2.10.34

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Question

Let

$$\mathbf{a} = \mathbf{i} - \mathbf{j}, \quad \mathbf{b} = \mathbf{j} - \mathbf{k}, \quad \mathbf{c} = \mathbf{k} - \mathbf{i}$$

Find the unit vector $\mathbf{d}$ such that

$$\mathbf{a}^T\mathbf{d} = 0 \quad \text{and} \quad [\mathbf{b}\ \mathbf{c}\ \mathbf{d}] = 0.$$

Solution

Calculate $\mathbf{b} \times \mathbf{c}$:

$$\mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \quad \Rightarrow \quad \mathbf{b} \times \mathbf{c} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Form matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{where} \quad \mathbf{A} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \times \mathbf{c} \end{pmatrix}^\top.$$

Solving $\mathbf{A}^\top\mathbf{d} = 0$

Form augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right) \xrightarrow{R_2 - R_1} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right)$$

Back-substitution gives:

$$\mathbf{d} = k \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

where k is a scalar.

Since $\mathbf{d}$ is a unit vector:

$$\|\mathbf{d}\| = 1 \implies k = \pm\frac{1}{\sqrt{6}}$$

$$\boxed{\mathbf{d} = \pm\frac{\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{\sqrt{6}}}$$