ANOMALIES IN NET PRESENT VALUE, RETURNS AND POLYNOMIALS, AND REGRET THEORY IN DECISION-MAKING

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Contents

Abbreviations

AIRR	Average Internal Rate of Return
APT	Arbitrage Pricing Theory
APV	Adjusted Present Value
AROI	Aggregate Return on Investment
CAPM	Capital Asset Pricing Model
CML	Capital Market Line
CPT	Cumulative Prospect Theory
CSIP	Consumption, Savings, Investment, and Production
DC	Defined Contribution
DCA	Decision Curve Analysis
DIM	Differential Importance Measure
EIS	Elasticity of Intertemporal Substitution
ETF	Exchange Traded Fund
EVA	Economic Value Added
FTA	Fundamental Theory of Algebra
IAPT	Intertemporal Arbitrage Pricing Theory
ICAPM	Intertemporal Capital Asset Pricing Model
IRR	Internal Rate of Return
LPVR	Least Present Value of the Revenues
MIRR	Modified Internal Rate of Return
MRIS	Marginal Rate of Intertemporal Substitution
MRIJS	Marginal Rate of Intertemporal Joint Substitution

NDRB	Non-Zero Discount Rate Bias
NFV	Net Future Value
NPV	Net Present Value
NPVv	NPV of the variance
PIH	Permanent Income Hypothesis
PT	Prospect Theory
PT^3	Third-Generation Prospect Theory
RRA	Relative Risk Aversion
ROFR	Rights of First Refusal
ROI	Return on Investment
SEG	Socio-Emotional Goods
SEHK	Hong Kong Stock Exchange
SEU	Social Expected Utility Model
SML	Security Market Line
SVA	Systemic Value Added
TCM	Transaction Costs and Monitoring Costs
TVM	Time Value of Money
UIWD	Unified Intertemporal Wealth-Allocation Decision
WACC	Weighted Average Cost of Capital
WTAL	Willingness To Accept Losses

List of Tables

1

Introduction

Polynomials are a well established branch of mathematics with wide applications in finance, physics, operations research, healthcare informatics, engineering, economics and other subject areas. However, most if not all the recent research on polynomials and a large portion of associated research in related fields are based on highly inaccurate theorems and methods. The NPV, APV, and IRR-MIRR models (and related discounting models) and compounded returns are classes of polynomials. Throughout this book, the NPV-IRR-MIRR model and compounded returns are used to illustrate the anomalies in polynomials specifically within the context of Regret Theory (that is, some of the biases and framing effects inherent in the NPV-IRR-MIRR model and compounded returns are explained using some elements of Regret Theory). This book also surveys the literature on how Regret Theory, as a growing cross-disciplinary subject area, has emerged as a viable alternative to the NPV-IRR model in decision-making (and Regret Theory has been used extensively in medical decision-making). Research in capital budgeting and the NPV-IRR model (and related approaches) is in a state of flux. Researchers from various fields (economics, computer science, Operations Research, Industrial

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Engineering, Engineering Economy, Applied Math and Finance) have addressed the topic from different approaches. The major obstacles seem to be as follows: (i) reliance on old math theorems which are inaccurate – such as the Fourier-Boudan theorem, the Descartes Sign Rule, Sturm's Theorem, Vincent's Theorem, and the Fundamental Theorem of Algebra; (ii) the NPV-IRR model is inherently problematic and generates Framing Effects, non-monotonic NPV and compounding anomalies; (iii) the NPV (and related approaches) is inherently inconsistent with IRR (and related approaches); (iv) researchers' focus on the uniqueness of IRR and conditions for monotonic-NPV don't solve the problems inherent in the NPV-IRR model; (v) the "new" approaches to capital budgeting (such as MIRR, AROI, AIRR, SIRR, EVA, NFV, SVA, GIRR, GNPV) suffer from the same or similar problems of NPV-IRR model. Research in this area has evolved from focusing on multiple IRRs and conditions for monotonic-NPV and unique IRR, to new alternatives to NPV-IRR, optimization models (of the NPV-IRR model); versions of the NPV-IRR model that account for stochastic cashflows and discount rates; and the consistency of various capital budgeting approaches.

The substantial gaps in the literature include the following:

(a) Documenting and proving new anomalies in the NPV-IRR model (and related approaches);
(b) Tests of the Descartes Sign Rule, Sturm's Theorem, Vincent's Theorem, and the Fundamental Theorem of Algebra;
(c) Tests of factoring in algebra;
(d) Analysis of framing effects inherent in the NPV-IRR model and related models (such as MIRR, AIRR, AROI, GNPV, APV, NFV, EVA, Profitability Index, etc.).
(e) Development of new and efficient alternatives to the NPV-IRR model.

Net Present Value (NPV)—and related approaches, such as Internal Rate Of Return (IRR), Modified Internal Rate Of Return (MIRR) and Adjusted Present Value (APV)—remains the most popular decision model among companies, individuals, and government agencies worldwide. The NPV-IRR-MIRR model is built into many large-scale decision models, such as: large securities trading systems; treasury operations of multinational companies; supply chain decisions; financial operations of government agencies; loan processing systems of banks, healthcare informatics systems; and so on. Compounded returns, the IRR, and the MIRR are used extensively in all areas of engineering, science and business to measure gains and performance, and are often built into reward systems, cost-allocation systems and performance measurement systems. In addition to capital budgeting, the NPV-IRR model and present-value models are used for evaluating the risk of, and calculating the yields for, more than US$100+ trillion worth of fixed income securities (bonds/bills/notes) and the annual percentage rates (APRs) and yields for more than US$40+ trillion worth of loans/mortgages that exist around the world (see https://www.federalreserve.gov/econresdata/releases/mortoutstand/current.htm). The NPV-IRR model and present-value/discounting models are also used for valuing interest rate swaps (which have a *gross market value* that exceeds US$13+ trillion and a *gross notional amount* that exceeds US$180 trillion). Correia (2012), Kester and Robbins (2011), Graham and Harvey (2002), Ryan and Ryan (2002), Graham et al. (2015: 463), Magni (2015), Jagannathan et al. (Aug. 2013), and Kester et al. (1999) noted that the NPV-IRR model is widely used by, and is pervasive in many companies and institutions around the world. Schaelemann et al. (2012) discussed the HAMP NPV model which the US government and the private-sector used extensively during and after 2009 for mortgage loan modifications. There are significant biases and computational errors inherent in methods of calculating returns (which can render scientific studies and business results useless).

Almost all existing asset pricing models have been based on a narrow set of parameters that are grossly inadequate to explain patterns of modern behavior and business. The net effect is that most existing asset pricing models are misspecified. That in turn, adversely affects NPV and IRR calculations.

Although most of the analysis in this book was done with discrete cashflows (instead of continuous cashflows) which are the most common format, some of the Framing Effects in Chapter 2 and anomalies in Chapter 7 also applies to continuous cashflows.

The principal audiences for this book are as follows:

1. Academics
2. Professionals that hold PhDs and work in government agencies, non-profit organizations and private companies, such as:
   - (i) technical management consultants
   - (ii) investment professionals, research analysts, portfolio managers, and insurance and banking professionals
   - (iii) strategic planning and business development professionals
   - (iv) economists and government-regulation professionals

1.1 The NPV-IRR Model and Related Approaches (Such As AIRR, MIRR, NFV, APV; GIRR; SIRR; AROI; etc.) are Wrong

The NPV-IRR model (and related approaches) is wrong in almost all circumstances primarily because of errors in the underlying theorems of polynomials, differential calculus and discounting; and because of the *framing effects* inherent in the NPV-IRR-MIRR model. The NPV-IRR-MIRR model (and related approaches; including almost all discounting models) don't not consider the many psychological biases inherent in human decision-making; and do not accurately incorporate time value; and do not account for Regret, which is a fundamentally significant element of both human and automated decision-making. NPV, APV, MIRR-IRR and related approaches also do not account for patterns of allocation of resources (i.e., consumption, investment, leisure, etc.) in individual, household, and corporate decision-making. NPV, MIRR, and related approaches do not accommodate the differences between compounded interest rates and simple interest rates and do not account for real options in decision-making. Stewart et al. (2014), Bleichrodt et al. (2008), Haven and Khrennikov (2016); Cantone et al. (2016); Guth et al. (2009); Gureckis and Love (2009); Tsai and Bockenholt (2008); De La Bruslerie (2015); Loewenstein and Prelec (1991, 1992); Frederick et al. (2002); Zauberman et al. (2009); Ouattara and De La Bruslerie

(2015); Scholten and Read (2006); and Frederick (2006) critiqued (or introduced theories that contravene) models of discounted utility and intertemporal choice.

The *Modified-IRR* (MIRR) has been criticized significantly in the literature, most notably by a series of articles by Magni some of which are cited in Chaps. 6 and 7 in this book; and also by Kulakov and Blaset Kastro (2015), Magni (2015) and Weber (2014). The MIRR formula is (or can be expressed as) a time-value-of-money (TVM) equation which has all the problems of IRR – and contrary to the literature and for a given cashflow series, there can be multiple MIRRs or no MIRR depending on the magnitudes of the present values of the cash outflows (PV_{MIRR}) and the future value of the cashflows (FV_{MIRR}). In the MIRR formula, the process of finding the FV_{MIRR} at a specific discount rate and then discounting the FV_{MIRR} to present value at another discount rate is somewhat redundant. An inspection of the MIRR formula shows that the occurrence of multiple MIRRs is more likely as FV_{MIRR} tends to zero; and as the PV_{MIRR} tends to zero. Magni (2013) mentioned eighteen flaws of the IRR and attempted to explain how *Average-IRR* (AIRR) does not incur the IRR problems. However, AIRR has many of the framing effects inherent in the NPV-IRR model (Types A, B, D, G & I Frames in Chap. 2), and the averaging implicit in AIRR raises the issue of an implicit reinvestment rate. AIRR is subject to the compounding anomalies mentioned in Chaps. 7 and 8 because the discount factor $(1+r)^n$ is in both the numerator and denominator of the AIRR formula (where r is a periodic rate-of-return) – see Eq. 3 in Altshuler and Magni (2012: 224). AIRR postulates that the decision maker can either explicitly estimate the values of interim-investments or use interim-values implied by the cost of capital (instead of IRR's implied interim-values) but that creates opportunities for manipulation, and interim-values may not be readily available; and the recommended estimation of interim-value involves calculating the NPV of future projected cash flows using the market-based cost-of-capital as the discount rate (which introduces biases and anomalies inherent in the NPV). Research in the literature (e.g., Jagannathan et al. 2013) has indicated that firms often use very high discount rates that are much greater than their true cost-of-capital. One version of the AIRR formula (Eq. 4 in Altshuler and Magni 2012: 224) involves heavy use of the NPV formula in both the denominator and numerator – and again that introduces all the problems of

NPV (eg. non-monotonic NPV; framing effects; and compounding effects). Given a specific time horizon and a cashflow series, a specific magnitude of AIRR can be achieved by selecting different combinations of interim-values and the estimated periodic investment rates. Thus the AIRR is not always unique and there can be multiple AIRRs if there is any change in the estimate of the cost of capital for various periods; or if interim-values are variable contingent values. The Selective-IRR ("SIRR"; introduced by Weber 2014) is subject to the compounding and framing anomalies mentioned in Chaps. 2 and 8 in this book. Given the process of deriving SIRR, it cannot be unique where a TVM equation has multiple roots (complex and or real IRRs) or no roots, in which case there can be multiple SIRRs or no SIRRs respectively – see the comments about multiple-IRRs and the usefulness of complex rates-of-return in Hazen (2003), Pierru (2010), and Ben-Horin and Kroll (2012). The Weber (2014) objective of, and excessive emphasis on the consistency of the SIRR with the NPV-rule is mis-placed (given the discussion in Chaps. 2 and 7 herein, and in Berkovitch and Isreal (2004), Magni (2009, 2002)) and does not necessarily make SIRR an effective decision making rule, and it doesn't solve the problems of non-monotonic NPV, compounding anomalies, framing anomalies and multiple IRRs. Thus, the SIRR is wrong. The Generalized-IRR (GIRR) (introduced by Kulakov and Blaset Kastro 2015; and Kulakova and Kulakov 2013) has the same inherent compounding and framing anomalies that are inherent in the NPV-IRR model, some of which are explained in Chaps. 2, 7 and 8 in this book. The adjusted-NPV ("APV"), the Modified-NPV ("MNPV"; McClure and Girma 2004) and the generalized NPV ("GNPV") (Beaves 1993; Blaset Kastro and Kulakov 2013) have the same anomalies and framing effects as the NPV-IRR model which are described in Chaps. 2 and 7 in this book. The AIRR, MIRR, GIRR and SIRR cannot help a decision-maker select from among competing projects when capital is constrained, primarily because of the weaknesses mentioned herein. The AIRR, MIRR, GIRR and SIRR also suffer from asymmetric risk-scaling which is explained in Chap. 2 herein. The *Aggregate Return On Investment* ("AROI"; introduced by Magni 2015) is subject to some of the Framing Effects mentioned in Chap. 2 in this book (Types A & G Frames). The compounding effects in AROI arises because there is an implicit rate-of-return and implicit re-investment of cashflows (just as in the NPV criterion) – Magni (2015)'s opposite conclusion contradicts its own admission that AROI indirectly incorporates the time-value of money. In the AROI formula, the process of calculating the Invested Capital for each period involves one-period (and implicitly, multi-period) compounding using a constant risk-adjusted cost-of-capital. There is significant research in the literature that concludes that the NPV model implicitly assumes that there is reinvestment of cashflows. Indeed, Magni (2015) concedes and explicitly shows that while AROI is based on un-discounted values, AROI indirectly incorporates the time-value of money; and that the AROI is a "modified-AIRR" where the cost-of-capital is equal to zero. For a given cashflow series that occurs in equal time-periods, there can be multiple AROIs if any of the following conditions exist: (a) the denominator or numerator includes variable contingent cashflows or contingent capital; (b) the risk-adjusted cost-of-capital is the IRR and there can be multiple IRRs; (c) when calculating the series of one-period Invested Capital, the risk-adjusted cost-of-capital can change. The AROI concept does not work where the total cashflows or the Invested Capital is less than zero (in which case the AROI will be negative). In Magni (2015), the definition of Invested Capital as the difference between the market value of the project and the wealth increase (NPV) is not entirely correct. The *Net Final Value* (NFV), *Economic Value Added* (EVA), *Systemic Value Added* (SVA), the DCF-based *Profitability Index* and Adjusted Present Value (APV) all have some of the disadvantages of the NPV (ie. Framing Effects and or compounding/discounting anomalies) some of which are explained in Chaps. 2 and 7 in this book. Magni (2005) discussed NFV and SVA. Padilla et al. (2013) concluded that some standard decision criteria (such as net final value; IRR; benefit–cost ratio; profitability index; equivalent annuity, discounted payback period and average payback period) lead to the same investment decision and same ranking as net present value rule. However, most of those methods have the same weaknesses as the NPV-IRR model and again, the emphasis on their consistency with the NPV-rule is misplaced given the discussions in Chaps. 2 and 7 herein, and in Berkovitch and Isreal (2004), Magni (2009, 2002); and doesnt solve the problems of non-monotonic NPV, compounding anomalies, framing anomalies and multiple IRRs. While several researchers (such as Leyman and Vanhoucke 2017, 2016; Wiesemann et al. 2010; and Creemers et al. 2010) have developed optimization models for NPV (wherein an objective function is maximized), such models are subject to the anomalies and framing-effects introduced in this book and thus are probably wrong. In the literature, the excessive emphasis on finding "unique IRRs" and conditions for monotonic-NPV, is un-warranted given the discussions in Chaps. 2 and 7 herein, and in Berkovitch and Isreal (2004), Magni (2009, 2002); and it doesn't solve the other problems inherent in NPV-IRR (such as Framing Effects and compounding anomalies). Developing the sufficient conditions of a "conventional project" (see Blaset Kastro and Kulakov 2016) has no basis and does not solve the anomalies and compounding problems mentioned in Chaps. 2 and 7 in this book. The analysis in Danielson (2016) and Osborne (2016) is wrong because of the anomalies and Framing Effects introduced in this book; and as mentioned in Nwogugu (2012: 324–330), Macaulay Duration is wrong. In the literature, the emphasis on developing discounting methods has not been successful; and Ouattara and De La Bruslerie (2015) noted that: (i) various discount functions such as the exponential, Hernstein, Harvey, proportional, Laibson, Rachlin, hyperbolic and generalized hyperbolic discount functions have been used in the literature to model individuals' time preferences; (ii) standard discounted utility theory is wrong and time preferences can't be characterized by an exponential discount function – that conclusion is consistent with other empirical studies and shows that the population is characterized by a decreasing impatience and by heterogenous psychological discount functions. All the discount functions mentioned in Ouattara and De La Bruslerie (2015) (e.g., exponential; Hernstein; Harvey; proportional; Laibson; Rachlin; hyperbolic; etc.) are conceptually similar and suffer from some of the Framing Effects and anomalies inherent in the NPV-IRR model which are explained in Chaps. 2 and 7 herein.

1.2 The NPV-IRR Model, Discounting and Macroeconomics

Unfortunately, the anomalies and framing effects inherent in discounting and the NPV-IRR model (and related approaches) and the pervasive use of these models significantly affect, translate and are aggregated into problems in major macroeconomic variables such as foreign direct investment (FDI) (reduced or increased) (see comments in Bekaert et al. 2015); balance-of-payments (reduced or increased), balance-of-trade (reduced or increased); M1/M2/M3/M4; corporate investment; capital expenditures; import substitution; government spending (reduced or increased); GDP/GNP; Financial Stability (reduced); systemic risk (increased); corporate bankruptcy rates (increased); liquidity in both the real and financial sectors (reduced); corporate credit ratings (reduced); consumer spending; consumer confidence; loan volumes; interest rates (increased); cross-border capital flows; demand for foreign currency (reduced or increased); etc.. Discounting and the NPV model are directly used in various areas of macroeconomic theory (eg. Investment; efficiency of Investment; marginal efficiency of capital; monetary economics; present value of debt; demand for capital; interest rates; regulatory capital for banks; analysis of systemic risk; etc.). Because of their pervasive use in industry and government around the world, the NPV-IRR model (and related approaches) are a major set of macroeconomic and international financial variables – the set includes average discount rates (by industry and organization-size); re-investment rates; average hurdle-rates (by industry and organization-size); exchange-rates; benchmark interest rates (interbank rates and discount-window rates); the average payback period; average coupon-rates for bonds/bills; and the average time-horizon for medium/large projects.

1.3 NPV-IRR Model (And Related Approaches) in Academia

Despite the critiques of the NPV-IRR model (and related approaches) and researchers' efforts during the past seventy years, and the significant importance of decisions made with the models (from investment decisions to capital allocation and divestment decisions in multinationals to major PPP infrastructure projects), the capital budgeting problem remains largely un-solved; academics and practitioners still continue to use the NPV-IRR model and its variants, and its being taught to millions of students annually. The following factors may account for this phenomenon: (1) Inertia by PHD researchers in academia and industry who haven't committed enough effort towards developing new capital budgeting methods (the financial/quantitative research departments at banks and financial institutions may not have allocated enough resources towards development of new and effective capital budgeting methods); (2) tenure-related anxiety in academia wherein professors and their research associates are wary of challenging the status quo in knowledge due to concerns about getting tenure at universities – this may be prevalent in economics/finance and operations research (but is much less likely in psychology where there continues to be active academic critique of theoretical and empirical research); (3) lack of adequate direction by editors of finance/economics/operations-research journals who haven't committed enough effort towards developing new capital budgeting methods; (4) group-think by academic researchers some of whom tend to follow trends (approaches, research-questions, theories, methods, literature reviews; etc.) in both empirical and theoretical research; (5) group-think and "conformance" by senior executives at companies and government agencies which reinforces the use of the NPV-IRR model (and related models such as EVA; APV; DCF-based profitability index; MIRR; etc.) – in such situations, the penalties for deviation/non-conformance can be substantial and can include loss of employment, reduced bonuses, transfers, etc.; (6) over-dependence on transferred knowledge wherein practitioners focus on using what they learnt in school, instead of researching and developing new methods (also the masters degree curricula in the economics/business/operations-research programs at most universities often don't encourage or teach independent research into theory); (7); over-dependence on the "top-journal syndrome" wherein academic researchers tend to accept as true and established (and tend to propagate), the theories, research questions, methods and empirical results that are published in "top-ranked journals" – where unfortunately, the rankings are based on popularity (citations); (8) lack of adequate direction by PHD committees in finance/economics/operations-research in PHD degree programs in universities – who arguably, should steer more PHD students into research in capital budgeting; (9) ironically, during the last twenty years, while the majority (or a significant percentage) of the academic research on intertemporal choice, time-valuation and discounting was done by the psychology faculties of universities, most of the practitioners that frequently use the NPV-IRR model and related approaches are trained by the business/economics/finance/operations-research/engineering faculties of universities whose members also deliver many of the internal corporate/government training programs where NPV-IRR is taught; (10) This psychological phenomenon can also be attributed to the concept of *Willingness-To-Accept-Losses* (WTAL), which is explained in Nwogugu (2006) and is different from Loss Aversion and Risk Aversion—NPV-IRR and related models/theories are deeply ingrained in academia and industry, and cancellation or replacement of those models will be a major psychological, economic, and socio-political loss to those practitioners and academicians.

1.4 The Un-Reliability of Empirical Methods/Models in Operations Research, Computer Science and Psychology

Various authors have noted the significant unreliability of empirical research methods/models in operations research, computer science, neuroscience, political economy and psychology. This issue was addressed in Open Science Collaboration (28 August 2015); Bohannon (August 28, 2015); Ioannidis (2005); Banks et al. (2015); Banks et al. (2016, in press); Bosco et al. (2016, in press); John et al. (2012); Kerr (1998); Masicampo and Lalande (2012); O'Boyle et al. (2016, in press); Schmidt and Hunter (2015); Bai, Zhang & Huang (2012); Fahimnia, Tang, Davarzani & Sarkis (2015); Uttal (2012); Fitzgerald, Matusall, Skewes & Roepstorff (May 30, 2014); Aven (2016); and BEC Crew (August 28, 2015).[1] Thus, the empirical studies cited in this book are subject to the limitations and issues mentioned in the foregoing articles.

[1] This article stated in part "A landmark study involving one hundred (100) scientists from around the world has tried to replicate the findings of 270 (two hundred and seventy) recent findings from highly ranked psychology journals and by one measure, only 36% (thirty six percent) turned up the same results. That means that for over half the studies, when scientists used the same methodology, they could not come up with the same results.… And earlier this year, a separate study found that the prevalence of irreproducible preclinical research exceeds 50% (fifty percent), "resulting in approximately US$28,000,000,000 (twenty-eight billion US dollars) per year spent on preclinical research that is not reproducible—in the United States alone."

1.5 The Chapters

Investment decision analysis using NPV and or the Mean-Variance model has become the primary method of investment evaluation. Chapter 2: (i) explains behavioral and psychological biases inherent in financing decisions, which contradict the NPV-APV-MIRR-IRR model; (ii) explains existing and new spatio-temporal framing effects inherent in NPV-MIRR-IRR and related models; (iii) critiques Iturbe-Ormaetxe et al. (2010); (iv) surveys the relevant literature on Regret Theory which explains how it can serve as an alternative to the NPV-MIRR model for decision-making; (v) explains the biases and framing effects inherent in the Mean-Variance model.

Researchers have since noted the many problems inherent in the one-period capital asset pricing model (CAPM) and the one-period arbitrage pricing theory, and have developed the intertemporal CAPM, the intertemporal APT, and the multifactor CAPM. The CML remains a somewhat popular but unchallenged proposition that has significant flaws. Chapter 3 introduces several empirically testable financial theories that provide insights, can be calibrated to real data and used to solve problems, and that contribute to the literature by: (i) explaining the conditions under which ICAPM/CAPM, IAPT, and CML may be accurate, why such conditions are not feasible, and why the existence of incomplete markets and dynamic unaggregated markets render CML, IAPT, and ICAPM inaccurate; (ii) explaining why the widely used *consumption-savings-investment-production framework* is insufficient for asset pricing and analysing changes in risk and asset values; (iii) introducing a unified approach to asset pricing that simultaneously considers six factors (consumption, taxes, investment, leisure, intangibles, and housing), and the conditions under which this approach will work; (iv) explaining why leisure, taxes, and housing are equally as important as consumption and investment in asset pricing; (v) introducing the *Marginal Rate of Intertemporal Joint Substitution* (MRIJS) among consumption, taxes, investment, leisure, intangibles, and housing—this model incorporates Regret Theory and captures features of reality that do not fit well into standard asset pricing models, and this framework can support specific or very general finance theories and very complicated models; (vi) explaining why the widely debated *Elasticity of Intertemporal Substitution* (EIS) is inaccurate and is insufficient for asset pricing and analysing investor preferences (within the context of capital budgeting and decision-making).

Chapter 4 proves that the *Descartes Sign Rule*—as interpreted by most academicians, such as Osborne (2010)—Sturm's Theorem, Vincent's Theorem and the *Fourier-Boudan Theorem* are wrong.

Chapter 5 proves that the Fundamental Theorem of Algebra (FTA) and the Binomial Theorem are wrong; and explains how root calculation in algebra may be misleading and introduces an alternative method for verifying real and complex roots of a polynomial. Both a six-degree and a nine-degree polynomial equation are solved by introducing new classes of invariants (MCN-2 invariants) and homomorphisms. Osborne (2010: 235) and many researchers have erroneously concluded that when the number of periods (n) in a TVM equation is greater than four, it is impossible to solve algebraically for (1+r); and NPV cannot be expressed algebraically in terms of IRR, or vice versa (where r is the IRR)—on the contrary, this contention is wrong as shown below. Burrus (2004), Sitton et al. (2003), Osborne (2010: 235) and Lei et al. (1996) concluded that polynomials with degrees that exceed six are impossible to factor and solve. These issues are applicable in nonlinear analysis, evolutionary computation and pattern-analysis—given the discussions in Yannacopoulos et al. (1996); Campos-Canton et al. (2015); Zheng et al. (2010); and Boyer and Goh (2007).

Chapter 6 surveys the literature on concepts of interest rates and discount rates; discusses the Weighted Average Cost of Capital (WACC); explains the conditions under which negative interest rates may be feasible and rational; and how human decisions differ from the standard NPV-IRR model and related approaches (most of which are variants of Samuelson's *Discounted Utility model*). The chapter also critiques Grosskopf & Roth (2009).

Investment decision analysis using NPV has become the primary method of investment evaluation. Chapter 7: (i) proves new anomalies and errors inherent in the NPV-IRR model; (ii) develops the necessary and sufficient conditions for monotonic NPV (well behaved NPV); (iii) develops the necessary and sufficient conditions for the anomalous behavior of NPV; (iv) shows that the power rule and the inverse function rule in differential calculus, are both wrong; (v) surveys the literature on anomalies in the NPV-MIRR-IRR model and related methods.

Chapter 8: (i) explains the biases and quasi framing effects inherent in the calculation of compounded returns for both single-assets and ETFs/Funds (distinct from human biases that can affect returns, and vice versa), which in turn, implies that various investment approaches (such as minimum variance investing; geometric mean maximization; etc.) are wrong and inaccurate; (ii) shows how some of the biases introduced herein are new types of evolutionary homomorphisms; and can affect the analysis of pattern formation, chaos, and adaptive systems—the discussions in Preis et al. (2012), Kenett et al. (2012), Preis (2011), Podobnik et al. (2009), Fenn, et al. (2011), Kriener et al. (2014), Hsieh (1993), Menna et al. (2002), and Hommes (2002) omitted such effects; and (iii) explains some of the effects of market volatility on compounded returns. The chapter also reviews Co and Labuszewski (July 2012), who attempted to use the Taylor Series to model the relationship between volatilty and returns.

Given the massive applications of NPV-IRR-MIRR and compounded returns in many areas of business government and science/engineering, an obvious application (of the issues discussed in this book) is in arbitrage, but that is not covered in this book. Rather, it is hoped that this book will: (i) provide professors with supplementary material; (ii) spur further research on new decision models; (iii) assist technology professionals charged with the reliability, engineering, and safety of large-scale computing systems to better understand the limitations of the models discussed herein.

1.6 Anomalies in Real Options Analysis (Used in Valuation or Capital Budgeting)

This book does not cover Real Options or financial options, both of which have emerged as capital budgeting tools. Prior studies (Denison 2009) noted that decision-makers that use Real Options display a lower escalation of commitment compared to decision-makers that use NPV. Liao and

Ho (Nov. 2010); Willigers and Hansen (2008); Brandão and Dyer (2005); and Mason and Merton (1985) analyzed real options. Lander and Pinches (1998); Schachter and Mancarella (2016); Baker et al. (2011); Triantis (2005); and Oppenheimer (Dec. 2002) critiqued Real Options analysis. The problems are that: (i) some projects don't have Real Options; and (ii) even when Real Options exist, it may be hard to forecast how and when such options can be exercised—this requires making assumptions; (iii) Real Options analysis involves discounting and present values and thus suffer from the same anomalies and framing effects as the NPV-IRR model (some of which are described in this book); (iv) in Real Options, items such as salvage-values and contract-rights require making forecasts for which data may not be available, and require discounting or calculation of future-values, all of which introduce anomalies and framing effects mentioned herein; and (v) Real Options analysis is relatively more complex than the NPV-IRR model and that discourages users (see Baker et al. (2011)).

Contrary to the literature, the NPV model can be used to estimate the value of the option to defer, expand, contract, or even abandon a project when market conditions change. For example, the value of the option to defer a project (V_d) that starts at time t but can be deferred by one period to time $t + 1$ is: $V_d = \text{NPV}_{(t+1)} - \text{NPV}_t$, where $\text{NPV}_{(t+1)}$ is the NPV of the project when started at time $t + 1$, but with the resulting NPV discounted to time t. NPV_t is the NPV if the project is started at time t. The issue of irreversibility of investments in capital projects is overemphasized because sometimes there are active markets for un-completed projects, equipment, and machinery parts, and companies can sell/assign their contract rights in big projects.

1.7 Financial Options Valuation in Capital Budgeting

Financial options have also been used to value projects. However, as noted in Nwogugu (2003) and Taleb (2009), most options pricing models are incorrect. Some of the financial options models use the binomial pricing model, but as explained herein, the binomial theorem is wrong (Smith and Nau (1995)).

Bibliography

Altshuler, D., & Magni, C. (2012). Why IRR is not the rate of return for your investment: Introducing AIRR to the Real Estate community. *Journal of Real Estate Portfolio Management, 18*(2), 219–223.

Ang, A., & Liu, J. (2004). How to discount cashflows with time-varying expected returns. *Journal of Finance, 59*(6), 2745–2783.

Aven, T. (2016). Risk assessment and risk management: Review of recent advances on their foundation. *European Journal of Operational Research, 253*(1), 1–13.

Bai, X., Zhang, H., & Huang, L. (2012). *Empirical research in software process modeling: A systematic literature review*. Working Paper.

Baker, H., Dutta, S., & Saadi, S. (2011). Management views on real options in capital budgeting. *Journal of Applied Finance, 21*, 18–29.

Banks, G. C., Kepes, S., & McDaniel, M. A. (2015). Publication bias: Understand the myths concerning threats to the advancement of science. In C. E. Lance & R. J. Vandenberg (Eds.), *More statistical and methodological myths and urban legends* (pp. 36–64). New York: Routledge.

Banks, G. C., O'Boyle Jr., E., Pollack, J. M., White, C. D., Batchelor, J. H., Whelpley, C. E., Abston, K. A., Bennett, A. A., & Adkins, C. L. (2016, in press). Questions about questionable research practices in the field of management: A guest commentary. *Journal of Management, 42,* 5–20.

Barcelona, R. (2015). Renewable energy with volatile prices: Why NPV fails to tell the whole story. *Journal of Applied Corporate Finance, 27*(1), 101–109.

Beaves, R. (1993). The case for a generalized net present value formula. *The Engineering Economist, 38*(2), 119–133.

BEC Crew. (2015, August 28). *Scientists tried to replicate 100 psychology experiments and 64% failed*. http://www.sciencealert.com/scientists-tried-to-replicate-100-psychology-experiments-and-64-failed

Bekaert, G., Harvey, C., Lundblad, C., & Siegel, S. (2015). *Political risk and international valuation* (Working paper). https://www0.gsb.columbia.edu/mygsb/faculty/research/pubfiles/13995/Bekaert_Political_Risk.pdf

Ben-Horin, M., & Kroll, Y. (2012). The limited relevance of the multiple IRRs. *The Engineering Economist, 57*, 101–118.

Berkovitch, E., & Isreal, R. (2004). Why the NPV criterion does not maximize NPV. *Review of Financial Studies, 17*(1), 239–255.

Blaset Kastro, A. N., & Kulakov, N. (2013). Evaluation of non-conventional projects. *The Engineering Economist, 52*(2), 137–148.

Blaset Kastro, A. N., & Kulakov, N. (2016). Definition of the concepts of conventional and non-conventional projects. *Business Informatics, 2*(36), 16–23.

Bleichrodt, H., Rohde, K., & Wakker, P. (2008). Koopmans' constant discounting for intertemporal choice: A simplification and a generalization. *Journal of Mathematical Psychology, 52*(6), 341–347.

Bohannon, J. (2015, August 28). Many psychology papers fail replication test. *Science, 349*(6251), 910–911. http://www.sciencemag.org/content/349/6251/910.full.pdf

Bosco, F. A., Aguinis, H., Field, J. G., Pierce, C. A., & Dalton, D. R. (2016, in press). Harking's threat to organizational research: Evidence from primary and meta-analytic sources. *Personnel Psychology, 69,* 709–750.

Boyer, R., & Goh, W. (2007). On the zero attractor of the Euler polynomials. *Advances in Applied Mathematics, 38*(1), 97–132.

Brandão, L., & Dyer, J. (2005). Decision analysis and real options: A discrete time approach to real option valuation. *Annals of Operation Research, 135*, 21–39.

Burrus, C.S. (2004, August 1–4). *Factoring high degree polynomials in signal processing*. 11th IEEE digital signal processing workshop, 2004 and the 3rd IEEE signal processing education workshop, 156–157.

Campos-Canton, E., Aguirre-Hernandez, B., Renteria, J., & Gonzalez, E. (2015). A polynomial approach for generating a monoparametric family of chaotic attractors via switched linear systems. *Chaos, Solitons & Fractals, 71*, 10–16. doi:10.1016/j.chaos.2014.12.012.

Cantone, D., Giarlotta, A., Greco, S., & Watson, S. (2016). (m,n)-rationalizable choices. *Journal of Mathematical Psychology, 73*, 12–27.

Co, R., & Labuszewski, J. (2012, July 26). Leveraged ETFs: Where is the missing performance? *Stock Market Landscape*, 1–4 |CME Group.

Correia, C. (2012). Capital budgeting practices in South Africa: A review. *South African Journal Of Business Management, 43*(2), 11–29.

Creemers, S., Leus, R., & Lambrecht, M. (2010). Scheduling Markovian PERT networks to maximize the net present value. *Operations Research Letters, 38*(1), 51–56.

Danielson, M. G. (2016). The IRR of a project with many potential outcomes. *The Engineering Economist, 61*(1), 44–56.

De La Bruslerie, H. (2015). Decreasing term structure of psychological discount rates: Experimental estimation and determinants. *Journal of Behavioral and Experimental Finance, 6*, 13–16.

Denison, C. (2009). Real options and escalation of commitment: A behavioral analysis of capital investment decisions. *The Accounting Review, 84*, 133–155.

Fahimnia, B., Tang, T., Davarzani, S., & Sarkis, J. (2015). Quantitative models for managing supply chain risks: A review. *European Journal of Operational Research, 247*, 1–15.

Fenn, D. J., et al. (2011). Temporal evolution of financial-market correlations. *Physics Review-E, 84*, 61–65.

Fitzgerald, D., Matusall, S., Skewes, J., & Roepstorff, A. (May 30, 2014). What's so critical about Critical Neuroscience? Rethinking experiment, enacting critique. *Frontiers in Human NeuroScience*. http://journal.frontiersin.org/article/10.3389/fnhum.2014.00365/full.

Frederick, S. (2006). Valuing future life and futures lives: A framework for understanding discounting. *Journal of Economic Psychology, 27*, 667–680.

Frederick, S., Loewenstein, G., & O'Donoghue, T. (2002). Time discounting and time preference: A critical review. *Journal of Economic Literature, 40*(1), 351–401.

Gerber, A., & Malhotra, N. (2008a). Do statistical reporting standards affect what is published? Publication bias in two leading political science journals. *Quarterly Journal of Political Science, 3*, 313–326.

Gerber, A. S., & Malhotra, N. (2008b). Publication bias in empirical sociological research- do arbitrary significance levels distort published results? *Sociological Methods & Research, 37*, 3–30.

Graham, J., & Harvey, C. (2002). How Do CFOs make capital budgeting and capital Structure Decisions? *Journal of Applied Corporate Finance, 15*(1), 8–23.

Graham, J., Harvey, C., & Puri, M. (2015). Capital allocation and delegation of decision-making authority within firms. *Journal of Financial Economics, 115*, 449–480.

Grosskopf, B., & Roth, A. E. (2009). If you are offered the right of first refusal, should you accept? An investigation of contract design. *Games and Economic Behavior, 65*, 176–204.

Gureckis, T., & Love, B. (2009). Learning in noise: Dynamic decision-making in a variable environment. *Journal of Mathematical Psychology, 53*(3), 180–193.

Guth, W., Levati, V., & Ploner, M. (2009). An experimental analysis of satisficing in saving decisions. *Journal of Mathematical Psychology, 53*(4), 265–272.

Haven, E., & Khrennikov, A. (2016). The use of action functionals within the quantum-like paradigm. *Journal of Mathematical Psychology*, In Press.

Hazen, G. B. (2003). A new perspective on multiple internal rates of return. *The Engineering Economist, 48*(1), 31–52.

Hommes, C. H. (2002). Modeling the stylized facts in finance through simple nonlinear adaptive systems. *PNAS, 99*, 7221–7228.

Hsieh, D. A. (1993). Implications of nonlinear dynamics for financial risk management. *Journal of Financial and Quantitative Analysis, 28*(1), 41–64.

Ioannidis, J. P. A. (2005). Why most published research findings are false. *PLoS Med, 2*(8), e124. doi:10.1371/journal.pmed.0020124 .http://journals.plos.org/plosmedicine/article?id=10.1371/journal.pmed.0020124

Iturbe-Ormaetxe, I., Ponti, G., Tomás, J., & Ubeda, L. (2010). Framing effects in public goods: Prospect theory and experimental evidence. *Games and Economic Behavior, 72*(2), 439–447.

Jagannathan, R., Matsa, D., Meier, I., & Tarhan, V. (2013). *Why do firms use high discount rates?* (Working paper). Kellogg School of Management at Northwestern University, Illinois. https://www.bus.miami.edu/_assets/files/faculty-and-research/conferences-and-seminars/finance-seminars/Jagannathan%20Paper.pdf. Accessed 31 Jan 2017.

John, L. K., Loewenstein, G., & Prelec, D. (2012). Measuring the prevalence of questionable research practices with incentives for truth telling. *Psychological Science, 23*, 524–532.

Kenett, D. Y., Preis, T., et al. (2012). Dependency network and node influence: Application to the study of financial markets. *International Journal of Bifurcation & Chaos, 22*, 1250181.

Kerr, N. L. (1998). Harking: Hypothesizing after the results are known. *Personality and Social Psychology Review, 2*, 196–217.

Kester, G., Chang, R., Echanis, E., Haikal, S., Isa, M., & Skully, M. (1999). Capital budgeting practices in the Asia-Pacific region: Australia, Hong Kong, Indonesia, Malaysia, Philippines and Singapore. *Financial Practice and Education, 9*(1), 25–33.

Kester, G., & Robbins, G. (2011). The capital budgeting practices of listed Irish companies: Insights from CFOs on their investment appraisal techniques. *Accountancy Ireland, 43*(1), 28–30.

Kriener, B., Helias, M., Rotter, S., et al. (2014). How pattern formation in ring networks of excitatory and inhibitory spiking neurons depends on the input current regime. *Frontiers of Computational Neuroscience, 7*, 187. doi:10.3389/fncom.2013.00187.

Kulakov, N., & Blaset Kastro, A. (2015). Evaluation of non-conventional projects: GIRR and GERR vs. MIRR. *The Engineering Economist, 60*(3), 183–196.

Kulakova, A. N., & Kulakov, N. J. (2013). Evaluation of non-conventional projects. *The Engineering Economist, 58*(2), 137–148.

Lander, D., & Pinches, G. (1998). Challenges to the practical implementation of modeling and valuing real options. *The Quarterly Review of Economics and Finance, 38*, 537–567.

Lei, Z., Blane, M. & Cooper, D. (1996, December 02–04). *3L Fitting of higher degree implicit polynomials*. Proceedings of the 3rd IEEE Workshop on Applications of Computer Vision (WACV '96), p. 148.

Leyman, P., & Vanhoucke, M. (2016). Payment models and net present value optimization for resource-constrained project scheduling. *Computers & Industrial Engineering, 91*, 139–153.

Leyman, P., & Vanhoucke, M. (2017). Capital- and resource-constrained project scheduling with net present value optimization. *European Journal of Operational Research, 256*(3), 757–776.

Liao, S., & Ho, S. (2010). *Investment project valuation based on the fuzzy real options approach*. 2010 International Conference on Technologies and Applications of Artificial Intelligence (TAAI).

Loewenstein, G., & Prelec, D. (1991). Negative time preferences. *American Economic Review, 81*(2), 347–352.

Loewenstein, G., & Prelec, D. (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *Quarterly Journal of Economics, 57*, 573–598.

Magni, C. (2002). Investment decisions in the theory of finance: Some antinomies and inconsistencies. *European Journal of Operational Research, 137*, 202–216.

Magni, C. A. (2005). On decomposing net final values: EVA, SVA and shadow project. *Theory and Decision, 59*(1), 51–95.

Magni, C. A. (2009). Investment decisions, NPV and bounded rationality. *Quantitative Finance, 9*(8), 967–979.

Magni, C. A. (2013). The Internal-Rate-of-Return approach and the AIRR paradigm: A refutation and a corroboration. *The Engineering Economist, 58*(2), 73–111.

Magni, C. (2015). Aggregate Return On Investment for investments under uncertainty. *International Journal of Production Economics, 165*, 29–37.

Masicampo, E. J., & Lalande, D. R. (2012). A peculiar prevalence of p values just below .05. *The Quarterly Journal of Experimental Psychology and Aging, 65*, 2271–2279.

Mason, S., & Merton, R. (1985). The role of contingent claim analysis in corporate finance. In E. Altman & M. Subrahmanyam (Eds.), *Recent advances in corporate finance.* Irwin.

McClure, K. & Girma, P. (2004). Modified Net Present Value (MNPV): A new technique for capital budgeting. *Zagreb International Review of Economics & Business*, Conference issue, pp. 67–82.

Menna, M., Rotundo, G., & Tirozzi, B. (2002). Distinguishing between chaotic and stochastic systems in financial time series. *International Journal of Modern Physics-C, 13*, 31–34.

Nwogugu, M. (2003). Decision-making under uncertainty: A critique of options pricing models. *Journal of Derivatives & Hedge funds, 9*(2), 164–178.

Nwogugu, M. (2006). Regret minimization, willingness-to-accept-losses and framing. *Applied Mathematics & Computation, 179*(2), 440–450.

Nwogugu, M. (2012). *Risk in the global real estate market*. Wiley.
O'Boyle, E. H., Banks, G. C., & Gonzalez-Mule, E. (2016, in press). The chrysalis effect: How ugly initial results metamorphosize into beautiful articles. *Journal of Management*.
Open Science Collaboration. (2015, August 28). Estimating the reproducibility of psychological science. *Science, 349*(6251), 943–945. http://www.sciencemag.org/content/349/6251/aac4716.full.pdf
Oppenheimer, P. (Dec. 2002). A critique of using real options pricing models in valuing real estate projects and contracts. *Briefings in Real Estate Finance, 2*(3), 221–233.
Osborne, M. (2010). A resolution to the NPV–IRR debate? *The Quarterly Review of Economics and Finance, 50*, 234–239.
Osborne, M. (2016). *A note on the IRR of a project with many potential outcomes*. Available at www.SSRN.com: https://ssrn.com/abstract=2830480 or http://dx.doi.org/10.2139/ssrn.2830480
Ouattara, A., & De La Bruslerie, H. (2015). *The term structure of psychological discount rate: characteristics and functional forms.* (Working paper). Centre Africain d'Etudes Superieures en Gestion, Senegal; and Universite Paris Dauphine, Paris. MPRA Paper No. 75111, MPRA, Germany. https://mpra.ub.uni-muenchen.de/75111/1/MPRA_paper_75111.pdf
Padilla, E., Jadotteb, E., & Pasqula, J. (2013). Technical note: Equivalence of different profitability criteria with the net present value. *International Journal of Production Economics, 142*(1), 205–210.
Pierru, A. (2010). The simple meaning of complex rates of return. *The Engineering Economist, 55*(2), 105–117.
Podobnik, B., Horvatic, D., Petersen, A. M., & Stanley, H. E. (2009). Cross-correlations between volume change and price change. *PNAS, 106*, 22079–22084.
Preis, T. (2011). Econophysics – Complex correlations and trend switchings in financial time series. *European Physical Journal, 194*, 5–86.
Preis, T., Kenett, D., et al. (2012). Quantifying the behavior of stock correlations under market stress. *Scientific Reports 2, 752*(2012). doi:10.1038/srep00752.
Ryan, P. A., & Ryan, G. P. (2002). Investment practices of the Fortune-1000: How have things changed? *Journal of Business and Management, 8*(4), 355–364.
Schachter, J., & Mancarella, P. (2016). A critical review of Real Options thinking for valuing investment flexibility in Smart Grids and low carbon energy systems. *Renewable and Sustainable Energy Reviews, 56*, 261–271.
Schaelemann, T., Singer, R., & Worth, J. (2012). The HAMP NPV model: Development and early performance. *Real Estate Economics, 40*(1), S32–S64.

Schmidt, F. L., & Hunter, J. E. (2015). *Methods of meta-analysis: Correcting error and bias in research findings* (3rd ed.). Newbury Park: Sage.

Scholten, M., & Read, D. (2006). Discounting by intervals: A generalized model of intertemporal choice. *Management Science, 52*(9), 1424–1436.

Sitton, G., Burrus, C. S., Fox, J., & Treitel, S. (2003). Factoring very high degree polynomials in signal processing. *Signal Processing Magazine, 6*(20), 27–42.

Smith, J., & Nau, R. (1995). Valuing risky projects: Option pricing theory and decision analysis. *Management Science, 41*(5), 795–816.

Stewart, N., Reimers, S., & Harris, A. (2014). On the origin of utility, weighting, and discounting functions: How they get their shapes and how to change their shapes. *Management Science, 61*(3), 687–705.

Taleb, N. (2009). Finiteness of variance is irrelevant in the practice of quantitative finance. *Complexity, 14*(3), 66–76.

Triantis, A. (2005). Realizing the potential of real options: Does theory meet practice? *Journal of Applied Corporate Finance, 17*, 8–16.

Tsai, R., & Bockenholt, U. (2008). On the importance of distinguishing between within- and between-subject effects in intransitive intertemporal choice. *Journal of Mathematical Psychology, 52*(1), 10–20.

Uttal, W. (2012). *Reliability in cognitive neuroscience: A meta-meta-analysis*. Cambridge: MIT Press.

Weber, T. (2014). On the (non-)equivalence of IRR and NPV. *Journal of Mathematical Economics, 52*, 25–39.

Wiesemann, W., Kuhn, D., & Rustem, B. (2010). Maximizing the net present value of a project under uncertainty. *European Journal of Operational Research, 202*(2), 356–367.

Willigers, B., & Hansen, T. (2008). Project valuation in the pharmaceutical industry: a comparison of least-squares Monte Carlo real option valuation and conventional approaches. *R&D Management, 38*(5), 520–537.

Yannacopoulos, A., Brindley, J., Merkin, A., & Pilling, M. (1996). Approximation of attractors using Chebyshev polynomials. *Physica D: Nonlinear Phenomena, 99*(2), 162–174.

Zauberman, G., Kim, K., Malkoc, S., & Bettman, J. (2009). Discounting time and time discounting: Subjective time perception and intertemporal preferences. *Journal of Marketing Research, XLVI*, 543–556.

Zheng, B., Takamatsu, J., & Ikeuchi, K. (2010). An adaptive and stable method for fitting implicit polynomial curves and surfaces. *IEEE Transactions on Pattern Analysis & Machine Intelligence, 32*(3), 561–568.

2

Spatio-Temporal Framing Anomalies in the NPV-MIRR-IRR Model and Related Approaches; and Regret Theory

This chapter: (1) explains existing and new spatio-temporal framing effects inherent in the NPV-MIRR model (and related models such as AROI, NFV; APV; etc.); (2) critiques Iturbe-Ormaetxe et al. (2010); (3) surveys the relevant literature on Regret Theory, which explains how it can serve as an alternative to the NPV-MIRR model for decision-making; (4) introduces framing effects inherent in the mean-variance model. Regret Theory can help avoid the inaccuracies and often distorting framing effects inherent in the NPV-MIRR-IRR models, although there is no generally accepted set of Regret-based decision models (unlike NPV/MIRR/APV/NFV/EVA and related models).

2.1 Existing Literature

The literature on the Framing Effects inherent in NPV, IRR, and related models (such as Average-IRR; Selective-IRR; Generalized-NPV; Generalized-IRR; Net Future Value; SVA; EVA; Modified-IRR) is somewhat limited and documents two issues: (1) that there are anomalies in the

M.C.I. Nwogugu, *Anomalies in Net Present Value, Returns and Polynomials, and Regret Theory in Decision-Making*,
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NPV and MIRR/IRR when signs of the project cash flows change; and (2) a few researchers (such as Magni (2002), Magni (2008), Magni (2009), Magni (2010) and Magni (2013)) have found a few framing effects in the NPV-IRR model, and when using CAPM with NPV - primarily about the non-additivity of discount-rates and NPV-values.

Magni (2002) concluded that using the NPV rule for making investment decisions can lead to inconsistencies and antinomies; and that the equivalent-risk principle (i.e., an investor needs to compare an investment opportunity with an asset of equivalent risk) is difficult to implement within the NPV model. Magni (2002) explained and critiqued the NPV model and described various anomalies and one type of framing effect inherent in the NPV-MIRR-IRR model (this framing effect involves combining different cash flows from different time periods and discounting them uniformly or separately). Magni (2010) critiqued NPV and described various CAPM related framing effects that are inherent in the NPV model. Fagley et al. (2010) analyzed framing effects within the context of decisions and choice. Ortendahl and Fries (2005) studied framing effects in financial decisions. Schindler and Pfattheicher (2017); Huang et al. (2015); Breuer and Soypak (2015); Stefan (2016); de Haan and van Veldhuizen (2015); and Barberis and Huang (2009) analyzed Framing Effects.

Shani et al. (2015); Foster and Vohra (1999); Viossat and Zapechelnyuk (2013); Bernstein, Mannor and Shimkin (2013), and Bhargave et al. (2015) studied issues relevant to Regret Theory and decision-making. Schindler and Pfattheicher (2017); Huang, Su and Chang (2015); Breuer and Soypak (2015); Stefan (2016); de Haan and van Veldhuizen (2015); Ortendahl & Fries (2005); Li & Chapman (2013); Hardin & Looney (2012); Milch, Weber, et. al. (2009); and Kirchler, Maciejovsky & Weber (2005) and Barberis and Huang (2009) analyzed various aspects of Framing Effects.

"*Recency effects*" and "*serial-order effects*" have been noted in the literature. Terry (2011) found that the order and/or orientation of the presentation of NPV-scenarios affect the capital budgeting decision. Lee and Terry (2010) noted that when making decisions under risk, individuals overweight the probability of the last potential outcome that is presented for each alternative; and less-riskier alternatives are chosen more frequently when potential outcomes for the choices are listed from best to worst than when they are given from worst to best. Levy and Levy (2005) found that when making portfolio decisions, individuals over-weight recent returns despite participants being informed beforehand that the computer-generated returns for the assets were i.i.d. through time. "*Presentation effects*" have also been noted in the literature – for example, Diacon and Hasseldine (2007) found that individuals are more likely to choose an equity-based fund over a fixed-interest fund when past performance is presented using fund value rather than annual returns. Kühberger et al. (1999) critiqued prospect theory, cumulative prospect theory, venture theory and Markowitz's utility theory, and noted that: (i) presenting outcomes as gains tends to induce risk aversion, while presenting outcomes as losses induces risk seeking; (ii) Risk preference depends on the size of the payoffs, on the probability levels, and on the type of good at stake (money/property vs. human lives), and generally, higher payoffs increase risk aversion, higher probabilities increase risk aversion for gains and increase risk seeking for losses; (iii) it is not probabilities or payoffs but the framing condition which explains most variance; (iv) no linear combination of formally relevant predictors is sufficient to capture the essence of the framing phenomenon. Levin et al. (1998) distinguished three types of *valence framing effects* (risky choice framing; attribute framing and goal framing). Read (2001) and Scholten and Read (2006) noted that the degree of "*delay-discounting*" is related to how the time interval is divided. Urminsky and Zauberman (2016) noted that: "*.........There are two aspects of temporal framing effects that have been documented. One pertains to the direction: whether the situation involves delaying a current outcome or expediting a future one. Loewenstein (1988) demonstrated that for a given time horizon, delaying a present outcome results in steeper discounting than when expediting a future outcome to the present. This effect was further established for losses as well as gains (Benzion et al. (1989); Shelly (1993)), and for the degree of hyperbolic discounting rather than just overall discount rates (Malkoc and Zauberman, 2006). The other facet concerns the manner by which the time horizon is being expressed, whether making the length of delay explicit or just providing the date, often referred to as the date-delay effect (LeBoeuf, 2006; Read et al., 2005). The two forms yield different elicited discount rates, indicating that the framing of* *the temporal context plays an important role in the decision process. This effect demonstrates higher discounting when a given future time horizon is framed as a delay (e.g., one month) versus a date marking the identical duration (June 27), as well as greater hyperbolic discounting. This shows that merely altering the way in which a time horizon is represented changes the valuation of outcomes over that duration*………".

Barcelona (2015) critiqued the NPV model. Ang and Liu (2004) noted that using a constant discount rate in Discounted Cashflow (DCF) models can result in significant mis-valuations—however, CAPM and related models have been criticized in the literature and in this book. Kruger et al. (2015) analyzed the investment distortions that can occur because of the use of a single discount rate within firms (instead of specific discount rates for different projects)—however, the study is based on Tobin's Q which has been criticized in the literature.

Bas (2013) and Remer and Nieto (1995) analyzed the decision rules of project evaluation techniques.

Magni (2005) decomposed Net Final Values (NFV) into Systemic Value Added (SVA), for use in decision-making, based on a *systemic* approach introduced in Magni (2003, 2004). Magni (2005) also analyzed similarities with other decomposition models such as Stewart's (1991) Economic Value Added (EVA) Model; and noted that the SVA index differs from Stewart's Economic Value Added (EVA) in that it is based on a different interpretation of excess profit and is formally connected with the EVA model by means of a *shadow* project; and that the SVA is formally based on economic, financial, and accounting considerations. Magni (2005) stated sufficient and necessary conditions for decomposing NFV; and explained the relationship between a project's SVA and its shadow project's EVA.

Magni (2013) explained why the IRR model is wrong; and why the average internal rate of return (AIRR) model may be more accurate. Magni (2013) noted the following errors in the IRR model: (1) it provides multiple rates of return; (2) in some cases it provides no rate of return; (3) it can be distorted by the use of varying costs of capital; (4) arbitrage strategies distort IRR; (5) mutually exclusive projects and project ranking; (6) rate of return on initial capital (or total investment cash flow) is distorted; (7) IRR produces various framing effects—present value vs. future value; expected value of stochastic IRR vs. IRR of expected investment; value additivity is not preserved by IRR; (8) IRR neglects a project's operating life; (9) IRR represents concocted capital; (10) IRR has only ad hoc consistency with NPV; (11) IRR provides multiple project balances and multiple excess returns; (12) intertemporal inconsistency; (13) IRR is not capable of summarizing accounting variables, in particular by accounting rates of return; (14) Makeham's Formula—IRR does not adequately summarize the information derived by the varying interest rates; (15) changes in capital—IRR is a cash flow based measure that neglects the actual operations involved in a project, and it remains constant when there are changes in capital; (16) computational issues—in MSExcel, multiple IRRs are not detected, and MATLAB and similar packages are cumbersome. Cuthbert and Magni (2016) critiqued IRR and Average Internal Rate Of Return (AIRR).

Almashat et al. (2008) described the framing effects inherent in patients' decision-making. The NPV-MIRR-IRR model and related models are also widely used in the healthcare industry (e.g., in healthcare informatics). Hall et al. (2012), McCabe et al. (2013), Califf et al. (2008), and Trusheim et al. (2011) analyzed the use of NPV and related models in medical decisions.

Booth (2003) found that standard compounding and the standard NPV model underestimate future cash flows when their growth rates (of the cash flows) are serially correlated.

Walthe (2010) analyzed anomalies in intertemporal choice, time-dependent uncertainty and expected utility.

Dybvig and Ingersoll (1982) attempted to prove that: (1) the *mean-variance separation theorem* is feasible in complete markets only if investors have quadratic utility; (2) the CAPM model is accurate for all assets in a complete market only if arbitrage opportunities exist; and the validity of this relationship is not affected by the distribution of returns of created financial assets. Senbet and Thompson (1978) compared various capital budgeting models based on mean-variance and concluded that all six models were very similar.

Rubinstein (1973), Senbet & Thompson (1978), and Dybvig & Ingersoll (1982), analyzed the simultaneous use of the Mean-Variance model and the NPV-IRR model (and the theories are applicable to related approaches such as APV and NFV). Martellini & Urosevic (2006) generalized—Markowitz analysis to the situations involving an uncertain exit time (inputs are now given by the generalized expressions for mean and variance covariance matrix involving moments of the random exit time in addition to the conditional moments of asset returns); and found that efficient frontiers in the—generalized analysis and the standard—Markowitz analysis may be similar on rare occasions; and efficient Markowitz-analysis portfolios can be inefficient in the generalized analysis sense and vice versa. Thus, an investor facing an uncertain time horizon and who imposes a specific investment horizon and exit will make suboptimal portfolio allocation decisions; and a significant efficiency loss can result from an improper use of standard mean-variance analysis.

Branch and Echevarria (1998), Strong (2006), Roll (1983), Liu and Strong (2008), Blume (1974), Cooper (2006), Jacquier et al. (2003), Indro and Lee (1997), Cheng and Deets (1971), Jacquier et al. (2005), Jean and Helms (1983), Dorfleitner (2003), Keim (1989), and Fisher et al. (2010) all found biases in the calculations of returns of both single assets and indices. (Chapter-8 in this book also introduces several new biases inherent in the calculation of returns). Some of those biases are framing effects, wherein the frames are defined by the time-horizon; the number and magnitude of the time-periods; and the magnitude of the interest rates.

The rest of this chapter discusses Framing Effects and the associated biases in NPV-IRR-MIRR model, related approaches and the mean-variance model.

2.2 Framing Effects in the Mean-Variance Framework

Theorem-3 and Theorem-4 in Nwogugu (2013) proved that correlation, covariance, variance, and semi-variance are irrelevant and misleading in risk analysis and in most statistical analysis. Those Nwogugu (2013) proofs included specific conditions under which correlation, covariance, variance, and semi-variance may be applicable and accurate (the "MV-conditions"). The MV-conditions are also evidence that correlation, covariance, variance and semi-variance are framing effects or have inherent framing effects; wherein the Frames are defined by the means, the time-horizons, the sample-sizes, and the "matching" of the samples.

2.3 Spatio-Temporal Cognition, Framing Effects, and Game Theory

Some researchers have documented behavioral issues and problems which indicate that human decision processes differ from the NPV-MIRR-IRR model; and that there are both framing and non-framing cognitive biases and deficiencies inherent in the use of the NPV-MIRR model. Similarly, Ellingsen et al. (2012); Caplin and Martin (2012); Bacharach and Stahl (2000); De Heus et al. (2010); Fleishman (1988); Gerlach and Jaeger (June 2016); Gerlagh and Van der Heijden (2015); and Dufwenberg et al. (2010) found framing effects in various types of game theory models. Tomasino et al. (2013) found framing effects in ultimatum games. The process of using the NPV-IRR model (and related approaches) for capital budgeting and risk management and the associated intra-firm decisions and politics can be expressed as game theory models (ie. coordination games, revelation games, ultimatum games, etc.) – some of the processes and games are described in Berkovitch and Isreal (2004) (however, the Berkovitch and Isreal (2004) conclusions about the validity of the NPV-IRR model are not correct). These processes often involve ultimatum games (because managers typically propose the positive-NPV and or high-IRR projects and if the firm rejects the project, the managers don't gain any performance bonuses and the firm doesn't gain anything) and rights of first refusal (ROFRs) (the firm – usually the corporate headquarters – typically has an ROFR on all positive-NPV and high-IRR projects that are proposed by its divisional or regional managers). That is, the decision-maker who uses the NPV-IRR-MIRR model faces the equivalent of ultimatum games and reverse ultimatum games when selecting among different projects with different NPVs. In the NPV-IRR-MIRR model, in many circumstances, the inherent choices pertaining to a potential project and the periodic project outcomes/cashflows involve a series of ROFRs and/or options on ROFRs. Like the NPV-IRR-MIRR model, ROFRs also involve sequences, time units, and project outcomes/cashflows. Thus, ROFRs affect interest rates, discount rates, and forward rates.

Friss and Speckbacher (1994) developed a modified present value decision rule for portfolio decisions under uncertainty based on a state-dependent valuation with opportunity costs. Their rule can be used to select a unique portfolio, which also maximizes the expected logarithmic utility of payoffs. Sandri et al. (2010) analyzed the disinvestment decision (project termination and liquidation of assets) within the context of entrepreneurial decision-making, and they found that the standard NPV decision model was not typical decision behavior. Sandri et al. (2010) found that most individual decision-makers understand the value of waiting; and their choices are weakly related to the disinvestment triggers derived from a formal optimal stopping benchmark consistent with real-options reasoning. Sandri et al. (2010) also observed a pronounced loss aversion, in that most individuals hold on to a losing project for even longer than real-options reasoning would predict—this effect has been documented in many prior empirical studies.

2.3.1 A Critique of Iturbe-Ormaetxe, Ponti, Tomás and Ubeda (2010)

As noted in Gintis (2005) and in the literature, decision models such as Cumulative Prospect theory, Prospect Theory, and Regret Theory are game theory models. Nwogugu (2005a,b,c, 2006a) critiqued prospect theory and cumulative prospect theory. Iturbe-Ormaetxe et al. (2010) erroneously concluded that two components of Prospect Theory (PT)—loss aversion and probability weighting—contribute to framing effects. Loss aversion and probability weighting, and many components of Cumulative Prospect Theory (CPT) and PT, were well established in the literature before CPT and PT were developed (as explained in Leland (2010)). As shown in Gonzalez et al. (2005) and in Fagley et al. (2010), framing effects are more attributable to perception and Regret, than to loss aversion and probability weighting. Thus, *Theorem-1* and *Theorem-2* in Iturbe-Ormaetxe et al. (2010) are wrong, and the findings are refuted by the following studies. Birnbaum (2006) conducted experiments to test five paradoxes that completely refuted CPT and PT with positive, negative, and mixed gambles (involving more than 600 participants and many formats for displaying gambles). Fagley et al. (2010) compared framing effects in men and women via experiments and found that larger framing effects observed for women in previous research may be due to differences in whether the individual spontaneously considers how he/she would feel (that is, to individual differences in affective perspective taking). Sher and McKenzie (2006) analyzed the equivalency of frames and found that framing effects and description invariance have not been completely conceptualized in the framing literature, and characterized information equivalence. Gonzalez et al. (2005) explained framing effects using cognitive information-processing principles. Gonzalez et al.'s (2005) theories are based on the cost-benefit tradeoffs inherent in contingent behavior, wherein individuals expend different levels of cognitive effort when examining various alternatives. They used brain activation functional magnetic resonance imaging (fMRI) to evaluate individuals who were asked to choose between one certain alternative and one risky alternative in response to problems framed as gains or losses. Gonzalez et al. (2005) found that the cognitive effort required to select a sure gain was considerably lower than the cognitive effort required to choose a risky gain; and conversely, the cognitive effort expended in choosing a sure loss was equal to the cognitive effort expended in choosing a risky loss. fMRI revealed that the cognitive functions used by the decision-makers in the study were localized in the prefrontal and parietal cortices of the brain, a finding that suggests the involvement of working memory and imagery in the selection process. Levin et al. (1998) concluded that there are at least three distinct types of framing effects (and that PT and CPT pertain to only one of the three types of framing). McElroy and Seta (2003) conducted experiments and found that individuals who engage in a decision task with (either induced or predisposed) an analytic/systematic versus holistic/heuristic processing style are especially insensitive to framing effects. Kontek (2009) conducted experiments using the same data used by Kahneman and Tversky (1979) and Tversky and Kahneman (1992) when they developed PT and CPT respectively, and found that other results were possible. The Kontek (2009) results show that, at best, CPT and PT are types of framing effects. Thus, contrary to Iturbe-Ormaetxe et al. (2010), those studies established that: (1) framing effects are attributable to perceptions and mental processes that are, or can be, distinct from loss aversion and probability weighting; (2) there are at least three different types of framing effects (and CPT/PT pertain to only one type); (3) framing effects are not universal and apply to only certain types of individuals and to some circumstances.

2.3.2 Rights of First Refusal (ROFRs) and a Critique of Grosskopf and Roth (2009)

Grosskopf and Roth (2009) is important because it is one of the few articles to analyze ROFRs as games, and to analyze BA-ROFRs. Grosskopf and Roth (2009) analyzed a combination of ex-ante ROFRs (a right-of-first-offer wherein the rights-holder is required to act before potential competitors) and ex-post ROFRs (traditional ROFR in which the rights-holder responds only after potential competitors make offers) in the context of sequential bargaining whereby the right-of-first-refusal is activated if the right-of-first-offer is violated (collectively, a "*Before and After Right of First Refusal*" or "BA-ROFR"). Grosskopf and Roth (2009) attempted to but could not develop conditions under which the BA-ROFR can be disadvantageous to its holder. Grosskopf and Roth (2009) concluded that where there is sequential bargaining with two buyers, the BA-ROFR can improve the seller's bargaining position but may not be beneficial to the BA-ROFR holder. ROFRs are a type of ultimatum game. Tomasino et al. (2013) found framing effects in ultimatum games. Straub and Murnighan (1995) noted that although there were popular hypotheses explaining the failure of sub-game-perfect equilibrium models to explain behavior in ultimatum games, evidence did not support earlier explanations for ultimatum anomalies, and there are conditions where sub-game-perfect models apply (some of which may be related to framing).

Generally, intertemporal choice involves the exercise of single or repeated joint ROFRs inherent in choices. The process of using the NPV-IRR model (and related approaches) for capital budgeting and risk management and the associated intra-firm decisions and politics can be expressed as games—some of the processes are described in Berkovitch and Isreal (2004). ROFRs are also important in intertemporal lending and leasing because the typical lender or lessor has an ROFR on some portion of the borrower's or lessee's cash flow. On each payment date for a loan, the borrower decides how much to pay a lender, and if the lender rejects the amount of interest/principal paid and walks away, both the lender and borrower may get nothing in bankruptcy (if there are many senior-priority creditors). A lender's entry into a new sub-prime market resembles ultimatum games wherein a lender decides how much loan principal to provide to a sub-prime borrower and both parties won't get anything if the prospective borrower walks away. The creditor's post-default rights include implied ROFRs (right to make claims, to accept/reject bankruptcy offers, etc.) which in turn can affect interest rate-setting processes. Shu and Morelli (2012) found that consumers are willing to incur costs to punish unfair offers (such as loans or mortgages) by a company as measured through a modified ultimatum game allocation task. ROFRs are also important in intertemporal savings and investments because the saver/investor typically has an implied ROFR on some portion of the cash flow and/or assets of the investee company or financial institution.

ROFRs are somewhat similar to Real Options, but the main differences are that (1) most ROFRs are contracted for at the beginning of the relationship; (2) ROFRs are typically not incidental, while Real Options may arise from factors outside the project; (3) the risk and value of the ROFR are closely related to the project, while the risk and value of a Real Option may not be related to the project; and (4) most ROFRs have definite pre-agreed lives, while the life of a Real Option may or may not be finite.

However, the analyses and all the theorems in Grosskopf and Roth (2009) are wrong because they omit critical issues, such as the following: (1) the current wealth of the holder of the ROFR affects the ROFR's feasibility and efficiency (a holder who does not have sufficient wealth or borrowing capacity to execute the ROFR can be just as passive as an unrelated third party); (2) the debt capacity of the ROFR holder; (3) the scope of the ROFR holder's opportunity set (the ROFR is not evaluated or exercised in a vacuum, but only relative to a perceived opportunity set); (4) the ROFR holder's substitution option (i.e., ability to substitute assets, human capital, and other resources); (5) ROFRs are often not affected by the path of negotiations (to buy or sell the subject property) and sometimes do not expressly incorporate the time value of money, although the perceived values of ROFRs are based on both factors—this divergence can cause mispricing and Regret; (6) in the ultimatum game and reverse ultimatum game described in Grosskopf and Roth (2009), if no agreement is reached, the owner of the asset will still retain his/her asset while both responders will have nothing; (7) the ultimatum game and reverse ultimatum game described in Grosskopf and Roth (2009) completely omit the possibility of collusion and information sharing, or of a parallel sub-game between responders, wherein they agree to share r_1 or r_2; (8) in Grosskopf and Roth (2009), another very feasible outcome of both games is that all three parties (owner and two responders) will each get a share of the 25 tokens; (9) in Grosskopf and Roth (2009), another feasible outcome of both games is lending/leasing, wherein a responder leases all or some of the tokens; (10) contrary to Grosskopf and Roth (2009), there is a substantial difference between agreeing to a zero payoff (25;0) and disagreement, and the likelihood that the owner or a responder will be indifferent to either outcome is very low; (11) contrary to Grosskopf and Roth (2009), neither the ultimatum game nor the reverse ultimatum game can have any Nash Equilibria because the responders can substitute the tokens, contract with third parties, produce their own tokens, or collude between themselves to share r_1 or r_2; the responders may be better off by not responding to the proposer's offers; and the owner of the tokens (proposer) can bargain with other pairs of responders; (12) Grosskopf and Roth (2009) do not consider the fact that the ROFR can be activated regardless of the magnitude of r_1 or r_2 (and regardless of information sharing among responders), because of altruism, intentional delay, hoarding, collusion, irrationality, information asymmetry, knowledge, player or non-player innovation, and so on; (13) contrary to Grosskopf and Roth (2009), the ROFR changes the payoffs at a sub-game-perfect equilibrium because the ROFR can be activated regardless of the magnitude of r_1 or r_2 (and regardless of information sharing between the two responders), due to altruism, fairness concerns, information asymmetry and knowledge, intentional delay, hoarding, collusion among responders, irrationality; (14) given the foregoing errors stated in this paragraph, Theorems 1 and 2 in Grosskopf and Roth (2009) are wrong; (15) the findings in Croson (1996) and Sanfey et al. (June 2003) directly prove that the findings and Theorems 1 and 2 in Grosskopf and Roth (2009) are wrong; (16) the experimental results obtained by Grosskopf and Roth (2009) indeed confirm that the payoffs in the reverse ultimatum game can vary over time (and will vary if the duration of each bargaining period is extended), and the apparent low variation of payoffs over time in the ultimatum game can be explained by other factors; and (17) Grosskopf and Roth (2009) does not consider framing effects. There are other feasible sub-games and scenarios that are not explored in Grosskopf and Roth (2009)—for example, there can be multi-stage games and allocations wherein the proposer/owner (p) can decide to divide the 25 tokens into n groups and make n offers to both responders (r_1 and r_2) in sequential sub-games at n distinct points in time, particularly where there are n number of events that may affect the ROFRs and the players' payoffs and utilities.

2.3.3 Framing Effects in the NPV-IRR Model (And Related Approaches)

The NPV-MIRR-IRR model and related models (such as APV, AIRR, Selective-IRR, Generalized-NPV; Generalized-IRR; EVA; NFV, etc.) are types of framing effects that distort true risk; and wherein the frames are defined by the time-horizons, the time-intervals (for discounting), and the discount rate; etc. Some specific spatio-temporal cognition effects (framing effects) that are not in the existing literature are introduced as follows and are also applicable in Game Theory analysis. Any changes in any of these frames will typically result in substantial changes in the NPV/IRR. These framing effects are different from the three types of framing effects that were observed in Levin et al. (1998) since they are spatio-temporal.

2.3.3.1 Type-A Frame

In the NPV/MIRR/IRR models and related models, the magnitude of the time periods is a type of framing effect (*Type-A frame*). Given a specific set of cash flows, changing the time intervals for discounting (i.e., from weekly to monthly, or semi-annually, or annually) produces non-trivial differences in the NPV and MIRR (and related models). This framing effect can be attributed partly to compounding (which affects both the discount factor and the reinvestment implicit in the NPV-IRR-MIRR model), and partly to the exact timing of cash flows within intervals and subintervals of time. The NPV-MIRR-IRR model erroenously assumes that each time interval has a value/relevance (to the decision-maker) that is determined by the applied discount factor. In reality, specific weeks or months may be much more important than others to the decision-maker in a way that differs from the "scaling" implied by the discount-factors. Furthermore, deadlines and time limits create substantial differences in the valuation of time units. Time based constraints on the ability to respond or on the firm's opportunity set also create substantial distortions in the NPV-MIRR model. Hence, a change to the magnitude of the time periods used in NPV-MIRR-IRR (and related) models will produce different results, mechanically and psychologically, due to the decision-makers' perceptions and valuations of time.

2.3.3.2 Type-B, Type-C, and Type-D Frames

In the NPV model and related models (APV, NFV, etc.), the use of discount rates versus absolute NPV (or APV, NFV; etc.) as a decision factor creates framing effects. The NPV model is based on using the actual calculated/absolute NPV as the sole and primary decision factor, as opposed to using: either the relative changes in the NPV for various discount rates (using an iterated discount rate as the primary decision factor wherein the user will continue to change the discount rate to observe the effect on the calculated NPV, henceforth referred to as a *Type-B frame*); or using the sensitivity to time of the absolute NPV (*Type-C frame*); or using the sensitivity of the NPV to the reinvestment rate (*Type-D frame*).

2.3.3.3 Type-E Frames

In the NPV/MIRR/IRR models and related models, the expectation of an initial investment is a type of framing effect (*Type-E frame*). In these models, there is usually an expected outflow of cash at time-zero, followed by the project's other cash flows in subsequent periods. However, this initial-period cash outflow can also be modeled as a reduction of cash-inflow in *period-1* without significant loss of accuracy (especially when the time intervals are short—such as days or weeks), and such change is a framing effect that will produce substantial differences in the calculated NPV and can substantially affect the user's perception of the risk and profitability of the project. See comments in Ehrhadrt and Wachowicz (Summer 2006). In many real world situations there are cash flow series that are either completely incidental to other events/projects (and thus do not incur any initial investment); and there are projects that do not require initial investment. In such circumstances the NPV-MIRR model often provides distorted results. In other circumstances, the initial investment (at time-zero) does not produce any tangible project benefits, but may produce: (1) intangible benefits that are difficult to quantify; and/or (2) multiplier effects. When the initial investment (at time-zero) produces only losses (or mostly negative project benefits), the NPV will become anomalous ($\partial N/\partial r > 0$)—this is a framing effect because, all else held constant, the behavior of the NPV/APV/NFV/MIRR can be changed by manipulating the amount and or timing of the percieved initial investment at time-zero.

2.3.3.4 Type-F Frames

In the NPV/MIRR/IRR models and related models, the time-value-of-money (TVM), and the effects of discounting/compounding (*Type-F frame*), are also frames because of the biases inherent in the NPV-IRR-MIRR model, which are explained in this book. The TVM formula is based entirely on mechanically discounting the time value of money, but does not consider the time discounting of specific individuals or companies, or the value of time to companies/individuals. Firstly, a specific week or day may be more valuable to a company than other days or weeks in the month/year. Secondly, a company may be functioning in an operating environment where time is a critical element of competition, or perceived quality, or perceived profitability, whereas another company may face opposite conditions (i.e., time does not matter). Thirdly, a company's employees may perceive time very differently from the standard individual, perhaps because: (1) the company operates worldwide on a 24/7 basis; or (2) most company employees are compensated based on their output, regardless of time spent. In such circumstances, the standard TVM formula will be inaccurate because it will not measure the company's or individual's true risk preferences within the context of their opportunity set. To be accurate, the opportunity set must be defined with respect to perceptions of time and the value of time. See the comments in Khwaja et al. (2007), Attema et al. (2010), and Ortendahl and Fries (2005).

The NPV-MIRR-IRR model and related approaches erroneously and implicitly assume that cash flows occurring in later time periods have low significance and low present values. For most positive discount rates, cash flows that occur after the fourth period have a minimal effect on NPVs and MIRRs, primarily because of the compounding inherent in the discount factor for each periodic project benefit. The NPV-MIRR model results in the selection of projects that generate more cash flow in earlier time periods, rather than projects that generate the bulk of their cash flows in later periods in the forecast period. This is because: (1) the NPV-MIRR model does not account for use-value of project cashflows/outcomes and assets, or the certainty of occurrence of each periodic project cashflows/outcomes (even when different discount rates are used for different periods); (2) the compounded discount factors in NPV-MIRR models typically underweight project cashflows/outcomes that occur in later time periods (the magnitude of the underweighting depends on economic factors); (3) temporal differences do not imply a declining utility or Regret-value of project cashflows/outcomes.

In reality, later-period cash flows can be substantial and more valuable (per dollar) than early-period cash flows if: (1) the yield curve remains inverted for a long period of time, (2) reinvestment rates in the early years are substantially less than both the reinvestment rates in later years and the discount rate; (3) macro-economic conditions change—

e.g., there is rampant deflation, recession, and so on; (4) prior reinvestment rates remain very low compared to the discount rate(s); (5) the decision-maker's opportunity set is reduced substantially over time; (6) the firm's or the project's tax position, tax rates, or tax statutes change in the future or are such that later-period cashflows are of greater value (on a per dollar basis) than earlier cashflows; (6) there are changes in the intertemporal value of cash (versus non-liquid assets), which increases the utility of cash in future periods. This Type-F frame can explain the non-additivity inherent in the NPV-MIRR model. The two types of non-additivity are: (1) *temporal non-additivity* (different from temporal framing effects mentioned in Urminsky & Zauberman (2016)); and (2) *lateral non-additivity*.

Assume that there are two projects, A and B, each of which last for n periods; and the combination of A and B produces another project, C. n can be subdivided into two periods, e and d, such that $e+d = n$. The initial cash outflows for A (K_{ao}) and B (K_{bo}) are the same. The NPV for cash flows from Project-A during periods e, d and n are NPV_{ae}, NPV_{ad} and NPV_{an} respectively. The NPV for cash flows from Project-B during periods e, d and n are NPV_{be}, NPV_{bd} and NPV_{bn} respectively. The NPV for cash flows from Project-C during periods e, d and n are NPV_{ce}, NPV_{cd} and NPV_{cn} respectively. Temporal non-additivity means that $NPV_{an} \neq NPV_{ae}+NPV_{ad}$; and $NPV_{bn} \neq NPV_{be}+NPV_{bd}$. Lateral non-additivity means that $NPV_{cn} \neq NPV_{an}+NPV_{bn}$; and $NPV_{ce} \neq NPV_{be}+NPV_{ae}$—and this condition typically exists where the separate cash flow series have different risk or discount rates.

2.3.3.5 Type-G Frames

NPV and related models (APV; NFV; MIRR; etc.) are *super-asymmetric* (*Type-G frame*) in at least two ways. The types of asymmetry are: (1) *temporal super-asymmetry*—whereby for a given series of cash flows C_1……… C_n, the absolute value of the NPV calculated using +r as discount rate ($|NPV_{+r}|$) will differ from the absolute-value of the NPV calculated with $-r$ as the discount rate ($|NPV_{-r}|$); and (2) *lateral super-asymmetry* means that for a given positive discount rate (+r) or negative discount rate

(−r), the absolute value of the NPV calculated using cash flows C_1…….. Cn ($|NPV_{+c}|$) will differ from the absolute-value of the NPV calculated if the signs of the cash flows C_1…….. C_n are changed ($|NPV_{-c}|$). This super-asymmetry arises partly because of the use of numbers that are between −1 and +1 as the discount-rates.

2.3.3.6 Type-H Frames

The NPV metric does not consider the relative performance of investments, and for a given horizon (which contains equal time periods) a specific magnitude of NPV can be achieved by different combinations of invested capital and positive rates-of-return. For example, an NPV of $20 can be achieved after two periods by either: (1) investing $100 at a rate-of-return r% (Investment-A); or (2) by investing $1,000 at a different but much lower rate-of-return y% (Investment-B); where r% > y%; and r%, y% > 0. Most rational investors would prefer Investment-A because the yield from Investment-B is substantially lower than the opportunity cost. Similarly, for a given horizon (which contains equal time periods) and a specific discount rate, a specific magnitude of NPV can be achieved by different combinations of positive and or negative cashflows that occur at different times. Depending on their valuations of time and capital constraints, different decision makers will prefer different cashflow combinations. In both capital constrained and non-constrained environments, this Type-H Framing Effect results in sub-optimal decisions. Thus, the NPV and related models (such as NFV and EVA) cannot choose between competing projects especially in capital-constrained situations. Similarly, for a given horizon (which contains equal time periods), a specific IRR can be achieved by different combinations of positive and or negative cashflows that occur at different times.

2.3.3.7 Type-I Frames

The NPV-MIRR-IRR model and related approaches cause the perception that decisions are made and value changing actions occur in specific sequences of defined time units (*Type-I frame*)—that is, activities in periods 3 and 4 generate cash flows in period-4 and so on. In reality, many decisions and their effects affect various parties over many time periods. Under an NPV-IRR regime, and for some projects, the timing and scope of decisions can be rearranged without significant loss of generality, and thus is a framing effect. All else held constant, an NPV-MIRR model based solely on matched decision-value pairs (i.e., matching each decision with the resulting gains/losses) produces drastically different results compared to a traditional NPV-MIRR model.

2.3.3.8 Type-J Frames

In some circumstances, the NPV-IRR model does not consider the actual economic value of periodic cashflows, and tradeoff between the time-value and the NPV, and or the discount rate and the cashflows. Thus, for a given horizon (which contains equal time periods) a specific magnitude of NPV can be achieved by different combinations of positive/negative invested capital and positive/negative rates-of-return. For example, an NPV of \$20 can be achieved over six periods by either: (1) a series of only negative cashflows (i.e. $-x_i$……..$-x_n$) and a negative discount rate; or (2) a series of only positive cashflows of same absolute magnitude but with opposite signs (i.e. x_i……..x_n), and a positive discount rate. That is:

$$\mathrm{NPV}_{20} = \sum_{i=n}\left[x_i / (1+r)^i\right] = \sum_{i=n}\left[-x_i / (1+y)^i\right]$$

Where: $0<r\%<\infty$; and $-\infty<y\%<0$; and $|y\%| > |r\%|$

During 2015–2016, several countries (such as Switzerland, Sweden and Japan) had negative benchmark interest rates, and their government bonds were trading at negative yields. This frame occurs because of an anomaly in the NPV-IRR model. In both capital constrained and non-constrained environments, this Type-J Framing Effect results in sub-optimal decisions.

2.3.3.9 Type-K Frames

For any two time periods *m* and *n* ($m > n$), each of which has equal time-periods, the same IRR can be achieved ($IRR_m = IRR_n$) by varying the cash flows in each of *m* and *n*.

2.3.3.10 Type-L Frames

In the NPV-IRR-MIRR model, the risk of a project can be adjusted by either changing the cash-flows/outcomes or changing the discount rate (*Type-L frame*), and combinations of either type of change can produce the same result. The NPV-MIRR model fosters the perception that all discount rates are strictly positive; and are a positive cost of capital or an opportunity cost. See Ortendahl and Fries (2005). In reality (and from a valuation perspective), the discount rate is an amalgam of many factors, including both positive and negative project benefits, inflation/deflation, income tax factors, capital gains tax factors, returns from alternative investments, and weighted average cost of capital (WACC). As explained above, the discount rate can be negative (less than zero) in certain circumstances. In the NPV-MIRR model and related approaches, the traditional way to adjust for reductions in risk or reductions of the direct benefits of projects has been to change the cash flows rather than reduce the discount rate to below 0%. This perception (about the discount rates always being positive) is reinforced by: (1) the popularity of the use of the WACC as the discount rate; (2) the generally accepted methods of constructing discount and capitalization rates as an amalgam of various rates, such a government bond rates, inflation, and so on. This framing effect (tradeoff between adjusting the cashflows or changing the discount-rate) can have a significant impact on the accuracy of the NPV-MIRR model.

2.3.3.11 Type-M Frames

The context in which the NPV-IRR model is used and "internal capital markets dynamics" can result in Framing Effects. For example, consider a conglomerate (B) and its wholly-owned division (B_d). B uses a firm-wide discount rate of 15% with a mandate to accept all projects with an NPV greater than \$2 Million. B_d's capital structure is different (from that of B) and it has several projects and its rule is to use a discount rate of 18% and to accept projects that have an NPV greater than \$1.5 million. All divisional projects above a specific dollar threshold must be approved by B; and any resources that B_d doesn't have must be internally purchased from B (at or below market value) before B_d can attempt to purchase it from external sources. One of B_d's projects can be executed by B or B_d and a choice has to be made about which entity shall execute the project. If the project is evaluated by B_d or B (two different frames), it will have an NPV of \$2.5 million or \$2.6 Million respectively (some of the difference in NPV occurs because of cost allocation), but the decision to accept the project will be the same for B or B_d. Such decisions sometime lead to internal politics and disputes among managers (of the parent company and or different divisions) but the reality is that both choices are essentially the same and have the same expected value to B. Regardless of whether B or B_d executes the project, it's the same group of shareholders (B's shareholders) that benefit, and B's consolidated financial statements and consolidated leverage will be the same.

2.3.3.12 Type-N Frame

Many borrowers who accepted HAMP mortgage modifications in the United States ended up in "under-water" status (the value of the home was less than the mortgage principal balance) immediately after the modification and for long periods of time. The HAMP model used the NPV model as the decision criteria for making mortgage modifications. See Romero (June 18, 2012); Gans (2012); Berry (Feb. 25, 2015); Korte (Jan. 27, 2011), and Schaelemann et al. (2012). The failures of a significant percentage of HAMP mortgage modifications and the improper rejection of tens of thousands of completed applications for HAMP modification is evidence of the weaknesses of the NPV model. According to a report by the Special Inspector General for the Troubled Asset Relief Program (TARP) which was presented to the US Congress in January 2015, of the homeowners that received loan modifications in 2009, 53% had re-defaulted by the end of 2014 (up from 46% at the end of 2013); and out of the nearly 1.4 million loans that had been modified between 2009 and 2014, 31% had re-defaulted by January 2015. Consider a borrower who learnt about the HAMP mortgage modification program (based on the HAMP NPV test). The borrower has the following four choices (four Frames): (i) apply for, and accept the HAMP modification proposal (Option-A), (ii) apply for, and reject the HAMP modification proposal and remain in default status (Option-B); (iii) do nothing and remain in default status (Option-C); (iv) execute a deed-in-lieu-of-foreclosure and leave the property (Option-D). Option-B may have significant information content because the lender may use the information gathered to make other decisions about the defaulted mortgage loan. Schindler and Pfattheicher (2017) noted that loss-framing can increase dishonest behavior. In this instance: (i) dishonesty by borrowers includes false disclosures/statements in the borrowers' HAMP applications and borrowers' lack of true intent or capacity to comply with the HAMP modification agreement if and when its executed; (ii) dishonesty by banks and servicers includes improper rejection of completed HAMP applications or the acceptance of false disclosures/statements in the documentation/processing of borrowers' HAMP applications. In many cases the HAMP mortgage modifications were loss-framing because: (i) the original (pre-modification) mortgages were non-performing and typically underwater, (ii) the resulting (post-modification) mortgages were usually underwater, (iii) the borrowers had inadequate or un-reliable incomes and the probability of borrower repayment was low; (iv) the HAMP NPV Test and associated mortgage modification were used to frame what were actual losses or potential losses to both the borrower and the lender, into possible gains; (v) the probability-of-success (of the mortgage modifications) could be varied and one of the major decision goals was to minimize losses incurred by both the borrower and the lender. See comments in Juliusson (2003) which noted that loss-framing tends to increase *Escalation* (towards negative outcomes), compared to gain-framing. For two types of such borrowers, all four choices (Options A, B, C & D) can have the same or near-equal economic values—and the two classes are as follows: (i) borrowers who don't have the intent and or capacity to execute the NPV-based HAMP plan (Class-A); (ii) borrowers whose mortgages are "underwater" and who don't have stable and or reliable income (Class-B). For Class-A borrowers, Options B & C will most certainly result in foreclosure and loss; and Option-D will result in loss of the property; and Option-A has a very high probability of foreclosure because it will likely result in re-default when the borrower does not perform his/her contractual obligations. While the borrower can remain in the subject property without paying rent during foreclosure, the borrower incurs other costs such as litigation costs, property maintenance costs, reduced credit-scores; lost work-hours; anxiety/depression; etc. For Class-B borrowers, Options B & C will most certainly result in foreclosure; and Option-D will result in loss of the property; and Option-A has a very high probability of foreclosure because an "underwater" mortgage provides significant incentives for strategic default by the borrower; and the borrower's un-stable or un-reliable income also will likely result in re-default. For Class-B borrowers, regardless of whether the HAMP NPV is positive (NPV>0) or is $-5{,}000 < \text{NPV} < 0$ or is $\text{NPV} < -5{,}000$, the economic value of the HAMP modification to the borrower can be the same and result in strategic default. For the lender, all four choices can have the same or near-equal present values for the same foregoing reasons. Thus, for decision-makers who don't have the intent or capacity to execute NPV-based plans, use of the NPV model may result in Framing Effects. Similarly, for decision-makers for whom the key decision-variable or decision-outcome (e.g. both ex-ante and ex-post "under-water" mortgage in HAMP mortgage modifications) is negative/detrimental regardless of whether or not NPV is positive, the use of the NPV model may result in Framing Effects.

2.3.4 Quasi-Frames and Some Framing-Related Inconsistencies in the NPV-IRR Model

The following are some quasi-Frames and Framing-related inconsistencies.

Assume that there are two cashflow series (C_a and C_b) where each is made up of positive and or negative real numbers; and each series has the same time-interval which is made up of equal time periods. C_c is the cashflow series that is obtained by combining/adding the cashflows from C_a and C_b in each time period. Assume that r_a is the IRR for C_a and

r_b is the IRR for C_b; and r_c is the IRR for C_c. For most cashflow series (and where the cashflows in C_a are not a constant multiple of matching cashflows in C_b), $r_c < r_a + r_b$; and there is no easily definable relationship between r_a on one hand, and the sum of r_b and r_c. This indicates that IRR is not additive (or is sub-additive) and that contrary to theory and practice, changes in the IRR don't necessarily reflect proportionate changes in real risk—whereas given that the IRR is a major indicator of the risk of a project, IRR should be additive (i.e. r_c should be equal to $r_a + r_b$). Similarly, AIRR, MIRR and GIRR are not additive. Magni (2013) mentioned eighteen flaws of IRR, and noted that IRR does not preserve value-additivity; and that IRR produces various framing effects—e.g. present value vs. future value; expected value of stochastic IRR vs. IRR of expected investment.

In theory and practice, when using the NPV-IRR model, one of the most common ways of adjusting the risk of a project is to increase or decrease the discount rate—which errorneously implies that the discount rates are additive with respect to their respective NPVs. Assume that there is a cashflow series made up of real numbers (positive and or negative), and a time-interval made up of equal time periods. For that cashflow series, if the discount rate (r_d) is divided into a risk-free component (r_f) and a risk premium (r_p), the NPV derived by using r_d (NPV_d) is not equal to the sum of the NPVs derived by using r_f (NPV_f) and r_p (NPV_p) as the discount rates respectively. That is, $NPV_d \neq NPV_f + NPV_p$; and typically for monotonic NPVs, $NPV_d < NPV_f + NPV_p$. If $r_d - r_f = r_p$, then in order to maintain consistent risk-scales, value-scales, utility and time-value, the condition [$NPV_d - NPV_f = NPV_p$] should exist but that is not the case in most instances (for both monotonic-NPV and non- monotonic NPV). This indicates that in the NPV-IRR model (and related approaches), the discount rates are not additive with respect to their respective NPVs, and that contrary to theory and practice, changes in the discount rate don't necessarily reflect proportionate changes in project risk. Similarly, in the APV, NFV, GNPV and EVA, the discount rates are not additive with respect to values; and the values are not additive with respect to cashflows. Magni (2008) concluded that contrary to Rubinstein (1973), *dis-equilibrium* NPV and economic profit for risky one-period projects are not equivalent partly because NPV-minded agents are subject to framing effects and to arbitrage losses, which imply violations of Modigliani and Miller's Proposition I; and that disequilibrium (present) value (a popular metric that was deductively derived from the CAPM) should therefore be dismissed. Magni (2008) also stated that (disequilibrium) NPV is non-additive and the same holds for the notion of disequilibrium value; and that decision-makers that jointly use NPV and CAPM suffer from a Framing Effect (non-additivity). Magni (2005, 2010) noted that the standard CAPM-based NPV cannot be used for investment valuations because the concepts of value inherent in the standard CAPM-based NPV criterion are not additive.

The IRR (and MIRR and GIRR) formula is non-homomorphic and results in asymmetric risk-scaling. Assume that there is a cashflow series made up of real numbers (positive and or negative), and a time-interval made up of equal time periods. For that cashflow series, assume that there are three possible IRRs ($r_b > r_c > r_d$), and $r_b - r_c = x_1 = r_c - r_d = x_2$. Although on the surface $x_1 = x_2$, the magnitude of real risk represented by x_1 is not equal to, and is often greater than the risk represented by x_2. Let: B = the sum of the cashflows in a series but without discounting; r_l = IRR in the domain of lower IRRs (i.e. x_2 range); r_h = IRR in the domain of higher IRRs (i.e. x_1 range). Generally given the NPV-IRR formula and for most cashflows for which there is an initial investment at time-zero, $\partial r_l / \partial B > \partial r_h / \partial B > 0$. This asymmetric risk-scaling occurs because of the compounding effect inherent in the IRR formula. That is, in the domain of higher IRRs (in the x_1 range), it takes a greater amount of increase in the sum-total cashflows to achieve a one percent increase in IRR, compared to the domain of lower IRRs (in the x_2 range).

The NPV-IRR model and related approaches do not distinguish between different types of economic benefits (such as cash and non-cash benefits), which have different illiquidity, realization rates, and use-values. Many non-cash project benefits are illiquid, and their use values and market values are substantially degraded by their illiquidity. Hence, if absolute risk is held constant, the classification of all or a part of a periodic project benefit (as cash, or non-cash benefits, or tax benefits, or economic income), may cause a framing effect that can materially change the values of the NPV/IRR.

The NPV-IRR model and related approaches do not incorporate Real options inherent in operational risk, such as the abandonment option. In the NPV/IRR model, risk is defined almost entirely only in terms of the discount rate, but the discount rate is insufficient to define operational risk, which includes: (1) elements of non-flexibility; (2) degree of inability to change investment terms; (3) comparison with a changing opportunity set. See the comments in Kaplan and Garrick (1981); Walls (2004); Jonkman et al. (2003); Jarrow (2008); Doraszelski (2001); Haley and Goldberg (1995); and Naim (2006).

The NPV-IRR model completely omits the value of substitution (of assets/resources over time) to the decision-maker. The substitution option changes the outcomes of an NPV-IRR analysis. Different assets may produce the same project benefits, but may have different use values (which is the typical criteria for project evaluation) during one or more time periods, and/or different accounting values (depreciation, cost, market value, etc.), and different risk-profiles. Incorporating the substitution option into a NPV-IRR analysis (where all else is held constant) changes the NPVs/IRRs, it is thus a major framing effect.

The NPV-IRR model and related approaches implicitly assume that the reinvestment rate is equal to the discount rate(s)—which may differ from the WACC. In reality, the reinvestment rate may differ substantially from both WACC and discount rate. This *constant reinvestment rate assumption* (or even the assumption that there are multiple reinvestment rates) also implies that there is a fixed term structure of forward rates—which is not accurate and maybe a Framing Effect. The NPV-IRR model implicitly and erroneously assumes that the firm's WACC or discount rate either remains constant over the forecast period, or is known at the beginning of the project, and that all else held constant, the cumulative project risk increases as time progresses. On the contrary, the WACC or the appropriate discount rate is dynamic over time; and for most break-even or profitable or slightly un-profitable projects, cumulative project risk declines over time as the project stabilizes and begins to generate positive cashflows.

The NPV-MIRR model causes the often false perception that decision steps and project cashflows/outcomes are irreversible (or non-transferable or non-assignable) and conditions remain constant during the project term. In reality, decision-makers can abandon projects; economic conditions can change; losses/profits can be transferred by contract (e.g., derivatives); and under certain tax regimes, losses can have economic value (used to offset other gains, etc.). Losses, ownership interests and contract rights can also be assigned or transferred pursuant to risk management contracts. If the decision-makers' abandonment option is incorporated into NPV-MIRR models, the results will be different from those obtained in traditional NPV-MIRR models.

2.4 Regret-Based Decision Models as an Alternative to the NPV-IRR Model (And Related Approaches)

Given the many problems inherent in the NPV-IRR model, Regret-based decision models are a viable alternative as decision model. Regret Theory can help avoid the often distorting framing effects inherent in the NPV-MIRR-IRR model (and related approaches) and in interest rates and forward rates. One hindrance is that there is no generally accepted set of Regret-based decision models (unlike NPV/MIRR/APV/NFV/EVA and related models). Regret Theory can incorporate many of the behavioral/psychological issues that the NPV-MIRR-IRR model (and related approaches) do not or cannot capture—such as flexibility, Real options, Regret minimization, willingness to accept losses, the value of waiting, reversibility of decisions, dynamic cost of capital, framing effects, and so on. This section surveys some critical literature on Regret Theory to show how it has been used in various contexts for project selection/evaluation (just as the NPV-IRR model is used).

Klauss (2006) investigated the potential effects of managerial overconfidence and Regret aversion in a corporate capital investment context and specifically analyzed project selection (accepting or rejecting a proposed investment), managerial effort, and project evaluation (continuing or abandoning a failing investment). Klauss (2006) developed a theoretical model that integrates these decisions with overconfidence and Regret aversion, and outlined the conditions under which a biased manager will make choices that are inefficient from a shareholder value perspective. A related survey of UK managers with capital investment responsibility found that overconfidence and Regret aversion were pervasive within the sample group; and there were indications for potential associations between these biases and certain capital investment decision choices.

Berns et al. (2008) analyzed three phenomena within the context of pain as incentive: negative discounting of future payoffs; non-linear probability weighting; and the experience of Regret and Rejoice when making a decision under risk. Previous experimental economic studies in the domain of losses have typically used monetary rewards. Berns et al. (2008) found evidence for negative time discounting of electric shocks, and for probability weighting in the domain of electric shocks, which is manifest at the neural level; and also found evidence, both behaviorally and neurally, for Regret and Rejoice functions for painful outcomes.

Nasiry and Popescu (2009) analyzed the effects of anticipated Regret on consumer decisions, firm profits and policies, in an advance selling context where buyers have uncertain valuations; and found that advance purchases trigger action Regret if valuations are lower than the price paid, and delaying purchase causes inaction Regret. Nasiry and Popescu (2009) developed a Regret threshold above which firms should only spot-sell to homogeneous markets, and otherwise advance selling is optimal. Nasiry and Popescu (2009) also found that the effect of Regret on profits depends on the type of Regret, market structure and the firm's pricing power—and Action Regret lowers the optimal profits of a price-setting firm in homogeneous markets, while inaction Regret has the opposite effect. According to Nasiry and Popescu (2009), firms can benefit from Regret by creating a buying frenzy, wherein consumers purchase in advance at negative surplus; and Action Regret can be profitable if high valuation consumers are more regretful, or if the firm is price-constrained.

Smith (1996) argued that Regret influences the valuation of alternative outcomes when making treatment decisions in healthcare; and that traditional valuation techniques rely on Expected Utility Theory (transitivity and independence), which causes misrepresentation of the respondents true preferences for treatment alternatives, and thus results in the potential for irrational decisions. Smith (1996) developed a modified version of Regret Theory and provided the results of a tentative empirical analysis to illustrate the importance of accounting for Regret in the valuation of health states.

Michenaud and Solnik (2008) applied Regret Theory to derive closed-form solutions to optimal currency hedging choices. They theorized that investors experience Regret in not having chosen the ex-post optimal hedging decision; and thus, investors anticipate their future experience of Regret and incorporate it in their objective function. Michenaud and Solnik (2008) derived a model of financial decision-making with two components of risk: traditional risk (volatility) and Regret risk; and their results contradicted traditional expected utility, loss aversion, and disappointment aversion theories.

Ghosh (1993) used Regret Theory to explain some survey results on managers' dividend policy decisions under uncertainty and postulates that: (1) the decision to pay dividends and simultaneously raise venture capital from external sources is attributable to managerial aversion for Regret at the failure of a risky investment opportunity implemented with internal funds generated by a conservative dividend policy; and (2) the decision to support dividends with borrowed funds when earnings are declining is motivated by the prospect that an improvement in the firm's financial condition will make the managers proud that their judgment has helped avert a potential crisis for the firm without any wealth loss to its shareholders.

Hayashi (2008) developed a Regret-based axiomatic model of decision-making under uncertainty, which allows for the coexistence of both Regret aversion and likelihood judgment over states. Hayashi (2008) also characterized Minimax Regret a smooth model of Regret aversion.

Larrick and Boles (1995) used a negotiation exercise to examine how the expectation of receiving feedback on a foregone alternative influenced negotiators' risk preferences. Consistent with the Regret Theory prediction, Larrick and Boles (1995) found that subjects were more risk averse in their negotiation decisions when they did not expect feedback on a foregone risky alternative than when they did; thus, negotiators who did not expect feedback on a foregone risky alternative were more likely to reach agreement in their negotiation than were negotiators who did expect feedback. As Larrick and Boles (1995) mentioned, the results show that motivational factors, such as Regret avoidance, are critical factors in project evaluation, decision theory and negotiation theory.

Muermann et al. (2006) showed that anticipated disutility from Regret can have a significant and drastic effect on investment choices. They analyzed and modeled how plan participants' asset allocation decisions in a defined contribution (DC) pension plan might vary with their preferences about risk and Regret. Muermann et al. (2006) found that the Regret averse investor will typically hold more stock than a risk averse investor when the equity premium is low, but less stock when the equity premium is high; and that Regret increases the Regret averse investor's willingness to pay for a guarantee when the portfolio is relatively risky, but decreases it when the portfolio is relatively safe.

Gertner and Schmutzler (2009) analyzed Regret within the context of decision-making in company mergers. Gertner and Schmutzler (2009) considered ex-post incentive compatible mechanisms, which use participants' (buyer and seller) reports to determine whether or not a merger will take place and what each participant will earn in each case. Gertner and Schmutzler (2009) found that when the outside option of at least one player is known, the efficient merger decision can be implemented by such a mechanism under plausible budget balance requirements. When neither outside option is known, the potential for Regret-free implementation is much more limited, unless the budget balance condition is relaxed to permit the burning of money in the case of false reports.

Brocas and Carrillo (2005) analyzed decision-making by a hyperbolic discounting agent and showed that the agent may rationally decide to consume with negative expected NPV only to prevent himself from consuming in the future, which could be profitable from a future perspective but highly detrimental from the current viewpoint. Comparative statistics reveal that the value of information is U-shaped.

Nwogugu (2006b) introduced new models of Regret and Willingness To Accept Losses (WTAL). The Nwogugu (2006b) Regret model can be used for project evaluation/selection.

Tsalatsanis et al. (2010) used Regret Theory to develop a clinical decision model that links intuition and analytical/deliberative processes, and to reformulate decision curve analysis (DCA). They analyzed a classic decision tree describing the three decision alternatives of treat, do not treat, and treat or no treat, based on a predictive model; and they then computed the expected Regret for each of these alternatives as the difference between the utility of the action taken and the utility of the action that, in retrospect, should have been taken; and for any pair of strategies, they measured the difference in Net Expected Regret. Tsalatsanis et al. (2010) used the concept of *acceptable Regret* to identify the circumstances under which a potentially wrong strategy is tolerable to a decision-maker; and developed a novel dual visual analog scale to describe the relationship between Regret associated with omissions (e.g., failure to treat) vs. commissions (e.g., treating unnecessary) and decision-maker's preferences as expressed in terms of threshold probability. Tsalatsanis et al. (2010) proved that the *Net Expected Regret Difference*, first presented in their paper, is equivalent to Net Benefits as described in the original DCA. Based on the concept of acceptable Regret, they identified the circumstances under which a decision-maker tolerates a potentially wrong decision and expresses it in terms of probability of disease.

Dodonova (2009), DeKay (2009), and Bleichrodt et al. (2010) all analyzed the use of Regret Theory for project evaluation and project selection. Mannor and Shimkin (2008); Mannor, Stoltz and Perchet (2011); Viossat and Zapechelnyuk (2013); Bernstein, Mannor and Shimkin (2013); Foster and Vohra (1999); Perchet (2011); Perchet (2009); Stoltz and Lugosi (2005); Zhang, Zhu, Liu and Chen (2016); Broll, Welzel and Wong (2016); Wong (2011); Wong (2014); Wong (2015); and Rakhlin, Sridharan and Tewari (2011) analyzed various aspects of Regret Theory. Chandrasekhar et al. (2008), Canessa et al. (2009), and Coricelli et al. (2007) analyzed neuro-biological Regret.

Gollier and Salanié (2006), Sagi and Friedland (2007), Wang et al. (2008), Huang et al. (2009), Laciana and Weber (2008); Sheng et al. (2014), Gollier and Salanié (2006), Clarke et al. (1994), Loomes and Sugden (1982) and, Michenaud and Solnik (2008), developed models of choice that were based on Regret Theory. Frehen et al. (2008) analyzed Regret aversion in the retirement investment decision of DC plan participants; and developed and priced a look-back option on a life annuity contract.

2.5 Conclusion

Clearly, the NPV-MIRR-IRR model (and related approaches such as AIRR, Selective-IRR, Net Future Value, SVA, etc.) and the mean-variance model are inaccurate, and are distorted by their inherent framing effects. The NPV-MIRR-IRR model (and related approaches) can be replaced with Regret based models wherein the main decision criterion is to accept projects that have the lowest Regret. The NPV-MIRR model (and related approaches) merely states the extent to which a project adds value, but cannot be used to choose among many positive-NPV projects where there are budget constraints. Regret can be minimized, and thus can be used to maximize objective functions in project evaluation/selection given specific criteria. Regret can capture inflexibility, Real options, framing effects, and other factors. Unlike Regret Theory, the NPV-MIRR-IRR model does not consider the consequences of mistakes in decisions; and that can be disastrous in some decision settings where pre-project evaluation costs are substantial and actions/investments are near irreversible — such as in surgery, large infrastructure projects, and so on.

Bibliography

Almashat, S., Ayotte, B., Edelstein, B., & Margrett, J. (2008). Framing effect de-biasing in medical decision making. *Patient Education and Counseling, 71*, 102–107.

Ang, A., & Liu, J. (2004). How to discount cashflows with time-varying expected returns. *Journal of Finance, 59*(6), 2745–2783.

Attema, A., Bleichrodt, H., Rohde, K., & Wakker, P. (2010). Time-tradeoff sequences for analyzing discounting and time inconsistency. *Management Science, 56*(11), 2015–2030.

Aven, T., & Flage, R. (2009). Use of decision criteria based on expected values to support decision-making in a production assurance and safety setting. *Reliability Engineering and System Safety, 94*, 1491–1498.

Bacharach, M., & Stahl, D. (2000). Variable-frame level-n theory. *Games & Economic Behavior, 32*, 220–246.

Bade, B. (2009). Comment on "The weighted average cost of capital is not quite right". *The Quarterly Review of Economics and Finance, 49*(4), 1476–1480.

Barberis, N., & Huang, M. (2009). Preferences with frames: A new utility specification that allows for the framing of risks. *Journal of Economic Dynamics and Control, 33*(8), 1555–1576.

Barcelona, R. (2015). Renewable energy with volatile prices: Why NPV fails to tell the whole story. *Journal of Applied Corporate Finance, 27*(1), 101–109.

Bas, E. (2013). A robust approach to the decision rules of NPV and IRR for simple projects. *Applied Mathematics and Computation, 219*(11), 5901–5908.

Berkovitch, E., & Isreal, R. (2004). Why the NPV criterion does not maximize NPV. *Review of Financial Studies, 17*(1), 239–255.

Berns, G. S., Capra, C. M., Moore, S., & Noussair, C. (2008). Three studies on the neuroeconomics of decision-making when payoffs are real and negative. In Grossman, M., Lindgren, B., Kaestner, R., & Kristian, B.__ (Eds.), *Neuroeconomics* (Advances in Health Economics and Health Services Research, 20, pp. 1–29). Bingley: Emerald Group Publishing Limited.

Bernstein, A., Mannor, S., & Shimkin, N. (2013). Opportunistic strategies for generalized no-regret problems. *Journal of Machine Learning Research: Workshop & Conference Proceedings, 30*, 158–171.

Berry, K. (2015). Re-defaults continue to plague HAMP. *National Mortgage News*. http://www.nationalmortgagenews.com/news/servicing/redefaults-continue-to-plague-hamp-1045414-1.html. Accessed 31 Jan 2017.

Bhargave, R., Chakravarti, A., & Guha, A. (2015). Two-stage decisions increase preference for hedonic options. *Organizational Behavior and Human Decision Processes, 130*, 123–135.

Birnbaum, M. (2006). Evidence against prospect theories in gambles with positive, negative, and mixed consequences. *Journal of Economic Psychology, 27*, 737–761.

Blaset Kastro, A. N., & Kulakov, N. (2016). Definition of the concepts of conventional and non-conventional projects. *Business Informatics, 2*(36), 16–23.

Bleichrodt, H., Cillo, A., & Diecidue, E. (2010). A quantitative measurement of regret theory. *Management Science, 56*(1), 161–175.

Blume, M. (1974). Unbiased estimators of long-run expected rates of return. *Journal of the American Statistical Association, 69*(347), 634–638.

Booth, L. (2003). Discounting expected values with parameter uncertainty. *Journal of Corporate Finance, 9*, 505–519.

