

VECTORS

Scalar

↓
Magnitude

Scalar quantity

- ⇒ Work
- ⇒ Speed
- ⇒ Distance

Vector



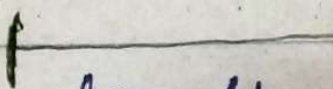
Magnitude

Direction

Vector quantity

- ⇒ Force
- ⇒ Velocity
- ⇒ Displacement

Representation of Vector

 length = Magnitude

$|A| = 4 \text{ unit}$ (Magnitude)

$\vec{A} = 4 \text{ unit south}$ (Direction)

Types of Vector

- ① Equal and Unequal.
- ② Parallel and antiparallel.
- ③ Collinear.
- ④ Concurrent.
- ⑤ Coplanar.
- ⑥ Zero
- ⑦ Unit.

⇒ Equal and Unequal Vector

For two vectors to be similar \vec{A} and \vec{B} should have equal magnitude and have same direction.

$$\vec{v}_1 = 5 \text{ m/sec East} \quad \vec{v}_2 = 5 \text{ m/sec West}$$

Unequal Vectors.

⇒ Parallel Vector

• Same direction

$$\theta = 0^\circ$$

⇒ Antiparallel Vector

• Opposite direction

$$\theta = 180^\circ$$

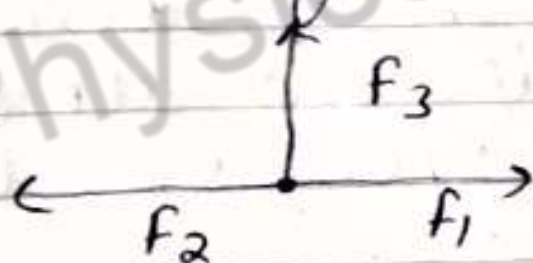
⇒ Collinear.
In a same line.
→ → →

⇒ coplanar (In a single plane)

- * 2 vectors are always coplanar.
- * 3 vectors may be coplanar or may be not. (They may lie or may not lie on a similar plane).

⇒ Concurrent Vector

* Forces (Acting at same point)



⇒ Zero vector

whose magnitude is 0. and direction is arbitrary. (It can take any direction).

⇒ Unit Vector $[\hat{A}]$.

\hat{x} in x -direction, \hat{y} in y -direction

Magnitude = 1
→ It gives direction

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x \hat{i} + A_y \hat{j} + A_z \hat{k}}{\sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}}$$

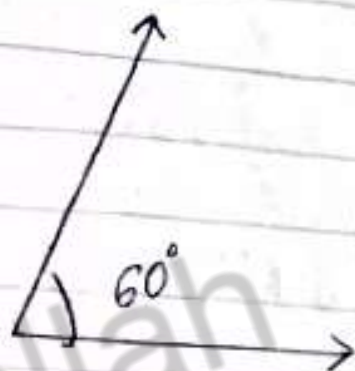
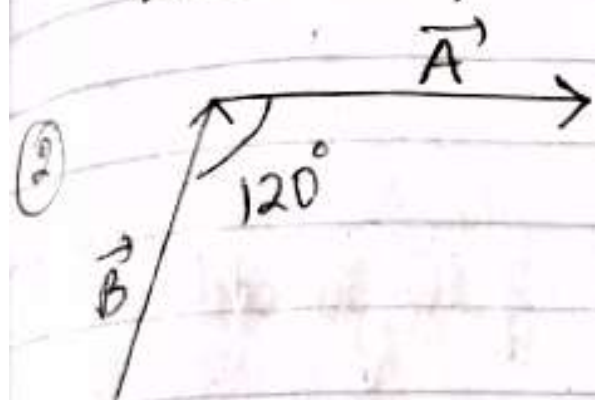
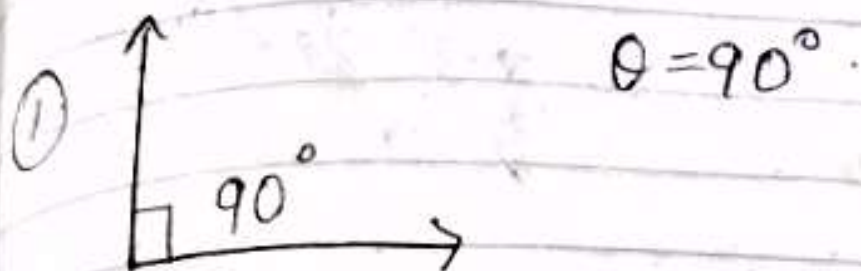
Parallel shift of vector.



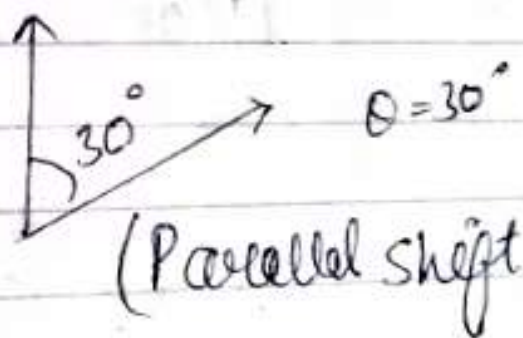
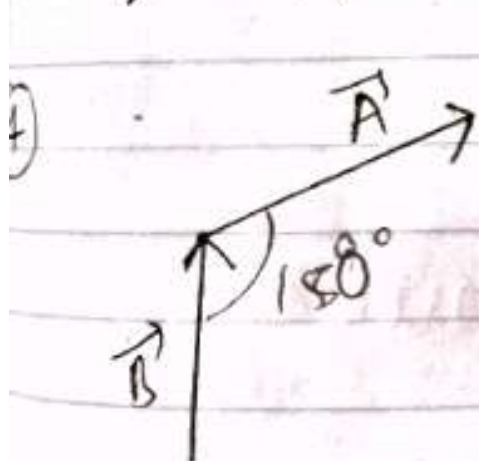
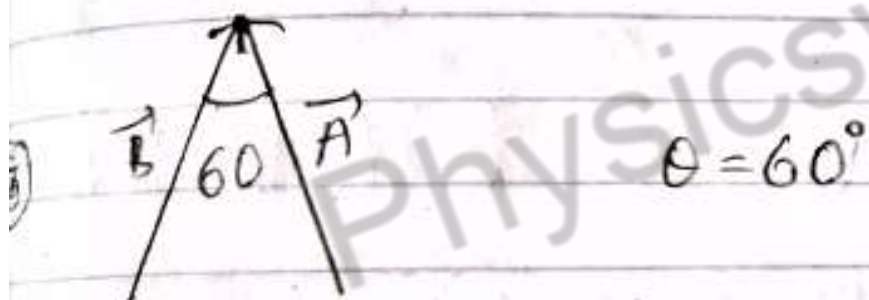
We can shift a vector, as it should be parallel shift and the shift should be on the same body.

If a force of 10N is applied on a body, it can be shifted parallel on the same body.

