The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

1. Solve the problem using lpsolve, or any other equivalent library in R.
2. Identify the shadow prices, dual solution, and reduced costs
3. Further, identify the sensitivity of the above prices and costs. That is, specify the range of shadow prices and reduced cost within which the optimal solution will not change.
4. Formulate the dual of the above problem and solve it. Does the solution agree with what you observed for the primal problem?

Answer:

1. **See code saved with file name WP_03**

2. Identify the shadow prices, dual solution, and reduced costs

Shadow prices:
0.00 0.00 0.00 12.00 20.00 60.00 0.00 0.00 0.00 -0.08 0.56

Dual solution:
0.00 0.00 0.00 12.00 20.00 60.00 0.00 0.00 0.00 -0.08 0.56

Reduced cost:
0 0 -24 -40 0 0 -360 -120 0

3.
```
      price         lower        upper
 [1,]  0.00 -1.000000e+30 1.000000e+30
 [2,]  0.00 -1.000000e+30 1.000000e+30
 [3,]  0.00 -1.000000e+30 1.000000e+30
```

```
 [4,] 12.00  1.122222e+04 1.388889e+04
 [5,] 20.00  1.150000e+04 1.250000e+04
 [6,] 60.00  4.800000e+03 5.181818e+03
 [7,]  0.00 -1.000000e+30 1.000000e+30
 [8,]  0.00 -1.000000e+30 1.000000e+30
 [9,]  0.00 -1.000000e+30 1.000000e+30
[10,] -0.08 -2.500000e+04 2.500000e+04
[11,]  0.56 -1.250000e+04 1.250000e+04

  cost          lower         upper
 [1,]    0 -1.000000e+30 1.000000e+30
 [2,]    0 -1.000000e+30 1.000000e+30
 [3,]  -24 -2.222222e+02 1.111111e+02
 [4,]  -40 -1.000000e+02 1.000000e+02
 [5,]    0 -1.000000e+30 1.000000e+30
 [6,]    0 -1.000000e+30 1.000000e+30
 [7,] -360 -2.000000e+01 2.500000e+01
 [8,] -120 -4.444444e+01 6.666667e+01
 [9,]    0 -1.000000e+30 1.000000e+30
```

4.

$x1$ = number of large units produced per day at Plant 1,
$x2$ = number of medium units produced per day at Plant 1,
$x3$ = number of small units produced per day at Plant 1,
$x4$ = number of large units produced per day at Plant 2,
$x5$ = number of medium units produced per day at Plant 2,
$x6$ = number of small units produced per day at Plant 2,
$x7$ = number of large units produced per day at Plant 3,
$x8$ = number of medium units produced per day at Plant 3,
$x9$ = number of small units produced per day at Plant 3.

Objective function:

Maximize $420\,x1 + 360\,x2 + 300\,x3 + 420\,x4 + 360\,x5 + 300\,x6 + 420\,x7 + 360\,x8 + 300\,x9,$

Subject to:

(Capacity)

$$\begin{array}{llllll}
x1 + x2 + x3 & & & \leq & 750 & \Rightarrow y1 \\
 & x4 + x5 + x6 & & \leq & 900 & \Rightarrow y2 \\
 & & x7 + x8 + x9 & \leq & 450 & \Rightarrow y3
\end{array}$$

(Square footage)

$$\begin{array}{llllll}
20\,x1 + 15\,x2 + 12\,x3 & & & \leq & 13000 & \Rightarrow y4 \\
 & 20x4 + 15\,x5 + 12\,x6 & & \leq & 12000 & \Rightarrow y5 \\
 & & 20\,x7 + 15\,x8 + 12x9 & \leq & 5000 & \Rightarrow y6
\end{array}$$

(Sales)

$$\begin{array}{llllll}
x1 + & x4 + & x7 & \leq & 900 & \Rightarrow y7 \\
x2 & + x5 & + x8 & \leq & 1200 & \Rightarrow y8
\end{array}$$

x3 + x6 + x9≤ 750 => y9

(Same percentage of capacity)

900 x1 +900 x2 +900 x3 - 750 x4 – 750 x5 – 750 x6 = 0 => y10

450 x1 + 450 x2 +450 x3 - 750 x7 – 750 x8 – 750 x9 = 0 => y11

Z = 750 y1+900 y2+450y3+13000y4+12000y5+5000y6+900y7+1200y8+750y9

y 1+20y4+y7+900y10+450y11 >=420
y1+15y4+y8+900y10+450y11>=360
y1+12y4+y9+900y10+450y11 >=300
y2+20y5+y7-750y10 >= 420
y2+15y5+y8-75010 >= 360
y2+12y5+y9-750y10 >= 300
y3+20y6+y7-750y11 >=420
y3+15y6+y8-75011 >= 360
y3+12y6+y9-750y11 >= 300

y1,y2,y3,y4,y5,y6,y7,y8,y9, >=0 y10,y11 unrestricted

After solving this dual by lpsolve solution agrees with primal problem (codes in **WP_Dual_03**)