Borgonovo, E., & Percocao, M. (2012). A note on the sensitivity analysis of the internal rate of return. *International Journal of Production Economics, 135*(1), 526–529.

Branch, B., & Echevarria, D. P. (1998). The bid-ask bias and the size effect: A test of the Blume-Stambaugh bid-ask bias effect hypothesis. *The Quarterly Review of Economics and Finance, 38*(1), 129–148.

Breuer, W., & Soypak, C. K. (2015). Framing effects in intertemporal choice tasks and financial implications. *Journal of Economic Psychology, 51*, 152–167.

Brocas, I., & Carrillo, J. D. (2005). A theory of haste. *Journal of Economic Behavior & Organization, 56*(1), 1–23.

Broll, U., Welzel, P., & Wong, K. P. (2016). Regret theory and the competitive firm revisited. *Eurasian Economic Review, 6*, 481–487.

Califf, R. M., Rasiel, E. B., & Schulman, K. A. (2008). Considerations of net present value in policy making regarding diagnostic and therapeutic technologies. *American Heart Journal, 156*(5), 879–885.

Canessa, N., Alemanno, F., Motterlini, F., et al. (2009). "Your regret and my decisions": A neuroimaging study of social influence on emotion-based decision making. *NeuroImage, 47*(1), S117.

Caplin, A., & Martin, D. (2012). *Framing effects and optimization* (Working paper). New York: New York University. http://www.martinonline.org/daniel/CaplinMartinFramingEffects20120415.pdf

Chandrasekhar, P., Capra, C. M., Moore, S., Noussair, C., & Berns, G. S. (2008). Neuro-biological regret and rejoice functions for aversive outcomes. *NeuroImage, 39*(3), 1472–1484.

Cheng, P., & Deets, M. K. (1971). Statistical biases and security rates of return. *The Journal of Financial and Quantitative Analysis, 6*(3), 977–994.

Cigola, M., & Peccati, L. (2005). On the comparison between the APV and the NPV computed via the WACC. *European Journal of Operational Research, 161*(2), 377–385.

Clarke, R., Krase, S., & Statman, M. (1994). Tracking errors, regret, and tactical asset allocation. *Journal of Portfolio Management, 20*(3), 16–24.

Cochrane, J. (2011). Presidential address: Discount rates. *The Journal of Finance, 66*(4), 1047–1108.

Cooper, I. (2006). Arithmetic versus geometric mean estimators: Setting discount rates for capital budgeting. *European Financial Management, 2*(2), 157–167.

Coricelli, G., Dolan, R. J., & Sirigu, A. (2007). Brain, emotion and decision making: The paradigmatic example of regret. *Trends in Cognitive Sciences, 11*(6), 258–265.

Correia, C. (2012). Capital budgeting practices in South Africa: A review. *South African Journal Of Business Management, 43*(2), 11–29.

Creemers, S., Leus, R., & Lambrecht, M. (2010). Scheduling Markovian PERT networks to maximize the net present value. *Operations Research Letters, 38*(1), 51–56.

Croson, R. (1996). Information in ultimatum games: An experimental study. *Journal of Economic Behavior & Organisation, 30*, 197–212.

Cuthbert, J. R., & Magni, C. (2016). Measuring the inadequacy of IRR in PFI schemes using profitability index and AIRR. *International Journal of Production Economics, 179*, 130–140.

de Haan, T., & van Veldhuizen, R. (2015). Willpower depletion and framing effects. *Journal of Economic Behavior & Organization, 117*, 47–61.

De Heus, P., Hoogervorst, N., & Van Dijk, E. (2010). Framing prisoners and chickens: Valence effects in the prisoner's dilemma and the chicken game. *Journal of Experimental Social Psychology, 46*, 736–742.

DeKay, M. L. (2009). Physicians' anticipated regret and diagnostic testing: Comment on Hozo and Djulbegovic (2008). *Medical Decision Making, 29*(3), 317–319.

Diacon, S., & Hasseldine, J. (2007). Framing effects and risk perception: The effect of prior performance presentation format on investment fund choice. *Journal of Economic Psychology, 28*, 31–52.

Dodonova, A. (2009). *Applications of regret theory to asset pricing* (Working paper). Henry B. Tippie College of Business, University of Iowa, Iowa.

Doraszelski, U. (2001). The net present value method versus the option value of waiting: A note on Farzin, Huisman & Kort (1998). *Journal of Economic Dynamics & Control, 25*, 1109–1115.

Dorfleitner, G. (2003). Why the return notion matters. *International Journal of Theoretical & Applied Finance, 6*, 73–86.

Dreber, A., Ellingsen, T., et al. (2013). Do people care about social context? Framing effects in dictator games. *Experimental Economics, 16*(3), 349–371.

Dufwenberg, M., Gächter, S., & Hennig-Schmidt, H. (2011). The framing of games and the psychology of play. *Games and Economic Behavior, 73*(2), 459–478.

Dybvig, P. H., & Ingersoll, J. (1982). Mean-variance theory in complete markets. *Journal of Business, 55*(2), 233–251.

Ehrhadrt, M., & Wachowicz, J. M. (Summer 2006). Capital budgeting and initial cash outlay (ICO) uncertainty. *Financial Decisions, 18*, 1–16.

Ellingsen, T., Johannesson, M., et al. (2012). Social framing effects: Preferences or beliefs? *Games & Economic Behavior, 76*, 117–130.

Fagley, N., Coleman, J., & Simon, A. (2010). Effects of framing, perspective taking, and perspective (affective focus) on choice. *Personality and Individual Differences, 48*(3), 264–269.

Fernandez, P. (2011). *WACC: Definition, misconceptions and errors*. (Working paper #WP-914). IESE Business School, Spain. http://www.iese.edu/research/pdfs/DI-0914-E.pdf

Fisher, L., Weaver, D., & Webb, G. (2010). Removing biases in computed returns. *Review of Quantitative Finance and Accounting, 21*, 137–151.

Fleishman, J. (1988). The effects of decision framing and others' behavior on cooperation in a social dilemma. *Journal of Conflict Resolution, 32*(1), 162–180.

Foster, D. P., & Vohra, R. (1999). Regret in the on-line decision problem. *Games & Economic Behavior, 29*, 7–35.

Frehen, R. G. P., Hoevenaars, M., Palm, F., & Schotman, P. (2008). Regret aversion and annuity risk in defined contribution pension plans. *Insurance: Mathematics and Economics, 42*(3), 1050–1061.

Friis, S., & Speckbacher, G. (1994). Present value based portfolio choice. *Economics Letters, 44*(3), 261–265.

Gans, M. (2012). HAMP: Doomed from the start. *Cornell Real Estate Review, 10*, 54–71.

Geiger, G. (2002). On the statistical foundations of non-linear utility theory: The case of status quo-dependent preferences. *European Journal of Operations Research, 136*(2), 449–465.

Gerlach, P., & Jaeger, B. (June 2016). Another frame, another game? Explaining framing effects in economic games. In *Conference: Second International Workshop on Norms, Actions and Games*, Toulouse.

Gerlagh, R., & Van der Heijden, E. (2015). *Going green: Framing effects in a dynamic coordination game* (Working paper), Tilburg University, Center For Economic Research, Netherlands.

Gertner, D., & Schmutzler, A. (2009). Merger negotiations and ex-post regret. *Journal of Economic Theory, 144*(4), 1636–1664.

Ghosh, C. (1993). A regret-theoretic explanation of corporate dividend policy. *Journal of Business Finance & Accounting, 20*(4), 559–573.

Gintis, H. (2005). Behavioral game theory and contemporary economic theory. *Analyse & Kritik – Journal of Philosophy and Social Theory, 27*(1). https://pdfs.semanticscholar.org/ec20/68ff9263c27b64eba7448784c37536ecdc7e.pdf

Gollier, C. (2010). Expected net present value, expected net future value, and the Ramsey rule. *Journal of Environmental Economics and Management, 59*, 142–148.

Gollier, C. & Salanié, B. (2006). *Individual decisions under risk, risk sharing, and asset prices with regret* (Working paper). University of Toulouse, France.

Gonzalez, C., Dana, J., Koshino, H., & Just, M. (2005). The framing effect and risky decisions: Examining cognitive functions with fMRI. *Journal of Economic Psychology, 26*, 1–20.

Graham, J. & Harvey, C. (2002). How Do CFOs Make Capital Budgeting And Capital Structure Decisions?. *Journal of Applied Corporate Finance, 15*(1), 8–23.

Haley, G., & Goldberg, S. (1995). Net present value techniques and their effects on new product research. *Industrial Marketing Management, 24*, 177–190.

Hall, P., Edlin, R., Kharroubi, S., et. al. (2012). Expected net present value of sample information: From burden to investment. *Medical Decision Making*, 32, E11–E21.

Handforth, F. (2004). Duration and convexity of mortgages in the context of real estate investment analysis. *Journal of Real Estate Portfolio Management, 10*(3), 187–192.

Hankins, K. W. (2009). How do financial firms manage risk? Unraveling the interaction of financial and operational hedging. *Management Science*, *57*(12), 2197–2212.

Hardin, A., & Looney, C. (2012). Myopic loss aversion: Demystifying the key factors influencing decision problem framing. *Organizational Behavior and Human Decision Processes, 117*(2), 311–331.

Hayashi, T. (2008). Regret aversion and opportunity dependence. *Journal of Economic Theory, 139*(1), 242–268.

Henrici, P. (1988). *Sign changes: The rule of descartes*. §6.2. In *Applied and computational complex analysis, Vol. 1: Power series-integration-conformal mapping-location of zeros* (pp. 439–443). New York: Wiley.

Huang, J., Tzeng, G., & Liu, H. (2009). A revised VIKOR model for multiple criteria decision making—The perspective of regret theory. *Communications in Computer and Information Science, 35*, 761–768.

Huang, H., Su, H., & Chang, C. (2015). The moderating effect of a no-choice option on risky framing effect. *Asia Pacific Management Review, 20*(1), 18–23.

Indro, D., & Lee, W. Y. (1997). Biases in arithmetic and geometric averages as estimates of long-run expected returns and risk premia. *Financial Management, 26*(4), 81–90.

Iturbe-Ormaetxe, I., Ponti, G., Tomás, J., & Ubeda, L. (2010). Framing effects in public goods: prospect theory and experimental evidence. *Games & Economic Behavior, 72*(2), 439–447.

Jacquier, E., Kane, A., & Marcus, A. (2003). Geometric or arithmetic mean: A re-consideration. *Financial Analysts Journal, 59*(6), 46–53.
Jacquier, E., Kane, A., & Marcus, A. (2005). Optimal estimation of the risk premium for the long run and asset allocation: A case of compounded estimation risk. *Journal of Financial Econometrics, 3*, 37–55.
Jarrow, R. (2008). Operational risk. *Journal of Banking & Finance, 32*, 870–879.
Jean, W., & Helms, W. P. (1983). Geometric mean approximations. *Journal of Financial and Quantitative Analysis, 18*, 287–293.
Jonkman, S. N., van Gelder, P., & Vrijling, J. K. (2003). An overview of quantitative risk measures for loss of life and economic damage. *Journal of Hazardous Materials, 99*(1), 1–30.
Juliusson, E. A. (2003). Effects of gain and loss frame on escalation. *Göteborg Psychological Reports, 33*(4). http://www.jmg.gu.se/digitalAssets/1286/1286065_gpr03_nr4.pdf
Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decisions under risk. *Econometrica, 47*, 313–327.
Kaplan, S., & Garrick, B. J. (1981). On the quantitative definition of risk. *Risk Analysis, 1*, 11–27.
Keef, S., Khaled, M., & Roush, M. (2012). A note resolving the debate on "The weighted average cost of capital is not quite right". *The Quarterly Review of Economics and Finance, 52*(4), 438–442.
Keim, D. (1989). Trading patterns, bid-ask spreads, and estimated security returns: The case of common stocks at calendar turning points. *Journal of Financial Economics, 25*(1), 75–97.
Kester, G. & Robbins, G. (2011). The capital budgeting practices of listed Irish companies: insights from CFOs on their investment appraisal techniques. *Accountancy Ireland, 43*(1), 28–30.
Kester, G., Chang, R., Echanis, E., Haikal, S., Isa, M., & Skully, M. (1999). Capital budgeting practices in the Asia-Pacific region: Australia, Hong Kong, Indonesia, Malaysia, Philippines and Singapore. *Financial Practice and Education, 9*(1), 25–33.
Khwaja, A., Silverman, D., & Sloan, F. (2007). Time preference, time discounting, and smoking decisions. *Journal of Health Economics, 26*(5), 927–949.
Kirchler, E., Maciejovsky, B., & Weber, M. (2005). Framing effects, selective information, and market behavior: An experimental analysis. *Journal of Behavioral Finance, 6*(2), 90–100.
Klauss, C. P. (2006). *Capital investment decisions with managerial overconfidence and regret aversion.* Ph.d thesis, University of Bath, England. Available at: http://opus.bath.ac.uk/18842/1/UnivBath_PhD_2006_Klauss_C.pdf

Klos, A., Weber, E., & Weber, M. (2005). Investment decisions and time horizon: Risk perception and risk behavior in repeated gambles. *Management Science, 51*(12), 1777–1787.

Kontek, K. (2009). On mental transformations. *Journal of Neuroscience Psychology and Economics, 4*(4), 235–253.

Korte, G. (2011). Inspector general: Mortgage modification program a 'failure'. *USA TODAY*. http://usatoday30.usatoday.com/money/economy/housing/2011-01-27-fed27_ST_N.htm. Accessed 31 Jan 2017.

Kruger, P., Landier, A., & Thesmar, D. (2015). The WACC fallacy: The real effects of using a unique discount rate. *Journal of Finance, 70*(30), 1253–1285.

Kühberger, A., Schulte-Mecklenbeck, M., & Perner, J. (1999). The effects of framing, reflection, probability, and payoff on risk preference in choice tasks. *Organizational Behavior and Human Decision Processes, 78*(3), 204–231.

Kulakov, N., & Blaset Kastro, A. (2015). Evaluation of nonconventional projects: GIRR and GERR vs MIRR. *The Engineering Economist, 60*(3), 183–196.

Kulakova, A. N., & Kulakov, N. J. (2013). Evaluation of non-conventional projects. *The Engineering Economist, 58*(2), 137–148.

Laciana, C. E., & Weber, E. U. (2008). Correcting expected utility for comparisons between alternative outcomes: A unified parameterization of regret and disappointment. *Journal of Risk & Uncertainty, 36*, 1–17.

Landsman, Z., & Sherris, M. (2001). Risk measures and insurance premium principles. *Insurance: Mathematics and Economics, 29*(1), 103–115.

Larrick, R., & Boles, T. L. (1995). Avoiding regret in decisions with feedback: A negotiation example. *Organizational Behavior and Human Decision Processes, 63*(1), 87–97.

Lee, J., & Terry, E. (2010). *Does the order of potential outcomes influence risky decision making?* (Working paper). Ryerson University, Canada.

Leland, J. (2010). The hunt for a descriptive theory of choice under risk—A view from the road not taken. *The Journal of Socio-Economics, 39*, 568–577.

Levin, I. P., Schneider, S. L., & Gaeth, G. J. (1998). All frames are not created equal: A typology and critical analysis of framing effects. *Organizational Behavior and Human Decision Processes, 76*, 149–188.

Levy, H., & Levy, M. (2005). Overweighting recent observations: Experimental results and economic implications. In R. Zwick & A. Rapoport (Eds.), *Experimental business research, volume III: Marketing, accounting and cognitive perspectives* (pp. 155–183). New York: Springer.

Leyman, P., & Vanhoucke, M. (2016). Payment models and net present value optimization for resource-constrained project scheduling. *Computers & Industrial Engineering, 91*, 139–153.
Leyman, P., & Vanhoucke, M. (2017). Capital- and resource-constrained project scheduling with net present value optimization. *European Journal of Operational Research, 256*(3), 757–776.
Li, M., & Chapman, G. (2013). A big fish or a small pond? Framing effects in percentages. *Organizational Behavior and Human Decision Processes, 122*(2), 190–199.
Liu, W., & Strong, N. (2008). Biases in decomposing holding-period portfolio returns. *The Review of Financial Studies, 21*(5), 2243–2274.
Loomes, G., & Sugden, R. (1982). Regret theory: An alternative theory of rational choice under uncertainty. *The Economic Journal, 92*(368), 805–824.
Lorenz, D., Kruschwitz, L., & Löffler, A. (2016). Are costs of capital necessarily constant over time and across states of nature?: Some remarks on the debate on 'WACC is not quite right'. *The Quarterly Review of Economics and Finance, 60*, 81–85.
Lumby, S. (1983). The case against WACC in investment appraisal. *Accountancy, 84*(1081), 116–118.
Magni, C. (2002). Investment decisions in the theory of finance: Some antinomies and inconsistencies. *European Journal of Operations Research, 137*, 202–216.
Magni, C. A. (2003). Decomposition of net final values: Systemic value added and residual income. *Bulletin of Economic Research, 55*(2), 149–176.
Magni, C. A. (2004). Modelling excess profit. *Economic Modelling, 21*, 595–617.
Magni, C. (2005). *Theoretical flaws in the use of CAPM for investment decisions* (Working paper). University of Modena and Reggio Emilia, Italy.
Magni, C. A. (2008). Economic profit, NPV, and CAPM: Biases and violations of Modigliani and Miller's Proposition I. *The ICFAI Journal of Applied Finance, 14*(10), 59–72.
Magni, C. (2009a). Splitting up value: A critical review of residual income theories. *European Journal of Operational Research, 198*, 1–22.
Magni, C. (2009b). Correct or incorrect application of CAPM? Correct or incorrect decisions with CAPM? *European Journal of Operational Research, 192*(2), 549–560.
Magni, C. (2010). CAPM and capital budgeting: Present/future, equilibrium/disequilibrium, decision/valuation. *IUP Journal of Financial Economics, 8*(1/2), 7–35.

Magni, C. A. (2013). The internal-rate-of-return approach and the AIRR paradigm: A refutation and a corroboration. *The Engineering Economist, 58*(2), 73–111.

Mannor, S., & Shimkin, N. (2008). Regret minimization in repeated matrix games with variable stage duration. *Games & Economic Behavior, 63*, 227–258.

Mannor, S., Stoltz, G., & Perchet, V. (2011). Robust approachability and regret minimization in games with partial monitoring. *Journal of Machine Learning Research: Workshop & Conference Proceedings, 19*, 515–536.

Martellini, L., & Urosevic, B. (2006). Static mean-variance analysis with uncertain time horizon. *Management Science, 52*(6), 955–964.

McCabe, C., Edlin, R., & Hall, P. (2013). Navigating time and uncertainty in health technology appraisal: Would a map help? *PharmacoEconomics, 31*(9), 731–737.

McElroy, T., & Seta, J. (2003). Framing effects: An analytic–holistic perspective. *Journal of Experimental Social Psychology, 39*, 610–617.

Merrett, A. J. (1965a). Net present value vs. the internal rate of return yet again. *Scottish Journal of Political Economy, 12*(1), 116–118.

Merrett, A. J. (1965b). A note on reinvestment assumptions in choosing between net present value and internal rate of return. *Journal of Finance, 27*(4), 907–915.

Michenaud, S., & Solnik, B. (2008). Applying regret theory to investment choices: Currency hedging decisions. *Journal of International Money and Finance, 27*(4), 677–694.

Milch, K., Weber, E., et al. (2009). From individual preference construction to group decisions: Framing effects and group processes. *Organizational Behavior and Human Decision Processes, 108*(2), 242–255.

Muermann, A., Mitchell, O., & Volkman, J. (2006). Regret, portfolio choice, and guarantees in defined contribution schemes. *Insurance: Mathematics and Economics, 39*(2), 219–229.

Naim, M. (2006). The impact of the net present value on the assessment of the dynamic performance of e-commerce enabled supply chains. *International Journal of Production Economics, 104*, 382–393.

Nasiry, J. & Popescu, I. (2009). The price of consumer regret (Working paper). INSEAD. http://web.uvic.ca/econ/research/seminars/Nasiry.pdf

Nwogugu, M. (2005a). Towards multifactor models of decision making and risk: Critique of prospect theory and related approaches, part one. *Journal of Risk Finance, 6*(2), 150–162.

Nwogugu, M. (2005b). Towards multifactor models of decision making and risk: Critique of prospect theory and related approaches, part two. *Journal of Risk Finance, 6*(2), 163–173.

Nwogugu, M. (2005c). Towards multifactor models of decision making and risk: Critique of prospect theory and related approaches, part three. *Journal of Risk Finance, 6*(3), 267–276.

Nwogugu, M. (2006a). A further critique of cumulative prospect theory and related approaches. *Applied Mathematics & Computation, 179*(2), 451–465.

Nwogugu, M. (2006b). Regret minimization, willingness-to-accept-losses and framing. *Applied Mathematics & Computation, 179*(2), 440–450.

Nwogugu, M. (2013). Decision-making, sub-additive recursive "matching" noise and biases in risk-weighted index calculation methods in in-complete markets with partially observable multi-attribute preferences. *Discrete Mathematics, Algorithms & Applications, 05*, 1350020. doi:10.1142/S1793830913500201.

Ortendahl, M., & Fries, J. F. (2005). Framing health messages based on anomalies in time. *Medical Science Monitor, 11*(8), 253–256.

Osborne, M. (2010). A resolution to the NPV–IRR debate? *The Quarterly Review of Economics and Finance, 50*, 234–239.

Padberg, M., & Wilczak, C. J. (1999). Optimal project selection when borrowing and lending rates differ. *Mathematical & Computer Modelling, 29*(3), 63–78.

Perchet, V. (2009). Calibration and internal no-regret with random signals. In *Algorithmic learning theory* (pp. 68–82), *Lecture notes in computer science*, 5809. Berlin: Springer.

Perchet, V. (2011). No-regret with partial monitoring: Calibration-based optimal algorithms. *Journal of Machine Learning Research: Workshop & Conference Proceedings, 12*, 1893–1921.

Rakhlin, A., Sridharan, K., & Tewari, A. (2011). Online learning: Beyond regret. *Journal of Machine Learning Research: Workshop & Conference Proceedings, 19*, 559–594.

Read, D. (2001). Is time-discounting hyperbolic or sub-additive? *Journal of Risk and Uncertainty, 23*(1), 5–32.

Remer, D., & Nieto, A. (1995). A compendium and comparison of 25 project evaluation techniques. Part 1: Net present value and rate of return methods. *International Journal of Production Economics, 42*(1), 79–96.

Rohde, K., & Wakker, P. P. (2010). Time-tradeoff sequences for analyzing discounting and time inconsistency. *Management Science, 56*(11), 2015–2030.

Roll, R. (1983). On computing mean returns and the small firm premium. *Journal of Financial Economics, 12*(3), 371–386.

Romero, C. (2012, June 18). *The net present value test's impact on the home affordable modification program—Report by the Special Inspector General for the Troubled Asset Relief Program (TARP)*. https://fraser.stlouisfed.org/files/docs/historical/fct/sigtarp_report_npv_20120618.pdf. Accessed 31 Jan 2017.

Rubinstein, M. (1973). A mean-variance synthesis of corporate financial theory. *The Journal of Finance, 28*(1), 241–250.

Ryan, P. A. & Ryan, G. P. (2002). Investment Practices Of The Fortune 1000: How Have Things Changed? *Journal of Business & Management, 8*(4), 355–364.

Sagi, A., & Friedland, N. (2007). The cost of richness: The effect of the size and diversity of decision sets on post-decision regret. *Journal of Personality and Social Psychology, 93*(4), 515–524.

Sandri, S., Schade, C., Mußhoff, O., & Odening, M. (2010). Holding on for too long? An experimental study on inertia in entrepreneurs' and non-entrepreneurs' disinvestment choices. *Journal of Economic Behavior & Organization, 76*(1), 30–44.

Sanfey, A., Rilling, J., Aronson, J., Nystrom, L., & Cohen, J. (2003, June). The neural basis of economic decision-making in the ultimatum game. *Science, 300*, 1755–1756.

Schaelemann, T., Singer, R., & Worth, J. (2012). The HAMP NPV model: Development and early performance. *Real Estate Economics, 40*(1), S32–S64.

Schindler, S., & Pfattheicher, S. (2017). The frame of the game: Loss-framing increases dishonest behavior. *Journal of Experimental Social Psychology, 69*, 172–177.

Scholten, M., & Read, D. (2006). Discounting by intervals: A generalized model of intertemporal choice. *Management Science, 52*(9), 1426–1438.

Schwab, B., & Lusztig, P. (1969). A comparative analysis of the net present value and the benefit-cost ratio as measures of the economic desirability of investments. *Journal of Finance, 24*(3), 507–516.

Senbet, L., & Thompson, H. (1978). The equivalence of alternative mean-variance capital budgeting models. *Journal of Finance, 33*(2), 395–401.

Shani, Y., Danziger, S., & Zeelenberg, M. (2015). Choosing between options associated with past and future regret. *Organizational Behavior and Human Decision Processes, 126*, 107–114.

Sheng, J., Wang, J. & Yang, J. (2014). Regret theory and equilibrium asset prices. *Mathematical Problems in Engineering, 2014*(2014), Article ID 912652; http://dx.doi.org/10.115C014/912652

Sher, S., & McKenzie, C. (2006). Information leakage from logically equivalent frames. *Cognition, 101*, 467–494.

Shu, S., & Morelli, S. (2012). *Applying fairness theories to financial decision making*. Los Angeles: UCLA. http://www.anderson.ucla.edu/faculty/suzanne.shu/Shu%20Morelli%20applying%20fairness.pdf

Smith, R. D. (1996). Is regret theory an alternative basis for estimating the value of healthcare interventions? *Health Policy, 37*(2), 105–115.

Smith, J., & McCardle, K. (1999). Options in the real world: Lessons learned in evaluating oil and gas investments. *Operations Research, 47*(1), 1–5.

Stefan, M. (2016). On the impact of semantic framing in experimental asset markets. *Journal of Behavioral and Experimental Finance, 9*, 81–87.

Stewart, G.B. (1991). *The quest for value: The EVA™ management guide*. Harper Collins Publishers, USA.

Stoltz, G., & Lugosi, G. (2005). Internal regret in on-line portfolio selection. *Machine Learning, 59*, 125–159.

Straub, P., & Murnighan, K. (1995). An experimental investigation of ultimatum games: Information, fairness, expectations, and lowest acceptable offers. *Journal of Economic Behavior & Organization, 27*(3), 345–364.

Strong, N. (2006). Modeling abnormal returns. *Journal of Business Finance & Accounting, 19*(4), 533–553.

Szego, G. (2005). Measures of risk. *European Journal of Operational Research, 163*(1), 5–19.

Terry, E. (2011). *The impact of scenario presentation on capital budgeting decisions* (Working paper). Ryerson University, Canada. https://www.wbiconpro.com/13.Eric.pdf

Tomasino, B., Lotto, L., et al. (2013). Framing the ultimatum game: The contribution of simulation. *Frontiers of Human Neuroscience, 7*, 337–339.

Trippi, R. (1989). A discount rate adjustment for calculation of net present values and internal rates of returns on investments whose lives are un-certain. *Journal of Economics & Business, 41*, 143–151.

Trusheim, M. R., Burgess, B., Hu, S. X., et al. (2011). Quantifying factors for the success of stratified medicine. *Nature Reviews. Drug Discovery, 10*(11), 817–833.

Tsalatsanis, A., Hozo, I., Vickers, A., & Djulbegovic, B. (2010). A regret theory approach to decision curve analysis: A novel method for eliciting decision-makers' preferences and decision-making. *BMC Medical Informatics and Decision Making, 10*, 51–61 Available at: http://webcache.googleusercontent.com/search?q=cache:YfBGWTc2aGoJ:www.biomedcentral.

com/1472-6947/10/51+regret+theory+and+net+present+value&cd=3&hl=en&ct=clnk&gl=us.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5*(4), 297–323.

Urminsky, O., & Zauberman, G. (2016). The psychology of intertemporal preferences. In G. Keren & G. Wu (Eds.), *Wiley-Blackwell handbook of judgment and decision making*. Wiley-Blackwell. http://home.uchicago.edu/ourminsky/Urminsky_Zauberman_2014.pdf

Viossat, Y., & Zapechelnyuk, A. (2013). No-regret dynamics and fictitious play. *Journal of Economic Theory, 148*, 825–842.

Walls, M. R. (2004). Combining decision analysis and portfolio management to improve project selection in the exploration and production firm. *Journal of Petroleum Science & Engineering, 44*(1), 55–65.

Walthe, H. (2010). Anomalies in inter-temporal choice, time-dependent uncertainty and expected utility—A common approach. *Journal of Economic Psychology, 31*(1), 114–130.

Wang, X., Triantaphyllou, E., & Kujawski, E. (2008). Communication on the paper "A reference-dependent regret model for deterministic tradeoff studies". *Systems Engineering, 11*(4), 360–364.

Wiesemann, W., Kuhn, D., & Rustem, B. (2010). Maximizing the net present value of a project under uncertainty. *European Journal of Operational Research, 202*(2), 356–367.

Wong, K. P. (2011). Regret theory and the banking firm: The optimal bank interest margin. *Economic Modelling, 28*, 2483–2487.

Wong, K. P. (2014). Regret theory of the competitive firm. *Economic Modelling, 36*, 172–175.

Wong, K. P. (2015). A regret theory of capital structure. *Finance Research Letters, 12*, 48–57.

Zhang, S., Zhu, J., Liu, X., & Chen, Y. (2016). Regret theory-based group decision-making with multidimensional preference and incomplete weight information. *Information Fusion, 31*, 1–13.

3

Regret Theory and Asset Pricing Anomalies in Incomplete Markets with Dynamic Unaggregated Preferences

Asset pricing is often implicated in the NPV-MIRR-IRR analysis (and related approaches such as NFV and APV) because the discount-rate or hurdle-rate is usually derived directly or indirectly from asset pricing models (such as CAPM, ICAPM and IAPT). Also, as mentioned in Chapter-2, some researchers noted some Framing Effects when CAPM was used with NPV. Myers & Turnbull (1977) and Magni (2008; 2009) critiqued the combined use of CAPM and the NPV-IRR model. Although the CML (capital market line), the intertemporal capital asset pricing model (ICAPM), the SML (security market line) and the Intertemporal Arbitrage Pricing Theory (IAPT) are widely used in portfolio management, valuation, and capital markets' financing, these theories are inaccurate and can adversely affect risk management and portfolio management processes. This chapter introduces several empirically testable financial theories that can be calibrated to real data and it contributes to the literature by: (1) explaining the conditions under which ICAPM/CAPM, IAPT, and CML may be accurate, and why such conditions are not feasible; and (2) explaining why the existence of incomplete markets and dynamic unaggregated markets render CML, IAPT, and ICAPM

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inaccurate; (3) explaining why the *consumption-savings-investment-production framework* is insufficient for asset pricing and analyses of changes in risk and asset values; and introducing a unified approach to asset pricing that considers six factors and the conditions under which this approach will work; (4) explaining why leisure, taxes, and housing are equally as important as consumption and investment in asset pricing; (5) introducing the *Marginal Rate of Intertemporal Joint Substitution* (MRIJS) among consumption, taxes, investment, leisure, intangibles, and housing—this model incorporates Regret Theory and captures features of reality that do not fit well into standard asset pricing models; and this MRIJS framework can support specific or very general finance theories and very complicated models; (6) showing why the *elasticity of intertemporal substitution* (EIS) is inaccurate and insufficient for asset pricing and analyses of investor preferences.

3.1 Existing Literature

Existing literature on the CML, IAPT, and ICAPM is extensive and centered on the consumption-investment-production debate. Balvers and Huang (2005) described the three main classes of research in asset pricing. The first is the consumption based approach of Breeden (1979), which uses a pricing kernel related to the marginal utility of consumption; and studies that use this approach include Campbell and Cochrane (1999), Lettau and Ludvigson (2001, 2002), and Parker and Julliard (2005). Initial versions of the consumption based approach have not worked well empirically; and its main problem is that consumption and its marginal utility are difficult to measure. Balvers and Huang (2005) state that there are asset pricing models that include a production sector but focus predominately on explaining the size of the equity premium. Boldrin et al. (2001) focused on limited intersectoral factor mobility and habit persistence to explain several aspects of the equity premium and key facts of the overall economy. The path breaking model by Gomes et al. (2003) provides a structural theory of the risk sensitivities of firms differing by value and size attributes. Balvers and Huang (2005) state that these approaches which focus on the equity premium belong to the consumption, rather than the production, based approach, because they use the marginal rate of intertemporal substitution (with or without a habit factor) as the pricing kernel.

The second line of research is the Merton (1973) marginal utility approach. However, the marginal utility of wealth is also affected by state variables that indicate how valuable wealth is in different states. This also raises the issue of the importance of state variables. Any variables that affect future risk or risk aversion are relevant, and examples include the variables governing habit persistence, irrationality, Regret, utility of investment/savings and the aggregate supply variables affecting productivity (Petkova and Zhang 2005).

The third line of research is based on production analysis. Liew and Vassalou (2000), Vassalou (2003), Peng and Shawky (1999), Cochrane (1991), Jermann (2005), Balvers et al. (1990), and Cechetti et al. (1990) argue that aggregate output is equal or proportionate to aggregate consumption and that the marginal utility of consumption can be evaluated at the observed level of output, and thus, aggregate output growth is the main asset pricing factor. The assumed advantage of this approach is that output growth is likely to be measured more accurately than consumption growth. Cochrane (1991, 1996) developed a different, production based perspective which explicitly derives an expression for investment returns and assumes that these can be used to price asset returns. Kim (2003) provides a different theoretical perspective by using duality theory in Cochrane's (1996) framework. Vassalou and Apedjinou (2003) empirically developed a corporate innovation factor and showed that this factor, when added to the market factor, absorbs the momentum effect in cross-sectional asset prices. The Balvers and Huang (2005) model is fundamentally different from the other production based asset pricing approaches because it derives a pricing kernel based on the state-contingent optimal reactions by firms to productivity shocks; and this kernel is shown to depend largely on factors determining the marginal value of capital. This approach assumes: (1) that in a competitive economy with complete financial markets, the marginal rate of intertemporal substitution is tied to a stochastic version of the marginal rate of intertemporal transformation; (2) perfect competition; (3) complete markets; (4) a neoclassical environment where supply shocks in the form of productivity shocks are important.

Dayala (2012); Lewellen, Nagel and Shanken (2012); Campbell and Cochrane (2000); Avramov and Chordia (2006); Bray (1994a, b);

Campbell and Cochrane (2000); Cho (2013); De Roon and Szymanowska (2012); Guo (May/June 2004); Hodrick and Zhang (2001); Lewellen and Nagel (2006); Lewis (1991); Llewellen, Nagel and Shanken (2007); and Roll (1977); critiqued asset pricing models and tests. Narayan, Phan, et. al. (2016); Kwan, Leung and Dong (2015); Ormos and Timotity (2016); Lee and Phillips (2016); Tong, Hu and Hu (2017); Elias (2016); Berk and Van Binsbergen (2016); Chiarella, He and Zwinkels (2014); Hansen (2015); Gorton, He and Huang (2014); Majumder (2014); Yang and Cai (2014); and Yang and Li (2013) discussed new approaches to asset pricing. Cho (2009) analyzed how Korean households make savings and portfolio decisions (housing plays a special role in the portfolios of households with collateral as a source of service flows as well as a source of potential capital gains or losses), and also analyzed the role of institutional features by comparing several alternative housing market arrangements to assess their impact on wealth accumulation, portfolio choices, and homeownership. Cho (2009) found that a lower down-payment increases both the homeownership ratio and the fraction of aggregate wealth held in housing assets, but lowers aggregate net worth with mixed demographic implications. Gonzalez, Jareno & Skinner (Mar. 18, 2016) found that: 1) investor behavior differs over time (according to the business cycle) and by sector; 2) in some financial and non-financial sectors, there is an insignificant relationship between stock returns and unexpected changes in real and nominal interest rates; while in some industries the relationship is consistent, significant and positive; and in others, the relationship is negative; 3) Gold has a negative relation to un-anticipated inflation in the overall sample and in the contraction and expansion sub-periods, and thus, its exposed to inflation risk.

Chambers et al. (2009a) analyzed how loan structure affects the borrower's selection of a mortgage contract and the aggregate economy, They found that the loan structure is a quantitatively significant factor in a household's housing finance decision, and is dependent on age and income, and that these effects are more important when inflation is low. Inflation reduces the real value of the mortgage payment and changes in the structure of mortgages have implications for risk sharing. Chambers et al. (2009b) found that although most countries have tax provisions and subsidies to promote homeownership (which generate an asymmetry in the tax treatment of owner and rental occupied housing, which affects the incentives to supply tenant-occupied housing), the progressivity of income taxation can amplify or mitigate the effects of the asymmetries with important implications for housing tenure, housing consumption, portfolio reallocation, and welfare that differ from those reported in the literature.

The asset pricing model developed in Albuquerque and Wang (2008) focuses on corporate governance issues within a consumption-production-investment framework but: (1) does not account for the costs of enforcing investor protection; (2) erroneously assumes that increases in investor protection automatically results in firm growth—differences in investor cognition, investor objectives, ability to process information, and the perceived value of investor protection may dampen positive investor perceptions about firm values; (3) overestimates and overemphasizes the corporate powers and operational powers of majority shareholders, which are quite limited in many common law jurisdictions, and which are also limited even when majority shareholders are managers/officers of the firm (statutorily limited by: the business judgment rule; fiduciary duty statutes that apply to officers/directors of corporate entities; threat of shareholder derivative lawsuits; Sarbanes Oxley Act and similar statutes; investor protection laws enacted by securities exchanges and by securities regulations. Also limited by common-law rules such as the duty of loyalty, fiduciary duties, fraud, usurpation of corporate opportunities, etc.); (4) omits the fact that even where there are majority shareholders, third party investors value independent boards (which theoretically and actually reduce the influence of majority shareholders); (5) makes unrealistic assumptions about the portfolio holdings of both minority and majority investors; (6) makes unrealistic assumptions about the path of growth of the firm's dividends and firm value. The utility functions introduced in Albuquerque and Wang (2008) are limited and unrealistic and do not define the scope and ramifications inherent in the agent's decision context.

Miao and Wang (2007) attempted to analyze the joint decisions of business investments, consumption/savings and portfolio selection for an entrepreneur within a Real Options framework. Their asset pricing analysis is flawed because: (1) as explained in this chapter, ICAPM/CAPM are incorrect, and thus the market portfolio does not exist in reality; and (2) entrepreneurs' intertemporal wealth allocation/reallocation decisions are often intertwined with (not separated from) the perceived risks of their ventures and often focus more on the short-term (which affects the entrepreneur's utility function and greater than normal Regret, neither of which were appropriately defined in Miao and Wang 2007); (3) the entrepreneur's utility function and preferences are time inconsistent because of a short-term focus. The utility functions introduced in Miao and Wang (2007) are limited and unrealistic and do not define the scope and ramifications inherent in the agent's decision context. However, Miao and Wang (2007) correctly noted that Real Options theory and approaches erroneously assume that: (1) the real investment opportunity is tradable; (2) its payoff can be spanned by existing traded assets; or (3) the agent is risk neutral.

Wang (2009) attempted to analyze an agent's optimal consumption saving and portfolio choice decisions when he/she cannot fully insure for income shocks and does not know the income growth rate. The relevance of the theories and conclusions in Wang (2009) to real world transactions/events is very limited. Most individuals know, or can reasonably estimate, expected growth in their labor income. Most individuals cannot, and do not seek to, hedge their labor income. Furthermore, there are very few and very limited insurance markets for hedging labor income (such markets do not exist in most countries). The utility functions introduced in Wang (2009) are limited and unrealistic and do not define the scope and ramifications inherent in the agent's decision context.

Nelson and Wu (1998) found that the standard intertemporal asset pricing model cannot predict risk premia with the correct sign.

Prono (June 2007, Jan. 2009), Green and Hollifield (1992), Guo (May/June 2004), Kumar and Ziemba (1993), Lewellen and Nagel (2006), Roll (1977), Flam (2010), Gharghori et al. (2007), Green and Hollifield (1992), Taleb (2009), Neely et al. (1999), and Mar et al. (2009), have shown that the ICAPM/CAPM are inaccurate[1] and thus,

[1] Avramov and Chordia (2006), Fama and French (1996), Gay and Jung (1999), Joyce and Vogel (1970), Moskowitz (2003), Seckin (2001), Brav and Heaton (2002), Brown and Cliff (2005), Hirshleifer (2001), Chen et al. (1997), Hong and Stein (2003), Llewellen et al. (2007), Lettau and Ludvigson (2001), Jaganathan and Wang (2002), Hodrick and Zhang (2001), Ferson et al. (2003),

the market portfolio is not the most efficient portfolio in terms of risk-reward trade-offs. Some of the issues raised in these articles are also applicable to IAPT, and imply that the IAPT is also inaccurate.

Cochrane (1996) developed an investment based asset pricing model, which performed as well as the CAPM and the Chen et al. (1986) factor model, and performed substantially better than a simple consumption based model. Attanasio and Paiella (2007) and Gerard and Wu (2006) concluded that intertemporal risk is very relevant in asset allocation. Detemple and Murthy (1997), Campbell et al. (1999), Gomes et al. (2006), Basak and Croitoru (2000), and Ou-Yang (2005) analyzed the conditions for equilibrium. Battig and Jarrow (1999) introduced a useful definition of market completeness that is relevant to asset pricing. Gomes et al. (2006) concluded that financing frictions provide an important common factor for the cross section of stock returns and that financial frictions are more important when economic conditions are relatively good.

The CML provides a formulation for the optimal portfolio. The major elements of the CML are as follows:

- It contains some risk-free assets.
- It is partly based on the risk-free rate.
- It assumes that risky assets earn a return equal to the risk premium—risk premium refers to the difference between the return on the market and the risk-free rate ($r_m - r_f$).

ICAPM and IAPT supposedly address the intertemporal nature of decisions and risk management, and provide the expected return for any asset. They are supposedly improvements on the CAPM and APT models. The major elements of ICAPM and IAPT are as follows:

- Expected return is based on the risk-free rate; investor's cost of capital is irrelevant.

Goval and Welch (2008), Campbell and Cochrane (2000), Shanken (1990), Fama (1998), Ehrhardt (Summer 1987), Bray (1994a, b), Campbell et al. (1999), Braun and Larrain (2005), Iacoviello and Pavan (Nov. 2009), Jamison and Wegener (Nov. 2009), Banerjee (2007), and Donihue and Avramenko (March 2007) documented various anomalies and errors in asset pricing models (mostly consumption and production based asset pricing models).

- Significant anchoring effects.
- Returns are directly proportional to risk.
- Expected return is largely determined by the returns on the market over multiple periods; or in the case of the APT, by the specified factors over multiple periods.
- All investors can earn the risk premium, $(r_m - r_f)$.
- Beta(s) (ICAPM) and regression coefficients (IAPT) truly reflect the relationship between the market return (ICAPM) or the factors (IAPT) and the subject asset.
- No utility from hedging.

SML and APT provide formulas for the expected return for any asset and their major elements are as follows:

- Expected return is based on the risk-free rate
- Significant anchoring effects.
- Returns are directly proportional to risk.
- Expected return is largely determined by the returns on the market; or, in the case of APT, by the specified factors.
- All investors can earn the risk premium, $(r_m - r_f)$.
- Beta(s) truly reflects the relationship between the market return and the subject asset.
- No utility from hedging.

3.2 Inaccurate Assumptions on Which ICAPM/IAPT and CML Are Based; And Why The Conditions Are Wrong and Not Feasible

The ICAPM/IAPT and CML (and many asset pricing models) are based on many erroneous assumptions (Error Conditions), some of which are analyzed as follows—since all of these assumptions are invalid, the CML/ICAPM/IAPT are invalid and all or most existing asset pricing models are inaccurate. This has substantial implications for asset management, dividend policy, capital budgeting, and risk management.

3.2.1 Error Condition #1: *There Is Continuous Trading and Portfolio Rebalancing*

ICAPM, IAPT, and the CML (and most asset pricing models) can be valid only if there is continuous portfolio rebalancing—that is, portfolio rebalancing is done at intervals of every one-hundredth of a second or even shorter. With today's technology and back office processing, continuous portfolio rebalancing is not possible because it takes time to make decisions and process transactions (even electronically processed transactions). Even if they are valid, CML, ICAPM, and IAPT are only a snapshot of the optimal portfolio at a specific point in time. The effective yield on a risk-free asset changes over time, and is equal to the stated yield only if the asset is held until maturity. Similarly, the effective yield on a risky asset changes over time. Hence, for CML, ICAPM, and IAPT to be valid, the proportions of the risky and risk-free asset have to be constantly changed (over micro-seconds intervals), as their yields change over time.

3.2.2 Error Condition #2: *There Are No Transaction Costs or Taxes*

The ICAPM/IAPT and the CML/SML models (and most asset pricing models) erroneously assume that there are no transaction costs. In a world with transaction costs, the expected return, yield on the risk-free asset, and the beta will be affected by transaction costs. These costs reduce the yields from the risk-free asset, distort the risk premium, and also distort the percentage of total assets invested in the risk-free and risky assets. Transaction costs affect the effective sensitivity to the market or to factors.

The ICAPM/IAPT and the CML/SML formulas/models (and many asset pricing models) erroneously assume that there are no income or capital gains taxes. In a world with either capital gains or income taxes: the effective risk-free rate will be distorted, and the risk premium will be distorted; the effective yields from risk-free and risky assets will be different from the stated yields; and taxes will distort the proportion of assets invested in both risky and risk-free assets.

3.2.3 Error Condition #3: *There Are No Synthetic Securities*

ICAPM/IAPT and CML/SML models/formulas (and most asset pricing models) erroneously assume that there are no synthetic securities. Synthetic securities can be used to replicate a position in the risk-free rate but with higher returns. The following portfolios are examples: (1) a high-yield bond (with a yield of 10–12%), plus a T-bond futures contract; (2) a high-yield bond plus a put option to sell the bond (or a forward contract) where the option counter-party is AAA-rated; or (3) a long-term commercial lease plus AAA-rated lease insurance. Similarly, synthetic securities can be used to replicate a long position in the market but with higher returns—such as a combination of index futures and a high-yield stock.

3.2.4 Error Condition #4: *There Cannot Be Any Hedging*

ICAPM/IAPT and the CML (and most asset pricing models) can be valid only if there cannot be any hedging. Intertemporal hedging transforms expected returns by placing a lower bound on returns. With hedging, the risk premium can or will always be positive, and the relationship between the beta (or in IAPT, any factor) and the expected return is truncated/bounded. Hedging can be costly or costless and this distinction renders IAPT/ICAPM and CML inaccurate. The combination of hedging and synthetic securities renders the assumptions of ICAPM/IAPT and CML meaningless.

3.2.5 Error Condition #5: *There Are No Framing Effects*

ICAPM/IAPT and CML models/formulas (and most asset pricing models) can be valid if and only if there are no framing effects. In the case of ICAPM, framing pertains to: (1) use of the risk-free rate and the market return to calculate the risk premium; (2) to the use of the risk-free rate and to the use of the beta. Framing effects have been defined in the literature.

3.2.6 Error Condition #6: *Losses Have No Utility and Do Not Cause Regret*

ICAPM/IAPT and CML models/formulas can be valid if and only if losses have no utility and do not cause Regret. On the contrary, some investors' objectives include obtaining losses. With the advent of derivative instruments, losses can be deferred, transferred, and sold. The enforcement of tax laws causes investors to focus on after-tax returns and consequences in financial decision-making. Hence, in a world in which losses have utility, the concept of risk premium is moot in some circumstances, because some investors may seek negative risk premia and beta is meaningless.

3.2.7 Error Condition #7: *Everybody Can Borrow Or Lend at the Risk-Free Rate*

ICAPM/IAPT and CML models/formulas (and many asset pricing models) can be valid if and only if everybody can borrow and/or lend at the risk-free rate. Unfortunately, only a very small percentage of capital markets participants can borrow/lend at the risk-free rate. The risk premium is erroneously defined only with reference to the risk-free rate, without reference to the investment opportunity set available to each capital market participant, or to the participant's actual cost of capital. There is a finite volume of risk-free assets that can be purchased or sold at any point in time, and so the risk-free rate is not available to all capital market investors—this volume of assets changes instantaneously and in most instances, cannot be controlled by any one capital markets participant.

3.2.8 Error Condition #8: *All Investors Can Earn the Risk-Premium*

ICAPM/IAPT and CML models/formulas (and most asset pricing models) can be valid if and only if all participants in the capital markets can earn the risk premium. The risk premium (the difference between the market and risk-free returns) changes instantaneously and cannot be controlled by any one capital markets participant. The risk premium changes for each time interval (daily, monthly, quarterly, and yearly) and the rate of change over different time intervals is not proportional and is distorted. Not all capital markets participants can earn the risk premium, because there is only a finite volume of securities/products that can be sold/purchased to achieve the risk premium. Furthermore, since most stock indices do not reflect the true nature of the underlying markets, the market return can be achieved only by owning accurate proportions of all securities traded in the market.

3.2.9 Error Condition #9: *All Risk-Free Assets (Typically Treasury Securities) Are Truly Risk Free; and the Risk-Free Rate Is Constant*

ICAPM/IAPT and CML models/formulas can be valid if and only if all risk-free assets are truly risk free. Risk-free assets earn the risk-free rate and function as risk-free assets if and only if they are held until maturity, which is not usually the case in real life. Furthermore, the events economic crisis and sovereign debt problems that occurred in Italy, Spain, and Greece during 2008–2013 show that there are no truely risk-free assets and that government bonds are risky and can default.

3.2.10 Error Condition #10: *Firms Investors Do Not Have Any Financing Constraints; and Can Borrow at Any Interest Rate*

ICAPM/IAPT and CML models/formulas can be valid if and only if all firms/investors can borrow any amount of money at any interest rate that they desire—which is inaccurate. Most asset pricing models erroneously assume that firms/investors do not have financing constraints and can borrow at any time, and at any interest rate above the risk-free rate. Constraints include: capital markets conditions, legal constraints (loan covenants, statutes, bankruptcy court rules, etc.), credit quality, availability of capital, usury statutes, and so on.

3.2.11 Error Condition #11: *The Risk-Free Rate and Beta Remain Constant During All Time Periods*

ICAPM/IAPT and CML/SML models/formulas erroneously can be correct if and only if the risk-free rate and the Beta (or IAPT factors) remain constant over all time periods, which is not possible. Furthermore, many researchers have documented significant problems in the various ways of calculating beta, for which there is no industry standard method. Time-varying betas are also inaccurate because there is no guarantee that the calculated beta will vary instantaneously with changes in the market—the models include critical but often false assumptions about the distribution of returns of the market.

3.2.12 Error Condition #12: *All Portfolios Are Superior to All Individual Assets*

The CML, IAPT and ICAPM models/formulas erroneously assume that all portfolios of assets are superior to holding only one single asset. This property follows from the assumed diversification property inherent in the Mean-Variance (M-V) framework. However, there are single assets that may have much better risk-reward profiles than some portfolios.

3.2.13 Error Condition #13: *All Assets Have the Same Duration*

ICAPM, IAPT and CML models/formulas (and most asset pricing models) erroneously assume that all assets have the same duration. The duration referred to is the same as the one used in fixed-income analysis. See Nwogugu (2012: 324–330) for a critique of the Duration formula. If all common stock of all companies do not have the same duration, then assumptions underlying the beta (and factors in IAPT) and risk premium are wrong. Risk-free assets (government bonds and AAA-rated securities) of the same maturity do not have the same duration—the causes of differences in duration include interest payments, call provisions, put rights, form of repayment (cash vs. common stock vs. pay-in-kind), etc. Also, shares of common stock of different companies do not have the same duration—the causes of differences in duration include dividends, corporate by-laws, anti take-over laws, warrants/options issued, trading rules, and perceived risk of the company.

3.2.14 Error Condition #14: *Interest Rates and the Yield Curve Remain Constant During All Investment Horizons*

ICAPM, IAPT and CML models/formulas erroneously assume that interest rates and the yield curve remain constant over any time interval. If interest rates change, then there may be arbitrage opportunities between short-term and long-term securities or across assets.

3.2.15 Error Condition #15: *All Investors in the Capital Markets Have Constant Amounts of Knowledge*

CML/SML erroneously assumes that all investors/participants in any given capital market have constant amounts of knowledge and the same amount of knowledge. This is not true because: (1) different market participants have different amounts of knowledge and perceptions of each company/security/asset; (2) different market participants have different information processing skills.

3.2.16 Error Condition #16: *There Are No Limits on the Volume of Short Positions an Investor Can Take*

CML/ICAPM/IAPT (and some asset pricing models) erroneously assume that there are no limits on the volume of short positions an investor can enter into. In reality, investors' possible short positions are limited by: (1) availability of securities to short; (2) the financing cost of short positions; (3) the limitations on available synthetic short positions.

3.2.17 Error Condition #17: *The Financing Cost of Short Positions Is Lower than the Risk-Free Rate and the Risk Premium*

ICAPM, IAPT, and CML models/formulas (and some asset pricing models) and erroneously assume that the financing cost of short positions is always lower than the risk-free rate and the risk premium. If the converse were true, then there would be arbitrage opportunities—the risk-free rate would be the wrong benchmark for calculating the risk-free premium; and investors could short securities and invest in the risk-free rate.

3.2.18 Error Condition #18: *Short Positions Are Always Profitable*

ICAPM and CML models/formulas implicitly and erroneously assume that all short positions are always profitable.

3.2.19 Error Condition #19: *There Is No Correlation Between the Risk-Free Asset and the Capital Market; and No Correlation Between the Risk-Free Asset and the Market Portfolio*

ICAPM, IAPT, and CML models/formulas (and some asset pricing models) can be accurate if and only if the risk-free asset is not correlated with the Market Portfolio—this is an error. Also, researchers have often observed a negative correlation between the short-term risk-free rates and various proxies for the stock market, such as stock indices.

3.2.20 Error Condition #20: *For Any Given Time Period, the Rate of Belief-Revision of Investors Is Higher than the Rate of Diffusion of Information in Capital Markets*

ICAPM, IAPT, and CML (and some asset pricing models) can be accurate if and only if the investors' average *rate of belief-revision* (R_B)

is greater than *rate of diffusion of information* (R_{ID}) in capital markets. Research has shown that many investors experience *inertia* when faced with critical news about their portfolios. *Anchoring effects* and *framing effects* also prevent, delay, or modify investors' belief revision. The R_{ID} is the rate at which price-changing information is disseminated among investors, market makers, and regulators in the market. Clearly, in today's computerized world, information travels very quickly and is sometimes immediately available to many market participants upon release. Also, various statutes and rules (such as Regulation FD in the USA) prevent or discourage insider trading. Hence, in most markets R_{ID} is always greater than R_B.

3.2.21 Error Condition #21: *The Rate of Substitution of (a Position in) the Market with (a Position in) Any Asset Is Directly Proportional to the Risk-Free Rate*

ICAPM, IAPT, and CML (and some asset pricing models) can be accurate if and only if the investors' average rate of substitution of (a position in) the market with (a position in) any asset (R_a, expressed as a return percentage) is directly proportional to the risk-free rate (R_f). This is a necessary condition because an investor can hold only a proxy of the market portfolio (referred to as market portfolio) and if $\partial R_a/\partial R_f < 0$, then: (1) increases in risk will not be matched by increases in expected return; (2) there may be arbitrage opportunities because arbitrageurs can short the market portfolio and buy risk-free securities when yields of risk-free securities are rising; (3) the market portfolio will no longer reflect the most optimal portfolio for the average investor; (4) increases in the risk-free rate without any asset substitution can increase the investor's margin costs and expected returns, and result in mispricing; (5) the investors' indifference between holding the market portfolio and holding other assets must be constant or relatively constant in order to derive the cost of capital via ICAPM and IAPT.

3.2.22 Error Condition #22: *The Rate of Intertemporal Substitution of (a Position in) the Market with (a Position in) Any Asset is Inversely Proportional to the Risk-Premium*

ICAPM, IAPT, and CML models/formulas can be accurate if and only if the *rate of substitution* (replacement) of a position in the market with (a position in) any asset (R_a) is inversely proportional to the risk premium (R_p). This is a necessary condition because in most instances, an investor can hold only a proxy of the "market portfolio" and if $\partial R_a/\partial R_p > 0$, then: (1) increases in risk will not be matched by increases in expected return; and (2) there may be arbitrage opportunities and arbitrageurs can short the market portfolio and buy combinations of securities and risk-free securities that will provide the highest risk premium when yields of risk-free securities are rising; and (3) the market portfolio will no longer reflect the optimal portfolio for the average investor. However, this condition can never occur because it implies that the average investor is not profit-maximizing.

3.2.23 Error Condition #23: *The Average Investor's Rate of Substitution of (a Position in) the Market with a Position in Any Asset Is Always Greater than the Average Investor's Rate of Substitution of an Equal Position in the Risk-Free Asset with (a Position in) Any Asset*

ICAPM, IAPT and CML models/formulas can be accurate if and only if the average investor's rate of substitution of a position in the market with a position in any asset (R_a, expressed as a percentage) is always greater than the rate of substitution of a position in the risk-free asset with a position in any asset (R_r; expressed as a percentage). This condition ensures and implies that any investor is generally risk averse and is more likely to switch from the market portfolio to an asset A, than from a risk-free asset to A in any market condition. Hence, the indifference curve of the risk-free asset and the market portfolio must be downward sloping.

3.2.24 Error Condition #24: *The Investor's Actual and Marginal Borrowing Rate Is Irrelevant to His/Her Expected Return from Any Asset/Portfolio*

ICAPM, IAPT and CML (and many asset pricing models) can be accurate if and only if the average investor's marginal borrowing rate is irrelevant to his/her expected return. ICAPM and CML do not incorporate investors' marginal borrowing costs. Most IAPT models also do not incorporate the average investor's marginal borrowing costs or changes in such costs over time. This condition cannot be feasible because any rational or irrational investor will typically consider his/her marginal borrowing costs (including margin costs in securities accounts)—such investments may require additional capital and thus borrowing (as in futures accounts, or real estate development projects, or acquisitions of assets).

3.2.25 Error Condition #25: *Investors and Traders Do Not Experience Regret; and Regret Does Not Affect Investor's Expected Returns*

ICAPM, IAPT, and CML (and many asset pricing models) can be accurate if and only if the average investor or trader does not experience Regret, and Regret does not affect their expected returns. ICAPM, IAPT, and CML completely omit Regret. There is a substantial literature on the effects of Regret on human decision-making—extendible to consumption choices and portfolio selection/rebalancing decisions. Hence, this condition is not feasible.

3.2.26 Error Condition #26: *For Any Asset, the Average Investor's Marginal Propensity-to-Substitute (for Any Other Asset) Is Irrelevant to the Calculation of His/Her Expected Return*

On the contrary, the average investor's marginal propensity-to-substitute any asset is highly relevant to both the investor's horizon, opportunity costs, indifference to other assets/opportunities, and thus, his/her expected returns. Hence, ICAPM, IAPT, and CML and related models/formulas are very inaccurate.

3.2.27 Error Condition #27: *Expected Returns Are Only in the Form of Cash; And Investors Do Not Experience Any Positive Utility from Holding Assets*

On the contrary, some expected returns are in the form of un-realized capital-gains; and it has been documented in the literature that investors can and do gain non-monetary utility from holding specific assets and specific combinations of assets. Such non-monetary utility is also part of an investor's return from holding the asset(s), but is not incorporated into ICAPM, CML, or most IAPT models.

3.2.28 Error Condition #28: *There Is No or Minimal Correlation Between the Asset Beta and the Risk-Free Rate*

On the contrary, the betas of some companies are highly correlated with the risk-free rate and this renders the ICAPM and CML inaccurate. Such companies include finance companies, banks, mortgage REITs, and so on. If the beta is correlated with the risk-free rate, then the risk premium and some of the terms in the ICAPM and CML formulas will be distorted.

3.2.29 Error Condition #29: *In the IAPT, the Nature of the Relationships Indicated by the Regression Coefficients Remains Constant Over Time; and for All Assets There Is Minimal Multi-Collinearity Among Factors*

On the contrary, the relationships indicated by the IAPT regression coefficients are very dynamic and change constantly—some change continuously. Also, for any asset there is often substantial multicollinearity among the IAPT regression factors. Hence, IAPT is inaccurate.

3.2.30 Error Condition #30: *In the IAPT, the First Derivative of the Number of Factors with Respect to the R^2 of the Equation Is Constant*

The IAPT models erroneously assume that the first derivative of the number of factors with the R^2 of the IAPT regression equation remains constant over time (i.e., that $\partial R^2/\partial n = 0$, where n is the number of factors).

3.2.31 Error Condition #31: *The Risk-Free Rate Always Adequately Incorporates Inflation Risk and Horizon Risk; or There Is No Inflation or Deflation*

On the contrary, the risk-free rate does not always incorporate inflation risk and horizon risk—such as inflation-protected US Treasury securities. Given such omissions, the risk premium will always be inaccurate and the ICAPM and CML are inaccurate. Most asset pricing models do not incorporate the effects of inflation, which in emerging economies can range from 10% to 3,000% annually. Inflation/deflation: (1) affects investor expectations of asset returns and asset prices; (2) inflation reduces investors' real returns; (3) affects the possibility and availability of an efficient hedge for investments; (4) affects investor current and future consumption, demand for goods and services, output, interest rates, and availability of capital. Hence, most asset pricing models are grossly misspecified.

3.2.32 Error Condition #32: *The Investor's Investment Horizon Does Not Matter; and the Changes in the Investor's Preferences and Risk Tolerance Do Not Matter; Intertemporal Risk and Benefits Can Be Defined Solely in Terms of Standard Deviation, Mean Return, and Consumption*

Investors' investment horizons matter and affect their expected returns because there are opportunity costs, Regret, and utility/disutility from holding assets. ICAPM, IAPT, and CML are defined only with respect to specific horizons, and do not account for changes in investor horizons. In most asset pricing models, the indicator of consumer/investor state and preferences is utility, which is expressed primarily in terms of standard deviation, means, and consumption. This approach is incorrect. Investors' preferences and states can also be expressed with other metrics such as Regret, opportunity costs, downside risk, willingness-to-accept-losses, and so on.

3.2.33 Error Condition #33: *All Markets Are Efficient in All Consecutive Periods; and the No-Arbitrage Condition Exists in Consecutive Time Intervals*

There has been significant research that has proved that markets can be, and are, inefficient. Research on market efficiency has always erroneously assumed constant knowledge and unity of opinions among market participants and regulators; and that all existing market inefficiencies are instantly recognized and taken advantage of by market participants. In reality, there are: (1) significant differences in knowledge of various classes of market participants—individual investors, traders, and so on; (2) not all market inefficiencies are identified and taken advantage of—due to knowledge limitations, availability of capital, time, regulations, Regret, risk aversion, inaccurate computer models, and so on; (3) the sheer volume and instantaneous changes in psychological states of investors and continuous revisions of investor beliefs, creates significant differences in opinion, and hence arbitrage opportunities.

3.2.34 Error Condition #34: *There Is Equilibrium in Financial Markets*

Most asset pricing models erroneously assume some degree of equilibrium in the economy and in some markets. Contrary to generally accepted assumptions, there cannot be any equilibrium in financial markets because: (1) there are instantaneous changes in investor perceptions, aspirations and beliefs about futures states, such that even dynamic equilibrium cannot exist; (2) the definitions of demand and supply are often based partly or wholly on the amount of capital in the market, but do not include the amount of capital that is potentially available to all market participants; (3) the existence of derivatives eliminates the possibility of equilibrium; (4) the possibility of shorting securities and the interest charge for short positions eliminate the possibility of equilibrium; (5) inflation (particularly in emerging market economies) has a significant but sometimes unrecognized effect on asset prices and asset values.

3.3 The Consumption-Savings-Investment-Production Dichotomy Is Inaccurate

During the last 100 years, economists and central bankers have built most of their models and analyses on the consumption-savings-investment-production (CSIP) dichotomy, which often treats each of the four factors almost as a unique domain of analysis (and does not focus on their interconnectedness); and which includes supply-side and demand-side analyses. However, the following economic catastrophes have proven that CSIP is inefficient and flawed in many ways:

(i) Asian financial crisis of the 1990s.
(ii) Collapse of the LDC debt market in the mid-1990s.
(iii) US recession of the early 1990s.
(iv) Stagnation of the growth of the Asian Tigers.

(v) Lack of improvement in the quality of life in most large developing countries (India, China, Brazil, and Indonesia) despite reported growth in their GDP/GNP.
(vi) Significant trade deficits in many countries over the last ten years.
(vii) Subprime mortgage crisis in the US during 2005–2010; and the collapse of asset securitization markets in the US during 2008–2009; and the insolvency of Fannie Mae and Freddie Mac in the US.
(viii) Global financial crisis that began in 2007.
(ix) Failure of government stimulus programs in the EU, US, Japan, and other countries during 2008–2012.
(x) Economic problems in Greece, Ireland, Iceland, and Spain during 2008–2012 (bailout of Greece by the EU; downgrading of Spain's debt to below-investment grade).
(xi) Economic problems in Latin American countries during the 1990s.
(xii) Significant increases in amounts of government debt in many countries over the last ten years.
(xiii) Unrecorded inflation in many countries; and continuing hyperinflation in many developing countries.
(xiv) Under-developed or non-existent real estate markets in many countries.
(xv) High unemployment in many countries during 1995–2015.
(xvi) Inaccuracy of major rating agencies, which contributed to many financial failures, and to market participants' inability to properly assess risk.
(xvii) Adverse effects of the fixed exchange rate of the Chinese currency (Yuan) (when exchange-rate was fixed).
(xviii) Loan losses incurred by Japanese banks during the 1990s.
(xix) Informal black-market economies in many developing countries.
(xx) Last-resort dollarization of economies of many developing countries.
(xxi) Russian financial crisis of 1998.
(xxii) Nigerian financial crisis of 2005–2012.
(xxiii) Crash of the US technology stock market in 2000.
(xxiv) Crash of Chinese stock markets during 2015.
(xxv) Sovereign debt crisis of Greece, Italy, and Spain during 2010–2015.

The CSIP dichotomy has always been used, and consumption has always been analyzed, from the perspective of the household without detailed analyses of the nuances of decision-making and the psychological benefits/costs of the allocation of wealth. Several studies that have analyzed the differences between consumption (of different goods/services) and investment conclusively show that consumption and investment/savings patterns are not uniform across goods/services, financial products, time, industries, and location; and that income and savings patterns are not uniform across time, region, industry, and age, in terms of the actual dollar amounts and the rationale for such behaviors, and the utility/disutility derived from such behaviors. Hence, the terms aggregate consumption, aggregate investment, and aggregate income may be misnomers within the context of asset pricing.

The CSIP framework and dichotomy/debate may have been useful in past eras where the following conditions existed:

1. Information was limited and its diffusion was much slower—there was no internet, and it was difficult to obtain and compare statistics about markets and products/services.
2. More uniformity of products and services—today modern technology enables companies to provide a much wider variety of goods and services at lower per-unit costs.
3. Payment systems were limited compared to today's proliferation
4. Entertainment was limited—today's forms, access, and pricing are drastically different with an exponential proliferation of type and volume.
5. For any given product the sources of utility/disutility were limited.
6. Loan volumes were much smaller compared to today's much larger volumes
7. There were Fewer complex financial instruments.
8. Stable and simpler taxation compared to today's complexity (personal, business, income, capital gains, etc.).
9. There were Fewer ways to hedge financial risk and operational risk.
10. Traditional central banking tools of monetary policy were more effective.
11. Government deficits were generally smaller.

3.3.1 Savings

Savings is much less of an important economic indicator for several reasons (See comments in Lindner (2015)). First, there are now a wider variety of sources of capital for investment and lending. Companies, foundations, pension funds, insurance companies, and governments (local, state, and federal) now have active treasury functions and routinely invest in all types of securities and instruments across different maturities and countries, and have essentially replaced household savings as the primary source of capital. Second, there has been a proliferation of new forms of financial products (such as 100% LTV loans; and long-term leasing) that do not require any equity investment by households or companies. These financial products have substantially reduced the incentive to save and the importance of savings. Third, the prevalence and rapid growth of online and non-internet social networks has reduced the need for household savings—if people can borrow short-term loans and obtain other goods (temporary housing, use of vehicles, etc.) from friends, then the need to save for emergencies and contingencies declines. Fourth, the growth of finance companies that provide short-term loans to individuals has also reduced the need for, and relevance of, savings as an economic indicator.

Fifth, there has been an increasing divergence between savings and consumption over the last twenty years—savings in the traditional sense does not equate to changes in consumption patterns in the future—rich and super-rich households and companies still borrow for various purposes. Hence the relationship between savings and consumption or investment has become much more tenuous than in the past due to changes in the financial services sector, and the utility of debt/borrowing.

Sixth, the utility of savings has declined over the last twenty years. In the past, savings provided some assurance of a safe retirement and the ability to manage contingencies and pay for household necessities, such as education and healthcare costs. However, unrecorded excess inflation in developed and developing countries (which has been non-uniform across industries and goods), higher taxes, market crashes (stocks; real estate; etc.) financial contagion, volatility in capital markets, and the declining financial stability of banks has reduced the utility derived from savings.

Seventh, the increase in government financial support for senior citizens and the increasing availability of loans, particularly in developed countries, has reduced the incentive to save and the utility of savings. Most of this government support is funded by increased taxes. Eighth, the proliferation of insurance and reinsurance products (for long-term care, senior citizen's housing, assisted living, etc.) has reduced the incentive to save money and the utility of savings. Ninth, lending by households (via the purchase of bonds and notes) is often not perceived as a form of savings (neither by households nor by central banks that calculate national accounts). There is increased blurring of the differences between household investment and household savings. Tenth, the perceived utility of a securities brokerage account is very different from the perceived utility of a traditional savings account at a bank (and from that of home equity). Given that the trend in consumer financial services has been a major shift away from savings accounts to brokerage accounts, the utility gained from "traditional" savings has clearly declined.

The transferability of the utility of savings has declined during the last 20 years primarily because of substantial divergences among individuals about:

(a) The value and utility of traditional savings.
(b) The utility of traditional measures of wealth (cash, securities, home equity, intangibles, savings accounts, gold/silver, etc.).

The utility of savings is derived as follows. Let:
U_s = utility of household savings.
I_s = PV of savings.
X_e = PV of expected future expenses not covered by savings.
X_u = PV of unexpected expenses.
X_i = PV of inflation effects.
L_f = negative effects of general instability of financial system on savings and home equity—similar to Regret.
V_h = present value of expected home equity.
t = time to death (years)
r = discount rate
I_g = PV of expected government support for senior citizens.
I_i = PV of third-party insurance benefits.

I_{su} = the estimated present value (PV) of short-term unsecured loans that the household can obtain in the future

$$U_s = \exp\left[\int_0^t \left(I_s - X_e - X_u - X_i - L_f + V_h - I_g - I_i\right)dt\right] \tag{3.1}$$

This implies that most utility functions derived for savings and investment in the existing literature are inaccurate and insufficient to describe the dynamics of real world conditions.

3.3.2 Aggregate Investment and Investment

Aggregate investment is also a much less useful economic indicator for many reasons since it lumps industry investment together with consumer investment. First, aggregate investment does not distinguish between differences in investment objectives and the structure of the investment (which is an increasingly critical point of differentiation). Second, aggregate investment is not adjusted for the duration of the investment, which is a critical element of the economy. Third, aggregate investment does not reflect government subsidies and incentives for both households and companies—this omission distorts the true nature of economic activity. Fourth, Aggregate Investment does not differentiate among capital investment, investment in securities, and investment in intangibles. See comments in Lindner (2015).

3.3.3 Intangibles: The Production and Consumption of Intangibles Differs from General Consumption and Traditional Production and Investment

Wyatt (2005) and Wong and Wong (2001) found significant economic and behavioral effects from accounting recognition/non-recognition of intangibles. According to Salinas (2009) (and other studies), intangible assets constitute 60–75% of the market capitalization value of the major stock indices in the world; and thus changes in the disclosed values of intangible assets can affect individual and group psychology. Corrado et al.

(2011) and Slaughter (2013) found that intangibles contribute substantially to the US economy. See comments in Bansal, Kiku, Shaliastovich and Yaron (2014), and Palacios (2015). At the individual level, intangibles also include social capital and personality traits that account for success or failure in business transactions and personal relationships. The consumption of intangibles also differs from traditional consumption in terms of timing, place, and frequency. Unlike traditional goods and traditional savings, intangibles consumption has substantial, sometimes irreversible, and very observable effects on the social networks of households. Unlike traditional goods, the consumption of intangibles occurs over many periods, and can yield future utility in the form of: home equity; social networks; social capital and reputation; peace of mind; reduced Regret; skills which improve labor mobility; second income; and so on.

The production of intangibles differs from traditional production. Substantial and increasing percentages of intangibles are produced in the services sector, or through services (non-manufacturing) activities, such as software development, advertising, promotions, and social media networks. In many instances, intangibles production or consumption is involuntary or incidental/tangential. The intangibles production decision is debated within the household and companies, and is sometimes a major component of the identity and self-worth of both units. The consumption of intangibles also differs from traditional consumption in terms of timing, place, and frequency. Hence, there are more psychosocial processes (internal and external) associated with intangibles consumption than is indicated in the existing literature.

Most analyses of consumption and savings do not account for the fact that behavioral factors and shocks (loss of income, ill health, property damage), and changes in tastes/preferences of households, and so on, are critical determinants of intangibles consumption.

De Roon and Szymanowska (2012) found that when US stock portfolios are sorted according to size, momentum, transaction costs, market to book, investment to assets, and return on assets (ROA) ratios, and industry classification, they show considerable levels and variations of return predictability inconsistent with asset pricing models, such that the risk premium predicted by asset pricing models is not sufficient compensation for systematic risk. In addition to short sales constraints, holding periods, and transaction costs, the asset pricing anomalies stated in De Roon and

Szymanowska (2012), Hodrick and Zhang (2001), Maslov and Rytchkov (2013), and Lewellen and Nagel (2006) can also be explained by differences in perceptions and amounts of intangibles (i.e., valuation, volatility, risk).

3.3.4 Leisure Differs from General Consumption, Investment and Production

The substantial differences between the consumption of leisure and traditional consumption have not been properly addressed in the existing literature on asset pricing. Unlike traditional goods and services, the consumption of leisure occurs over many periods, and can yield future utility in the form of: social networks; social capital; peace of mind; reputation; reduced Regret; increased cognition and productivity; second income—from additional skills; and so on. In many instances the leisure consumption decision is debated within the household, and is a major component of the identity and self-worth of the household. Hence, there are more psycho-social processes (internal and external) associated with leisure consumption than is indicated in the existing literature. The nature and timing of leisure activities have changed substantially since the mid-1990s and the advent of Broadband Internet. Information about more leisure opportunities are now available on the Internet, which also provides various platforms for matching individuals/households with low-cost and unique leisure opportunities. Movies, games, group memberships, and social networks are readily available on the Internet. These trends have generally resulted in the segmentation of leisure, and declining costs of leisure for many classes/types of leisure activities.

Also, more leisure activities are, or can become, income producing activities or be produced at home. This is partly because: (1) issues like marketing, advertising, distribution, customer services, and quality have all been made cheaper and more available to small businesses by the growth of the Internet; (2) uneven growth in income and living expenses (across industries, regions, households) and shocks (such as the subprime crisis) have compelled adults to seek second incomes. See Aguiar and Hurst (2007), Gelber and Mitchell (2012), Dittmar, Palomino and Yang (2016); Ramey and Francis (2009); and Gronau (1977). Unlike traditional goods/services and traditional savings, leisure consumption can have substantial, sometimes irreversible, and very observable effects on the social networks, future allocations and consumption choices of households. Furthermore, most analyses of consumption, investment and savings do not account for the fact that behavioral factors and shocks (loss of income, ill health, property damage), and changes in tastes/preferences of households, and so on, are critical determinants of leisure consumption.

In addition to short sales constraints and transaction costs, the asset pricing anomalies stated in De Roon and Szymanowska (2012), Hodrick and Zhang (2001), Maslov and Rytchkov (2013), and Lewellen and Nagel (2006) can also be explained by differences in perceptions and amounts of leisure at the individual, household, and company levels (i.e., value of leisure, gains/losses from leisure activities, risk, etc.).

3.3.5 The Consumption of Housing Differs from General Consumption and Traditional Savings

The substantial differences between the consumption of housing on one hand, and traditional consumption and saving have not been properly addressed in the existing literature on asset pricing. Housing accounts for 25–40% of the economies of many developed and Third World countries. Furthermore, housing accounts for more than half of the total wealth of most households in most developed countries, and is the biggest investment decision made by many households (El-Attar and Poschke 2011). Housing-related monthly expenditures (ie. rent; maintenance; mortgage principal/interest; utilities; property taxes and property insurance) account for 20–40% of the total monthly household expenditures in most countries. The concept of housing can also be extended to companies, for which occupancy costs (rent; maintainance; overages; utilities; property taxes; property insurance; fees; etc.) account for 15–50% of monthly corporate operating expenses. Yang (2009) contrasted the consumption of housing to that of traditional goods. In addition to those noted by Yang (2009), the following are other critical differences among the consumption of housing and the consumption of traditional goods/services, and traditional savings.

Unlike traditional goods, the housing unit is typically fixed in time, space, and form. The buyer usually cannot change the configuration of, or move most types of housing units. The transaction and search costs for buying/selling housing units are relatively large, and can vary drastically across geographical regions and time. The purchase of a housing unit almost always involves a mortgage loan (which incurs additional cost for search, processing, reputation, household dynamics, and commitment). In most jurisdictions, the purchase of a housing unit incurs future property taxes. The consumption of housing is highly regulated at local, state, and federal levels—by zoning laws, building codes, environmental laws, ordinances, mortgage laws, and so on.

Unlike traditional goods/services, the purchase of housing typically can involve at least as much consumption as the sale of the same housing unit—that is, both buyer and seller simultaneously experience consumption as a result of the sale transaction. This is henceforth referred to as *asymmetrical two-sided housing transaction consumption*.

Unlike traditional goods/services, the consumption of housing occurs over many periods, and can yield future utility in the form of: home equity; social networks; social capital; peace of mind; reduced Regret; etc. In many instances, the housing consumption decision is debated within the household and is a major component of the identity and self-worth of the household. Hence, there are more psycho-social processes (internal and external) associated with housing consumption than is indicated in the existing literature. Unlike traditional goods/services and traditional savings, housing consumption has substantial, sometimes irreversible and very observable effects on the social networks of households.

Furthermore, most analyses of consumption, investment and savings do not account for the fact that behavioral factors shocks (loss of income; ill health, property damage), and changes in tastes/preferences of households, and so on, are critical determinants of housing consumption. Traditional analyses of consumption and savings do not differentiate among housing as physical space/shelter, housing as expectations and investment, housing as conformance/status, and housing as a bundle of psychological/social goods. Traditional analyses of consumption and savings do not analyze the various effects of debt and access to credit on the consumption of housing and non-housing goods.

Housing remains a basic element of human need and economic activity, but its ramifications and somewhat unique results are not fully understood. There remain several paradoxes in housing markets that transcend location, time, wealth, and household structure, such as the following: (1) why households that rent housing units are not taxed, but households that own homes are taxed without any consideration of how the household financed the home purchase, and regardless of whether or not homeownership is less or more beneficial for the economy; (2) for some wealthy households, the stock of housing is potentially infinite if it is assumed that exchange costs (costs to change housing units) are relatively minimal, and the utility of exchanges is substantial and transferable; (3) rent controls are not enforced fully, such that even middle-income households benefit from rent control; (4) rent control/stabilization laws are almost permanent in most large cities in developed countries, and many of the specific rent control mechanisms are not designed to vary significantly with economic cycles or time; (5) in most countries federal/central governments continue to delegate construction/maintenance of housing almost entirely to the private sector, even though it is apparent that private companies do not have sufficient incentives to provide truly affordable housing; (6) federal and state governments give out tax credits to developers to build affordable housing, but do not give tax credits to households to limit or optimize their consumption of housing (there is huge housing waste caused by "empty nesters" and persistent mis-matches of actual housing needs and housing supply); (7) in many countries, despite the critical nature of housing, governments are hesitant to participate directly in the housing sector, and government activity in the housing sector is limited to income support (housing vouchers, free emergency housing, tax credits), offering free land, and, to a lesser extent, low-cost financing; (8) with all other factors assumed to be constant and similar across housing types (including price), the consumption of housing varies drastically among different types of housing (condos vs. coops. vs. townhouses vs. rental units vs. single family homes

In addition to short-sales constraints and transaction costs, the asset pricing anomalies stated in De Roon and Szymanowska (2012), Hodrick and Zhang (2001), Maslov and Rytchkov (2013), and Lewellen and Nagel (2006) can also be explained by differences in perceptions of, and amounts of housing at the individual and household levels (i.e., value of housing; gains/losses from housing; risk; etc.); and the occupancy costs for commercial real estate at the company level.

3.3.6 Regret Theory and Behavior Based Asset Pricing Models

Given the many problems inherent in existing asset-pricing models, Regret Theory is a viable alternative. This is because Regret Theory incorporates many of the behavioral/psychological issues that ICAPM and IAPT models do not, or cannot capture—such as flexibility, Real Options, Regret minimization, dynamic cost-of-capital, framing effects, and so on. Bonomo et al. (2011), Hirshleifer (2001), Ray and Robson (2012), Jamison and Wegener (Nov. 2009), Han and Yang (2013), Barberis et al. (2001), Korniotis and Kumar (2011), El-Attar and Poschke (2011), Berkelaar and Kouwenberg (2009), Boldrin et al. (2001); Hirshleifer (2001); Ray and Robson (2012); Ou-Yang (2005); Saltari and Ticchi (2007); and Solnik and Zuo (2012) analyzed the effect of individual and group behaviors on asset pricing. Lia and Yang (2013), Dodonova and Khoroshilov (2005), Barberis and Huang (2008), Hung and Wang (2005), De Giorgi et al. (2007), Yogo (2008), and Barberis et al. (2001), developed behavior based asset pricing models and choice models, which were based on, or inspired by Prospect Theory - however Nwogugu (2005a,b,c) and Nwogugu (2006) show that Prospect Theory and Cumulative Prospect Theory are invalid.

Nwogugu (2006) introduced new models of Regret and willingness to accept losses (WTAL). The Nwogugu (2006) Regret model can be used for project evaluation/selection. Sheng et al. (2014), Gollier and Salanié (2006); Solnik and Zuo (2012); Bonomo et al. (2011); Solnik (2006); Dodonova and Khoroshilov (2005); Bell (1983); Dodonova (2009); and Jiliang (2012) developed asset pricing models that were based on Regret Theory. Clarke et al. (1994), DeKay (2009); Bleichrodt et al. (2010); Bell (1982); Frehen et al. (2008); Michenaud and Solnik (2008); and Loomes and Sugden (1982) developed decision models that were based on Regret Theory. Mannor and Shimkin (2008); Mannor, Stoltz and Perchet (2011); Viossat and Zapechelnyuk (2013); Bernstein, Mannor and Shimkin (2013); Foster and Vohra (1999); Perchet (2011); Perchet (2009); Stoltz and Lugosi (2005); Rakhlin, Sridharan and Tewari (2011); Sagi and Friedland (2007); Wang et al. (2008); Huang et al. (2009); Laciana and Weber (2008); Chorus (2012); Ramos, Daamen and Hoogendoorn (2014); Zhang, Zhu, Liu and Chen (2016); and Perchet (2014) analyzed various aspects of Regret Theory.

Chandrasekhar et al. (2008), Canessa et al. (2009), and Coricelli et al. (2007) analyzed neuro-biological Regret.

3.4 The *Unified Intertemporal Wealth-Allocation Decision* (UIWD)

Definitions

Definition 1: CSIP is irrelevant because all four asset-pricing approaches are elements of one *unified intertemporal wealth-allocation decision* (UIWD) by an investor or household. UIWD encompasses all the elements of the household's or the firm's simultaneous decisions about consumption, savings, investment, taxation, production, leisure, intangibles, and housing. The investor or head of the household typically budgets for each period (weekly, monthly, or annually) either mentally, or in physical form (written budgets). Such budgets are often discussed with others—including spouses/partners, employees, investment advisors, or other members of the household. The investor/household head decides how much of his/her wealth to allocate to consumption, savings, investment, production, leisure, intangibles, and housing. Such decisions may be revised/updated during the subject period, and the initial allocation decision is followed almost immediately by more detailed suballocations within each of the six domains: *Consumption*—household necessities, luxuries, and so on; *Investment*—production, amount of work time, overtime, bonuses, savings, amounts to allocate to or withdraw from various savings programs, tax considerations, and so on; *Taxes*—capital gains, income, saless, property, and so on; *Leisure*—entertainment, vacation, and so on; *Intangibles*—training, conferences, professional certifications, cost of patents/trademarks, and so on; *Housing*—rent, mortgage payments, maintenance, and so on.

Definition 2: Total wealth (W(.)) includes monetary and non-monetary wealth (time; intangibles; intellectual property rights; social capital; contingent rights; utility from deferral of obligations; etc.). Investable wealth includes wealth that can be readily converted into cash or exchanged for other assets.

Definition 3: Housing for the agent refers to all costs necessary for tenancy or ownership of a housing unit (individuals) or office space (companies)—such as rent, property taxes, mortgage payments, home insurance, maintenance costs, and so on. Housing for the corporate entity refers to all costs necessary for physical facilities for its operations, and include rent, property taxes, mortgage payments, insurance, maintenance costs, and so on.

Definition 4: Leisure for the agent includes all costs for leisure and entertainment that are not necessary for day-to-day living, and include entertainment costs, vacation time, and so on. Leisure for the corporate entity includes all costs for entertainment and for employee leisure, such as corporate events, wellness programs, employee vacation time, and so on.

Definition 5: For the agent, intangibles includes all costs for developing or changing intangible property, such as training expenses, costs for networking and special events, personal debt capacity, patent/trademark costs, costs for preparing proprietary data, and so on. For the firm, intangibles includes all costs for developing or changing intangible property, such as training expenses, corporate debt capacity, costs for networking and special events, patent/trademark costs, costs for preparing proprietary data, and so on.

Definition 6: For the agent, investment includes all traditional investment and savings activities and all forms of production and service activities because the agent is, in effect, investing time and human capital, which provides returns in the form of salaries, royalties, fees or other remuneration. For the firm, investment includes all traditional investment and savings activities and all forms of non-leisure production and services because the firm is, in effect, investing money, time, human capital, equipment, and other resources.

Definition 7: For the agent, taxes includes all traditional income taxes, capital gains taxes, property taxes, and other taxes. For the firm, taxes includes all traditional income taxes, capital gains taxes, property taxes, and other taxes. For the government, taxes includes all tax incentives and tax abatements for traditional income taxes, capital gains taxes, property taxes, and other taxes. Taxes can be positive, as when wealth is allocated for payment of taxes or when government provides tax incentives/benefits; or negative, as when an agent avoids/defers payment of taxes or when government eliminates existing tax incentives/benefits.

Definition 8: c, t, i, l, b, and h are distinct single units of consumption, taxes, investment, leisure, intangibles and housing respectively.

$-\infty < v, y, x, z, s$ and $r < +\infty$ are the numbers of units of total wealth allocated to consumption, taxes, investment, leisure, intangibles and housing respectively, in the periodic wealth allocation process. These reallocations are effected with single period or multiperiod contracts. Thus, the agent/investor can decide to forgo regular scheduled consumption (v can be negative); and the investor can decide to delay payment of taxes due, or implement tax reduction strategies (y can be negative); and can decide to withdraw funds from his/her investment securities account (x can be negative); and can decide to take a vacation, reduce regular work hours, or spend more leisure time instead of working (z can be negative).

Definition 9: Markets are incomplete because there is intertemporal uncertainty (imperfect information about future states and preferences), there are contract enforcement costs, and the set of available contracts which can be used to transfer or reallocate wealth across time is limited to those contracts that may match uncertain future states; and agents trade in both sequential spot markets and multiperiod forward markets. T is a block of time that contains discrete units of time, each of which is t.

Theorem 3.1 The consumption-investment-savings-production dichotomy is irrelevant because all four factors are elements of one decision process and are insufficient for defining real world situations; and any asset pricing model based solely on one or all of these four factors is inaccurate.

Proof Prior theoretical and empirical asset pricing studies have unnecessarily focused on the four approaches, which are erroneously assumed to be different: consumption based; savings (or consumption-savings) based; investment based; and production based. Basu (Oct. 2002). Garnier et al. (2007). Crossley and Low (2011). See the discussion above on the feasibility conditions for the CSIP framework.

The UIWD is valid where the following conditions exist:

1. Individuals' time and knowledge have both monetary and non-monetary value. The individual's/investor's time and knowledge are part of his/her total wealth.
2. The capital markets are composed of firms that employ individuals based wholly or partly on their time, knowledge, and effort.

3. For any individual and for any time interval, production, investment, savings, and consumption can produce the same types and magnitudes of utility and wealth.
4. There are or may be opportunity costs for every allocation of an individual's resources/total wealth to either consumption, savings, production, or investment.
5. The individual's or investor's marginal rate of intertemporal substitution (MRIS) among any of production, investment, savings, or consumption, changes in some proportion to his/her: (1) total wealth; (2) total investable wealth; (3) horizon. Here the MRIS is the rate at which the individual reallocates one unit of total wealth (which includes investible wealth, non-monetary wealth, and monetary wealth) among consumption, savings, investment, or production.
6. Both capital and labor markets are incomplete.
7. There is never pure equilibrium in either capital or labor markets.
8. The household/investor derives the same types of utility/disutility from investments, products, and traditional goods, such that there is minimal distinction between consumption, investment, and savings.
9. The household/investor derives the same types of utility/disutility from investment products, traditional goods, and earnings from work, such that there is minimal distinction between consumption, investment, savings, or production.
10. The indifference curve between any pair of the investor's four allocation decision factors (consumption, savings, production, and investment) is always downward-sloping.

All of the above-mentioned conditions exist simultaneously in many markets. ■

3.5 The Elasticity of Intertemporal Substitution Is Inaccurate

Several authors have noted the inaccuracy and inapplicability of the EIS. Guvenen (2006) attempted to reconcile two opposing views on EIS—empirical studies using aggregate consumption data typically find the EIS to be close to zero, whereas calibrated models designed to match growth and fluctuations facts typically require it to be close to one. Guvenen (2006) noted that this contradiction is resolved when two kinds of heterogeneity are acknowledged: one, the majority of households do not participate in stock markets; and two, that the EIS increases with wealth. When Guvenen (2006) introduced these two features into a standard real business cycle model, it was noted that limited participation creates substantial wealth inequality, as in the US data; and as such the properties of aggregates directly linked to wealth (e.g., investment and output) are mainly determined by the (high-EIS) stockholders; and since consumption is much more evenly distributed than is wealth, estimation from aggregate consumption uncovers the low EIS of the majority (i.e., the poor).

Havranek (2015) analyzed 2,735 estimates of EIS in consumption that were derived in 169 published articles and found evidence of strong selective reporting wherein authors frequently discarded negative and insignificant estimates, which in turn increased the mean estimate by about 0.5. Havranek (2015) noted that this *reporting bia*s was more dominant than the effects of empirical methods used (with the exception of the choice between micro and macro data). Most importantly, when Havranek (2015) corrected the mean for the reporting bias for macro estimates the EIS obtained was zero, even though the reported average t-statistics was two; but the corrected mean of micro estimates of the EIS for asset holders was around 0.3–0.4. Havranek (2015) concluded that estimated EIS that are greater than 0.8 are inconsistent with the bulk of the empirical evidence, and thus are wrong. Havranek, Horvath, Irsova and Rusnak (2015) compared estimates of EIS that were derived using data from various countries.

Wallenius (2011) considered two different skill accumulation technologies: learning by doing; and Ben-Porath type training. Wallenius (2011) noted that the effect of human capital accumulation in the form of learning by doing is to increase the labor supply elasticity estimate by a factor of 2.1 relative to the estimate that ignores human capital accumulation—all of which biases estimates of the EIS of labor. Okubo (2008) used a model with non-separable and non-homothetic preferences to estimate the EIS, and found that while the assumption of homotheticity is strongly rejected, the estimated EIS is positive and significant.

Such empirical rejection of homotheticity is prime evidence that the EIS is wrong—because homotheticity is a major assumption underlying EIS. Lybbert and McPeak (2012) estimated risk aversion and intertemporal substitution as distinct preferences using data from Kenyan herders, and based on an assumption of the existence of Epstein and Zin recursive preferences. Epstein and Zin (1989) and Lybbert and McPeak (2012) found that the assumption implicit in additive expected utility models was that *relative risk aversion* (RRA) is the inverse of the EIS and that is wrong. Lybbert and McPeak (2012) also stated that their RRA and EIS estimates are consistent with a preference for the early resolution of uncertainty, which is caused by the instrumental value of early resolution of uncertainty; and this same preference pattern is consistent with asset smoothing in response to a dynamic asset threshold. Such early resolution and asset smoothing trends/preferences render the EIS inaccurate because of the assumptions inherent in the definitions and calculation of EIS. Garcia et al. (2006) noted that although in the canonical CCAPM, the coefficient of RRA is constrained to be the inverse of the EIS; for theoretical and empirical reasons the EIS and the RRA should be disentangled; and such disentangling may be achieved by replacing the future consumption stream, not by a certainty equivalent of future utility as in the recursive utility model of Epstein and Zin (1989), but by an exogenous reference level of consumption, which, in a recursive way, assesses the expected future consumption. Garcia et al.'s (2006) observations imply that those methods of calculating the EIS by direct or indirect reference to the RRA, are wrong.

Giulano and Turnovsky (2003) noted that the constant elasticity utility function implies that the EIS is the inverse of the coefficient of RRA, but empirical evidence suggests that this relationship may or may not hold, and thus studies of risk and growth should decouple these two parameters. Giulano and Turnovsky (2003) analyzed the equilibrium of a stochastically growing small open economy under general recursive preferences, and attempted to show that errors committed by using the constant elasticity utility function, even for small violations of the compatibility condition, can be substantial. The Giulano and Turnovsky (2003) results cast substantial doubt on the validity of both the EIS, and the EIS–RRA relationship.

Saltari and Ticchi (2007) analyzed the role of risk aversion and intertemporal substitution in a simple dynamic general equilibrium model of investment and savings, and found that risk aversion cannot, by itself, explain a negative relationship between aggregate investment and aggregate uncertainty, because the effect of increased uncertainty on investment also depends on the intertemporal elasticity of substitution. They also noted that the relationship between aggregate investment and aggregate uncertainty is positive, even if agents are very risk averse, as long as the elasticity of intertemporal substitution is low, but this statement is a contradiction to their other observations. Saltari and Ticchi (2007) stated that a negative investment–uncertainty relationship requires that the RRA and the EIS are both relatively high or both relatively low—but either condition contravenes the very definition of both RRA and EIS.

Lewis (1991) noted that empirical studies of the restrictions implied by the ICAPM across different asset markets have found conflicting evidence, and using data on foreign exchange, bonds, and equity returns, Lewis (1991) found that the tendency to reject the intertemporal consumption based asset pricing relationship depends upon the inadequacy of an auxiliary assumption (that covariances of returns with consumption move in constant proportion over time), not necessarily the relationship itself. Stern (1997) noted that ecological economics is characterized by arguments concerning limits to substitution between inputs in production (energy, natural capital, etc., vs. manufactured capital, labor, etc.) and the implications these have for sustainability; and that various authors have also expressed concerns regarding limits to substitution in consumption, either between environmental assets and other goods or between basic needs commodities and other goods. Stern (1997) stated that the underlying theme is that individual commodities and other inputs have unique physical or other properties which make them poor substitutes, and that many authors have argued for irreversibilities in consumption behavior. Kim and Lee (2007) analyzed on-the-job human capital accumulation, from the perspective of time invested for acquiring skills and learning by doing, in an RBC model and show that the inability to account for human capital accumulation leads to a substantial bias in conventional estimates of EIS. Kim and Lee (2007) stated that their main results were based on the standard intuition that the opportunity cost of time invested in acquiring human capital moves pro-cyclically, such that on-the-job time invested in acquiring human capital is counter-cyclical; and the true wage rate becomes less pro-cyclical, while production hours become more pro-cyclical than total hours at work.

Lee (2008) noted that estimates of EIS obtained from standard life-cycle models are subject to a downward bias because they neglect the life-cycle and demographic patterns of on-the-job human capital investment. Lee (2008) stated that there was statistically significant evidence that conventional estimates of EIS for total hours at work are biased downward by about 20 % at the intensive margin; and the corresponding EIS estimates for production hours are biased downward even more, which provides an explanation for why output fluctuation is greater than hours/employment fluctuation over the business cycle. Taking into account the fact that part of a worker's time at work goes to acquiring human capital, in addition to his main task of producing goods, Lee (2008) attempted to extend the standard life-cycle model to include time spent on investing in on-the-job human capital and proposed a new framework for identifying the EIS. Kapoor and Ravi (2016); Cashin and Unayama (2016); and Gruber (2013) analyzed various aspects of EIS.

Theorem 3.2: The EIS is inaccurate.

Proof: Many studies have empirically and theoretically derived very different estimates for the EIS. Giuliano and Turnovsky (2003). Crossley and Low (2011). The EIS is deficient because it only addresses substitution between two factors/goods in only one domain (typically consumption, production, or investment). The EIS does not address the UIWD which refers to reallocation of wealth among consumption, production, investment and/or savings. The EIS does not account for the finiteness of the factors being substituted—that is, a human being can work for only a finite number of hours in each day/month/year; a household has a finite amount of wealth that it can spend (even when borrowing and leasing are considered); and so on.

Even if the EIS is meaningful in the consumption-savings domain, or the investment domain or the production domain it is never constant because of the following reasons. Labor markets are incomplete and constantly evolving and, in most circumstances, UIWD prevails. EIS does not account for different perceptions of time by different people/groups; and the different values of different types of time (i.e., leisure; work; family) to different people/groups—this is a critical factor in intertemporal analysis. EIS addresses only changes in two periods, which is, or can be, very misleading because many decisions and preferences are substantially multiperiod in nature (i.e., involve more than three time periods). EIS is defined completely without reference to investible wealth or total wealth. This is an error, because there are many behavioral biases (anchoring effects and framing effects) that are intentional (advertising from brokerage firms), or unintentional (discussion with spouse/partners), and that are observed or unobserved, in the wealth reallocation decision process. Most of these framing effects are based on total wealth, or investible wealth, or disposable wealth or future wealth. Furthermore, CSIP is best defined with reference to some form of wealth or total wealth.

The EIS does not consider Regret at either individual or group levels.

Giuliano and Turnovsky (2003) note that the EIS (emphasized by Hall (1978, 1988), and Mankiw et al. (1985) and others) focuses on intertemporal preferences and is well defined in the absence of risk. This is an error because the investor's wealth reallocation decision processes almost always involves some analysis of risk, which is manifested in part by the actual reallocation process. Giuliano and Turnovsky (2003) note that a natural definition of the EIS is in terms of the percentage change in intertemporal consumption in response to a given percentage change in the intertemporal price. Hence, EIS is erroneously defined primarily in terms of consumption and prices, which by themselves are insufficient to fully capture the investor's preferences.

EIS does not account for the fact that the average investor's labor income is subject to various shocks; and that the investor is able to observe most of the components of his/her labor income in the short run (1–15 days). EIS does not account for anchoring effects or framing effects. EIS does not account for the fact that the average investor's propensity to substitute is partly based on his/her total wealth, wealth available for reallocation, and perceived risk of both opportunity costs and returns from abstinence.

EIS may be accurate only if the permanent income hypothesis (PIH) (introduced by Friedman (1957)) is correct. The PIH states that consumption is equal to the annuity value of total wealth calculated as the sum of the discounted expected value of future income (discounted using the risk-free rate), plus the agent's human wealth, plus financial wealth (cumulative savings). Contrary to the existing literature, and given the analysis herein and above (including the UIWD framework and the invalidity of the CSIP framework), the individual agent's optimal consumption rule is not governed by the PIH. The conditions under which the PIH rule is feasible are unrealistic, and the PIH erroneously implies that changes in an individual agent's consumption are not predictable (Hall (1978)). The PIH is based on the erroneous assumptions that: (1) utility is quadratic utility; and (2) there are no precautionary savings by the average individual; (3) the individual's future labor income is riskless; and (4) the subjective discount rate and the prevailing interest rate are equal. Guvenen (2006) and Havranek (2015) also noted that there is conflicting evidence about the validity of EIS. ■

3.6 Relationships Among the Factors: The *Marginal Rate of Intertemporal Joint Substitution*

Theorem 3.3: For any time interval or successive time intervals, each of c, t, i, l, b, and h are non-monotonic, and the relationship between each of c, t, i, l, b and h on one hand and total wealth on the other hand, is also non-monotonic, where total wealth is finite, worker effort is rewarded with monetary benefits and utility/disutility provided by consumption/taxes/investment/leisure/intangibles/housing is non-monotonic.

Proof: For any time interval or any series of successive time intervals, the relationship between total wealth and each of *c, t, i, l, b and h* is not monotonic and can change because there may be frictions, surprises, and other considerations that may cause the relationship to change, such as: new knowledge; fairness; pressure from spouse/partner; willingness to defer gains/losses; need for leisure time; involuntary changes in work conditions; temporary or permanent layoffs; advertising; change of job; and so on. Therefore, di/dW, dp/dW, dt/dW, dl/dW, dh/dW and dc/dW are all non-monotonic. ■

Theorem 3.4: For any time interval or successive time intervals, each of c, t, i, l, b, and h is non-additive, and the relationship between total wealth and each of c, t, i, l, b and h is also non-additive, where total wealth is finite, worker effort is rewarded with monetary benefits and utility/disutility provided by consumption/taxes/investment/leisure/intangibles/housing is non-monotonic.

Proof: Each of *c, t, i, l, b,* and *h* is non-additive in any time interval—for example, where $c(.)$ is a consumption function, $c(x) + c(y) \neq c(x+y)$. First, allocations of wealth to *c, t, i, l, b,* and *h* are done primarily at the beginning of the subject period (and also during the subject period), and any additional allocations require either reduction of allocations to the other five factors or a change in total wealth, which, in turn, causes changes in anchoring, framing, and preferences, all of which changes the utility/disutility of the sum of the added allocations $\{c(x+y)\}$. The second reason is that the average investor derives different amounts of utility from, and assigns different risk profiles for each of *c, t, i, l, b,* and *h*, and such utility is dynamic. Thus, when $c(x)$ is added to $c(y)$ the relative risk of *c, t, i, l, b,* and *h* is very likely to change, such that the sum will not be $c(x+y)$. It also follows automatically that the relationship between total wealth and each of *c, t, i, l, b,* and *h* is also non-additive. ■

Theorem 3.5: For any time interval or successive time intervals, the relationship between total wealth, and each of c, t, i, l, b, and h is recursive, where total wealth is finite, worker effort is rewarded with monetary benefits, and the utility/disutility provided by consumption/taxes/investment/leisure/intangibles/housing is non-monotonic.

Proof: $-\infty < v, y, x, z, s, r < +\infty$ are the numbers of units of total wealth allocated to consumption, taxes, investment, leisure, intangibles and housing respectively, in the periodic wealth allocation process. Because of UIWD, in every subperiod t, the amount of total wealth allocated to a factor is a direct function of:

(a) amounts of wealth allocated to all six factors in the prior period—the utility/disutility gained from such prior allocations shape decisions about future allocations and expected Regret.
(b) amounts of Wealth allocated to the other five factors in the current period (three of $c_{(t+1)}$, $t_{(t+1)}$, $i_{(t+1)}$, $l_{(t+1)}$, $b_{(t+1)}$, $h_{(t+1)}$)—which also affected reallocations in the current period.
(c) the portion of total wealth that is available for allocation in the current period ($w_{(t+1)}$).
(d) total wealth (W_T).

Therefore, *c, t, i, l, b* and *h* are recursive, and:

$$x_{(t+1)} = f\left(x_t, v_t, y_t, z_t, r_t, s_t, w_{(t+1)}, v_{(t+1)}, y_{(t+1)}, z_{(t+1)}, r_{(t+1)}, s_{(t+1)}, W_T\right) \quad (3.2)$$

$$v_{(t+1)} = f\left(x_t, v_t, y_t, z_t, r_t, s_t, w_{(t+1)}, x_{(t+1)}, y_{(t+1)}, z_{(t+1)}, r_{(t+1)}, s_{(t+1)}, W_T\right) \quad (3.3)$$

$$y_{(t+1)} = f\left(x_t, v_t, y_t, z_t, r_t, s_t, w_{(t+1)}, v_{(t+1)}, x_{(t+1)}, z_{(t+1)}, r_{(t+1)}, s_{(t+1)}, W_T\right) \quad (3.4)$$

$$z_{(t+1)} = f\left(x_t, v_t, y_t, z_t, r_t, s_t, w_{(t+1)}, v_{(t+1)}, y_{(t+1)}, i_{(t+1)}, r_{(t+1)}, s_{(t+1)}, W_T\right) \quad (3.5)$$

$$r_{(t+1)} = f\left(x_t, v_t, y_t, z_t, r_t, s_t, w_{(t+1)}, v_{(t+1)}, y_{(t+1)}, i_{(t+1)}, x_{t+1)}, s_{(t+1)}, W_T\right) \quad (3.6)$$

$$s_{(t+1)} = f\left(x_t, v_t, y_t, z_t, r_t, s_t, w_{(t+1)}, v_{(t+1)}, y_{(t+1)}, i_{(t+1)}, r_{(t+1)}, x_{(t+1)}, W_T\right) \quad (3.7)$$

Also, the relationship between total wealth (W) and each of *c, t, i, l, b*, and h is recursive in any time interval or a series of successive time intervals—thus where *a, b, d, e, j, k* are variables with defined formulas:

$dx_{(t+1)}/dW_{(t+1)} = a^*f\{dx_t/dW_t\}$ because for most investors, investment returns in the first period and the change in total wealth has a direct effect on the amount of wealth that is allocated to investments in the second period.

$dv_{(t+1)}/dW_{(t+1)} = b^*f\{dx_t/dW_t\}$ because for most investors, the utility/disutility gained from consumption in the first period and the change in total wealth have a direct effect on the amount of wealth that is allocated to consumption in the second period and subsequent periods.

$dy_{(t+1)}/dW_{(t+1)} = d^*\{dy_t/dW_t\}$ because for most investors, the amount of wealth allocated to taxes and the associated returns in the first period and the change in total wealth both have a direct effect on the amount of wealth that is allocated to taxes in the second and subsequent periods.

$dz_{(t+1)}/dW_{(t+1)} = e^*\{dz_t/dW_t\}$ because for most investors, the amount of wealth allocated to leisure and the associated returns in the first period and the change in total wealth both have a direct effect on the amount of wealth that is allocated to leisure in the second period and subsequent time periods. ■

Theorem 3.6: (a) The marginal rate of intertemporal joint substitution (MRIJS) measures the propensity for an investor/household to substitute units of consumption, taxes, investment, leisure, intangibles, and housing (factors), where total wealth is finite and limited in each time interval, and the effects of such substitution may increase or decrease the other five factors; (b) the MRIJS measures the investor's propensity to substitute/reallocate one unit of total wealth from any of five factors (c, l, t, i, b or h) to a sixth factor.

Proof: $-\infty < v, y, x, z, s, r < +\infty$ are the numbers of units of total wealth allocated to consumption, taxes, investment, leisure, intangibles, and housing respectively, in the periodic wealth allocation process. $W(x)$ is a wealth function; and w_t denotes the periodic total wealth to be allocated at the beginning of the budget period. The utility gained by the household from such wealth is a function defined as:

$$U(c, t, I, l, b, h) = F\{W(c, t, I, l, b, h)\} \tag{3.8}$$

$$w_t = vc + yt + xi + zl + sb + rh \tag{3.9}$$

$$v = (W_t - xi_t - sb_t - l_t z_t - yt_t - rh_t) \tag{3.10}$$

$$x = (W_t - vc_t - sb_t - lz_t - yt_t - rh_t) \tag{3.11}$$

$$y = (W_t - vc_t - i_t x - sb_t - l_t z - rh_t) \tag{3.12}$$

$$z = (W_t - vc_t - sb_t - i_t x - yt - rh_t) \tag{3.13}$$

$$s = (W_t - vc_t - i_t x - l_t z - yt_t - rh_t) \tag{3.14}$$

$$r = \left(W_t - vc_t - sb_t - i_t x - yt_t - l_t z\right) \quad (3.15)$$

Let:

C'' = the rate of substitution of consumption with respect to the other five factors—that is, the average change in consumption, as a result of simultaneous changes in the other five factors.

I'' = the rate of substitution of investment with respect to the other five factors—that is, the average change in investment, as a result of simultaneous changes in the other five factors.

T'' = the rate of substitution of taxes with respect to the other five factors—that is, the average change in taxes, as a result of simultaneous changes in the other five factors.

L'' = the rate of substitution of leisure with respect to the other five factors—that is, the average change in leisure expenditures, as a result of simultaneous changes in the other five factors.

B'' = the rate of substitution of intangibles with respect to the other five factors—that is, the average change in expenditure on intangibles, as a result of simultaneous changes in the other five factors.

H'' = the rate of substitution of housing with respect to the other five factors—that is, the average change in housing expenditures as a result of simultaneous changes in the other five factors.

C'', I'', T'', L'', B'' and H'' are measured over one or more periods and, thus, there is some averaging.

Then:

$$C'' = \partial v / \partial W_t - \partial v / \partial x_t - \partial v / \partial y_t - \partial v / \partial z_t - \partial v / \partial s_t - \partial v / \partial s_t \quad (3.16)$$

$$L'' = \partial z / \partial W_t - \partial z / \partial x_t - \partial z / \partial y_t - \partial z / \partial v_t - \partial z / \partial s_t - \partial z / \partial r_t \quad (3.17)$$

$$T'' = \partial y / \partial W_t - \partial y / \partial x - \partial y / \partial v_t - \partial y / \partial z_t - \partial y / \partial s_t - \partial y / \partial r_t \quad (3.18)$$

$$I'' = \partial x / \partial W_t - \partial x / \partial v_t - \partial x / \partial y_t - \partial x / \partial z_t - \partial x / \partial s_t - \partial x / \partial r_t \quad (3.19)$$

$$B'' = \partial s / \partial W_t - \partial s / \partial v_t - \partial s / \partial y_t - \partial s / \partial z_t - \partial s / \partial x_t - \partial s / \partial r_t \quad (3.20)$$

$$H'' = \partial x / \partial W_t - \partial r / \partial v_t - \partial r / \partial y_t - \partial r / \partial z_t - \partial r / \partial s_t - \partial r / \partial x_t \quad (3.21)$$

C'', L'', T'', I'', B'', H'' are referred to as the marginal rates of substitution.