Angles between two vectors
(Tail to tail or head to head)

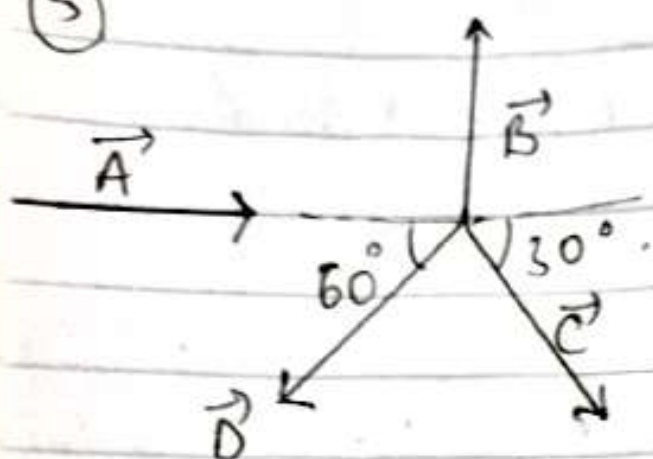


(Parallel shift)

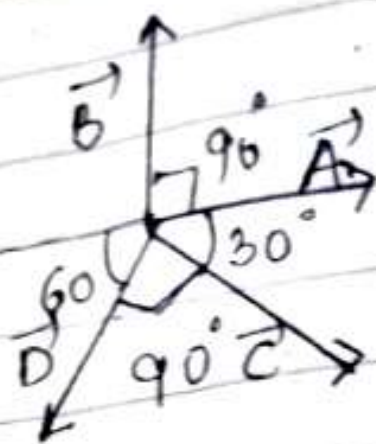


(Parallel shift)

⑤



$$(0 \leq \theta \leq 180)$$

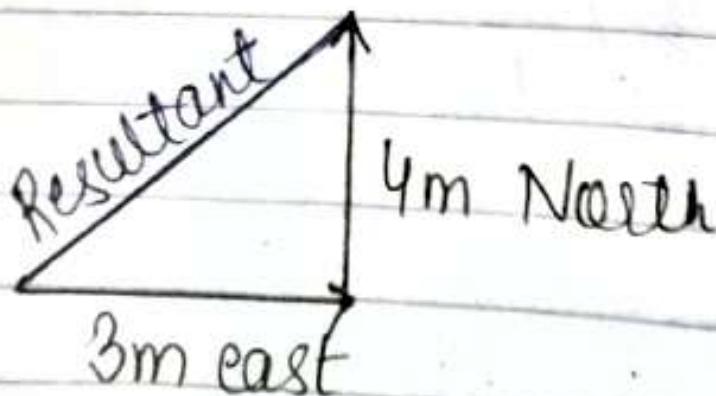


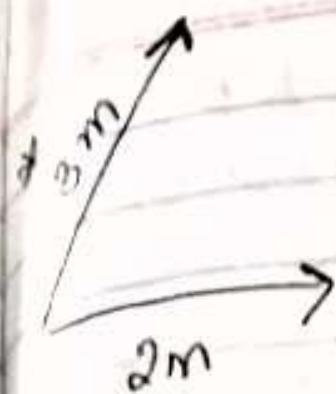
(Tail to Tail)

Smaller angle will be preferred.

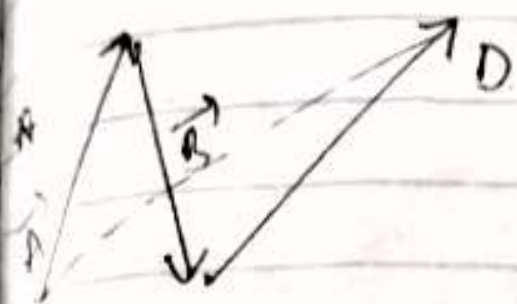
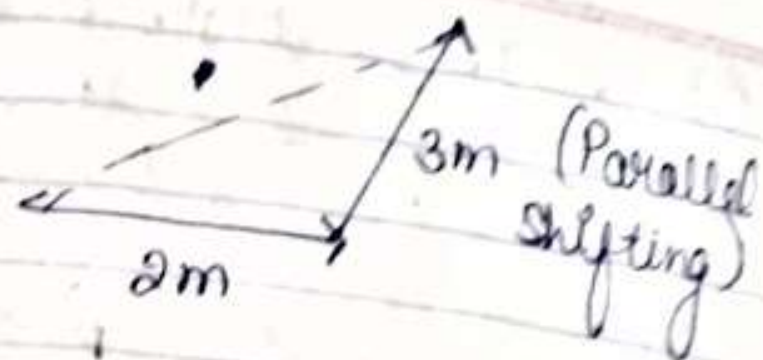
Addition AND SUBTRACTION.

(Head-Tail Method)

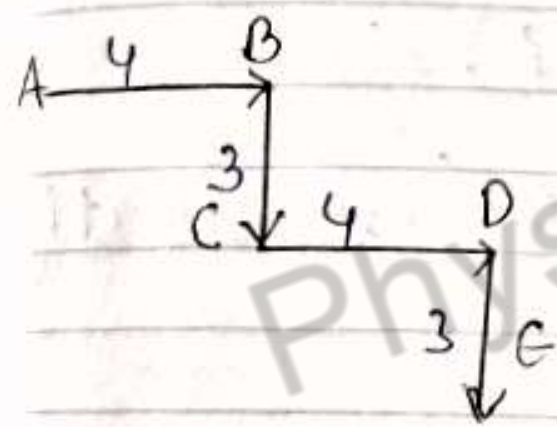




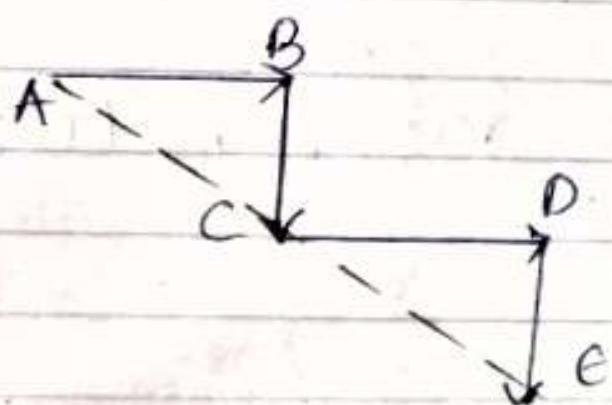
\Rightarrow



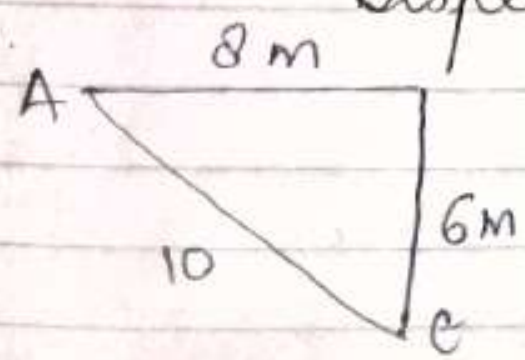
Resultant = Displacement =
1st vector tail +
last vector head



Displacement = ?

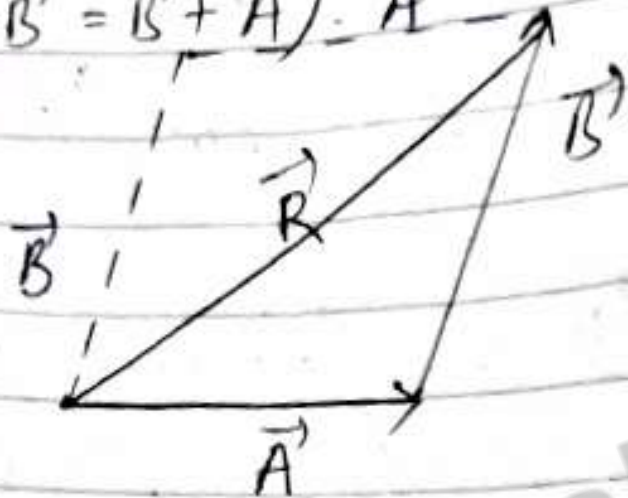


Displacement AE



Is Vector Commutative $(\vec{A} + \vec{B} = \vec{B} + \vec{A})$

$$\Rightarrow (\vec{A} + \vec{B} = \vec{B} + \vec{A}) \cdot \vec{A}$$



We know that,

$$\vec{A} + \vec{B} = \vec{R} \quad \text{--- (1)}$$

$$\vec{B} + \vec{A} = \vec{R} \quad \text{--- (2) (Parallel shifting)}$$

So, $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Hence vectors are commutative

⇒ Vectors can not be added as all scalar quantities are added.

