2.10.34

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Question

Let $\mathbf{a} = \mathbf{i} - \mathbf{j}$, $\mathbf{b} = \mathbf{j} - \mathbf{k}$, $\mathbf{c} = \mathbf{k} - \mathbf{i}$. If $\mathbf{d}$ is a unit vector such that $\mathbf{a} \cdot \mathbf{d} = 0 = [\mathbf{b}\ \mathbf{c}\ \mathbf{d}]$, then $\mathbf{d}$ equals

1. $\pm\frac{\mathbf{i}+\mathbf{j}-2\mathbf{k}}{\sqrt{6}}$
2. $\pm\frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$
3. $\pm\frac{\mathbf{i}+\mathbf{j}-\mathbf{k}}{\sqrt{3}}$
4. $\pm\mathbf{i} + \mathbf{j} - \mathbf{k}$

Solution

Given:

$$[\mathbf{b}\ \mathbf{c}\ \mathbf{d}] = 0 \tag{1}$$

This implies $\mathbf{d}$ is linearly dependant on $\mathbf{b}$ and $\mathbf{c}$

$$\implies \mathbf{d} = k \begin{pmatrix} \mathbf{b} & \mathbf{c} \end{pmatrix} \begin{pmatrix} \lambda \\ 1 \end{pmatrix} \tag{2}$$

Now, since $\mathbf{a}^T\mathbf{d} = 0$,

$$k\mathbf{a}^T(\mathbf{c} + \lambda\mathbf{b}) = 0 \tag{3}$$

$$\implies \mathbf{a}^T\mathbf{c} + \lambda\mathbf{a}^T\mathbf{b} = 0 \tag{4}$$

$$\lambda = -\frac{\mathbf{a}^T\mathbf{c}}{\mathbf{a}^T\mathbf{b}} \tag{5}$$

Substitute this value of λ back:

$$\mathbf{d} = k(\mathbf{c} - \frac{\mathbf{a}^T\mathbf{c}}{\mathbf{a}^T\mathbf{b}}\mathbf{b}) \tag{6}$$

To find k:

$$\|\mathbf{d}| \ = 1 \tag{7}$$

$$k = \pm\frac{1}{\|\mathbf{c} - \frac{\mathbf{a}^T\mathbf{c}}{\mathbf{a}^T\mathbf{b}}\mathbf{b}\|} \tag{8}$$

$$\mathbf{d} = \pm\frac{(\mathbf{c} - \frac{\mathbf{a}^T\mathbf{c}}{\mathbf{a}^T\mathbf{b}}\mathbf{b})}{\|\mathbf{c} - \frac{\mathbf{a}^T\mathbf{c}}{\mathbf{a}^T\mathbf{b}}\mathbf{b}\|} \tag{9}$$

Substituting Numbers

Given:

$$\mathbf{a} = \mathbf{i} - \mathbf{j}, \quad \mathbf{b} = \mathbf{j} - \mathbf{k}, \quad \mathbf{c} = \mathbf{k} - \mathbf{i} \tag{10}$$

So,

$$\mathbf{d} = \pm\frac{\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{\sqrt{6}} \tag{11}$$

Answer: Option 1

$$\boxed{\mathbf{d} = \pm\frac{\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{\sqrt{6}}}$$