MRIJS is the investor's propensity to substitute/reallocate one unit of total wealth from any of five factors (*c, l, t, i, b* or *h*) to a sixth factor. MRIJS ε (0, 1).

$$\mathrm{MRIJS} = \exp[\mathrm{Min}\{0, -(C^{\ddot{}\,'} + L^{\ddot{}\,'} + T^{\ddot{}\,'} + I^{\ddot{}\,'} + B^{\ddot{}\,'} + H^{\ddot{}\,'}\}] \tag{3.22}$$

Thus, MRIJS is a measure of both a person's ability-to-repay and willingness-to-repay an obligation. The greater a person's total wealth, the greater his/her ability to reallocate such wealth among the six factors.

The MRIJS implicitly incorporates Regret, because the person's reallocation of wealth among the six factors implicitly includes a Regret minimization process. ■

3.7 Other Recent Research on Asset Pricing

Given the foregoing analysis, the theories and models introduced and discussed in the following articles, are either moot or inaccurate: Meghir and Weber (1996), Ray and Robson (2012), Chen and Epstein (2002), Ozsoylev and Walden (2011), Hugonnier (2012), Adam and Marcet (2011), Epstein and Zin (1989), Duffie and Strulovici (2012), Bossaerts et al. (2007); Alvarez and Jermann (2000, 2005); Narayan, Phan, et. al. (2016); Kwan, Leung and Dong (2015); Ormos and Timotity (2016); Lee and Phillips (2016); Tong, Hu and Hu (2017); Elias (2016); Berk and Van Binsbergen (2016); Chiarella, He and Zwinkels (2014); Hansen (2015); Gorton, He and Huang (2014); Prono (2015); Majumder (2014); Yang and Cai (2014); and Yang and Li (2013).

3.8 Conclusion

In most markets, investors' preferences diverge substantially, all existing asset pricing models are inaccurate because the underlying assumptions are not realistic. Obviously, this has important ramifications for asset management and capital budgeting. EIS is very inaccurate, especially where markets are incomplete and investors' preferences are both dynamic and multifaceted. The focus on consumption and price as the definition of investors' preferences and constraints is very limited and misleading—the investors' decision problem is much broader in scope.

Given the problems and inaccuracies inherent in the CSIP dichotomy, the more the unified asset-pricing approach introduced herein (UIWD) is more likely to result in better policy decisions. The MRIJS is distribution free, does not require the use of any specific utility functions, implicitly accounts for risk (by multi-faceted wealth allocation) and provides a more unified and accurate indication/analysis of the average investor's wealth allocation decisions. MRIJS and Theorems 3.4, 3.5, and 3.6 and the theories introduced herein are also testable hypotheses, which can become the foundation for further detailed models that are better able to reflect reality and economic transactions.

Bibliography

Adam, K., & Marcet, A. (2011). Internal rationality, imperfect market knowledge and asset prices. *Journal of Economic Theory, 146*(3), 1224–1252.

Aguiar, M., & Hurst, E. (2007). Measuring trends in leisure: The allocation of time over five decades. *The Quarterly Journal of Economics, 122*(3), 969–1006.

Albuquerque, R., & Wang, N. (2008). Agency conflicts, investment and asset pricing. *Journal of Finance, 63*(1), 1–40.

Alvarez, F., & Jermann, U. (2000). Efficiency, equilibrium, and asset pricing with risk of default. *Econometrica, 68*(4), 775–797.

Alvarez, F., & Jermann, U. J. (2005). Using asset prices to measure the persistence of the marginal utility of wealth. *Econometrica, 73*(6), 1977–2016.

Attanasio, O., & Paiella, M. (2007). *Intertemporal consumption choices, transaction costs and limited participation in financial markets: Reconciling data and theory*. Temi di discussione (Economic working papers) 620, Bank of Italy, Economic Research Department, Italy.

Avramov, D., & Chordia, T. (2006). Asset pricing models and financial market anomalies. *Review of Financial Studies, 19*, 1001–1040.

Bai, J., & Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica, 70*, 191–221.

Balvers, R., Cosimano, T., & McDonald, W. (1990). Predicting stock returns in an efficient market. *Journal of Finance, 45*(4), 1109–1128.

Balvers, R., & Huang, D. (2007). Productivity-based asset pricing: Theory and evidence. *Journal of Financial Economics, 86*(2), 405–445.
Balvers, R., & Huang, D. (2009). Money and the (C)CAPM: Theory and evaluation. *Journal of Financial and Quantitative Analysis, 44*, 337–368.
Banerjee, S. (2007). *Learning from prices and dispersion in beliefs* (Working paper).
Bansal, R., Kiku, D., Shaliastovich, I., & Yaron, A. (2014). Volatility, the macroeconomy, and asset prices. *Journal of Finance, 69*, 2471–2511.
Barberis, N., & Huang, M. (2008). Stocks as lotteries: The implications of probability weighting for security prices. *The American Economic Review, 98*(5), 2066–2100.
Barberis, N., Huang, M., & Santos, T. (2001). Prospect theory and asset prices. *The Quarterly Journal of Economics, 116*(1), 1–53.
Basak, S., & Croitoru, B. (2000). Equilibrium mispricing in a capital market with portfolio constraints. *Review of Financial Studies, 13*, 715–748.
Basu, S., & Kimball, M. (2002, October). *Long run labor supply and the elasticity of intertemporal substitution for consumption*. Available at: https://files.nyu.edu/cps272/public/ReadingGroup/Shenghao030207.pdf; and http://www-personal.umich.edu/~mkimball/pdf/cee_oct02-3.pdf
Battig, R. J., & Jarrow, R. A. (1999). The second fundamental theorem of asset pricing: A new approach. *Review of Financial Studies, 12*, 1219–1235.
Bell, D. E. (1982). Regret in decision making under uncertainty. *Operations Research, 30*(5), 961–981.
Bell, D. E. (1983). Risk premiums for decision regret. *Management Science, 29*(10), 1156–1166.
Berk, J., & Van Binsbergen, J. (2016). Assessing asset pricing models using revealed preference. *Journal of Financial Economics, 119*(1), 1–23.
Berkelaar, A. B., & Kouwenberg, R. (2009). From boom till boost: How loss aversion affects asset prices. *Journal of Banking & Finance, 33*(6), 1005–1013.
Bernstein, A., Mannor, S., & Shimkin, N. (2013). Opportunistic strategies for generalized no-regret problems. *Journal of Machine Learning Research: Workshop & Conference Proceedings, 30*, 158–171.
Bleichrodt, H., Cillo, A., & Diecidue, E. (2010). A quantitative measurement of regret theory. *Management Science, 56*(1), 161–175.
Boldrin, M., Christiano, L., & Fisher, J. (2001). Habit persistence, asset returns, and the business cycle. *American Economic Review, 91*, 149–166.
Bonomo, M., Garcia, R., Meddahi, N., & Tédongap, R. (2011). Generalized disappointment aversion, long-run volatility risk, and asset prices. *Review of Financial Studies, 24*(1), 82–122.

Booth, L. (2003). Discounting expected values with parameter uncertainty. *Journal of Corporate Finance, 9*, 505–519.
Bossaerts, P., Plott, C., & Zame, W. R. (2007). Prices and portfolio choices in financial markets: Theory, econometrics, experiments. *Econometrica, 75*(4), 993–1038.
Braun, M., & Larrain, B. (2005). *Supply matters for asset prices: Evidence from IPOs in emerging markets* (Working paper #06-4). Boston: Federal Reserve Bank of Boston.
Brav, A., & Heaton, J. B. (2002). Competing theories of financial anomalies. *Review of Financial Studies, 15*, 575–606.
Bray, M. (1994a, January). *The arbitrage pricing theory is not robust—Part 1: Variance matrices and portfolio theory in pictures* (Discussion paper #178). Financial Markets Group, London School of Economics, United Kingdom.
Bray, M. (1994b, January). *The arbitrage pricing theory is not robust—part 2: Factor structures and factor pricing* (Discussion paper #179). Financial Markets Group, London School of Economics, United Kingdom.
Breeden, D. (1979). An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics, 7*, 265–296.
Brocas, I., & Carrillo, J. D. (2005). A theory of haste. *Journal of Economic Behavior & Organization, 56*(1), 1–23.
Brown, G., & Cliff, M. (2005). Investor sentiment and asset valuation. *Journal of Business, 78*, 405–440.
Campbell, J. (1993). Intertemporal asset pricing without consumption data. *American Economic Review, 3*, 487–512.
Campbell, J., & Cochrane, J. (2000). Explaining the poor performance of consumption based asset pricing models. *Journal of Finance, 28*, 63–78.
Campbell, C. J., Kazemi, H., & Nanisetty, P. (1999). Time-varying risk and return in the bond market: A test of a new equilibrium pricing model. *Review of Financial Studies, 12*, 631–642.
Canessa, N., Alemanno, F., Motterlini, F., et al. (2009). "Your Regret and my decisions": A neuroimaging study of social influence on emotion-based decision making. *NeuroImage, 47*(1), S117.
Cashin, D., & Unayama, T. (2016). Measuring intertemporal substitution in consumption: Evidence from a VAT increase in Japan. *The Review of Economics and Statistics, 98*(2), 285–297.
Cechetti, S., Lam, P., & Mark, N. (1990). Mean reversion in equilibrium asset prices. *American Economic Review, 80*(3), 195–227.

Chambers, M., Garriga, C., & Schlagenhauf, D. E. (2009a). The loan structure and housing tenure decisions in an equilibrium model of mortgage choice. *Review of Economic Dynamics, 12*(3), 444–468.

Chambers, M., Garriga, C., & Schlagenhauf, D. E. (2009b). Housing policy and the progressivity of income taxation. *Journal of Monetary Economics, 56*(8), 1116–1134.

Chandrasekhar, P., Capra, C. M., Moore, S., Noussair, C., & Berns, G. S. (2008). Neuro-biological regret and rejoice functions for aversive outcomes. *NeuroImage, 39*(3), 1472–1484.

Chen, C. D., Eun, C. S., & Senbet, L. W. (1986). International arbitrage pricing theory: An empirical investigation. *Journal of Finance, 41*(2), 313–329.

Chen, Z., & Epstein, L. (2002). Ambiguity, risk, and asset returns in continuous time. *Econometrica, 70*(4), 1403–1443.

Chen, S., Cheng-Ho, H., & Jordan, B. (1997). Real estate and the arbitrage pricing theory: Macro-variables vs derived factors. *AREUEA Journal of Real Estate Economics, 25*, 505–523.

Chiarella, C., He, X., & Zwinkels, R. (2014). Heterogeneous expectations in asset pricing: Empirical evidence from the S&P-500. *Journal of Economic Behavior and Organization, 105*, 1–16.

Cho, S. (2009). Household wealth accumulation and portfolio choices in Korea. *Journal of Housing Economics, 19*(1), 13–25.

Cho, S. (2013). New return anomalies and new-Keynesian ICAPM. *International Review of Financial Analysis, 29*, 87–106.

Chorus, C. (2012). Regret theory-based route choices and traffic equilibria. *Transportmetrica, 8*(4), 291–305.

Clarke, R., Krase, S., & Statman, M. (1994). Tracking errors, regret, and tactical asset allocation. *Journal of Portfolio Management, 20*(3), 16–24.

Cochrane, J. (1991). Production-based asset pricing and the link between stock returns and economic fluctuations. *The Journal of Finance, 46*, 209–237.

Cochrane, J. (1996). A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy, 104*, 572–621.

Coricelli, G., Dolan, R. J., & Sirigu, A. (2007). Brain, emotion and decision making: The paradigmatic example of regret. *Trends in Cognitive Sciences, 11*(6), 258–265.

Corrado, C., Hulten, C., & Sichel, D. (2009). Intangible capital and US economic growth. *Review of Income and Wealth, 55*, 661–685.

Cox, J., Ingersoll, J., & Ross, S. (1985). An intertemporal general equilibrium model of asset prices. *Econometrica, 53*, 385–408.

Crossley, F., & Low, H. W. (2011). Is the elasticity of intertemporal substitution constant? *Journal of the European Economic Association, 9*(1), 87–105.

Dayala, R. (2012). The capital asset pricing model, a fundamental critique. *Business Valuation Review, 31*(1), 23–34.

De Giorgi, E., Hens, T., & Mayer, J. (2007). Computational aspects of prospect theory with asset pricing applications. *Computational Economics, 29*(3–4), 267–281.

DeKay, M. L. (2009). Physicians' anticipated regret and diagnostic testing: Comment on Hozo & Djulbegovic (2008). *Medical Decision Making, 29*(3), 317–319.

De Roon, F., & Szymanowska, M. (2012). Asset pricing restrictions on predictability: Frictions matter. *Management Science, 58*(10), 1916–1932.

Detemple, J., & Murthy, S. (1997). Equilibrium asset prices and no-arbitrage with portfolio constraints. *Review of Financial Studies, 10*, 1133–1174.

Dittmar, R., Palomino, F., & Yang, W. (2016). Leisure preferences, long-run risks, and human capital returns. *Review of Asset Pricing Studies, 6*(1), 90–96.

Dodonova, A., & Khoroshilov, Y. (2005). (Working paper). Telfer School of Management, University of Ottawa, Canada.

Dodonova, A. (2009). *Applications of regret theory to asset pricing* (Working paper). Henry B. Tippie College of Business, University of Iowa, Iowa.

Dodonova, A., & Khoroshilov, Y. (2005). *Applications of regret theory to asset pricing* (Working paper). Ottawa: Telfer School of Management, University of Ottawa.

Donihue, M. & Avramenko, A. (2007, March). *Decomposing consumer wealth effects: Evidence on the role of real estate assets following the wealth cycle of 1990–2002* (Working paper #06-15). Boston: Federal Reserve Bank of Boston.

Duffie, D., & Strulovici, B. (2012). Capital mobility and asset pricing. *Econometrica, 80*(6), 2469–2509.

Ehrhardt, M. C. (1987, Summer). Arbitrage pricing models: The sufficient number of factors and equilibrium conditions. *Journal of Financial Research, 10*, 111–120.

El-Attar, M., & Poschke, M. (2011). Trust and the choice between housing and financial assets: Evidence from Spanish households. *Review of Finance,15*(4), 727–756.

Elias, C. (2016). Asset pricing with expectation shocks. *Journal of Economic Dynamics and Control, 65*, 68–82.

Epstein, L., & Zin, S. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns I: A theoretical framework. *Econometrica, 57*, 937–969.

Fama, E. (1998). Determining the number of priced state variables in the ICAPM. *Journal of Financial & Quantitative Analysis, 33*, 217–231.

Fama, E., & French, K. R. (1996). Multifactor explanations of asset pricing anomalies. *Journal of Finance, 51*(1), 55–84. doi:10.1111/j.1540-6261.1996.tb05202.x.

Ferson, E., Sarkissian, S., & Simin, T. (2003). Spurious regressions in financial economics? *Journal of Finance, 58*, 1393–1414.

Flam, S. (2010). Portfolio management without probabilities or statistics. *Annals of Finance, 6*(3), 357–368.

Foster, D. P., & Vohra, R. (1999). Regret in the on-line decision problem. *Games and Economic Behavior, 29*, 7–35.

Frehen, R. G. P., Hoevenaars, M., Palm, F., & Schotman, P. (2008). Regret aversion and annuity risk in defined contribution pension plans. *Insurance: Mathematics and Economics, 42*(3), 1050–1061.

Friedman, M. (1957). *A theory of the consumption function*. Princeton, NJ, USA.

Garcia, R., Renault, E., & Semenov, A. (2006). Disentangling risk aversion and intertemporal substitution through a reference level. *Finance Research Letters, 3*(3), 181–193.

Garnier, J., Nishimura, K., & Venditti, A. (2007). Intertemporal substitution in consumption, labor supply elasticity and sunspot fluctuations in continuous-time models. *International Journal of Economic Theory, 3*(4), 235–259.

Gay, G., & Jung, D. (1999). A further look at transaction costs, short sale restrictions and futures market efficiency: The case of Korea stock index. *Journal of Futures Markets, 19*(2), 153–172.

Gelber, A., & Mitchell, J. (2012). Taxes and time allocation: Evidence from single women and men. *Review of Economic Studies, 79*(3), 863–897.

Gerard, B., & Wu, G. (2006). How important is intertemporal risk for asset allocation? *Journal of Business, 79*(4), 2203–2210.

Gharghori, P., Chan, H., & Faff, R. (2007). Are the Fama-French factors proxying default risk? *Australian Journal of Management, 32*, 223–249.

Ghosh, C. (1993). A regret-theoretic explanation of corporate dividend policy. *Journal of Business Finance & Accounting, 20*(4), 559–573.

Giuliano, P., & Turnovsky, S. (2003). Intertemporal substitution, risk aversion, and economic performance in a stochastically growing open economy. *Journal of International Money and Finance, 22*(4), 529–556.

Gollier, C., & Salanié, B. (2006). *Individual decisions under risk, risk sharing and asset prices with regret* (Working paper). University of Toulouse, France.

Gomes, J. F., Yaron, A., & Zhang, L. (2006). Asset pricing implications of firms' financing constraints. *Review of Financial Studies, 19*, 1321–1356.

Gonzalez, M., Jareno, F., & Skinner, F. (2016, March 18). Interest and inflation risk: Investor behavior. *Frontiers In Psychology*. https://doi.org/10.3389/fpsyg.2016.00390; http://journal.frontiersin.org/article/10.3389/fpsyg.2016.00390/full

Gorton, G., He, P., & Huang, L. (2014). Agency-based asset pricing. *Journal of Economic Theory, 149*, 311–349.

Goval, A., & Welch, I. (2008). A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies, 21*(4), 1455–1508.

Green, R., & Hollifield, B. (1992). When will mean-variance portfolios be well diversified? *Journal of Finance, 47*, 1785–1809.

Gronau, R. (1977). Leisure, home production and work—The theory of the allocation of time revisited. *Journal of Political Economy, 85*(6), 1099–1123.

Gruber, J. (2013). A tax-based estimate of the elasticity of intertemporal substitution. *Quarterly Journal of Finance*, 03, 1350001 (2013) [20 pages] doi: http://dx.doi.org/10.1142/S2010139213500018.

Guo, H. (2004). A rational pricing explanation for the failure of the CAPM. *Federal Reserve Bank of St. Louis, 86*(3), 23–33.

Guvenen, F. (2006). Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective. *Journal of Monetary Economics, 53*, 1451–1472.

Hall, R. E. (1978). Stochastic implications of the life cycle-permanent income hypothesis: Theory and evidence. *Journal of Political Economy, 91*(6), 249–265.

Han, B., & Yang, L. (2013). Social networks, information acquisition, and asset prices. *Management Science, 59*(6), 1444–1457.

Hansen, S. (2015). Cross-sectional asset pricing with heterogeneous preferences and beliefs. *Journal of Economic Dynamics and Control, 58*, 125–151.

Havranek, T. (2015). Measuring intertemporal substitution: The importance of method choices and selective reporting. *Journal of the European Economic Association, 13*(6), 1180–1204.

Havranek, T., Horvath, R., Irsova, Z., & Rusnak, M. (2015). Cross-country heterogeneity in intertemporal substitution. *Journal of International Economics, 96*(1), 100–118.

Hayashi, T. (2008). Regret aversion and opportunity dependence. *Journal of Economic Theory, 139*(1), 242–268.

Hirshleifer, D. (2001). Investor psychology and asset pricing. *Journal of Finance, 56*(4), 1533–1598.

Hodrick, R., & Zhang, X. (2001). Evaluating the specification errors of asset pricing models. *Journal of Financial Economics, 62*, 327–376.

Hong, H., & Stein, J. C. (2003). Differences of opinion, short-sales constraints, and market crashes. *The Review of Financial Studies, 16*, 487–525.

Huang, J., Tzeng, G., & Liu, H. (2009). A revised VIKOR model for multiple criteria decision making – The perspective of regret theory. *Communications in Computer and Information Science, 35*, 761–768.

Hugonnier, J. (2012). Rational asset pricing bubbles and portfolio constraints. *Journal of Economic Theory, 147*(6), 2260–2302.

Hung, M., & Wang, J. (2005). Asset prices under prospect theory and habit formation. *Review of Pacific Basin Financial Markets & Policy, 8*(1), 1–29.

Iacoviello, M., & Pavan, M. (2009, November). *Housing and debt over the life cycle and over the business cycle* (Working paper #09-12). Boston: Federal Reserve Bank of Boston.

Jaganathan, R., & Wang, Z. (2002). Empirical evaluation of asset pricing models: A comparison of the SDF and Beta method. *Journal of Finance, 57*, 2337–2367.

Jamison, J., & Wegener, J. (2009, November). *Multiple selves in intertemporal choice* (Working paper #09-17). Boston: Federal Reserve Bank of Boston.

Jermann, U. (2005, March). *The equity premium implied by production* (Working paper). University of Pennsylvania, Penssylvannia, USA.

Jiliang, S. (2012, August 18–21). *Behavioral asset pricing model based on regret theory*. In *2012 Fifth International Conference on Business Intelligence and Financial Engineering (BIFE)*, Washington, DC: IEEE Computer Society.

Joyce, J., & Vogel, R. (1970). The uncertainty in risk: Is variance unambiguous? *Journal of Finance, 25*(1), 127–134.

Kapoor, M., & Ravi, S. (2016). Elasticity of intertemporal substitution in consumption in the presence of inertia: Empirical evidence from a natural experiment. *Management Science*. In press.

Kim, Y. H. (2003). Intertemporal production and asset pricing: A duality approach. *Oxford Economic Papers, 55*, 344–379.

Kim, D. H., & Lee, C. I. (2007). On-the-job human capital accumulation in a real business cycle model: Implications for intertemporal substitution elasticity and labor hoarding. *Review of Economic Dynamics, 10*(3), 494–518.

Korniotis, G., & Kumar, A. (2011). Do behavioral biases adversely affect the macro-economy? *Review of Financial Studies, 24*(5), 1513–1559.

Kumar, V., & Ziemba, W. (1993). The effect of errors in means, variances, and covariances on optimal portfolio choice. *Journal of Portfolio Management, 19*(2), 6–11.

Kwan, Y., Leung, C., & Dong, J. (2015). Comparing consumption-based asset pricing models: The case of an Asian city. *Journal of Housing Economics, 28*, 18–41.

Laciana, C. E., & Weber, E. U. (2008). Correcting expected utility for comparisons between alternative outcomes: A unified parameterization of regret and disappointment. *Journal of Risk & Uncertainty, 36*, 1–17.

Larrick, R., & Boles, T. L. (1995). Avoiding regret in decisions with feedback: A negotiation example. *Organizational Behavior and Human Decision Processes, 63*(1), 87–97.

Lee, C. I. (2008). On-the-job human capital investment and inter-temporal substitution: New evidence on intertemporal substitution elasticity. *Journal of Economic Dynamics and Control, 32*(10), 3350–3375.

Lee, J., & Phillips, P. (2016). Asset pricing with financial bubble risk. *Journal of Empirical Finance, 38*(B), 590–622.

Lettau, M., & Ludvigson, S. C. (2001). Resurrecting the (C)CAPM: A cross-sectional test when risk Premia are time-varying. *Journal of Political Economy, 109*, 1238–1287.

Lewellen, J., & Nagel, S. (2006). The conditional CAPM does not explain asset-pricing anomalies. *Journal of Financial Economics, 82*, 289–314.

Lewellen, J., Nagel, S., & Shanken, J. (2012). A skeptical appraisal of asset pricing tests. *Journal of Financial Economics, 96*, 175–194.

Lewis, K. K. (1991). Should the holding period matter for the intertemporal consumption-based CAPM? *Journal of Monetary Economics, 28*(3), 365–389.

Lia, Y., & Yang, L. (2013). Prospect theory, the disposition effect, and asset prices. *Journal of Financial Economics, 107*(3), 715–739.

Liew, J., & Vassalou, M. (2000). Can book-to-market, size and momentum Be risk factors that predict economic growth? *Journal of Financial Economics, 57*, 221–245.

Lindner, F. (2015). Does saving increase the supply of credit? A critique of the loanable funds theory. *World Economic Review, 4*, 1–26.

Llewellen, J., Nagel, S. & Shanken, J. (2007). *A skeptical appraisal of asset-pricing tests* (Working paper). Cambridge, MA: National Bureau of Economic Research.

Loomes, G., & Sugden, R. (1982). Regret theory: An alternative theory of rational choice under uncertainty. *The Economic Journal, 92*(368), 805–824.

Lybbert, T., & McPeak, J. (2012). Risk and intertemporal substitution: Livestock portfolios and off-take among Kenyan pastoralists. *Journal of Development Economics, 97*(2), 415–426.

Magni, C. A. (2008). Economic profit, NPV, and CAPM: Biases and violations of Modigliani and Miller's proposition I. *The ICFAI Journal of Applied Finance, 14*(10), 59–72.

Magni, C. (2009). Correct or incorrect application of CAPM? Correct or incorrect decisions with CAPM? *European Journal of Operational Research, 192*(2), 549–560.

Majumder, D. (2014). Asset pricing for inefficient markets: Evidence from China and India. *The Quarterly Review of Economics and Finance, 54*(2), 282–291.

Mankiw, N., Rotemberg, J., & Summers, L. (1985). Intertemporal substitutions in macroeconomics. *Quarterly Journal of Economics, 100*, 225–251.

Mannor, S., & Shimkin, N. (2008). Regret minimization in repeated matrix games with variable stage duration. *Games and Economic Behavior, 63*, 227–258.

Mannor, S., Stoltz, G., & Perchet, V. (2011). Robust approachability and regret minimization in games with partial monitoring. *Journal of Machine Learning Research: Workshop & Conference Proceedings, 19*, 515–536.

Mar, J., Bird, R., Casaveccia, L., & Yeung, D. (2009). Fundamental indexation: An Australian investigation. *Australian Journal of Management, 34*, 1–20.

Maslov, D. & Rytchkov, O. (2013). *Robustness and monotonicity of asset pricing anomalies* (Working paper). University of Texas at Austin, Texas, USA. http://faculty.mccombs.utexas.edu/~maslov/paper2.pdf

Meghir, C., & Weber, G. (1996). Intertemporal non-separability or borrowing restrictions? A disaggregate analysis using a U.S. consumption panel. *Econometrica, 64*(5), 1151–1181.

Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica, 41*, 867–887.

Miao, J., & Wang, N. (2007). Investment, consumption and hedging under incomplete markets. *Journal of Financial Economics, 86*, 608–642.

Michenaud, S., & Solnik, B. (2008). Applying regret theory to investment choices: Currency hedging decisions. *Journal of International Money and Finance, 27*(4), 677–694.

Moskowitz, T. (2003). An analysis of covariance risk and pricing anomalies. *Review of Financial Studies, 16*, 417–457.

Muermann, A., Mitchell, O. S., & Volkman, J. M. (2006). Regret, portfolio choice, and guarantees in defined contribution schemes. *Insurance: Mathematics and Economics, 39*(2), 219–229.

Myers, S., & Turnbull, S. (1977). Capital budgeting and the capital asset pricing model: Good news and bad news. *Journal of Finance, 32*(2), 321–333.

Naim, M. (2006). The impact of the net present value on the assessment of the dynamic performance of e-commerce enabled supply chains. *International Journal of Production Economics, 104*, 382–393.

Narayan, P., Phan, D., et al. (2016). Price discovery and asset pricing. *Pacific-Basin Finance Journal, 40*(A), 224–235.

Nasiry, J., & Popescu, I. (2009). *The price of consumer regret* (Working paper). INSEAD. http://web.uvic.ca/econ/research/seminars/Nasiry.pdf

Neely, C., Roy, A., & Whiteman, C. (1999). *Risk aversion versus intertemporal substitution: Identification failure in the intertemporal consumption CAPM* (Working paper). Federal Reserve Bank of St. Louis, USA.

Nelson, D., & Wu, Y. (1998). Rethinking deviations from uncovered interest parity: The role of covariance risk and noise. *Economic Journal, 108*, 1686–1706.

Nwogugu, M. (2005a). Towards multifactor models of decision making and risk: Critique of Prospect Theory and related approaches, part one. *Journal of Risk Finance, 6*(2), 150–162.

Nwogugu, M. (2005b). Towards multifactor models of decision making and risk: Critique of prospect theory and related approaches, part two. *Journal of Risk Finance, 6*(2), 163–173.

Nwogugu, M. (2005c). Towards multifactor models of decision making and risk: Critique of Prospect Theory and related approaches, part three. *Journal of Risk Finance, 6*(3), 267–276.

Nwogugu, M. (2006). A further critique of cumulative Prospect Theory and related approaches. *Applied Mathematics and Computation, 179*(2), 451–465.

Nwogugu, M. (2006a). Regret minimization, willingness-to-accept-losses and framing. *Applied Mathematics & Computation, 179*(2), 440–450.

Nwogugu, M. (2006b). A further critique of cumulative prospect theory and related approaches. *Applied Mathematics & Computation, 179*(2), 451–465.

Nwogugu, M. (2007). *Correlation, covariance, variance and semi-variance are irrelevant in risk analysis, portfolio management and mechanics* (Working paper). Available at www.ssrn.com

Nwogugu, M. (2012). *Risk in the global real estate market*. Hoboken: John Wiley & Sons.

Okubo, M. (2008). Intertemporal substitution and nonhomothetic preferences. *Economics Letters, 98*(1), 41–47.

Ormos, M., & Timotity, D. (2016). Generalized asset pricing: Expected downside risk-based equilibrium modeling. *Economic Modelling, 52*(B), 967–980.

Ortalo-Magné, F., & Rady, S. (2002). Tenure choice and the riskiness of non-housing consumption. *Journal of Housing Economics, 11*(3), 266–279.
Ou-Yang, H. (2005). An equilibrium model of asset pricing and moral hazard. *Review of Financial Studies, 18*, 1253–1303.
Ozsoylev, H. N., & Walden, J. (2011). Asset pricing in large information networks. *Journal of Economic Theory, 146*(6), 2252–2280.
Palacios, M. (2015). Human capital as an asset class: Implications from a general equilibrium model. *Review of Financial Studies, 28*, 978–1023.
Parker, J., & Julliard, C. (2005). Consumption risk and the cross section of expected returns. *Journal of Political Economy, 113*, 185–222.
Peng, Y., & Shawky, H. (1999). Stochastic discount rates, productivity shocks and capital asset pricing. *Review of Quantitative Finance and Accounting, 12*(1), 21–34.
Perchet, V. (2009). Calibration and internal no-regret with random signals. In *Algorithmic learning theory* (Lecture notes in computer science, Vol. 5809, pp. 68–82). Berlin: Springer.
Perchet, V. (2011). No-regret with partial monitoring: Calibration-based optimal algorithms. *Journal of Machine Learning Research: Workshop & Conference Proceedings, 12*, 1893–1921.
Perchet, V. (2014). Approachability, regret and calibration: Implications and equivalences. *Journal of Dynamics & Games, 2*, 181–254.
Petkova, R., & Zhang, L. (2005). Is value riskier than growth? *Journal of Financial Economics, 78*, 187–202.
Prono, T. (2007, June). *GARCH-based identification of triangular systems with an application to the CAPM: Still living with the roll critique* (Working papers #07-1). Boston: Federal Reserve Bank of Boston. http://www.bos.frb.org/economic/wp/wp2007/wp0701.pdf
Prono, T. (2009, June). *Market proxies, correlation and relative mean-variance efficiency: Still living with the roll critique* (Working paper # QAU09-3). Boston: Federal Reserve Bank of Boston.
Prono, T. (2015). Market proxies as factors in linear asset pricing models: Still living with the roll critique. *Journal of Empirical Finance, 31*, 36–53.
Rakhlin, A., Sridharan, K., & Tewari, A. (2011). Online learning: Beyond regret. *Journal of Machine Learning Research: Workshop & Conference Proceedings, 19*, 559–594.
Ramey, V. A., & Francis, N. (2009). A century of work and leisure. *American Economic Journal: Macroeconomics, 1*, 189–224.

Ramos, G., Daamen, W., & Hoogendoorn, S. (2014). A state-of-the-art review: Developments in utility theory, prospect theory and regret theory to investigate travellers' behaviour in situations involving travel time uncertainty. *Transportation Review, 34*(1), 46–67.
Ray, D., & Robson, A. (2012). Status, intertemporal choice, and risk-taking. *Econometrica, 80*(4), 1505–1531.
Roll, R. (1977). A critique of the asset pricing theory's tests. *Journal of Financial Economics, 4*, 129–176.
Sagi, A., & Friedland, N. (2007). The cost of richness: The effect of the size and diversity of decision sets on post-decision regret. *Journal of Personality and Social Psychology, 93*(4), 515–524.
Salinas, G. (2009). *The international brand valuation manual* (1st ed.). UK: John Wiley & Sons, Ltd.
Saltari, E., & Ticchi, D. (2007). Risk aversion, intertemporal substitution, and the aggregate investment–uncertainty relationship. *Journal of Monetary Economics, 54*(3), 622–648.
Sanfey, A., Rilling, J., Aronson, J., Nystrom, L., & Cohen, J. (2003, June). The neural basis of economic decision-making in the Ultimatum game. *Science, 300*, 1755–1756.
Seckin, A. (2001). Consumption–leisure choice with habit formation. *Economics Letters, 70*, 115–120.
Shanken, J. (1990). Intertemporal asset pricing: An empirical investigation. *Journal of Econometrics, 45*, 99–120.
Sheng, J., Wang, J. & Yang, J. (2014). Regret theory and equilibrium asset prices. *Mathematical Problems in Engineering, 2014*(2014), Article ID 912652. http://dx.doi.org/10.1155/2014/912652
Slaughter, M. (2013). *The contributions of intangible property to the U.S. economy*. http://www.tiecoalition.com/wp-content/uploads/2013/06/IP-White-Paper-Final-Draft-June-2013.pdf
Solnik, B. (2006). *Equity home bias and regret: An international equilibrium model* (Working paper). HEC Paris.
Solnik, B., & Zuo, L. (2012). A global equilibrium asset pricing model with home preference. *Management Science, 58*(2), 273–292.
Stern, D. (1997). Limits to substitution and irreversibility in production and consumption: A neoclassical interpretation of ecological economics. *Ecological Economics, 21*(3), 197–215.
Stoltz, G., & Lugosi, G. (2005). Internal regret in on-line portfolio selection. *Machine Learning, 59*, 125–159.

Taleb, N. (2009). Finiteness of variance is irrelevant in the practice of quantitative finance. *Complexity, 14*(3), 66–76.

Tong, J., Hu, J., & Hu, J. (2017). Computing equilibrium prices for a capital asset pricing model with heterogeneous beliefs and margin-requirement constraints. *European Journal of Operational Research, 256*(1), 24–34.

Tsalatsanis, A., Hozo, I., Vickers, A., & Djulbegovic, B. (2010). A regret theory approach to decision curve analysis: A novel method for eliciting decision makers' preferences and decision-making. *BMC Medical Informatics and Decision Making, 10*, 51–61.

Vassalou, M. (2003). News related to future GDP growth as a risk factor in equity returns. *Journal of Financial Economics, 68*, 47–73.

Vassalou, M. & Apedjinou, K. (2003). *Corporate innovation and its effects on equity returns* (Working paper). New York: Columbia University.

Viossat, Y., & Zapechelnyuk, A. (2013). No-regret dynamics and fictitious play. *Journal of Economic Theory, 148*, 825–842.

Wallenius, J. (2011). Human capital accumulation and the intertemporal Elasticity of substitution of labor: How large is the bias? *Review of Economic Dynamics, 14*(4), 577–591.

Wang, N. (2005). Precautionary saving and partially observed income. *Journal of Monetary Economics, 51*, 1645–1681.

Wang, X., Triantaphyllou, E., & Kujawski, E. (2008). Communication On The Paper "A Reference-Dependent Regret Model For Deterministic Tradeoff Studies". *Systems Engineering, 11*(4), 360–364.

Wang, N. (2009). Optimal consumption and asset allocation with unknown income growth. *Journal of Monetary Economics, 56*(4), 524–534.

Wong, J., & Wong, N. (2001). The investment opportunity set and acquired goodwill. *Contemporary Accounting Research, 18*, 173–196.

Wyatt, A. (2005). Accounting recognition of intangible assets: Theory and evidence on economic determinants. *The Accounting Review, 80*, 967–1003.

Yang, F. (2009). Consumption over the life cycle: How different is housing? *Review of Economic Dynamics, 12*(3), 423–443.

Yang, C., & Cai, C. (2014). Higher order expectations in sentiment asset pricing model. *Economic Modelling, 39*, 95–100.

Yang, C., & Li, J. (2013). Investor sentiment, information and asset pricing model. *Economic Modelling, 35*, 436–442.

Yogo, M. (2008). Asset prices under habit formation and reference-dependent preferences. *Journal of Business and Economic Statistics, 26*, 131–143.

Zhang, S., Zhu, J., Liu, X., & Chen, Y. (2016). Regret theory-based group decision-making with multidimensional preference and incomplete weight information. *Information Fusion, 31C*, 1–13.

4

The Descartes' Sign Rule, Sturm's Theorem, Vincent's Theorem and the Fourier-Budan Theorem Are Wrong

This chapter shows that the *Descartes' Sign Rule* (as interpreted by most academicians—such as Osborne (2010)), the *Fourier-Boudan Theorem*, Sturm's Theorem and Vincent's Theorem are wrong. Even if the Descartes Sign Rule, the Fourier-Boudan Theorem and Sturm's Theorem are right, they are used to find the numbers of only real roots; whereas the NPV-IRR TVM equation (and related models) often produces both real and complex roots, all of which matter (see comments about multiple-IRRs and the usefulness of complex rates-of-return in Hazen (2003), Pierru (2010), and Ben-Horin & Kroll (2012)). These issues are also applicable in non-linear analysis, evolutionary computation, and pattern-analysis—given the discussions in Yannacopoulos et al. (1996); Campos-Canton et al. (2015); Zheng et al. (2010); and Boyer and Goh (2007).

4.1 Existing Literature

The literature on biases and anomalies in the Descartes' Sign Rule, the Fourier-Boudan Theorem, Sturm's Theorem and Vincent's Theorem is scant (partly because of their widespread acceptance). Most of the existing

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research documents that the maximum number of number roots of the NPV formula does not exceed the number of time periods (this conclusion was also mentioned by Descartes more than 300 years ago, but was later supposedly proved by another). However, there are still substantial gaps in the literature such as tests of these theorems and the Fundamental Theorem of Algebra (FTA).

Anderson et al. (1998), Gauss (1828), Grabiner (1999), Henrici (1988), Itenberg and Roy (1996), and Levin (Nov. 2002), analyzed the Descartes' Sign Rule and related theories. On the Fourier-Boudan Theorem, see Akritas (1982), Panton (1999), and Hewitt (2012). On Vincent's Theorem, see Akritas (2008). On Sturm's Theorem, see Panton & Verdini (1984), Bernhard (1967); Hewitt (2012); and Petersen (1991). On other methods for solving roots of polynomials, see Akritas (2009); Farahmand & Sambandham (2003); Farahmand & Sambandham (2009); Farahmand & Stretch (2008); Farahmand (2008); Farahmand, Grigorash & McGuinness (2008); Farahmand (2007); Farahmand & Nezakati (2006); Farahmand & Jahangiri (2005); Farahmand & Nezakati (2005a); and Farahmand & Nezakati (2005b). Sitton et al. (2003) reviewed various "modern" methods for factoring higher order polynomials—however, most of these methods are based on the FTA and Descartes' Sign Rule. Simerska (2008) surveyed the literature on mathematical methods used in polynomials. Nwogugu (2010) showed that the Descartes Sign Rule and the Fourier-Budan Theorem are wrong.

The rest of this chapter discusses the inherent problems in the Descartes' Sign Rule, Sturm's Theorem, Vincent's Theorem and the Fourier-Budan Theorem.

4.2 The Descartes' Sign Rule and Fourier-Boudan Theorem Are Wrong

Theorem 4.1: *For all polynomials, and for all TVMs and associated series of time periods of any magnitude, and for all project cashflows that are real numbers or complex numbers, and for all discount rates, the Descartes' Sign Rule and the Fourier-Budan Theorem are wrong.*

Proof: Descartes' Sign Rule states that: 1) the number of changes of signs in the coefficients of a polynomial is equal to (or exceeds by an even integer) number of negative signs as a percentage of all signs and not the number the maximum number of real, positive roots; and 2) that if the signs are reversed on all the coefficients attached to odd powers, then the number of changes of signs in the coefficients (project cash flows) is equal to (or exceeds by an even integer) the maximum number of real, negative roots; 3) the number of positive roots differs from the number of sign changes by an even integer. See comments in. Anderson et al. (1998)[1], Grabiner (1999), Henrici (1988), and Itenberg and Roy (1996). The reality is that the number of sign changes has nothing to do with the number of real roots. Descartes' Rule is silent about the sequence of sign changes, but as applied by researchers to NPV-IRR, the sequence of sign changes matter—see the comments in Levin (2002), Gauss (1828), Anderson et al. (1998), and Basu et al. (2008). The important issue is the absolute number of negative signs as a percentage of all signs and not the number of sign changes—and the sequence of occurrence of the sign changes does not determine the number of positive roots. In the case of NPV, the nature of the roots is determined by the discount factor (which is $\{1+r\}$ and is completely independent of: the sequence and number of sign changes of the coefficients); both the sign and magnitude of the coefficients (project outcomes/cashflows); and the percentage of sign changes (number of sign changes as a percentage of total signs, which is completely independent of the sequence of sign changes).

The standard TVM polynomial and some associated derivatives are as follows (subject to the limitations on derivatives which were stated in Nwogugu (2012:330-335)):

[1] Anderson et al. (1998) states in part "As it has come to be stated, if a real polynomial is arranged in ascending or descending powers, its number of positive roots is no more than the number of sign variations in consecutive coefficients, and differs from this upper bound by an even integer. In 1807, Budan extended Descartes' Rule of Signs to determine an upper bound on the number of real roots in any given interval (p, q). It extends Descartes' Rule of Signs by substituting $x' = x-p$ and $x'' = x-q$ and counting the sign variations lost in the sequence of coefficients between the resulting transformed polynomials. This forms the upper bound; the actual number of roots differs by an even number. Budan's theorem, largely ignored, was restated by Fourier in 1820 and this better known formulation is usually referred to as the Fourier-Budan Theorem. In 1829 Descartes' Rule of Signs and related estimates were seemingly eclipsed by Sturm's Sign Sequence Theorem, which provides a precise measure for how many real roots lie in any interval. Nevertheless, due to the computational complexity of generating the Sturm sequences, less precise estimates like Descartes' remain useful to this day."

$$N = -a + \sum_{i=n} \left[\left\{ x_i * (1+p) \right\} / (1+r)^i \right] \tag{4.1}$$

$$N = -a + \sum \left\{ (x_i + x_i p) * (1+r)^{(n-i)} \right\} * (1+r)^{-n} \tag{4.2}$$

$$N = -a + \left\{ c * (1+r)^{-n} \right\} \tag{4.3}$$

$$\partial N / \partial B = \sum_{i=n} \left[x_i p * \left\{ (1+r)^{(n-i)} \right\} \right] * \left\{ (1+r)^{-n} \right\} \tag{4.4}$$

$$\partial^2 N / \partial B \partial r = \sum_{i=n} \left[x_i p * \left\{ -i / (1+r)^{(i-1)} \right\} \right] * \left\{ -n / (1+r)^{(n-1)} \right\} \tag{4.5}$$

$$\partial N / \partial c = \sum_{i=n} \left[x_i p * \left\{ -i (1+r)^{(i-1)} \right\} \right] * \left\{ (1+r)^{-n} \right\} \tag{4.6}$$

$$\text{Where } c = \sum \left\{ (x_i + x_i p) * (1+r)^{(n-i)} \right\} \tag{4.7}$$

and where:

a = initial investment;
B = the sum total of the periodic project benefits without regard to time value of money, x_i..... x_n ε B;
n = number of total time periods;
i = the specific time period (i.e., 1, 2, 3, 4,.........n; and i ε n);
r = the discount rate;
p = the periodic percentage change in each periodic project benefit, and p is equal to zero in the original TVM equation.

The IRR is simply the discount rate that equates the sum of the discounted project outcomes/cashflows to zero. The TVM equation $N = -a + \sum_{i=n} \left[\left\{ x_i * (1+p) \right\} / (1+r)^{-i} \right]$, is a special case of the polynomial equation $y = a_1 x^{-1} + a_2 x^{-2} + a_3 x^{-3} \ldots\ldots + a_n x^{-n}$ where $x = (1 + r)$; and r is the discount rate in the TVM equation. A combined sign is the net sign derived by multiplying the sign of the coefficient (x_i, x_i, x_i,.......x_n) and the sign of the discount rate (r). Let s_c = number of negative signs of only the coefficients. s_r = number of negative combined-signs (i.e., the combined signs of both the discounts rates (r can be less than zero) and the coefficients). ϕ = total number of combined-signs in the TVM equation. B is defined herein and above. For a given TVM equation, as $s_c \to \infty$, $s_c/\phi \to 1$, and $-\infty < B < +\infty$. But as $s_r \to \infty$, $s_r/\phi \to 1$, and $B \to -\infty$. When $r < -1$, $s_r < s_c$ *iff* the polynomial has an even number of degrees, and $s_r > s_c$ *iff* the polynomial has an odd number of degrees. When $-1 < r < 0$, $s_r = s_c$ if the polynomial has an even or odd number of degrees. When $0 < r < 1$, $s_r = s_c$ *iff* the polynomial has an odd or even number of degrees. When $r > 1$, $s_r > s_c$ *iff* the polynomial has an even number of degrees, and $s_r < s_c$ *iff* the polynomial has an odd number of degrees. As $r \to +\infty$, the sequence of combined signs becomes less relevant because $(1+r)$ becomes much larger while the numerator remains constant. Because of the differences in s_r and s_c when r changes, the TVM equation may not have a root, or may have that number of roots that is less than or greater than its degree, if $\partial N/\partial s_r < 0$ and/or $\partial s_r/\partial r > 0$. Thus, the Descartes Rule is wrong for all polynomials (with real or complex roots).

Let B_p^+ and B_m^- represent the sum of positive and negative coefficients respectively. When $-1 < r < 0$, dB/dr >0, if $s_r > s_c$ and $B_p^+ < B_m^-$, and dB/dr <0, if $s_r \leq s_c$, and $B_p^+ < B_m^-$. When $r < -1$, dB/dr >0, if $s_r > s_c$ and $B_p^+ < B_m^-$. When and dB/dr >0, if $s_r \leq s_c$.

Also, B reflects all sign changes. The standard TVM polynomial equation, Eq. 4.1b will have a root only when:

$-a + \sum_{i=n} \left[\{x_i * (1+p)\} / (1+r)^i \right] = -a + \left\{ c * (1+r)^{-n} \right\} = 0;$ and given that x_i ....x_n will remain constant in Eq. 4.1, a root will occur *iff* $y = -a + \sum_{i=n} \left[(1+r)^{-i} \right] = 0$.

$$y = -a + \left\{ c * (1+r)^{-n} \right\} = 0 \tag{4.8}$$

$$\left\{ c * (1+r)^{-n} \right\} = a \tag{4.9}$$

$$(1+r) =^n \sqrt{(c/a)} \tag{4.10}$$

$$r = \left\{ {}^{n}\sqrt{\left(c / a\right)} \right\} - 1; \text{ or} \tag{4.11}$$

$$r = \{ {}^{n}\sqrt{(} \sum_{i=n} \left[\left\{ x_i * \left(1 + p\right) \right\} / \left(1 + r\right)^{-i}] / a \right) \} - 1 \tag{4.12}$$

$$a\left(1 + r\right)^{n} = \sum_{i=n} \left[\left\{ x_i * \left(1 + p\right) \right\} / \left(1 + r\right)^{-i} \right] \tag{4.13}$$

The above solution of r shows that when r = −1 (ie. 100%), then $\{{}^{n}\sqrt{(c/a)}\} =$ 0; and $c/a = {}^{n}\sqrt{0}$; and c=0; and

$$\sum_{i=n} \left[\left\{ x_i * \left(1 + p\right) \right\} / \left(1 + r\right)^{-i} \right] = 0; \tag{4.14}$$

These equations show that:

1. The number of sign changes are partly captured in B, and are better captured in *c*.
2. The number of sign changes of the project cashflows also interacts with the sign of the discount rate (*r*), particularly when *r* is less than zero.
3. It is primarily the magnitude of the sign changes (magnitude of periodic project cashflows) and its interaction with the sign of the discount rate (*r*) (but only when *r* is less than zero) that determines the number of roots.

Each of Tables A1, A2, A3, A4 and B4 (in Chart-4A in Chapter-4) and Tables B6, B7, and B8 (in Chart 4B in Chapter-4) contain a TVM equation (in the same form as Eq. 4.1b) with NPVs calculated for various discount rates. The cashflows in the TVM equation in each such Table and also in Tables B9 & B10 (in Chart-4C in Chapter-4) are based in part on a cashflow series stated in Oehmke (2000). In each of these tables, the roots if any can be identified by: 1) pairs of consecutive IRRs that produce NPVs that have opposite signs and that are closest to zero (Intermediate Value Theorem); or 2) local maxima/minima. There seems to be a discrepancy between NPV calculated with the NPV function in MS-Excel, and the NPV that is calculated manually in MS-Excel. These discrepancies may be related to end-of-period and beginning-of-period cashflow conventions implicit in MS-Excel.

The polynomial equations in each of Tables B6, B7, and B8 (in Chart 4B) contain two or more sign changes but in each case and within the interval (of discount rates) shown, the number of sign changes exceeds the number of positive roots (in some cases by a number that is not an even integer - which is contrary to the Descartes Sign Rule). Similarly, the TVM equation in each of Tables A1, A2, A3 and A4 (in Chart 4A) contains four or more sign changes but in each TVM equation and within the interval (discount rates) shown, the difference between the number of sign changes and the number of positive roots is not an even integer. The TVM equation in Table-B9 (in Chart 4C) does not contain any sign change, but has at least one root. The simulations of these TVM equations prove are evidence that the Descartes' Sign Rule and the Fourier-Budan Theorem are Wrong. In Tables B1, B2, and B5 (in Chart-4A), there are some local maxima and minima in the NPV series where the curve of the TVM equation reverses direction (is equal to or near zero but does not cross or touch the x-axis); and in each such table, the number of sign changes differs from the number of positive real roots by an odd integer. There are also local maxima/minima in Tables B6, B7, and B8 (in Chart 4B).

Contrary to Descartes' Sign Rule (as interpreted by most academicians—such as Osborne (2010)) a negative root of a TVM equation does not imply the existence of a discount rate that is less than minus 100%. This is illustrated in the simulation in Table-A2 and Table-A1 (in Chart 4A), where the TVM equation has negative roots (local minima/maxima) where r is between −60% and −95%. Akritas (2008) noted that the Descartes' sign rule provides the exact number of positive roots only in two special cases - when the number of sign changes is zero or one (in which cases the number of positive roots are zero and one respectively). This contention is wrong as illustrated in: 1) Lines 12A and 12B in Tables 2-7 in Chart-5A where the TVM equation (Line 12A) has only one sign change but has two positive roots one of which is around r=100% and the other is when r is between 300% and 500% (local maxima/minima); 2) Lines 1A and 13A in Tables 2-7 in Chart-5A where each TVM equation has zero sign changes but may have a positive root when the discount rate is above r=1,000,000,000%. Also the Fourier-Boudan Theorem is wrong because as explained in Chapter-7 in this book and in Nwogugu (2012: 330-335), the inverse function rule and the power rule in traditional differentiation (calculus) are either wrong or may be applicable only for very small numbers around zero. ■

4.3 Sturm's Theorem and Vincent's Theorem Are Wrong

Theorem 4.2: The Sturm Theorem Is Wrong.

Proof: Sturm's theorem is explained in Akritas (1982), Panton and Verdini (1981), and Petersen (1991). Hewitt (2012) noted that the Sturm's Theorem is applicable only to polynomials that don't have multiple roots (also see: https://en.wikipedia.org/wiki/Sturm%27s_theorem). Sturm's Theorem in wrong because as noted in Nwogugu (2012: 330–335) and in Chap. 7 in this book, the *Inverse Function rule* and the *power rule* in traditional differentiation (calculus) are either wrong or may be applicable only to very small/minute numbers around zero (on the contrary, in business engineering and government, the NPV-IRR model is used with relatively very large numbers). The number of roots in a range for the polynomial $f(x) = x^4 + x^3 - x - 1$ is given by the Sturm Theorem as follows.

$$f_0(x) = x^4 + x^3 - x - 1 \tag{4.15}$$

$$f_1(x) = f'(x) = 4x^3 + 3x^2 - 1 \tag{4.16}$$

If f_0 is divided by p_1 (using polynomial long division), the remainder is $-3/16x^2 - 3/4x - 15/16$ which when multiplied by -1, is $f_2(x) = 3/16x^2 + 3/4x + 15/16$. If f_1 is divided by f_2, and the result is multiplied by -1, the result is $f_3(x) = -32x - 64$. If f_2 is divided by f_3, and the result is multiplied by −1, the result is $f_4(x) = -3/16$. The complete chain of Sturm Polynomials is as follows:

$$f_0(x) = x^4 + x^3 - x - 1 \tag{4.17}$$

$$f_1(x) = 4x^3 + 3x^2 - 1 \tag{4.18}$$

$$f_2(x) = 3/16x^2 + 3/4x + 15/16 \tag{4.19}$$

$$f_3(x) = -32x - 64 \tag{4.20}$$

$$f_4(x) = -3/16 \tag{4.21}$$

In order to find the number of roots between $-\infty$ and ∞, f_0, f_1, f_2, f_3 and f_4 are then evaluated at both $-\infty$ and ∞; and the results are three sign changes (with the sequence +−++−) for $-\infty$, and one sign change (with the sequence +++−−) for ∞ respectively. Thus, according to Sturm's Theorem, in the interval $(-\infty; \infty)$, the number of real roots of the original polynomial $(x^4 + x^3 - x - 1)$ is $3 - 1 = 2$ (two roots). However, under traditional root calculation methods, $x^4 + x^3 - x - 1$ can be factored into $(x^3 - 1)(x + 1) = 0$; and $(x^3 - 1) = 0$, and $(x + 1) = 0$. Thus, the original polynomial $(x^4 + x^3 - x - 1)$ has only one real root at $x = -1$; and Sturm's theorem is wrong. Alternatively, the original polynomial $(x^4 + x^3 - x - 1)$ can also be factored into $(x^2 - 1)(x_2 + x + 1)$; where $(x^2 - 1)$ appears to have two roots at −1 and 1, and $x^2 + x + 1$ has no real roots. But if $x = 1$ is substituted in $x^4 + x^3 - x - 1$, the result is −2, and thus $x^4 + x^3 - x - 1$ does not have any root at $x = 1$; and has only one real root at $x = -1$. This second factoring also confirms (as stated in Chap. 5), that traditional factoring is sometimes problematic. ■

Theorem 4.3: Vincent's Theorem Is Wrong.

Proof: Vincent's Theorem is explained in Akritas (2008). Vincent's Theorem was substantially based on the Descartes Sign Rule and the Fourier-Boudan Theorem both of which are wrong. Thus, Vincent's Theorem is also wrong. Also see Tables B6, B7 & B8 in Chart 4B. ■

Chart 4A

Table-A1: Eleven Terms

Project Outcome/Cashflow:

	0	1	2	3	4	5	6	7	8	9	10	11
	(815.00)	900.00	(100.00)	1,200.00	(1,200.00)	3,000.00	(15,000.00)	1,000.00	(10,000.00)	17,000.00	(200.00)	15,000.00

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

1	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
2	250000.00%	(0.3257)	2000.00%	(36.7736)	−300.00%	965.3613	0.00%	10,785.0000	95.00%	(233.1896)	190.00%	(170.7001)
3	246000.00%	(0.3310)	1960.00%	(37.4473)	−290.00%	1,136.5090	1.00%	9,127.4961	96.00%	(231.0381)	191.00%	(170.4544)
4	242000.00%	(0.3365)	1920.00%	(38.1461)	−280.00%	1,379.1128	2.00%	7,697.4529	97.00%	(228.9743)	192.00%	(170.2097)
5	238000.00%	(0.3421)	1880.00%	(38.8714)	−270.00%	1,738.5318	3.00%	6,463.7464	98.00%	(226.9944)	193.00%	(169.9657)
6	234000.00%	(0.3480)	1840.00%	(39.6246)	−260.00%	2,298.3493	4.00%	5,399.6293	99.00%	(225.0947)	194.00%	(169.7225)
7	230000.00%	(0.3540)	1800.00%	(40.4075)	−250.00%	3,221.1839	5.00%	4,482.0910	100.00%	(223.2715)	195.00%	(169.4801)
8	226000.00%	(0.3603)	1760.00%	(41.2217)	−240.00%	4,843.6572	6.00%	3,691.3163	101.00%	(221.5215)	196.00%	(169.2384)
9	222000.00%	(0.3668)	1720.00%	(42.0693)	−230.00%	7,914.0791	7.00%	3,010.2247	102.00%	(219.8413)	197.00%	(168.9973)
10	218000.00%	(0.3735)	1680.00%	(42.9523)	−220.00%	14,239.4098	8.00%	2,424.0793	103.00%	(218.2280)	198.00%	(168.7569)
11	214000.00%	(0.3805)	1640.00%	(43.8730)	−210.00%	28,626.6381	9.00%	1,920.1546	104.00%	(216.6783)	199.00%	(168.5172)
12	210000.00%	(0.3877)	1600.00%	(44.8337)	−200.00%	65,415.0000	10.00%	1,487.4523	105.00%	(215.1896)	200.00%	(168.2780)
13	206000.00%	(0.3952)	1560.00%	(45.8372)	−190.00%	173,659.5925	11.00%	1,116.4594	106.00%	(213.7590)	201.00%	(168.0393)
14	202000.00%	(0.4030)	1520.00%	(46.8863)	−180.00%	551,579.9179	12.00%	798.9417	107.00%	(212.3839)	202.00%	(167.8012)
15	198000.00%	(0.4112)	1480.00%	(47.9843)	−170.00%	2,183,877.3142	13.00%	527.7668	108.00%	(211.0619)	203.00%	(167.5635)
16	194000.00%	(0.4196)	1440.00%	(49.1344)	−160.00%	11,438,498.8438	14.00%	296.7525	109.00%	(209.7904)	204.00%	(167.3264)
17	190000.00%	(0.4285)	1400.00%	(50.3407)	−150.00%	86,809,230.0000	**15.00%**	100.5373	110.00%	(208.5673)	205.00%	(167.0897)
18	186000.00%	(0.4377)	1360.00%	(51.6071)	−140.00%	1.11E+09	**16.00%**	(65.5316)	111.00%	(207.3904)	206.00%	(166.8534)
19	182000.00%	(0.4473)	1320.00%	(52.9382)	−130.00%	3.18E+10	17.00%	(205.4938)	112.00%	(206.2575)	207.00%	(166.6176)
20	178000.00%	(0.4573)	1280.00%	(54.3392)	−120.00%	3.86E+12	18.00%	(322.8579)	113.00%	(205.1666)	208.00%	(166.3821)
21	174000.00%	(0.4678)	1240.00%	(55.8154)	−110.00%	1.52E+16	19.00%	(420.6729)	114.00%	(204.1159)	209.00%	(166.1471)
22	170000.00%	(0.4788)	1200.00%	(57.3732)	−100.00%	−8.15E+02	20.00%	(501.5890)	115.00%	(203.1035)	210.00%	(165.9124)
23	166000.00%	(0.4903)	1160.00%	(59.0193)	−90.00%	1.51E+16	21.00%	(567.9108)	116.00%	(202.1276)	211.00%	(165.6780)
24	162000.00%	(0.5024)	1120.00%	(60.7613)	−80.00%	3.80E+12	22.00%	(621.6423)	117.00%	(201.1867)	212.00%	(165.4440)
25	158000.00%	(0.5151)	1080.00%	(62.6077)	−70.00%	3.04E+10	23.00%	(664.5268)	118.00%	(200.2791)	213.00%	(165.2103)
26	154000.00%	(0.5285)	1040.00%	(64.5679)	−60.00%	1.01E+09	24.00%	(698.0809)	119.00%	(199.4032)	214.00%	(164.9768)
27	150000.00%	(0.5426)	1000.00%	(66.6526)	−50.00%	71,828,370.0000	25.00%	(723.6237)	120.00%	(198.5577)	215.00%	(164.7437)
28	146000.00%	(0.5574)	960.00%	(68.8736)	−40.00%	8,237,483.4478	26.00%	(742.3030)	121.00%	(197.7412)	216.00%	(164.5109)
29	142000.00%	(0.5731)	920.00%	(71.2443)	−30.00%	1,286,558.2075	27.00%	(755.1167)	122.00%	(196.9522)	217.00%	(164.2784)
30	138000.00%	(0.5897)	880.00%	(73.7799)	−20.00%	245,108.2019	28.00%	(762.9328)	123.00%	(196.1897)	218.00%	(164.0461)
31	134000.00%	(0.6073)	840.00%	(76.4975)	−10.00%	51,889.0144	29.00%	(766.5057)	124.00%	(195.4522)	219.00%	(163.8140)
32	130000.00%	(0.6259)	800.00%	(79.4165)	0.00%	10,785.0000	30.00%	(766.4909)	125.00%	(194.7388)	220.00%	(163.5822)
33	126000.00%	(0.6457)	760.00%	(82.5589)	10.00%	1,487.4523	31.00%	(763.4576)	126.00%	(194.0483)	221.00%	(163.3507)
34	122000.00%	(0.6669)	720.00%	(85.9498)	20.00%	(501.5890)	32.00%	(757.8998)	127.00%	(193.3796)	222.00%	(163.1194)
35	118000.00%	(0.6894)	680.00%	(89.6176)	30.00%	(766.4909)	33.00%	(750.2458)	128.00%	(192.7318)	223.00%	(162.8883)
36	114000.00%	(0.7136)	640.00%	(93.5945)	40.00%	(663.5754)	34.00%	(740.8667)	129.00%	(192.1038)	224.00%	(162.6575)
37	110000.00%	(0.7395)	600.00%	(97.9171)	50.00%	(521.4916)	35.00%	(730.0833)	130.00%	(191.4948)	225.00%	(162.4268)
38	106000.00%	(0.7673)	560.00%	(102.6262)	60.00%	(409.0098)	36.00%	(718.1732)	131.00%	(190.9038)	226.00%	(162.1964)
39	102000.00%	(0.7974)	520.00%	(107.7668)	70.00%	(330.9530)	37.00%	(705.3754)	132.00%	(190.3301)	227.00%	(161.9662)
40	98000.00%	(0.8298)	480.00%	(113.3876)	80.00%	(279.2189)	38.00%	(691.8961)	133.00%	(189.7729)	228.00%	(161.7361)
41	94000.00%	(0.8651)	440.00%	(119.5383)	90.00%	(245.4126)	39.00%	(677.9122)	134.00%	(189.2314)	229.00%	(161.5063)
42	90000.00%	(0.9034)	400.00%	(126.2648)	100.00%	(223.2715)	40.00%	(663.5754)	135.00%	(188.7048)	230.00%	(161.2767)
43	86000.00%	(0.9454)	360.00%	(133.6000)	110.00%	(208.5673)	41.00%	(649.0153)	136.00%	(188.1925)	231.00%	(161.0473)
44	82000.00%	(0.9914)	320.00%	(141.5467)	120.00%	(198.5577)	42.00%	(634.3423)	137.00%	(187.6938)	232.00%	(160.8181)
45	78000.00%	(1.0421)	280.00%	(150.0522)	130.00%	(191.4948)	43.00%	(619.6500)	138.00%	(187.2080)	233.00%	(160.5890)
46	74000.00%	(1.0982)	240.00%	(158.9910)	140.00%	(186.2731)	44.00%	(605.0174)	139.00%	(186.7347)	234.00%	(160.3602)
47	70000.00%	(1.1608)	200.00%	(168.2780)	150.00%	(182.1962)	45.00%	(590.5108)	140.00%	(186.2731)	235.00%	(160.1315)
48	66000.00%	(1.2309)	160.00%	(178.8266)	160.00%	(178.8266)	**46.00%**	(576.1855)	141.00%	(185.8227)	236.00%	(159.9031)
49	62000.00%	(1.3101)	120.00%	(198.5577)	170.00%	(175.8901)	**47.00%**	(562.0871)	142.00%	(185.3831)	237.00%	(159.6748)
50	58000.00%	(1.4001)	80.00%	(279.2189)	180.00%	(173.2163)	48.00%	(548.2529)	143.00%	(184.9537)	238.00%	(159.4467)

51	54000.00%	(1.5034)	40.00%	(663.5754)	190.00%	(170.7001)	49.00%	(534.7131)	144.00%	(184.5340)	239.00%	(159.2188)
52	50000.00%	(1.6232)	0.00%	10,785.0000	200.00%	(168.2780)	50.00%	(521.4916)	145.00%	(184.1236)	240.00%	(158.9910)
53	46000.00%	(1.7637)	−40.00%	8,237,483.4478	210.00%	(165.9124)	51.00%	(508.6069)	146.00%	(183.7221)	241.00%	(158.7635)
54	42000.00%	(1.9308)	−80.00%	3.7981E+12	220.00%	(163.5822)	52.00%	(496.0729)	147.00%	(183.3290)	242.00%	(158.5362)
55	38000.00%	(2.1329)	−120.00%	3.8590E+12	230.00%	(161.2767)	53.00%	(483.8995)	148.00%	(182.9439)	243.00%	(158.3090)
56	34000.00%	(2.3823)	−160.00%	11,438,498.8438	240.00%	(158.9910)	54.00%	(472.0931)	149.00%	(182.5664)	244.00%	(158.0820)
57	30000.00%	(2.6977)	−200.00%	65,415.0000	250.00%	(156.7242)	55.00%	(460.6574)	150.00%	(182.1962)	245.00%	(157.8553)
58	26000.00%	(3.1094)	−240.00%	4,843.6572	260.00%	(154.4773)	56.00%	(449.5934)	151.00%	(181.8330)	246.00%	(157.6287)
59	22000.00%	(3.6694)	−280.00%	1,379.1128	270.00%	(152.2525)	57.00%	(438.9002)	152.00%	(181.4764)	247.00%	(157.4023)
60	18000.00%	(4.4753)	−320.00%	744.6095	280.00%	(150.0522)	58.00%	(428.5749)	153.00%	(181.1261)	248.00%	(157.1761)
61	14000.00%	(5.7349)	−360.00%	520.7213	290.00%	(147.8792)	59.00%	(418.6132)	154.00%	(180.7817)	249.00%	(156.9501)
62	10000.00%	(7.9812)	−400.00%	407.0751	300.00%	(145.7358)	60.00%	(409.0098)	155.00%	(180.4431)	250.00%	(156.7242)
63	6000.00%	(13.1191)	−440.00%	336.8332	310.00%	(143.6244)	61.00%	(399.7580)	156.00%	(180.1099)	251.00%	(156.4986)
64	2000.00%	(36.7736)	−480.00%	288.3122	320.00%	(141.5467)	62.00%	(390.8507)	157.00%	(179.7819)	252.00%	(156.2732)
65	−2000.00%	45.4122	−520.00%	252.4350	330.00%	(139.5043)	63.00%	(382.2797)	158.00%	(179.4589)	253.00%	(156.0480)
66	−6000.00%	14.0727	−560.00%	224.6779	340.00%	(137.4985)	64.00%	(374.0367)	159.00%	(179.1405)	254.00%	(155.8230)
67	−10000.00%	8.3243	−600.00%	202.4975	350.00%	(135.5302)	65.00%	(366.1127)	160.00%	(178.8266)	255.00%	(155.5982)
68	−14000.00%	5.9099	−640.00%	184.3356	360.00%	(133.6000)	66.00%	(358.4987)	161.00%	(178.5169)	256.00%	(155.3736)
69	−18000.00%	4.5812	−680.00%	169.1761	370.00%	(131.7082)	67.00%	(351.1851)	162.00%	(178.2113)	257.00%	(155.1492)
70	−22000.00%	3.7402	−720.00%	156.3242	380.00%	(129.8551)	68.00%	(344.1627)	163.00%	(177.9095)	258.00%	(154.9250)
71	−26000.00%	3.1601	−760.00%	145.2867	390.00%	(128.0407)	69.00%	(337.4218)	164.00%	(177.6114)	259.00%	(154.7011)
72	−30000.00%	2.7358	−800.00%	135.7029	400.00%	(126.2648)	70.00%	(330.9530)	165.00%	(177.3167)	260.00%	(154.4773)
73	−34000.00%	2.4120	−840.00%	127.3024	410.00%	(124.5270)	71.00%	(324.7468)	166.00%	(177.0254)	261.00%	(154.2538)
74	−38000.00%	2.1567	−880.00%	119.8786	420.00%	(122.8270)	72.00%	(318.7939)	167.00%	(176.7372)	262.00%	(154.0305)
75	−42000.00%	1.9502	−920.00%	113.2703	430.00%	(121.1643)	73.00%	(313.0850)	168.00%	(176.4521)	263.00%	(153.8075)
76	−46000.00%	1.7799	−960.00%	107.3501	440.00%	(119.5383)	74.00%	(307.6111)	169.00%	(176.1697)	264.00%	(153.5846)
77	−50000.00%	1.6369	−1000.00%	102.0159	450.00%	(117.9483)	75.00%	(302.3633)	170.00%	(175.8901)	265.00%	(153.3620)
78	−54000.00%	1.5152	−1040.00%	97.1849	460.00%	(116.3937)	76.00%	(297.3328)	171.00%	(175.6130)	266.00%	(153.1396)
79	−58000.00%	1.4103	−1080.00%	92.7891	470.00%	(114.8737)	77.00%	(292.5113)	172.00%	(175.3384)	267.00%	(152.9175)
80	−62000.00%	1.3190	−1120.00%	88.7724	480.00%	(113.3876)	78.00%	(287.8905)	173.00%	(175.0662)	268.00%	(152.6956)
81	−66000.00%	1.2388	−1160.00%	85.0879	490.00%	(111.9347)	79.00%	(283.4623)	174.00%	(174.7961)	269.00%	(152.4739)
82	−70000.00%	1.1678	−1200.00%	81.6959	500.00%	(110.5141)	80.00%	(279.2189)	175.00%	(174.5281)	270.00%	(152.2525)
83	−74000.00%	1.1045	−1240.00%	78.5632	510.00%	(109.1251)	81.00%	(275.1529)	176.00%	(174.2621)	271.00%	(152.0313)
84	−78000.00%	1.0477	−1280.00%	75.6610	520.00%	(107.7668)	82.00%	(271.2569)	177.00%	(173.9981)	272.00%	(151.8104)
85	−2000.00%	0.9965	−1320.00%	72.9650	530.00%	(106.4386)	83.00%	(267.5238)	178.00%	(173.7358)	273.00%	(151.5897)
86	−86000.00%	0.9500	−1360.00%	70.4539	540.00%	(105.1396)	84.00%	(263.9469)	179.00%	(173.4752)	274.00%	(151.3693)
87	−90000.00%	0.9077	−1400.00%	68.1094	550.00%	(103.8690)	85.00%	(260.5196)	180.00%	(173.2163)	275.00%	(151.1491)
88	−94000.00%	0.8690	−1440.00%	65.9154	560.00%	(102.6262)	86.00%	(257.2355)	181.00%	(172.9589)	276.00%	(150.9292)
89	−98000.00%	0.8334	−1480.00%	63.8580	570.00%	(101.4103)	87.00%	(254.0885)	182.00%	(172.7030)	277.00%	(150.7096)
90	−102000.00%	0.8007	−1520.00%	61.9248	580.00%	(100.2206)	88.00%	(251.0727)	183.00%	(172.4484)	278.00%	(150.4902)
91	−106000.00%	0.7704	−1560.00%	60.1049	590.00%	(99.0564)	89.00%	(248.1826)	184.00%	(172.1952)	279.00%	(150.2711)
92	−110000.00%	0.7423	−1600.00%	58.3886	600.00%	(97.9171)	90.00%	(245.4126)	185.00%	(171.9432)	280.00%	(150.0522)
93	−114000.00%	0.7162	−1640.00%	56.7674	610.00%	(96.8019)	91.00%	(242.7576)	186.00%	(171.6924)	281.00%	(149.8336)
94	−118000.00%	0.6919	−1680.00%	55.2336	620.00%	(95.7102)	92.00%	(240.2125)	187.00%	(171.4428)	282.00%	(149.6153)
95	−122000.00%	0.6692	−1720.00%	53.7802	630.00%	(94.6413)	93.00%	(237.7725)	188.00%	(171.1942)	283.00%	(149.3973)
	−126000.00%	0.6479	−1760.00%	52.4013	640.00%	(93.5945)	94.00%	(235.4330)	189.00%	(170.9467)	284.00%	(149.1796)

(*continued*)

Chart 4A (continued)

Table-A2: Nine Terms

Project Cash Flow:

0	1	2	3	4	5	6	7	8	9
(815.00)	900.00	(100.00)	1,200.00	(1,200.00)	100.00	(10,000.00)	6,000.00	(7,000.00)	11,000.00

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.3257)	2000.00%	(36.7736)	−300.00%	885.0391	**0.00%**	85.0000	95.00%	(220.4047)	190.00%	(171.6540)
2	246000.00%	(0.3310)	1960.00%	(37.4474)	−290.00%	1,022.3388	**1.00%**	(144.6046)	96.00%	(218.9245)	191.00%	(171.4047)
3	242000.00%	(0.3365)	1920.00%	(38.1462)	−280.00%	1,212.3453	2.00%	(335.7804)	97.00%	(217.5020)	192.00%	(171.1559)
4	238000.00%	(0.3421)	1880.00%	(38.8714)	−270.00%	1,486.8244	3.00%	(493.9814)	98.00%	(216.1345)	193.00%	(170.9077)
5	234000.00%	(0.3480)	1840.00%	(39.6247)	−260.00%	1,902.9937	4.00%	(623.9071)	99.00%	(214.8196)	194.00%	(170.6600)
6	230000.00%	(0.3540)	1800.00%	(40.4075)	−250.00%	2,569.0340	5.00%	(729.6083)	100.00%	(213.5547)	195.00%	(170.4129)
7	226000.00%	(0.3603)	1760.00%	(41.2218)	−240.00%	3,700.8387	6.00%	(814.5780)	101.00%	(212.3376)	196.00%	(170.1662)
8	222000.00%	(0.3668)	1720.00%	(42.0694)	−230.00%	5,756.2668	7.00%	(881.8289)	102.00%	(211.1660)	197.00%	(169.9200)
9	218000.00%	(0.3735)	1680.00%	(42.9524)	−220.00%	9,775.9166	8.00%	(933.9600)	103.00%	(210.0380)	198.00%	(169.6742)
10	214000.00%	(0.3805)	1640.00%	(43.8731)	−210.00%	18,321.2963	9.00%	(973.2138)	104.00%	(208.9514)	199.00%	(169.4289)
11	210000.00%	(0.3877)	1600.00%	(44.8338)	−200.00%	38,315.0000	10.00%	(1,001.5250)	**105.00%**	(207.9044)	200.00%	(169.1840)
12	206000.00%	(0.3952)	1560.00%	(45.8373)	−190.00%	90,664.9837	11.00%	(1,020.5628)	**106.00%**	(206.8951)	201.00%	(168.9395)
13	202000.00%	(0.4030)	1520.00%	(46.8865)	−180.00%	247,639.6292	12.00%	(1,031.7669)	107.00%	(205.9219)	202.00%	(168.6953)
14	198000.00%	(0.4112)	1480.00%	(47.9844)	−170.00%	804,668.0093	13.00%	(1,036.3788)	108.00%	(204.9830)	203.00%	(168.4515)
15	194000.00%	(0.4196)	1440.00%	(49.1346)	−160.00%	3,259,407.1149	**14.00%**	(1,035.4687)	109.00%	(204.0768)	204.00%	(168.2081)
16	190000.00%	(0.4285)	1400.00%	(50.3409)	−150.00%	17,734,030.0000	**15.00%**	(1,029.9583)	110.00%	(203.2020)	205.00%	(167.9650)
17	186000.00%	(0.4377)	1360.00%	(51.6073)	−140.00%	147,063,546.2891	**16.00%**	(1,020.6412)	111.00%	(202.3569)	206.00%	(167.7223)
18	182000.00%	(0.4473)	1320.00%	(52.9385)	−130.00%	2,356,466,284.0430	17.00%	(1,008.2000)	112.00%	(201.5403)	207.00%	(167.4798)
19	178000.00%	(0.4573)	1280.00%	(54.3395)	−120.00%	1.2422E+11	18.00%	(993.2210)	113.00%	(200.7507)	208.00%	(167.2377)
20	174000.00%	(0.4678)	1240.00%	(55.8159)	**−110.00%**	1.1770E+14	19.00%	(976.2075)	114.00%	(199.9871)	209.00%	(166.9959)
21	170000.00%	(0.4788)	1200.00%	(57.3737)	**−100.00%**	−8.1500E+02	20.00%	(957.5903)	115.00%	(199.2482)	210.00%	(166.7544)
22	166000.00%	(0.4903)	1160.00%	(59.0199)	**−90.00%**	1.0350E+14	21.00%	(937.7384)	116.00%	(198.5327)	211.00%	(166.5131)
23	162000.00%	(0.5024)	1120.00%	(60.7621)	−80.00%	9.5311E+10	22.00%	(916.9661)	117.00%	(197.8398)	212.00%	(166.2721)
24	158000.00%	(0.5151)	1080.00%	(62.6086)	−70.00%	1,552,742,626.8615	23.00%	(895.5414)	118.00%	(197.1682)	213.00%	(166.0314)
25	154000.00%	(0.5285)	1040.00%	(64.5690)	−60.00%	81,209,178.3203	24.00%	(873.6915)	119.00%	(196.5171)	214.00%	(165.7910)
26	150000.00%	(0.5426)	1000.00%	(66.6539)	−50.00%	7,924,370.0000	25.00%	(851.6087)	120.00%	(195.8855)	215.00%	(165.5508)
27	146000.00%	(0.5574)	960.00%	(68.8753)	−40.00%	1,121,244.8860	26.00%	(829.4549)	121.00%	(195.2724)	216.00%	(165.3109)
28	142000.00%	(0.5731)	920.00%	(71.2464)	−30.00%	197,690.3962	27.00%	(807.3659)	122.00%	(194.6770)	217.00%	(165.0712)
29	138000.00%	(0.5897)	880.00%	(73.7826)	−20.00%	38,211.7254	28.00%	(785.4547)	123.00%	(194.0986)	218.00%	(164.8318)
30	134000.00%	(0.6073)	840.00%	(76.5009)	−10.00%	6,563.6244	29.00%	(763.8148)	124.00%	(193.5362)	219.00%	(164.5926)
31	130000.00%	(0.6259)	800.00%	(79.4208)	0.00%	85.0000	30.00%	(742.5231)	125.00%	(192.9893)	220.00%	(164.3536)
32	126000.00%	(0.6457)	760.00%	(82.5645)	10.00%	(1,001.5250)	31.00%	(721.6416)	126.00%	(192.4569)	221.00%	(164.1149)
33	122000.00%	(0.6669)	720.00%	(85.9571)	20.00%	(957.5903)	32.00%	(701.2201)	127.00%	(191.9386)	222.00%	(163.8764)
34	118000.00%	(0.6894)	680.00%	(89.6272)	30.00%	(742.5231)	33.00%	(681.2978)	128.00%	(191.4336)	223.00%	(163.6382)
35	114000.00%	(0.7136)	640.00%	(93.6075)	40.00%	(557.4697)	34.00%	(661.9043)	129.00%	(190.9412)	224.00%	(163.4001)
36	110000.00%	(0.7395)	600.00%	(97.9348)	50.00%	(427.6667)	35.00%	(643.0619)	130.00%	(190.4610)	225.00%	(163.1623)
37	106000.00%	(0.7673)	560.00%	(102.6506)	60.00%	(342.2512)	36.00%	(624.7857)	131.00%	(189.9923)	226.00%	(162.9247)
38	102000.00%	(0.7974)	520.00%	(107.8012)	70.00%	(287.1967)	37.00%	(607.0855)	132.00%	(189.5346)	227.00%	(162.6874)
39	98000.00%	(0.8298)	480.00%	(113.4370)	80.00%	(251.7759)	38.00%	(589.9661)	133.00%	(189.0875)	228.00%	(162.4502)
40	94000.00%	(0.8651)	440.00%	(119.6105)	90.00%	(228.7727)	39.00%	(573.4284)	134.00%	(188.6503)	229.00%	(162.2133)
41	90000.00%	(0.9034)	400.00%	(126.3727)	100.00%	(213.5547)	40.00%	(557.4697)	135.00%	(188.2226)	230.00%	(161.9766)
42	86000.00%	(0.9454)	360.00%	(133.7647)	110.00%	(203.2020)	41.00%	(542.0847)	136.00%	(187.8041)	231.00%	(161.7402)
43	82000.00%	(0.9914)	320.00%	(141.8031)	120.00%	(195.8855)	42.00%	(527.2655)	137.00%	(187.3942)	232.00%	(161.5039)
44	78000.00%	(1.0421)	280.00%	(150.4563)	130.00%	(190.4610)	43.00%	(513.0026)	138.00%	(186.9925)	233.00%	(161.2679)
45	74000.00%	(1.0982)	240.00%	(159.6220)	140.00%	(186.2125)	44.00%	(499.2846)	139.00%	(186.5988)	234.00%	(161.0321)
46	70000.00%	(1.1608)	200.00%	(169.1840)	150.00%	(182.6918)	45.00%	(486.0991)	140.00%	(186.2125)	235.00%	(160.7965)
47	66000.00%	(1.2309)	160.00%	(179.6189)	160.00%	(179.6189)	46.00%	(473.4327)	141.00%	(185.8334)	236.00%	(160.5612)
48	62000.00%	(1.3101)	120.00%	(195.8855)	170.00%	(176.8195)	47.00%	(461.2711)	142.00%	(185.4610)	237.00%	(160.3260)
49	58000.00%	(1.4001)	80.00%	(251.7759)	180.00%	(174.1863)	48.00%	(449.5997)	143.00%	(185.0952)	238.00%	(160.0911)
50	54000.00%	(1.5034)	40.00%	(557.4697)	190.00%	(171.6540)	49.00%	(438.4033)	144.00%	(184.7355)	239.00%	(159.8565)

51	50000.00%	(1.6232)	0.00%	85.0000	200.00%	(169.1840)	50.00%	(427.6667)	145.00%	(184.3818)	240.00%	(159.6220)
52	46000.00%	(1.7637)	−40.00%	1,121,244.8860	210.00%	(166.7544)	51.00%	(417.3744)	146.00%	(184.0336)	241.00%	(159.3878)
53	42000.00%	(1.9308)	−80.00%	9.5311E+10	220.00%	(164.3536)	52.00%	(407.5111)	147.00%	(183.6907)	242.00%	(159.1538)
54	38000.00%	(2.1329)	−120.00%	1.2422E+11	230.00%	(161.9766)	53.00%	(398.0613)	148.00%	(183.3530)	243.00%	(158.9201)
55	34000.00%	(2.3823)	−160.00%	3,259,407.1149	240.00%	(159.6220)	54.00%	(389.0099)	149.00%	(183.0201)	244.00%	(158.6866)
56	30000.00%	(2.6977)	−200.00%	38,315.0000	250.00%	(157.2906)	55.00%	(380.3420)	150.00%	(182.6918)	245.00%	(158.4533)
57	26000.00%	(3.1094)	−240.00%	3,700.8387	260.00%	(154.9842)	56.00%	(372.0428)	151.00%	(182.3679)	246.00%	(158.2203)
58	22000.00%	(3.6694)	−280.00%	1,212.3453	270.00%	(152.7053)	57.00%	(364.0980)	152.00%	(182.0482)	247.00%	(157.9875)
59	18000.00%	(4.4753)	−320.00%	702.1530	280.00%	(150.4563)	58.00%	(356.4936)	153.00%	(181.7325)	248.00%	(157.7549)
60	14000.00%	(5.7349)	−360.00%	506.3625	290.00%	(148.2397)	59.00%	(349.2157)	154.00%	(181.4205)	249.00%	(157.5226)
61	10000.00%	(7.9812)	−400.00%	401.2895	300.00%	(146.0575)	60.00%	(342.2512)	155.00%	(181.1122)	250.00%	(157.2906)
62	6000.00%	(13.1191)	−440.00%	334.1989	310.00%	(143.9114)	61.00%	(335.5870)	156.00%	(180.8073)	251.00%	(157.0588)
63	2000.00%	(36.7736)	−480.00%	286.9976	320.00%	(141.8031)	62.00%	(329.2108)	157.00%	(180.5057)	252.00%	(156.8272)
64	−2000.00%	45.4121	−520.00%	251.7300	330.00%	(139.7335)	63.00%	(323.1103)	158.00%	(180.2072)	253.00%	(156.5959)
65	−6000.00%	14.0727	−560.00%	224.2772	340.00%	(137.7036)	64.00%	(317.2739)	159.00%	(179.9116)	254.00%	(156.3649)
66	−10000.00%	8.3243	−600.00%	202.2585	350.00%	(135.7139)	65.00%	(311.6902)	160.00%	(179.6189)	255.00%	(156.1341)
67	−14000.00%	5.9099	−640.00%	184.1871	360.00%	(133.7647)	66.00%	(306.3485)	161.00%	(179.3289)	256.00%	(155.9036)
68	−18000.00%	4.5812	−680.00%	169.0807	370.00%	(131.8561)	67.00%	(301.2382)	162.00%	(179.0414)	257.00%	(155.6733)
69	−22000.00%	3.7402	−720.00%	156.2610	380.00%	(129.9881)	68.00%	(296.3491)	163.00%	(178.7563)	258.00%	(155.4433)
70	−26000.00%	3.1601	−760.00%	145.2436	390.00%	(128.1604)	69.00%	(291.6718)	164.00%	(178.4735)	259.00%	(155.2136)
71	−30000.00%	2.7358	−800.00%	135.6729	400.00%	(126.3727)	70.00%	(287.1967)	165.00%	(178.1930)	260.00%	(154.9842)
72	−34000.00%	2.4120	−840.00%	127.2812	410.00%	(124.6244)	71.00%	(282.9150)	166.00%	(177.9145)	261.00%	(154.7550)
73	−38000.00%	2.1567	−880.00%	119.8632	420.00%	(122.9151)	72.00%	(278.8181)	167.00%	(177.6380)	262.00%	(154.5261)
74	−42000.00%	1.9502	−920.00%	113.2590	430.00%	(121.2440)	73.00%	(274.8977)	168.00%	(177.3634)	263.00%	(154.2975)
75	−46000.00%	1.7799	−960.00%	107.3416	440.00%	(119.6105)	74.00%	(271.1460)	169.00%	(177.0906)	264.00%	(154.0692)
76	−50000.00%	1.6369	−1000.00%	102.0095	450.00%	(118.0138)	75.00%	(267.5554)	170.00%	(176.8195)	265.00%	(153.8411)
77	−54000.00%	1.5152	−1040.00%	97.1800	460.00%	(116.4532)	76.00%	(264.1186)	171.00%	(176.5501)	266.00%	(153.6134)
78	−58000.00%	1.4103	−1080.00%	92.7853	470.00%	(114.9279)	77.00%	(260.8288)	172.00%	(176.2822)	267.00%	(153.3859)
79	−62000.00%	1.3190	−1120.00%	88.7695	480.00%	(113.4370)	78.00%	(257.6793)	173.00%	(176.0157)	268.00%	(153.1588)
80	−6000.00%	1.2388	−1160.00%	85.0855	490.00%	(111.9797)	79.00%	(254.6636)	174.00%	(175.7507)	269.00%	(152.9319)
81	−70000.00%	1.1678	−1200.00%	81.6941	500.00%	(110.5552)	80.00%	(251.7759)	175.00%	(175.4870)	270.00%	(152.7053)
82	−74000.00%	1.1045	−1240.00%	78.5617	510.00%	(109.1627)	81.00%	(249.0102)	176.00%	(175.2246)	271.00%	(152.4790)
83	−78000.00%	1.0477	−1280.00%	75.6598	520.00%	(107.8012)	82.00%	(246.3611)	177.00%	(174.9633)	272.00%	(152.2530)
84	−82000.00%	0.9965	−1320.00%	72.9640	530.00%	(106.4701)	83.00%	(243.8231)	178.00%	(174.7033)	273.00%	(152.0273)
85	−86000.00%	0.9500	−1360.00%	70.4531	540.00%	(105.1685)	84.00%	(241.3913)	179.00%	(174.4443)	274.00%	(151.8020)
86	−90000.00%	0.9077	−1400.00%	68.1087	550.00%	(103.8956)	85.00%	(239.0608)	180.00%	(174.1863)	275.00%	(151.5769)
87	−94000.00%	0.8690	−1440.00%	65.9149	560.00%	(102.6506)	86.00%	(236.8269)	181.00%	(173.9294)	276.00%	(151.3522)
88	−98000.00%	0.8334	−1480.00%	63.8575	570.00%	(101.4328)	87.00%	(234.6853)	182.00%	(173.6733)	277.00%	(151.1277)
89	−102000.00%	0.8007	−1520.00%	61.9244	580.00%	(100.2413)	88.00%	(232.6317)	183.00%	(173.4182)	278.00%	(150.9036)
90	−106000.00%	0.7704	−1560.00%	60.1045	590.00%	(99.0756)	89.00%	(230.6621)	184.00%	(173.1639)	279.00%	(150.6798)
91	−110000.00%	0.7423	−1600.00%	58.3883	600.00%	(97.9348)	90.00%	(228.7727)	185.00%	(172.9104)	280.00%	(150.4563)
92	−114000.00%	0.7162	−1640.00%	56.7671	610.00%	(96.8182)	91.00%	(226.9597)	186.00%	(172.6578)	281.00%	(150.2332)
93	−118000.00%	0.6919	−1680.00%	55.2333	620.00%	(95.7253)	92.00%	(225.2197)	187.00%	(172.4058)	282.00%	(150.0104)
94	−122000.00%	0.6692	−1720.00%	53.7801	630.00%	(94.6552)	93.00%	(223.5493)	188.00%	(172.1545)	283.00%	(149.7879)
95	−126000.00%	0.6479	−1760.00%	52.4011	640.00%	(93.6075)	94.00%	(221.9453)	189.00%	(171.9039)	284.00%	(149.5657)

(*continued*)

Chart 4A (continued)

Table-A3

Project Cash Flow:

0	1	2	3	4	5	6
(815.00)	900.00	(100.00)	1,200.00	(1,200.00)	100.00	(100.00)

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.33)	2000.00%	(36.77)	–300.00%	759.84	0.00%	(15.00)	95.00%	(153.43)	190.00%	(166.89)
2	246000.00%	(0.33)	1960.00%	(37.45)	–290.00%	836.62	1.00%	(9.37)	96.00%	(154.13)	191.00%	(166.75)
3	242000.00%	(0.34)	1920.00%	(38.15)	–280.00%	930.09	2.00%	(4.72)	97.00%	(154.81)	192.00%	(166.61)
4	238000.00%	(0.34)	1880.00%	(38.87)	–270.00%	1,045.96	**3.00%**	(0.95)	98.00%	(155.47)	193.00%	(166.47)
5	234000.00%	(0.35)	1840.00%	(39.62)	–260.00%	1,192.58	**4.00%**	2.04	99.00%	(156.10)	194.00%	(166.32)
6	230000.00%	(0.35)	1800.00%	(40.41)	–250.00%	1,382.66	5.00%	4.32	100.00%	(156.72)	195.00%	(166.17)
7	226000.00%	(0.36)	1760.00%	(41.22)	–240.00%	1,636.03	6.00%	5.96	101.00%	(157.32)	196.00%	(166.02)
8	222000.00%	(0.37)	1720.00%	(42.07)	–230.00%	1,984.99	7.00%	7.03	102.00%	(157.89)	197.00%	(165.87)
9	218000.00%	(0.37)	1680.00%	(42.95)	–220.00%	2,484.39	8.00%	7.60	103.00%	(158.45)	198.00%	(165.72)
10	214000.00%	(0.38)	1640.00%	(43.87)	–210.00%	3,232.33	9.00%	7.70	104.00%	(159.00)	199.00%	(165.56)
11	210000.00%	(0.39)	1600.00%	(44.83)	–200.00%	4,415.00	10.00%	7.40	105.00%	(159.52)	200.00%	(165.40)
12	206000.00%	(0.40)	1560.00%	(45.84)	–190.00%	6,412.28	11.00%	6.74	106.00%	(160.02)	201.00%	(165.24)
13	202000.00%	(0.40)	1520.00%	(46.89)	–180.00%	10,070.42	12.00%	5.76	107.00%	(160.51)	202.00%	(165.08)
14	198000.00%	(0.41)	1480.00%	(47.98)	–170.00%	17,494.62	13.00%	4.48	108.00%	(160.99)	203.00%	(164.92)
15	194000.00%	(0.42)	1440.00%	(49.13)	–160.00%	34,728.25	14.00%	2.96	109.00%	(161.44)	204.00%	(164.75)
16	190000.00%	(0.43)	1400.00%	(50.34)	–150.00%	82,830.00	**15.00%**	1.21	110.00%	(161.88)	205.00%	(164.59)
17	186000.00%	(0.44)	1360.00%	(51.61)	–140.00%	258,736.72	**16.00%**	(0.73)	111.00%	(162.30)	206.00%	(164.42)
18	182000.00%	(0.45)	1320.00%	(52.94)	–130.00%	1,252,817.26	17.00%	(2.85)	112.00%	(162.71)	207.00%	(164.25)
19	178000.00%	(0.46)	1280.00%	(54.34)	–120.00%	13,914,075.00	18.00%	(5.11)	113.00%	(163.11)	208.00%	(164.08)
20	174000.00%	(0.47)	1240.00%	(55.82)	–110.00%	1,232,198,150.00	19.00%	(7.50)	114.00%	(163.49)	209.00%	(163.90)
21	170000.00%	(0.48)	1200.00%	(57.37)	–100.00%	(815.00)	20.00%	(10.00)	115.00%	(163.85)	210.00%	(163.73)
22	166000.00%	(0.49)	1160.00%	(59.02)	–90.00%	(1,008,018,150.00)	21.00%	(12.60)	116.00%	(164.20)	211.00%	(163.55)
23	162000.00%	(0.50)	1120.00%	(60.76)	–80.00%	(9,244,075.00)	22.00%	(15.28)	117.00%	(164.54)	212.00%	(163.38)
24	158000.00%	(0.52)	1080.00%	(62.61)	–70.00%	(662,172.54)	23.00%	(18.02)	118.00%	(164.86)	213.00%	(163.20)
25	154000.00%	(0.53)	1040.00%	(64.57)	–60.00%	(104,908.59)	24.00%	(20.82)	119.00%	(165.17)	214.00%	(163.02)
26	150000.00%	(0.54)	1000.00%	(66.65)	–50.00%	(24,430.00)	25.00%	(23.65)	120.00%	(165.47)	215.00%	(162.84)
27	146000.00%	(0.56)	960.00%	(68.87)	–40.00%	(6,923.03)	26.00%	(26.52)	121.00%	(165.76)	216.00%	(162.66)
28	142000.00%	(0.57)	920.00%	(71.25)	–30.00%	(2,125.34)	27.00%	(29.42)	122.00%	(166.03)	217.00%	(162.47)
29	138000.00%	(0.59)	880.00%	(73.78)	–20.00%	(635.60)	28.00%	(32.32)	123.00%	(166.29)	218.00%	(162.29)
30	134000.00%	(0.61)	840.00%	(76.50)	–10.00%	(155.75)	29.00%	(35.24)	124.00%	(166.54)	219.00%	(162.10)
31	130000.00%	(0.63)	800.00%	(79.42)	0.00%	(15.00)	30.00%	(38.16)	125.00%	(166.78)	220.00%	(161.92)
32	126000.00%	(0.65)	760.00%	(82.56)	10.00%	7.40	31.00%	(41.07)	126.00%	(167.01)	221.00%	(161.73)
33	122000.00%	(0.67)	720.00%	(85.95)	20.00%	(10.00)	32.00%	(43.97)	127.00%	(167.23)	222.00%	(161.54)
34	118000.00%	(0.69)	680.00%	(89.62)	30.00%	(38.16)	33.00%	(46.86)	128.00%	(167.43)	223.00%	(161.35)
35	114000.00%	(0.71)	640.00%	(93.60)	40.00%	(66.36)	34.00%	(49.73)	129.00%	(167.63)	224.00%	(161.16)
36	110000.00%	(0.74)	600.00%	(97.92)	50.00%	(91.02)	35.00%	(52.57)	130.00%	(167.82)	225.00%	(160.97)
37	106000.00%	(0.77)	560.00%	(102.63)	60.00%	(111.33)	36.00%	(55.39)	131.00%	(167.99)	226.00%	(160.78)
38	102000.00%	(0.80)	520.00%	(107.78)	70.00%	(127.48)	37.00%	(58.18)	132.00%	(168.16)	227.00%	(160.58)
39	98000.00%	(0.83)	480.00%	(113.40)	80.00%	(140.03)	38.00%	(60.94)	133.00%	(168.31)	228.00%	(160.39)
40	94000.00%	(0.87)	440.00%	(119.54)	90.00%	(149.60)	39.00%	(63.67)	134.00%	(168.46)	229.00%	(160.20)
41	90000.00%	(0.90)	400.00%	(126.26)	100.00%	(156.72)	40.00%	(66.36)	135.00%	(168.60)	230.00%	(160.00)
42	86000.00%	(0.95)	360.00%	(133.56)	110.00%	(161.88)	41.00%	(69.01)	136.00%	(168.73)	231.00%	(159.81)
43	82000.00%	(0.99)	320.00%	(141.42)	120.00%	(165.47)	42.00%	(71.63)	137.00%	(168.85)	232.00%	(159.61)
44	78000.00%	(1.04)	280.00%	(149.70)	130.00%	(167.82)	43.00%	(74.20)	138.00%	(168.96)	233.00%	(159.41)
45	74000.00%	(1.10)	240.00%	(158.01)	140.00%	(169.16)	44.00%	(76.73)	139.00%	(169.07)	234.00%	(159.21)
46	70000.00%	(1.16)	200.00%	(165.40)	150.00%	(169.72)	45.00%	(79.22)	140.00%	(169.16)	235.00%	(159.01)
47	66000.00%	(1.23)	160.00%	(169.66)	160.00%	(169.66)	46.00%	(81.67)	141.00%	(169.25)	236.00%	(158.81)
48	62000.00%	(1.31)	120.00%	(165.47)	170.00%	(169.10)	47.00%	(84.07)	142.00%	(169.33)	237.00%	(158.61)
49	58000.00%	(1.40)	80.00%	(140.03)	180.00%	(168.15)	48.00%	(86.44)	143.00%	(169.41)	238.00%	(158.41)
50	54000.00%	(1.50)	40.00%	(66.36)	190.00%	(166.89)	49.00%	(88.75)	144.00%	(169.47)	239.00%	(158.21)

51	50000.00%	(1.62)	0.00%	(15.00)	200.00%	(165.40)	50.00%	(91.02)	145.00%	(169.53)	240.00%	(158.01)
52	46000.00%	(1.76)	–40.00%	(6,923.03)	210.00%	(163.73)	51.00%	(93.25)	146.00%	(169.58)	241.00%	(157.81)
53	42000.00%	(1.93)	–80.00%	(9,244,075.00)	220.00%	(161.92)	52.00%	(95.44)	147.00%	(169.63)	242.00%	(157.61)
54	38000.00%	(2.13)	–120.00%	13,914,075.00	230.00%	(160.00)	53.00%	(97.57)	148.00%	(169.67)	243.00%	(157.40)
55	34000.00%	(2.38)	–160.00%	34,728.25	240.00%	(158.01)	54.00%	(99.67)	149.00%	(169.70)	244.00%	(157.20)
56	30000.00%	(2.70)	–200.00%	4,415.00	250.00%	(155.97)	55.00%	(101.72)	150.00%	(169.72)	245.00%	(157.00)
57	26000.00%	(3.11)	–240.00%	1,636.03	260.00%	(153.89)	56.00%	(103.73)	151.00%	(169.74)	246.00%	(156.79)
58	22000.00%	(3.67)	–280.00%	930.09	270.00%	(151.80)	57.00%	(105.69)	152.00%	(169.75)	247.00%	(156.59)
59	18000.00%	(4.48)	–320.00%	641.59	280.00%	(149.70)	58.00%	(107.61)	153.00%	(169.76)	248.00%	(156.38)
60	14000.00%	(5.73)	–360.00%	489.09	290.00%	(147.61)	59.00%	(109.49)	154.00%	(169.76)	249.00%	(156.18)
61	10000.00%	(7.98)	–400.00%	395.31	300.00%	(145.53)	60.00%	(111.33)	155.00%	(169.76)	250.00%	(155.97)
62	6000.00%	(13.12)	–440.00%	331.81	310.00%	(143.47)	61.00%	(113.12)	156.00%	(169.75)	251.00%	(155.76)
63	2000.00%	(36.77)	–480.00%	285.93	320.00%	(141.42)	62.00%	(114.87)	157.00%	(169.73)	252.00%	(155.56)
64	–2000.00%	45.41	–520.00%	251.21	330.00%	(139.41)	63.00%	(116.59)	158.00%	(169.71)	253.00%	(155.35)
65	–6000.00%	14.07	–560.00%	224.01	340.00%	(137.43)	64.00%	(118.26)	159.00%	(169.69)	254.00%	(155.14)
66	–10000.00%	8.32	–600.00%	202.11	350.00%	(135.48)	65.00%	(119.89)	160.00%	(169.66)	255.00%	(154.94)
67	–14000.00%	5.91	–640.00%	184.10	360.00%	(133.56)	66.00%	(121.48)	161.00%	(169.62)	256.00%	(154.73)
68	–18000.00%	4.58	–680.00%	169.03	370.00%	(131.68)	67.00%	(123.04)	162.00%	(169.58)	257.00%	(154.52)
69	–22000.00%	3.74	–720.00%	156.23	380.00%	(129.84)	68.00%	(124.56)	163.00%	(169.53)	258.00%	(154.31)
70	–26000.00%	3.16	–760.00%	145.22	390.00%	(128.03)	69.00%	(126.04)	164.00%	(169.48)	259.00%	(154.10)
71	–30000.00%	2.74	–800.00%	135.66	400.00%	(126.26)	70.00%	(127.48)	165.00%	(169.43)	260.00%	(153.89)
72	–34000.00%	2.41	–840.00%	127.27	410.00%	(124.53)	71.00%	(128.89)	166.00%	(169.37)	261.00%	(153.69)
73	–38000.00%	2.16	–880.00%	119.86	420.00%	(122.83)	72.00%	(130.26)	167.00%	(169.31)	262.00%	(153.48)
74	–42000.00%	1.95	–920.00%	113.25	430.00%	(121.17)	73.00%	(131.60)	168.00%	(169.24)	263.00%	(153.27)
75	–46000.00%	1.78	–960.00%	107.34	440.00%	(119.54)	74.00%	(132.90)	169.00%	(169.17)	264.00%	(153.06)
76	–50000.00%	1.64	–1000.00%	102.01	450.00%	(117.95)	75.00%	(134.17)	170.00%	(169.10)	265.00%	(152.85)
77	–54000.00%	1.52	–1040.00%	97.18	460.00%	(116.40)	76.00%	(135.41)	171.00%	(169.02)	266.00%	(152.64)
78	–58000.00%	1.41	–1080.00%	92.78	470.00%	(114.88)	77.00%	(136.61)	172.00%	(168.93)	267.00%	(152.43)
79	–62000.00%	1.32	–1120.00%	88.77	480.00%	(113.40)	78.00%	(137.78)	173.00%	(168.85)	268.00%	(152.22)
80	–66000.00%	1.24	–1160.00%	85.08	490.00%	(111.94)	79.00%	(138.92)	174.00%	(168.76)	269.00%	(152.01)
81	–70000.00%	1.17	–1200.00%	81.69	500.00%	(110.52)	80.00%	(140.03)	175.00%	(168.66)	270.00%	(151.80)
82	–74000.00%	1.10	–1240.00%	78.56	510.00%	(109.13)	81.00%	(141.12)	176.00%	(168.57)	271.00%	(151.59)
83	–78000.00%	1.05	–1280.00%	75.66	520.00%	(107.78)	82.00%	(142.17)	177.00%	(168.47)	272.00%	(151.38)
84	–82000.00%	1.00	–1320.00%	72.96	530.00%	(106.45)	83.00%	(143.19)	178.00%	(168.36)	273.00%	(151.17)
85	–86000.00%	0.95	–1360.00%	70.45	540.00%	(105.15)	84.00%	(144.19)	179.00%	(168.26)	274.00%	(150.96)
86	–90000.00%	0.91	–1400.00%	68.11	550.00%	(103.88)	85.00%	(145.15)	180.00%	(168.15)	275.00%	(150.75)
87	–94000.00%	0.87	–1440.00%	65.91	560.00%	(102.63)	86.00%	(146.09)	181.00%	(168.03)	276.00%	(150.54)
88	–98000.00%	0.83	–1480.00%	63.86	570.00%	(101.42)	87.00%	(147.01)	182.00%	(167.92)	277.00%	(150.33)
89	–102000.00%	0.80	–1520.00%	61.92	580.00%	(100.23)	88.00%	(147.89)	183.00%	(167.80)	278.00%	(150.12)
90	–106000.00%	0.77	–1560.00%	60.10	590.00%	(99.06)	89.00%	(148.76)	184.00%	(167.68)	279.00%	(149.91)
91	–110000.00%	0.74	–1600.00%	58.39	600.00%	(97.92)	90.00%	(149.60)	185.00%	(167.55)	280.00%	(149.70)
92	–114000.00%	0.72	–1640.00%	56.77	610.00%	(96.81)	91.00%	(150.41)	186.00%	(167.43)	281.00%	(149.49)
93	–118000.00%	0.69	–1680.00%	55.23	620.00%	(95.72)	92.00%	(151.20)	187.00%	(167.30)	282.00%	(149.28)
94	–122000.00%	0.67	–1720.00%	53.78	630.00%	(94.65)	93.00%	(151.97)	188.00%	(167.16)	283.00%	(149.08)
95	–126000.00%	0.65	–1760.00%	52.40	640.00%	(93.60)	94.00%	(152.71)	189.00%	(167.03)	284.00%	(148.87)

(*continued*)

Chart 4A (continued)

Table-A4
Project Outcome/Cashflow:

0	1	2	3	4
(815.00)	900.00	(100.00)	1,200.00	(1,200.00)

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.326)	2000.00%	(36.774)	−300.00%	757.500	0.00%	(15.000)	95.00%	(154.316)	190.00%	(167.003)
2	246000.00%	(0.331)	1960.00%	(37.447)	−290.00%	833.378	1.00%	(10.306)	96.00%	(154.993)	191.00%	(166.862)
3	242000.00%	(0.336)	1920.00%	(38.146)	−280.00%	925.521	2.00%	(6.462)	97.00%	(155.650)	192.00%	(166.718)
4	238000.00%	(0.342)	1880.00%	(38.871)	−270.00%	1,039.377	3.00%	(3.386)	98.00%	(156.286)	193.00%	(166.572)
5	234000.00%	(0.348)	1840.00%	(39.625)	−260.00%	1,182.898	**4.00%**	(1.000)	99.00%	(156.903)	194.00%	(166.424)
6	230000.00%	(0.354)	1800.00%	(40.408)	−250.00%	1,368.025	**5.00%**	0.764	100.00%	(157.500)	195.00%	(166.274)
7	226000.00%	(0.360)	1760.00%	(41.222)	−240.00%	1,613.261	6.00%	1.970	101.00%	(158.078)	196.00%	(166.122)
8	222000.00%	(0.367)	1720.00%	(42.069)	−230.00%	1,948.332	7.00%	2.674	102.00%	(158.638)	197.00%	(165.968)
9	218000.00%	(0.373)	1680.00%	(42.952)	−220.00%	2,422.994	8.00%	2.928	103.00%	(159.179)	198.00%	(165.812)
10	214000.00%	(0.380)	1640.00%	(43.873)	−210.00%	3,124.564	9.00%	2.780	104.00%	(159.702)	199.00%	(165.654)
11	210000.00%	(0.388)	1600.00%	(44.834)	−200.00%	4,215.000	10.00%	2.272	105.00%	(160.208)	200.00%	(165.494)
12	206000.00%	(0.395)	1560.00%	(45.837)	−190.00%	6,015.041	11.00%	1.442	106.00%	(160.698)	201.00%	(165.332)
13	202000.00%	(0.403)	1520.00%	(46.886)	−180.00%	9,212.109	**12.00%**	0.327	107.00%	(161.170)	202.00%	(165.169)
14	198000.00%	(0.411)	1480.00%	(47.984)	−170.00%	15,430.365	**13.00%**	(1.041)	108.00%	(161.626)	203.00%	(165.004)
15	194000.00%	(0.420)	1440.00%	(49.135)	−160.00%	29,012.654	14.00%	(2.635)	109.00%	(162.066)	204.00%	(164.837)
16	190000.00%	(0.428)	1400.00%	(50.341)	−150.00%	63,630.000	15.00%	(4.426)	110.00%	(162.491)	205.00%	(164.669)
17	186000.00%	(0.438)	1360.00%	(51.607)	−140.00%	173,287.500	16.00%	(6.392)	111.00%	(162.901)	206.00%	(164.499)
18	182000.00%	(0.447)	1320.00%	(52.938)	−130.00%	658,395.679	17.00%	(8.509)	112.00%	(163.295)	207.00%	(164.328)
19	178000.00%	(0.457)	1280.00%	(54.339)	−120.00%	4,539,075.000	18.00%	(10.759)	113.00%	(163.675)	208.00%	(164.155)
20	174000.00%	(0.468)	1240.00%	(55.816)	−110.00%	132,198,150.000	19.00%	(13.124)	114.00%	(164.041)	209.00%	(163.981)
21	170000.00%	(0.479)	1200.00%	(57.374)	−100.00%	(815.000)	20.00%	(15.586)	115.00%	(164.393)	210.00%	(163.805)
22	166000.00%	(0.490)	1160.00%	(59.020)	−90.00%	(108,018,150.000)	21.00%	(18.132)	116.00%	(164.731)	211.00%	(163.628)
23	162000.00%	(0.502)	1120.00%	(60.762)	−80.00%	(2,994,075.000)	22.00%	(20.748)	117.00%	(165.056)	212.00%	(163.450)
24	158000.00%	(0.515)	1080.00%	(62.608)	−70.00%	(342,099.383)	23.00%	(23.420)	118.00%	(165.368)	213.00%	(163.270)
25	154000.00%	(0.528)	1040.00%	(64.569)	−60.00%	(68,287.500)	24.00%	(26.140)	119.00%	(165.667)	214.00%	(163.090)
26	150000.00%	(0.543)	1000.00%	(66.654)	−50.00%	(18,030.000)	25.00%	(28.896)	120.00%	(165.954)	215.00%	(162.908)
27	146000.00%	(0.557)	960.00%	(68.875)	−40.00%	(5,494.136)	26.00%	(31.680)	121.00%	(166.229)	216.00%	(162.725)
28	142000.00%	(0.573)	920.00%	(71.246)	−30.00%	(1,761.061)	27.00%	(34.482)	122.00%	(166.492)	217.00%	(162.540)
29	138000.00%	(0.590)	880.00%	(73.782)	−20.00%	(540.234)	28.00%	(37.297)	123.00%	(166.743)	218.00%	(162.355)
30	134000.00%	(0.607)	840.00%	(76.500)	−10.00%	(134.840)	29.00%	(40.117)	124.00%	(166.983)	219.00%	(162.168)
31	130000.00%	(0.626)	800.00%	(79.419)	0.00%	(15.000)	30.00%	(42.937)	125.00%	(167.211)	220.00%	(161.981)
32	126000.00%	(0.646)	760.00%	(82.562)	10.00%	2.272	31.00%	(45.750)	126.00%	(167.429)	221.00%	(161.792)
33	122000.00%	(0.667)	720.00%	(85.954)	20.00%	(15.586)	32.00%	(48.553)	127.00%	(167.636)	222.00%	(161.603)
34	118000.00%	(0.689)	680.00%	(89.622)	30.00%	(42.937)	33.00%	(51.340)	128.00%	(167.833)	223.00%	(161.412)
35	114000.00%	(0.714)	640.00%	(93.600)	40.00%	(70.154)	34.00%	(54.109)	129.00%	(168.020)	224.00%	(161.221)
36	110000.00%	(0.739)	600.00%	(97.924)	50.00%	(93.951)	35.00%	(56.855)	130.00%	(168.197)	225.00%	(161.029)
37	106000.00%	(0.767)	560.00%	(102.635)	60.00%	(113.562)	36.00%	(59.576)	131.00%	(168.364)	226.00%	(160.836)
38	102000.00%	(0.797)	520.00%	(107.777)	70.00%	(129.186)	37.00%	(62.268)	132.00%	(168.522)	227.00%	(160.642)
39	98000.00%	(0.830)	480.00%	(113.398)	80.00%	(141.342)	38.00%	(64.930)	133.00%	(168.671)	228.00%	(160.447)
40	94000.00%	(0.865)	440.00%	(119.547)	90.00%	(150.602)	39.00%	(67.559)	134.00%	(168.810)	229.00%	(160.252)
41	90000.00%	(0.903)	400.00%	(126.264)	100.00%	(157.500)	40.00%	(70.154)	135.00%	(168.941)	230.00%	(160.055)
42	86000.00%	(0.945)	360.00%	(133.571)	110.00%	(162.491)	41.00%	(72.712)	136.00%	(169.063)	231.00%	(159.858)
43	82000.00%	(0.991)	320.00%	(141.439)	120.00%	(165.954)	42.00%	(75.234)	137.00%	(169.176)	232.00%	(159.661)
44	78000.00%	(1.042)	280.00%	(149.729)	130.00%	(168.197)	43.00%	(77.716)	138.00%	(169.282)	233.00%	(159.462)
45	74000.00%	(1.098)	240.00%	(158.057)	140.00%	(169.469)	44.00%	(80.159)	139.00%	(169.379)	234.00%	(159.263)
46	70000.00%	(1.161)	200.00%	(165.494)	150.00%	(169.968)	45.00%	(82.562)	140.00%	(169.469)	235.00%	(159.064)
47	66000.00%	(1.231)	160.00%	(169.855)	160.00%	(169.855)	46.00%	(84.924)	141.00%	(169.550)	236.00%	(158.863)
48	62000.00%	(1.310)	120.00%	(165.954)	170.00%	(169.258)	47.00%	(87.244)	142.00%	(169.625)	237.00%	(158.663)
49	58000.00%	(1.400)	80.00%	(141.342)	180.00%	(168.280)	48.00%	(89.522)	143.00%	(169.691)	238.00%	(158.461)
50	54000.00%	(1.503)	40.00%	(70.154)	190.00%	(167.003)	49.00%	(91.757)	144.00%	(169.751)	239.00%	(158.259)

