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NCDS – Unit 2 HW

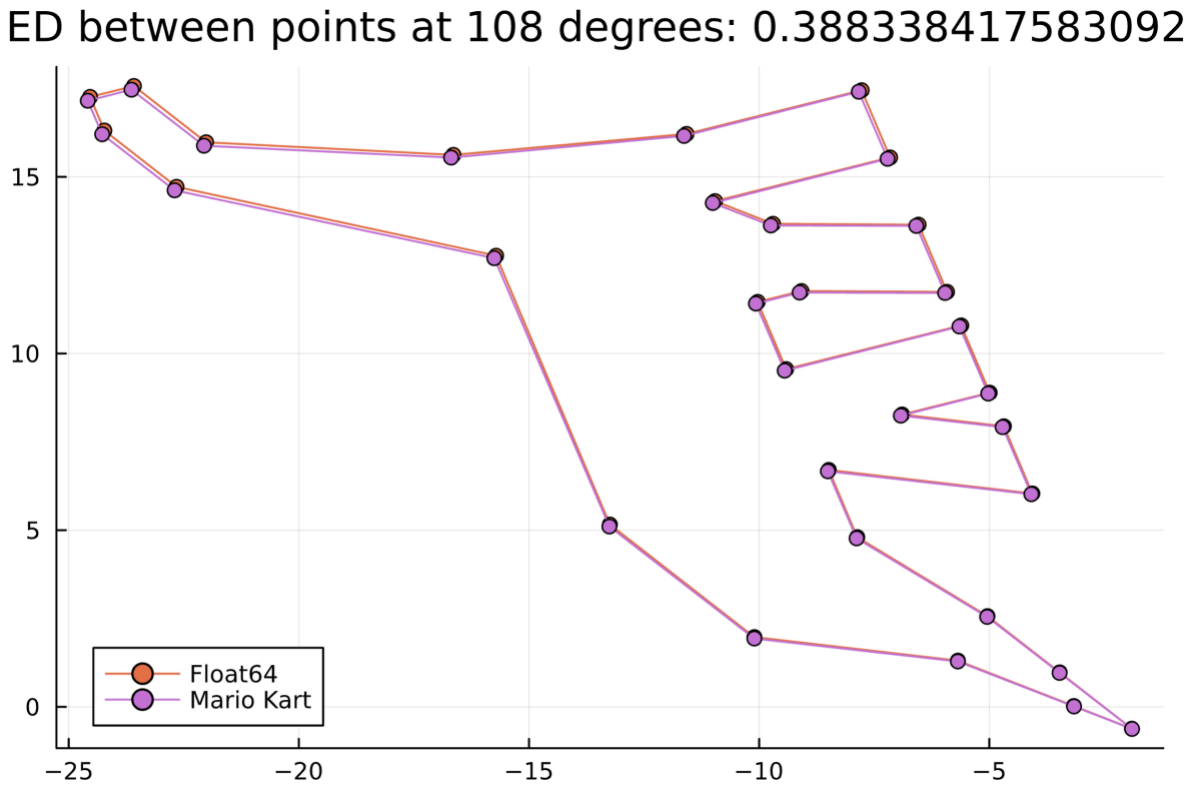
Q1.)
To solve, I modified Professor Gleich's fixed-point.jl code to include a function to find the mean Euclidean distance between the sets of points (meanEucDist), and adding a for loop to iterate rotation between 1 and 359 degrees:

```
##
using Printf
using FixedPointNumbers
using FixedPointDecimals
using LinearAlgebra

##
x = FixedDecimal{Int,2}(5)
## Emulate the Mario Kart system to rotate a dinosaur
MKFP = Fixed{Int16,8} # use 16 bits for a signed integer and 8 bits for fraction
val = MKFP(5.25)

## Function to determine average square drift between 2 sets of equal length
function meanEucDist(act, ref)
    med = sqrt(sum((act .- ref).^2))
    return med
end

## Get some data off the internet and plot it
data = [24,18, 24,17, 22,16, 20,11, 19,6, 19,2, 17,2, 17,6,
16,5, 15,2, 13,2, 14,5, 14,6, 12,6, 12,2, 10,2, 10,4,
9,2, 7,2, 9,6, 7,6, 4,4, 2,3, 0,2, 1,3, 3,5, 5,9,
9,11, 17,11, 21,17, 23,18, 24,18]
P = reshape(data,2,div(length(data),2))

using Plots
driftDict = Dict{Int64, Float64}()

## Rotate 359 degrees
for deg in 1:359
    local plt = plot()
```

```
    global theta = deg/180*pi
    R(theta) = [cos(theta) -sin(theta); sin(theta) cos(theta)]
    global Pp = R(theta)*P # True
    plot!(plt, Pp[1,:],Pp[2,:],linestyle = :solid,marker = :circle,color=2,lab="Float64")
    # Rotate in fixed Point
    global Pmk = MKFP.(R(MKFP(theta)))*MKFP.(P)
    plot!(plt, Pmk[1,:],Pmk[2,:],linestyle = :solid,marker = :circle,color=4,lab="Mario Kart")
    # Calculate mean euclidean distance between points
    local err = meanEucDist(Pp, Float64.(Pmk))
    title!("ED between points at $deg degrees: $err")
    push!(driftDict, deg => err)
    display(plt)
end

## Find the maximum drift value
maxDeg, maxDrift, = findmax(driftDict)
```

The output of this is: (0.3883384175830925, 108)
Translated, this means that the maximum mean Euclidean distance is 0.3883… at a 108-degree rotation.

Plotting these points, we see that this is a good estimate, as there are distinct visual differences between the points on the graph.


