Unit :- 4

May-Jun 2023

Q.1]

Q.3] (b) Given :- $m = 400\ Kg$

$\delta t = 0.0025\ m$

$m_0 = 5\ Kg$

$e = \frac{\text{Stroke}}{2} = \frac{120}{2} = 60\ mm = 0.060\ m$

& $c = 20 \times 10^{+3}\ \frac{N\text{-Sec}}{m}$

To Find :- (i) X at $\omega = 540$ rpm

(ii) F_{T}

(iii) N

⇒ (i) For X Amplitude :-

$\omega = \frac{2\pi \times N}{60} = \frac{2\pi \times 540}{60}$

$\boxed{\omega = 56.84\ rad/sec}$

(ii) natural frequency

$\omega_n = \sqrt{\frac{g}{\delta t}} = \sqrt{\frac{9.81}{0.0025}}$

$\boxed{\omega_n = 62.64\ rad/sec}$

(iii) Damping factor :-

$\xi = \frac{c}{c_c}$ $\quad [c = 2m\omega_n]$

$= \frac{20 \times 10^3}{2 \times 400 \times 62.642}$

$\boxed{\xi = 0.399}$

(iv) Frequency ratio $\frac{\omega}{\omega_n} = 0.9074$

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(v) Hence now for amplitude

$$\frac{mX}{Me} = \frac{(w/w_n)^2}{\sqrt{[1-(w/w_n)^2]^2 + \left[2\xi \times \frac{w}{w_n}\right]^2}}$$

$$\frac{400 \times X}{5 \times 0.06} = \frac{0.9074^2}{\sqrt{[1-0.9074^2]^2 + \left[2 \times 0.344 \times \frac{0.9074}{0.9074}\right]^2}}$$

$$1333.33X = \frac{0.90 \quad 0.8233}{0.1767 \rightarrow 0.5243}$$

$$\boxed{X = 0.8089 \text{ mm}}$$

(VI) Force

$F_0 = m_0 \times e \times \omega_n^2$

$= 1177.13 N$

$F_{tr} = F_0 \sqrt{\frac{1+4\xi^2}{4\xi^2}}$

$= 1887.33 N$

(VII) Speed

$\omega_n = \frac{2\pi \times N}{60}$

$62.64 = \frac{2\pi \times N}{60}$

$\boxed{N = 598.1678 \text{ rpm}}$

Q.2]

Q.4] b] Given :- $\delta_{st} = 100 mm = 0.1 m$

$Y = 80 mm = 0.08$

$\lambda = 1 m$

$\xi = 0.05$

$V = 75 Km/hr = 20.833 m/s$

To find :- (I) Critical Speed

$V_{cr} = \frac{\omega_n \times \lambda}{2\pi}$

(II) Amplitude. X.

$\Rightarrow$ (I) $\omega_n = \frac{2\pi N}{60} \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{9.81}{0.01}}$

$\boxed{\omega_n = 9.4045 \text{ rad/s}}$

(II) Critical Speed

$$V_{cr} = \frac{\omega_n \times \lambda}{2\pi}$$

$$= \frac{9.9045 \times 16}{2\pi}$$

$$\boxed{V_{cr} = 25.22 \text{ m/sec}}$$

$$= 25.22 \times \frac{3600}{1000}$$

$$\boxed{V_{cr} = 90.74 \text{ Km/hr}}$$

(III) Frequency

$$\omega = \frac{2\pi \times V}{\lambda}$$

$$= 2\pi \times 20.8$$

$$\boxed{\omega = 8.181 \text{ rad/s}}$$

(IV) $\frac{\omega}{\omega_n} = \frac{8.181}{9.4045} = 0.826$

(V) amplitude

$$X = Y \left[\frac{\sqrt{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2}} \right]$$

$$= 0.08 \left[\frac{\sqrt{1 + (0.0826)^2}}{\sqrt{0.9864 + (0.0826)^2}} \right]$$

$$\boxed{X = 805.45 \text{ mm}}$$

$$\boxed{X = 244.5 \text{ mm}}$$

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Q.8]

