4.10.3

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Question

Find the vector equation of the plane passing through the intersection of the planes

$$\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6 \quad \text{and} \quad \mathbf{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5,$$

and the point $(1, 1, 1)$.

Solution

Given planes:

$$\mathbf{n_1}^\top \mathbf{x} = c_1, \quad \mathbf{n_2}^\top \mathbf{x} = c_2,$$

with point $\mathbf{P}$.

General solution for the plane passing through the intersection:

$$\mathbf{n_1}^\top \mathbf{x} - c_1 + \lambda(\mathbf{n_2}^\top \mathbf{x} - c_2) = 0$$

where

$$\lambda = \frac{c_1 - \mathbf{n_1}^\top \mathbf{P}}{\mathbf{n_2}^\top \mathbf{P} - c_2}$$

Given:

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad c_1 = 6, \quad \mathbf{n_2} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \quad c_2 = -5, \quad \mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Evaluate:

$$\mathbf{n_1}^\top \mathbf{P} = 3, \quad \mathbf{n_2}^\top \mathbf{P} = 9$$

$$\lambda = \frac{6-3}{9+5} = \frac{3}{14}$$

Substituting all into the general family and simplifying:

$$\mathbf{n_1}^\top \mathbf{x} - 6 + \frac{3}{14}(\mathbf{n_2}^\top \mathbf{x} + 5) = 0$$

Expanding:

$$\left(\begin{pmatrix}1\\1\\1\end{pmatrix} + \frac{3}{14}\begin{pmatrix}2\\3\\4\end{pmatrix}\right)^\top \mathbf{x} = 6 - \frac{15}{14}$$

That is,

$$\begin{pmatrix}\frac{20}{14}\\\frac{23}{14}\\\frac{26}{14}\end{pmatrix}^\top \mathbf{x} = \frac{69}{14}$$

Or, in simplest integer form, the equation of plane is:

$$\boxed{(20 \quad 23 \quad 26)\mathbf{x} = 69}$$