**Eigenvalues Applications Manufacturing & Supply Chain**

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In this posting I will be exploring the use of eigenvalues & eigenvectors and their practical applications in supply chain and manufacturing.

**Intro to Eigenvalues & Eigen Vectors**:

Eigenvalues & Eigen vectors are used the analysis of linear transformations. They represent a special set of scalar values that is associated with a set of linear equations. The EigenVector is a non-zero vector that can by its eigenvalue. ‘Eigen’ comes from German and means proper or characteristic, as they can also be known as characteristic or latent roots. represent characteristic value, roots.

Consider a square matrix A of size nxn. An eigen vector of this matrix is a non-zero vector v such that when A multples v, the result is a scalar of multiple of V. This scalar is called an eigenvalue denoted by λ.

Given Matrix A and Eigen Vector, we can start setting up our Equations:



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Setting up the equation

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Take the determinant

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Solve for the Eigen Value – Not this gives us our Polynomial Roots.



Then Substitute back to find corresponding eigenvector

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At this point its clear to see that the roots of a polynomial(λ) =0 are equal to the eigenvalues and the eigenvector corresponding to each eigenvalue is a non-zero vector v that satisfies the equation Av=λv were A is a square matrix and λ is an eigenvalue. If we think of the matrix as a transformation that stretches, shrinks, or rotates vectors. Eigenvalues tell you by what factor the vector is stretched or shrunk (magnitude change), while eigenvectors indicate the direction along which this transformation occurs.

**Eigenvalues in Manufacturing Capacity Planning**

**Covariance Matrix:** This matrix represents the covariance (a measure of how much two random variables change together) between pairs of variables in the dataset. It is commonly used in Principal Component Analysis (PCA), as we can use it to identify the principal components that capture the most variance in the data.

**Eigenvalues and Eigenvectors**

* **Eigenvalues**: Indicate the amount of variance captured by each principal component. Larger eigenvalues correspond to components that capture more variance in the data.
* **Eigenvectors**: Provide the direction of the principal components in the original feature space. These components are linear combinations of the original variables and are orthogonal to each other.

**Possible Uses Cases:**

**Demand Forecasting**:

* + **Application**: Use PCA to reduce the dimensionality of demand data, identifying key patterns and trends.
  + **Benefit**: Enhances the accuracy of demand forecasts by focusing on the most significant factors.

**Inventory Management**:

* + **Application**: Analyze the variability and correlation between different inventory items to optimize stock levels.
  + **Benefit**: Helps in better stock management and reordering strategies, reducing stockouts and overstocking.

**Adjacency Matrix**

Represents the connections between nodes in a network, where each element indicates whether a direct route exists between two nodes and the cost associated with that route.

* **Eigenvalues**: Provide insights into the connectivity and criticality of the network. Smaller eigenvalues indicate routes whose optimization will significantly impact overall network efficiency.
* **Eigenvectors**: Show the influence and direction of each route within the network, helping to identify the most significant routes for optimization.

**Use Cases:**

1. **Route Optimization**:
   * **Application**: Identify and optimize critical transportation routes to minimize costs and improve delivery times.
   * **Benefit**: Reduces transportation costs and improves logistics efficiency.
2. **Network Robustness Analysis**:
   * **Application**: Evaluate the robustness of the supply chain network to identify weak points and critical nodes.
   * **Benefit**: Enhances the resilience and reliability of the supply chain.

**Performance Matrix**

Represents various performance metrics (ex: cost, quality, reliability) for different suppliers or processes.

**Eigenvalues and Eigenvectors:**

* **Eigenvalues**: Indicate significant performance factors affecting the supply chain. Larger eigenvalues highlight the most impactful metrics.
* **Eigenvectors**: Show the contribution of each supplier or process to the performance factors, aiding in selection and optimization efforts.

**Use Cases:**

1. **Supplier Performance Evaluation**:
   * **Application**: Evaluate and rank suppliers based on performance metrics to select the most reliable and cost-effective ones.
   * **Benefit**: Improves supplier selection and management, reducing risks and costs.
2. **Process Optimization**:
   * **Application**: Analyze performance metrics to identify key factors driving process efficiency.
   * **Benefit**: Optimizes manufacturing processes for better efficiency and quality.

**Transition Matrix**

**Transition Matrix**: Represents the probabilities of transitioning from one state to another in a Markov chain model. Used for modeling and predicting future states based on current data.

**Eigenvalues**: Provide insights into the stability and long-term behavior of the system. The largest eigenvalue (typically 1 for a probability transition matrix) and its corresponding eigenvector indicate the steady-state distribution.

**Eigenvectors**: Indicate the transition dynamics between states, helping to understand how the system evolves over time.

**Use Cases:**

1. **Scenario Analysis for Disruptions**:
   * **Application**: Model and analyze different disruption scenarios in the supply chain to predict their impact and develop mitigation strategies.
   * **Benefit**: Enhances preparedness and resilience against potential disruptions.
2. **Capacity Planning**:
   * **Application**: Use Markov chain models to predict future capacity needs based on current usage patterns.
   * **Benefit**: Helps in proactive planning and resource allocation, ensuring adequate capacity to meet demand.

**Summary**

By leveraging eigenvalues and eigenvectors in our various types of matrices (covariance, adjacency, performance, and transition matrices) we can use this simple math tool to help us extract interesting and valuable insights, making this another simple tool to help us enable data-driven decisions that enhance efficiency, reduce costs, and improve overall performance across the difference types of analysis.

References:

[Eigenvalues ( Definition, Properties, Examples) | Eigenvectors (byjus.com)](https://byjus.com/maths/eigen-values/#:~:text=Eigenvalues%20are%20the%20special%20set%20of%20scalars%20associated,root%2C%20proper%20values%20or%20latent%20roots%20as%20well.)

Feedback:

Reviewer: Aditya Patel

Strengths:

* Your blog is very good at detailing the definitions of the eigenvalues and eigenvectors, and explaining their significance for linear transforms, and is at a level that is understood by the average reader.
* Your examples of practical applications, such as inventory management and process optimization are great, but could use a bit more detail.
* The math is broken down and is good at explaining the detail of how to calculate the eigenvectors and eigenvalues.

Suggestions:

* Make sure to double check your grammar for spelling mistakes and typos
* Make sure that your math notation is consistent all the way through the document
* Be more detailed about how eigenvectors/values would be applied in each of your usecases.