QUIZ 4

purdue university · cs 51500

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matrix computations

*June 27, 2023*

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AP Any behavior consistent with academic dishonesty (i.e. cheating) *will* *not be tolerated and may result in a 0 even a failing grade in the class*.

AP You have until the Gradescope or Blackboard submission closes to com plete and submit the exam. No exceptions.

Write your name, PUID, and sign below to indicate you agree with the statement:

*The remainder of this exam represents my own work.*

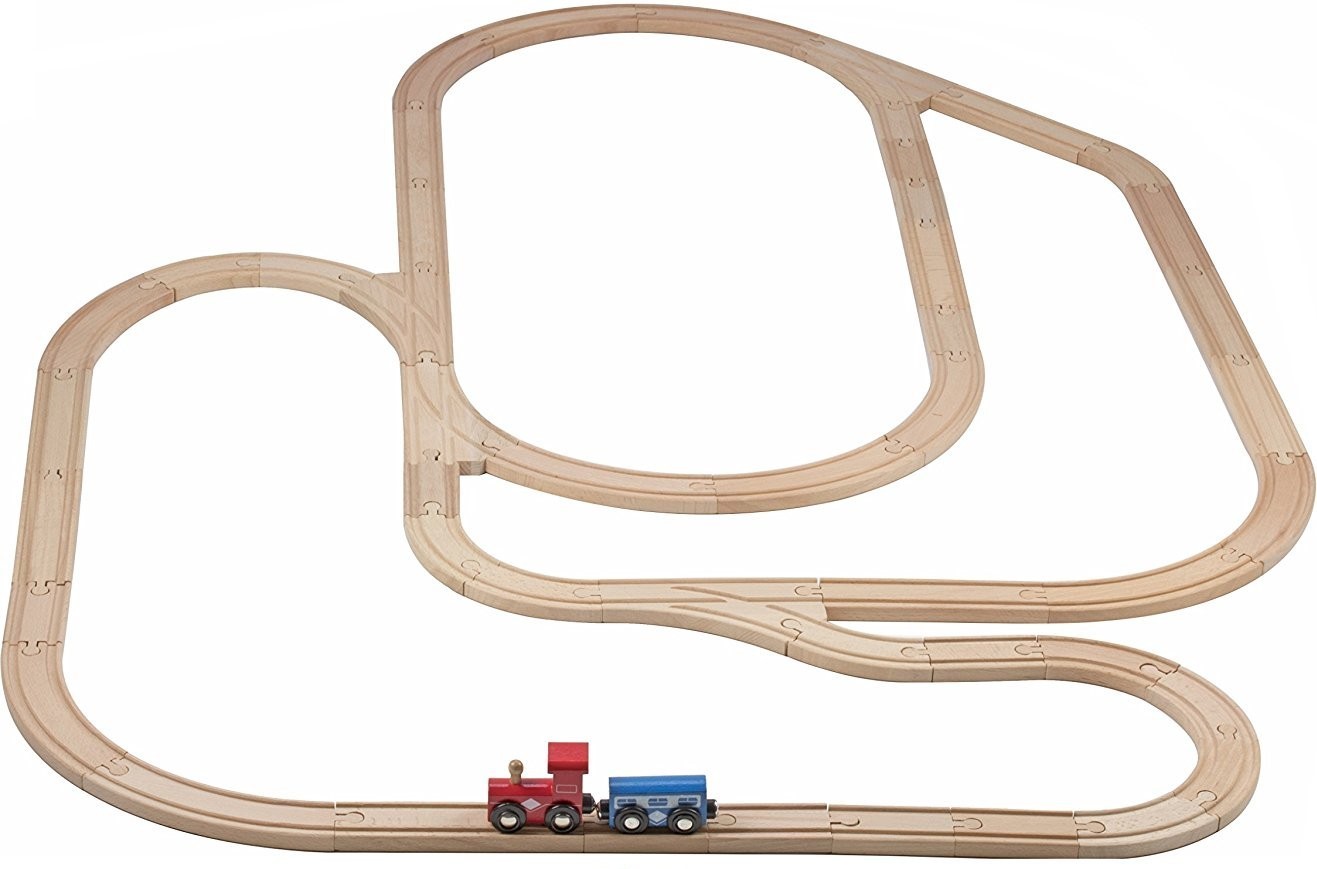
(Your Name) Aditya Patel (PUID) 0035573514

Aditya Patel

(Signature)

