Experiment 4

Power Spectrum Estimation

Welch's Non Parametric Method

4.1.1 Aim:

To study and estimate the power spectrum of a random sequence x (n) using Welch's Non Parametric Method in MATLAB.

4.1.2 Theory:

An improved estimator of the PSD is the one proposed by Welch. The method consists of dividing the time series data into (possibly overlapping) segments, computing a modified periodogram of each segment, and then averaging the PSD estimates. The result is Welch's PSD estimate.

Welch's method is implemented in the Signal Processing Toolbox by the pwelch function. By default, the data is divided into eight segments with 50% overlap between them. A Hamming window is used to compute the modified periodogram of each segment.

The averaging of modified periodograms tends to decrease the variance of the estimate relative to a single periodogram estimate of the entire data record. Although overlap between segments tends to introduce redundant information, this effect is diminished by the use of a nonrectangular window, which reduces the importance or *weight* given to the end samples of segments (the samples that overlap).

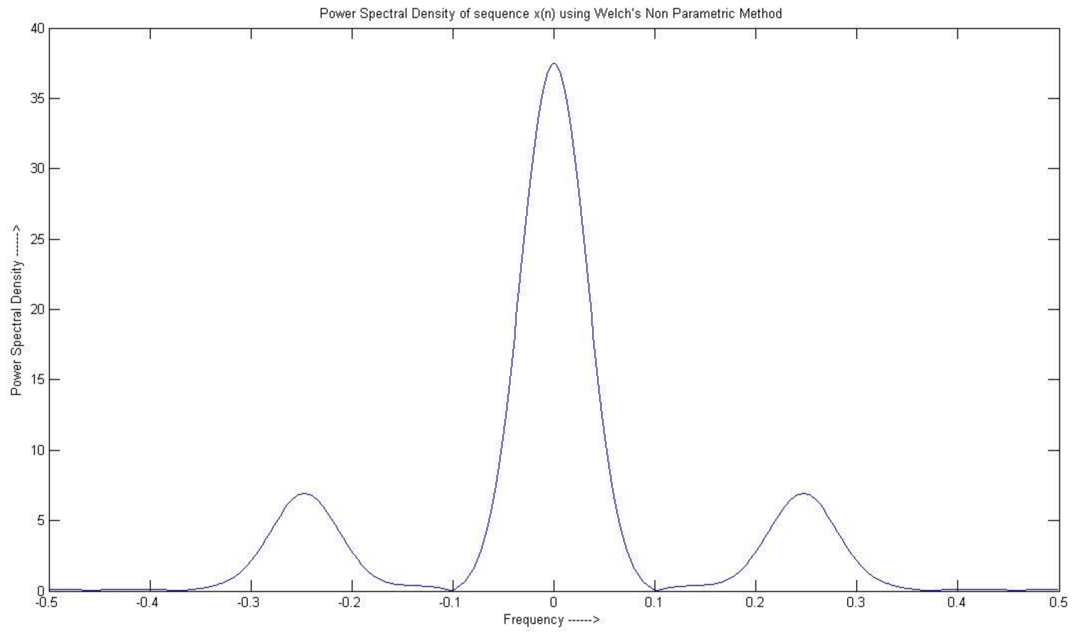
However, as mentioned above, the combined use of short data records and nonrectangular windows results in reduced resolution of the estimator. In summary, there is a trade-off between variance reduction and resolution. One can manipulate the parameters in Welch's method to obtain improved estimates relative to the periodogram, especially when the SNR is low.

4.1.3 Source Code:

```
close all;clear all;clc;

N=128;
N_point_dft=512;
y=normrnd(0,1,1,N);
B=1;
A=[1 -0.9 0.81 -0.729];

x=filter(B,A,y);
```

```
x_size=size(x);

L=8;
D=2;
M=ceil((x_size(1,2)+(L-1)*D)/L);
k=1;
x_samples=zeros(L,M);

for Rs=1:L
    for j=1:M
        if k <= x_size(1,2)
            x_samples(Rs,j)=x(1,k);
        end
        k=k+1;
    end
    k=k-D;
end

n=0:M-1;
sum=0;
for N=1:M
    w(1,N)=0.54-0.46*cos(2*pi*n(1,N)/(M-1));
    sum=sum+w(1,N)*w(1,N);
end

U=sum/M;

for Rs=1:L
    for n=1:M
        W(Rs,n)=x_samples(Rs,n).*w(1,n);
    end
    dft_sq(Rs,:)=fft(W(Rs,:),N_point_dft);
end


for Rs=1:L
    for n=1:N_point_dft
        p(Rs,n)=(dft_sq(Rs,n).*dft_sq(Rs,n))./(M*U);
    end
end


P=zeros(1,N_point_dft);
for n=1:N_point_dft
    for Rs=1:L
        P(1,n)= P(1,n)+p(Rs,n);
    end
    P(1,n)=P(1,n)./L;
end

plot(-.5:1/N_point_dft:.5-1/N_point_dft,fftshift(abs(P)));
title('Power Spectral Density of sequence x(n) using Welch's Non
Parametric Method ');
xlabel('Frequency ------>');ylabel('Power Spectral Density ------>')
```

4.1.4 RESULT:



Yule-Walker AR Parametric Method

4.2.1 AIM:

To study and estimate the power spectrum of a random sequence x (n) using Yule-Walker AR Parametric Method in MATLAB.

4.2.2 THEORY:

Parametric methods can yield higher resolutions than nonparametric methods in cases when the signal length is short. These methods use a different approach to spectral estimation; instead of trying to estimate the PSD directly from the data, they model the data as the output of a linear system driven by white noise, and then attempt to estimate the parameters of that linear system.

