega_n(t_1+t_p)}\sin[\omega_d t_1+\phi]}$$
$$= e^{-\xi\omega_n(t_1-t_1-t_p)}$$

or
$$\frac{x_1}{x_2} = e^{\xi\omega_n t_p} \quad ...(d)$$

Hence, the logarithmic decrement is given by,

$$\delta = \log_e\left(\frac{x_1}{x_2}\right) \quad ...(e)$$

Substituting Equation (d) in Equation (e), we get

$$\delta = \log_e(e^{\xi\omega_n t_p}) = \xi\omega_n t_p$$
$$= \xi\omega_n\frac{2\pi}{\omega_d} \quad ...\left(\because t_p = \frac{2\pi}{\omega_d}\right)$$
$$\therefore \delta = \xi\omega_n\frac{2\pi}{\left(\sqrt{1-\xi^2}\right)\omega_n}$$
$$...\left[\because \omega_d = \left(\sqrt{1-\xi^2}\right)\omega_n\right]$$
$$\therefore \delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad ...(2)$$

or
$$\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} \quad ...(2(a))$$

- **Logarithmic decrement alternative expression (δ) :**
The logarithmic decrement can also be determined as follows :

$$\delta = \log_e\left[\frac{X_0}{X_1}\right] = \log_e\left[\frac{X_1}{X_2}\right]$$

$$= \log_e\left[\frac{X_2}{X_3}\right] = ... = \log_e\left[\frac{X_{n-1}}{X_n}\right]$$

$$\therefore \quad n\delta = \log_e\left[\frac{X_0}{X_1}\right] + \log_e\left[\frac{X_1}{X_2}\right] + \log_e\left[\frac{X_2}{X_3}\right] + \log_e\left[\frac{X_{n-1}}{X_n}\right]$$

$$= \log_e\left[\frac{X_0}{X_1}\cdot\frac{X_1}{X_2}\cdot\frac{X_2}{X_3}\cdots\frac{X_{n-1}}{X_n}\right]$$

$$\text{or} \quad n\delta = \log_e\left[\frac{X_0}{X_n}\right]$$

$$\therefore \quad \delta = \frac{1}{n}\log_e\left[\frac{X_0}{X_n}\right] \quad ...(3)$$

Where, X_0 = amplitude at the starting position,

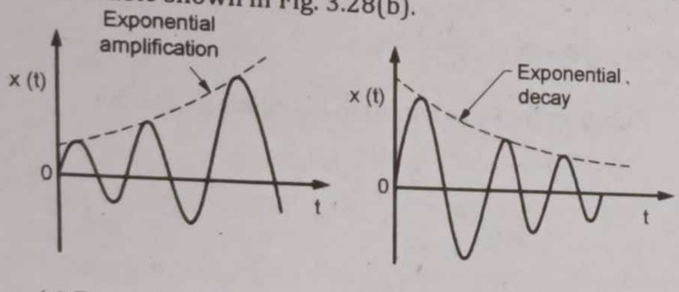
X_n = amplitude after '**n**' cycles

Q.24 Explain the significance of negative damping.

SPPU : May 17

Ans. : Negative Damping

- In case of positive damping, vibrating body gradually loose their energy and thus loose their amplitude of vibration and gradually die away.
- But in case of negative damping, vibrating body gradually gains their energy and amplitude of vibration will goes on increasing and become infinitely large in time.
- A system with positive damping is called to be dynamically stable shown in Fig. 3.28(a), whereas one with negative damping is known as dynamically unstable shown in Fig. 3.28(b).



(a) Dynamically unstable **(b) Dynamically stable**

Fig. 3.28

- In case of positive damping, the damping force does negative work, the mechanical energy is converted into heat usually in the dashpot fluid.
- The energy is taken from the vibrating system. Each successive vibration has less amplitude and less kinetic energy.
- In case of negative damping the damping force (which is now a driving force) does positive work on the system.
- The work done by that force during a cycle is converted into the additional kinetic energy of the increased vibration.
- A negative damping phenomenon occurs due to reduction in dry frictional forces with increase in rubbing velocity.
- The friction coefficient dictates the frictional forces, which in turn influences the damping conditions in the system.
- It is seen in Fig. 3.28(c) that the coefficient of friction reduces with increase in velocity at low speeds and again rises at higher speeds.

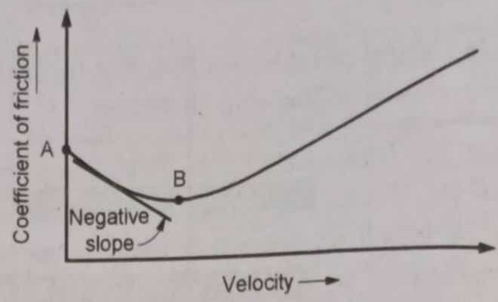


Fig. 3.28(c) : Coefficient of Friction Versus Velocity

- It can be seen that the plot has region AB where the slope of the curve is negative and hence in this region the damping coefficient would be negative. In this case, the governing equation of motion would be modified to

$$m\ddot{x} - C\dot{x} + Kx = 0$$

- In the region AB, the negative damping coefficient creates a propulsive force instead of a damping force. This force is also proportional to velocity and leads to unstable motion.

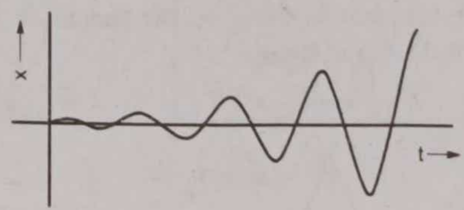


Fig. 3.28(d) : Unstable motion

- An unstable motion is the case where the disturbance causes the amplitude to be built up with time. Fig. 3.28(d) shows an unstable motion.
- As the damping coefficient is negative, the damping factor would be negative and the study of equation of motions for different values of ξ indicate that instead of decay in amplitude, the motion builds up and hence the system would be inherently unstable.
- Oscillations of this type can often be observed in belt and pulley absorption brakes. The action of a bow on the strings of a violin.

Q.25 A wheel is mounted on a steel shaft (G = 83 × 10^9 N/m^2) of length 1.5 m and radius 0.80 cm the wheel is rotated 5° and released. The period of oscillation is observed as 2.3 seconds. Determine the mass moment of inertia of the wheel. **SPPU : May 17, 4 Marks**

Ans. :

Given : $G = 83 \times 10^9\ N/m^2$, $l = 1.5\ m$,

$r = 0.80\ cm = 0.008 m$

$\therefore d = 0.016$

According to D'Alembert's principle,

$$\therefore I\ddot{\theta} + K_t\theta = 0$$

$$\therefore \ddot{\theta} + \frac{K_t}{I}\theta = 0$$

The fundamental equation of simple harmonic motion is,

$$\ddot{\theta} + \omega_n^2\theta = 0\ ; \qquad \omega_n^2 = \frac{K_t}{I}$$

$$\omega_n = \sqrt{\frac{K_t}{I}}\ , \text{rad/s}$$

But $$\omega_n = \frac{2\pi}{t_p} = \frac{2\pi}{2.3} = 2.73\ \text{rad/sec}$$

$$I = \frac{K_t}{\omega_n} = \frac{GJ}{l\,\omega_n^2} = \frac{\pi \times (0.016)^4 \times 83 \times 10^9}{32 \times 1.5 \times 2.73^2}$$

$$= \mathbf{47.77\ kg.m^2}$$ ...Ans.

Q.26 The restroom door shown in Fig. 3.29 is equipped with a torsional spring with 25 Nm/rad as stiffness and a torsional viscous damper. The door has a mass of 60 kg and a centroidal moment of inertia about an axis parallel to the axis of the door 's rotation is 7.2 kg-m^2. Assuming that the system is critically damped, determine the damping coefficient. **SPPU - May 17, 6 Marks**

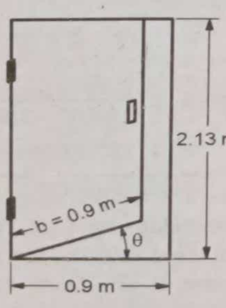


Fig. 3.29

Ans. :

Given : $K_t = 25\ Nm/rad$, $m = 60\ kg$,

$I_c = 7.2\ kg.m^2$, $b = 0.9\ m$

$$\therefore \quad I = I_c + \frac{m\,b^2}{4} = 7.2 + \frac{60 \times 0.9^2}{4}$$

$$= \mathbf{19.35\ kg.m^2}$$

System is critically damped, $\xi = 1$

But $\xi = \frac{c_t}{c_{ct}}$ $\qquad \therefore c_t = c_{ct}$

$$\therefore c_{ct} = 2\sqrt{K_t I} = 2 \times \sqrt{25 \times 19.35}$$

$$= \mathbf{43.99\ Nm.s/rad}$$ ...Ans.

Q.27 An under-damped shock absorber is to be designed for a motorcycle of mass 200 kg, such that during a road bump, the damped period of vibration is limited to 2 seconds and the amplitude of vibration should reduce to one sixteenth in one cycle. Find the spring stiffness and damping coefficient of the shock absorber. **SPPU - Dec. 19, 5 Marks**

Ans. :

Given : $m = 200\ kg$; $t_p = 2\ s$;

$\frac{x_0}{x_1} = 16$; $n = 1$.

1. Logarithmic Decrement :

$$\delta = \frac{1}{n}\log_e\left(\frac{x_0}{x_n}\right) = 1\log_e\left(\frac{x_0}{x_1}\right) = 1\log_e(16)$$

or $$\delta = 2.7725$$

2. Damping Factor :

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$2.7725 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\sqrt{1-\xi^2} = 2.2665\,\xi$$

$$1-\xi^2 = 5.136\,\xi^2$$

$$1 = 6.136\,\xi^2$$

$$\therefore \xi = 0.4037$$

3. Stiffness of Spring :

$$\omega_d = \frac{2\pi}{t_p} = \frac{2\pi}{2} = 3.14\ \text{rad/sec}$$

Again, $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\therefore \quad 3.14 = \omega_n\sqrt{1-(0.4037)^2}$$

$$\therefore \quad \omega_n = 3.43\ \text{rad/sec}$$

Now, $$\omega_n = \sqrt{\frac{K}{m}}$$

$$\therefore \quad 3.43 = \sqrt{\frac{K}{200}}$$

$$\therefore \quad \mathbf{K = 2352.98\ N/m}$$ ...Ans.

4. Damping Coefficient :

$$\xi = \frac{c}{c_c}$$

or

$$\xi = \frac{c}{2\,m\,\omega_n}$$

$$\therefore \quad 0.4037 = \frac{c}{2 \times 200 \times 3.43}$$

$$\therefore \quad c = \mathbf{553.87\ N\text{-}sec/m}$$ ...Ans..

Q.28 A shock absorber is to be designed so that its overshoot is 10% of the initial displacement when released. Determine the damping factor. Also find the overshoot if the damping factor is reduced to 50%.

SPPU - Dec. 16, Dec. 17, 6 Marks

Ans. : Given : $\frac{x_2}{x_1} = 0.1$

(i) Damping factor :

$$\frac{x_1}{x_2} = 10$$

$$\delta = \log_e\left(\frac{x_1}{x_2}\right) = \log_e(10)$$

$$\delta = 2.30258$$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}};$$

$$2.30258 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\sqrt{1-\xi^2} = 2.729\,\xi$$

$$\therefore 1-\xi^2 = 7.447\,\xi^2$$

$$1 = 8.447\,\xi^2$$

$$\boldsymbol{\xi = 0.344}$$ ...Ans.

(ii) Damping factor reduced to 50% :

$$\therefore \xi' = \frac{\xi}{2} = 0.172$$

$$\delta' = \frac{2\pi\xi'}{\sqrt{1-\xi^2}} = \frac{2\pi \times 0.172}{\sqrt{1-0.172^2}} = \frac{1.808}{0.985}$$

$$\delta' = 1.095$$

Now,

$$\delta' = \log_e\left(\frac{x_1}{x_2}\right)$$

$$1.095 = \log_e\left(\frac{x_1}{x_2}\right)$$

$$\frac{x_1}{x_2} = 2.989$$

$$\therefore \frac{x_2}{x_1} = 0.3345$$

$$\text{overshoot} = \mathbf{33.45\ \%}$$ ...Ans.

Q.29 An under damped shock absorber is to be designed for a motorcycle of mass 200 kg such that during a road bump, the damped period of vibration is limited to 2 sec and amplitude of vibration should reduce to one-sixth in one cycle. Find spring stiffness and the damping coefficient of shock absorber.

SPPU - Oct. 16 (In Sem), 6 Marks

Ans. :

Given : $m = 200$ kg ; $t_p = 2s$;

$\frac{x_0}{x_1} = 6$; $n = 1$.

1. Logarithmic Decrement :

$$\delta = \frac{1}{n}\log_e\left(\frac{x_0}{x_n}\right) = 1\log_e\left(\frac{x_0}{x_1}\right) = 1\log_e(6)$$

or $\delta = 1.79$

2. Damping Factor :

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

or

$$\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{1.79}{\sqrt{4\pi^2+1.79^2}} = 0.2739$$

3. Stiffness of Spring :

$$\omega_d = \frac{2\pi}{t_p} = \frac{2\pi}{2} = 3.1415 \text{ rad/sec}$$

Again,

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\therefore 3.1415 = \omega_n\sqrt{1-(0.2739)^2}$$

$$\therefore \omega_n = 3.26 \text{ rad/sec}$$

Now,

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$\therefore 3.26 = \sqrt{\frac{K}{200}}$$

$$\therefore \mathbf{K = 2133.89\ N/m}$$ ...Ans.

4. Damping Coefficient :

$$\xi = \frac{c}{c_c} \quad \text{or} \quad \xi = \frac{c}{2\,m\,\omega_n}$$

$$\therefore 0.2739 = \frac{c}{2 \times 200 \times 3.26}$$

$$\therefore \mathbf{c = 357.16\ N\text{-}sec/m}$$ ...Ans.

Q.30 In a single degree freedom viscously damped vibrating system, the suspended mass of 20 kg makes 50 oscillations in 20 seconds. The amplitude of natural vibrations decreases to one fourth of the initial value after 4 oscillations. Determine :

(i) The logarithmic decrement.

(ii) Damping factor

(iii) Damping coefficient.

SPPU - May 18, 6 Marks

Ans. :

Given : Mass, m = 20 kg, Number of oscillations, n = 4

1. **Logarithmic decrement :**

Let, x_0 = Initial amplitude,

x_4 = Final amplitude after 4 oscillations

$$\therefore \quad x_4 = \frac{x_0}{4}$$

or $$\frac{x_0}{x_4} = 4$$

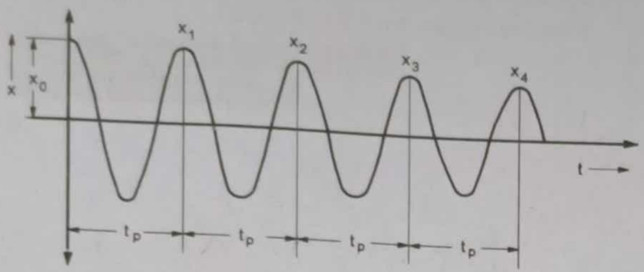


Fig. 3.30

The logarithmic decrement is,

$$\delta = \frac{1}{n}\log_e\left(\frac{x_0}{x_n}\right) = \frac{1}{4}\log_e\left(\frac{x_0}{x_4}\right) \quad \text{...(when n = 4)}$$

$$= \frac{1}{4}\log_e(4)$$

or $\delta = 0.3465$ **...Ans.**

2. **Damping factor :**

The damping factor is,

$$\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{0.3465}{\sqrt{4\pi^2+(0.3465)^2}}$$

or $\xi = 0.055$ **...Ans.**

3. **Spring stiffness :**

The natural frequency of damped vibrations is,

$$f_d = \frac{N}{t_p} = \frac{50}{20}$$

$$\text{or } f_d = 2.5 \text{ Hz}$$

The natural circular frequency of damped vibrations is,

$$\omega_d = f_d \times 2\pi = 2.5 \times 2\pi = 15.70 \text{ rad/s}$$

but, $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\therefore \quad 15.70 = \omega_n\sqrt{1-(0.055)^2}$$

$$\therefore \quad \omega_n = 15.72 \text{ rad/s}$$

Now, $$\omega_n = \sqrt{\frac{K}{m}}$$

$$\therefore \quad 15.72 = \sqrt{\frac{K}{20}}$$

$\therefore$ **K = 4942.36 N/m** **...Ans.**

4. **Damping coefficient :**

$$\xi = \frac{c}{c_c}$$

or $$\xi = \frac{c}{2m\,\omega_n}$$

or $$0.055 = \frac{c}{2 \times 20 \times 15.72}$$

or **c = 34.58 N-s/m** **...Ans.**

Q.31 A body of mass 5 kg is supported on a spring of stiffness 1960 N/m and has dashpot connected to it, which produces a resistance of 1.96 N at a velocity of 1 m/sec.In what ratio will be amplitude of vibration be reduced after 5 cycles ? **SPPU - Dec. 18, 6 Marks**

Ans. :

Given : m = 5 kg ; K = 1960 N/m ; F = 1.96 N ; V = 1m/s ; n = 5

Ratio of amplitude of vibration after 5 cycles = $\frac{x_0}{x_5}$

- **Natural circular frequency :**

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1960}{5}} = 19.8 \text{ rad/sec}$$

- **Damping coefficient**

$$c = \frac{F}{V} = \frac{1.96}{1} = 1.96 \text{ N-s/m}$$

- **Critical damping coefficient :**

$$c_c = 2\,m\,\omega_n = 2 \times 5 \times 1.98 = 198$$

- **Damping factor :**

$$\xi = \frac{c}{c_c} = \frac{1.96}{198} = 9.89 \times 10^{-3}$$

- **Logarithmic Decrement :**

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi \times 9.89 \times 10^{-3}}{\sqrt{1-(9.89 \times 10^{-3})^2}}$$

$$\delta = 0.0621$$

- **Ratio of amplitudes :**

$$\delta = \frac{1}{n}\log_e\frac{x_0}{x_n}$$

$$0.0621 = \frac{1}{5}\log_e\left[\frac{x_0}{x_5}\right]$$

$$\therefore \quad \log_e\left[\frac{x_0}{x_5}\right] = 0.3105$$

$$\frac{x_0}{x_5} = e^{0.3105}$$

$\therefore$ $\frac{x_0}{x_5}$ = **1.364** **...Ans.**

Q.32 A vibrating system is defined by the following parameters : m = 3 kg, k = 100 N/m and c = 3 N-s/m

Determine :

(i) Critical damping Coefficient;

(ii) the damping factor ;

(iii) the natural frequency of damped vibration ;

(iv) the logarithmic decrement ;

(v) the ratio of two consecutive amplitudes ; and

(vi) the number of cycles after which the original amplitude is reduced to 20%.

SPPU - May 15, Oct. 16 (In Sem.), 5 Marks, Oct. 18 (In Sem.), Oct. 19(In Sem.), 6 Marks

Ans. :

Given : $m = 3$ kg ; $K = 100$ N/m ; $C = 3$ N-s/m ; $x_n = 0.2\, x_0$.

1. Natural Circular Frequency of System :

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{100^3}{3}} = \mathbf{5.77\ rad/sec} \qquad \text{... Ans.}$$

2. Critical damping Coefficient

$$c_c = 2m\omega_n = 2 \times 3 \times 5.77 = \mathbf{34.62} \qquad \text{... Ans.}$$

2. Damping Factor :

$$\xi = \frac{c}{c_c} = \frac{c}{2m\omega_n} = \frac{3}{2 \times 3 \times 5.77} = \mathbf{0.058} \qquad \text{...Ans.}$$

3. Natural Circular Frequency of Damped Vibration

$$\omega_d = \omega_n \sqrt{1-\xi^2} = 5.77\sqrt{1-(0.086)^2}$$

$$= 5.74 \text{ rad/sec}$$

4. Natural Frequency of Damped Vibration :

$$f_d = \frac{\omega_d}{2\pi} = \frac{5.74}{2\pi} = \mathbf{0.914\ Hz} \qquad \text{... Ans.}$$

5. Logarithmic Decrement :

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi \times 0.086}{\sqrt{1-(0.086)^2}} = \mathbf{0.5423} \qquad \text{...Ans.}$$

6. Ratio of Two Consecutive Amplitudes :

$$\delta = \frac{1}{n}\log_e\left(\frac{X_0}{X_n}\right) = 1 \times \log_e\left(\frac{x_0}{x_1}\right) \therefore$$

$$\frac{x_0}{x_1} = e^{\delta} = e^{0.5423} = \mathbf{1.71} \qquad \text{...Ans.}$$

7. Number of cycles after which the original amplitude is reduced to 20% :

$$\delta = \frac{1}{n}\log_e\left(\frac{X_0}{X_n}\right) = \frac{1}{n}\log_e\left(\frac{X_0}{0.2\,X_0}\right) = \frac{1}{n}\log_e\left(\frac{1}{0.2}\right)$$

$$\delta = \frac{1}{n} \times 1.6094$$

$$\therefore\ 0.5423 = \frac{1}{n} \times 1.6094$$

$$n = 2.96$$

$$n \simeq 3 \text{ cycles} \qquad \text{...Ans.}$$

Q.33 Derive an equation of motion for the system shown in Fig. 3.31(a). If m = 1.5 kg, K = 4900 N/m, a = 6 cm and b = 14 cm, determine the value of c for which the system is critically damped.

SPPU - Aug 15 (In Sem), 4 Marks, Oct 16 (In Sem), 5 Marks

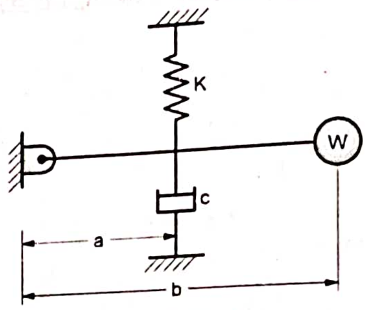
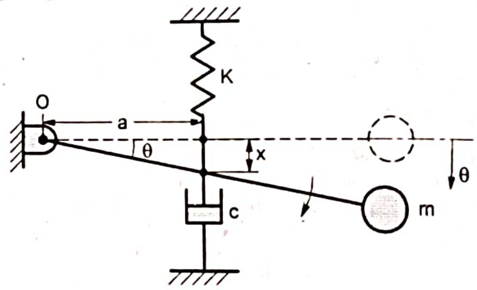


Fig. 3.31(a)

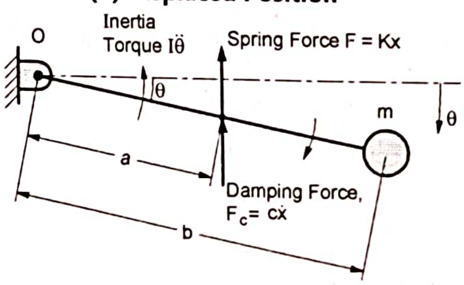
Ans. :

1. Differential Equation of Motion :

Fig. 3.31 shows the system when mass 'm' is deflected through a angle 'θ' , due to which the spring 'K' will be stretched by a distance $x = a\,\theta$.



(b) Displaced Position



(c) F.B.D

Fig. 3.31

From Fig. 3.31(c) ;

$$\Sigma\,[\text{ Inertia torque + External torques }] = 0$$

$$\therefore \quad I_0 \ddot{\theta} + c \dot{x} a + K x a = 0$$

$$\therefore \quad I_0 \ddot{\theta} + c a^2 \dot{\theta} + K a^2 \theta = 0 \qquad \text{...(a)}$$

$$\text{... [} \because \ x = a\theta, \text{ and } \dot{x} = a\dot{\theta} \text{]}$$

or

$$\ddot{\theta} + \left(\frac{c a^2}{I_o}\right)\dot{\theta} + \left(\frac{K a^2}{I_o}\right)\theta = 0 \qquad \text{...(b)}$$

- This is the fundamental differential equation of motion for single degree of freedom of a system having viscous damping. This is a linear differential equation of the second order and its solution can be written as substituting :

$$\theta = e^{St}$$

$$\dot{\theta} = S e^{St}$$

and $$\ddot{\theta} = S^2 e^{St}$$

Therefore Equation (b) can be written as,

$$S^2 e^{St} + \frac{c a^2}{I_o} S e^{St} + \frac{K a^2}{I_o} e^{St} = 0$$

$$\therefore \quad S^2 + \left(\frac{c a^2}{I_o}\right) S + \left(\frac{K a^2}{I_o}\right) = 0 \qquad \text{...(c)}$$

- **Two roots of equation :** The Equation (c) is the quadric equation for which the two roots are :

$$S_{1,2} = \frac{\left(\frac{c a^2}{I_o}\right) \pm \sqrt{\left(\frac{c a^2}{I_o}\right)^2 - 4\left(\frac{K a^2}{I_o}\right)}}{2}$$

$$S_{1,2} = \frac{c a^2}{2 I_o} \pm \sqrt{\left(\frac{c a^2}{2 I_o}\right)^2 - \left(\frac{K a^2}{I_o}\right)}$$

2. **Critical Damping Coefficient :** The system is critically damped when,

$$\left(\frac{c a^2}{2 I_o}\right)^2 = \frac{K a^2}{I_o}$$

$$\therefore \quad \left(\frac{c_c a^2}{2 I_o}\right)^2 = \frac{K a^2}{I_o} \qquad \text{...[} \because \ c = c_c \text{]}$$

$$\therefore \quad \frac{c_c^2 a^4}{4 I_o^2} = \frac{K a^2}{I_o}$$

$$\therefore \quad c_c^2 = \frac{K a^2}{I_o} \times \frac{4 I^2}{a^4} = \frac{4 K I}{a^2}$$

$$\therefore \quad c_c = \frac{2\sqrt{K I}}{a} = \frac{2\sqrt{K m l^2}}{a} \qquad \text{...[} \because \ I = m b^2 \text{]}$$

or $$c_c = \frac{2 b}{a}\sqrt{K m} \qquad \text{...Ans.}$$

But, $$\xi = \frac{c}{c_c} = \frac{c}{\frac{2 b}{a}\sqrt{K m}} = \frac{c a}{2 b \sqrt{K m}}$$

3. **Frequency of Damped Vibrations :**

- **Circular damped frequency :**

$$\omega_d = \omega_n \sqrt{1 - \xi^2} = \sqrt{\omega_n^2 - \omega_n^2 \xi^2}$$

$$= \sqrt{\frac{K a^2}{m b^2} - \frac{K a^2}{m b^2} \times \left(\frac{c a}{2 b \sqrt{K m}}\right)^2}$$

$$\text{...}\left[\because \ \omega_n = \sqrt{\frac{K a^2}{m b^2}}\right]$$

$$= \sqrt{\frac{K a^2}{m b^2} - \frac{K a^2}{m b^2} \times \frac{c^2 a^2}{4 b^2 K m}}$$

$$= \sqrt{\frac{K a^2}{m b^2} - \frac{c^2 a^4}{4 m^2 b^4}}$$

or $$\omega_d = \frac{a}{b}\sqrt{\frac{K}{m} - \frac{c^2 a^2}{4 m^2 b^2}}$$

- **Frequency of damped vibration :**

$$f_d = \frac{\omega_d}{2\pi}$$

or $$f_d = \frac{1}{2\pi}\frac{a}{b}\sqrt{\frac{K}{m} - \frac{c^2 a^2}{4 m^2 b^2}}, \ \text{Hz} \qquad \text{...Ans.}$$

4. **Damping Coefficient :**

When m = 1.5 kg, K = 4900 N/m, b = 0.14 m and a = 0.06 m.

The critical damping coefficient is,

$$c_c = \frac{2b}{a}\sqrt{K m} = \frac{2 \times 0.14}{0.06}\sqrt{4900 \times 1.5}$$

or $$c_c = 400 \text{ N-s/m} \qquad \text{...Ans.}$$

5. **Alternate Solution of Equation (a) :**

From Equation (a),

$$I_o \ddot{\theta} + c a^2 \dot{\theta} + K a^2 \theta = 0 \qquad \text{...(d)}$$

Equation (d) can be written as,

$$I_o \ddot{\theta} + c_{te} \dot{\theta} + K_{te} \theta = 0$$

where $I_o = m b^2$; $c_{te} = c a^2$

and $K_{te} = K a^2$

For critically damped torsional vibrations,

$$c_{te} = 2\sqrt{K_{te} I_o}$$

$$\therefore \quad c_c a^2 = 2\sqrt{K a^2 m b^2}$$

$$\therefore \quad c_c = \frac{2}{a^2}\sqrt{K a^2 m b^2} = \frac{2 a b}{a^2}\sqrt{K m}$$

or $$c_c = \frac{2 b}{a}\sqrt{K m} \qquad \text{...Ans.}$$

Q.34 What is Coulomb Damping ? SPPU : May 19

Ans. : Coulomb Damping :

Coulomb damping is the damping that occurs when two dry or unlubricated surfaces slide against each other. The frictional force, which is constant in magnitude, acts as damping force. Therefore, coulomb or dry friction damping is also known as **constant damping**.

Spring-Mass System with Coulomb Damping :

Consider a spring-mass system, show in Fig. 3.32, with mass 'm' sliding on a dry surface. Let μ be the coefficient of dry friction between the two surfaces. In equilibrium position, the spring us unstretched and no fractional force acts on the mass.

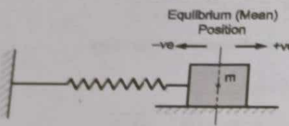


Fig. 3.32 : Spring-mass System with Coulomb or Dry Friction Damping

Q.35 Derive a relation to determine the loss of amplitude per cycle in case of Coulomb damping. MU : May 14

OR Derive that the loss of amplitude per cycle for coloumb damping is given by 4 F/K. MU : Dec. 14, May 19

Ans. : Fig. 3.33 shows a rate of decay of vibrations i.e. loss of amplitude per cycle with coulomb damping. Consider cycle starting from point A (i.e. extreme left position).

- **Total energy of system at point A (strain energy)**

$$U_A = \frac{1}{2} K X_A^2 \qquad \text{... (m)}$$

At point A, the velocity of mass is zero, hence, the kinetic energy is zero.

- **Total energy of system at point B :**

  As the system moves from point A to point B (i.e. from extreme left position to extreme right position), because of coulomb damping, there is decay of vibrations.

  At point B, the amplitude reduces to 'X_B'. At point B, the velocity of mass 'm' in again zero, hence the total energy of the system, which is strain energy is given by,

$$U_B = \frac{1}{2} K X_B^2 \qquad \text{... (n)}$$

- **Loss of energy in half cycle from point A to point B :**

$$\Delta U = U_A - U_B = \frac{1}{2} K X_A^2 - \frac{1}{2} K X_B^2$$

or $$\Delta U = \frac{1}{2} K (X_A^2 - X_B^2) \qquad \text{... (1)}$$

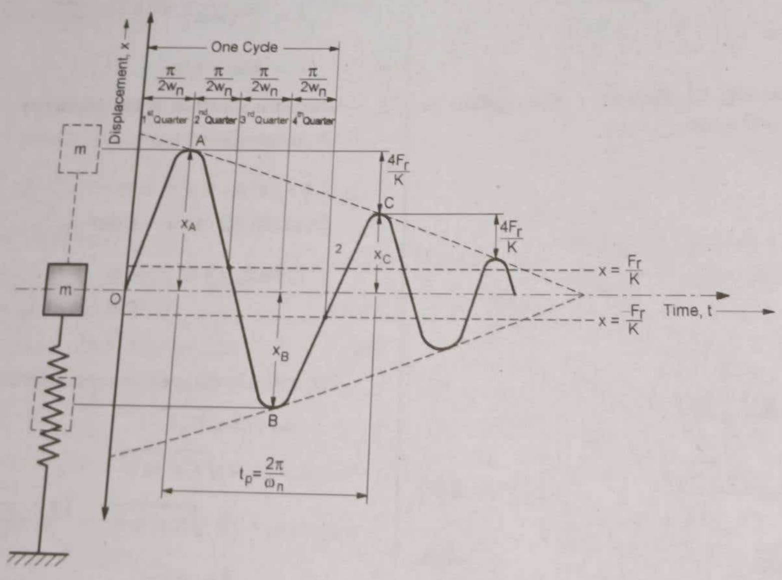


Fig. 3.33 : Displacement vs Time Plot for Coulomb or Dry Friction Damping

- **Work done against frictional force in half cycle from point A to point B :**

The loss of energy is due to work done against frictional force. This work done against frictional force is given by,

$$W = \text{Force} \times \text{Distance traveled}$$

or $$W = F_r (X_A + X_B) \quad \text{... (2)}$$

- **Loss of amplitude in half cycle form point A to point B :**

From Equations (1) and (2),

$$\text{Loss of energy} = \text{Work done against friction}$$

$$\Delta U = W$$

$$\frac{1}{2} K (X_A^2 - X_B^2) = F_r (X_A + X_B)$$

$$\frac{1}{2} K (X_A - X_B)(X_A + X_B) = F_r (X_A + X_B)$$

$$\therefore X_A - X_B = \frac{2 F_r}{K}$$

The loss of amplitude in half cycle from point A to point B is,

$$X_A - X_B = \frac{2 F_r}{K} \quad \text{...(3)}$$

- **Loss of amplitude in one cycle :**

Similarly the loss of amplitude in half cycle from point B to point C is,

$$X_B - X_C = \frac{2 F_r}{K} \quad \text{... (4)}$$

Hence total loss of amplitude in one cycle is,

$$\Delta = X_A - X_C = (X_A - X_B) + (X_B - X_C)$$

$$= \frac{2 F_r}{K} + \frac{2 F_r}{K}$$

or $$\Delta = \frac{4 F_r}{K} \quad \text{...(5)}$$

Thus, in coulomb damping, the difference between any two successive amplitudes is constant and is given by,

$$\Delta = \frac{4 F_r}{K}, \text{ m} \quad \text{... (6)}$$

Q.36 A horizontal spring mass system with coulomb damping has a mass of 5 kg attached to a spring of stiffness 980 N/m. If the coefficient of friction is 0.25, calculate :

(i) the frequency of free oscillations;

(ii) the number of cycles corresponding to 50% reduction in amplitude if the initial amplitude is 5 cm; and

(iii) the time taken to achieve this 50% reduction

SPPU - May 10, Dec. 16, Dec. 17, 6 Marks

Ans. :

$m = 5$ kg ; $K = 980$ N/m ;

$\mu = 0.25$; $x_o = 5$ cm $= 0.05$ m.

1. Frequency of Free Vibrations :

- **Natural circular frequency :**

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{980}{5}}$$

or $$\omega_n = 14 \text{ rad/s} \quad \text{...Ans.}$$

- **Natural frequency :**

$$f_n = \frac{\omega_n}{2\pi} = \frac{14}{2\pi} = 2.23 \text{ Hz} \quad \text{...Ans.}$$

2. Number of Cycles for 50 % Reduction in Amplitude :

- **Loss of amplitude per cycle :**

$$\Delta = \frac{4 F_r}{K} = \frac{4 \mu R_N}{K} = \frac{4 \mu m g}{K}$$

$$\Delta = \frac{4 \times 0.25 \times 5 \times 9.81}{980} = 0.05 \text{ m}$$

- **Amplitude at 50% of the initial amplitude :**

$$x_n = 0.5 \times x_o = 0.5 \times 5 = 2.5 \text{ cm} = 0.025 \text{ m}$$

- **Number of cycles corresponding to 50% reduction in amplitude :**

We know that,

$$x_n = x_o - n\Delta$$

$$\therefore 0.025 = 0.05 - n \times 0.05$$

$$n = 0.5 \text{ cycles} \quad \text{...Ans.}$$

3. Time Taken to Achieve 50% Reduction :

$$t = n\frac{2\pi}{\omega_n} = 0.5 \times \frac{2\pi}{14}$$

or $$t = 0.22 \text{ s} \quad \text{...Ans.}$$

Q.37 A body of mass 100 kg is suspended on a leaf spring. The system is then made to vibrate and its natural frequency measured is 7 rad/s. It is observed that if the initial amplitude is 48 mm, the subsequent amplitudes are 32 mm and 20 mm. Determine spring stiffness and coulomb damping force. **SPPU - Dec. 15, 6 Marks**

Ans. :

Given : $m = 100$ kg ; $\omega_n = 7$ rad/s ;

$x_0 = 48$ mm ; $x_{1/2} = 32$ mm ;

$x_1 = 20$ mm.

1. Stiffness of Spring :

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$\therefore 7 = \sqrt{\frac{K}{100}}$$

$$\therefore K = 4900 \text{ N/m} = 4.9 \text{ N/mm} \quad \text{...Ans.}$$

2. **Damping Force :**

The loss of amplitude per half cycle is,

$$\frac{\Delta}{2} = x_0 - x_{1/2} = 48 - 32 = 16 \text{ mm}$$

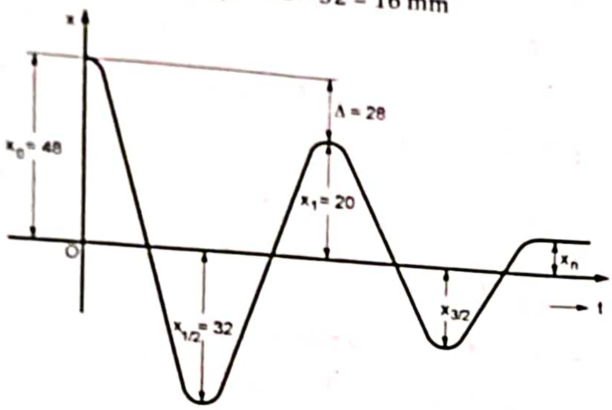


Fig. 3.34

The loss of amplitude per cycle is,

$$\Delta = x_0 - x_1 = 48 - 20 = 28 \text{ mm}$$

Therefore $\Delta = \frac{4 F_r}{K}$

$$\therefore \quad 28 = \frac{4 \times F_r}{4.9}$$

$$F_r = \mathbf{34.3\ N} \quad \text{...Ans.}$$

Chapter 4 : Single Degree of Freedom Systems : Forced Vibrations

Q.1 Define forced vibrations with example.

Ans. :

- **Forced vibrations :** *If the system vibrates under the influence of external periodic force, the vibrations are known as* **forced vibrations.**
- **Examples of forced Vibrations :** Vibrations of air compressors, I.C. engines, turbines, pumps, etc.

