4.13.10

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Question

The vertices of a triangle are

$$\mathbf{A} = \begin{pmatrix} -1 \\ -7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 \\ 10 \end{pmatrix}.$$

Find the equation of the bisector of the angle $\angle ABC$.

Solution

Define vectors

$$\mathbf{D} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} -1-5 \\ -7-1 \end{pmatrix} = \begin{pmatrix} -6 \\ -8 \end{pmatrix}, \tag{1}$$

$$\mathbf{E} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 1-5 \\ 10-1 \end{pmatrix} = \begin{pmatrix} -4 \\ 9 \end{pmatrix}. \tag{2}$$

Compute magnitudes:

$$\|\mathbf{D}\| = \sqrt{(-6)^2 + (-8)^2} = 10, \tag{3}$$

$$\|\mathbf{E}\| = \sqrt{(-4)^2 + 9^2} = \sqrt{97}. \tag{4}$$

Normalize:

$$\mathbf{e}_D = \frac{1}{10}\begin{pmatrix} -6 \\ -8 \end{pmatrix}, \tag{5}$$

$$\mathbf{e}_E = \frac{1}{\sqrt{97}}\begin{pmatrix} -4 \\ 9 \end{pmatrix}. \tag{6}$$

The angle bisector vector $\mathbf{L}$ is along the sum of normalized vectors:

$$\mathbf{L} = \mathbf{e}_D + \mathbf{e}_E = \begin{pmatrix} -\frac{6}{10} - \frac{4}{\sqrt{97}} \\ -\frac{8}{10} + \frac{9}{\sqrt{97}} \end{pmatrix}. \tag{7}$$

The line along the bisector passes through $\mathbf{B}$, so the vector equation is:

$$\mathbf{x} = \mathbf{B} + \lambda \mathbf{L}, \quad \lambda \in \mathbb{R}. \tag{8}$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -\frac{6}{10} - \frac{4}{\sqrt{97}} \\ -\frac{8}{10} + \frac{9}{\sqrt{97}} \end{pmatrix}. \tag{9}$$

This is the equation of the bisector of $\angle ABC$.

Plot