Q.3] a] Given :- $m = 1\,kg$ $K = 1000\ N/m$

$F_0 = 10\ N$ $C = 40\ \frac{N\text{-}s}{m}$

To find :- (i) ω
(ii) ϕ
(iii) X
(iv) ω_p
(v) ω_d

⇒ Solu :- (i) Resonant frequency

$$\omega = \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1000}{1}}$$

$$\boxed{\omega = 31.628\ rad/sec}$$

(ii) Phase angle

$$\phi = \tan^{-1}\left[\frac{2\xi \times \omega/\omega_n}{1-(\omega/\omega_n)^2}\right] \qquad \frac{\omega}{\omega_n} = 1$$

$$\xi = \frac{C}{C_c} = \frac{40}{2 \times 1 \times 31.62}$$

$$\boxed{\xi = 0.6325}$$

$$\phi = \tan^{-1}\left(\frac{2 \times 0.6325}{1-1}\right)$$

$$\boxed{\phi = 90^\circ}$$

(iii) $$X = \frac{F_0/K}{\sqrt{\left[1-(\omega/\omega_n)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$= \frac{10/1000}{\sqrt{2 \times 0.6325}}$$

$$\boxed{X = 0.00841\ m}$$

(iv) $\omega_P = \sqrt{1-2\zeta^2} \times \omega_n$

$= \sqrt{1-2\times 0.6325^2} \times 31.62$

$\boxed{\omega_P = 14.137 \text{ rad/sec}}$

(v) $\omega_d = \sqrt{1-\zeta^2} \times \omega_n$

$\boxed{\omega_d = 24.5 \text{ rad/sec}}$

Q.4 Q.4

a] $\delta = 0.1\ m$ $\quad \lambda = 14\ m$

$Y = 0.08\ m$ $\quad V = 60\ km/hr = 16.667\ m/s.$

To find :- (i) ω/ω_n (ii) X (iii) V_c

⇒ (i) Frequency ratio $\dfrac{\omega}{\omega_n}$

$\omega_n = \sqrt{\dfrac{g}{\delta}} = \sqrt{\dfrac{9.81}{0.1}} = 9.90 \text{ rad/sec}$

$\omega = \dfrac{2\pi V}{\lambda} = \dfrac{2\pi \times 16.667}{14}$

$\boxed{\omega = 7.48 \text{ rad/s}}$

$\dfrac{\omega}{\omega_n} = 0.7545$

(ii) $X = Y \left[\dfrac{\sqrt{1+\left[2\zeta\dfrac{\omega}{\omega_n}\right]^2}}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\dfrac{\omega}{\omega_n}\right]^2}} \right]$ Take $\zeta = 0$

$= 0.08 \left[\dfrac{\sqrt{1}}{\sqrt{1-0.7545^2}} \right]$

$= 0.12189\ m.$

$\boxed{X = 121.8\ mm}$

(iii) $\omega_d = \frac{2\pi V_c}{\lambda}$

$9.90 = 2\pi \times \frac{V_c}{11}$

$V_c = 22.05$ m/sec

$\therefore$ 79.411 Km/hr

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Q.5

Q.3. b] Given :- $m = 1000$ Kg

$F_0 = 2450$ N

$\omega = \frac{2\pi \times 1500}{60} = 157.08$ rad/s.

$\delta = 2 \times 10^{-3}$ m

$\zeta = 0.2$

(i)

To find :- (I) X

(II) F_T

(III) ϕ

(IV) α

(V) Np

Soln :- (I) Spring Stiffness

$K = \frac{mg}{\delta}$

$= \frac{1000 \times 9.81}{2 \times 10^{-3}}$

$K = 4.905 \times 10^6$ N/m

(II) Natural Frequency

$\omega_n = \sqrt{\frac{K}{m}}$

$\omega_n = 70.036$ rad/s

(iii) $\frac{\omega}{\omega_n} = \frac{157.08}{70.031} = 2.243$

(iv) $X = \frac{F_0/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$

$= 1.2095 \times 10^{-4}$ m

$X = 0.12095$ mm

(v) $F_T = F_0 \left[\frac{\sqrt{1+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}\right]$