Q.2 Derive an expression for magnification factor for steady state amplitude of vibration subjected to external excitation F sin ωt. **SPPU : May 14, May 16**

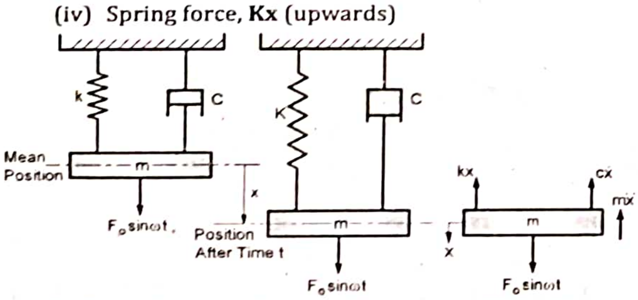
Ans. : Spring-Mass-Damper System Excited by Harmonic Force :

- Consider the system having spring-mass-damper system excited by a harmonic force $F_0 \sin \omega t$, as shown in Fig. 4.1.

where, F_o = magnitude of external exciting harmonic force, N

ω = circular frequency of external exciting force, rad/s

- **Forces acting on mass in displaced position [Fig. 4.1] :**
  - (i) External harmonic force, $F_o \sin \omega t$, (downwards)
  - (ii) Inertia force, $m\ddot{x}$ (upwards)
  - (iii) Damping force, $\dot{x}$ (upwards)
  - (iv) Spring force, Kx (upwards)



(a) Equilibrium Position (b) Displaced Position (c) F.B.D. of Mass

Fig. 4.1

Differential Equation of Motion for Forced Damped Vibrations :

Consider F.B.D. of mass, shown in Fig. 4.1(c). According to D' Alembert's principle,

$$\Sigma\,[\text{Inertia force} + \text{External force}] = 0$$

$$m\ddot{x} + c\dot{x} + Kx - F_o \sin \omega t = 0$$

$$m\ddot{x} + c\dot{x} + Kx = F_o \sin \omega t \quad ...(1)$$

Q.3 Plot magnification factor versus frequency ratio curve for different damping conditions and write concluding remarks. **SPPU : Dec. 15, May 16**

OR Explain with neat sketch, effect of damping on magnification factor for different forcing frequencies and hence justify that dynamic systems are to be operated at high speed as is possible. **SPPU : May 18**

OR Explain frequency response curve with neat labelled diagram. **SPPU : Dec. 18, May 19, Oct. 19(In Sem.), Dec. 19**

Ans. : Frequency Response Curves :

- **Frequency response curve :** The plot of magnification factor **(M.F.)** versus frequency ratio (ω / ω_n) is known as **frequency response curve** (Fig. 4.2).
- Frequency response curves is plotted using Equation,

$$\text{M. F.} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi \frac{\omega}{\omega_n}\right]^2}}$$

- **Observations made from frequency response curve :**
  - (i) The magnification factor (M.F.) is maximum when $(\omega/\omega_n) = 1$. This condition is known as **resonance**
  - (ii) As the damping factor (ξ) decreases, the maximum value of magnification factor (M.F.) increases.
  - (iii) When there is no damping ($\xi = 0$), the magnification factor (M.F.) reaches infinity at $(\omega/\omega_n) = 1$. However, the system may go into destruction much before that.
  - (iv) At zero frequency of excitation (i.e. at $\omega = 0$) the magnification factor (M.F.) is unity for all values of damping factors. In other words, the damping does not have any effect on magnification factor at zero frequency of excitation.
  - (v) At very high frequency of excitation, the magnification factor (M.F.) tends to zero.
  - (vi) For damping factor (ξ) more than 0.707, the magnification factor is below unity.

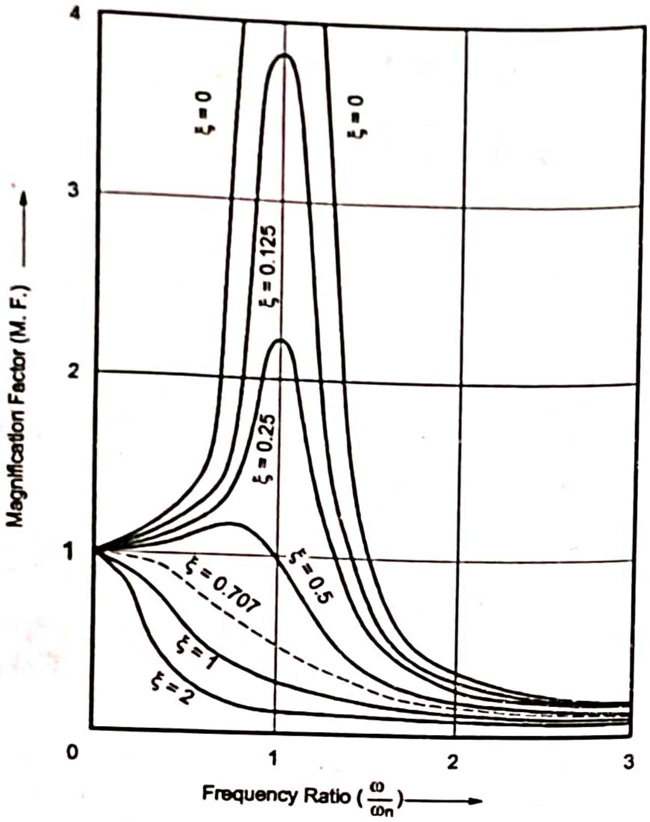


Fig. 4.2 : Frequency Response Curve For Different Damping Conditions

Q.4 Discuss the response curve and variation of phase angle with frequency ratio. **SPPU : May 19**

Ans. : Plot of Phase Angle (ϕ) Versus Frequency Ratio (ω / ω_n)

- The plot of phase angle (ϕ) versus frequency ratio (ω/ω_n) for different damping conditions is shown in Fig. 4.3. These curves are plotted using Equation,

$$\phi = \tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

- **Observations made from phase angle vs frequency ratio curves :**

  i) The phase angle varies from 0° at low frequency ratio to 180° at very high frequency ratio.

  ii) At resonance frequency (i.e. at $\omega = \omega_n$) the phase angle is 90° and damping does not have any effect on phase angle.

  (iii) At frequency ratio (ω / ω_n) less than unity, higher the damping factor-higher is the phase angle; whereas at frequency (ω / ω_n) greater than unity, higher the damping factor-lower is the phase angle. In other words, below resonance frequency, the phase angle increases with increase in damping factor; whereas above resonance frequency, the phase angle decreases with increase in damping factor.

  (iv) The variation in phase angle is because of damping. If there is no damping, ($\xi = 0$) the phase angle is either 0° or 180° and at resonance the phase angle suddenly changes from 0° to 180°.

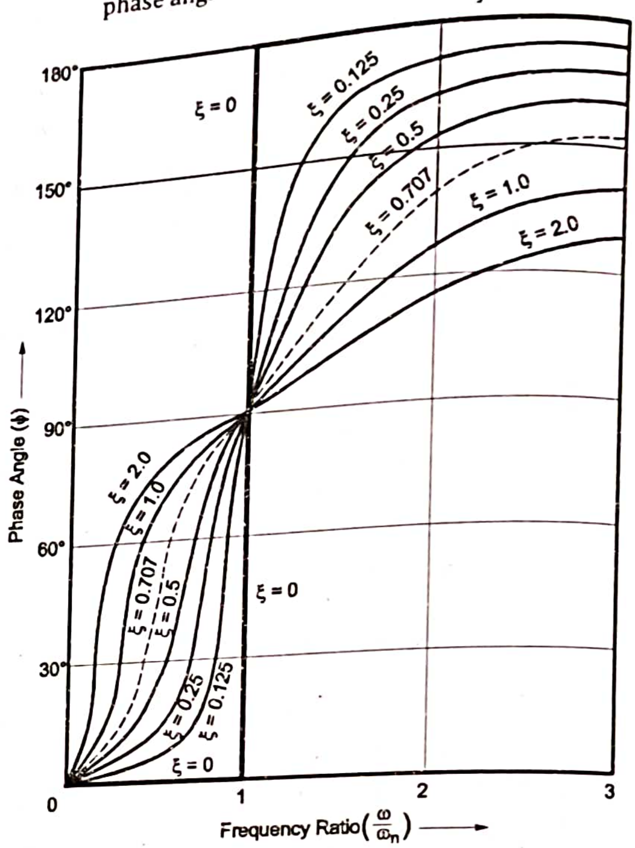


Fig. 4.3 : Phase Angle Versus Frequency Ratio for Different Damping Conditions

Q.5 A spring-mass-damper systems is subjected to a harmonic force. The amplitude is found to be 0.02 m at resonance and 0.01 m at a frequency 0.75 times the resonant frequency. Find the damping ratio of the system.
SPPU : Aug. 15 (In Sem), Oct. 16 (In Sem), 4 Marks

Ans. :

Given : $X = 0.02$ m at $\omega = \omega_n$

and $X = 0.01$ m at $\omega = 0.75\,\omega_n$

$$X = \frac{F_0 / K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\therefore \quad X = \frac{X_{st}}{\sqrt{[1-r^2]^2 + [2\xi r]^2}} \qquad ...(a)$$

where, $r = \frac{\omega}{\omega_n}$

1. At $\omega = \omega_n$:

$$r = \frac{\omega}{\omega_n} = 1$$

$\therefore$ Equation (a) becomes,

$$X = \frac{X_{st}}{2\xi}$$

$$\therefore 0.02 = \frac{X_{st}}{2\xi} \qquad ...(b)$$

2. At $\omega = 0.75\ \omega_n$:

$$\therefore \quad r = \frac{\omega}{\omega_n} = 0.75$$

$\therefore$ Equation (a) becomes,

$$X = \frac{X_{st}}{\sqrt{[1-r^2]^2 + [2\xi r]^2}}$$

$$0.01 = \frac{X_{st}}{\sqrt{[1-(0.75)^2]^2 + [2 \times \xi \times 0.75]^2}}$$

$$0.01 = \frac{X_{st}}{\sqrt{0.191 + 2.25\xi^2}} \qquad ...(c)$$

3. Damping ratio (ξ) :

Dividing equation (b) by equation (c) ;

$$\frac{0.02}{0.01} = \frac{X_{st}}{2\xi} \times \frac{\sqrt{0.191 + 2.25\xi^2}}{X_{st}}$$

$$\therefore \quad 4\xi = \sqrt{0.191 + 2.25\xi^2}$$

$$16\xi^2 = 0.191 + 2.25\,\xi^2$$

$$13.75\,\xi^2 = 0.1914$$

$$\xi = \mathbf{0.118} \qquad ...\text{Ans.}$$

Q.6 A vibrating system having mass 1 kg is suspended by a spring of stiffness 1000 N/m and it is put to harmonic excitation of 10 N. Assuming viscous damping, determine :

(i) the resonant frequency

(ii) the phase angle at resonance

(iii) the amplitude at resonance

(iv) the corresponding to the peak amplitude and

(v) damped frequency

Take damping coefficient, C = 40 N-S/m.

SPPU - Oct. 19(In Sem.), 6 Marks

Ans. :

Given : $m = 1$ kg ; $K = 1000$ N/m ; $F_0 = 10$ N ; $c = 40$ N-sec/m .

1. Resonance Frequency (ω_n) :

- **Natural circular frequency :**

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1000}{1}} = 31.62 \text{ rad/s}$$

- **Resonant frequency :**

$$\omega = \omega_n = \mathbf{31.62\ rad / s} \qquad ...\text{Ans.}$$

2. Phase Angle of Resonance (ϕ) :

$$\phi = \tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right] = \tan^{-1}\left[\frac{2\xi \times 1}{1-(1)^2}\right]$$

$$= \tan^{-1}[\infty]$$

or $\quad \phi = \mathbf{90°} \qquad$ **...Ans.**

3. Amplitude of resonance (X) :

- **Damping factor :**

$$\xi = \frac{c}{c_c} = \frac{c}{2\,m\,\omega_n} = \frac{40}{2 \times 1 \times 31.62} = 0.6325$$

- **Amplitude at resonance :**

$$X = \frac{F_0 / K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{10 / 1000}{\sqrt{[1-(1)^2]^2 + [2 \times 0.6325 \times 1]^2}}$$

or $\quad X = \mathbf{7.90 \times 10^{-3}\ m} \qquad$ **...Ans.**

4. Frequency Corresponding to Peak Amplitude :

- The frequency at which the amplitude becomes maximum is called as **frequency of peak amplitude (ω_p).**

- Now, $$X = \frac{F_o / K}{\sqrt{\left[1-\left(\frac{\omega_p}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega_p}{\omega_n}\right]^2}}$$

where, ω_p = frequency at peak amplitude, rad/s

$$X = \frac{X_{st}}{\sqrt{\left[1-\left(\frac{\omega_p}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega_p}{\omega_n}\right]^2}}$$

$$\frac{X}{X_{st}} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega_p}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega_p}{\omega_n}\right]^2}} \qquad ...(a)$$

- Putting $\frac{\omega_p}{\omega_n}$ = a, in Equation (a), we get,

$$\frac{X}{X_{st}} = \frac{1}{\sqrt{[1-a^2]^2+[2\xi a]^2}} \quad ...(b)$$

For maximum amplitude,

$$\frac{d}{da}\left(\frac{X}{X_{st}}\right) = 0$$

$$\frac{d}{da}\left(\frac{X}{X_{st}}\right) = \frac{2(1-a^2)(-2a)+2(2\xi a)(2\xi)}{2[(1-a^2)^2+(2\xi a)^2]^{3/2}} = 0$$

$$\therefore \quad 0 = \frac{-4a+4a^3+8a\xi^2}{2[(1-a^2)^2+(2\xi a)^2]^{3/2}} = -4a+4a^3+8a\xi^2$$

$$0 = -1+a^2+2\xi^2$$

$$\therefore \quad a^2 = 1-2\xi^2$$

$$\therefore \quad a = \sqrt{1-2\xi^2}$$

$$\therefore \quad \frac{\omega_p}{\omega_n} = \sqrt{1-2\xi^2}$$

$$\therefore \quad \omega_p = \omega_n\sqrt{1-2\xi^2} = 31.62\sqrt{1-2(0.6325)^2}$$

or $\quad \omega_p = 14.13 \text{ rad/s}$

5. Damped Natural Circular Frequency :

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 31.62\sqrt{1-(0.6325)^2}$$

$\therefore \quad \omega_d = 24.49 \text{ rad/sec}$ ...Ans.

Q.7 A machine part of mass 4 kg vibrates in a viscous fluid. Find the damping coefficient when a harmonic exciting force of 50 N results in resonant amplitude of 250 mm with a period of 0.4 sec. If the excitation frequency is 2 Hz, find the percentage increase in the amplitude of forced vibration when the damper is removed.
SPPU - Aug. 15(In Sem), Oct. 16(In Sem), 6 Marks

Ans. :

Given : $m = 4$ kg ; $F_o = 50$ N ;
$X = 250$ mm $= 0.25$ m ;
$t_p = 0.4$ s ; $f_1 = 2$ Hz.

1. Damping Coefficient (c) :

- **Circular frequency of forced vibrations :**

$$\omega = \frac{2\pi}{t_p} = \frac{2\pi}{0.4} = 15.70 \text{ rad/s}$$

- **At resonance :**

$$\therefore \quad \omega = \omega_n = 15.70 \text{ rad/s}$$

- **Spring stiffness :**

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$\therefore \quad 15.70 = \sqrt{\frac{K}{4}}$$

$$\therefore \quad K = 986.96 \text{ N/m}$$

- **Damping coefficient :**

$$X = \frac{F_0/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{F_0/K}{\sqrt{[1-(1)^2]^2+[2\xi\times 1]^2}} \quad [\because \omega = \omega_n]$$

$$= \frac{F_0/K}{2\xi} = \frac{F_0/K}{2\times\frac{c}{2m\omega_n}} \quad ...\left[\because \xi = \frac{c}{2m\omega_n}\right]$$

$$\text{Or} \quad X = \frac{F_0 \cdot 2m\omega_n}{2Kc}$$

$$\therefore \quad 0.250 = \frac{50\times 2\times 4\times 15.70}{2\times 986.96\times c}$$

$\therefore \quad c = 12.72$ N-s/m ...Ans.

- **Damping factor :**

$$\xi = \frac{c}{2m\omega_n} = \frac{12.72}{2\times 4\times 15.70} = 0.10$$

2. Amplitude of Forced Vibrations Under New Condition :

- **Circular frequency of forced vibrations :**

$$f_1 = \frac{\omega_1}{2\pi}$$

$$\therefore \quad 2 = \frac{\omega_1}{2\pi}$$

$$\therefore \quad \omega_1 = 12.56 \text{ rad/s}$$

- **Amplitude of forced vibrations at ω_1 = 31.41 rad/s and with damper :**

$$X_1 = \frac{F_0/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{50/986.96}{\sqrt{\left[1-\left(\frac{12.56}{15.70}\right)^2\right]^2+\left[2\times 0.1\times\frac{12.56}{15.70}\right]^2}}$$

or $\quad X_1 = 0.128$ ...Ans.

- **Amplitude of forced vibration at ω_1 = 31.41 rad/s and without damper :**

$$X_2 = \frac{F_0/K}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \frac{50/986.96}{1-\left(\frac{12.56}{15.70}\right)^2}$$

or $\quad X_2 = 0.140$ m. ...Ans.

- **Percentage increase in amplitude of vibrations when damper is removed :**

$$= \frac{X_2-X_1}{X_1}\times 100 = \frac{0.140-0.128}{0.128}\times 100$$

$= 9.37\%$ ...Ans.

Q.8 A 45 kg machine is mounted on four parallel springs each of stiffness 2×10^5 N/m. When the machine operates at 32 Hz, the machine's steady- state amplitude is measured as 1.5 mm. What is the magnitude of the excitation force provided to the machine at this speed ? **SPPU - May 19, 5 Marks**

Ans. :

$$m = 45\ \text{kg}\ ;\ K = 2 \times 10^5\ \text{N/m}$$

$$K_e = 4K = 4 \times 2 \times 10^5\ \text{N/m}$$

$$f = 32\ \text{Hz}$$

$$X = 1.5\ \text{mm} = 1.5 \times 10^{-3}\ \text{m}$$

$$\therefore \omega = f \times 2\pi = 32 \times 2\pi = 201.06\ \text{rad/s}$$

1. **undamped natural Circular Frequency (ω_n) :**

$$\omega_n = \sqrt{\frac{K_e}{m}} = \sqrt{\frac{4 \times 2 \times 10^5}{45}}$$

or $\omega_n = 133.33$ rad/s ...Ans.

2. **Frequency ratio (r) :**

$$r = \frac{\omega}{\omega_n} = \frac{201.06}{133.33} = 1.50$$

3. **Amplitude of Forced Vibration (X) :**

$$X = \frac{F_0 / K}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$X = \frac{F_0 / K_e}{1 - r^2}\ \text{or}\ \frac{F_0 / K_e}{r^2 - 1}$$

[$\because$ Assuming No damping]

$$1.5 \times 10^{-3} = \frac{F_0 / 4 \times 2 \times 10^5}{(1.50)^2 - 1}$$

$\therefore$ **F_0 = 1500 N** ...Ans.

Q.9 Write a short note on : Forced vibrations due to reciprocating unbalance. **SPPU : Dec. 16**

Ans.:Forced vibrations due to reciprocating unbalance

- **Examples of reciprocating unbalance :** The unbalance in reciprocating machines such as : I.C engines, reciprocating compressors, reciprocating pumps, etc is an another source of forced vibrations.
- **Reciprocating machine (Fig. 4.3) :**

Let, m = total mass of the reciprocating machine, kg

m_o = mass of reciprocating parts, kg

l = length of connecting rod, m

r = length of crank, m

n = obliquity ratio = l / r

ω = angular velocity of crank, rad/s

ωt = angle made by crank with horizontal reference axis.

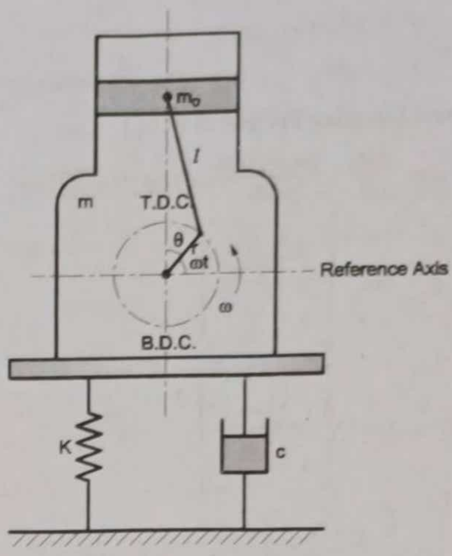


Fig. 4.3 : Reciprocating Unbalance

- **Inertia force due to mass of reciprocating parts :** The inertia force due to the mass of reciprocating parts acts as the excitation force in reciprocating machines. The inertia force due to mass of reciprocating parts is given by,

$$F_I = m_o r \omega^2 \left[\sin \omega t + \frac{\sin 2\omega t}{n}\right] \quad ...(1)$$

As 'n' is large $\left[\frac{\sin 2\omega t}{n}\right]$ is very small and hence, can neglected.

$$\therefore F_I = m_o r \omega^2 \sin \omega t \quad ...(2)$$

From Equation (2) it is seen that, the exciting force due to reciprocating unbalance is same as due to rotating unbalance, discussed in earlier section.

Conclusion : Therefore, analysis of the rotating unbalance is also applicable to reciprocating unbalance.

Q.10 A system having rotating unbalance has total mass of 25 kg. The unbalanced mass of 1 kg rotates with a radius 0.04 m. It has been observed that, at a speed of 1000 rpm, the system and eccentric mass have a phase difference of 90° and the corresponding amplitude is 0.015 m. Determine :

(i) the natural frequency of the system ;

(ii) the damping factor

(iii) the amplitude at 1500 r.p.m. ; and

(iv) the phase angle at 1500 r.p.m.

SPPU - May 15, 8 Marks, Oct 16 (In Sem), 4 Marks

Ans. :

Given : $m = 25$ kg ; $m_o = 1$ kg ;

$e = 40$ mm $= 0.04$ m ;

$N = 1000$ r.p.m.;

$\phi = 90°$; $X = 15$ mm $= 0.015$ m.

1. Natural Circular Frequency (ω_n) :

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1000}{60} = 104.71 \text{ rad/sec}$$

$$\phi = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$\therefore\ 90° = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$\therefore \tan(90°) = \left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$\therefore\ \infty = \left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$\therefore\ 1-\left(\frac{\omega}{\omega_n}\right)^2 = 0 \qquad \therefore\ \left(\frac{\omega}{\omega_n}\right)^2 = 1$$

$$\therefore\ \frac{\omega}{\omega_n} = 1 \qquad \therefore\ \omega_n = \omega$$

or $\omega_n = 104.71$ **rad/s.** ...Ans.

2. Damping Factor (ξ) :

$$\frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

At resonance $\omega = \omega_n$, hence

$$\therefore\ \frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{1}{2\xi}$$

$$\xi = \frac{m_o e}{2\,m\,X} = \frac{1\times 0.04}{2\times 25\times 0.015} = \mathbf{0.0533}$$ ...Ans.

3. Amplitude at 1500 r.p.m. (X_1) :

$$\omega_1 = \frac{2\pi N}{60} = \frac{2\pi\times 1500}{60} = 157.07 \text{ rad/s.}$$

$$X_1 = \frac{\left(\frac{m_o e}{m}\right)\left(\frac{\omega_1}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega_1}{\omega_n}\right)^2\right]^2+\left[2\xi\left(\frac{\omega_1}{\omega_n}\right)\right]^2}}$$

$$= \frac{\left(\frac{1\times 0.04}{25}\right)\left(\frac{157.07}{104.71}\right)^2}{\sqrt{\left[1-\left(\frac{157.07}{104.71}\right)^2\right]^2+\left[2\times 0.0533\times\left(\frac{157.07}{104.71}\right)\right]^2}}$$

or $X_1 = $ **1.79 mm** ...Ans.

4. Phase Angle at 1500 r.p.m. (ϕ_1) :

$$\phi_1 = \tan^{-1}\left[\frac{2\,\xi\frac{\omega_1}{\omega_n}}{1-\left(\frac{\omega_1}{\omega_n}\right)^2}\right] = \tan^{-1}\left[\frac{2\times 0.533\times\left(\frac{157.01}{104.71}\right)}{1-\left(\frac{157.01}{104.71}\right)^2}\right]$$

$$= \tan^{-1}[-0.12792]$$

$\therefore\ \phi_1 =$ **172.71° or 352.71°** ...Ans.

Q.11 A single cylinder vertical petrol engine of total mass 320 kg is mounted on a steel chassis and causes a vertical static deflection of 2 mm. The reciprocating parts of the engine have a mass of 24 kg and move through a vertical stroke of 150 mm with SHM. A dashpot attached to the system offers a resistance of 490 N at a velocity of 0.3 m/s. Determine :

(i) the speed of driving shaft at resonance; and

(ii) the amplitude of steady state vibrations when the driving shaft of the engine rotates at 480r.p.m.

SPPU - Dec. 16, Aug 17 (In Sem), 6 Marks

Ans. :

$m = 320$ kg ; $\delta = 2$ mm $= 0.002$ m

$m_o = 24$ kg ; $S = 150$ mm $= 0.15$ m;

$F = 490$ N ; $v = \dot{x} = 0.3$ m/s ;

$N = 480$ r. p. m ; $s = 2r = 0.15$ m

$\therefore\ r = 0.075$ m

- **Damping resistance :**

$$c = \frac{F}{v} = \frac{490}{0.3} = 1633.3 \text{ N-sec. m}$$

- **Natural circular frequency :**

$$\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{0.002}} = 70.04 \text{ rad/sec}$$

- **Damping factor (ξ) :**

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times 480}{60} = 50.272 \text{ rad/sec}$$

and $\dfrac{\omega}{\omega_n} = \dfrac{50.272}{70.04} = 0.717$

$$\therefore \xi = \frac{c}{2m\,\omega_n} = \frac{1633.3}{2 \times 320 \times 70.04}$$

$$\therefore \xi = 0.0364$$

1. **Speed of driving shaft at resonance :**

$$N_n = \frac{\omega_n \times 60}{2\pi} = \frac{70.04 \times 60}{2\pi}$$

$$\mathbf{N_n = 668.75\ rpm.}$$ ...**Ans.**

2. **Amplitude of steady vibration (X) :**

$$X = \frac{\left(\frac{m_o r}{m}\right)\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$X = \frac{\left(\frac{24 \times 0.075}{320}\right)(0.717)^2}{\sqrt{[1-0.717^2]^2 + [2 \times 0.0364 \times 0.717]^2}}$$

$$= \frac{2.891 \times 10^{-3}}{\sqrt{0.236 + 0.00272}}$$

$$\mathbf{X = 5.917 \times 10^{-3}\ m}$$

OR $\mathbf{X = 5.917mm}$

Q.12 Explain the term : Force transmissibility.

SPPU : Dec. 12, Dec. 13, May 15, May 16

Ans. : Force Transmissibility (T_r) :

- **Force Transmissibility :** Force transmissibility is defined as *the ratio of the force transmitted to the supporting structure or foundation,* $\mathbf{F_T}$ *to that force impressed upon the system,* $\mathbf{F_0}$. Force transmissibility measures the effectiveness of the vibration isolating material.

$$\text{Force Transmissibility} = \frac{\text{Force transmitted to the foundation}}{\text{Force impressed upon the system}}$$

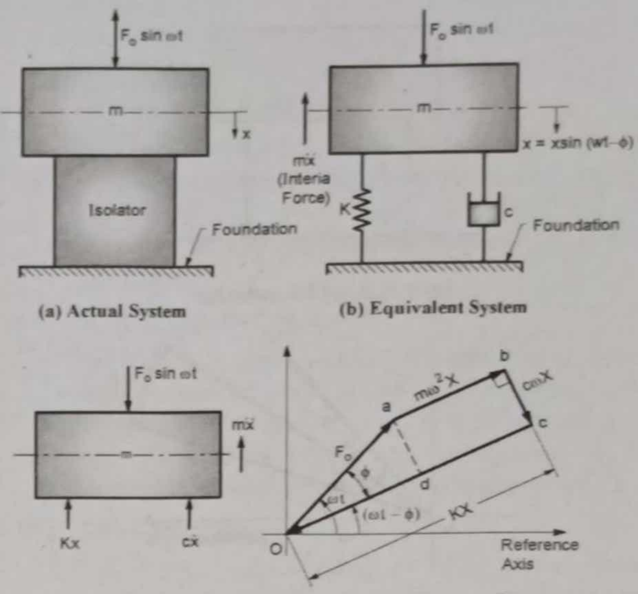
$$\therefore T_r = \frac{F_T}{F_0} \quad ...(1)$$

- **Spring Mass-Damper system :** Consider the mass '**m**' is supported on the foundation by means of an isolator and excited by the external force $F_0 \sin \omega t$, as shown in Fig. 4.4(a).

  The displacement of mass is,

$$x = X \sin(\omega t - \phi)$$

Hence,
$$\dot{x} = X\omega \cos(\omega t - \phi)$$
$$= -X\omega \sin\left(\omega t - \phi + \frac{\pi}{2}\right)$$
$$\ddot{x} = -X\omega^2 \sin(\omega t - \phi)$$



(a) Actual System (b) Equivalent System

(c) F.B.D. of Mass (d) Vector Diagram of forces Acting on Mass

Fig. 4.4 : Forces Acting on Mass

Let, K = equivalent stiffness of isolator, N/m

c = damping coefficient of isolator, N-s/m

Force Acting on Mass :

(i) External impressed force, $F_0 \sin \omega t$ (downwards)

(ii) Inertia force, $m\ddot{x} = m\omega^2 X \sin(\omega t - \phi)$ (upward)

(iii) Damping force,

$$c\dot{x} = -\left(\omega X \sin\left(\omega t - \phi + \frac{\pi}{2}\right)\right) \text{ (upwards)}$$

(iv) Spring force, Kx (upwards) = $KX \sin(\omega t - \phi)$(upwards)

Fig. 4.4(c) shows the F.B.D. of mass and Fig. 4.4(d) shows the vector diagram of forces acting on the mass.

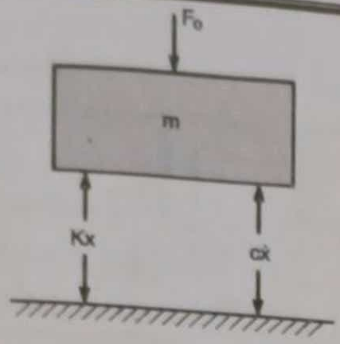
- **Force Transmitted to Foundation :**

  Out of four forces acting on mass, the following two forces are transmitted to the foundation [Fig. 4.5(a)]

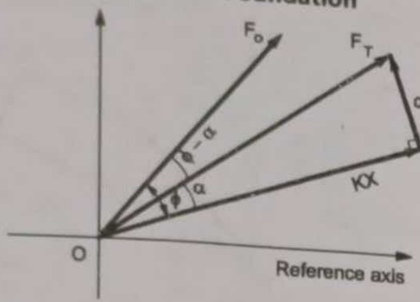
  (i) Spring force Kx (downwards).

  (ii) Damping force, $c\dot{x}$ (downwards).

  The total force transmitted to the foundation, $\mathbf{F_T}$ is the vector sum of these two forces acting on the foundation. These two forces i.e. spring force and damping force are 90° out of phase with each other, as shown in Fig. 4.5(b). Fig. 4.5(b) shows the force transmitted to the foundation '$\mathbf{F_T}$' and force impressed upon the mass, '$\mathbf{F_0}$'.



(a) F.B.D. of Foundation



(b) Vector Diagram of Transmitted and Impressed Forces

Fig. 4.5 : Force Transmitted to Foundation

- From Fig. 4.5(b), force transmitted to foundation is given by,

$$F_T = \sqrt{(KX)^2 + (c\,\omega\,X)^2}$$

or

$$F_T = X\sqrt{K^2 + (c\,\omega)^2} \qquad \text{...(a)}$$

$$X = \frac{F_0 / K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\,\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}} \qquad \text{(1(a))}$$

Substituting the value of X form Equation (1(a)) in Equation (a), we get,

$$F_T = \left[\frac{F_o/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}\right]\sqrt{K^2 + (c\omega)^2}$$

$$= \frac{F_o\sqrt{1+\left(\frac{c\omega}{K}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\text{or } F_T = \frac{F_o\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}} \qquad \text{...(b)}$$

$$\left[\because \frac{c\omega}{K} = 2\xi\frac{\omega}{\omega_n}\right]$$

- **Force Transmissibility (T_r) :**

The force transmissibility is given by,

$$T_r = \frac{F_T}{F_o}$$

From Equation (b),

$$T_r = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}} \qquad \text{...(2)}$$

- **Angle of Lag ($\phi - \alpha$) :**

The angle through which the transmitted force $\mathbf{F_T}$ lags the impressed force $\mathbf{F_o}$ is ($\phi - \alpha$)

where, $\alpha = \tan^{-1}\left[\frac{c\,\omega\,X}{KX}\right] = \tan^{-1}\left[\frac{c\,\omega}{K}\right]$

$$\text{or} \quad \alpha = \tan^{-1}\left[2\xi\frac{\omega}{\omega_n}\right] \qquad \text{...(3)}$$

Therefore, the angle of lag is given by,

$$(\phi - \alpha) = \tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right] - \tan^{-1}\left[2\xi\frac{\omega}{\omega_n}\right] \qquad \text{...(4)}$$

Q.13 Explain significance of Force transmissibility versus Frequency ratio curve. **Dec. 14, Dec. 15**

OR Draw transmissibility curves for different damping conditions . Give the significance of these curves. **SPPU : Oct. 19(In Sem.)**

Ans. : Transmissibility Versus Frequency Ratio :

The plot of transmissibility ($\mathbf{T_r}$) versus frequency ratio ($\boldsymbol{\omega/\omega_n}$) for different damping conditions is shown in Fig. 4.6. The curves are plotted using Equation (2(From above question)).

- **Observation From Transmissibility Versus Frequency Ratio Curves :**

1. All the curves start from the unit value of transmissibility. At frequency ratio $\left(\frac{\omega}{\omega_n}\right) = 0$, transmissibility, $T_r = 1$.
2. When $(\omega/\omega_n) = 1$, the transmissibility is maximum. In this conditions, damping controls the transmissibility.
3. When $(\omega/\omega_n) < \sqrt{2}$, the transmissibility is greater than one. This means, the transmitted force is always greater than the impressed exciting force. In this range, the greater amount of damping gives lower transmissibility.

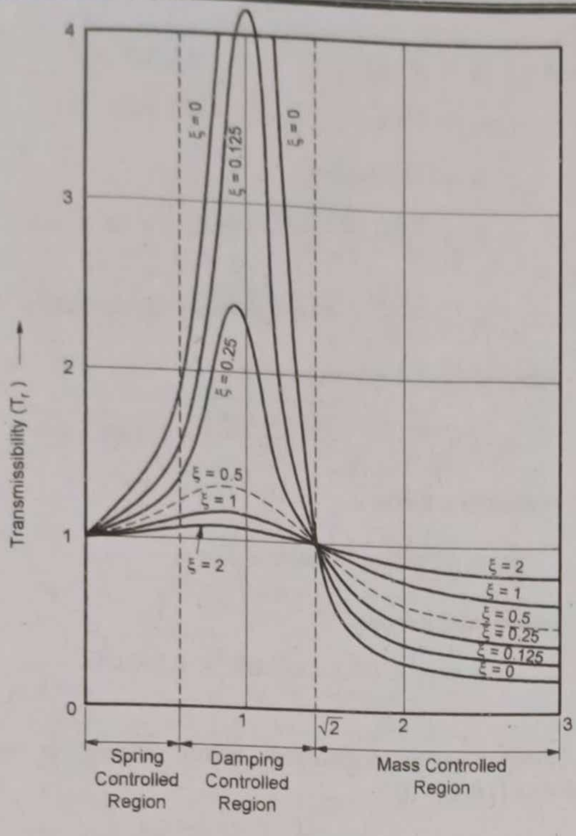


Fig. 4.6 : Transmissibility Versus Frequency Ratio

4. When $(\omega/\omega_n) = \sqrt{2}$, the transmissibility is equal to one. This means, the transmitted force is equal to the impressed exciting force, irrespective of damping. At this value of frequency ratio (ω/ω_n), the transmissibility is independent of the damping.
5. When $(\omega/\omega_n) > \sqrt{2}$, the transmissibility is less than one. This means, the transmitted force is always less than the impressed exciting force. The better vibration isolation is possible in this range. In this range, greater amount of damping gives greater transmissibility, and hence, damping is unfavorable.
6. The transmissibility, T_r tends to zero as the frequency ratio $\left(\frac{\omega}{\omega_n}\right)$ tends to infinity.
7. In order to have low value of transmissibility, the operation of vibrating system generally kept in the range $(\omega/\omega_n) > \sqrt{2}$. In this range, zero damping will be ideally suitable as this would give extremely low value of transmissibility. But since the system has to pass through the resonance (i.e. $\omega = \omega_n$) in reaching the operating point and zero damping will give very high transmissibility (though for a moment only), some amount of damping is generally incorporated in the system.

- **Regions of Transmissibility Vs Frequency Ratio Curve :** The transmissibility curve can be divided into three distinct frequency regions, as shown in Fig. 4.6.

(i) Spring controlled region :

The region where (ω/ω_n) is small, is called **spring controlled region**. In this region, the larger value of spring stiffness gives high value of natural frequency (ω_n) and consequently lower frequency ratio (ω/ω_n).

(ii) Damping controlled region :

The middle region is called **damping controlled region** which should be generally avoided. When damping is zero and $\omega = \omega_n$, the transmissibility tends to infinity. Therefore, some amount of damping is generally incorporated in the system.

(iii) Mass controlled region :

The region where (ω/ω_n) is large called **mass controlled region**. The larger value of mass gives lower value of natural frequency (ω_n) and consequently higher frequency ratio (ω/ω_n).

Q.14 A mass 25 kg is placed on an elastic foundation. A sinusoidal force of magnitude 25 N is applied to the machine. A frequency sweep reveals that the maximum steady state amplitude of 1.3 mm occurs when the period of response is 0.22 seconds. Determine the equivalent stiffness and damping ratio of the foundation. **SPPU - May 17, 6 Marks**

Ans. :

Given : $F_0 = 25$ N ; $m = 25$ kg ;

$X_{max} = 1.3$ mm ; $t_p = 0.22$ sec

For linear system, the frequency of response is same as frequency of excitation.

$\therefore$ Excitation frequency $= \omega = 2\pi f$

$= 2\pi / t_p = 28.6$ rad/sec

Thus X_{max} = occurs when $\omega = 28.6$ rad/s

- Condition for maximum amplitude to occur

$$r = \sqrt{1 - 2\xi^2} = \frac{\omega}{\omega_n}$$

$$\therefore \omega_n = \frac{\omega}{\sqrt{1-\xi^2}} = \frac{28.6}{\sqrt{1-\xi^2}} \quad ...(1)$$

- Also we have,

$$\frac{X}{X_{st}} = \frac{1}{\sqrt{[1-r^2]^2 + [2\xi r]^2}}$$

For $x_{max} = \quad r = \sqrt{1 - 2\xi^2}$

$$\frac{X_{max}}{X_{st}} = \frac{1}{\sqrt{[1-(1-2\xi^2)]^2 + [4\xi^2(1-2\xi^2)}}$$

$$= \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\frac{X_{max}K}{F_0} = \frac{1}{2\xi\sqrt{1-\xi^2}} \; ;$$

$$\frac{X_{max}m\omega_n^2}{F_0} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$25 \times 0.013 \times \frac{\omega_n^2}{25} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

Now substitute for ω_n^2 from Equation (1),

$$0.013 \times \frac{28.6}{\sqrt{1-2\xi^2}} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\frac{1.0633}{\sqrt{1-2\xi^2}} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

Squaring and rearranging,

$$\xi^4 - \xi^2 + 0.117 = 0$$

$$Z^2 - Z + 0.117 = 0$$

Where $\xi^2 = Z$

Solving the quadratic equation

$$\xi = 0.368, 0.93$$

The larger value of ξ is to be discarded because the amplitude would be maximum only for $\xi < 0.707$

$$\therefore \quad \xi = \mathbf{0.368}$$

$\therefore$ Natural frequency $\omega_n = \dfrac{\omega}{\sqrt{(1-2(0.368)^2)}}$

$$= 3.5 \text{ rad/sec}$$

Stiffness of the foundation,

$$K = m\omega_n^2 = 25\,(33.5)^2$$

$$= \mathbf{28.05 \times 10^3 \; N/m} \quad \text{...Ans.}$$

Q.15 An electric motor weighs 25 kg and is mounted on a rubber pad which deflects by 1 mm to motor weight. The rotor weighs 5 kg, has an eccentricity of 0.1 mm and rotates at 1500 r.p.m. Find the amplitude of vibration of the motor and the force transmitted to the foundation under the following conditions :

(i) there is no damping.