ED between points at 108 degrees: 0.388338417583092
15
10
5
0
−25
−20
−15
−10
−5
Float64
Mario Kart

2a)
Three methods of calculating roots are:

1. Utilizing the quadratic formula $x_1, x_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
2. Utilizing the Newton-Rhapson formula and solving $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

   For a quadratic function $ax^2 + bx + c$, we solve $x_{n+1} = x_n - \frac{ax_n^2+bx_n+c}{2ax_n+b}$. We guess some value of x that we believe to be the 0, and then iterate until convergence.
3. Utilizing the completing the square method.

   Using the form $ax^2 + bx + c$:

   1. Divide all terms by a: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
   2. Move the constant term to the other side: $x^2 + \frac{b}{a}x = -\frac{c}{a}$
   3. Add and subtract $\left(\frac{b}{2a}\right)^2$ from the left side: $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 = -\frac{c}{a}$

$$\Rightarrow x^2 + \frac{b}{a} + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\Rightarrow \left(x - \frac{b}{2a}\right) = \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

$$x_1 = \frac{b}{2a} + \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}, x_2 = \frac{b}{2a} - \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

2b.
Converting the methods to Julia code and running values of a, b, and c from -10 to +10:

```
##
using Printf
using LinearAlgebra

## Define functions for the three solution methods
function quadraticFormula(a, b, c)
    if b^2 > 4*a*c
        return (-b + sqrt(b^2 - 4*a*c)) / (2*a), (-b - sqrt(b^2 - 4*a*c)) / (2*a)
    end
end

function completeTheSquare(a, b, c)
    ## Normalize all terms:
    c = c/a
    b = b/a
    a = a/a

    ## Constant term to right side:
    c = -c

    ## Add/Subtract b to make the perfect trinomial
    b_half_sqr = (b/2)^2

    ## Squared Binomial
    rhs = c + b_half_sqr

    ## Solve for X
    if rhs > 0
        ## Real Roots
        roots = -b/2 + (sqrt(rhs)), -b/2 - (sqrt(rhs))
        return roots
    end
end

function NewtonRaphsonMethod(a, b, c, itr, guess)
```

```
    ## Define f(x) and f'(x)
    f(x, a, b, c) = a*x^2 + b*x + c
    f_prime(x, a, b) = 2*a*x + b

    x = guess
    for n in 1:itr
        f_x = f(x, a, b, c)
        f_xp = f_prime(x, a, b)
        if abs(f_xp) < 1e-10
            return x
        end
        x_new = x - f_x/f_xp
        if abs(x_new - x) < 1e-10
            return x_new
        end
        x = x_new
    end
    return x
end

## Execute with Float16 values for a, b, c, and x_guess

for i in -10:10
    for j in -10:10
        for k in -10:10
            local a = Float16(i)
            local b = Float16(j)
            local c = Float16(k)
            local x_guess = Float16((sum([i, j, k])/3))

            quadRoot = quadraticFormula(a, b, c)
            ctsRoot = completeTheSquare(a, b, c)
            newtonRoot = NewtonRaphsonMethod(a, b, c, 1000, x_guess)

            if  !isnothing(quadRoot) && !isnothing(ctsRoot) ## Only take real zeros
                if quadRoot[1] != ctsRoot[1] != newtonRoot ## Check for where they are not equal
                    println("Root 1 with a=$i, b=$j, c=$k:\t", quadRoot[1], "\t\t\t", ctsRoot[1], "\t\t\t", newtonRoot)
```

```
            end
          end
        end
      end
    end
```

We get the following results:

```
Root 1 with a=-10, b=-10, c=-2: -0.7236          -0.2764          -0.724
Root 1 with a=-10, b=-9, c=-2:  -0.5             -0.3994          -0.4976
Root 1 with a=-10, b=-9, c=-1:  -0.7705          -0.1296          -0.77
Root 1 with a=-10, b=-7, c=-1:  -0.5             -0.1998          -0.5005
Root 1 with a=-10, b=-5, c=0:   -0.5             0.0              -0.4998
Root 1 with a=-9, b=-10, c=-2:  -0.8496          -0.2612          -0.8506
Root 1 with a=-9, b=-9, c=-2:   -0.6665          -0.333           -0.667
Root 1 with a=-9, b=-9, c=-1:   -0.8726          -0.1272          -0.873
Root 1 with a=-9, b=-9, c=0:    -1.0             0.0              -0.9995
Root 1 with a=-9, b=-8, c=0:    -0.8887          0.0              -0.889
Root 1 with a=-9, b=-7, c=-1:   -0.5894          -0.1885          -0.589
Root 1 with a=-9, b=-4, c=0:    -0.4443          0.0              -0.4446
Root 1 with a=-9, b=-2, c=0:    -0.2222          0.0              -0.2223
Root 1 with a=-9, b=-1, c=0:    -0.1111          0.0              -0.11115
Root 1 with a=-8, b=-10, c=-2:  -1.0             -0.25            -0.9995
Root 1 with a=-8, b=-9, c=-2:   -0.8203          -0.3047          -0.821
Root 1 with a=-8, b=-6, c=-1:   -0.5             -0.25            -0.5005
Root 1 with a=-8, b=-6, c=0:    -0.75            0.0              -0.7495
Root 1 with a=-7, b=-10, c=-3:  -1.0             -0.4285          -1.001
Root 1 with a=-7, b=-8, c=-2:   -0.7734          -0.3694          -0.773
Root 1 with a=-7, b=-7, c=-1:   -0.827           -0.1726          -0.828
Root 1 with a=-7, b=-6, c=-1:   -0.6304          -0.2266          -0.631
Root 1 with a=-6, b=-10, c=-4:  -1.0             -0.665           -0.9995
Root 1 with a=-6, b=-10, c=-2:  -1.435           -0.2324          -1.434
Root 1 with a=-6, b=-10, c=0:   -1.667           0.0              -1.666
Root 1 with a=-6, b=-9, c=-3:   -1.0             -0.5             -0.9995
Root 1 with a=-6, b=-9, c=-2:   -1.2295          -0.2712          -1.23
Root 1 with a=-6, b=-8, c=-1:   -1.194           -0.1396          -1.193
Root 1 with a=-6, b=-7, c=-2:   -0.6665          -0.498           -0.666
Root 1 with a=-6, b=-5, c=-1:   -0.5             -0.3325          -0.4995
Root 1 with a=-6, b=-5, c=0:    -0.8335          0.0              -0.834
Root 1 with a=-6, b=-4, c=0:    -0.6665          0.0              -0.667
Root 1 with a=-6, b=-2, c=0:    -0.3333          0.0              -0.3335
Root 1 with a=-6, b=-1, c=0:    -0.1666          0.0              -0.1667
Root 1 with a=-5, b=-10, c=-4:  -1.447           -0.5527          -1.448
```

