9.4.17

EE25BTECH11005 - Aditya Mishra

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Question

Find the roots of the quadratic equation graphically

$$2x^2 + x - 4 = 0$$

Matrix Representation

Write the quadratic as a conic in matrix form:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$$

where

$$\mathbf{V} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}, \quad f = -4$$

Roots via Line Intersection

The roots correspond to intersection points of the conic with the x-axis:

$$\mathbf{x} = \mathbf{h} + k\mathbf{m}$$

where

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Solving for k

The value of k can be found by solving the line and conic equation:

$$(\mathbf{h} + k\mathbf{m})^\top \mathbf{V}(\mathbf{h} + k\mathbf{m}) + 2\mathbf{u}^\top(\mathbf{h} + k\mathbf{m}) + f = 0 \tag{1}$$

$$\implies k^2\mathbf{m}^\top\mathbf{V}\mathbf{m} + 2k\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) + g(\mathbf{h}) = 0 \tag{2}$$

Solution for k

Solving the quadratic formula:

$$k = \frac{1}{\mathbf{m}^\top\mathbf{V}\mathbf{m}}\left[-\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{(\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}))^2 - g(\mathbf{h})(\mathbf{m}^\top\mathbf{V}\mathbf{m})}\right] \tag{3}$$

Substitute values for our quadratic:

$$\mathbf{V} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}, \quad f = -4, \quad \mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{m}^\top\mathbf{V}\mathbf{m} = 2, \quad \mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) = \frac{1}{2}, \quad g(\mathbf{h}) = -4$$

$$\therefore k = \frac{1}{2}\Big[-\frac{1}{2} \pm \sqrt{\frac{1}{4} + 8}\Big] = \frac{1}{2}\Big[-\frac{1}{2} \pm \sqrt{\frac{33}{4}}\Big]$$

$$k_1 = \frac{-1 + \sqrt{33}}{4} \approx 1.186, \quad k_2 = \frac{-1 - \sqrt{33}}{4} \approx -1.686$$

Roots of the Quadratic

The intersection points:

$$\mathbf{x}_1 = \mathbf{h} + k_1\mathbf{m} = \begin{pmatrix} 1.186 \\ 0 \end{pmatrix}, \quad \mathbf{x}_2 = \mathbf{h} + k_2\mathbf{m} = \begin{pmatrix} -1.686 \\ 0 \end{pmatrix}$$

$\Rightarrow$ The roots of $2x^2 + x - 4 = 0$ are $x \approx 1.186$ and $x \approx -1.686$

Graphical Representation

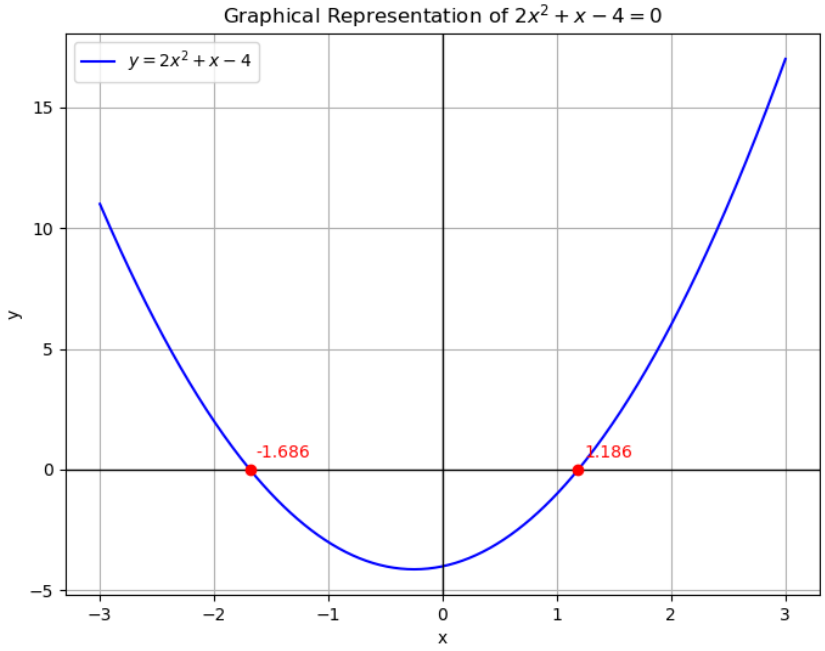

Figure 1: Graph of $2x^2 + x - 4 = 0$ showing its roots.