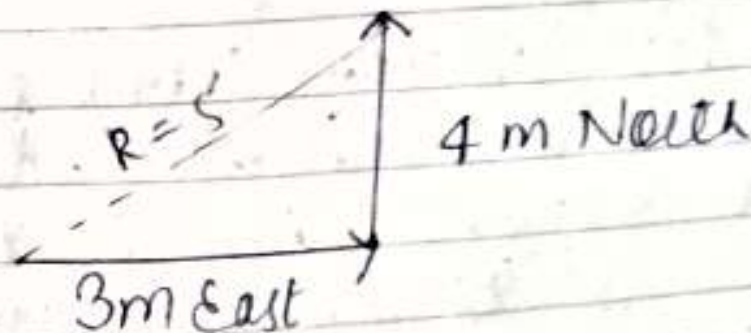
VECTOR ADDITION

- ① Head-Tail Method.
- ② Parallelogram Law.
- ③ Triangle Law.

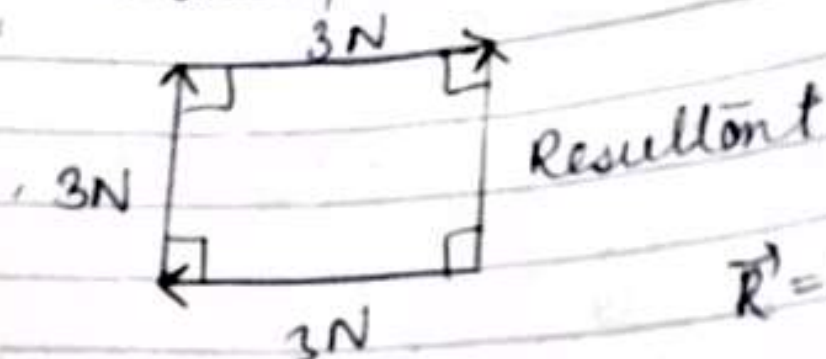
① Head-tail method

Join tail of next vector with Head of previous vector

$$3\text{m East} + 4\text{m North} = R$$



⇒ Add 3 vectors
 3N West, 3N North, 3N East



$$\vec{R} = 3 [90^\circ]$$

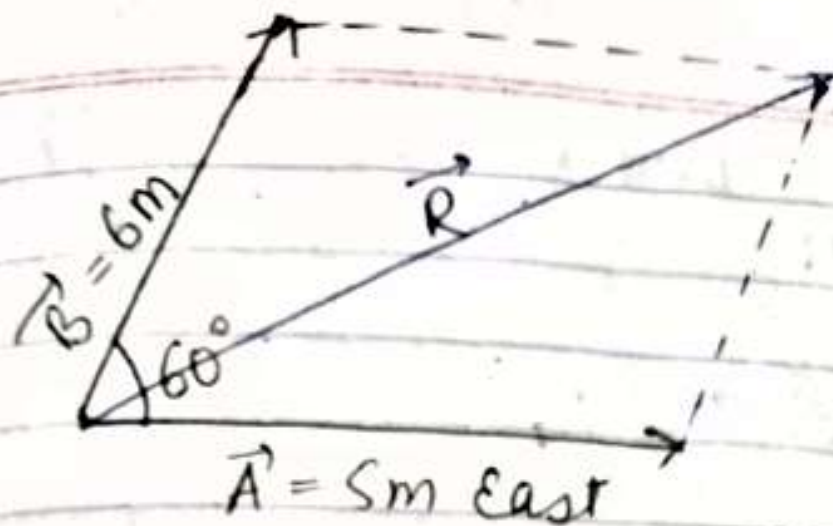
⇒ 5m East then 5m at 60° from East

This law fails here

Parallelogram Law

It allows us to add any kind of vector

⇒ Join two vectors from tail to tail as the two adjacent sides of parallelogram. $\vec{A} = 5 \text{ East}$, $\vec{B} = 6 \text{ m, } 60^\circ \text{ from East}$.
 (Imagine complete MgM)



\vec{R} = diagonal of 11gm from common point.

$$\vec{R} = \vec{A} + \vec{B}$$

$$R^2 = A^2 + B^2 + 2AB \cos \theta \quad (\text{Magnitude})$$

θ = angle between 2 vectors

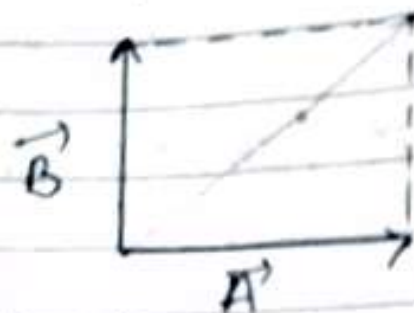
$$R^2 = 5^2 + 6^2 + 2 \times 5 \times 6 \times \frac{1}{2}$$

$$R^2 = 25 + 36 + 30:$$

$$R = 9.$$

Quesb- Add two vectors 6 units, 8 units
at 90°

$$\vec{A} = 6, \vec{B} = 8, \theta = 90^\circ$$

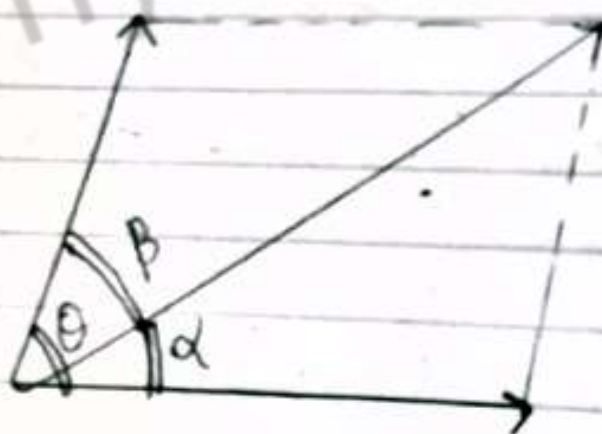


$$\vec{R} = \vec{A} + \vec{B}$$

$$R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$R^2 = 36 + 64 + 2 \times 6 \times 8 \times 0$$

$$R = 10$$



$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

(Direction of resultant)