Root 1 with a=-5, b=-9, c=-4:	-1.0	-0.799	-0.995
Root 1 with a=-5, b=-9, c=-3:	-1.357	-0.4417	-1.358
Root 1 with a=-5, b=-9, c=-2:	-1.541	-0.2593	-1.54
Root 1 with a=-5, b=-9, c=-1:	-1.682	-0.11865	-1.681
Root 1 with a=-5, b=-8, c=-3:	-1.0	-0.601	-1.001
Root 1 with a=-5, b=-8, c=-2:	-1.29	-0.31	-1.289
Root 1 with a=-5, b=-8, c=-1:	-1.463	-0.1367	-1.464
Root 1 with a=-5, b=-5, c=-1:	-0.7236	-0.2764	-0.724
Root 1 with a=-4, b=-10, c=-5:	-1.809	-0.691	-1.811
Root 1 with a=-4, b=-10, c=-4:	-2.0	-0.5	-1.999
Root 1 with a=-4, b=-9, c=-5:	-1.25	-1.0	-1.251
Root 1 with a=-4, b=-9, c=-3:	-1.844	-0.4067	-1.842
Root 1 with a=-4, b=-7, c=-3:	-1.0	-0.75	-0.9985
Root 1 with a=-3, b=-10, c=-2:	-3.12	-0.2129	-3.121
Root 1 with a=-3, b=-10, c=-1:	-3.229	-0.1035	-3.23
Root 1 with a=-3, b=-9, c=-6:	-2.0	-1.0	-1.999
Root 1 with a=-3, b=-9, c=-4:	-2.459	-0.5425	-2.457
Root 1 with a=-3, b=-8, c=-3:	-2.217	-0.4512	-2.215
Root 1 with a=-3, b=-8, c=-2:	-2.389	-0.2793	-2.387
Root 1 with a=-3, b=-7, c=-3:	-1.769	-0.565	-1.77
Root 1 with a=-3, b=-7, c=-1:	-2.18	-0.1523	-2.182
Root 1 with a=-3, b=-6, c=-2:	-1.578	-0.4224	-1.576
Root 1 with a=-3, b=-5, c=-2:	-1.0	-0.665	-1.001
Root 1 with a=-3, b=-2, c=0:	-0.6665	0.0	-0.667
Root 1 with a=-2, b=-10, c=-10:	-3.617	-1.382	-3.621
Root 1 with a=-2, b=-10, c=-9:	-3.824	-1.177	-3.82
Root 1 with a=-2, b=-10, c=-2:	-4.79	-0.209	-4.793
Root 1 with a=-2, b=-9, c=-10:	-2.5	-2.0	-2.498
Root 1 with a=-2, b=-9, c=-9:	-3.0	-1.5	-2.998
Root 1 with a=-2, b=-9, c=-7:	-3.5	-1.0	-3.498
Root 1 with a=-2, b=-9, c=-4:	-4.0	-0.5	-3.998
Root 1 with a=-2, b=-8, c=-3:	-3.582	-0.419	-3.58
Root 1 with a=-2, b=-7, c=-6:	-2.0	-1.5	-2.004
Root 1 with a=-2, b=-7, c=-5:	-2.5	-1.0	-2.498
Root 1 with a=-2, b=-7, c=-2:	-3.188	-0.3135	-3.186
Root 1 with a=-2, b=-6, c=-4:	-2.0	-1.0	-2.002
Root 1 with a=-2, b=-6, c=-3:	-2.367	-0.634	-2.365
Root 1 with a=-2, b=-6, c=-1:	-2.824	-0.1768	-2.82
Root 1 with a=-2, b=-5, c=-3:	-1.5	-1.0	-1.497
Root 1 with a=-2, b=-5, c=-2:	-2.0	-0.5	-1.999
Root 1 with a=-2, b=-3, c=-1:	-1.0	-0.5	-1.001
Root 1 with a=-2, b=-3, c=0:	-1.5	0.0	-1.499
Root 1 with a=-1, b=-10, c=-4:	-9.58	-0.418	-5.0
Root 1 with a=-1, b=-10, c=-3:	-9.69	-0.3086	-0.3096

