4.13.10

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October 3, 2025

Question

$$\mathbf{A} = \begin{pmatrix} -1 \\ -7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 \\ 10 \end{pmatrix}$$

Equation of the internal bisector of $\angle ABC$.

Solution

$$\text{Given lines } \mathbf{n}_1^\top \mathbf{x} = c_1, \quad \mathbf{n}_2^\top \mathbf{x} = c_2,$$

$$\text{Angle bisectors satisfy } \frac{|\mathbf{n}_1^\top \mathbf{x} - c_1|}{\|\mathbf{n}_1\|} = \frac{|\mathbf{n}_2^\top \mathbf{x} - c_2|}{\|\mathbf{n}_2\|}.$$

$$\Rightarrow \quad \frac{\mathbf{n}_1^\top \mathbf{x} - c_1}{\|\mathbf{n}_1\|} = \pm \frac{\mathbf{n}_2^\top \mathbf{x} - c_2}{\|\mathbf{n}_2\|}.$$

Internal bisector: sign chosen so that for $\mathbf{u} = \dfrac{\mathbf{A}-\mathbf{B}}{\|\mathbf{A}-\mathbf{B}\|} + \dfrac{\mathbf{C}-\mathbf{B}}{\|\mathbf{C}-\mathbf{B}\|}$,

we have $\operatorname{sgn}(\mathbf{n}_1^\top \mathbf{u}) = -\operatorname{sgn}(\mathbf{n}_2^\top \mathbf{u})$

Normals:

$$\mathbf{n}_1 = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$$
$$\mathbf{n}_2 = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$$

Norms:

$$\|\mathbf{n}_1\| = 10$$
$$\|\mathbf{n}_2\| = \sqrt{97}$$

Solution

Equation of angle bisector:

$$\frac{\mathbf{n}_1^\top(\mathbf{x}-\mathbf{B})}{10} = -\frac{\mathbf{n}_2^\top(\mathbf{x}-\mathbf{B})}{\sqrt{97}}$$

$$\sqrt{97}\,\mathbf{n}_1^\top(\mathbf{x}-\mathbf{B}) + 10\,\mathbf{n}_2^\top(\mathbf{x}-\mathbf{B}) = 0$$

Solution

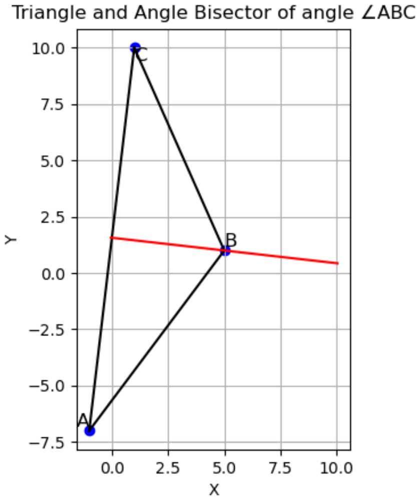
With $\mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$, equation of Angle Bisector is:

$$\boxed{\begin{pmatrix} 90 - 8\sqrt{97} & 40 + 6\sqrt{97} \end{pmatrix}^T \mathbf{x} + (34\sqrt{97} - 490) = 0}$$

Or,

$$\boxed{\mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 40 + 6\sqrt{97} \\ -(90 - 8\sqrt{97}) \end{pmatrix}, \quad \lambda \in \mathbb{R}}$$

Plot



Triangle and Angle Bisector of angle ∠ABC
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
Y
0.0
2.5
5.0
7.5
10.0
X
A
B
C

Codes

For Codes, refer to the URL below:
https://github.com/Aditya-Mishra11005/ee1030-2025/tree/temp/ee25btech11005/matgeo/4.13.10/Codes