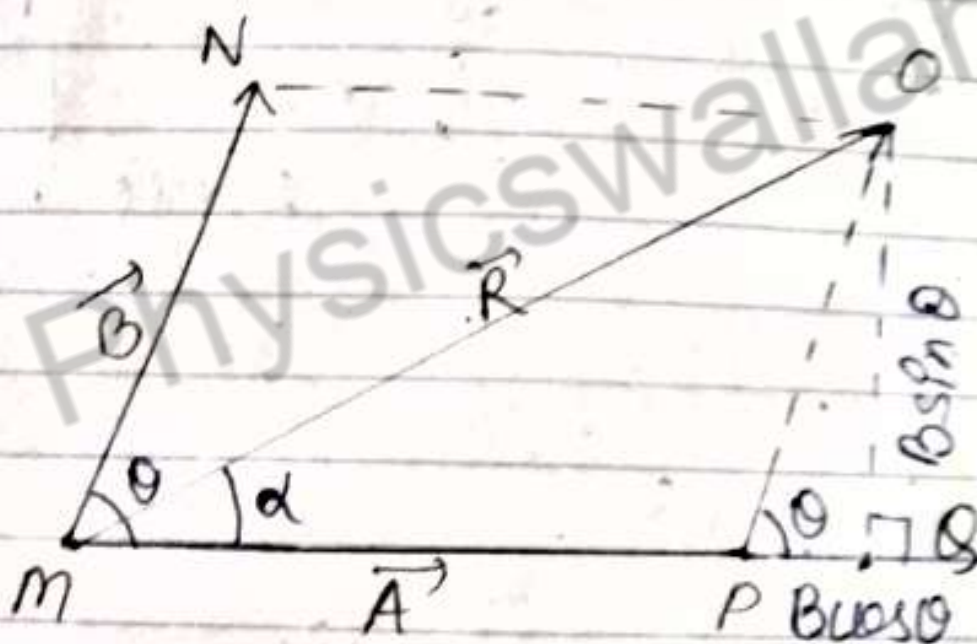
$$\tan \beta = \frac{A \sin \theta}{B + A \cos \theta}$$

$$(\beta = \theta - \alpha)$$

\vec{R} direction is from vector \vec{A} (α)

\vec{R} direction from \vec{B} (β)

Derive: $R^2 = A^2 + B^2 + 2AB \cos \theta$



Parallelogram's pair of opp sides is parallel and equal.

$\triangle POQ$

$$\cos \theta = \frac{B}{H}$$

$$\cos \theta = \frac{PQ}{PO}$$

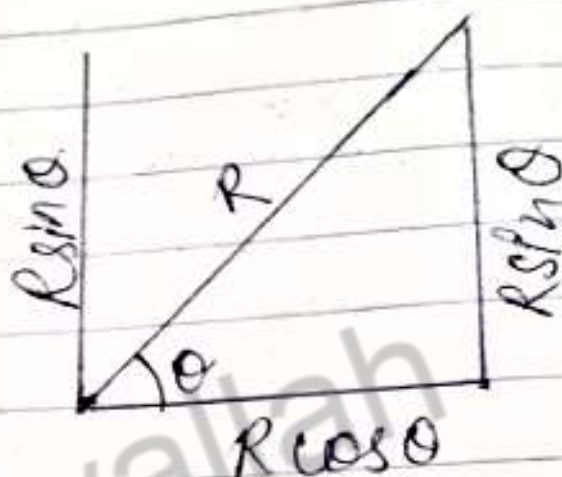
$$PQ = PO \cos \theta$$

$$PQ = B \cos \theta$$

$$\sin \theta = \frac{P}{H}$$

$$\sin \theta = \frac{OQ}{PO} = \frac{OQ}{B}$$

$$OQ = B \sin \theta$$



In $\triangle OQm$

$$(Om)^2 = (OQ)^2 + (mq)^2$$

$$R^2 = (B \sin \theta)^2 + (A + B \cos \theta)^2$$

$$R^2 = B^2 \sin^2 \theta + A^2 + B^2 \cos^2 \theta + 2AB \cos \theta$$

$$R^2 = A^2 + B^2 \sin^2 \theta + B^2 \cos^2 \theta + 2AB \cos \theta$$

$$R^2 = A^2 + B^2 (\sin^2 \theta + \cos^2 \theta) + 2AB \cos \theta$$

$$[R^2 = A^2 + B^2 + 2AB \cos \theta]$$

Direction of Resultant

in $\triangle OQM$

$$\tan \alpha = \frac{P}{B}$$

$$\tan \alpha = \frac{OQ}{MQ} = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\boxed{\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}}$$

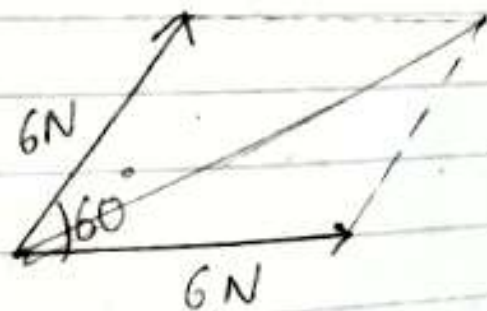
Ques 6 - Two forces of magnitude 6N each at a point as shown. Find the resultant.

$$R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$R^2 = 36 + 36 + 2 \times 36 \cos 60^\circ$$

$$R^2 = 108$$

$$R = 6\sqrt{3}$$



$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\tan \alpha = \frac{1}{\sqrt{3}}, \alpha = 30^\circ$$

If 2 vectors are equal in magnitude the resultant will pass through the angle between them.

Quesb - Two vectors of equal magnitude are added to give resultant, which is of same magnitude as the 2 vectors. Find the angle between them.

$$R = A = B = x$$

$$R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$x^2 = x^2 + x^2 + 2x^2 \cos \theta$$

$$-x^2 = 2x^2 \cos \theta$$

$$\cos \theta = \frac{-x^2}{2x^2} = -\frac{1}{2}$$

$$\cos \theta = -\frac{1}{2}$$

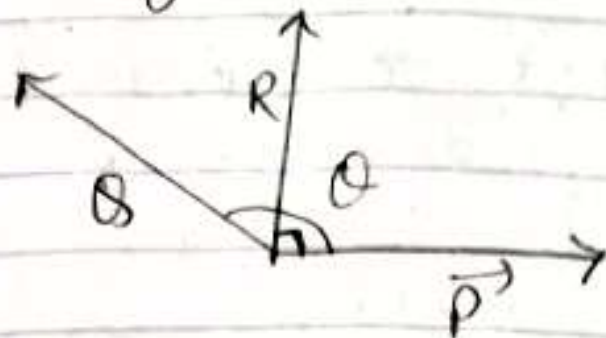
$$\theta = 120^\circ$$

Quesb - Two vectors P (smaller one) & Q as a sum of 18 and their resultant is 12. The resultant is

1 to smaller of two vector. Find the value of P & angle between them.

$$P + Q = 18$$

$$P' + Q' = 12$$



$$R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$12^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$12^2 = P^2 + Q^2 + 2P(-P)$$

$$12^2 = P^2 + Q^2 - 2P^2$$

$$12^2 = Q^2 - P^2$$

$$12^2 = 13^2 - P^2$$

$$\boxed{P = 5}$$

$$\begin{cases} (Q-P)(Q+P) = 144 \\ Q-P(18) = 144 \\ Q-P = 8 \\ + P+Q = 18 \\ \hline 2Q = 26, \boxed{Q = 13} \end{cases}$$

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\tan 90^\circ = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$P + Q \cos \theta = 0$$

$$Q \cos \theta = -P$$

$$Q \cos \theta = -5$$

$$\boxed{\cos \theta = \frac{-5}{13}}$$

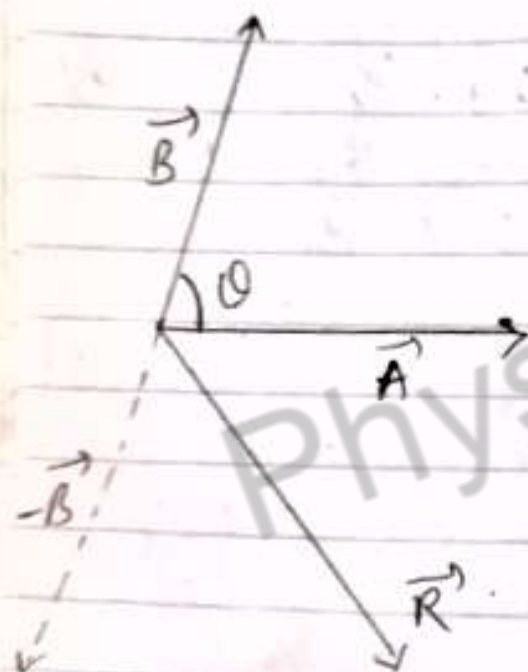
Subtraction of Vector

(Means negative of vector (opp in direction)

⇒ Vectors can only be added.