Root 1 with a=-1, b=-10, c=-2:	-9.8	-0.2031	-0.2043
Root 1 with a=-1, b=-10, c=-1:	-9.9	-0.10156	-0.10095
Root 1 with a=-1, b=-9, c=-10:	-7.703	-1.299	-7.7
Root 1 with a=-1, b=-9, c=-9:	-7.85	-1.146	-7.855
Root 1 with a=-1, b=-9, c=-5:	-8.41	-0.5957	-8.4
Root 1 with a=-1, b=-9, c=-4:	-8.53	-0.4688	-8.52
Root 1 with a=-1, b=-9, c=-3:	-8.66	-0.3477	-0.3467
Root 1 with a=-1, b=-9, c=-2:	-8.77	-0.2266	-0.228
Root 1 with a=-1, b=-9, c=-1:	-8.89	-0.1133	-0.11255
Root 1 with a=-1, b=-8, c=-10:	-6.45	-1.551	-6.445
Root 1 with a=-1, b=-8, c=-6:	-7.164	-0.838	-7.16
Root 1 with a=-1, b=-8, c=-3:	-7.605	-0.3945	-4.0
Root 1 with a=-1, b=-8, c=-2:	-7.742	-0.2578	-0.2583
Root 1 with a=-1, b=-8, c=-1:	-7.875	-0.127	-0.1271
Root 1 with a=-1, b=-7, c=-9:	-5.305	-1.697	-5.3
Root 1 with a=-1, b=-7, c=-7:	-5.79	-1.209	-5.797
Root 1 with a=-1, b=-7, c=-5:	-6.195	-0.8066	-6.19
Root 1 with a=-1, b=-7, c=-3:	-6.54	-0.459	-6.543
Root 1 with a=-1, b=-7, c=-2:	-6.703	-0.2988	-0.2986
Root 1 with a=-1, b=-7, c=-1:	-6.85	-0.1465	-0.1459
Root 1 with a=-1, b=-6, c=-6:	-4.734	-1.268	-4.73
Root 1 with a=-1, b=-6, c=-4:	-5.234	-0.7637	-5.24
Root 1 with a=-1, b=-6, c=-2:	-5.65	-0.3535	-3.0
Root 1 with a=-1, b=-6, c=-1:	-5.83	-0.1719	-0.1716
Root 1 with a=-1, b=-5, c=-6:	-3.0	-2.0	-2.998
Root 1 with a=-1, b=-5, c=-5:	-3.617	-1.382	-3.615
Root 1 with a=-1, b=-5, c=-4:	-4.0	-1.0	-3.998
Root 1 with a=-1, b=-5, c=-1:	-4.79	-0.209	-0.2087
Root 1 with a=-1, b=-4, c=-3:	-3.0	-1.0	-2.998
Root 1 with a=-1, b=-4, c=-1:	-3.732	-0.2676	-2.0
Root 1 with a=-1, b=-2, c=0:	-2.0	0.0	-1.0
Root 1 with a=3, b=5, c=2:	-0.6665	-0.665	-0.6675
Root 1 with a=3, b=7, c=1:	-0.153	-0.1523	-0.1528
Root 1 with a=3, b=7, c=2:	-0.3333	-0.333	-0.3335
Root 1 with a=3, b=9, c=1:	-0.1159	-0.11523	-0.11554
Root 1 with a=3, b=9, c=2:	-0.2416	-0.2412	-0.2417
Root 1 with a=3, b=10, c=1:	-0.10284	-0.1035	-0.1032
Root 1 with a=3, b=10, c=2:	-0.2135	-0.2129	-0.2136
Root 1 with a=3, b=10, c=3:	-0.3333	-0.333	-0.3335
Root 1 with a=3, b=10, c=8:	-1.333	-1.33	-1.336
Root 1 with a=5, b=7, c=1:	-0.1614	-0.1616	-0.1615
Root 1 with a=5, b=7, c=2:	-0.4	-0.3997	-0.4004
Root 1 with a=5, b=8, c=3:	-0.6	-0.601	-0.5996
Root 1 with a=5, b=9, c=1:	-0.11914	-0.11865	-0.119

Root 1 with a=5, b=9, c=4:	-0.8	-0.799	-0.802
Root 1 with a=5, b=10, c=2:	-0.2253	-0.2251	-0.2255
Root 1 with a=6, b=5, c=1:	-0.3333	-0.3325	-0.3337
Root 1 with a=6, b=7, c=1:	-0.1666	-0.1665	-0.1667
Root 1 with a=6, b=7, c=2:	-0.5	-0.498	-0.4998
Root 1 with a=6, b=9, c=1:	-0.1208	-0.1206	-0.12085
Root 1 with a=6, b=10, c=1:	-0.10675	-0.10645	-0.1069
Root 1 with a=6, b=10, c=4:	-0.6665	-0.665	-0.6655
Root 1 with a=7, b=9, c=1:	-0.12274	-0.12256	-0.1228
Root 1 with a=7, b=9, c=2:	-0.2856	-0.2854	-0.286
Root 1 with a=7, b=10, c=1:	-0.1083	-0.1084	-0.1082
Root 1 with a=9, b=8, c=1:	-0.1504	-0.1506	-0.1505
Root 1 with a=9, b=9, c=2:	-0.3333	-0.333	-0.3335
Root 1 with a=10, b=7, c=1:	-0.2	-0.1998	-0.2001
Root 1 with a=10, b=9, c=2:	-0.4	-0.3994	-0.4001