51	50000.00%	(1.623)	0.00%	(15.000)	200.00%	(165.494)	50.00%	(93.951)	145.00%	(169.804)	240.00%	(158.057)
52	46000.00%	(1.764)	−40.00%	(5,494.136)	210.00%	(163.805)	51.00%	(96.101)	146.00%	(169.850)	241.00%	(157.854)
53	42000.00%	(1.931)	−80.00%	(2,994,075.000)	220.00%	(161.981)	52.00%	(98.209)	147.00%	(169.889)	242.00%	(157.650)
54	38000.00%	(2.133)	−120.00%	4,539,075.000	230.00%	(160.055)	53.00%	(100.275)	148.00%	(169.922)	243.00%	(157.447)
55	34000.00%	(2.382)	−160.00%	29,012.654	240.00%	(158.057)	54.00%	(102.298)	149.00%	(169.948)	244.00%	(157.242)
56	30000.00%	(2.698)	−200.00%	4,215.000	250.00%	(156.008)	55.00%	(104.279)	150.00%	(169.968)	245.00%	(157.038)
57	26000.00%	(3.109)	−240.00%	1,613.261	260.00%	(153.928)	56.00%	(106.219)	151.00%	(169.982)	246.00%	(156.832)
58	22000.00%	(3.669)	−280.00%	925.521	270.00%	(151.831)	57.00%	(108.116)	152.00%	(169.990)	247.00%	(156.627)
59	18000.00%	(4.475)	−320.00%	640.307	280.00%	(149.729)	58.00%	(109.972)	153.00%	(169.992)	248.00%	(156.421)
60	14000.00%	(5.735)	−360.00%	488.647	290.00%	(147.632)	59.00%	(111.787)	154.00%	(169.989)	249.00%	(156.215)
61	10000.00%	(7.981)	−400.00%	395.123	300.00%	(145.547)	60.00%	(113.562)	155.00%	(169.979)	250.00%	(156.008)
62	6000.00%	(13.119)	−440.00%	331.726	310.00%	(143.481)	**61.00%**	(115.297)	156.00%	(169.965)	251.00%	(155.801)
63	2000.00%	(36.774)	−480.00%	285.892	320.00%	(141.439)	**62.00%**	(116.991)	157.00%	(169.945)	252.00%	(155.594)
64	−2000.00%	45.412	−520.00%	251.192	330.00%	(139.424)	63.00%	(118.647)	158.00%	(169.920)	253.00%	(155.387)
65	−6000.00%	14.073	−560.00%	223.997	340.00%	(137.440)	64.00%	(120.264)	159.00%	(169.890)	254.00%	(155.179)
66	−10000.00%	8.324	−600.00%	202.104	350.00%	(135.488)	65.00%	(121.843)	160.00%	(169.855)	255.00%	(154.971)
67	−14000.00%	5.910	−640.00%	184.098	360.00%	(133.571)	66.00%	(123.385)	161.00%	(169.816)	256.00%	(154.763)
68	−18000.00%	4.581	−680.00%	169.027	370.00%	(131.689)	67.00%	(124.889)	162.00%	(169.771)	257.00%	(154.554)
69	−22000.00%	3.740	−720.00%	156.227	380.00%	(129.844)	68.00%	(126.357)	163.00%	(169.722)	258.00%	(154.346)
70	−26000.00%	3.160	−760.00%	145.222	390.00%	(128.035)	69.00%	(127.790)	164.00%	(169.668)	259.00%	(154.137)
71	−30000.00%	2.736	−800.00%	135.659	400.00%	(126.264)	70.00%	(129.186)	165.00%	(169.610)	260.00%	(153.928)
72	−34000.00%	2.412	−840.00%	127.272	410.00%	(124.530)	71.00%	(130.548)	166.00%	(169.548)	261.00%	(153.719)
73	−38000.00%	2.157	−880.00%	119.857	420.00%	(122.832)	72.00%	(131.876)	167.00%	(169.482)	262.00%	(153.509)
74	−42000.00%	1.950	−920.00%	113.254	430.00%	(121.172)	73.00%	(133.171)	168.00%	(169.411)	263.00%	(153.300)
75	−46000.00%	1.780	−960.00%	107.338	440.00%	(119.547)	74.00%	(134.432)	169.00%	(169.337)	264.00%	(153.090)
76	−50000.00%	1.637	−1000.00%	102.007	450.00%	(117.958)	75.00%	(135.661)	170.00%	(169.258)	265.00%	(152.881)
77	−54000.00%	1.515	−1040.00%	97.178	460.00%	(116.404)	76.00%	(136.859)	171.00%	(169.176)	266.00%	(152.671)
78	−58000.00%	1.410	−1080.00%	92.784	470.00%	(114.884)	77.00%	(138.025)	172.00%	(169.090)	267.00%	(152.461)
79	−62000.00%	1.319	−1120.00%	88.768	480.00%	(113.398)	78.00%	(139.160)	173.00%	(169.001)	268.00%	(152.251)
80	−66000.00%	1.239	−1160.00%	85.085	490.00%	(111.945)	79.00%	(140.266)	174.00%	(168.908)	269.00%	(152.041)
81	−70000.00%	1.168	−1200.00%	81.693	500.00%	(110.525)	80.00%	(141.342)	175.00%	(168.811)	270.00%	(151.831)
82	−74000.00%	1.104	−1240.00%	78.561	510.00%	(109.135)	81.00%	(142.389)	176.00%	(168.712)	271.00%	(151.621)
83	−78000.00%	1.048	−1280.00%	75.659	520.00%	(107.777)	82.00%	(143.407)	177.00%	(168.609)	272.00%	(151.410)
84	−82000.00%	0.996	−1320.00%	72.964	530.00%	(106.448)	83.00%	(144.398)	178.00%	(168.502)	273.00%	(151.200)
85	−86000.00%	0.950	−1360.00%	70.453	540.00%	(105.149)	84.00%	(145.362)	179.00%	(168.393)	274.00%	(150.990)
86	−90000.00%	0.908	−1400.00%	68.109	550.00%	(103.878)	85.00%	(146.299)	180.00%	(168.280)	275.00%	(150.780)
87	−94000.00%	0.869	−1440.00%	65.915	560.00%	(102.635)	86.00%	(147.210)	181.00%	(168.165)	276.00%	(150.569)
88	−98000.00%	0.833	−1480.00%	63.857	570.00%	(101.419)	87.00%	(148.095)	182.00%	(168.047)	277.00%	(150.359)
89	−102000.00%	0.801	−1520.00%	61.924	580.00%	(100.229)	88.00%	(148.955)	183.00%	(167.925)	278.00%	(150.149)
90	−106000.00%	0.770	−1560.00%	60.104	590.00%	(99.064)	89.00%	(149.791)	184.00%	(167.801)	279.00%	(149.939)
91	−110000.00%	0.742	−1600.00%	58.388	600.00%	(97.924)	90.00%	(150.602)	185.00%	(167.675)	280.00%	(149.729)
92	−114000.00%	0.716	−1640.00%	56.767	610.00%	(96.809)	91.00%	(151.390)	186.00%	(167.545)	281.00%	(149.519)
93	−118000.00%	0.692	−1680.00%	55.233	620.00%	(95.717)	92.00%	(152.155)	187.00%	(167.414)	282.00%	(149.309)
94	−122000.00%	0.669	−1720.00%	53.780	630.00%	(94.647)	93.00%	(152.897)	188.00%	(167.279)	283.00%	(149.099)
95	−126000.00%	0.648	−1760.00%	52.401	640.00%	(93.600)	94.00%	(153.618)	189.00%	(167.142)	284.00%	(148.889)

(continued)

Chart 4A (continued)

Table-B1

Project Outcome/Cashflow:

	0	1	2	3	4	5	6	7	8	9	10	11
	(815.00)	900.00	(100.00)	1,200.00	(1,200.00)	3,000.00	(20,000.00)	1,000.00	(10,000.00)	17,000.00	(200.00)	15,000.00
	Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):											
	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.33)	2000.00%	(36.77)	−300.00%	1,004.42	0.00%	5,785.00	95.00%	(279.83)	190.00%	(173.60)
2	246000.00%	(0.33)	1960.00%	(37.45)	−290.00%	1,192.45	1.00%	4,463.91	96.00%	(276.03)	191.00%	(173.28)
3	242000.00%	(0.34)	1920.00%	(38.15)	−280.00%	1,460.78	2.00%	3,344.65	97.00%	(272.40)	192.00%	(172.97)
4	238000.00%	(0.34)	1880.00%	(38.87)	−270.00%	1,860.38	3.00%	2,398.29	98.00%	(268.90)	193.00%	(172.66)
5	234000.00%	(0.35)	1840.00%	(39.62)	−260.00%	2,484.61	4.00%	1,600.04	99.00%	(265.55)	194.00%	(172.36)
6	230000.00%	(0.35)	1800.00%	(40.41)	−250.00%	3,513.82	5.00%	928.68	100.00%	(262.33)	195.00%	(172.05)
7	226000.00%	(0.36)	1760.00%	(41.22)	−240.00%	5,317.98	6.00%	366.03	101.00%	(259.24)	196.00%	(171.75)
8	222000.00%	(0.37)	1720.00%	(42.07)	−230.00%	8,710.91	7.00%	(103.52)	102.00%	(256.28)	197.00%	(171.45)
9	218000.00%	(0.37)	1680.00%	(42.95)	−220.00%	15,634.82	8.00%	(493.37)	103.00%	(253.42)	198.00%	(171.15)
10	214000.00%	(0.38)	1640.00%	(43.87)	−210.00%	31,192.43	9.00%	(815.02)	104.00%	(250.68)	199.00%	(170.86)
11	210000.00%	(0.39)	1600.00%	(44.83)	−200.00%	70,415.00	10.00%	(1,078.34)	105.00%	(248.05)	200.00%	(170.56)
12	206000.00%	(0.40)	1560.00%	(45.84)	−190.00%	184,113.35	11.00%	(1,291.83)	106.00%	(245.52)	201.00%	(170.27)
13	202000.00%	(0.40)	1520.00%	(46.89)	−180.00%	575,421.78	12.00%	(1,462.80)	107.00%	(243.09)	202.00%	(169.98)
14	198000.00%	(0.41)	1480.00%	(47.98)	−170.00%	2,244,590.60	13.00%	(1,597.54)	108.00%	(240.75)	203.00%	(169.70)
15	194000.00%	(0.42)	1440.00%	(49.13)	−160.00%	11,617,111.10	14.00%	(1,701.43)	109.00%	(238.49)	204.00%	(169.41)
16	190000.00%	(0.43)	1400.00%	(50.34)	−150.00%	87,449,230.00	15.00%	(1,779.15)	110.00%	(236.33)	205.00%	(169.13)
17	186000.00%	(0.44)	1360.00%	(51.61)	−140.00%	1.11E+09	16.00%	(1,834.68)	111.00%	(234.24)	206.00%	(168.84)
18	182000.00%	(0.45)	1320.00%	(52.94)	−130.00%	3.18E+10	17.00%	(1,871.47)	112.00%	(232.24)	207.00%	(168.56)
19	178000.00%	(0.46)	1280.00%	(54.34)	−120.00%	3.86E+12	18.00%	(1,892.48)	113.00%	(230.30)	208.00%	(168.28)
20	174000.00%	(0.47)	1240.00%	(55.82)	−110.00%	1.52E+16	19.00%	(1,900.26)	114.00%	(228.44)	209.00%	(168.01)
21	170000.00%	(0.48)	1200.00%	(57.37)	−100.00%	−8.15E+02	20.00%	(1,897.00)	115.00%	(226.65)	210.00%	(167.73)
22	166000.00%	(0.49)	1160.00%	(59.02)	−90.00%	1.51E+16	21.00%	(1,884.57)	116.00%	(224.92)	211.00%	(167.45)
23	162000.00%	(0.50)	1120.00%	(60.76)	−80.00%	3.80E+12	22.00%	(1,864.59)	117.00%	(223.25)	212.00%	(167.18)
24	158000.00%	(0.52)	1080.00%	(62.61)	−70.00%	3.04E+10	23.00%	(1,838.44)	118.00%	(221.65)	213.00%	(166.91)
25	154000.00%	(0.53)	1040.00%	(64.57)	−60.00%	1.00E+09	24.00%	(1,807.30)	119.00%	(220.10)	214.00%	(166.64)
26	150000.00%	(0.54)	1000.00%	(66.65)	−50.00%	71,188,370.00	25.00%	(1,772.20)	120.00%	(218.60)	215.00%	(166.37)
27	146000.00%	(0.56)	960.00%	(68.87)	−40.00%	8,058,871.19	26.00%	(1,733.99)	121.00%	(217.16)	216.00%	(166.10)
28	142000.00%	(0.57)	920.00%	(71.24)	−30.00%	1,225,844.92	27.00%	(1,693.42)	122.00%	(215.77)	217.00%	(165.83)
29	138000.00%	(0.59)	880.00%	(73.78)	−20.00%	221,266.34	28.00%	(1,651.11)	123.00%	(214.42)	218.00%	(165.57)
30	134000.00%	(0.61)	840.00%	(76.50)	−10.00%	41,435.26	29.00%	(1,607.59)	124.00%	(213.12)	219.00%	(165.30)
31	130000.00%	(0.63)	800.00%	(79.42)	0.00%	5,785.00	30.00%	(1,563.32)	125.00%	(211.87)	220.00%	(165.04)
32	126000.00%	(0.65)	760.00%	(82.56)	10.00%	(1,078.34)	31.00%	(1,518.67)	126.00%	(210.65)	221.00%	(164.77)
33	122000.00%	(0.67)	720.00%	(85.95)	20.00%	(1,897.00)	32.00%	(1,473.96)	127.00%	(209.48)	222.00%	(164.51)
34	118000.00%	(0.69)	680.00%	(89.62)	30.00%	(1,563.32)	33.00%	(1,429.46)	128.00%	(208.34)	223.00%	(164.25)
35	114000.00%	(0.71)	640.00%	(93.60)	40.00%	(1,137.90)	34.00%	(1,385.39)	129.00%	(207.24)	224.00%	(163.99)
36	110000.00%	(0.74)	600.00%	(97.92)	50.00%	(814.13)	35.00%	(1,341.92)	130.00%	(206.18)	225.00%	(163.73)
37	106000.00%	(0.77)	560.00%	(102.64)	60.00%	(595.27)	36.00%	(1,299.20)	131.00%	(205.15)	226.00%	(163.47)
38	102000.00%	(0.80)	520.00%	(107.78)	70.00%	(452.80)	37.00%	(1,257.36)	132.00%	(204.15)	227.00%	(163.22)
39	98000.00%	(0.83)	480.00%	(113.41)	80.00%	(360.89)	38.00%	(1,216.48)	133.00%	(203.18)	228.00%	(162.96)
40	94000.00%	(0.87)	440.00%	(119.58)	90.00%	(301.35)	39.00%	(1,176.64)	134.00%	(202.25)	229.00%	(162.70)
41	90000.00%	(0.90)	400.00%	(126.33)	100.00%	(262.33)	40.00%	(1,137.90)	135.00%	(201.34)	230.00%	(162.45)
42	86000.00%	(0.95)	360.00%	(133.71)	110.00%	(236.33)	41.00%	(1,100.29)	136.00%	(200.46)	231.00%	(162.20)
43	82000.00%	(0.99)	320.00%	(141.76)	120.00%	(218.60)	42.00%	(1,063.83)	137.00%	(199.60)	232.00%	(161.94)
44	78000.00%	(1.04)	280.00%	(150.49)	130.00%	(206.18)	43.00%	(1,028.55)	138.00%	(198.77)	233.00%	(161.69)
45	74000.00%	(1.10)	240.00%	(159.94)	140.00%	(197.17)	44.00%	(994.45)	139.00%	(197.96)	234.00%	(161.44)
46	70000.00%	(1.16)	200.00%	(170.56)	150.00%	(190.39)	45.00%	(961.53)	140.00%	(197.17)	235.00%	(161.19)
47	66000.00%	(1.23)	160.00%	(185.05)	160.00%	(185.05)	46.00%	(929.78)	141.00%	(196.41)	236.00%	(160.94)
48	62000.00%	(1.31)	120.00%	(218.60)	170.00%	(180.67)	47.00%	(899.18)	142.00%	(195.67)	237.00%	(160.69)
49	58000.00%	(1.40)	80.00%	(360.89)	180.00%	(176.92)	48.00%	(869.72)	143.00%	(194.95)	238.00%	(160.44)
50	54000.00%	(1.50)	40.00%	(1,137.90)	190.00%	(173.60)	49.00%	(841.38)	144.00%	(194.24)	239.00%	(160.19)

51	50000.00%	(1.62)	0.00%	5,785.00	200.00%	(170.56)	50.00%	(814.13)	145.00%	(193.56)	240.00%	(159.94)
52	46000.00%	(1.76)	−40.00%	8,058,871.19	210.00%	(167.73)	51.00%	(787.95)	146.00%	(192.89)	241.00%	(159.70)
53	42000.00%	(1.93)	−80.00%	3.80E+12	220.00%	(165.04)	52.00%	(762.80)	147.00%	(192.24)	242.00%	(159.45)
54	38000.00%	(2.13)	−120.00%	3.86E+12	230.00%	(162.45)	53.00%	(738.66)	148.00%	(191.61)	243.00%	(159.20)
55	34000.00%	(2.38)	−160.00%	11,617,111.10	240.00%	(159.94)	54.00%	(715.50)	149.00%	(190.99)	244.00%	(158.96)
56	30000.00%	(2.70)	−200.00%	70,415.00	250.00%	(157.50)	55.00%	(693.28)	150.00%	(190.39)	245.00%	(158.71)
57	26000.00%	(3.11)	−240.00%	5,317.98	260.00%	(155.12)	56.00%	(671.97)	151.00%	(189.80)	246.00%	(158.47)
58	22000.00%	(3.67)	−280.00%	1,460.78	270.00%	(152.78)	57.00%	(651.55)	152.00%	(189.22)	247.00%	(158.23)
59	18000.00%	(4.48)	−320.00%	764.65	280.00%	(150.49)	58.00%	(631.98)	153.00%	(188.66)	248.00%	(157.99)
60	14000.00%	(5.73)	−360.00%	526.95	290.00%	(148.24)	59.00%	(613.23)	154.00%	(188.11)	249.00%	(157.74)
61	10000.00%	(7.98)	−400.00%	409.36	300.00%	(146.04)	60.00%	(595.27)	155.00%	(187.57)	250.00%	(157.50)
62	6000.00%	(13.12)	−440.00%	337.79	310.00%	(143.88)	61.00%	(578.07)	156.00%	(187.05)	251.00%	(157.26)
63	2000.00%	(36.77)	−480.00%	288.75	320.00%	(141.76)	62.00%	(561.60)	157.00%	(186.53)	252.00%	(157.02)
64	−2000.00%	45.41	−520.00%	252.65	330.00%	(139.69)	63.00%	(545.83)	158.00%	(186.03)	253.00%	(156.78)
65	−6000.00%	14.07	−560.00%	224.79	340.00%	(137.66)	64.00%	(530.73)	159.00%	(185.54)	254.00%	(156.54)
66	−10000.00%	8.32	−600.00%	202.56	350.00%	(135.66)	65.00%	(516.28)	160.00%	(185.05)	255.00%	(156.30)
67	−14000.00%	5.91	−640.00%	184.37	360.00%	(133.71)	66.00%	(502.45)	161.00%	(184.58)	256.00%	(156.06)
68	−18000.00%	4.58	−680.00%	169.20	370.00%	(131.81)	67.00%	(489.21)	162.00%	(184.11)	257.00%	(155.83)
69	−22000.00%	3.74	−720.00%	156.34	380.00%	(129.94)	68.00%	(476.54)	163.00%	(183.65)	258.00%	(155.59)
70	−26000.00%	3.16	−760.00%	145.30	390.00%	(128.11)	69.00%	(464.41)	164.00%	(183.21)	259.00%	(155.35)
71	−30000.00%	2.74	−800.00%	135.71	400.00%	(126.33)	70.00%	(452.80)	165.00%	(182.76)	260.00%	(155.12)
72	−34000.00%	2.41	−840.00%	127.31	410.00%	(124.58)	71.00%	(441.70)	166.00%	(182.33)	261.00%	(154.88)
73	−38000.00%	2.16	−880.00%	119.88	420.00%	(122.88)	72.00%	(431.07)	167.00%	(181.91)	262.00%	(154.64)
74	−42000.00%	1.95	−920.00%	113.27	430.00%	(121.21)	73.00%	(420.89)	168.00%	(181.49)	263.00%	(154.41)
75	−46000.00%	1.78	−960.00%	107.35	440.00%	(119.58)	74.00%	(411.16)	169.00%	(181.08)	264.00%	(154.18)
76	−50000.00%	1.64	−1000.00%	102.02	450.00%	(117.98)	75.00%	(401.84)	170.00%	(180.67)	265.00%	(153.94)
77	−54000.00%	1.52	−1040.00%	97.19	460.00%	(116.42)	76.00%	(392.92)	171.00%	(180.27)	266.00%	(153.71)
78	−58000.00%	1.41	−1080.00%	92.79	470.00%	(114.90)	77.00%	(384.38)	172.00%	(179.88)	267.00%	(153.48)
79	−62000.00%	1.32	−1120.00%	88.77	480.00%	(113.41)	78.00%	(376.20)	173.00%	(179.49)	268.00%	(153.24)
80	−66000.00%	1.24	−1160.00%	85.09	490.00%	(111.95)	79.00%	(368.38)	174.00%	(179.11)	269.00%	(153.01)
81	−70000.00%	1.17	−1200.00%	81.70	500.00%	(110.53)	80.00%	(360.89)	175.00%	(178.73)	270.00%	(152.78)
82	−74000.00%	1.10	−1240.00%	78.56	510.00%	(109.14)	81.00%	(353.72)	176.00%	(178.36)	271.00%	(152.55)
83	−78000.00%	1.05	−1280.00%	75.66	520.00%	(107.78)	82.00%	(346.85)	177.00%	(177.99)	272.00%	(152.32)
84	−82000.00%	1.00	−1320.00%	72.97	530.00%	(106.45)	83.00%	(340.27)	178.00%	(177.63)	273.00%	(152.09)
85	−86000.00%	0.95	−1360.00%	70.45	540.00%	(105.15)	84.00%	(333.97)	179.00%	(177.27)	274.00%	(151.86)
86	−90000.00%	0.91	−1400.00%	68.11	550.00%	(103.88)	85.00%	(327.94)	180.00%	(176.92)	275.00%	(151.63)
87	−94000.00%	0.87	−1440.00%	65.92	560.00%	(102.64)	86.00%	(322.16)	181.00%	(176.57)	276.00%	(151.40)
88	−98000.00%	0.83	−1480.00%	63.86	570.00%	(101.42)	87.00%	(316.62)	182.00%	(176.23)	277.00%	(151.17)
89	−102000.00%	0.80	−1520.00%	61.92	580.00%	(100.23)	88.00%	(311.31)	183.00%	(175.89)	278.00%	(150.94)
90	−106000.00%	0.77	−1560.00%	60.10	590.00%	(99.06)	89.00%	(306.22)	184.00%	(175.55)	279.00%	(150.72)
91	−110000.00%	0.74	−1600.00%	58.39	600.00%	(97.92)	90.00%	(301.35)	185.00%	(175.22)	280.00%	(150.49)
92	−114000.00%	0.72	−1640.00%	56.77	610.00%	(96.81)	91.00%	(296.68)	186.00%	(174.89)	281.00%	(150.26)
93	−118000.00%	0.69	−1680.00%	55.23	620.00%	(95.72)	92.00%	(292.20)	187.00%	(174.56)	282.00%	(150.04)
94	−122000.00%	0.67	−1720.00%	53.78	630.00%	(94.65)	93.00%	(287.90)	188.00%	(174.24)	283.00%	(149.81)
95	−126000.00%	0.65	−1760.00%	52.40	640.00%	(93.60)	94.00%	(283.78)	189.00%	(173.92)	284.00%	(149.59)

(*continued*)

Chart 4A (continued)

Table-B2

Project Outcome/Cashflow:

0	1	2	3	4	5	6	7	8	9
(815.00)	900.00	(100.00)	1,200.00	(1,200.00)	100.00	(10,000.00)	6,000.00	(7,000.00)	11,000.00

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.33)	2000.00%	(36.77)	−300.00%	885.04	0.00%	85.00	95.00%	(220.40)	190.00%	(171.65)
2	246000.00%	(0.33)	1960.00%	(37.45)	−290.00%	1,022.34	1.00%	(144.60)	96.00%	(218.92)	191.00%	(171.40)
3	242000.00%	(0.34)	1920.00%	(38.15)	−280.00%	1,212.35	2.00%	(335.78)	97.00%	(217.50)	192.00%	(171.16)
4	238000.00%	(0.34)	1880.00%	(38.87)	−270.00%	1,486.82	3.00%	(493.98)	98.00%	(216.13)	193.00%	(170.91)
5	234000.00%	(0.35)	1840.00%	(39.62)	−260.00%	1,902.99	4.00%	(623.91)	99.00%	(214.82)	194.00%	(170.66)
6	230000.00%	(0.35)	1800.00%	(40.41)	−250.00%	2,569.03	5.00%	(729.61)	100.00%	(213.55)	195.00%	(170.41)
7	226000.00%	(0.36)	1760.00%	(41.22)	−240.00%	3,700.84	6.00%	(814.58)	101.00%	(212.34)	196.00%	(170.17)
8	222000.00%	(0.37)	1720.00%	(42.07)	−230.00%	5,756.27	7.00%	(881.83)	102.00%	(211.17)	197.00%	(169.92)
9	218000.00%	(0.37)	1680.00%	(42.95)	−220.00%	9,775.92	8.00%	(933.96)	103.00%	(210.04)	198.00%	(169.67)
10	214000.00%	(0.38)	1640.00%	(43.87)	−210.00%	18,321.30	9.00%	(973.21)	104.00%	(208.95)	199.00%	(169.43)
11	210000.00%	(0.39)	1600.00%	(44.83)	−200.00%	38,315.00	10.00%	(1,001.53)	105.00%	(207.90)	200.00%	(169.18)
12	206000.00%	(0.40)	1560.00%	(45.84)	−190.00%	90,664.98	11.00%	(1,020.56)	106.00%	(206.90)	201.00%	(168.94)
13	202000.00%	(0.40)	1520.00%	(46.89)	−180.00%	247,639.63	12.00%	(1,031.77)	107.00%	(205.92)	202.00%	(168.70)
14	198000.00%	(0.41)	1480.00%	(47.98)	−170.00%	804,668.01	13.00%	(1,036.38)	108.00%	(204.98)	203.00%	(168.45)
15	194000.00%	(0.42)	1440.00%	(49.13)	−160.00%	3,259,407.11	14.00%	(1,035.47)	109.00%	(204.08)	204.00%	(168.21)
16	190000.00%	(0.43)	1400.00%	(50.34)	−150.00%	17,734,030.00	15.00%	(1,029.96)	110.00%	(203.20)	205.00%	(167.97)
17	186000.00%	(0.44)	1360.00%	(51.61)	−140.00%	147,063,546.29	16.00%	(1,020.64)	111.00%	(202.36)	206.00%	(167.72)
18	182000.00%	(0.45)	1320.00%	(52.94)	−130.00%	2.36E+09	17.00%	(1,008.20)	112.00%	(201.54)	207.00%	(167.48)
19	178000.00%	(0.46)	1280.00%	(54.34)	−120.00%	1.24E+11	18.00%	(993.22)	113.00%	(200.75)	208.00%	(167.24)
20	174000.00%	(0.47)	1240.00%	(55.82)	−110.00%	1.18E+14	19.00%	(976.21)	114.00%	(199.99)	209.00%	(167.00)
21	170000.00%	(0.48)	1200.00%	(57.37)	−100.00%	−8.15E+02	20.00%	(957.59)	115.00%	(199.25)	210.00%	(166.75)
22	166000.00%	(0.49)	1160.00%	(59.02)	−90.00%	1.03E+14	21.00%	(937.74)	116.00%	(198.53)	211.00%	(166.51)
23	162000.00%	(0.50)	1120.00%	(60.76)	−80.00%	9.53E+10	22.00%	(916.97)	117.00%	(197.84)	212.00%	(166.27)
24	158000.00%	(0.52)	1080.00%	(62.61)	−70.00%	1.55E+09	23.00%	(895.54)	118.00%	(197.17)	213.00%	(166.03)
25	154000.00%	(0.53)	1040.00%	(64.57)	−60.00%	81,209,178.32	24.00%	(873.69)	119.00%	(196.52)	214.00%	(165.79)
26	150000.00%	(0.54)	1000.00%	(66.65)	−50.00%	7,924,370.00	25.00%	(851.61)	120.00%	(195.89)	215.00%	(165.55)
27	146000.00%	(0.56)	960.00%	(68.88)	−40.00%	1,121,244.89	26.00%	(829.45)	121.00%	(195.27)	216.00%	(165.31)
28	142000.00%	(0.57)	920.00%	(71.25)	−30.00%	197,690.40	27.00%	(807.37)	122.00%	(194.68)	217.00%	(165.07)
29	138000.00%	(0.59)	880.00%	(73.78)	−20.00%	38,211.73	28.00%	(785.45)	123.00%	(194.10)	218.00%	(164.83)
30	134000.00%	(0.61)	840.00%	(76.50)	−10.00%	6,563.62	29.00%	(763.81)	124.00%	(193.54)	219.00%	(164.59)
31	130000.00%	(0.63)	800.00%	(79.42)	0.00%	85.00	30.00%	(742.52)	125.00%	(192.99)	220.00%	(164.35)
32	126000.00%	(0.65)	760.00%	(82.56)	10.00%	(1,001.53)	31.00%	(721.64)	126.00%	(192.46)	221.00%	(164.11)
33	122000.00%	(0.67)	720.00%	(85.96)	20.00%	(957.59)	32.00%	(701.22)	127.00%	(191.94)	222.00%	(163.88)
34	118000.00%	(0.69)	680.00%	(89.63)	30.00%	(742.52)	33.00%	(681.30)	128.00%	(191.43)	223.00%	(163.64)
35	114000.00%	(0.71)	640.00%	(93.61)	40.00%	(557.47)	34.00%	(661.90)	129.00%	(190.94)	224.00%	(163.40)
36	110000.00%	(0.74)	600.00%	(97.93)	50.00%	(427.67)	35.00%	(643.06)	130.00%	(190.46)	225.00%	(163.16)
37	106000.00%	(0.77)	560.00%	(102.65)	60.00%	(342.25)	36.00%	(624.79)	131.00%	(189.99)	226.00%	(162.92)
38	102000.00%	(0.80)	520.00%	(107.80)	70.00%	(287.20)	37.00%	(607.09)	132.00%	(189.53)	227.00%	(162.69)
39	98000.00%	(0.83)	480.00%	(113.44)	80.00%	(251.78)	38.00%	(589.97)	133.00%	(189.09)	228.00%	(162.45)
40	94000.00%	(0.87)	440.00%	(119.61)	90.00%	(228.77)	39.00%	(573.43)	134.00%	(188.65)	229.00%	(162.21)
41	90000.00%	(0.90)	400.00%	(126.37)	100.00%	(213.55)	40.00%	(557.47)	135.00%	(188.22)	230.00%	(161.98)
42	86000.00%	(0.95)	360.00%	(133.76)	110.00%	(203.20)	41.00%	(542.08)	136.00%	(187.80)	231.00%	(161.74)
43	82000.00%	(0.99)	320.00%	(141.80)	120.00%	(195.89)	42.00%	(527.27)	137.00%	(187.39)	232.00%	(161.50)
44	78000.00%	(1.04)	280.00%	(150.46)	130.00%	(190.46)	43.00%	(513.00)	138.00%	(186.99)	233.00%	(161.27)
45	74000.00%	(1.10)	240.00%	(159.62)	140.00%	(186.21)	44.00%	(499.28)	139.00%	(186.60)	234.00%	(161.03)
46	70000.00%	(1.16)	200.00%	(169.18)	150.00%	(182.69)	45.00%	(486.10)	140.00%	(186.21)	235.00%	(160.80)
47	66000.00%	(1.23)	160.00%	(179.62)	160.00%	(179.62)	46.00%	(473.43)	141.00%	(185.83)	236.00%	(160.56)
48	62000.00%	(1.31)	120.00%	(195.89)	170.00%	(176.82)	47.00%	(461.27)	142.00%	(185.46)	237.00%	(160.33)
49	58000.00%	(1.40)	80.00%	(251.78)	180.00%	(174.19)	48.00%	(449.60)	143.00%	(185.10)	238.00%	(160.09)
50	54000.00%	(1.50)	40.00%	(557.47)	190.00%	(171.65)	49.00%	(438.40)	144.00%	(184.74)	239.00%	(159.86)

51	50000.00%	(1.62)	0.00%	85.00	200.00%	(169.18)	50.00%	(427.67)	145.00%	(184.38)	240.00%	(159.62)
52	46000.00%	(1.76)	−40.00%	1,121,244.89	210.00%	(166.75)	51.00%	(417.37)	146.00%	(184.03)	241.00%	(159.39)
53	42000.00%	(1.93)	−80.00%	9.53E+10	220.00%	(164.35)	52.00%	(407.51)	147.00%	(183.69)	242.00%	(159.15)
54	38000.00%	(2.13)	−120.00%	1.24E+11	230.00%	(161.98)	53.00%	(398.06)	148.00%	(183.35)	243.00%	(158.92)
55	34000.00%	(2.38)	−160.00%	3,259,407.11	240.00%	(159.62)	54.00%	(389.01)	149.00%	(183.02)	244.00%	(158.69)
56	30000.00%	(2.70)	−200.00%	38,315.00	250.00%	(157.29)	55.00%	(380.34)	150.00%	(182.69)	245.00%	(158.45)
57	26000.00%	(3.11)	−240.00%	3,700.84	260.00%	(154.98)	56.00%	(372.04)	151.00%	(182.37)	246.00%	(158.22)
58	22000.00%	(3.67)	−280.00%	1,212.35	270.00%	(152.71)	57.00%	(364.10)	152.00%	(182.05)	247.00%	(157.99)
59	18000.00%	(4.48)	−320.00%	702.15	280.00%	(150.46)	58.00%	(356.49)	153.00%	(181.73)	248.00%	(157.75)
60	14000.00%	(5.73)	−360.00%	506.36	290.00%	(148.24)	59.00%	(349.22)	154.00%	(181.42)	249.00%	(157.52)
61	10000.00%	(7.98)	−400.00%	401.29	300.00%	(146.06)	60.00%	(342.25)	155.00%	(181.11)	250.00%	(157.29)
62	6000.00%	(13.12)	−440.00%	334.20	310.00%	(143.91)	61.00%	(335.59)	156.00%	(180.81)	251.00%	(157.06)
63	2000.00%	(36.77)	−480.00%	287.00	320.00%	(141.80)	62.00%	(329.21)	157.00%	(180.51)	252.00%	(156.83)
64	−2000.00%	45.41	−520.00%	251.73	330.00%	(139.73)	63.00%	(323.11)	158.00%	(180.21)	253.00%	(156.60)
65	−6000.00%	14.07	−560.00%	224.28	340.00%	(137.70)	64.00%	(317.27)	159.00%	(179.91)	254.00%	(156.36)
66	−10000.00%	8.32	−600.00%	202.26	350.00%	(135.71)	65.00%	(311.69)	160.00%	(179.62)	255.00%	(156.13)
67	−14000.00%	5.91	−640.00%	184.19	360.00%	(133.76)	66.00%	(306.35)	161.00%	(179.33)	256.00%	(155.90)
68	−18000.00%	4.58	−680.00%	169.08	370.00%	(131.86)	67.00%	(301.24)	162.00%	(179.04)	257.00%	(155.67)
69	−22000.00%	3.74	−720.00%	156.26	380.00%	(129.99)	68.00%	(296.35)	163.00%	(178.76)	258.00%	(155.44)
70	−26000.00%	3.16	−760.00%	145.24	390.00%	(128.16)	69.00%	(291.67)	164.00%	(178.47)	259.00%	(155.21)
71	−30000.00%	2.74	−800.00%	135.67	400.00%	(126.37)	70.00%	(287.20)	165.00%	(178.19)	260.00%	(154.98)
72	−34000.00%	2.41	−840.00%	127.28	410.00%	(124.62)	71.00%	(282.92)	166.00%	(177.91)	261.00%	(154.76)
73	−38000.00%	2.16	−880.00%	119.86	420.00%	(122.92)	72.00%	(278.82)	167.00%	(177.64)	262.00%	(154.53)
74	−42000.00%	1.95	−920.00%	113.26	430.00%	(121.24)	73.00%	(274.90)	168.00%	(177.36)	263.00%	(154.30)
75	−46000.00%	1.78	−960.00%	107.34	440.00%	(119.61)	74.00%	(271.15)	169.00%	(177.09)	264.00%	(154.07)
76	−50000.00%	1.64	−1000.00%	102.01	450.00%	(118.01)	75.00%	(267.56)	170.00%	(176.82)	265.00%	(153.84)
77	−54000.00%	1.52	−1040.00%	97.18	460.00%	(116.45)	76.00%	(264.12)	171.00%	(176.55)	266.00%	(153.61)
78	−58000.00%	1.41	−1080.00%	92.79	470.00%	(114.93)	77.00%	(260.83)	172.00%	(176.28)	267.00%	(153.39)
79	−62000.00%	1.32	−1120.00%	88.77	480.00%	(113.44)	78.00%	(257.68)	173.00%	(176.02)	268.00%	(153.16)
80	−66000.00%	1.24	−1160.00%	85.09	490.00%	(111.98)	79.00%	(254.66)	174.00%	(175.75)	269.00%	(152.93)
81	−70000.00%	1.17	−1200.00%	81.69	500.00%	(110.56)	80.00%	(251.78)	175.00%	(175.49)	270.00%	(152.71)
82	−74000.00%	1.10	−1240.00%	78.56	510.00%	(109.16)	81.00%	(249.01)	176.00%	(175.22)	271.00%	(152.48)
83	−78000.00%	1.05	−1280.00%	75.66	520.00%	(107.80)	82.00%	(246.36)	177.00%	(174.96)	272.00%	(152.25)
84	−82000.00%	1.00	−1320.00%	72.96	530.00%	(106.47)	83.00%	(243.82)	178.00%	(174.70)	273.00%	(152.03)
85	−86000.00%	0.95	−1360.00%	70.45	540.00%	(105.17)	84.00%	(241.39)	179.00%	(174.44)	274.00%	(151.80)
86	−90000.00%	0.91	−1400.00%	68.11	550.00%	(103.90)	85.00%	(239.06)	180.00%	(174.19)	275.00%	(151.58)
87	−94000.00%	0.87	−1440.00%	65.91	560.00%	(102.65)	86.00%	(236.83)	181.00%	(173.93)	276.00%	(151.35)
88	−98000.00%	0.83	−1480.00%	63.86	570.00%	(101.43)	87.00%	(234.69)	182.00%	(173.67)	277.00%	(151.13)
89	−102000.00%	0.80	−1520.00%	61.92	580.00%	(100.24)	88.00%	(232.63)	183.00%	(173.42)	278.00%	(150.90)
90	−106000.00%	0.77	−1560.00%	60.10	590.00%	(99.08)	89.00%	(230.66)	184.00%	(173.16)	279.00%	(150.68)
91	−110000.00%	0.74	−1600.00%	58.39	600.00%	(97.93)	90.00%	(228.77)	185.00%	(172.91)	280.00%	(150.46)
92	−114000.00%	0.72	−1640.00%	56.77	610.00%	(96.82)	91.00%	(226.96)	186.00%	(172.66)	281.00%	(150.23)
93	−118000.00%	0.69	−1680.00%	55.23	620.00%	(95.73)	92.00%	(225.22)	187.00%	(172.41)	282.00%	(150.01)
94	−122000.00%	0.67	−1720.00%	53.78	630.00%	(94.66)	93.00%	(223.55)	188.00%	(172.15)	283.00%	(149.79)
95	−126000.00%	0.65	−1760.00%	52.40	640.00%	(93.61)	94.00%	(221.95)	189.00%	(171.90)	284.00%	(149.57)

(*continued*)

Chart 4A (continued)

Table-B3

Project Outcome/Cashflow:

0	1	2	3	4	5	6
(815.00)	900.00	(100.00)	1,200.00	(1,200.00)	100.00	(100.00)

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.33)	2000.00%	(36.77)	−300.00%	759.84	0.00%	(15.00)	95.00%	(153.43)	190.00%	(166.89)
2	246000.00%	(0.33)	1960.00%	(37.45)	−290.00%	836.62	1.00%	(9.37)	96.00%	(154.13)	191.00%	(166.75)
3	242000.00%	(0.34)	1920.00%	(38.15)	−280.00%	930.09	2.00%	(4.72)	97.00%	(154.81)	192.00%	(166.61)
4	238000.00%	(0.34)	1880.00%	(38.87)	−270.00%	1,045.96	3.00%	(0.95)	98.00%	(155.47)	193.00%	(166.47)
5	234000.00%	(0.35)	1840.00%	(39.62)	−260.00%	1,192.58	4.00%	2.04	99.00%	(156.10)	194.00%	(166.32)
6	230000.00%	(0.35)	1800.00%	(40.41)	−250.00%	1,382.66	5.00%	4.32	100.00%	(156.72)	195.00%	(166.17)
7	226000.00%	(0.36)	1760.00%	(41.22)	−240.00%	1,636.03	6.00%	5.96	101.00%	(157.32)	196.00%	(166.02)
8	222000.00%	(0.37)	1720.00%	(42.07)	−230.00%	1,984.99	7.00%	7.03	102.00%	(157.89)	197.00%	(165.87)
9	218000.00%	(0.37)	1680.00%	(42.95)	−220.00%	2,484.39	8.00%	7.60	103.00%	(158.45)	198.00%	(165.72)
10	214000.00%	(0.38)	1640.00%	(43.87)	−210.00%	3,232.33	9.00%	7.70	104.00%	(159.00)	199.00%	(165.56)
11	210000.00%	(0.39)	1600.00%	(44.83)	−200.00%	4,415.00	10.00%	7.40	105.00%	(159.52)	200.00%	(165.40)
12	206000.00%	(0.40)	1560.00%	(45.84)	−190.00%	6,412.28	11.00%	6.74	106.00%	(160.02)	201.00%	(165.24)
13	202000.00%	(0.40)	1520.00%	(46.89)	−180.00%	10,070.42	12.00%	5.76	107.00%	(160.51)	202.00%	(165.08)
14	198000.00%	(0.41)	1480.00%	(47.98)	−170.00%	17,494.62	13.00%	4.48	108.00%	(160.99)	203.00%	(164.92)
15	194000.00%	(0.42)	1440.00%	(49.13)	−160.00%	34,728.25	14.00%	2.96	109.00%	(161.44)	204.00%	(164.75)
16	190000.00%	(0.43)	1400.00%	(50.34)	−150.00%	82,830.00	15.00%	1.21	110.00%	(161.88)	205.00%	(164.59)
17	186000.00%	(0.44)	1360.00%	(51.61)	−140.00%	258,736.72	16.00%	(0.73)	111.00%	(162.30)	206.00%	(164.42)
18	182000.00%	(0.45)	1320.00%	(52.94)	−130.00%	1,252,817.26	17.00%	(2.85)	112.00%	(162.71)	207.00%	(164.25)
19	178000.00%	(0.46)	1280.00%	(54.34)	−120.00%	13,914,075.00	18.00%	(5.11)	113.00%	(163.11)	208.00%	(164.08)
20	174000.00%	(0.47)	1240.00%	(55.82)	−110.00%	1.23E+09	19.00%	(7.50)	114.00%	(163.49)	209.00%	(163.90)
21	170000.00%	(0.48)	1200.00%	(57.37)	−100.00%	−8.15E+02	20.00%	(10.00)	115.00%	(163.85)	210.00%	(163.73)
22	166000.00%	(0.49)	1160.00%	(59.02)	−90.00%	−1.01E+09	21.00%	(12.60)	116.00%	(164.20)	211.00%	(163.55)
23	162000.00%	(0.50)	1120.00%	(60.76)	−80.00%	(9,244,075.00)	22.00%	(15.28)	117.00%	(164.54)	212.00%	(163.38)
24	158000.00%	(0.52)	1080.00%	(62.61)	−70.00%	(662,172.54)	23.00%	(18.02)	118.00%	(164.86)	213.00%	(163.20)
25	154000.00%	(0.53)	1040.00%	(64.57)	−60.00%	(104,908.59)	24.00%	(20.82)	119.00%	(165.17)	214.00%	(163.02)
26	150000.00%	(0.54)	1000.00%	(66.65)	−50.00%	(24,430.00)	25.00%	(23.65)	120.00%	(165.47)	215.00%	(162.84)
27	146000.00%	(0.56)	960.00%	(68.87)	−40.00%	(6,923.03)	26.00%	(26.52)	121.00%	(165.76)	216.00%	(162.66)
28	142000.00%	(0.57)	920.00%	(71.25)	−30.00%	(2,125.34)	27.00%	(29.42)	122.00%	(166.03)	217.00%	(162.47)
29	138000.00%	(0.59)	880.00%	(73.78)	−20.00%	(635.60)	28.00%	(32.32)	123.00%	(166.29)	218.00%	(162.29)
30	134000.00%	(0.61)	840.00%	(76.50)	−10.00%	(155.75)	29.00%	(35.24)	124.00%	(166.54)	219.00%	(162.10)
31	130000.00%	(0.63)	800.00%	(79.42)	0.00%	(15.00)	30.00%	(38.16)	125.00%	(166.78)	220.00%	(161.92)
32	126000.00%	(0.65)	760.00%	(82.56)	10.00%	7.40	31.00%	(41.07)	126.00%	(167.01)	221.00%	(161.73)
33	122000.00%	(0.67)	720.00%	(85.95)	20.00%	(10.00)	32.00%	(43.97)	127.00%	(167.23)	222.00%	(161.54)
34	118000.00%	(0.69)	680.00%	(89.62)	30.00%	(38.16)	33.00%	(46.86)	128.00%	(167.43)	223.00%	(161.35)
35	114000.00%	(0.71)	640.00%	(93.60)	40.00%	(66.36)	34.00%	(49.73)	129.00%	(167.63)	224.00%	(161.16)
36	110000.00%	(0.74)	600.00%	(97.92)	50.00%	(91.02)	35.00%	(52.57)	130.00%	(167.82)	225.00%	(160.97)
37	106000.00%	(0.77)	560.00%	(102.63)	60.00%	(111.33)	36.00%	(55.39)	131.00%	(167.99)	226.00%	(160.78)
38	102000.00%	(0.80)	520.00%	(107.78)	70.00%	(127.48)	37.00%	(58.18)	132.00%	(168.16)	227.00%	(160.58)
39	98000.00%	(0.83)	480.00%	(113.40)	80.00%	(140.03)	38.00%	(60.94)	133.00%	(168.31)	228.00%	(160.39)
40	94000.00%	(0.87)	440.00%	(119.54)	90.00%	(149.60)	39.00%	(63.67)	134.00%	(168.46)	229.00%	(160.20)
41	90000.00%	(0.90)	400.00%	(126.26)	100.00%	(156.72)	40.00%	(66.36)	135.00%	(168.60)	230.00%	(160.00)
42	86000.00%	(0.95)	360.00%	(133.56)	110.00%	(161.88)	41.00%	(69.01)	136.00%	(168.73)	231.00%	(159.81)
43	82000.00%	(0.99)	320.00%	(141.42)	120.00%	(165.47)	42.00%	(71.63)	137.00%	(168.85)	232.00%	(159.61)
44	78000.00%	(1.04)	280.00%	(149.70)	130.00%	(167.82)	43.00%	(74.20)	138.00%	(168.96)	233.00%	(159.41)
45	74000.00%	(1.10)	240.00%	(158.01)	140.00%	(169.16)	44.00%	(76.73)	139.00%	(169.07)	234.00%	(159.21)
46	70000.00%	(1.16)	200.00%	(165.40)	150.00%	(169.72)	45.00%	(79.22)	140.00%	(169.16)	235.00%	(159.01)
47	66000.00%	(1.23)	160.00%	(169.66)	160.00%	(169.66)	46.00%	(81.67)	141.00%	(169.25)	236.00%	(158.81)
48	62000.00%	(1.31)	120.00%	(165.47)	170.00%	(169.10)	47.00%	(84.07)	142.00%	(169.33)	237.00%	(158.61)
49	58000.00%	(1.40)	80.00%	(140.03)	180.00%	(168.15)	48.00%	(86.44)	143.00%	(169.41)	238.00%	(158.41)
50	54000.00%	(1.50)	40.00%	(66.36)	190.00%	(166.89)	49.00%	(88.75)	144.00%	(169.47)	239.00%	(158.21)

51	50000.00%	(1.62)	0.00%	(15.00)	200.00%	(165.40)	50.00%	(91.02)	145.00%	(169.53)	240.00%	(158.01)
52	46000.00%	(1.76)	−40.00%	(6,923.03)	210.00%	(163.73)	51.00%	(93.25)	146.00%	(169.58)	241.00%	(157.81)
53	42000.00%	(1.93)	−80.00%	(9,244,075.00)	220.00%	(161.92)	52.00%	(95.44)	147.00%	(169.63)	242.00%	(157.61)
54	38000.00%	(2.13)	−120.00%	13,914,075.00	230.00%	(160.00)	53.00%	(97.57)	148.00%	(169.67)	243.00%	(157.40)
55	34000.00%	(2.38)	−160.00%	34,728.25	240.00%	(158.01)	54.00%	(99.67)	149.00%	(169.70)	244.00%	(157.20)
56	30000.00%	(2.70)	−200.00%	4,415.00	250.00%	(155.97)	55.00%	(101.72)	150.00%	(169.72)	245.00%	(157.00)
57	26000.00%	(3.11)	−240.00%	1,636.03	260.00%	(153.89)	56.00%	(103.73)	151.00%	(169.74)	246.00%	(156.79)
58	22000.00%	(3.67)	−280.00%	930.09	270.00%	(151.80)	57.00%	(105.69)	152.00%	(169.75)	247.00%	(156.59)
59	18000.00%	(4.48)	−320.00%	641.59	280.00%	(149.70)	58.00%	(107.61)	153.00%	(169.76)	248.00%	(156.38)
60	14000.00%	(5.73)	−360.00%	489.09	290.00%	(147.61)	59.00%	(109.49)	154.00%	(169.76)	249.00%	(156.18)
61	10000.00%	(7.98)	−400.00%	395.31	300.00%	(145.53)	60.00%	(111.33)	155.00%	(169.76)	250.00%	(155.97)
62	6000.00%	(13.12)	−440.00%	331.81	310.00%	(143.47)	61.00%	(113.12)	156.00%	(169.75)	251.00%	(155.76)
63	2000.00%	(36.77)	−480.00%	285.93	320.00%	(141.42)	62.00%	(114.87)	157.00%	(169.73)	252.00%	(155.56)
64	−2000.00%	45.41	−520.00%	251.21	330.00%	(139.41)	63.00%	(116.59)	158.00%	(169.71)	253.00%	(155.35)
65	−6000.00%	14.07	−560.00%	224.01	340.00%	(137.43)	64.00%	(118.26)	159.00%	(169.69)	254.00%	(155.14)
66	−10000.00%	8.32	−600.00%	202.11	350.00%	(135.48)	65.00%	(119.89)	160.00%	(169.66)	255.00%	(154.94)
67	−14000.00%	5.91	−640.00%	184.10	360.00%	(133.56)	66.00%	(121.48)	161.00%	(169.62)	256.00%	(154.73)
68	−18000.00%	4.58	−680.00%	169.03	370.00%	(131.68)	67.00%	(123.04)	162.00%	(169.58)	257.00%	(154.52)
69	−22000.00%	3.74	−720.00%	156.23	380.00%	(129.84)	68.00%	(124.56)	163.00%	(169.53)	258.00%	(154.31)
70	−26000.00%	3.16	−760.00%	145.22	390.00%	(128.03)	69.00%	(126.04)	164.00%	(169.48)	259.00%	(154.10)
71	−30000.00%	2.74	−800.00%	135.66	400.00%	(126.26)	70.00%	(127.48)	165.00%	(169.43)	260.00%	(153.89)
72	−34000.00%	2.41	−840.00%	127.27	410.00%	(124.53)	71.00%	(128.89)	166.00%	(169.37)	261.00%	(153.69)
73	−38000.00%	2.16	−880.00%	119.86	420.00%	(122.83)	72.00%	(130.26)	167.00%	(169.31)	262.00%	(153.48)
74	−42000.00%	1.95	−920.00%	113.25	430.00%	(121.17)	73.00%	(131.60)	168.00%	(169.24)	263.00%	(153.27)
75	−46000.00%	1.78	−960.00%	107.34	440.00%	(119.54)	74.00%	(132.90)	169.00%	(169.17)	264.00%	(153.06)
76	−50000.00%	1.64	−1000.00%	102.01	450.00%	(117.95)	75.00%	(134.17)	170.00%	(169.10)	265.00%	(152.85)
77	−54000.00%	1.52	−1040.00%	97.18	460.00%	(116.40)	76.00%	(135.41)	171.00%	(169.02)	266.00%	(152.64)
78	−58000.00%	1.41	−1080.00%	92.78	470.00%	(114.88)	77.00%	(136.61)	172.00%	(168.93)	267.00%	(152.43)
79	−62000.00%	1.32	−1120.00%	88.77	480.00%	(113.40)	78.00%	(137.78)	173.00%	(168.85)	268.00%	(152.22)
80	−66000.00%	1.24	−1160.00%	85.08	490.00%	(111.94)	79.00%	(138.92)	174.00%	(168.76)	269.00%	(152.01)
81	−70000.00%	1.17	−1200.00%	81.69	500.00%	(110.52)	80.00%	(140.03)	175.00%	(168.66)	270.00%	(151.80)
82	−74000.00%	1.10	−1240.00%	78.56	510.00%	(109.13)	81.00%	(141.12)	176.00%	(168.57)	271.00%	(151.59)
83	−78000.00%	1.05	−1280.00%	75.66	520.00%	(107.78)	82.00%	(142.17)	177.00%	(168.47)	272.00%	(151.38)
84	−82000.00%	1.00	−1320.00%	72.96	530.00%	(106.45)	83.00%	(143.19)	178.00%	(168.36)	273.00%	(151.17)
85	−86000.00%	0.95	−1360.00%	70.45	540.00%	(105.15)	84.00%	(144.19)	179.00%	(168.26)	274.00%	(150.96)
86	−90000.00%	0.91	−1400.00%	68.11	550.00%	(103.88)	85.00%	(145.15)	180.00%	(168.15)	275.00%	(150.75)
87	−94000.00%	0.87	−1440.00%	65.91	560.00%	(102.63)	86.00%	(146.09)	181.00%	(168.03)	276.00%	(150.54)
88	−98000.00%	0.83	−1480.00%	63.86	570.00%	(101.42)	87.00%	(147.01)	182.00%	(167.92)	277.00%	(150.33)
89	−102000.00%	0.80	−1520.00%	61.92	580.00%	(100.23)	88.00%	(147.89)	183.00%	(167.80)	278.00%	(150.12)
90	−106000.00%	0.77	−1560.00%	60.10	590.00%	(99.06)	89.00%	(148.76)	184.00%	(167.68)	279.00%	(149.91)
91	−110000.00%	0.74	−1600.00%	58.39	600.00%	(97.92)	90.00%	(149.60)	185.00%	(167.55)	280.00%	(149.70)
92	−114000.00%	0.72	−1640.00%	56.77	610.00%	(96.81)	91.00%	(150.41)	186.00%	(167.43)	281.00%	(149.49)
93	−118000.00%	0.69	−1680.00%	55.23	620.00%	(95.72)	92.00%	(151.20)	187.00%	(167.30)	282.00%	(149.28)
94	−122000.00%	0.67	−1720.00%	53.78	630.00%	(94.65)	93.00%	(151.97)	188.00%	(167.16)	283.00%	(149.08)
95	−126000.00%	0.65	−1760.00%	52.40	640.00%	(93.60)	94.00%	(152.71)	189.00%	(167.03)	284.00%	(148.87)

(*continued*)

Chart 4A (continued)

Table-B4
Project Outcome/Cashflow:

0	1	2	3	4
(815.00)	900.00	(100.00)	1,200.00	(1,200.00)

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.33)	2000.00%	(36.77)	−300.00%	757.50	0.00%	(15.00)	95.00%	(154.32)	190.00%	(167.00)
2	246000.00%	(0.33)	1960.00%	(37.45)	−290.00%	833.38	1.00%	(10.31)	96.00%	(154.99)	191.00%	(166.86)
3	242000.00%	(0.34)	1920.00%	(38.15)	−280.00%	925.52	2.00%	(6.46)	97.00%	(155.65)	192.00%	(166.72)
4	238000.00%	(0.34)	1880.00%	(38.87)	−270.00%	1,039.38	3.00%	(3.39)	98.00%	(156.29)	193.00%	(166.57)
5	234000.00%	(0.35)	1840.00%	(39.62)	−260.00%	1,182.90	4.00%	(1.00)	99.00%	(156.90)	194.00%	(166.42)
6	230000.00%	(0.35)	1800.00%	(40.41)	−250.00%	1,368.02	5.00%	0.76	100.00%	(157.50)	195.00%	(166.27)
7	226000.00%	(0.36)	1760.00%	(41.22)	−240.00%	1,613.26	6.00%	1.97	101.00%	(158.08)	196.00%	(166.12)
8	222000.00%	(0.37)	1720.00%	(42.07)	−230.00%	1,948.33	7.00%	2.67	102.00%	(158.64)	197.00%	(165.97)
9	218000.00%	(0.37)	1680.00%	(42.95)	−220.00%	2,422.99	8.00%	2.93	103.00%	(159.18)	198.00%	(165.81)
10	214000.00%	(0.38)	1640.00%	(43.87)	−210.00%	3,124.56	9.00%	2.78	104.00%	(159.70)	199.00%	(165.65)
11	210000.00%	(0.39)	1600.00%	(44.83)	−200.00%	4,215.00	10.00%	2.27	105.00%	(160.21)	200.00%	(165.49)
12	206000.00%	(0.40)	1560.00%	(45.84)	−190.00%	6,015.04	11.00%	1.44	106.00%	(160.70)	201.00%	(165.33)
13	202000.00%	(0.40)	1520.00%	(46.89)	−180.00%	9,212.11	12.00%	0.33	107.00%	(161.17)	202.00%	(165.17)
14	198000.00%	(0.41)	1480.00%	(47.98)	−170.00%	15,430.37	13.00%	(1.04)	108.00%	(161.63)	203.00%	(165.00)
15	194000.00%	(0.42)	1440.00%	(49.13)	−160.00%	29,012.65	14.00%	(2.63)	109.00%	(162.07)	204.00%	(164.84)
16	190000.00%	(0.43)	1400.00%	(50.34)	−150.00%	63,630.00	15.00%	(4.43)	110.00%	(162.49)	205.00%	(164.67)
17	186000.00%	(0.44)	1360.00%	(51.61)	−140.00%	173,287.50	16.00%	(6.39)	111.00%	(162.90)	206.00%	(164.50)
18	182000.00%	(0.45)	1320.00%	(52.94)	−130.00%	658,395.68	17.00%	(8.51)	112.00%	(163.30)	207.00%	(164.33)
19	178000.00%	(0.46)	1280.00%	(54.34)	−120.00%	4,539,075.00	18.00%	(10.76)	113.00%	(163.68)	208.00%	(164.16)
20	174000.00%	(0.47)	1240.00%	(55.82)	−110.00%	132,198,150.00	19.00%	(13.12)	114.00%	(164.04)	209.00%	(163.98)
21	170000.00%	(0.48)	1200.00%	(57.37)	−100.00%	−8.1500E+02	20.00%	(15.59)	115.00%	(164.39)	210.00%	(163.81)
22	166000.00%	(0.49)	1160.00%	(59.02)	−90.00%	−1.0802E+08	21.00%	(18.13)	116.00%	(164.73)	211.00%	(163.63)
23	162000.00%	(0.50)	1120.00%	(60.76)	−80.00%	(2,994,075.00)	22.00%	(20.75)	117.00%	(165.06)	212.00%	(163.45)
24	158000.00%	(0.52)	1080.00%	(62.61)	−70.00%	(342,099.38)	23.00%	(23.42)	118.00%	(165.37)	213.00%	(163.27)
25	154000.00%	(0.53)	1040.00%	(64.57)	−60.00%	(68,287.50)	24.00%	(26.14)	119.00%	(165.67)	214.00%	(163.09)
26	150000.00%	(0.54)	1000.00%	(66.65)	−50.00%	(18,030.00)	25.00%	(28.90)	120.00%	(165.95)	215.00%	(162.91)
27	146000.00%	(0.56)	960.00%	(68.87)	−40.00%	(5,494.14)	26.00%	(31.68)	121.00%	(166.23)	216.00%	(162.72)
28	142000.00%	(0.57)	920.00%	(71.25)	−30.00%	(1,761.06)	27.00%	(34.48)	122.00%	(166.49)	217.00%	(162.54)
29	138000.00%	(0.59)	880.00%	(73.78)	−20.00%	(540.23)	28.00%	(37.30)	123.00%	(166.74)	218.00%	(162.35)
30	134000.00%	(0.61)	840.00%	(76.50)	−10.00%	(134.84)	29.00%	(40.12)	124.00%	(166.98)	219.00%	(162.17)
31	130000.00%	(0.63)	800.00%	(79.42)	0.00%	(15.00)	30.00%	(42.94)	125.00%	(167.21)	220.00%	(161.98)
32	126000.00%	(0.65)	760.00%	(82.56)	10.00%	2.27	31.00%	(45.75)	126.00%	(167.43)	221.00%	(161.79)
33	122000.00%	(0.67)	720.00%	(85.95)	20.00%	(15.59)	32.00%	(48.55)	127.00%	(167.64)	222.00%	(161.60)
34	118000.00%	(0.69)	680.00%	(89.62)	30.00%	(42.94)	33.00%	(51.34)	128.00%	(167.83)	223.00%	(161.41)
35	114000.00%	(0.71)	640.00%	(93.60)	40.00%	(70.15)	34.00%	(54.11)	129.00%	(168.02)	224.00%	(161.22)
36	110000.00%	(0.74)	600.00%	(97.92)	50.00%	(93.95)	35.00%	(56.86)	130.00%	(168.20)	225.00%	(161.03)
37	106000.00%	(0.77)	560.00%	(102.63)	60.00%	(113.56)	36.00%	(59.58)	131.00%	(168.36)	226.00%	(160.84)
38	102000.00%	(0.80)	520.00%	(107.78)	70.00%	(129.19)	37.00%	(62.27)	132.00%	(168.52)	227.00%	(160.64)
39	98000.00%	(0.83)	480.00%	(113.40)	80.00%	(141.34)	38.00%	(64.93)	133.00%	(168.67)	228.00%	(160.45)
40	94000.00%	(0.87)	440.00%	(119.55)	90.00%	(150.60)	39.00%	(67.56)	134.00%	(168.81)	229.00%	(160.25)
41	90000.00%	(0.90)	400.00%	(126.26)	100.00%	(157.50)	40.00%	(70.15)	135.00%	(168.94)	230.00%	(160.06)
42	86000.00%	(0.95)	360.00%	(133.57)	110.00%	(162.49)	41.00%	(72.71)	136.00%	(169.06)	231.00%	(159.86)
43	82000.00%	(0.99)	320.00%	(141.44)	120.00%	(165.95)	42.00%	(75.23)	137.00%	(169.18)	232.00%	(159.66)
44	78000.00%	(1.04)	280.00%	(149.73)	130.00%	(168.20)	43.00%	(77.72)	138.00%	(169.28)	233.00%	(159.46)
45	74000.00%	(1.10)	240.00%	(158.06)	140.00%	(169.47)	44.00%	(80.16)	139.00%	(169.38)	234.00%	(159.26)
46	70000.00%	(1.16)	200.00%	(165.49)	150.00%	(169.97)	45.00%	(82.56)	140.00%	(169.47)	235.00%	(159.06)
47	66000.00%	(1.23)	160.00%	(169.86)	160.00%	(169.86)	46.00%	(84.92)	141.00%	(169.55)	236.00%	(158.86)
48	62000.00%	(1.31)	120.00%	(165.95)	170.00%	(169.26)	47.00%	(87.24)	142.00%	(169.62)	237.00%	(158.66)
49	58000.00%	(1.40)	80.00%	(141.34)	180.00%	(168.28)	48.00%	(89.52)	143.00%	(169.69)	238.00%	(158.46)
50	54000.00%	(1.50)	40.00%	(70.15)	190.00%	(167.00)	49.00%	(91.76)	144.00%	(169.75)	239.00%	(158.26)

51	50000.00%	(1.62)	0.00%	(15.00)	200.00%	(165.49)	50.00%	(93.95)	145.00%	(169.80)	240.00%	(158.06)
52	46000.00%	(1.76)	−40.00%	(5,494.14)	210.00%	(163.81)	51.00%	(96.10)	146.00%	(169.85)	241.00%	(157.85)
53	42000.00%	(1.93)	−80.00%	(2,994,075.00)	220.00%	(161.98)	52.00%	(98.21)	147.00%	(169.89)	242.00%	(157.65)
54	38000.00%	(2.13)	−120.00%	4,539,075.00	230.00%	(160.06)	53.00%	(100.28)	148.00%	(169.92)	243.00%	(157.45)
55	34000.00%	(2.38)	−160.00%	29,012.65	240.00%	(158.06)	54.00%	(102.30)	149.00%	(169.95)	244.00%	(157.24)
56	30000.00%	(2.70)	−200.00%	4,215.00	250.00%	(156.01)	55.00%	(104.28)	150.00%	(169.97)	245.00%	(157.04)
57	26000.00%	(3.11)	−240.00%	1,613.26	260.00%	(153.93)	56.00%	(106.22)	151.00%	(169.98)	246.00%	(156.83)
58	22000.00%	(3.67)	−280.00%	925.52	270.00%	(151.83)	57.00%	(108.12)	152.00%	(169.99)	247.00%	(156.63)
59	18000.00%	(4.48)	−320.00%	640.31	280.00%	(149.73)	58.00%	(109.97)	153.00%	(169.99)	248.00%	(156.42)
60	14000.00%	(5.73)	−360.00%	488.65	290.00%	(147.63)	59.00%	(111.79)	154.00%	(169.99)	249.00%	(156.21)
61	10000.00%	(7.98)	−400.00%	395.12	300.00%	(145.55)	60.00%	(113.56)	155.00%	(169.98)	250.00%	(156.01)
62	6000.00%	(13.12)	−440.00%	331.73	310.00%	(143.48)	61.00%	(115.30)	156.00%	(169.96)	251.00%	(155.80)
63	2000.00%	(36.77)	−480.00%	285.89	320.00%	(141.44)	62.00%	(116.99)	157.00%	(169.95)	252.00%	(155.59)
64	−2000.00%	45.41	−520.00%	251.19	330.00%	(139.42)	63.00%	(118.65)	158.00%	(169.92)	253.00%	(155.39)
65	−6000.00%	14.07	−560.00%	224.00	340.00%	(137.44)	64.00%	(120.26)	159.00%	(169.89)	254.00%	(155.18)
66	−10000.00%	8.32	−600.00%	202.10	350.00%	(135.49)	65.00%	(121.84)	160.00%	(169.86)	255.00%	(154.97)
67	−14000.00%	5.91	−640.00%	184.10	360.00%	(133.57)	66.00%	(123.38)	161.00%	(169.82)	256.00%	(154.76)
68	−18000.00%	4.58	−680.00%	169.03	370.00%	(131.69)	67.00%	(124.89)	162.00%	(169.77)	257.00%	(154.55)
69	−22000.00%	3.74	−720.00%	156.23	380.00%	(129.84)	68.00%	(126.36)	163.00%	(169.72)	258.00%	(154.35)
70	−26000.00%	3.16	−760.00%	145.22	390.00%	(128.04)	69.00%	(127.79)	164.00%	(169.67)	259.00%	(154.14)
71	−30000.00%	2.74	−800.00%	135.66	400.00%	(126.26)	70.00%	(129.19)	165.00%	(169.61)	260.00%	(153.93)
72	−34000.00%	2.41	−840.00%	127.27	410.00%	(124.53)	71.00%	(130.55)	166.00%	(169.55)	261.00%	(153.72)
73	−38000.00%	2.16	−880.00%	119.86	420.00%	(122.83)	72.00%	(131.88)	167.00%	(169.48)	262.00%	(153.51)
74	−42000.00%	1.95	−920.00%	113.25	430.00%	(121.17)	73.00%	(133.17)	168.00%	(169.41)	263.00%	(153.30)
75	−46000.00%	1.78	−960.00%	107.34	440.00%	(119.55)	74.00%	(134.43)	169.00%	(169.34)	264.00%	(153.09)
76	−50000.00%	1.64	−1000.00%	102.01	450.00%	(117.96)	75.00%	(135.66)	170.00%	(169.26)	265.00%	(152.88)
77	−54000.00%	1.52	−1040.00%	97.18	460.00%	(116.40)	76.00%	(136.86)	171.00%	(169.18)	266.00%	(152.67)
78	−58000.00%	1.41	−1080.00%	92.78	470.00%	(114.88)	77.00%	(138.02)	172.00%	(169.09)	267.00%	(152.46)
79	−62000.00%	1.32	−1120.00%	88.77	480.00%	(113.40)	78.00%	(139.16)	173.00%	(169.00)	268.00%	(152.25)
80	−66000.00%	1.24	−1160.00%	85.08	490.00%	(111.95)	79.00%	(140.27)	174.00%	(168.91)	269.00%	(152.04)
81	−70000.00%	1.17	−1200.00%	81.69	500.00%	(110.52)	80.00%	(141.34)	175.00%	(168.81)	270.00%	(151.83)
82	−74000.00%	1.10	−1240.00%	78.56	510.00%	(109.14)	81.00%	(142.39)	176.00%	(168.71)	271.00%	(151.62)
83	−78000.00%	1.05	−1280.00%	75.66	520.00%	(107.78)	82.00%	(143.41)	177.00%	(168.61)	272.00%	(151.41)
84	−82000.00%	1.00	−1320.00%	72.96	530.00%	(106.45)	83.00%	(144.40)	178.00%	(168.50)	273.00%	(151.20)
85	−86000.00%	0.95	−1360.00%	70.45	540.00%	(105.15)	84.00%	(145.36)	179.00%	(168.39)	274.00%	(150.99)
86	−90000.00%	0.91	−1400.00%	68.11	550.00%	(103.88)	85.00%	(146.30)	180.00%	(168.28)	275.00%	(150.78)
87	−94000.00%	0.87	−1440.00%	65.91	560.00%	(102.63)	86.00%	(147.21)	181.00%	(168.16)	276.00%	(150.57)
88	−98000.00%	0.83	−1480.00%	63.86	570.00%	(101.42)	87.00%	(148.10)	182.00%	(168.05)	277.00%	(150.36)
89	−102000.00%	0.80	−1520.00%	61.92	580.00%	(100.23)	88.00%	(148.96)	183.00%	(167.93)	278.00%	(150.15)
90	−106000.00%	0.77	−1560.00%	60.10	590.00%	(99.06)	89.00%	(149.79)	184.00%	(167.80)	279.00%	(149.94)
91	−110000.00%	0.74	−1600.00%	58.39	600.00%	(97.92)	90.00%	(150.60)	185.00%	(167.67)	280.00%	(149.73)
92	−114000.00%	0.72	−1640.00%	56.77	610.00%	(96.81)	91.00%	(151.39)	186.00%	(167.55)	281.00%	(149.52)
93	−118000.00%	0.69	−1680.00%	55.23	620.00%	(95.72)	92.00%	(152.16)	187.00%	(167.41)	282.00%	(149.31)
94	−122000.00%	0.67	−1720.00%	53.78	630.00%	(94.65)	93.00%	(152.90)	188.00%	(167.28)	283.00%	(149.10)
95	−126000.00%	0.65	−1760.00%	52.40	640.00%	(93.60)	94.00%	(153.62)	189.00%	(167.14)	284.00%	(148.89)

(continued)

Chart 4A (continued)

Table-B5
Project Outcome/Cashflow:

0	1	2	3	4	5	6
(815.00)	900.00	(100.00)	1,200.00	(1,200.00)	100.00	150.00

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.33)	2000.00%	(36.77)	−300.00%	757.89	0.00%	235.00	95.00%	(151.10)	190.00%	(166.75)
2	246000.00%	(0.33)	1960.00%	(37.45)	−290.00%	833.83	1.00%	223.81	96.00%	(151.88)	191.00%	(166.61)
3	242000.00%	(0.34)	1920.00%	(38.15)	−280.00%	926.01	2.00%	212.92	97.00%	(152.64)	192.00%	(166.47)
4	238000.00%	(0.34)	1880.00%	(38.87)	−270.00%	1,039.86	3.00%	202.33	98.00%	(153.37)	193.00%	(166.33)
5	234000.00%	(0.35)	1840.00%	(39.62)	−260.00%	1,183.27	4.00%	192.02	99.00%	(154.08)	194.00%	(166.19)
6	230000.00%	(0.35)	1800.00%	(40.41)	−250.00%	1,368.02	5.00%	181.99	100.00%	(154.77)	195.00%	(166.05)
7	226000.00%	(0.36)	1760.00%	(41.22)	−240.00%	1,612.31	6.00%	172.22	101.00%	(155.43)	196.00%	(165.90)
8	222000.00%	(0.37)	1720.00%	(42.07)	−230.00%	1,945.14	7.00%	162.72	102.00%	(156.07)	197.00%	(165.75)
9	218000.00%	(0.37)	1680.00%	(42.95)	−220.00%	2,414.62	8.00%	153.47	103.00%	(156.69)	198.00%	(165.60)
10	214000.00%	(0.38)	1640.00%	(43.87)	−210.00%	3,104.04	9.00%	144.46	104.00%	(157.29)	199.00%	(165.44)
11	210000.00%	(0.39)	1600.00%	(44.83)	−200.00%	4,165.00	10.00%	135.69	105.00%	(157.88)	200.00%	(165.29)
12	206000.00%	(0.40)	1560.00%	(45.84)	−190.00%	5,889.60	11.00%	127.16	106.00%	(158.44)	201.00%	(165.13)
13	202000.00%	(0.40)	1520.00%	(46.89)	−180.00%	8,878.32	12.00%	118.84	107.00%	(158.98)	202.00%	(164.97)
14	198000.00%	(0.41)	1480.00%	(47.98)	−170.00%	14,458.95	13.00%	110.75	108.00%	(159.50)	203.00%	(164.81)
15	194000.00%	(0.42)	1440.00%	(49.13)	−160.00%	25,797.63	14.00%	102.87	109.00%	(160.01)	204.00%	(164.65)
16	190000.00%	(0.43)	1400.00%	(50.34)	−150.00%	50,830.00	15.00%	95.20	110.00%	(160.49)	205.00%	(164.48)
17	186000.00%	(0.44)	1360.00%	(51.61)	−140.00%	106,148.83	16.00%	87.73	111.00%	(160.96)	206.00%	(164.32)
18	182000.00%	(0.45)	1320.00%	(52.94)	−130.00%	109,698.83	17.00%	80.45	112.00%	(161.41)	207.00%	(164.15)
19	178000.00%	(0.46)	1280.00%	(54.34)	−120.00%	(5,617,175.00)	18.00%	73.37	113.00%	(161.85)	208.00%	(163.98)
20	174000.00%	(0.47)	1240.00%	(55.82)	−110.00%	(1,267,801,850.00)	19.00%	66.48	114.00%	(162.27)	209.00%	(163.81)
21	170000.00%	(0.48)	1200.00%	(57.37)	−100.00%	($815.00)	20.00%	59.77	115.00%	(162.67)	210.00%	(163.64)
22	166000.00%	(0.49)	1160.00%	(59.02)	−90.00%	1,491,981,850.00	21.00%	53.23	116.00%	(163.06)	211.00%	(163.46)
23	162000.00%	(0.50)	1120.00%	(60.76)	−80.00%	10,287,175.00	22.00%	46.87	117.00%	(163.44)	212.00%	(163.29)
24	158000.00%	(0.52)	1080.00%	(62.61)	−70.00%	480,945.88	23.00%	40.67	118.00%	(163.80)	213.00%	(163.11)
25	154000.00%	(0.53)	1040.00%	(64.57)	−60.00%	47,679.30	24.00%	34.65	119.00%	(164.14)	214.00%	(162.94)
26	150000.00%	(0.54)	1000.00%	(66.65)	−50.00%	7,570.00	25.00%	28.78	120.00%	(164.47)	215.00%	(162.76)
27	146000.00%	(0.56)	960.00%	(68.87)	−40.00%	2,007.58	26.00%	23.06	121.00%	(164.79)	216.00%	(162.58)
28	142000.00%	(0.57)	920.00%	(71.25)	−30.00%	910.32	27.00%	17.50	122.00%	(165.09)	217.00%	(162.40)
29	138000.00%	(0.59)	880.00%	(73.78)	−20.00%	556.49	28.00%	12.09	123.00%	(165.38)	218.00%	(162.21)
30	134000.00%	(0.61)	840.00%	(76.50)	−10.00%	366.94	29.00%	6.82	124.00%	(165.66)	219.00%	(162.03)
31	130000.00%	(0.63)	800.00%	(79.42)	0.00%	235.00	30.00%	1.69	125.00%	(165.93)	220.00%	(161.84)
32	126000.00%	(0.65)	760.00%	(82.56)	10.00%	135.69	31.00%	(3.31)	126.00%	(166.18)	221.00%	(161.66)
33	122000.00%	(0.67)	720.00%	(85.95)	20.00%	59.77	32.00%	(8.17)	127.00%	(166.42)	222.00%	(161.47)
34	118000.00%	(0.69)	680.00%	(89.62)	30.00%	1.69	33.00%	(12.90)	128.00%	(166.65)	223.00%	(161.28)
35	114000.00%	(0.71)	640.00%	(93.60)	40.00%	(42.64)	34.00%	(17.50)	129.00%	(166.87)	224.00%	(161.09)
36	110000.00%	(0.74)	600.00%	(97.92)	50.00%	(76.39)	35.00%	(21.98)	130.00%	(167.08)	225.00%	(160.90)
37	106000.00%	(0.77)	560.00%	(102.63)	60.00%	(102.01)	36.00%	(26.34)	131.00%	(167.28)	226.00%	(160.71)
38	102000.00%	(0.80)	520.00%	(107.77)	70.00%	(121.39)	37.00%	(30.58)	132.00%	(167.47)	227.00%	(160.52)
39	98000.00%	(0.83)	480.00%	(113.40)	80.00%	(135.95)	38.00%	(34.71)	133.00%	(167.64)	228.00%	(160.33)
40	94000.00%	(0.87)	440.00%	(119.54)	90.00%	(146.80)	39.00%	(38.73)	134.00%	(167.81)	229.00%	(160.14)
41	90000.00%	(0.90)	400.00%	(126.26)	100.00%	(154.77)	40.00%	(42.64)	135.00%	(167.97)	230.00%	(159.94)
42	86000.00%	(0.95)	360.00%	(133.56)	110.00%	(160.49)	41.00%	(46.45)	136.00%	(168.12)	231.00%	(159.75)
43	82000.00%	(0.99)	320.00%	(141.41)	120.00%	(164.47)	42.00%	(50.15)	137.00%	(168.25)	232.00%	(159.55)
44	78000.00%	(1.04)	280.00%	(149.68)	130.00%	(167.08)	43.00%	(53.75)	138.00%	(168.38)	233.00%	(159.36)
45	74000.00%	(1.10)	240.00%	(157.96)	140.00%	(168.62)	44.00%	(57.26)	139.00%	(168.51)	234.00%	(159.16)
46	70000.00%	(1.16)	200.00%	(165.29)	150.00%	(169.31)	45.00%	(60.67)	140.00%	(168.62)	235.00%	(158.96)
47	66000.00%	(1.23)	160.00%	(169.34)	160.00%	(169.34)	46.00%	(63.99)	141.00%	(168.72)	236.00%	(158.76)
48	62000.00%	(1.31)	120.00%	(164.47)	170.00%	(168.86)	47.00%	(67.22)	142.00%	(168.82)	237.00%	(158.56)
49	58000.00%	(1.40)	80.00%	(135.95)	180.00%	(167.96)	48.00%	(70.36)	143.00%	(168.91)	238.00%	(158.36)
50	54000.00%	(1.50)	40.00%	(42.64)	190.00%	(166.75)	49.00%	(73.42)	144.00%	(168.99)	239.00%	(158.16)

51	50000.00%	(1.62)	0.00%	235.00	200.00%	(165.29)	50.00%	(76.39)	145.00%	(169.06)	240.00%	(157.96)
52	46000.00%	(1.76)	−40.00%	2,007.58	210.00%	(163.64)	51.00%	(79.29)	146.00%	(169.12)	241.00%	(157.76)
53	42000.00%	(1.93)	−80.00%	10,287,175.00	220.00%	(161.84)	52.00%	(82.10)	147.00%	(169.18)	242.00%	(157.56)
54	38000.00%	(2.13)	−120.00%	(5,617,175.00)	230.00%	(159.94)	53.00%	(84.84)	148.00%	(169.23)	243.00%	(157.36)
55	34000.00%	(2.38)	−160.00%	25,797.63	240.00%	(157.96)	54.00%	(87.50)	149.00%	(169.28)	244.00%	(157.16)
56	30000.00%	(2.70)	−200.00%	4,165.00	250.00%	(155.93)	55.00%	(90.09)	150.00%	(169.31)	245.00%	(156.95)
57	26000.00%	(3.11)	−240.00%	1,612.31	260.00%	(153.86)	56.00%	(92.61)	151.00%	(169.34)	246.00%	(156.75)
58	22000.00%	(3.67)	−280.00%	926.01	270.00%	(151.78)	57.00%	(95.06)	152.00%	(169.37)	247.00%	(156.54)
59	18000.00%	(4.48)	−320.00%	640.59	280.00%	(149.68)	58.00%	(97.44)	153.00%	(169.38)	248.00%	(156.34)
60	14000.00%	(5.73)	−360.00%	488.78	290.00%	(147.59)	59.00%	(99.76)	154.00%	(169.40)	249.00%	(156.14)
61	10000.00%	(7.98)	−400.00%	395.19	300.00%	(145.51)	60.00%	(102.01)	155.00%	(169.40)	250.00%	(155.93)
62	6000.00%	(13.12)	−440.00%	331.76	310.00%	(143.45)	61.00%	(104.21)	156.00%	(169.40)	251.00%	(155.73)
63	2000.00%	(36.77)	−480.00%	285.91	320.00%	(141.41)	62.00%	(106.34)	157.00%	(169.40)	252.00%	(155.52)
64	−2000.00%	45.41	−520.00%	251.20	330.00%	(139.40)	63.00%	(108.41)	158.00%	(169.38)	253.00%	(155.31)
65	−6000.00%	14.07	−560.00%	224.00	340.00%	(137.42)	64.00%	(110.42)	159.00%	(169.37)	254.00%	(155.11)
66	−10000.00%	8.32	−600.00%	202.11	350.00%	(135.47)	65.00%	(112.38)	160.00%	(169.34)	255.00%	(154.90)
67	−14000.00%	5.91	−640.00%	184.10	360.00%	(133.56)	66.00%	(114.29)	161.00%	(169.32)	256.00%	(154.69)
68	−18000.00%	4.58	−680.00%	169.03	370.00%	(131.68)	67.00%	(116.14)	162.00%	(169.28)	257.00%	(154.49)
69	−22000.00%	3.74	−720.00%	156.23	380.00%	(129.83)	68.00%	(117.94)	163.00%	(169.25)	258.00%	(154.28)
70	−26000.00%	3.16	−760.00%	145.22	390.00%	(128.03)	69.00%	(119.69)	164.00%	(169.21)	259.00%	(154.07)
71	−30000.00%	2.74	−800.00%	135.66	400.00%	(126.26)	70.00%	(121.39)	165.00%	(169.16)	260.00%	(153.86)
72	−34000.00%	2.41	−840.00%	127.27	410.00%	(124.52)	71.00%	(123.04)	166.00%	(169.11)	261.00%	(153.65)
73	−38000.00%	2.16	−880.00%	119.86	420.00%	(122.83)	72.00%	(124.65)	167.00%	(169.05)	262.00%	(153.45)
74	−42000.00%	1.95	−920.00%	113.25	430.00%	(121.17)	73.00%	(126.21)	168.00%	(168.99)	263.00%	(153.24)
75	−46000.00%	1.78	−960.00%	107.34	440.00%	(119.54)	74.00%	(127.72)	169.00%	(168.93)	264.00%	(153.03)
76	−50000.00%	1.64	−1000.00%	102.01	450.00%	(117.95)	75.00%	(129.20)	170.00%	(168.86)	265.00%	(152.82)
77	−54000.00%	1.52	−1040.00%	97.18	460.00%	(116.40)	76.00%	(130.63)	171.00%	(168.78)	266.00%	(152.61)
78	−58000.00%	1.41	−1080.00%	92.78	470.00%	(114.88)	77.00%	(132.02)	172.00%	(168.71)	267.00%	(152.40)
79	−62000.00%	1.32	−1120.00%	88.77	480.00%	(113.40)	78.00%	(133.37)	173.00%	(168.63)	268.00%	(152.19)
80	−66000.00%	1.24	−1160.00%	85.08	490.00%	(111.94)	79.00%	(134.68)	174.00%	(168.54)	269.00%	(151.99)
81	−70000.00%	1.17	−1200.00%	81.69	500.00%	(110.52)	80.00%	(135.95)	175.00%	(168.45)	270.00%	(151.78)
82	−74000.00%	1.10	−1240.00%	78.56	510.00%	(109.13)	81.00%	(137.19)	176.00%	(168.36)	271.00%	(151.57)
83	−78000.00%	1.05	−1280.00%	75.66	520.00%	(107.77)	82.00%	(138.39)	177.00%	(168.27)	272.00%	(151.36)
84	−82000.00%	1.00	−1320.00%	72.96	530.00%	(106.45)	83.00%	(139.55)	178.00%	(168.17)	273.00%	(151.15)
85	−86000.00%	0.95	−1360.00%	70.45	540.00%	(105.15)	84.00%	(140.68)	179.00%	(168.07)	274.00%	(150.94)
86	−90000.00%	0.91	−1400.00%	68.11	550.00%	(103.88)	85.00%	(141.78)	180.00%	(167.96)	275.00%	(150.73)
87	−94000.00%	0.87	−1440.00%	65.91	560.00%	(102.63)	86.00%	(142.85)	181.00%	(167.85)	276.00%	(150.52)
88	−98000.00%	0.83	−1480.00%	63.86	570.00%	(101.42)	87.00%	(143.88)	182.00%	(167.74)	277.00%	(150.31)
89	−102000.00%	0.80	−1520.00%	61.92	580.00%	(100.23)	88.00%	(144.88)	183.00%	(167.63)	278.00%	(150.10)
90	−106000.00%	0.77	−1560.00%	60.10	590.00%	(99.06)	89.00%	(145.86)	184.00%	(167.51)	279.00%	(149.89)
91	−110000.00%	0.74	−1600.00%	58.39	600.00%	(97.92)	90.00%	(146.80)	185.00%	(167.39)	280.00%	(149.68)
92	−114000.00%	0.72	−1640.00%	56.77	610.00%	(96.81)	91.00%	(147.71)	186.00%	(167.27)	281.00%	(149.47)
93	−118000.00%	0.69	−1680.00%	55.23	620.00%	(95.72)	92.00%	(148.60)	187.00%	(167.14)	282.00%	(149.26)
94	−122000.00%	0.67	−1720.00%	53.78	630.00%	(94.65)	93.00%	(149.46)	188.00%	(167.01)	283.00%	(149.05)
95	−126000.00%	0.65	−1760.00%	52.40	640.00%	(93.60)	94.00%	(150.29)	189.00%	(166.88)	284.00%	(148.85)

Chart 4B

Table-B6

Project Outcome/CashFlow:

0	1	2	3	4	5	6
(815.00)	900.00	(100.00)	1,200.00	(1,200.00)	100.00	(12,000.00)

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.33)	2000.00%	(36.77)	−300.00%	852.81	0.00%	(11,915.00)	95.00%	(264.43)	190.00%	(173.79)
2	246000.00%	(0.33)	1960.00%	(37.45)	−290.00%	969.75	1.00%	(11,108.72)	96.00%	(261.22)	191.00%	(173.49)
3	242000.00%	(0.34)	1920.00%	(38.15)	−280.00%	1,124.47	2.00%	(10,364.39)	97.00%	(258.15)	192.00%	(173.19)
4	238000.00%	(0.34)	1880.00%	(38.87)	−270.00%	1,335.96	3.00%	(9,676.74)	98.00%	(255.21)	193.00%	(172.89)
5	234000.00%	(0.35)	1840.00%	(39.62)	−260.00%	1,635.89	4.00%	(9,040.98)	99.00%	(252.39)	194.00%	(172.59)
6	230000.00%	(0.35)	1800.00%	(40.41)	−250.00%	2,079.14	5.00%	(8,452.79)	100.00%	(249.69)	195.00%	(172.29)
7	226000.00%	(0.36)	1760.00%	(41.22)	−240.00%	2,764.92	6.00%	(7,908.22)	101.00%	(247.10)	196.00%	(172.00)
8	222000.00%	(0.37)	1720.00%	(42.07)	−230.00%	3,881.45	7.00%	(7,403.69)	102.00%	(244.61)	197.00%	(171.71)
9	218000.00%	(0.37)	1680.00%	(42.95)	−220.00%	5,805.46	8.00%	(6,935.94)	103.00%	(242.22)	198.00%	(171.42)
10	214000.00%	(0.38)	1640.00%	(43.87)	−210.00%	9,338.91	9.00%	(6,502.00)	104.00%	(239.93)	199.00%	(171.13)
11	210000.00%	(0.39)	1600.00%	(44.83)	−200.00%	16,315.00	10.00%	(6,099.18)	105.00%	(237.73)	200.00%	(170.84)
12	206000.00%	(0.40)	1560.00%	(45.84)	−190.00%	31,292.23	11.00%	(5,724.99)	106.00%	(235.62)	201.00%	(170.56)
13	202000.00%	(0.40)	1520.00%	(46.89)	−180.00%	66,814.04	12.00%	(5,377.20)	107.00%	(233.59)	202.00%	(170.27)
14	198000.00%	(0.41)	1480.00%	(47.98)	−170.00%	161,992.23	13.00%	(5,053.74)	108.00%	(231.63)	203.00%	(169.99)
15	194000.00%	(0.42)	1440.00%	(49.13)	−160.00%	459,825.41	14.00%	(4,752.72)	109.00%	(229.76)	204.00%	(169.71)
16	190000.00%	(0.43)	1400.00%	(50.34)	−150.00%	1,606,030.00	15.00%	(4,472.44)	110.00%	(227.95)	205.00%	(169.43)
17	186000.00%	(0.44)	1360.00%	(51.61)	−140.00%	7,521,920.31	16.00%	(4,211.30)	111.00%	(226.21)	206.00%	(169.15)
18	182000.00%	(0.45)	1320.00%	(52.94)	−130.00%	55,665,254.39	17.00%	(3,967.87)	112.00%	(224.54)	207.00%	(168.88)
19	178000.00%	(0.46)	1280.00%	(54.34)	−120.00%	9.44E+08	18.00%	(3,740.82)	113.00%	(222.93)	208.00%	(168.60)
20	174000.00%	(0.47)	1240.00%	(55.82)	−110.00%	1.20E+11	19.00%	(3,528.92)	114.00%	(221.38)	209.00%	(168.33)
21	170000.00%	(0.48)	1200.00%	(57.37)	−100.00%	−8.15E+02	20.00%	(3,331.08)	115.00%	(219.89)	210.00%	(168.05)
22	166000.00%	(0.49)	1160.00%	(59.02)	−90.00%	−1.20E+11	21.00%	(3,146.24)	116.00%	(218.45)	211.00%	(167.78)
23	162000.00%	(0.50)	1120.00%	(60.76)	−80.00%	−9.39E+08	22.00%	(2,973.48)	117.00%	(217.06)	212.00%	(167.51)
24	158000.00%	(0.52)	1080.00%	(62.61)	−70.00%	(55,074,609.67)	23.00%	(2,811.92)	118.00%	(215.72)	213.00%	(167.24)
25	154000.00%	(0.53)	1040.00%	(64.57)	−60.00%	(7,368,092.19)	24.00%	(2,660.76)	119.00%	(214.43)	214.00%	(166.97)
26	150000.00%	(0.54)	1000.00%	(66.65)	−50.00%	(1,547,630.00)	25.00%	(2,519.26)	120.00%	(213.18)	215.00%	(166.70)
27	146000.00%	(0.56)	960.00%	(68.88)	−40.00%	(432,020.20)	26.00%	(2,386.75)	121.00%	(211.98)	216.00%	(166.44)
28	142000.00%	(0.57)	920.00%	(71.25)	−30.00%	(146,622.96)	27.00%	(2,262.58)	122.00%	(210.81)	217.00%	(166.17)
29	138000.00%	(0.59)	880.00%	(73.78)	−20.00%	(57,379.22)	28.00%	(2,146.19)	123.00%	(209.69)	218.00%	(165.91)
30	134000.00%	(0.61)	840.00%	(76.50)	−10.00%	(25,035.69)	29.00%	(2,037.03)	124.00%	(208.60)	219.00%	(165.64)
31	130000.00%	(0.63)	800.00%	(79.42)	0.00%	(11,915.00)	30.00%	(1,934.62)	125.00%	(207.55)	220.00%	(165.38)
32	126000.00%	(0.65)	760.00%	(82.57)	10.00%	(6,099.18)	31.00%	(1,838.48)	126.00%	(206.53)	221.00%	(165.12)
33	122000.00%	(0.67)	720.00%	(85.96)	20.00%	(3,331.08)	32.00%	(1,748.21)	127.00%	(205.54)	222.00%	(164.86)
34	118000.00%	(0.69)	680.00%	(89.63)	30.00%	(1,934.62)	33.00%	(1,663.39)	128.00%	(204.59)	223.00%	(164.60)
35	114000.00%	(0.71)	640.00%	(93.61)	40.00%	(1,195.25)	34.00%	(1,583.68)	129.00%	(203.66)	224.00%	(164.34)
36	110000.00%	(0.74)	600.00%	(97.94)	50.00%	(787.50)	35.00%	(1,508.74)	130.00%	(202.77)	225.00%	(164.08)
37	106000.00%	(0.77)	560.00%	(102.66)	60.00%	(554.64)	36.00%	(1,438.24)	131.00%	(201.90)	226.00%	(163.82)
38	102000.00%	(0.80)	520.00%	(107.81)	70.00%	(417.48)	37.00%	(1,371.90)	132.00%	(201.05)	227.00%	(163.56)
39	98000.00%	(0.83)	480.00%	(113.45)	80.00%	(334.41)	38.00%	(1,309.46)	133.00%	(200.23)	228.00%	(163.30)
40	94000.00%	(0.87)	440.00%	(119.63)	90.00%	(282.72)	39.00%	(1,250.65)	134.00%	(199.44)	229.00%	(163.05)
41	90000.00%	(0.90)	400.00%	(126.41)	100.00%	(249.69)	40.00%	(1,195.25)	135.00%	(198.67)	230.00%	(162.79)
42	86000.00%	(0.95)	360.00%	(133.84)	110.00%	(227.95)	41.00%	(1,143.03)	136.00%	(197.91)	231.00%	(162.54)
43	82000.00%	(0.99)	320.00%	(141.94)	120.00%	(213.18)	42.00%	(1,093.81)	137.00%	(197.18)	232.00%	(162.29)
44	78000.00%	(1.04)	280.00%	(150.74)	130.00%	(202.77)	43.00%	(1,047.38)	138.00%	(196.47)	233.00%	(162.03)
45	74000.00%	(1.10)	240.00%	(160.28)	140.00%	(195.11)	44.00%	(1,003.58)	139.00%	(195.78)	234.00%	(161.78)
46	70000.00%	(1.16)	200.00%	(170.84)	150.00%	(189.22)	45.00%	(962.24)	140.00%	(195.11)	235.00%	(161.53)
47	66000.00%	(1.23)	160.00%	(184.47)	160.00%	(184.47)	46.00%	(923.21)	141.00%	(194.45)	236.00%	(161.28)
48	62000.00%	(1.31)	120.00%	(213.18)	170.00%	(180.47)	47.00%	(886.35)	142.00%	(193.81)	237.00%	(161.03)
49	58000.00%	(1.40)	80.00%	(334.41)	180.00%	(176.97)	48.00%	(851.53)	143.00%	(193.19)	238.00%	(160.78)
50	54000.00%	(1.50)	40.00%	(1,195.25)	190.00%	(173.79)	49.00%	(818.62)	144.00%	(192.58)	239.00%	(160.53)

51	50000.00%	(1.62)	0.00%	(11,915.00)	200.00%	(170.84)	50.00%	(787.50)	145.00%	(191.99)	240.00%	(160.28)
52	46000.00%	(1.76)	−40.00%	(432,020.20)	210.00%	(168.05)	51.00%	(758.08)	146.00%	(191.41)	241.00%	(160.03)
53	42000.00%	(1.93)	−80.00%	(938,931,575.00)	220.00%	(165.38)	52.00%	(730.24)	147.00%	(190.84)	242.00%	(159.78)
54	38000.00%	(2.13)	−120.00%	943,601,575.00	230.00%	(162.79)	53.00%	(703.90)	148.00%	(190.29)	243.00%	(159.53)
55	34000.00%	(2.38)	−160.00%	459,825.41	240.00%	(160.28)	54.00%	(678.97)	149.00%	(189.75)	244.00%	(159.29)
56	30000.00%	(2.70)	−200.00%	16,315.00	250.00%	(157.82)	55.00%	(655.36)	150.00%	(189.22)	245.00%	(159.04)
57	26000.00%	(3.11)	−240.00%	2,764.92	260.00%	(155.41)	56.00%	(632.99)	151.00%	(188.70)	246.00%	(158.80)
58	22000.00%	(3.67)	−280.00%	1,124.47	270.00%	(153.06)	57.00%	(611.81)	152.00%	(188.19)	247.00%	(158.55)
59	18000.00%	(4.48)	−320.00%	689.30	280.00%	(150.74)	58.00%	(591.73)	153.00%	(187.70)	248.00%	(158.31)
60	14000.00%	(5.73)	−360.00%	503.91	290.00%	(148.48)	59.00%	(572.69)	154.00%	(187.21)	249.00%	(158.06)
61	10000.00%	(7.98)	−400.00%	400.75	300.00%	(146.25)	60.00%	(554.64)	155.00%	(186.73)	250.00%	(157.82)
62	6000.00%	(13.12)	−440.00%	334.08	310.00%	(144.08)	61.00%	(537.51)	156.00%	(186.26)	251.00%	(157.58)
63	2000.00%	(36.77)	−480.00%	286.97	320.00%	(141.94)	62.00%	(521.26)	157.00%	(185.80)	252.00%	(157.33)
64	−2000.00%	45.41	−520.00%	251.73	330.00%	(139.85)	63.00%	(505.84)	158.00%	(185.35)	253.00%	(157.09)
65	−6000.00%	14.07	−560.00%	224.28	340.00%	(137.80)	64.00%	(491.20)	159.00%	(184.91)	254.00%	(156.85)
66	−10000.00%	8.32	−600.00%	202.26	350.00%	(135.80)	65.00%	(477.30)	160.00%	(184.47)	255.00%	(156.61)
67	−14000.00%	5.91	−640.00%	184.19	360.00%	(133.84)	66.00%	(464.09)	161.00%	(184.04)	256.00%	(156.37)
68	−18000.00%	4.58	−680.00%	169.08	370.00%	(131.92)	67.00%	(451.54)	162.00%	(183.62)	257.00%	(156.13)
69	−22000.00%	3.74	−720.00%	156.26	380.00%	(130.04)	68.00%	(439.61)	163.00%	(183.21)	258.00%	(155.89)
70	−26000.00%	3.16	−760.00%	145.25	390.00%	(128.21)	69.00%	(428.27)	164.00%	(182.80)	259.00%	(155.65)
71	−30000.00%	2.74	−800.00%	135.67	400.00%	(126.41)	70.00%	(417.48)	165.00%	(182.40)	260.00%	(155.41)
72	−34000.00%	2.41	−840.00%	127.28	410.00%	(124.66)	71.00%	(407.23)	166.00%	(182.00)	261.00%	(155.18)
73	−38000.00%	2.16	−880.00%	119.86	420.00%	(122.94)	72.00%	(397.47)	167.00%	(181.61)	262.00%	(154.94)
74	−42000.00%	1.95	−920.00%	113.26	430.00%	(121.27)	73.00%	(388.18)	168.00%	(181.23)	263.00%	(154.70)
75	−46000.00%	1.78	−960.00%	107.34	440.00%	(119.63)	74.00%	(379.33)	169.00%	(180.85)	264.00%	(154.46)
76	−50000.00%	1.64	−1000.00%	102.01	450.00%	(118.03)	75.00%	(370.91)	170.00%	(180.47)	265.00%	(154.23)
77	−54000.00%	1.52	−1040.00%	97.18	460.00%	(116.47)	76.00%	(362.89)	171.00%	(180.10)	266.00%	(153.99)
78	−58000.00%	1.41	−1080.00%	92.79	470.00%	(114.94)	77.00%	(355.25)	172.00%	(179.74)	267.00%	(153.76)
79	−62000.00%	1.32	−1120.00%	88.77	480.00%	(113.45)	78.00%	(347.97)	173.00%	(179.38)	268.00%	(153.52)
80	−66000.00%	1.24	−1160.00%	85.09	490.00%	(111.99)	79.00%	(341.03)	174.00%	(179.02)	269.00%	(153.29)
81	−70000.00%	1.17	−1200.00%	81.69	500.00%	(110.57)	80.00%	(334.41)	175.00%	(178.67)	270.00%	(153.06)
82	−74000.00%	1.10	−1240.00%	78.56	510.00%	(109.17)	81.00%	(328.10)	176.00%	(178.32)	271.00%	(152.82)
83	−78000.00%	1.05	−1280.00%	75.66	520.00%	(107.81)	82.00%	(322.07)	177.00%	(177.98)	272.00%	(152.59)
84	−82000.00%	1.00	−1320.00%	72.96	530.00%	(106.48)	83.00%	(316.33)	178.00%	(177.64)	273.00%	(152.36)
85	−86000.00%	0.95	−1360.00%	70.45	540.00%	(105.17)	84.00%	(310.84)	179.00%	(177.30)	274.00%	(152.13)
86	−90000.00%	0.91	−1400.00%	68.11	550.00%	(103.90)	85.00%	(305.60)	180.00%	(176.97)	275.00%	(151.89)
87	−94000.00%	0.87	−1440.00%	65.91	560.00%	(102.66)	86.00%	(300.60)	181.00%	(176.64)	276.00%	(151.66)
88	−98000.00%	0.83	−1480.00%	63.86	570.00%	(101.44)	87.00%	(295.83)	182.00%	(176.31)	277.00%	(151.43)
89	−102000.00%	0.80	−1520.00%	61.92	580.00%	(100.25)	88.00%	(291.26)	183.00%	(175.98)	278.00%	(151.20)
90	−106000.00%	0.77	−1560.00%	60.10	590.00%	(99.08)	89.00%	(286.90)	184.00%	(175.66)	279.00%	(150.97)
91	−110000.00%	0.74	−1600.00%	58.39	600.00%	(97.94)	90.00%	(282.72)	185.00%	(175.35)	280.00%	(150.74)
92	−114000.00%	0.72	−1640.00%	56.77	610.00%	(96.82)	91.00%	(278.73)	186.00%	(175.03)	281.00%	(150.52)
93	−118000.00%	0.69	−1680.00%	55.23	620.00%	(95.73)	92.00%	(274.92)	187.00%	(174.72)	282.00%	(150.29)
94	−122000.00%	0.67	−1720.00%	53.78	630.00%	(94.66)	93.00%	(271.27)	188.00%	(174.41)	283.00%	(150.06)
95	−126000.00%	0.65	−1760.00%	52.40	640.00%	(93.61)	94.00%	(267.77)	189.00%	(174.10)	284.00%	(149.83)

(continued)

Chart 4B (continued)

Table-B7:
Cash Flows:

0	1	2
(815.00)	900.00	(12,000.00)