⇒ $\vec{A} - \vec{B}$ (x)

⇒ $\vec{A} + (-\vec{B})$ (✓)



$$\vec{R} = \vec{A} + (-\vec{B})$$

$$\text{angle} = (180 - \theta)$$

$$R^2 = A^2 + B^2 + 2AB \cos \theta$$

$$R^2 = A^2 + B^2 + 2AB \cos (180 - \theta)$$

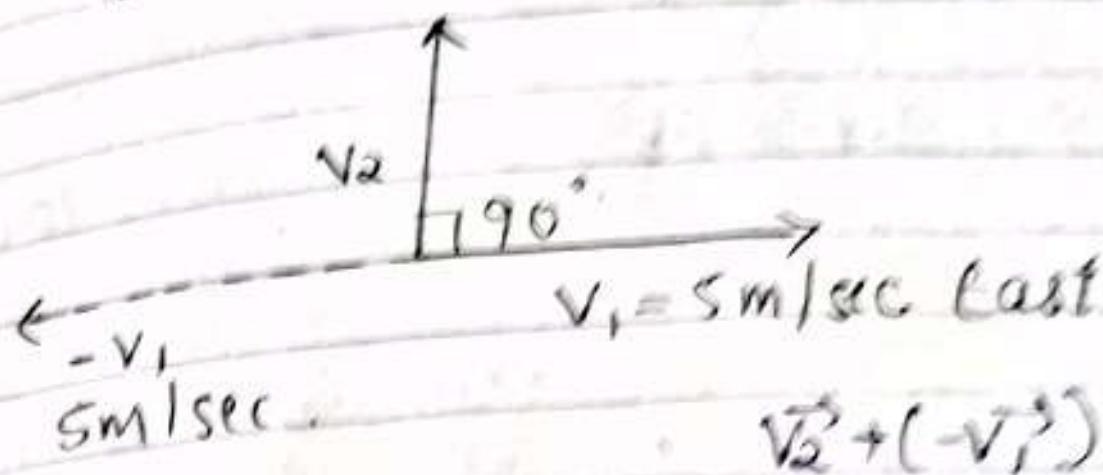
$$R^2 = A^2 + B^2 + 2AB (-\cos \theta)$$

$$\cos (180 - \theta) = -\cos \theta$$

$$\boxed{R^2 = A^2 + B^2 - 2AB \cos \theta}$$

Ques 6 - A car runs at 5m/sec East
a sharp turn to North and continues
at 5m/sec. Find the change in
velocity of car.

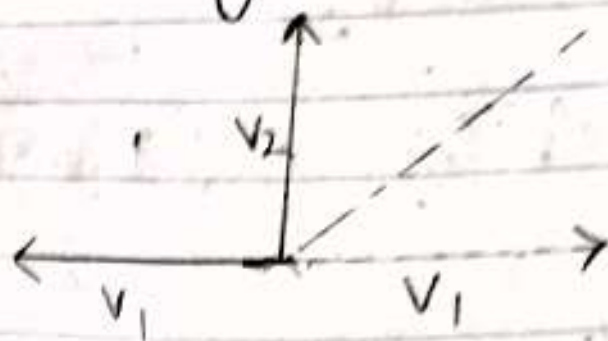
Δv = change in velocity $\Rightarrow \vec{v}_2 - \vec{v}_1$



$$R^2 = A^2 + B^2 - 2AB \cos 90^\circ$$

$$[R = 5\sqrt{2}] \text{ North west.}$$

Quesb - A car running at 10 m/sec (west) takes a sharp turn towards north and continues at 10 m/sec . If it takes 2 sec in turning. find acc of car.



$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t}$$

$$R^2 = A^2 + B^2 - 2AB \cos \theta$$

$$R^2 = 200 - 200 \times 0$$

$$R = 10\sqrt{2} \text{ NE}$$

$$a = \frac{\Delta v}{t} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \text{ NE m/sec}^2$$

Ques B - A plane moving with velocity v turns by ' θ ' angle & its speed remains ' v ', find the change in velocity of plane.

$$\vec{v}_2 - \vec{v}_1$$

$$\text{Ans: } (R)^2 = A^2 + B^2 - 2AB \cos \theta$$

$$(R)^2 = v^2 + v^2 - 2v \times v \cos \theta$$

$$(R)^2 = 2v^2 - 2v^2 \cos \theta$$

$$(R)^2 = 2v^2 (1 - \cos \theta)$$

$$(R)^2 = 2v^2 \cdot 2 \sin^2 \theta / 2$$

$$(R)^2 = 4v^2 \sin^2 \theta / 2$$

$$R = 2v \sin \theta / 2$$

$$\left[\begin{aligned} 1 - \cos \theta &= 2 \sin^2 \theta / 2 \end{aligned} \right]$$

Quesb- The difference of 2 unit vectors is a unit vector - find the angle between 2 vectors.

$$\vec{A} = 1, \vec{B} = 1, R = 1$$

$$R^2 = A^2 + B^2 - 2AB \cos \theta$$

$$1^2 = 1^2 + 1^2 - 2 \cos \theta$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

Quesb- The sum and difference are equal in magnitude. Find the angle b/w vectors

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

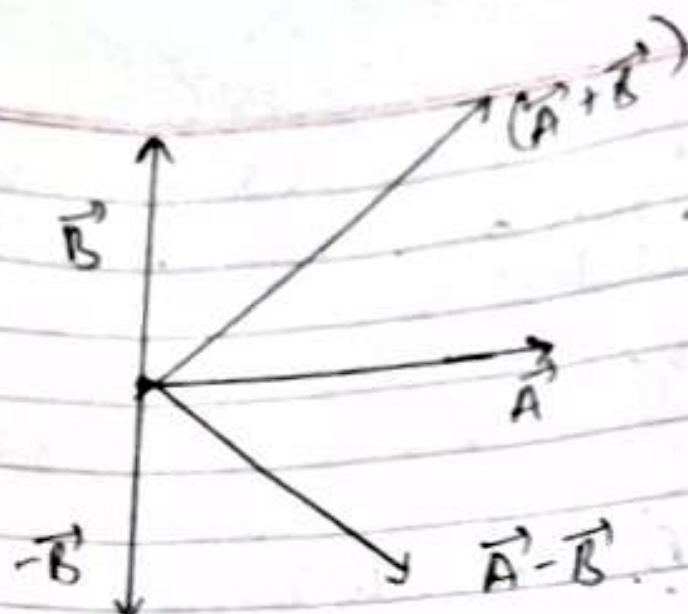
$$\text{Let } |\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$$

$$A^2 + B^2 + 2AB \cos \theta = A^2 + B^2 - 2AB \cos \theta$$

$$4AB \cos \theta = 0$$

$$\cos \theta = 0$$

$$\theta = 90^\circ$$



Ques 6 - 3 vectors $\vec{A} + \vec{B} + \vec{C} = 0$, if $|\vec{A}| = 12$,
 $|\vec{B}| = 5$, $|\vec{C}| = 13$.
 find angle between \vec{A} and \vec{B} .