Using Float64:
Root 1 with a=-10, b=-10, c=-1: -0.8872983346207416 -0.1127016653792583 -0.8872983346207417
Root 1 with a=-10, b=-9, c=-1: -0.7701562118716424 -0.12984378812835756 -0.7701562118716423
Root 1 with a=-10, b=-9, c=0: -0.9 0.0 -0.8999999999999999
Root 1 with a=-10, b=-7, c=-1: -0.5 -0.20000000000000004 -0.4999999999999999
Root 1 with a=-10, b=-6, c=0: -0.6 0.0 -0.5999999999999999
Root 1 with a=-10, b=-4, c=0: -0.4 0.0 -0.39999999999999997
Root 1 with a=-10, b=-2, c=0: -0.2 0.0 -0.19999999999999998
Root 1 with a=-10, b=-1, c=0: -0.1 0.0 -0.09999999999999999
Root 1 with a=-9, b=-9, c=-1: -0.872677996249965 -0.12732200375003505 -0.8726779962499648
Root 1 with a=-9, b=-8, c=-1: -0.73841681234051 -0.1504720765483788 -0.7384168123405102
Root 1 with a=-9, b=-7, c=-1: -0.5891972930813327 -0.188580484696445 -0.5891972930813326
Root 1 with a=-9, b=-4, c=0: -0.4444444444444444 0.0 -0.4444444444444445
Root 1 with a=-9, b=-1, c=0: -0.1111111111111111 0.0 -0.11111111111111112
Root 1 with a=-8, b=-10, c=-3: -0.75 -0.5 -0.7499999999999997
Root 1 with a=-8, b=-10, c=-1: -1.1403882032022077 -0.10961179679779243 -1.1403882032022075
Root 1 with a=-8, b=-9, c=-1: -1.0 -0.125 -0.9999999999999999
Root 1 with a=-8, b=-7, c=-1: -0.6951941016011038 -0.1798058983988962 -0.6951941016011037
Root 1 with a=-8, b=-6, c=0: -0.75 0.0 -0.7499999999999999
Root 1 with a=-8, b=-3, c=0: -0.375 0.0 -0.37500000000000006
Root 1 with a=-7, b=-10, c=-1: -1.3203772410170405 -0.10819418755438781 -1.3203772410170407
Root 1 with a=-7, b=-9, c=-1: -1.1628649920914655 -0.12284929362282004 -1.1628649920914653
Root 1 with a=-7, b=-9, c=0: -1.2857142857142858 0.0 -1.2857142857142856
Root 1 with a=-7, b=-6, c=-1: -0.6306019374818707 -0.2265409196609864 -0.6306019374818708
Root 1 with a=-7, b=-6, c=0: -0.8571428571428571 0.0 -0.8571428571428572
Root 1 with a=-7, b=-5, c=0: -0.7142857142857143 0.0 -0.7142857142857142
Root 1 with a=-6, b=-10, c=-4: -1.0 -0.6666666666666663 -1.0000000000000002

Root 1 with a=-6, b=-10, c=-3: -1.2742918851774319 -0.3923747814892348 -1.2742918851774314
Root 1 with a=-6, b=-10, c=-2: -1.434258545910665 -0.2324081207560017 -1.4342585459106654
Root 1 with a=-6, b=-10, c=-1: -1.5598164905901122 -0.10685017607655434 -1.5598164905901124
Root 1 with a=-6, b=-10, c=0: -1.6666666666666667 0.0 -1.666666666666667
Root 1 with a=-6, b=-9, c=-3: -1.0 -0.5 -0.9999999999999999
Root 1 with a=-6, b=-9, c=-1: -1.379152869605896 -0.12084713039410411 -1.3791528696058957
Root 1 with a=-6, b=-8, c=-1: -1.1937129433613967 -0.1396203899719367 -1.1937129433613969
Root 1 with a=-6, b=-7, c=-2: -0.6666666666666666 -0.4999999999999995 -0.666666666666667
Root 1 with a=-6, b=-7, c=-1: -1.0 -0.16666666666666663 -0.9999999999999999
Root 1 with a=-6, b=-6, c=-1: -0.7886751345948128 -0.21132486540518708 -0.788675134594813
Root 1 with a=-6, b=-5, c=-1: -0.5 -0.33333333333333315 -0.4999999999999998
Root 1 with a=-6, b=-3, c=0: -0.5 0.0 -0.49999999999999994
Root 1 with a=-5, b=-10, c=-4: -1.4472135954999579 -0.5527864045000421 -1.4472135954999577
Root 1 with a=-5, b=-10, c=-3: -1.632455532033676 -0.3675444679663241 -1.6324555320336758
Root 1 with a=-5, b=-10, c=-2: -1.7745966692414832 -0.2254033307585166 -1.7745966692414834
Root 1 with a=-5, b=-9, c=-4: -1.0 -0.7999999999999999 -1.0000000000000004
Root 1 with a=-5, b=-9, c=-3: -1.3582575694955838 -0.4417424305044159 -1.358257569495584
Root 1 with a=-5, b=-9, c=0: -1.8 0.0 -1.8000000000000003
Root 1 with a=-5, b=-8, c=0: -1.6 0.0 -1.5999999999999999
Root 1 with a=-5, b=-7, c=-2: -1.0 -0.4000000000000001 -0.9999999999999998
Root 1 with a=-5, b=-7, c=-1: -1.2385164807134506 -0.16148351928654958 -1.2385164807134503
Root 1 with a=-5, b=-5, c=-1: -0.7236067977499789 -0.27639320225002106 -0.723606797749979
Root 1 with a=-5, b=-2, c=0: -0.4 0.0 -0.39999999999999997
Root 1 with a=-4, b=-10, c=-6: -1.5 -1.0 -1.5000000000000004
Root 1 with a=-4, b=-10, c=-5: -1.8090169943749475 -0.6909830056250525 -1.809016994374947