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.33)	2000.00%	(38.06)	–300.00%	2,132.50	0.00%	(11,915.00)	95.00%	(1,799.63)	190.00%	(666.04)
2	246000.00%	(0.33)	1960.00%	(38.81)	–290.00%	2,427.78	1.00%	(11,571.75)	96.00%	(1,775.26)	191.00%	(660.76)
3	242000.00%	(0.34)	1920.00%	(39.60)	–280.00%	2,788.17	2.00%	(11,241.84)	97.00%	(1,751.38)	192.00%	(655.54)
4	238000.00%	(0.34)	1880.00%	(40.41)	–270.00%	3,233.33	3.00%	(10,924.63)	98.00%	(1,727.96)	193.00%	(650.39)
5	234000.00%	(0.35)	1840.00%	(41.26)	–260.00%	3,790.63	4.00%	(10,619.51)	99.00%	(1,705.01)	194.00%	(645.30)
6	230000.00%	(0.35)	1800.00%	(42.15)	–250.00%	4,498.89	5.00%	(10,325.92)	100.00%	(1,682.50)	195.00%	(640.28)
7	226000.00%	(0.36)	1760.00%	(43.08)	–240.00%	5,414.50	6.00%	(10,043.30)	101.00%	(1,660.43)	196.00%	(635.32)
8	222000.00%	(0.37)	1720.00%	(44.05)	–230.00%	6,621.46	7.00%	(9,771.16)	102.00%	(1,638.78)	197.00%	(630.43)
9	218000.00%	(0.37)	1680.00%	(45.07)	–220.00%	8,248.61	8.00%	(9,509.01)	103.00%	(1,617.55)	198.00%	(625.60)
10	214000.00%	(0.38)	1640.00%	(46.14)	–210.00%	10,500.49	9.00%	(9,256.40)	104.00%	(1,596.73)	199.00%	(620.82)
11	210000.00%	(0.39)	1600.00%	(47.27)	–200.00%	13,715.00	10.00%	(9,012.89)	105.00%	(1,576.30)	200.00%	(616.11)
12	206000.00%	(0.40)	1560.00%	(48.45)	–190.00%	18,477.57	11.00%	(8,778.07)	106.00%	(1,556.26)	201.00%	(611.46)
13	202000.00%	(0.40)	1520.00%	(49.70)	–180.00%	25,862.50	12.00%	(8,551.57)	107.00%	(1,536.59)	202.00%	(606.86)
14	198000.00%	(0.41)	1480.00%	(51.02)	–170.00%	37,986.44	13.00%	(8,333.01)	108.00%	(1,517.30)	203.00%	(602.32)
15	194000.00%	(0.42)	1440.00%	(52.41)	–160.00%	59,413.89	14.00%	(8,122.05)	109.00%	(1,498.36)	204.00%	(597.84)
16	190000.00%	(0.43)	1400.00%	(53.89)	–150.00%	101,230.00	15.00%	(7,918.36)	110.00%	(1,479.77)	205.00%	(593.41)
17	186000.00%	(0.44)	1360.00%	(55.46)	–140.00%	195,162.50	16.00%	(7,721.63)	111.00%	(1,461.52)	206.00%	(589.03)
18	182000.00%	(0.45)	1320.00%	(57.12)	–130.00%	457,161.11	17.00%	(7,531.57)	112.00%	(1,443.61)	207.00%	(584.71)
19	178000.00%	(0.46)	1280.00%	(58.90)	–120.00%	1,526,575.00	18.00%	(7,347.88)	113.00%	(1,426.03)	208.00%	(580.44)
20	174000.00%	(0.47)	1240.00%	(60.80)	–110.00%	12,098,150.00	19.00%	(7,170.32)	114.00%	(1,408.76)	209.00%	(576.22)
21	170000.00%	(0.48)	1200.00%	(62.83)	–100.00%	($815.00)	20.00%	(6,998.61)	115.00%	(1,391.81)	210.00%	(572.06)
22	166000.00%	(0.49)	1160.00%	(65.01)	–90.00%	(11,918,150.00)	21.00%	(6,832.53)	116.00%	(1,375.16)	211.00%	(567.94)
23	162000.00%	(0.50)	1120.00%	(67.36)	–80.00%	(1,481,575.00)	22.00%	(6,671.84)	117.00%	(1,358.81)	212.00%	(563.87)
24	158000.00%	(0.52)	1080.00%	(69.91)	–70.00%	(437,161.11)	23.00%	(6,516.32)	118.00%	(1,342.75)	213.00%	(559.85)
25	154000.00%	(0.53)	1040.00%	(72.67)	–60.00%	(183,912.50)	24.00%	(6,365.78)	119.00%	(1,326.97)	214.00%	(555.88)
26	150000.00%	(0.54)	1000.00%	(75.67)	–50.00%	(94,030.00)	25.00%	(6,220.00)	120.00%	(1,311.48)	215.00%	(551.96)
27	146000.00%	(0.56)	960.00%	(78.95)	–40.00%	(54,413.89)	26.00%	(6,078.80)	121.00%	(1,296.25)	216.00%	(548.08)
28	142000.00%	(0.57)	920.00%	(82.56)	–30.00%	(34,312.97)	27.00%	(5,942.01)	122.00%	(1,281.29)	217.00%	(544.24)
29	138000.00%	(0.59)	880.00%	(86.54)	–20.00%	(23,050.00)	28.00%	(5,809.45)	123.00%	(1,266.59)	218.00%	(540.45)
30	134000.00%	(0.61)	840.00%	(90.96)	–10.00%	(16,255.35)	29.00%	(5,680.95)	124.00%	(1,252.14)	219.00%	(536.71)
31	130000.00%	(0.63)	800.00%	(95.91)	0.00%	(11,915.00)	30.00%	(5,556.37)	125.00%	(1,237.94)	220.00%	(533.01)
32	126000.00%	(0.65)	760.00%	(101.46)	10.00%	(9,012.89)	31.00%	(5,435.55)	126.00%	(1,223.99)	221.00%	(529.35)
33	122000.00%	(0.67)	720.00%	(107.77)	20.00%	(6,998.61)	32.00%	(5,318.36)	127.00%	(1,210.27)	222.00%	(525.73)
34	118000.00%	(0.69)	680.00%	(114.98)	30.00%	(5,556.37)	33.00%	(5,204.65)	128.00%	(1,196.78)	223.00%	(522.16)
35	114000.00%	(0.71)	640.00%	(123.31)	40.00%	(4,496.14)	34.00%	(5,094.30)	129.00%	(1,183.53)	224.00%	(518.62)
36	110000.00%	(0.74)	600.00%	(133.05)	50.00%	(3,698.89)	35.00%	(4,987.18)	130.00%	(1,170.49)	225.00%	(515.13)
37	106000.00%	(0.77)	560.00%	(144.56)	60.00%	(3,087.50)	36.00%	(4,883.18)	131.00%	(1,157.67)	226.00%	(511.68)
38	102000.00%	(0.80)	520.00%	(158.39)	70.00%	(2,610.49)	37.00%	(4,782.18)	132.00%	(1,145.07)	227.00%	(508.26)
39	98000.00%	(0.83)	480.00%	(175.27)	80.00%	(2,232.61)	38.00%	(4,684.07)	133.00%	(1,132.67)	228.00%	(504.88)
40	94000.00%	(0.87)	440.00%	(196.27)	90.00%	(1,929.17)	39.00%	(4,588.76)	134.00%	(1,120.48)	229.00%	(501.54)
41	90000.00%	(0.90)	400.00%	(223.00)	100.00%	(1,682.50)	40.00%	(4,496.14)	135.00%	(1,108.49)	230.00%	(498.24)
42	86000.00%	(0.95)	360.00%	(257.93)	110.00%	(1,479.77)	41.00%	(4,406.11)	136.00%	(1,096.69)	231.00%	(494.98)
43	82000.00%	(0.99)	320.00%	(305.00)	120.00%	(1,311.48)	42.00%	(4,318.59)	137.00%	(1,085.09)	232.00%	(491.75)
44	78000.00%	(1.04)	280.00%	(370.84)	130.00%	(1,170.49)	43.00%	(4,233.49)	138.00%	(1,073.67)	233.00%	(488.56)
45	74000.00%	(1.10)	240.00%	(467.16)	140.00%	(1,051.39)	44.00%	(4,150.72)	139.00%	(1,062.44)	234.00%	(485.40)
46	70000.00%	(1.16)	200.00%	(616.11)	150.00%	(950.00)	45.00%	(4,070.21)	140.00%	(1,051.39)	235.00%	(482.28)
47	66000.00%	(1.23)	160.00%	(863.07)	160.00%	(863.07)	46.00%	(3,991.87)	141.00%	(1,040.51)	236.00%	(479.19)
48	62000.00%	(1.31)	120.00%	(1,311.48)	170.00%	(788.06)	47.00%	(3,915.64)	142.00%	(1,029.81)	237.00%	(476.13)
49	58000.00%	(1.40)	80.00%	(2,232.61)	180.00%	(722.92)	48.00%	(3,841.45)	143.00%	(1,019.28)	238.00%	(473.11)
50	54000.00%	(1.50)	40.00%	(4,496.14)	190.00%	(666.04)	49.00%	(3,769.22)	144.00%	(1,008.91)	239.00%	(470.12)

51	50000.00%	(1.62)	0.00%	(11,915.00)	200.00%	(616.11)	50.00%	(3,698.89)	145.00%	(998.70)	240.00%	(467.16)
52	46000.00%	(1.76)	−40.00%	(54,413.89)	210.00%	(572.06)	51.00%	(3,630.40)	146.00%	(988.66)	241.00%	(464.24)
53	42000.00%	(1.93)	−80.00%	(1,481,575.00)	220.00%	(533.01)	52.00%	(3,563.68)	147.00%	(978.77)	242.00%	(461.34)
54	38000.00%	(2.13)	−120.00%	1,526,575.00	230.00%	(498.24)	53.00%	(3,498.69)	148.00%	(969.03)	243.00%	(458.48)
55	34000.00%	(2.38)	−160.00%	59,413.89	240.00%	(467.16)	54.00%	(3,435.36)	149.00%	(959.44)	244.00%	(455.65)
56	30000.00%	(2.70)	−200.00%	13,715.00	250.00%	(439.27)	55.00%	(3,373.65)	150.00%	(950.00)	245.00%	(452.85)
57	26000.00%	(3.11)	−240.00%	5,414.50	260.00%	(414.15)	56.00%	(3,313.49)	151.00%	(940.70)	246.00%	(450.07)
58	22000.00%	(3.67)	−280.00%	2,788.17	270.00%	(391.43)	57.00%	(3,254.84)	152.00%	(931.55)	247.00%	(447.33)
59	18000.00%	(4.48)	−320.00%	1,683.38	280.00%	(370.84)	58.00%	(3,197.66)	153.00%	(922.53)	248.00%	(444.62)
60	14000.00%	(5.74)	−360.00%	1,129.35	290.00%	(352.10)	59.00%	(3,141.89)	154.00%	(913.65)	249.00%	(441.93)
61	10000.00%	(7.99)	−400.00%	816.11	300.00%	(335.00)	60.00%	(3,087.50)	155.00%	(904.90)	250.00%	(439.27)
62	6000.00%	(13.17)	−440.00%	622.87	310.00%	(319.35)	61.00%	(3,034.44)	156.00%	(896.29)	251.00%	(436.64)
63	2000.00%	(38.06)	−480.00%	495.49	320.00%	(305.00)	62.00%	(2,982.67)	157.00%	(887.80)	252.00%	(434.04)
64	−2000.00%	47.14	−520.00%	407.04	330.00%	(291.79)	63.00%	(2,932.14)	158.00%	(879.43)	253.00%	(431.46)
65	−6000.00%	14.13	−560.00%	342.99	340.00%	(279.61)	64.00%	(2,882.84)	159.00%	(871.19)	254.00%	(428.91)
66	−10000.00%	8.34	−600.00%	295.00	350.00%	(268.35)	65.00%	(2,834.70)	160.00%	(863.07)	255.00%	(426.39)
67	−14000.00%	5.91	−640.00%	258.00	360.00%	(257.93)	66.00%	(2,787.71)	161.00%	(855.07)	256.00%	(423.89)
68	−18000.00%	4.58	−680.00%	228.77	370.00%	(248.24)	67.00%	(2,741.83)	162.00%	(847.19)	257.00%	(421.42)
69	−22000.00%	3.74	−720.00%	205.22	380.00%	(239.24)	68.00%	(2,697.02)	163.00%	(839.42)	258.00%	(418.97)
70	−26000.00%	3.16	−760.00%	185.89	390.00%	(230.84)	69.00%	(2,653.25)	164.00%	(831.76)	259.00%	(416.54)
71	−30000.00%	2.74	−800.00%	169.78	400.00%	(223.00)	70.00%	(2,610.49)	165.00%	(824.22)	260.00%	(414.15)
72	−34000.00%	2.41	−840.00%	156.18	410.00%	(215.66)	71.00%	(2,568.72)	166.00%	(816.78)	261.00%	(411.77)
73	−38000.00%	2.16	−880.00%	144.57	420.00%	(208.79)	72.00%	(2,527.90)	167.00%	(809.44)	262.00%	(409.42)
74	−42000.00%	1.95	−920.00%	134.54	430.00%	(202.34)	73.00%	(2,488.01)	168.00%	(802.21)	263.00%	(407.09)
75	−46000.00%	1.78	−960.00%	125.80	440.00%	(196.27)	74.00%	(2,449.02)	169.00%	(795.09)	264.00%	(404.79)
76	−50000.00%	1.64	−1000.00%	118.13	450.00%	(190.56)	75.00%	(2,410.90)	170.00%	(788.06)	265.00%	(402.51)
77	−54000.00%	1.52	−1040.00%	111.34	460.00%	(185.17)	76.00%	(2,373.64)	171.00%	(781.13)	266.00%	(400.25)
78	−58000.00%	1.41	−1080.00%	105.28	470.00%	(180.08)	77.00%	(2,337.20)	172.00%	(774.30)	267.00%	(398.01)
79	−62000.00%	1.32	−1120.00%	99.86	480.00%	(175.27)	78.00%	(2,301.56)	173.00%	(767.56)	268.00%	(395.80)
80	−66000.00%	1.24	−1160.00%	94.97	490.00%	(170.71)	79.00%	(2,266.71)	174.00%	(760.92)	269.00%	(393.61)
81	−70000.00%	1.17	−1200.00%	90.54	500.00%	(166.39)	80.00%	(2,232.61)	175.00%	(754.37)	270.00%	(391.43)
82	−74000.00%	1.10	−1240.00%	86.52	510.00%	(162.29)	81.00%	(2,199.26)	176.00%	(747.90)	271.00%	(389.28)
83	−78000.00%	1.05	−1280.00%	82.84	520.00%	(158.39)	82.00%	(2,166.62)	177.00%	(741.53)	272.00%	(387.16)
84	−82000.00%	1.00	−1320.00%	79.46	530.00%	(154.68)	83.00%	(2,134.68)	178.00%	(735.24)	273.00%	(385.05)
85	−86000.00%	0.95	−1360.00%	76.35	540.00%	(151.15)	84.00%	(2,103.42)	179.00%	(729.04)	274.00%	(382.96)
86	−90000.00%	0.91	−1400.00%	73.48	550.00%	(147.78)	85.00%	(2,072.82)	180.00%	(722.92)	275.00%	(380.89)
87	−94000.00%	0.87	−1440.00%	70.82	560.00%	(144.56)	86.00%	(2,042.87)	181.00%	(716.89)	276.00%	(378.84)
88	−98000.00%	0.83	−1480.00%	68.35	570.00%	(141.49)	87.00%	(2,013.54)	182.00%	(710.93)	277.00%	(376.81)
89	−102000.00%	0.80	−1520.00%	66.05	580.00%	(138.55)	88.00%	(1,984.83)	183.00%	(705.06)	278.00%	(374.80)
90	−106000.00%	0.77	−1560.00%	63.90	590.00%	(135.74)	89.00%	(1,956.71)	184.00%	(699.26)	279.00%	(372.81)
91	−110000.00%	0.74	−1600.00%	61.89	600.00%	(133.05)	90.00%	(1,929.17)	185.00%	(693.54)	280.00%	(370.84)
92	−114000.00%	0.72	−1640.00%	60.00	610.00%	(130.46)	91.00%	(1,902.19)	186.00%	(687.89)	281.00%	(368.88)
93	−118000.00%	0.69	−1680.00%	58.23	620.00%	(127.98)	92.00%	(1,875.76)	187.00%	(682.32)	282.00%	(366.95)
94	−122000.00%	0.67	−1720.00%	56.56	630.00%	(125.60)	93.00%	(1,849.87)	188.00%	(676.83)	283.00%	(365.03)
95	−126000.00%	0.65	−1760.00%	54.99	640.00%	(123.31)	94.00%	(1,824.49)	189.00%	(671.40)	284.00%	(363.13)

(*continued*)

Chart 4B (continued)

Table-B8:
Project Outcome/Cashflows:

0	1	2	3	4	5	6	7
(815.00)	900.00	(100.00)	1,200.00	(1,200.00)	100.00	(119,980.00)	(12,000.00)

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.33)	2000.00%	(36.77)	−300.00%	1,649.53	0.00%	(131,895.00)	95.00%	(1,328.99)	190.00%	(238.79)
2	246000.00%	(0.33)	1960.00%	(37.45)	−290.00%	2,107.10	1.00%	(122,905.41)	96.00%	(1,288.06)	191.00%	(236.93)
3	242000.00%	(0.34)	1920.00%	(38.15)	−280.00%	2,779.32	2.00%	(114,609.36)	97.00%	(1,248.78)	192.00%	(235.11)
4	238000.00%	(0.34)	1880.00%	(38.87)	−270.00%	3,795.42	3.00%	(106,947.27)	98.00%	(1,211.09)	193.00%	(233.34)
5	234000.00%	(0.35)	1840.00%	(39.62)	−260.00%	5,379.06	4.00%	(99,865.19)	99.00%	(1,174.90)	194.00%	(231.61)
6	230000.00%	(0.35)	1800.00%	(40.41)	−250.00%	7,930.73	5.00%	(93,314.23)	100.00%	(1,140.16)	195.00%	(229.92)
7	226000.00%	(0.36)	1760.00%	(41.22)	−240.00%	12,195.26	6.00%	(87,250.04)	101.00%	(1,106.79)	196.00%	(228.27)
8	222000.00%	(0.37)	1720.00%	(42.07)	−230.00%	19,618.75	7.00%	(81,632.32)	102.00%	(1,074.73)	197.00%	(226.66)
9	218000.00%	(0.37)	1680.00%	(42.95)	−220.00%	33,149.88	8.00%	(76,424.46)	103.00%	(1,043.93)	198.00%	(225.09)
10	214000.00%	(0.38)	1640.00%	(43.87)	−210.00%	59,151.63	9.00%	(71,593.16)	104.00%	(1,014.34)	199.00%	(223.55)
11	210000.00%	(0.39)	1600.00%	(44.83)	−200.00%	112,295.00	10.00%	(67,108.08)	105.00%	(985.89)	200.00%	(222.05)
12	206000.00%	(0.40)	1560.00%	(45.84)	−190.00%	229,174.90	11.00%	(62,941.59)	106.00%	(958.54)	201.00%	(220.58)
13	202000.00%	(0.40)	1520.00%	(46.89)	−180.00%	510,177.23	12.00%	(59,068.47)	107.00%	(932.24)	202.00%	(219.14)
14	198000.00%	(0.41)	1480.00%	(47.98)	−170.00%	1,264,996.48	13.00%	(55,465.70)	108.00%	(906.95)	203.00%	(217.73)
15	194000.00%	(0.42)	1440.00%	(49.14)	−160.00%	3,602,686.64	14.00%	(52,112.27)	109.00%	(882.62)	204.00%	(216.36)
16	190000.00%	(0.43)	1400.00%	(50.34)	−150.00%	12,355,470.00	15.00%	(48,988.94)	110.00%	(859.20)	205.00%	(215.01)
17	186000.00%	(0.44)	1360.00%	(51.61)	−140.00%	55,117,135.16	16.00%	(46,078.12)	111.00%	(836.68)	206.00%	(213.70)
18	182000.00%	(0.45)	1320.00%	(52.94)	−130.00%	366,502,017.08	17.00%	(43,363.70)	112.00%	(814.99)	207.00%	(212.41)
19	178000.00%	(0.46)	1280.00%	(54.34)	−120.00%	4.69E+09	18.00%	(40,830.90)	113.00%	(794.11)	208.00%	(211.15)
20	174000.00%	(0.47)	1240.00%	(55.82)	−110.00%	3.22E+07	19.00%	(38,466.19)	114.00%	(774.01)	209.00%	(209.92)
21	170000.00%	(0.48)	1200.00%	(57.38)	−100.00%	(815.00)	20.00%	(36,257.13)	115.00%	(754.65)	210.00%	(208.71)
22	166000.00%	(0.49)	1160.00%	(59.02)	−90.00%	−2.40E+12	21.00%	(34,192.30)	116.00%	(736.00)	211.00%	(207.53)
23	162000.00%	(0.50)	1120.00%	(60.76)	−80.00%	−1.41E+10	22.00%	(32,261.21)	117.00%	(718.03)	212.00%	(206.37)
24	158000.00%	(0.52)	1080.00%	(62.61)	−70.00%	−7.32E+08	23.00%	(30,454.20)	118.00%	(700.72)	213.00%	(205.23)
25	154000.00%	(0.53)	1040.00%	(64.57)	−60.00%	(91,584,400.78)	24.00%	(28,762.39)	119.00%	(684.03)	214.00%	(204.12)
26	150000.00%	(0.54)	1000.00%	(66.66)	−50.00%	(18,441,070.00)	25.00%	(27,177.58)	120.00%	(667.95)	215.00%	(203.03)
27	146000.00%	(0.56)	960.00%	(68.88)	−40.00%	(5,003,779.46)	26.00%	(25,692.23)	121.00%	(652.43)	216.00%	(201.96)
28	142000.00%	(0.57)	920.00%	(71.26)	−30.00%	(1,665,946.87)	27.00%	(24,299.39)	122.00%	(637.48)	217.00%	(200.92)
29	138000.00%	(0.59)	880.00%	(73.80)	−20.00%	(643,793.56)	28.00%	(22,992.62)	123.00%	(623.05)	218.00%	(199.89)
30	134000.00%	(0.61)	840.00%	(76.52)	−10.00%	(278,671.73)	29.00%	(21,766.01)	124.00%	(609.13)	219.00%	(198.88)
31	130000.00%	(0.63)	800.00%	(79.44)	0.00%	(131,895.00)	30.00%	(20,614.06)	125.00%	(595.70)	220.00%	(197.90)
32	126000.00%	(0.65)	760.00%	(82.60)	10.00%	(67,108.08)	31.00%	(19,531.72)	126.00%	(582.74)	221.00%	(196.93)
33	122000.00%	(0.67)	720.00%	(86.00)	20.00%	(36,257.13)	32.00%	(18,514.28)	127.00%	(570.23)	222.00%	(195.98)
34	118000.00%	(0.69)	680.00%	(89.69)	30.00%	(20,614.06)	33.00%	(17,557.42)	128.00%	(558.15)	223.00%	(195.05)
35	114000.00%	(0.71)	640.00%	(93.70)	40.00%	(12,251.84)	34.00%	(16,657.10)	129.00%	(546.49)	224.00%	(194.13)
36	110000.00%	(0.74)	600.00%	(98.07)	50.00%	(7,575.54)	35.00%	(15,809.61)	130.00%	(535.23)	225.00%	(193.24)
37	106000.00%	(0.77)	560.00%	(102.86)	60.00%	(4,856.60)	36.00%	(15,011.48)	131.00%	(524.35)	226.00%	(192.35)
38	102000.00%	(0.80)	520.00%	(108.12)	70.00%	(3,220.99)	37.00%	(14,259.52)	132.00%	(513.84)	227.00%	(191.49)
39	98000.00%	(0.83)	480.00%	(113.95)	80.00%	(2,207.05)	38.00%	(13,550.73)	133.00%	(503.68)	228.00%	(190.64)
40	94000.00%	(0.87)	440.00%	(120.46)	90.00%	(1,561.38)	39.00%	(12,882.36)	134.00%	(493.87)	229.00%	(189.80)
41	90000.00%	(0.90)	400.00%	(127.82)	100.00%	(1,140.16)	40.00%	(12,251.84)	135.00%	(484.38)	230.00%	(188.98)
42	86000.00%	(0.95)	360.00%	(136.37)	110.00%	(859.20)	41.00%	(11,656.77)	136.00%	(475.21)	231.00%	(188.18)
43	82000.00%	(0.99)	320.00%	(146.75)	120.00%	(667.95)	42.00%	(11,094.94)	137.00%	(466.34)	232.00%	(187.39)
44	78000.00%	(1.04)	280.00%	(160.46)	130.00%	(535.23)	43.00%	(10,564.27)	138.00%	(457.76)	233.00%	(186.61)
45	74000.00%	(1.10)	240.00%	(181.51)	140.00%	(441.44)	44.00%	(10,062.83)	139.00%	(449.47)	234.00%	(185.84)
46	70000.00%	(1.16)	200.00%	(222.05)	150.00%	(374.00)	45.00%	(9,588.83)	140.00%	(441.44)	235.00%	(185.09)
47	66000.00%	(1.23)	160.00%	(324.66)	160.00%	(324.66)	46.00%	(9,140.59)	141.00%	(433.68)	236.00%	(184.35)
48	62000.00%	(1.31)	120.00%	(667.95)	170.00%	(287.95)	47.00%	(8,716.55)	142.00%	(426.16)	237.00%	(183.62)
49	58000.00%	(1.40)	80.00%	(2,207.05)	180.00%	(260.17)	48.00%	(8,315.26)	143.00%	(418.88)	238.00%	(182.90)
50	54000.00%	(1.50)	40.00%	(12,251.84)	190.00%	(238.79)	49.00%	(7,935.34)	144.00%	(411.84)	239.00%	(182.20)

51	50000.00%	(1.62)	0.00%	(131,895.00)	200.00%	(222.05)	50.00%	(7,575.54)	145.00%	(405.02)	240.00%	(181.51)
52	46000.00%	(1.76)	−40.00%	(5,003,779.46)	210.00%	(208.71)	51.00%	(7,234.66)	146.00%	(398.42)	241.00%	(180.82)
53	42000.00%	(1.93)	−80.00%	−1.41E+10	220.00%	(197.90)	52.00%	(6,911.60)	147.00%	(392.02)	242.00%	(180.15)
54	38000.00%	(2.13)	−120.00%	4.69E+09	230.00%	(188.98)	53.00%	(6,605.31)	148.00%	(385.82)	243.00%	(179.49)
55	34000.00%	(2.38)	−160.00%	3,602,686.64	240.00%	(181.51)	54.00%	(6,314.82)	149.00%	(379.82)	244.00%	(178.84)
56	30000.00%	(2.70)	−200.00%	112,295.00	250.00%	(175.13)	55.00%	(6,039.22)	150.00%	(374.00)	245.00%	(178.20)
57	26000.00%	(3.11)	−240.00%	12,195.26	260.00%	(169.62)	56.00%	(5,777.66)	151.00%	(368.36)	246.00%	(177.57)
58	22000.00%	(3.67)	−280.00%	2,779.32	270.00%	(164.77)	57.00%	(5,529.35)	152.00%	(362.89)	247.00%	(176.95)
59	18000.00%	(4.48)	−320.00%	1,100.33	280.00%	(160.46)	58.00%	(5,293.53)	153.00%	(357.59)	248.00%	(176.33)
60	14000.00%	(5.73)	−360.00%	632.60	290.00%	(156.57)	59.00%	(5,069.50)	154.00%	(352.45)	249.00%	(175.73)
61	10000.00%	(7.98)	−400.00%	448.29	300.00%	(153.03)	60.00%	(4,856.60)	155.00%	(347.46)	250.00%	(175.13)
62	6000.00%	(13.12)	−440.00%	353.96	310.00%	(149.77)	61.00%	(4,654.22)	156.00%	(342.62)	251.00%	(174.55)
63	2000.00%	(36.77)	−480.00%	296.14	320.00%	(146.75)	62.00%	(4,461.78)	157.00%	(337.93)	252.00%	(173.97)
64	−2000.00%	45.41	−520.00%	256.29	330.00%	(143.92)	63.00%	(4,278.73)	158.00%	(333.37)	253.00%	(173.40)
65	−6000.00%	14.07	−560.00%	226.70	340.00%	(141.27)	64.00%	(4,104.56)	159.00%	(328.95)	254.00%	(172.84)
66	−10000.00%	8.32	−600.00%	203.62	350.00%	(138.76)	65.00%	(3,938.79)	160.00%	(324.66)	255.00%	(172.28)
67	−14000.00%	5.91	−640.00%	184.98	360.00%	(136.37)	66.00%	(3,780.96)	161.00%	(320.49)	256.00%	(171.73)
68	−18000.00%	4.58	−680.00%	169.56	370.00%	(134.10)	67.00%	(3,630.66)	162.00%	(316.45)	257.00%	(171.20)
69	−22000.00%	3.74	−720.00%	156.56	380.00%	(131.92)	68.00%	(3,487.48)	163.00%	(312.52)	258.00%	(170.66)
70	−26000.00%	3.16	−760.00%	145.44	390.00%	(129.83)	69.00%	(3,351.04)	164.00%	(308.70)	259.00%	(170.14)
71	−30000.00%	2.74	−800.00%	135.80	400.00%	(127.82)	70.00%	(3,220.99)	165.00%	(304.99)	260.00%	(169.62)
72	−34000.00%	2.41	−840.00%	127.37	410.00%	(125.89)	71.00%	(3,097.00)	166.00%	(301.39)	261.00%	(169.11)
73	−38000.00%	2.16	−880.00%	119.92	420.00%	(124.02)	72.00%	(2,978.75)	167.00%	(297.88)	262.00%	(168.60)
74	−42000.00%	1.95	−920.00%	113.30	430.00%	(122.21)	73.00%	(2,865.94)	168.00%	(294.48)	263.00%	(168.10)
75	−46000.00%	1.78	−960.00%	107.37	440.00%	(120.46)	74.00%	(2,758.29)	169.00%	(291.17)	264.00%	(167.61)
76	−50000.00%	1.64	−1000.00%	102.03	450.00%	(118.76)	75.00%	(2,655.54)	170.00%	(287.95)	265.00%	(167.12)
77	−54000.00%	1.52	−1040.00%	97.20	460.00%	(117.11)	76.00%	(2,557.45)	171.00%	(284.82)	266.00%	(166.64)
78	−58000.00%	1.41	−1080.00%	92.80	470.00%	(115.51)	77.00%	(2,463.77)	172.00%	(281.77)	267.00%	(166.16)
79	−62000.00%	1.32	−1120.00%	88.78	480.00%	(113.95)	78.00%	(2,374.28)	173.00%	(278.81)	268.00%	(165.69)
80	−66000.00%	1.24	−1160.00%	85.09	490.00%	(112.43)	79.00%	(2,288.77)	174.00%	(275.93)	269.00%	(165.23)
81	−70000.00%	1.17	−1200.00%	81.70	500.00%	(110.96)	80.00%	(2,207.05)	175.00%	(273.12)	270.00%	(164.77)
82	−74000.00%	1.10	−1240.00%	78.57	510.00%	(109.52)	81.00%	(2,128.92)	176.00%	(270.39)	271.00%	(164.32)
83	−78000.00%	1.05	−1280.00%	75.66	520.00%	(108.12)	82.00%	(2,054.22)	177.00%	(267.73)	272.00%	(163.87)
84	−82000.00%	1.00	−1320.00%	72.97	530.00%	(106.76)	83.00%	(1,982.77)	178.00%	(265.15)	273.00%	(163.43)
85	−86000.00%	0.95	−1360.00%	70.46	540.00%	(105.42)	84.00%	(1,914.41)	179.00%	(262.63)	274.00%	(162.99)
86	−90000.00%	0.91	−1400.00%	68.11	550.00%	(104.13)	85.00%	(1,849.00)	180.00%	(260.17)	275.00%	(162.56)
87	−94000.00%	0.87	−1440.00%	65.92	560.00%	(102.86)	86.00%	(1,786.39)	181.00%	(257.78)	276.00%	(162.13)
88	−98000.00%	0.83	−1480.00%	63.86	570.00%	(101.62)	87.00%	(1,726.44)	182.00%	(255.45)	277.00%	(161.70)
89	−102000.00%	0.80	−1520.00%	61.93	580.00%	(100.41)	88.00%	(1,669.04)	183.00%	(253.18)	278.00%	(161.28)
90	−106000.00%	0.77	−1560.00%	60.11	590.00%	(99.23)	89.00%	(1,614.06)	184.00%	(250.96)	279.00%	(160.87)
91	−110000.00%	0.74	−1600.00%	58.39	600.00%	(98.07)	90.00%	(1,561.38)	185.00%	(248.80)	280.00%	(160.46)
92	−114000.00%	0.72	−1640.00%	56.77	610.00%	(96.94)	91.00%	(1,510.91)	186.00%	(246.70)	281.00%	(160.05)
93	−118000.00%	0.69	−1680.00%	55.23	620.00%	(95.84)	92.00%	(1,462.52)	187.00%	(244.65)	282.00%	(159.65)
94	−122000.00%	0.67	−1720.00%	53.78	630.00%	(94.76)	93.00%	(1,416.14)	188.00%	(242.65)	283.00%	(159.25)
95	−126000.00%	0.65	−1760.00%	52.40	640.00%	(93.70)	94.00%	(1,371.65)	189.00%	(240.69)	284.00%	(158.86)

(*continued*)

Chart 4B (continued)

Table-B9:
Project Outcome/Cashflows:

0	1	2	3	4	5	6
200.00	4.0000E+09	200.00	30,000,000.00	400.00	2.000000000E+12	500,000,000.00

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	639.57	2000.00%	9,093,778.10	–300.00%	32,247,968,612.50	0.00%	2.00453E+12	95.00%	3.74353E+10	190.00%	3.83868E+09
2	246000.00%	660.53	1960.00%	9,452,307.12	–290.00%	43,616,433,392.36	1.00%	1.88851E+12	96.00%	3.63249E+10	191.00%	3.76667E+09
3	242000.00%	682.53	1920.00%	9,832,589.88	–280.00%	60,031,646,756.08	2.00%	1.78025E+12	97.00%	3.52533E+10	192.00%	3.69633E+09
4	238000.00%	705.66	1880.00%	10,236,438.66	–270.00%	84,233,875,327.32	3.00%	1.67917E+12	98.00%	3.42189E+10	193.00%	3.62761E+09
5	234000.00%	729.98	1840.00%	10,665,860.89	–260.00%	1.207577E+11	4.00%	1.58473E+12	99.00%	3.32202E+10	194.00%	3.56047E+09
6	230000.00%	755.57	1800.00%	11,123,085.42	–250.00%	1.773374E+11	5.00%	1.49644E+12	100.00%	3.22558E+10	195.00%	3.49487E+09
7	226000.00%	782.54	1760.00%	11,610,592.95	–240.00%	2.676218E+11	6.00%	1.41384E+12	101.00%	3.13244E+10	196.00%	3.43076E+09
8	222000.00%	810.98	1720.00%	12,131,151.73	–230.00%	4.166501E+11	7.00%	1.33651E+12	102.00%	3.04247E+10	197.00%	3.36811E+09
9	218000.00%	841.00	1680.00%	12,687,859.23	–220.00%	6.724487E+11	8.00%	1.26408E+12	103.00%	2.95554E+10	198.00%	3.30688E+09
10	214000.00%	872.72	1640.00%	13,284,191.44	–210.00%	1.132018E+12	9.00%	1.19620E+12	104.00%	2.87154E+10	199.00%	3.24703E+09
11	210000.00%	906.26	1600.00%	13,924,061.05	–200.00%	2.003530E+12	10.00%	1.13253E+12	105.00%	2.79036E+10	200.00%	3.18853E+09
12	206000.00%	941.78	1560.00%	14,611,886.70	–190.00%	3.767291E+12	11.00%	1.07279E+12	106.00%	2.71188E+10	201.00%	3.13133E+09
13	202000.00%	979.43	1520.00%	15,352,675.77	–180.00%	7.633334E+12	12.00%	1.01670E+12	107.00%	2.63601E+10	202.00%	3.07542E+09
14	198000.00%	1,019.38	1480.00%	16,152,123.91	–170.00%	1.700194E+13	13.00%	9.64001E+11	108.00%	2.56265E+10	203.00%	3.02074E+09
15	194000.00%	1,061.82	1440.00%	17,016,735.40	–160.00%	4.286042E+13	14.00%	9.14469E+11	109.00%	2.49169E+10	204.00%	2.96728E+09
16	190000.00%	1,106.97	1400.00%	17,953,969.68	–150.00%	1.2795E+14	15.00%	8.67885E+11	110.00%	2.42306E+10	205.00%	2.91500E+09
17	186000.00%	1,155.07	1360.00%	18,972,420.89	–140.00%	4.8800E+14	16.00%	8.24051E+11	111.00%	2.35666E+10	206.00%	2.86386E+09
18	182000.00%	1,206.37	1320.00%	20,082,039.66	–130.00%	2.7412E+15	17.00%	7.82782E+11	112.00%	2.29241E+10	207.00%	2.81385E+09
19	178000.00%	1,261.16	1280.00%	21,294,409.04	–120.00%	3.1211E+16	18.00%	7.43908E+11	113.00%	2.23023E+10	208.00%	2.76493E+09
20	174000.00%	1,319.78	1240.00%	22,623,090.98	–110.00%	1.9950E+18	19.00%	7.07272E+11	114.00%	2.17005E+10	209.00%	2.71707E+09
21	170000.00%	1,382.57	1200.00%	24,084,065.30	–100.00%	200.0	20.00%	6.72728E+11	115.00%	2.11179E+10	210.00%	2.67025E+09
22	166000.00%	1,449.96	1160.00%	25,696,291.49	–90.00%	2.0050E+18	21.00%	6.40139E+11	116.00%	2.05538E+10	211.00%	2.62444E+09
23	162000.00%	1,522.40	1120.00%	27,482,435.39	–80.00%	3.1289E+16	22.00%	6.09381E+11	117.00%	2.00075E+10	212.00%	2.57962E+09
24	158000.00%	1,600.41	1080.00%	29,469,820.40	–70.00%	2.7458E+15	23.00%	5.80337E+11	118.00%	1.94785E+10	213.00%	2.53576E+09
25	154000.00%	1,684.57	1040.00%	31,691,688.14	–60.00%	4.8861E+14	24.00%	5.52899E+11	119.00%	1.89660E+10	214.00%	2.49283E+09
26	150000.00%	1,775.54	1000.00%	34,188,892.13	–50.00%	1.2808E+14	25.00%	5.26965E+11	120.00%	1.84695E+10	215.00%	2.45082E+09
27	146000.00%	1,874.09	960.00%	37,012,207.25	–40.00%	4.28961E+13	26.00%	5.02443E+11	121.00%	1.79885E+10	216.00%	2.40970E+09
28	142000.00%	1,981.08	920.00%	40,225,528.88	–30.00%	1.70141E+13	27.00%	4.79244E+11	122.00%	1.75223E+10	217.00%	2.36945E+09
29	138000.00%	2,097.50	880.00%	43,910,381.50	–20.00%	7.63810E+12	28.00%	4.57289E+11	123.00%	1.70704E+10	218.00%	2.33005E+09
30	134000.00%	2,224.50	840.00%	48,172,391.59	–10.00%	3.76938E+12	29.00%	4.36501E+11	124.00%	1.66324E+10	219.00%	2.29147E+09
31	130000.00%	2,363.38	800.00%	53,150,768.41	0.00%	2.00453E+12	30.00%	4.16809E+11	125.00%	1.62077E+10	220.00%	2.25370E+09
32	126000.00%	2,515.69	760.00%	59,032,493.99	10.00%	1.13253E+12	31.00%	3.98150E+11	126.00%	1.57959E+10	221.00%	2.21672E+09
33	122000.00%	2,683.21	720.00%	66,074,063.47	20.00%	6.72728E+11	32.00%	3.80460E+11	127.00%	1.53966E+10	222.00%	2.18050E+09
34	118000.00%	2,868.04	680.00%	74,635,645.54	30.00%	4.16809E+11	33.00%	3.63682E+11	128.00%	1.50092E+10	223.00%	2.14504E+09
35	114000.00%	3,072.65	640.00%	85,236,249.03	40.00%	2.67717E+11	34.00%	3.47764E+11	129.00%	1.46335E+10	224.00%	2.11031E+09
36	110000.00%	3,299.96	600.00%	98,645,503.67	50.00%	1.77396E+11	35.00%	3.32655E+11	130.00%	1.42689E+10	225.00%	2.07629E+09
37	106000.00%	3,553.47	560.00%	116,041,393.31	60.00%	1.20795E+11	36.00%	3.18309E+11	131.00%	1.39152E+10	226.00%	2.04296E+09
38	102000.00%	3,837.34	520.00%	139,291,149.87	70.00%	8.42582E+10	37.00%	3.04682E+11	132.00%	1.35719E+10	227.00%	2.01032E+09
39	98000.00%	4,156.65	480.00%	171,471,482.86	80.00%	6.00480E+10	38.00%	2.91732E+11	133.00%	1.32387E+10	228.00%	1.97834E+09
40	94000.00%	4,517.53	440.00%	217,874,977.66	90.00%	4.36276E+10	39.00%	2.79423E+11	134.00%	1.29153E+10	229.00%	1.94700E+09
41	90000.00%	4,927.54	400.00%	288,054,441.73	100.00%	3.22558E+10	40.00%	2.67717E+11	135.00%	1.26013E+10	230.00%	1.91631E+09
42	86000.00%	5,396.01	360.00%	400,211,896.51	110.00%	2.42306E+10	41.00%	2.56581E+11	136.00%	1.22964E+10	231.00%	1.88622E+09
43	82000.00%	5,934.60	320.00%	591,238,505.60	120.00%	1.84695E+10	42.00%	2.45984E+11	137.00%	1.20003E+10	232.00%	1.85674E+09
44	78000.00%	6,558.05	280.00%	941,441,129.40	130.00%	1.42689E+10	43.00%	2.35895E+11	138.00%	1.17127E+10	233.00%	1.82785E+09
45	74000.00%	7,285.17	240.00%	1.6410E+09	140.00%	1.11620E+10	44.00%	2.26288E+11	139.00%	1.14334E+10	234.00%	1.79954E+09
46	70000.00%	8,140.28	200.00%	3.1885E+09	150.00%	8.83359E+09	45.00%	2.17136E+11	140.00%	1.11620E+10	235.00%	1.77179E+09
47	66000.00%	9,155.28	160.00%	7.0673E+09	160.00%	7.06725E+09	46.00%	2.08415E+11	141.00%	1.08983E+10	236.00%	1.74458E+09
48	62000.00%	10,372.66	120.00%	1.8470E+10	170.00%	5,712,089,013.70	47.00%	2.00101E+11	142.00%	1.06421E+10	237.00%	1.71791E+09
49	58000.00%	11,850.05	80.00%	6.0048E+10	180.00%	4,661,384,951.54	48.00%	1.92174E+11	143.00%	1.03931E+10	238.00%	1.69177E+09
50	54000.00%	13,667.13	40.00%	2.6772E+11	190.00%	3,838,681,302.90	49.00%	1.84612E+11	144.00%	1.01511E+10	239.00%	1.66614E+09

51	50000.00%	15,936.59	0.00%	2.0045E+12	200.00%	3,188,527,739.19	50.00%	1.77396E+11	145.00%	9.91586E+09	240.00%	1.64100E+09
52	46000.00%	18,822.10	–40.00%	4.2896E+13	210.00%	2,670,251,521.78	51.00%	1.70509E+11	146.00%	9.68714E+09	241.00%	1.61636E+09
53	42000.00%	22,568.62	–80.00%	3.1289E+16	220.00%	2,253,701,840.47	52.00%	1.63933E+11	147.00%	9.46475E+09	242.00%	1.59219E+09
54	38000.00%	27,556.14	–120.00%	3.1211E+16	230.00%	1,916,305,135.97	53.00%	1.57652E+11	148.00%	9.24849E+09	243.00%	1.56849E+09
55	34000.00%	34,400.02	–160.00%	4.2860E+13	240.00%	1,641,002,791.46	54.00%	1.51652E+11	149.00%	9.03816E+09	244.00%	1.54524E+09
56	30000.00%	44,150.29	–200.00%	2.0035E+12	250.00%	1,414,790,352.82	55.00%	1.45918E+11	150.00%	8.83359E+09	245.00%	1.52245E+09
57	26000.00%	58,719.82	–240.00%	2.6762E+11	260.00%	1,227,671,768.74	56.00%	1.40437E+11	151.00%	8.63458E+09	246.00%	1.50008E+09
58	22000.00%	81,899.34	–280.00%	6.0032E+10	270.00%	1,071,903,214.54	57.00%	1.35195E+11	152.00%	8.44096E+09	247.00%	1.47814E+09
59	18000.00%	122,097.58	–320.00%	1.8466E+10	280.00%	941,441,129.40	58.00%	1.30182E+11	153.00%	8.25257E+09	248.00%	1.45662E+09
60	14000.00%	201,198.87	–360.00%	7.0660E+09	290.00%	831,535,712.10	59.00%	1.25385E+11	154.00%	8.06925E+09	249.00%	1.43551E+09
61	10000.00%	392,122.57	–400.00%	3.1881E+09	300.00%	738,429,008.59	60.00%	1.20795E+11	155.00%	7.89083E+09	250.00%	1.41479E+09
62	6000.00%	1,075,024.11	–440.00%	1.6408E+09	310.00%	659,128,882.13	61.00%	1.16401E+11	156.00%	7.71717E+09	251.00%	1.39446E+09
63	2000.00%	9,093,778.10	–480.00%	941,353,615.16	320.00%	591,238,505.60	62.00%	1.12193E+11	157.00%	7.54812E+09	252.00%	1.37452E+09
64	–2000.00%	11,123,063.19	–520.00%	591,195,027.80	330.00%	532,826,807.22	63.00%	1.08162E+11	158.00%	7.38354E+09	253.00%	1.35494E+09
65	–6000.00%	1,149,141.59	–560.00%	400,188,859.68	340.00%	482,329,355.43	64.00%	1.04301E+11	159.00%	7.22329E+09	254.00%	1.33573E+09
66	–10000.00%	408,122.04	–600.00%	288,041,558.27	350.00%	438,472,033.61	65.00%	1.00600E+11	160.00%	7.06725E+09	255.00%	1.31687E+09
67	–14000.00%	207,027.54	–640.00%	217,867,432.19	360.00%	400,211,896.51	66.00%	9.70531E+10	161.00%	6.91528E+09	256.00%	1.29837E+09
68	–18000.00%	124,839.02	–680.00%	171,466,882.70	370.00%	366,691,066.43	67.00%	9.36520E+10	162.00%	6.76727E+09	257.00%	1.28020E+09
69	–22000.00%	83,400.21	–720.00%	139,288,243.98	380.00%	337,200,588.35	68.00%	9.03900E+10	163.00%	6.62309E+09	258.00%	1.26237E+09
70	–26000.00%	59,628.64	–760.00%	116,039,498.12	390.00%	311,151,937.58	69.00%	8.72609E+10	164.00%	6.48263E+09	259.00%	1.24486E+09
71	–30000.00%	44,741.57	–800.00%	98,644,231.05	400.00%	288,054,441.73	70.00%	8.42582E+10	165.00%	6.34579E+09	260.00%	1.22767E+09
72	–34000.00%	34,805.93	–840.00%	85,235,371.00	410.00%	267,497,298.92	71.00%	8.13764E+10	166.00%	6.21244E+09	261.00%	1.21080E+09
73	–38000.00%	27,846.68	–880.00%	74,635,024.10	420.00%	249,135,187.44	72.00%	7.86098E+10	167.00%	6.08250E+09	262.00%	1.19422E+09
74	–42000.00%	22,783.63	–920.00%	66,073,612.80	430.00%	232,676,695.61	73.00%	7.59531E+10	168.00%	5.95586E+09	263.00%	1.17795E+09
75	–46000.00%	18,985.62	–960.00%	59,032,159.42	440.00%	217,874,977.66	74.00%	7.34015E+10	169.00%	5.83242E+09	264.00%	1.16197E+09
76	–50000.00%	16,063.79	–1000.00%	53,150,514.33	450.00%	204,520,174.51	75.00%	7.09501E+10	170.00%	5.71209E+09	265.00%	1.14628E+09
77	–54000.00%	13,768.00	–1040.00%	48,172,194.34	460.00%	192,433,240.75	76.00%	6.85945E+10	171.00%	5.59478E+09	266.00%	1.13087E+09
78	–58000.00%	11,931.37	–1080.00%	43,910,225.06	470.00%	181,460,896.43	77.00%	6.63305E+10	172.00%	5.48040E+09	267.00%	1.11573E+09
79	–62000.00%	10,439.15	–1120.00%	40,225,402.22	480.00%	171,471,482.86	78.00%	6.41539E+10	173.00%	5.36888E+09	268.00%	1.10086E+09
80	–66000.00%	9,210.32	–1160.00%	37,012,102.67	490.00%	162,351,547.69	79.00%	6.20610E+10	174.00%	5.26012E+09	269.00%	1.08625E+09
81	–70000.00%	8,186.35	–1200.00%	34,188,804.15	500.00%	154,003,020.71	80.00%	6.00480E+10	175.00%	5.15404E+09	270.00%	1.07190E+09
82	–74000.00%	7,324.11	–1240.00%	31,691,612.81	510.00%	146,340,869.95	81.00%	5.81115E+10	176.00%	5.05058E+09	271.00%	1.05781E+09
83	–78000.00%	6,591.26	–1280.00%	29,469,754.87	520.00%	139,291,149.87	82.00%	5.62482E+10	177.00%	4.94965E+09	272.00%	1.04395E+09
84	–82000.00%	5,963.13	–1320.00%	27,482,377.52	530.00%	132,789,370.54	83.00%	5.44548E+10	178.00%	4.85119E+09	273.00%	1.03035E+09
85	–86000.00%	5,420.70	–1360.00%	25,696,239.70	540.00%	126,779,130.77	84.00%	5.27286E+10	179.00%	4.75512E+09	274.00%	1.01697E+09
86	–90000.00%	4,949.04	–1400.00%	24,084,018.41	550.00%	121,210,968.84	85.00%	5.10664E+10	180.00%	4.66138E+09	275.00%	1.00383E+09
87	–94000.00%	4,536.37	–1440.00%	22,623,048.07	560.00%	116,041,393.31	86.00%	4.94658E+10	181.00%	4.56991E+09	276.00%	990,918,229.52
88	–98000.00%	4,173.24	–1480.00%	21,294,369.41	570.00%	111,232,063.28	87.00%	4.79240E+10	182.00%	4.48063E+09	277.00%	978,226,704.95
89	–102000.00%	3,852.03	–1520.00%	20,082,002.76	580.00%	106,749,093.06	88.00%	4.64386E+10	183.00%	4.39350E+09	278.00%	965,752,957.99
90	–106000.00%	3,566.52	–1560.00%	18,972,386.29	590.00%	102,562,460.70	89.00%	4.50072E+10	184.00%	4.30844E+09	279.00%	953,492,545.96
91	–110000.00%	3,311.62	–1600.00%	17,953,937.04	600.00%	98,645,503.67	90.00%	4.36276E+10	185.00%	4.22541E+09	280.00%	941,441,129.40
92	–114000.00%	3,083.10	–1640.00%	17,016,704.45	610.00%	94,974,487.43	91.00%	4.22977E+10	186.00%	4.14434E+09	281.00%	929,594,469.44
93	–118000.00%	2,877.44	–1680.00%	16,152,094.42	620.00%	91,528,235.71	92.00%	4.10154E+10	187.00%	4.06518E+09	282.00%	917,948,425.12
94	–122000.00%	2,691.70	–1720.00%	15,352,647.57	630.00%	88,287,812.66	93.00%	3.97788E+10	188.00%	3.98789E+09	283.00%	906,498,950.95
95	–126000.00%	2,523.37	–1760.00%	14,611,859.63	640.00%	85,236,249.03	94.00%	3.85860E+10	189.00%	3.91240E+09	284.00%	895,242,094.40

Chart 4C

Table-B10:
Project Outcome/Cashflows:

0	1	2	3	4	5	6	7	8	9	10	11
(815.00)	900.00 (0.3257)	(100.00)	1,200.00	(1,200.00)	100.00	(119,980.00)	(12,000.00)	12,000.0000	12,000.0000	12,000.0000	12,000.0000

Net Present values (For The Cash-Flow Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV	Guess Rate	NPV
1	250000.00%	(0.33)	2000.00%	(36.77)	–300.00%	1,634.88	0.00%	(83,895.00)	95.00%	(1,272.75)	190.00%	(237.54)
2	246000.00%	(0.33)	1960.00%	(37.45)	–290.00%	2,084.60	1.00%	(79,664.62)	96.00%	(1,234.56)	191.00%	(235.72)
3	242000.00%	(0.34)	1920.00%	(38.15)	–280.00%	2,744.13	2.00%	(75,611.04)	97.00%	(1,197.87)	192.00%	(233.95)
4	238000.00%	(0.34)	1880.00%	(38.87)	–270.00%	3,739.34	3.00%	(71,735.53)	98.00%	(1,162.63)	193.00%	(232.21)
5	234000.00%	(0.35)	1840.00%	(39.62)	–260.00%	5,288.00	4.00%	(68,037.24)	99.00%	(1,128.76)	194.00%	(230.52)
6	230000.00%	(0.35)	1800.00%	(40.41)	–250.00%	7,780.44	5.00%	(64,513.77)	100.00%	(1,096.21)	195.00%	(228.87)
7	226000.00%	(0.36)	1760.00%	(41.22)	–240.00%	11,944.65	6.00%	(61,161.43)	101.00%	(1,064.92)	196.00%	(227.25)
8	222000.00%	(0.37)	1720.00%	(42.07)	–230.00%	19,203.09	7.00%	(57,975.66)	102.00%	(1,034.84)	197.00%	(225.67)
9	218000.00%	(0.37)	1680.00%	(42.95)	–220.00%	32,493.09	8.00%	(54,951.19)	103.00%	(1,005.91)	198.00%	(224.13)
10	214000.00%	(0.38)	1640.00%	(43.87)	–210.00%	58,306.62	9.00%	(52,082.28)	104.00%	(978.09)	199.00%	(222.62)
11	210000.00%	(0.39)	1600.00%	(44.83)	–200.00%	112,295.00	10.00%	(49,362.89)	105.00%	(951.32)	200.00%	(221.14)
12	206000.00%	(0.40)	1560.00%	(45.84)	–190.00%	236,865.31	11.00%	(46,786.79)	106.00%	(925.57)	201.00%	(219.70)
13	202000.00%	(0.40)	1520.00%	(46.89)	–180.00%	567,453.57	12.00%	(44,347.65)	107.00%	(900.78)	202.00%	(218.29)
14	198000.00%	(0.41)	1480.00%	(47.98)	–170.00%	1,652,532.69	13.00%	(42,039.18)	108.00%	(876.93)	203.00%	(216.91)
15	194000.00%	(0.42)	1440.00%	(49.14)	–160.00%	6,601,608.44	14.00%	(39,855.13)	109.00%	(853.96)	204.00%	(215.56)
16	190000.00%	(0.43)	1400.00%	(50.34)	–150.00%	43,075,470.00	15.00%	(37,789.37)	110.00%	(831.85)	205.00%	(214.24)
17	186000.00%	(0.44)	1360.00%	(51.61)	–140.00%	552,935,128.32	16.00%	(35,835.92)	111.00%	(810.55)	206.00%	(212.95)
18	182000.00%	(0.45)	1320.00%	(52.94)	–130.00%	17,595,131,347.52	17.00%	(33,989.00)	112.00%	(790.03)	207.00%	(211.68)
19	178000.00%	(0.46)	1280.00%	(54.34)	–120.00%	2.4422E+12	18.00%	(32,242.99)	113.00%	(770.27)	208.00%	(210.45)
20	174000.00%	(0.47)	1240.00%	(55.82)	–110.00%	1.0908E+16	19.00%	(30,592.53)	114.00%	(751.22)	209.00%	(209.23)
21	170000.00%	(0.48)	1200.00%	(57.38)	–100.00%	–8.1500E+02	20.00%	(29,032.45)	115.00%	(732.87)	210.00%	(208.05)
22	166000.00%	(0.49)	1160.00%	(59.02)	–90.00%	1.3330E+16	21.00%	(27,557.81)	116.00%	(715.17)	211.00%	(206.88)
23	162000.00%	(0.50)	1120.00%	(60.76)	–80.00%	3.6422E+12	22.00%	(26,163.93)	117.00%	(698.12)	212.00%	(205.74)
24	158000.00%	(0.52)	1080.00%	(62.61)	–70.00%	31,264,316,630.37	23.00%	(24,846.31)	118.00%	(681.67)	213.00%	(204.63)
25	154000.00%	(0.53)	1040.00%	(64.57)	–60.00%	1,069,990,916.60	24.00%	(23,600.69)	119.00%	(665.80)	214.00%	(203.53)
26	150000.00%	(0.54)	1000.00%	(66.66)	–50.00%	73,718,930.00	25.00%	(22,423.05)	120.00%	(650.50)	215.00%	(202.46)
27	146000.00%	(0.56)	960.00%	(68.88)	–40.00%	6,991,907.74	26.00%	(21,309.55)	121.00%	(635.74)	216.00%	(201.41)
28	142000.00%	(0.57)	920.00%	(71.26)	–30.00%	530,091.66	27.00%	(20,256.56)	122.00%	(621.49)	217.00%	(200.38)
29	138000.00%	(0.59)	880.00%	(73.80)	–20.00%	(128,306.52)	28.00%	(19,260.66)	123.00%	(607.74)	218.00%	(199.37)
30	134000.00%	(0.61)	840.00%	(76.52)	–10.00%	(132,553.87)	29.00%	(18,318.62)	124.00%	(594.47)	219.00%	(198.38)
31	130000.00%	(0.63)	800.00%	(79.44)	0.00%	(83,895.00)	30.00%	(17,427.36)	125.00%	(581.65)	220.00%	(197.41)
32	126000.00%	(0.65)	760.00%	(82.60)	10.00%	(49,362.89)	31.00%	(16,584.02)	126.00%	(569.28)	221.00%	(196.45)
33	122000.00%	(0.67)	720.00%	(86.00)	20.00%	(29,032.45)	32.00%	(15,785.85)	127.00%	(557.33)	222.00%	(195.52)
34	118000.00%	(0.69)	680.00%	(89.69)	30.00%	(17,427.36)	33.00%	(15,030.30)	128.00%	(545.79)	223.00%	(194.60)
35	114000.00%	(0.71)	640.00%	(93.70)	40.00%	(10,748.19)	34.00%	(14,314.95)	129.00%	(534.64)	224.00%	(193.70)
36	110000.00%	(0.74)	600.00%	(98.07)	50.00%	(6,824.08)	35.00%	(13,637.52)	130.00%	(523.86)	225.00%	(192.81)
37	106000.00%	(0.77)	560.00%	(102.86)	60.00%	(4,461.99)	36.00%	(12,995.86)	131.00%	(513.45)	226.00%	(191.94)
38	102000.00%	(0.80)	520.00%	(108.12)	70.00%	(3,004.67)	37.00%	(12,387.94)	132.00%	(503.38)	227.00%	(191.09)
39	98000.00%	(0.83)	480.00%	(113.95)	80.00%	(2,083.90)	38.00%	(11,811.87)	133.00%	(493.65)	228.00%	(190.25)
40	94000.00%	(0.87)	440.00%	(120.45)	90.00%	(1,488.90)	39.00%	(11,265.85)	134.00%	(484.24)	229.00%	(189.42)
41	90000.00%	(0.90)	400.00%	(127.82)	100.00%	(1,096.21)	40.00%	(10,748.19)	135.00%	(475.14)	230.00%	(188.62)
42	86000.00%	(0.95)	360.00%	(136.36)	110.00%	(831.85)	41.00%	(10,257.30)	136.00%	(466.34)	231.00%	(187.82)
43	82000.00%	(0.99)	320.00%	(146.71)	120.00%	(650.50)	42.00%	(9,791.70)	137.00%	(457.82)	232.00%	(187.04)
44	78000.00%	(1.04)	280.00%	(160.36)	130.00%	(523.86)	43.00%	(9,349.96)	138.00%	(449.58)	233.00%	(186.27)
45	74000.00%	(1.10)	240.00%	(181.23)	140.00%	(433.89)	44.00%	(8,930.77)	139.00%	(441.61)	234.00%	(185.51)
46	70000.00%	(1.16)	200.00%	(221.14)	150.00%	(368.89)	45.00%	(8,532.88)	140.00%	(433.89)	235.00%	(184.77)
47	66000.00%	(1.23)	160.00%	(321.15)	160.00%	(321.15)	46.00%	(8,155.11)	141.00%	(426.42)	236.00%	(184.04)
48	62000.00%	(1.31)	120.00%	(650.50)	170.00%	(285.50)	47.00%	(7,796.35)	142.00%	(419.19)	237.00%	(183.32)
49	58000.00%	(1.40)	80.00%	(2,083.90)	180.00%	(258.43)	48.00%	(7,455.58)	143.00%	(412.18)	238.00%	(182.61)
50	54000.00%	(1.50)	40.00%	(10,748.19)	190.00%	(237.54)	49.00%	(7,131.79)	144.00%	(405.40)	239.00%	(181.91)

51	50000.00%	(1.62)	0.00%	(83,895.00)	200.00%	(221.14)	50.00%	(6,824.08)	145.00%	(398.82)	240.00%	(181.23)
52	46000.00%	(1.76)	–40.00%	6,991,907.74	210.00%	(208.05)	51.00%	(6,531.56)	146.00%	(392.46)	241.00%	(180.55)
53	42000.00%	(1.93)	–80.00%	3.64E+12	220.00%	(197.41)	52.00%	(6,253.43)	147.00%	(386.29)	242.00%	(179.89)
54	38000.00%	(2.13)	–120.00%	2.44E+12	230.00%	(188.62)	53.00%	(5,988.90)	148.00%	(380.31)	243.00%	(179.24)
55	34000.00%	(2.38)	–160.00%	6,601,608.44	240.00%	(181.23)	54.00%	(5,737.25)	149.00%	(374.51)	244.00%	(178.59)
56	30000.00%	(2.70)	–200.00%	112,295.00	250.00%	(174.92)	55.00%	(5,497.79)	150.00%	(368.89)	245.00%	(177.96)
57	26000.00%	(3.11)	–240.00%	11,944.65	260.00%	(169.46)	56.00%	(5,269.88)	151.00%	(363.44)	246.00%	(177.33)
58	22000.00%	(3.67)	–280.00%	2,744.13	270.00%	(164.65)	57.00%	(5,052.91)	152.00%	(358.16)	247.00%	(176.72)
59	18000.00%	(4.48)	–320.00%	1,093.79	280.00%	(160.36)	58.00%	(4,846.29)	153.00%	(353.03)	248.00%	(176.11)
60	14000.00%	(5.73)	–360.00%	631.04	290.00%	(156.49)	59.00%	(4,649.49)	154.00%	(348.06)	249.00%	(175.51)
61	10000.00%	(7.98)	–400.00%	447.84	300.00%	(152.97)	60.00%	(4,461.99)	155.00%	(343.23)	250.00%	(174.92)
62	6000.00%	(13.12)	–440.00%	353.81	310.00%	(149.72)	61.00%	(4,283.32)	156.00%	(338.55)	251.00%	(174.34)
63	2000.00%	(36.77)	–480.00%	296.08	320.00%	(146.71)	62.00%	(4,113.01)	157.00%	(334.00)	252.00%	(173.77)
64	–2000.00%	45.41	–520.00%	256.27	330.00%	(143.89)	63.00%	(3,950.64)	158.00%	(329.59)	253.00%	(173.20)
65	–6000.00%	14.07	–560.00%	226.69	340.00%	(141.24)	64.00%	(3,795.79)	159.00%	(325.31)	254.00%	(172.65)
66	–10000.00%	8.32	–600.00%	203.61	350.00%	(138.74)	65.00%	(3,648.08)	160.00%	(321.15)	255.00%	(172.10)
67	–14000.00%	5.91	–640.00%	184.98	360.00%	(136.36)	66.00%	(3,507.16)	161.00%	(317.11)	256.00%	(171.55)
68	–18000.00%	4.58	–680.00%	169.56	370.00%	(134.08)	67.00%	(3,372.67)	162.00%	(313.18)	257.00%	(171.02)
69	–22000.00%	3.74	–720.00%	156.56	380.00%	(131.91)	68.00%	(3,244.29)	163.00%	(309.37)	258.00%	(170.49)
70	–26000.00%	3.16	–760.00%	145.44	390.00%	(129.82)	69.00%	(3,121.72)	164.00%	(305.66)	259.00%	(169.97)
71	–30000.00%	2.74	–800.00%	135.80	400.00%	(127.82)	70.00%	(3,004.67)	165.00%	(302.06)	260.00%	(169.46)
72	–34000.00%	2.41	–840.00%	127.37	410.00%	(125.88)	71.00%	(2,892.86)	166.00%	(298.56)	261.00%	(168.95)
73	–38000.00%	2.16	–880.00%	119.92	420.00%	(124.01)	72.00%	(2,786.03)	167.00%	(295.16)	262.00%	(168.45)
74	–42000.00%	1.95	–920.00%	113.30	430.00%	(122.20)	73.00%	(2,683.94)	168.00%	(291.85)	263.00%	(167.95)
75	–46000.00%	1.78	–960.00%	107.37	440.00%	(120.45)	74.00%	(2,586.35)	169.00%	(288.63)	264.00%	(167.46)
76	–50000.00%	1.64	–1000.00%	102.03	450.00%	(118.75)	75.00%	(2,493.05)	170.00%	(285.50)	265.00%	(166.98)
77	–54000.00%	1.52	–1040.00%	97.20	460.00%	(117.11)	76.00%	(2,403.82)	171.00%	(282.45)	266.00%	(166.50)
78	–58000.00%	1.41	–1080.00%	92.80	470.00%	(115.50)	77.00%	(2,318.48)	172.00%	(279.49)	267.00%	(166.03)
79	–62000.00%	1.32	–1120.00%	88.78	480.00%	(113.95)	78.00%	(2,236.82)	173.00%	(276.60)	268.00%	(165.56)
80	–66000.00%	1.24	–1160.00%	85.09	490.00%	(112.43)	79.00%	(2,158.69)	174.00%	(273.80)	269.00%	(165.10)
81	–70000.00%	1.17	–1200.00%	81.70	500.00%	(110.96)	80.00%	(2,083.90)	175.00%	(271.06)	270.00%	(164.65)
82	–74000.00%	1.10	–1240.00%	78.57	510.00%	(109.52)	81.00%	(2,012.30)	176.00%	(268.40)	271.00%	(164.20)
83	–78000.00%	1.05	–1280.00%	75.66	520.00%	(108.12)	82.00%	(1,943.74)	177.00%	(265.81)	272.00%	(163.75)
84	–82000.00%	1.00	–1320.00%	72.97	530.00%	(106.76)	83.00%	(1,878.07)	178.00%	(263.29)	273.00%	(163.31)
85	–86000.00%	0.95	–1360.00%	70.46	540.00%	(105.42)	84.00%	(1,815.16)	179.00%	(260.83)	274.00%	(162.88)
86	–90000.00%	0.91	–1400.00%	68.11	550.00%	(104.12)	85.00%	(1,754.89)	180.00%	(258.43)	275.00%	(162.44)
87	–94000.00%	0.87	–1440.00%	65.92	560.00%	(102.86)	86.00%	(1,697.12)	181.00%	(256.10)	276.00%	(162.02)
88	–98000.00%	0.83	–1480.00%	63.86	570.00%	(101.62)	87.00%	(1,641.74)	182.00%	(253.82)	277.00%	(161.60)
89	–102000.00%	0.80	–1520.00%	61.93	580.00%	(100.41)	88.00%	(1,588.65)	183.00%	(251.61)	278.00%	(161.18)
90	–106000.00%	0.77	–1560.00%	60.11	590.00%	(99.23)	89.00%	(1,537.74)	184.00%	(249.44)	279.00%	(160.77)
91	–110000.00%	0.74	–1600.00%	58.39	600.00%	(98.07)	90.00%	(1,488.90)	185.00%	(247.34)	280.00%	(160.36)
92	–114000.00%	0.72	–1640.00%	56.77	610.00%	(96.94)	91.00%	(1,442.05)	186.00%	(245.28)	281.00%	(159.96)
93	–118000.00%	0.69	–1680.00%	55.23	620.00%	(95.84)	92.00%	(1,397.09)	187.00%	(243.27)	282.00%	(159.56)
94	–122000.00%	0.67	–1720.00%	53.78	630.00%	(94.76)	93.00%	(1,353.94)	188.00%	(241.32)	283.00%	(159.16)
95	–126000.00%	0.65	–1760.00%	52.40	640.00%	(93.70)	94.00%	(1,312.52)	189.00%	(239.41)	284.00%	(158.77)

4.4 Conclusion

The Descartes' Sign Rule the Fourier-Boudan Theorem, Sturm's Theorem and Vincent's Theorem are wrong, and that has significant implications for research professionals in mathematics, finance, Computer Science, operations research, and all areas of fundamental research that rely on algebra.

Bibliography

Akritas, A. (1982). Reflections on a pair of theorems by Budan and Fourier. *Mathematics Magazine, 55*(5), 292–298.

Akritas, A. G. (2008). *A new look at one of the bisection methods derived from Vincent's theorem or there is no Descartes*' method. In M. Wester & M. Beaudin (Eds.), *Computer algebra in education* (pp. 19–35). AullonaPress. http://inf-server.inf.uth.gr/~akritas/articles/71.pdf.

Akritas, A. (2009). Linear and quadratic complexity bounds on the values of the positive roots of polynomials. *Journal of Universal Computer Science, 15*(3), 523–537.

Anderson, B., Jackson, J., & Sitharam, M. (1998). Descartes' rule of signs revisited. *American Mathematical Monthly, 105*, 447–451.

Basu, S., Bhatnagar, N., Gopalan, P., & Lipton, R. (2008). Polynomials that sign represent parity and descartes' rule of signs. *Computational Complexity, 17*(3), 377–406.

Ben-Horin, M., & Kroll, Y. (2012). The limited relevance of the multiple IRRs. *The Engineering Economist, 57*, 101–118.

Bernhard, R. (1967). On inconsistency of Soper and Sturm-Kaplan conditions for uniqueness of internal rate of return. *Journal of Industrial Engineering, 18*(8), 498–502.

Boyer, R., & Goh, W. (2007). On the zero attractor of the Euler polynomials. *Advances in Applied Mathematics, 38*(1), 97–132.

Campos-Canton, E., Aguirre-Hernandez, B., J., R., & Gonzalez, E. (2015). A polynomial approach for generating a monoparametric family of chaotic attractors via switched linear systems. *Chaos, Solitons & Fractals, 71*, 10–16. doi:10.1016/j.chaos.2014.12.012.

Farahmand, K. (2007). On zeros of self-reciprocal random algebraic polynomials. *Journal of Applied Mathematics & Stochastic Analysis, 2007*, 43091. http://dx.doi.org/10.1155/2007/43091.

Farahmand, K. (2008). Real zeros of algebraic polynomials with stable random coefficients. *Journal of the Australian Mathematical Society, 85*(1), 81–86.

Farahmand, K., & Jahangiri, J. (2005). Algebraic polynomials with non-identical random coefficients. *Journal of Theoretical Probability, 18*(4), 827–835.

Farahmand, K., & Nezakati, A. (2005a). Algebraic polynomials with dependent random coefficients. *Communications in Applied Analysis, 9*(1), 95–104.

Farahmand, K., & Nezakati, A. (2005b). Algebraic polynomials with non-identical random coefficients. *Proceedings of the American Mathematical Society, 133*(1), 275–283.

Farahmand, K., & Nezakati, A. (2006). Covariance of the number of real zeros of a random algebraic polynomial with binomial elements. *Stochastic Analysis and Applications, 24*(2), 329–337.

Farahmand, K., & Sambandham, M. (2003). Real zeros of classes of random algebraic polynomials. *Journal of Applied Mathematics and Stochastic Analysis, 16*(3), 249–255.

Farahmand, K., & Sambandham, M. (2009). Algebraic polynomials with random coefficients with binomial and geometric progressions. *Journal of Applied Mathematics and Stochastic Analysis, 2009*, 725260. http://dx.doi.org/10.1155/2009/725260.

Farahmand, K., & Stretch, C. T. (2008). Algebraic polynomials with random non-symmetric coefficients. *Statistics & Probability Letters, 78*(11), 1305–1313.

Farahmand, K., Grigorash, A., & McGuinness, B. (2008). On different classes of algebraic polynomials with random coefficients. *Journal of Applied Mathematics & Stochastic Analysis, 2008*, 189675. http://dx.doi.org/10.1155/2008/189675.

Gauss, C. F. (1828). Proof by gauss of descartes rule of signs. *Journal fur die reine und angewandte Mathematik, 3*, 1–4 http://sepwww.stanford.edu/oldsep/stew/gausstrn.pdf

Grabiner, D. J. (1999). Descartes' rule of signs: Another construction. *American Mathematical Monthly, 106*, 854–855.

Hazen, G. B. (2003). A new perspective on multiple internal rates of return. *The Engineering Economist, 48*(1), 31–52.

Henrici, P. (1988). Sign changes: The rule of descartes. §6.2. In *Applied and computational complex analysis, Vol. 1: Power series-integration-conformal mapping-location of zeros* (pp. 439–443). New York: Wiley.

Hewitt, C. (2012). *Real roots of univariate polynomials with real coefficients* (Working paper). http://www4.ncsu.edu/~aszanto/MA722/ln-05.pdf.

Itenberg, U., & Roy, M. F. (1996). Multivariate descartes' rule. *Beiträge Algebra Geom, 37*, 337–346.

Levin, S. (2002, November). *Descartes' rule of signs—How hard can it be?* (Working paper). http://sepwww.stanford.edu/oldsep/stew/descartes.pdf

Magni, C. (2002). Investment decisions in the theory of finance: Some antinomies and inconsistencies. *European Journal of Operations Research, 137*, 202–216.

Magni, C. (2009). Splitting up value: A critical review of residual income theories. *European Journal of Operational Research, 198*, 1–22.

Nwogugu, M. (2010). *On regret theory, and anomalies in polynomials and net-present-value decisions* (Working paper). Available in: www.ssrn.com. http://dx.doi.org/10.2139/ssrn.1925458; http://dx.doi.org/10.2139/ssrn.1925458.

Nwogugu, M. (2012). *Risk in the global real estate market: International risk regulation, mechanism design, foreclosures, title systems and REITs*. Hoboken: Wiley.

Oehmke, J. F. (2000). Anomalies in net present value calculations. *Economics Letters, 67*, 349–351.

Osborne, M. (2010). A resolution to the NPV–IRR debate? *The Quarterly Review of Economics and Finance, 50*, 234–239.

Panton, D. (1999). Descartes, Budan and Sturm on multiple internal rates of return. *Journal of Financial Education, 25*, 12–17.

Panton, D., & Verdini, W. (1981). A fortran program for applying Sturm's theorem in counting internal rates of return. *Journal of Financial & Quantitative Analysis, 16*(3), 381–388.

Panton, D., & Verdini, W. (1984). On implementing the Sturm-Kaplan procedure for counting internal rates of return. *The Engineering Economist, 29*(4), 300–302.

Petersen, P. (1991). Multivariate Sturm theory. *Lecture Notes in Computer Science, 539*, 318–332.

Pierru, A. (2010). The simple meaning of complex rates of return. *The Engineering Economist, 55*(2), 105–117.

Simerska, C. (2008, June 1–6). Remarks on the economic criterion-the internal rate of return. In J. Chleboun, P. Přikryl, K. Segeth, & T. Vejchodský (Eds.), *Programs and algorithms of numerical mathematics, proceedings of seminar* (pp. 170–176). Dolní Maxov. Institute of Mathematics AS CR (Institute of Mathematics of the Czech Academy of Science), Prague, Czech Republic. http://dml.cz/bitstream/handle/10338.dmlcz/702871/PANM_14-2008-1_24.pdf

Sitton, G., Burrus, C. S., Fox, J., & Treitel, S. (2003). Factoring very high degree polynomials in signal processing. *Signal Processing Magazine, 6*(20), 27–42.

Yannacopoulos, A., Brindley, J., Merkin, A., & Pilling, M. (1996). Approximation of attractors using Chebyshev polynomials. *Physica D: Nonlinear Phenomena, 99*(2), 162–174.

Zheng, B., Takamatsu, J., & Ikeuchi, K. (2010). An adaptive and stable method for fitting implicit polynomial curves and surfaces. *IEEE Transactions on Pattern Analysis & Machine Intelligence, 32*(3), 561–568.

5

MN-2 Invariants and Homomorphisms for Solving Polynomials; And Anomalies in the Binomial Theorem and the Fundamental Theorem Of Algebra

In this chapter: (1) it is proved that the *Fundamental Theorem Of Algebra* (FTA) and the *Binomial Theorem* are wrong; (2) how traditional root calculation in algebra may lead to inaccurate conclusions is explained; and an alternative method for verifying real and complex roots of a polynomial is introduced; (3) a six-degree polynomial equation and a nine-degree polynomial equation are solved, by introducing classes of invariants (*MN-2 invariants*) and homomorphisms Osborne (2010), Burrus (2004), Sitton et al. (2003), and Lei et al. (1996), had concluded that such higher-order polynomials were impossible to solve). These issues are also applicable in non-linear analysis, evolutionary computation, and pattern analysis—given the discussions in Yannacopoulos et al. (1996), Campos-Canton et al. (2015), Zheng et al. (2010), and Boyer and Goh (2007).

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5.1 Existing Literature

The literature on biases and anomalies in the FTA and the Binomial Theorem is scant. However, there are still substantial gaps in the literature such as the following tests for the Fundamental Theorem of Algebra and the Binomial Theorem; and tests of factoring in algebra—and comparing factoring to analysis of changes in curves in calculus.

Farahmand and Sambandham (2003, 2009), Farahmand and Stretch (2008), Farahmand (2007, 2008), Farahmand et al. (2008), Nezakati and Farahmand (2006), Farahmand and Jahangiri (2005), and Farahmand and Nezakati (2005a, b, 2006) analyzed polynomials and algebraic structures. Sitton et al. (2003) reviewed various "modern" methods for factoring higher order polynomials—however, most of these methods are based on the FTA and Descartes' Sign Rule (which are shown to be wrong in this book).

The rest of this chapter discusses the inherent problems and biases in the Fundamental Theorem of Algebra (FTA) and the Binomial Theorem; and the use of invariants (MN2 invariants) and homomorphisms to solve higher order polynomials.

5.2 Factoring of Some Higher Order Polynomials Using a New Approach (Invariants and Homomorphisms)

This section provides some new solutions to a six-degree and a nine-degree polynomial (using the standard TVM polynomial in the NPV/IRR model), using a new approach that involves invariants and homomorphisms. Osborne (2010: 235) and many researchers have erroneously concluded that when the number of periods (n) in a TVM equation is greater than four, it is impossible to solve algebraically for $(1+r)$; and NPV cannot be expressed algebraically in terms of IRR, or vice versa (where r is the IRR)—on the contrary, this contention is wrong, as shown below. Burrus (2004), Sitton et al. (2003), and Lei et al. (1996) also concluded that polynomials with degrees that exceed six are impossible to factor and solve.

5.2.1 Solving a Six-Degree Polynomial

Consider the following typical six-degree TVM equation, which is set equal to zero in order to determine the root (IRR):

$$\begin{aligned}\text{NPV} = -a + x_1(1+r)^{-1} + x_2(1+r)^{-2} + x_3(1+r)^{-3} + x_4(1+r)^{-4} \\ + x_5(1+r)^{-5} + x_6(1+r)^{-6} = 0\end{aligned} \tag{5.1}$$

Eq. 5.1 can be manipulated to Eq. 5.2 by moving a to the other side of the equation; and changed to Eq. 5.3 by multiplying both sides of the equation by $(1+r)^6$

$$\begin{aligned}x_1(1+r)^{-1} + x_2(1+r)^{-2} + x_3(1+r)^{-3} + x_4(1+r)^{-4} + x_5(1+r)^{-5} \\ + x_6(1+r)^{-6} = a\end{aligned} \tag{5.2}$$

$$\begin{aligned}x_1(1+r)^5 + x_2(1+r)^4 + x_3(1+r)^3 + x_4(1+r)^2 + x_5(1+r)^1 \\ + x_6 = a(1+r)^6\end{aligned} \tag{5.3}$$

Eq. 5.4 is derived by expanding the terms.

$$\begin{aligned}x_1\left(r^5 + 5r^4 + 10r^3 + 9r^2 + 4r + 1\right) + x_2\left(r^4 + 4r^3 + 6r^2 + 3r + 1\right) \\ + x_3\left(r^3 + 3r^2 + 3r + 1\right) + x_4\left(r^2 + 2r + 1\right) + x_5(1+r)^1 + x_6 \\ = a\left(r^6 + 6r^5 + 15r^4 + 19r^3 + 13r^2 + 5r + 1\right)\end{aligned} \tag{5.4}$$

Eq. 5.5 is derived by rearranging the terms of Eq. 5.4:

$$\begin{aligned}&r^5(x_1)+r^4(5x_1+x_2)+r^3(10x_1+4x_2+x_3)+r^2(9x_1+6x_2+3x_3+x_4)\\&+r(4x_1+3x_2+3x_3+2x_4+x_5)+(x_6+x_5+x_4+x_3+x_2+x_1)\\&=ar^6+\mathrm{a}6r^5+a15r^4+a19r^3+a13r^2+a5r+a\end{aligned}\tag{5.5}$$

Eq. 5.6 is derived by rearranging the terms of Eq. 5.5:

$$\begin{aligned}&r^5(x_1-6a)+r^4(5x_1+x_2-15a)+r^3(10x_1+4x_2+x_3-19a)\\&+r^2(9x_1+6x_2+3x_3+x_4-13a)+r(4x_1+3x_2+3x_3+2x_4+x_5-5a)\\&+(x_6+x_5+x_4+x_3+x_2+x_1)-ar^6-a=0\end{aligned}\tag{5.6}$$

Eq. 5.7 is derived by rewriting Eq. 5.6 as:

$$-ar^6+br^5+\mathrm{c}r^4+dr^3+er^2+fr+s-a=0\tag{5.7}$$

Where

$$b=(x_1-6a);\tag{5.7a}$$

$$c=(5x_1+x_2-15a);\tag{5.7b}$$

$$d=(10x_1+4x_2+x_3-19a);\tag{5.7c}$$

$$e=(9x_1+6x_2+3x_3+x_4-13a);\tag{5.7d}$$

$$f=(4x_1+3x_2+3x_3+2x_4+x_5-5a);\tag{5.7e}$$

$$s=(x_6+x_5+x_4+x_3+x_2+x_1)\tag{5.7f}$$

Eq. 5.8 is derived by factoring and:

$-ar^6+br^5+cr^4+dr^3+er^2+fr+s-a=0$ can be factored into the product of three terms:

$$-a\{r^3+(-1/a)gr^2+(-1/a)hr+(-1/a)j\}\{r^3+r^2-a\}\tag{5.8}$$

When these three terms are multiplied, the result is as follows:

$$-a\begin{bmatrix} r^6+(-1/a)gr^5+(-1/a)hr^4+(-1/a)jr^3+r^5+(-1/a)\mathrm{g}r^4 \\ +(-1/a)\mathrm{h}r^3+(-1/a)\mathrm{j}r^2-ar^3+gr^2+hr+j \end{bmatrix} \tag{5.9}$$

$$\begin{aligned} &= -ar^6+gr^5+hr^4+jr^3-ar^5+gr^4+hr^3+jr^2 \\ &\quad +a^2r^3-\mathrm{ag}r^2-\mathrm{ahr}-aj \end{aligned} \tag{5.9a}$$

By using invariants and homomorphisms (functions that preserve relationships), Eq. 5.10 is derived by rearranging the terms in Eq. 5.9a as follows:

$$\begin{aligned} &-ar^6+r^5\{g-a\}+r^4\{h+g\}+r^3\{j+h+a^2\} \\ &\qquad +r^2\{j-\mathrm{ag}\}+r(-\mathrm{ah})+a^2j \end{aligned} \tag{5.10}$$

Where g, j, h, u, v_1, v_2, y_1, y_2, y_3, w_1, w_2, w_3, and q are real numbers, and:

$$g*u=b; \tag{5.11a}$$

$$h*v_1=\mathrm{g}*v_2=c; \tag{5.11b}$$

$$h*y_1=j*y_3=d; \tag{5.11c}$$

$$g*w_2=j*w_3=e; \tag{5.11d}$$

$$h*q=f; \tag{5.11e}$$

$$\mathrm{a}^2j=s-a \tag{5.11f}$$

where:

$$-\infty<u,v_1,v_2,y_1,y_2,y_3,w_1,w_2,w_3,q<+\infty \tag{5.11g}$$

Thus:

$$b=\{g-a\} \tag{5.11h}$$

$$c=\{h+g\} \tag{5.11i}$$

$$d = \left\{ j + h + a^2 \right\} \tag{5.11j}$$

$$e = \left\{ j - \left(ag \right) \right\} \tag{5.11k}$$

$$f = \left(-ah \right) \tag{5.11l}$$

The two equations implicit in Eq. 5.8 can be further solved as follows. The first equation is decomposed as:

$$\begin{aligned} &-a\left\{ r^3 + \left(-1/a\right)gr^2 + \left(-1/a\right)hr + \left(-1/a\right)j \right\} \\ &= -ar^3 + gr^2 + hr + j \;\; = \left(-ar^2 + p \right)\left(r + q \right) \end{aligned} \tag{5.12a}$$

The second implicit equation is decomposed as:

$$\left\{ r^3 + r^2 - a \right\} = \{ r^2 - m)\left(r + k \right); \tag{5.12b}$$

Where *k, m, p, i, z,* and *q* are real numbers and are all multiples of *a*:

$$k * z_1 = m * z_2 = p * i_1 = q * i_2 = a; \tag{5.12c}$$

$$-\infty < k, m, z_1, z_2, p, i_1, i_2, \text{and } q < +\infty \tag{5.13}$$

5.2.2 Solving a Nine-Degree Polynomial Using Invariants and Homomorphisms

Consider a traditional nine-degree TVM equation:

$$\begin{aligned} -a + x_1\left(1+r\right)^{-1} + x_2\left(1+r\right)^{-2} + x_3\left(1+r\right)^{-3} + x_4\left(1+r\right)^{-4} + x_5\left(1+r\right)^{-5} \\ + x_6\left(1+r\right)^{-6} + x_7\left(1+r\right)^{-7} + x_8\left(1+r\right)^{-8} + x_9\left(1+r\right)^{-9} = 0 \end{aligned} \tag{5.14}$$

Eq. 5.14 can be changed to Eq. 5.15 by moving *a* to the other side of the equation; and changed to Eq. 5.16 by multiplying both sides of the equation by $(1+r)^9$.

$$x_1(1+r)^{-1}+x_2(1+r)^{-2}+x_3(1+r)^{-3}+x_4(1+r)^{-4}+x_5(1+r)^{-5}$$
$$+x_6(1+r)^{-6}+x_7(1+r)^{-7}+x_8(1+r)^{-8}+x_9(1+r)^{-9}=a \quad (5.15)$$

$$x_1(1+r)^{8}+x_2(1+r)^{7}+x_3(1+r)^{6}+x_4(1+r)^{5}+x_5(1+r)^{4}$$
$$+x_6(1+r)^{3}+x_7(1+r)^{2}+x_8(1+r)+x_9=a(1+r)^{9} \quad (5.16)$$

Eq. 5.17 is derived by expanding the terms as follows:

$$x_1\left(r^8+8r^7+28r^6+54r^5+65r^4+50r^3+24r^2+7r+1\right)$$
$$+x_2\left(r^7+7r^6+21r^5+33r^4+32r^3+18r^2+6r+1\right)$$
$$+x_3\left(r^6+6r^5+15r^4+19r^3+13r^2+5r+1\right)$$
$$+x_4\left(r^5+5r^4+10r^3+9r^2+4r+1\right)$$
$$+x_5\left(r^4+4r^3+6r^2+3r+1\right)$$
$$+x_6\left(r^3+3r^2+3r+1\right)+x_7\left(r^2+2r+1\right)+x_8(1+r)$$
$$+x_9=a\begin{pmatrix}r^9+9r^8+34r^7+82r^6+109r^5\\+115r^4+74r^3+31r^2+8r+1\end{pmatrix} \quad (5.17)$$

Eq. 5.18 is derived by rearranging the terms of Eq. 5.17:

$$r^8(x_1)+r^7(8x_1+x_2)+r^6(28x_1+7x_2+x_3)$$
$$+r^5(54x_1+21x_2+6x_3+x_4)$$
$$+r^4(65x_1+33x_2+15x_3+5x_4+x_5)$$
$$+r^3(50x_1+32x_2+19x_3+10x_4+4x_5+x_6)$$
$$+r^2(24x_1+18x_2+13x_3+9x_4+6x_5+3x_6+x_7)$$
$$+r(7x_1+6x_2+5x_3+4x_4+3x_5+3x_6+2x_7+x_8)$$
$$+(x_9+x_8+x_7+x_6+x_5+x_4+x_3+x_2+x_1)$$
$$=ar^9+a9r^8+a34r^7+a82r^6$$
$$+a109r^5+a115r^4+a74r^3+a31r^2+a8r+a \quad (5.18)$$

Eq. 5.19 is derived by rearranging the terms of Eq. 5.18:

$$\begin{aligned}
-ar^9 + r^8(x_1 - 92a) + r^7(8x_1 + x_2 - 34a)\\
+ r^6(28x_1 + 7x_2 + x_3 - 82a)\\
+ r^5(54x_1 + 21x_2 + 6x_3 + x_4 - 109a)\\
+ r^4(65x_1 + 33x_2 + 15x_3 + 5x_4 + x_5 - 115a)\\
+ r^3(50x_1 + 32x_2 + 19x_3 + 10x_4 + 4x_5 + x_6 - 74a)\\
+ r^2(24x_1 + 18x_2 + 13x_3 + 9x_4 + 6x_5 + 3x_6 + x_7 - 31a)\\
+ r(7x_1 + 6x_2 + 5x_3 + 4x_4 + 3x_5 + 3x_6 + 2x_7 + x_8 - 8a)\\
+ (x_9 + x_8 + x_7 + x_6 + x_5 + x_4 + x_3 + x_2 + x_1) - a = 0
\end{aligned} \tag{5.19}$$

Using invariants and homomorphisms, Eqs. 5.20a, 5.20b, 5.20c, 5.20d, 5.20e, 5.20f, 5.20g, and 5.20h are derived by rewriting Eq. 5.19 as:

$$\begin{aligned}
&-ar^9 + b_1r^8 + b_2r^7 + b_3r^6 + b_4r^5 + b_5r^4\\
&+b_6r^3 + b_7r^2 + b_8r + b_9 - a = 0
\end{aligned} \tag{5.20}$$

Where

$$b_1 = (x_1 - 92a); \tag{5.20a}$$

$$b_2 = (8x_1 + x_2 - 34a); \tag{5.20b}$$

$$\begin{aligned}
b_3 = (28x_1 + 7x_2 + x_3 - 82a); b_4\\
= (54x_1 + 21x_2 + 6x_3 + x_4 - 109a);
\end{aligned} \tag{5.20c}$$

$$b_5 = (65x_1 + 33x_2 + 15x_3 + 5x_4 + x_5 - 115a); \tag{5.20d}$$

$$b_6 = (50x_1 + 32x_2 + 19x_3 + 10x_4 + 4x_5 + x_6 - 74a); \tag{5.20e}$$

$$b_7 = (24x_1 + 18x_2 + 13x_3 + 9x_4 + 6x_5 + 3x_6 + x_7 - 31a); \tag{5.20f}$$

$$b_8(7x_1 + 6x_2 + 5x_3 + 4x_4 + 3x_5 + 3x_6 + 2x_7 + x_8 - 8a); \tag{5.20g}$$

and

$$b_9 = (x_9 + x_8 + x_7 + x_6 + x_5 + x_4 + x_3 + x_2 + x_1) \tag{5.20h}$$

By using invariants and homomorphisms (functions that preserve relationships) and operations that resemble traditional factoring, Eqs. 5.21, 5.22 and 5.23 are derived as follows:

$-ar^9 + b_1r^8 + b_2r^7 + b_3r^6 + b_4r^5 + b_5r^4 + b_6r^3 + b_7r^2 + b_8r + b_9 - a = 0$, is equivalent to the product of the following three terms:

$$-a\begin{Bmatrix} r^5 + (-1/a)cr^4 + (-1/a)dr^3 \\ +(-1/a)er^2 + (-1/a)fr + 1 \end{Bmatrix}\{r^4 + r + 1 - a\} \tag{5.21}$$

When these three terms are multiplied, the result is as follows:

$$-a\begin{bmatrix} r^9 + (-1/a)cr^8 + (-1/a)dr^7 + (-1/a)er^6 + (-1/a)fr^5 + r^4 + r^6 \\ + (-1/a)cr^5 + (-1/a)dr^4 + (-1/a)er^3 + (-1/a)fr^2 + r \\ + r^5(-1/a)cr^4 + (-1/a)dr^3 + (-1/a)er^2 \\ + (-1/a)fr + 1 - ar^5 + cr^4 + dr^3 + er^2 + fr - a \end{bmatrix}$$
$$\begin{aligned} &= -ar^9 + cr^8 + dr^7 + er^6 + fr^5 - ar^4 - ar^6 + cr^5 + dr^4 \\ &\quad + er^3 + fr^2 - ar - ar^5 + cr^4 + dr^3 + wwer^2 + fr - a + a^2r^5 \\ &\quad - acr^4 - adr^3 - aer^2 - afr + a^2 \end{aligned} \tag{5.22}$$

$$\begin{aligned} &= -ar^9 + r^8(c) + r^7(d) + r^6(e - a) + r^5(f + c - a + a^2) \\ &\quad + r^4(-a + d + c - ac) + r^3(e + d - ad) + r^2(f + e - ae) \\ &\quad + r(-a + f - af) - a + a^2 \end{aligned} \tag{5.23}$$

Where g, h, u, j_1, j_2, k_1, k_2, k_4, k_4, m_1, m_2, m_3, m_4, n_1, n_2, n_3, p_1, p_2, p_3, q_1, q_2, q_3, and s are real numbers and are all multiples of associated coefficients in the equation—that is:

$$c * h = b_1 \tag{5.23a}$$

$$d * g = b_2 \tag{5.23b}$$

$$e * j_1 = a * j_2 = b_3 \tag{5.23c}$$

$$\mathrm{f} * k_1 = \mathrm{c} * k_2 = \mathrm{a} * k_3 = \mathrm{a}^2 * k_4 = \mathrm{b}_4; \tag{5.23d}$$

$$a * m_1 = d * m_2 = c * m_3 = ac * m_4 = b_5; \tag{5.23e}$$

$$e * n_1 = d * n_2 = ad * n_3 = b_6; \tag{5.23f}$$

$$f * p_1 = e * p_2 = ae * p_3 = b_7; \tag{5.23g}$$

$$-a * q_1 = f * q_2 = -af * q_3 = b_8; \tag{5.23h}$$

$$a^2 * s = b_9; \tag{5.23i}$$

and

$$-\infty < h_1,\ g_1,\ j_1;\ j_2;\ k_1;\ k_2;\ k_3;\ k_4;\ m_1;\ m_2;\ m_3;\ m_4;\ n_1;\ n_2;\ n_3;$$
$$p_1;\ p_2;\ p_3;\ q_1;\ q_2;\ q_3;\ s < +\infty; \tag{5.24}$$

Thus,

$$b_1 = c \tag{5.25}$$

$$b_2 = d \tag{5.26}$$

$$b_3 = (e - a) \tag{5.27}$$

$$b_4 = \left(f + c - a + a^2\right) \tag{5.28}$$

$$b_5 = (-a + d + c - ac) \tag{5.29}$$

$$b_6 = (e + d - ad) \tag{5.30}$$

$$b_7 = (f + e - ae) \tag{5.31}$$

$$b_8 = (-a + f - af) \tag{5.32}$$

$$b_9 = a^2 \tag{5.33}$$

The two equations implicit in Eq. 5.21 can be further factored as follows.

The first section of Eq. 5.21 can be further factored as follows:

$$-\mathrm{a}\left\{r^5 + (-1/a)cr^4 + (-1/a)dr^3 + (-1/a)\mathrm{er}^2 + (-1/a)\mathrm{fr} + 1\right\}$$
$$= -\mathrm{a}r^5 + cr^4 + \mathrm{dr}^3 + \mathrm{er}^2 + \mathrm{fr} - \mathrm{a} = \left(-ar^3 + \mathrm{p}\right)\left(r^2 - \mathrm{q}\right)(\mathrm{r} + \mathrm{v}) \tag{5.34}$$

When these three factors are multiplied, the result is:

$$\begin{aligned}&-ar^5+pr^2+qar^3-pq-ar^4+rp-var^3+pv+r^3-rq+vr^2-qv\\&\quad=-ar^5-ar^4+r^3\left(qa-va+1\right)+r^2\left(p+v\right)+r\left(p-q\right)\\&\quad-pq+rp+pv-rq-qv\end{aligned}\tag{5.35}$$

Where p, q, and v are real numbers and are multiples of a.
Alternatively,

$$-ar^5+cr^4+dr^3+er^2+fr-a=\left(-ar^4-r\right)\left(r-1\right)\left(1/r-1\right)\tag{5.35a}$$

$$\begin{aligned}&-ar^5-r^2+ar^4+r-ar^3-1+ar^4+r+1-1/r-r\\&\quad+1=-ar^5+2ar^4-ar^3+-r^2+2r+3\end{aligned}\tag{5.35b}$$

$$-ar^5+cr^4+dr^3+er^2+fr-a=\left(-ar^4-1\right)\left(r-1\right)\left(1-1/r\right)\tag{5.35c}$$

$$\begin{aligned}&-ar^5-r-ar^4+1-ar^4+1+ar^3+1/r+r-1-1+1/r\\&\quad=-ar^5-ar^4+r^3+-2r^2-2r+3\end{aligned}\tag{5.35d}$$

The second component of Eq. 5.21 can be further decomposed.
Where $z = (1-a)$, then:

$$\left\{r^4+r+1-a\right\}=\left\{r^4+r+z\right\}=\left(r^3+2\right)\left(r-t\right)\tag{5.36a}$$

$$\begin{aligned}&\left\{r^4+r+1-a\right\}=\left\{r^4+r+z\right\}=\left(-r^2-1\right)\left(-r^2+\left[1/r\right]-z\right)\\&\quad=r^4-r^2-r-\left[1/r\right]+\left[\left(1-a\right)*r^2\right]-\left(1-a\right)\end{aligned}\tag{5.36b}$$

The rest of the solution is straightforward.

5.3 The Binomial Theorem Is Wrong

Theorem 5.1 For all real and complex numbers, the Binomial Theorem is wrong.

Proof The proof is straightforward. The *Binomial Theorem* states that:

$$\left(x_1+b\right)^{\mathrm{n}}=\sum_{k=n}\left[\left\{n!/\left(k!\left(n-k\right)!\right)\right\}b^{\left(n-k\right)}x_1^{\ k}\right] \tag{5.37}$$

The expansion of $(x_1+b)^n$ with the Binomial Theorem differs substantially from the result using manual expansion of terms. Note that $(1+r)^n$ can be expanded as follows:

$$\left(1+r\right)^2=\left(1+r\right)*\left(1+r\right)=r^2+2r+1 \tag{5.38}$$

$$\begin{aligned}\left(1+r\right)^3&=\left(r^2+2r+1\right)*\left(1+r\right)=r^2+r^3+2r+2r^2+1+\mathrm{r}\\&=r^3+3r^2+3r+1\end{aligned} \tag{5.39}$$

$$\begin{aligned}\left(1+r\right)^4&=\left(r^3+3\mathrm{r}^2+3r+1\right)*\left(1+r\right)\\&=r^3+r^4+3r^2+3r^3+3r+3r^2+1+r\\&=r^4+4r^3+6r^2+3r+1+r\end{aligned} \tag{5.40}$$

$$\begin{aligned}\left(1+r\right)^5&=\left(r^4+4r^3+6r^2+3r+1\right)*\left(1+r\right)\\&=r^4+r^5+4r^3+4r^4+6r^2+6r^3+3r+3r^2+1+\mathrm{r}\\&=r^5+5r^4+10r^3+9r^2+4r+1\end{aligned} \tag{5.41}$$

$$\begin{aligned}\left(1+r\right)^6&=\left(r^5+5r^4+10r^3+9r^2+4r+1\right)*\left(1+r\right)\\&=r^5+r^6+5r^4+5r^5+10r^3+10r^4+9r^2+9r^3+4r+4r^2+1+r\\&=r^6+6r^5+15r^4+19r^3+13r^2+5r+1\end{aligned} \tag{5.42}$$

$$\begin{aligned}\left(1+r\right)^7&=\left(r^6+6r^5+15r^4+19r^3+13r^2+5r+1\right)*\left(1+r\right)\\&=r^6+6r^5+15r^4+19r^3+13r^2+5r+1+r^7+6r^6\\&\quad+15r^5+19r^4+13r^3+5r^2+r\\&=r^7+7r^6+21r^5+33r^4+32r^3+18r^2+6r+1\end{aligned} \tag{5.43}$$

$$\begin{aligned}\left(1+r\right)^8&=\left(r^7+7r^6+21r^5+33r^4+32r^3+18r^2+6r+1\right)*\left(1+r\right)\\&=r^7+7r^6+21r^5+33r^4+32r^3+18r^2+6r+1+r^8+7r^7\\&\quad+21r^6+33r^5+32r^4+18r^3+6r^2+r\\&=r^8+8r^7+28r^6+54r^5+65r^4+50r^3+24r^2+7r+1\end{aligned} \tag{5.44}$$

$$\begin{aligned}(1+r)^9 &= \left(r^8+8r^7+28r^6+54r^5+65r^4+50r^3+24r^2+7r+1\right)*(1+r)\\ &= r^8+8r^7+28r^6+54r^5+65r^4+50r^3+24r^2+7r+1+r^9+8r^8\\ &\quad +28r^7+54r^6+65r^5+50r^4+24r^3+7r^2+r\\ &= r^9+9r^8+34r^7+82r^6+109r^5+115r^4+74r^3+31r^2+8r+1 \qquad (5.45)\end{aligned}$$

But using the Binomial Theorem and assuming that x_1= 1, x; and b = y, r; so that (x_1+b) = (1+r) or (x+y); the following are the equivalent expansions of (x+y):

$$\begin{aligned}(x+y)^3 &= x^3+3x^2y+3xy^2+y^3,\\ (x+y)^4 &= x^4+4x^3y+6x^2y^2+4xy^3+y^4,\\ (x+y)^5 &= x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5,\\ (x+y)^6 &= x^6+6x^5y+15x^4y^2+20x^3y^3+15x^2y^4+6xy^5+y^6,\\ (x+y)^7 &= x^7+7x^6y+21x^5y^2+35x^4y^3+35x^3y^4+21x^2y^5+7xy^6+y^7.\end{aligned}$$

Thus, there is a difference between the expansion of the polynomial with the Binomial Theorem, and the manual expansion of the polynomial, which indicates that the Binomial Theorem is not accurate. ■

5.4 The Fundamental Theorem Of Algebra Is Wrong

Theorem 5.2: Contrary to established theorems, a polynomial equation of n degrees does not have exactly n roots; and thus, the Fundamental Theorem of Algebra is wrong.

Proof: The proof is straightforward. The mere existence of the TVM equations is the proof. In the TVM equations all but one term is raised to a degree of a negative number:

$$\text{NPV} = -a + x_1(1+r)^{-1} + x_2(1+r)^{-2} + \ldots\ldots + x_n(1+r)^{-n} \tag{5.46}$$

there cannot be a negative number of roots, and the TVM equation does not have exactly n roots. Even if the established theorems are read to mean that the TVM equation has n negative roots, the above-mentioned TVM equation can have at least one positive root, as amply illustrated in Tables 1–11 (in Chart 5A).

Tables A1, A2, A3, A4, B1, B2, B3, and B4 (in Chart 4A in Chapter-4) also prove this theorem. Tables A1, A2, A3 and A4 contain a series of cash flows which contain 11, nine, six, and four sign changes respectively, and also have 11, nine, six, and four degrees respectively. Also Tables B1, B2, B3 and B4 contain a series of cash flows which contain 11, nine, six, and four sign changes respectively, and also have 11, nine, six, and four degrees respectively. Also Tables B1, B2, B3 and B4 contain a series of cash flows which contain 11, nine, six, and four sign changes respectively, and also have 11, nine, six, and four degrees respectively. However, in Tables A1, A2, A3, A4, B1, B2, B3, and B4 (in Chart 4A), and in Tables B6, B7, and B8 (in Chart-4B in Chapter-4), the number of degrees of each TVM equation is not equal to the number of roots of such equation, for all possible and typical values of r. Each of the TVM equations in each of Tables B6, B7, and B8 (in Chart-4B in Chapter-4) have more than four degrees but do not have any IRRs.

Although the number of degrees of the transformed Eq. 5.7 is six, if a sufficient number of coefficients (a, b, c, d, e, or f) in Eq. 5.7 are equal to zero, then the number of degrees of the polynomial equation declines to below six, and the number of roots will not be equal to the number of degrees of the equation. Similarly, if a sufficient number of coefficients (a, b, c, d, e, or f) and/or the constant (a) in Eq. 5.7 are less than zero, then for all values of r, there cannot be any root because the curve of Eq. 5.7 can never cross the x-axis (in a graph of r on the x-axis and NPV on the y-axis) and will always be negative. In such circumstance, any calculated root will be imaginary and only a local maxima/minima that occurs below zero.

Hence, Eq. 4.6 (in Chap. 4) is a simple formula that finds all the roots of the TVM equation in a finite number of steps by:

(i) changing the r by regular small increments—this is demonstrated in Tables B1, B2, B3, and B4 (in Table-4A in Chapter-4); or
(ii) solving Eq. 4.6 (in Chap. 4) as a regular polynomial. ■

Theorem 5.3: (a) Any polynomial of any (even or odd numbered) degree does not have any roots, if negative sign magnitude dominance and sign negative dominance exist simultaneously in the polynomial; (b) and thus, the Fundamental Theorem of Algebra is wrong.

Proof: The Fundamental Theorem of Algebra states that a polynomial that has n degrees must have n roots. Although the number of degrees of the transformed equation (Eq. 5.7) is six, if a sufficient number of coefficients ($a, b, c, d, e,$ or f) in Eq. 5.7 are equal to zero, then the polynomial equation will never be equal to zero. For example, if coefficients b, c, d, e, and f in Eq. 5.7 are equal to zero, then the equation will be equal to a negative number and will never have any roots because the curve of the equation will never cross the x-axis (in a graph of r on the x-axis and NPV on the y-axis) and will always be less than zero. That is:

$$-ar^6 + br^5 + cr^4 + dr^3 + er^2 + fr + s - a = 0 \tag{5.47}$$

If $b, c, d, e, f = 0$, then:

$$-ar^6 + s - a = 0 \tag{5.47a}$$

$$-ar^6 = -s + a \tag{5.47b}$$

$$r^6 = (-s + a) / -a = (-s / -a) - 1 \tag{5.47c}$$

$$r =^6 \sqrt{\{(-s / -a) - 1\}} \tag{5.47d}$$

From Eq. 5.7f,

$$s = (x_6 + x_5 + x_4 + x_3 + x_2 + x_1) \tag{5.7f}$$

thus, r will always be less than zero if $s < a$.

Similarly, if a sufficient number of coefficients (a, b, c, d, e, or f) and/or the constant (a) in Eq. 5.7 are less than zero, then for some values of r, the TVM equation cannot have any root because the curve of Eq. 5.7 can never cross the x-axis (in a graph of r on the x-axis and NPV on the y-axis). For example, if b, c, d, e, and f are each less than zero, then in some circumstances, the TVM equation may be equal to a negative number and will never have any roots because the curve of the equation will never cross the x-axis (in a graph of r on the x-axis and NPV on the y-axis) and will always be less than zero. That is, if b, c, d, e, and f are each less than zero, then:

$$-ar^6 + br^5 + cr^4 + dr^3 + er^2 + fr + s - a = -ar^6 - br^5 - cr^4 - dr^3 - er^2 - fr + s - a \quad (5.48)$$

$$-ar^6 - br^5 - cr^4 - dr^3 - er^2 - fr = -s + a \quad (5.49)$$

If both sides are multiplied by -1, then:

$$ar^6 + br^5 + \mathrm{c}r^4 + dr^3 + er^2 + fr = s - a \quad (5.50)$$

If b, c, d, e, and f are all less than zero; and r is greater than zero; and:

(i) $a > 0$ and $a > s$; then Eq. 5.7 will always be equal to a negative number and will not have any roots; or if
(ii) $a > 0$ and $a < s$; then Eq. 5.7 will not always be equal to a negative number and <u>may not</u> have and roots; or if
(iii) $a < 0$ and $a > s$; then Eq. 5.7 will always be equal to a negative number and will not have any roots; or if
(iv) $a < 0$ and $a < x_6 <0$ then Eq. 5.7 will always be equal to a negative number and will not have any roots; or if
(v) $a < 0$ and a, $0 < x_6$; then Eq. 5.7 will not always be equal to a negative number and may not have any roots for some values of x_6.

If b, c, d, e, and f are all less than zero; and r is less than zero (Eq. 5.7 becomes: $-\mathrm{a}r^6+br^5 -cr^4 +dr^3 -er^2 +fr +s-a$) and:

(i) if $a > 0$ and $a > s$; then Eq. 5.7. may be equal to a negative number and may not have any roots depending on the values of *b*, *c*, *d*, *e,* and *f*;
(ii) if $a > 0$ and $a < s$; then Eq. 5.7 will not always be equal to a negative number and <u>may not</u> have any roots depending on the values of *b*, *c*, *d*, *e,* and *f*;
(iii) if $a < 0$ and $a > s$; then Eq. 5.7 will not always be equal to a negative number and <u>may not</u> have any roots depending on the values of *b*, *c*, *d*, *e, f,* and *s*;
(iv) if $a < 0$ and $a < s < 0$ then Eq. 5.7 will not always be equal to a negative number and <u>may not</u> have any roots depending on the values of *b*, *c*, *d*, *e, f,* and *s*;
(v) if $a < 0$ and a, $0 < s$; then Eq. 5.7 will not always be equal to a negative number and <u>may not</u> have any roots depending on the values of *b*, *c*, *d*, *e, f,* and *s*.

As mentioned, using existing differentiation methods, $\partial N/\partial r = \sum_{i=n}[(-\mathrm{i} * x_{\mathrm{i}})/(1+r)^{(i-1)}\}]$; but a better approximation is $\partial N/\partial r = \Sigma_{i=n}[(-i)/(1+r)^{(i-1)}\}]$; where N is the NPV and r is the discount rate; and x_i is the periodic project benefit in time period i; and n is the total number of time periods; and i ε n. Anomalous behavior of NPV is defined as only when $\partial N/\partial r > 0$, which occurs when $\Sigma_{i=n}\{-i/(1+r)^{(i-1)}\} > 0$, which occurs only when $\Sigma[-x_i(1+r)^i] > 0$, (negative sign magnitude dominance), and or when $\{-i/(1+r)^{(i-1)}\} < 0$ for a majority of time periods (negative r-sign dominance, which is common when the TVM equation has an even-numbered degree or when r is less than zero). Thus, a polynomial of an odd-numbered degree or an even-numbered degree (and all TVM equations) will not have any roots *iff* all the following conditions exist:

1. The absolute magnitude of combined negative periodic project outcome/cashflows is at least X% of the absolute magnitude of positive periodic project cashflows with regard to the time value of money (this state is henceforth referred to as negative sign magnitude dominance while the opposite is positive sign magnitude dominance); where X% is large and always greater than one (100%); and is larger for odd-number degree polynomials than for even-number degree polynomials (X% generally exceeds 150% for even-numbered degree polynomials; and 1,000% for odd-numbered degree polynomials).

2. The number of negative signs of periodic project cashflows are greater than the number of positive signs of periodic project cashflows without regard to the magnitude of the project cashflows (this state is referred to as *sign negative dominance* while the opposite is sign positive dominance).
3. All coefficients (project cashflows) are real numbers.

Condition 1 is evident in Tables B6, B7, and B8 (in Chart 4B in Chapter-4) where negative sign magnitude dominance exists and ensures that for most discount rates (and for all positive discount rates), the sum of all project outcome/cashflows will always be less than zero.

Under *sign negative dominance*, the signs of r are less relevant (i.e., whether the number of degrees is even or odd is less relevant), and in a limited number of cases, the TVM equation will never be equal to zero. The dynamics of the signs of the project cashflows differ from the dynamics of the signs of r. Hence, condition 2 is valid.

5.5 Traditional Root Calculation Can Lead to Inaccurate Results

Observation-1: Root calculation can lead to inaccurate results.

Root calculation (the traditional way of finding roots of polynomials) is wrong because it does not distinguish between a true root (wherein $f(x) = 0$, and the equation line crosses the zero line or x-axis—where r is plotted on the x-axis and y is plotted on the y-axis) and a local maxima/minima (an inflection point wherein the equation line reverses direction, and $f(x) = 0$). Root calculation is comparable to the procedure used in calculus when finding the turning points of an equation—in those cases, the first derivative of the equation is set equal to zero ($dy/dx = 0$), and then the roots of the resulting post-differentiation equation are found. But in traditional algebra, the polynomial (such as a TVM equation) is usually not a first derivative of any equation.

