4.10.3

Aditya Mishra- EE25BTECH

September 30, 2025

Question

Find the vector equation of the plane passing through the intersection of the planes

$$\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6 \quad \text{and} \quad \mathbf{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5,$$

and the point $(1, 1, 1)$.

Solution

Given planes:

$$\mathbf{n_1}^\top \mathbf{x} = c_1, \quad \mathbf{n_2}^\top \mathbf{x} = c_2,$$

with point

$$\mathbf{P} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}.$$

General form:

$$\mathbf{n_1}^\top \mathbf{x} - c_1 + \lambda(\mathbf{n_2}^\top \mathbf{x} - c_2) = 0, \quad \lambda = \frac{c_1 - \mathbf{n_1}^\top \mathbf{P}}{\mathbf{n_2}^\top \mathbf{P} - c_2}.$$

Solution

Given:

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad c_1 = 6, \quad \mathbf{n_2} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \quad c_2 = -5,$$

$$\mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Evaluate:

$$\mathbf{n_1}^\top \mathbf{P} = 3, \quad \mathbf{n_2}^\top \mathbf{P} = 9, \quad \lambda = \frac{3}{14}.$$

Solution

Plane equation:

$$\mathbf{n_1}^\top \mathbf{x} - 6 + \frac{3}{14}\left(\mathbf{n_2}^\top \mathbf{x} + 5\right) = 0.$$

Simplified:

$$\left(\begin{pmatrix}1\\1\\1\end{pmatrix} + \frac{3}{14}\begin{pmatrix}2\\3\\4\end{pmatrix}\right)^\top \mathbf{x} = 6 - \frac{15}{14}.$$

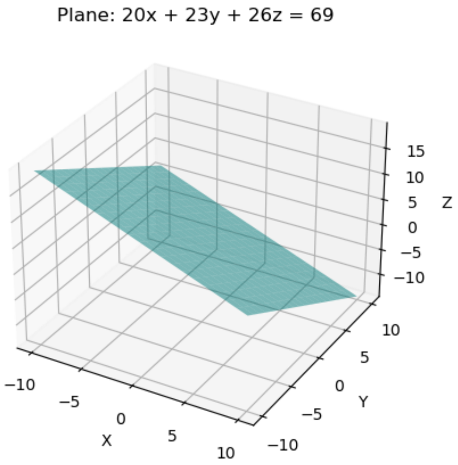
Or

$$\begin{pmatrix}\frac{20}{14}\\\frac{23}{14}\\\frac{26}{14}\end{pmatrix}^\top \mathbf{x} = \frac{69}{14},$$

which gives integer form

$$\boxed{\begin{pmatrix}20 & 23 & 26\end{pmatrix}\mathbf{x} = 69}.$$

Plot



Codes

For Codes, refer to the URL below:

`https://github.com/Aditya-Mishra11005/ee1030-2025/tree/main/ee25btech11005/matgeo/4.10.3/Codes`