(ii) damping factor = 0.1

SPPU - Oct. 18(In sem),6 Marks

Ans. :

Given : $m = 25$ kg ; $\delta = 1 \times 10^3$ m ;

$m_o = 5$ Kg ; $e = 0.1 \times 10^{-3}$ m ;

$N = 1500$ r.p.m.

$$K = \frac{mg}{\delta} = \frac{25 \times 9.81}{1 \times 10^{-3}} = 245.25 \times 10^3 \text{ N / m} ;$$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1500}{60} = 157.08 \text{ rad/s}$$

1. Natural Circular Frequency :

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{245.25 \times 10^3}{25}} = 99.045 \text{ rad/s}$$

2. Frequency Ratio :

$$\frac{\omega}{\omega_n} = \frac{157.08}{99.045} = 1.586$$

3. Impressed Force :

$$F_0 = m\omega^2 e = 5 \times (157.08)^2 \times 0.1 \times 10^{-3}$$

$$= 12.33 \text{ N}$$

4. Amplitude of Vibration When There is No Damping (i.e. $\xi = 0$) :

$$X = \frac{F_0}{K\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$X = \frac{F_0}{K\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2}} \quad \text{...(when } \xi = 0)$$

$$X = \frac{12.33}{245.25 \times 10^3\sqrt{(1-(1.586)^2)^2}}$$

$$= 3.31 \times 10^{-5} \text{ m}$$

$$X = \mathbf{0.0331 \; mm} \quad \text{...Ans.}$$

5. Amplitude of Vibration When Damping $\xi = 0.1$:

$$X = \frac{12.33}{245.25 \times 10^3\sqrt{[1-(1.586)^2]^2 + [2 \times 0.1 \times 1.586]^2}}$$

$$= 3.25 \times 10^{-5} \text{ m}$$

$$X = \mathbf{0.0325 \; mm} \quad \text{...Ans.}$$

6. Force Transmitted When There is No Damping (i.e. $\xi = 0$) :

$$T_r = \frac{F_T}{F_0} = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$\frac{F_T}{F_r} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}} \quad \text{...(when } \xi = 0)$$

$$\frac{F_T}{12.33} = \frac{1}{\sqrt{[1-(1.586)^2]^2}}$$

$$F_T = \mathbf{8.13\ N}$$ **...Ans.**

7. Force Transmitted When $\xi = 0.1$:

$$\frac{F_T}{12.33} = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$= \frac{\sqrt{1+(2\times0.1\times1.586)^2}}{\sqrt{[1-(1.586)^2]^2+[2\times0.1\times1.586]^2}}$$

$$F_T = \mathbf{8.36\ N}$$ **...Ans.**

Q.16 An exhaust fan, rotating at 1000 rpm, is to be supported by four springs, each having a stiffness of K. If only 10 percent of the unbalanced force of the fan is to be transmitted to the base, what should be the value of K? Assume the mass of the exhaust fan to be 40 kg.

SPPU - May 18, 8 Marks

Ans. : Given :

Mass of refrigerator, $m = 40$ kg

Speed of refrigerator unit, $N = 1000$ r.p.m.

$$\therefore \quad \omega = \frac{2\pi\times1000}{60} = 104.71 \text{ rad/sec.}$$

Transmissibility, $T_r = 10\% = 0.1$

1. Natural Circular Frequency (ω_n) :

The transmissibility is,

$$T_R = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}} \quad \text{...(a)}$$

- Since damping is not present (i.e. $\xi = 0$), Equation (a) becomes,

$$T_r = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}} = \frac{1}{\sqrt{\left[\left(\frac{\omega}{\omega_n}\right)^2-1\right]^2}}$$

or $$T_r = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2-1}$$

$$\therefore \quad 0.1 = \frac{1}{\left(\frac{104.71}{\omega_n}\right)^2-1}$$

$$\therefore \quad \frac{1096.41}{\omega_n^2} - 0.1 = 1$$

$$\therefore \quad \omega_n^2 = 996.73$$

$$\therefore \quad \omega_n = 31.57 \text{ rad/s.}$$

2. Spring Constant (K) :

- Four springs having stiffness K are used. Assuming they are in parallel, the equivalent spring stiffness is,

$$K_e = 4K$$

- Therefore, the natural circular frequency of the system is,

$$\omega_n = \sqrt{\frac{K_e}{m}}$$

or $$\omega_n = \sqrt{\frac{4K}{m}}$$

$$\therefore \quad 31.57 = \sqrt{\frac{4K}{30}}$$

$$\therefore \quad \mathbf{K = 7474.98\ N/m}$$ **...Ans.**

Q.17 A single cylinder vertical petrol engine of total mass 400 kg is mounted upon a steel chassis frame and causes a vertical static deflection of 2.5 mm. The reciprocating parts of the engine have a mass of 5 kg and move through a vertical stroke of 120 mm with SHM. A dashpot provided, the damping resistance of which is directly proportional to the velocity and amounts to 20 N at 1m/s, if a steady state vibrations has been reached. determine :

a) The amplitude of forced vibration when the driving shaft of engine rotates at 540 rpm

b) The maximum dynamic force transmitted to the ground through chassis frame (which behaves as a spring), through the dashpot and through the chassis frame and dashpot together.

c) The driving shaft speed at which resonance will occur.

SPPU - Oct. 18(In sem), 10 Marks

Ans. :

$m = 400$ kg ; $\delta = 2.5$ mm $= 0.0025$ m ;

$m_o = 5$ kg ; $S = 120$ mm $= 0.12$ m;

$F = 20$ kN $= 20\times10^3$ N;

$v = 1$ m/s ; $N = 540$ r. p. m.;

$$r = \frac{S}{2} = \frac{0.12}{2} = 0.06 \text{ m}$$

- **Damping resistance :**

$$c = \frac{F}{v} = \frac{20 \times 10^3}{1} = 20 \times 10^3 \text{ N-sec. m}$$

- **Load taken by each spring = $\frac{mg}{4}$**
- **Stiffness of each spring :**

$$K = \frac{mg/4}{\delta} = \frac{400 \times 9.81}{4 \times 0.0025} = 392.4 \text{ N/m}$$

- **Equivalent stiffness of four springs :**

$$K_e = 4K = 4 \times 392.4$$

or $$K_e = 15.696 \times 10^5 \text{ N/m}$$

- **Natural circular frequency :**

$$\omega_n = \sqrt{\frac{K_e}{m}} = \sqrt{\frac{15.696 \times 10^5}{400}} = 62.64 \text{ rad/sec}$$

- **Damping factor (ξ) :**

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 540}{60} = 56.54 \text{ rad/sec}$$

and $$\frac{\omega}{\omega_n} = \frac{56.54}{62.64} = 0.90$$

$$\therefore \quad \xi = \frac{C}{2m\,\omega_n} = \frac{20 \times 10^3}{2 \times 400 \times 62.64}$$

$$\therefore \xi = 0.399$$

1. **Speed of driving shaft at resonance :**

$$N_n = \frac{\omega_n \times 60}{2\pi} = \frac{62.64 \times 60}{2\pi}$$

$$\mathbf{N_n = 598.16 \text{ rpm.}}$$ ...Ans.

2. **Amplitude of steady vibration (X) :**

$$X = \frac{\left(\frac{m_o r}{m}\right)\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$X = \frac{\left(\frac{5 \times 0.06}{400}\right)(0.90)^2}{\sqrt{[1 - 0.90^2]^2 + [2 \times 0.399 \times 0.90]^2}}$$

$$= \frac{6.075 \times 10^{-4}}{\sqrt{0.0361 + 0.5158}}$$

$$\mathbf{X = 8.17 \times 10^{-4} \text{ m}}$$

OR $$\mathbf{X = 0.8177 \text{ mm}}$$

3) **The maximum force transmitted to the ground is**

$$F_t = KX = 15.696 \times 10^5 \times 8.17 \times 10^{-4} \text{ m}$$

$$= \mathbf{1282.36 \text{ N}}$$ ...Ans.

The dynamic force transmitted to the foundation through dashpot is,

$$F_d = c\dot{x} = c\omega \times = 20 \times 10^3 \times 56.54 \times 8.17 \times 10^{-4}$$

$$= 923.86 \text{ N}$$

The spring force F_t & the force in dashpot F_d are out of phase by 90° and therefore maximum force transmitted to the ground through chassis & dashpot both is,

$$F = \sqrt{F_t^2 + F_d^2}$$

$$= \sqrt{(1282.36)^2 + (923.86)^2}$$

$$\mathbf{F = 1580.49 \text{ N}}$$ ... **Ans.**

Q.18 The static deflection of an automobile spring under its weight is 10 cm. Find the critical speed when the trailer is traveling over a road with a profile approximated by a sine wave of amplitude 8 cm and wavelength of 16 m. If damping factor is $\xi = 0.05$, what will be the amplitude of vibration at 75 km/hr ?

SPPU - Dec.14, Dec.15, Dec. 18(In sem). 10 Marks

Ans. :

Given : $\delta = 10 \text{ cm} = 0.1 \text{ m}$; $Y = 8 \text{ cm} = 0.08 \text{ m}$;

$\lambda = 16 \text{ m}$; $\xi = 0.05$.

$$v = 70 \text{ km/hr} = \frac{70 \times 1000}{3600} = 19.44 \text{ m/s}$$

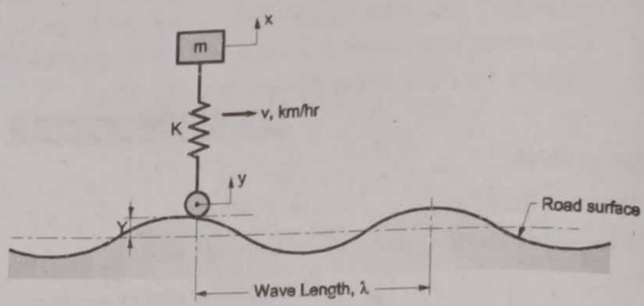


Fig. 4.7

1. **Frequency Ratio$\left(\frac{\omega}{\omega_n}\right)$:**

$$\therefore \omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{0.1}} = 9.90 \text{ rad/s}$$

$$\text{Time period} = \frac{\text{Wavelength}}{\text{Velocity}}$$

$$\therefore \quad t_p = \frac{\lambda}{v}$$

$$\therefore \quad \frac{2\pi}{\omega} = \frac{\lambda}{v}$$

$$\therefore \quad \frac{2\pi}{\omega} = \frac{16}{19.44}$$

$$\therefore \quad \omega = 8.82 \text{ rad/s.}$$

$$\therefore \frac{\omega}{\omega_n} = \frac{8.82}{990} = 0.89$$

2. **Steady - state Amplitude (X) :**

For forced vibrations due to excitation of support, the steady-state amplitude is given by,

$$X = \frac{Y\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}} \quad ...(a)$$

$$= \frac{0.08\times\sqrt{1+(2\times0.05\times0.89)^2}}{\sqrt{[1-(0.89)^2]^2+[2\times0.05\times0.89]^2}}$$

$= 0.3551$

$\mathbf{X = 355.1\ mm}$ ...Ans.

3. **Critical Speed of Vehicle (v_c) :**

At the critical speed of vehicle, $\omega = \omega_n$, therefore,

$$\frac{2\pi}{\omega_n} = \frac{\lambda}{v_c}$$

$$\therefore \quad \frac{2\pi}{9.90} = \frac{16}{v_c}$$

$$\therefore \quad v_c = 25.21\ m/sec = \frac{25.21\times3600}{1000}$$

or $\mathbf{v_c = 90.75\ km/hr}$ ...Ans.

Q.19 An automobile, weighing 9.8 kN when fully loaded and 2.45 kN when empty, vibrates in a vertical direction while travelling at 96 km/hr on a rough road having a sinusoidal wave form with an amplitude Y and wavelength 4.88m. Assuming that the automobile can be modeled as a single degree-of-freedom system with stiffness 350 kN/m and the damping factor 0.5. Determine the amplitude ratio of the vehicle when fully loaded and when empty.

SPPU : Oct 16 (In Sem), 6 Marks

Ans. :

Given : $m_f = \frac{9.8\times10^3}{9.8}$ kg = 1000 kg ;

$m_e = \frac{2.45\times10^3}{9.8}$ =250 kg;

$v = 96\ km/hr = \frac{96\times10^3}{3600} = 26.66\ m/sec.$;

$\lambda = 4.88\ m$;

$K = 350\ kN/m = 350\times10^3\ N/m$;

$\xi = 0.5.$

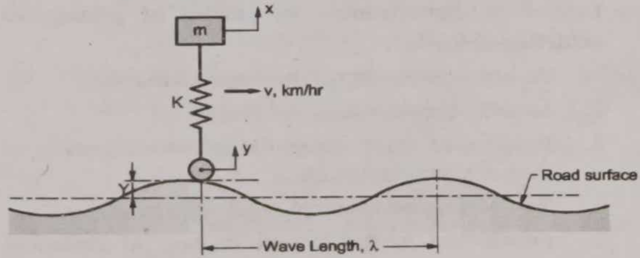


Fig. 4.8

1. **Forced Circular Frequency (ω) :**

- **Time period :**

$$t_p = \frac{\text{Wavelength}}{\text{Velocity}} = \frac{\lambda}{v} = \frac{4.88}{26.66} = 0.183\ s$$

- **Forced Circular Frequency :**

$$\omega = \frac{2\pi}{t_p} = \frac{2\pi}{0.183} = 34.32\ rad/s.$$

2. **Case I :Vehicle Fully Loaded :**

- **Natural circular frequency :**

$$\omega_n = \sqrt{\frac{K}{m_f}} = \sqrt{\frac{350\times10^3}{1000}} = 18.70\ rad/s$$

- **Ratio of amplitude due to excitation of support :**

$$\frac{X}{Y} = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{\sqrt{1+\left(2\times0.5\times\frac{34.32}{18.70}\right)^2}}{\sqrt{\left[1-\left(\frac{34.32}{18.70}\right)^2\right]^2+\left[2\times0.5\times\left(\frac{34.32}{18.70}\right)\right]^2}}$$

or $\frac{X}{Y} = \mathbf{0.69\ m}$ ...Ans.

3. **Case II : Vehicle is Empty :**

- **Natural circular frequency of system :**

$$\omega_n = \sqrt{\frac{K}{m_e}} = \sqrt{\frac{350\times10^3}{250}} = 37.41\ rad/s.$$

- **Ratio of amplitude due to excitation to support :**

$$\frac{X}{Y} = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{\sqrt{1+\left(2\times0.5\times\frac{34.32}{37.41}\right)^2}}{\sqrt{\left[1-\left(\frac{34.32}{37.41}\right)^2\right]^2+\left[2\times0.5\times\frac{34.32}{37.41}\right]^2}}$$

or $\frac{X}{Y} = \mathbf{1.45}$ ...Ans.

Q.20 An instrument of 50 kg mass is located in an airplane cabin which vibrates at 2000 r.p.m. with an amplitude of 0.1 mm, Determine the stiffness of the four springs to be used as supports for the instrument to reduce its amplitude to 0.005 mm. Also calculate the maximum total load for which each spring must be designed.

SPPU - May 16

Ans. :

Given : $m = 50\text{ kg}$; $N = 2000\text{ r.p.m.}$; $Y = 0.1\text{ mm}$; $X = 0.005\text{ mm}$

1. **Transmissibility :**

and $$T_r = \frac{X}{Y} = \frac{0.005}{0.1} = 0.05$$

2. **Natural Circular Frequency (ω_n) :**

$$\therefore \omega = \frac{2\pi \times 2000}{60} = 209.43\text{ rad/sec.}$$

$$T_r = \frac{\sqrt{1 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}} \quad \text{...(a)}$$

- Since damping is not present (i.e. $\xi = 0$), Equation (a) becomes,

$$T_r = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2}} = \frac{1}{\sqrt{\left[\left(\frac{\omega}{\omega_n}\right)^2 - 1\right]^2}}$$

or $$T_r = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\therefore 0.05 = \frac{1}{\left(\frac{209.43}{\omega_n}\right)^2 - 1}$$

$$\therefore \frac{2193.04}{\omega_n^2} - 0.05 = 1 ;$$

$$\therefore \omega_n^2 = 2088.60$$

$$\therefore \omega_n = 45.70\text{ rad/s.}$$

3. **Spring Constant (K) :**

- Four springs having stiffness K are used. Assuming the springs are in parallel, the equivalent spring stiffness is,

$$K_e = 4K$$

$$\omega_n = \sqrt{\frac{K_e}{m}}$$

or $$\omega_n = \sqrt{\frac{4K}{m}}$$

$$\therefore 45.70 = \sqrt{\frac{4K}{30}}$$

$$\therefore \mathbf{K = 15663.67\ N/m} \quad \text{...Ans.}$$

4. **Relative Amplitude (Z) :**

$$Z = \frac{Y\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{Y\left(\frac{\omega}{\omega_n}\right)^2}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} \quad [\because \xi = 0]$$

$$= \frac{0.1 \times 10^{-3} \times 4.58^2}{(4.58)^2 - 1} \quad \left[\because \frac{\omega}{\omega_n} = \frac{209.43}{45.70} = 4.58\right]$$

$$\therefore Z = 1.05 \times 10^{-4}\text{ m}$$

5. **Dynamic Load on Each Isolator (F_s) :**

The spring force is,

$$F_s = KZ = 15663.67 \times 1.05 \times 10^{-4}$$

or $$\mathbf{F_s = 1.64\ N} \quad \text{...Ans.}$$

Q.21 What do you mean by whirling of shaft ?

SPPU : May 15, May 18

OR Explain critical speed of shaft carrying single rotor.

SPPU : Oct. 18(In sem)

Ans. : Critical Speed of Shafts

- **Whirling of shaft :** When a rotor is mounted on a shaft, its center of gravity usually does not coincide with the axis of rotation of the shaft. This center of gravity is normally displaced from the axis of rotation, although the amount of displacement may be very small. As a result of this initial eccentricity of the center of gravity from the axis of rotation, shaft is subjected to a centrifugal force when it begins to rotate.
- This centrifugal force acts radially outwards, which makes the shaft to bend in the direction of eccentricity of the C. G. This further increases the eccentricity, and hence the magnitude of centrifugal force.
- In this way the effect is cumulative and ultimately the shaft may fail. Because of this unbalanced centrifugal force, a shaft starts vibrating violently in the direction perpendicular to the axis of the shaft. This phenomenon is known as **whirling of shaft**.
- **Critical speed (whirling speed) :** The speed at which the shaft starts to vibrate violently in the direction perpendicular to the axis of the shaft is known as **critical speed** or **whirling speed**.
- **Causes of displacement of centre of gravity of Whirling of shafts :**
  1. Eccentric mounting of the rotor on the shaft
  2. Lack of straightness of the shaft,
  3. Bending of shaft under the action of gravity in case of horizontal shaft,
  4. Non-homogeneous rotor material, and
  5. Unbalanced magnetic pull in case of electrical machinery.

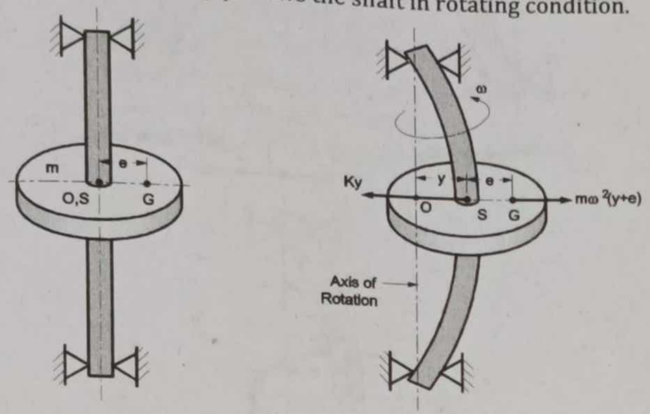
Q.22 Obtain expression for excessive transverse vibrations of a simply supported shaft rotating at N rpm with a rotor of mass 'm' having eccentricity 'e'.

SPPU : May 18

Ans. : Critical Speed of Shaft Carrying Single Rotor Without Damping

Vertical Rotating Shaft :

- Consider a vertical shaft having negligible inertia and carrying a single rotor, as shown in Fig. 4.9.
- Fig. 4.9(a) shows the shaft in stationary condition, while Fig. 4.9(b) shows the shaft in rotating condition.



(a) Shaft in Stationary Condition **(b) Shaft in Rotating Condition**

Fig. 4.9

Let, O = point of intersection of bearing centre line with the rotor. It is the point on the axis of rotation,

S = geometric centre of the rotor

G = centre of gravity of the rotor

m = mass of the rotor, kg

e = eccentricity of the rotor i.e. distance between the C.G. of rotor and geometric center 'S', m

y = deflection of geometric center 'S' from point 'O' due to centrifugal force, m

K = transverse stiffness of the shaft, N/m

ω = angular speed of the shaft, rad/s

ω_n = natural circular frequency of lateral or transverse vibrations of the shaft, rad/s

- **Forces Acting on Shaft in Rotating condition :** There are two forces acting on the rotating shaft :
  1. **Centrifugal Force = m ω^2 (y + e)** (radially outward through point G)
  2. **Restoring Force = Ky** (radially inward through point G)

Equation of Motion :

- **In equilibrium condition,**

$$\text{Centrifugal force} = \text{Restoring force}$$

$$\therefore \quad m\omega^2 (y + e) = Ky$$

$$\therefore \quad m\omega^2 y + m\omega^2 e = Ky$$

$$Ky - m\omega^2 y = m\omega^2 e$$

$$y (K - m\omega^2) = m\omega^2 e$$

$$\therefore \quad y = \frac{m\omega^2 e}{K - m\omega^2} = \frac{\frac{m\omega^2 e}{K}}{1 - \frac{m\omega^2}{K}} = \frac{\frac{\omega^2 e}{K/m}}{1 - \left(\frac{\omega^2}{K/m}\right)}$$

$$\text{or} \quad y = \frac{\left(\frac{\omega}{\omega_n}\right)^2 e}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \qquad \left[\because \omega_n^2 = \frac{K}{m}\right] \qquad \text{...(1)}$$

- **Observations from Equation (1) :**
  i) As angular speed of shaft 'ω' increases, the deflection of shaft '**y**' increases.
  ii) When $\omega = \omega_n$, the deflection of shaft **y** becomes infinity.

Critical Speed or Whirling Speed (ω_c) :

- *The speed of shaft at which the deflection of the shaft tends to be infinity is known as* **critical speed** *or* **whirling speed.**
- Critical speed or whirling speed of shaft is given by,

$$\omega_c = \omega_n$$

$$\left.\begin{aligned} \text{or} \quad \omega_c &= \sqrt{\frac{K}{m}}\ \text{, rad/s} \\ \text{or} \quad \omega_c &= \sqrt{\frac{g}{\delta}}\ \text{, rad/s} \end{aligned}\right\} \qquad \text{...(2)}$$

$$\text{or} \quad N_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$\text{or} \quad N_c = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}\ \text{r.p.s.} \qquad \text{...(3)}$$

Where, N_c = Critical speed, in r.p.s.

δ = Static deflection of the shaft, m

- **Deflection of Shaft :** Equation (1) can be written as,

$$y = \frac{(\omega / \omega_c)^2 e}{1 - (\omega / \omega_c)^2} \qquad \text{...(4)}$$

Q.23 Discuss the effect of shaft speed $\omega < \omega_c$, $\omega = \omega_c$ and $\omega > \omega_c \cdot \omega_c$ is the critical speed of the shaft.

Ans. :

- **Deflection of Shaft :**

$$y = \frac{(\omega / \omega_c)^2 e}{1 - (\omega / \omega_c)^2}$$

- From above Equation, it is seen that, there are three ranges of shaft speed 'ω' :

1. **Shaft speed (ω) < Critical speed (ω_c)**

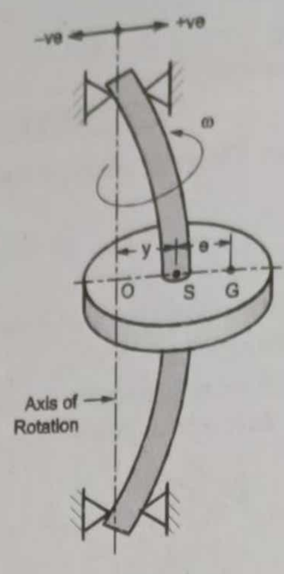
- When the speed of shaft is less than the critical speed (i.e. $\omega < \omega_c$), the deflection of shaft '**y**' is positive.
- In this speed range, the deflection of shaft '**y**' and eccentricity '**e**' are on opposite side of the geometric centre of the rotor '**S**'.
- This means, the rotor rotates with heavy side outwards, as shown in Fig. 4.10 (a).
- In this speed range, the deflection of shaft '**y**' increases with shaft speed 'ω'

2. **Shaft speed (ω) = Critical speed (ω_c) :**

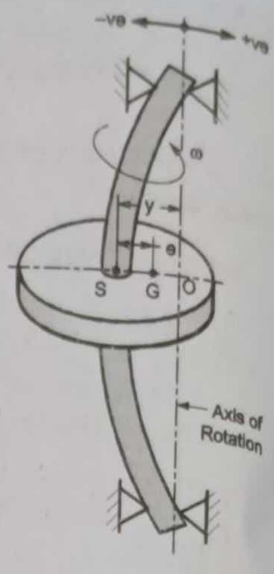
When the speed of shaft is equal to the critical speed (i.e. $\omega = \omega_c$), the deflection of shaft '**y**' tends to be infinity and the shaft vibrates with large amplitude. This may lead to the failure of the shaft.

3. **Shaft speed (ω) > Critical speed (ω_c) :**

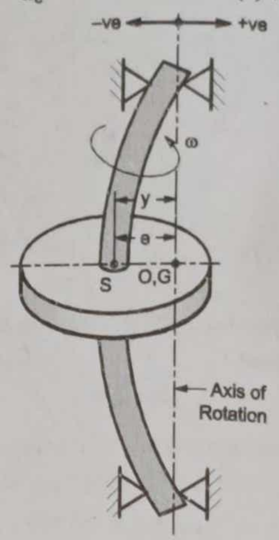
- When the speed of shaft is greater than the critical speed (i.e. $\omega > \omega_c$), the deflection of shaft '**y**' is negative.
- In this speed range, the deflection of shaft '**y**' and eccentricity '**e**' are on the same side of the geometric centre of the rotor '**S**'. This means, the rotor rotates with light side outwards, as shown in Fig. 4.10(b).
- In this speed range, $|y| > |e|$. As the shaft speed 'ω' increases, the deflection of shaft 'y' approaches – e.
- When $\omega >> \omega_c$, $y = -e$; which means that the centre of gravity of rotor '**G**' approaches the axis of rotation 'O' and the rotor rotates about its C.G, as shown in Fig. 4.10(c). This principle is used in running high speed turbines by speeding up the rotor rapidly beyond the critical speed. When 'y' approaches the value of ' – e', the rotor runs steadily.



(a) When $\omega < \omega_c$



(b) When $\omega > \omega_c$



(c) When $\omega >> \omega_c$

Fig. 4.10 : Ranges of Shaft Speed

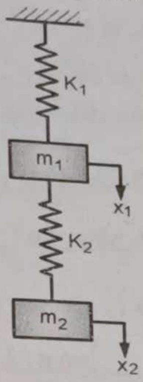
Q.1 Explain two degree of freedom system with any two practical examples. **SPPU : May 13**

OR What is 2 DOF systems? **SPPU : May 19**

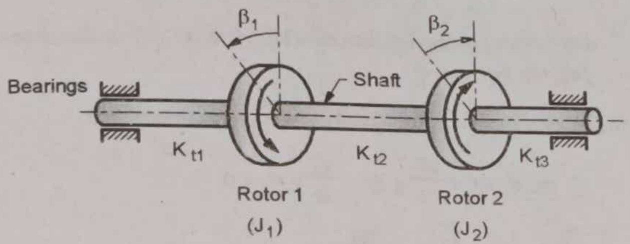
Ans. : Two Degrees of Freedom Systems

- **Two degrees of freedom system :** The system which requires independent two co-ordinates to specify its motion or configuration at any instant is known as **two degrees of freedom system.**
- **Examples of two degrees of freedom systems :**

i) Two springs and two masses system, which requires two independent co-ordinates x_1 and x_2 to specify the motion from equilibrium position [Fig. 5.1(a)].



(a) Two Masses and Two Springs System



(b) Two Rotor System

Fig. 5.1 : Two Degrees of Freedom Systems

(ii) Two rotors mounted an a shaft, which requires two angular displacements θ_1 and θ_2 to specify the motion from equilibrium position. [Fig. 5.1(b)].

Q.1 Find the natural frequency of oscillations of the double pendulum as shown in Fig.5.2. where $m_1 = m_2 = m$ and $l_1 = l_2 = 1$. Draw mode shapes and locate the nodes for each mode of vibration. **SPPU : Dec. 18**

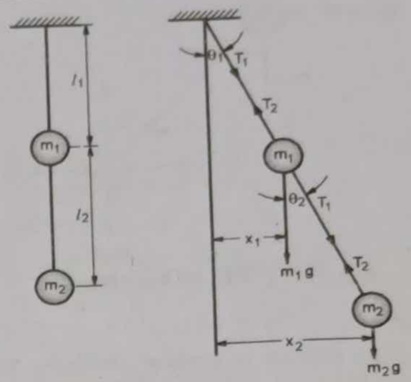


Fig. 5.2

Ans. :

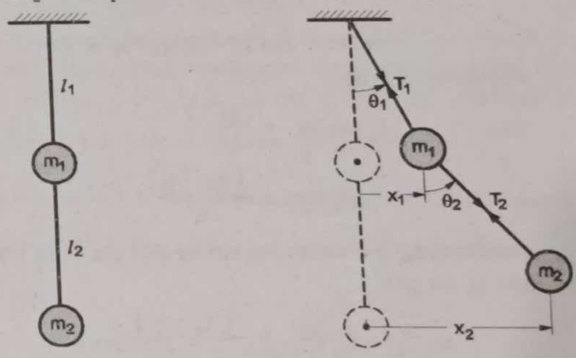
- Consider double pendulum having two point masses m_1 and m_2 suspended by inextensible strings of lengths l_1 and l_2 respectively as shown in Fig. 5.3. The masses m_1 and m_2 are considered to have only horizontal motion.

Let, θ_1, θ_2 = angles of upper and lower strings with the vertical,

x_1 and x_2 = horizontal displacements of masses m_1 and m_2 from equilibrium position.

T_1, T_2 = tensions in upper and lower strings

X_1 and X_2 = amplitudes of masses m_1 and m_2



(a) In Equilibrium Position **(b) In Displaced Position**

Fig. 5.3 : Double Pendulum

Differential Equations of Motion :

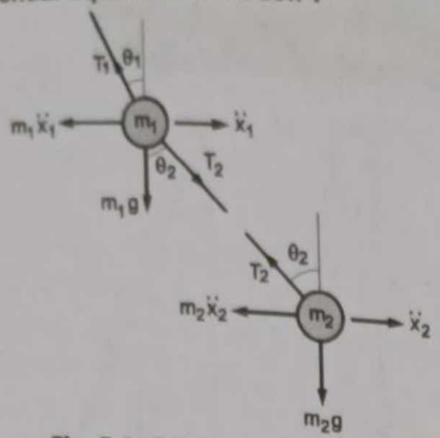


Fig. 5.4 : F.B.D. of Two Masses

- $\sum F_v = 0$:

There is no motion in vertical direction. The vertical components of forces acting on each mass must be balanced.

$$T_1 \cos\theta_1 = m_1 g + T_2 \cos\theta_2 \quad ...(a)$$

$$\text{and } T_2 \cos\theta_2 = m_2 g \quad ...(b)$$

$$\cos\theta_1 = \cos\theta_2 \approx 1 \quad [\text{As } \theta_1 \text{ and } \theta_2 \text{ are small}] \quad ...(c)$$

Substituting Equation (c) in Equations (a) and (b), we get,

$$\therefore \quad T_1 = m_1 g + T_2 \quad ...(d)$$

$$\text{and } T_2 = m_2 g \quad ...(e)$$

- Substituting Equation (e) in Equation (d), we get,

$$\therefore \quad T_1 = m_1 g + m_2 g$$

$$\text{or } T_1 = (m_1 + m_2) g \quad ...(f)$$

- $\sum F_H = 0$

$$m_1 \ddot{x}_1 + T_1 \sin\theta_1 - T_2 \sin\theta_2 = 0 \quad ...(1)$$

$$\text{and } m_2 \ddot{x}_2 + T_2 \sin\theta_2 = 0 \quad ...(2)$$

From Fig. 5.3(b),

$$\sin\theta_1 = \frac{x_1}{l_1} \quad ...(g)$$

$$\text{and } \sin\theta_2 = \frac{(x_2 - x_1)}{l_2} \quad ...(h)$$

Substituting the values of $\sin\theta_1$ and $\sin\theta_2$ in Equation (1) and (2), we get,

$$m_1 \ddot{x}_1 + T_1 \frac{x_1}{l_1} - T_2 \frac{(x_2 - x_1)}{l_2} = 0$$

$$\text{and } m_2 \ddot{x}_2 + T_2 \frac{(x_2 - x_1)}{l_2} = 0$$

Rearranging the above equations,

$$m_1 \ddot{x}_1 + \left(\frac{T_1}{l_1} + \frac{T_2}{l_2}\right) x_1 - \frac{T_2 x_2}{l_2} = 0 \quad ...(i)$$

$$m_2 \ddot{x}_2 + \frac{T_2 x_2}{l_2} - \frac{T_2 x_1}{l_2} = 0 \quad ...(j)$$

Substituting values of T_1 and T_2 from Equations (f) and (e) in Equations (i) and (j), we get,

$$m_1 \ddot{x}_1 + \left[\frac{m_1 + m_2}{l_1} + \frac{m_2}{l_2}\right] g x_1 - \frac{m_2}{l_2} g x_2 = 0 \quad ...(3)$$

$$m_2 \ddot{x}_2 + \frac{m_2}{l_2} g x_2 - \frac{m_2}{l_2} g x_1 = 0 \quad ...(4)$$

Solution of Differential Equation :

- **Solutions for x_1 and x_2 :**

$$x_1 = X_1 \sin\omega t \quad ...(k)$$

$$x_2 = X_2 \sin\omega t \quad ...(l)$$

where,

X_1, X_2 = amplitudes of vibration of the two masses m_1 and m_2

ω = frequency of vibration rad/s

Therefore,

$$\ddot{x}_1 = -X_1 \omega^2 \sin\omega t \quad ...(m)$$

$$\ddot{x}_2 = -X_2 \omega^2 \sin\omega t \quad ...(n)$$

Substituting Equations (k), (*l*) and (m) in Equation (3) we get,

$$-m_1 X_1 \omega^2 \sin\omega t + \left[\frac{m_1 + m_2}{l_1} + \frac{m_2}{l_2}\right] g X_1 \sin\omega t - \frac{m_2}{l_2} g X_2 \sin\omega t = 0$$

$$\therefore -m_1 X_1 \omega^2 + \left[\frac{m_1 + m_2}{l_1} + \frac{m_2}{l_2}\right] g X_1 - \frac{m_2}{l_2} g X_2 = 0$$

$$\therefore \left\{\left[\frac{m_1 + m_2}{l_1} + \frac{m_2}{l_2}\right] g - m_1 \omega^2\right\} X_1 = \frac{m_2}{l_2} g X_2$$

$$\therefore \frac{X_1}{X_2} = \frac{m_2 g / l_2}{\left[\frac{m_1 + m_2}{l_1} + \frac{m_2}{l_2}\right] g - m_1 \omega^2} \quad ...(5)$$

- On substituting Equations (k), (*l*) and (n) in Equation (4), we get,

$$-m_2 X_2 \omega^2 \sin\omega t + \frac{m_2}{l_2} g X_2 \sin\omega t - \frac{m_2}{l_2} g X_1 \sin\omega t = 0$$

$$\therefore -m_2 X_2 \omega^2 + \frac{m_2}{l_2} g X_2 - \frac{m_2}{l_2} g X_1 = 0$$

$$\therefore \left(\frac{m_2}{l_2} g - m_2 \omega^2\right) X_2 = \frac{m_2}{l_2} g X_1$$

$$\therefore \frac{X_1}{X_2} = \frac{m_2 g / l_2 - m_2 \omega^2}{m_2 g / l_2} \quad ...(6)$$

- Equations (5.) or (6), give the mode shapes of the system.
- **Frequency equations :**

From Equations (5) and (6),

$$\frac{m_2 g / l_2}{\left[\frac{m_1 + m_2}{l_1} + \frac{m_2}{l_2}\right] g - m_1 \omega^2} = \frac{\left[\frac{m_2 g}{l_2} - m_2 \omega^2\right]}{m_2 g / l_2}$$

$$\therefore \left\{\left[\frac{m_1+m_2}{l_1}+\frac{m_2}{l_2}\right]g - m_1\omega^2\right\}\left\{\frac{m_2 g}{l_2}-m_2\omega^2\right\} = \left[\frac{m_2 g}{l_2}\cdot\frac{m_2 g}{l_2}\right]$$

$$\left(\frac{m_1 m_2 + m_2^2}{l_1 l_2}\right)g^2 - \frac{\left(m_1 m_2 + m_2^2\right)}{l_1} g\omega^2 + \frac{m_2^2 g^2}{l_2^2} - \frac{m_2^2 g\omega^2}{l_2} - \frac{m_1 m_2 g\omega^2}{l_2} + m_1 m_2 \omega^4 = \frac{m_2^2 g^2}{l_2^2}$$

$$\mathbf{m_1 m_2 \omega^4 - [m_1 m_2 + m_2^2]\left[\frac{1}{l_1}+\frac{1}{l_2}\right] g\omega^2 + \left[\frac{m_1 m_2}{l_1 l_2}+\frac{m_2^2}{l_2^2}\right] g^2 = 0} \quad ...7)$$

The above Equation (7) is quadratic in ω^2 and gives two values of ω^2 (Two positive values of ω and two negative values of ω). The two positive values of ω give the natural frequencies ω_{nl} and ω_{n2} of the system. Therefore, Equation (7) is known as **frequency equation**.

Special Case of Double Pendulum ($m_1 = m_2 = m$ and $l_1 = l_2 = l$) :

$$\frac{X_1}{X_2} = \frac{mg/l}{\left(\frac{2m}{l}+\frac{m}{l}\right)g - m\omega^2}$$

and $$\frac{X_1}{X_2} = \frac{mg/l - m\omega^2}{mg/l}$$

or $$\frac{X_1}{X_2} = \frac{g/l}{\frac{3g}{l}-\omega^2} \quad ...(o)$$

and $$\frac{X_1}{X_2} = \frac{g/l-\omega^2}{g/l} \quad ...(p)$$

1. **Natural Frequencies :** Substituting $m_1 = m_2 = m$ and $l_1 = l_2 = l$ in Equation (7), we get,

$$m^2\omega^2 - [m^2+m^2]\left[\frac{2}{l}\right] g\omega^2 + \left[\frac{m^2}{l^2}+\frac{m^2}{l^2}\right] g^2 = 0$$

$$\therefore m^2\omega^4 - \frac{4m^2}{l} g\omega^2 + \frac{2m^2 g^2}{l^2} = 0$$

$$\therefore \omega^4 - \frac{4g}{l}\omega^2 + \frac{2g^2}{l^2} = 0$$

$$\therefore \omega^2 = \frac{+\frac{4g}{l^2} \pm \sqrt{\frac{16g^2}{l^2} - \frac{4\times 2g^2}{l^2}}}{2}$$

$$\therefore \omega^2 = \frac{4g}{2l} \pm \frac{\sqrt{\frac{16g^2}{l^2}-\frac{8g^2}{l^2}}}{2} = \frac{2g}{l} \pm \frac{\sqrt{\frac{8g^2}{l^2}}}{2}$$

$$\therefore \omega^2 = \frac{2g}{l} \pm \frac{2\sqrt{2}}{2}\frac{g}{l} = \frac{2g}{l} \pm \sqrt{2}\frac{g}{l}$$

$$\therefore \omega_{n1}^2 = \left(2-\sqrt{2}\right)\frac{g}{l} \text{ and } \omega_{n2}^2 = \left(2+\sqrt{2}\right)\frac{g}{l}$$

$$\therefore \omega_{n1}^2 = 0.58\frac{g}{l} \text{ and } \omega_{n2}^2 = 3.4142\frac{g}{l}$$

$$\left.\begin{aligned} \therefore \omega_{n1} &= 0.7615\sqrt{\frac{g}{l}}, \text{ rad/s and} \\ \omega_{n2} &= 1.8477\sqrt{\frac{g}{l}}, \text{ rad/s} \end{aligned}\right\} \quad ...(q)$$

- Therefore, ω_{nl} and ω_{n2} are two natural frequencies of the double pendulum shown in Fig. 5.3.