Furthermore, assume there is a regular polynomial Y_1, where

$$Y_1 = (x-1)(x+2) = x^2 - x + 2x - 2 = x^2 + x - 2 \tag{5.51}$$

Using traditional root calculation, in Eq. 5.51, Y_1 appears to have two roots which are at $x = 1$, and at $x = -2$. For root calculation to be correct, the following conditions must exist simultaneously when Y_1 is equal to zero:

Condition 1: $Y_1 = x^2+x-2 = 0$; and thus, $Y_1-x^2-x+2 = 0$
Condition 2: $Y_1 = (x-1) = 0$; and thus, $Y_1-x+1 = 0$;
Condition 3: $Y_1 = (x+2) = 0$;

If Condition 3 $\{Y_1 = (x+2)\}$ is substituted in condition 1, then condition 1 becomes:

$$(x+2) - x^2 - x + 2 = 0; \text{ and solving for } x,$$

$$-x^2 + 4 = 0; \text{ or } x^2 = 4; \text{ or } x = 2, \text{ or } -2$$

Thus, Conditions 1, 2, and 3 do not exist simultaneously, and what is deemed a root is only a local maxima/minima; and the root calculation method is wrong.

Also assume there is another regular polynomial Y_2, where:

$$Y_2 = (x-4)(x+1) = x^2 - 4x + x - 4 = x^2 - 3x - 4 \tag{5.52}$$

Using traditional root calculation methods, in Eq. 5.52, Y_2 appears to have two roots which are at $x = 4$, and at $x = -1$. For root calculation to be correct, the following conditions must exist simultaneously when Y_2 is equal to zero:

Condition 1: $Y_2 = x^2-3x-4 = 0$; and $Y_2-x^2+3x+4 = 0$;
Condition 2: $Y_2 = (x-4) = 0$; and
Condition 3: $Y_2 = (x+1) = 0$

If $Y_2 = (x-4)$ is substituted in condition 1, then condition 1 becomes: $(x-4) - x^2+3x+4 = 0$; and solving for x,

$$x - x^2 + 3x = 0;\ \text{or} - x^2 + 4x = 0;$$

$$x = \surd 4x$$

Again, conditions 1, 2, and 3 do not exist simultaneously, and what is deemed a root is only a local maxima/minima; and the root calculation method is wrong.

Consider a polynomial equation:

$$Y_3 = (x-m)(x+n)(x-p)(x+q) \tag{5.53}$$

In order to comply with the factoring requirement that $(x-m)$ $(x+n)$ $(x-p)$ $(x+q) = 0$; only one factor has to be equal to zero. Thus, while $(x-m) = 0$; $(x+n)$, $(x-p)$ and $(x+q)$ may not be equal to zero, although traditional factoring erroneously assumes that all four factors are equal to zero. This makes root calculation very inaccurate.

To distinguish between a true root and a local maxima/minima, another procedure has to be used. A true root occurs when the equation curve passes zero (x-axis) <u>and</u> goes to both 0.00001 and −0.00001. To find a true root, the polynomial is set equal to both 0.00001 (Result 1; $f(x) = 0.00001$) and −0.00001 (Result 2; $f(x) = -0.00001$), and if the resulting values of the polynomial for both results 1 and 2 are close, then there is a true root at $f(x) = 0$.

Thus, assume that for a polynomial, $f(x) = 0$ at four points A(3), B(−2), C(8) and D(−9). Also assume that at $f(x) = 0.00001$, the values of x are A+(3.01), B+(4), C+(5) and D+(−9.2); and at $f(x) = -0.00001$, the values of x are A−(2.95), B−4.5), C(6) and D(−8.90); then the polynomial has only two roots (at A and D)—because these are the only two points where the polynomial line crosses the x-axis as indicated by the changes in the values of x when $f(x)$ is equal to 0.00001, 0 and −0.0001. ■

Theorem 5.5: Contrary to established theorems, a polynomial of any degree can never be positive semi–definite or non-negative for all real and complex values of x (or {1+r} in TVM equations).

Proof: Table B9 (in Chart 4B in Chapter-4) illustrates the proof of this *Non-Negative Impossibility Theorem*. For any type of polynomial, even when all the coefficients (project cash flows) are positive, for negative values of x (or r in TVM equations), the polynomial can have negative values, because of the effect of the negative signs of x (where x is $\{1+r\}$)—that is, the negative signs of x (x is $\{1+r\}$ in TVM equations) will ensure that the polynomial will have negative values. ■

5.6 Conclusion

The Binomial Theorem and the Fundamental Theorem of Algebra are wrong; and that has significant implications for research professionals in mathematics, finance, operations research, and all areas of fundamental research that rely on algebra.

Chart 5A ■

Table-1:
Project Outcomes/CashFlows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(100,000,000,000)	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)
2A.	(100,000,000,000)	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	100,000,000,000
3A.	(100,000,000,000)	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	100,000,000,000	100,000,000,000
4A.	(100,000,000,000)	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000
5A.	(100,000,000,000)	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000
6A.	(100,000,000,000)	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000
7A.	(100,000,000,000)	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000
8A.	(100,000,000,000)	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000
9A.	(100,000,000,000)	100,000,000,000	(100,000,000,000)	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000
10A.	(100,000,000,000)	100,000,000,000	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000
11A.	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000
12A.	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000
13A.	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000
14A	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000
15A	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	(100,000,000,000)
16A	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	(100,000,000,000)	(100,000,000,000)
17A	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)
18A	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)
19A	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)
20A	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)
21A	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)
22A	(100,000,000,000)	100,000,000,000	100,000,000,000	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)
23A	(100,000,000,000)	100,000,000,000	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)
24A	(100,000,000,000)	100,000,000,000	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)	(100,000,000,000)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	–250%	–200%	–150%	–126%	–100%	–10%	–5.00%	0.250%	5%	50%	100%	300%
1B.	128,580,595,024	200,000,000,000	(272,200,000,000,000)	–8.3166360E+17	(100,000,000,000)	(2,293,792,581,225)	(1,479,628,964,244)	(981,721,646,591)	(704,919,267,953)	(109,569,641,785)	(49,975,585,938)	(20,833,331,347)
2B.	130,122,064,350	400,000,000,000	547,000,000,000,000	1.2641363E+18	(100,000,000,000)	(1,585,651,348,931)	(1,109,505,403,000)	(787,625,273,512)	(593,551,784,318)	(108,028,172,459)	(49,926,757,813)	(20,833,319,426)
3B.	127,809,860,361	200,000,000,000	137,400,000,000,000	7.1922834E+17	(100,000,000,000)	(948,324,239,866)	(757,888,019,818)	(593,043,659,500)	(476,615,926,500)	(105,715,968,471)	(49,829,101,563)	(20,833,271,742)
4B.	131,278,166,344	400,000,000,000	342,200,000,000,000	8.6090442E+17	(100,000,000,000)	(374,729,841,707)	(423,851,505,794)	(397,975,591,452)	(353,833,275,792)	(102,247,662,487)	(49,633,789,063)	(20,833,081,007)
5B.	126,075,707,369	200,000,000,000	239,800,000,000,000	8.2406864E+17	(100,000,000,000)	141,505,116,635	(106,516,817,473)	(202,419,853,235)	(224,911,492,549)	(97,045,203,513)	(49,243,164,063)	(20,832,318,068)
6B.	133,879,395,831	400,000,000,000	291,000,000,000,000	8.3364594E+17	(100,000,000,000)	606,116,579,144	194,951,136,433	(6,375,225,673)	(89,543,620,143)	(89,241,515,051)	(48,461,914,063)	(20,829,266,310)
7B.	122,173,863,138	200,000,000,000	265,400,000,000,000	8.3115584E+17	(100,000,000,000)	1,024,266,895,401	481,345,692,644	190,159,513,459	52,592,645,883	(77,535,982,357)	(46,899,414,063)	(20,817,059,278)
8B.	139,732,162,178	400,000,000,000	278,200,000,000,000	8.3180327E+17	(100,000,000,000)	1,400,602,180,033	753,420,521,044	387,185,589,439	201,835,725,210	(59,977,683,318)	(43,774,414,063)	(20,768,231,153)
9B.	113,394,713,618	200,000,000,000	271,800,000,000,000	8.3163494E+17	(100,000,000,000)	1,739,303,936,202	1,011,891,608,024	584,704,230,608	358,540,958,504	(33,640,234,758)	(37,524,414,063)	(20,572,918,653)
10B.	152,900,886,458	400,000,000,000	275,000,000,000,000	8.3167871E+17	(100,000,000,000)	2,044,135,516,753	1,257,439,140,655	782,716,668,380	523,081,453,462	5,865,938,082	(25,024,414,063)	(19,791,668,653)
11B.	93,641,627,199	200,000,000,000	273,400,000,000,000	8.3166733E+17	(100,000,000,000)	2,318,483,939,250	1,490,709,296,654	981,224,137,247	695,848,973,169	65,125,197,341	(24,414,063)	(16,666,668,653)
12B.	93,641,627,199	200,000,000,000	273,400,000,000,000	8.3166733E+17	(100,000,000,000)	2,318,483,939,250	1,490,709,296,654	981,224,137,247	695,848,973,169	65,125,197,341	(24,414,063)	(16,666,668,653)
13B.	(39,691,706,135)	0	273,000,000,000,000	8.3166656E+17	100,000,000,000	2,540,706,161,472	1,701,235,612,443	1,180,725,384,130	886,325,163,645	198,458,530,674	99,975,585,938	33,333,331,347
14B	93,641,627,199	200,000,000,000	273,400,000,000,000	8.3166733E+17	(100,000,000,000)	2,318,483,939,250	1,490,709,296,654	981,224,137,247	695,848,973,169	65,125,197,341	(24,414,063)	(16,666,668,653)
15B	92,100,157,873	0	(545,800,000,000,000)	–1.2641326E+18	(100,000,000,000)	1,610,342,706,956	1,120,585,735,410	787,127,764,168	584,481,489,533	63,583,728,015	(73,242,188)	(16,666,680,574)
16B	94,412,361,861	200,000,000,000	(136,200,000,000,000)	–7.1922462E+17	(100,000,000,000)	973,015,597,891	768,968,352,227	592,546,150,156	467,545,631,716	61,271,524,026	(170,898,438)	(16,666,728,258)
17B	90,944,055,878	0	(341,000,000,000,000)	–8.6090069E+17	(100,000,000,000)	399,421,199,732	434,931,838,204	397,478,082,109	344,762,981,008	57,803,218,043	(366,210,938)	(16,666,918,993)
18B	96,146,514,853	200,000,000,000	(238,600,000,000,000)	–8.2406491E+17	(100,000,000,000)	(116,813,758,611)	117,597,149,883	201,922,343,891	215,841,197,764	52,600,759,068	(756,835,938)	(16,667,681,932)
19B	88,342,826,391	0	(289,800,000,000,000)	–8.3364221E+17	(100,000,000,000)	(581,425,221,119)	(183,870,804,023)	5,877,716,329	80,473,325,358	44,797,070,606	(1,538,085,938)	(16,670,733,690)
20B	100,048,359,084	200,000,000,000	(264,200,000,000,000)	–8.3115212E+17	(100,000,000,000)	(999,575,537,377)	(470,265,360,234)	(190,657,022,803)	(61,662,940,668)	33,091,537,913	(3,100,585,938)	(16,682,940,722)
21B	82,490,060,044	0	(277,000,000,000,000)	–8.3179954E+17	(100,000,000,000)	(1,375,910,822,008)	(742,340,188,634)	(387,683,098,782)	(210,906,019,995)	15,533,238,873	(6,225,585,938)	(16,731,768,847)
22B	108,827,508,604	200,000,000,000	(270,600,000,000,000)	–8.3163121E+17	(100,000,000,000)	(1,714,612,578,177)	(1,000,811,275,614)	(585,201,739,952)	(367,611,253,289)	(10,804,209,686)	(12,475,585,938)	(16,927,081,347)
23B	69,321,335,764	0	(273,800,000,000,000)	–8.3167498E+17	(100,000,000,000)	(2,019,444,158,729)	(1,246,358,808,245)	(783,214,177,724)	(532,151,748,247)	(50,310,382,526)	(24,975,585,938)	(17,708,331,347)
24B	128,580,595,024	200,000,000,000	(272,200,000,000,000)	–8.3166360E+17	(100,000,000,000)	(2,293,792,581,225)	(1,479,628,964,244)	(981,721,646,591)	(704,919,267,953)	(109,569,641,785)	(49,975,585,938)	(20,833,331,347)

Table-2:
Project Outcomes/Cashflows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)
2A.	(1,000,000)	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)
3A.	(1,000,000)	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	1,000,000
4A.	(1,000,000)	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	1,000,000	1,000,000
5A.	(1,000,000)	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	1,000,000	1,000,000	1,000,000
6A.	(1,000,000)	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000
7A.	(1,000,000)	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
8A.	(1,000,000)	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
9A.	(1,000,000)	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
10A.	(1,000,000)	1,000,000	(1,000,000)	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
11A.	(1,000,000)	1,000,000	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
12A.	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
13A	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
14A	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000
15A	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	(1,000,000)
16A	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	(1,000,000)	(1,000,000)
17A	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)
18A	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)
19A	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)
20A	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)
21A	(1,000,000)	1,000,000	1,000,000	1,000,000	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)
22A	(1,000,000)	1,000,000	1,000,000	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)
23A	(1,000,000)	1,000,000	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)
24A	(1,000,000)	1,000,000	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)	(1,000,000)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	–250%	–200%	–150%	–126%	–100%	–10%	–5.00%	0.250%	5%	50%	100%	300%
1B.	396,917	0	(2,730,000,000)	–8.317E+12	(1,000,000)	(25,407,062)	(17,012,356)	(11,807,254)	(8,863,252)	(1,984,585)	(999,756)	(333,333)
2B.	1,285,806	2,000,000	(2,722,000,000)	–8.317E+12	(1,000,000)	(22,937,926)	(14,796,290)	(9,817,216)	(7,049,193)	(1,095,696)	(499,756)	(208,333)
3B.	1,301,221	4,000,000	5,470,000,000	1.264E+13	(1,000,000)	(15,856,513)	(11,095,054)	(7,876,253)	(5,935,518)	(1,080,282)	(499,268)	(208,333)
4B.	1,278,099	2,000,000	1,374,000,000	7.192E+12	(1,000,000)	(9,483,242)	(7,578,880)	(5,930,437)	(4,766,159)	(1,057,160)	(498,291)	(208,333)
5B.	1,312,782	4,000,000	3,422,000,000	8,609,044,195,474	(1,000,000)	(3,747,298)	(4,238,515)	(3,979,756)	(3,538,333)	(1,022,477)	(496,338)	(208,331)
6B.	1,260,757	2,000,000	2,398,000,000	8,240,686,401,051	(1,000,000)	1,415,051	(1,065,168)	(2,024,199)	(2,249,115)	(970,452)	(492,432)	(208,323)
7B.	1,338,794	4,000,000	2,910,000,000	8,336,459,427,601	(1,000,000)	6,061,166	1,949,511	(63,752)	(895,436)	(892,415)	(484,619)	(208,293)
8B.	1,221,739	2,000,000	2,654,000,000	8,311,558,440,698	(1,000,000)	10,242,669	4,813,457	1,901,595	525,926	(775,360)	(468,994)	(208,171)
9B.	1,397,322	4,000,000	2,782,000,000	8,318,032,697,293	(1,000,000)	14,006,022	7,534,205	3,871,856	2,018,357	(599,777)	(437,744)	(207,682)
10B.	1,133,947	2,000,000	2,718,000,000	8,316,349,390,578	(1,000,000)	17,393,039	10,118,916	5,847,042	3,585,410	(336,402)	(375,244)	(205,729)
11B.	1,529,009	4,000,000	2,750,000,000	8,316,787,050,324	(1,000,000)	20,441,355	12,574,391	7,827,167	5,230,815	58,659	(250,244)	(197,917)
12B.	936,416	2,000,000	2,734,000,000	8,316,673,258,790	(1,000,000)	23,184,839	14,907,093	9,812,241	6,958,490	651,252	(244)	(166,667)
13B	(396,917)	0	2,730,000,000	8,316,665,566,482	1,000,000	25,407,062	17,012,356	11,807,254	8,863,252	1,984,585	999,756	333,333
14B	936,416	2,000,000	2,734,000,000	8.317E+12	(1,000,000)	23,184,839	14,907,093	9,812,241	6,958,490	651,252	(244)	(166,667)
15B	921,002	0	(5,458,000,000)	–1.264E+13	(1,000,000)	16,103,427	11,205,857	7,871,278	5,844,815	635,837	(732)	(166,667)
16B	944,124	2,000,000	(1,362,000,000)	–7.192E+12	(1,000,000)	9,730,156	7,689,684	5,925,462	4,675,456	612,715	(1,709)	(166,667)
17B	909,441	0	(3,410,000,000)	–8.609E+12	(1,000,000)	3,994,212	4,349,318	3,974,781	3,447,630	578,032	(3,662)	(166,669)
18B	961,465	2,000,000	(2,386,000,000)	–8.241E+12	(1,000,000)	(1,168,138)	1,175,971	2,019,223	2,158,412	526,008	(7,568)	(166,677)
19B	883,428	0	(2,898,000,000)	–8.336E+12	(1,000,000)	(5,814,252)	(1,838,708)	58,777	804,733	447,971	(15,381)	(166,707)
20B	1,000,484	2,000,000	(2,642,000,000)	–8.312E+12	(1,000,000)	(9,995,755)	(4,702,654)	(1,906,570)	(616,629)	330,915	(31,006)	(166,829)
21B	824,901	0	(2,770,000,000)	–8.318E+12	(1,000,000)	(13,759,108)	(7,423,402)	(3,876,831)	(2,109,060)	155,332	(62,256)	(167,318)
22B	1,088,275	2,000,000	(2,706,000,000)	–8.316E+12	(1,000,000)	(17,146,126)	(10,008,113)	(5,852,017)	(3,676,113)	(108,042)	(124,756)	(169,271)
23B	693,213	0	(2,738,000,000)	–8.317E+12	(1,000,000)	(20,194,442)	(12,463,588)	(7,832,142)	(5,321,517)	(503,104)	(249,756)	(177,083)
24B	1,285,806	2,000,000	(2,722,000,000)	–8.317E+12	(1,000,000)	(22,937,926)	(14,796,290)	(9,817,216)	(7,049,193)	(1,095,696)	(499,756)	(208,333)

Table-3:
Project Outcomes/Cashflows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)
2A.	(100,000)	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)
3A.	(100,000)	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	100,000
4A.	(100,000)	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	100,000	100,000
5A.	(100,000)	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	100,000	100,000	100,000
6A.	(100,000)	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	100,000	100,000	100,000	100,000
7A.	(100,000)	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	100,000	100,000	100,000	100,000	100,000
8A.	(100,000)	100,000	(100,000)	(100,000)	(100,000)	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000
9A.	(100,000)	100,000	(100,000)	(100,000)	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000	100,000
10A.	(100,000)	100,000	(100,000)	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
11A.	(100,000)	100,000	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
12A.	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
13A	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
14A	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
15A	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	(100,000)
16A	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	(100,000)	(100,000)
17A	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	(100,000)	(100,000)	(100,000)
18A	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000	100,000	(100,000)	(100,000)	(100,000)	(100,000)
19A	(100,000)	100,000	100,000	100,000	100,000	100,000	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)
20A	(100,000)	100,000	100,000	100,000	100,000	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)
21A	(100,000)	100,000	100,000	100,000	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)
22A	(100,000)	100,000	100,000	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)
23A	(100,000)	100,000	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)
24A	(100,000)	100,000	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)	(100,000)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	–250%	–200%	–150%	–126%	–100%	–10%	–5.00%	0.250%	5%	50%	100%	300%
1B.	39,692	0	(273,000,000)	(831,666,556,648)	(100,000)	(2,540,706)	(1,701,236)	(1,180,725)	(886,325)	(198,459)	(99,976)	(33,333)
2B.	128,581	200,000	(272,200,000)	(831,663,598,068)	(100,000)	(2,293,793)	(1,479,629)	(981,722)	(704,919)	(109,570)	(49,976)	(20,833)
3B.	130,122	400,000	547,000,000	1,264,136,324,685	(100,000)	(1,585,651)	(1,109,505)	(787,625)	(593,552)	(108,028)	(49,927)	(20,833)
4B.	127,810	200,000	137,400,000	719,228,344,769	(100,000)	(948,324)	(757,888)	(593,044)	(476,616)	(105,716)	(49,829)	(20,833)
5B.	131,278	400,000	342,200,000	860,904,419,547	(100,000)	(374,730)	(423,852)	(397,976)	(353,833)	(102,248)	(49,634)	(20,833)
6B.	126,076	200,000	239,800,000	824,068,640,105	(100,000)	141,505	(106,517)	(202,420)	(224,911)	(97,045)	(49,243)	(20,832)
7B.	133,879	400,000	291,000,000	833,645,942,760	(100,000)	606,117	194,951	(6,375)	(89,544)	(89,242)	(48,462)	(20,829)
8B.	122,174	200,000	265,400,000	831,155,844,070	(100,000)	1,024,267	481,346	190,160	52,593	(77,536)	(46,899)	(20,817)
9B.	139,732	400,000	278,200,000	831,803,269,729	(100,000)	1,400,602	753,421	387,186	201,836	(59,978)	(43,774)	(20,768)
10B.	113,395	200,000	271,800,000	831,634,939,058	(100,000)	1,739,304	1,011,892	584,704	358,541	(33,640)	(37,524)	(20,573)
11B.	152,901	400,000	275,000,000	831,678,705,032	(100,000)	2,044,136	1,257,439	782,717	523,081	5,866	(25,024)	(19,792)
12B.	93,642	200,000	273,400,000	831,667,325,879	(100,000)	2,318,484	1,490,709	981,224	695,849	65,125	(24)	(16,667)
13B	(39,692)	0	273,000,000	831,666,556,648	100,000	2,540,706	1,701,236	1,180,725	886,325	198,459	99,976	33,333
14B	93,642	200,000	273,400,000	831,667,325,879	(100,000)	2,318,484	1,490,709	981,224	695,849	65,125	(24)	(16,667)
15B	92,100	0	(545,800,000)	–1.264E+12	(100,000)	1,610,343	1,120,586	787,128	584,481	63,584	(73)	(16,667)
16B	94,412	200,000	(136,200,000)	(719,224,616,959)	(100,000)	973,016	768,968	592,546	467,546	61,272	(171)	(16,667)
17B	90,944	0	(341,000,000)	(860,900,691,737)	(100,000)	399,421	434,932	397,478	344,763	57,803	(366)	(16,667)
18B	96,147	200,000	(238,600,000)	(824,064,912,294)	(100,000)	(116,814)	117,597	201,922	215,841	52,601	(757)	(16,668)
19B	88,343	0	(289,800,000)	(833,642,214,949)	(100,000)	(581,425)	(183,871)	5,878	80,473	44,797	(1,538)	(16,671)
20B	100,048	200,000	(264,200,000)	(831,152,116,259)	(100,000)	(999,576)	(470,265)	(190,657)	(61,663)	33,092	(3,101)	(16,683)
21B	82,490	0	(277,000,000)	(831,799,541,919)	(100,000)	(1,375,911)	(742,340)	(387,683)	(210,906)	15,533	(6,226)	(16,732)
22B	108,828	200,000	(270,600,000)	(831,631,211,247)	(100,000)	(1,714,613)	(1,000,811)	(585,202)	(367,611)	(10,804)	(12,476)	(16,927)
23B	69,321	0	(273,800,000)	(831,674,977,222)	(100,000)	(2,019,444)	(1,246,359)	(783,214)	(532,152)	(50,310)	(24,976)	(17,708)
24B	128,581	200,000	(272,200,000)	(831,663,598,068)	(100,000)	(2,293,793)	(1,479,629)	(981,722)	(704,919)	(109,570)	(49,976)	(20,833)

Table-4:

Project Outcomes/Cashflows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(10,000)	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)
2A.	(10,000)	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	10,000
3A.	(10,000)	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	10,000	10,000
4A.	(10,000)	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	10,000	10,000	10,000
5A.	(10,000)	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	10,000	10,000	10,000	10,000
6A.	(10,000)	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	10,000	10,000	10,000	10,000	10,000
7A.	(10,000)	10,000	(10,000)	(10,000)	(10,000)	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000
8A.	(10,000)	10,000	(10,000)	(10,000)	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	10,000
9A.	(10,000)	10,000	(10,000)	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
10A.	(10,000)	10,000	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
11A.	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
12A	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
13A	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
14A	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
15A	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	(10,000)
16A	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	(10,000)	(10,000)
17A	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	(10,000)	(10,000)	(10,000)
18A	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	10,000	(10,000)	(10,000)	(10,000)	(10,000)
19A	(10,000)	10,000	10,000	10,000	10,000	10,000	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)
20A	(10,000)	10,000	10,000	10,000	10,000	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)
21A	(10,000)	10,000	10,000	10,000	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)
22A	(10,000)	10,000	10,000	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)
23A	(10,000)	10,000	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)
24A	(10,000)	10,000	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	–250%	–200%	–150%	–126%	–100%	–10%	–5.00%	0.250%	5%	50%	100%	300%
1B.	12,858.06	20,000.00	(27,220,000.00)	–8.3166E+10	(10,000.00)	(229,379.26)	(147,962.90)	(98,172.16)	(70,491.93)	(10,956.96)	(4,997.56)	(2,083.33)
2B.	13,012.21	40,000.00	54,700,000.00	1.2641E+11	(10,000.00)	(158,565.13)	(110,950.54)	(78,762.53)	(59,355.18)	(10,802.82)	(4,992.68)	(2,083.33)
3B.	12,780.99	20,000.00	13,740,000.00	71,922,834,476.93	(10,000.00)	(94,832.42)	(75,788.80)	(59,304.37)	(47,661.59)	(10,571.60)	(4,982.91)	(2,083.33)
4B.	13,127.82	40,000.00	34,220,000.00	86,090,441,954.74	(10,000.00)	(37,472.98)	(42,385.15)	(39,797.56)	(35,383.33)	(10,224.77)	(4,963.38)	(2,083.31)
5B.	12,607.57	20,000.00	23,980,000.00	82,406,864,010.51	(10,000.00)	14,150.51	(10,651.68)	(20,241.99)	(22,491.15)	(9,704.52)	(4,924.32)	(2,083.23)
6B.	13,387.94	40,000.00	29,100,000.00	83,364,594,276.01	(10,000.00)	60,611.66	19,495.11	(637.52)	(8,954.36)	(8,924.15)	(4,846.19)	(2,082.93)
7B.	12,217.39	20,000.00	26,540,000.00	83,115,584,406.98	(10,000.00)	102,426.69	48,134.57	19,015.95	5,259.26	(7,753.60)	(4,689.94)	(2,081.71)
8B.	13,973.22	40,000.00	27,820,000.00	83,180,326,972.93	(10,000.00)	140,060.22	75,342.05	38,718.56	20,183.57	(5,997.77)	(4,377.44)	(2,076.82)
9B.	11,339.47	20,000.00	27,180,000.00	83,163,493,905.78	(10,000.00)	173,930.39	101,189.16	58,470.42	35,854.10	(3,364.02)	(3,752.44)	(2,057.29)
10B.	15,290.09	40,000.00	27,500,000.00	83,167,870,503.24	(10,000.00)	204,413.55	125,743.91	78,271.67	52,308.15	586.59	(2,502.44)	(1,979.17)
11B.	9,364.16	20,000.00	27,340,000.00	83,166,732,587.90	(10,000.00)	231,848.39	149,070.93	98,122.41	69,584.90	6,512.52	(2.44)	(1,666.67)
12B.	9,364.16	20,000.00	27,340,000.00	83,166,732,587.90	(10,000.00)	231,848.39	149,070.93	98,122.41	69,584.90	6,512.52	(2.44)	(1,666.67)
13B	(3,969.17)	0.00	27,300,000.00	83,166,655,664.82	10,000.00	254,070.62	170,123.56	118,072.54	88,632.52	19,845.85	9,997.56	3,333.33
14B	9,364.16	20,000.00	27,340,000.00	8.3167E+10	(10,000.00)	231,848.39	149,070.93	98,122.41	69,584.90	6,512.52	(2.44)	(1,666.67)
15B	9,210.02	0.00	(54,580,000.00)	–1.2641E+11	(10,000.00)	161,034.27	112,058.57	78,712.78	58,448.15	6,358.37	(7.32)	(1,666.67)
16B	9,441.24	20,000.00	(13,620,000.00)	–7.1922E+10	(10,000.00)	97,301.56	76,896.84	59,254.62	46,754.56	6,127.15	(17.09)	(1,666.67)
17B	9,094.41	0.00	(34,100,000.00)	–8.6090E+10	(10,000.00)	39,942.12	43,493.18	39,747.81	34,476.30	5,780.32	(36.62)	(1,666.69)
18B	9,614.65	20,000.00	(23,860,000.00)	–8.2406E+10	(10,000.00)	(11,681.38)	11,759.71	20,192.23	21,584.12	5,260.08	(75.68)	(1,666.77)
19B	8,834.28	0.00	(28,980,000.00)	–8.3364E+10	(10,000.00)	(58,142.52)	(18,387.08)	587.77	8,047.33	4,479.71	(153.81)	(1,667.07)
20B	10,004.84	20,000.00	(26,420,000.00)	(83,115,211,625.91)	(10,000.00)	(99,957.55)	(47,026.54)	(19,065.70)	(6,166.29)	3,309.15	(310.06)	(1,668.29)
21B	8,249.01	0.00	(27,700,000.00)	–8.3180E+10	(10,000.00)	(137,591.08)	(74,234.02)	(38,768.31)	(21,090.60)	1,553.32	(622.56)	(1,673.18)
22B	10,882.75	20,000.00	(27,060,000.00)	–8.3163E+10	(10,000.00)	(171,461.26)	(100,081.13)	(58,520.17)	(36,761.13)	(1,080.42)	(1,247.56)	(1,692.71)
23B	6,932.13	0.00	(27,380,000.00)	–8.3167E+10	(10,000.00)	(201,944.42)	(124,635.88)	(78,321.42)	(53,215.17)	(5,031.04)	(2,497.56)	(1,770.83)
24B	12,858.06	20,000.00	(27,220,000.00)	–8.3166E+10	(10,000.00)	(229,379.26)	(147,962.90)	(98,172.16)	(70,491.93)	(10,956.96)	(4,997.56)	(2,083.33)

Table-5:
Project Outcomes/Cashflows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(1,000.00)	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)
2A.	(1,000.00)	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	1,000.00
3A.	(1,000.00)	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	1,000.00	1,000.00
4A.	(1,000.00)	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	1,000.00	1,000.00	1,000.00
5A.	(1,000.00)	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00
6A.	(1,000.00)	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
7A.	(1,000.00)	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
8A.	(1,000.00)	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
9A.	(1,000.00)	1,000.00	(1,000.00)	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
10A.	(1,000.00)	1,000.00	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
11A.	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
12A	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
13A	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
14A	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
15A	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	(1,000.00)
16A	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	(1,000.00)	(1,000.00)
17A	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)
18A	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)
19A	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)
20A	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)
21A	(1,000.00)	1,000.00	1,000.00	1,000.00	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)
22A	(1,000.00)	1,000.00	1,000.00	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)
23A	(1,000.00)	1,000.00	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)
24A	(1,000.00)	1,000.00	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)	(1,000.00)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	–250%	–200%	–150%	–126%	–100%	–10%	–5.00%	0.250%	5%	50%	100%	300%
1B.	1,285.81	2,000.00	(2,722,000.00)	–8.3166E+09	(1,000.00)	(22,937.93)	(14,796.29)	(9,817.22)	(7,049.19)	(1,095.70)	(499.76)	(208.33)
2B.	1,301.22	4,000.00	5,470,000.00	1.2641E+10	(1,000.00)	(15,856.51)	(11,095.05)	(7,876.25)	(5,935.52)	(1,080.28)	(499.27)	(208.33)
3B.	1,278.10	2,000.00	1,374,000.00	7,192,283,447.69	(1,000.00)	(9,483.24)	(7,578.88)	(5,930.44)	(4,766.16)	(1,057.16)	(498.29)	(208.33)
4B.	1,312.78	4,000.00	3,422,000.00	8,609,044,195.47	(1,000.00)	(3,747.30)	(4,238.52)	(3,979.76)	(3,538.33)	(1,022.48)	(496.34)	(208.33)
5B.	1,260.76	2,000.00	2,398,000.00	8,240,686,401.05	(1,000.00)	1,415.05	(1,065.17)	(2,024.20)	(2,249.11)	(970.45)	(492.43)	(208.32)
6B.	1,338.79	4,000.00	2,910,000.00	8,336,459,427.60	(1,000.00)	6,061.17	1,949.51	(63.75)	(895.44)	(892.42)	(484.62)	(208.29)
7B.	1,221.74	2,000.00	2,654,000.00	8,311,558,440.70	(1,000.00)	10,242.67	4,813.46	1,901.60	525.93	(775.36)	(468.99)	(208.17)
8B.	1,397.32	4,000.00	2,782,000.00	8,318,032,697.29	(1,000.00)	14,006.02	7,534.21	3,871.86	2,018.36	(599.78)	(437.74)	(207.68)
9B.	1,133.95	2,000.00	2,718,000.00	8,316,349,390.58	(1,000.00)	17,393.04	10,118.92	5,847.04	3,585.41	(336.40)	(375.24)	(205.73)
10B.	1,529.01	4,000.00	2,750,000.00	8,316,787,050.32	(1,000.00)	20,441.36	12,574.39	7,827.17	5,230.81	58.66	(250.24)	(197.92)
11B.	936.42	2,000.00	2,734,000.00	8,316,673,258.79	(1,000.00)	23,184.84	14,907.09	9,812.24	6,958.49	651.25	(0.24)	(166.67)
12B	936.42	2,000.00	2,734,000.00	8,316,673,258.79	(1,000.00)	23,184.84	14,907.09	9,812.24	6,958.49	651.25	(0.24)	(166.67)
13B	(396.92)	0.00	2,730,000.00	8,316,665,566.48	1,000.00	25,407.06	17,012.36	11,807.25	8,863.25	1,984.59	999.76	333.33
14B	936.42	2,000.00	2,734,000.00	8,316,673,258.79	(1,000.00)	23,184.84	14,907.09	9,812.24	6,958.49	651.25	(0.24)	(166.67)
15B	921.00	0.00	(5,458,000.00)	################	(1,000.00)	16,103.43	11,205.86	7,871.28	5,844.81	635.84	(0.73)	(166.67)
16B	944.12	2,000.00	(1,362,000.00)	(7,192,246,169.59)	(1,000.00)	9,730.16	7,689.68	5,925.46	4,675.46	612.72	(1.71)	(166.67)
17B	909.44	0.00	(3,410,000.00)	(8,609,006,917.37)	(1,000.00)	3,994.21	4,349.32	3,974.78	3,447.63	578.03	(3.66)	(166.67)
18B	961.47	2,000.00	(2,386,000.00)	(8,240,649,122.94)	(1,000.00)	(1,168.14)	1,175.97	2,019.22	2,158.41	526.01	(7.57)	(166.68)
19B	883.43	0.00	(2,898,000.00)	(8,336,422,149.49)	(1,000.00)	(5,814.25)	(1,838.71)	58.78	804.73	447.97	(15.38)	(166.71)
20B	1,000.48	2,000.00	(2,642,000.00)	(8,311,521,162.59)	(1,000.00)	(9,995.76)	(4,702.65)	(1,906.57)	(616.63)	330.92	(31.01)	(166.83)
21B	824.90	0.00	(2,770,000.00)	(8,317,995,419.19)	(1,000.00)	(13,759.11)	(7,423.40)	(3,876.83)	(2,109.06)	155.33	(62.26)	(167.32)
22B	1,088.28	2,000.00	(2,706,000.00)	(8,316,312,112.47)	(1,000.00)	(17,146.13)	(10,008.11)	(5,852.02)	(3,676.11)	(108.04)	(124.76)	(169.27)
23B	693.21	0.00	(2,738,000.00)	(8,316,749,772.22)	(1,000.00)	(20,194.44)	(12,463.59)	(7,832.14)	(5,321.52)	(503.10)	(249.76)	(177.08)
24B	1,285.81	2,000.00	(2,722,000.00)	(8,316,635,980.68)	(1,000.00)	(22,937.93)	(14,796.29)	(9,817.22)	(7,049.19)	(1,095.70)	(499.76)	(208.33)

Table-6:
Project Outcomes/Cashflows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(100.00)	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
2A.	(100.00)	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	100.00
3A.	(100.00)	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	100.00	100.00
4A.	(100.00)	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	100.00	100.00	100.00
5A.	(100.00)	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	100.00	100.00	100.00	100.00
6A.	(100.00)	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	100.00	100.00	100.00	100.00	100.00
7A.	(100.00)	100.00	(100.00)	(100.00)	(100.00)	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00
8A.	(100.00)	100.00	(100.00)	(100.00)	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	100.00
9A.	(100.00)	100.00	(100.00)	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10A.	(100.00)	100.00	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
11A.	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
12A	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
13A	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
14A	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
15A	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	(100.00)
16A	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	(100.00)	(100.00)
17A	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	(100.00)	(100.00)	(100.00)
18A	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	(100.00)	(100.00)	(100.00)	(100.00)
19A	(100.00)	100.00	100.00	100.00	100.00	100.00	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
20A	(100.00)	100.00	100.00	100.00	100.00	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
21A	(100.00)	100.00	100.00	100.00	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
22A	(100.00)	100.00	100.00	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
23A	(100.00)	100.00	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)
24A	(100.00)	100.00	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

	–250%	–200%	–150%	–126%	–100%	–10%	–5.00%	0.250%	5%	50%	100%	300%
1B.	128.58	200.00	(272,200.00)	(831,663,598.07)	(100.00)	(2,293.79)	(1,479.63)	(981.72)	(704.92)	(109.57)	(49.98)	(20.83)
2B.	130.12	400.00	547,000.00	1,264,136,324.69	(100.00)	(1,585.65)	(1,109.51)	(787.63)	(593.55)	(108.03)	(49.93)	(20.83)
3B.	127.81	200.00	137,400.00	719,228,344.77	(100.00)	(948.32)	(757.89)	(593.04)	(476.62)	(105.72)	(49.83)	(20.83)
4B.	131.28	400.00	342,200.00	860,904,419.55	(100.00)	(374.73)	(423.85)	(397.98)	(353.83)	(102.25)	(49.63)	(20.83)
5B.	126.08	200.00	239,800.00	824,068,640.11	(100.00)	141.51	(106.52)	(202.42)	(224.91)	(97.05)	(49.24)	(20.83)
6B.	133.88	400.00	291,000.00	833,645,942.76	(100.00)	606.12	194.95	(6.38)	(89.54)	(89.24)	(48.46)	(20.83)
7B.	122.17	200.00	265,400.00	831,155,844.07	(100.00)	1,024.27	481.35	190.16	52.59	(77.54)	(46.90)	(20.82)
8B.	139.73	400.00	278,200.00	831,803,269.73	(100.00)	1,400.60	753.42	387.19	201.84	(59.98)	(43.77)	(20.77)
9B.	113.39	200.00	271,800.00	831,634,939.06	(100.00)	1,739.30	1,011.89	584.70	358.54	(33.64)	(37.52)	(20.57)
10B.	152.90	400.00	275,000.00	831,678,705.03	(100.00)	2,044.14	1,257.44	782.72	523.08	5.87	(25.02)	(19.79)
11B.	93.64	200.00	273,400.00	831,667,325.88	(100.00)	2,318.48	1,490.71	981.22	695.85	65.13	(0.02)	(16.67)
12B	93.64	200.00	273,400.00	831,667,325.88	(100.00)	2,318.48	1,490.71	981.22	695.85	65.13	(0.02)	(16.67)
13B	(39.69)	0.00	273,000.00	831,666,556.65	100.00	2,540.71	1,701.24	1,180.73	886.33	198.46	99.98	33.33
14B	93.64	200.00	273,400.00	831,667,325.88	(100.00)	2,318.48	1,490.71	981.22	695.85	65.13	(0.02)	(16.67)
15B	92.10	0.00	(545,800.00)	(1,264,132,596.87)	(100.00)	1,610.34	1,120.59	787.13	584.48	63.58	(0.07)	(16.67)
16B	94.41	200.00	(136,200.00)	(719,224,616.96)	(100.00)	973.02	768.97	592.55	467.55	61.27	(0.17)	(16.67)
17B	90.94	0.00	(341,000.00)	(860,900,691.74)	(100.00)	399.42	434.93	397.48	344.76	57.80	(0.37)	(16.67)
18B	96.15	200.00	(238,600.00)	(824,064,912.29)	(100.00)	(116.81)	117.60	201.92	215.84	52.60	(0.76)	(16.67)
19B	88.34	0.00	(289,800.00)	(833,642,214.95)	(100.00)	(581.43)	(183.87)	5.88	80.47	44.80	(1.54)	(16.67)
20B	100.05	200.00	(264,200.00)	(831,152,116.26)	(100.00)	(999.58)	(470.27)	(190.66)	(61.66)	33.09	(3.10)	(16.68)
21B	82.49	0.00	(277,000.00)	(831,799,541.92)	(100.00)	(1,375.91)	(742.34)	(387.68)	(210.91)	15.53	(6.23)	(16.73)
22B	108.83	200.00	(270,600.00)	(831,631,211.25)	(100.00)	(1,714.61)	(1,000.81)	(585.20)	(367.61)	(10.80)	(12.48)	(16.93)
23B	69.32	0.00	(273,800.00)	(831,674,977.22)	(100.00)	(2,019.44)	(1,246.36)	(783.21)	(532.15)	(50.31)	(24.98)	(17.71)
24B	128.58	200.00	(272,200.00)	(831,663,598.07)	(100.00)	(2,293.79)	(1,479.63)	(981.72)	(704.92)	(109.57)	(49.98)	(20.83)

Table-7:
Project Outcomes/Cashflows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(10)	10	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)
2A.	(10)	10	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)	10
3A.	(10)	10	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)	10	10
4A.	(10)	10	(10)	(10)	(10)	(10)	(10)	(10)	(10)	10	10	10
5A.	(10)	10	(10)	(10)	(10)	(10)	(10)	(10)	10	10	10	10
6A.	(10)	10	(10)	(10)	(10)	(10)	(10)	10	10	10	10	10
7A.	(10)	10	(10)	(10)	(10)	(10)	10	10	10	10	10	10
8A.	(10)	10	(10)	(10)	(10)	10	10	10	10	10	10	10
9A.	(10)	10	(10)	(10)	10	10	10	10	10	10	10	10
10A.	(10)	10	(10)	10	10	10	10	10	10	10	10	10
11A.	(10)	10	10	10	10	10	10	10	10	10	10	10
12A.	(10)	10	10	10	10	10	10	10	10	10	10	10
13A.	10	10	10	10	10	10	10	10	10	10	10	10
14A	(10)	10	10	10	10	10	10	10	10	10	10	10
15A	(10)	10	10	10	10	10	10	10	10	10	10	(10)
16A	(10)	10	10	10	10	10	10	10	10	10	(10)	(10)
17A	(10)	10	10	10	10	10	10	10	10	(10)	(10)	(10)
18A	(10)	10	10	10	10	10	10	10	(10)	(10)	(10)	(10)
19A	(10)	10	10	10	10	10	10	(10)	(10)	(10)	(10)	(10)
20A	(10)	10	10	10	10	10	(10)	(10)	(10)	(10)	(10)	(10)
21A	(10)	10	10	10	10	(10)	(10)	(10)	(10)	(10)	(10)	(10)
22A	(10)	10	10	10	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)
23A	(10)	10	10	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)
24A	(10)	10	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

NPV	–250%	–200%	–150%	–126%	–100%	–10%	–5.00%	0.250%	5%	50%	100%	300%
1B.	12.86	20.00	(27,220.00)	(83,166,359.81)	(10.0)	(229.38)	(147.96)	(98.17)	(70.49)	(10.96)	(5.00)	(2.08)
2B.	13.01	40.00	54,700.00	126,413,632.47	(10.0)	(158.57)	(110.95)	(78.76)	(59.36)	(10.80)	(4.99)	(2.08)
3B.	12.78	20.00	13,740.00	71,922,834.48	(10.0)	(94.83)	(75.79)	(59.30)	(47.66)	(10.57)	(4.98)	(2.08)
4B.	13.13	40.00	34,220.00	86,090,441.95	(10.0)	(37.47)	(42.39)	(39.80)	(35.38)	(10.22)	(4.96)	(2.08)
5B.	12.61	20.00	23,980.00	82,406,864.01	(10.0)	14.15	(10.65)	(20.24)	(22.49)	(9.70)	(4.92)	(2.08)
6B.	13.39	40.00	29,100.00	83,364,594.28	(10.0)	60.61	19.50	(0.64)	(8.95)	(8.92)	(4.85)	(2.08)
7B.	12.22	20.00	26,540.00	83,115,584.41	(10.0)	102.43	48.13	19.02	5.26	(7.75)	(4.69)	(2.08)
8B.	13.97	40.00	27,820.00	83,180,326.97	(10.0)	140.06	75.34	38.72	20.18	(6.00)	(4.38)	(2.08)
9B.	11.34	20.00	27,180.00	83,163,493.91	(10.0)	173.93	101.19	58.47	35.85	(3.36)	(3.75)	(2.06)
10B.	15.29	40.00	27,500.00	83,167,870.50	(10.0)	204.41	125.74	78.27	52.31	0.59	(2.50)	(1.98)
11B.	9.36	20.00	27,340.00	83,166,732.59	(10.0)	231.85	149.07	98.12	69.58	6.51	(0.00)	(1.67)
12B.	9.36	20.00	27,340.00	83,166,732.59	(10.0)	231.85	149.07	98.12	69.58	6.51	(0.00)	(1.67)
13B.	(3.97)	0.00	27,300.00	83,166,655.66	10.0	254.07	170.12	118.07	88.63	19.85	10.00	3.33
14B	9.36	20.00	27,340.00	83,166,732.59	(10.0)	231.85	149.07	98.12	69.58	6.51	(0.00)	(1.67)
15B	9.21	0.00	(54,580.00)	(126,413,259.69)	(10.0)	161.03	112.06	78.71	58.45	6.36	(0.01)	(1.67)
16B	9.44	20.00	(13,620.00)	(71,922,461.70)	(10.0)	97.30	76.90	59.25	46.75	6.13	(0.02)	(1.67)
17B	9.09	0.00	(34,100.00)	(86,090,069.17)	(10.0)	39.94	43.49	39.75	34.48	5.78	(0.04)	(1.67)
18B	9.61	20.00	(23,860.00)	(82,406,491.23)	(10.0)	(11.68)	11.76	20.19	21.58	5.26	(0.08)	(1.67)
19B	8.83	0.00	(28,980.00)	(83,364,221.49)	(10.0)	(58.14)	(18.39)	0.59	8.05	4.48	(0.15)	(1.67)
20B	10.00	20.00	(26,420.00)	(83,115,211.63)	(10.0)	(99.96)	(47.03)	(19.07)	(6.17)	3.31	(0.31)	(1.67)
21B	8.25	0.00	(27,700.00)	(83,179,954.19)	(10.0)	(137.59)	(74.23)	(38.77)	(21.09)	1.55	(0.62)	(1.67)
22B	10.88	20.00	(27,060.00)	(83,163,121.12)	(10.0)	(171.46)	(100.08)	(58.52)	(36.76)	(1.08)	(1.25)	(1.69)
23B	6.93	0.00	(27,380.00)	(83,167,497.72)	(10.0)	(201.94)	(124.64)	(78.32)	(53.22)	(5.03)	(2.50)	(1.77)
24B	12.86	20.00	(27,220.00)	(83,166,359.81)	(10.0)	(229.38)	(147.96)	(98.17)	(70.49)	(10.96)	(5.00)	(2.08)

Table-8:
Project Outcomes/Cashflows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(1)	1	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)
2A.	(1)	1	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	1
3A.	(1)	1	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	1	1
4A.	(1)	1	(1)	(1)	(1)	(1)	(1)	(1)	(1)	1	1	1
5A.	(1)	1	(1)	(1)	(1)	(1)	(1)	(1)	1	1	1	1
6A.	(1)	1	(1)	(1)	(1)	(1)	(1)	1	1	1	1	1
7A.	(1)	1	(1)	(1)	(1)	(1)	1	1	1	1	1	1
8A.	(1)	1	(1)	(1)	(1)	1	1	1	1	1	1	1
9A.	(1)	1	(1)	(1)	1	1	1	1	1	1	1	1
10A.	(1)	1	(1)	1	1	1	1	1	1	1	1	1
11A.	(1)	1	1	1	1	1	1	1	1	1	1	1
12A.	(1)	1	1	1	1	1	1	1	1	1	1	1
13A.	1	1	1	1	1	1	1	1	1	1	1	1
14A	(1)	1	1	1	1	1	1	1	1	1	1	1
15A	(1)	1	1	1	1	1	1	1	1	1	1	(1)
16A	(1)	1	1	1	1	1	1	1	1	1	(1)	(1)
17A	(1)	1	1	1	1	1	1	1	1	(1)	(1)	(1)
18A	(1)	1	1	1	1	1	1	1	(1)	(1)	(1)	(1)
19A	(1)	1	1	1	1	1	1	(1)	(1)	(1)	(1)	(1)
20A	(1)	1	1	1	1	1	(1)	(1)	(1)	(1)	(1)	(1)
21A	(1)	1	1	1	1	(1)	(1)	(1)	(1)	(1)	(1)	(1)
22A	(1)	1	1	1	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)
23A	(1)	1	1	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)
24A	(1)	1	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

NPV	–250%	–200%	–150%	–126%	–100%	–10%	–5.00%	0.250%	5%	50%	100%	300%
1B.	1.29	2.00	(2,722.00)	(8,316,635.98)	(1.0)	(22.94)	(14.80)	(9.82)	(7.05)	(1.10)	(0.50)	(0.21)
2B.	1.30	4.00	5,470.00	12,641,363.25	(1.0)	(15.86)	(11.10)	(7.88)	(5.94)	(1.08)	(0.50)	(0.21)
3B.	1.28	2.00	1,374.00	7,192,283.45	(1.0)	(9.48)	(7.58)	(5.93)	(4.77)	(1.06)	(0.50)	(0.21)
4B.	1.31	4.00	3,422.00	8,609,044.20	(1.0)	(3.75)	(4.24)	(3.98)	(3.54)	(1.02)	(0.50)	(0.21)
5B.	1.26	2.00	2,398.00	8,240,686.40	(1.0)	1.42	(1.07)	(2.02)	(2.25)	(0.97)	(0.49)	(0.21)
6B.	1.34	4.00	2,910.00	8,336,459.43	(1.0)	6.06	1.95	(0.06)	(0.90)	(0.89)	(0.48)	(0.21)
7B.	1.22	2.00	2,654.00	8,311,558.44	(1.0)	10.24	4.81	1.90	0.53	(0.78)	(0.47)	(0.21)
8B.	1.40	4.00	2,782.00	8,318,032.70	(1.0)	14.01	7.53	3.87	2.02	(0.60)	(0.44)	(0.21)
9B.	1.13	2.00	2,718.00	8,316,349.39	(1.0)	17.39	10.12	5.85	3.59	(0.34)	(0.38)	(0.21)
10B.	1.53	4.00	2,750.00	8,316,787.05	(1.0)	20.44	12.57	7.83	5.23	0.06	(0.25)	(0.20)
11B.	0.94	2.00	2,734.00	8,316,673.26	(1.0)	23.18	14.91	9.81	6.96	0.65	(0.00)	(0.17)
12B.	0.94	2.00	2,734.00	8,316,673.26	(1.0)	23.18	14.91	9.81	6.96	0.65	(0.00)	(0.17)
13B.	(0.40)	0.00	2,730.00	8,316,665.57	1.0	25.41	17.01	11.81	8.86	1.98	1.00	0.33
14B	0.94	2.00	2,734.00	8,316,673.26	(1.0)	23.18	14.91	9.81	6.96	0.65	(0.00)	(0.17)
15B	0.92	0.00	(5,458.00)	(12,641,325.97)	(1.0)	16.10	11.21	7.87	5.84	0.64	(0.00)	(0.17)
16B	0.94	2.00	(1,362.00)	(7,192,246.17)	(1.0)	9.73	7.69	5.93	4.68	0.61	(0.00)	(0.17)
17B	0.91	0.00	(3,410.00)	(8,609,006.92)	(1.0)	3.99	4.35	3.97	3.45	0.58	(0.00)	(0.17)
18B	0.96	2.00	(2,386.00)	(8,240,649.12)	(1.0)	(1.17)	1.18	2.02	2.16	0.53	(0.01)	(0.17)
19B	0.88	0.00	(2,898.00)	(8,336,422.15)	(1.0)	(5.81)	(1.84)	0.06	0.80	0.45	(0.02)	(0.17)
20B	1.00	2.00	(2,642.00)	(8,311,521.16)	(1.0)	(10.00)	(4.70)	(1.91)	(0.62)	0.33	(0.03)	(0.17)
21B	0.82	0.00	(2,770.00)	(8,317,995.42)	(1.0)	(13.76)	(7.42)	(3.88)	(2.11)	0.16	(0.06)	(0.17)
22B	1.09	2.00	(2,706.00)	(8,316,312.11)	(1.0)	(17.15)	(10.01)	(5.85)	(3.68)	(0.11)	(0.12)	(0.17)
23B	0.69	0.00	(2,738.00)	(8,316,749.77)	(1.0)	(20.19)	(12.46)	(7.83)	(5.32)	(0.50)	(0.25)	(0.18)
24B	1.29	2.00	(2,722.00)	(8,316,635.98)	(1.0)	(22.94)	(14.80)	(9.82)	(7.05)	(1.10)	(0.50)	(0.21)

Table-9:
Project Outcomes/Cashflows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(0.10)	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
2A.	(0.10)	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	0.10
3A.	(0.10)	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	0.10	0.10
4A.	(0.10)	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	0.10	0.10	0.10
5A.	(0.10)	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	0.10	0.10	0.10	0.10
6A.	(0.10)	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	0.10	0.10	0.10	0.10	0.10
7A.	(0.10)	0.10	(0.10)	(0.10)	(0.10)	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10
8A.	(0.10)	0.10	(0.10)	(0.10)	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	0.10
9A.	(0.10)	0.10	(0.10)	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
10A.	(0.10)	0.10	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
11A.	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
12A.	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
13A.	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
14A	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
15A	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	(0.10)
16A	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	(0.10)	(0.10)
17A	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	(0.10)	(0.10)	(0.10)
18A	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	0.10	(0.10)	(0.10)	(0.10)	(0.10)
19A	(0.10)	0.10	0.10	0.10	0.10	0.10	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
20A	(0.10)	0.10	0.10	0.10	0.10	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
21A	(0.10)	0.10	0.10	0.10	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
22A	(0.10)	0.10	0.10	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
23A	(0.10)	0.10	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
24A	(0.10)	0.10	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

NPV	−250%	−200%	−150%	−126%	−100%	−10%	−5.00%	0.250%	5%	50%	100%	300%
1B.	0.13	0.20	(272.2000)	(831,663.5981)	(0.1000)	(2.2938)	(1.4796)	(0.9817)	(0.7049)	(0.1096)	(0.0500)	(0.0208)
2B.	0.13	0.40	547.0000	1,264,136.3247	(0.1000)	(1.5857)	(1.1095)	(0.7876)	(0.5936)	(0.1080)	(0.0499)	(0.0208)
3B.	0.13	0.20	137.4000	719,228.3448	(0.1000)	(0.9483)	(0.7579)	(0.5930)	(0.4766)	(0.1057)	(0.0498)	(0.0208)
4B.	0.13	0.40	342.2000	860,904.4195	(0.1000)	(0.3747)	(0.4239)	(0.3980)	(0.3538)	(0.1022)	(0.0496)	(0.0208)
5B.	0.13	0.20	239.8000	824,068.6401	(0.1000)	0.1415	(0.1065)	(0.2024)	(0.2249)	(0.0970)	(0.0492)	(0.0208)
6B.	0.13	0.40	291.0000	833,645.9428	(0.1000)	0.6061	0.1950	(0.0064)	(0.0895)	(0.0892)	(0.0485)	(0.0208)
7B.	0.12	0.20	265.4000	831,155.8441	(0.1000)	1.0243	0.4813	0.1902	0.0526	(0.0775)	(0.0469)	(0.0208)
8B.	0.14	0.40	278.2000	831,803.2697	(0.1000)	1.4006	0.7534	0.3872	0.2018	(0.0600)	(0.0438)	(0.0208)
9B.	0.11	0.20	271.8000	831,634.9391	(0.1000)	1.7393	1.0119	0.5847	0.3585	(0.0336)	(0.0375)	(0.0206)
10B.	0.15	0.40	275.0000	831,678.7050	(0.1000)	2.0441	1.2574	0.7827	0.5231	0.0059	(0.0250)	(0.0198)
11B.	0.09	0.20	273.4000	831,667.3259	(0.1000)	2.3185	1.4907	0.9812	0.6958	0.0651	(0.0000)	(0.0167)
12B.	0.09	0.20	273.4000	831,667.3259	(0.1000)	2.3185	1.4907	0.9812	0.6958	0.0651	(0.0000)	(0.0167)
13B.	(0.04)	0.00	273.0000	831,666.5566	0.1000	2.5407	1.7012	1.1807	0.8863	0.1985	0.1000	0.0333
14B	0.09	0.20	273.4000	831,667.3259	(0.1000)	2.3185	1.4907	0.9812	0.6958	0.0651	(0.0000)	(0.0167)
15B	0.09	0.00	(545.8000)	(1,264,132.5969)	(0.1000)	1.6103	1.1206	0.7871	0.5845	0.0636	(0.0001)	(0.0167)
16B	0.09	0.20	(136.2000)	(719,224.6170)	(0.1000)	0.9730	0.7690	0.5925	0.4675	0.0613	(0.0002)	(0.0167)
17B	0.09	0.00	(341.0000)	(860,900.6917)	(0.1000)	0.3994	0.4349	0.3975	0.3448	0.0578	(0.0004)	(0.0167)
18B	0.10	0.20	(238.6000)	(824,064.9123)	(0.1000)	(0.1168)	0.1176	0.2019	0.2158	0.0526	(0.0008)	(0.0167)
19B	0.09	0.00	(289.8000)	(833,642.2149)	(0.1000)	(0.5814)	(0.1839)	0.0059	0.0805	0.0448	(0.0015)	(0.0167)
20B	0.10	0.20	(264.2000)	(831,152.1163)	(0.1000)	(0.9996)	(0.4703)	(0.1907)	(0.0617)	0.0331	(0.0031)	(0.0167)
21B	0.08	0.00	(277.0000)	(831,799.5419)	(0.1000)	(1.3759)	(0.7423)	(0.3877)	(0.2109)	0.0155	(0.0062)	(0.0167)
22B	0.11	0.20	(270.6000)	(831,631.2112)	(0.1000)	(1.7146)	(1.0008)	(0.5852)	(0.3676)	(0.0108)	(0.0125)	(0.0169)
23B	0.07	0.00	(273.8000)	(831,674.9772)	(0.1000)	(2.0194)	(1.2464)	(0.7832)	(0.5322)	(0.0503)	(0.0250)	(0.0177)
24B	0.13	0.20	(272.2000)	(831,663.5981)	(0.1000)	(2.2938)	(1.4796)	(0.9817)	(0.7049)	(0.1096)	(0.0500)	(0.0208)

Table-10:
Project Outcomes/Cashflows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(0.01)	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
2A.	(0.01)	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	0.01
3A.	(0.01)	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	0.01	0.01
4A.	(0.01)	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	0.01	0.01	0.01
5A.	(0.01)	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	0.01	0.01	0.01	0.01
6A.	(0.01)	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	0.01	0.01	0.01	0.01	0.01
7A.	(0.01)	0.01	(0.01)	(0.01)	(0.01)	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01
8A.	(0.01)	0.01	(0.01)	(0.01)	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	0.01
9A.	(0.01)	0.01	(0.01)	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10A.	(0.01)	0.01	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
11A.	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
12A.	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
13A.	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
14A	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
15A	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	(0.01)
16A	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	(0.01)	(0.01)
17A	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	(0.01)	(0.01)	(0.01)
18A	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	(0.01)	(0.01)	(0.01)	(0.01)
19A	(0.01)	0.01	0.01	0.01	0.01	0.01	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
20A	(0.01)	0.01	0.01	0.01	0.01	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
21A	(0.01)	0.01	0.01	0.01	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
22A	(0.01)	0.01	0.01	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
23A	(0.01)	0.01	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
24A	(0.01)	0.01	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

NPV	−250%	−200%	−150%	−126%	−100%	−10%	−5.00%	0.250%	5%	50%	100%	300%
1B.	0.01	0.02	(27.2200)	(83,166.3598)	(0.0100)	(0.2294)	(0.1480)	(0.0982)	(0.0705)	(0.0110)	(0.0050)	(0.0021)
2B.	0.01	0.04	54.7000	126,413.6325	(0.0100)	(0.1586)	(0.1110)	(0.0788)	(0.0594)	(0.0108)	(0.0050)	(0.0021)
3B.	0.01	0.02	13.7400	71,922.8345	(0.0100)	(0.0948)	(0.0758)	(0.0593)	(0.0477)	(0.0106)	(0.0050)	(0.0021)
4B.	0.01	0.04	34.2200	86,090.4420	(0.0100)	(0.0375)	(0.0424)	(0.0398)	(0.0354)	(0.0102)	(0.0050)	(0.0021)
5B.	0.01	0.02	23.9800	82,406.8640	(0.0100)	0.0142	(0.0107)	(0.0202)	(0.0225)	(0.0097)	(0.0049)	(0.0021)
6B.	0.01	0.04	29.1000	83,364.5943	(0.0100)	0.0606	0.0195	(0.0006)	(0.0090)	(0.0089)	(0.0048)	(0.0021)
7B.	0.01	0.02	26.5400	83,115.5844	(0.0100)	0.1024	0.0481	0.0190	0.0053	(0.0078)	(0.0047)	(0.0021)
8B.	0.01	0.04	27.8200	83,180.3270	(0.0100)	0.1401	0.0753	0.0387	0.0202	(0.0060)	(0.0044)	(0.0021)
9B.	0.01	0.02	27.1800	83,163.4939	(0.0100)	0.1739	0.1012	0.0585	0.0359	(0.0034)	(0.0038)	(0.0021)
10B.	0.02	0.04	27.5000	83,167.8705	(0.0100)	0.2044	0.1257	0.0783	0.0523	0.0006	(0.0025)	(0.0020)
11B.	0.01	0.02	27.3400	83,166.7326	(0.0100)	0.2318	0.1491	0.0981	0.0696	0.0065	(0.0000)	(0.0017)
12B.	0.01	0.02	27.3400	83,166.7326	(0.0100)	0.2318	0.1491	0.0981	0.0696	0.0065	(0.0000)	(0.0017)
13B.	(0.00)	0.00	27.3000	83,166.6557	0.0100	0.2541	0.1701	0.1181	0.0886	0.0198	0.0100	0.0033
14B	0.01	0.02	27.3400	83,166.7326	(0.0100)	0.2318	0.1491	0.0981	0.0696	0.0065	(0.0000)	(0.0017)
15B	0.01	0.00	(54.5800)	(126,413.2597)	(0.0100)	0.1610	0.1121	0.0787	0.0584	0.0064	(0.0000)	(0.0017)
16B	0.01	0.02	(13.6200)	(71,922.4617)	(0.0100)	0.0973	0.0769	0.0593	0.0468	0.0061	(0.0000)	(0.0017)
17B	0.01	0.00	(34.1000)	(86,090.0692)	(0.0100)	0.0399	0.0435	0.0397	0.0345	0.0058	(0.0000)	(0.0017)
18B	0.01	0.02	(23.8600)	(82,406.4912)	(0.0100)	(0.0117)	0.0118	0.0202	0.0216	0.0053	(0.0001)	(0.0017)
19B	0.01	0.00	(28.9800)	(83,364.2215)	(0.0100)	(0.0581)	(0.0184)	0.0006	0.0080	0.0045	(0.0002)	(0.0017)
20B	0.01	0.02	(26.4200)	(83,115.2116)	(0.0100)	(0.1000)	(0.0470)	(0.0191)	(0.0062)	0.0033	(0.0003)	(0.0017)
21B	0.01	0.00	(27.7000)	(83,179.9542)	(0.0100)	(0.1376)	(0.0742)	(0.0388)	(0.0211)	0.0016	(0.0006)	(0.0017)
22B	0.01	0.02	(27.0600)	(83,163.1211)	(0.0100)	(0.1715)	(0.1001)	(0.0585)	(0.0368)	(0.0011)	(0.0012)	(0.0017)
23B	0.01	0.00	(27.3800)	(83,167.4977)	(0.0100)	(0.2019)	(0.1246)	(0.0783)	(0.0532)	(0.0050)	(0.0025)	(0.0018)
24B	0.01	0.02	(27.2200)	(83,166.3598)	(0.0100)	(0.2294)	(0.1480)	(0.0982)	(0.0705)	(0.0110)	(0.0050)	(0.0021)

Table-11:
Project Outcomes/Cashflows:

	0	1	2	3	4	5	6	7	8	9	10	11
1A.	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)
2A.	(0.0010)	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	0.0010
3A.	(0.0010)	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	0.0010	0.0010
4A.	(0.0010)	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	0.0010	0.0010	0.0010
5A.	(0.0010)	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	0.0010	0.0010	0.0010	0.0010
6A.	(0.0010)	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010
7A.	(0.0010)	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
8A.	(0.0010)	0.0010	(0.0010)	(0.0010)	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
9A.	(0.0010)	0.0010	(0.0010)	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
10A.	(0.0010)	0.0010	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
11A.	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
12A.	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
13A.	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
14A	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
15A	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	(0.0010)
16A	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	(0.0010)	(0.0010)
17A	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	(0.0010)	(0.0010)	(0.0010)
18A	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)
19A	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)
20A	(0.0010)	0.0010	0.0010	0.0010	0.0010	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)
21A	(0.0010)	0.0010	0.0010	0.0010	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)
22A	(0.0010)	0.0010	0.0010	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)
23A	(0.0010)	0.0010	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)
24A	(0.0010)	0.0010	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)	(0.0010)

Net Present values (For Each Of The Cash Flows Above At The Discount Rates Below; Calculated With The NPV Function In MS-Excel Except For Where The Discount rate is -100%):

NPV	–250%	–200%	–150%	–126%	–100%	–10%	–5.00%	0.250%	5%	50%	100%	300%
1B.	0.0004	0.0000	(2.7300)	(8,316.6656)	(0.0010)	(0.0254)	(0.0170)	(0.0118)	(0.0089)	(0.0020)	(0.0010)	(0.0003)
2B.	0.0013	0.0040	5.4700	12,641.3632	(0.0010)	(0.0159)	(0.0111)	(0.0079)	(0.0059)	(0.0011)	(0.0005)	(0.0002)
3B.	0.0013	0.0020	1.3740	7,192.2834	(0.0010)	(0.0095)	(0.0076)	(0.0059)	(0.0048)	(0.0011)	(0.0005)	(0.0002)
4B.	0.0013	0.0040	3.4220	8,609.0442	(0.0010)	(0.0037)	(0.0042)	(0.0040)	(0.0035)	(0.0010)	(0.0005)	(0.0002)
5B.	0.0013	0.0020	2.3980	8,240.6864	(0.0010)	0.0014	(0.0011)	(0.0020)	(0.0022)	(0.0010)	(0.0005)	(0.0002)
6B.	0.0013	0.0040	2.9100	8,336.4594	(0.0010)	0.0061	0.0019	(0.0001)	(0.0009)	(0.0009)	(0.0005)	(0.0002)
7B.	0.0012	0.0020	2.6540	8,311.5584	(0.0010)	0.0102	0.0048	0.0019	0.0005	(0.0008)	(0.0005)	(0.0002)
8B.	0.0014	0.0040	2.7820	8,318.0327	(0.0010)	0.0140	0.0075	0.0039	0.0020	(0.0006)	(0.0004)	(0.0002)
9B.	0.0011	0.0020	2.7180	8,316.3494	(0.0010)	0.0174	0.0101	0.0058	0.0036	(0.0003)	(0.0004)	(0.0002)
10B.	0.0015	0.0040	2.7500	8,316.7871	(0.0010)	0.0204	0.0126	0.0078	0.0052	0.0001	(0.0003)	(0.0002)
11B.	0.0009	0.0020	2.7340	8,316.6733	(0.0010)	0.0232	0.0149	0.0098	0.0070	0.0007	(0.0000)	(0.0002)
12B.	0.0009	0.0020	2.7340	8,316.6733	(0.0010)	0.0232	0.0149	0.0098	0.0070	0.0007	(0.0000)	(0.0002)
13B.	(0.0004)	0.0000	2.7300	8,316.6656	0.0010	0.0254	0.0170	0.0118	0.0089	0.0020	0.0010	0.0003
14B	0.0009	0.0020	2.7340	8,316.6733	(0.0010)	0.0232	0.0149	0.0098	0.0070	0.0007	(0.0000)	(0.0002)
15B	0.0009	0.0000	(5.4580)	(12,641.3260)	(0.0010)	0.0161	0.0112	0.0079	0.0058	0.0006	(0.0000)	(0.0002)
16B	0.0009	0.0020	(1.3620)	(7,192.2462)	(0.0010)	0.0097	0.0077	0.0059	0.0047	0.0006	(0.0000)	(0.0002)
17B	0.0009	0.0000	(3.4100)	(8,609.0069)	(0.0010)	0.0040	0.0043	0.0040	0.0034	0.0006	(0.0000)	(0.0002)
18B	0.0010	0.0020	(2.3860)	(8,240.6491)	(0.0010)	(0.0012)	0.0012	0.0020	0.0022	0.0005	(0.0000)	(0.0002)
19B	0.0009	0.0000	(2.8980)	(8,336.4221)	(0.0010)	(0.0058)	(0.0018)	0.0001	0.0008	0.0004	(0.0000)	(0.0002)
20B	0.0010	0.0020	(2.6420)	(8,311.5212)	(0.0010)	(0.0100)	(0.0047)	(0.0019)	(0.0006)	0.0003	(0.0000)	(0.0002)
21B	0.0008	0.0000	(2.7700)	(8,317.9954)	(0.0010)	(0.0138)	(0.0074)	(0.0039)	(0.0021)	0.0002	(0.0001)	(0.0002)
22B	0.0011	0.0020	(2.7060)	(8,316.3121)	(0.0010)	(0.0171)	(0.0100)	(0.0059)	(0.0037)	(0.0001)	(0.0001)	(0.0002)
23B	0.0007	0.0000	(2.7380)	(8,316.7498)	(0.0010)	(0.0202)	(0.0125)	(0.0078)	(0.0053)	(0.0005)	(0.0002)	(0.0002)
24B	0.0013	0.0020	(2.7220)	(8,316.6360)	(0.0010)	(0.0229)	(0.0148)	(0.0098)	(0.0070)	(0.0011)	(0.0005)	(0.0002)

Bibliography

Boyer, R., & Goh, W. (2007). On the zero attractor of the Euler polynomials. *Advances in Applied Mathematics, 38*(1), 97–132.

Burrus, C. S. (2004). *Factoring high degree polynomials in signal processing*. 11th IEEE digital signal processing workshop, 2004 and the 3rd IEEE signal processing education workshop. 1–4 Aug. 2004. pp. 156–157.

Campos-Canton, E., Aguirre-Hernandez, B., Renteria, J., & Gonzalez, E. (2015). A polynomial approach for generating a monoparametric family of chaotic attractors via switched linear systems. *Chaos, Solitons & Fractals, 71*, 10–16. doi:10.1016/j.chaos.2014.12.012.

Farahmand K. (2007). On zeros of self-reciprocal random algebraic polynomials. *Journal of Applied Mathematics & Stochastic Analysis*, *2007*(2007), Article ID 43091, 10.1155/2007/43091

Farahmand, K. (2008). Real zeros of algebraic polynomials with stable random coefficients. *Journal of the Australian Mathematical Society, 85*(1), 81–86.

Farahmand, K., & Jahangiri, J. (2005). Algebraic polynomials with non-identical random coefficients. *Journal of Theoretical Probability, 18*(4), 827–835.

Farahmand, K., & Nezakati, A. (2005a). Algebraic polynomials with dependent random coefficients. *Communications in Applied Analysis, 9*(1), 95–104.

Farahmand, K., & Nezakati, A. (2005b). Algebraic polynomials with non-identical random coefficients. *Proceedings of the American Mathematical Society, 133*(1), 275–283.

Farahmand, K., & Nezakati, A. (2006). Covariance of the number of real zeros of a random algebraic polynomial with binomial elements. *Stochastic Analysis & Applications, 24*(2), 329–337.

Farahmand, K., & Sambandham, M. (2003). Real zeros of classes of random algebraic polynomials. *Journal of Applied Mathematics and Stochastic Analysis, 16*(3), 249–255.

Farahmand, K., & Sambandham, M. (2009). Algebraic polynomials with random coefficients with binomial and geometric progressions. *Journal of Applied Mathematics and Stochastic Analysis*, *2009*(2009), Article ID 725260, doi. 10.1155/2009/725260.

Farahmand, K., & Stretch, C. T. (2008). Algebraic polynomials with random non-symmetric coefficients. *Statistics & Probability Letters, 78*(11), 1305–1313.

Farahmand K., Grigorash, A. & McGuinness B. (2008). On different classes of algebraic polynomials with random coefficients. *Journal Of Applied Mathematics & Stochastic Analysis*, *2008*(2008), Article ID 189675, 10.1155/2008/189675

Lei, Z., Blane, M., & Cooper, D. (1996). *3L fitting of higher degree implicit polynomials*. Proceedings of the 3rd IEEE Workshop on Applications of Computer Vision (WACV '96), p. 148, December 02–04, 1996.

Nezakati, A., & Farahmand, K. (2006). Real zeros of random algebraic polynomials with binomial elements. *Journal of Applied Mathematics and Stochastic Analysis*, *2006*(2006), Article ID 13980, 10.1155/JAMSA/2006/13980.

Nwogugu, M. (2010a). *Anomalies in polynomials, the descartes sign rule and the fundamental theorem of algebra* (Working paper). Available in www.ssrn.com

Nwogugu, M. (2010b). *Anomalies in polynomials and the NPV model* (Working paper). Available in www.ssrn.com

Oehmke, J. F. (2000). Anomalies in net present value calculations. *Economics Letters, 67*, 349–351.

Osborne, M. (2010). A resolution to the NPV–IRR debate? *The Quarterly Review of Economics and Finance, 50*, 234–239.

Sitton, G., Burrus, C. S., Fox, J., & Treitel, S. (2003). Factoring very high degree polynomials in signal processing. *Signal Processing Magazine, 6*(20), 27–42.

Yannacopoulos, A., Brindley, J., Merkin, A., & Pilling, M. (1996). Approximation of attractors using Chebyshev polynomials. *Physica D: Nonlinear Phenomena, 99*(2), 162–174.

Zheng, B., Takamatsu, J., & Ikeuchi, K. (2010). An adaptive and stable method for fitting implicit polynomial curves and surfaces. *IEEE Transactions on Pattern Analysis & Machine Intelligence, 32*(3), 561–568.

6

The Historical and Current Concepts of "Plain" Interest Rates, Forward Rates and Discount Rates Are or Can Be Misleading

This chapter: (1) surveys the literature on anomalies in discounting and intertemporal choice, and critiques of the NPV-IRR model (and related approaches); and discusses the varied treatment of discount rates in analyses of NPV/MIRR/IRR; (2) explains why the NPV-IRR-MIRR model (and related approaches) differs from human behavior; (3) explains the conditions under which negative discount rates may be feasible and rational; (4) explains why the weighted average cost of capital (WACC) may be misleading; (5) explains why the concept of plain interest rates and forward rates (unadjusted for behavioral biases, taxes, Regret and transaction costs) may not be entirely correct; (6) explains the differences between willingness-to-accept-losses (WTAL) on one hand, and Loss Aversion, Risk Aversion and Regret Aversion.

6.1 Existing Literature

Cochrane (2011) surveyed the literature on discount rates. Osborne (2014) analyzed interest rates. Walther (2010) analyzed anomalies in intertemporal choice, time-dependent uncertainty and expected utility.

M.C.I. Nwogugu, *Anomalies in Net Present Value, Returns and Polynomials, and Regret Theory in Decision-Making*,
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Frederick et al. (2002), Ortendahl and Fries (2005), Walther (2010), and Booth (2003) made some useful comments about framing and intertemporal discounting. Frederick et al. (2002) noted that empirically observed discount rates (in intertemporal choice experiments) tend to decline over time (hyperbolic discounting); and that sequences of multiple outcomes are discounted differently from single outcomes; and that gains and losses are often discounted differently. Barry et al. (1996) commented on the use of time in intertemporal analysis. Forstater and Mosler (2005), Aspromorgous (2000), Palley (2015), and Frederick et al. (2002) all critiqued the concepts of interest rates and discount rates. The Office of the President of the United States (July 2015) surveyed the literature on interest rates, forward rates, and discount rates. Thornton and Valente (2012) found that the information content of forward rates does not provide investors with higher excess returns compared to an investment strategy that is derived from current information.

Jarrow (2008) developed an economic and mathematical characterization of operational risk that can be useful for clarifying the issues related to estimation and the determination of economic capital.

Van Groenendaal (1998), Hanafizadeh and Vahideh (2011), Wiesemann et al. (2010), Simerska (2008), Leyman and Vanhoucke (2016), Pasqual et al. (2013), and Ramsey (1970) analyzed various aspects of project selection with the NPV/IRR model. Chituc (2011) and Saleh and Marais (2006) analyzed NPV decisions within the context of software reliability engineering. Magni (2009a) analyzed the evolution and properties of residual income theories and the construction of performance metrics. Lohmann (1988), Biondi (2006), Aven and Flage (2009), Hazen (2003), and Kharabe and Rimbach (1989) analyzed the deficiencies inherent in IRR and NPV, and related models.

Magni (2005a) concluded that the standard CAPM-based NPV cannot be used for investment valuations because: (1) the concepts of value inherent in the standard CAPM-based NPV criterion are not additive; (2) the standard CAPM-NPV procedure for project valuation violates the principle of *description invariance* (has some framing effects which arise in valuation and decisions, which, in turn, result in different choices); (3) NPV is an ambiguous notion and thus, the CAPM-based NPV is a theoretically inaccurate method for valuation and investment decisions, even if the CAPM assumptions exist. Magni (2008a) concluded that, contrary to Rubinstein (1973), *disequilibrium* NPV and economic profit for risky one-period projects are not equivalent, partly because NPV-minded agents are subject to framing effects and to arbitrage losses, which imply violations of Modigliani and Miller's Proposition I; and that disequilibrium (present) value (a popular metric that was deductively derived from the CAPM) should therefore be dismissed. Magni (2008a) also states that (disequilibrium) NPV is non-additive, and the same holds for the notion of disequilibrium value. Magni (2008b) analyzed the CAPM-derived capital budgeting criterion, and Rubinstein's (1973) criterion, according to which a project is profitable if the project rate of return is greater than the risk-adjusted cost of capital (which depends on the project's disequilibrium systematic risk), and concluded that the disequilibrium NPV implied by this criterion is non-additive. Magni (2008b) concluded that the disequilibrium NPV model should not be used, and provided proofs showing that it is incompatible with additivity and does not fulfil the principle of description invariance; and that CAPM-minded evaluators may incur arbitrage losses.

Magni (2007a, c) analyzed the simultaneous use of CAPM and the NPV-IRR model (and the theories are applicable to related approaches, such as APV and NFV). Cigola and Peccati (2005) compared APV and NPV.

Magni (2005b) decomposed NFVs into *systemic value added* (SVA) for use in decision-making, based on a systemic approach introduced in Magni (2003, 2004). Magni (2005b) also analyzed similarities with other decomposition models, such as Stewart's economic value added (EVA); and noted that the SVA index differs from Stewart's EVA in that it is based on a different interpretation of excess profit and is formally connected with the EVA model by means of a *shadow* project; and the SVA is formally based on economic, financial, and accounting considerations. Magni (2005b) stated sufficient and necessary conditions for decomposing NFV; and explained the relationship between a project's SVA and its shadow project's EVA; and provided proofs for all results of Pressacco and Stucchi (1997) by using the systemic approach. Magni

(2007b) developed a decomposition of the NFV of a project under certainty; and developed the SVA profitability index, which is amenable to a periodic decomposition into an economic concept of residual income. Magni (2007b) noted that the SVA and the NFV have some common elements and that the systemic partition of SVA is shown to differ from the NFV decomposition model proposed by Peccati (1992), which in turn is very similar to Stewart's (1991) EVA model. Magni (2002) concluded that using the NPV rule for making investment decisions can lead to inconsistencies and antinomies; and that the equivalent-risk principle (i.e., an investor needs to compare an investment opportunity with an asset of equivalent risk) is difficult to implement within the NPV model. Magni (2002) explained and critiqued the NPV model and described various anomalies and one type of framing effect inherent in the NPV-IRR model. Magni (2009c) analyzed the NPV maximization rule and concluded that: (1) discounting cash flows with a hurdle rate differs from the cost of capital normatively suggested (e.g., CAPM, arbitrage pricing); (2) given that decision-makers' aspiration levels matter and are subjectively determined, the hurdle rate rule can be deemed to be a boundedly rational approach to investment decisions; (3) some empirical evidence shows that the bounded-rationality approach is ecologically rational, domain-specific, and psychological plausible; and the hurdle rate rule results in near-optimal solutions when confronted with the expanded NPV; (4); the hurdle rate method is consistent with the Real Options approach, resource-based theory, top management team literature, agency theory, and strategy literature given that the hurdle rate is affected by: uncertainty, future opportunities, rationing of managerial skills, strategic considerations, agency costs, and costs of external financing; (5) the NPV maximizing model may take several different forms; some of which are inapplicable and/or deviate from accepted standards of rationality (additivity, no arbitrage, description invariance); (6) individually, both unboundedly rational NPV and boundedly rational NPV have inherent inaccuracies; but an interaction between the two may improve performance (see Gigerenzer and Regier 1996).

Rubinstein (1973), Senbet and Thompson (1978), and Dybvig and Ingersoll (1982), analyzed the simultaneous use of the mean-variance and NPV-IRR models (and the theories are applicable to related approaches such as APV and NFV).

De Reyck et al. (2008) concluded that NPV can be valid if correctly applied in the case of project flexibility to value flexibility in projects by deriving the appropriate discount rates that prevail at each chance node. Smith and McCardle (1999) found that using WACC-based IRR and/or NPV may cause errors when applied to projects that are significantly different from the firm as a whole (i.e., different discount rates should be used for different projects, on the basis of their own cost of capital). Jagannathan, Matsa, Meier & Tarhan (Aug. 2013) analyzed why firms use discount rates that are significantly greater than their cost-of-capital. Berkovitch and Isreal (2004) attempted to explain why the NPV is incorrect and how information asymmetry and agency problems prevent or hinder the application of the optimal capital budgeting outcomes; and also defined the conditions under which IRR and NPV can be used. However, as explained in this book, NPV and IRR are inaccurate under almost all circumstances. Berkovitch and Isreal (2004) did not analyze the methodological accuracy of NPV or IRR or the inherent framing effects, but focus on agency and information asymmetry problems within the context of parent–subsidiary company relations and headquarters–branch–office relations.

Branch and Echevarria (1998), Strong (2006), Roll (1983), Liu and Strong (2008), Blume (1974), Cooper (2006), Jacquier et al. (2003), Indro and Lee (1997), Cheng and Deets (1971), Jacquier et al. (2005), Jean and Helms (1983), Dorfleitner (2003), Keim (1989), and Fisher et al. (2010) all found inherent biases in the formulas for the calculations of returns of both single-assets and indices—and those biases contradict the findings in Barry and Robison (2014). Sometimes such returns are used as discount rates (or components of discount rates) in the NPV model (and related approaches).

6.2 Concepts of "Plain" Interest Rates, Forward Rates and Discount Rates Can Be Misleading

The historical and current concepts of interest rates and associated forward and discount rates were partly derived from *indifference curves,* consumption models, research by Fisher and from Paul Samuelson's *discounted utility model*; all which have been erroneously treated in the literature as complete specifications of human behavior and intertemporal choice. See Fisher (1930) and Samuelson (1937a, b). More recently, researchers have developed intertemporal choice models that are conceptually very similar to, and thus have the same weaknesses as Paul Samuelson's discounted utility model—such as APV, MIRR, AIRR; EVA; SIRR; NFV, the discount factors mentioned in Ouattara & De La Bruslerie (2015) and Scholten & Read (2006); and so on. The reality is that psychological biases of individuals can cause substantial departures from "rational" economic CSIP models and discounting. As explained in Chap. 3, traditional CSIP models are misleading and remain grossly inadequate to address human behaviors, time, changes in wealth, and institutions (e.g., regulations, differential compliance, taxes; etc.). Distorting factors that cause or amplify departures from rationality (as predicted in discounting and the traditional CSIP models) include: impulsive behavior; fairness; habit; addictions; peer pressure; social network trends; fear of the future; hoarding behavior; emergency expenses; the tax benefits of losses; WTAL; Regret; differential valuation of time; ability to hedge (existence of visible or undisclosed hedging contracts); concerns about social welfare (such as environmental degradation); differential valuation of risk; perceived stability of savings and pension assets; discounting of future utility; loss aversion; etc. Some of these distorting factors are primary behaviors, while others are derivative behaviors or higher-order behaviors. Bernard (1926) distinguished between primary and derivative attitudes and ideals. Bernard (1936) analyzed conflicts between primary group attitudes and derivative group ideals. Hullian Theory in psychology also distinguishes between direct and derivative human (individual and group) behaviors.

Deck and Schlesinger (2014), Noussair et al. (2013), and other articles have analyzed a few higher-order risk preferences. It is noteworthy that the literature on discounting, capital budgeting and asset pricing does not address such primary and derivative behaviors.

The existence of income taxes and capital gains taxes can also cause substantial deviations from the rational consumption/investment models; and completely changes or nullifies the concept of interest rates. That is, foregoing consumption today may not necessarily result in availability of cash/capital for future consumption in the presence of some types of tax. Also, short-term losses and capital losses can have substantial value in certain tax regimes and can be deferred/transferred—which can substantially change the individual's perception of opportunity costs and, hence, cost of capital and the value of consumption in different time units.

Transaction costs and monitoring costs (TCM) can also cause substantial deviations from the rational consumption/investment/production models and completely nullify the concept of interest rates. High expected TCM (which may exceed real or nominal interest rates) will affect (and, in most cases, probably reduce) the individual's propensity to defer consumption, and increase his/her opportunity cost. Conversely, extremely low, or even negative, expected TCM will tend to increase the individual's indifference to consumption in various contiguous time periods, and can reduce the individual's opportunity cost, and also reduce his/her cost of capital to below zero.

The acts of saving and investment by themselves are, or can be, forms of consumption because persons (companies and individuals) can derive specific utility or disutility from those acts. That nullifies, or can nullify, the theoretical foundations of interest rates as explained in the literature.

As explained in Chap. 3, the risk-free rate is not entirely risk-free; and the risk premium under IAPT and ICAPM are not accurate. As demonstrated by the economic/financial crisis in the US, Greece, Spain and other countries during 2007–2012, the designation of interest rates of government securities as "risk-free" rates is not really correct. Furthermore, even if the credit quality of such government securities is risk-free, investors can earn the publicized yields only if they hold the securities to maturity (and without including the effects of inflation and currency risks).

These foregoing distorting factors that can cause individuals, groups, and households to consume more than necessary today, and produce and invest less than is necessary today can also reduce the individual's discount rates to below zero (regardless of whether or not the individual perceives his/her discount rate to be negative). That is, the person is either completely indifferent to consumption of two or more distinct units of value in various contiguous time periods, or his/her preference for consumption in different time periods is independent of cost of capital considerations and is driven by emotions, tax factors, altruism, commitment, regret, or other factors.

Fullwiler (2006) noted that the relevance and effectiveness of the traditional rate-setting function of central banks continues to decline and will be made increasingly redundant by the dominance of electronic funds transfers. Tymoigne (2006) concluded that Fisher's theory of the rate of interest cannot be applied to macro or microeconomic problems.

6.2.1 The Weak "Money-Rate" Relationship

Contrary to the literature and in the modern world, interest rate is increasingly less of a direct function of money demand and money supply for the following reasons, most of which significantly reduce money demand (or supply) and the relationship between interest rates and money/capital (the "money-rate relationship"). Joint ventures, strategic alliances, licensing, installment sales contracts and partnerships are increasingly popular financing methods which reduce current and future money demand, and can dampen monetary transmission. The rapid growth of the global swaps/options/futures markets reduces the money-rate relationship. Empirical research has shown that the personal discount rates implicit in intertemporal choices of consumers/borrowers differ substantially from, and don't seem to be affected by prevailing interest rates. See Warner and Pleeter (2001); Loewenstein and Prelec (1991); and De La Bruslerie (2015). Usury statutes, due diligence costs, and collateral verification costs and availability can limit money supply. Around the world (and specifically in the US, China, Japan, and Europe), the availability of credit (without regard to interest rates) seems to have a strong positive relationship with the volumes of debt-financed discretionary spending by households and companies, and this relationship isn't substantially affected by the level of interest rates—and is partly attributable to impulse buying, peer pressure, Regret, optimism, overconfidence, ego, indiscipline, individuals' aspirations, perceived fairness of prices, and other human psychological biases. Partly due to concerns about regulation, profitability, stock price performance, and credit ratings, the interest rates charged (for loans, etc.) and the savings deposit rates offered by many non-bank intermediaries (credit card companies, finance companies, etc.) and banks are increasingly less a function of money demand/supply, and increasingly more a function of the cost structure, cost allocation/recovery methods, local/regional relationships, target profits, and internal politics of those financial institutions (see Das (2015: 8–10); Kitamura et al. (July 2015); Thornton (2015); and Carbo-Valverde et al. (2011)). The growth of credit chains in many industries can impose another layer of psychological bias of borrowers and lenders, and weakens the money-rate relationship (the criteria for extending credit and making payments in those credit chains often differ from those intended by, or implicit in, monetary policies). Most multinationals and large companies have their own *internal capital markets* where projects are selected, priced, and financed, and transfer pricing is implemented (see Campello (2002), and the policies of such markets can weaken the money-rate relationship. The volume of financial regulations around the world has increased—there are now non-trivial transaction costs, potential penalties, human capital costs, reporting requirements, and compliance costs associated with raising capital, making investments, and monitoring investments, all of which are distinct from the money demand/supply issue and are priced into, and distort the capital allocation, policy reaction, and rate-setting decisions of securities issuers and investors. Fixed income and currency asset managers around the world control more than US$20 trillion of capital and affect interest rates in most countries, but their compensation contracts provide significant incentives for them to weaken the relationship between interest rates and money supply/demand and for them not to respond to monetary policy (they are paid either a percentage of Assets-Under-Management or the 2%-and-20%

or 1%-and-10% model of hedge funds). Around the world and on a frequent basis, specialized auctions are used for the sale, refinancing, or periodic interest rate resetting of more than US$15 trillion of relatively illiquid auction-based securities (which are corporate, municipal, and government bills/notes/bonds), and those auction processes and the illiquidity reduce and distort the relationship between interest rates and money supply/demand—many empirical studies and government reports have concluded that there is collusion and underpricing in both uniform price and discriminatory auctions of securities. Primary dealers over-bid/underbid and overbuy/underbuy in those auctions depending on their inventory estimates, profitability targets, traders' aspirations, ego, indiscipline, internal controls, primary dealers' arbitrage/hedging needs, client relationships, conditions in the when-issued market, etc. (most of which have little to do with true money demand/supply). See Goldreich (2007); Malvey et al. (2014); Guth et al. (2009); and Sundaresan (1994). Das (2015) noted that the percentage of bank loans that have fixed interest and banks' use of average-cost-of-funds (rather than the marginal-cost-of-funds) to set loan interest rates reduces monetary transmission. In both the global fixed income primary markets and secondary markets, a significant percentage of bonds/notes are purchased (e.g., treasury notes/bonds/bills; corporate bonds) and sold (e.g., structured notes; currency-linked notes, corporate bonds and treasury bonds/notes) solely for arbitrage or hedging purposes, rather than to raise capital that will be deployed in the real sector for economic growth—thus there is a growing disconnect between monetary policy and real sector activities. Experienced or anticipated Regret can affect both intertemporal choices and the relationship between interest rates and money demand/supply, and thus borrowers with default histories may be hesitant to obtain new loans, and lenders/investors who have experienced credit losses will likely be hesitant to provide new loans (see Raeva et al. (2010) and Marcatto et al. (2015)). The credit default swap markets affect the demand/supply of bonds, and distort the relationship between interest rates and money demand/supply (see Oehmke and Zawadowski (2015, 2016)). Friedman (1966) critiqued the elasticity of demand for money with respect to interest rates. Contrary to the literature, the productivity of capital has a substantial and direct impact in determining the rate of interest—the demand for capital investment depends upon the marginal revenue productivity of capital or the marginal efficiency of capital, and companies ship goods and perform services on consignment or based on deferred-payment arrangements, which are forms of "quasi" money supply; and higher productivity encourages lending/investment. The hoarding of cash by both governments and companies (e.g., during 2014–2017 in the US) who at the same time were borrowing large amounts of money is evidence of weakening of the money-rate relationship. Hetzel (1986) critiqued theories of money stock determination. Garner (1986) noted that there was little evidence that interest rate volatility affected the demand for M1. King and Lin (2005) noted that the Monetarist Critique of interest rate rules may be valid. The monetary policy objective of price stability sometimes does not have political support and this weakens the money-rate relationship. The low monetary transmission that was prevalent in both developed and developing countries during 2010–2017 is often asymmetrical, and when the central bank raises short-term interest rates this tends to increase the rates that financial institutions charge on credit cards, home equity lines of credit, and some other types of loans, but doesn't increase deposits at the same time—most depositors are price-takers with respect to rates. Farazmand et al. (2016) surveyed different theories of money demand and found that inflation and exchange rates have a negative effect on money demand, and that income has a positive effect on changes in money demand in the MENA region.

6.2.2 Excessive Dependence On Interest Rate Policy Is Detrimental

Contrary to the literature (see Archibald and Hunter (2001); Brainard (2015); Evans et al. (2015); Thornton (June 2015); Engen et al. (2015); Palley (2016); and Gomme et al. (2015)), the excessive and detrimental dependence of many central banks on interest rate policy is not warranted, especially in emerging markets countries and for the following reasons. Emerging markets economies dont have the same financial infrastructure (financial regulations; payment systems; mortgage/housing finance systems; secondary markets; credit cards; consumer finance systems;

credit scoring systems; customer identification systems; etc.), derivatives markets and technical financial professionals as developed economies. As mentioned herein, interest rates is increasingly less of a direct function of money demand and money supply—Bindseil et al. (2015) critiqued central bank policies, and King and Lin (2005) noted that the Monetarist Critique of interest rate rules may be valid. Interest rates is increasingly less of a direct function of resource utilization in the economy—due to joint ventures; strategic alliances; partnerships; licensing; outsourcing; arbitrage/hedging; direct government or corporate spending on projects without borrowing; human capital; impact of labor unions; the cost structure of companies; competition and the nature of demand in product markets; the age/condition of capital equipment, etc. As explained in this book, the modern models of present values/discounting don't fully or accurately reflect the actual behavior of investors/savers, market intermediaries, and regulators, and hence perceptions and interpretations of benchmark interest rates by decision-makers are likely to be heterogenous and to differ from patterns intended by regulators. Many market participants (investors; lenders; borrowers) don't react to interest rates in ways that are anticipated/intended by central banks. That is, monetary policies of central banks are often not transmitted in the real and financial sectors due to uncertainty; managers' concerns about compensation, incentives, and career prospects; capital constraints and reserve requirements; securitization; credit chains in industries; industry norms; corporate strategy; the percentage of loans that have fixed interest rates; labor union actions; and the methods that banks use to set interest rates for lending and savings deposits (see Das (2015: 8–10); Kitamura et al. (July 2015); Carbo-Valverde et al. (2011); and Thornton (2015)). During the last thirty years, many large private companies and financial institutions with global operations (e.g., Citicorp, HSBC, Mitsubishi, etc.) have emerged as quasi-central banks in their own right and have their own internal capital markets and *internal interest rate policies* which sometimes conflict with those of central banks (see Campello (2002)). As mentioned in Lindner (2015), the much theorized relationship between savings and investment is not entirely correct and saving doesn't always increase the supply of credit. The way that interest rates are used in pervasive decision models (such as NPV-IRR, EVA, AIRR, NFV, discounted cashflow, etc.) often differs from human behavior and actual perceptions of risk and time; and the anomalies/biases in those pervasive decision models distort decision-makers’ responses to governments’ interest rate policies—the NPV-IRR model and present value models are used for calculating the yields for more than US$100+ trillion worth of fixed income securities (bonds/bills/notes) and more than US$40+ trillion worth of loans/mortgages that exist around the world. The four generally accepted channels through which the government’s monetary policy can affect output (the interest rate channel, the credit channel, the asset-price channel, and the exchange-rate channel) are all significantly affected by, or are based on, or are measured with present value/time-discounting models which, as explained in this book, suffer from many anomalies. Various empirical research has concluded that during 1997–2016 and in many countries, there has been slow and low monetary policy transmission, especially in developing countries (see Mishra and Montiel (2013); Das (2015); Engen et al. (2015); and Beck et al. (2014)). In countries that don’t have well-developed capital markets, financial institutions may not have the knowledge or financial instruments with which to hedge if they respond to monetary policies. The deliberations for setting and or responding to governments’ monetary policies can result in, or are subject to, framing effects (see Cheng and Chiou (Feb. 2008); Nwogugu (2006); and Milch et al. (2009)), such as those related to speeding up or delaying monetary policy or reactions to monetary policy; demarcation of time intervals; polarization of groups, etc. As noted in Gomme et al. (2015), the rate-of-return on capital investments in industry is a more appropriate indicator of the real interest rate than benchmark interest rates and government bond/note/bill rates (which are major elements of monetary policy). The efforts expended by central banks to develop and implement monetary policies (e.g., purchases/sales of bonds/notes/bills; interest costs; research/monitoring costs; opportunity costs, etc.) and the efforts and opportunity costs of companies and financial institutions that try to react to, or comply with, monetary policies is, or can be, significant and wasteful. See Breedon and Turner (July 2016). Gonzalez et al. (2016) found that investor behavior differs over time (according to the business cycle) and by sector; and in some financial and non-financial sectors, there is an insignificant relationship between stock returns and unexpected changes in real and nominal interest rates. In some industries, the relationship is consistent, significant, and positive, and in others the relationship is negative. Around the world and on a frequent basis, specialized auctions are used for the sale of auction-based government securities, and those auction processes can significantly reduce monetary transmission. As mentioned above, many studies have concluded that there is collusion and underpricing in both uniform price and discriminatory auctions. Primary dealers overbid/underbid and overbuy/underbuy in those auctions depending on various factors that have little relationship to the true money demand/supply). See Goldreich (2007); Malvey et al. (2014); Guth et al. (2009); and Sundaresan (1994). As mentioned above, in both the global fixed income primary markets and secondary markets, a significant percentage of bonds/notes/bills are purchased and sold solely for arbitrage or hedging purposes, rather than to raise capital that will be deployed in the real sector for economic growth—thus there is a growing disconnect between monetary policy on one hand investor/market reactions and activities in the real sector on the other (see Engen et al. (2015)). The aspirations and social networks of individuals and firms affect and distort their intertemporal choices, the interest rate setting process, and their reactions to monetary policies, and sometimes result in irrationality (see Guth et al. (2009)).

6.2.3 The Relationship Between Interest Rates and Stock Markets

Contrary to the literature and in most countries, the much theorized relationship between interest rates and stock markets has been shown to be tenuous in recent times, and this can be attributed to the following reasons. Taxes, risk aversion, uncertainty, excess wealth, and knowledge may change investors' perceptions and cause them to have *individual inverted yield curves*. There is the concept of the *term structure of psychological interest rates*—and the term structure of personal interest rates inherent in intertemporal choices of investors may differ from the publicized term structure of interest rates (see De La Bruslerie (2015); Warner and Pleeter (2001); and Loewenstein and Prelec (1991)). The investor's propensity to substitute between the stock market and fixed income securities can fluctuate significantly, and may be significantly reduced by the foregoing factors as well as uncertainty; *Willingness-to-Accept-Losses* ("WTAL"; see Nwogugu (2006)); inertia; valuation of time; concerns about retirement income; framing effects, etc. The investor's intertemporal choices with respect to stock markets and fixed income securities is less likely to be affected by time-discounting (present values) than by risk perceptions, uncertainty, opportunity costs, and retirement concerns. The supposed/apparent inverse relationship between interest rates and stock markets can be attributed almost entirely to the pervasive practice of using present-value/DCF models for valuing stocks and bonds and the resulting self-fulfilling prophecies wherein when after interest rates decline, stocks begin to look cheap (while bonds look expensive), and money flows out of the bond markets and inflates equity markets. The NPV-IRR formula (and its inherent anomalies and framing effects) is used for calculating bond/note/bill/loan/mortgage yields (see Chapters 2 and 8), and the formulas for Macaulay Duration and Convexity are wrong (see Nwogugu (2012: 330–335)), and if DCF-present value models are used for valuation, when interest rates decline, it creates larger than justified increases in actual bond prices and "implied" equity valuations, whereas when interest rates rise, it creates smaller than justified decreases in bond prices and "implied" equity valuations. The statistical methods used to evaluate the relationship between interest rates and stock markets in the past may not be accurate. Gonzalez et al. (2016) found that investor behavior differs over time (according to the business cycle) and by sector; and in some financial and non-financial sectors, there is an insignificant relationship between stock returns and unexpected changes in real and nominal interest rates; while in some industries the relationship is consistent, significant, and positive, and in others the relationship is negative.

6.2.4 Interest Rate Parity Does Not Exist In Many Circumstances

Contrary to the economics/finance literature, *interest rate parity* does not hold, and during 2013–2017 there was a substantial disconnect between currency exchange rates and interest rates globally, especially in emerging markets and countries that don't have well-developed capital markets. During 2014–2017, some emerging markets countries (such as Nigeria, Turkey, Mexico and South Africa) increased their benchmark interest rates but the values of their currencies decreased against major currencies. On the other hand, during 2014–2017, some developed countries (such as the US and the UK) decreased their benchmark interest rates and their currencies strengthened against those of emerging markets countries and some developed countries. This non-existence of interest rate parity can be attributed to the following factors. There are substantial differences between interest rate theories in the literature (which some traders and regulators try to apply) on one hand, and actual human behavior and the impacts of regulation and government policies on the other. Uncertainty and investors' perceptions of uncertainty can be major factors. There are differences in time valuation among investors, companies, and regulators in different countries (see Havranek et al. (2015))—time valuation may be a function of regulation, availability of capital, corruption, opportunity costs, human capital, personal aspirations, demand/supply in product markets, etc. There are often differences in individuals' knowledge processing capabilities and their perceptions of fairness of interest rate levels and exchange rates in different countries. The ways that decision-makers use interest rates in pervasive capital budgeting models and present value models (e.g., NPV, IRR, EVA, NFV, DCF, etc.) for cross-border investment analysis, and the inherent anomalies and framing effects in those models, are likely to create violations of interest rate parity (see Bekaert et al. (Sept. 2015)). Capital controls can limit cross-border capital flows, and the ability to make foreign investments and or react to trends in foreign markets (see Song et al. (2014)). Investors', issuers' and regulators' perceptions of, and anxiety about, current/future and absolute/relative credit quality (of government and corporate bonds and credit card portfolios) in both domestic and foreign markets, and associated intertemporal choices, liquidity in domestic and or foreign markets, balances of trade, political risk, taxation, and/or the ability to repatriate profits can cause violations of interest rate parity. Mohanty and Rishabh (March 2016) noted that exchange rates have become much more volatile than can be justified by measures of interest rate differentials, and that the growth of currency mismatches associated with the expansion of unhedged dollar borrowing by emerging market economies has meant that currency depreciation can be contractionary (and or disastrous) for those economies.

6.2.5 The *Marginal Utility Of Wealth* and The *Marginal Utility Of Consumption* are Almost Irrelevant In The Formation Of Equilibrium Interest Rates

One strand common to the economics and psychology approaches in the analysis of equilibrium interest rates is the marginal rate of intertemporal substitution between two points in time, which is referred to as time-discounting in psychology.

Contrary to the economics and operations research literatures, the *marginal utility of wealth* and the *marginal utility of consumption* are almost irrelevant in the formation of equilibrium interest rates for the following reasons. Clearly when investors/savers and issuers/borrowers make decisions about (or that affect) interest rates, they don't calculate their marginal utility of wealth or their marginal utility of consumption—in most instances, they focus on relative values and spreads and portfolio characteristics, none of which are included in most academic models of the marginal utilities of both consumption and wealth (this inability to reflect real-world human and organizational behaviors and psychological processes remains a major weakness of economic models). The typical smart or professional investor often assesses the impact of adding or selling or swapping a bond/bill/note on their overall "standard" portfolio characteristics (e.g., portfolio duration/convexity, tax, arbitrage opportunities, portfolio variance, portfolio income, hedging opportunities, etc.), which implies that total wealth and changes in total wealth are the key issue (and not the marginal utility of wealth) and that "wealth" also includes the aforementioned "portfolio characteristics" even though such measures may be wrong. When making decisions about consumption, households/organizations are likely to consider their experiences when consuming the goods/services being considered and how the intended consumption will affect their overall well-being, Regret, opportunity costs, future budgets, overall "maintenance" costs, social capital and their wealth (existing "portfolio" of goods/services) are collectively referred to as "Unified Consumption," which is distinct from and much more important than the marginal utility of consumption. The methods that banks and financial institutions use to set interest rates for loans and deposits are not derived from the marginal utilities of wealth or consumption of investors/savers and issuer/borrowers; rather, these decisions are usually based on the financial institution's cost structure, cost recovery/allocation methods, local/regional relationships, and target profitability (see Das (2015: 8–10); Kitamura et al. (July 2015); Thornton (2015); and Carbo-Valverde et al. (2011)). As stated herein, tax considerations, perceived high risk, high inflation, uncertainty, inertia, deferred opportunities, bias toward the status quo, high WTAL, etc. may compel a person to forego the equivalent of interest/returns and time-discounting (i.e., the marginal utility of wealth is zero or negative and/or the marginal utility of consumption is zero or negative). For some classes of households and organizations, the rate of substitution between investment and consumption may not be affected by wealth, expected wealth, current consumption, or the marginal utility of consumption—this may be true for households and organizations that have any of the following characteristics: they expect shocks (or are subject to constant shocks); have budget constraints; are subject to severe framing effects or cognitive dissonance; have high conformity; have changing cost structures; are subject to intense regulation; have changing income constraints; have concentrated portfolios (concentrated in a few assets); have low/narrow social networks; or have very high economic or social transaction costs. The permanent income hypothesis is wrong—partly due to imperfect capital markets, risk aversion, etc. Contrary to the economics and operations research literature, "consumption" isn't a single homogenous unit, but contains diverse goods/services for which both the allocated amounts differ and the flexibility of allocated amounts differs. Contrary to the economics, operations research, and psychology literatures, decision-makers tend to first form aspirations for a few relevant scenarios, and then search for consumption plans and wealth targets that guarantee such aspirations (see Guth et al. (2009)). Noise and intentional/unintentional arbitrage can distort the marginal utilities of wealth and consumption (see Haven and Khrennikov (2016) and Gureckis and Love (2009)).

6.2.6 On Discount Rates

In the literature, interest rates have been analyzed from the following view-points: (i) as a price of risk (finance and operations research approach); (ii) as a price of time (the psychology approach); (iii) as an opportunity cost for the use of capital (the economics approach); (iv) as a contractual obligation (legal approach); (v) as a composite of various factors such as inflation and risk premia (the appraisal/valuation approach). Discount rates have also been analyzed along similar lines. Although many researchers (such as Frederick et al. (2002)) have noted that the standard discounted utility model was never intended to serve as a descriptive model of behavior, that model has become the core of many popular decision models such as NPV-IRR, APV, NFV, etc. The literature has documented anomalies in both intertemporal choice and discounting; and has noted that human valuation of time, perceptions of the time-value-of-money and discounting patterns differ from what has been theorized in various journal articles; and thus the models of discounting, utility and interest rates developed by Fisher (1930) and Samuelson (1937a, b) (which have been the foundations for modern theories) are not entirely correct. Fisher (1930) claimed that "…the value of any property, or rights to wealth, is its value *as a source of income* and is found by discounting that expected income…"; and most modern present-value/discounting models are based on this theory and are conceptually similar and are wrong because of the following reasons: (1) there are many assets that don't produce any cashflow or rights to cashflow but have significant non-cash value (such as social capital, reputational capital, goodwill; rights of first refusal, intangibles, etc.); (2) decision-makers may be completely indifferent between current consumption and future consumption—this may be the case where the individual's wealth is substantial, or interest rates (and returns on other investments) are expected to be much less than the inflation rate, or prospective investments are risky or the individual can afford not to earn returns; (3) as mentioned herein and contrary to Fisher (1930) and most modern discounting theories/formulas, people may be willing to forego the equivalent of interest due to tax reasons, high perceived risk of investments, expectations of future broader opportunity-set, ability to afford loss of interest/returns, etc.; (4)

time-discounting is usually justified by the combination of an implied opportunity cost (of capital), expectations of gain on capital, and the value of time, all of which may not exist or occur or have any value, and all of which can be negated where they are less than the inflation rate; (5) the postulated non-economic factors that affect interest rates (foresight/intelligence, self-control, habit, life expectancy, the love of one's children, and fashion) are or can be substantially modified/truncated by other factors (e.g., budget constraints, capital constraints, personality, inter-personal skills, education level and social networks), to the extent that they can become non-factors; (6) the relationship between interest rates and capital value is not always inverse—because that model does not fully reflect the behaviors of buyers/intermediaries/regulators, or that relationship substantially depends on opportunity costs which are not properly accounted for in present-value models; (7) experienced or anticipated regret can affect both intertemporal choices and the valuation of assets in ways that contravene Fisher (1930) and modern present-value theories (see Raeva et al. (2010) and Marcatto et al. (2015). See comments in: Gonzalez et al. (2016), De La Bruslerie (2015), Loewenstein and Prelec (1991, 1992), Frederick et al. (2002), Tymoigne (2006), Soman et al. (2005), Warner and Pleeter (2001), Zauberman et al. (2009), Urminsky and Zauberman (2016), Loewenstein (1992), Kable (2013), Ouattara and De La Bruslerie (2015), Stewart et al. (2014); Guth et al. (2009); Gureckis and Love (2009); Scholten and Read (2006); and Frederick (2006). Dubra (2009) developed two representation theorems for time preferences which cover various time preference models in intertemporal choice as special cases (such as exponential, quasi-hyperbolic and hyperbolic discounting). Masatlioglu & Ok (2009) analyzed discounting.