Root 1 with a=-4, b=-10, c=-3: -2.1513878188659974 -0.3486121811340027 -2.151387818865997
Root 1 with a=-4, b=-10, c=-1: -2.3956439237389597 -0.10435607626104004 -2.39564392373896
Root 1 with a=-4, b=-10, c=0: -2.5 0.0 -2.4999999999999996
Root 1 with a=-4, b=-9, c=-5: -1.25 -1.0 -1.2500000000000009
Root 1 with a=-4, b=-9, c=-4: -1.6403882032022077 -0.6096117967977924 -1.6403882032022075
Root 1 with a=-4, b=-8, c=-3: -1.5 -0.5 -1.4999999999999998
Root 1 with a=-4, b=-8, c=-2: -1.7071067811865475 -0.2928932188134524 -1.7071067811865477
Root 1 with a=-4, b=-7, c=-3: -1.0 -0.75 -1.0000000000000002
Root 1 with a=-4, b=-7, c=-2: -1.3903882032022077 -0.3596117967977924 -1.3903882032022075
Root 1 with a=-4, b=-7, c=-1: -1.5930703308172536 -0.15692966918274642 -1.5930703308172534
Root 1 with a=-4, b=-6, c=-2: -1.0 -0.5 -1.0000000000000002
Root 1 with a=-4, b=-5, c=-1: -1.0 -0.25 -0.9999999999999999
Root 1 with a=-3, b=-10, c=-6: -2.5485837703548637 -0.7847495629784697 -2.5485837703548633
Root 1 with a=-3, b=-10, c=-5: -2.720759220056127 -0.6125741132772069 -2.7207592200561264
Root 1 with a=-3, b=-10, c=-4: -2.86851709182133 -0.4648162415120034 -2.8685170918213294
Root 1 with a=-3, b=-10, c=-3: -3.0 -0.33333333333333326 -2.9999999999999996
Root 1 with a=-3, b=-10, c=-2: -3.1196329811802244 -0.21370035215310867 -3.119632981180225
Root 1 with a=-3, b=-10, c=-1: -3.23013858660781 -0.10319474672552342 -3.2301385866078096
Root 1 with a=-3, b=-7, c=-4: -1.3333333333333333 -0.999999999999999 -1.3333333333333335
Root 1 with a=-3, b=-6, c=-2: -1.5773502691896255 -0.42264973081037416 -1.5773502691896257
Root 1 with a=-3, b=-6, c=0: -2.0 0.0 -1.9999999999999998
Root 1 with a=-3, b=-5, c=-1: -1.434258545910665 -0.2324081207560017 -1.4342585459106647
Root 1 with a=-3, b=-1, c=0: -0.3333333333333333 0.0 -0.33333333333333337
Root 1 with a=-2, b=-10, c=-10: -3.618033988749895 -1.381966011250105 -3.6180339887498953
Root 1 with a=-2, b=-10, c=-9: -3.8228756555322954 -1.1771243444677046 -3.822875655532295

Root 1 with a=-2, b=-10, c=-5: -4.436491673103708 -0.5635083268962915 -4.436491673103709
Root 1 with a=-2, b=-10, c=-4: -4.561552812808831 -0.4384471871911697 -4.56155281280883
Root 1 with a=-2, b=-10, c=-2: -4.7912878474779195 -0.20871215252208009 -4.79128784747792
Root 1 with a=-2, b=-9, c=-10: -2.5 -2.0 -2.4999999999999996
Root 1 with a=-2, b=-9, c=-5: -3.850781059358212 -0.6492189406417879 -3.8507810593582126
Root 1 with a=-2, b=-9, c=-3: -4.1374586088176875 -0.36254139118231254 -4.137458608817687
Root 1 with a=-2, b=-9, c=-1: -4.386000936329383 -0.1139990636706174 -4.386000936329382
Root 1 with a=-2, b=-8, c=-7: -2.7071067811865475 -1.2928932188134525 -2.707106781186548
Root 1 with a=-2, b=-8, c=-5: -3.224744871391589 -0.7752551286084111 -3.2247448713915894
Root 1 with a=-2, b=-8, c=-3: -3.58113883008419 -0.41886116991581024 -3.5811388300841895
Root 1 with a=-2, b=-8, c=-1: -3.8708286933869704 -0.12917130661302934 -3.870828693386971
Root 1 with a=-2, b=-7, c=-6: -2.0 -1.5 -1.9999999999999991
Root 1 with a=-2, b=-7, c=-5: -2.5 -1.0 -2.4999999999999996
Root 1 with a=-2, b=-7, c=-3: -3.0 -0.5 -2.9999999999999996
Root 1 with a=-2, b=-5, c=-3: -1.5 -1.0 -1.4999999999999998
Root 1 with a=-2, b=-5, c=-1: -2.2807764064044154 -0.21922359359558485 -2.280776406404415
Root 1 with a=-1, b=-10, c=-10: -8.872983346207416 -1.127016653792583 -8.872983346207418
Root 1 with a=-1, b=-10, c=-7: -9.242640687119284 -0.7573593128807152 -9.242640687119286
Root 1 with a=-1, b=-10, c=-4: -9.582575694955839 -0.41742430504416017 -5.0
Root 1 with a=-1, b=-10, c=-3: -9.69041575982343 -0.30958424017657027 -0.3095842401765705
Root 1 with a=-1, b=-10, c=-2: -9.79583152331272 -0.2041684766872809 -0.20416847668728044
Root 1 with a=-1, b=-10, c=-1: -9.898979485566356 -0.10102051443364424 -0.1010205144336438
Root 1 with a=-1, b=-10, c=0: -10.0 0.0 3.944304526105059e-30
Root 1 with a=-1, b=-9, c=-10: -7.701562118716424 -1.2984378812835757 -7.701562118716425
Root 1 with a=-1, b=-9, c=-5: -8.405124837953327 -0.594875162046673 -8.405124837953329

