

5.13.31

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Question

Let \mathbf{A} and \mathbf{B} be 3×3 matrices of real numbers, where \mathbf{A} is symmetric, \mathbf{B} is skew-symmetric and

$$(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B}) = (\mathbf{A} - \mathbf{B})(\mathbf{A} + \mathbf{B}).$$

If

$$(\mathbf{AB})^T = (-1)^k \mathbf{AB},$$

where $(\mathbf{AB})^T$ is the transpose of the matrix \mathbf{AB} , find the value(s) of k .

Matrix Properties

Given:

A is symmetric: $\mathbf{A}^T = \mathbf{A}$

B is skew-symmetric: $\mathbf{B}^T = -\mathbf{B}$

Expanding the Given Equation

Expand both sides of

$$(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B}) = (\mathbf{A} - \mathbf{B})(\mathbf{A} + \mathbf{B})$$

$$\mathbf{A}^2 - \mathbf{AB} + \mathbf{BA} - \mathbf{B}^2 = \mathbf{A}^2 + \mathbf{AB} - \mathbf{BA} - \mathbf{B}^2$$

$$2\mathbf{BA} = 2\mathbf{AB}$$

$$\implies \mathbf{AB} = \mathbf{BA}$$

So \mathbf{A} and \mathbf{B} commute.

Transpose of the Product

We have:

$$(\mathbf{AB})^{\top} = \mathbf{B}^{\top} \mathbf{A}^{\top}$$

Using properties,

$$= (-\mathbf{B})\mathbf{A} = -\mathbf{BA}$$

But since they commute:

$$= -\mathbf{AB}$$

Thus,

$$(\mathbf{AB})^{\top} = -\mathbf{AB}$$

Solving for k

Given,

$$(\mathbf{AB})^T = (-1)^k \mathbf{AB}$$

So,

$$-\mathbf{AB} = (-1)^k \mathbf{AB}$$

If $\mathbf{AB} \neq 0$, comparing:

$$-1 = (-1)^k$$

So k must be odd.

k is any odd integer.

For Codes, refer to the URL below:

<https://github.com/Aditya-Mishra11005/ee1030-2025/tree/main/ee25btech11005/matgeo/5.13.31/Codes>