Soman et al. (2005) noted that other research in the literature critiques the standard discounted utility model, and that: (i) substantial research has concluded that for discounting of single outcomes, the discount rate is not stable and varies as a function of several contextual factors; (ii) the *magnitude effect* implies that the discount rates are higher for smaller dollar amounts relative to larger ones; (iii) the *direction effect* implies that the discount rate obtained by increasing delay to an outcome is greater than that obtained by reducing the delay; (iv) the *sign effect* implies that discount rates are lower for losses than for gains, although there have been demonstrations of the reverse effect as well; (v) the *delay effect* implies that the discount rate is smaller for larger delays; (vi) the *interval effect* implies that the discount rate depends on the time interval between the two outcomes used to impute the discount rate–the greater the temporal interval, the smaller the discount rate; (vii) both the *resource slack theory* and the *temporal construal theory* affect the utility of the outcome as a function of time; (viii) Loewenstein and Prelec (1991) show that an improving sequence can be preferred over a declining sequence of the same ingredient outcomes, which implies that there is "*negative discounting*". Note that both the *resource slack theory* and the *temporal construal theory* can contradict the standard discounted utility model. Ouattara and De La Bruslerie (2015) found that: (i) various discount functions such as the exponential, Hernstein, Harvey, proportional, Laibson, Rachlin, hyperbolic and generalized hyperbolic discount functions have been used in the literature to model individuals' time preferences – these functions are summarized in Scholten and Read (2006) and Ouattara and De La Bruslerie (2015); (ii) standard discounted utility theory is wrong and time preferences cant be characterized by an exponential discount function – this conclusion is consistent with other empirical studies and shows that the population is characterized by a decreasing impatience and by heterogenous psychological discount functions. All of the discount functions mentioned in Ouattara and De La Bruslerie (2015) and Scholten and Read (2006) (eg. the exponential, Hernstein, Harvey, proportional, Laibson, Rachlin, hyperbolic; etc.) suffer from some of the framing effects and anomalies that are inherent in the NPV-IRR model and which are explained in Chaps. 2 and 7 in this book. Ouattara and De La Bruslerie (2015) also noted that discounted utility theory suffers from at least twelve anomalies which are widely documented and are as follows: (i) hyperbolic decreasing of the discount rate ("*hyperbolic discounting*"), (ii) the sensitivity of the psychological discount rate to a common difference ("*common difference effect*"), (iii) the influence of the reward size ('*magnitude effect*'); (iv) the influence of the sign of the expected flux on the psychological discount rate ('*sign effect*'); (v) the psychological discount rate difference by trade-off media ('*domain effect*'); (vi) the difference of psychological discount rate between delaying or speeding up ('*delay–speed up asymmetry*'); (vii) the preference for an increasing sequence ('*preference for improving sequences*'); (viii) the violation of the independence assumption ('violations of independence and preference spread'); (ix) strong sub-additivity ('*strong sub-additivity*'); (x) the separability ('*separability*'). Ouattara and De La Bruslerie (2015) mentioned that these anomalies in discounted utility theory led to three strands of research: (i) the improvement of the basic model by changing the instantaneous utility function; (ii) the improvement of the basic model by changing the psychological discount rate function; (iii) he questioning of the original framework and the establishment of a more suitable framework than discounted utility theory.

De La Bruslerie (2015) found that: (i) there is a difference between the "*subjective interest rate*" which is a variable used by an agent to discount his future utility, and the time preference hypothesis (the time preference hypothesis is the concept that an agent prefers immediate utility instead of a future utility); (ii) the term structure of psychological rates depends strongly on gender, but is not related to life expectancy; (iii) there may not be a cross relationship between risk aversion and time preference which theoretically, are two different and independent dimensions of choice; iv) empirically, both time preference attitude and slope seem to be directly affected by risk attitude. Allais (1974) developed various theories of psychological interest rates. Gonzalez et al. (2016) concluded that: (1) investor behavior differs over time (according to the business cycle) and by sector; (2) in some financial and non-financial sectors, there is an insignificant relationship between stock returns and unexpected changes in real and nominal interest rates; while in some industries the relationship is consistent, significant and positive; and in others, the relationship is negative; (3) Gold has a negative relation to un-anticipated inflation in the overall sample and in the contraction and expansion sub-periods, and thus, its exposed to inflation risk.

Knetsch et al. (2012) and Komlos (2014) critiqued indifference curves and found that: (1) indifference curves dont indicate the reference point or the current level of consumption and it errorneously assumes that decision-makers don't have prior knowledge/experience of the two goods being compared, and that choices along indifference curves are reversible; (2) the endowment effect implies that there is an additional disutility caused by foregoing something, in addition to the loss of the pleasure associated with consuming it; (3) there is a kink in the behavioral indifference curves and the utility function is not differentiable everywhere, preferences are not homothetic and these factors may explain the stickiness in adjustment to changes in wages, prices and interest rates (which are noted in Andersen (1998), and Ausubel (1991)). Furthermore, indifference curves, discounting and the liquidity premium theory implicitly and erroneously assume that benefits received (expenses paid) in nearer time periods are more valuable (more expensive) than those received (paid) in future periods. The contrary may be the case because of the following: (1) the decision-maker's opportunity set expands in future periods such that more profitable opportunities exist in future periods (and future events/outcomes have greater value than earlier events/outcomes); (2) tax regimes and tax considerations can cause cash/benefits received in future periods to be more valuable than if received in the present; (3) the decision-maker is loss-seeking and or can afford to forgo what is usually regarded as the time-value of money; (4) there are non-cash and non-tax benefits/objectives (eg. strategic benefits; synergies; logistics; human capital; Intangibles) that can be gained by forgoing the equivalent of interest.

Some implications are as follows. First, the concept of interest rates as a price of risk may be misleading – investors may focus on tax factors or may intentionally seek losses or may be willing to take un-compensated short term risk in exchange for future gains; or there may be non-cash benefits/outcomes that may justify loss-seeking. Second, the concept of interest rates derived from indifference curves and intertemporal choice theories may be misleading – any two decision-makers may have different scales for measuring compared goods or the time-value of money; or the decision-maker may be willing to forego time-value of money in exchange for tax benefits or non-cash benefits or because of altruism or to reduce his/her Regret; or the decision-maker may be subject to the endowment effect and or reference-points. Third, in order to maintain acceptable levels of economic activity and for borrowers to survive, the upper bound for interest rates should be the borrower's (firm; household) "total profits" (including periodic realizable capital gains and non-cash benefits that can be readily converted into cash) while the lower bound should be zero or a negative interest rate that is near zero. This implies that: (i) interest rates for loans should provide borrowers with psychological, economic, political and social incentives to maximize their total profits; (ii) there is an opportunity cost (which can increase social welfare to a certain level) for capital; and thus interest rates should compensate lenders for providing capital, if not for taking risk; (iii) interest rates should vary according to the borrower's unique circumstances; and (iv) fixed interest rates or floating rates that exceed the borrower's total-profit reduces social welfare; (v) "two-tiered" interest rates (wherein there is lower-than-usual fixed "base interest rate" plus an additional "participating interest rate" that is a function of the borrower's total profits) may improve social welfare more than the usual single-tiered fixed or floating interest rates (see Ebrahim et al. (2009) and Chapter 15 in Nwogugu (2012))—and two-tiered interest rates facilitate better risk sharing; reduces default/workout costs; provides lenders some measure of safety; and provides incentives for borrowers to perform; (vi) negative interest rates may be justified for all or part of the loan-term (in the case where the interest rate is negative for the entire loan term, the lender can recover any "lost interest" through tax benefits; or additional advisory business and or loans to the borrower; etc.). Fourth, the national and worldwide aggregation of the "inefficient" and wrong decisions made with the NPV-IRR model (and related methods) and discounting is a major determinant of interest rates, monetary transmission, competition in industries, international capital flows, systemic risk and Financial Stability. Fifth, forward rates (as they are presently derived) are not accurate indicators of future interest rates; and the term structure theory of interest rates is also not entirely correct - and the analysis in Czaja et al. (2009), Memmel (2011) and similar articles is wrong. Fama and Bliss (1987) concluded that forward rate forecasts for near-term changes in interest rates (for less than one year) are usually not correct; and Hamburger and Platt (1975) and Shiller et al. (1983) concluded that forward rates don't have any forecast power. Thornton and Valente (2012) noted that the information content of forward rates does not provide investors with higher excess returns compared to an investment strategy that is derived from current information. For the following different reasons, forward rates dont have any forecast power: (i) there is entropy, and economic conditions, spot rates, investors' preferences/capabilities (ie. intertemporal choices, information processing capabilities and available capital; etc.), and demand/supply conditions in fixed income markets change frequently over time; (ii) the no-arbitrage conditions and implicit "equilibrium" under which the forward rates formula is derived cannot hold if income/capital-gains taxes, transaction costs, Regret, inflation and capital-constraints are introduced – thus, forward rates are inaccurate; (iii) the no-arbitrage conditions and implicit "equilibrium" under which the forward rates formula is derived can be circumvented using swaps/options; (iv) forward rates erroneously imply that most investors have the same valuation of time, and that events in some time periods are more valuable than the same in other time periods – see the research cited in this chapter about time preferences and valuation. Sixth, firms are more likely than not to use higher-than-warranted discount rates because of the following reasons: (i) differences in risk perception and the valuation of time and opportunity costs among senior managers that make decisions; (ii) uncertainty; (iii) fear of anticipated penalties for failure; (iv) risk slack – wherein managers intentionally increase hurdle rates and discount rates in order to build in perceived cushions against failure; (v) framing effects; (vi) differences in estimates of internal cost of capital among managers; (vi) profit-seeking and risk-seeking behavior wherein managers will tend to prefer positive-NPV projects that have the highest discount rates.

The *liquidity peference theory* is wrong or isn't entirely correct for the following reasons. The occurrence of negative interest rates during 2014–2017 for the government bonds or benchmark interest rates of some countries/regions (e.g., Sweden, Japan, Demark, ECB, etc.) indicated that in some economic conditions, uncertainty and concerns about credit quality far outweigh liquidity preference. During the last forty years, in many countries there have been instances of inverted or flat yield curves in both the corporate bond and the government bond markets. Taxes, expectations of future higher inflation rates, or announced government policies may cause investors to enter into swap contracts that produce synthetic inverted or flat yield curves for any given fixed income portfolio. As noted in this chapter, in modern times the relationship between interest rates and money demand/supply is tenuous or non-existent. Taxes, risk-aversion, uncertainty, excess wealth, and knowledge may change investors' perceptions and cause them to have *individual* inverted or flat yield curves (different from publicized yield curves). There is the concept of the *term structure of* *psychological interest rates*—and the term structure of personal discount rates inherent in intertemporal choices of investors may differ from the term structure of interest rates predicted by the *liquidity preference theory* or from the prevailing term structure (see De La Bruslerie (2015); Warner and Pleeter (2001); Urminsky and Zauberman (2016); and Loewenstein and Prelec (1991)). Experienced or anticipated Regret can affect both investors' intertemporal choices and the valuation of assets in ways that contravene the *liquidity preference theory*, with the result being inverted, partially-inverted or flat yield curves (see Raeva et al. (2010) and Marcatto et al. (2015)). In modern times, the price of liquidity and the opportunity costs of liquidity (which include non-monetary benefits) in fixed income/currency/equity markets have declined significantly as the global swaps/futures/options markets have grown and become more standardized—these markets have enabled investors to achieve synthetic liquidity and to transfer various types of risks at much lower overall (i.e., transaction, monitoring, tax, etc.) costs than usual. Significant differences among investors' combined valuations of time, risk and opportunity costs, or temporary imbalances in supply and demand in fixed income markets can result in inverted or flat yield curves. Friedman (1966) critiqued the *liquidity preference theory* for different reasons. The *liquidity preference theory* does not explain the existence of different rates of interest in the same market at the same time for similar securities/instruments.

On the *Expectations Hypothesis*, see Carpenter and Demiralp (2011), who state, "First, the expectations hypothesis is likely to be rejected in money markets if the realized federal funds rate is studied instead of an appropriate measure of the expected federal funds rate. Second, we find that lower volatility in the bank funding markets, all else equal, leads to a lower term premium and thus longer-term rates for a given setting of the overnight rate. The results appear to hold for the US as well as the Euro Area and the UK. The results have implications for the design of operational frameworks for the implementation of monetary policy and for the interpretation of the changes in the Libor-OIS spread during the financial crisis. We also demonstrate that the expectations hypothesis is more likely to hold the more closely linked the short- and long-term interest rates are…"

6.3 The NPV-MIRR-IRR Model (And Related Approaches) Differs from Actual Human Decision Processes

The foregoing discussion and the critiques of Fisher's and Samuelson's theories and modern discounting models confirms this. The NPV-IRR model (and related models) erroneously assumes that actual market values are exactly equal to actual present values; and also that perceived market values are exactly equal to perceived present values of assets. However, so long as market values differ from present values, the NPV and IRR models will be inaccurate. Hence, the use of the NPV/IRR implicates issues of cognition/perception by decision-makers and framing effects (and ability to decipher intended and unintended frames).

Rayman (2007) tried to establish that the Hicksian concept of income is fundamentally flawed by what may be called the present value fallacy. Rayman (2007) contrasted the Hicksian and Fisherian conceptual models and concluded that a logical extension of Fisher's Theory suggests that agent/managers disclose the return on investment they are planning to deliver to their principal/owners, which will be helpful for the efficient operation of capital markets and for removing the accounting incentive to focus on short-term company financial results. However, Rayman's (2007) concurrence that, in an ideal capital market, market values should equal present values is misplaced because: (1) it is the divergences of opinion that create markets; (2) the dynamics of supply/demand often differ from the dynamics of true asset values and of present values—while markets attempt to reconcile these differences, the unity of market values and present values is an impractical, unfeasible, and perhaps unnecessary state that may provide the wrong incentives to market participants. These wrong incentives include: the incentive not to hedge; the incentive to use only equity financing; and the incentive to reduce/cease otherwise potentially useful and innovative search (such as R&D; pricing strategies; alliance/JV partners; etc.).

Kogut (1990) empirically studied search patterns of undergraduate students and found that individuals were making decisions based on the total return from searching, rather than the marginal return from another draw (from a known distribution of prices). Kogut's (1990) findings contradict standard economic theory, which predicts that individuals should continue to search until the expected gain of another search is less than the marginal search cost; and that once individuals have made a draw (or have selected an item from a known distribution) and then continue, they should never return to a selected price (or item) drawn earlier. These findings are critical and are proof that the NPV/IRR model is substantially flawed. The NPV/IRR model relies on cut-off points without regard to the following critical factors: (1) the psychological gains/losses from the search (the project); (2) the individual's/group's perception of the magnitude of the search costs (both the initial investment and intermittent investments); (3) the perceived value of sunk costs; (4) the Regret associated with sunk costs (which can affect project benefits—employee motivation; calculation of ROI; awards of employee incentives and performance bonuses; career concerns; the project termination decision; allocation of costs among departments; etc.).

Frey and Heggli (1989) found that traditional decision theory often does not account for business decisions like entering a business or a profession, or choosing the type of investment; and they analyzed a revised model of human behavior that provides a new explanation of business behavior by comparing the objective possibility set with the personal (ipsative) possibility set of decision-makers, both of which systematically deviate from each other. Frey and Heggli (1989) suggested that institutions such as research departments within firms and outside consulting firms may emerge to overcome the (ipsative) limitations of individual decision-making.

Smith (1996) argues that Regret influences the valuation of alternative outcomes when making treatment decisions in healthcare; and that valuation techniques rely on Expected Utility Theory (transitivity and independence), which causes misrepresentation of the respondents' true preferences over treatment alternatives, and thus results in the potential for "irrational" decisions. Smith (1996) developed a modified version of Regret Theory and provided the results of a tentative empirical analysis to illustrate the importance of accounting for Regret in the valuation of health states. The NPV-IRR model does not account for Regret.

Poole (2000) analyzed and identified the effects of human perception of time on financial decision-making. Hilton (2001) and Baker and Nofsinger (2002) also analyzed psychological factors inherent in financial decision-making. Almashat et al. (2008) analyzed framing effects inherent in patients' decision-making. All the factors noted in the foregoing articles are not reflected in the NPV-IRR model (and related approaches).

Bade (2009) and Padberg and Wilczak (1999) analyzed discount rates and interest rates. Walther (2010) studied anomalies in intertemporal choice—such anomalies (and those described in Chap. 3 in this book) are a basis for challenging the validity of concepts of interest rates. Fagley et al. (2010) analyzed framing effects within the context of decisions and choice. Magni (2002) critiqued NPV and described various anomalies and one framing effect inherent in NPV (this framing effect involves combining different cash flows from different time periods and discounting them uniformly or separately). Magni (2010) critiqued NPV and described various CAPM-related framing effects that are inherent in the NPV model, and noted that there is very little in the literature about the way a discount rate should be computed where there is uncertainty. Haley and Goldberg (1995) documented several short-term biases inherent in the NPV-MIRR model, and noted that the main weaknesses of NPV are that it erroneously assumes that: (1) any decisions made can be reversed without penalty; (2) different investments have equal or similar effects on a firm's ability to make investments; (3) the results of NPV analyses are objective and unbiased. Gollier (2010) found that, contrary to Weitzman (2007), when future interest rates are uncertain, using the expected net present value implies a term structure of discount rates that is decreasing to the smallest possible interest rate; while using the expected net future value criteria implies an increasing term structure of discount rates up to the largest possible interest rate. Gollier (2010) showed that if the aggregate consumption path is optimized and made flexible to news about future interest rates, risk aversion and utility maximization are equivalent. Almashat et al. (2008) and Ortendahl and Fries (2005) analyzed framing effects in medical and financial decisions. Hilton (2001) and Baker and Nofsinger (2002) analyzed the biases inherent in financial decisions. Robison et al. (2010) provided evidence that several important expected utility model paradoxes can be explained by accounting for socio-emotional goods (SEGs) embedded in word and symbolic frames. To account for the influence of SEGs under uncertainty, Robison et al.

(2010) introduced a social expected utility (SEU) model (a variation of the expected utility model) which was then used to resolve the following paradoxes: the Allais paradox, the insurance paradox, the Ellsberg paradox, coalescing and FSD paradox, SSD paradox, and the cash segregation paradox. In the process of resolving the cash segregation paradox, Robison et al. (2010) tried to show that apparent risk preference for losses is consistent with risk aversion once one accounts for the influence of SEGs. Again, most of the factors noted in the foregoing articles are not reflected in the NPV-IRR model (and related approaches).

6.4 The NPV-IRR Model (And Related Approaches Such as NFV, APV, AIRR; etc.) Does Not Account for Wllingness-To-Accept-Losses (WTAL)

Willingness To Accept Losses ("WTAL"; which was introduced in Nwogugu (2006a)) refers to: (i) the built-in and/or conditioned and/or learned tolerance (of an individual or group) for risk and losses; and (ii) the built-in and/or conditioned/learned reluctance (of an individual or group) to acknowledge and/or use unrealized, realized, or deferred losses. WTAL is different from: (i) *Loss Aversion* (a general aversion to losses where such aversion is often preferred to gains of equivalent amounts, e.g., it's better to not lose \$6,000 than to gain \$6,000); (ii) Risk Aversion (aversion to quantifiable risk within defined parameters, e.g., the utility value of a sure opportunity with a lower return is deemed higher than the utility value of a risky opportunity with a higher return); and (iii) *Regret Aversion* (a general aversion to Regret; e.g., the decision-maker (DM) will pay a "Regret premium" in order to avoid future Regret); and (iv) *ambiguity aversion* (the tendency to avoid making decisions when relevant information is missing or unclear). Nwogugu (2006b: 1740–1746) noted that: "…contrary to existing theory, in trading, losses most probably do not loom larger than corresponding gains in all situations … . At the group level, tolerance for risk/losses can be conditioned by groups dynamics, group think, group objectives, peer pressure, qualifications and affiliations of group members, member relations outside the group setting, institutional constraints, ability to hedge losses, substitution costs, cost of revising decisions, current and future budget constraints, time constraints, public policy considerations, socio-economic background of group members, environment in which the group operates (private vs. governmental, etc.), propensity to act irrationally in the face of excess information, etc…" (see the discussion on primary and higher-order risk attitudes in Deck and Schlesinger (2014); Noussair et al. (2013); and Bernard (1926, 1936)). WTAL is a more comprehensive and efficient measure of human behavior under more varied circumstances than Loss Aversion, Risk Aversion, and Regret Aversion as explained as follows:

(i) *Loss Aversion*: If it exists, Loss Aversion is conjectured to depend on, but does not have a constant relationship with, the DM's WTAL. Loss Aversion does not account for the DM's tax preferences (losses can be valuable and can be sold/hedged/transferred/deferred); arbitrage (losses can be used in arbitrage); noise; differences in knowledge; inertia; aspirations (which may crowd out any loss aversion); framing effects, etc. Loss Aversion (if it exists) can be significantly affected by framing Effects. WTAL is a primary behavioral trait while Loss Aversion is a higher-order risk attitude. Each of Erev et al. (2008); Ert and Erev (2013); Yechiam et al. (2014); Walasek and Stewart (2015); Harinck et al. (2007); Kermer et al. (2006); Gal (2006); and Nicolau (2012) didn't find any Loss Aversion in several experiments. Losses may affect attention but not the weighting of outcomes, as suggested by *Loss Attention* (Loss Attention is the idea that losses lead to more autonomic arousal than gains even in the absence of Loss Aversion; see Yechiam and Hochman (2013)). Researchers have noted that Loss Aversion appears to be magnitude-dependent such that for low magnitudes of losses, there is no Loss Aversion (see Mukherjee et al. (2017)). Researchers have noted that Loss Aversion is sometimes confused with the *Endowment Effect*.

(ii) *Risk Aversion*: Risk Aversion is conjectured to depend on, but does not have a constant relationship, with WTAL. Risk Aversion does not account for tax preferences (risk/volatility and associated losses can be valuable and can be sold/hedged/transferred/deferred); arbitrage; noise; aspirations; inertia; framing effects, etc. WTAL is a primary behavioral trait while Risk Aversion is a higher-order risk attitude. Christopoulos et al. (2009) and Knoch et al. (2006) have shown that Risk Aversion is a direct function of the magnitude and duration of stimulation of a specific area of the human brain—thus, if it exists, an individual's or group's Risk Aversion is likely to be context-dependent (and can vary drastically over time and in different circumstances). Stewart et al. (2014) critiqued both expected utility and discounted utility, which have been the most frequently used theories in the development of Risk Aversion models. Nwogugu (2005: 8–10) critiqued Risk Aversion and noted that: (1) risk may be in the form of non-monetary factors (while most studies of Risk Aversion are based on monetary factors), and there are situations in which risk can be defined in terms of gains (i.e. the greater the gains, the riskier the outcome); (2) the DM's preferences may remain constant when expected values and probabilities and decision weights change if the DM can transfer/sell/modify/defer risk and losses (which also drastically eliminates or moots Risk Aversion)—indeed in most developed financial markets, volatility, down-side risk, credit-risk, and interest rate risk can now be sold/modified/transferred/deferred; (3) Risk Aversion can be distorted by regulation or public policy or constraints on resources; (4) variance does not fully account for the nature/characteristics of risk and losses, and for any given level of variance, there is more than one possible probability distribution and risk profile; (5) probability does not fully account for the characteristics of risk; (6) some DMs are risk-seekers (and any traditional investor or trader has some inherent risk-seeking desire); (7) Prospect Theory and Cumulative Prospect Theory do not explain Risk Aversion because the value function suggests that there are different utility curves for different people depending on their income, the size of the benefit (and risk) at stake, and time factors (i.e., Risk Aversion appears to be magnitude-dependent such that for low magnitudes of risk or losses, there is no Risk Aversion; and for high magnitudes of risk or losses, there is disproportionately high Risk Aversion); (8) an investor considering two outcomes may choose the one that has higher expected losses and higher variance over a five-year horizon for several reasons (explained in the article); and (9) for any given decision, the existence of multiple or changing reference points can cause the disutility of a loss to be much less than the utility of a gain of equivalent magnitude (such that "classic" Risk Aversion is muted or negligible). Given that, as mentioned above, many researchers concluded that Loss Aversion doesn't exist, then true Risk Aversion is likely to be negligible or non-existent in humans. Within the realm of human feelings of risk, the more accurate and comprehensive measure is *Risk Demand*, which is a new measure. Risk Demand refers to an individual's or group's "total propensity" (not marginal propensity) to take risks. In this context, total propensity refers to the tendency to seek risk within the context of various considerations such as capital constraints, framing effects, the expected effect of the additional risk on portfolio wealth and portfolio characteristics, tax factors, etc. If Risk Aversion exists, an individual's Risk Demand is not inversely proportional to his/her Risk Aversion in all circumstances and the relationship depends on his/her tax position, framing effects, inertia, and other factors. An individual's Risk Demand is not directly proportional to his/her WTAL in all circumstances and the relationship can be time-varying and context-dependent, and depends on his/her tax-position, framing effects, inertia, and other factors. Risk Demand is very different from both WTAL and *Risk Appetite*. Risk Appetite is defined in the literature as the amount of capital and resources that the DMis willing to put at risk within a specific time frame. Risk Demand is an inherent mental state and is not measured only in terms of capital/resources at stake, but rather with respect to: (i) utility/disutility; (ii) economic and psychological impacts of changes in perception; (iii) ability to transfer/defer/sell risk; (iv) economic, social capital, and psychological impacts of disclosed/undisclosed expectations; (v) changes in the DM's wealth/conditions and third-party perceptions and valuations of such wealth/conditions; (vi) the magnitude of deviation from internal controls; (vii) multiplier effects of the decision; (viii) the value of time; and (ix) propensity and opportunities to substitute assets and resources.

(iii) *Regret Aversion*: Regret Aversion is conjectured: (1) to depend on, but does not have a constant relationship, with WTAL; (2) to be magnitude-dependent such that for low magnitudes of risk or losses, there is no Regret Aversion, and for high magnitudes of risk or losses, there is disproportionately high Regret Aversion; (3) to be directly proportional to both the opportunity cost of the decision and anticipated Regret, but the problem is that for many individuals and in non-professional settings and for low-risk/low-loss situations, ex-ante estimation of opportunity costs is likely to be crudely/inaccurately estimated such that Regret Aversion is distorted; (4) to be closely related to Loss Aversion—and if there isn't any Loss Aversion (as confirmed by the abovementioned researchers), it's likely that Regret Aversion is low or unpredictable. Van de Ven and Zeelenberg (2010) critiqued Regret Aversion. Seiler et al. (2008). WTAL is a primary behavioral trait while Regret Aversion is a higher-order risk attitude. Regret Aversion does not account for tax preferences (Regret and associated losses can be valuable and can be sold/hedged/transferred/deferred), arbitrage, noise, aspirations, inertia, framing effects, etc. *Status Quo Bias* (i.e., a tendency to choose the default option) and the *Endowment Effect* (i.e., possession of a good immediately increases its utility value) can be confused with Regret Aversion. For any given decision, the existence of multiple or changing reference points can distort the estimation of opportunity costs, such that "classic" Regret Aversion is muted or negligible.

WTAL and Regret Aversion contradict the NPV-IRR model (and related models). This is because of the nature of the NPV rule and the way IRR is used. For example, the NPV rule says DMs should accept only positive NPV projects or projects that have the highest NPVs, even though the DM may have high WTAL that would cause selection of low or negative NPV projects that subseqently become profitable. Similarly, a Regret-averse DM is likely to choose a low- or negative-NPV project/scenario, which has a high probability of occurrence, over a positive- or high-NPV project/scenario that has a low probability of occurrence simply to minimize his or her Regret. Sandri et al. (2010) found that the NPV model is not the typical human decision model.

WTAL affects intertemporal choice and the interest rate-setting processes of DMs and the following are applicable conjectures. WTAL affects the decision-maker’s perceptions of smaller-and-sooner-gains (SSG); larger-and-sooner-gains (LSG); smaller-and-later-gains (SLG); and larger-and-later-gains (LLG); smaller-and-sooner-losses (SSL), larger-and-sooner-losses (LSL), smaller-and-later-losses (SLL); larger-and-later-losses (LLL); and thus affects the magnitude and variation of his/her discount rates for various types of gains and losses. Part of the problem is that empirical research in intertemporal choice and discounting have not comprehensively covered most of these foregoing types of gains and losses in any one study. WTAL can affect the decision-maker’s explicit or implicit discount rates in situations where the discounting process is revealed or not revealed respectively, in the presentation of choices. WTAL will also affect the DM’s perceptions of risks inherent in the addition of any of the above mentioned types of losses (SSL, SLL, LSL, or LLL) to the DM’s current portfolio of choices/assets. In both real life and laboratory intertemporal choice situations, there are various dimensions such as monetary rewards, non-monetary rewards; type of payment; losses; time; opportunity costs that are not explicitly included in the framing of choices; implementability of choices; trust; etc.; and WTAL can affect each of these dimensions. Low-WTAL DMs will be more likely to defer consumption and leisure and housing expenses if future returns are more assured, and are more likely to prefer lower but more certain returns. Low-WTAL DMs are more likely to have a higher propensity for intertemporal substitution among housing, consumption, investment, and leisure than high-WTAL DMs.

Investors that have low WTAL will tend to focus on the certainty of returns rather than the magnitude of returns; will tend to overestimate interest rates and credit risks and to include “certainty cushions” in their estimates of interest rates; and will be less amendable to flat/inverted yield curves. Investors that have high WTAL will tend to focus on the magnitude of returns rather than the certainty of returns; will tend to prefer higher interest rates, and to underestimate interest rates; and will be more amendable to flat/inverted yield curves. Issuers (of securities and financial instruments) that have low WTAL will tend to underestimate interest rates and will be less amendable to flat/inverted yield curves.

6.5 The Use of WACC Further Distorts the NPV-MIRR-IRR (and Similar) Models and Can Create Additional Frames

The *weighted average cost of capital* ("WACC"; or the WACC plus an interest margin) is typically used as the discount rate in the NPV-IRR model (and related approaches). See comments and the criticisms of WACC in Lumby (September 1983); Lorenz et al. (2016); Magni (2009); Fernandez (2011); Cigola and Peccati (2005); Keef et al. (2012); and Hankins (2009). Magni (2008, 2009) noted that decision-makers that jointly use NPV and CAPM-based WACC can suffer from Framing Effects. Wong (2015) developed theories of capital structure within the context of Regret. Bade (2009), Cochrane (2011) and Padberg and Wilczak (1999) analyzed discount rates and interest rates. Bekaert et al. (2015) introduced a new method for calculating the cost-of-capital for international investments – but that method is associated and used with the NPV-IRR model. Smith and McCardle (1999) found that using WACC-based IRR and or NPV may cause errors when applied to projects that are significantly different from the firm as a whole (ie. different discount rates should be used for different projects, on the basis of their own cost of capital).

The firm's or the project's true operational risk (and true blended cost of capital) may be significantly different from the WACC in various circumstances, such as any of the following:

(a) The firm's (or the project-entity's) credit quality has not been updated by rating agencies—it takes between three and 30 months for some credit rating agencies to update changes in the risk profiles of some companies. In some instances (such as the cases of Lehman Brothers and AIG during 2006–2008), the credit rating agencies never updated the credit ratings until the firms suddenly declared bankruptcy due to financial distress.

(b) The firm or the project-entity either has substantial goodwill or a very bad reputation in the industry, which is reflected in its contracting terms with suppliers, but not in its loan covenants or credit ratings.
(c) The firm (or the project-entity) has off-balance sheet arrangements which are considered as debt by credit rating agencies; but are viewed differently by industry suppliers and some lenders.
(d) The firm (or the project-entity) has substantial intangible assets (like most firms in the industry), and thus, has substantial negative net tangible assets, which is viewed as a credit negative by rating agencies and some lenders, but not by industry suppliers.
(e) The firm (or the project-entity) offers sales terms to customers that are financially expensive and constrain its short-term liquidity, but such terms are not reflected in its loan covenants or credit ratings. Some examples are: (1) no late fees or interest payments for overdue accounts receivables; (2) the firm pays for shipping/storage costs; (3) the firm offers generous product return policies; and or generous product warranty terms.
(f) The firm offers cash incentives to its staff and or customers and or external sales people, which constrain its short-term liquidity, and such constraints are masked by the use of bank overdrafts or revolving loans, and are not reflected in its cost of debt or loan covenants.
(g) While the WACC is partly derived from the firm's income tax rate, that rate is typically based on its income statement, which includes many accounting distortions (e.g., non-cash charges, mis-allocation of costs; line-items that should have been capitalized; extraordinary items, etc.) and does not reflect its true cash position or its true operational risk.

In these circumstances, there is likely to be substantial divergences between lenders' and suppliers' perceptions of the firm's cost of capital—and both the WACC and the true operational risk-based cost of capital will provide different NPVs/IRRs for any set of cash flows. The WACC used in NPV-MIRR-IRR models (and related approaches) is assumed to remain constant; whereas the firm's true risk is dynamic over time.

Given the foregoing, the use of the WACC can create various frames that are based on, and vary depending on:

(1) Changes in the absolute magnitude of the WACC – when used as the discount rate in the NPV-IRR model, the WACC is non-additive;
(2) Changes in the components of short-term debt of the company or project – perceived differences among loans, bank-overdrafts, short-term guarantees, commercial paper and other short term instruments which may otherwise have equivalent value, can cause framing effects;
(3) Changes in the actual and perceived risk of leases, preferred stock, contingent debt and contingent equity securities/interests.

6.6 Conditions for Negative Discount Rates

As shown in Tables 1–11 (in Chart 5A in Chapter-5), negative discount rates (and associated biases) completely distort the usefulness of the NPV-IRR-MIRR model (and by extension, the usefulness of related models such as APV, NFV, etc.). Most decision-makers never use negative discount rates (in NPV, IRR and related models); rather, risk and uncertainty are measured solely in terms of the magnitude of positive discount rates and by adjustment of cash flows. Given the pervasive use of the NPV-IRR, its reasonable to conclude that decision-makers do not believe that a discount rate should be below zero and they are accustomed to positive discount rates because: (1) they perceive risk and or uncertainty as a cost that reduces, and never as a benefit or as a cost that improves (sometimes there is no distinction between a good or a bad risk); (2) people believe or are cognizant that there is inflation (which is typically perceived to be greater than zero—deflation is not popular, and is often not a major economic concern); (3) they are trained to reason this way—most finance/valuation/appraisal courses focus on constructing positive discount and capitalization rates by adding various positive discount rates. This tendency to believe that a discount rate can never be less than zero is henceforth referred to as *non-zero discount rate bias* (NDRB). See the comments in Hankins (2009). However, the negative benchmark interest rate environment that prevailed in some countries (such as Switzerland, Sweden and Japan) during 2014-2016 shows that under some conditions, discount rates can indeed be negative. See comments in Brainard (2015); Evans et al. (2015); Thornton (June 2015); Engen et al. (2015); Palley (June 2016); Loewenstein and Prelec (1991); Gust et al. (2015); Guth et al. (2009); and Gomme et al. (2015). It appears that negative interest rates (as implemented by several central banks during 2014–2017) were an alternative to relatively expensive monetary policies such as quantitative easing (which can result in significant and unjustifiable wealth transfers and unexpected effects).

However, a negative discount rate may be justified if any of the following conditions exist:

(i) There is rampant deflation, and the deflation rate exceeds government benchmark interest rates (e.g., Fed Funds rate in the US). Significant deflation occurred in US real estate markets during 2007–2010, wherein property values declined by more than 20% annually in some states, and some operating expenses also declined.
(ii) The risks of the project are extremely low such that project benefits far outweigh costs. By investing in the project, the company is paid periodic benefits that exceed periodic costs.
(iii) Forward-rate curves are steeply inverted, and remain so for several years.
(iv) The project outcomes/cashflows in each period are riskless, and risk eliminating benefits. After the initial investment, each periodic project outcomes/cashflows eliminates firm-risk and/or project-risk in that and subsequent periods. The risk-eliminating feature then requires elimination of the traditional components of the discount rate—such as the government bond rate, inflation, and so on.
(v) The project sponsor has completely hedged the project (via contracts with third parties), such that each periodic project outcomes/cashflows is not only riskless, but generates negative risk. The concept of negative risk is introduced here, and refers to situations where a project, event, item, or process generates risk-eliminating benefits, which can be in the form of money, tax benefits, government credits, derivatives hedge, grants, and other types of monetary or non-monetary

benefits. For example, a new project involves building 100 new bus stops in each period. Each bus stop costs $400,000 but will generate tax benefits of $600,000 in the years of construction and $100,000 of tax benefits in the two subsequent years for the contractor. The traditional way to model such benefits has been to include the benefits as part of the cash flow projections, which is inaccurate because the benefits are not subject to any implied reinvestment rate inherent in NPV. The alternative, and perhaps more accurate, method is to reduce the discount rate below zero.

(vi) The applicable growth rate of the periodic project outcomes/cash-flows far exceeds the Weighted-Average-Cost-of-Capital (WACC) or discount rate. For example, where the discount rate is 7% and the periodic growth rate is 10%, then the discount rate is −3% or lower.

(vii) The WACC for the firm or project is the lower of 0% or minus the percentage benefits. A hypothetical firm, whose only assets are government bonds, plus third party AAA-rated insurance of such bonds, plus tax benefits, can have a negative WACC.

(viii) Due to tax laws, lending to the firm generates substantial tax benefits for lenders and for the firm, such that lenders do not charge the firm any interest rates, and the firm's low cost of equity is much less than the tax benefit generated by the firm from borrowing. In such circumstances, the firm's WACC should be negative.

(ix) Significant uncertainty can cause investors/savers to accept negative interest rates for fixed income securities and bank accounts.

Central banks may reduce benchmark interest rates below zero (as in the case of the central banks of Switzerland, Sweden, Denmark and Japan and the European Central Bank during 2014–2016 - Wigglesworth, Lewis & McCrum (February 17, 2016)) if any of the following conditions occur or are likely:

(i) There is deflation and the absolute amount of the benchmark government interest rate is less than the deflation rate.

(ii) The absolute amount of the actual, or perceived, or expected rate of currency devaluation over the medium term (12–36 months) exceeds the average projected aggregate returns on individual and corporate investments.

(iii) The central bank believes that negative interest rates will cause banks to reduce lending interest rates which in turn, will spur economic activity - however, the opposite occurred in Japan during 2013–2016.

The foregoing implies that:

(i) The regimes for interest rates for savings, borrowing, and leasing can be different.
The regimes include amount; timing; relationship to other assets; and so on.
(ii) Taxation should have different effects on interest rates for savings, borrowing, and leasing.
(iii) Transaction costs and monitoring costs should have different effects on interest rates for savings, borrowing, and leasing.

6.7 Conclusion

Clearly, the concepts of interest rates, discount rates and forward rates can be misleading (depending on the context). Also, the treatment of interest rates in the NPV-MIRR model (and related approaches such as APV; AIRR; MIRR; net future value; SVA; EVA) is at best misleading, and can also be distorted by human biases and framing effects.

Bibliography

Aho, T., & Virtanen, I. (1983). On relationships between ROI and IRR under inflation: A constant real cash-flow case. *European Journal of Operational Research, 13*(3), 256–267.

Allais, M. (1974). The psychological rate of interest. *Journal of Money, Credit and Banking, 6*(3), 285–331.

Almashat, S., Ayotte, B., Edelstein, B., & Margrett, J. (2008). Framing effect de-biasing in medical decision making. *Patient Education and Counseling, 71*, 102–107.

Andersen, T. M. (1998). Persistence in sticky price models. *European Economic Review, 42*, 593–603.

Archibald, J., & Hunter, L. (2001). What is the neutral real interest rate, and how can we use it? *Reserve Bank of New Zealand: Bulletin, 64*(3), 15–19. http://www.rbnz.govt.nz/-/media/ReserveBank/Files/Publications/Bulletins/2001/2001sep64-3ArchibaldHunter.pdf

Aspromorgous, T. (2000). Can (and should) monetary policy pursue a zero real interest rate, permanently? *Metroeconomica, 62*(4), 635–655.

Attema, A., Bleichrodt, H., Rohde, K., & Wakker, P. P. (2010). Time-tradeoff sequences for analyzing discounting and time inconsistency. *Management Science, 56*(11), 2015–2030.

Ausubel, L. (1991). The failure of competition in the credit card market. *The American Economic Review, 81*, 50–81.

Aven, T., & Flage, R. (2009). Use of decision criteria based on expected values to support decision-making in a production assurance and safety setting. *Reliability Engineering and System Safety, 94*, 1491–1498.

Bade, B. (2009). Comment on "The weighted average cost of capital is not quite right". *The Quarterly Review of Economics and Finance, 49*(4), 1476–1480.

Baker, H., & Nofsinger, J. (2002). Psychological biases of investors. *Financial Services Review, 11*(2), 97–116.

Barry, P., & Robison, L. (2014). Technical note: Economic rates of return and investment analysis. *Engineering Economist, 59*(3), 231–236.

Barry, P., Robison, L., & Nartea, G. (1996). Changing time attitudes in intertemporal analysis. *American Journal of Agricultural Economics, 78*(4), 972–981.

Bas, E. (2013). A robust approach to the decision rules of NPV and IRR for simple projects. *Applied Mathematics and Computation, 219*(9), 5901–5908.

Beck, T., Colciago, A., & Pfajfar, D. (2014). The role of financial intermediaries in monetary policy transmission. *Journal of Economic Dynamics & Control, 43*, 1–11.

Bekaert, G., Harvey, C., Lundblad, C., & Siegel, S. (2015). *Political risk and international valuation*. (Working paper). Columbia University.

Berkovitch, E., & Isreal, R. (2004). Why the NPV criterion does not maximize NPV. *Review of Financial Studies, 17*(1), 239–255.

Bernard, L. (1926). Primary and derivative attitudes and ideals. In *An introduction to social psychology* (Chap. 27, pp. 425–437). New York: Henry Holt and Co. Available at: https://www.brocku.ca/MeadProject/Bernard/1926/1926_27.html

Bernard, L. (1936). The conflict between primary group attitudes and derivative group ideals in modern society. *American Journal of Sociology, 41*(5), 611–623.

Berns, G. S., Capra, C. M., Moore, S. & Noussair, C. (2008). Three studies on the neuroeconomics of decision-making when payoffs are real and negative. In Grossman, M., Lindgren, B., Kaestner, R., & __.Kristian, B. (Eds.), *Neuroeconomics* (Advances in Health Economics and Health Services Research, Vol. 20, pp. 1–29). Bingley: Emerald Group Publishing Limited, UK.

Bindseil, U., Domnick, C., & Zeuner, J. (2015). *Critique of accommodating central bank policies and the 'expropriation of the saver': A review* (European Central Bank, Working paper #161). https://www.ecb.europa.eu/pub/pdf/scpops/ecbop161.en.pdf

Blume, M. (1974). Unbiased estimators of long-run expected rates of return. *Journal of the American Statistical Association, 69*(347), 634–638.

Blume, M. E., & Stambaugh, R. F. (1983). Biases in computed returns: An application to the size effect. *Journal of Financial Economics, 12*, 387–404.

Booth, L. (2003). Discounting expected values with parameter uncertainty. *Journal of Corporate Finance, 9*, 505–519.

Borgonovo, E., & Percocao, M. (2012). A note on the sensitivity analysis of the internal rate of return. *International Journal of Production Economics, 135*(1), 526–529.

Brainard, L. (2015, December). *Normalizing monetary policy when the neutral interest rate is low*. Speech given at the Stanford Institute for Economic Policy Research, California, Stanford. https://www.federalreserve.gov/newsevents/speech/brainard20151201a.htm

Branch, B., & Echevarria, D. P. (1998). The bid-ask bias and the size effect: A test of the Blume-Stambaugh bid-ask bias effect hypothesis. *Quarterly Review of Economics & Finance, 38*(1), 129–148.

Breedon, F., & Turner, P. (2016). *On the transactions costs of quantitative easing*. (BIS Working papers no 571). Bank For International Settlements. http://www.bis.org/publ/work571.pdf

Brocas, I., & Carrillo, J. D. (2005). A theory of haste. *Journal of Economic Behavior & Organization, 56*(1), 1–23.

Califf, R. M., Rasiel, E. B., & Schulman, K. A. (2008). Considerations of net present value in policy making regarding diagnostic and therapeutic technologies. *American Heart Journal, 156*(5), 879–885.

Campello, M. (2002). Internal capital markets in financial conglomerates: Evidence from small bank responses to monetary policy. *The Journal of Finance, 57*(6), 2773–2805.

Carbo-Valverde, S., Hannan, T., & Rodriguez-Fernandez, F. (2011). Exploiting old customers and attracting new ones: The case of bank deposit pricing. *European Economic Review, 55*, 903–915.

Carpenter, S., & Demiralp, S. (2011). *Volatility, money market rates, and the transmission of monetary policy*. Finance and Economics Discussion Series 2011–22, Board of Governors of the Federal Reserve System (U.S.).

Carpenter S., Demiralp, S. & Senyuz, Z. (2012). *Money market dynamics and the role of federal funds rate volatility during the 2007–2009 financial crisis*. Un-published manuscript, US Federal Reserve Board.

Cheng, P., & Chiou, W. (2008). Framing effects in group investment decision making: Role of group polarization. *Psychology Reports, 102*(1), 283–292.

Cheng, P., & Deets, M. K. (1971). Statistical biases and security rates of return. *The Journal of Financial and Quantitative Analysis, 6*(3), 977–994.

Chituc, C.-M. (2011). Towards assessing performance in service computing. *Lecture Notes in Computer Science* (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics).

Christopoulos, G., Tobler, P., Bossaerts, P., et. al. (2009). *Neural correlates of value, risk, and risk aversion contributing to decision making under risk. Journal Of Neuroscience, 26*(24), 6469–6472.

Cigola, M., & Peccati, L. (2005). On the comparison between the APV and the NPV computed via the WACC. *European Journal of Operational Research, 161*(2), 377–385.

Cochrane, J. (2011). Presidential address: Discount rates. *The Journal of Finance, 66*(4), 1047–1108.

Croson, R. (1996). Information in ultimatum games: An experimental study. *Journal of Economic Behavior & Organisation, 30*, 197–212.

Czaja, M., Scholz, H., & Wilkens, M. (2009). Interest rate risk of German financial institutions: The impact of level, slope, and curvature of the term structure. *Review of Quantitative Finance and Accounting, 33*, 1–26.

Das, S. (2015). *Monetary policy in India: Transmission to bank interest rates* (IMF Working paper WP/15/129). Washington DC: IMF. https://www.imf.org/external/pubs/ft/wp/2015/wp15129.pdf

De La Bruslerie, H. (2015). Decreasing term structure of psychological discount rates: Experimental estimation and determinants. *Journal of Behavioral and Experimental Finance, 6*, 13–26.

De Reyck, B., Degraeve, Z., & Vandenborre, R. (2008). Project options valuation with net present value and decision tree analysis. *European Journal of Operational Research, 184*, 341–355.

Deck, C., & Schlesinger, H. (2014). Consistency of higher order risk preferences. *Econometrica, 82*(5), 1913–1943.

Dixit, A.K., & Pindyck, R. S. (1995, May–June). The options approach to capital investment. *Harvard Business Review*, *73*, 105–115

Doraszelski, U. (2001). The net present value method versus the option value of waiting: A note on Farzin, Huisman and Kort (1998). *Journal of Economic Dynamics & Control, 25*, 1109–1115.

Dorfleitner, G. (2003). Why the return notion matters. *International Journal of Theoretical & Applied Finance, 6*, 73–86.

Duarte, B. P. M., & Reis, A. (2006). Developing a projects evaluation system based on multiple attribute value theory. *Computers & Operations Research, 33*(5), 1488–1504.

Dubra, J. (2009). A theory of time preferences over risky outcomes. *Journal of Mathematical Economics, 45*(9–10), 576–588.

Dybvig, P. H., & Ingersoll, J. (1982). Mean-variance theory in complete markets. *Journal of Business, 55*(2), 233–251.

Ebrahim, M., Shackleton, M., & Wojakowski, R. (2009). Participating mortgages and the efficiency of financial intermediation. *Journal of Banking & Finance, 35*(11), 3042–3054.

Ehrhadrt, M., & Wachowicz, J. M. (2006, Summer). Capital budgeting and Initial Cash Outlay (ICO) uncertainty. *Financial Decisions, 18*(1), 1–16.

Engen, E., Laubach, T., & Reifschneider, D. (2015). *The macroeconomic effects of the federal reserve's unconventional monetary policies*. Finance and Economics Discussion Series 2015-005. Board of Governors of the Federal Reserve System, Washington, DC.

Erev, I., Ert, E., & Yechiam, E. (2008). Loss aversion, diminishing sensitivity, and the effect of experience on repeated decisions. *Journal of Behavioral Decision Making, 21*, 575–597.

Ert, E., & Erev, I. (2013). On the descriptive value of loss aversion in decisions under risk: Six clarifications. *Judgment & Decision Making, 8*, 214–235.

Evans, C., Fisher, J., Gourio, F. & Krane, S. (2015). Risk management for monetary policy near the zero lower bound. *Brookings Papers on Economic Activity*, Spring.

Fama, E., & Bliss, R. (1987). The information in long-maturity forward rates. *The American Economic Review, 77*(4), 680–692.

Farazmand, H., Ansari, M., & Moradi, M. (2016). What determines money demand?: Evidence from MENA. *Economic Review, 45*, 151–157. https://www.euba.sk/department-for-research-and-doctoral-studies/economic-review/volume-44,-2015/preview-file/er2_2016_farazmand_ansari_moradi-23508.pdf

Fernandez, P. (2011). *WACC: Definition, misconceptions and errors* (Working paper #WP-914). IESE Business School. http://www.iese.edu/research/pdfs/DI-0914-E.pdf

Fisher, I. (1930). *The theory of interest*. New York: Macmillan.

Fisher, L., Weaver, D., & Webb, G. (2010). Removing biases in computed returns. *Review of Quantitative Finance and Accounting, 21*, 137–151.
Forstater, M., & Mosler, W. (2005). The natural rate of interest is zero. *Journal of Economic Issues, 39*(2), 535–542.
Frederick, S. (2006). Valuing future life and futures lives: A framework for understanding discounting. *Journal of Economic Psychology, 27*, 667–680.
Frederick, S., Loewenstein, G., & O'Donoghue, T. (2002). Time discounting and time preference: A critical review. *Journal of Economic Literature, 40*(1), 351–401.
Frey, B., & Heggli, B. (1989). An ipsative theory of business behavior. *Journal of Economic Psychology, 10*(1), 1–20.
Friedman, M. (1966). Interest rates and the demand for money. *The Journal of Law & Economics, 9*, 71–85.
Fullwiler, S. (2006). Setting interest rates in the modern money era. *Journal of Post Keynesian Economics, 28*, 495–505.
Gal, D. (2006). A psychological law of inertia and the illusion of loss aversion. *Judgment & Decision Making, 1*, 23–32.
Garner, A. (1986). *Does interest rate volatility affect money demand*? (Working paper). Federal Reserve Bank of Kansas. https://www.kansascityfed.org/~/media/files/publicat/econrev/econrevarchive/1986/1q86garn.pdf
Gigerenzer, G., & Regier, T. (1996). How do we tell an association from a rule? Comment on Sloman. *Psychological Bulletin, 119*, 23–26.
Goldreich, D. (2007). Under-pricing in discriminatory and uniform-price treasury auctions. *Journal of Financial and Quantitative Analysis, 42*(2), 443–466.
Gollier, C. (2010). Expected net present value, expected net future value, and the Ramsey rule. *Journal of Environmental Economics and Management, 59*, 142–148.
Gomme, P., Ravikumar, B., & Rupert, P. (2015). Secular stagnation and returns on capital. *Federal Reserve Bank of St. Louis – Economic Synopses* (USA), no. 19, August 18.
Gonzalez, M., Jareno, F., & Skinner, F. (2016). Interest and inflation risk: Investor behavior. *Frontiers in Psychology*. https://doi.org/10.3389/fpsyg.2016.00390; http://journal.frontiersin.org/article/10.3389/fpsyg.2016.00390/full
Graham, J., & Harvey, C. (2001). The theory and practice of corporate finance: Evidence from the field. *Journal of Financial Economics, 60*, 187–243.
Groenendaal, W. (1998). Estimating NPV variability for deterministic models. *European Journal of Operational Research, 107*, 202–213.

Grosskopf, B., & Roth, A. E. (2009). If you are offered the right of first refusal, should you accept? An investigation of contract design. *Games and Economic Behavior, 65*, 176–204.

Gureckis, T., & Love, B. (2009). Learning in noise: Dynamic decision-making in a variable environment. *Journal of Mathematical Psychology, 53*(3), 180–193.

Gust, C., Johannsen, B., & López-Salido, D. (2015). *Monetary policy, incomplete information, and the zero lower bound*. Finance and Economics Discussion Series 2015-099. Board of Governors of the Federal Reserve System, Washington, DC.

Guth, W., Levati, V., & Ploner, M. (2009). An experimental analysis of satisficing in saving decisions. *Journal of Mathematical Psychology, 53*(4), 265–272.

Haley, G., & Goldberg, S. (1995). Net present value techniques and their effects on new product research. *Industrial Marketing Management, 24*, 177–190.

Hall, P., Edlin, R., Kharroubi, S., et al. (2012). Expected net present value of sample information: From burden to investment. *Medical Decision Making, 32*, E11–E21.

Hamburger, M., & Platt, E. (1975). The expectations hypothesis and the efficiency of the treasury bill market. *Review of Economics and Statistics, 57*, 190–199.

Hanafizadeh, P., & Vahideh, L. (2011). Robust net present value. *Mathematical and Computer Modelling, 54*(1–2), 233–242.

Hankins, K. W. (2009). How do financial firms manage risk? Unraveling the interaction of financial and operational hedging. *Management Science, 57*(12), 2197–2212.

Harinck, F., Van Dijk, E., Van Beest, I., & Mersmann, P. (2007). When gains loom larger than losses: Reversed loss aversion for small amounts of money. *Psychological Science, 18*(12), 1099–1105.

Haven, E. & Khrennikov, A. (2016). The use of action functionals within the quantum-like paradigm. *Journal of Mathematical Psychology*, In Press.

Havranek, T., Horvath, R., Irsova, Z., & Rusnak, M. (2015). Cross-country heterogeneity in intertemporal substitution. *Journal of International Economics, 96*(1), 100–118.

Hazen, G. B. (2003). A new perspective on multiple internal rates of return. *The Engineering Economist, 48*(1), 31–52.

Hetzel, R. (1986). *A critique of theories of money stock determination* (Working paper 86-6). Federal Reserve Bank of Richmond. https://www.richmondfed.org/-/media/richmondfedorg/publications/research/working_papers/1986/pdf/wp86-6.pdf

Hilton, D. (2001). The psychology of financial decision making: Applications to trading, dealing and investment analysis. *Journal of Psychology & Financial Markets, 2*(1), 37–53.

Indro, D., & Lee, W. Y. (1997). Biases in arithmetic and geometric averages as estimates of long-run expected returns and risk premia. *Financial Management, 26*(4), 81–90.

Jacquier, E., Kane, A., & Marcus, A. (2003). Geometric or arithmetic mean: A re-consideration. *Financial Analysts Journal, 59*(6), 46–53.

Jacquier, E., Kane, A., & Marcus, A. (2005). Optimal estimation of the risk premium for the long run and asset allocation: A case of compounded estimation risk. *Journal of Financial Econometrics, 3*, 37–55.

Jagannathan, R., Matsa, D., Meier, I., & Tarhan, V. (2013, August). *Why do firms use high discount rates?* (Working Paper). Kellogg School of Management at Northwestern University, Illinois. https://www.bus.miami.edu/_assets/files/faculty-and-research/conferences-and-seminars/finance-seminars/Jagannathan%20Paper.pdf. Accessed 31 Jan 2017.

Jean, W., & Helms, W. P. (1983). Geometric mean approximations. *Journal of Financial and Quantitative Analysis, 18*, 287–293.

Jonkman, S. N., van Gelder, P., & Vrijling, J. K. (2003). An overview of quantitative risk measures for loss of life and economic damage. *Journal of Hazardous Materials, 99*(1), 1–30.

Kable, J. W. (2013). Valuation, intertemporal choice and self-control. In P. Glimcher & E. Fehr (Eds.), *Neuroeconomics* (2nd ed., pp. 173–192). New York: Academic Press.

Kaplan, S., & Garrick, B. J. (1981). On the quantitative definition of risk. *Risk Analysis, 1*, 11–27.

Keef, S., Khaled, M., & Roush, M. (2012). A note resolving the debate on "The weighted average cost of capital is not quite right". *The Quarterly Review of Economics and Finance, 52*(4), 438–442.

Keim, D. (1989). Trading patterns, bid-ask spreads, and estimated security returns: The case of common stocks at calendar turning points. *Journal of Financial Economics, 25*(1), 75–97.

Kermer, D., Driver-Linn, E., Wilson, T., & Gilbert, D. (2006). Loss aversion is an affective forecasting error. *Psychological Science, 17*(8), 649–653.

Khwaja, A., Silverman, D., & Sloan, F. (2007). Time preference, time discounting, and smoking decisions. *Journal of Health Economics, 26*(5), 927–949.

Kierfulff, H. (2008). MIRR: A better measure. *Business Horizons, 51*, 321–329.

King, R., & Lin, M. (2005). Re-examining the monetarist critique of interest rate rules. *Federal Reserve Bank of St. Louis Review, 87*(4), 513–530. http://

citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.581.4837&rep=rep1&type=pdf

Kitamura, T., Muto, I., & Takei, I. (2015). *How do Japanese banks set loan interest rates?: Estimating pass-through using bank-level data* (Working paper series). Bank of Japan. https://www.boj.or.jp/en/research/wps_rev/wps_2015/data/wp15e06.pdf

Klauss, C. P. (2006). *Capital investment decisions with managerial overconfidence and regret aversion.* Phd thesis. University of Bath, England. http://opus.bath.ac.uk/18842/1/UnivBath_PhD_2006_Klauss_C.pdf

Knetsch, J. L., Riyanto, J., & Zong, J. (2012). Gain and loss domains and the choice of welfare measure of positive and negative changes. *Journal of Benefit-Cost Analysis, 3*. Article 1.

Knoch, D., Gianotti, L., Pascual-Leone, A., et. al. (2006). *Disruption of right prefrontal cortex by low-frequency repetitive transcranial magnetic stimulation induces risk-taking behavior. Journal of Neuroscience, 26*(24), 6469–6472.

Kogut, C. A. (1990). Consumer search behavior and sunk costs. *Journal of Economic Behavior & Organization, 14*(3), 381–392.

Komlos, J. (July 2014). Sticky prices and behavioural indifference curves. VOX Portal. http://voxeu.org/article/sticky-prices-and-behavioural-indifference-curves

Kruger, P., Landier, A., & Thesmar, D. (2015). The WACC fallacy: The real effects of using a unique discount rate. *Journal of Finance, 70*(30), 1253–1285.

Leland, J. (2010). The hunt for a descriptive theory of choice under risk—A view from the road not taken. *The Journal of Socio-Economics, 39*, 568–577.

Leyman, P., & Vanhoucke, M. (2016). Payment models and net present value optimization for resource-constrained project scheduling. *Computers & Industrial Engineering, 91*, 139–153.

Lindner, F. (2015). Does saving increase the supply of credit? A critique of the loanable funds theory. *World Economic Review, 4*, 1–26.

Liu, W., & Strong, N. (2008). Biases in decomposing holding-period portfolio returns. *The Review of Financial Studies, 21*(5), 2243–2274.

Liu, L., Rettenmaier, A. J., & Saving, T. (2004). Present value criterion: The case of differing borrowing and lending rates. *Economics Letters, 84*, 427–432.

Loewenstein, G. (1992). The fall and rise of psychological explanations in the economics of intertemporal choice. In G. Loewenstein & J. Elster (Eds.), *Choice over time*. New York: Russell Sage Foundation.

Loewenstein, G., & Prelec, D. (1991). Negative time preferences. *American Economic Review, 81*(2), 347–352.

Loewenstein, G., & Prelec, D. (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *Quarterly Journal of Economics, 57*, 573–598.

Lohmann, J. R. (1988). The IRR, NPV and the fallacy of the reinvestment rate assumption. *The Engineering Economist, 33*(4), 303–330.

Lorenz, D., Kruschwitz, L., & Löffler, A. (2016). Are costs of capital necessarily constant over time and across states of nature? Some remarks on the debate on 'WACC is not quite right'. *The Quarterly Review of Economics and Finance, 60*, 81–85.

Lumby, S. (1983, September). The case against WACC in investment appraisal. *Accountancy, 84*(1081), 116–118.

Magni, C. (2002). Investment decisions in the theory of finance: Some antinomies and inconsistencies. *European Journal of Operations Research, 137*, 202–216.

Magni, C. A. (2003). Decomposition of net final values: Systemic value added and residual income. *Bulletin of Economic Research, 55*(2), 149–176.

Magni, C. A. (2004). Modelling excess profit. *Economic Modelling, 21*, 595–617.

Magni, C. (2005a). *Theoretical flaws in the use of CAPM for investment decisions* (Working paper). University of Modena and Reggio Emilia. http://mpra.ub.uni-muenchen.de/6330/1/MPRA_paper_6330.pdf

Magni, C. A. (2005b). On decomposing net final values: EVA, SVA and shadow project. *Theory & Decision, 59*(1), 51–95.

Magni, C. A. (2007a). Project selection and equivalent CAPM-based investment criteria. *Applied Financial Economics Letters, 3*(3), 165–168.

Magni, C. A. (2007b). Correct or incorrect application of CAPM? Correct or incorrect decisions with CAPM? *European Journal of Operational Research, 192*(2), 549–560.

Magni, C. A. (2007c). Project valuation and investment decisions: CAPM versus arbitrage. *Applied Financial Economics Letters, 3*(1), 137–140.

Magni, C. A. (2008). Economic profit, NPV, and CAPM: Biases and violations of Modigliani and Miller's proposition I. *The ICFAI Journal of Applied Finance, 14*(10), 59–72.

Magni, C. A. (2008a). Economic profit, NPV, and CAPM: Biases and violations of Modigliani and Miller's Proposition I. *The ICFAI Journal of Applied Finance, 14*(10), 59–72.

Magni, C. A. (2008b). CAPM-based capital budgeting and non additivity. *Journal of Property Investment and Finance, 26*(5), 388–398.

Magni, C. (2009). Correct or incorrect application of CAPM? Correct or incorrect decisions with CAPM? *European Journal of Operational Research, 192*(2), 549–560.

Magni, C. (2009a). Splitting up value: A critical review of residual income theories. *European Journal of Operational Research, 198*, 1–22.

Magni, C. (2009b). Correct or incorrect application of CAPM? Correct or incorrect decisions with CAPM? *European Journal of Operational Research, 192*(2), 549–560.

Magni, C. A. (2009c). Investment decisions, NPV and bounded rationality. *Quantitative Finance, 9*(8), 967–979.

Magni, C. (2010). CAPM and capital budgeting: Present/future, equilibrium/disequilibrium, decision/valuation. *IUP Journal of Financial Economics, 8*(1/2), 7–35.

Magni, C. A. (2011). Addendum to "average internal rate of return and investment decisions: A new perspective". *The Engineering Economist, 56*(2), 140–169.

Magni, C. A. (2013). The internal-rate-of-return approach and the AIRR paradigm: A refutation and a corroboration. *The Engineering Economist, 58*(2), 73–111.

Magni, C. (2015). Aggregate return on investment for investments under uncertainty. *International Journal of Production Economics, 165*, 29–37.

Malvey, P., Archibald, C., & Flynn, S. (2014). *Uniform-price auctions: Evaluation of the treasury experience*. Office of Market Finance, U.S. Treasury, Washington, DC. http://www.treasury.gov/resource-center/fin-mkts/Documents/final.pdf

Marcatto, F., Cosulich, A. & Ferrante, D. (2015). Once bitten, twice shy: Experienced regret and non-adaptive choice switching. *PeerJ, 3*, e1035. https://doi.org/10.7717/peerj.1035

Martellini, L., & Urosevic, B. (2006). Static mean-variance analysis with uncertain time horizon. *Management Science, 52*(6), 955–964.

Masatlioglu, Y., & Ok, E. (2009). A theory of (relative) discounting. *Journal of Economic Theory*.

McCabe, C., Edlin, R., & Hall, P. (2013). Navigating time and uncertainty in health technology appraisal: Would a map help? *PharmacoEconomics, 31*(9), 731–737.

Mellichamp, D. (2013). New discounted cash flow method: Estimating plant profitability at the conceptual design level while compensating for business risk/uncertainty. *Computers & Chemical Engineering, 48*(10), 251–263.

Memmel, C. (2011). Banks' exposure to interest rate risk, their earnings from term transformation, and the dynamics of the term structure. *Journal of Banking & Finance, 35*, 282–289.

Milch, K., Weber, E., et al. (2009). From individual preference construction to group decisions: Framing effects and group processes. *Organizational Behavior and Human Decision Processes, 108*(2), 242–255.

Mishra, P., & Montiel, P. (2013). How effective is monetary transmission in developing countries? A survey of the empirical evidence. *Economic Systems, 37*(2), 187–216.

Mohanty, M., & Rishabh, K. (2016). *Financial intermediation and monetary policy transmission in EMEs: What has changed post-2008 crisis?* (BIS Working papers #546). Bank for International Settlements. http://www.bis.org/publ/work546.pdf

Mukherjee, S., Sahay, A., Pammi, V., & Srinivasan, N. (2017). Is loss-aversion magnitude-dependent? Measuring prospective affective judgments regarding gains and losses. *Judgment and Decision Making, 12*(1), 81–89.

Naim, M. (2006). The impact of the net present value on the assessment of the dynamic performance of e-commerce enabled supply chains. *International Journal Of Production Economics, 104*, 382–393.

Nicolau, J. (2012). Asymmetric tourist response to price: Loss aversion segmentation. *Journal of Travel Research, 51*(5).

Noussair, C. N., Trautmann, S., & Kuilen, G. (2013). Higher order risk attitudes, demographics and saving. *Review of Economic Studies, 81*(1), 325–355.

Nwogugu, M. (2005). Towards multi-factor models of decision making and risk: A critique of Prospect Theory and related approaches, part-I. *Journal of Risk Finance, 6*(2), 150–162.

Nwogugu, M. (2006). Regret minimization, willingness-to-accept-losses and framing. *Applied Mathematics & Computation, 179*(2), 440–450.

Nwogugu, M. (2006a). Regret minimization, willingness-to-accept-losses and framing. *Applied Mathematics & Computation, 179*(2), 440–450.

Nwogugu, M. (2006b). Further critique of GARCH/ARMA/VAR/EVT/Stochastic-Volatility models and related approaches. *Applied Mathematics and Computation, 182*, 1735–1748.

Nwogugu, M. (2010). *On anomalies in the polynomials and net present value*. Available at www.ssrn.com

Nwogugu, M. (2012). *Risk in the global real estate market.* Hoboken: John Wiley & Sons.

Oehmke, J. F. (2000). Anomalies in net present value calculations. *Economics Letters, 67*, 349–351.

Oehmke, M., & Zawadowski, A. (2015). Synthetic or real? The equilibrium effects of credit default swaps on bond markets. *Review of Financial Studies, 28*(12), 3303–3337.

Oehmke, M., & Zawadowski, A. (2016). The anatomy of the CDS market. *Review of Financial Studies,* Forthcoming.

Office of the President of the United States. (2015, July). *Long-term interest rates: A survey*. https://www.whitehouse.gov/sites/default/files/docs/interest_rate_report_final_v2.pdf

Ortendahl, M., & Fries, J. F. (2005). Framing health messages based on anomalies in time. *Medical Science Monitor, 11*(8), 253–256.

Osborne, M. (2010). A resolution to the NPV–IRR debate? *The Quarterly Review of Economics and Finance, 50*, 234–239.

Osborne, M. J. (2014). *Multiple interest rate analysis: Theory and applications*. Basingstoke/New York: Palgrave MacMillan.

Ouattara, A., & De La Bruslerie, H. (2015). *The term structure of psychological discount rate: Characteristics and functional forms. Working paper.* Centre Africain d'Etudes Superieures en Gestion, Senegal; and Universite Paris Dauphine, Paris, France. MPRA paper no. 75111, MPRA. https://mpra.ub.uni-muenchen.de/75111/1/MPRA_paper_75111.pdf

Padberg, M., & Wilczak, C. J. (1999). Optimal project selection when borrowing and lending rates differ. *Mathematical and Computer Modelling, 29*(3), 63–78.

Padilla, E., Jadotteb, E., & Pasqula, J. (2013). Technical note: Equivalence of different profitability criteria with the net present value. *International Journal of Production Economics, 142*(1), 205–210.

Palley, T. (2015). Money, fiscal policy, and interest rates: A critique of modern monetary theory. *Review of Political Economy, 27*(1), 1–23.

Palley, T. (June 2016). *Why ZLB Economics and Negative Interest Rate Policy(NIRP) are wrong: A theoretical critique* (Working paper). IMK Macroeconomic Policy Institute. http://www.boeckler.de/pdf/p_imk_wp_172_2016.pdf

Pasqual, J., Padilla, E., & Evans, J. (2013). Technical note: Equivalence of different profitability criteria with the net present value. *International Journal of Production Economics, 142*(1), 205–210.

Poole, B. (2000). On time: Contribution from the social sciences. *Financial Services Review, 9*(4), 375–387.

Poterba, J. M., & Summers, L. H. (1995). A CEO survey of US companies' time horizons and hurdle rates. *Sloan Management Review, 37*(1), 43–53.

Pressacco, F. & Stucchi, P. (1997). *Rivista di Matematica per le Scienze Economiche e Sociali, 20*, 165–185.

Promislow, D. S., & Spring, D. (1996). Postulate for an internal rate of return for an investment project. *Journal of Mathematical Economics, 26*, 335–345.

Raeva, D., Mittone, L., & Schwarzbach, J. (2010). Regret now, take it now: On the role of experienced regret on intertemporal choice. *Journal of Economic Psychology, 31*(4), 634–642.

Ramsey, J. (1970). The marginal efficiency of capital, the internal rate of return, and net present value: An analysis of investment criteria. *Journal of Political Economy, 78*(5), 1017–1027.

Rayman, R. A. (2007). Fair value accounting and the present value fallacy: The need for an alternative conceptual framework. *The British Accounting Review, 39*, 211–225.

Robison, L. J., & Barry, P. J. (1996). *Present value models and investment analysis*. Northport AL: The Academic Page.

Robison, L., Shupp, R., & Myers, R. (2010). Expected utility paradoxes. *Journal of Socio-Economics, 39*(2), 187–193.

Roll, R. (1983). On computing mean returns and the small firm premium. *Journal of Financial Economics, 12*(3), 371–386.

Ross, S., Spatt, C., & Dybvig, P. (1980). Present values and internal rates of return. *Journal of Economic Theory, 23*, 66–70.

Rubinstein, M. (1973). A mean-variance synthesis of corporate financial theory. *The Journal of Finance, 28*(1), 241–250.

Saak, A., & Hennesey, D. (2001). Well-behaved cash flows. *Economic Letters, 73*, 81–88.

Saleh, J., & Marais, K. (2006). Reliability: How much is it worth? Beyond its estimation or prediction, the (net) present value of reliability. *Reliability Engineering & System Safety, 91*(6), 665–673.

Samuelson, P. A. (1937a). Some aspects of the pure theory of capital. *Quarterly Journal of Economics, 51*(3), 469–496.

Samuelson, P. A. (1937b). A note on measurement utility. *Review of Economic Studies, 4*, 155–161.

Sandri, S., Schade, C., Mußhoff, O., & Odening, M. (2010). Holding on for too long? An experimental study on inertia in entrepreneurs' and non-entrepreneurs' disinvestment choices. *Journal of Economic Behavior & Organization, 76*(1), 30–44.

Scholten, M., & Read, D. (2006). Discounting by intervals: A generalized model of intertemporal choice. *Management Science, 52*(9), 1424–1436.

Seiler, M., Seiler, V., Traub, S., & Harrison, D. (2008). Regret aversion and false reference points in residential real estate. *Journal of Real Estate Research, 30*(4), 461–474.

Senbet, L., & Thompson, H. (1978). The equivalence of alternative mean-variance capital budgeting models. *Journal of Finance, 33*(2), 395–40l.

Sher, S., & McKenzie, C. (2006). Information leakage from logically equivalent frames. *Cognition, 101*, 467–494.

Shiller, R., Campbell, J., & Schoenholz, K. (1983). Forward rates and future policy: Interpreting the term structure of interest rates. *Brookings Papers on Economic Activity, 1*, 173–217.

Simerska, C. (2008). Remarks on the economic criterion-the internal rate of return. *Programs and Algorithms of Numerical Mathematics, 14*, 170–176.

Smith, R. D. (1996). Is regret theory an alternative basis for estimating the value of healthcare interventions? *Health Policy, 37*(2), 105–115.

Smith, J., & McCardle, K. (1999). Options in the real world: Lessons learned in evaluating oil and gas investments. *Operations Research, 47*(1), 1–5.

Song, Z., Storesletten, K., & Zilibotti, F. (2014). Growing (with Capital Controls) like China. *IMF Economic Review, 62*(3), 327–370.

Soman, D., Ainslie, G., et al. (2005). The psychology of intertemporal discounting: Why are distant events valued differently from proximal ones? *Marketing Letters, 16*(3/4), 347–360.

Stewart, G.B. (1991), *The quest for value: The EVA™ management guide*. Harper Collins, Publishers Inc., USA.

Stewart, N., Reimers, S., & Harris, A. (2014). On the origin of utility, weighting, and discounting functions: How they get their shapes and how to change their shapes. *Management Science, 61*(3), 687–705.

Strong, N. (2006). Modeling abnormal returns. *Journal of Business Finance & Accounting, 19*(4), 533–553.

Sundaresan, S. (1994). An empirical analysis of U.S. treasury auctions: Implications for auction and term structure theories. *Journal of Fixed Income, 4*(2), 35–50.

Thornton, D. L., & Valente, G. (2012). Out-of-sample predictions of bond excess returns and forward rates: An asset allocation perspective. *Review of Financial Studies, 25*(10), 3141–3168.

Thornton, M. (2015). *The meaning of financial repression*. Mises Institute. https://mises.org/library/meaning-financial-repression

Trippi, R. (1989). A discount rate adjustment for calculation of net present values and internal rates of returns on investments whose lives are uncertain. *Journal of Economics & Business, 41*, 143–151.

Trusheim, M. R., Burgess, B., Hu, S. X., et al. (2011). Quantifying factors for the success of stratified medicine. *Nature Reviews Drug Discovery, 10*(11), 817–833.

Tsao, C. (2012). Fuzzy net present values for capital investments in an uncertain environment. *Computers & Operations Research, 39*(8), 1885–1892.

Tymoigne, E. (2006). *Fisher's theory of interest rates and the notion of real: A critique* (Levy Economics Institute working paper no. 483). Levy Economics Institute, US.

Urminsky, O., & Zauberman, G. (2016). The psychology of intertemporal preferences. In G. Keren & G. Wu (Eds.), *Wiley-Blackwell handbook of judgment and decision making*. Hoboken: Wiley-Blackwell. http://home.uchicago.edu/ourminsky/Urminsky_Zauberman_2014.pdf

Van de Ven, N., & Zeelenberg, M. (2010). Regret aversion and the reluctance to exchange lottery tickets. *Journal of Economic Psychology, 32*, 194–200.

Van Groenendaal, W. (1998). Estimating NPV variability for deterministic models. *European Journal of Operational Research, 107*(1), 202–213.
Vassallo, J. (2010). The role of the discount rate in tendering highway concessions under the LPVR approach. *Transportation Research Part A, 44*, 806–814.
Vuina, D., Lozina, Z., & Vlak, F. (2010). NPV-based decision support in unobjective design using evolutionary algorithms. *Engineering Applications of Artificial Intelligence, 23*(1), 48–60.
Walasek, L., & Stewart, N. (2015). How to make loss aversion disappear and reverse: Tests of the decision by sampling origin of loss aversion. *Journal Of Experimental Psychology: General, 144*(1), 7–11.
Walls, M. R. (2004). Combining decision analysis and portfolio management to improve project selection in the exploration and production firm. *Journal of Petroleum Science & Engineering, 44*(1), 55–65.
Walther, H. (2010). Anomalies in inter-temporal choice, time-dependent uncertainty and expected utility—A common approach. *Journal of Economic Psychology, 31*(1), 114–130.
Warner, J., & Pleeter, S. (2001). The personal discount rate: Evidence from military downsizing programs. *American Economic Review, 91*, 33–53.
Weber, T. (2014). On the (non-)equivalence of IRR and NPV. *Journal of Mathematical Economics, 52*, 25–39.
Weitzman, M. L. (2009). On modeling and interpreting the economics of catastrophic climate change. *Review of Economics & Statistics, 91*(1), 1–19.
Wiesemann, W., Kuhn, D., & Rustem, N. B. (2010). Maximizing the net present value of a project under uncertainty. *European Journal of Operational Research, 202*(2), 356–367.
Wigglesworth, R., Lewis, L., & McCrum, D. (2016, February 17). Central banks: Negative thinking – Sub-zero rates are a sign that policymakers are running out of options to invigorate the economy, say critics. *Financial Times* (UK). https://www.ft.com/content/7333e92a-d4a2-11e5-829b-8564e7528e54
Wong, K. P. (2015). A regret theory of capital structure. *Finance Research Letters, 12*, 48–57.
Yechiam, E., & Hochman, G. (2013). Loss-aversion or loss-attention: The impact of losses on cognitive performance. *Cognitive Psychology, 66*(2), 212–231.
Yechiam, E., Telpaz, A., & Hochman, G. (2014). The complaint bias in subjective evaluations of incentives. *Decision, 1*, 147–160.
Zauberman, G., Kim, K., Malkoc, S., & Bettman, J. (2009). Discounting time and time discounting: Subjective time perception and intertemporal preferences. *Journal of Marketing Research, XLVI*, 543–556.

7

On Algebraic Anomalies in Polynomials and Net Present Value Decisions

This chapter (1) simulates and proves new biases and anomalies inherent in the NPV-MIRR-IRR model (and related approaches such as SIRR, AIRR, APV, EVA, NFV; etc.); (2) develops the necessary and sufficient conditions for monotonic NPV ("well behaved" NPV); and for the anomalous behavior of NPV; (3) explains how the power rule and the inverse function rule in differential calculus, are both wrong.

7.1 Existing Literature

The literature on biases and anomalies in NPV, MIRR/IRR and related models (such as average-IRR, Selective-IRR, APV, NFV, SVA, EVA, etc.) is substantial, but there are still gaps in the literature with regard to specific phenomena such as: (1) documenting and proving new anomalies in the NPV-MIRR model; (2) development and testing of the conditions for anomalous behavior of NPV-MIRR; and monotonic NPV. Most of the existing research documents two issues: (1) that there are anomalies in the NPV and MIRR-IRR models when the signs of the project cash flows

M.C.I. Nwogugu, *Anomalies in Net Present Value, Returns and Polynomials, and Regret Theory in Decision-Making*,
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change (the current research focuses on non-monotonic NPVs, multiple IRRs and conditions for a "unique" IRR); and (2) the maximum number of number roots of the NPV formula does not exceed the number of time periods (this conclusion was also mentioned by Descartes more than 300 years ago, but was supposedly proved later).

Magni (2010) noted that the NPV/IRR model has been used extensively in the construction industry, for project scheduling problems, production inventory problems, advance manufacturing technology, and in logistics for several decades. The NPV-MIRR-IRR model and related approaches are also widely used in the healthcare industry. Hall et al. (2012) analyzed NPV-based decisions. McCabe et al. (2013) introduced the *net benefit probability map*, which compares net health benefit versus time, and attempts to quantify decision-makers' uncertainty about the benefit of technologies at different points in time (such as: the payback period for innovative technologies; how methodological choices on discount rates affect results; how alternative payment mechanisms can reduce the risk for decision-makers within the context of innovative technologies). Califf et al. (2008) analyzed the symbiotic relationship between policies and NPV when pharmaceutical and medical device industries make technology decisions that can be beneficial to society. Trusheim et al. (2011) analyzed the use of NPV in medical decisions.

Groenendaal (1998) developed new methods to analyze the variability of NPV. Liu et al. (2004) analyzed NPVs within the context of a capital market with differing borrowing and lending rates. Vassallo (2010) analyzed the use of least present value of the revenues (LPVR) in the award of highway concessions. De Reyck et al. (2008) developed an alternative approach for valuing Real Options based on the certainty-equivalent version of the NPV formula, which eliminates the need to identify market-priced twin securities to value Real Options, which is not practical. Trigeorgis (1996) states that NPV and IRR cannot cope with the operation flexibility options and other strategic aspects of various projects, and that Real Options approaches are more accurate. Dixit and Pindyck (1995) found that NPV makes the false assumption that the investment is either reversible or that it cannot be delayed. Bas (2013) proposed decision rules of NPV and IRR by developing a relationship between the robust optimization approach proposed in the literature and the decision rules of NPV and IRR by considering uncertainty in cash flows. Mellichamp (2013) developed a new discounted cash flow method—however, this method is subject to the same weaknesses as NPV.

Padberg and Wilczak (1999) developed mathematical programming decision models for project approval in capital budgeting where the borrowing/lending rates for capital are constant and time dependent, respectively. Remer and Nieto (1995) analyzed various classes of project evaluation decision models. Haley and Goldberg (1995) analyzed the effect of NPV techniques on new product research. Naim (2006) analyzed the effect of using NPV on parameter selection in the ordering policy of a production planning and control system; and introduced the NPV of the variance (NPVv) measure. Trippi (1989) developed adjustments for the calculation of NPVs and IRRs.

Padilla et al. (2013) concluded that some standard decision criteria (such as net final value; IRR; benefit–cost ratio; profitability index; equivalent annuity, discounted payback period and average payback period) lead to the same investment decision and same ranking as net present value. Borgonovo and Percocao (2012) concluded that for sensitivity analysis, partial derivatives can be misleading in the identification of key drivers of a project's performance; and that the sensitivity analysis results for NPV and IRR may differ; and they proposed the use of an alternative sensitivity measure called the differential importance measure (DIM). The DIM is wrong because: (1) DIM does not consider the biases introduced in this chapter-8 in this book; (2) DIM does not function when the change in two or more variables are non-uniform; (3) by its very nature, and contrary to Perocco and Borgonovo (2012), DIM is or can be non-additive—especially when the inputs are highly correlated, and/or are conditioned on the occurrence of each other; and/or change when they interact with each other.

Nwogugu (2010): (1) proved that Descartes' Sign Rule and the Fourier-Boudan Theorem are wrong; (2) proved that the Fundamental Theorem of Algebra is wrong; (in part by doing sensitivity analysis of NPVs as discount rates change) (3) explained how root calculation in algebra is wrong; (4) analyzed framing effects inherent in the NPV-IRR model (and related approaches such as AIRR, Selected-IRR, MIRR, NFV, SVA, EVA, etc.) and the mean-variance model; (5) analyzed algebraic biases/anomalies inherent in the calculation of investment returns (in part by doing sensitivity analysis). Walthe (2010) analyzed anomalies in intertemporal choice, time-dependent uncertainty and expected utility.

Lohmann (1988), Biondi (2006), Aven & Flage (2009), and Hazen (2003) analyzed the deficiencies inherent in IRR, NPV and related models.

Oehmke (2000) found that the NPV may exhibit anomalous behavior if the signs (positive or negative) of the annual net benefits of the project change more than once; and, in particular, that it is possible for the NPV to increase as the discount rate increases, or to decrease as the discount rate falls. Oehmke (2000) found that when investment in the project occurs entirely at time 0, and net benefits are positive thereafter, then the sign of $\partial N/\partial r$ is unambiguously negative: an increase in the discount rate decreases the NPV of the project. When the net benefits, $\{B: t = 1, 2,\ldots\}$, take on both positive and negative values, then the sign of $\partial N/\partial r$ is indeterminate a priori (an increase or decrease in the discount rate can increase or decrease the NPV; where N is the NPV and r is the discount rate). However, as shown in this chapter, and contrary to Oehmke (2000), it is not in all circumstances where the IRR shows anomalies, that the NPV also has anomalies. Saak and Hennessy (2001) developed necessary and sufficient conditions that preclude anomalous behaviors of IRR and NPV when cash flows change sign more than once. The analysis in Saak and Hennessy (2001) differs substantially from that in Ross et al. (1980) and in Oehmke (2000) because Saak and Hennessy (2001) focused on continuous cash flows. Kierfulff (2008) explained various anomalies and errors inherent in NPV and IRR models, and demonstrated how the MIRR solves some of these problems.

Kierfulff (2008); Rocabert, Tarrio & Perez (2005); Domingo (2001); Merlo (2016); Magni (2002); Sen (1975), Nwogugu (Sept. 2010), Ben-Horin and Kroll (2012); Altshuler and Magni (2002); Pressacco, Magni and Stucchi, (2014); Tsao (2012); Simerska (2008); Magni (2015); Magni (2002); Leyman and Vanhoucke (2016); Hazen (2003); Hanafizadeh and Vahideh (2011); Groenendaal (1998); Barry and Robison (2014); Magni (2009a, b); Magni (2013) and Mellichamp (2013) discussed various anomalies inherent in the NPV-MIRR-IRR and related models - but most of the research focuses on conditions for monotonic/non-monotonic NPV and multiple/unique IRRs.

Haley and Goldberg (1995) documented several weaknesses and short-term biases inherent in the NPV-MIRR model, and noted that the main weaknesses of NPV are that it erroneously assumes that: (1) any decisions made can be reversed without penalty; (2) different investments have equal or similar effects on a firm's ability to make investments; (3) the results of NPV analyses are objective and unbiased. Promislow and Spring (1996) analyzed conditions for the monotonicity of IRR and scale invariance. However, the Promislow and Spring (1996) optimal characteristics for IRRs (e.g., monotonicity, scale invariance, and continuity) are not feasible or appropriate. Booth (2003) analyzed the effects of serial correlation of cash flows on NPV and IRR and developed methods to incorporate the effects of serial correlation in capital budgeting. Booth (2003) found that the standard compounding and the standard NPV models underestimate future cash flows when growth rates are serially correlated (i.e., when growth rates of expected cash flows are serially correlated, standard compounding underestimates future cash flows).

Magni (2013) explained why the IRR model is wrong and why the AIRR model may be more accurate. Magni (2013) noted the following errors in the IRR model: (1) IRR provides multiple rates of return; (2) in some cases, the IRR model can provide no rate of return; (3) the IRR can be distorted by use of varying costs of capital; (4) arbitrage strategies distort IRR; (5) mutually exclusive projects and project ranking; (6) rate of return on initial capital (or total investment cash flow) is distorted; (7) IRR produces framing effects such as present value vs. future value; expected value of stochastic IRR vs. IRR of expected investment; value additivity is not preserved by IRR; (8) IRR neglects Project's Operating Life; (9) IRR represents concocted capital; (10) IRR has only ad hoc consistency with NPV; (11) IRR provides multiple project balances, and multiple excess returns; (12) IRR has intertemporal inconsistency; (13) IRR is not capable of summarizing accounting variables, in particular by accounting rates of return; (14) Makeham's formula does not adequately summarize the information derived by the varying interest rates; (15) changes in capital—IRR is a cash flow based measure that neglects the actual operations involved in a project, it remains constant under changes in capital (i.e., any two projects with the same cash flow have the same IRR, no matter what the invested capital actually is); (16) computational issues—in MSExcel, multiple IRRs are not detected, and MATLAB and similar packages are cumbersome. Lohmann (1988) analyzed problems and inaccuracies inherent in IRR and AIRR. Magni (2011) is an addendum to a prior article that notes that the IRR has the following flaws: (1) there can be multiple real-valued IRRs or complex-valued IRRs may arise; (2) the IRR is incompatible with NPV in accept/reject decisions, and the IRR ranking of projects is, in general, different from the NPV ranking; (3) the IRR criterion is not applicable when costs of capital varies. Magni (2011) argued that the AIRR implicit in a project is a reliable estimate of a project's profitability and correctly ranks competing projects; and AIRR eliminates complex-valued numbers and associated computational problems. Doraszelski (2001), Sugden and Williams (1978), and Robison and Barry (1996) analyzed the IRR measure and found that when the project cash flows and benefits change sign more than once, there may be more than one IRR.

Weber (2014) analyzed the (non-)equivalence of IRR and NPV; and introduced the "Selective-IRR". Ross et al. (1980), Saak & Hennesey (2001) and Sen (1975) attempted to develop the complete set of conditions under which the present value of a return stream decreases as interest rates rise. However, as is shown and proved in this book and contrary to Ross et al. (1980), the present value of a return stream decreases as interest rates rise only when some conditions exist: (1) the number of negative project cashflows are less than the number of positive periodic project cashflows; (2) discount rates are greater than zero. Saak and Hennessy (2001) developed necessary and sufficient conditions that preclude anomalous behavior of Internal rate of return and net present value when cashflows change sign more than once. The analysis in Saak and Hennessy (2001) differs substantially from that in Ross, Spatt and Dybvig (1980) and in Oehmke (2000) because Saak and Hennessy (2001) focused on continuous cash flows. Dybvig (1983) analyzed models which have present value decreasing in interest rates, and conditions for well behaved IRR. Shea, Tang & Tso (2000) analyzed the sequential execution of projects, associated a "priority value" with each project and concluded that projects should be ordered in decreasing priority values, and that this priority value rule is equivalent to the replication rule in the literature; and compared the priority value rule and the ordinary present value rule.

Magni (2009b) analyzed the NPV maximization Rule and concluded that: (1) discounting cash flows with a hurdle rate differs from the cost of capital normatively suggested (e.g. CAPM, arbitrage pricing); (2) given that decision makers' aspiration levels matter and are subjectively determined, the hurdle-rate rule can be deemed to be a boundedly rational approach to investment decisions; (3) Some empirical evidence show that the bounded-rationality approach is ecologically rational, domain-specific, and psychological plausible; and the Hurdle-rate Rule results in near-optimal solutions when confronted with the expanded NPV; (4) given that the Hurdle rate is affected by: uncertainty, future opportunities, rationing of managerial skills, strategic considerations, agency costs, and costs of external financing; the Hurdle Rate method is consistent with the Real-options approach, resource-based theory, Top Management Team literature, agency theory, and strategy literature; (5) The NPV maximizing model may take several different forms; some of which are inapplicable and/or deviate from accepted standards of rationality (additivity, no-arbitrage, description invariance); (6) individually, both unboundedly rational NPV and boundedly rational NPV have inherent inaccuracies; but an interaction between the two may improve performance.

The rest of this chapter discusses the inherent problems and algebraic biases in the NPV-IRR model (which can also occur in related approaches such as AIRR, MIRR, Selective-IRR, NFV, SVA, and EVA).

7.2 Biases in the NPV-IRR-MIRR Model and Related Approaches

Theorem 7.1: For any series of cash flows during any series of equal time intervals, a negative cash flow has a greater effect on the NPV than a positive cash flow of the same magnitude.

Proof: Proof of the ***negative-returns NPV effect*** is also evident in Tables 1–10 (in Chart 5A in Chapter-5), each of which contain a series of time value of money (TVM) equations (lines1A-24A), which are a special class of polynomials. In those tables the signs of the cash flows in consecutive TVM equations are changed so that the only difference between any two adjacent TVM equations is one sign change. The corresponding NPVs (with different discount rates) are shown immediately below the TVM equations, and a visual inspection of the changes in the NPVs shows that a negative project cashflow ($-x_i$) causes a greater change in the NPV, than a positive project cashflow ($+x_i$) of the same magnitude—that is, $|x_i|$ is same for positive and negative Project cashflow. The standard TVM equation for NPV is the following polynomial:

$$N = -a + \sum_{i=n} \left\{ x_i / (1+r)^i \right\}; \text{or} \tag{7.1}$$

Where:

a = initial investment.

B = the sum total of the periodic project outcome/cashflow (polynomial coefficients) without regard to the TVM; $x_i \ldots x_n$ ε B.

n = number of total time periods.

i = the specific time period, (i.e., 1, 2, 3, 4,…n); and i ε n.

r = the discount rate.

p = the periodic percentage change in each periodic project cashflow, p is equal to zero in the original TVM equation which can be rewritten as:

$$N = -a + \sum_{\mathrm{n=i}} \left[\left\{ x_i * (1+p) \right\} / (1+r)^{-\mathrm{i}} \right] \tag{7.1a}$$

If the lowest common factor is found, then:

$$N = -a + \sum_{\mathrm{n=i}} \left\{ (x_i + x_i p) * (1+r)^{(n-i)} \right\} * (1+r)^{-n} \tag{7.2}$$

$$N = -a + \sum_{\mathrm{n=i}} \left\{ c * (1+r)^{-n} \right\} \tag{7.3}$$

Where $c = \sum_{\mathrm{n=i}} \left\{ (x_i + x_i p) * (1+r)^{(n-i)} \right\}$

Using existing differentiation methods (which are valid only for very small changes around the number zero):

$$\partial N / \partial B = \sum_{i=n} \left[B_i p * \left\{ (1+r)^{(n-i)} \right\} \right] * \left\{ (1+r)^{-n} \right\} \tag{7.4}$$

$$\partial^2 N / \partial B \partial r = \sum_{i=n} \left[B_i p * \left\{ -i / \left(1+r\right)^{(i-1)} \right\} \right] * \left\{ -n / \left(1+r\right)^{(n-1)} \right\} \tag{7.5}$$

$$\partial N / \partial c = \sum_{i=n} \left[x_i p * \left\{ -i \left(1+r\right)^{(i-1)} \right\} \right] * \left\{ \left(1+r\right)^{-n} \right\} \tag{7.6}$$

As noted in Nwogugu (2012: 325–330), the *power rule* and the *inverse function rule* in differentiation are inaccurate and Eqs. 7.4, 7.5, and 7.6, are presented for illustration purposes. For any series of project outcomes/cashflows during any series of equal time intervals, and for all discount rates, and for roughly equal periodic project benefits, where *sign magnitude positive dominance* (the total negative project cashflows exceeds the total positive project cashflows with adjustment for the TVM) and *sign positive dominance* (the number of negative signs exceeds the number of positive signs) exists, as the number of negative project cashflows increases (as c decreases), the change in the NPV decreases, regardless of whether or not the NPV is normal (monotonic) or exhibits anomalies. That is, within the range of discount rates where NPV is monotonic, as $B \rightarrow -\infty$, $\partial N/\partial B \rightarrow -\infty$. ■

Theorem 7.2: For any series of project outcomes/cashflows during any series of equal time intervals, and for all discount rates that exceed 100%, as the discount rate increases the NPV becomes much less sensitive to the signs of periodic project cashflows.

Proof: The *discount rate NPV effect* occurs because when the interest rate exceeds 100%, it becomes large enough such that the effect of the coefficient of each term in the TVM equation, and the effect of the magnitude of the signs, become much less dominant for the NPV value. The *discount rate NPV effect* increases as the average number of integers in the project cashflows increases. The typical TVM equation has the following form:

$$NPV = -a + x_1 \left(1+r\right)^{-1} + x_2 \left(1+r\right)^{-2} + \ldots\ldots + x_n \left(1+r\right)^{-n} \tag{7.7}$$

Using existing differentiation rules,

$$\partial N / \partial r = \sum_{i=n} \left[\left(-i * x_i \right) / \left(1+r \right)^{(i-1)} \} \right]. \tag{7.8}$$

As explained below and in Nwogugu (2012: 325–330), although this formula is widely used in NPV and modified-duration models it is not correct, and a more accurate approximation is:

$$\partial N / \partial r = \sum_{i=n} \left[-i / \left(1+r \right)^{(i-1)} \} \right]. \tag{7.9}$$

Where N is the NPV, r is the discount rate, a is the initial investment, and B_i is the periodic project outcome/cashflow in time period i. Thus, when r is greater than +100%, the compounding effect of $[\{-i/\{(1+r)^{(i-1)}\}]$ increases exponentially and the term $\sum_{i=n} \left[-i / \{ \left(1+r \right)^{(i-1)} \} \right]$ is smaller; and, as the discount rate increases, the NPV becomes increasingly less sensitive to changes in the signs of the project benefits. The *discount rate NPV effect* is evident in lines 1A–13A and the corresponding lines 1B–13B in Tables 1–10 in Chart 5A in Chapter-5. ■

Theorem 7.3: The Contra NPV Theorem (Non-Monotonic NPV)—for any series of equal periodic (annuity-type) cashflows during any series of equal time intervals, and for all discount rates that are greater than 0%, the NPV will increase as the discount rate increases, iff a simple majority of the polynomial coefficients (periodic project cashflows) have negative signs; and the sum total of the cashflows (without considering time-value) is negative.

Proof: When the discount rate exceeds 0%, the signs of the coefficients in the TVM equation become dominant, and together with the negative exponent in the term $\{(1+r)^{-n}\}$, are the primary cause of the *contra NPV effect*.

$$N = -a + \sum \left\{ \left(x_i + x_i p \right) * \left(1+r \right)^{(n-i)} \right\} * \left(1+r \right)^{-n} \tag{7.10}$$

$$N = -a + \sum \left\{ c * \left(1+r \right)^{-n} \right\} \tag{7.11}$$

Where

$$c = \sum \left\{ \left(x_i + x_i p \right) * \left(1 + r \right)^{(n-i)} \right\} \tag{7.12}$$

Using existing differentiation methods (which are valid only for extremely minute small numbers around the number zero):

$$\partial N / \partial B = \sum_{i=n} \left[x_i p * \left\{ \left(1 + r \right)^{(n-i)} \right\} \right] * \left\{ \left(1 + r \right)^{-n} \right\} \tag{7.13}$$

$$\partial^2 N / \partial B \partial r = \sum_{i=n} \left[x_i p * \left\{ -i / \left(1 + r \right)^{(i-1)} \right\} \right] * \left\{ -n / \left(1 + \mathrm{r} \right)^{(n-1)} \right\} \tag{7.14}$$

$$\partial N / \partial c = \sum_{i=n} \left[x_i p * \left\{ -i \left(1 + r \right)^{(i-1)} \right\} \right] * \left\{ \left(1 + r \right)^{-n} \right\} \tag{7.15}$$

From the above equations, its evident that for very small numbers around zero, and for all positive discount rates [*r*], the signs of the polynomial coefficients (x_i) are the dominant factors in the TVM polynomial that affect the NPV. The *Contra-NPV effect* is evident in Lines 1A-6A and 22A-24A and the corresponding lines 1B–16B and 22B-24B in Tables 1–10 (in Chart 5A in Chapter-5). In each of Tables 1-11, there are cashflows in rows 1A……24A followed by NPVs for each cashflow under different discount rates (in the columns) in rows 1B……24B. The Contra NPV Effect may also occur in TVM equations where the periodic cashflows are not equal (but a majority of the cash flows have negative signs, and the sum total of the cashflows is negative) and or where the discount rate is less than zero-percent. For example in Tables B6, B7 and B8 (in Chart-4C in Chapter-4), in the intervals of discount rates between -300 and -110%, and between 0% and 189%, the NPV increases as the discount-rate increases. The Contra-NPV Effect can occur in related models such as AFV, EVA and Generalized-NPV. The Contra-NPV Effect can also occur in the NFV models wherein as the rate-of-return or reinvestment rate increases, the NFV declines. ■

Theorem 7.4: The Oehmke (2000) Theorem/Proposition which states that a necessary condition for anomalous behavior of NPV is that p(r) (the polynomial that is the NPV formula) should have at least two real roots; and that a sufficient condition is that p(r) have (at least) two distinct real roots, is wrong.

Proof: Contrary to Oehmke (2000), and as mentioned above, an approximation is $\partial N / \partial r \approx \sum_{i=n} \left\{ -i / \Delta (1+r)^{(i-1)} \right\}$. Where N is the NPV, r is the discount rate, and x_n is the periodic project benefit in time period n. Anomalous behavior of NPV is defined as when $\partial N/\partial r > 0$; which occurs when $\sum_{i=n} \left\{ -i / \Delta (1+r)^{(i-1)} \right\} > 0$, which occurs only when $\sum [-B_n] > 0$ (*negative sign magnitude dominance*), and or when $\{-i/(1+r)^{(i-1)}\} < 0$ for a majority of time periods (*negative r-sign dominance*), which, in turn, is common when the TVM equation has an even-numbered degree or when r is less than zero. Thus, the anomalous behavior of NPV occurs because of the interaction of: (1) the dynamics of the signs of r; and (2) the dynamics of signs of the project cashflows (sign negative dominance or sign positive dominance). Anomalous behavior of NPV does not necessarily depend on the number of time periods in the TVM equation.

The necessary conditions for anomalous behavior of NPV were proved in a foregoing theorem, and are illustrated in Tables 1–11 (in Chart 5A in Chapter-5) where anomalous behavior occurs only when there is simultaneous sign negative dominance and negative sign magnitude dominance. The proof is also evident in Tables B6, B7 (in Chart-4B), where the NPV exhibits anomalous behavior but there are no IRRs (roots) in the intervals shown. Contrary to Oehmke (2000) (and other researchers), the cases in which the NPV may exhibit anomalous behavior are not always those cases in which there are multiple IRRs – for example, in the TVM equations in each of Tables B6, B7 and B8 (in Chart-4B in Chapter-4), the NPV is non-monotonic when the discount rate is above zero percent, but each such TVM equation does not have an IRR. In the TVM equation in Table B10 (in Chart 4C in Chapter-4) has no IRR, but the NPVs exhibit anomalous behavior when *r* is less than 0% (when *r* is between –300% and –100%) and when r is greater than zero. Similarly, in the TVM Equation in line 1A in each of Tables 1–11 (in Chart 5A in Chapter-5), their NPVs exhibit anomalous behavior when *r* is between −250% and 50% (with the exception of r = –100%). Thus, the Oehmke (2000) Theorem/Proposition is wrong. ■

Theorem 7.5: The present value of a return stream decreases as interest rates rise only iff: (1) the number of negative periodic project cashflows are less than the number of positive periodic project cashflows; and (2) the absolute magnitude of combined negative periodic project cashflows are less than the number of positive periodic project cashflows without regard to the TVM; (3) interest rates are greater than zero.

Proof: An NPV is monotonic *iff* it decreases as the discount rate increases; regardless of whether or not other factors change. Ross et al. (1980) attempted to, but did not develop the complete set of conditions under which the present value of a return stream decreases as the discount rate rates rise. One of the Ross et al. (1980) conditions was that the discount rate must be greater than −100%. However, with interest rates between −99.9% and 0%, the NPV will be non-monotonic for any return stream wherein the number of negative project cashflows exceeds the number of positive project cashflows—this is clearly shown in Tables 1–11 (in Chart 5A in Chapter-5), where for all cash flows in lines 1A–13A, and for the discount rates of −10%, −5%, 0.25%, and 50%, the NPV is non-monotonic and increases as the discount rate increases where sign negative dominance exists. Contrary to Ross et al. (1980), the monotonic NPV exists *iff* the following conditions exist:

Condition 1: The absolute magnitude of combined negative periodic project cashflows are less than the absolute magnitude of the positive periodic project cashflows with regard to the TVM (referred to as positive sign magnitude dominance while the opposite is negative sign magnitude dominance);

Condition 2: The number of negative periodic project cashflows are less than the number of positive periodic project cashflows (this state is referred to as sign positive dominance while the opposite is sign negative dominance);

Condition 3: The discount/interest rates are greater than zero.

Condition-1 is evident in Tables 1–11 (in Chart 5A in Chapter-5), where for all discount rates above zero, the NPV is monotonic and decreases when interest rates rise if positive sign magnitude dominance exists. This *monotonic NPV effect* exists because with positive sign magnitude dominance, the terms in the TVM formula with positive signs will dominate and ensure monotonicity. Condition-2 is necessary because with sign negative dominance, the NPV will tend to increase as the discount rate increases. Condition-1 is evident in Tables 1–11 (in Chart 5A in Chapter-5), where for all discount rates between −99.99% and 50% inclusive, the NPV is not monotonic when sign negative dominance exists. Condition-3 is self-evident in Tables 1–11 (in Chart 5A in Chapter-5) because for all project outcomes/cashflows, when the discount rate is between −99% and 0%, the $(1+r)$ in each term in the TVM equation works in the opposite way to increase each term $(x_n(1+r)^{-n})$ in the TVM equation. That is, $\partial N/\partial r < 0$. When the discount rate is −100%, the NPV is equal to the first investment at time zero (i.e., $\partial N/\partial r = 0$). When the interest rate is between −101% and −124%, the *−101%/–124% NPV effect* (which is described herein and below) occurs. ■

Theorem 7.6: The Osborne (2010: 239) proof which states that $\sum|b_i| = \prod |z_i|$ *(where* b_i *is the periodic project outcomes/cashflows and* z_i *is the discount rate), is wrong.*

Proof: In most circumstances, each periodic project outcomes/cashflows (e.g., cash flow) is either greater than one or less than minus one (−1), while most discount rates are less than 1.5 (150%), hence, the relationship $\sum|b_i| = \prod |z_i|$ does not exist in most circumstances. In Osborne (2010), changing the definition of Z from $Z = (1+z)$ to $Z=1$, is inconsistent. ■

Theorem 7.7: For any series of equal periodic (annuity-type) project cashflows during any series of equal time intervals, and where most of the project outcomes/cashflows are six digit numbers (between 999,999 and −999,999), and for all discount rates that are between –250% and 300%, if the signs of all the cashflows are reversed, the sign of the NPV will also reverse.

Proof: The proof is observable in lines 1B and 13B in Tables 1–10 (in Chart 5A in Chapter-5). This *Mirror NPV* effect occurs because of the combination of the equality of the annuity-type cashflows and the change of signs. ■

Theorem 7.8: For any series of equal periodic (annuity-type) project cashflows during any series of equal time intervals, and where most of the project cashflows are 1-,2-,3-,4-,5- or six digit numbers (between 999,999 and −999,999), the discount rates that are between but not including −100% and −124%, produce the most extreme positive or negative NPVs (absolute NPV without regards to signs) than any other negative discount rate.

Proof: The proof is straightforward and is partially observable in lines 1A–13A and the corresponding lines 1B-13B in Tables 1–10 (in Chart 5A in Chapter-5). This –100%/–124% NPV Effect does not occur when the discount rate (r) is a negative number that is not between –100% and –124%. This *−100%/−124% NPV effect* occurs because this range of discount rates *(between but not including –100% and –124%)* are those that provide the highest minimization or maximization of the numerator of each term in the NPV TVM equation (after taking compounding into consideration), which then results in the most extreme negative or positive NPVs respectively. This –100%/–124% NPV Effect occurs because of both the often significant effects of compounding and the absolute-magnitude of the compounded discount rates. When the discount rate (r) is between –0.00001% and –99.99%, a portion of the denominator (ie. {1+r}) in each term of the TVM polynomial becomes equal to between .99999 and 0.000001 (the "Series-2 numbers") and the compounding of any Series-2 number will result in a positive number. When the discount rate is less than –124%, a portion of the denominator (ie. {1+r}) in each term of the TVM polynomial becomes less than the number –0.24 (the "Series-3 numbers") but compounding of such Series-3 number will result in a negative or positive number, depending on the degree of the TVM polynomial and the time-period. When the discount rate is between –100% and –124%, a portion of the denominator (ie. {1+r}) in each term of the TVM polynomial is between zero and –0.24 (the "Series-1 numbers") and compounding of such Series-1 number will result in a negative or positive number depending on the degree of the TVM polynomial and the time period. Both the compounding effect and absolute-magnitude (without signs) of compounded Series-3 numbers are greater than those of Series-1 numbers. Furthermore, for any specific time-period or term in a TVM polynomial: 1) a negative Series-3 number will result in a lower NPV than a negative Series-1 number; and 2) a positive Series-3 number will result in a lower NPV than a positive Series-1 number; 3) a positive Series-2 number will result in a lower NPV than a positive or negative Series-1 number. The net effect is that all else held constant, any term or time-period in any TVM polynomial where a portion of the denominator (ie. {1+r}) contains a Series-2 number or Series-3 number will always produce a lower NPV than a similar denominator where the {1+r} contains a Series-1 number. ■

Theorem 7.9: *For any series of project outcomes/cashflows during any series of equal time intervals, and for all discount rates that are* −*100%, the NPVs will always be less than zero or zero if the initial investment (at time zero) is a negative cashflow.*

Proof: The proof is straightforward. The −*100% NPV effect* occurs because when the discount rate is −100%, the denominator for all the terms in the TVM equation (i.e., {1+*r*} compounded) except the initial investment, will be zero. Thus, the only remaining non-zero term will be the initial investment (if any, or a positive cash flow) in period zero. This is also illustrated in Tables 1–10 (in Chart 5A in Chapter-5) and Tables A1, A2, A3, and A4 (in Chart 4A in Chapter-4). Thus, Theorem-2 in Hazen (2003) is wrong; and when the discount rate is zero, all the other terms in the TVM equation except the initial investment (or cash inflow) at time-zero will be equal to their un-discounted equivalents. ■

Theorem 7.10: *For any series of project outcomes/cashflows during any series of equal time intervals, and for all discount rates that are between 0% and 100%, the NPVs will always increase as the number of positive project outcomes/cashflows in the series increases; but the rate of change of the NPVs slows down as the discount rate increases to 100%.*

Proof: The proof is straightforward. This *NPV transition effect* does not occur when discount rate is not in the 0%–100% range. This NPV Transition Effect is evident in Tables 1–10 in Chart-5A (in Chapter-5). The NPV Transition Effect occurs because with discount rates that are between 0% and 100%, as the number of positive signs (of the cashflows) increases, the NPV will increase; but the rate of change in the NPV declines as the discount rates tend to 100% because of the compounding effect of the greater magnitude of the higher positive discount rates. In other words, the effect of (1+*r*) in each term in the TVM polynomial increases as the discount rate increases to 100%, causing the rate of growth of the NPV to decline. This *NPV Transition Effect* also occurs because of the often significant effects of compounding. When the discount rate is between 0% and 100%, a portion of the denominator (ie. the {1+r}) in each term of the TVM polynomial becomes equal to between one and two (the "Series-1 numbers"). When the discount rate (r) is greater than 100%, a portion of the denominator (ie. the {1+r}) in each term of the TVM polynomial becomes greater than the number two (the "Series-2 numbers"). When the discount rate is less than zero percent, a portion of the denominator (ie. the {1+r}) in each term of the TVM polynomial becomes less than the number one (the "Series-3 numbers"). The compounding of some Series-3 Numbers will result in a negative number (depending on the time-period and the degree of the TVM polynomial). The compounding of any Series-1 Number or Series-2 Number will result in a positive number. The compounding of any Series-2 Number can have a significantly disproportionate "reducing effect" on project cashflows as the number of time-periods increases such that an increase in the number of positive signs of the cashflows may not decrease the rate of change of the NPV as the discount rate is increased. The compounding of any Series-3 Number can have a significantly disproportionate reducing or increasing effect (depending on the time period and degree of the TVM polynomial) on project cashflows as the number of time-periods increases such that an increase in the number of positive signs of the cashflows may not decrease the rate of increase of the NPVs as the discount rate is increased. The key factor is that the compounding effects of all Series-2 Numbers and some Series-3 Numbers (those less than –200%) are much greater than the compounding effect of Series-1 Numbers. ■

Theorem 7.11: *For any TVM equation, Multiple IRR occurs only when the TVM equation either has more than one root, and/or has multiple maxima/minima; and the multiple-IRR phenomenon is not directly related to the anomalous behavior of NPV (for any TVM equation, both phenomena can occur separately).*

Proof: The proof is straightforward. The IRR/MIRR is the discount rate at which the NPV polynomial (TVM equation) is equal to zero. A TVM equation can have multiple IRRs (multiple real roots and/or local maxima/minima). As explained in Chapter-5 (in this book), the traditional root calculation methods sometimes confuses true roots with local maxima/minima. Also, anomalous NPV occurs only when there is simultaneous *negative sign magnitude dominance* and *sign negative dominance*. However, simultaneous negative sign magnitude dominance and sign negative dominance does not always result in multiple roots and or multiple maxima/minima for polynomials—it was shown above that simultaneous negative sign magnitude dominance and sign negative dominance can results in the subject TVM polynomial not having any roots. Thus, multiple IRR phenomena and anomalous NPV phenomena are not directly related. ■

Theorem 7.12: *The inverse function rule and the power rule in traditional differentiation (calculus) are wrong.*

Proof: See Nwogugu (2012: 324–330) where this proof is stated. Under existing differentiation rules, the derivative of the function $h(x) = a\, f(x) + b\, g(x)$ with respect to x is:

$$h'(x) = af'(x) + bg'(x). \tag{7.16}$$

That is:

$$\frac{\mathrm{d}(af + bg)}{\mathrm{d}x} = a\frac{\mathrm{d}f}{\mathrm{d}x} + b\frac{\mathrm{d}g}{\mathrm{d}x}. \tag{7.17}$$

The derivative of $h(x) = 1/f(x)$ is:

$$h'(x) = -\frac{f'(x)}{f(x)^2}. \tag{7.18}$$

This inverse function rule is written as:

$$\frac{\mathrm{d}(1/f)}{\mathrm{d}x} = -\frac{1}{f^2}\frac{\mathrm{d}f}{\mathrm{d}x}. \tag{7.19}$$

$$\text{D'}\left[k * f(x)\right] = k * f(x)\left(\text{Constant Multiple Rule}\right) \tag{7.20}$$

$$\text{If } y = ax^n, \text{then } \mathrm{d}y/\mathrm{d}x = anx^{(n-1)} \left(\text{Power Rule}\right) \tag{7.21}$$

Applying the same constant multiple rule and power rule to the TVM equation, the result is:

$$\partial N / \partial r = \sum_{i=n}\left[\{x_i * -i\} / (1+r)^{(i-1)}\right] \tag{7.22}$$

Eq. 7.16 is very inaccurate, as is confirmed by the simulations in Tables A1, A2, A3, A4, B1, B2, B3, and B4 (in Chart 4A in Chapter-4). In sections of each of these tables, the discount rate (r) is changed by 1%, and the resulting values of NPV do not match the formula in Eq. 7.16—in these tables, a 1% change in r causes a quite different change in NPV to that predicted by the power rule and the inverse function rule.

