# 4.13.10

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## Question

$$\mathbf{A} = \begin{pmatrix} -1 \\ -7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 \\ 10 \end{pmatrix}$$

Equation of the internal bisector of $\angle ABC$.

## Solution

$$\text{Given lines } \mathbf{n}_1^\top \mathbf{x} = c_1, \quad \mathbf{n}_2^\top \mathbf{x} = c_2,$$

$$\text{Angle bisectors satisfy } \frac{|\mathbf{n}_1^\top \mathbf{x} - c_1|}{\|\mathbf{n}_1\|} = \frac{|\mathbf{n}_2^\top \mathbf{x} - c_2|}{\|\mathbf{n}_2\|}.$$

$$\Rightarrow \quad \frac{\mathbf{n}_1^\top \mathbf{x} - c_1}{\|\mathbf{n}_1\|} = \pm \frac{\mathbf{n}_2^\top \mathbf{x} - c_2}{\|\mathbf{n}_2\|}.$$

Internal bisector: sign chosen so that for $\mathbf{u} = \dfrac{\mathbf{A} - \mathbf{B}}{\|\mathbf{A} - \mathbf{B}\|} + \dfrac{\mathbf{C} - \mathbf{B}}{\|\mathbf{C} - \mathbf{B}\|}$, we have $\operatorname{sgn}(\mathbf{n}_1^\top \mathbf{u}) = -\operatorname{sgn}(\mathbf{n}_2^\top \mathbf{u})$

Normals:

$$\mathbf{n}_1 = \begin{pmatrix} -8 \\ 6 \end{pmatrix} \tag{1}$$

$$\mathbf{n}_2 = \begin{pmatrix} 9 \\ 4 \end{pmatrix} \tag{2}$$

Norms:

$$\|\mathbf{n}_1\| = \sqrt{(-8)^2 + 6^2} = 10 \tag{3}$$

$$\|\mathbf{n}_2\| = \sqrt{9^2 + 4^2} = \sqrt{97} \tag{4}$$

Equation of angle bisector:

$$\frac{\mathbf{n}_1^\top(\mathbf{x} - \mathbf{B})}{10} = -\frac{\mathbf{n}_2^\top(\mathbf{x} - \mathbf{B})}{\sqrt{97}}$$

Thus,

$$\sqrt{97}\,\mathbf{n}_1^\top(\mathbf{x} - \mathbf{B}) + 10\,\mathbf{n}_2^\top(\mathbf{x} - \mathbf{B}) = 0$$

With $\mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$, equation of Angle Bisector is:

$$\boxed{\begin{pmatrix} 90 - 8\sqrt{97} \\ 40 + 6\sqrt{97} \end{pmatrix}^T \mathbf{x} + (34\sqrt{97} - 490) = 0}$$

Or,

$$\boxed{\mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 40 + 6\sqrt{97} \\ -(90 - 8\sqrt{97}) \end{pmatrix}, \quad \lambda \in \mathbb{R}}$$

**Plot**