Assignment Instructions: Assignment 5

Directions

1. Consider the following activity-on-arc project network, where the 12 arcs (arrows) represent the 12 activities (tasks) that must be performed to complete the project and the network displays the order in which the activities need to be performed. The number next to each arc (arrow) is the time required for the corresponding activity. Consider the problem of finding the longest path (the largest total time) through this network from start (node 1) to finish (node 9), since the longest path is the critical path.

   Formulate and solve the binary integer programming (BIP) model for this problem using library *lpsolve* or equivalent in R.

Solution :

LP Formulations : 5x12+3x13+3x35+2x25+4x24+4x47+x46+1x58+6x57+5x69+4x79+7x89

Subject to :

x13+x12=1 origin node
x12-x25-x24=0 intermediate node1
x13-x35=0 intermediate node2
x25+x35-x57-x58=0 intermediate node3
x24-x47-x46=0 intermediate node4
x46-x69=0 intermediate node5
x47+x57-x79=0 intermediate node6
x58-x89=0 intermediate node7
x89+x79+x69=1 Destination Node

the longest path is the critical path is 17

After running this problem on any LP solver, the results are:

"x12" "1"
"x25" "1"
"x57" "1"
"x79" "1"

2. **Selecting an Investment Portfolio** An investment manager wants to determine an optimal portfolio for a wealthy client. The fund has $2.5 million to invest, and its objective is

to maximize total dollar return from both growth and dividends over the course of the coming year. The client has researched eight high-tech companies and wants the portfolio to consist of shares in these firms only. Three of the firms (S1 – S3) are primarily software companies, three (H1–H3) are primarily hardware companies, and two (C1–C2) are internet consulting companies. The client has stipulated that no more than 40 percent of the investment be allocated to any one of these three sectors. To assure diversification, at least $100,000 must be invested in each of the eight stocks. Moreover, the number of shares invested in any stock must be a multiple of 1000.

The table below gives estimates from the investment company's database relating to these stocks. These estimates include the price per share, the projected annual growth rate in the share price, and the anticipated annual dividend payment per share.

1) Determine the maximum return on the portfolio. What is the optimal number of shares to buy for each of the stocks? What is the corresponding dollar amount invested in each stock?
2) Compare the solution in which there is no integer restriction on the number of shares invested. By how much (in percentage terms) do the integer restrictions alter the value of the optimal objective function? By how much (in percentage terms) do they alter the optimal investment quantities?

Solution

In the given problem, the objective function includes the price per share, the projected annual growth rate in the share price, and the anticipated annual dividend payment per share. The expression is as below
Returns = (Price per share) * (Growth rate of share) + (Dividend per share)
Hence the objective function is
Max, Z = 4 XS1 + 6.5 XS2 + 5.9 XS3 + 5.4 XH1 + 5.15 XH2 + 10 XH3 + 8.4 XC1 + 6.25 XC2
Subject to the constraints,
Investment constraint:
40 XS1 + 50 XS2 + 80 XS3 + 60 XH1 + 45 XH2 + 60 XH3 + 30 XC1 + 25 XC2 <= 2500000
The number of shares invested in any stock must be a multiple of 1000
1000 XSJ >= 0 (J = 1,2,3)
1000 XHJ >= 0 (J = 1,2,3)
1000 XCJ >= 0 (J = 1,2)
At least $100,000 must be invested in each of the eight stocks
40 XS1 >= 100000; 50 XS2 >= 100000; 80 XS3 >= 100000; 60 XH1 >= 100000;
45 XH2 >= 100000; 60 XH3 >= 100000; 30 XC1 >= 100000; 25 XC2 >= 100000
The client has stipulated that no more than 40 percent of the investment be allocated to any one of these three sectors
40 XS1 + 50 XS2 + 80 XS3 <= 1000000
60 XH1 + 45 XH2 + 60 XH3 <= 1000000
30 XC1 + 25 XC2 <= 1000000
Where XSJ, XHJ, XCJ >= 0 are integers.

Using lpsolve with integer restriction we get the objective function, maximum returns as 487145.2 and number of stocks are S1= 2500, S2= 6000, S3= 1250, H1= 1667, H2= 2223, H3= 3332, C1= 30000, C2= 4000.
The amount invested in each stock S1= 100000, S2= 300000, S3= 100000, H1= 100020, H2= 100035, H3= 799920, C1= 900000, C2= 100000.

Q 2b:

Using lpsolve without integer restriction we get the objective function, maximum returns as 487152.8 and number of stocks are S1= 2500.0, S2= 6000.0, S3= 1250.0, H1= 1667.667, H2= 2222.222, H3= 13333.333, C1= 30000.0, C2= 4000.0
The amount invested in each stock S1= 100000, S2= 300000, S3= 100000, H1= 100000, H2= 100000, H3= 800000, C1= 900000, C2= 100000.
Percentage difference in objective functions with and without integer restriction is 0.00156