2. **Mode Shapes :**

From Equation (6), $$\frac{X_1}{X_2} = \frac{\frac{m_2 g}{l_2} - m_2^2\omega^2}{\frac{m_2 g}{l_2}} \quad ...(r)$$

Substituting $m_1 = m_2 = m$ and $l_1 = l_2 = l$ in Equation (r),

$$\frac{X_1}{X_2} = \frac{\frac{mg}{l} - m\omega^2}{\frac{mg}{l}}$$

or $$\frac{X_1}{X_2} = \frac{\frac{g}{l}-\omega^2}{\frac{g}{l}} \quad ...(s)$$

- **First mode shape (first ratio of amplitudes) :** Substituting $\omega^2 = \omega_{n1}^2 = 0.58\frac{g}{l}$ in Equation (s), we get,

$$\left(\frac{X_1}{X_2}\right)_1 = \frac{\frac{g}{l} - 0.58\frac{g}{l}}{\frac{g}{l}}$$

$$\therefore \left(\frac{X_1}{X_2}\right)_1 = \mathbf{0.42} \quad ...(t)$$

The first ratio of amplitudes of the two masses is + 0.414, which means that the two motions are in phase, i.e. the two masses move left or right together such that $\left(\frac{X_1}{X_2}\right)_1 = 0.414$, and frequency is ω_{n1} [Fig. 5.4(b)].

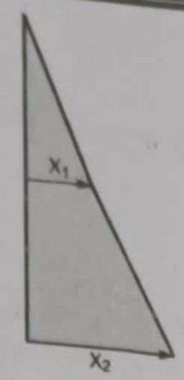
- **Second mode shape (Second ratio of amplitudes)**

Again substituting $\omega^2 = \omega_{n2}^2 = 3.4142\frac{g}{l}$ in Equation (t), we get,

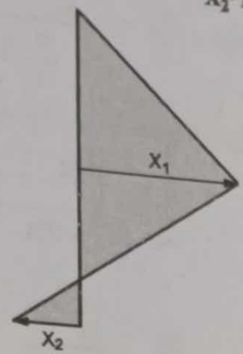
$$\left(\frac{X_1}{X_2}\right)_2 = \frac{\frac{g}{l} - 3.4142\frac{g}{l}}{\frac{g}{l}} \therefore \left(\frac{X_1}{X_2}\right)_2 = \mathbf{-2.4142} \quad ...(u)$$

The second ratio of amplitudes of the two masses is – 2.4142 which means that the two motions are out of phase, i.e. when mass moves left the other mass moves right or vice- versa such that $\left(\frac{X_1}{X_2}\right)_2 = -2.4142$, and frequency is ω_{n2} [Fig. 5.4(c)].

(a) Equilibrium Position



(b) First Mode Shape at ω_{n_1} for $\left(\frac{X_1}{X_2}\right)_1 = 0.414$



(c) Second Mode Shape at ω_{n_2} for $\left(\frac{X_1}{X_2}\right)_2 = -2.4142$

Fig. 5.4 : Principal Mode Shapes for Double Pendulum

Q.2 Explain degenerate system with any two examples.

SPPU : Dec. 15, May 18

OR What is semi definite system ?

SPPU : Oct. 19(In Sem.)

Ans. : Semi-definite System :

- If one of the natural frequency of the system is zero, then system will not vibrate. In other words there is no relative motion between two masses and the system can be moved as a rigid body. Such systems are known as semi-definite system or degenerate systems.
- The example of such type of system os shown in Fig. 5.5
- The two differential equations of motion are,

$m_1 \ddot{x}_1 - k(x_2 - x_1) = 0$ and $m_2 \ddot{x}_2 + k(x_2 - x_1) = 0$

The natural frequencies are, $\omega_1 = 0$ and

$$\omega_2 = \sqrt{\frac{(m_1 + m_2)\,k}{m_1 m_2}} \text{ rad/s}$$

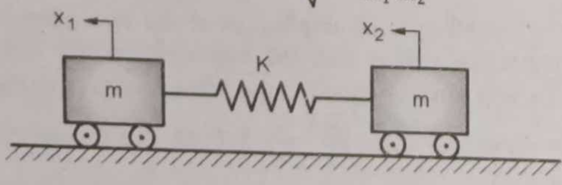


Fig. 5.5

- Another example of degenerate system is shown in Fig. 5.6.

The natural frequencies for the system are,

$$\omega_{n1} = 0 \quad \text{and}$$

$$\omega_{n2} = \sqrt{\frac{(m_1 + m_2)\,g}{m_1 l}} \text{ rad/s}$$

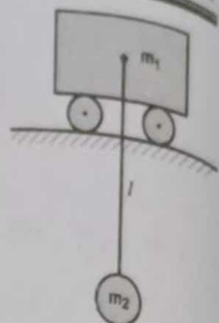


Fig. 5.6

Q.3 Find natural frequencies and mode shapes for the system shown. Consider m_1 = 25 kg, m_2 = 20 kg and k = 2000 N/m. **SPPU - Oct. 19(In Sem.), 10 Marks**

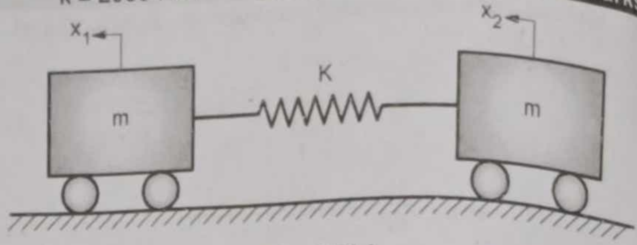
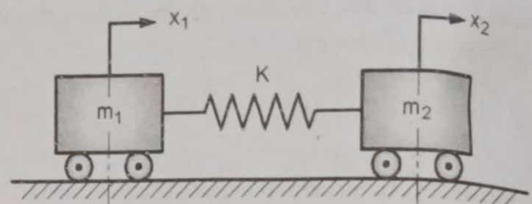


Fig. 5.7(a)

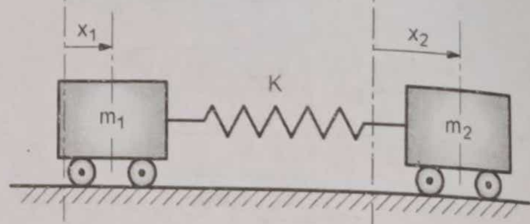
Ans. :

1. Frequency Equation :

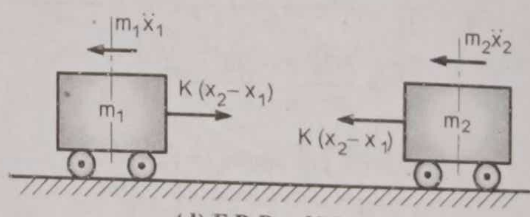
- **Assumption : : $x_2 > x_1$**



(b) Equilibrium Position



(c) Displaced Position



(d) F.B.D. of Masses

Fig. 5.7

- **Two differential equations of motion :**

From Fig. 5.7(d),

$$m_1 \ddot{x}_1 - K(x_2 - x_1) = 0$$

and $m_2 \ddot{x}_2 + K(x_2 - x_1) = 0$

or

$$m_1 \ddot{x}_1 + K x_1 - K x_2 = 0 \quad ...(a)$$

and $m_2 \ddot{x}_2 - K x_1 + K x_2 = 0 \quad ...(b)$

- **Solutions for x_1 and x_2 under steady state conditions :**

$$\left.\begin{aligned} x_1 &= X_1 \sin \omega t \\ x_2 &= X_2 \sin \omega t \end{aligned}\right\} \quad ...(c)$$

Therefore,

$$\left.\begin{aligned} \ddot{x}_1 &= -X_1 \omega^2 \sin \omega t \\ \ddot{x}_2 &= -X_2 \omega^2 \sin \omega t \end{aligned}\right\} \quad ...(d)$$

- Substituting Equations (c) and (d) in Equation (a),

$$-m_1 X_1 \omega^2 \sin \omega t + K X_1 \sin \omega t - K X_2 \sin \omega t = 0$$

$$\therefore -m_1 X_1 \omega^2 + K X_1 - K X_2 = 0$$

$$\therefore (K - m_1 \omega^2) X_1 = K X_2$$

$$\therefore \frac{X_1}{X_2} = \frac{K}{(K - m_1 \omega^2)} \quad ...(e)$$

- Substituting Equations (c) and (d) in Equation (b),

$$-m_2 X_2 \omega^2 \sin \omega t - K X_1 \sin \omega t + K X_2 \sin \omega t = 0$$

$$\therefore -m_2 X_2 \omega^2 - K X_1 + K X_2 = 0$$

$$\therefore (K - m_2 \omega^2) X_2 = K X_1$$

$$\therefore \frac{X_1}{X_2} = \frac{(K - m_2 \omega^2)}{K} \quad ...(f)$$

- **Frequency equation :**

From Equations (e) and (f),

$$\frac{K}{(K - m_1 \omega^2)} = \frac{(K - m_2 \omega^2)}{K}$$

$$\therefore (K - m_1 \omega^2)(K - m_2 \omega^2) = K^2$$

$$\therefore K^2 - K m_2 \omega^2 - K m_1 \omega^2 + m_1 m_2 \omega^4 = K^2$$

$$\therefore m_1 m_2 \omega^4 - (m_1 + m_2) K \omega^2 = 0$$

$$\therefore \omega^4 - \frac{(m_1 + m_2) K}{m_1 m_2} \omega^2 = 0 \quad ...(g)$$

This Equation (g) is called as **frequency equation.**

2. Two Natural Frequencies :

- Substituting $m_1 = 25$ kg, $m_2 = 20$ kg and $K = 2000$ N / m in Equation (g), we get,

$$\omega^4 - \frac{(25 + 20)\, 2000}{25 \times 20} \omega^2 = 0$$

$$\therefore \omega^4 - 180 \omega^2 = 0$$

$$\therefore \omega^2 = \frac{+180 \pm \sqrt{(180)^2 - 0}}{2}$$

$$\therefore \omega^2 = \frac{180}{2} \pm \frac{180}{2}$$

$$\therefore \omega_{n1}^2 = 0 \text{ and } \omega_{n2}^2 = 180$$

$$\therefore \omega_{n1} = 0 \text{ rad/s and } \omega_{n2} = 13.41 \text{ rad/s} \quad ...\textbf{Ans.}$$

- It is seen that as if one of the natural frequencies of the system is zero, the system is not vibrating. There is no relative motion between masses m_1 and m_2 and system can be moved as a rigid body. Such systems are known as **semi-definite systems or degenerate systems**.

3. Ratio of Amplitudes :

- **For first mode shape :**

$$\frac{X_1}{X_2} = \frac{K}{K - m_1 \omega^2}$$

$$\therefore \left(\frac{X_1}{X_2}\right) = \frac{K}{K - m_1 \omega_{n1}^2} = \frac{2000}{2000 - 25 \times 0}$$

$$\text{or } \left(\frac{X_1}{X_2}\right)_1 = 1 \quad ...\textbf{Ans.}$$

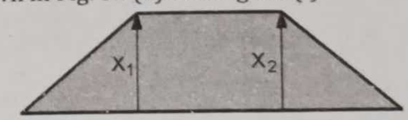
- **For second mode shape :**

$$\left(\frac{X_1}{X_2}\right)_2 = \frac{K}{K - m_1 \omega_{n2}^2} = \frac{2000}{2000 - 25 \times 180}$$

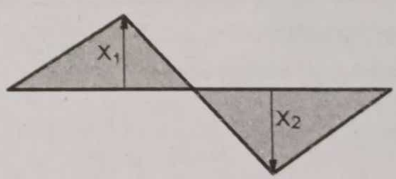
$$\text{Or } \left(\frac{X_1}{X_2}\right)_2 = -0.8 \quad ...\textbf{Ans.}$$

4. Principal Mode Shapes :

- The two principal mode shapes for given system are shown in Fig. 5.7(e) and Fig. 5.7(f).



(c) First Mode Shape at $\omega_{n1} = 0$ rad/s for $\left(\frac{X_1}{X_2}\right)_2 = 1$



(f) Second Mode Shape at $\omega_{n2} = 31.41$ rad/s for $\left(\frac{X_1}{X_2}\right)_2 = -0.8$

Fig. 5.7

Q.4 Determine the natural frequency and corresponding mode shapes of given system, shown in Fig. 5.8(a) Assume each spring stiffness as K.

SPPU - Dec. 15, 12 Marks

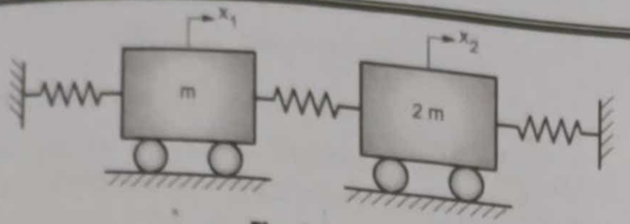


Fig. 5.8(a)

Ans. :

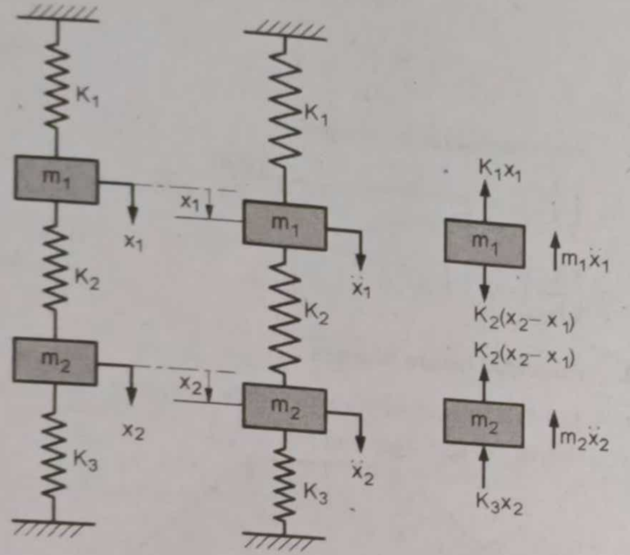
1. **Frequency Equation :**

Assumptions :

(i) The masses are constrained to move in the direction of spring axis.

(ii) The springs are weightless.

(iii) $x_2 > x_1$



(b) Equilibrium Position **(c) Displaced Position** **(d) F.B.D. of Masses**

Fig. 5.8 : Spring Mass System having Two Degrees of Freedom

- **Two differential equations of motion :**

From Fig. 5.8(d),

$$m_1 \ddot{x}_1 + K_1 x_1 - K_2 (x_2 - x_1) = 0 \quad ...(a)$$

$$m_2 \ddot{x}_2 + K_2 (x_2 - x_1) + K_3 x_2 = 0 \quad ...(b)$$

- By rearranging the terms of Equations (a) and (b) we get,

$$m_1 \ddot{x}_1 + (K_1 + K_2) x_1 - K_2 x_2 = 0 \quad ...(c)$$

$$m_2 \ddot{x}_2 + (K_2 + K_3) x_2 - K_2 x_1 = 0 \quad ...(d)$$

- **Solutions for x_1 and x_2 under steady state condition :**

$$x_1 = X_1 \sin \omega t \quad ...(e)$$

$$x_2 = X_2 \sin \omega t \quad ...(f)$$

Therefore,

$$\ddot{x}_1 = -X_1 \omega^2 \sin \omega t \quad ...(g)$$

$$\ddot{x}_2 = -X_2 \omega^2 \sin \omega t \quad ...(h)$$

- Substituting Equations (e), (f) and (g) in Equation (c), we get

$$-m_1 X_1 \omega^2 \sin \omega t + (K_1 + K_2) X_1 \sin \omega t - K_2 X_2 \sin \omega t = 0$$

$$\therefore \quad -m_1 X_1 \omega^2 + (K_1 + K_2) X_1 - K_2 X_2 = 0$$

$$\therefore \quad [-m_1 \omega^2 + (K_1 + K_2)] X_1 - K_2 X_2 = 0$$

$$\therefore \quad [(K_1 + K_2) - m_1 \omega^2] X_1 = K_2 X_2$$

$$\therefore \quad \frac{X_1}{X_2} = \frac{K_2}{[(K_1 + K_2) - m_1 \omega^2]} \quad ...(i)$$

- Substituting Equations (e), (f) and (h) in Equation (d) we get,

$$-m_2 \omega^2 X_2 \sin \omega t + (K_2 + K_3) X_2 \sin \omega t - K_2 X_1 \sin \omega t = 0$$

$$[-m_2 \omega^2 + (K_2 + K_3)] X_2 - K_2 X_1 = 0$$

$$\therefore \quad \frac{X_1}{X_2} = \frac{[(K_2 + K_3) - m_2 \omega^2]}{K_2} \quad ...(j)$$

- **Frequency equation :**

From Equations (i) and (j) we get,

$$\frac{K_2}{[(K_1 + K_2) - m_1 \omega^2]} = \frac{[(K_2 + K_3) - m_2 \omega^2]}{K_2}$$

$$\therefore \quad [(K_1 + K_2) - m_1 \omega^2][(K_2 + K_3) - m_2 \omega^2] = K_2^2$$

$$\therefore \quad (K_1 + K_2)(K_2 + K_3) - m_1 \omega^2 (K_2 + K_3)\, m_2 \omega^2 (K_1 + K_2) + m_1 m_2 \omega^4 = K_2^2$$

$$\therefore \quad K_1 K_2 + K_1 K_3 + K_2^2 + K_2 K_3 - [m_1 (K_2 + K_3) + m_2 (K_1 + K_2)] \omega^2 + m_1 m_2 \omega^4 = K_2^2$$

$$\therefore \quad \mathbf{m_1 m_2 \omega^4 - [m_1 (K_2 + K_3) + m_2 (K_1 + K_2)] \omega^2 + [K_1 K_2 + K_2 K_3 + K_3 K_1] = 0} \quad ...(k)$$

- The above Equation (k) is quadratic in ω^2 and gives two values of ω^2, (two positive values of ω and two negative values of ω). The two positive values of ω give natural frequencies ω_{n1} and ω_{n2} of the system.
- Therefore, Equation (k) is called as **frequency equation**.

2. Two Natural Frequencies :

- Refer Fig. 5.8(e),
- Substituting, $m_1 = m$, $m_2 = 2m$, $K_1 = K_3 = K$ and $K_2 = 2K$, in Equation (k),

$$m \cdot 2m\,\omega^4 - [m(K+K) + 2m(K+K)]\,\omega^2 + [KK + KK + KK] = 0$$

$$\therefore \quad 2m^2\omega^4 - [2\,Km + 4\,Km]\omega^2 + [K^2 + K^2 + K^2] = 0$$

$$\therefore \quad 2m^2\omega^4 - 6\,Km\,\omega^2 + 3K^2 = 0$$

$$\therefore \omega^2 = \frac{6\,Km \pm \sqrt{36\,K^2m^2 - 4\,(2\,m^2\,3\,K^2)}}{4m^2}$$

$$\therefore \omega^2 = \frac{6Km}{4m^2} \pm \frac{\sqrt{36\,K^2m^2 - 24K^2m^2}}{4m^2}$$

$$\therefore \omega^2 = \frac{6K}{4m} \pm \frac{\sqrt{12}\,Km}{4m^2} = \frac{6K}{4m} \pm \frac{\sqrt{12}}{4}\frac{K}{m}$$

$$\therefore \omega^2 = \left(\frac{6}{4} \pm \frac{\sqrt{12}}{4}\right)\frac{K}{m} = (1.5 \pm 0.86)\frac{K}{m}$$

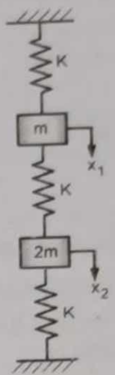


Fig. 5.8(e)

$$\therefore \quad \omega_{n1}^2 = (0.64)\frac{K}{m} \text{ and } \omega_{n2}^2 = (2.36)\frac{K}{m}$$

$$\therefore \quad \omega_{n1} = 0.80\sqrt{\frac{K}{m}}, \text{ rad/s}$$

$$\text{and } \omega_{n2} = 1.53\sqrt{\frac{K}{m}}, \text{ rad/s} \qquad ...(l)$$

- Therefore ω_{n1} and ω_{n2} are the two natural frequencies of the system shown in Fig. 5.8(e)

3. Ratio of Amplitudes : :

- From Equation (i),

$$\frac{X_1}{X_2} = \frac{K_2}{[(K_1 + K_2) - m_1\omega^2]} \qquad ...(m)$$

- Substituting, $m_1 = m$ and $K_1 = K_2 = K$ in Equation (m)

$$\frac{X_1}{X_2} = \frac{K}{[(K + K) - m\,\omega^2]}$$

$$\therefore \quad \frac{X_1}{X_2} = \frac{K}{2K - m\omega^2} \qquad ....(n)$$

For first mode shape :

- Substituting $\omega^2 = \omega_{n1}^2 = 0.64\frac{K}{m}$ in Equation (n), we get,

$$\left(\frac{X_1}{X_2}\right)_1 = \frac{K}{2K - m\left(0.64\frac{K}{m}\right)} = \frac{K}{2K - 0.64\,K}$$

$$\therefore \quad \left(\frac{X_1}{X_2}\right)_1 = \frac{1}{1.36}$$

$$\therefore \quad \left(\frac{X_1}{X_2}\right)_1 = 0.73 \qquad ...(o)$$

- **For second mode shape :**
- Substituting $\omega^2 = \omega_{n2}^2 = 2.36\frac{K}{m}$ in Equation (n), we get,

$$\left(\frac{X_1}{X_2}\right)_2 = \frac{K}{2K - m\left(2.36\frac{K}{m}\right)}$$

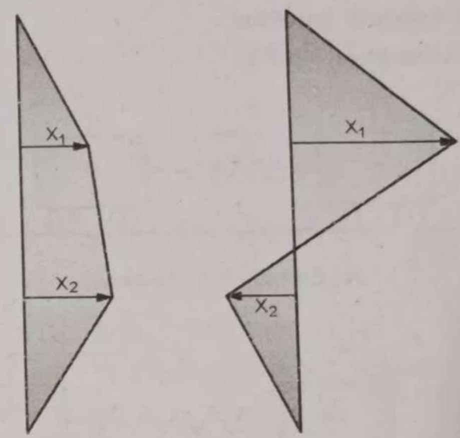
$$= \frac{K}{2K - 2.36\,K}$$

$$\therefore \quad \left(\frac{X_1}{X_2}\right)_2 = \frac{K}{-0.36\,K} = \frac{1}{-0.36}$$

$$\therefore \quad \left(\frac{X_1}{X_2}\right)_2 = -2.77 \qquad ...(p)$$

4. Principal Mode Shapes :

The two principal mode shapes II are shown in Fig. 5.8(f) and Fig. P. 5.8(g)



(f) First Mode Shape at ω_{n_1} for $\left(\frac{X_1}{X_2}\right)_1 = 0.73$ (g) Second Mode Shape at ω_{n_2} for $\left(\frac{X_1}{X_2}\right)_2 = -2.77$

Fig. 5.8 : Principal Mode Shapes for Case III

Q.5 What do you mean by semi definite system ? An electric train made of two cars each of mass 2000 kg is connected by couplings of stiffness equal to 40×10^6 N/m as shown in Fig. 5.9(a). Determine the natural frequency of the system.

SPPU : Oct.18(In sem),10 Marks

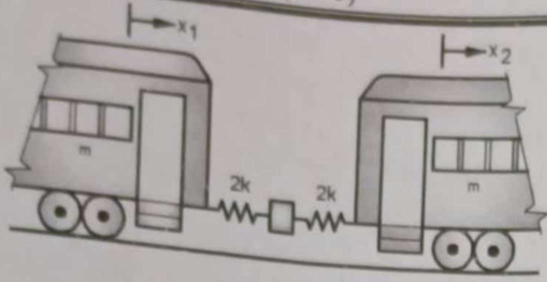


Fig. 5.9(a)

1. **Equivalent Spring :**

- The two coaches are connected by coupling where, the two springs are in series. Therefore equivalent spring stiffness is,

$$\frac{1}{K_e} = \frac{1}{2K} + \frac{1}{2K} = \frac{2}{2K}$$

$$\therefore \quad K_e = K = 40 \times 10^6$$

$$\text{or} \quad K_e = 40 \times 10^6 \text{ N/m}$$

The equivalent system is shown in Fig. 5.9(b)

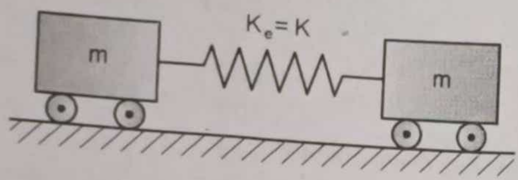
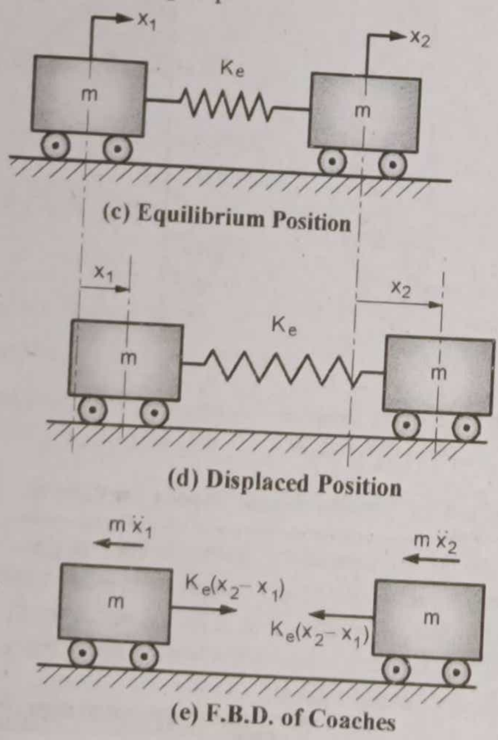


Fig. 5.9(b): Equivalent System

2. **Frequency Equation :**

- **Assumption : $x_2 > x_1$**



(c) Equilibrium Position

(d) Displaced Position

(e) F.B.D. of Coaches

Fig. 5.9

- **Two differential equations of motion :**

From Fig. 5.9(e)

$$m\ddot{x}_1 - K_e(x_2 - x_1) = 0$$

$$\text{and} \quad m\ddot{x}_2 + K_e(x_2 - x_1) = 0$$

or

$$m\ddot{x}_1 + K_e x_1 - K_e x_2 = 0 \qquad \text{...(a)}$$

$$\text{and} \quad m\ddot{x}_2 - K_e x_1 + K_e x_2 = 0 \qquad \text{...(b)}$$

- **Solutions for x_1 and x_2 under steady state conditions :**

$$\left.\begin{aligned} x_1 &= X_1 \sin \omega t \\ x_2 &= X_2 \sin \omega t \end{aligned}\right\} \qquad \text{...(c)}$$

Therefore,

$$\left.\begin{aligned} \ddot{x}_1 &= -X_1 \omega^2 \sin \omega t \\ \ddot{x}_2 &= -X_2 \omega^2 \sin \omega t \end{aligned}\right\} \qquad \text{...(d)}$$

- Substituting Equations (c) and (d) in Equation (a),

$$-m X_1 \omega^2 \sin \omega t + K_e X_1 \sin \omega t - K_e X_2 \sin \omega t = 0$$

$$\therefore \quad -m X_1 \omega^2 + K_e X_1 - K_e X_2 = 0$$

$$\therefore \quad (K_e - m\omega^2) X_1 = K_e X_2$$

$$\therefore \quad \frac{X_1}{X_2} = \frac{K_e}{(K_e - m\omega^2)} \qquad \text{...(e)}$$

- Substituting Equations (c) and (d) in Equation (b),

$$-m X_2 \sin \omega t - K_e X_1 \sin \omega t + K_e X_2 \sin \omega t = 0$$

$$\therefore \quad -m X_2 \omega^2 - K_e X_1 + K_e X_2 = 0$$

$$\therefore \quad (K_e - m\omega^2) X_2 = K_e X_1$$

$$\therefore \quad \frac{X_1}{X_2} = \frac{(K_e - m\omega^2)}{K_e} \qquad \text{...(f)}$$

- **Frequency equation :**

- From Equation (e) and (f),

$$\frac{K_e}{(K_e - m\omega^2)} = \frac{(K_e - m\omega^2)}{K_e}$$

$$\therefore \quad (K_e - m\omega^2)(K_e - m\omega^2) = K^2$$

$$\therefore \quad K_e^2 - 2K_e m\omega^2 + m^2\omega^4 = K_e^2$$

$$\therefore \quad m^2\omega^4 - 2K_e m^2 \omega^2 = 0$$

$$\therefore \quad \omega^4 - \frac{2K_e}{m}\omega^2 = 0 \qquad \text{...(g)}$$

3. **Two Natural Frequencies :**

Substituting m = 2000 kg and $K_e = 40 \times 10^6$ N/m in Equation (g),

$$\therefore \quad \omega^4 - \frac{2 \times 40 \times 10^6}{2000}\omega^2 = 0$$

$\therefore \quad \omega^4 - 40000\,\omega^2 = 0$

$$\therefore \quad \omega^2 = \frac{+40000 \pm \sqrt{(40000)^2 - 0}}{2}$$

$$\therefore \quad \omega^2 = \frac{40000}{2} \pm \frac{40000}{2}$$

$$\therefore \quad \omega_{n1}^2 = 0 \text{ and } \omega_{n2}^2 = 40000$$

$$\therefore \quad \omega_{n1} = 0 \text{ rad/s}$$

and $\quad \omega_{n2} = 200 \text{ rad/s}$ ...Ans.

- **Since one of the natural frequency is zero, system is not vibrating. There is no relative motion between two coaches, and system can be moved as a rigid body.**

4. Ratio of Amplitudes :

- **First mode shape :**

$$\frac{X_1}{X_2} = \frac{K}{K - m\,\omega^2}$$

$$\therefore \quad \left(\frac{X_1}{X_2}\right)_1 = \frac{K}{K - m\,\omega_{n1}^2} = \frac{40 \times 10^6}{40 \times 10^6 - 2000 \times 0}$$

$$\text{or} \quad \left(\frac{X_1}{X_2}\right)_1 = 1$$ ...Ans.

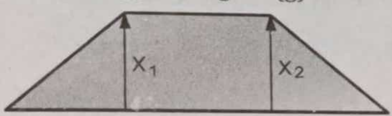
- **For second mode shape :**

$$\left(\frac{X_1}{X_2}\right)_2 = \frac{K}{K - m\,\omega_{n2}^2} = \frac{40 \times 10^6}{40 \times 10^6 - 2000 \times 40000}$$

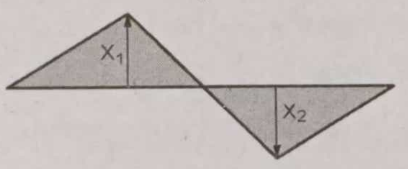
$$\text{or} \quad \left(\frac{X_1}{X_2}\right)_2 = -1$$ ...Ans.

5. Principal Mode Shapes :

- The two principal mode shapes for given system are shown in Fig. 5.9(f) and Fig. 5.9(g).



(f) First Mode Shape at $\omega_{n1} = 0$ rad/s for $\left(\frac{X_1}{X_2}\right)_1 = 1$



(g) Second Mode Shape at $\omega_{n2} = 200$ rad/s for $\left(\frac{X_1}{X_2}\right)_2 = -1$

Fig. 5.9

Q.6 Write governing differential equations for system given below **SPPU -May 19, 5 Marks**

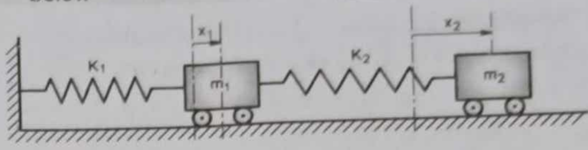
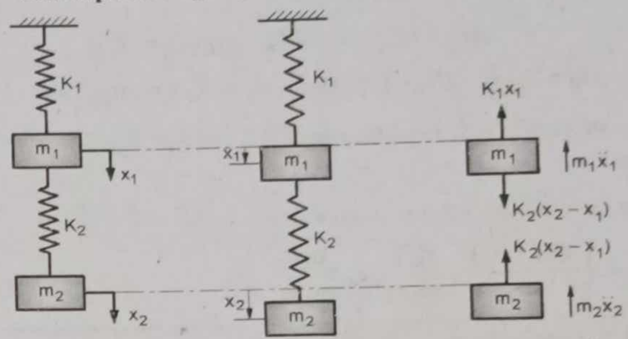


Fig. 5.10(a)

Ans. : Systems that require two independent Coordinates to describe their motion are called two degree of freedom systems.

1. Frequency Equation :

- **Assumption : $x_2 > x_1$.**



(b) Equilibrium Position **(c) Displaced Position** **(d) F.B.D. of Masses**

Fig. 5.10(A)

- **Two differential equations of motion :**

From Fig. 5.10(A)(d),

$$m_1\ddot{x}_1 + K_1x_1 - K_2(x_2 - x_1) = 0 \quad \text{...(a)}$$

and
$$m_2\ddot{x}_2 + K_2(x_2 - x_1) = 0 \quad \text{...(b)}$$

Rearranging the terms in Equation (a) and (b)

or
$$m_1\ddot{x}_1 + (K_1 + K_2)x_1 - K_2x_2 = 0 \quad \text{...(c)}$$

and
$$m_2\ddot{x}_2 - K_2x_1 + K_2x_2 = 0 \quad \text{...(d)}$$

- **Solutions for x_1 and x_2 under steady state conditions :**

$$\left.\begin{aligned} x_1 &= X_1 \sin \omega t \\ x_2 &= X_2 \sin \omega t \end{aligned}\right\} \quad \text{...(e)}$$

Therefore,

$$\left.\begin{aligned} \ddot{x}_1 &= -X_1\omega^2 \sin \omega t \\ \ddot{x}_2 &= -X_2\omega^2 \sin \omega t \end{aligned}\right\} \quad \text{...(f)}$$

Substituting Equations (e) and (f) in Equation (c),

$$-m_1X_1\omega^2 \sin \omega t + (K_1 + K_2)X_1 \sin \omega t - K_2X_2 \sin \omega t = 0$$

$$\therefore \quad -m_1X_1\omega^2 + (K_1 + K_2)X_1 - K_2X_2 = 0$$

$$\therefore \quad [(K_1 + K_2) - m_1\omega^2]X_1 = K_2X_2$$

$$\therefore \frac{X_1}{X_2} = \frac{K_2}{[(K_1 + K_2) - m_1\,\omega^2]} \quad ...(g)$$

Substituting Equations (e) and (f) in Equation (d),

$$- m_2 X_2 \omega^2 \sin \omega t - K_2 X_1 \sin \omega t + K_2 X_2 \sin \omega t = 0$$

$$\therefore - m_2 X_2 \omega^2 - K_2 X_1 + K_2 X_2 = 0$$

$$\therefore (K_2 - m_2 \omega^2) X_2 = K_2 X_1$$

$$\therefore \frac{X_1}{X_2} = \frac{(K_2 - m_2\omega^2)}{K_2} \quad ...(h)$$

- **Frequency equation :**

From Equations (g) and (h),

$$\frac{K_2}{[(K_1 + K_2) - m_1\,\omega^2]} = \frac{(K_2 - m_2\omega^2)}{K_2}$$

$$\therefore [(K_1 + K_2) - m_1\omega^2](K_2 - m_2\omega^2) = K_2^2$$

$$(K_1 + K_2) K_2 - (K_1 + K_2) m_2\omega^2 - m_1\omega^2 K_2 + m_1 m_2\omega^4 = K_2^2$$

$$\therefore m_1 m_2\omega^4 - [(K_1 + K_2) m_2 + m_1 K_2]\,\omega^2 + (K_1 + K_2) K_2 - K_2^2 = 0$$

$$m_1 m_2\omega^4 - [(K_1 + K_2) m_2 + m_1 K_2]\,\omega^2 + K_1 K_2 + K_2^2 - K_2^2 = 0$$

$$\omega^4 - \left[\frac{(K_1 + K_2)}{m_1} + \frac{K_2}{m_2}\right]\omega^2 + \frac{K_1 K_2}{m_1 m_2} = 0 \quad ...(i)$$

Q.7 Determine the natural frequencies of the system shown in Fig. 5.11. k = 90 N/m, I = 25 m, m_1 = 2 kg, m_2 = 0.5 kg.

SPPU - May 17, 12 Marks

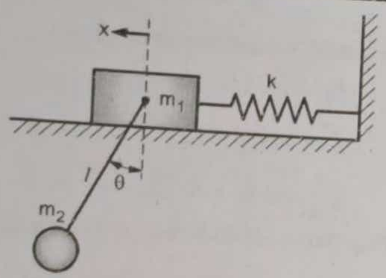


Fig. 5.11

Ans. :

- Two natural frequencies for given case is given by

$$\omega^2 = \frac{(m_1 + m_2)\,g + kl \pm \sqrt{[(m_1 + m_2)\,g + kl]^2 - 4 m_1\, l\, kg}}{2\, m_1 l}$$

Put value of K = 90 N/m, l = 0.25 m,

m_1 = 2kg, m_2 = 0.5 kg

$$\omega^2 = \frac{(2 + 0.5)\,9.81 + 90 \times 0.25 \pm \sqrt{[(2 + 0.5)\,9.81 + 90 \times 2.5]^2 - 4 \times 2 \times 0.25 \times 90 \times 9.81}}{2 \times 2 \times 0.25}$$

$$\omega^2 = 24.5 + 22.5 \pm \sqrt{(24.5 + 22.5)^2 - 1764}$$

$$= 47 \pm \sqrt{2209 + 1764} = 47 \pm 21.09$$

$$\therefore \omega_1 = \mathbf{8.25\ rad/s}$$

and ω_2 = **5.08 rad/s** ...Ans.