$$|\vec{A} + \vec{B}|^2 = |(-\vec{C})|^2$$

$$A^2 + B^2 + 2AB \cos \theta = C^2$$

$$144 + 25 + 120 \cos \theta = 169$$

$$\cos \theta = 0$$

$$\theta = 90^\circ$$

\Rightarrow We have 2 vectors 3 & 4, then resultant cannot be

(a) 2

(b) 6

(c) 8

(d) 4

Max value of any vector

$$R = |A + B|$$

Min value of any vector

$$R = |A - B|$$

Multiplication of Vector

- ① Scalar \times Vector
- ② Vector \times Vector = scalar
- ③ Vector \times Vector = Vector

$$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$$

Cartesian form

$$3\vec{A} = 6\hat{i} - 3\hat{j} + 3\hat{k}$$

Vector \times Vector = Scalar

$$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{B} = 3\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\vec{A} \cdot \vec{B} = C \quad (3, 4, 5 \dots)$$

\downarrow scalar
(dot product)

$$V \times V = V$$

$$\vec{A} \times \vec{B} = \vec{C} \quad (\text{vector product})$$

\downarrow

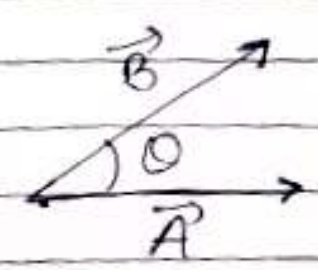
(cross product)

Q2) If the angle between A & B are greater than 90° , the dot product will be -ve

(vectors ^{with} length will always be +ve)
Dot Product

$$\vec{A} \cdot \vec{B} = c$$

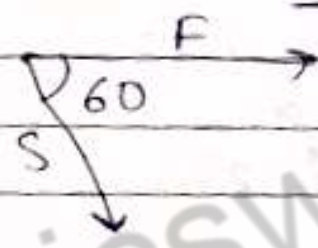
$$\text{Work} = \vec{F} \cdot \vec{S}$$



$$\vec{A} \cdot \vec{B} = |A| \times |B| \times \cos \theta$$

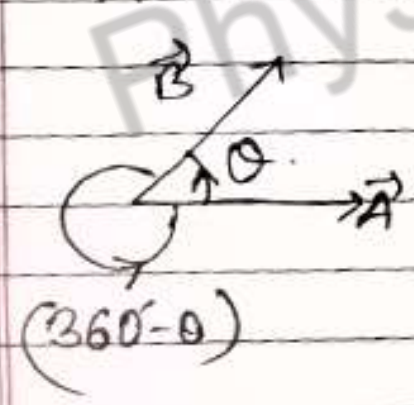
$$\vec{F} = 10 \text{ N}$$

$$\vec{S} = 5 \text{ m}$$



$$\begin{aligned} \text{Work} &= \vec{F} \cdot \vec{S} \\ \text{Work} &= 25 \text{ J} \end{aligned}$$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$



(A wrt B angle)

$$\vec{A} \cdot \vec{B} = |A| |B| \cos \theta$$

(B wrt A angle)

$$\vec{B} \cdot \vec{A} = |B| |A| \cos(360 - \theta)$$

$$\begin{aligned} \cos(360 - \theta) \\ = \cos \theta \end{aligned}$$

* If $\vec{A} \perp \vec{B}$

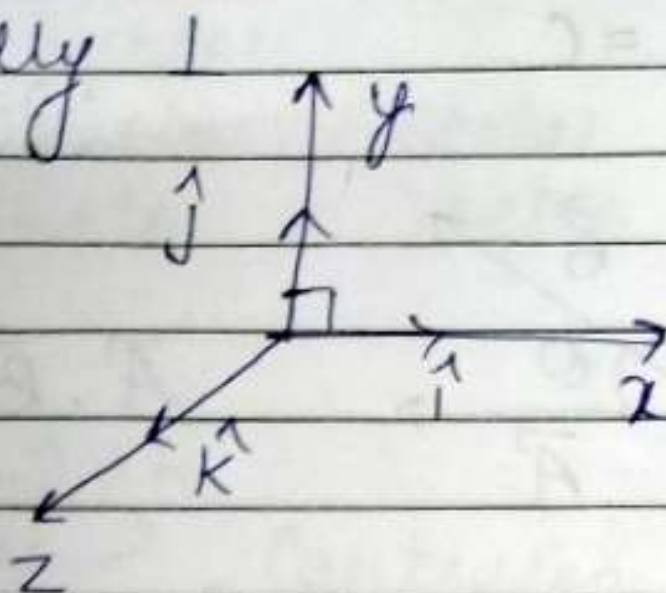
$$\theta = 90^\circ$$

$$\vec{A} \cdot \vec{B} = |A| |B| \cos 90^\circ$$

$$\vec{A} \cdot \vec{B} = 0$$

Orthogonal unit vectors

mutually



\hat{i} = whose mag is 1 and is in the direction of x .
 similarly \hat{j} & \hat{k}

$$\hat{i} \cdot \hat{j} = 0$$

$$\hat{i} \cdot \hat{i} = 1$$

$$\hat{i} \cdot \hat{k} = 0$$

$$\hat{j} \cdot \hat{j} = 1$$

$$\hat{j} \cdot \hat{k} = 0$$

$$\hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{i} = |\hat{i}| |\hat{i}| \cos 0^\circ$$

$$\Rightarrow 1$$

$$0 = 8 - 12 + 8a$$

$$0 = -4 + 8a$$

$$4 = 8a$$

$$a = \frac{1}{2}$$

Q- Angle between two vectors

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\left| \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right|$$

$$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{B} = \hat{i} - \hat{j}$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\vec{A} \cdot \vec{B} = (2 + 1)$$

$$\Rightarrow 3$$

$$|\vec{A}| |\vec{B}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$\Rightarrow \sqrt{4+1+1} \sqrt{1+1}$$

$$\Rightarrow \sqrt{6} \sqrt{2}$$

$$\Rightarrow 2\sqrt{3}$$

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ 3 & 2 \end{array}$$

$$\boxed{\cos \theta = \frac{3}{2\sqrt{3}}}$$

Quest- $\vec{P} = 2\hat{i} + \hat{j} - \hat{k}$, find θ
 $\vec{Q} = \hat{i} - \hat{j}$

$$\cos \theta = \frac{2-1}{\sqrt{6} \sqrt{2}}$$

$$\cos \theta \Rightarrow \frac{1}{2\sqrt{3}}$$

Quest- $\vec{R} = \hat{i} + \hat{j}$ (find θ)
 $\vec{S} = \hat{i} - \hat{j}$

$$\cos \theta = \frac{1-1}{\sqrt{2} \sqrt{2}} \Rightarrow \frac{0}{\sqrt{4}} \Rightarrow \frac{0}{2} = 0$$

Quest- Find the angle that $\vec{A} = \hat{i} + \hat{j}$ makes with x-axis.