The most commonly used linear system model is the all-pole model, a filter with all of its zeroes at the origin in the z-plane. The output of such a filter for white noise input is an autoregressive (AR) process. For this reason, these methods are sometimes referred to as AR methods of spectral estimation.

The Yule-Walker AR method of spectral estimation computes the AR parameters by forming a biased estimate of the signal's autocorrelation function, and solving the least squares minimization of the forward prediction error. This results in the Yule-Walker equations

$$\begin{bmatrix} r(1) & r(2)^* & \cdots & r(p)^* \\ r(2) & r(1) & \cdots & r(p-1)^* \\ \vdots & \ddots & \ddots & \vdots \\ r(p) & \cdots & r(2) & r(1) \end{bmatrix} \begin{bmatrix} a(2) \\ a(3) \\ \vdots \\ a(p+1) \end{bmatrix} = \begin{bmatrix} -r(2) \\ -r(3) \\ \vdots \\ -r(p+1) \end{bmatrix}$$

The use of a biased estimate of the autocorrelation function ensures that the autocorrelation matrix above is positive definite. Hence, the matrix is invertible and a solution is guaranteed to exist. Moreover, the AR parameters thus computed always result in a stable all-pole model.

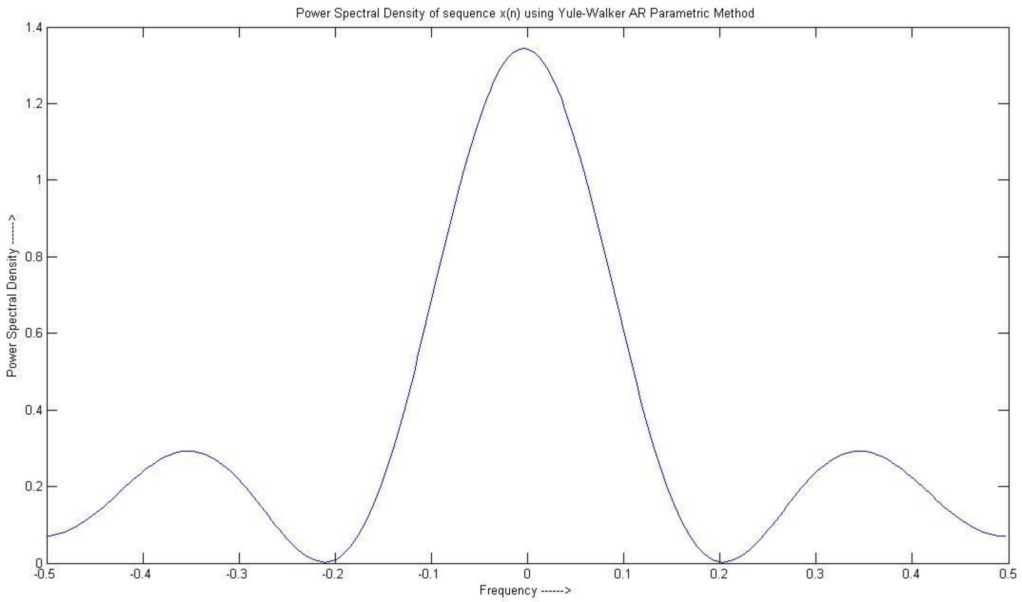
The toolbox function pyulear implements the Yule-Walker AR method.

4.1.3 Source Code:

```
close all;clc;clear all;

N=6;
p=512;
N_point_dft=256;

x=rand(1,N);
x_size=size(x);
r=zeros(2*p,1);

for m=1:2*p
    for n=1:N-m
        r(m,1)=r(m,1)+x(n)*x(n+m-1);
    end
    r(m,1)= (r(m,1)./N);
end

R=zeros(p+1,p+1);
for i=1:p+1
    for j=1:p+1
        R(i,j)= (r(abs(i-j)+1,1));
    end
end

R_auto=R(1:p,1:p);
C=R(2:p+1,1);

A=R_auto\(-1.*C);

var=R_auto(1,1);
for k=1:p
    var=var+A(k,1).*C(k,1);
```

```
end

sum=ones(1,N_point_dft);

for f=1:N_point_dft
    for k=1:p
        sum(1,f)=sum(1,f)+A(k,1).*exp(-1i*2*pi*f/N_point_dft*k);
    end
end

P=var./(sum.*conj(sum));
plot(-.5:1/N_point_dft:.5-1/N_point_dft,fftshift(P));
title('Power Spectral Density of sequence x(n) using Yule-Walker AR
Parametric Method ');
xlabel('Frequency ------>');ylabel('Power Spectral Density ------
>');
```

4.2.4 Result:



Discussion:

Deep Kiran Shroti

- The power density estimation is done by two methods i.e. parametric and non-parametric. For parametric estimation we used Yule-Walker AR method and for non-parametric we used Welch’s Non Parametric Method.

- In Yule Walker method we predicted the coefficients and then found the PSD.

- The Welch’s method consists of dividing the time series data into (possibly overlapping) segments, computing a modified periodogram of each segment, and then averaging the PSD estimates.

DISCUSSION: VED PRAKASH TIWARI

- For parametric estimation of power spectral density we have to model the source with help of coefficients obtained using autocorrelation values.

- In Yule-Walker method, the processing of the random signal has its own limitations with increasing size of the signal.

- We can have a proper overlapping interval ideally being 50% of the interval chosen to have smaller variance of the value of PSD in Welch method.

- In Welch method, choosing the length of interval and percentage of overlap is trade-off between frequency resolution and variance.