Q.8 Find the natural frequencies of the system shown in Fig. 5.12.

m_1 = 10 kg ; m_2 = 12 kg ;

r_1 = 0.10 m ; r_2 = 0.11 kg ;

$K_1 = 40 \times 10^3$ N/m ; $K_2 = 50 \times 10^3$ N/m ;

$K_3 = 60 \times 10^3$ N/m;

SPPU - Dec. 16, Dec. 17, 12 Marks

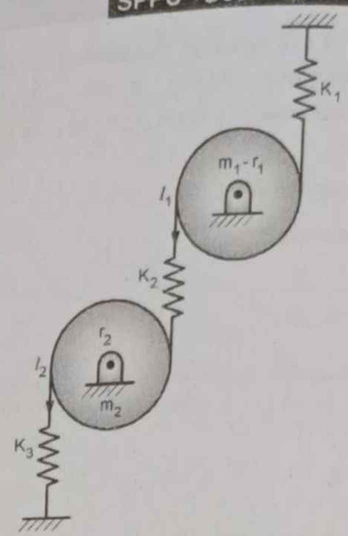


Fig. 5.12

Ans. :

m_1 = 10 kg ; m_2 = 12 kg;

r_1 = 0.10 m ; r_2 = 0.11m ;

$K_1 = 40 \times 10^3$ N/m ; $K_2 = 50 \times 10^3$ N/m;

$K_3 = 60 \times 10^3$ N/m

1. **Frequency Equation :**

- **Two differential equations of motion :**

$$I_1 \ddot{\theta}_1 + \left(K_1 r_1^2 + K_2 r_1^2\right)\theta_1 - K_2 r_1 r_2 \theta_2 = 0 \quad ...(a)$$

$$I_2 \ddot{\theta}_2 + \left(K_3 r_2^2 + K_2 r_2^2\right)\theta_2 - K_2 r_1 r_2 \theta_1 = 0 \quad ...(b)$$

- **Solutions for θ_1 and θ_2 under steady state conditions :**

Assume,

$$\left.\begin{aligned} \theta_1 &= \phi_1 \sin \omega t \\ \theta_2 &= \phi_2 \sin \omega t \end{aligned}\right\}$$

$$\left.\begin{aligned} \ddot{\theta}_1 &= -\omega^2 \phi_1 \sin \omega t ; \\ \ddot{\theta}_2 &= -\omega\, \phi_2 \sin \omega t \end{aligned}\right\} \quad ...(d)$$

$$\left.\begin{aligned} I_1 &= \frac{1}{2} m_1 r_1^2 ; \\ I_2 &= \frac{1}{2} m_2 r_2^2 \end{aligned}\right\} \quad ...(e)$$

Substituting Equations (c), (d) and (e) in Equation (a),

$$m_1 r_1^2 [-\omega^2 \phi_1 \sin \omega t] + \left(K_1 r_1^2 + K_2 r_1^2\right)\phi_1 \sin \omega t$$

$$-K_2 r_1 r_2 \phi_2 \sin \omega t = 0$$

$$\frac{1}{2} m_1 r_1^2 (-\omega^2)\, \phi_1 + \left(K_1 r_1^2 + K_2 r_1^2 \right) \phi_1 - K_2 r_1 r_2 \theta_1 = 0$$

$$\frac{\phi_1}{\phi_2} = \frac{2 K_2 r_1 r_2}{(-m_1 r_1^2 \omega^2) + 2\left(K_1 r_1^2 + K_2 r_1^2 \right)} \quad \text{...(f)}$$

- Substituting Equations (c), (d) and (e) in Equation (b),

$$\frac{1}{2} m_2 r_2^2 (-\omega^2 \phi_2 \sin \omega t) + \left(K_3 r_2^2 + K_2 r_2^2 \right) \phi_2 \sin \omega t - K_2 r_1 r_2 \phi_1 \sin \omega t = 0$$

$$\frac{1}{2} m_2 r_2^2 (-\omega^2)\, \phi_2 + \left(K_3 r_2^2 + K_2 r_2^2 \right) \phi_2 - K_2 r_1 r_2 \phi_1 = 0$$

$$\frac{\phi_1}{\phi_2} = \frac{\left(\omega^2 m_2 r_2^2\right) + 2\left(K_3 r_2^2 + K_2 r_2^2 \right)}{2 K_2 r_1 r_2} \quad \text{...(g)}$$

- **Frequency equation :** From Equations (f) and (g),

$$\frac{2 K_2 r_1 r_2}{\left[-m_1 r_1^2 \omega^2 + 2\left(K_1 r_1^2 + K_2 r_2^2 \right) \right]} = \frac{\left(-m_2 r_2^2 \omega^2 \right) + 2\left(K_3 r_2^2 + K_2 r_2^2 \right)}{2 K_2 r_1 r_2}$$

$$\omega^4 - \left(\frac{2K_3}{m_2} + \frac{2K_2}{m_2} + \frac{2K_1}{m_1} + \frac{2K_2}{m_1} \right) \omega^2 + \frac{4}{m_1 m_2} (K_1K_3 + K_2K_3 + K_1K_2) = 0$$

2. **Two natural frequencies :**

Substituting the values of m_1, m_2, K_1, K_2, and K_3

$$\omega^4 - \left(\frac{2 \times 60 \times 10^3}{12} + \frac{2 \times 50 \times 10^3}{12} + \frac{2 \times 40 \times 10^3}{10} + \frac{2 \times 50 \times 10^3}{10} \right)$$

$$\omega^2 + \frac{4}{10 \times 12} \times (40 \times 10^3 \times 60 \times 10^3 + 50 \times 10^3 \times 60 \times 10^3 + 40 \times 10^3 \times 50 \times 10^3) = 0 \quad \text{...(e)}$$

$$\omega^4 - (18333.3 + 18000)\, \omega^2 + 246 \times 10^6 = 0$$

$$\omega^4 - 36333.3\, \omega^2 + 246 \times 10^6 = 0$$

$$\omega^2 = \frac{36333.3 \pm \sqrt{(-36333.3)^2 - 4 \times 246 \times}}{2 \times 1} \quad \text{...(f)}$$

$$\omega^2 = \frac{36333.3 \pm 18333.26}{2}$$

$$\omega_{n1}^2 = 9000 \text{ and } \omega_{n2}^2 = 27333.3 \quad \text{...Ans.}$$

$$\boldsymbol{\omega_{n1} = 94.86 \text{ rad/s and } \omega_{n2} = 165.2 \text{ rad/s}}$$

...Ans.

Q.9 Find the natural frequencies and mode shapes for the torsional system shown in Fig. 5.13(a). Assume $J_1 = J_0$, $J_2 = 2J_0$ and stiffness for each spring as K_t.

SPPU - Dec. 16, Dec. 17, May 18, 12 Marks

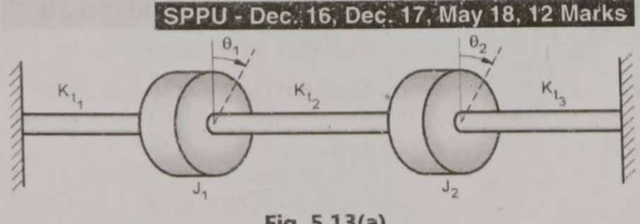


Fig. 5.13(a)

Ans. :

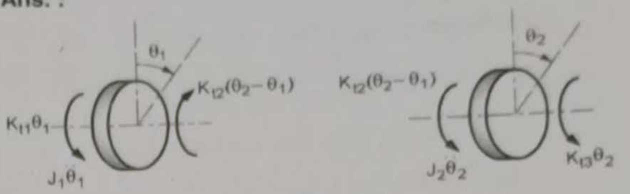


Fig. 5.13(b) : F.B.D

1. **Frequency Equation :**

- **Two differential equations of motion :**

From Fig. 5.13(b) ;

$$J_1 \ddot{\theta}_1 + K_{t1} \theta_1 - K_{t2}(\theta_2 - \theta_1) = 0 \quad \text{...(a)}$$

and
$$J_2 \ddot{\theta}_2 + K_{t2}(\theta_2 - \theta_1) + K_{t3}\theta_2 = 0 \quad \text{...(b)}$$

- Substituting $J_1 = J_0$, $J_2 = 2J_0$ and $K_{t1} = K_{t2} = K_{t3} = K_t$ in Equations (a) and (b)

$$J_0 \ddot{\theta}_1 + K_t\theta_1 - K_t(\theta_2 - \theta_1) = 0$$

And
$$2J_0 \ddot{\theta}_2 + K_t(\theta_2 - \theta_1) + K_t\theta_2 = 0$$

$$\therefore\ J_0 \ddot{\theta}_1 + 2K_t \theta_1 - K_t \theta_2 = 0 \quad \text{...(c)}$$

And
$$2J_0 \ddot{\theta}_1 - K_t \theta_1 + 2K_t \theta_2 = 0 \quad \text{...(d)}$$

- **Solutions for θ_1 and θ_2 under steady state conditions**

$$\left.\begin{aligned} \theta_1 &= \phi_1 \sin \omega t \\ \theta_2 &= \phi_2 \sin \omega t \end{aligned}\right\}$$

Therefore,

$$\left.\begin{aligned} \ddot{\theta}_1 &= -\phi_1 \omega^2 \sin \omega t \\ \ddot{\theta}_2 &= -\phi_2 \omega^2 \sin \omega t \end{aligned}\right\}$$

- Substituting Equations (e) and (f) in Equations (c),

$$J_0(-\phi_1 \omega^2 \sin \omega t) + 2K_t \phi_1 \sin \omega t - K_t \phi_2 \sin \omega t = 0$$

$$-J_0\omega^2 \phi_1 + 2K_t \cdot \phi_1 - K_t\phi_2 = 0$$

$$(2K_t - J_0 \omega^2)\, \phi_1 = K_t \phi_2$$

$$\frac{\phi_1}{\phi_2} = \frac{K_t}{(2K_t - J_0\omega^2)} \quad \text{...(g)}$$

- Substituting Equations (e) and (f) in Equation (d),

$$2J_0(-\phi_2 \omega^2 \sin \omega t) - K_t \phi_1 \sin \omega t + 2K_t \phi_2 \sin \omega t = 0$$

$$(2K_t - 2J_0 \omega^2)\, \phi_2 = K_t \phi_1$$

$$\frac{\phi_1}{\phi_2} = \frac{2K_t - 2J_0\omega^2}{K_t} \quad \text{...(h)}$$

- **Frequency equation :**

From Equations (g) and (h),

$$\frac{K_t}{(2K_t - J_0\omega^2)} = \frac{2K_t - 2J_0}{K_t}$$

$$(2K_t - J_0\,\omega^2)\,(2K_t - 2J_0\,\omega^2) = K_t^2$$

$$4K_t^2 - 4J_0\,K_t\,\omega^2 - 2\,J_0\,K_t\,\omega^2 + 2\,J_0^2\,\omega^4 = K_t^2$$

$$2J_0^2\,\omega^4 - 6\,J_0\,K_t\,\omega^2 + 3K_t^2 = 0$$

$$\omega^4 - 3\frac{K_t}{J_0}\omega^2 + 1.5\left(\frac{K_t}{J_0}\right)^2 = 0 \qquad ...(i)$$

2. **Two Natural Frequencies :**

$$\omega^2 = +\frac{\frac{3K_t}{J_0} \pm \sqrt{\left(\frac{-3K_t}{J_0}\right)^2 - 4\times 1.5\times\left(\frac{K_t}{J_0}\right)^2}}{2\times 1}$$

$$= \frac{\frac{3K_t}{J_0} \pm \sqrt{3\left(\frac{K_t}{J_0}\right)^2}}{2}$$

$$\omega^2 = \left[\frac{+3\pm\sqrt{3}}{2}\right]\frac{K_t}{J_0}$$

$$\therefore\ \omega_{n1}^2 = \left[\frac{3-\sqrt{3}}{2}\right]\frac{K_t}{J_0} \text{ and } \omega_{n2}^2 = \left[\frac{3+\sqrt{3}}{2}\right]\frac{K_t}{J_0}$$

$$\therefore\ \omega_{n1} = \sqrt{\left[\frac{3-\sqrt{3}}{2}\right]\frac{K_t}{J_0}} \text{ and}$$

$$\omega_{n2} = \sqrt{\left[\frac{3+\sqrt{3}}{2}\right]\frac{K_t}{J_0}} \qquad ...\text{Ans.}$$

Q.10 An automobile of mass 2000 kg has a wheel base of 2.5 m. Its C.G. is located 1.5 m behind the front wheel axle and has a radius of gyration about C.G. 1.2 m. the front springs have a combined stiffness of 4000 N/m and rear springs 4500 N/m. Refer Fig. 5.14(a).

Determine :

(i) The natural frequency

(ii) Amplitude ratio for two modes of vibration.

SPPU - May 18, 12 Marks

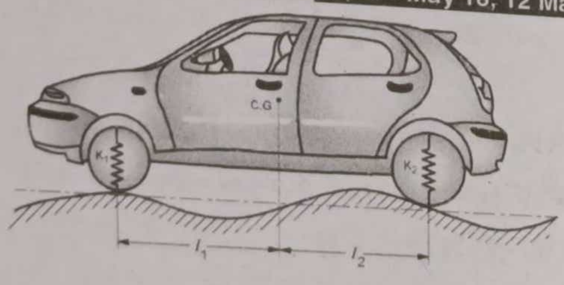


Fig. 5.14(a)

Ans. :

Given : Mass of automobile, m = 2000 kg

Wheel base, $l = l_1 + l_2 = 2.5$ m, $l_1 = 1.5$ m, $\therefore\ l_2 = 1$ m

Radius of gyration, k = 1.2 m

$\therefore$ M.I. of automobile,

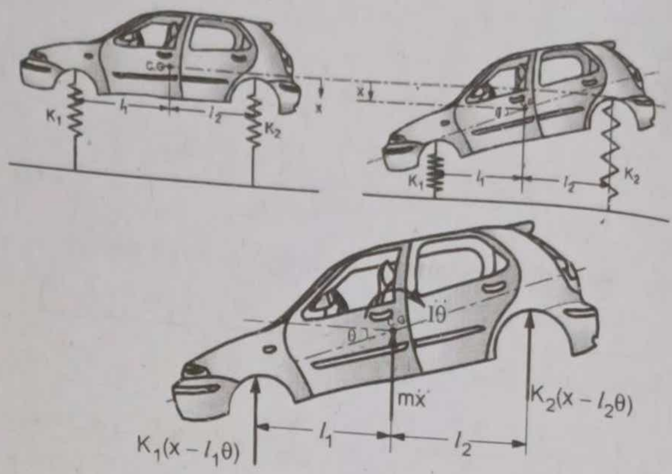
$$I = mk^2 = 2000 \times (1.2)^2 = 2880 \text{ kg-m}^2$$

Equivalent spring stiffness for front wheel,

$$K_1 = 4000 \text{ N / m}$$

Equivalent spring stiffness for rear wheel,

$$K_2 = 4500 \text{ N / m}$$



(b) Equivalent Position **(c) Displaced Position**

(d)

Fig. 5.14 : F.B.D of System

Fig. 5.14 (b) shows equivalent system, Fig. 5.14 (c) shows displaced position and 5.14(d) shows F.B.D. of system

1. **Differential Equations :**

From Fig. 5.14(b), the differential equations of motion for a given system are,

$$\left.\begin{aligned} m\ddot{x} + K_1(x + l_1\theta) + K_2(x - l_2\theta) &= 0 \\ \text{and } I\ddot{\theta} + K_1(x + l_1\theta)\,l_1 - K_2(x - l_2\theta)\,l_2 &= 0 \end{aligned}\right\} \quad ...(a)$$

or

$$\left.\begin{aligned} m\ddot{x} + (K_1 + K_2)\,x + (K_1 l_1 - K_2 l_2)\theta &= 0 \\ \text{and } I\ddot{\theta} + (K_1 l_1 - K_2 l_2)\,x + (K_1 l_1^2 + K_2 l_2^2)\,\theta &= 0 \end{aligned}\right\} \quad ...(b)$$

Substituting values in Equations (b),

$$2000 \times \ddot{x} + (4000 + 4500)\,x + (4000 \times 1.5 - 4500 \times 1)\,\theta = 0$$

and $2880\,\ddot{\theta} + (4000 \times 1.5 - 4500 \times 1)\,x + (4000 \times 1.5^2 + 4500 \times 1^2)\,\theta = 0$

or $$2000\,\ddot{x} + 8500\,x + 1500\,\theta = 0$$

and $$2880\,\ddot{\theta} + 1500\,x + 13500\,\theta = 0$$

or $$\ddot{x} + 4.25\,x + 0.75\,\theta = 0 \qquad ...(c)$$

and $\qquad \ddot{\theta} + 0.52083\, x + 4.6875\, \theta = 0 \qquad$ ...(d)

2. **Frequency Equation :** The solutions for x and θ under steady state conditions are,

$$\left.\begin{aligned} x &= X \sin \omega t \\ \text{and} \quad \theta &= \phi \sin \omega t \end{aligned}\right\} \qquad \text{...(e)}$$

$$\left.\begin{aligned} \text{Therefore, } \ddot{x} &= -X \omega^2 \sin \omega t \\ \text{and} \quad \ddot{\theta} &= -\phi \omega^2 \sin \omega t \end{aligned}\right\} \qquad \text{...(f)}$$

- Substituting Equations (e) and (f) in Equation (c),

$$-X \omega^2 \sin \omega t + 4.25 X \sin \omega t + 0.75 \phi \sin \omega t = 0$$

$$\therefore \quad -X\omega^2 + 4.25 X + 0.75 \phi = 0$$

$$\therefore \quad (\omega^2 - 4.25) X + 0.75 \phi = 0$$

$$(\omega^2 - 4.25) X = 0.75 \phi$$

$$\therefore \quad \mathbf{\frac{X}{\phi} = \frac{0.75}{(\omega^2 - 4.25)}} \qquad \text{...(g)}$$

Substituting Equations (e) and (f) in Equation (d),

$$-\phi \omega^2 \sin \omega t + 0.52083\, X \sin \omega t + 4.6875\, \phi \sin \omega t = 0$$

$$\therefore \quad -\phi \omega^2 + 0.52083 X + 4.6875 \phi = 0$$

$$\therefore \quad -(\omega^2 - 4.6875) \phi + 0.52083 X = 0$$

$$\therefore \quad (\omega^2 - 4.6875) \phi = 0.52083 X$$

$$\therefore \quad \mathbf{\frac{X}{\phi} = \frac{(\omega^2 - 4.6875)}{0.52083}} \qquad \text{...(h)}$$

From Equations (g) and (h),

$$\frac{0.75}{(\omega^2 - 4.25)} = \frac{(\omega^2 - 4.6875)}{0.52083}$$

$$\therefore \quad (\omega^2 - 4.6875)(\omega^2 - 4.25) = 0.3906$$

$$\therefore \quad \omega^4 - 4.25\,\omega^2 - 4.6875\,\omega^2 + 19.9219 = 0.3906$$

$$\mathbf{\omega^4 - 8.9375\,\omega^2 + 19.53 = 0} \qquad \text{...(i)}$$

- Equation (i) is known as frequency Equation

3. **Natural Frequencies :** From Equation (i),

$$\omega^2 = \frac{8.9375 \pm \sqrt{(8.9375)^2 - 4 \times 1 \times 19.53}}{2}$$

or $$\omega^2 = \frac{8.9375 \pm 1.3262}{2}$$

$$\therefore \quad \omega_{n1}^2 = \frac{8.9375 - 1.3262}{2} = 3.8056$$

and $$\omega_{n2}^2 = \frac{8.9375 + 1.3262}{2} = 5.1318$$

$$\therefore \quad \omega_{n1} = \sqrt{3.8056} = 1.95 \text{ rad/s}$$

and $$\omega_{n2} = \sqrt{5.1318} = 2.265 \text{ rad/s}$$

$$\mathbf{\omega_{n1} = 1.95 \text{ rad/s}}$$

$$\mathbf{\omega_{n2} = 2.265 \text{ rad/s}}$$

4. **Ratio of Amplitudes :**

The amplitude ratio for first mode of vibrations is,

$$\left(\frac{X}{\phi}\right)_1 = \frac{0.75}{\omega_{n1}^2 - 4.25} = \frac{0.75}{(1.95)^2 - 4.25} = \mathbf{-1.68}$$

The amplitude ratio for second mode of vibrations is,

$$\left(\frac{X}{\phi}\right)_2 = \frac{0.75}{\omega_{n2}^2 - 4.25} = \frac{0.75}{(2.265)^2 - 4.25}$$

or $$\left(\frac{X}{\phi}\right)_2 = \mathbf{0.852}$$

Q.11 Define the term : Node point

SPPU : Dec. 16, Dec. 17, Dec. 18, Dec. 19

Ans. : Node point : There is a point or a section of the shaft which remains untwisted. *This point or section where amplitude of vibration is zero is known as* **node point or nodal section.**

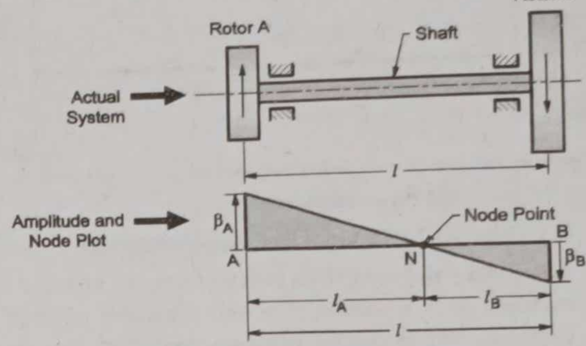


Fig. 5.15 : Two Rotor System

Q.12 Define the term : Zero frequency

SPPU : Dec. 16, Dec. 17, Dec. 18, Dec. 19

Ans. : Zero Frequency Vibration

- If two rotors A and B rotate in same direction, then the shaft is said to vibrate with **zero frequency**. Such behaviour is called as **zero node behaviour**, as the amplitudes of vibration at both ends will be in the same direction as shown in Fig. 5.16.

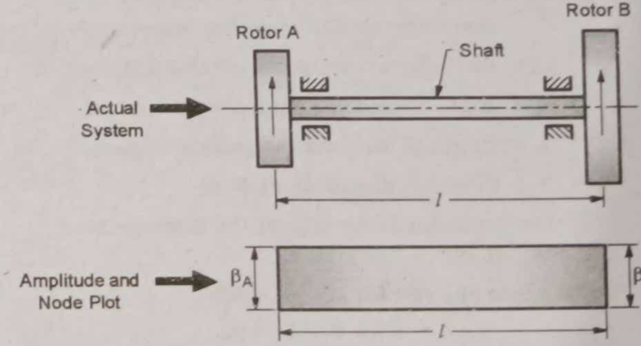


Fig. 5.16 : Two Rotor System
(When Two Rotors Rotates in Same Direction)

Q.13 Explain the concept of torsionally equivalent shaft.

SPPU : May 15, Dec. 16, Dec. 17, May 18, Dec. 19

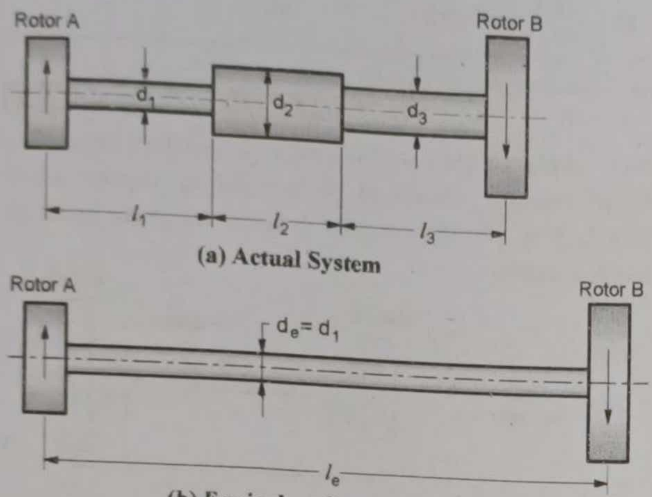
Q. Explain the concept of torsionally equivalent shaft and derive the equation for its equivalent length.

SPPU : Dec. 13, May 16

Q. Explain Torsionally equivalent shaft with respect to 2 DOF free vibration.

SPPU : May 19

Ans. : Torsionally Equivalent Shaft



(a) Actual System

(b) Equivalent System

Fig. 5.17 : Torsionally Equivalent Shaft

- Torsional equations shaft : In two rotor and three rotor systems it is assumed that the diameter of shaft is uniform. But in actual practice, the shaft may have different diameters for different lengths. If the shafts of different diameters are replaced by a theoretically equivalent shaft of a uniform diameter, such shaft is called as torsionally equivalent shaft. [Fig. 5.17]

Let,

d_1, d_2 and d_3 = diameters of shaft for the lengths l_1, l_2, and l_3 respectively, m.

θ_1, θ_2 and θ_3 = angles of twist for shaft lengths l_1, l_2 and l_3 respectively, rad

J_1, J_2 and J_3 = polar moment of inertia of shaft of diameters d_1 and d_2 and d_3 respectively, m^4

d_e = diameter of torsionally equivalent shaft, m

θ_e = angle of twist for equivalent shaft, rad

l_e = length of torsionally equivalent shaft, m

T = torque acting on shaft, N-m

G = modulus of rigidity for the shaft material, N/m^2

- **Total angle of twist for actual shaft :**

$$\theta = \theta_1 + \theta_2 + \theta_3 \qquad ...(a)$$

or

$$\theta = \frac{Tl_1}{GJ_1} + \frac{Tl_2}{GJ_2} + \frac{Tl_3}{GJ_3} \qquad ...(b)$$

- **Angle of twist of equivalent shaft :**

$$\therefore \quad \theta_e = \frac{Tl_e}{GJ_e} \qquad ...(c)$$

- Angle of twist of equivalent shaft = Total angle of twist of actual shaft

$$\theta_e = \theta_1 + \theta_2 + \theta_3 \qquad ...(d)$$

- **From Equations (b) and (c),**

$$\frac{Tl_e}{GJ_e} = \frac{Tl_1}{GJ_1} + \frac{Tl_2}{GJ_2} + \frac{Tl_3}{GJ_3}$$

$$\therefore \quad \frac{l_e}{J_e} = \frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}$$

$$\therefore \quad \frac{l_e}{\frac{\pi}{32}d_e^4} = \frac{l_1}{\frac{\pi}{32}d_1^4} + \frac{l_2}{\frac{\pi}{32}d_2^4} + \frac{l_3}{\frac{\pi}{32}d_3^4}$$

$$\therefore \quad \frac{l_e}{d_e^4} = \frac{l_1}{d_1^4} + \frac{l_2}{d_2^4} + \frac{l_3}{d_3^4}$$

$$\therefore \quad l_e = l_1\left(\frac{d_e}{d_1}\right)^4 + l_2\left(\frac{d_e}{d_2}\right)^4 + l_3\left(\frac{d_e}{d_3}\right)^4 \qquad ...(1)$$

In actual practice, the diameter of equivalent shaft is taken as one of the diameters of the actual shaft. Let us assume that $d_e = d_1$, then the length of equivalent shaft is given by,

$$l_e = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4 \qquad ...(2)$$

Q.14 The flywheel of an engine driving a dynamo has mass of 200 kg and has a radius of gyration of 300 mm. The shaft at the flywheel end has an effective length of 250 mm and is 50 mm Diameter the arc mature mass is 225 kg and has a radius of gyration of 255 mm. The dynamo shaft has a diameter of 43.75 mm and a length of 255 mm. Neglecting the inertia of the shaft and coupling calculate the frequency of the torsional vibrations and position of node. Take the modulus of rigidity for shaft material as 80 GPa.

SPPU : May 16, 12 Marks

Ans. : Mass movement of inertia for rotor A,

$$I_A = 200 \times 0.3^2 = 18 \text{ kg-m}^2$$

Mass movement of inertia for rotor B,

$$I_B = 225 \times 0.255^2 = 14.63 \text{ kg-m}^2$$

Modulus of rigidity for shaft material,

$$G = 80 \text{ GPa} = 80 \times 10^9 \text{ N/m}^2$$

1. **Equivalent Length :**

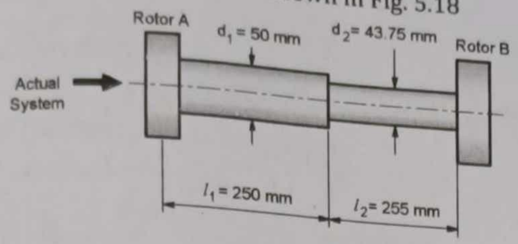
The equivalent length of shaft having diameter d_1 (50 mm) is,

$$l_e = l_1\left(\frac{d_e}{d_1}\right)^4 + l_2\left(\frac{d_e}{d_2}\right)^4 = l_1\left(\frac{d_1}{d_1}\right)^4 + l_2\left(\frac{d_1}{d_2}\right)^4$$

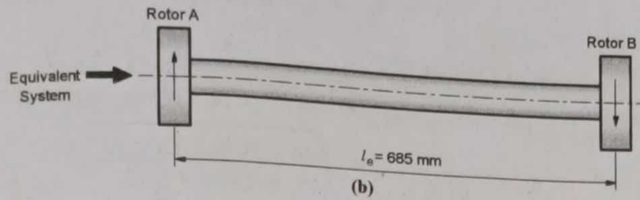
$$= l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 = 250 + 255\left(\frac{50}{43.75}\right)^4$$

$$\therefore \quad l_e = \mathbf{685\ mm}$$

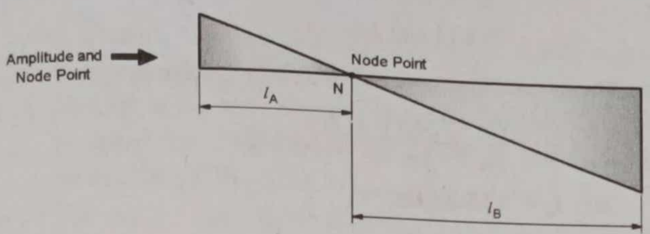
The equivalent system is shown in Fig. 5.18



(a)



(b)



(c)

Fig. 5.18

2. Position of Node :

We know that,

$$l_A I_A = l_B I_B$$

$$\therefore \quad l_A = \frac{l_B I_B}{I_A} = \frac{l_B \times 14.63}{18}$$

or $\quad l_A = 0.812\, l_B$

But $\quad l_e = l_A + l_B$;

$$685 = 0.812\, l_B + l_B$$

$$\therefore \quad l_B = 377.87 \text{ m}$$

$$l_A = 0.812 \times 377.87 = \mathbf{306.83\ m}$$ ...Ans.

3. Natural Frequency :

The natural circular frequency of two rotor system is,

$$\omega_n = \sqrt{\frac{GJ}{l_A I_A}}$$

$$\text{or} \quad \sqrt{\frac{GJ}{l_B I_B}} = \sqrt{\frac{G \times \frac{\pi}{32} d_e^4}{l_A I_A}}$$

$$= \sqrt{\frac{80 \times 10^9 \times \frac{\pi}{32}(0.05)^4}{0.30683 \times 18}}$$

or $\quad \omega_n = \mathbf{94.27\ rad/s}$ ...Ans.

The natural frequency of two rotor system is,

$$f_n = \frac{\omega_n}{2\pi} = \frac{94.27}{2\pi}$$

or $\quad f_n = \mathbf{15\ Hz}$ ...Ans.

Q.15 Two equal masses of weight 400 N each and radius of gyration 400 mm are keyed to the opposite ends of a shaft 600 mm long. The shaft is 750 mm diameter for the first 250 mm of its length, 125 mm diameter for the next 125 mm and 85 mm diameter for the remaining of its length. Find the frequency of free torsional vibrations of the system and position of node. Assume $G = 0.84 \times 10^{11}$ N/m^2

SPPU - Dec. 14, 8 Marks, Oct. 18(In-sem), Oct. 19(In Sem.), 10 Marks

Ans. : Given :

Mass of rotor A, $m_A = 400 \text{ N} = \frac{400}{9.81} = 40.77$ kg

Radius of gyration for rotor A,

$$K_A = 400 \text{ mm} = 0.4 \text{ m}$$

Mass of rotor B, $\quad m_B = 400 \text{ N} = \frac{400}{9.81} = 40.77$ kg

Radius of gyration for rotor B

$$K_B = 400 \text{ mm} = 0.4 \text{ m}$$

1. Equivalent Length

- The equivalent length of shaft having diameter d_3 = 0.0875 m is,

$$l_e = l_1\left[\frac{d_e}{d_1}\right]^4 + l_2\left[\frac{d_e}{d_2}\right]^4 + l_3\left[\frac{d_e}{d_3}\right]^4$$

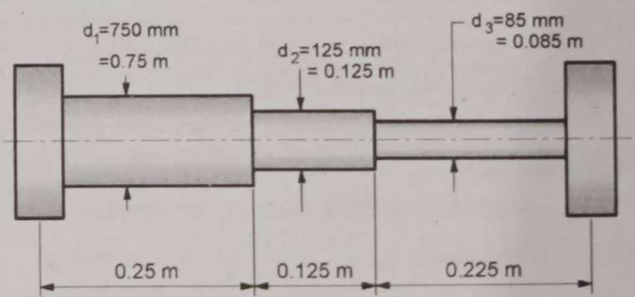
$$= 0.25$$

$$\left[\frac{0.085}{0.75}\right]^4 + 0.125\left[\frac{0.085}{0.125}\right]^4 + 0.225\left[\frac{0.085}{0.085}\right]^4 = 0.2517 \text{ m}$$

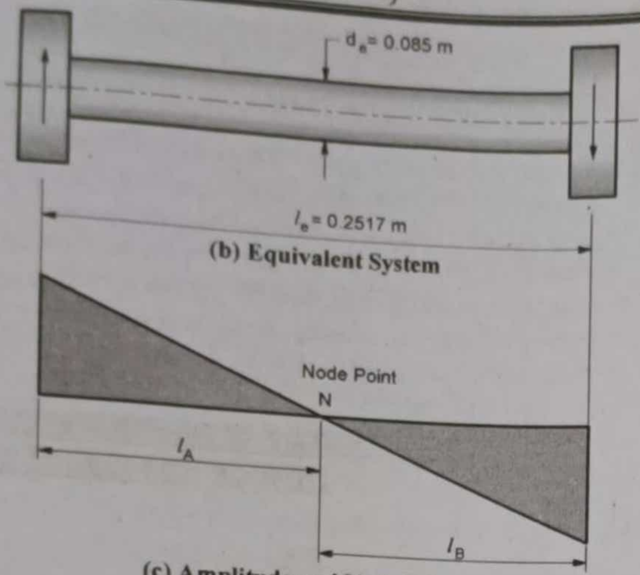
- The polar moment of inertia of equivalent shaft is,

$$J_e = \frac{\pi}{32}(0.085)^4 = 5.12 \times 10^{-6} \text{ m}^4$$

2. Position of Node



(a) Actual System



(b) Equivalent System

(c) Amplitude and Node Point

Fig. 5.19

For rotors A and B,

$$I_A = m_A K_A^2 = 40.77 \times 0.4^2 = 6.52 \text{ kg.m}^2$$

$$I_B = m_B K_B^2 = 40.77 \times 0.4^2 = 6.52 \text{ kg.m}^2$$

We have $\quad l_A I_A = l_B I_B$

Therefore, $\quad l_A = \dfrac{l_B I_B}{I_A}$

or $\quad l_A = l_B$

But, $\quad l_e = l_A + l_B$

$\therefore \quad 0.2517 = 2\, l_A$ ...($\because l_A = l_B$)

$\quad$ **$l_A = l_B = 0.1258$ m** ...**Ans.**

3. **Natural Frequency**

- The natural circular frequency of two rotor system is,

$$\omega_n = \sqrt{\frac{GJ}{l_A I_A}} = \sqrt{\frac{84 \times 10^9 \times 5.12 \times 10^{-6}}{0.1258 \times 6.52}}$$

or, $\quad \omega_n = 724.11$ rad / s

- The natural frequency of two rotor system is,

$$f_n = \frac{\omega_n}{2\pi} = \frac{724.11}{2\pi} = 115.24 \text{ Hz}$$

or **f_n = 115.24 Hz**

□□□

Chapter 6 : Measurement and Control of Vibrations and Introduction To Noise

Q.1 List the vibration measuring devices.

Ans. : The various vibration measuring devices are :

1. Vibrometers
2. Velometers
3. Acclerometers
4. Frequency Measuring Instruments

Q.1 Write short note on : Seismic instruments.

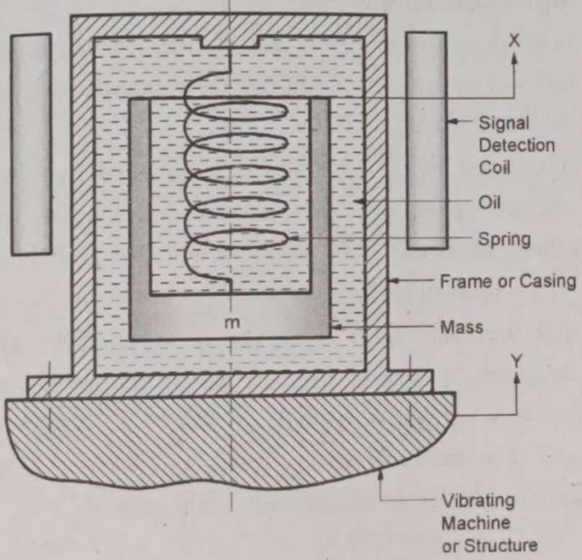
SPPU : Dec. 15

OR Explain with diagram the working principle of Seismic instrument.

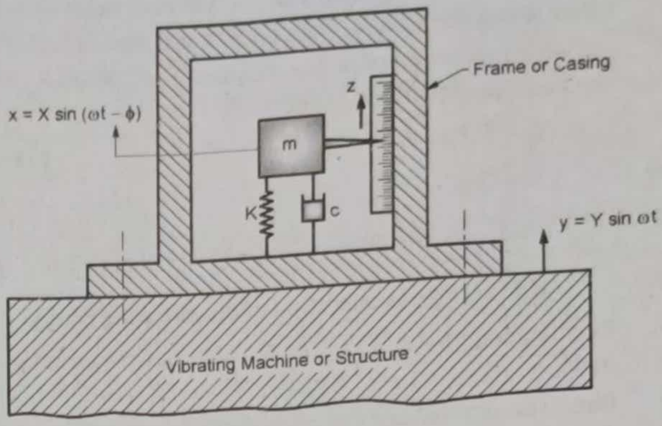
SPPU : May 16, May 19

Ans. : Seismic Instrument or Seismometer or Vibration Pick-up :

- A seismic instrument consists of a spring-mass-damper system in a frame or casing, which is mounted (fastened) on the vibrating machine or structure to measure the displacement or amplitude of vibratory motion, as shown in Fig. 6.1.
- The mass '**m**' is supported in a frame or casing by means of spring having stiffness '**K**' and dashpot of damping coefficient '**c**'.
- The seismometer, shown in Fig. 6.1(a), is equivalent to a spring-mass-damper system shown in Fig. 6.1(b), having base or support excitation.



(a)



(b)

Fig. 6.1 : Seismic Instrument or Seismometer

- The relative amplitude Z can be measured by strain sensing transducer which is rigidly fixed to the seismic mass.
- The output voltage from the transducer is proportional to the displacement y of vibrating body. Hence, such instrument is called as **vibration pick-ups** [Fig. 6.2]

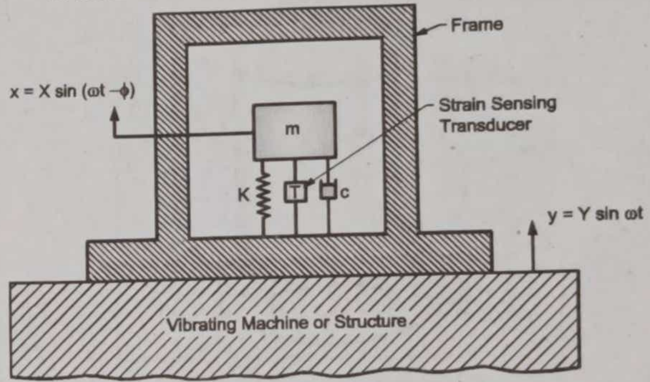


Fig. 6.2 : Vibration Pick-up

Q.2 Write short note on : Piezoelectric accelerometer.

SPPU : Dec. 11, Dec. 13, May 15

OR Explain the working principle of Accelerometer.

SPPU : May 19

Ans. : Accelerometer (Acceleration Pick-Up) :

- **Acceleration pick-up** or **accelerometer** is an instrument that measures the acceleration of a vibrating body.
- For seismic instrument, steady state relative amplitude is given by,

$$Z = \frac{Y(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}} \quad ...(1)$$

- When, the frequency ratio (ω / ω_n) is very small or in other words the natural frequency of instrument 'ω_n' is very high as compare to the frequency of vibration of the body 'ω', Equation (1) becomes,

$$Z \simeq Y(\omega/\omega_n)^2 \quad ...(2)$$

or $Z \simeq Y\omega^2$

....[$\because$ ω_n is constant for a given instrument]

or $Z \simeq -\ddot{y}$ (i.e. acceleration of the vibrating body)

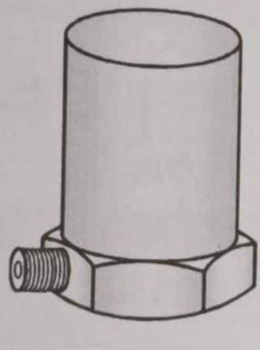
- Relative amplitude('Z') $\propto$ acceleration of the vibrating body.
- Once the acceleration is recorded, the velocity and displacement is obtained by integrating.