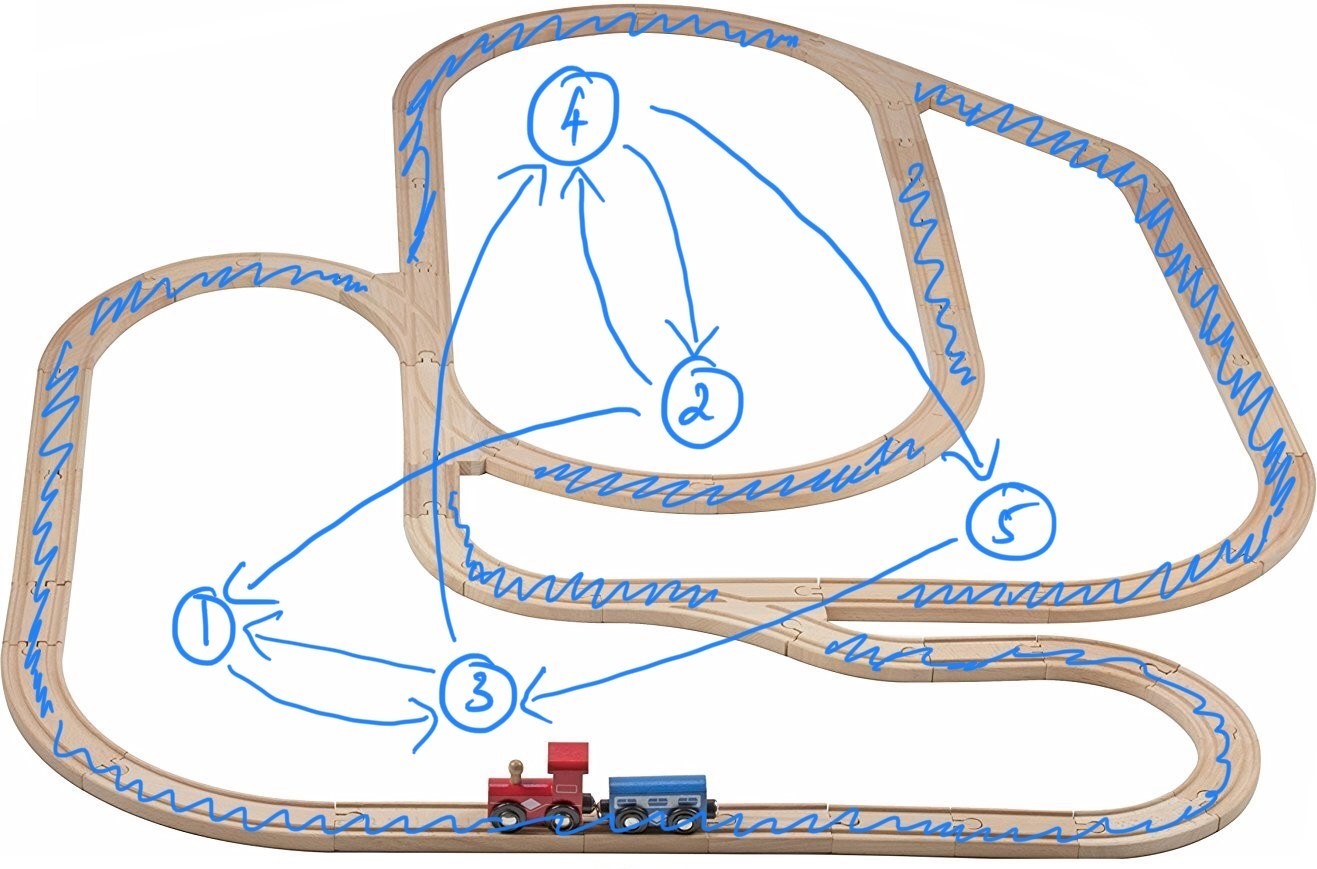
**Problem 1** *(30 points)*

In class we saw random walks and random surfers. Here, we are going to investigate random play. A common household toy in the US is a wooden train track. An example from a Walmart advertisement[[1]](#footnote-1) is



Imagine a child playing with this train set where the train starts as indicated in the picture and moves to the left (i.e. in the direction of the red car). At any intersection where there is a choice, the child will randomly pick a direction. Note that an intersection may not offer a choice based on which way the train is going. We are going to use a graph to build a model to determine how often each section of track between intersections is visited under the child’s random play model.

A random walk on the graph in following picture illustrates a simple model of what happens.



(10 points) Use that graph model to find the long term fraction of time that the train will spend on each segment 1-5 based on what you learned in class. Justify any entries of 0 based on the strong component structure.

The long-term fractions of time can be determined by using the following code in Julia.

using LinearAlgebra

A = [0 0 1 0 0

1 0 0 1 0

1 0 0 1 0

0 1 0 0 1

0 0 1 0 0]

## Get the inverse of the diagonal degree matrix

d = vec(sum(A, dims=2))

D\_inv = Diagonal(1.0 ./ d)

## Find P

P = Matrix(A' \* D\_inv)

vals, vecs = eigen(P)

## Find the eigenvector corresponding to the largest eigenvalue

max\_val\_ind = findmax(abs.(vals))[2]

println("Largest Eigenvector:")

probs = Float64.(vecs[:, max\_val\_ind]/sum(vecs[:, max\_val\_ind]))

This results in the following results corresponding to each node:

0.22222222222222252

0.11111111111111088

0.3333333333333332

0.2222222222222225

0.11111111111111092

(10 points) Now suppose that the train starts in the opposite direction. Give the new graph model and show the long term probability. Justify any entries of 0 based on the strong component structure.