$$\vec{A} = \hat{i} + \hat{j}$$

$$\vec{B} = \hat{i}$$

(any vector along x-axis)
 can be $2\hat{i}, 0.5\hat{i}$... anything

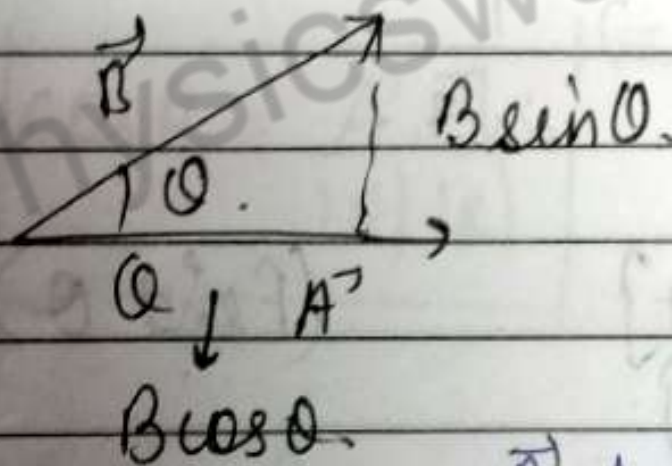
$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\cos \theta \Rightarrow \frac{1 \times 1 \times 0}{\sqrt{2} \sqrt{1}}$$

$$\cos \theta \Rightarrow \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$



\vec{B} projection on \vec{A}

$B \cos \theta$ is along \vec{A} .

$$\vec{A} \cdot \vec{B} = |\vec{A}| (|\vec{B}| \cos \theta)$$

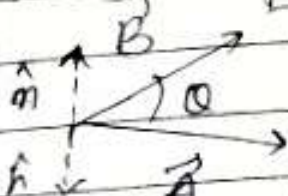
Cross / Vector Product

$$\vec{A} \times \vec{B} = \vec{C} \text{ vector}$$

$$\text{Torque / moment of force} = \vec{r} \times \text{displacement}$$

(This is \perp to \vec{A} & \vec{B})

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta \cdot \hat{n}$$



direction

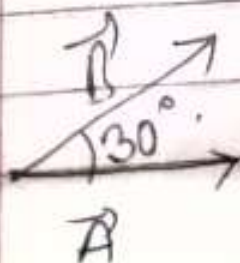
To give direction to vectors we use unit vector

$$\begin{aligned} \vec{C} &\perp \vec{A} \\ \vec{C} &\perp \vec{B} \end{aligned}$$

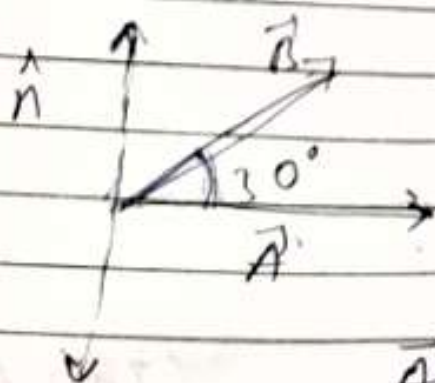
Ques $\Rightarrow \vec{A} = 5, \theta = 30^\circ$
 $\vec{B} = 2,$ Find $\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta$
 $\vec{B} \times \vec{A} =$

$$\vec{A} \times \vec{B} = 5$$

$$\vec{B} \times \vec{A} = |\vec{B}| |\vec{A}| \sin \theta = 5$$



$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$



(Right hand thumb rule)

$\vec{A} \times \vec{B}$ = while curling from \vec{A} to \vec{B} using R.H. Thumb rule the thumb is upwards so the \hat{n} will be upwards

$\vec{B} \times \vec{A}$ = while curling from \vec{B} to \vec{A} using R.H. Thumb rule the thumb will be downwards

(as ~~we~~ we will take the smaller angle between the vectors).

$\vec{A} \times \vec{B}$ = 5 upwards

$\vec{B} \times \vec{A}$ = 5 downwards

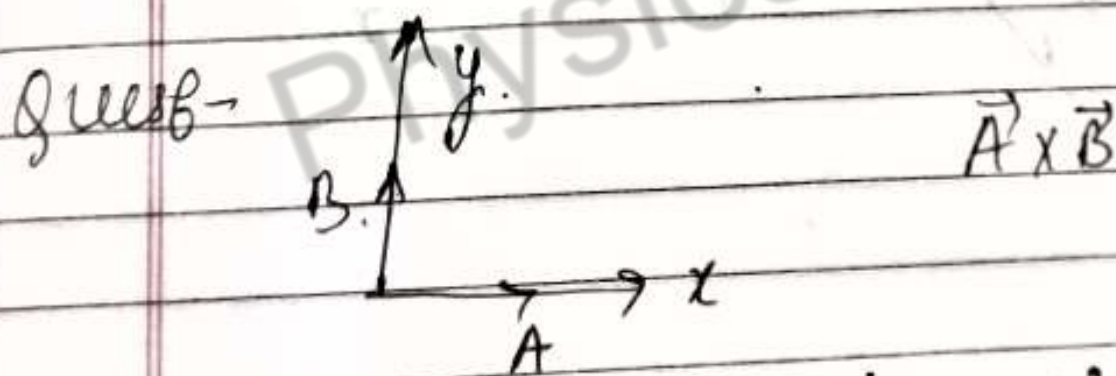
so, $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

OR Screw Rule.

(Add both the vectors tail to tail; move the screw from \vec{A} to \vec{B} , so the direction is upwards)

and move the screw from \vec{B} to \vec{A} so the screw will go downwards.

Commutative rule is not valid.
for cross product.



what will be the direction of $\vec{A} \times \vec{B}$
upwards (outwards)

$\vec{B} \times \vec{A}$ (Inwards) downwards

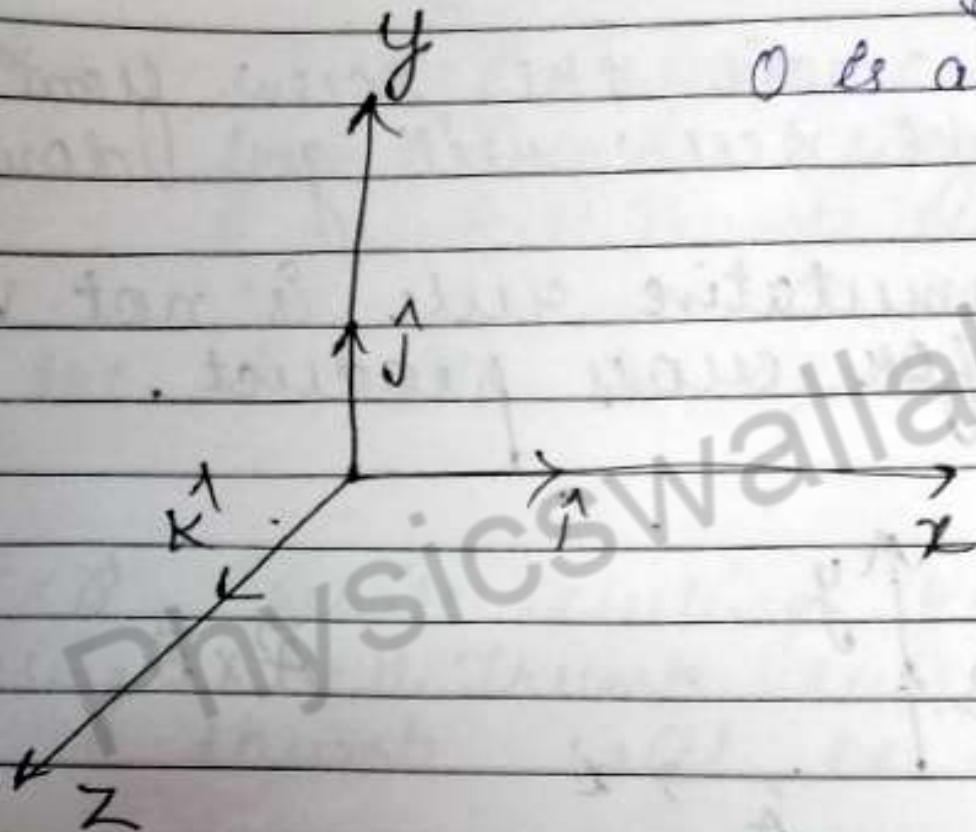
Orthogonal unit vectors:

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

1.



$$\hat{i} \times \hat{i} = |\hat{i}| |\hat{i}| \sin 0 = 0$$

$$\hat{j} \times \hat{j} = |\hat{j}| |\hat{j}| \sin 0 = 0$$

$$\hat{k} \times \hat{k} = 0$$

\Downarrow

$\vec{0}$

$\vec{0}$ is a vector

$$\hat{i} \times \hat{j} = |\hat{i}| |\hat{j}| \sin 90^\circ$$

$\Rightarrow 1 \hat{n} \rightarrow$ (unit vector outwards).