Conditions for Using Seismometer as Acceleration Measuring Instrument :

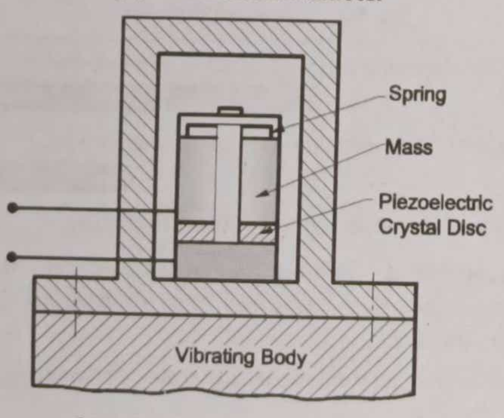
(i) It should have very high natural frequency (ω_n); and

(ii) It should generate output signal proportional to the relative acceleration of the vibrating body.

Piezoelectric Accelerometer :



(a) Actual Accelerometer



(b) Schematic of Accelerometer

Fig. 6.3 : Accelerometer

- The natural frequency of the instrument should be very large as compared to the frequency of vibrating body. This is possible only when the seismic mass 'm' is small and the spring has large value of stiffness 'K' is large (i.e. short spring).
- Fig. 6.3 shows piezoelectric crystal accelerometer, which is used to measure the acceleration of vibrating body with higher frequencies.
- However a piezoelectric crystal is capable of generating a measurable signal even for a small deformation. If the signals are very weak, amplifier is used to amplify them.

Q.3 Write short note on : Vibration exciter **SPPU : May 15**

OR What are vibration exciters? Explain any one Exciter **SPPU : May 19**

Ans. :

Vibration Exciters :

- ***Vibration exciters*** *or* ***shakers*** *are used to produce the required cyclic excitation force at a required frequency.* The cyclic excitation force produced by the exciter can be applied to the machine or structure so as to study its dynamic characteristics
- The following three types of vibration exciters are commonly used :

1. Mechanical Exciters :

(i) Scotch yoke mechanism :

- In mechanical exciters, a scotch yoke mechanism can be used to produce the harmonic excitation force, as shown in Fig. 6.4.
- In this system, the crank of mechanism is driven with constant speed or variable speed by the motor. The other end of the crank slides in a slotted rod which reciprocates in the vertical guide.
- The harmonic force that can be applied on the structure by the exciter, to which the exciter is attached, is given by,

$$F = m\omega^2 r \sin \omega t \quad ...(1)$$

Where, m = mass attached to the scotch yoke mechanism, kg

ω = crank speed, rad/s

r = radius of crank, m

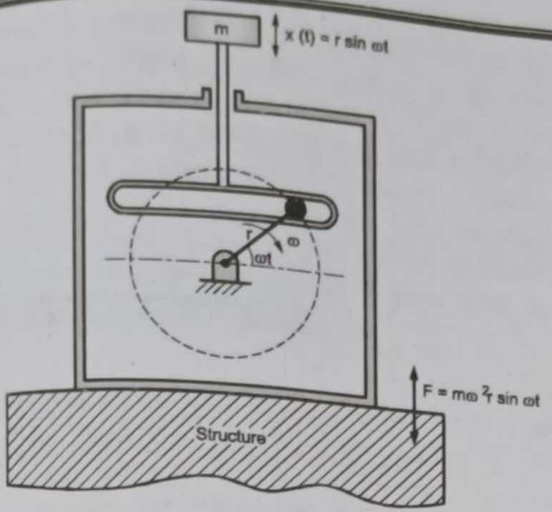


Fig. 6.4 : Mechanical Exciter (Scotch Yoke Mechanism)

(ii) Two rotating masses :

- The harmonic force can also be created by means of two masses rotating at the same speed but in opposite directions as shown in Fig. 6.5.

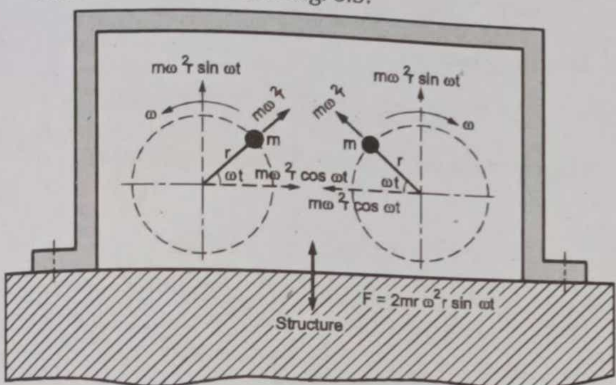


Fig. 6.5 : Mechanical Exciter (Exciting Force Due to Two Rotating Masses)

- When two masses of equal magnitude, rotate at an angular velocity 'ω' at radius '**r**', then the vertical sinusoidal force generated is given by,

$$F = 2m\,\omega^2\,r \sin \omega t \qquad ...(2)$$

Where, m = rotating unbalanced mass, kg

ω = crank speed, rad/s ,

r = Radius of crank, m

- The horizontal components of two unbalanced forces act opposite to each other, hence the resultant horizontal force will be zero.

2. Electrodynamic Exciters :

- Fig. 6.6 shows an **electrodynamic exciter or shaker** which is also known as **electromagnetic exciter**. It consists of : heavy magnet, coil, moving element, flexible support and exciter table.
- When the current passes through the coil placed in magnetic field, a force is developed which is proportional to current and magnetic flux. This force accelerates the component placed on the exciter table.
- The magnitude of force developed is given by,

$$F = BI\,l \qquad ...(3)$$

where, B = magnetic flux intensity in Wb / m^2

I = current in coil, A

l = length of coil, m

- The magnitude of accelerating force produced depends upon the maximum current, and the mass of moving element.

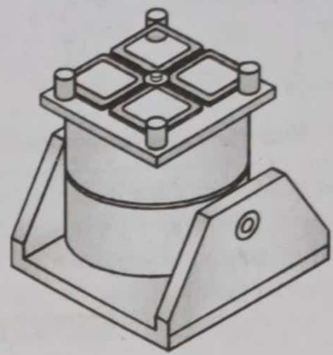
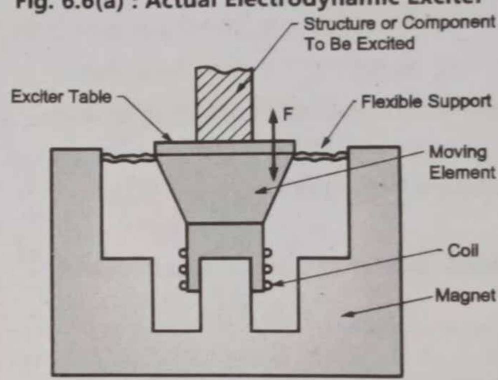


Fig. 6.6(a) : Actual Electrodynamic Exciter



(b) Schematic of Electrodynamic Exciter

Fig. 6.6: Electrodynamic Exciter

- If the current passing through the coil varies harmonically with time (i.e. A.C. current), the force produced also varies harmonically.
- The electrodynamic exciters are used to generate the forces upto 30 kN and displacements upto 25 mm and frequencies in the range of 5 Hz to 20 Hz.

3. Hydraulic and Pneumatic Exciters :

- The mechanical and electrodynamic exciters are generally used for limited force and limited frequency range. When exciters are to be used for larger force capacity and wider frequency range, the **hydraulic** or **pneumatic exciters** are preferred.

- Fig. 6.7 shows the block diagram of a hydraulic or pneumatic exciter.
- In this arrangement, an electrically actuated servo valve operates a main control valve to regulate the flow of fluid (i.e. oil in case of hydraulic exciter or air in case of pneumatic exciter) to each end of the cylinder.
- The exciting force of high magnitude and frequency (upto 400 Hz) can be obtained by using such exciters.

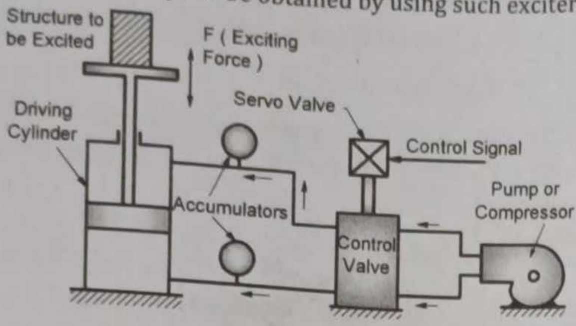


Fig. 6.7 : Hydraulic and Pneumatic Exciter

Q.4 In a seismic instrument if mass m = 0.1 kg, stiffness of spring, K = 1 N/mm and damping ratio, ξ = 0.5, determine the amplitude of recorded motion if the motion of vibrating body is 3 sin 200 t (mm).

SPPU - May 19, 6 Marks

Ans. :

Given : $m = 0.1\text{ kg}$; $K = 1\text{ N/mm} = 1000\text{ N/m}$;

$\xi = 0.5$; $y = 3\sin 200t$.

- **Maximum displacement of vibrating body :**

$$Y = 3\text{ mm}$$

- **Excitation frequency :**

$$\omega = 200\text{ rad/s.}$$

- **Natural circular frequency of instrument :**

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1000}{0.1}} = 100\text{ rad/s.}$$

- **Frequency ratio :**

$$r = \frac{\omega}{\omega_n} = \frac{200}{100} = 2$$

- **Steady-state relative amplitude of recorded motion**

$$Z = \frac{Y(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$= \frac{Y(r)^2}{\sqrt{[1-r^2]^2 + [2\xi r]^2}}$$

$$= \frac{3(2)^2}{\sqrt{[1-(2)^2]^2 + [2\times 0.5\times 2]^2}}$$

or $\mathbf{Z = 3.3283\text{ mm}}$ ...Ans.

Q.5 A device used to measure tensional acceleration consist of a ring having a moment of inertia of 0.049 kg-m^2 connected to a shaft by a spiral spring having a scale of 0.98 N-m/rad and a viscous damper having a constant of 0.11 N-m sec/rad. When the shaft vibrates with a frequency of 15 cpm, the relative amplitude between the ring and the shaft is found to be 2°. What is the maximum acceleration of the shaft ?

SPPU - Dec. 10, May 19, 6 Marks

Ans. :

Given : $I = 0.049\text{ kg-m}^2$; $K = 0.98\text{ N-m/rad}$;

$c = 0.11\text{ Nm-s/rad}$; $N = 15\text{ cpm} = 15\text{ r.p.m.}$

$$Z = 2° = 2\times\frac{\pi}{180} = 0.03490\text{ rad.}$$

- **Excitation frequency :**

$$\therefore \quad \omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 15}{60} = 1.57\text{ rad/s}$$

- **Natural circular frequency of system :**

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{0.98}{0.049}} = 4.47\text{ rad/s}$$

- **Damping factor :**

$$\therefore \quad \xi = \frac{c}{2I\omega_n} = \frac{0.11}{2\times(0.049)\times(4.47)} = 0.25$$

- **Steady-state relative amplitude of vibration :**

$$Z = \frac{Y\left(\frac{\omega}{\omega n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega n}\right]^2}}$$

$$0.03490 = \frac{\left(\frac{1.57}{4.47}\right)^2}{\sqrt{\left[1-\left(\frac{1.57}{4.47}\right)^2\right]^2 + \left[2\,'\,0.25\,'\left(\frac{1.57}{4.47}\right)\right]^2}}$$

$\therefore \quad Y = 0.253\text{ rad}$

- **Maximum acceleration of shaft :**

$$\ddot{y}_{max} = \omega^2\cdot Y = (1.57)^2\times(0.253)$$

$$\ddot{y}_{max} = \mathbf{0.6236\text{ rad/s}^2}$$ ...Ans.

Q.6 A seismic instrument is used to find the displacement, velocity and acceleration of a machine running at 250 r.p.m. If the natural frequency of the instrument is 5 Hz and it records the displacement 5 mm, determine the displacement, velocity and acceleration of the vibrating machine assuming no damping.

SPPU - May 14, 8 Marks, Dec. 18, 6 Marks

Ans. :

Given : $N = 250\text{ r.p.m.}$; $f_n = 5\text{ Hz}$;

$Z = 5\text{ mm}$; $\xi = 0.$

- **Excitation frequency :**

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 250}{60} = 26.17 \text{ rad/s.}$$

- **Natural circular frequency of instrument :**

$$\omega_n = f_n \times 2\pi = 5 \times 2\pi = 31.41 \text{ rad/s.}$$

- **Steady-state relative of vibrating machine :**

$$Z = \frac{Y(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\therefore \quad Z = \frac{Y(\omega/\omega_n)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2} \quad \text{... [} \therefore \xi = 0 \text{]}$$

$$\therefore \quad 5 = \frac{Y\left(\frac{26.17}{31.41}\right)^2}{1-\left(\frac{26.17}{31.41}\right)^2}$$

$$\therefore \quad \mathbf{Y = 2.20 \text{ mm}} \quad \text{...Ans.}$$

$\therefore$ Displacement of vibrating machine = 2.20 mm

- **Velocity of vibrating machine :**

$$\dot{y} = \omega \cdot Y = 26.17 \times 2.20 = 57 \text{ mm/s} \quad \text{...Ans.}$$

- **Acceleration of vibrating machine :**

$$\ddot{y} = \omega^2 Y = (26.17)^2 \times 2.20$$

$$= \mathbf{1506.7 \text{ mm/s}^2} \quad \text{...Ans.}$$

Q.7 An accelerometer has a suspended mass of 0.01 kg with a damped natural frequency of vibration of 150 Hz. It is mounted on an engine running at 6000 rpm and undergoes an acceleration of 1g. The instrument records an acceleration of 9.5 m/s^2. Find the damping constant and the spring stiffness of the accelerometer.

SPPU - Dec. 16, Dec. 17, 8 Marks

Ans. :

Given : $m = 0.01$ kg ; $f_d = 150$ Hz ; $N = 6000$ r.p.m.,

$$\therefore \omega = \frac{2\pi \times 6000}{60} = 628.30 \text{ rad/s}$$

Actual Acceleration, $Y\omega^2 = 9.81$ m/s^2

Measured Acceleration , $Z\omega_n^2 = 9.5$ m/s^2

The damped natural frequency is,

$$f_d = \frac{\omega d}{2\pi}$$

$$\therefore \quad \omega_d = 150 \times 2\pi = 942.48 \text{ rad/s}$$

The steady-state amplitude of vibration is,

$$Z = \frac{Y(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$\therefore \quad Z = \frac{Y(r)^2}{\sqrt{[1-r^2]^2 + [2\xi r]^2}} \quad \text{...}\left[\because \frac{\omega}{\omega_n} = r\right]$$

$$\frac{Z\omega_n^2}{Y\omega^2} = \frac{1}{\sqrt{[1-r^2]^2 + [2\xi r]^2}}$$

$$\left(\frac{9.5}{9.81}\right) = \frac{1}{([1-r^2]^2 + [2\xi r]^2)}$$

$$\left(\frac{9.5}{9.81}\right)^2 = \frac{1}{\sqrt{[1-r^2]^2 + [2\xi r]^2}}$$

$$\text{or} \quad [1-r^2]^2 + [2\xi r]^2 = 1.066 \quad \text{...(1)}$$

Now, $$\frac{\omega}{\omega_d} = \frac{\omega}{\omega_n\sqrt{1-\xi^2}}$$

or $$\frac{\omega}{\omega_d} = \frac{\left(\frac{\omega}{\omega_n}\right)}{\sqrt{1-\xi^2}} \quad \therefore \quad \frac{628.32}{942.48} = \frac{r}{\sqrt{1-\xi^2}}$$

$$\therefore \quad r = 0.6667\sqrt{1-\xi^2}$$

$$\text{or} \quad r^2 = 0.4444(1-\xi^2) \quad \text{...(2)}$$

From Equation (1) and (2), we get

$$\xi^4 - 1.4375\,\xi^2 + 0.4794 = 0$$

$$\therefore \quad \xi^2 = 0.5260 \text{ and } 0.9115$$

$$\text{or} \quad \xi = 0.7253 \text{ and } 0.9547$$

$$\text{Taking} \quad \xi = 0.7253$$

$$\text{Now,} \quad \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$942.48 = \omega_n\sqrt{1-(0.7253)^2}$$

$$\omega_n = 1368.90 \text{ rad/s}$$

The spring stiffness is,

$$K = m\,\omega_n^2 = 0.01 \times (1368.89)^2$$

$$\mathbf{K = 18738.56 \text{ N/m}} \quad \text{...Ans.}$$

The damping constant is,

$$c = 2m\omega_n\xi = 2 \times 0.01 \times 1368.90 \times 0.7253$$

$$\mathbf{c = 19.86 \text{ N.s/m}} \quad \text{...Ans.}$$

Q.8 Vibrations of a machine tool structure subjected to an excitation at 2 Hz is measured using a seismic instrument whose natural frequency is 5 Hz. The relative displacement shown is 0.4 μm. Determine the acceleration of the machine tool structure.

SPPU - May 17, 8 Marks

Ans. :

Given : $Z = 0.4\ \mu m = 0.4 \times 10^{-3}$ m,

$f_n = 5$ Hz, $f = 2$ Hz

Natural frequency of system is,

$$\therefore \quad \omega_n = 2\pi \times f_n = 2\pi \times 5$$

$$\omega_n = 31.41 \text{ rad/s}$$

and, $$\omega = 2\pi \times f = 2\pi \times 2 = 12.56 \text{ rad/s}$$

But, $\frac{\omega}{\omega_n} = \frac{12.56}{31.41} = 0.399$

The steady state relative amplitude of vibration,

$$Z = \frac{Y(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\therefore \quad Z = \frac{Y(\omega/\omega_n)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2} \qquad ...[\because \xi = 0]$$

$$\therefore 0.4\times10^{-3} = \frac{Y(0.399)^2}{1-(0.399)^2}$$

$$Y = 2.11\times10^{-3}\,m = 2.11\,mm$$

The acceleration of vibrating machine is,

$$\ddot{y} = \omega^2 Y = (12.56)^2\times(2.11\times10^{-3})$$

$$= \mathbf{0.333\ m/s^2}$$

Q.9 For finding vibration parameters of a machine running at 260 rpm, a seismic instrument is used. The natural frequency of the instrument is 7 Hz and the recorded displacement is 6 mm. Determine the displacement, velocity and acceleration of the vibrating machine assuming no damping.

SPPU - Dec. 16, Dec. 17, 8 Marks

Ans. :

Given : Speed of machine, N = 260 r.p.m.

$\therefore$ Excitation frequency, $\omega = \frac{2\pi\times260}{60} = 27.22$ rad/s.

Natural frequency of instrument, $f_n = 7$ Hz.

Amplitude of vibration, Z = 6 mm

Damping ratio, $\xi = 0$

- Natural circular frequency of instrument is,

$$\omega_n = f_n\times2\pi = 7\times2\pi = 43.98\ rad/s.$$

- The steady-state relative amplitude of vibration,

$$Z = \frac{Y(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\therefore \quad Z = \frac{Y(\omega/\omega_n)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2} \qquad ...[\therefore \xi = 0]$$

$$\therefore \quad 6 = \frac{Y\left(\frac{27.22}{43.98}\right)^2}{1-\left(\frac{27.22}{43.98}\right)^2}$$

$\therefore$ **Y = 9.66 mm** ...Ans.

$\therefore$ Displacement of vibrating machine is 9.66 mm

- The velocity of vibrating machine is,

$$\dot{y} = \omega\cdot Y = 27.22\times9.66 = 262.94\ \mathbf{mm/s}$$ ...Ans.

- The acceleration of vibrating machine is,

$$\ddot{y} = \omega^2 Y = (27.22)^2\times9.66$$

$$\ddot{y} = \mathbf{7157.36\ mm/s^2}$$ ...Ans.

$$\ddot{y} = \mathbf{7.15\ m/s^2}$$ ...Ans.

Q.10 It is required to measure the maximum acceleration of a machine, which vibrates violently with the frequency of 700 cycles per min. Accelerometer with negligible damping is attached to it and the indicator travels by 8.2 mm. If the accelerometer weighs 0.5 kg and has a spacing rate of 17500 N/m, what is the maximum amplitude and maximum acceleration of the part ?

SPPU - Dec. 19, 6 Marks

Ans. :

Given : Speed of machine, N = 260 r.p.m.

$\therefore$ Excitation frequency, f = 700 cycle/mm

$$= \frac{700}{60} = 11.16\ Hz$$

Excitation frequency $\omega = 11.16\times2\pi$

$= 70.12$ rad/s

Amplitude of vibration, Z = 8.2 mm

Damping ratio, $\xi = 0$

Mass of accelerometer, m = 0.5 kg

Spring constant, K = 17500

- Natural circular frequency of instrument is,

$$\omega_n = \frac{K}{m} = \frac{17500}{0.5} = 187.08\ rad/s$$

- The steady-s tate relative amplitude of vibration,

$$Z = \frac{Y(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\therefore \quad Z = \frac{Y(\omega/\omega_n)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2} \qquad ...[\therefore \xi = 0]$$

$$\therefore \quad 8.2 = \frac{Y\left(\frac{70.12}{187.08}\right)^2}{1-\left(\frac{70.12}{187.08}\right)^2}$$

$\therefore$ **Y = 50.34 mm** ...Ans.

$\therefore$ Displacement of vibrating machine is 50.34 mm

The velocity of vibrating machine is,

$$\dot{y} = \omega\cdot Y = 70.12\times50.34 = 3530.05\ \mathbf{mm/s}$$ ...Ans.

The acceleration of vibrating machine is,

$$\ddot{y} = \omega^2 Y = (70.12)^2 \times 50.34$$

$$\ddot{y} = 247.51 \times 10^3 \text{ mm/s}^2$$ ...Ans.

$$\ddot{y} = 247.51 \text{ m/s}^2$$ ...Ans.

Q.11 A vibrometer with a natural frequency of 2 Hz and with negligible damping is attached to a vibrating system which performs a harmonic excitation. Assuming the difference between the maximum and minimum recorded values are 0.6 mm. Determine the amplitude of motion of the vibrating system when its frequency is 20 Hz and 4 Hz. **SPPU - Dec. 18, Dec. 19, 6 Marks**

Ans. :

Given : $f_n = 2$ Hz ;

$\therefore \omega_n = 2\pi f_n = 2\pi \times 2 = 12.56$ rad/s

Difference between maximum and minimum recorded value is 0.6 mm, therefore taking steady-state relative amplitude is 0.3 mm i.e. z = 0.3 mm.

1) When system relative at frequency 20 Hz

$\therefore f = 20$ Hz

or $\omega = 2\pi \times f = 2\pi \times 20 = 125.66$ rad/s

Steady – state relative amplitude of vibration is

$$Z = \frac{Y\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

or $$Z = \frac{Y\left(\frac{\omega}{\omega_n}\right)^2}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} \qquad [\because \xi = 0]$$

$$\therefore 0.3 = \frac{Y\left(\frac{125.66}{12.56}\right)^2}{\left(\frac{125.66}{12.56}\right)^2 - 1} = \frac{Y(10)^2}{(10)^2 - 1}$$

$\therefore Y = 0.297$ mm ... Ans.

The amplitude of motion of the vibrating system is 0.297 mm.

2) When system vibrates at frequency 4 Hz

$\therefore f = 4$ Hz

or $\omega = 2\pi \times 4 = 25.13$ rad/s

Steady state relative amplitude of vibration when no damping i.e $\xi = 0$ is,

$$Z = \frac{Y\left(\frac{\omega}{\omega_n}\right)^2}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\therefore 0.3 = \frac{Y\left(\frac{25.13}{12.56}\right)^2}{\left(\frac{25.13}{12.56}\right)^2 - 1} = \frac{Y(2)^2}{(2)^2 - 1}$$ ... Ans.

$\therefore Y = 0.225$ mm

The amplitude of motion of the vibrating system is 0.225 mm

Q.12 A vibrometer has a natural frequency of 5 rad/sec and a damping factor of 0.2. An instrument is used to measure a vibrations of a body having a harmonic frequency of 45 rad/sec. The difference between the maximum and minimum reading is 7 mm. Find the amplitude of motion of vibrating body. **SPPU - Dec. 19, 6 Marks**

Ans. : Given : $\omega_n = 5$ rad/s, $\xi = 0.2$

Difference between maximum and minimum recorded value is 7 mm, therefore taking steady-state relative amplitude is 0.35 mm i.e. z = 0.35 mm

When system relative at frequency 45 rad/s

$\therefore \omega = 45$ rad/s

Steady – state relative amplitude of vibration is

$$Z = \frac{Y\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$0.35 = \frac{Y\left(\frac{45}{5}\right)^2}{\sqrt{\left[1-\left(\frac{45}{5}\right)^2\right]^2 + \left[2 \times 0.2 \times \frac{45}{5}\right]^2}}$$

$\therefore Y = 0.3460$ mm ... Ans.

The amplitude of motion of the vibrating system is 0.3460 mm.

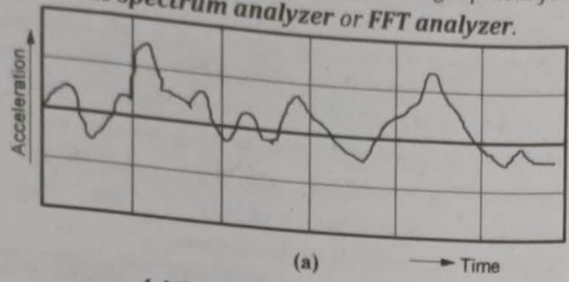
Q.13 Write short note on : FFT spectrum analyzer **SPPU : May 15, Dec. 15, Dec. 17, May 18**

Q. Explain the working of FFT Analyzer. **SPPU : Dec. 18, Dec. 19**

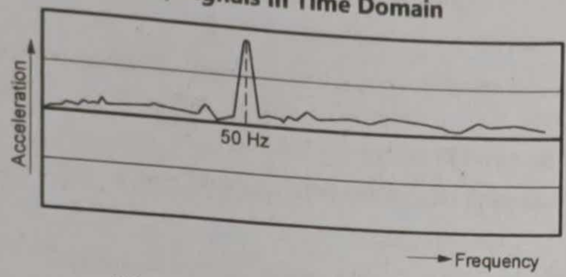
Ans. : FFT Spectrum Analyzer

- **Fourier transform :** ***Fourier transform** is a mathematical procedure to obtain the spectrum of a given input signal.* A signal which is represented by an equation or a graph or a set of data points with time as an independent variable is transformed into another equation or graph or a set of data points where frequency is the independent variable, by using fourier transform.

- **Fast Fourier Transform (FFT) :** *The method to obtain the spectrum using a computer is called as fast fourier transform (FFT).*
- **FFT analyzer :** *The instrument which converts the input signal, with time as an independent variable, into frequency spectrum and displays it in graphical form is called as* ***spectrum analyzer*** *or* ***FFT analyzer****.*



(a) Signals in Time Domain



(b) Signals in Frequency Domain

Fig. 6.8 : Representation of Signals In Different Form

- **Analysis using FFT Analyzer :** Consider the acceleration time plot of a machine structure that is subjected to the excessive vibrations, shown in Fig. 6.8(a). From this plot, it is very difficult to identify the cause of vibrations.
- If the acceleration-time plot is transformed into the acceleration-frequency plot, the resulting frequency spectrum is shown in Fig. 6.8(b). From frequency spectrum it can be seen that, the peak response of the system is at 50 Hz. This frequency can easily be related, for example, to the rotational speed of the motor. Thus, the frequency spectrum shows a strong evidence that, the motor might be the cause of vibrations.
- Hence, by changing the motor or by changing its speed of operation, the resonance condition can be avoided.
- **Working of FFT Analyzer :** Fig. 6.9 shows the block diagram of FFT analyzer. The various elements of FFT analyzer are as follows :

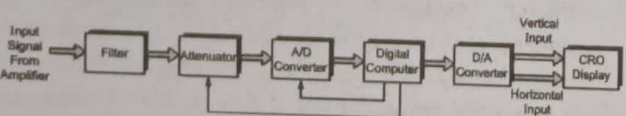


Fig. 6.9 : Block Diagram of FFT Analyzer

(i) Filter : The input signals are supplied through a filter. The filter is used to reject unwanted signals.

(ii) Attenuator : The next part is attenuator which sets the level of the signals to be fed to the analog to digital (A/D) converter. This is necessary to prevent the over loading of the converter.

(iii) A/D Converter : The digital computer requires the data in the digital form. Therefore, analog signals from attenuator are converted into digital signals in A/D converter.

(iv) Digital Computer : The converted data is stored in the memory of the computer. The input signals are sampled for a specific period of time called window. The sample rate, the window time and starting time are determined from the front panel controls of FFT analyzer. Once the sampling period is over and all the samples are digitised, the computer starts the calculations. The computer has a programme for the calculation and using this programme, all the spectral components are calculated and all the values are stored in the computer memory.

(v) D/A Converter : All digitised values stored in the computer memory are given to the digital to analog (D/A) converter, which converts the digital signals from computer into analog signals and send to the cathode ray oscilloscope (CRO) to display the spectrum.

(vi) CRO display : CRO displays the spectrum.

Q.14 Write short note on Condition monitoring of machines

SPPU : Dec. 17, Dec. 18, Dec. 19

Ans. : Condition Monitoring :

- ***Condition monitoring*** *of machine implies the determination of condition of a machine and its change with time.*
- The condition of the machines may be determined by measuring the physical parameters like : vibration, noise, temperature, wear debris, oil contamination etc. The changes in these parameters are called as signatures. The signatures indicate the change in condition or health of a machine. The analysis of signatures helps in predicting and preventing the failure of the machine.

☞ **Vibration Monitoring :**

- The vibration monitoring is most commonly used for machine condition monitoring. The vibration signature of a machine are seen to be very much related to the health of a machine.
- Thus the measurement of vibration levels of machine component can provide useful information regarding the faults like : unbalance, misalignment, lack of oil, wear, etc.

- Fig. 6.10 shows the frequency spectrum of vibration in ball bearings for original new and old ball bearings. The increased level of vibrations and additional peaks indicate the bearing is defected.

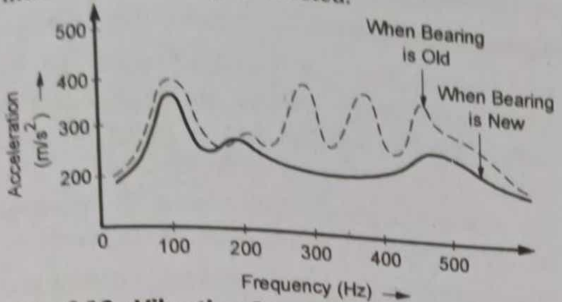


Fig. 6.10 : Vibration Spectrum For Ball Bearings

Q.15 What is meant by time domain and frequency domain analysis ? Explain how frequency spectrum can be used to detect vibration related faults in a system.

SPPU : May 15

Q. Explain different techniques for Vibration Monitoring.

SPPU : Dec. 18, Dec. 19

Ans. : Vibration Monitoring Techniques:

- The vibration monitoring techniques are classified as shown in Fig. 6.11.

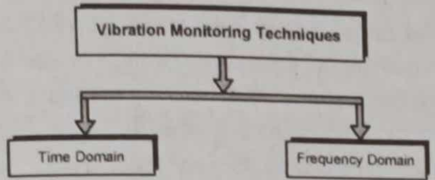
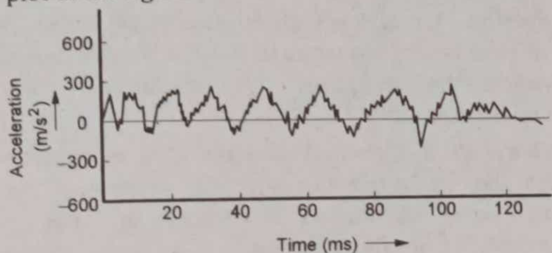


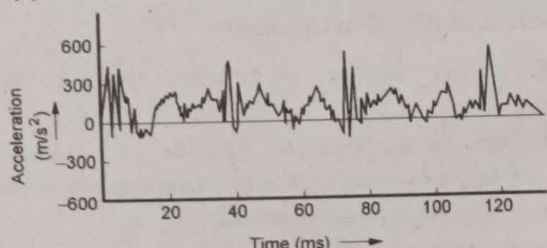
Fig. 6.11 : Vibration Monitoring Techniques

1. Time Domain Analysis :

- The time domain analysis uses the acceleration-time plot of the signal.



(a) Time Domain Spectrum of a Good Gear Box



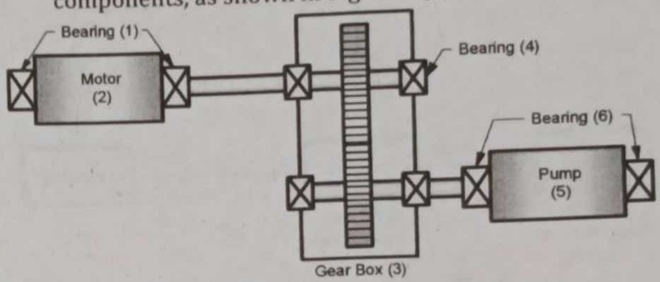
(b) Time Domain Spectrum of a Bad Gear Box

Fig. 6.12 : Time Domain Analysis

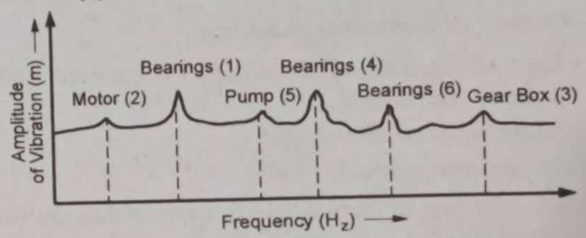
- The damages such as broken teeth in gears can be identified easily from the acceleration-time plot of the casing of a gear box.
- For example, Fig. 6.12 shows the acceleration-time signal of a single stage gear box.

2. Frequency Domain Analysis :

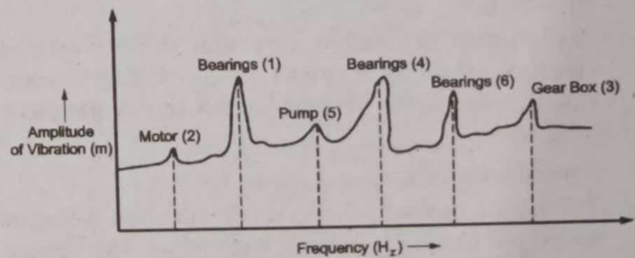
- The frequency domain analysis or frequency spectrum is the plot of amplitude of vibrations versus frequency, which is converted from time domain signals by using FFT spectrum analyzer.
- When machine starts developing faults, shape of frequency spectrum changes accordingly. By comparing frequency spectrum of machine when it is in good condition, with the frequency spectrum when it is in damaged condition, the nature and location of fault can easily be detected.
- **Advantage of frequency spectrum :** Each element in machine has identifiable frequency, therefore the change in the spectrum at given frequency can be attributed directly to that corresponding machine component, as shown in Fig. 6.13.
- The peaks in frequency spectrum are related to various components, as shown in Fig. 6.13(b).



(a) Machine Elements or Components



(b) Frequency Spectrum of Machine When it is in Good Condition



(c) Frequency Spectrum of Machine When it is in Bad Condition

Fig. 6.13

- The analytical equations are available to find the fault frequencies of standard components like : bearings, gear boxes, motors, pumps, fans, etc.

Q.16 What are the various methods of vibration control ? Explain any one. **SPPU : May 16, Dec. 19**

Ans. : Methods of Vibration Control

- The various methods of vibration control aim at modifying the source or the system or the transmission path from source to the system. The various methods of vibration control are broadly classified, as shown in Fig. 6.14.

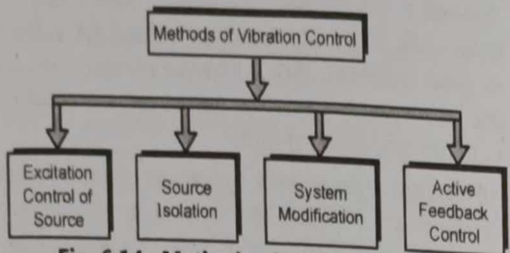


Fig. 6.14 : Methods of Vibration Control

- The various methods of vibration control at various phases are shown in Fig. 6.15.

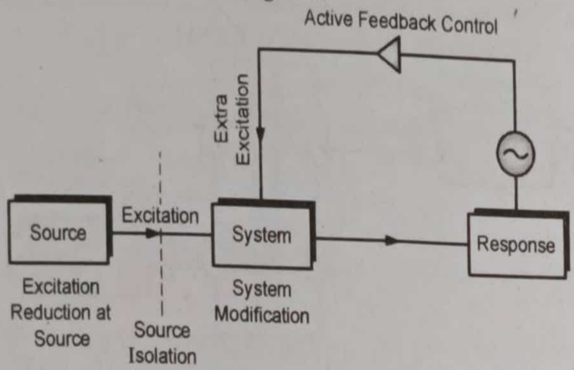


Fig. 6.15 : Methods of Vibration Control at Various Phases

1. **Control of Excitation at Source :**

   The various methods to reduce the excitation level at the source are :

(i) Balancing of inertia forces.

(ii) Proper lubrication of joints.

(iii) Smoothening of fluid flows in case of flow-induced vibrations.

(iv) Modification in surface finish in case of self excited vibrations.

(v) The chatter of cutting tool due to self excited vibration may be reduced by providing coolant which reduces the friction at the tool-work piece interface.

2. **Source Isolation :**

- The second method to control the vibration is known as source isolation. In such method, an appropriate suspension system or an anti-vibration mountings are inserted in the path of vibration transmission from the source to the system.
- Sometimes, a machine which creates significant vibration during its normal operation may be supported upon isolators to protect other machinery and workers from shock and vibrations.

3. **System Modification :**

- A large number of methods exist in system modification. In this method, the system parameters namely inertia, stiffness and damping are suitably chosen or modified to reduce the vibration.
- The system parameters are effectively controlled by both geometry and material of a vibrating member.
- When designing a system, care should be taken that, its natural frequencies lie outside the excitation frequency range. This can be done by suitably adjusting the mass and stiffness parameters. This process of avoiding the resonance is called **detuning**.

4. **Active Feedback Control :**

- In this method, a signal generated by the response is suitably processed to produce a desired amplification of the phase change in the signal. This processed signal drives an actuator which in turn provides an additional excitation.
- This excitation is then fed back to the vibratory system which shifts the excitation frequency away from its natural frequencies, and hence, resonance is avoided. Sometime, both vibration absorber and isolators are also used to minimise the excitation level.

Q.17 Explain the term : Vibration Isolation .

SPPU : Dec. 12, May 15, Dec. 15, May 16, May 18

Ans. : Vibration Isolation

- **Vibration isolation :** Vibrations are produced in machines having unbalanced masses or forces. These vibrations are transmitted to the foundation upon which the machines are mounted, which is undesirable.
- Therefore, it is essential to isolate the machines from foundations so that the adjoining structure is not set into heavy vibrations. This process of isolating the machines from the foundations is known as **vibration isolation.**

Objectives of vibration isolation :

The basic objectives of vibration isolation are as follows :

(i) To protect the delicate machine (e.g. measuring instruments) from excessive vibrations transmitted to it from its supporting structure.

(ii) To prevent vibratory forces generated by machine from being transmitted to its supporting structure.

- **Force isolation and motion isolation :** The effectiveness of isolation may be measured in terms of the force or motion transmitted to that in existence. Accordingly it is known as **force isolation or motion isolation**. The lesser the force or motion transmitted the greater is the isolation.

Q.18 What are the methods of vibration isolation ?

Ans. : Methods of Vibration Isolation

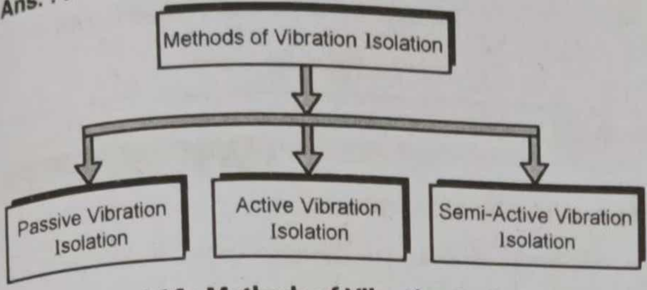


Fig. 6.16 : Methods of Vibration Isolation

Q.19 Explain in detail Active vibration control.