Taking the transpose of A in the Julia code above to turn the in-edges into out-edges (reversing the graph directions) and then running the code once more, we see the following results corresponding to each node.

0.11111111111111116

0.22222222222222215

0.2222222222222222

0.3333333333333335

0.11111111111111113

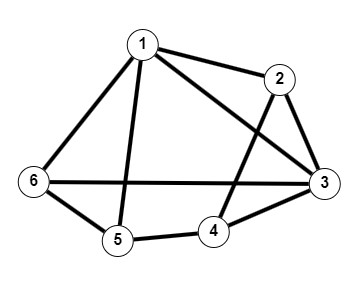
(10 points) Now imagine we have two children playing. One of whom wants to have the train go, and the other who keeps picking the train up off the train and putting it back randomly. Let’s approximate the behavior of what happens with a PageRank model. Suppose that 90% of the time, the train-child is allowed to move the train in a fixed direction. The other child will come with probability 1*/*8 and move the train somewhere new with a random direction. Use a PageRank model with *α* = 0*.*875 to find the long-term fraction of time on each segment of track going each of the two directions.

Using the simple pagerank algorithm with alpha = 0.875, we observe the following results:

0.21852551984877125  
0.12060491493383743  
0.32173913043478264  
0.21852551984877125  
0.12060491493383743

**Problem 2** *(20 points)*

Consider this graph



(10 points) Give the Laplacian matrix for this graph.

using LinearAlgebra

using MatrixNetworks

using SparseArrays

A = [0 1 1 0 1 1;

1 0 1 1 0 0;

1 1 0 1 0 1;

0 1 1 0 1 0;

1 0 0 1 0 1;

1 0 1 0 1 0;]

D = Diagonal(vec(sum(A, dims=2)))

L = D – A

Which gives the following Laplacian Matrix:

4 -1 -1 0 -1 -1

-1 3 -1 -1 0 0

-1 -1 4 -1 0 -1

0 -1 -1 3 -1 0

-1 0 0 -1 3 -1

-1 0 -1 0 -1 3

(5 points) Give the incidence matrix for this graph.

To find the incidence matrix B, we can use this Julia code:

A = [0 1 1 0 1 1

1 0 1 1 0 0

1 1 0 1 0 1

0 1 1 0 1 0

1 0 0 1 0 1

1 0 1 0 1 0]

# Get the number of vertices

num\_vertices = size(A, 1)

# Find all edges in the graph

edges = []

for i in 1:num\_vertices

for j in i+1:num\_vertices

if A[i, j] == 1

push!(edges, (i, j))

end

end

end

# Get the number of edges

num\_edges = length(edges)

# Initialize the incidence matrix B

B = zeros(Int, num\_edges, num\_vertices)

# Fill the incidence matrix B

for (k, (i, j)) in enumerate(edges)

B[k, i] = 1

B[k, j] = -1

end

# Print the incidence matrix B

print(B)

Which results in the following B matrix

1 -1 0 0 0 0

1 0 -1 0 0 0

1 0 0 0 -1 0

1 0 0 0 0 -1

0 1 -1 0 0 0

0 1 0 -1 0 0

0 0 1 -1 0 0

0 0 1 0 0 -1

0 0 0 1 -1 0

0 0 0 0 1 -1

(5 points) Give the Fiedler vector (the eigenvector used for spectral clustering) for this graph with the first entry *x*1 positive.

To find the Fiedler vector, we can use this Julia code:

using LinearAlgebra

using MatrixNetworks

using SparseArrays

A = [0 1 1 0 1 1;

1 0 1 1 0 0;

1 1 0 1 0 1;

0 1 1 0 1 0;

1 0 0 1 0 1;

1 0 1 0 1 0;]

f2 = fiedler\_vector(sparse(A))[1]

f2 .\*= sign(f2[1])

The fiedler vector f2 is then

6-element Vector{Float64}:

0.11619967077072714

-0.304407036570377

-0.1161996707707274

-0.23664291968450613

0.23664291968450415

0.30440703657037743

1. https://www.walmart.com/ip/Trains-All-PCS-Railway-Set-Is-With-Toys-Sets-52-

   Thomas-Tracks-For-Bonus-Compatible-Kids-Systems-2-Wooden-Major-Car-By-Play22-

   Brands-Train-Original-Toy/85696313 [↑](#footnote-ref-1)