

7.4.12

Aditya Mishra — EE25BTECH11005

October 12, 2025

Question

The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. Find the locus of any point \mathbf{x} in the set.

Solution

For a circle,

$$\mathbf{u} = -\mathbf{c}, \quad f = \|\mathbf{u}\|^2 - r^2 \quad (1)$$

Hence, the general equation is

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2)$$

For this family,

$$\|\mathbf{u}\| = 5, \quad r = 3, \quad f = 25 - 9 = 16 \quad (3)$$

Thus,

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^T \mathbf{x} + 16 \leq 0 \quad (4)$$

$$2\mathbf{u}^T \mathbf{x} \leq -\|\mathbf{x}\|^2 - 16 \quad (5)$$

Since $\|\mathbf{u}\| = 5$, by Cauchy-Schwarz inequality:

$$-5\|\mathbf{x}\| \leq \mathbf{u}^T \mathbf{x} \leq 5\|\mathbf{x}\| \quad (6)$$

The minimum value of $2\mathbf{u}^T \mathbf{x}$ is $-10\|\mathbf{x}\|$. Hence, existence requires

$$-10\|\mathbf{x}\| \leq -\|\mathbf{x}\|^2 - 16 \quad (7)$$

$$\Rightarrow \|\mathbf{x}\|^2 - 10\|\mathbf{x}\| + 16 \leq 0 \Rightarrow 4 \leq \mathbf{x}^T \mathbf{x} \leq 64 \quad (8)$$

$$\boxed{4 \leq x^2 + y^2 \leq 64}$$

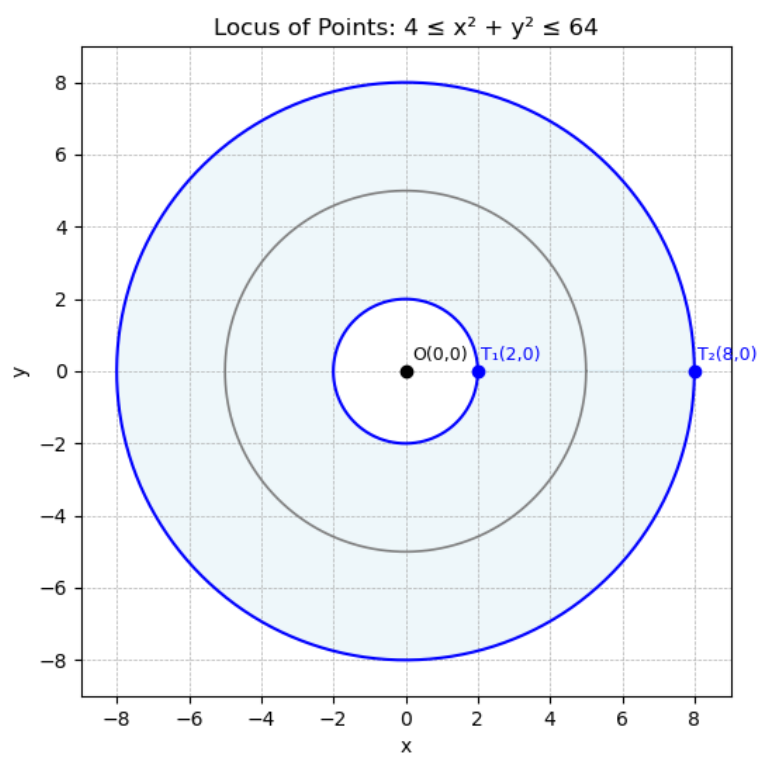


Figure 1: Locus of points for circles of radius 3 with centers on $x^2 + y^2 = 25$.