

## 5.3.39

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### Question

Using matrices, solve the following system of equations:

$$\begin{aligned}x + y + z &= 6 \\x + 2z &= 7 \\3x + y + z &= 12\end{aligned}$$

### Solution

Forming the augmented matrix,

$$\left( \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 0 & 2 & 7 \\ 3 & 1 & 1 & 12 \end{array} \right)$$

Perform row operations to reduce to row echelon form:

$$\left( \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 0 & 2 & 7 \\ 3 & 1 & 1 & 12 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 - R_1} \left( \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 3 & 1 & 1 & 12 \end{array} \right)$$

$$\xrightarrow{R_3 \rightarrow R_3 - 3R_1} \left( \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & -2 & -2 & -6 \end{array} \right)$$

$$\xrightarrow{R_3 \rightarrow R_3 - 2R_2} \left( \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -4 & -8 \end{array} \right)$$

From the third row:

$$-4z = -8 \implies z = 2$$

From the second row:

$$-y + z = 1 \implies -y + 2 = 1 \implies y = 1$$

From the first row:

$$x + y + z = 6 \implies x + 1 + 2 = 6 \implies x = 3$$

Thus, the solution is :

$$\boxed{\mathbf{x} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}}$$

# Plot

Intersection of Three Planes