Root 1 with a=-1, b=-9, c=-2: -8.772001872658766 -0.2279981273412348 -0.2279981273412344
Root 1 with a=-1, b=-9, c=-1: -8.887482193696062 -0.1125178063039387 -0.11251780630393897
Root 1 with a=-1, b=-9, c=0: -9.0 0.0 2.8398992587956425e-29
Root 1 with a=-1, b=-8, c=-6: -7.16227766016838 -0.8377223398316205 -7.162277660168379
Root 1 with a=-1, b=-8, c=-3: -7.60555127546399 -0.3944487245360109 -4.0
Root 1 with a=-1, b=-8, c=-2: -7.741657386773941 -0.25834261322605867 -0.2583426132260586
Root 1 with a=-1, b=-8, c=-1: -7.872983346207417 -0.12701665379258298 -0.12701665379258312
Root 1 with a=-1, b=-8, c=0: -8.0 0.0 3.218552493301728e-28
Root 1 with a=-1, b=-7, c=-10: -5.0 -2.0 -4.999999999999999
Root 1 with a=-1, b=-7, c=-9: -5.302775637731995 -1.6972243622680054 -5.302775637731994
Root 1 with a=-1, b=-7, c=-2: -6.701562118716424 -0.29843788128357573 -0.2984378812835756
Root 1 with a=-1, b=-7, c=-1: -6.854101966249685 -0.1458980337503153 -0.14589803375031546
Root 1 with a=-1, b=-7, c=0: -7.0 0.0 7.169167906648555e-27
Root 1 with a=-1, b=-6, c=-6: -4.732050807568877 -1.2679491924311228 -4.732050807568878
Root 1 with a=-1, b=-6, c=-2: -5.645751311064591 -0.3542486889354093 -3.0
Root 1 with a=-1, b=-6, c=-1: -5.82842712474619 -0.1715728752538097 -0.17157287525380988
Root 1 with a=-1, b=-6, c=0: -6.0 0.0 4.493351716138883e-25
Root 1 with a=-1, b=-5, c=-5: -3.618033988749895 -1.381966011250105 -3.6180339887498953
Root 1 with a=-1, b=-5, c=-1: -4.7912878474779195 -0.20871215252208009 -0.20871215252208
Root 1 with a=-1, b=-5, c=0: -5.0 0.0 1.4431070514782548e-22
Root 1 with a=-1, b=-4, c=-2: -3.414213562373095 -0.5857864376269049 -3.4142135623730954
Root 1 with a=-1, b=-4, c=-1: -3.732050807568877 -0.2679491924311228 -2.0
Root 1 with a=-1, b=-3, c=0: -3.0 0.0 4.658949397362878e-25
Root 1 with a=-1, b=-2, c=0: -2.0 0.0 -1.0
Root 1 with a=3, b=5, c=1: -0.2324081207560018 -0.2324081207560017 -0.23240812075600176
Root 1 with a=3, b=7, c=2: -0.3333333333333333 -0.33333333333333326 -0.33333333333333337

Root 1 with a=3, b=7, c=4: -1.0 -0.999999999999999 -0.9999999999999994
Root 1 with a=3, b=8, c=1: -0.13148290817867028 -0.13148290817867014 -0.13148290817867023
Root 1 with a=3, b=8, c=2: -0.2792407799438735 -0.2792407799438734 -0.2792407799438736
Root 1 with a=3, b=9, c=2: -0.24169426078820835 -0.24169426078820822 -0.2416942607882084
Root 1 with a=3, b=10, c=5: -0.6125741132772068 -0.6125741132772069 -0.612574113277207
Root 1 with a=3, b=10, c=6: -0.7847495629784698 -0.7847495629784697 -0.7847495629784699
Root 1 with a=3, b=10, c=7: -1.0 -0.9999999999999999 -1.0000000000000002
Root 1 with a=3, b=10, c=8: -1.3333333333333333 -1.3333333333333326 -1.3333333333333337
Root 1 with a=5, b=7, c=1: -0.16148351928654964 -0.16148351928654958 -0.1614835192865496
Root 1 with a=5, b=8, c=1: -0.13667504192892005 -0.13667504192892 -0.13667504192892002
Root 1 with a=5, b=8, c=2: -0.31010205144336445 -0.31010205144336433 -0.3101020514433644
Root 1 with a=5, b=8, c=3: -0.6 -0.5999999999999996 -0.6000000000000001
Root 1 with a=5, b=9, c=1: -0.1189750324093346 -0.11897503240933449 -0.11897503240933456
Root 1 with a=5, b=9, c=4: -0.8 -0.7999999999999999 -0.8000000000000004
Root 1 with a=5, b=10, c=1: -0.10557280900008408 -0.10557280900008414 -0.10557280900008413
Root 1 with a=6, b=5, c=1: -0.3333333333333333 -0.33333333333333315 -0.3333333333333332
Root 1 with a=6, b=6, c=1: -0.21132486540518713 -0.21132486540518708 -0.2113248654051871
Root 1 with a=6, b=7, c=1: -0.16666666666666666 -0.16666666666666663 -0.16666666666666669
Root 1 with a=6, b=7, c=2: -0.5 -0.4999999999999995 -0.4999999999999999
Root 1 with a=6, b=8, c=1: -0.13962038997193674 -0.1396203899719367 -0.13962038997193676
Root 1 with a=6, b=9, c=1: -0.12084713039410418 -0.12084713039410411 -0.12084713039410419
Root 1 with a=6, b=10, c=2: -0.2324081207560018 -0.2324081207560017 -0.23240812075600176

Root 1 with a=6, b=10, c=4: -0.6666666666666666 -0.6666666666666663 -0.6666666666666665
Root 1 with a=7, b=9, c=1: -0.12284929362282014 -0.12284929362282004 -0.12284929362282011
Root 1 with a=7, b=9, c=2: -0.2857142857142857 -0.28571428571428564 -0.28571428571428575
Root 1 with a=7, b=10, c=1: -0.1081941875543879 -0.10819418755438781 -0.10819418755438784
Root 1 with a=9, b=9, c=2: -0.3333333333333333 -0.33333333333333326 -0.3333333333333334
Root 1 with a=10, b=7, c=1: -0.2 -0.20000000000000004 -0.19999999999999998
Root 1 with a=10, b=8, c=1: -0.15505102572168222 -0.15505102572168217 -0.1550510257216822
Root 1 with a=10, b=9, c=2: -0.4 -0.39999999999999997 -0.39999999999999974

2c)
We see that when Float64 arithmetic is used over Float16, such as with the case of

(Float16) Root 1 with a=-10, b=-9, c=-1: -0.7705 -0.1296 -0.77

vs.