$F_T = 797.04$N

(vi) $\phi = \tan^{-1}\left[\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$

$\phi = 167.45°$

(vii) $\alpha = \phi - \tan^{-1}\left(2\zeta\frac{\omega}{\omega_n}\right)$

$= 167.45 - 41.848$

$\alpha = 125.55°$

(viii) $\omega_p = \sqrt{1-2\zeta^2}\,\omega_n$

$= 67.976$ rad/s

(ix) $\omega_p = \frac{2\pi N_p}{60} \Rightarrow N_p = 641.40$ rpm

Q.6]

Q.45] $W = mg = 125\,N$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1500}{60} = 157.08 \text{ rad/sec}$$

$W_M = m_M g = 35\,N$

$e = 0.05 \times 10^{-2}\,m$

$F_T = \left(\frac{1}{11}\right) F_0$

To find → (i) K
(ii) ω_n
(iii) F_T

(i) For stiffness

$$\frac{F_T}{F_0} = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\frac{1}{11} = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\boxed{\frac{\omega}{\omega_n} = 3.40}$$

$$\frac{157.08}{\omega_n} = 3.40$$

$\omega_n = 45.3482$ rad/sec

$$\sqrt{\frac{K}{m}} = 45.3488$$

$$\sqrt{\frac{K}{160/9.81}} = 45.3481$$

$$\therefore \boxed{K_t = 33615.51\ N/m}$$

$$K = \frac{K_t}{5}$$

$$\boxed{K = 6723.102\ N/m}$$

(ii) Natural Frequency $\omega_n = 45.3486$

(iii) $\frac{F_T}{F_0} = TR$

$$= TR \times m e \omega^2$$

$$= \frac{1}{11}\left[\frac{35}{4.81} \times 0.05 \times 10^{-2} \times 157.082\right]$$

$$\boxed{F_T = 4\ N}$$

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Q.7]

Q.3 b] $m = 4Kg$ $F_0 = 50\ N$

$X = 0.25m$ $t_p = 0.4\ s$

$f = 25\ Hz$

To find $= \frac{x_2 - x_1}{x_1}$

① Damping Coefficient :-

$$\omega = \frac{2\pi}{t_p} = \frac{2\pi}{0.4} = 15.707\ rad/s$$

$\omega = \omega_n = 15.7\ rad/s$

$$\omega_n = \sqrt{\frac{K}{m}} \Rightarrow 15.7 = \sqrt{\frac{K}{4}}$$

$$\boxed{K = 985.96\ N/m}$$

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Damping

$$X = \frac{F_0/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$0.25 = \frac{50}{\frac{985.96}{\sqrt{[1-1]^2+[2\times\xi]^2}}}$$

$$\boxed{\xi = 0.101423}$$

$$\xi = \frac{C}{C_c}$$

$$0.101423 = \frac{C}{2\times 4\times 15.7}$$

$$\boxed{C = 12.7387\ N\text{-}s/m}$$

$$f_1 = \frac{\omega_1}{2\pi} \Rightarrow 2 = \frac{\omega_1}{2\pi}$$

$$\omega = 4\pi$$

$$\boxed{\omega = 12.568\ rad/s}$$

$$X_1 = \frac{F_0/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{50}{\frac{985.96}{\sqrt{\left[1-\left(\frac{12.56}{15.7}\right)^2\right]^2+\left[2\times 0.101423\times\frac{12.73}{15.7}\right]^2}}}$$

$$x_1 = 0.12812\,m$$

$$x_2 = \frac{F_0}{\frac{k}{1-\left(\frac{\omega}{\omega_n}\right)^2}}$$

$$= \frac{50}{\frac{985.96}{1-\left(\frac{12.56}{15.7}\right)^2}}$$

$$x_2 = 0.1408\,m$$

$$\therefore \frac{x_2 - x_1}{x_1} \times 100 = \frac{0.1408 - 0.128}{0.128} \times 100$$

$$= 9.86969\,\%$$

—x—