SPPU : May 18

Ans. : Active Vibration Isolation

- **Active vibration isolations :** Active vibration isolation system is requires an external power source to perform its function.
- A typical active vibration isolation system, shown in Fig. 6.17, uses sensors, controller and actuators to achieve the vibration isolation. The sensor is used to detect the vibrations to be controlled, the controller is used to interpret the vibrations detected by the sensor and to execute commands on the actuators, where as the actuators are used to reposition the mass.
- **Advantages of active vibration isolation :**

(i) It can adapt to different vibrating bodies. If can be programmed to a targeted specifications and vibrating body.

(ii) It is more effective to suppress low-frequency vibrations (< 70 Hz).

(iii) Typical actuators, sensors, and controller used in active vibration isolation can provide quick isolation response within milliseconds.

(iv) The repositioning accuracy can be controlled within 10 μm.

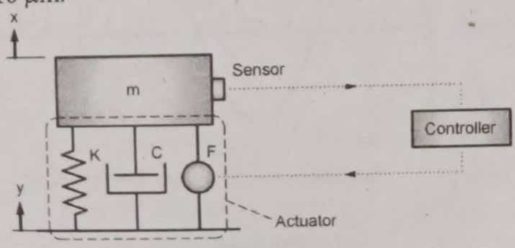


Fig. 6.17 : Schematic Diagram of Typical Active Vibration Isolation System

Q.20 Explain working of Magneto - Rheological dampers with neat sketch and application.

SPPU : Dec. 18, May 19, Dec. 19

Ans. : Magneto-Rheological Fluids (MR Fluids) :

- ***MR fluids** are smart fluids which change their viscosity by application of magnetic field.*
- MR fluid is a type of fluid, when subjected to magnetic field, the fluid greatly increases its viscosity. The viscosity depends upon the strength of magnetic field.
- The fluid basically consists of particles that are held in suspension by a non-conducting fluid. The suspension or carrier fluid is hydrocarbon or silicone oil. The particles dispersed in this fluid are commonly metal oxides, alumino silicates, silica, organics or polymers.
- In particular, the particles are very small and have sufficiently low concentration to allow the fluid to maintain a relatively low viscosity in the absence of an applied electric field.

Use of ER and MR Fluids in Vibration Isolation :

- A typical use of ER fluid for vibration isolation is shown in Fig. 6.18.
- The vibration isolator consists of a parallel combination of a helical spring and a piston-cylinder with ER fluid or MR fluid. It has been observed that with suitable combination of particle concentration, carrier fluid and the strength of the electric/magnetic field using high voltage amplifier, the damping characteristics of the dashpot can be varied over a wide range. At high field strength even Coulomb damping characteristic also achieved.

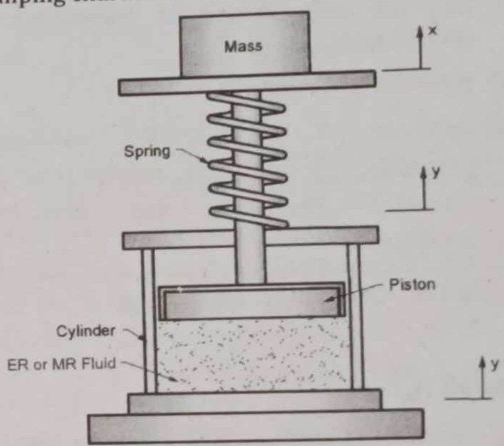


Fig. 6.18 : Isolator with ER fluid

- ERF can be used for both passive and active vibration isolation. In case of passive isolation, a fixed field could be applied to achieve Coulomb damping characteristics.

- However, for optimal use over a broad frequency range, one can activity control the electric field to the ERF mount for vibration control.
- Such types of dampers are used in semi-active vehicle suspensions which are monitored through sensor and adjust according to road conditions. Also such dampers are used in heavy industry to damp out the vibrations of heavy motors.

Q.21 Explain ISO standards used in vibration.

SPPU : Dec. 18

Ans. :

ISO Standards for Vibration Monitoring and Analysis :

- In the field of machinery vibration monitoring and analysis, a variety of relevant standards are developed and published by ISO (International Organization for Standardization). ISO is a worldwide federation of national standards bodies from 145 countries, and considers itself a bridge between the public and private sectors.
- The scope of standardization in the field of mechanical vibration and shock, monitoring and analysis of machines include
  - (i) terminology;
  - (ii) excitation by sources, such as machines and vibration / shock testing devices;
  - (iii) elimination, reduction and control of vibration and shock, especially by balancing, isolation and damping;
  - (iv) measurement and evaluation of human exposure to vibration and shock;
  - (v) methods and means of measurement and calibration;
  - (vi) methods of testing;
  - (vii) methods of measurement, handling and processing of the data required to perform condition monitoring and diagnostics of machines.

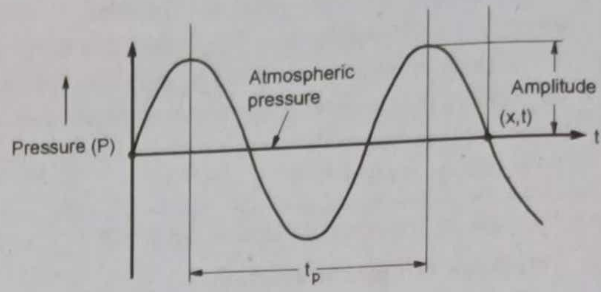
Q.22 Define Time period of sound sources.

Ans. :

Time period (t_p) :

Time period *of the sinusoidal wave is the time interval required for one complete cycle, as depicted in Fig. 6.19.* The time period is given by,

$$t_p = \frac{1}{f}\text{, second}$$



Pressure Variation with Respect to Time

Fig. 6.19

Q.23 Explain the term : Velocity of sound.

SPPU : Dec. 16, Dec. 17

Ans. : Velocity of sound (c) :

- **Velocity of sound** *is identical to the velocity of wave propagation (c).*

Velocity or speed of sound in air :

$$c = \sqrt{\frac{\gamma p_a}{\rho}}\text{, m/sec.}$$

where, $\gamma = \dfrac{\text{specific heat at constant pressure}}{\text{specific heat at constant volume}}$

$= 1.4$ for air

p_a = ambient or equilibrium pressure, N/m^2

ρ = ambient or equilibrium density of the medium, kg/m^3

Q.24 Explain the term : Wavelength.

SPPU : Dec. 16, Dec. 17

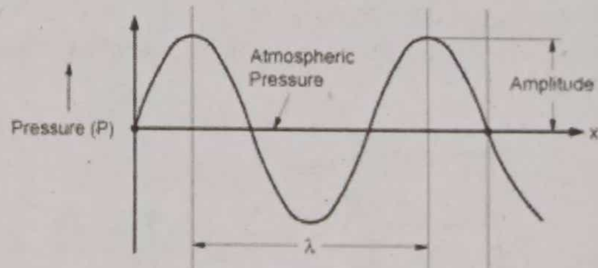
Ans. : Wavelength (λ) :

Wavelength *is the distance between two successive peaks in the waveform [Fig. 6.20].*

$$\lambda = \frac{c}{f} \text{ or } ct_p\text{, meter}$$

where, c = velocity of sound, m/s

f = frequency of sound wave, Hz



Instantaneous Pressure Distribution

Fig. 6.20

Q.25 Explain the term : Decibel scale.

SPPU : Dec. 13, Dec. 14, Dec. 15, Dec. 16, May 17, Dec. 17

Ans. : Need of Decibel Scale :

The range of audible sound pressure to which human ear is likely to be subjected is between 2×10^{-5} N/m^2 to 200 N/m^2. Because of the wide range of sound pressure, it is convenient to describe the sound level through the use of logarithmic scales known as **decibel scale**. The use of logarithmic scale helps in covering the entire sound level range by a small scale of numbers rather than the extremely large scale of numbers.

Decibel :

Decibel is the ten times the logarithm to the base 10 of the ratio of the quantity of sound measured to an arbitrarily chosen reference quantity.

$$\text{Decibel level} = 10 \log_{10}\left(\frac{\text{quantity measured}}{\text{reference quantity}}\right), \text{dB}$$

Parameters Used for Measurement of Sound :

The various parameters used for measurement of sound by using decibel scale are as follows :

Parameters used for measurement of sound

- 1. Sound Pressure Level (L_p)
- 2. Sound Power Level (L_W)
- 3. Sound (Acoustic) Intensity (L_i)

Q.26 Explain the term : Sound pressure level.

SPPU : Dec. 15, Dec. 16, May 17, Dec. 17, May 19

Ans. : Sound Pressure Level (L_p) :

- **Sound pressure level (SPL)** is the most common decibel scale used for measurement of sound and is measured directly on sound level meter.
- The sound pressure level is expressed in decibels as,

$$L_p = 10 \log_{10}\left(\frac{p_{rms}^2}{p_{ref}^2}\right), \text{dB} \quad (1)$$

$$\text{or} \quad L_p = 10 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right)^2, \text{dB} \quad (2)$$

$$\text{or} \quad L_p = 20 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right), \text{dB} \quad (3)$$

where, p_{rms} = root mean square (rms) sound pressure in given source, N/m^2

p_{ref} = reference sound pressure(usually 2×10^{-5} N/m^2)

Q.27 Explain the term : Sound power level.

SPPU : Dec. 13, Dec. 15, Dec. 16, Dec. 17, May 19

Ans. : Sound Power Level (L_W) :

- **Sound power level** *describes the acoustical power radiated by a given source with respect to the international reference sound power of 10^{-12} W.*
- The sound power level is given by,

$$L_W = 10 \log_{10}\left(\frac{W}{W_{ref}}\right), \text{dB} \quad (1)$$

where, W = Sound power of the given source ;

W_{ref} = 10^{-12} W (reference sound power)

Q.28 Explain the term : Sound intensity .

SPPU : Dec. 15, May 17

Ans. : Sound (Acoustic) Intensity (I) :

- **Sound intensity (I) :** Sound intensity at a given point in a sound field, in a given direction is defined as the average sound power, W passing through a unit area perpendicular to the given direction at that point.

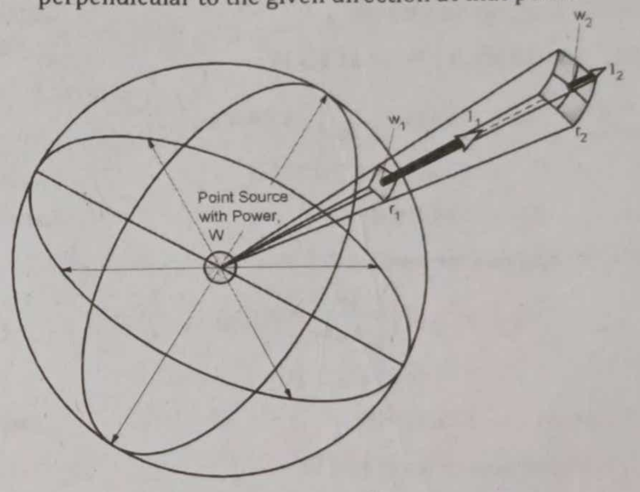


Fig. 6.21 : Basic Parameters of Sound

- The sound intensity is given by,

$$\therefore \quad I = \frac{W}{S}, \text{W/m}^2$$

where, W = sound power, W

S = area perpendicular to sound wave, m^2

$$\text{Again, } I = \frac{p_{rms}^2}{\rho c}, \text{W/m}^2$$

where, p = root mean square (rms) sound pressure, N/m^2 ;

ρ = density of the medium, kg/m^3, and

c = velocity of sound in medium, m/s

Q.29 Explain the term : Sound Intensity level.

SPPU : Dec. 12, Dec. 15

Ans. : Sound Intensity level (L_I) :

$$L_I = 10 \log_{10}\left(\frac{I}{I_{ref}}\right), \text{ dB}$$

where, I = Sound intensity, W/m²

I_{ref} = reference sound intensity (usually 10^{-12} W/m²)

Q.30 Determine the sound power level of a source that generate sound power of :

(i) 0.5 W ; (ii) 1.5 W ;

(iii) 2.2 W; and (iv) 3 W of sound power.

SPPU - Dec. 16, Dec. 17, 8 Marks

Ans. :

$$W_{ref} = 10^{-12} \text{ W}$$

(i) Sound power level at 0.5 W :

$$L_W = 10 \log_{10}\left(\frac{W}{W_{ref}}\right) = 10 \log_{10}\left(\frac{0.5}{10^{-12}}\right)$$

$$= 10 \log_{10}(5 \times 10^{11})$$

or $\mathbf{L_W = 116.98 \text{ dB}}$ ...Ans.

(ii) Sound power level at 1.5 W :

$$L_W = 10 \log_{10}\left(\frac{W}{W_{ref}}\right) = 10 \log_{10}\left(\frac{1.5}{10^{-12}}\right)$$

$$= 10 \log_{10}(1.5 \times 10^{12})$$

or $\mathbf{L_W = 121.76 \text{ dB}}$ ...Ans.

(iii) Sound power level at 2.2 W :

$$L_W = 10 \log_{10}\left(\frac{W}{W_{ref}}\right) = 10 \log_{10}\left(\frac{2.2}{10^{-12}}\right)$$

$$= 10 \log_{10}(2.2 \times 10^{12})$$

or $\mathbf{L_W = 123.42 \text{ dB}}$ ...Ans.

(iv) Sound power level at 3 W :

$$L_W = 10 \log_{10}\left(\frac{W}{W_{ref}}\right) = 10 \log_{10}\left(\frac{3}{10^{-12}}\right)$$

$$= 10 \log_{10}(3 \times 10^{12})$$

or $\mathbf{L_W = 124.77 \text{ dB}}$ ...Ans.

Q.31 Determine the sound power of a machine whose specified sound power level is 125 dB.

Ans. :

Given : $L_W = 125$ dB ;

$W_{ref} = 10^{-12}$ W (reference).

The sound power level is,

$$L_W = 10 \log_{10}\left(\frac{W}{W_{ref}}\right)$$

$$125 = 10 \log_{10}\left(\frac{W}{10^{-12}}\right)$$

$$\frac{125}{10} = \log_{10}\left(\frac{W}{10^{-12}}\right)$$

$$3.16 \times 10^{-12} = \frac{W}{10^{-12}}$$

$\therefore$ $\mathbf{W = 3.16 \text{ W}}$ ...Ans.

Q.32 Determine the sound pressure level for a following rms sound pressure :

(i) 0.33 N/m² ; (ii) 2.97 N/m² ;

(iii) 1.5 N/m² ; and (iv) 2 N/m².

Ans. : $p_{ref} = 2 \times 10^{-5}$ N/m² (reference)

(i) Sound pressure level at 0.33 N/m² :

$$L_p = 20 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) = 20 \log_{10}\left(\frac{0.33}{2 \times 10^{-5}}\right)$$

$$= 20 \log_{10}(16500)$$

or $\mathbf{L_p = 84.34 \text{ dB}}$ ...Ans.

ii) Sound pressure level at 2.97 N/m² :

$$L_p = 20 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) = 20 \log_{10}\left(\frac{2.97}{2 \times 10^{-5}}\right)$$

$$= 20 \log_{10}(148500)$$

or $\mathbf{L_p = 103.43 \text{ dB}}$ ...Ans.

iii) Sound pressure level at 1.5 N/m² :

$$L_p = 20 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) = 20 \log_{10}\left(\frac{1.5}{2 \times 10^{-5}}\right)$$

$$= 20 \log_{10}(75000)$$

or $\mathbf{L_p = 97.50 \text{ dB}}$ ...Ans.

iv) Sound pressure level at 2 N/m² :

$$L_p = 20 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) = 20 \log_{10}\left(\frac{2}{2 \times 10^{-5}}\right)$$

$$= 20 \log_{10}(100 \times 10^3)$$

or $\mathbf{L_p = 100 \text{ dB}}$ ...Ans.

Q.33 Determine the sound pressure level for a sound with rms sound pressure of 2 N/m² and 0.4 N/m².

SPPU - May 17, 6 Marks

Ans. : Take $p_{ref} = 2 \times 10^{-5}$ N/m² (reference).

Given :

(i) The sound pressure level at 2 N/m² is, i.e. $p_{rms} = 2$ N/m²

The sound pressure level is,

$$L_p = 20 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) = 20 \log_{10}\left(\frac{2}{2 \times 10^{-5}}\right)$$

$$= 20 \log_{10}(100 \times 10^3)$$

$$L_p = 100 \text{ dB}$$

...Ans.

(ii) The sound pressure level at 0.4 N/m^2 is, i.e. p_{rms} = 0.4 N/m^2

The sound pressure level is,

$$L_p = 20 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) = 20 \log_{10}\left(\frac{0.4}{2 \times 10^{-5}}\right)$$

$$= 20 \log_{10}(20 \times 10^3)$$

$$L_p = 86.02 \text{ dB}$$

...Ans.

Q.34 Determine different levels

i) Sound pressure level, if rms sound pressure is 1 Pa,

ii) Sound intensity level, if sound intensity is 1 W / m^2 and

iii) Sound Power level of a source generating 1 W of Sound Power.

SPPU - May 18, 6 Marks

Ans. :

(i) Sound pressure level, if rms sound pressure is 1 Pa

Given : $p_{rms} = 1 \text{ N/m}^2$;

$p_{ref} = 2 \times 10^{-5} \text{ N/m}^2$ (reference).

The sound pressure level is,

$$L_p = 20 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) = 20 \log_{10}\left(\frac{1}{2 \times 10^{-5}}\right)$$

$$= 20 \log_{10}(0.5 \times 10^5)$$

or $$L_p = 93.97 \text{ dB}$$

...Ans.

(ii) Sound intensity level, if sound intensity is 1 W / m^2

Given : $I = 1 \text{ W/m}^2$;

$I_{ref} = 10^{-12} \text{ W/m}^2$ (reference).

The sound power level is,

$$L_i = 10 \log_{10}\left(\frac{I}{I_{ref}}\right)$$

$$L_i = 10 \log_{10}\left(\frac{1}{10^{-12}}\right)$$

$$L_i = 10 \log_{10} 10^{12}$$

$$L_i = 10 \times 12$$

$$L_i = 120 \text{ dB}$$

...Ans.

(iii) Sound Power level of a source generating 1 W of Sound Power

Given : $W = 1$ W; $W_{ref} = 10^{-12}$ W (reference).

The sound power level is,

$$L_W = 10 \log_{10}\left(\frac{W}{W_{ref}}\right) = 10 \log_{10}\left(\frac{1}{10^{-12}}\right)$$

$$= 10 \log_{10}(10^{12}) = 10 \times 12$$

or $$L_W = 120 \text{ dB}$$

...Ans.

Q.35 Determine the sound pressure level of a source that generate a following rms sound pressure.

(i) 1.7 N/m^2 (ii) 0.7 Pa **SPPU - Dec. 18, 4 Marks**

Ans. :

(i) Sound pressure level at 1.7 N/m^2 :

$$L_p = 20 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) = 20 \log_{10}\left(\frac{1.7}{2 \times 10^{-5}}\right)$$

$$= 20 \log_{10}(85000)$$

or $$L_p = 98.58 \text{ dB}$$

...Ans.

ii) Sound pressure level at 0.7 Pa or 0.7 N/m^2:

$$L_p = 20 \log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) = 20 \log_{10}\left(\frac{0.7}{2 \times 10^{-5}}\right)$$

$$= 20 \log_{10}(35000)$$

or $$L_p = 90.88 \text{ dB}$$

Q.36 Determine the sound power level of a source that generate a sound power of (i) 1.0 W (ii) 3.0 W

SPPU - May 19, 4 Marks

Ans. :

(i) 1.0 W :

Sound Power level of a source generating 1.0 W of Sound Power

Given : $W = 1$ W; $W_{ref} = 10^{-12}$ W (reference).

The sound power level is,

$$L_W = 10 \log_{10}\left(\frac{W}{W_{ref}}\right) = 10 \log_{10}\left(\frac{1}{10^{-12}}\right)$$

$$= 10 \log_{10}(10^{12}) = 10 \times 12$$

or $$L_W = 120 \text{ dB}$$

(ii) 3.0W :

- **Sound power level at 3.0 W**

$$L_W = 10 \log_{10}\left(\frac{W}{W_{ref}}\right) = 10 \log_{10}\left(\frac{3}{10^{-12}}\right)$$

$$= 10 \log_{10}(3 \times 10^{12})$$

or $$L_W = 124.77 \text{ dB}$$

Q.37 Derive a relation between sound intensity level and sound pressure level.

SPPU : Dec. 12, May 13, May 16

Ans. : Sound intensity level of sound wave :

$$L_i = 10 \log_{10}\left(\frac{I}{I_{ref}}\right), \text{dB} \qquad ...(a)$$

where, I = sound intensity, $W/m^2 = \frac{p_{rms}^2}{\rho c}$ ...(b)

I_{ref} = reference sound intensity, W/m^2

$$= \frac{p_{ref}^2}{\rho c} \quad ...(c)$$

Substituting Equation (b) in Equation (a), we get,

$$L_i = 10 \log_{10}\left(\frac{p_{rms}^2}{\rho c\, I_{ref}}\right)$$

$$= 10 \log_{10}\left[\left(\frac{p_{rms}^2}{p_{ref}^2}\right) \cdot \left(\frac{p_{ref}^2}{\rho c\, I_{ref}}\right)\right]$$

$$\text{or} \quad L_i = 10 \log_{10}\left(\frac{p_{rms}^2}{p_{ref}^2}\right) + 10 \log_{10}\left(\frac{p_{ref}^2}{\rho c\, I_{ref}}\right) \quad ...(d)$$

- **Relationship between sound pressure level and sound intensity level :**

Substituting Equation ($L_p = 10 \log_{10}\left(\frac{p_{rms}^2}{p_{ref}^2}\right)$, dB) in Equation (d), we get,

$$L_i = L_p + 10 \log_{10}\left(\frac{p_{ref}^2}{\rho c\, I_{ref}}\right)$$

Taking, $p_{ref} = 2 \times 10^{-5}$ N/m², $I_{ref} = 10^{-12}$ W/m²,

ρ = 1.21 kg/m²; and c = 344 kg/m², we get,

$$L_i = L_p + 10 \log_{10}\left[\frac{(2 \times 10^{-5})^2}{1.21 \times 344 \times 10^{-12}}\right]$$

$$\text{or} \quad L_i = L_p - 0.167 \quad ...(1)$$

For all practical purpose, the sound intensity level is taken to be numerically equal to the sound pressure level. Therefore,

$$\therefore \quad L_i \approx L_p$$

$$\textbf{or} \quad \mathbf{L_p = L_i} \quad ...(2$$

Q.38 Show that when the distance from point of source is doubled the sound intensity level decreases by 6 dB.

SPPU - Dec. 12, May 14, Dec. 15, May 18, 6 Marks

Ans. :

Let, L_{i1} = Sound intensity level at radius r

L_{i2} = Sound intensity level at radius 2r

- **Surface area of spherical wave at distances r :**

$$S_1 = 4\pi r^2$$

- **Surface area of spherical wave at distance of 2r :**

$$S_2 = 4\pi (2r^2) = 16\pi r^2$$

- **Sound power level at distance r and 2r from same source :**

$$L_W = L_{i1} + 10 \log_{10}(S_1) = L_{i1} + 10 \log_{10}(4\pi r^2) \quad ...(a)$$

and, $$L_W = L_{i2} + 10 \log_{10}(S_2) = L_{i2} + 10 \log_{10}(16\pi r^2) \quad ...(b)$$

- **Difference of sound intensity levels :**

From Equations (a) and (b),

$$L_{i1} + 10 \log_{10}(4\pi r^2) = L_{i2} + 10 \log_{10}(16\pi r^2)$$

$$\therefore \quad L_{i2} - L_{i1} = 10 \log_{10}(4\pi r^2) - 10 \log_{10}(16\pi r^2)$$

$$= 10 \log_{10}\left(\frac{4\pi r^2}{16\pi r^2}\right)$$

$$L_{i2} - L_{i1} = -6.02 \text{ dB}$$

$$\text{or} \quad \mathbf{L_{i2} - L_{i1} = -6\ dB} \quad ...\text{Ans.}$$

Q.39 Show that if sound pressure is doubled, the sound pressure level increases by six decibels.

SPPU - Dec. 16, Dec. 17, Dec.18, 6 Marks

Ans. :

Given :

Let, P_{rms1} = Sound pressure at condition 1, N/m²

P_{rms2} = Sound pressure at condition 2, N/m²

L_{P1} = Sound pressure level at condition 1

L_{P2} = Sound pressure level at condition 2

$$P_{rms2} = 2 P_{rms1} \quad ...(a)$$

- **Sound pressure levels :**

$$L_{P1} = 20 \log_{10}\left(\frac{P_{rms1}}{P_{ref}}\right) \quad ...(b)$$

$$L_{P2} = 20 \log_{10}\left(\frac{P_{rms2}}{P_{ref}}\right) \quad ...(c)$$

- **Difference of sound pressure levels :**

$$L_{P2} - L_{P1} = 20 \log_{10}\left(\frac{P_{rms2}}{P_{ref}}\right) - 20 \log_{10}\left(\frac{P_{rms1}}{P_{ref}}\right)$$

$$= 20 \log_{10}\left(\frac{P_{rms2}}{P_{ref}} \times \frac{P_{ref}}{P_{rms1}}\right)$$

$$= 20 \log_{10}\left(\frac{P_{rms2}}{P_{rms1}}\right) = 20 \log_{10}\left(\frac{2 P_{rms1}}{P_{rms1}}\right)$$

$$\textbf{or} \quad \mathbf{LP_2 - LP_1 = 6} \quad ...\text{Ans.}$$

Q.40 Show that if the sound power is doubled, then the sound power level increases by approximately 3 dB.

SPPU - Dec,19, 4 Marks

Ans. : Given :

Let, W_1 = Sound power at condition 1, N/m²

W_2 = Sound power at condition 2, N/m²

L_{W_1} = Sound power level at condition 1

L_{W_2} = Sound power level at condition 2

$$W_2 = 2 W_1 \quad ...(a)$$

- **Sound pressure levels :**

$$L_{W_1} = 10 \log_{10}\left(\frac{W_1}{W_{ref}}\right) \quad ...(b)$$

$$L_{W_2} = 10 \log_{10}\left(\frac{W_2}{W_{ref}}\right) \quad ...(c)$$

- **Difference of sound pressure levels :**

$$L_{W_2} - L_{W_1} = 10 \log_{10}\left(\frac{W_2}{W_{ref}}\right) - 10 \log_{10}\left(\frac{W_1}{W_{ref}}\right)$$

$$= 10 \log_{10}\left(\frac{W_2}{W_{ref}} \times \frac{W_{ref}}{W_1}\right)$$

$$= 10 \log_{10}\left(\frac{W_2}{W_1}\right) = 10 \log_{10}\left(\frac{2 W_1}{W_1}\right)$$

or $L_{W_1} - L_{W_2} = 3\text{dB}$ ...Ans.

Q.41 Calculate the total noise, if there are 4 sourcing of noise having magnitudes 45 dB, 54 dB, 53 dB, and 52 dB. What would be effect on total noise, if 45 dB noise is switched off ? **SPPU - Dec. 19, 4 Marks**

Ans. :

Given : $L_{p1} = 45$ dB ; $L_{p2} = 54$ dB ; $L_{p3} = 53$ dB ; $L_{p4} = 52$ dB ; $n = 4$.

(i) When all five machines are turned ON :

$$L_p = 10 \log_{10}\left[\sum_{n=1}^{n} 10^{(L_{pn}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(L_{p1}/10)} + 10^{(L_{p2}/10)} + 10^{(L_{p3}/10)} + 10^{(L_{p4}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(45/10)} + 10^{(54/10)} + 10^{(53/10)} + 10^{(52/10)}\right]$$

$$= 10 \log_{10}\left[10^{4.5} + 10^{5.4} + 10^{5.3} + 10^{8.8} + 10^{5.2}\right]$$

or $L_p = 58.06$ dB ...Ans.

(ii) When machine 1 is turned OFF (i.e. $L_{p1} = 0$) :

$$L_p = 10 \log_{10}\left[\sum_{n=1}^{n} 10^{L_{pn}/10}\right]$$

$$= 10 \log_{10}\left[10^{(L_{p2}/10)} + 10^{(L_{p3}/10)} + 10^{(L_{p4}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(54/10)} + 10^{(53/10)} + 10^{(52/10)}\right]$$

$$= 10 \log_{10}\left[10^{5.4} + 10^{5.3} + 10^{5.2}\right]$$

or $L_p = 57.84$ dB ...Ans.

Q.42 A customer care centre containing six offices, individually makes noise level of 60, 56, 62, 53, 51 and 54 dB respectively. Add the noise levels when :

(i) All officers are working ; and

(ii) When first and second officers are not working.

SPPU - May 13, May 19, 6 Marks

Ans. :

Given : $L_{p1} = 60$ dB ; $L_{p2} = 56$ dB ; $L_{p3} = 62$ dB ; $L_{p4} = 53$ dB ; $L_{p5} = 51$ dB ; $L_{p6} = 54$ dB ; $n = 6$.

(i) When all six offices are working :

$$L_p = 10 \log_{10}\left[\sum_{n=1}^{n} 10^{(L_{pn}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(L_{p1}/10)} + 10^{(L_{p2}/10)} + 10^{(L_{p3}/10)} + 10^{(L_{p4}/10)} + 10^{(L_{p5}/10)} + 10^{(L_{p6}/10)}\right]$$

$[\therefore n = 5]$

$$= 10 \log_{10}\left[10^{(60/10)} + 10^{(56/10)} + 10^{(62/10)} + 10^{(51/10)} + 10^{(83/10)} + 10^{(54/10)}\right]$$

$$= 10 \log_{10}\left[10^{6} + 10^{5.6} + 10^{6.2} + 10^{5.3} + 10^{5.1} + 10^{5.4}\right]$$

or $L_p = 62.95$ dB ...Ans.

(ii) When offices 1 and 2 are not working

(i.e. $L_{p1} = L_{p2} = 0$) :

$$L_p = 10 \log_{10}\left[\sum_{n=1}^{n} 10^{(L_{pn}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(L_{p3}/10)} + 10^{(L_{p4}/10)} + 10^{(L_{p5}/10)} + 10^{(L_{p6}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(62/10)} + 10^{(53/10)} + 10^{(51/10)} + 10^{(54/10)}\right]$$

$$= 10 \log_{10}\left[10^{6.2} + 10^{5.3} + 10^{5.1} + 10^{5.4}\right]$$

or $L_p = 63.34$ dB ...Ans.

Q.43 Noise at the construction site is contributed by a few construction activities such as piling work : 104 dB, Scraper : 93 dB, Bulldozer : 94 dB, Mobile compressor : 73 dB and Mechanical Shovel : 76 dB on a weighing network. What is the overall sound pressure level? **SPPU - Dec. 18, 4 Marks**

Ans. : Given :

$L_{P1} = 104$ dB, $L_{P2} = 93$ dB $L_{P3} = 94$ dB,

$L_{P4} = 73$ dB, $L_{P5} = 76$ dB ; $n = 5$.

$$L_P = 10 \log_{10}\left[\sum_{n=1}^{n} 10\,(L_{P_{n/10}})\right]$$

$$= 10 \log_{10}\left[10^{(L_{P1}/10)} + 10^{(L_{P2}/10)} + 10^{(L_{P3}/10)} + 10^{(L_{P4}/10)} + 10^{(L_{P5}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(104/10)} + 10^{(93/10)} + 10^{(94/10)} + 10^{(73/10)} + 10^{(76/10)}\right]$$

$$= 10 \log_{10}\left[10^{10.4} + 10^{9.3} + 10^{9.4} + 10^{7.3} + 10^{7.6}\right]$$

$L_p = 104.72$ dB

Q.44 Give four machines producing 100 dB, 91 dB, 90 dB and 89 dB. What is the total sound pressure level ? **SPPU - May 17, 4 Marks**

Ans. : Given : $L_{p1} = 100$ dB, $L_{p2} = 91$ dB, $L_{p3} = 90$ dB,

$L_{p4} = 89$ dB, $n = 4$

Thus total sound pressure level is,

$$L_p = 10 \log_{10}\left[\sum_{n=1}^{n} 10^{(L_{pn}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(L_{p1}/10)} + 10^{(L_{p2}/10)} + 10^{(L_{p3}/10)} + 10^{(L_{p4}/10)}\right]$$

...[∴ n = 4]

$$= 10 \log_{10}\left[10^{(100/10)} + 10^{(91/10)} + 10^{(90/10)} + 10^{(89/10)}\right]$$

$$= 10 \log_{10}\left[10^{10} + 10^{9.1} + 10^{9} + 10^{8.9}\right]$$

or $\mathbf{L_p = 101.16\ dB}$ ...Ans

Q.45 If two machines are producing 80 dB each what will be the overall sound pressure level? Derive the equation you use. **SPPU - May 18, 6 Marks**

Ans. : Given :

$L_{p1} = 80$ dB, $L_{p2} = 80$ dB, $n = 2$

Thus total sound pressure level is,

$$L_p = 10 \log_{10}\left[\sum_{n=1}^{n} 10^{(L_{pn}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(L_{p1}/10)} + 10^{(L_{p2}/10)}\right] \quad [\therefore n = 2]$$

$$= 10 \log_{10}\left[10^{(80/10)} + 10^{(80/10)}\right]$$

$$= 10 \log_{10}\left[10^{8} + 10^{8}\right]$$

or $\mathbf{L_p = 83.01\ dB}$ ...Ans.

Q.46 When operating independently in the presence of back ground noise, measurement at a given location of the sound pressure level for machines 1, 2 and 3 are respectively 88 dB, 90 dB and 87 dB. When the machines are turned off, the sound pressure level at the same point is 86 dB. Determine the overall sound pressure level (SPL) of the three machines independent of the background noise. **SPPU - Dec. 12, May 16, 6 Marks**

Ans. : Given : L_{p1} for machine 1 = 88 dB,

L_{p1} for machine 2 = 90 dB

L_{p1} for machine 3 = 87 dB

L_{p2} i.e. ambient or background noise = 86 dB.

- **Sound pressure level for machine 1 independent of background noise :**

$$L_p = 10 \log_{10}\left[10^{88/10} - 10^{86/10}\right] = 10 \log_{10}\left[10^{88} - 10^{86}\right]$$

or $L_p = 83.67$ dB

- **Sound pressure level for machine 2 independent of background noise :**

$$L_p = 10 \log_{10}\left[10^{90/10} - 10^{86/10}\right] = 10 \log_{10}\left[10^{90} - 10^{86}\right]$$

or $L_p = 87.79$ dB

- **Sound pressure level for machine 3 independent of background noise :**

$$L_p = 10 \log_{10}\left[10^{87/10} - 10^{86/10}\right] = 10 \log_{10}\left[10^{87} - 10^{86}\right]$$

or $L_p = 80.13$ dB

- **Overall sound pressure level of three machines independent of background noise :**

$$L_p = 10 \log_{10}\left[10^{83.67/10} + 10^{87.79/10} + 10^{80.13/10}\right]$$

$$= 10 \log_{10}\left[10^{8.367} + 10^{8.779} + 10^{8.013}\right]$$

or $\mathbf{L_p = 89.71\ dB}$ ...Ans.

Q.47 Explain the various types of sound fields in the vicinity of a sound source. **SPPU : May 15, Dec. 18, Dec. 19**

OR Explain different sound field for sound measurements. **SPPU : May 18**

Ans. : Types of Sound Fields :

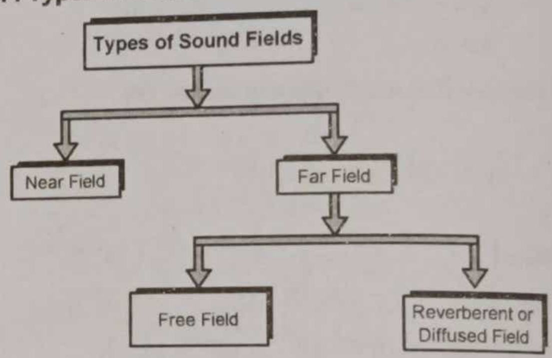


Fig. 6.22 : Types of Sound Fields

- Fig. 6.23 shows the various sound fields regions.

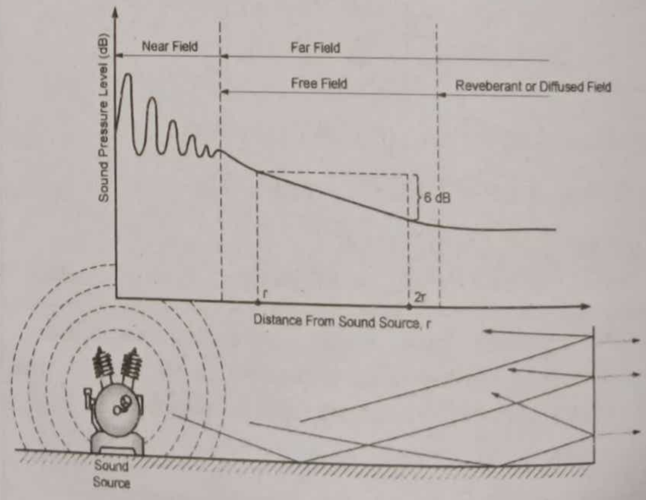


Fig. 6.23 : Sound Fields in Vicinity of a Sound Source

Q.44 Give four machines producing 100 dB, 91 dB, 90 dB and 89 dB. What is the total sound pressure level ? **SPPU - May 17, 4 Marks**

Ans. : Given : $L_{p1} = 100$ dB, $L_{p2} = 91$ dB, $L_{p3} = 90$ dB,

$L_{p4} = 89$ dB, $n = 4$

Thus total sound pressure level is,

$$L_p = 10 \log_{10}\left[\sum_{n=1}^{n} 10^{(L_{pn}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(L_{p1}/10)} + 10^{(L_{p2}/10)} + 10^{(L_{p3}/10)} + 10^{(L_{p4}/10)}\right] \quad ...[\therefore n = 4]$$

$$= 10 \log_{10}\left[10^{(100/10)} + 10^{(91/10)} + 10^{(90/10)} + 10^{(89/10)}\right]$$

$$= 10 \log_{10}\left[10^{10} + 10^{9.1} + 10^{9} + 10^{8.9}\right]$$

or $\quad L_p = \mathbf{101.16\ dB}$ ...Ans

Q.45 If two machines are producing 80 dB each what will be the overall sound pressure level? Derive the equation you use. **SPPU - May 18, 6 Marks**

Ans. : Given :

$L_{p1} = 80$ dB, $L_{p2} = 80$ dB, $n = 2$

Thus total sound pressure level is,

$$L_p = 10 \log_{10}\left[\sum_{n=1}^{n} 10^{(L_{pn}/10)}\right]$$

$$= 10 \log_{10}\left[10^{(L_{p1}/10)} + 10^{(L_{p2}/10)}\right] \quad [\therefore n = 2]$$

$$= 10 \log_{10}\left[10^{(80/10)} + 10^{(80/10)}\right]$$

$$= 10 \log_{10}\left[10^{8} + 10^{8}\right]$$

or $\quad L_p = \mathbf{83.01\ dB}$ ...Ans.

Q.46 When operating independently in the presence of back ground noise, measurement at a given location of the sound pressure level for machines 1, 2 and 3 are respectively 88 dB, 90 dB and 87 dB. When the machines are turned off, the sound pressure level at the same point is 86 dB. Determine the overall sound pressure level (SPL) of the three machines independent of the background noise. **SPPU - Dec. 12, May 16, 6 Marks**

Ans. : Given : L_{p1} for machine 1 = 88 dB,

L_{p1} for machine 2 = 90 dB

L_{p1} for machine 3 = 87 dB

L_{p2} i.e. ambient or background noise = 86 dB.

