2.10.34

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Question

Let $\mathbf{a} = \mathbf{i} - \mathbf{j}$, $\mathbf{b} = \mathbf{j} - \mathbf{k}$, and $\mathbf{c} = \mathbf{k} - \mathbf{i}$. If $\mathbf{d}$ is a unit vector such that

$$\mathbf{a} \cdot \mathbf{d} = 0 = [\mathbf{b}\ \mathbf{c}\ \mathbf{d}],$$

then $\mathbf{d}$ equals:

Given Condition

We have:

$$[\mathbf{b}\ \mathbf{c}\ \mathbf{d}] = 0$$

This implies that **d** is linearly dependent on **b** and **c**:

$$\mathbf{d} = k\left(\mathbf{b}\ \mathbf{c}\right)\begin{pmatrix}\lambda \\ 1\end{pmatrix}$$

Using $\mathbf{a}^T\mathbf{d} = 0$

Since $\mathbf{a}^T\mathbf{d} = 0$:

$$k\mathbf{a}^T(\mathbf{c} + \lambda\mathbf{b}) = 0$$
$$\implies \mathbf{a}^T\mathbf{c} + \lambda\mathbf{a}^T\mathbf{b} = 0$$

Thus,

$$\lambda = -\frac{\mathbf{a}^T\mathbf{c}}{\mathbf{a}^T\mathbf{b}}$$

Substitute λ back

Substituting for λ:

$$\mathbf{d} = k\left(\mathbf{c} - \frac{\mathbf{a}^T\mathbf{c}}{\mathbf{a}^T\mathbf{b}}\mathbf{b}\right)$$

For a unit vector $\mathbf{d}$:

$$\|\mathbf{d}\| = 1 \quad \implies \quad k = \pm\frac{1}{\left\|\mathbf{c} - \frac{\mathbf{a}^T\mathbf{c}}{\mathbf{a}^T\mathbf{b}}\mathbf{b}\right\|}$$

Final Vector Form

Therefore,

$$\mathbf{d} = \pm \frac{\left(\mathbf{c} - \frac{\mathbf{a}^T\mathbf{c}}{\mathbf{a}^T\mathbf{b}}\mathbf{b}\right)}{\left\|\mathbf{c} - \frac{\mathbf{a}^T\mathbf{c}}{\mathbf{a}^T\mathbf{b}}\mathbf{b}\right\|}$$

Substituting Values

Given:

$$\mathbf{a} = \mathbf{i} - \mathbf{j}, \quad \mathbf{b} = \mathbf{j} - \mathbf{k}, \quad \mathbf{c} = \mathbf{k} - \mathbf{i}$$

Hence,

$$\boxed{\mathbf{d} = \pm\frac{\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{\sqrt{6}}}$$

Codes

For Codes, refer to the URL below:
https://github.com/Aditya-Mishra11005/ee1030-2025/tree/main/ee25btech11005/matgeo/2.10.34/Codes