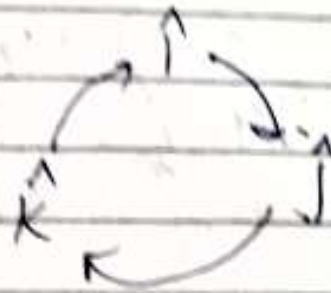
$$\Rightarrow \hat{k}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

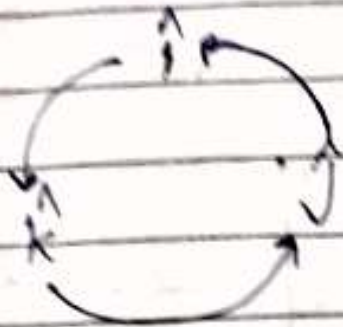
$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$



Clockwise = +ve



anticlockwise = -ve

ques B $\vec{A} = 5\hat{i}$
 $\vec{B} = 2\hat{k}$

$$\vec{A} \times \vec{B} = 5\hat{i} \times 2\hat{k}$$

$$= -10\hat{j}$$

$$\vec{B} \times \vec{A} = 10\hat{j}$$

$$\boxed{\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$$

Ques 6- $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$
 $\vec{B} = 3\hat{i} + 2\hat{j} + 3\hat{k}$

$$\vec{A} \times \vec{B} = 6(0) + 4\hat{k} + 6(-\hat{j}) - 9(\hat{k}) + 6(0) + 9\hat{j} - 12\hat{j} + 8(-\hat{i}) + 12(0)$$

$$\vec{A} \times \vec{B} = -\hat{i} + 6\hat{j} - 5\hat{k}$$

Short cut

	\hat{i}	\hat{j}	\hat{k}
A	2	3	4
B	3	2	3

$$\vec{A} \times \vec{B} = \hat{i}(9-8) - \hat{j}(6-12) + \hat{k}(4-9)$$

$$\vec{A} \times \vec{B} = \hat{i} - (-6)\hat{j} + (-5\hat{k})$$

$$(\vec{A} \times \vec{B} = \hat{i} + 6\hat{j} - 5\hat{k})$$

Ques 6- Find the mag. of $\vec{A} \times \vec{B}$ if $A = 2\hat{i} + \hat{j} + \hat{k}$
 and $B = 6\hat{i} + 3\hat{j} - 3\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 6 & 3 & -3 \end{vmatrix}$$

$$\Rightarrow \hat{i}(-3+3) - \hat{j}(-6+6) + \hat{k}(6-6)$$

$$\Rightarrow \hat{i}(0) - \hat{j}(0) + \hat{k}(0)$$

$$\Rightarrow 0$$

$$|\vec{A} \times \vec{B}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Mag B - If $\vec{A} \times \vec{B} = (3\hat{i} + 2\hat{j} + 4\hat{k})$

$$|\vec{A} \times \vec{B}| = \sqrt{3^2 + 2^2 + 4^2} \quad (\text{Mag})$$

* If $\vec{A} \times \vec{B} = 0$

Either $A = 0$ or $B = 0$

or

$$|A||B|\sin\theta = 0$$

$$\sin\theta = 0$$

$$\theta = 0^\circ$$

$$\vec{A} \cdot \vec{B} = 0$$

$$\theta = 90^\circ \text{ or } 90^\circ$$

$$\vec{A} \times \vec{B} = 0$$

$$\theta = 0^\circ$$

Ques:- $|\vec{A}| = 5$ (mag)
 $|\vec{B}| = 6$
 $|\vec{A} \times \vec{B}| = 15$ (Mag)

Find angle b/w A & B

$$|\vec{A} \times \vec{B}| = |A||B|\sin\theta$$

$$15 = 5 \times 6 \sin\theta$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = 30^\circ \text{ and } 150^\circ$$

Find angle between \vec{A} & \vec{B} .

$$|\vec{A} \times \vec{B}| = |\vec{A} \cdot \vec{B}|$$

$$|A||B|\sin\theta = |A||B|\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1$$

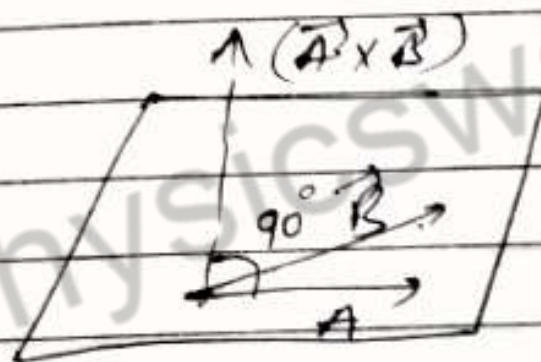
$$\tan\theta = 1$$

$$\theta = 45^\circ$$

$$\text{or } \frac{1}{4} \Rightarrow 180^\circ$$

$$\vec{A} \cdot (\vec{A} \times \vec{B}) = ?$$

$$\vec{A} \cdot (\vec{A} \times \vec{B})$$



$$\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$$

(\perp) to $(\vec{A} \times \vec{B})$

If $\vec{A} \cdot \vec{B} = 0$

$\vec{A} \cdot \vec{C} = 0$

Then \vec{A} is \parallel to

(a) \vec{C}

(c) $\vec{B} \times \vec{C}$

(b) \vec{B}

(d) $\vec{B} \cdot \vec{C}$

Unit Vectors

egs: weight & not vector,

⇒ Magnitude = 1

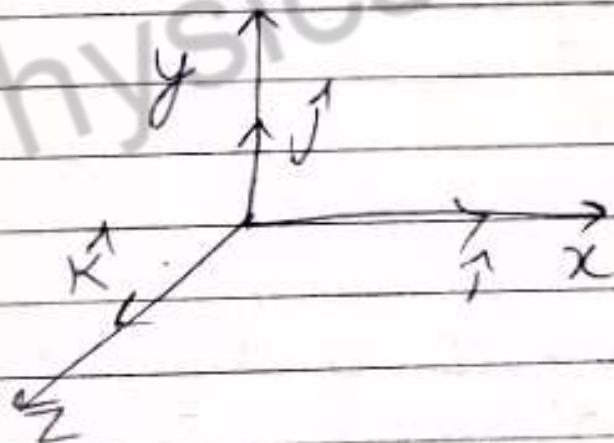
⇒ It gives direction

 $\vec{A} = \text{Magnitude} \times \text{direction}$

$$\vec{A} = |\vec{A}| \times \hat{A}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Orthogonal unit vector



Ques:- A force 10 N is in x direction.
Represent it in vector form

$$\vec{F} = 10\hat{i}$$

$$\vec{F} = 10\hat{i}$$