

4.10.3

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Question

Find the vector equation of the plane passing through the intersection of the planes

$$\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6 \quad \text{and} \quad \mathbf{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5,$$

and the point $(1, 1, 1)$.

Solution

Given planes:

$$\mathbf{n}_1^\top \mathbf{x} = c_1, \quad \mathbf{n}_2^\top \mathbf{x} = c_2,$$

with point

$$\mathbf{P} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}.$$

General form:

$$\mathbf{n}_1^\top \mathbf{x} - c_1 + \lambda(\mathbf{n}_2^\top \mathbf{x} - c_2) = 0, \quad \lambda = \frac{c_1 - \mathbf{n}_1^\top \mathbf{P}}{\mathbf{n}_2^\top \mathbf{P} - c_2}.$$

Solution

Given:

$$\mathbf{n}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad c_1 = 6, \quad \mathbf{n}_2 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \quad c_2 = -5,$$

$$\mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Evaluate:

$$\mathbf{n}_1^\top \mathbf{P} = 3, \quad \mathbf{n}_2^\top \mathbf{P} = 9, \quad \lambda = \frac{3}{14}.$$

Solution

Plane equation:

$$\mathbf{n}_1^\top \mathbf{x} - 6 + \frac{3}{14} (\mathbf{n}_2^\top \mathbf{x} + 5) = 0.$$

Simplified:

$$\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \frac{3}{14} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \right)^\top \mathbf{x} = 6 - \frac{15}{14}.$$

Or

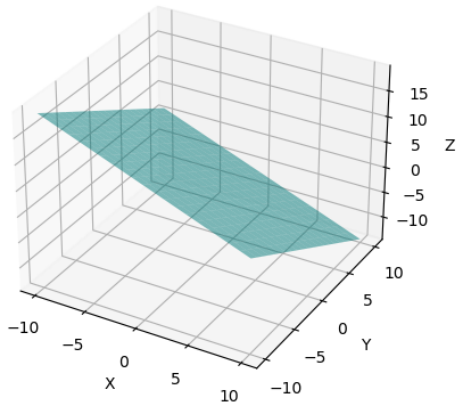
$$\begin{pmatrix} \frac{20}{14} \\ \frac{23}{14} \\ \frac{26}{14} \end{pmatrix}^\top \mathbf{x} = \frac{69}{14},$$

which gives integer form

$$\boxed{(20 \quad 23 \quad 26)\mathbf{x} = 69}.$$

Plot

$$\text{Plane: } 20x + 23y + 26z = 69$$



For Codes, refer to the URL below:

[https://github.com/Aditya-Mishra11005/ee1030-2025/tree/
temp/ee25btech11005/matgeo/4.10.3/Codes](https://github.com/Aditya-Mishra11005/ee1030-2025/tree/temp/ee25btech11005/matgeo/4.10.3/Codes)