7.4.12

Aditya Mishra — EE25BTECH11005

October 12, 2025

Question

The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. Find the locus of any point $\mathbf{x}$ in the set.

Solution

For a circle,

$$\mathbf{u} = -\mathbf{c}, \quad f = \|\mathbf{u}\|^2 - r^2 \tag{1}$$

Hence, the general equation is

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2}$$

For this family, since the centre lies on a circle, $\|\mathbf{u}\| = 5$, $r = 3$, $f = 25 - 9 = 16$. Thus, any point on such a circle satisfies

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^T\mathbf{x} + 16 = 0 \tag{3}$$

$$2\mathbf{u}^T\mathbf{x} = -\|\mathbf{x}\|^2 - 16 \tag{4}$$

Since $\|\mathbf{u}\| = 5$, by Cauchy-Schwarz inequality:

$$-5\|\mathbf{x}\| \le \mathbf{u}^T\mathbf{x} \le 5\|\mathbf{x}\| \tag{5}$$

The minimum value of $2\mathbf{u}^T\mathbf{x}$ is $-10\|\mathbf{x}\|$. Hence, existence requires

$$-10\|\mathbf{x}\| \le -\|\mathbf{x}\|^2 - 16 \tag{6}$$

$$\Rightarrow \|\mathbf{x}\|^2 - 10\|\mathbf{x}\| + 16 \le 02 \le \|\mathbf{x}\| \le 8 \quad \Rightarrow \quad 4 \le \mathbf{x}^T\mathbf{x} \le 64 \tag{7}$$

$$\boxed{4 \le x^2 + y^2 \le 64}$$

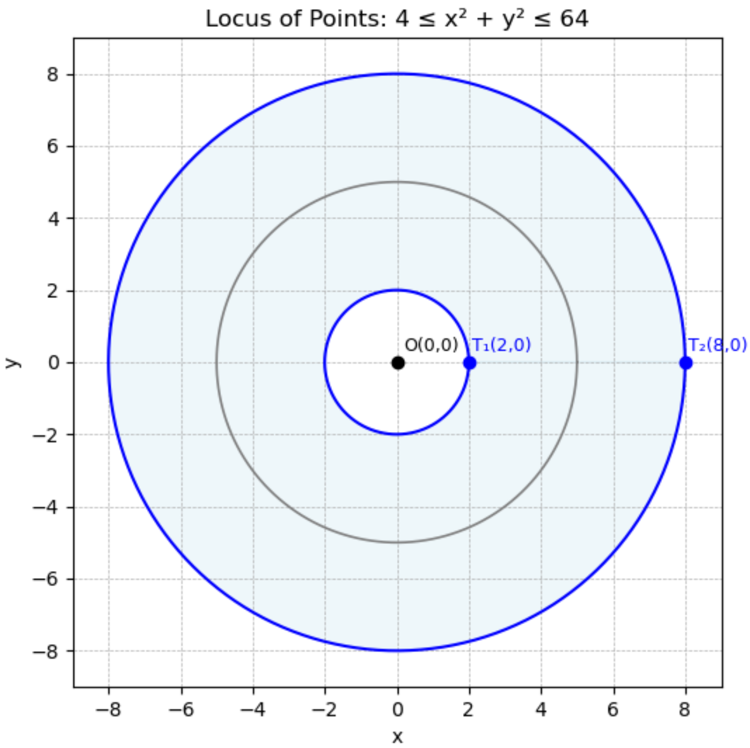

Figure 1: Locus of points for circles of radius 3 with centers on $x^2 + y^2 = 25$.