(Float64) Root 1 with a=-10, b=-9, c=-1: -0.7701562118716424 -0.12984378812835756 -0.7701562118716423

We observe similar results, but with different degrees of accuracy. In the float16 example, we see that the A value and the C value have a difference of 0.0005, whereas with the Float64 sample, we see that there's a difference of 0.0000000000000001. However, the interesting part is the comparison of the value of the completing the square value. Between Float16 and Float64, we observe that not only is there a difference of 0.0002…, but there is a consistent difference of 0.6… between the other methods as well.

3a)

- **Data** (Input) – The data in the random artists problem is the directed graph of musical artists

- **Model** (Mathematical model) – The model in the random artists problem is the PageRank model, or the linear system of equations and matrices defined by PageRank, including the out-edge adjacency matrix, the diagonal matrix D, and the system of equations defined by $(\boldsymbol{I} - \alpha \boldsymbol{A}^T \boldsymbol{D}^{-1})\boldsymbol{x} = \frac{1-\alpha}{n}\boldsymbol{e}$.

- **Algorithm** (Code and arithmetic) – The algorithm used in the random artists problem is the code to solve the $(\boldsymbol{I} - \alpha \boldsymbol{A}^T \boldsymbol{D}^{-1})\boldsymbol{x} = \frac{1-\alpha}{n}\boldsymbol{e}$ system of equations.

3b)

Having two separate people implementing the same program and end up with different results stems from identifying error in the data, the model, and the algorithm.

Considering the problem, and under the assumption that the data is the same between the two implementations, the model used by both programs solve the same linear system of equations, and barring that the differences between the two rankings is the result of any bugs, we can then look to the algorithm to identify the issue. Three examples that I can think of are:

- Rounding issues – Despite Julia's 10e-16 precision, we know that Julia is not immune from issues with working with floating-point numbers. There may have been differences with the way that both programs use values to solve the equation, which may have caused the

- Preprocessing Differences – If one program preprocessed the data (normalization, scaling, etc.,) prior to solving the system of equations, versus if the other program didn't preprocess or preprocessed the data differently, could lead to implications in the results of the ranking.

- Tie handling – The method of handling ties could also impact the resultant ranking. One algorithm could break ties using alphabetical ranking, vs the other could break ties based on the list-order priority.

3c)

Having two separate people implementing the same program and end up with different results stems from identifying error in the data, the model, and the algorithm.

Considering the problem, and under the assumption that the data is the same between the two implementations and barring that the differences between the two rankings is the result of any bugs, we can then look to the model and the algorithm to identify the issue. Three examples that I can think of are:

- Model issue: Variability of Random Sampling – Because Monte Carlo processes are reliant on random sampling to estimate their results, the randomness of the model inherently introduces differences. If the scores for the 5^{th} and 6^{th} artists are close, the differences due to randomness could interfere with the ordering.

- Algorithm issue: Number of Trials affecting Convergence – Monte Carlo simulations grow more accurate with the number of trials that take place. The accuracy scores may not have converged fully if enough trials were not used, which leads to variability in the rankings.

- Algorithm issue: Seed differences – If the two programs are not using the same seed for the random operations, this directly affects the generation of random numbers that would be generated by the code.

4)

Julia code to solve a Linear System of equation for x given matrices A and b, and save b to a file

```
## import packages
using Random
using LinearAlgebra
using DelimitedFiles

## Function to create a square matrix of fractions
function ijmat(n::Integer)
    A = zeros(n,n)
    for j=1:n
        for i=1:n
            A[i,j] = 1/(i+j-1)
        end
    end
    return A
end

## Function to generate a vector of random values
function randVec(n::Integer)
    b = zeros(n,1)
    for i=1:n
        b[i, 1] = randn()
    end
    return b
end

## solve linear system
A = ijmat(10)
b = randVec(10)

x = A \ b

## Write b to a files
writedlm("b-matrix.csv", b, ",")
```

Python function to read the file containing b, create the A matrix, and solve for x

```
## Import statements
import numpy as np

## Function to create A matrix
def ijmat(n:int):
    a = np.zeros((n, n))
    for j in range(0, n):
        for i in range(0, n):
            a[i][j] = 1/(i+j+1)
    return a

## Read Matrix from files
b = np.loadtxt("b-matrix.csv", delimiter=',')
a = ijmat(10)

## Solve Linear System of Equations
x = np.linalg.solve(a,b)

print("{}x{} Matrix[{}]".format(x.shape[0], 1, type(x[0])))
print(x)
```

Results:

Julia	Python
[1.1679478871319711e7; -9.922291147728866e8; 2.0880921003386864e10; -1.8812416864386414e11; 8.91056570604955e11; -2.435858305880686e12; 3.9783045458550254e12; -3.830009203030495e12; 2.0042902293131746e12; -4.395674439527591e11;]	[1.16796042e+07 -9.92239980e+08 2.08811531e+10 -1.88126284e+11 8.91066683e+11 -2.43588616e+12 3.97835031e+12 -3.83005349e+12 2.00431351e+12 -4.39572568e+11]

We see that therefor the most part, both languages solve the system of equations to a similar accuracy, however there are some differences between the two, possibly due to the way that the files are read into python and stored. When looking at the b matrix in python we get the following values:

[-0.33403393 0.17410968 0.6433672 -0.93453359 0.52022645 -1.49928572
 -0.18298165 0.22038015 0.8657362 -0.21727673]

Versus the values stored in the file:

[-0.3340339323081479 0.17410968487380635 0.6433672025329462 -0.9345335863585456
0.5202264501717059 -1.4992857198925658 -0.18298164669857492 0.22038014842594486
0.8657362023180324 -0.21727673171628686]

We see that the values encoded in the file have greater significant digits than the values read into python, which would impact rounding and the display of the number.