- **Sound pressure level for machine 1 independent of background noise :**

$$L_p = 10 \log_{10}\left[10^{88/10} - 10^{86/10}\right] = 10 \log_{10}\left[10^{88} - 10^{86}\right]$$

or $L_p = 83.67$ dB

- **Sound pressure level for machine 2 independent of background noise :**

$$L_p = 10 \log_{10}\left[10^{90/10} - 10^{86/10}\right] = 10 \log_{10}\left[10^{90} - 10^{86}\right]$$

or $L_p = 87.79$ dB

- **Sound pressure level for machine 3 independent of background noise :**

$$L_p = 10 \log_{10}\left[10^{87/10} - 10^{86/10}\right] = 10 \log_{10}\left[10^{87} - 10^{86}\right]$$

or $\quad L_p = 80.13$ dB

- **Overall sound pressure level of three machines independent of background noise :**

$$L_p = 10 \log_{10}\left[10^{83.67/10} + 10^{87.79/10} + 10^{80.13/10}\right]$$

$$= 10 \log_{10}\left[10^{8.367} + 10^{8.779} + 10^{8.013}\right]$$

or $\quad L_p = \mathbf{89.71\ dB}$ ...Ans.

Q.47 Explain the various types of sound fields in the vicinity of a sound source. **SPPU : May 15, Dec. 18, Dec. 19**

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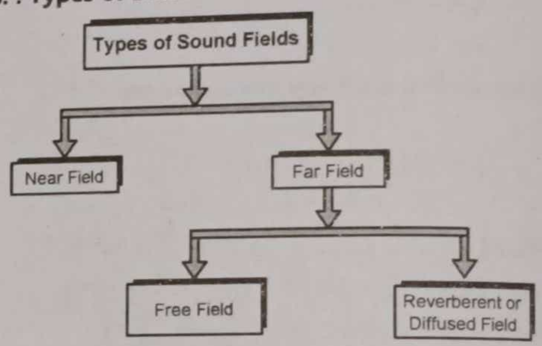


Fig. 6.22 : Types of Sound Fields

- Fig. 6.23 shows the various sound fields regions.

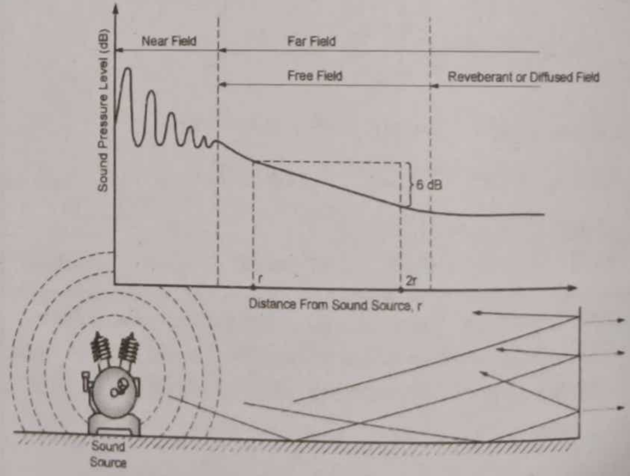


Fig. 6.23 : Sound Fields in Vicinity of a Sound Source

1. **Near Field :**

- **Near field** : *The* ***near field*** *region is located within a few wavelengths of the source of sound or within a few diameters of the source of sound.*
- In near field, the sound pressure level may be either more or less than that predicted by the inverse-square law. Therefore, the sound pressure level measured within the near field cannot be used to predict the sound pressure level.

2. **Far Field :**

- **Far field** : *The region beyond the near field is called as* ***far field***. The far field is the region suitable for recording sound pressure level of sound source.
- When one moves out of the near field and enter into the far field, the sound pressure levels drop off at the rate of 6 dB per doubling the distance from a point source.
- According to the inverse-square law, the sound pressure level in far field is given by,

$$L_{p2} = L_{p1} - \left[20 \log_{10}\left(\frac{r_2}{r_1}\right) \right] \quad ...(1)$$

where, L_{p1} = Sound pressure level at the distance r_1 from sound source, dB

L_{p2} = Sound pressure level at the distance r_2 from sound source, dB

Types of far field :

(i) Free field :

- The region in which Equation (1) is valid is known as **free field**.
- The free field conditions exist in large open outdoor spaces or in rooms having highly absorptive surfaces in which there are no obstructers or barriers in the sound travel path between source and listener.

(ii) Reverberant or diffused field :

- *The sound field which is formed by multiple reflections from obstacles or barriers like wall, floor and celling surfaces etc. in the room is called as* **reverberant or diffused field**.
- As sound pressure levels decay from sources within a room, they eventually drop to a relatively constant level which is determined by the amount of reflected sound within a room.

Q.48 Explain, in brief, the term : Sound reflection coefficient.

SPPU : Dec. 12, May 19

OR Define the term : Reflection coefficient.

SPPU : Dec. 16, Dec. 17

Ans. : Reflection of Sound Wave :

- If a sound wave in air encounters a large heavy and rigid wall, the sound wave will be reflected back. A sound wave from a given source, reflected by a plane surface, appears to come from the image of the source in that surface, as shown in Fig. 6.24.

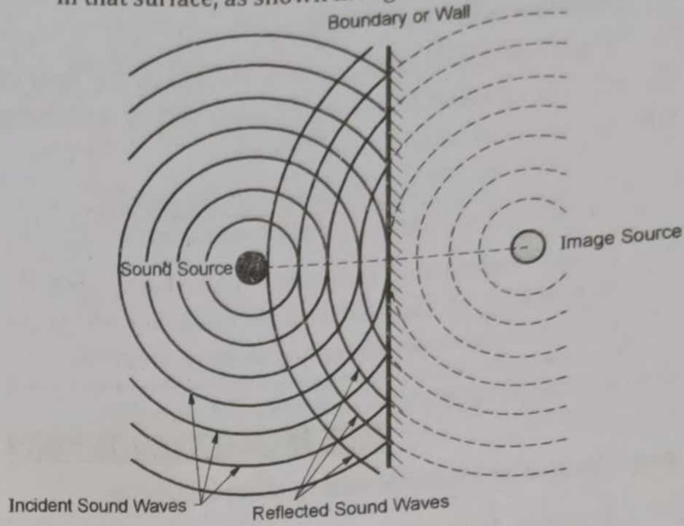


Fig. 6.24 : Reflection of Sound Wave from Plane Surface

☞ **Reflection Coefficient :**

Reflection coefficient *'γ' is defined as the ratio of the intensity of sound that is reflected from the boundary material to the intensity of sound incident on it.* Therefore,

$$\gamma = \frac{I_r}{I_e} = \frac{\text{Intensity of sound reflected}}{\text{Intensity of sound incident}}$$

Q.49 Define the term : Absorption coefficient .

SPPU : Dec. 14, Dec. 16, Dec. 17, May 18, May 19

Ans. : Absorption of Sound Wave :

- The absorption of sound is the process by which sound energy is diminished in passing through a medium or in striking on a surface. In the absorption of sound wave, the mechanism is usually the conversion of sound into other form of energy.

☞ **Absorption Coefficient :**

Absorption coefficient 'α' *is defined as the ratio of the intensity of sound absorbed from the boundary or material to the intensity of sound incident on it.* Therefore,

$$\alpha = \frac{I_a}{I_e} = \frac{\text{Intensity of sound absorbed}}{\text{Intensity of incident}}$$

Q.50 Explain the term : Sound transmission.

SPPU : May 12, May 15

OR Define the term : Transmission coefficient

SPPU : Dec. 12, Dec. 16, Dec. 17, May 18, May 19

Ans. : Transmission of a Sound Wave :

- Sound transmission occurs when sound energy passes through partition or boundary. In practically all reflectors or sound absorbers, some of the sound energy is transmitted through the wall.

Transmission Coefficient :

Transmission coefficient 'τ' *is defined as the ratio of the intensity of sound transmitted from boundary wall to the intensity of sound incident on it. Therefore,*

$$\tau = \frac{I_T}{I_e} = \frac{\text{Intensity of sound transmitted}}{\text{Intensity of sound incident}}$$

- Sound transmission is significantly reduced when the dimensions of the partition or boundary are larger than the largest wavelength of incident sound wave.

Q.51 Explain acoustic material and its characteristics.

SPPU : Dec. 18, May 19, Dec. 19

Ans. : Acoustic Material and Its Characteristics

Acoustic Materials :

- Acoustic or Sound absorbing materials are the materials which absorb and transmit the maximum sound waves and reflect the minimum sound waves.
- A material that could absorb and transmit more sound waves than it reflects, is considered a good sound absorbing material.

Factors Influencing Acoustic Properties of Material

The following factors influence the acoustic properties of materials :

1. **Material thickness :**

- One of the factors that influence a sound absorption by a material is the thickness of the material.
- The thickness of the materials is relevant or has direct relationship at low frequency range (100-2000 Hz) and is insignificant at high frequency (> 2000 Hz).
- Increase in the thickness provides better absorption of the wave and reflect less energy. As the thickness of the samples increases, the sound absorption coefficient increases. The reason is, at low frequency waves have higher wavelength, which means the thicker material contributes in better absorption.

2. **Material density :**

- The density of the material is another factor that influences sound absorption by a material.
- A high density material increases the sound absorption.

3. **Material porosity :**

- The presence of pores of voids plays a crucial part as they act as the medium of sound wave dissipation.
- Open pores with continuous channels provides better sound absorption because of the multiple reactions between the sound wave and the walls of the pores.

Q.52 Explain, with neat diagram, the working of human hearing mechanism.

SPPU : Dec. 14, May 15, Dec. 16, Dec. 17

Ans. : Human Hearing Mechanism

Function of Human Ear :

The function of the ear is to convert the physical vibration into an encoded nervous impulse signal which reaches the brain.

Components of Human Ear :

The human ear is commonly divided into three main components [Fig. 6.25] :

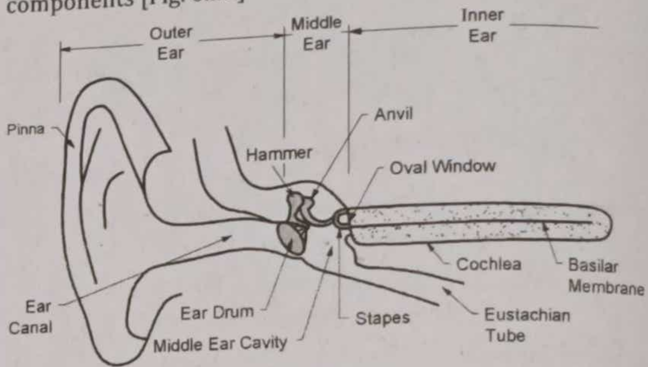


Fig. 6.25 : Schematic of Human Hearing Mechanism

1. **Outer Ear :**

The outer ear composed of (i) pinna, (ii) ear canal, and (iii) ear drum

(i) Pinna : The pinna projects from the side of the head skin. The pinna collects sound and channels it into the ear canal. The pinna is angled so that it catches sounds that come from front more than that from behind.

(ii) Ear canal : The ear canal is a tubular passage of about 25 mm long and 5-7 mm in diameter through which sound waves pass to the ear drum.

(ii) Ear drum : The ear drum is a very shallow cone of about 7 mm in diameter with its apex directed inwards. The ear drum separates the middle ear from the outer ear. It's purpose is to vibrate according to the frequency and amplitude of sounds that strike on it.

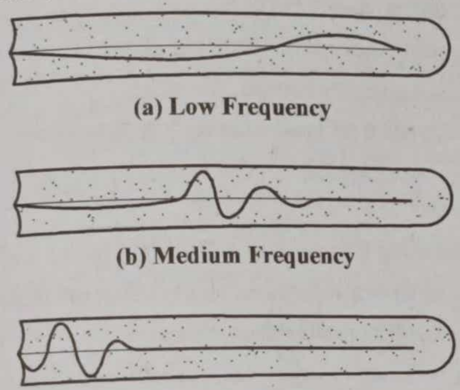
2. **Middle Ear :**

- The purpose of the middle ear is to transmit and amplify sounds from the ear drum to the oval window.

- The middle ear is an air filled space connected to the back of the nose by a long, thin tube called the **eustachian tube**. It transmits the sound from the ear drum to the inner ear.
- The ear drum is
- 13 times larger than the oval window, giving an amplification of about 1:13 compared to the oval window.

3. **Inner Ear :**

- In the inner ear, the cochlea is the main component where the actual reception of sound takes place. [Fig. 6.26]
- The cochlea which is located in extremely hard temporal bone, is divided almost its entire length by the **basilar membrane**.
- At the end of the cochlea, the two canals are connected by the **helicontrema**, which allows for the flow of the lymphatic fluid between the two sections. The basilar membrane which is 30 mm long and 0.2 mm wide has about 24,000 nerve ends.



(a) Low Frequency

(b) Medium Frequency

(c) High Frequency

Fig. 6.26 : Schematic Representation of Sound Waves Travelling Along Basilar Membrane

- Low frequency sound results in maximum amplitude near the distant end of the basilar membrane and high frequency sound produces peaks near the oval window. For a complex signal such as music or speech, many momentary peaks are produced, constantly shifting in amplitude and position along the basilar membrane.
- Fig. 6.26 shows schematic representation of sound waves travelling along the basilar membrane.

Q.53 Explain in brief : Various sources of noise and how to control same. **SPPU : Dec. 18, Dec. 19**

Ans. : Sources of Noise

- **Categories of sources of noise :**
  The sources of noise are categorized in the following major groups :
  1. Industrial Noise
  2. Home Appliances
  3. Construction Equipment
  4. Road, Air and Rail and Air Transportation
- **Noise levels in industrial machinery :**
  The following tables gives typical noise levels in dB for industrial machinery, home appliances and construction equipment.

Table 6.1 : Machinery Noise Level in Industry

Industrial Machinery	Range of Noise Level, dB
Rivetting	105 to 125
Presses	110 to 120
Pneumatic power tool	90 to 115
Hydraulic machine tool	85 to 105
Grinding machine	99 to 105
Air compressor	90 to 100
Fluid pumps	80 to 90
Lathes and milling machine	80 to 90

Table 6.2 : Noise Level for Home Appliances

Hone Appliances	Range of Noise Level, dB
Food blander	62 to 88
Vacuum cleaner	62 to 85
Food mixer	50 to 80
Dish washer	55 to 75
Fan	38 to 70
Electric shaver	50 to 65
Hair drier	55 to 62
Air conditioner	50 to 60
Clothes dryer	50 to 60
Refrigerator	35 to 52

Table 6.3 : Noise Level for Construction Equipment (measured at 15m distance from equipment)

Construction Equipment	Range of Noise Level, dB
Rock drill	80 to 100
Scrapper and graders	80 to 95
Pneumatic wrenches	83 to 93
Concrete mixer	75 to 88
Concrete pump	80 to 83
Movable cranes	75 to 85

Construction Equipment	Range of Noise Level, dB
Front loader	72 to 84
Generators	70 to 83
Pumps	68 to 70
Compressors	65 to 70

- The major source of the noise discussed in the chapter is industrial noise (both within and around the industrial promises).

Q.54 Write short note on : Noise control in industries.

SPPU : May 16

Ans. : Industrial Noise Control

Basic Elements in Noise Control System (S - P - R) :

1. **Source** of noise, 2. **Path** of noise, and
3. **Receiver** of noise

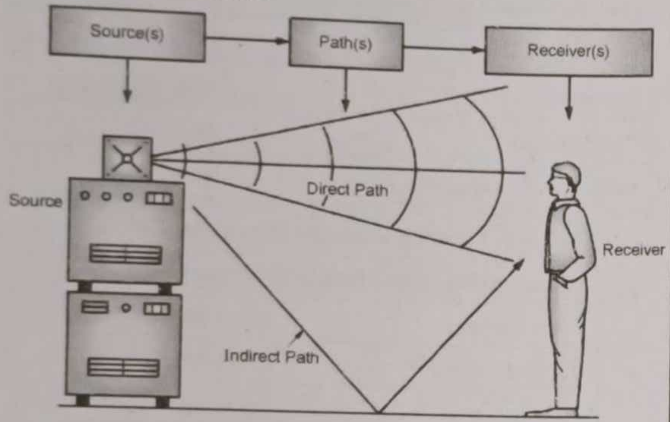


Fig. 6.27 : Elements of Noise Control System (S-P-R)

1. **Source of Noise :**

   The ***source*** *of noise may be a machine or any number of mechanical devices, vibrating surface, mechanical shock, mechanical friction, fluid flow, flame burst or an explosion.* Identification of source of noise usually helps to reduce or eliminate the noise. Therefore, acoustic engineer should identify all possible noise sources when considering a solution for a noise problem.

2. **Path of Noise :**

   The ***path*** *implies course or direction taken by the noise to reach the listener.* The path may be direct or indirect. The direct path for the sound may be the air between the source and receiver, as is the case for machinery noise transmitted directly to the operator's ears. The indirect path is the noise which is being reflected by a wall or flooring to a person in the room.

3. **Receiver of Noise :**

   The ***receiver*** *in the noise control system is usually the human.*

Q.55 Explain the various methods of industrial noise control.

SPPU : May 16, Dec.18

Ans. : Methods of Industrial Noise Control

There are three basic methods of industrial noise control :

1. **Noise Control at Source :** It is more economical and efficient to control the noise at the source. Modifications at the source of sound are usually considered to be the best solution for an industrial noise control problem.

☞ **Steps in Noise Control at Source :**

(i) Maintenance :

- Balancing of unbalanced equipment.
- Lubrication of moving parts.
- Replacement or adjustment of worn or loose parts.
- Tensioning driven belts.
- Tightening loose and vibrating screws or bolts.
- Replacement of worn out bearings.

(ii) Substitution of manufacturing processes :

- Using mechanical ejectors rather than pneumatic ejectors.
- Hot working rather than cold working.
- Pressing operation rather than rolling or forging.
- Using welding rather than stoppers.
- Using welding or squeeze stoppers rather than impact stoppers.
- Using cutting fluid in machining processes.
- Change from impact action (e.g. hammering a metal bar) to progressive pressure action (e.g. bending metal bar with pliers) or increase of time during which a force is applied.
- Replacing circular saw blades with damped blades.
- Replacing mechanical limit a stoppers with micro-switches.

(iii) Substitution of equipment :

- Use electric equipment instead of pneumatic equipment (e.g. hand tools)
- Using stepped dies instead of single-operation dies.
- Using rotating shears instead of square shears.
- Using hydraulic presses instead of mechanical presses.
- Using presses instead of hammers.
- Using electric motors instead of internal combustion engines or gas turbines.

- Using belt conveyors instead of roller conveyors.
- Using belts or hydraulic power transmissions instead of gear boxes.

iv) Modification of parts of equipment :

- Replacing gear drives with belt drives.
- Modification of gear teeth, by replacing spur gears with helical gears.
- Replacing straight edged cutters with spiral cutters.
- Using properly shaped and sharpened cutting tools.
- Use of proper dampers for vibrating machines.
- Use of complete or partial enclosure around noisy machines.
- Use of proper mountings and isolation of vibratory machines or equipments.
- Providing machines with adequate cooling fins so that noisy fans are no longer needed.
- Using centrifugal rather than propeller fans.
- Locating fans in smooth, undisturbed air flow, or using optimized fan blades.

v) Substitution of materials :

- Replacing metal gears with plastic gears. However, it requires additional maintenance.
- Replacing steel or solid wheels with pneumatic tyres.
- Replacement of steel sprockets in chain drives with polyamide plastics sprockets.

vi) Change of work methods :

- In building demolition, replace use of ball machine with selective demolition.
- By changing manufacturing methods, such as moulding holes in concrete rather than cutting after production of concrete component.
- Instead of using an air jet to remove debris from a manufactured part, rotating cleaning brushes may be used.
- Keep noisy operations in the same area and separated from non-noisy processes.
- Select slowest machine speed appropriate for a job. Also select large slow machines rather than smaller faster ones.
- Minimise width of tools in contact with workpiece.

vii Noise enclosure :

An effective noise control procedure is to enclose the sound source in an acoustic enclosure.

2. Noise Control in Path :

Modifying the path through which the noise is propagated is often used when modification of the noise source is not possible, not practical, or not economically feasible.

☞ Steps in Noise Control in Path :

(i) For noise sources located outdoors, one simple approach for noise control would be to move the sound source farther away from the receiver. Keep noisy machines in the farthest corner of the industrial premises.

(ii) For noise sources located outdoors or indoors, the transmission path may be modified by placing a wall or barrier between the source and the receiver.

(iii) If the sound transmitted indirectly to the receiver through reflections from the room surfaces is significant, the noise may be reduced by applying acoustic absorbing materials on the walls of the room or by placing additional acoustic absorbing surfaces in the room.

(iv) The noise from metal cut-off saws can be reduced to acceptable levels by enclosing the saw in an acoustically treated box.

(v) The exhaust noise from engines, fans, and turbines is often controlled by using mufflers or silencers in the exhaust line for the device.

3. Noise Control at Receiver :

In some cases, when first two methods fail, it may be necessary to apply noise control to the receiver. The human ear is the usual receiver for noise, and there can be a limited amount of modification possible.

☞ Steps in Noise Control at Receiver :

(i) Limiting the time during which the person is exposed to high noise levels.

(ii) Rotating the job between the workers working at a particular extreme noise source.

(iii) Generally, the noise level of 90 dB for more than 8 hr continuous exposure is prohibited. The schedule of the workers should be planned in such a way that, they should not be over exposed to the high noise levels.

(iv) Using protectors such as earplugs, acoustic muffs, or active noise cancelling headphones.

(v) A very effective, although sometimes expensive, noise control procedure is to enclose the receiver in a personnel booth. If an equipment or process can be remotely operated, a personnel booth is usually an effective solution in reducing the workers noise exposure.

Q.56 What is sound enclosure ? Describe the two types of sound enclosures. **SPPU : May 15, May 17**

Ans. : Noise Enclosure :

The noise level of any machines is reduced by providing a suitable enclosures. There are two types of enclosures, used for reducing the noise level :

1. Complete Enclosures :

- Fig. 6.28 shows a typical complete enclosure. Such enclosure is essentially a sealed box with stiffened walls. The panels of the walls have damping treatment applied to them to control their resonant vibration.
- The interior of the box is covered with absorptive treatment such as open cell foam or glass-fiber mat, to prevent the buildup of reverberant sound in the interior. All the openings are carefully sealed or provided with sound absorbing ducts of mufflers.
- The machine is kept on vibration isolation mount so that the machine does not excite the wall of the enclosure.

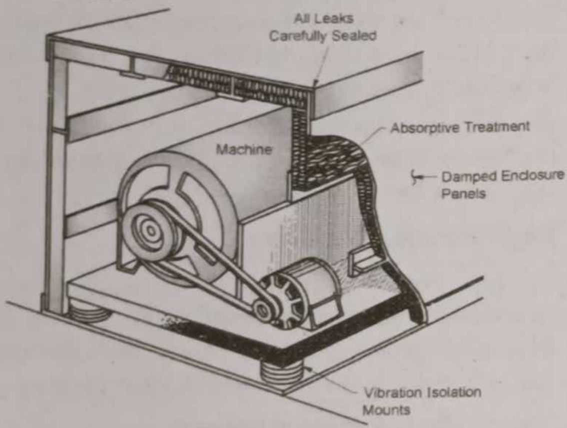


Fig. 6.28 : Simple Complete Enclosure

2. Partial Enclosures :

- The complete enclosure cannot be used, where opening for cooling air is provided. The mufflers can be employed to prevent noise from escaping from these opening, as shown in Fig. 6.29.

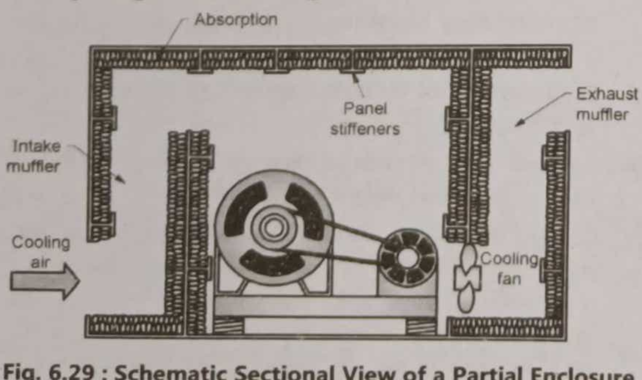


Fig. 6.29 : Schematic Sectional View of a Partial Enclosure with Mufflers

- With properly designed mufflers, a partial enclosure can be used as effective as complete enclosure.

Q.57 What are the noise measuring instruments.

Ans. :

- The noig. 6.30, are :
  1. Microphone
  2. Sound Level Meter
  3. Data Storage Facilities

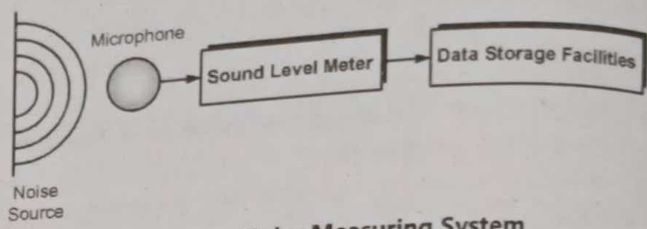


Fig. 6.30 : Noise Measuring System

Q.58 Explain the working of a microphone.

SPPU : May 12, Dec. 13, May 15, May 16

Ans. : Microphone

Microphone *is a device which converts acoustical energy (sound waves) into electrical energy (the audio signal).*

☞ **Working Principle of Microphone :**

- The microphone is the interface between the sound field and the measuring system. It responds to sound pressure and transforms it into an electric signal, which can be interpreted by the measuring instrument (e.g. the sound level meter).
- The microphone (Fig. 6.31) contains a diaphragm. The diaphragm (membrane) is a thin piece of material such as paper, plastic or aluminum which vibrates when it is struck by sound waves. When the diaphragm vibrates, it causes other components in the microphone to vibrate. These vibrations are converted into an electrical current.

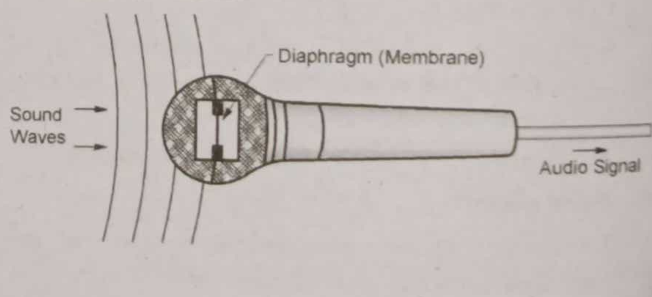


Fig. 6.31 : Microphone

Types of Microphones :

- The different types of microphones are as shown in Fig. 6.32.

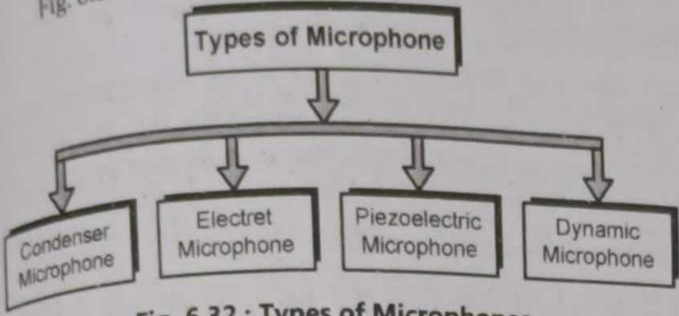


Fig. 6.32 : Types of Microphones

Q.59 Explain, with neat, sketch the working of sound level meter. **SPPU : Dec. 14, May 17, May 19**

Ans. : Sound Level Meter :

- **Sound level meter :** One of the most useful and important instrument in the analysis of noise is the sound level meter, shown in Fig. 6.34. ***Sound level meter*** *amplifies the very small output signal from the microphone and make it available for processing and for visual display by a meter contained within the unit.*
- **Elements of sound level meter :** The various elements of sound level meter (Fig. 6.33) are as follows

i) Pre-amplifier ii) Attenuator

iii) Amplifier iv) Weighting network or filter

v) Rectifier

- The electrical signal from the microphone is fed to the pre-amplifier of the sound level meter and it is further fed to a weighted filter over a specified range of frequencies. From filler, the signals are fed to amplifier for amplification. From amplifier the signals enter :

  (a) Rectifier and then digital display meter; or

  (b) Analog meter
- The output of the rectifier is given to other instruments such as a tape recorder, graphic level recorder or for rectification and direct reading on the meter.
- In some cases, the sound level meter does not include a logarithmic converter. The scale on the indicating device is then exponential. In this case, the dynamic range of the display is usually restricted to 10 to 16 dB. When a log converter is used, the display scale is linear in dB and its dynamic range is usually much greater.

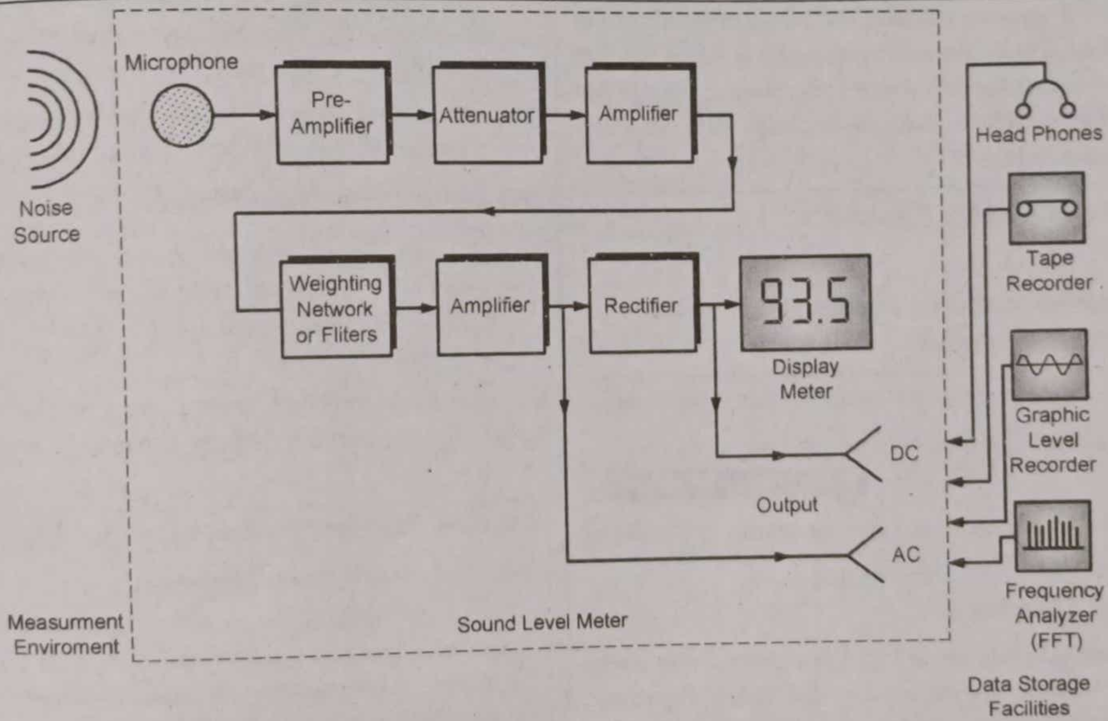


Fig. 6.33 : Sound Level Meter

Q.60 Wite a short note on Frequency Analyzer.

Ans. :

- ***Frequency analyzer*** *is a device that provides the capability for analysis of a noise signal in the frequency domain by electronically separating the signal energy into various frequency band. This separation is performed by means of a set of filters.*

- The objective of frequency analysis is to determine how the overall sound level is distributed over a range of frequencies. The simplest type of sound frequency analyzer contains sets of passive filters (octave or one third octave) that can be inserted between the two amplifiers.
- These are sequential instruments making measurements in one band at a time. This strongly restricts their use, as the noise must be constant both in amplitude and in frequency during the 5 to 10 minutes of the analysis.

Fig. 6.34 : Sound Frequency Analyzer

- More sophisticated analyzers have the possibility to make the frequency analysis in all desired bands at the same time. These are analyzers using a set of parallel filters or using the fast fourier transform (FFT) of the input signal before recombining the data into the desired bands [Fig. 6.34].

Q.61 What are the Types of Recorders ?

Ans. :

1. Graphic level recorder
2. Magnetic tape recorder

Q.62 Write a short note on noise measurement environment.

OR Explain anechoic chamber and reverberant chamber.

SPPU : May 16, Dec. 19

Ans. : Measurement of noise can be made in following environments :

(i) Anechoic Chamber :

- This is special chamber or room in which all side walls, ceiling and floor are covered by the sound absorbing materials for maximum absorption of sound during measurement.
- The ideal anechoic chamber will have no reflecting surface.
- The anechoic chamber measurement can yield the information like, sound radiation pattern and narrow band frequency data.

(ii) Semi-anechoic Chamber :

- In this chamber or room all side walls and ceilings have sound absorption materials but the floors is having reflecting material.
- The microphone response varies with the angle of incidence of the sound wave on the microphone it is called as ***"directivity"***. Due to reflection of the floor some directivity is caused in semi-anechoic chamber.

(iii) Reverberant Chamber :

- In this chamber all the walls, ceiling and floor are acoustically hard reflected surface. There is reflections and re-reflections which result in diffuse pressure field.
- Reverberant chambers are used to take average of energy radiated from different machines. In reverberant chamber, the effects of all sound sources are averaged out and therefore no information is obtained about phase relationship, directionality and location of different sources of sound.
- To increase diffusion of sound, the reverberant rooms are usually made of slightly irregular shape and paddle like turning vanes are provided for the same purpose. To minimize the effect of the vibration of the nearby structures, the foundations of such room are isolated from these structures.
- For sound power measurement, the reverberant chamber volume should be 200 m^3 minimum.

(iv) Out of Doors Environment :

- The out of doors environment can be treated as Semi-anechoic chamber if there are no reflecting surfaces within 35 meters from the microphone or from the machine under test.
- Building, publicity board or banners, and hillsides are reflecting surfaces, which should be avoided during test.

Q.63 Write short note on : Pass-by-noise. **SPPU : May 18**

Ans. : Pass-By Noise Measurement :

- A pass-by noise measurement is defined as the method of measuring the noise emission of a road vehicle under acceleration conditions, with various gear positions in a certain measurement range.
- The pass - by noise measurement is mandatory for automotive manufacturers in terms of product certification. The, ISO (Internal Organization for Standardization) has norms for measurement, analysis and reporting format of pass-by-noise measurements test.

- The pass-by noise measurement should be performed in a large open space for type approval of commercial vehicles, or measured during the official test station's manufacturing stage. Therefore, it is very important that the certification of emission noise measurement is performed before mass production starts.

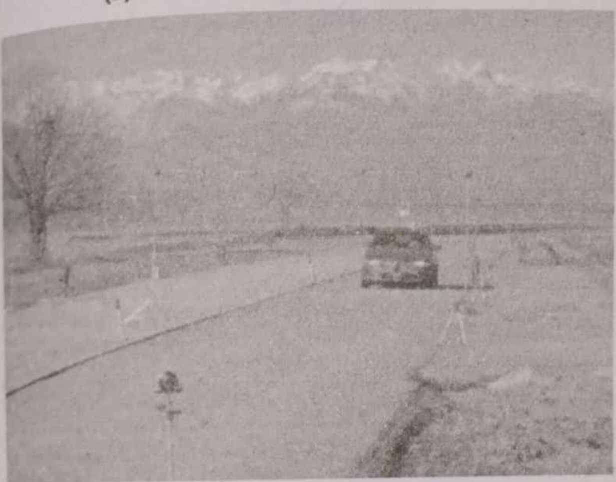
Two Types of Techniques for Pass-By-Noise Measurement :

1. **Field Pass-By-Noise Measurement :**

   In field pass-by-noise measurement, the test vehicle is made to pass by two stationary microphones. The sound is measured by two stationary microphones.

2. **Indoor Pass-By-Noise Measurement :**

(a) Indoor Pass By Noise Testing



(b) Outdoor Pass By Noise Testing

Fig. 6.35 : Pass-by noise measurement techniques

- In some cases, pass-by-noise measurement cannot be taken in the field because of bad weather or bad test-track conditions. In such cases, the indoor simulated pass-by-noise measurement is done. The indoor pass-by-noise measurement is considered as the conformance test, together with the field pass-by noise test.
- Fig. 6.35 shows indoor pass-by noise and field pass-by noise measurement technique.
- In indoor pass-by setups, two rows of microphones placed alongside the vehicle. The vehicle runs on a chassis dynamometer and is accelerated in the same way it would be for a filed pass-by measurement. Time histories are measured by the microphones along with vehicle parameters and dynamometer drum speed. A sophisticated algorithm uses information from the dynamometer to calculate a vehicle's position relative to the microphones as a function of time. This is then used extract the contributing portions of the time histories that correspond to when the vehicle would have passed the standard microphone positions had it been moving.

Q.64 A worker is exposed to noise according to the following schedule :

Exposure level [dB]	92	95	97	102
Period of exposure [hrs]	3	2	2	1

Does the daily noise dose is exceeded as per OSHA standards. **SPPU : Dec. 15, 6 Marks**

Ans. :

1. **Permissible Time Duration for Noise Exposure at Four Given SPL :**

Exposure Level (dBA)	92	95	97	102
Period of Exposure (t)	3 Hrs.	2 Hrs.	2 Hrs.	1 Hrs.
Permissible Duration from OSHA Standard (T)	1 Hr.35 min i.e. 1.58 Hrs.	48 min i.e. 0.8 Hrs.	30 min i.e. 0.5 Hrs.	9 min i.e. 0.15 Hrs.

2. **Daily Noise Dose (D) :**

   The daily does 'D' is given by,

$$D = \frac{t_1}{T_1} + \frac{t_1}{T_2} + \frac{t_3}{T_3} + \ldots\ldots + \frac{t_n}{T_n}$$

$$D = \sum_n \frac{t_n}{T_n}$$

where, t_n = duration in hours for which a man is exposed to a given SPL.

T_n = maximum exposure time limit in hours permitted at that SPL

$$\therefore \quad D = \frac{3}{1.58} + \frac{2}{0.8} + \frac{2}{0.5} + \frac{1}{0.15}$$

$$\text{or} \quad D = 1.89 + 2.5 + 4 + 6.66 = 15 \geq 1$$

As per OSHA standard, the daily noise dose (D) must be less than or equal to 1.

Thus, the daily dose is exceeded. **...Ans.**

Q.65 Write short note on : octave bands. **SPPU : May 18**

Ans. : Octave Bands

- In noise analysis, the frequency analysis in made by using spectrum analysers.
- The frequency range is divided into sets of frequencies known as frequency bands. Each frequency band contains a specific range of frequencies. The frequency bands are defined by scale called octave band.
- **Octave band** *is defined as a range of frequencies in which the highest frequency (f_h) is twice the lowest frequency (f_l) within the range. Thus, for one octave band,*

$$f_h = 2f_l \qquad \text{...(a)}$$

- For noise analysis, fixed octave bands have been internationally agreed.
- **Central frequency of octave band :** Since central frequency f_c is the arithmetic mean of f_h and f_l on logarithmic scale, we can write

$$\log f_c = \frac{1}{2} (\log f_h + \log f_l)$$

$$\text{or} \quad \log f_c = \log (f_h\, f_l)^{1/2}$$

$$\therefore \quad f_c = (f_h\, f_l)^{1/2} = \sqrt{f_h\, f_l} \qquad \text{...(b)}$$

From Equation (a) and (b) we can write,

$$f_c = \sqrt{2}\, f_l \text{ and } f_c = \frac{f_h}{\sqrt{2}} \qquad \text{...(c)}$$

- Using above equations, the octave band frequencies can be determined. (Table 6.4)
- In general, if f_h and f_l are separated by n octaves, then

$$f_h = 2^n f_l \qquad \text{...(d)}$$

If n = 1, the band is termed as one octave band.

If $n = \frac{1}{2}$, the band is termed as one – half octave band

If $n = \frac{1}{3}$, the band is termed as one third octave band.

Table 6.4 : The Centre, Lower Cut-Off and Upper Cut-Off Frequencies of All the Octave Bands

One-octave bands		
f_c	f_l	f_h
16	11	22
31.5	22	44
63	44	88
125	88	177
250	177	355
500	355	710
1000	710	1420
2000	1420	2840
4000	2840	5680
8000	5680	11360
16,000	11,360	22,720

❑❑❑