If the inverse function rule is applied to differentiate the TVM equation, the result is:

$$\begin{aligned} \partial N / \partial r &= \sum_{i=n} \left[\left\{ -i / (1+r)^{(i-1)} \right\} / \left\{ (1+r)^{(i*2)} \right\} \right] \\ &= \sum_{i=n} \left[\left\{ -i / (1+r)^{(i-1)} \right\} * \left\{ (1+r)^{(-i*2)} \right\} \right] \end{aligned} \tag{7.23}$$

This result is not correct, as indicated by the simulations in Tables A1, A2, A3, A4, B1, B2, B3, and B4 (in Chart-4A in Chapter-4). Thus, the formulas for both modified duration and convexity of bonds are also inaccurate. Compare comments in Handforth (2004).

However, a closer approximation is:

$$\partial N / \partial r = \sum_{i=n} \left[-i / (1+r)^{(i-1)} \} \right] = \sum_{i=n} \left[(-i) * (1+r)^{(-i-1)} \} \right] \tag{7.24}$$

Eq. 7.24 is not entirely accurate but is the closest to the results of the simulations in Tables A1, A2, A3, A4, B1, B2, B3, and B4 (in Chart 4A in Chapter-4). Eq. 7.18 is more accurate because elimination of the periodic project benefit (x_i) from the numerator of the result is required in the TVM formula, for each minute change in r, the periodic project benefit (x_i) does not change, and thus should removed from the differentiation result.

Similarly, consider an equation $y = x^3$. Under the power rule, $dy/dx = 3x$, Chart 7A shows the various values of x, x^3, $3x$, and the actual values of dy/dx. However, for each one unit increase in x, y changes by more than $3x$.

Chart 7B also shows the values of the differentiation of $y = x^4+3x$. Here, $dy/dx = 4x^3+3$. However, for each one unit increase in x, y changes by more than $4x^3+3$. The table assumes that the x in $4x^3+3$ refers to the magnitude of the change in x, and not to the initial or ending values of x (which is a liberal interpretation of the power rule).

These simulations show that the power rule is inaccurate. A new derivation of the power rule is:

$$Let\ y = ax^n; \tag{7.25}$$

Assume that x_1 is the starting value which changes to x_2; and dx=x_2–x_1 = b;

$$\mathrm{d}y = (a)\left\{(x_1 + b)^n - x_1^{\ n}\right\} \tag{7.26}$$

The only change in the right side of Eq. 7.20 is the change in x. If both sides of 7.20 are divided by dx, then:

$$\mathrm{d}y / \mathrm{d}x = \left[(a)\left\{(x_1 + b)^n - x_1^{\ n}\right\}\right] / b = \mathrm{a}\left[\left\{(x_1 + b)^n - x_1^{\ n}\right\} / b\right] \tag{7.27}$$

But as $b \to +\infty$ and as $b \to -\infty$, d$y \to ab^n$; and dy/d$x \to ab^{(n-1)}$; this rule provides a vastly different result when x changes by very large amounts. The other issue in the standard power rule, is the expansion of $(x_1+b)^n$ with the *Binomial Theorem*, which states that:

$$(x_1 + b)^n = \sum_{k=n}\left[\left\{n!/\left(k!(n-k)!\right)\right\} b^{(n-k)} x_1^{\ k}\right] \tag{7.28}$$

As shown in Chapter-5, the Binomial Theorem is inaccurate, and, the expansion of $(x_1+b)^n$ with the Binomial Theorem differs substantially from the result achieved by using a manual expansion of terms. See Purcell and Varberg (1987: 103), which explains the standard, generally accepted proof of the power rule and focuses on limits when b approaches zero, and so their solution approximates reality only for very small values of b that are around zero.

But as $b \to 0$, if the Binomial Theorem is used for expansion of $(x_1+b)^n$, then

$$\mathrm{d}y / \mathrm{d}x = \lim_{b \to 0} \mathrm{an}\left\{x_1^{\ (n-1)}\right\}; \tag{7.29}$$

(as noted in Purcell & Varberg (1987:103)) which is the standard power rule. That may be an accurate approximation only for extremely minute numbers around zero. Also, the standard power rule is problematic in at least two ways: (1) the x_1 in $nx_1^{(n-1)}$, can be interpreted to refer to the initial value of x, and not to the magnitude of the actual change in x; (2) the standard power rule and the inverse function rule provide vastly different results for the differentiation of $y= ax^{-n}$, which is the standard component of TVM equation. Ideally, both the power rule and the inverse function rule should provide the same differentiation results. ■

Theorem 7.13: For all TVM equations with equal time periods, and for cashflows in which the number of negative signs exceeds positive signs and the sum of the cashflows (without regards to time-value) is negative; where the discount-rates are between –∞% and 101% (between but not including –99% and 0%), the NPV is positive (negative) and the rate of increase of the NPV increases (decreases) significantly as the discount rate increases.

Proof: This anomaly is evident in Tables B6, B7 and B8 in Chart-4B in Chapter-4. When the discount rates are in the interval (–∞%; –101%), the NPVs are positive and increasing and this anomaly occurs because the absolute-magnitude of the compounded discount factor [i.e., $|(1+r)^i|$] declines as r→ –101%; and the compounding effect of (1+r) declines more rapidly as the discount rate increases within that interval (as r →–101%, the rate of change of [$(1+r)^i$] increases); and the signs of the compounded discount factors interact with the signs of the cashflows to produce positive NPVs. In the interval of discount rates between but not including –99% and 0%, the NPVs are negative and declining and this anomaly occurs because the compounded discount factor [ie. $(1+r)^i$] is less than one and is always positive and increases as r →0%; and the compounding effect of $(1+r)^i$ increases more rapidly as the discount rate r →0%; and the signs of the compounded discount factors interact with the signs of the cashflows to produce negative NPVs. ■

Theorem 7.14: Contrary to Saak & Hennesey (2001) (and other researchers), a set of cashflows may exhibit anomalous NPV when there is only one *or zero sign changes – and for all TVM equations with equal (annuity type) periodic cashflows and one or zero sign changes, the NPV can be anomalous when the discount rates are between −250% and −126%.*

Proof: This is evident in Lines 12A-13A and 12B-13B in Tables 1-10 (in Chart-5A in Chapter-5) where each such TVM equation has only one or no sign change but exhibits anomalous NPV when the discount rates are between −250% and −126%. Most of the cashflows in each TVM equations Lines 12A-13A are positive, and the anomalous (non-monotonic) NPVs are caused by the combined effect of the signs of the discount rates, and the declining absolute magnitude of the discount rates (ie. $|r|$; or 2.50……1.26). The necessary and sufficient conditions for monotonic NPV which were supposedly developed in Saak & Hennesey (2001) are not valid because of compounding; and the conditions are not valid in the domain of negative interest rates. Even if those conditions were valid, they are very rare in real life such that the theories are really not applicable. ■

Theorem 7.15: Contrary to Blaset Kastro & Kulakov (2016) and Kulakova & Kulakov (2013) (and many other researchers – eg. those cited in Magni (2016) - who have developed necessary and sufficient conditions), a project cashflow that has only one sign change (a "conventional project") does not always have a unique IRR.

Proof: This is evident in the TVM equation in Line-14A and the NPVs in Line-14B in Tables 2, 3 & 4 in Chart-5A in Chapter-5 – the TVM equation has only one sign change but has at least two IRRs which occur at around r=100%, and also when r is between 300% and 500% (local maxima/minima). The multiple IRRs for such project cashflows occur because of the combination of the one change of the signs of the cash flows and compounding effects. Blaset Kastro & Kulakov (2016) listed the many researchers that developed sufficient conditions of a "conventional" project. Doraszelski (2001), Osborne (2010), Sugden & Williams (1978) and Robison & Barry (1996) and other researchers also erroneously concluded that when the project cash flows change sign more than once, there may be multiple IRRs. For example the TVM equation in each of Tables B6, B7 and B8 (in Chart-4B in Chapter-4) changes sign more than once but in each case, there is no IRR (using MS-Excel). Similarly, contrary to the literature, where there is a series of only negative cash flows followed by a series of only positive ones (or a series of only positive cash flows followed by a series of only negative cashflows), there isnt always a unique IRR. The argument for the existence of a unique IRR is that the resulting TVM equation is continuous and monotonically decreasing from positive infinity (when the rate of return approaches −100%) to the value of the first cash flow (when the rate of return approaches infinity), and so there is a unique IRR. Again, in the domain of positive discount rates, due to compounding effects, and or at sufficiently high discount rates its possible that the resulting NPV = 0 and or $\partial N/\partial r = 0$ at more than one point, and there can be multiple-IRRs. In the domain of negative discount rates, due to compounding effects, and or at sufficiently high discount rates, and or because of the number of time periods (odd or even), its possible that the resulting NPV = 0 and or $\partial N/\partial r = 0$ at more than one point. However for both monotonic and non-monotonic NPVs, as the discount rate increases towards infinity, $\partial N/\partial r \to 0$, and multiple IRRs are likely. ■

Theorem 7.16: Contrary to Oehmke (2000), when investment in the project occurs entirely at time-zero, and net cashflows are positive thereafter, the sign of $\partial N/\partial r$ (the rate of change of NPV with respect to the discount rate) is not unambiguously negative, and an increase in the discount rate doesn't always decrease the NPV of the project.

Proof: This is evident in the TVM equation in Line-14A and the NPVs in Line-14B in Tables 2, 3 & 4 in Chart-5A in Chapter-5. The TVM equation has only one sign change (the initial investment at time zero, and thereafter, positive cashflows) and the NPV decreases when the discount rate increases from −10% to 100% but also starts increasing as the discount rate increases from 300% to 500%. The TVM equation has at least two IRRs which occur at around r=100%, and also when r is between 300% and 500% (local maxima/minima). The changes in the direction of the NPV for such project cashflows occur because of the relatively high magnitude of the discount rates (100% and above) and compounding effects. ■

Theorem 7.17: For all TVM equations with equal time periods, multiple positive IRRs (positive roots - real roots or local maxima/minima) are more likely as the sum of the periodic cashflows (without time value) tends to a lower negative number; and where the majority of cash flows have negative signs; and the average of the cashflows is a negative number; and as the number of sign changes (of consecutive-period cashflows) tends to the number of time periods.

Proof : This is evident by comparing Tables A3, A4, B3 & B4 in Chart-4A in Chapter-4 (each has at least four roots/IRRs and at least four time periods) with Tables B6, B7, B8 & B9 in Chart-4B and Table B10 in Chart-4C (in Chapter-4) each of which doesn't have any positive IRRs within the intervals of discount rates shown (calculated with MS Excel). An IRR occurs where NPV=0, or $\partial N/\partial r = 0$; where N is the NPV and r is the discount rate. By simple inspection of the standard TVM equation and for monotonic NPVs, its easy to see that: 1) in most TVM equations, given a constant set of cashflows but changing discount rates, as NPV→0, $|\partial N/\partial r| \rightarrow 0$; and as NPV→−∞, $|\partial N/\partial r| \rightarrow 0$ (and this is evident in Tables A1, A2 & A3 in Chart-4A in Chapter-4); and 2) for most TVM equations and most positive discount rates and given a constant discount rate, as B (the sum of all the periodic cashflows without time value) tends to a lower negative number (that is, as B → (−y; 0), where −y depends on the magnitude of B), NPV→ (0,−∞), and $\partial N/\partial B \rightarrow 0$ (and this is evident in Lines 14B-23B in Tables 1-10 in Chart-5A in Chapter-5); and 3) as the number of cashflows with negative signs increases to a simple majority, B →0, and NPV→-∞, and for most positive discount rates, $|\partial N/\partial r| \rightarrow 0$;

and 4) as the average of all the periodic cashflows (without time value) tends to (0,−∞), NPV→0, and $|\partial N/\partial r| \rightarrow 0$; and 5) as the number of sign changes (of consecutive-period cashflows) increases to the number of time periods, in most TVM equations, B →0, and NPV→0, and for most positive discount rates, $|\partial N/\partial r| \rightarrow 0$.

By simple inspection of the standard TVM equation, its easy to see that for non-monotonic NPVs: 1) in most TVM equations, given a constant set of cashflows but changing discount rates, as NPV→0, $|\partial N/\partial r| \rightarrow 0$; and as NPV→−∞, $|\partial N/\partial r| \rightarrow 0$ (and this is evident in Tables B6, B7 & B8 Chart-4B in Chapter-4); and 2) for most TVM equations, and most positive discount rates and given a constant discount rate, as B (the sum of all the periodic cashflows without time value) tends to a lower negative number (that is, as B → (−y; 0), where −y depends on the magnitude of B), NPV→ (0,−∞), and $\partial N/\partial B \rightarrow 0$ (and this is evident in Lines 1B-6B in Tables 1-10 in Chart-5A in Chapter-5); and 3) as the number of cashflows with negative signs increases to a simple majority, B → (0,−∞), and NPV→(0,−∞), and for most positive discount rates, $|\partial N/\partial r| \rightarrow 0$; and 4) as the average of all the periodic cashflows (without time value) tends to zero, NPV→0, and $|\partial N/\partial r| \rightarrow 0$; and 5) generally, as the number of sign changes (of consecutive-period cashflows) increases to the number of time periods, in most TVM equations, B →0, and NPV→0, and for most positive discount rates, $|\partial N/\partial r| \rightarrow 0$. Thus contrary to Ng & Beruvides (2015), Multiple-IRRs are not unusual and are more likely to occur as the number of sign changes in a TVM equation increases. Many authors have developed "necessary and sufficient conditions" for a unique non-negative IRR, but even if they are valid, such conditions are often unusual or even rare such that those theories are not really applicable (for example, see Russell & Rickard (1982) and Weber (2014)). ■

Chart 7A

		First Derivative Of X^3	
X	X^3	Predicted (3x)	Actual
1	1		
2	8	3	7
3	27	3	19
4	64	3	37
5	125	3	61
6	216	3	91
7	343	3	127
8	512	3	169
9	729	3	217
10	1,000	3	271
11	1,331	3	331
12	1,728	3	397
13	2,197	3	469
14	2,744	3	547
15	3,375	3	631
16	4,096	3	721
28	21,952	36	17,856
148	3,241,792	360	3,219,840
1,448	3,036,027,392	3,900	3,032,785,600
2,448	14,670,139,392	3,000	11,634,112,000
3,448	40,992,251,392	3,000	26,322,112,000
12,448	1,928,851,259,392	27,000	1,887,859,008,000
828,448	568,585,475,534,651,000	2,448,000	568,583,546,683,392,000
1,926,558	7,150,662,238,058,930,000	3,294,330	6,582,076,762,524,280,000
3,966,558	62,408,167,007,926,600,000	6,120,000	55,257,504,769,867,700,000

Chart 7B

x	$x^4 + 3x$	First Derivative Of X^4+3x Predicted ($4x^3$+3)	Actual
1	1		
2	8	7	7
3	27	7	19
4	64	7	37
5	125	7	61
6	216	7	91
7	343	7	127
8	512	7	169
9	729	7	217
10	1,000	7	271
11	1,331	7	331
12	1,728	7	397
13	2,197	7	469
14	2,744	7	547
15	3,375	7	631
16	4,096	7	721
28	21,952	6,915	17,856
148	3,241,792	6,912,003	3,219,840
1,448	3,036,027,392	8,788,000,003	3,032,785,600
2,448	14,670,139,392	4,000,000,003	11,634,112,000
3,448	40,992,251,392	4,000,000,003	26,322,112,000
12,448	1,928,851,259,392	2,916,000,000,003	1,887,859,008,000
828,448	568,585,475,534,651,000	2,173,353,984,000,000,000	568,583,546,683,392,000
1,926,558	7,150,662,238,058,930,000	5,296,604,324,714,920,000	6,582,076,762,524,280,000
3,966,558	62,408,167,007,926,600,000	33,958,656,000,000,000,000	55,257,504,769,867,700,000

7.3 Conclusion

Clearly, there are significant anomalies in the NPV-MIRR model (and related approaches such as APV, NFV, SVA, EVA) that arise due to compounding; the signs/magnitude of the discount rates; and the signs of the cashflows (in addition to Framing Effects). The NPV-MIRR-IRR model can be replaced with Regret-based models. The main disadvantage is that currently, there is no unified method for calculating Regret (which can address taxes, transaction costs, and human biases inflexibility, Real Options, framing effects, and other factors).

Bibliography

Altshuler, D., & Magni, C. A. (2012). Why IRR is not the rate of return on your investment: Introducing the AIRR to the real estate community. *Journal of Real Estate Portfolio Management, 18*(2), 219–224. http://dx.doi.org/10.2139/ssrn.1825544.

Aven, T., & Flage, R. (2009). Use of decision criteria based on expected values to support decision-making in a production assurance and safety setting. *Reliability Engineering and System Safety, 94*, 1491–1498.

Barry, P., & Robison, L. (2014). Technical note: Economic rates of return and investment analysis. *The Engineering Economist, 59*(3), 231–236.

Bas, E. (2013). A robust approach to the decision rules of NPV and IRR for simple projects. *Applied Mathematics and Computation, 219*(9), 5901–5908.

Ben-Horin, M., & Kroll, Y. (2012). The limited relevance of the multiple IRRs. *The Engineering Economist, 57*(2), 101–118.

Biondi, Y. (2006). The double emergence of the modified internal rate of return: The neglected financial work of Duvillard (1755–1832) in a comparative perspective. *The European Journal of the History of Economic Thought, 13*(3), 311–335.

Blaset Kastro, A., & Kulakov, N. (2016). Definition of the concepts of conventional and non-conventional projects. *Business Informatics, 2*(36), 16–23.

Borgonovo, E., & Percocao, M. (2012). A note on the sensitivity analysis of the internal rate of return. *International Journal of Production Economics, 135*(1), 526–529.

Booth, L. (2003). Discounting expected values with parameter uncertainty. *Journal of Corporate Finance, 9*, 505–519.

Califf, R. M., Rasiel, E. B., & Schulman, K. A. (2008). Considerations of net present value in policy making regarding diagnostic and therapeutic technologies. *American Heart Journal, 156*(5), 879–885.

De Reyck, B., Degraeve, Z., & Vandenborre, R. (2008). Project options valuation with net present value and decision tree analysis. *European Journal of Operational Research, 184*, 341–355.

Dixit, A. K., & Pindyck, R. S. (1995, May–June). The options approach to capital investment. *Harvard Business Review, 73*, 105–115.

Domingo, C. J. (2001). Anomalies in net present value calculations? *Economics Letters, 72*, 127–129.

Doraszelski, U. (2001). The net present value method versus the option value of waiting: A note on Farzin, Huisman and Kort (1998). *Journal of Economic Dynamics & Control, 25*, 1109–1115.

Dybvig, P. (1983). Duality, interest rates, and the theory of present value. *Journal of Economic Theory, 30*(1), 98–114.

Groenendaal, W. (1998). Estimating NPV variability for deterministic models. *European Journal of Operational Research, 107*, 202–213.

Haley, G., & Goldberg, S. (1995). Net present value techniques and their effects on new product research. *Industrial Marketing Management, 24*, 177–190.

Hall, P., Edlin, R., Kharroubi, S., et al. (2012). Expected net present value of sample information: From burden to investment. *Medical Decision Making, 32*, E11–E21.

Hanafizadeh, P., & Vahideh, L. (2011). Robust net present value. *Mathematical and Computer Modelling, 54*(1–2), 233–242.

Handforth, F. (2004). Duration and convexity of mortgages in the context of real estate investment analysis. *Journal of Real Estate Portfolio Management, 10*(3), 187–192.

Hazen, G. B. (2003). A new perspective on multiple internal rates of return. *The Engineering Economist, 48*(1), 31–52.

Kulakov, N., & Blaset Kastro, A. (2015). Evaluation of nonconventional projects: GIRR and GERR vs MIRR. *The Engineering Economist, 60*(3), 183–196.

Kulakova, A. N., & Kulakov, N. J. (2013). Evaluation of non-conventional projects. *The Engineering Economist, 58*(2), 137–148.

Leyman, P., & Vanhoucke, M. (2016). Payment models and net present value optimization for resource-constrained project scheduling. *Computers & Industrial Engineering, 91*, 139–153.

Liu, L., Rettenmaier, A. J., & Saving, T. (2004). Present value criterion: The case of differing borrowing and lending rates. *Economics Letters, 84*, 427–432.

Lohmann, J. R. (1988). The IRR, NPV and the fallacy of the reinvestment rate assumption. *The Engineering Economist, 33*(4), 303–330.

Magni, C. (2002). Investment decisions in the theory of finance: Some antinomies and inconsistencies. *European Journal of Operations Research, 137*, 202–216.

Magni, C. A. (2005). On decomposing net final values: EVA, SVA and shadow project. *Theory and Decision, 59*(1), 51–95.

Magni, C. A. (2007). Project valuation and investment decisions: CAPM versus arbitrage. *Applied Financial Economics Letters, 3*(1), 137–140.

Magni, C. (2009a). Splitting up value: A critical review of residual income theories. *European Journal of Operational Research, 198*, 1–22.

Magni, C. A. (2009b). Investment decisions, NPV and bounded rationality. *Quantitative Finance, 9*(8), 967–979.

Magni, C. (2010). CAPM and capital budgeting: Present/future, equilibrium/disequilibrium, decision/valuation. *IUP Journal of Financial Economics, 8*(1/2), 7–35.

Magni, C. A. (2011). Addendum to "average internal rate of return and investment decisions: A new perspective". *The Engineering Economist, 56*(2), 140–169.

Magni, C. A. (2013). The Internal-Rate-of-Return approach and the AIRR paradigm: A refutation and a corroboration. *The Engineering Economist, 58*(2), 73–111.

Magni, C. (2015). Aggregate Return On Investment for investments under uncertainty. *International Journal of Production Economics, 165*, 29–37.

Magni, C. (2016). Capital depreciation and the underdetermination of rate of return: A unifying perspective. *Journal of Mathematical Economics, 67*, 54–79.

McCabe, C., Edlin, R., & Hall, P. (2013). Navigating time and uncertainty in health technology appraisal: Would a map help? *PharmacoEconomics, 31*(9), 731–737.

Mellichamp, D. (2013). New discounted cash flow method: Estimating plant profitability at the conceptual design level while compensating for business risk/uncertainty. *Computers & Chemical Engineering, 48*(10), 251–263.

Merlo, P. (2016). Consequences of the absence of monotonicity of the NPV function to the assessment of effectiveness of investment projects. *Engineering Economics, 27*(1), 39–46.

Naim, M. (2006). The impact of the net present value on the assessment of the dynamic performance of e-commerce enabled supply chains. *International Journal of Production Economics, 104*, 382–393.

Ng, E., & Beruvides, M. (2015). Multiple internal rate of return revisited: Frequency of occurrences. *The Engineering Economist, 60*(1), 75–87.

Nwogugu, M. (2010). *On regret theory, and anomalies in polynomials and net-present-value decisions*. www.ssrn.com. September 10, http://dx.doi.org/10.2139/ssrn.1925458.

Nwogugu, M. (2012). *Risk in the global real estate market*. Hoboken: John Wiley & Sons.

Oehmke, J. F. (2000). Anomalies in net present value calculations. *Economics Letters, 67*, 349–351.

Osborne, M. (2010). A resolution to the NPV–IRR debate? *The Quarterly Review of Economics and Finance, 50*, 234–239.

Padberg, M., & Wilczak, M. J. (1999). Optimal project selection when borrowing and lending rates differ. *Mathematical and Computer Modelling, 29*, 63–78.

Padilla, E., Jadotteb, E., & Pasqula, J. (2013). Technical note: Equivalence of different profitability criteria with the net present value. *International Journal of Production Economics, 142*(1), 205–210.

Pressacco, F., Magni, C., & Stucchi, P. (2014). A quasi-IRR for a project without IRR. *Frontiers in Finance & Economics, 11*(2), 1–23.

Promislow, D. S., & Spring, D. (1996). Postulate for an internal rate of return for an investment project. *Journal of Mathematical Economics, 26*, 335–345.

Purcell, E. J., & Varberg, D. (1987). *Calculus with analytic geometry* (5th ed.). Prentice Hall, USA.

Remer, D., & Nieto, A. (1995). A compendium and comparison of 25 project evaluation techniques. Part 2: Ration, payback, and accounting methods. *International Journal of Production Economics, 42*(2), 101–129.

Robison, L. J., & Barry, P. J. (1996). *Present value models and investment analysis*. Northport: The Academic Page.

Rocabert, J., Tarrio, J., & Perez, M. (2005). Anomalies in net present value calculations. A solution. *Revista de Economía Pública, 173*(2), 47–60. https://core.ac.uk/download/pdf/6835918.pdf.

Ross, S., Spatt, C., & Dybvig, P. (1980). Present values and internal rates of return. *Journal of Economic Theory, 23*, 66–70.

Russell, A., & Rickard, J. (1982). Uniqueness of non-negative internal rate of return. *Journal of the Institute of Actuaries, 109*, 435–445.

Saak, A., & Hennesey, D. (2001). Well behaved cashflows. *Economic Letters, 73*, 81–88.

Sen, A. (1975). Minimal conditions for monotonicity of capital values. *Journal of Economic Theory, 11*, 340–343.

Shea, K., Tang, T., & Tso, P. (2000). Optimal investment sequence. *Economic Theory, 15*(1), 215–219.

Simerska, C. (2008). Remarks on the economic criterion-the internal rate of return. In: J. Chleboun, P. Přikryl, K. Segeth, & T. Vejchodský (Eds.): *Programs and algorithms of numerical mathematics, Proceedings of seminar*, Dolní Maxov, June 1–6, 2008 (pp. 170–176). Prague: Institute of Mathematics of the Czech Academy of Science. http://dml.cz/bitstream/handle/10338.dmlcz/702871/PANM_14-2008-1_24.pdf.

Simerska, C. (2008). Remarks on the economic criterion-the Internal rate of return. *Programs and Algorithms of Numerical Mathematics, 14*, 170–176.

Sugden, R., & Williams, A. (1978). *The principles of practical cost-benefit analysis*. Oxford: Oxford University Press.

Trigeorgis. (1996). *Real options*. Cambridge: MIT Press.

Trippi, R. (1989). A discount rate adjustment for calculation of net present values and internal rates of returns on investments whose lives are uncertain. *Journal of Economics and Business, 41*, 143–151.

Trusheim, M. R., Burgess, B., Hu, S. X., et al. (2011). Quantifying factors for the success of stratified medicine. *Nature Reviews Drug Discovery, 10*(11), 817–833.

Tsao, C. (2012). Fuzzy net present values for capital investments in an uncertain environment. *Computers & Operations Research, 39*(8), 1885–1892.

Vassallo, J. (2010). The role of the discount rate in tendering highway concessions under the LPVR approach. *Transportation Research Part A, 44*, 806–814.

Walthe, H. (2010). Anomalies in inter-temporal choice, time-dependent uncertainty and expected utility—A common approach. *Journal of Economic Psychology, 31*(1), 114–130.

Weber, T. (2014). On the (non-)equivalence of IRR and NPV. *Journal of Mathematical Economics, 52*, 25–39.

8

Some Biases and Evolutionary Homomorphisms Implicit in the Calculation of Returns

Investment returns are calculated in different ways and over different horizons and, regardless of the method of computation, they often contain substantial biases, which distort results in the end-uses. These biases tend to have adverse multiplier effects because investment returns are the foundation or core elements in various statistical analyses such as: (1) developing distributions of expected returns; (2) time series analysis; (3) developing options pricing models; (4) estimating various types of risk; (5) developing pricing models for insurance; (6) project evaluation and capital-budgeting. Some of those biases are Framing Effects with the Frames defined by the time horizon, the magnitude of the time-periods and the magnitude of interest rates (ie. positive or negative interest rates). The two most popular methods for calculating long-term returns are simple returns and compounded returns (sometimes expressed as natural logs), which yield vastly different results that have significant computational consequences. This chapter: (1) explains the biases and effects inherent in the calculation of compounded returns (distinct from human biases that can affect returns, and vice versa), which, in turn, implies that various investment approaches (such as minimum variance investing;

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geometric mean maximization; etc) are wrong and inaccurate (some of the biases introduced herein are new types of evolutionary homomorphisms); (2) shows how biases in returns and the calculation methods, can affect the analysis of pattern formation, chaos and adaptive systems—the discussions in Preis et al. (2012), Kenett et al. (2012), Preis (2011), Podobnik et al. (2009), Fenn, et al. (2011), Kriener et al. (2014), Hsieh (1993), Menna et al. (2002), and Hommes (2002) omitted such effects; and (3) explains some of the possible effects of market volatility on compounded returns.

8.1 Existing Literature

The literature on biases in returns is substantial, but it contains gaps on the subject and on the characterization of specific phenomena in the calculation of returns.

Blume and Stambaugh (1983) found that previous estimates of a "size effect" based on daily returns data were biased, and that the use of quoted closing prices in computing returns on individual stocks imparts an upward bias. They noted that Returns computed for buy-and-hold portfolios largely avoid the bias induced by closing prices; and based on such buy-and-hold returns, the full-year size effect is half as large as previously reported, and all of the full-year effect is, on average, due to the month of January. Roll (1983) found that the mean return computational method has a substantial effect on the estimated small-firm premium; and that the buy-and-hold method produces an estimated small-firm premium only one-half as large as the arithmetic and rebalanced methods often used in empirical studies. Hillion and Suominen (2004) found that before the introduction of a call auction at the close, the last minute of trading at the Paris Bourse was the most active of the whole day; and although the bid–ask spread increased substantially, the probability of large and aggressive orders increased, as did price volatility, one-minute returns, and the proportion of partially hidden orders. Hillion and Suominen (2004) developed an agency-based model of closing price manipulation (which can account for these phenomena) and the optimal closing price mechanism under manipulation.

Comerton-Forde and Putniņš (2010) quantified the effects of closing price manipulation on trading characteristics and stock price accuracy using a unique sample of prosecuted manipulation cases. Based on these findings they developed an index of the probability and intensity of closing price manipulation.

In many fields—such as engineering, physics, finance, mathematics and psychology—returns are sometimes expressed in terms of natural logarithms. Graff (2014) noted that most psychophysical investigations measure stimuli or performance in Système International units and use relative differences between them for comparison. Graff (2014) proposed the ratio's natural logarithm or the difference between the Napierian logarithms as a desirable measure of relative differences between two psychophysical quantities (and stated that it can be conveniently expressed as a percentage). Graff (2014) concluded that this measure satisfies all three of the following properties: *symmetry* (i.e., agreement between inverted units, such as hertz); *additivity*, and compliance with the *Weber–Fechner* and *Stevens Laws*.

Keim (1989) found that returns computed with closing bid or ask prices that may not represent "true" prices introduce measurement error into portfolio returns if there are systematic patterns in investor buying and selling. Kallunki and Martikainen (1997) analyzed the Finnish stock markets and found that the accommodation of price adjustment delays of three days roughly doubles the covariation in daily returns for the most frequently traded stocks, but for the sample of most infrequently traded stocks, the increase is as high as 360–490%. Handa et al. (1989) found that the size effect is sensitive to the length of the return interval used in estimating betas; and that beta changes with the return interval because an asset's covariance with the market and the market's variance do not change proportionately as the return interval is changed; and that the size effect becomes statistically insignificant when risk is measured by betas estimated using annual returns. Choe and Shin (1993) studied both interday and intraday return volatility of the Korea Stock Price Index and noted that open-to-open return variance is consistently greater than close-to-close return variance; and that open-to-open return volatility is greater than interday return volatility measured at any other time of the day. Antoniou et al. (2004) found that for some stocks the statistical distribution of closing prices normalized by the corresponding traded volumes, fits a log-normal law; but for other stocks, the log-normal law is obtained after application of a de-trending procedure. Ahn et al. (2005) observed an abnormally high frequency of even and integer prices in trade and quote prices for all tick size groups on the Hong Kong Stock Exchange (SEHK), and found that the deeper quotes displayed greater clustering than the best quotes, which implies that the distance of the quotes from the best queue is inversely proportional to the amount of information they carry. Ahn et al. (2005) also found that an extremely fine tick size hinders the price resolution process; and that short sale prohibitions imposed on the majority of stocks listed on the SEHK causes a significant bias in clustering towards the ask side of the limit order book.

Dorfleitner (2003) explained the basis of, and differences between, compounded log returns (continuously compounded returns) and compounded simple-interest returns, and found that for small returns around zero, $\ln(1 + R_t)$ is approximately equal to R_t (for instance $\ln(1 + 0.02) = 0.0198$). Dorfleitner (2003) explained that although it is often claimed that simple returns can generally be approximated by log returns, both types of returns may be used interchangeably only in a few special circumstances. However, the examples (portfolio analysis and calculation of beta) introduced and discusssed in Dorfleitner (2003) are based on normally distributed log-returns and the mean-variance framework, which are very unrealistic.

Goetzmann et al. (2007) showed that circumvention of performance measurement systems has a substantial impact on popular measures, even in the presence of high transactions costs; and they characterize the conditions under which a manipulation-proof measure exists (that performance measure has been criticized in the literature).

Booth (2002) found that the standard compounding and the standard NPV models underestimate future cash flows when growth rates are serially correlated—i.e., standard compounding underestimates future cash flows. The Booth (2002) results explain a relevant but unrecognized distortion in returns.

The optimality criteria and the concept of optimal geometric returns that are described in Zhang (2012) are inaccurate and have no basis in empirical or theoretical finance because: (1) the binomial option pricing model and continuous stochastic models have repeatedly been found to be inappropriate and inaccurate—see Taleb (2009) and the discussion about the Binomial Theorem in chapter-5 in this book; (2) rationality implies that investors will try to maximize their returns (even where there may be other factors such as taxation)—and there is no need for the returns from a stock and its option to be the same, and any such occurrence does not imply or guarantee a maximum theoretical or market value for the option or the stock; (3) while Bickel (1969) discovered the relationship between optimal long-run growth rate and the efficient portfolios based on the minimum variance criterion, Scherer (2011) noted the errors and problems inherent in minimum-variance investing; (4) MacLean et al. (1992) summarized the good and bad properties of the Kelly Criterion; (5) in Zhang (2012), equations 2.1 and 2.3 are wrong; (6) the existing literature on geometric mean maximization as an investment approach is wrong because of the biases/anomalies introduced in this chapter.

Although the *leverage effect* (the generally negative correlation between an asset's return and changes in its volatility) is well described in the literature, Ait-Sahalia et al. (2013) found that there was near-zero correlation between daily returns and changes in daily volatilities (estimated from high frequency data) for various types of assets (which they called the leverage effect puzzle). Ait-Sahalia et al. (2013) analyzed the different asymptotic biases that are involved in high-frequency estimation of the leverage effect (such as biases due to discretization errors; to smoothing errors in estimating spot volatilities; to estimation errors; and to market microstructure noise) and proposed bias-correction methods for estimating the leverage effect. However, those bias-correction methods of Ait-Sahalia et al. (2013) and Fisher et al. (2009, 2010), are inaccurate for the following reasons: (1) the biases introduced in this article affect the calculation of periodic (intraday and daily) returns and volatilities; (2) Taleb (2009) has shown that stochastic volatility models are wrong; and (3) Nwogugu (2013) has shown that variance, covariance and correlation are irrelevant (or not as useful as previously thought) in risk analysis.

Furthermore, the *leverage effect* puzzle may be explained by other phenomena that were not studied by Ait-Sahalia et al. (2013), such as: (1) price-clustering (described in Ahn et al. 2005); (2) the bid-ask bias and the size effect (described in Branch and Echevarria 1998); (3) the biases mentioned in Dorfleitner (2003); (4) commonality effects (described in Chae and Yang 2013); (5) statistical biases described in Cheng and Deets (1971), Indro and Lee (1997), and Conrad and Kaul (1993); and (6) the statistical biases developed/introduced in this chapter.

Branch and Echevarria (1998); Strong (2006); Roll (1983); Liu and Strong (2008); Blume (1974); Cooper (2006); Jacquier et al. (2003, 2005); Indro and Lee (1997); Cheng and Deets (1971), Jean and Helms (1983); Fisher et al. (2009, 2010); Dorfleitner (2003); Keim (1989); Wong (2009) (modeling with diffusion processes); Verwaeren et al. (2013) (negative search bias); Zhao and Fang (2013), Ait-Sahalia et al. (2013), Cowan and Sergeant (2001), Zhu (2002), Asparouhova et al. (2010), and Scherer (2011) all, analyzed various issues pertaining to biases inherent in the calculation of returns of both single-assets and indices.

Wen et al. (2007), Vamvakoussi et al. (2012), Akhtar et al. (2011), Cao and Shan (2013) (bi-random returns), Benos and Jochec (2012), Ortoleva (2010), and Chae and Yang (2013) analyzed various human behavioral biases that can affect investment returns, and can amplify the statistical biases introduced in this chapter.

Gerrard and Haberman (1996), Bakken et al. (2006), and Guillen et al. (2006), analyzed returns within the context of insurance and pensions.

The rest of this chapter discusses the inherent anomalies and biases in compounded returns and introduces new theorems pertaining to compounding and volatility.

8.2 Biases in Compounded Returns

Many studies have documented biases in returns calculated using arithmetic or geometric means. These biases tend to be magnified as the time horizon increases (Roll 1983; Lakonishok and Smidt 1984; Keim 1989; Kallunki and Martikainen 1997; Handa et al. (1989), Ahn et al. (2005), and Booth (2002). Apart from biases that arise solely from the mathematical methods of computing returns, other sources of biases in returns include transaction costs, rebalancing costs, fees, and taxes.

Theorem 8.1: Positive-returns Effect-For all cash flows with uncorrelated growth rates, when compounding returns obtained during equal time intervals (such as daily returns), and for all returns series of any magnitude, a one-period negative return (loss) has a lower effect on the long-term compounded return of the return series, than a one-period positive return (gain) of the same absolute magnitude.

Proof: Assume that cash flows are the same, and absolute-returns are the same - that is, $|r^-| = |r^+|$. Let R be the compounded return; and r is the periodic return and n is the number of periods so that $R = (1+r)^n$. When r is negative (r^-), $(1+r^-)$ will be less than one, and if $n > 0$, then $(1+r^-)^n \to 0$, as $n \to \infty$. Conversely, when r is positive (r^+), $(1+r^+)$ will be greater than one, and if $n>0$, then $(1+r^+)^n \to \infty$, as $n \to \infty$. The key issue is that as n increases, the rate at which $(1+r^-)^n$ approaches zero, is less than the rate at which $(1+r^+)^n$ approaches positive infinity. That is, $\partial\{(1+r^-)^n\}/\partial n < \partial\{(1+r^+)^n\}/\partial n$. The *positive returns effect* is illustrated in Chart 8A where $r^- = -10\%$, and $r^+=10\%$. The correct measure of the effect of a negative or a positive periodic return is the deviation of the compounded return from one, because if r is zero, the compounded return will be one. In Chart 8A, the deviations from one for the 10% return far exceed the deviations from one for the −10% return.

This positive returns effect persists in both low and high interest rate environments, and also persists in both upward and downward trending markets. ■

Theorem 8.2: The Downward Bias Effect-For all cash flows with uncorrelated growth rates, when compounding short-term returns obtained during equal time intervals (such as daily returns), and for all returns series of any magnitude, there is a downward bias in the compounded long-term return (which increases as the compounding horizon increases), if the number of negative returns exceeds the number of positive returns, and the absolute magnitude of total negative periodic returns ($\Sigma|r^-|$) is equal to or exceeds the total positive periodic returns ($\Sigma|r^+|$).

Proof: Let R_c be the compounded return and R_3 the three-year simple return; and r is the periodic return and n is the number of periods, so that

$R_c = (1+r)^n$. Let p and m be the number of positive and negative returns respectively in the returns series, so that $p+m = n$. When r is negative (r^-), $(1+r^-)$ will be less than one, and if $n > 0$, then $(1+r^-)^n \to 0$, as $n \to \infty$. Conversely, when r is positive (r^+), $(1+r^+)$ will be greater than one, and if $n>0$, then $(1+r^+)^n \to \infty$, as $n \to \infty$. If $m>p$, and $\Sigma|r^-| \geq \Sigma|r^+|$, then $(1+r)^n \to 0$, as $n \to \infty$. As n increases, the rate at which $(1+r^-)^n$ approaches zero, is less than the rate at which $(1+r^+)^n$ approaches positive infinity. That is, $\partial\{(1+r^-)^n\}/\partial n < \partial\{(1+r^+)^n\}/\partial n$. As m increases, the rate at which $(1+r)^n$ approaches zero, is less than the rate at which $(1+r)^n$ approaches positive infinity as p increases. That is, $\partial\{(1+r)^n\}/\partial m > \partial\{(1+r)^n\}/\partial p$.

Secondly, compounding does not consider the sequence of the occurrence of the returns, and hence, does not fully account for reinvestment and time value of money. Chart 8B illustrates the downwards bias effect—wherein the compounded return for 36 months (R_c) is always substantially less than the three-year return (R_3). The periodic returns are measured with reference to the portfolio value at the beginning of Period 1. ■

Theorem 8.3: The Negative Returns Effect-For all cash flows with uncorrelated growth rates, when compounding returns obtained during equal time intervals (such as daily returns), and for all returns series whose returns are single-digit and/or double-digit returns (between −99% and 99%), as the number of negative returns increases, the rate of change of the compounded return declines.

Proof: The *negative returns effect* occurs because the numbers involved are all between negative infinity and 0.99, and negative returns are expressed as either negative numbers ($-r$) or numbers that are less than one [i.e., $(1-r)$] (where r is the one-period return). This negative returns effect persists in both low and high interest rate environments, and also persists in both upward and downward trending markets, and can be illustrated by comparing hypothetical leveraged/inverse ETFs with daily compounded positive and negative returns, but with the same magnitude and timing. The negative returns effect is illustrated in Table 1 (of Chart 8C) where the Time Periods 1.....12 are equal, and the series S1.....S39 are returns series. In Table 1 (of Chart 8C), the absolute magnitude of the first cross-sectional difference between the compounded returns of consecutive series declines ($|R_{n+1}–R_n|> |R_n–R_{n−1}|$; and ($|R_1–R_2|>$ ($|R_{12}–R_{13}|$) as the number of negative returns in the series increases. R_1.....R_{26} in Table 1 (of Chart 8C) shows that the negative returns effect is valid only for, but is not proportional across, single-digit and double-digit returns (the negative returns effect does not occur when returns are triple-digit returns, as shown in R_{27}.....R_{39}). Let R be the compounded return, r is the periodic return, and n is the number of periods, so that $R = (1+r)^n$. When r is negative (r^-), $(1+r^-)$ will be less than one, and if $n>0$, then $(1+r^-)^n \rightarrow 0$, as $n \rightarrow \infty$. Conversely, when r is positive (r^+), $(1+r^+)$ will be greater than one, and if $n>0$, then $(1+r^+)^n \rightarrow \infty$, as $n \rightarrow \infty$. Assume that cash flows are the same, and $|r^-|= |r^+|$. The key issue is that as n increases, the rate at which $(1+r^-)^n$ approaches zero, is greater than the rate at which $(1+r^+)^n$ approaches positive infinity. That is, $\partial\{(1+r^-)^n\}/\partial n < \partial\{(1+r^+)^n\}/\partial n$. ■

Theorem 8.4: The Asymmetrical Returns Effect-When compounding returns (for which cash flows/benefits are uncorrelated) obtained during equal time intervals (such as daily returns), and for all returns series whose returns are single-digit and/or double-digit returns (between −99% and 99%), an all-negative returns series will not have a long-term compounded return that is not proportional to an all-positive return series with the same term.

Proof: Let R be the compounded return, r is the periodic return, and n is the number of periods, so that $R = (1+r)^n$. When r is negative (r^-), $(1+r^-)$ will be less than one, and if $n>0$, then $(1+r^-)^n \rightarrow 0$, as $n \rightarrow \infty$. Conversely, when r is positive (r^+), $(1+r^+)$ will be greater than one, and if $n>0$, then $(1+r^+)^n \rightarrow \infty$, as $n \rightarrow \infty$. This divergence is the main cause of the asymmetrical returns effect. Assume that cash flows are the same, and $|r^-|= |r^+|$. The key issue is that as n increases, the rate at which $(1+r^-)^n$ approaches zero, is greater than the rate at which $(1+r^+)^n$ approaches positive infinity. That is $\partial\{(1+r^-)^n\}/\partial n > \partial\{(1+r^+)^n\}/\partial n$. The proof of the asymmetrical returns effect is also illustrated in Table 1 (of Chart 8C), where the return series-13 and series-26 consists of only losses, and the return series-1 and series-14 consist of only gains, but R_1 and R_{14} are not proportional or mirror-images of R_{13} and R_{26} respectively. A comparison of returns in Table 1 (of Chart 8C) shows that the asymmetrical returns effect occurs only when returns are mostly single-digit or double-digit returns (between −99% and 99%). In Table 2 (of Chart 8C), the *asymmetrical returns effect* is evident by comparing lines R_1 and lines R_{13}. ■

Theorem 8.5: When compounding uncorrelated returns (for which the growth rates of cashflows/outcomes are uncorrelated) obtained during equal time intervals (such as daily returns), and for all returns series whose returns are single-digit and/or double-digit returns (between −99% and 99%), the ratio of the rate of increase of the periodic return (in each time period) to the rate of increase of the long-term compounded return is greater than zero but less than one for all single-digit and/or double-digit returns.

Proof: Let R be the compounded return, r is the periodic return, and n is the number of periods so that $R = (1+r)^n$. The *proportionality effect* is that $0<\partial R/\partial r<1$. For all $-.99<r<.99$ and $n>0$, as $r \to 0$, $R \to 1$; and as $r \to +\infty$, $R \to \infty +$; and as $r \to -\infty$, $R \to -\infty$. For all $r<1$ and $n>0$, as $r \to 0$, $R \to 1$; and as $r \to +\infty$, $R \to \infty +$. For all $-1>r$ and $n>0$, as $r \to 0$, the change in R will depend on whether n is an odd or even number. The proof of the proportionality effect is straightforward and is evident by comparing the values of R_1, R_2, $R_{12,}$ and R_{13} in Table 1 (of Chart 8C). The proportionality effect occurs only when returns are all single-digit and/or double-digit returns. ■

Theorem 8.6: When compounding uncorrelated returns (for which growth rates of cash flows/benefits are uncorrelated) obtained during equal time intervals (such as daily returns or weekly returns), for all returns series whose returns are not single-digit or double-digit returns (not between −99% and 99%), an odd number of negative returns will always result in a negative compounded series return, regardless of the magnitude and timing of other returns in the series; and an even number of negative returns will always result in a positive compounded return regardless of the magnitude and timing of other returns in the series. This effect will persist until the number of negative returns in the series exceeds the number of positive returns.

Proof: Let R be the compounded return, r is the periodic return, and n is the number of periods so that $R = (1+r)^n$. The odd-even effect occurs because: (1) when $-1>r$ or $r>1$, r dominates 1 in $(1+r)$ such that the $(1+r)$ is heavily dependent on the value r; and (2) the multiplication of the odd-numbered number of negative signs of $(1+r)$ will result in a negative compounded return (which occurs when there is an odd number of negative returns); and (3) the multiplication of the even-numbered number of negative signs of $(1+r)$ will result in a positive compounded return (which occurs when there is an even number of negative returns). The proof of the odd-even returns effect is straightforward and is illustrated in Table 1 (of Chart 8C), by a comparison of R_{27}...... R_{34}. The *odd-even effect* does not occur when the return series consists of single-digit and/or double-digit returns. ■

Theorem 8.7: When compounding uncorrelated returns (for which the growth rates of cash flows/outcomes are uncorrelated) that are derived during equal time intervals (such as daily returns), the unneutral effect inherent in compounding distorts the true returns.

Proof: The proof of the *timing effect* is straightforward and is evident by comparing rows R_1.....R_{13} and R_{14}......R_{26} in Tables 1, 2, and 3 (of Chart 8C). In compounding, the sequence/timing of the returns (gains and losses) does not matter and hence does not affect the long-term compounded return. Thus, compounding does not fully capture the time value of money and reinvestment. Compounding contrasts with other measures, such as the NPV or IRR, where the timing of gains/losses affects the long-term NPV or IRR respectively. Hence, given the time value of money, the timing effect distorts the reported compounded returns (and this distortion is particularly severe for instruments such as leveraged/inverse exchange traded funds). ■

Theorem 8.8: When compounding uncorrelated returns obtained during equal time intervals (such as hourly, daily, or weekly returns), and for all series in which all the returns are triple-digit returns (not between −99% and 99%), as the number of negative returns in the series increases, there is a specific number of negative returns which is an integer (known as the negative returns apex), after which any increases in the number of negative returns will result in no change or minimal change in the long-term compounded return of the series.

Proof: Let R be the compounded return, r is the periodic return, and n is the number of periods so that $R = (1+r)^n$. Assume that $|r^-| = |r^+|$. Let m, p, and n be the number of negative returns, positive returns, and total returns in the series, respectively. The negative returns apex effect occurs because: (1) when $-1>r$ or $r>1$, r dominates 1 in $(1+r)$ such that the $(1+r)$ is heavily dependent on the value r; and (2) when the negative returns apex (A) is reached, the multiplication of additional odd-numbered number of negative signs of $(1+r)$ will result in a negative compounded return and the multiplication of additional even-numbered number of negative signs of $(1+r)$ will result in negative compounded return; (3) when r is negative (r^-), $(1+r^-)$ will be less than one, and if $n > 0$, then $(1+r^-)^n \rightarrow 0$, as $n \rightarrow \infty$; and conversely, when r is positive (r^+), $(1+r^+)$ will be greater than one, and if $n>0$, then $(1+r^+)^n \rightarrow \infty$, as $n \rightarrow \infty$; and (4) as m increases, the rate at which $(1+r)^n$ approaches zero, slows down considerably, and is less than the rate at which $(1+r)^n$ approaches positive infinity as p increases—that is, $\partial\{(1+r)^n\}/\partial m < \partial\{(1+r^+)^n\}/\partial p$.

The proof is straightforward and is evident by comparing rows R_{27}....... R_{39}, in Tables 1 and 2 (of Chart 8C), where the negative returns apex is six (occurs in R_{32}). ■

Theorem 8.9: When compounding returns obtained during equal time intervals (such as hourly, daily, or weekly returns), and for all series in which all the returns are either single-digit or double-digit returns (between −99% and 99%), the rate of change of the compounded return is inversely related to the number of negative returns in the series; and the ratio of the change in the compounded return to the change in the number of negative returns will always be less than −1.

Proof: Let R be the compounded return, r is the periodic return, and n is the number of periods so that $R = (1+r)^n$. Assume that $|r^-| = |r^+|$; and $-.99<r<.99$. Let m, p, and n be the number of negative returns, positive returns, and total returns in the series respectively. The *volume loss effect* states that $\partial R/\partial m < -1$, and it occurs because: (1) $(1+r)$ is not dominated by r; (2) if $-.99<r<.99$, and if $n>0$, then $(1+r)^n \rightarrow 0$, as $m \rightarrow \infty$; and (3) as m increases, the rate at which $(1+r)^n$ approaches zero, slows down considerably, and is less than the rate at which $(1+r)^n$ approaches positive infinity as p increases—that is, $\partial\{(1+r)^n\}/\partial m < \partial\{(1+r^+)^n\}/\partial p$.

The proof of the *Volume-Loss Theorem* is straightforward and is evident by comparing rows R_1....... R_{13} and R_{14}....... R_{26} in Table 1 (of Chart 8C). ■

Theorem 8.10: When compounding returns obtained during equal time intervals (such as hourly, daily, or weekly returns), and for all series in which all the returns are triple-digit or higher digit returns (not between −99% and 99%), the relationship between the rate of change of the compounded return and the number of negative returns in the series will depend on the number of negative returns in the series.

Proof: The proof of the *Abnormal Volume-Loss Theorem* is straightforward and is evident in Table 1 (of Chart 8C). ■

Theorem 8.11: When compounding returns obtained during equal time intervals (such as daily returns or weekly returns), and for all series in which all the returns are triple-digit or higher digit returns (not between −99% and 99%), as the number of negative or positive double digit returns increases, there is a specific number of negative or positive double digit returns (the transition apex) after which there is minimal or no change in the compounded return.

Proof: The proof of the *Transition Loss Theorem* is straightforward and is evident by comparing rows R_{14}....... R_{26} and R_{27}....... R_{39} in Table 3 (of Chart 8C), where the transition apex is six (R_{33} and R_{20}). ■

8.3 Volatility has Minimal Effects on the Downward Returns Bias

Co and Labuszewski (July 2012) attempted to provide explanations for the *downward returns bias* (downward drifts in long-term compounded returns of leveraged/inverse ETFs); and erroneously attributed the downward returns bias in the returns of leveraged/inverse ETFs to the effects of intraday volatility and/or long-term volatility of the underlying index. Co and Labuszewski (July 2012) calculated the value of the ETF portfolio after N days as:

$$V = \Pi^{N}{}_{i=1}\left(1+\beta r_i\right);\text{ and thus }\; V = \exp\left[\sum_{i=1}^{n}\mathrm{Ln}\left(1+\beta r_i\right)\right]$$

Using the Taylor Series expansion, Co and Labuszewski (July 2012) decomposed the Value of the ETF portfolio to the following:

$$V = \left(1+\beta r\right)^{N} * \exp\left[\left(-0.5\right) * \left\{\beta^2 / \left(1+\beta r\right)^2\right\} * N * \sigma^2\right]$$

Where:

V = value of the leveraged/inverse ETF.
r = the average daily return of the index.
β = the leverage or inverse amount/beta (typically 2X, −2X, 3X or −3X).
N = number of days.
VolEff = the term $\exp[(-0.5)*\{\beta^2/(1+\beta r)^2\}*N*\sigma^2]$
BetaCoeff = the term $\{\beta^2/(1+\beta r)^2\}$

Co and Labuszewski (July 2012) postulated that the VolEff is a modifier to the first term $(\{(1+\beta r)^N\}$, which is simply the compounded return based on only the mean daily return and is dominated by the volatility during the holding period. Other than the *constant-leverage effect* (which has been documented in the literature), the conclusions in Co and Labuszewski (July 2012) are wrong. The key issue is not to confuse intraday volatility (not relevant) with end-of-period index prices/values, which are relevant. Furthermore, volatility and variance are typically less than one, and N and B are typically greater than or equal to one; and BetaCoeff is typically greater than one; so that for leveraged/inverse ETFs that have daily, weekly, or monthly compounding, the effect of volatility in VolEff is very small or negligible in almost all market conditions.

Theorem 8.12: The volatility of returns has minimal effect on long-term compounded returns.

Proof: Assuming that the decomposed portfolio-value formula in Co and Labuszewski (July 2012) is correct, although the term $\exp[(-0.5)*\{\beta^2/(1+\beta r)^2\}*N*\sigma^2]$ will typically be less than one, in most market conditions the effects of the magnitude of the daily returns of the ETF, compounding, and the leverage/beta (i.e., 2X,−2X, 3X, etc.) are the dominant factors in VolEff, in order of importance. Volatility may have some slight noticeable adverse effects only when daily returns of the ETFs are relatively high. Furthermore, volatility and variance are typically less than one, and N and B are typically greater than or equal to one; and BetaCoeff is typically greater than one (except for some negative double-digit returns and all negative triple-digit returns); so that for leveraged/inverse ETFs that have daily, weekly, or monthly compounding, the effect of volatility in VolEff is very small or negligible in most market conditions.

These trends are most evident by comparing Tables 1, 2 and 3 (in Chart-8D), where as in Co and Labuszewski (July 2012):

N = number of trading days.
β = leverage or inverse effect/beta (typically 2X,−2X, 3X,−3X).
σ = standard deviation of daily returns of the leveraged/inverse ETF.
r = the average daily return of the leveraged/inverse ETF.
VolEff = the term $\exp[(-0.5)*\{\beta^2/(1+\beta r)^2\}*N*\sigma^2]$
BetaCoeff = the term $\{\beta^2/(1+\beta r)^2\}$

Table 1 (of Chart 8D) shows the change in VolEff over different numbers of trading days, 2X/−2X leverage, single-digit daily returns, and low/high volatility. Table 2 (of Chart 8D) shows the change in VolEff over different numbers of trading days, 3X/−3X leverage, single-digit daily returns, and low/high volatility. Table 3 (of Chart 8D) shows the change in VolEff over different numbers of trading days, 2X/−2X leverage, and double-digit and triple-digit daily returns, and low/high volatility. ■

8.4 Conclusion

Clearly, there are substantial statistical biases that are caused by the compounding of periodic returns. These biases tend to increase over time, and also tend to increase if the growth rates of the associated periodic cash flows/benefits are serially correlated, as explained in Booth (2002). Where daily data are used, there are also other biases caused by differences in the calculation of prices, and possible manipulation of closing prices. These biases have critical implications for portfolio management, capital budgeting, insurance and risk management. However, the leverage effect puzzle (discussed in Ait-Sahalia et al. 2013) can also be partly attributed to some of the biases introduced herein—as the data frequency increases, the periodic returns become smaller and more subject to these biases. Similarly, substantial portions of the leverage effect (which has been documented in the literature) can be attributed to some of the biases introduced above—such as the *downward-bias effect*, the *negative-returns effect*, the *proportionality effect*, and the *volume-loss effect*.

Chart 8A ■

	Un-Correlated			
	Compounded Returns		Deviation From One	
Period	−10.0%	10.0%	−10.0%	10.0%
1	0.9000	1.1000	0.1000	0.1000
2	0.8100	1.2100	0.1900	0.2100
3	0.7290	1.3310	0.2710	0.3310
4	0.6561	1.4641	0.3439	0.4641
5	0.59049	1.61051	0.40951	0.61051
6	0.531441	1.771561	0.468559	0.771561
7	0.4782969	1.9487171	0.5217031	0.9487171
8	0.43046721	2.14358881	0.56953279	1.14358881
9	0.387420489	2.357947691	0.612579511	1.357947691
10	0.34867844	2.59374246	0.65132156	1.59374246

Chart 8B The downward bias effect

	1	2	3	4	5	6
1	–7.00%	–1.40%	–1.40%	–1.00%	–1.00%	–7.00%
2	–0.50%	–1.20%	–1.20%	–3.00%	–3.00%	–0.50%
3	–2.00%	–2.00%	**15.00%**	–12.00%	–12.00%	–2.00%
4	–3.00%	–3.00%	–3.00%	–3.30%	–3.30%	–3.00%
5	–1.00%	**11.00%**	–7.00%	**20.00%**	–3.00%	–1.00%
6	–1.00%	–1.00%	–0.50%	–2.00%	–2.00%	–1.00%
7	–3.00%	–3.00%	–2.00%	–3.00%	–3.00%	–3.00%
8	–7.00%	–7.00%	–3.00%	–7.00%	**25.00%**	–7.00%
9	–0.50%	–0.50%	–1.00%	–0.50%	–0.50%	–0.50%
10	–2.00%	–3.30%	–7.00%	–2.00%	–2.00%	–2.00%
11	–3.00%	**20.00%**	–0.50%	–3.00%	–3.00%	–3.00%
12	–1.00%	–2.00%	–3.30%	–20.00%	–20.00%	–1.00%
13	–3.00%	–3.00%	–3.00%	–3.00%	–3.00%	–3.00%
14	–7.00%	–7.00%	–7.00%	–7.00%	–7.00%	–7.00%
15	–0.50%	–0.50%	**10.00%**	–0.50%	–0.50%	–0.50%
16	–2.00%	–2.00%	–2.80%	–2.00%	–2.00%	–2.00%
17	0.00%	–0.10%	–3.00%	0.00%	**59.00%**	0.00%
18	–1.00%	–1.00%	–1.00%	–1.00%	–1.00%	–1.00%
19	–3.00%	–3.00%	–3.00%	–3.00%	–3.00%	–3.00%
20	–7.00%	–7.00%	**23.00%**	–1.00%	–1.00%	–7.00%
21	–0.50%	–0.50%	–0.50%	**59.00%**	–3.00%	–0.50%
22	**98.00%**	–0.20%	–0.20%	–7.00%	–7.00%	**98.00%**
23	–3.00%	–3.00%	–3.00%	–0.50%	–0.50%	–3.00%
24	–1.00%	–1.00%	–3.00%	–1.00%	–1.00%	–1.00%
25	–3.00%	**49.00%**	–7.00%	–3.00%	–3.00%	–3.00%
26	–7.00%	–7.00%	–0.50%	**43.00%**	–3.00%	–7.00%
27	–0.50%	–0.50%	–2.00%	–1.00%	–1.00%	–0.50%
28	–2.00%	–2.00%	–3.00%	–2.00%	–2.00%	–2.00%
29	–1.00%	–3.00%	–1.00%	–1.00%	–1.00%	–1.00%
30	–3.00%	–0.10%	–0.10%	–3.00%	**47.00%**	–3.00%
31	–7.00%	–1.00%	–1.00%	14.30%	14.30%	–7.00%
32	–0.50%	–3.00%	–3.00%	–0.50%	–0.50%	–0.50%
33	–2.00%	–7.00%	**33.00%**	–30.00%	–30.00%	–2.00%
34	–3.00%	–0.50%	–1.00%	–3.00%	–3.00%	**98.00%**
35	–3.00%	–0.20%	–3.00%	–3.00%	–3.00%	–3.00%
36	–7.00%	–3.00%	–3.00%	–7.00%	–7.00%	–7.00%
R_c =	–27.45%	–12.44%	–9.51%	–28.06%	–22.07%	48.09%
R_3 =	0.00%	0.00%	0.00%	0.00%	10.00%	101.00%

Chart 8C ■

Table-1

	Time Period											
	1	2	3	4	5	6	7	8	9	10	11	12
1	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
2	10%	10%	10%	10%	−10%	10%	10%	10%	10%	10%	10%	10%
3	10%	10%	10%	10%	−10%	−10%	10%	10%	10%	10%	10%	10%
4	10%	10%	10%	10%	−10%	−10%	−10%	10%	10%	10%	10%	10%
5	10%	10%	10%	10%	−10%	−10%	−10%	−10%	10%	10%	10%	10%
6	10%	10%	10%	10%	−10%	−10%	−10%	−10%	−10%	10%	10%	10%
7	10%	10%	10%	10%	−10%	−10%	−10%	−10%	−10%	−10%	10%	10%
8	10%	10%	10%	10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	10%
9	10%	10%	10%	10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%
10	10%	10%	10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%
11	10%	10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%
12	10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%
13	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%	−10%
14	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%	5%
15	5%	5%	5%	5%	−5%	5%	5%	5%	5%	5%	5%	5%
16	5%	5%	5%	5%	−5%	−5%	5%	5%	5%	5%	5%	5%
17	5%	5%	5%	5%	−5%	−5%	−5%	5%	5%	5%	5%	5%
18	5%	5%	5%	5%	−5%	−5%	−5%	−5%	5%	5%	5%	5%
19	5%	5%	5%	5%	−5%	−5%	−5%	−5%	−5%	5%	5%	5%
20	5%	5%	5%	5%	−5%	−5%	−5%	−5%	−5%	−5%	5%	5%
21	5%	5%	5%	5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	5%
22	5%	5%	5%	5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%
23	5%	5%	5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%
24	5%	5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%
25	5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%
26	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%	−5%
27	110%	110%	110%	110%	110%	110%	110%	110%	110%	110%	110%	110%
28	110%	110%	110%	110%	−110%	110%	110%	110%	110%	110%	110%	110%
29	110%	110%	110%	110%	−110%	−110%	110%	110%	110%	110%	110%	110%
30	110%	110%	110%	110%	−110%	−110%	−110%	110%	110%	110%	110%	110%
31	110%	110%	110%	110%	−110%	−110%	−110%	−110%	110%	110%	110%	110%
32	110%	110%	110%	110%	−110%	−110%	−110%	−110%	−110%	110%	110%	110%
33	110%	110%	110%	110%	−110%	−110%	−110%	−110%	−110%	−110%	110%	110%
34	110%	110%	110%	110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	110%
35	110%	110%	110%	110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%
36	110%	110%	110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%
37	110%	110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%
38	110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%
39	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%	−110%

	Returns	First Cross-Sectional Difference {(R(n + 1) − R(n)}	Standard Deviation	First Cross-Sectional Difference {(σ(n + 1) − σ(n)}
R1	213.8%	0.0%	0.000%	0.000%
R2	156.8%	57.1%	5.774%	−5.774%
R3	110.1%	46.7%	7.785%	−2.011%
R4	71.9%	38.2%	9.045%	−1.260%
R5	40.6%	31.3%	9.847%	−0.802%
R6	15.1%	25.6%	10.299%	−0.451%
R7	−5.9%	20.9%	10.445%	−0.146%
R8	−23.0%	17.1%	10.299%	0.146%
R9	−37.0%	14.0%	9.847%	0.451%
R10	−48.4%	11.5%	9.045%	0.802%
R11	−57.8%	9.4%	7.785%	1.260%
R12	−65.5%	7.7%	5.774%	2.011%
R13	−71.8%	6.3%	0.000%	5.774%
R14	79.6%	0.0%	0.000%	0.00%
R15	62.5%	17.1%	2.887%	−2.89%
R16	47.0%	15.5%	3.892%	−1.01%
R17	33.0%	14.0%	4.523%	−0.63%
R18	20.3%	12.7%	4.924%	−0.40%
R19	8.9%	11.5%	5.149%	−0.23%
R20	−1.5%	10.4%	5.222%	−0.07%
R21	−10.9%	9.4%	5.149%	0.07%
R22	−19.4%	8.5%	4.924%	0.23%
R23	−27.0%	7.7%	4.523%	0.40%
R24	−34.0%	6.9%	3.892%	0.63%
R25	−40.3%	6.3%	2.887%	1.01%
R26	−46.0%	5.7%	0.000%	2.89%
R27	735482.8%	0.0%	0.00%	0.00%
R28	−35127.8%	770610.5%	63.51%	−63.51%
R29	1568.0%	−36695.7%	85.63%	−22.13%
R30	−179.4%	1747.4%	99.50%	−13.86%
R31	−96.218%	−83.210%	108.32%	−8.82%
R32	−100.180%	3.962%	113.28%	−4.96%
R33	−99.991%	−0.18869%	114.89%	−1.61%
R34	−100.000%	0.00899%	113.28%	1.61%
R35	−100.000%	−0.00043%	108.32%	4.96%
R36	−100.000%	0.00002%	99.50%	8.82%
R37	−100.000%	0.00000%	85.63%	13.86%
R38	−100.000%	0.00000%	63.51%	22.13%
R39	−100.000%	0.00000%	0.00%	63.51%

Table-2

	Time Period											
	1	2	3	4	5	6	7	8	9	10	11	12
Section-A:												
1	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
2	5.0%	5.0%	5.0%	5.0%	10.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
3	5.0%	5.0%	5.0%	5.0%	10.0%	10.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
4	5.0%	5.0%	5.0%	5.0%	10.0%	10.0%	10.0%	5.0%	5.0%	5.0%	5.0%	5.0%
5	5.0%	5.0%	5.0%	5.0%	10.0%	10.0%	10.0%	10.0%	5.0%	5.0%	5.0%	5.0%
6	5.0%	5.0%	5.0%	5.0%	10.0%	10.0%	10.0%	10.0%	10.0%	5.0%	5.0%	5.0%
7	5.0%	5.0%	5.0%	5.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	5.0%	5.0%
8	5.0%	5.0%	5.0%	5.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	5.0%
9	5.0%	5.0%	5.0%	5.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
10	5.0%	5.0%	5.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
11	5.0%	5.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
12	5.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
13	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
14	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%
15	−5.0%	−5.0%	−5.0%	−5.0%	−10.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%
16	−5.0%	−5.0%	−5.0%	−5.0%	−10.0%	−10.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%
17	−5.0%	−5.0%	−5.0%	−5.0%	−10.0%	−10.0%	−10.0%	−5.0%	−5.0%	−5.0%	−5.0%	−5.0%
18	−5.0%	−5.0%	−5.0%	−5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−5.0%	−5.0%	−5.0%	−5.0%
19	−5.0%	−5.0%	−5.0%	−5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−5.0%	−5.0%	−5.0%
20	−5.0%	−5.0%	−5.0%	−5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−5.0%	−5.0%
21	−5.0%	−5.0%	−5.0%	−5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−5.0%
22	−5.0%	−5.0%	−5.0%	−5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
23	−5.0%	−5.0%	−5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
24	−5.0%	−5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
25	−5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
26	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
27	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
28	5.0%	5.0%	5.0%	5.0%	−10.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
29	5.0%	5.0%	5.0%	5.0%	−10.0%	−10.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
30	5.0%	5.0%	5.0%	5.0%	−10.0%	−10.0%	−10.0%	5.0%	5.0%	5.0%	5.0%	5.0%
31	5.0%	5.0%	5.0%	5.0%	−10.0%	−10.0%	−10.0%	−10.0%	5.0%	5.0%	5.0%	5.0%
32	5.0%	5.0%	5.0%	5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	5.0%	5.0%	5.0%
33	5.0%	5.0%	5.0%	5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	5.0%	5.0%
34	5.0%	5.0%	5.0%	5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	5.0%
35	5.0%	5.0%	5.0%	5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
36	5.0%	5.0%	5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
37	5.0%	5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
38	5.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
39	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%

	Time Period											
	1	2	3	4	5	6	7	8	9	10	11	12
Section-B: Random Numbers:												
1	5.0%	3.0%	9.0%	6.0%	1.0%	3.0%	7.0%	9.0%	2.0%	4.0%	6.0%	1.0%
2	2.0%	8.0%	2.0%	9.0%	40.0%	3.0%	2.0%	5.0%	4.0%	1.0%	1.0%	9.0%
3	4.0%	1.0%	9.0%	6.0%	10.0%	11.0%	4.0%	1.0%	7.0%	8.0%	8.0%	3.0%
4	5.0%	3.0%	5.0%	7.0%	36.0%	15.0%	10.0%	8.0%	5.0%	6.0%	3.0%	7.0%
5	8.0%	1.0%	9.0%	6.0%	27.0%	40.0%	12.0%	13.0%	9.0%	2.0%	7.0%	4.0%
6	5.0%	6.0%	3.0%	3.0%	19.0%	11.0%	19.0%	10.0%	10.0%	4.0%	4.0%	2.0%
7	9.0%	3.0%	9.0%	6.0%	10.0%	24.0%	10.0%	34.0%	22.0%	11.0%	6.0%	8.0%
8	5.0%	1.0%	6.0%	1.0%	12.0%	10.0%	35.0%	10.0%	11.0%	30.0%	10.0%	5.0%
9	1.0%	3.0%	9.0%	6.0%	10.0%	32.0%	14.0%	65.0%	19.0%	10.0%	19.0%	17.0%
10	5.0%	3.0%	2.0%	13.0%	34.0%	11.0%	23.0%	15.0%	41.0%	22.0%	15.0%	21.0%
11	6.0%	3.0%	15.0%	46.0%	53.0%	56.0%	78.0%	10.0%	13.0%	10.0%	49.0%	34.0%
12	5.0%	22.0%	11.0%	22.0%	10.0%	23.0%	11.0%	11.0%	10.0%	11.0%	28.0%	10.0%
13	10.0%	17.0%	10.0%	13.0%	16.0%	11.0%	17.0%	31.0%	12.0%	23.0%	19.0%	18.0%
Random Numbers:												
14	−5.0%	−9.0%	−2.0%	−6.0%	−5.0%	−1.0%	−4.0%	−3.0%	−8.0%	−6.0%	−2.0%	−9.0%
15	−2.0%	−9.0%	−1.0%	−4.0%	−10.0%	−1.0%	−1.0%	−7.0%	−2.0%	−1.0%	−7.0%	−2.0%
16	−5.0%	−1.0%	−8.0%	−6.0%	−11.0%	−32.0%	−4.0%	−3.0%	−8.0%	−6.0%	−5.0%	−9.0%
17	−6.0%	−9.0%	−2.0%	−9.0%	−10.0%	−10.0%	−23.0%	−9.0%	−4.0%	−3.0%	−2.0%	−5.0%
18	−5.0%	−8.0%	−4.0%	−6.0%	−34.0%	−13.0%	−10.0%	−56.0%	−8.0%	−9.0%	−1.0%	−9.0%
19	−9.0%	−9.0%	−2.0%	−8.0%	−11.0%	−10.0%	−15.0%	−10.0%	−63.0%	−6.0%	−2.0%	−7.0%
20	−5.0%	−3.0%	−6.0%	−6.0%	−10.0%	−15.0%	−10.0%	−14.0%	−10.0%	−78.0%	−9.0%	−9.0%
21	−1.0%	−9.0%	−2.0%	−1.0%	−44.0%	−10.0%	−10.0%	−10.0%	−20.0%	−10.0%	−34.0%	−1.0%
22	−5.0%	−6.0%	−1.0%	−6.0%	−10.0%	−65.0%	−16.0%	−77.0%	−14.0%	−14.0%	−10.0%	−22.0%
23	−3.0%	−9.0%	−2.0%	−10.0%	−33.0%	−10.0%	−10.0%	−11.0%	−10.0%	−10.0%	−19.0%	−10.0%
24	−5.0%	−5.0%	−10.0%	−22.0%	−10.0%	−34.0%	−33.0%	−10.0%	−11.0%	−19.0%	−10.0%	−19.0%
25	−4.0%	−10.0%	−14.0%	−10.0%	−13.0%	−22.0%	−10.0%	−16.0%	−10.0%	−10.0%	−33.0%	−11.0%
26	−33.0%	−11.0%	−35.0%	−12.0%	−10.0%	−10.0%	−15.0%	−10.0%	−22.0%	−31.0%	−10.0%	−29.0%
Random Numbers:												
27	5.0%	7.0%	4.0%	3.0%	9.0%	2.0%	5.0%	8.0%	7.0%	4.0%	6.0%	3.0%
28	2.0%	5.0%	1.0%	1.0%	−10.0%	2.0%	5.0%	1.0%	11.0%	1.0%	6.0%	3.0%
29	5.0%	7.0%	4.0%	9.0%	−11.0%	−19.0%	5.0%	6.0%	2.0%	6.0%	6.0%	3.0%
30	6.0%	4.0%	9.0%	3.0%	−16.0%	−14.0%	−34.0%	8.0%	5.0%	8.0%	6.0%	3.0%
31	5.0%	7.0%	4.0%	1.0%	−19.0%	−20.0%	−11.0%	−16.0%	7.0%	4.0%	5.0%	3.0%
32	8.0%	2.0%	5.0%	3.0%	−13.0%	−65.0%	−19.0%	−22.0%	−44.0%	2.0%	9.0%	3.0%
33	5.0%	7.0%	4.0%	5.0%	−10.0%	−17.0%	−33.0%	−18.0%	−10.0%	−13.0%	6.0%	3.0%
34	1.0%	8.0%	3.0%	3.0%	−11.0%	−61.0%	−19.0%	−10.0%	−27.0%	−18.0%	−22.0%	3.0%
35	5.0%	7.0%	4.0%	7.0%	−13.0%	−10.0%	−20.0%	−10.0%	−18.0%	−44.0%	−13.0%	−17.0%
36	9.0%	3.0%	8.0%	−13.0%	−10.0%	−19.0%	−10.0%	−11.0%	−10.0%	−22.0%	−11.0%	−19.0%
37	5.0%	7.0%	−20.0%	−14.0%	−18.0%	−10.0%	−15.0%	−10.0%	−56.0%	−10.0%	−34.0%	−23.0%
38	2.0%	−10.0%	−11.0%	−53.0%	−10.0%	−16.0%	−10.0%	−22.0%	−10.0%	−12.0%	−13.0%	−10.0%
39	−10.0%	−10.0%	−10.0%	−10.0%	−13.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%

	Section-A Returns	First Cross-Sectional Difference {(R(n + 1) − R(n)}	Section-A Standard Deviation	First Cross-Sectional Difference {(σ(n + 1) − σ(n)}
R1	79.59%	0.00%	0.000%	0.00%
R2	88.14%	−8.55%	1.443%	−1.44%
R3	97.10%	−8.96%	1.946%	−0.50%
R4	106.48%	−9.39%	2.261%	−0.32%
R5	116.31%	−9.83%	2.462%	−0.20%
R6	126.61%	−10.30%	2.575%	−0.11%
R7	137.41%	−10.79%	2.611%	−0.04%
R8	148.71%	−11.31%	2.575%	0.04%
R9	160.55%	−11.84%	2.462%	0.11%
R10	172.96%	−12.41%	2.261%	0.20%
R11	185.96%	−13.00%	1.946%	0.32%
R12	199.58%	−13.62%	1.443%	0.50%
R13	213.84%	−14.27%	0.000%	1.44%
R14	−45.96%	0.00%	0.000%	0.00%
R15	−48.81%	2.84%	1.443%	−1.44%
R16	−51.50%	2.69%	1.946%	−0.50%
R17	−54.05%	2.55%	2.261%	−0.32%
R18	−56.47%	2.42%	2.462%	−0.20%
R19	−58.76%	2.29%	2.575%	−0.11%
R20	−60.93%	2.17%	2.611%	−0.04%
R21	−62.99%	2.06%	2.575%	0.04%
R22	−64.94%	1.95%	2.462%	0.11%
R23	−66.78%	1.85%	2.261%	0.20%
R24	−68.53%	1.75%	1.946%	0.32%
R25	−70.19%	1.66%	1.443%	0.50%
R26	−71.76%	1.57%	0.000%	1.44%
R27	79.59%	0.00%	0.000%	0.00%
R28	53.93%	25.66%	4.330%	−4.33%
R29	31.94%	21.99%	5.839%	−1.51%
R30	13.09%	18.85%	6.784%	−0.95%
R31	−3.06%	16.16%	7.385%	−0.60%
R32	−16.91%	13.85%	7.724%	−0.34%
R33	−28.78%	11.87%	7.833%	−0.11%
R34	−38.96%	10.17%	7.724%	0.11%
R35	−47.68%	8.72%	7.385%	0.34%
R36	−55.15%	7.47%	6.784%	0.60%
R37	−61.56%	6.41%	5.839%	0.95%
R38	−67.05%	5.49%	4.330%	1.51%
R39	−71.76%	4.71%	0.000%	4.33%

	Section-B (Random Numbers) Returns	First Cross-Sectional Difference {(R(n + 1) − R(n)}	Section-B (Random Numbers): Standard Deviation	First Cross-Sectional Difference {(σ(n + 1) − σ(n)}
R1	79.59%	0.00%	2.807%	0.00%
R2	88.14%	−8.55%	10.760%	−7.95%
R3	97.10%	−8.96%	3.384%	7.38%
R4	106.48%	−9.39%	9.064%	−5.68%
R5	116.31%	−9.83%	11.229%	−2.17%
R6	126.61%	−10.30%	5.954%	5.27%
R7	137.41%	−10.79%	9.139%	−3.18%
R8	148.71%	−11.31%	10.603%	−1.46%
R9	160.55%	−11.84%	17.270%	−6.67%
R10	172.96%	−12.41%	11.935%	5.34%
R11	185.96%	−13.00%	24.766%	−12.83%
R12	199.58%	−13.62%	7.180%	17.59%
R13	213.84%	−14.27%	6.097%	1.08%
R14	−45.96%	0.00%	2.730%	0.00%
R15	−48.81%	2.84%	3.397%	−0.67%
R16	−51.50%	2.69%	7.987%	−4.59%
R17	−54.05%	2.55%	5.726%	2.26%
R18	−56.47%	2.42%	15.710%	−9.98%
R19	−58.76%	2.29%	16.261%	−0.55%
R20	−60.93%	2.17%	20.273%	−4.01%
R21	−62.99%	2.06%	13.667%	6.61%
R22	−64.94%	1.95%	24.385%	−10.72%
R23	−66.78%	1.85%	7.982%	16.40%
R24	−68.53%	1.75%	9.893%	−1.91%
R25	−70.19%	1.66%	7.489%	2.40%
R26	−71.76%	1.57%	10.260%	−2.77%
R27	79.59%	0.00%	2.179%	0.00%
R28	53.93%	25.66%	4.887%	−2.71%
R29	31.94%	21.99%	8.284%	−3.40%
R30	13.09%	18.85%	13.266%	−4.98%
R31	−3.06%	16.16%	10.673%	2.59%
R32	−16.91%	13.85%	23.240%	−12.57%
R33	−28.78%	11.87%	12.831%	10.41%
R34	−38.96%	10.17%	19.205%	−6.37%
R35	−47.68%	8.72%	14.727%	4.48%
R36	−55.15%	7.47%	10.235%	4.49%
R37	−61.56%	6.41%	16.714%	−6.48%
R38	−67.05%	5.49%	13.263%	3.45%
R39	−71.76%	4.71%	0.866%	12.40%

Table-3

	Time Period 1	2	3	4	5	6	7	8	9	10	11	12
Section-A:												
1	110.0%	110.0%	110.0%	110.0%	110.0%	110.0%	110.0%	110.0%	110.0%	110.0%	110.0%	110.0%
2	110.0%	110.0%	110.0%	110.0%	10.0%	110.0%	110.0%	110.0%	110.0%	110.0%	110.0%	110.0%
3	110.0%	110.0%	110.0%	110.0%	10.0%	10.0%	110.0%	110.0%	110.0%	110.0%	110.0%	110.0%
4	110.0%	110.0%	110.0%	110.0%	10.0%	10.0%	10.0%	110.0%	110.0%	110.0%	110.0%	110.0%
5	110.0%	110.0%	110.0%	110.0%	10.0%	10.0%	10.0%	10.0%	110.0%	110.0%	110.0%	110.0%
6	110.0%	110.0%	110.0%	110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	110.0%	110.0%	110.0%
7	110.0%	110.0%	110.0%	110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	110.0%	110.0%
8	110.0%	110.0%	110.0%	110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	110.0%
9	110.0%	110.0%	110.0%	110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
10	110.0%	110.0%	110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
11	110.0%	110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
12	110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
13	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
14	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%
15	−110.0%	−110.0%	−110.0%	−110.0%	10.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%
16	−110.0%	−110.0%	−110.0%	−110.0%	10.0%	10.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%
17	−110.0%	−110.0%	−110.0%	−110.0%	10.0%	10.0%	10.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%
18	−110.0%	−110.0%	−110.0%	−110.0%	10.0%	10.0%	10.0%	10.0%	−110.0%	−110.0%	−110.0%	−110.0%
19	−110.0%	−110.0%	−110.0%	−110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	−110.0%	−110.0%	−110.0%
20	−110.0%	−110.0%	−110.0%	−110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	−110.0%	−110.0%
21	−110.0%	−110.0%	−110.0%	−110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	−110.0%
22	−110.0%	−110.0%	−110.0%	−110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
23	−110.0%	−110.0%	−110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
24	−110.0%	−110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
25	−110.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
26	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%
27	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%
28	−110.0%	−110.0%	−110.0%	−110.0%	−10.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%
29	−110.0%	−110.0%	−110.0%	−110.0%	−10.0%	−10.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%
30	−110.0%	−110.0%	−110.0%	−110.0%	−10.0%	−10.0%	−10.0%	−110.0%	−110.0%	−110.0%	−110.0%	−110.0%
31	−110.0%	−110.0%	−110.0%	−110.0%	−10.0%	−10.0%	−10.0%	−10.0%	−110.0%	−110.0%	−110.0%	−110.0%
32	−110.0%	−110.0%	−110.0%	−110.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−110.0%	−110.0%	−110.0%
33	−110.0%	−110.0%	−110.0%	−110.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−110.0%	−110.0%
34	−110.0%	−110.0%	−110.0%	−110.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−110.0%
35	−110.0%	−110.0%	−110.0%	−110.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
36	−110.0%	−110.0%	−110.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
37	−110.0%	−110.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
38	−110.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%
39	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%	−10.0%

	Time Period 1	2	3	4	5	6	7	8	9	10	11	12
Section-B: Random Numbers:												
1	110.0%	121.0%	101.0%	133.0%	154.0%	171.0%	119.0%	144.0%	162.0%	107.0%	132.0%	191.0%
2	121.0%	189.0%	185.0%	110.0%	10.0%	135.0%	211.0%	110.0%	227.0%	162.0%	421.0%	103.0%
3	110.0%	111.0%	122.0%	171.0%	11.0%	35.0%	128.0%	201.0%	162.0%	153.0%	125.0%	222.0%
4	156.0%	121.0%	101.0%	133.0%	34.0%	12.0%	61.0%	181.0%	341.0%	107.0%	132.0%	112.0%
5	110.0%	231.0%	132.0%	222.0%	28.0%	10.0%	15.0%	40.0%	113.0%	129.0%	220.0%	118.0%
6	298.0%	121.0%	101.0%	133.0%	14.0%	34.0%	56.0%	23.0%	28.0%	105.0%	141.0%	191.0%
7	110.0%	322.0%	151.0%	237.0%	65.0%	10.0%	11.0%	26.0%	34.0%	12.0%	132.0%	126.0%
8	132.0%	121.0%	101.0%	133.0%	16.0%	22.0%	40.0%	34.0%	23.0%	72.0%	10.0%	184.0%
9	110.0%	117.0%	333.0%	191.0%	10.0%	15.0%	22.0%	30.0%	32.0%	78.0%	34.0%	16.0%
10	221.0%	121.0%	101.0%	11.0%	25.0%	32.0%	45.0%	22.0%	34.0%	17.0%	19.0%	22.0%
11	110.0%	112.0%	10.0%	32.0%	16.0%	54.0%	22.0%	32.0%	44.0%	13.0%	30.0%	13.0%
12	175.0%	10.0%	11.0%	18.0%	30.0%	52.0%	41.0%	36.0%	10.0%	11.0%	15.0%	56.0%
13	10.0%	12.0%	36.0%	60.0%	15.0%	25.0%	40.0%	70.0%	20.0%	32.0%	11.0%	28.0%
Random Numbers:												
14	−110.0%	−121.0%	−101.0%	−133.0%	−154.0%	−171.0%	−119.0%	−144.0%	−162.0%	−107.0%	−132.0%	−191.0%
15	−121.0%	−189.0%	−185.0%	−110.0%	10.0%	−135.0%	−211.0%	−110.0%	−227.0%	−162.0%	−421.0%	−103.0%
16	−110.0%	−111.0%	−122.0%	−171.0%	11.0%	35.0%	−128.0%	−201.0%	−162.0%	−153.0%	−125.0%	−222.0%
17	−156.0%	−121.0%	−101.0%	−133.0%	34.0%	12.0%	61.0%	−181.0%	−341.0%	−107.0%	−132.0%	−112.0%
18	−110.0%	−231.0%	−132.0%	−222.0%	28.0%	10.0%	15.0%	40.0%	−162.0%	−129.0%	−220.0%	−118.0%
19	−298.0%	−121.0%	−101.0%	−133.0%	14.0%	34.0%	56.0%	23.0%	28.0%	−107.0%	−141.0%	−191.0%
20	−110.0%	−322.0%	−151.0%	−237.0%	65.0%	10.0%	11.0%	26.0%	34.0%	12.0%	−132.0%	−126.0%
21	−132.0%	−121.0%	−101.0%	−133.0%	16.0%	22.0%	40.0%	34.0%	23.0%	72.0%	10.0%	−184.0%
22	−110.0%	−117.0%	−333.0%	−191.0%	10.0%	15.0%	22.0%	30.0%	32.0%	78.0%	34.0%	16.0%
23	−221.0%	−121.0%	−101.0%	11.0%	25.0%	32.0%	45.0%	22.0%	34.0%	17.0%	19.0%	22.0%
24	−110.0%	−112.0%	10.0%	32.0%	16.0%	54.0%	22.0%	32.0%	44.0%	13.0%	30.0%	13.0%
25	−175.0%	10.0%	11.0%	18.0%	30.0%	52.0%	41.0%	36.0%	10.0%	11.0%	15.0%	56.0%
26	10.0%	12.0%	36.0%	60.0%	15.0%	25.0%	40.0%	70.0%	20.0%	32.0%	11.0%	28.0%
Random Numbers:												
27	−110.0%	−121.0%	−101.0%	−133.0%	−154.0%	−171.0%	−119.0%	−144.0%	−162.0%	−107.0%	−132.0%	−191.0%
28	−121.0%	−189.0%	−101.0%	−133.0%	−10.0%	−171.0%	−133.0%	−111.0%	−131.0%	−333.0%	−231.0%	−111.0%
29	−110.0%	−111.0%	−101.0%	−133.0%	−11.0%	−35.0%	−151.0%	−182.0%	−206.0%	−210.0%	−177.0%	−214.0%
30	−156.0%	−121.0%	−101.0%	−133.0%	−34.0%	−12.0%	−61.0%	−195.0%	−302.0%	−192.0%	−191.0%	−178.0%
31	−110.0%	−231.0%	−101.0%	−133.0%	−28.0%	−10.0%	−15.0%	−40.0%	−156.0%	−131.0%	−118.0%	−142.0%
32	−298.0%	−121.0%	−101.0%	−133.0%	−14.0%	−34.0%	−56.0%	−23.0%	−28.0%	−107.0%	−132.0%	−102.0%
33	−110.0%	−322.0%	−101.0%	−133.0%	−65.0%	−10.0%	−11.0%	−26.0%	−34.0%	−12.0%	−156.0%	−241.0%
34	−132.0%	−121.0%	−101.0%	−133.0%	−16.0%	−22.0%	−40.0%	−34.0%	−23.0%	−72.0%	−10.0%	−191.0%
35	−110.0%	−117.0%	−101.0%	−133.0%	−10.0%	−15.0%	−22.0%	−30.0%	−32.0%	−78.0%	−34.0%	−16.0%
36	−221.0%	−121.0%	−101.0%	−11.0%	−25.0%	−32.0%	−45.0%	−22.0%	−34.0%	−17.0%	−19.0%	−22.0%
37	−110.0%	−112.0%	−10.0%	−32.0%	−16.0%	−54.0%	−22.0%	−32.0%	−44.0%	−13.0%	−30.0%	−13.0%
38	−175.0%	−10.0%	−11.0%	−18.0%	−30.0%	−52.0%	−41.0%	−36.0%	−10.0%	−11.0%	−15.0%	−56.0%
39	−10.0%	−12.0%	−36.0%	−60.0%	−15.0%	−25.0%	−40.0%	−70.0%	−20.0%	−32.0%	−11.0%	−28.0%

	Section-A Returns	First Cross-Sectional Difference {(R(n + 1) − R(n)}	Section-A Standard Deviation	First Cross-Sectional Difference {(σ(n + 1) − σ(n)}
R1	735482.8%	0.0%	0.00%	0.00%
R2	385205.3%	350277.5%	28.87%	−28.87%
R3	201726.6%	183478.7%	38.92%	−10.06%
R4	105618.7%	96107.9%	45.23%	−6.30%
R5	55276.4%	50342.2%	49.24%	−4.01%
R6	28906.7%	26369.7%	51.49%	−2.26%
R7	15094.0%	13812.7%	52.22%	−0.73%
R8	7858.8%	7235.2%	51.49%	0.73%
R9	4068.9%	3789.9%	49.24%	2.26%
R10	2083.7%	1985.2%	45.23%	4.01%
R11	1043.8%	1039.9%	38.92%	6.30%
R12	499.2%	544.7%	28.87%	10.06%
R13	213.8%	285.3%	0.00%	28.87%
R14	−100.000%	0.000%	0.00%	0.00%
R15	−100.000%	0.000%	34.64%	−34.64%
R16	−100.000%	0.000%	46.71%	−12.07%
R17	−100.000%	0.000%	54.27%	−7.56%
R18	−100.000%	0.000%	59.08%	−4.81%
R19	−100.000%	0.000%	61.79%	−2.71%
R20	−100.000%	0.000%	62.67%	−0.88%
R21	−100.002%	0.002%	61.79%	0.88%
R22	−99.979%	−0.023%	59.08%	2.71%
R23	−100.236%	0.257%	54.27%	4.81%
R24	−97.406%	−2.830%	46.71%	7.56%
R25	−128.531%	31.125%	34.64%	12.07%
R26	213.843%	−342.374%	0.00%	34.64%
R27	−100.000%	0.000%	0.00%	0.00%
R28	−100.000%	0.000%	28.87%	−28.87%
R29	−100.000%	0.000%	38.92%	−10.06%
R30	−100.000%	0.000%	45.23%	−6.30%
R31	−100.000%	0.000%	49.24%	−4.01%
R32	−100.000%	0.000%	51.49%	−2.26%
R33	−100.000%	0.000%	52.22%	−0.73%
R34	−100.000%	0.001%	51.49%	0.73%
R35	−99.996%	−0.005%	49.24%	2.26%
R36	−100.039%	0.043%	45.23%	4.01%
R37	−99.651%	−0.387%	38.92%	6.30%
R38	−103.138%	3.487%	28.87%	10.06%
R39	−71.757%	−31.381%	0.00%	28.87%

	Section-B (Random Numbers) Returns	First Cross-Sectional Difference {(R(n + 1) − R(n)}	Section-B (Random Numbers): Standard Deviation	First Cross-Sectional Difference {(σ(n + 1) − σ(n)}
R1	2927104.78%	0.00%	27.930%	0.00%
R2	5847495.29%	−2920390.51%	99.652%	−71.72%
R3	1316465.01%	4531030.28%	60.882%	38.77%
R4	807640.79%	508824.22%	83.715%	−22.83%
R5	400435.70%	407205.09%	80.148%	3.57%
R6	222098.20%	178337.50%	82.767%	−2.62%
R7	149635.50%	72462.71%	98.704%	−15.94%
R8	42033.49%	107602.00%	58.316%	40.39%
R9	41973.66%	59.83%	96.315%	−38.00%
R10	10415.39%	31558.27%	62.409%	33.91%
R11	4345.26%	6070.14%	35.390%	27.02%
R12	3188.62%	1156.64%	46.038%	−10.65%
R13	1964.16%	1224.46%	19.256%	26.78%
R14	−100.000%	0.000%	27.930%	0.00%
R15	−100.005%	0.005%	102.609%	−74.68%
R16	−99.993%	−0.012%	76.187%	26.42%
R17	−100.000%	0.008%	107.218%	−31.03%
R18	−99.550%	−0.451%	101.716%	5.50%
R19	−100.013%	0.464%	109.454%	−7.74%
R20	−95.083%	−4.930%	122.081%	−12.63%
R21	−100.115%	5.032%	88.392%	33.69%
R22	−73.588%	−26.527%	121.120%	−32.73%
R23	−101.874%	28.286%	83.320%	37.80%
R24	−88.018%	−13.856%	55.153%	28.17%
R25	−996.896%	908.878%	60.476%	−5.32%
R26	1964.156%	−2961.052%	19.256%	41.22%
R27	−100.000%	0.000%	27.930%	0.00%
R28	−100.000%	0.000%	79.076%	−51.15%
R29	−99.999%	−0.001%	66.631%	12.45%
R30	−100.011%	0.012%	80.940%	−14.31%
R31	−100.000%	−0.011%	66.487%	14.45%
R32	−100.000%	0.000%	77.692%	−11.20%
R33	−99.993%	−0.007%	99.213%	−21.52%
R34	−100.001%	0.008%	59.538%	39.68%
R35	−100.000%	−0.001%	46.031%	13.51%
R36	−100.017%	0.017%	62.409%	−16.38%
R37	−99.955%	−0.062%	35.390%	27.02%
R38	−101.872%	1.917%	46.038%	−10.65%
R39	−99.189%	−2.683%	19.256%	26.78%

Chart 8D

Table-1

2X; Low Volatility; Single Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	2	0.500%	0.600%	0.99964716	3.9211842
5	2	0.750%	0.800%	0.99937897	3.8826470
5	2	1.000%	1.200%	0.99861687	3.8446751
5	2	1.500%	1.570%	0.99767929	3.7703836
5	2	2.000%	2.300%	0.99512104	3.6982249
5	2	2.500%	2.577%	0.99399459	3.6281179
20	2	0.500%	0.600%	0.99858937	3.9211842
20	2	0.750%	0.800%	0.99751819	3.8826470
20	2	1.000%	1.200%	0.99447897	3.8446751
20	2	1.500%	1.570%	0.99074943	3.7703836
20	2	2.000%	2.300%	0.98062652	3.6982249
20	2	2.500%	2.577%	0.97619387	3.6281179
120	2	0.500%	0.600%	0.99156601	3.9211842
120	2	0.750%	0.800%	0.98520123	3.8826470
120	2	1.000%	1.200%	0.96732767	3.8446751
120	2	1.500%	1.570%	0.94576447	3.7703836
120	2	2.000%	2.300%	0.88924574	3.6982249
120	2	2.500%	2.577%	0.86539914	3.6281179
250	2	0.500%	0.600%	0.98250944	3.9211842
250	2	0.750%	0.800%	0.96941627	3.8826470
250	2	1.000%	1.200%	0.93313616	3.8446751
250	2	1.500%	1.570%	0.89032365	3.7703836
250	2	2.000%	2.300%	0.78306066	3.6982249
250	2	2.500%	2.577%	0.73994759	3.6281179

−2X; Low Volatility; Single Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	−2	0.500%	0.600%	0.99963276	4.0812162
5	−2	0.750%	0.800%	0.99934058	4.1227550
5	−2	1.000%	1.200%	0.99850175	4.1649313
5	−2	1.500%	1.570%	0.9973837	4.2512488
5	−2	2.000%	2.300%	0.99427643	4.3402778
5	−2	2.500%	2.577%	0.99266864	4.4321330
20	−2	0.500%	0.600%	0.99853184	4.0812162
20	−2	0.750%	0.800%	0.99736491	4.1227550
20	−2	1.000%	1.200%	0.99402045	4.1649313
20	−2	1.500%	1.570%	0.98957581	4.2512488
20	−2	2.000%	2.300%	0.97730151	4.3402778
20	−2	2.500%	2.577%	0.97099547	4.4321330
120	−2	0.500%	0.600%	0.99122331	4.0812162
120	−2	0.750%	0.800%	0.98429328	4.1227550
120	−2	1.000%	1.200%	0.96465476	4.1649313
120	−2	1.500%	1.570%	0.93906233	4.2512488
120	−2	2.000%	2.300%	0.87130742	4.3402778
120	−2	2.500%	2.577%	0.83811422	4.4321330
250	−2	0.500%	0.600%	0.98180214	4.0812162
250	−2	0.750%	0.800%	0.96755594	4.1227550
250	−2	1.000%	1.200%	0.92777247	4.1649313
250	−2	1.500%	1.570%	0.8772298	4.2512488
250	−2	2.000%	2.300%	0.75051108	4.3402778
250	−2	2.500%	2.577%	0.69217359	4.4321330

−2X; Low Volatility; Negative Single Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	−2	−0.500%	0.600%	0.99964716	3.9211842
5	−2	−0.750%	0.800%	0.99937897	3.8826470
5	−2	−1.000%	1.200%	0.99861687	3.8446751
5	−2	−1.500%	1.570%	0.99767929	3.7703836
5	−2	−2.000%	2.300%	0.99512104	3.6982249
5	−2	−2.500%	2.577%	0.99399459	3.6281179
20	−2	−0.500%	0.600%	0.99858937	3.9211842
20	−2	−0.750%	0.800%	0.99751819	3.8826470
20	−2	−1.000%	1.200%	0.99447897	3.8446751
20	−2	−1.500%	1.570%	0.99074943	3.7703836
20	−2	−2.000%	2.300%	0.98062652	3.6982249
20	−2	−2.500%	2.577%	0.97619387	3.6281179
120	−2	−0.500%	0.600%	0.99156601	3.9211842
120	−2	−0.750%	0.800%	0.98520123	3.8826470
120	−2	−1.000%	1.200%	0.96732767	3.8446751
120	−2	−1.500%	1.570%	0.94576447	3.7703836
120	−2	−2.000%	2.300%	0.88924574	3.6982249
120	−2	−2.500%	2.577%	0.86539914	3.6281179
250	−2	−0.500%	0.600%	0.98250944	3.9211842
250	−2	−0.750%	0.800%	0.96941627	3.8826470
250	−2	−1.000%	1.200%	0.93313616	3.8446751
250	−2	−1.500%	1.570%	0.89032365	3.7703836
250	−2	−2.000%	2.300%	0.78306066	3.6982249
250	−2	−2.500%	2.577%	0.73994759	3.6281179

2X; High Volatility; Single-Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	2	0.500%	1.050%	0.9989198	3.9211842
5	2	0.750%	1.640%	0.9973927	3.8826470
5	2	1.000%	2.300%	0.9949283	3.8446751
5	2	1.500%	3.750%	0.9868322	3.7703836
5	2	2.000%	4.500%	0.9814519	3.6982249
5	2	2.500%	6.250%	0.9651895	3.6281179
20	2	0.500%	1.050%	0.9956862	3.9211842
20	2	0.750%	1.640%	0.9896116	3.8826470
20	2	1.000%	2.300%	0.9798671	3.8446751
20	2	1.500%	3.750%	0.9483601	3.7703836
20	2	2.000%	4.500%	0.9278464	3.6982249
20	2	2.500%	6.250%	0.8678613	3.6281179
120	2	0.500%	1.050%	0.9743949	3.9211842
120	2	0.750%	1.640%	0.9392660	3.8826470
120	2	1.000%	2.300%	0.8851218	3.8446751
120	2	1.500%	3.750%	0.7275110	3.7703836
120	2	2.000%	4.500%	0.6380527	3.6982249
120	2	2.500%	6.250%	0.4272696	3.6281179
250	2	0.500%	1.050%	0.9473953	3.9211842
250	2	0.750%	1.640%	0.8776261	3.8826470
250	2	1.000%	2.300%	0.7755141	3.8446751
250	2	1.500%	3.750%	0.5154254	3.7703836
250	2	2.000%	4.500%	0.3921491	3.6982249
250	2	2.500%	6.250%	0.1700705	3.6281179

−2X; High Volatility; Single-Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	−2	0.500%	1.050%	0.9988757	4.0812162
5	−2	0.750%	1.640%	0.9972317	4.1227550
5	−2	1.000%	2.300%	0.9945070	4.1649313
5	−2	1.500%	3.750%	0.9851653	4.2512488
5	−2	2.000%	4.500%	0.9782670	4.3402778
5	−2	2.500%	6.250%	0.9576408	4.4321330
20	−2	0.500%	1.050%	0.9955106	4.0812162
20	−2	0.750%	1.640%	0.9889727	4.1227550
20	−2	1.000%	2.300%	0.9782085	4.1649313
20	−2	1.500%	3.750%	0.9419687	4.2512488
20	−2	2.000%	4.500%	0.9158610	4.3402778
20	−2	2.500%	6.250%	0.8410281	4.4321330
120	−2	0.500%	1.050%	0.9733639	4.0812162
120	−2	0.750%	1.640%	0.9356336	4.1227550
120	−2	1.000%	2.300%	0.8761702	4.1649313
120	−2	1.500%	3.750%	0.6985845	4.2512488
120	−2	2.000%	4.500%	0.5901705	4.3402778
120	−2	2.500%	6.250%	0.3538857	4.4321330
250	−2	0.500%	1.050%	0.9453082	4.0812162
250	−2	0.750%	1.640%	0.8705701	4.1227550
250	−2	1.000%	2.300%	0.7592637	4.1649313
250	−2	1.500%	3.750%	0.4736485	4.2512488
250	−2	2.000%	4.500%	0.3333265	4.3402778
250	−2	2.500%	6.250%	0.1148501	4.4321330

−2X; High Volatility; Negative Single-Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	−2	−0.500%	1.050%	0.9989198	3.9211842
5	−2	−0.750%	1.640%	0.9973927	3.8826470
5	−2	−1.000%	2.300%	0.9949283	3.8446751
5	−2	−1.500%	3.750%	0.9868322	3.7703836
5	−2	−2.000%	4.500%	0.9814519	3.6982249
5	−2	−2.500%	6.250%	0.9651895	3.6281179
20	−2	−0.500%	1.050%	0.9956862	3.9211842
20	−2	−0.750%	1.640%	0.9896116	3.8826470
20	−2	−1.000%	2.300%	0.9798671	3.8446751
20	−2	−1.500%	3.750%	0.9483601	3.7703836
20	−2	−2.000%	4.500%	0.9278464	3.6982249
20	−2	−2.500%	6.250%	0.8678613	3.6281179
120	−2	−0.500%	1.050%	0.9743949	3.9211842
120	−2	−0.750%	1.640%	0.9392660	3.8826470
120	−2	−1.000%	2.300%	0.8851218	3.8446751
120	−2	−1.500%	3.750%	0.7275110	3.7703836
120	−2	−2.000%	4.500%	0.6380527	3.6982249
120	−2	−2.500%	6.250%	0.4272696	3.6281179
250	−2	−0.500%	1.050%	0.9473953	3.9211842
250	−2	−0.750%	1.640%	0.8776261	3.8826470
250	−2	−1.000%	2.300%	0.7755141	3.8446751
250	−2	−1.500%	3.750%	0.5154254	3.7703836
250	−2	−2.000%	4.500%	0.3921491	3.6982249
250	−2	−2.500%	6.250%	0.1700705	3.6281179

Table-2

3X; Low Volatility; Single-Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	3	0.50%	0.60%	0.9992141	8.7359557
5	3	0.75%	0.80%	0.9986236	8.6082699
5	3	1.00%	1.20%	0.9969506	8.4833632
5	3	1.50%	1.57%	0.9949342	8.2415696
5	3	2.00%	2.30%	0.9894627	8.0099680
5	3	2.50%	2.58%	0.9871534	7.7879935
20	3	0.50%	0.60%	0.9968600	8.7359557
20	3	0.75%	0.80%	0.9945059	8.6082699
20	3	1.00%	1.20%	0.9878583	8.4833632
20	3	1.50%	1.57%	0.9798903	8.2415696
20	3	2.00%	2.30%	0.9585124	8.0099680
20	3	2.50%	2.58%	0.9495952	7.7879935
120	3	0.50%	0.60%	0.9813073	8.7359557
120	3	0.75%	0.80%	0.9674846	8.6082699
120	3	1.00%	1.20%	0.9293255	8.4833632
120	3	1.50%	1.57%	0.8852476	8.2415696
120	3	2.00%	2.30%	0.7755085	8.0099680
120	3	2.50%	2.58%	0.7332144	7.7879935
250	3	0.50%	0.60%	0.9614509	8.7359557
250	3	0.75%	0.80%	0.9334516	8.6082699
250	3	1.00%	1.20%	0.8583867	8.4833632
250	3	1.50%	1.57%	0.7757437	8.2415696
250	3	2.00%	2.30%	0.5888056	8.0099680
250	3	2.50%	2.58%	0.5238793	7.7879935

−3X; Low Volatility; Single-Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	−3	0.50%	0.60%	0.9991655	9.2761988
5	−3	0.75%	0.80%	0.9984941	9.4190907
5	−3	1.00%	1.20%	0.9965624	9.5653098
5	−3	1.50%	1.57%	0.9939375	9.8681505
5	−3	2.00%	2.30%	0.9866199	10.1856043
5	−3	2.50%	2.58%	0.9826882	10.5186267
20	−3	0.50%	0.60%	0.9966661	9.2761988
20	−3	0.75%	0.80%	0.9939899	9.4190907
20	−3	1.00%	1.20%	0.9863204	9.5653098
20	−3	1.50%	1.57%	0.9759694	9.8681505
20	−3	2.00%	2.30%	0.9475441	10.1856043
20	−3	2.50%	2.58%	0.9325305	10.5186267
120	−3	0.50%	0.60%	0.9801628	9.2761988
120	−3	0.75%	0.80%	0.9644770	9.4190907
120	−3	1.00%	1.20%	0.9206786	9.5653098
120	−3	1.50%	1.57%	0.8642061	9.8681505
120	−3	2.00%	2.30%	0.7237631	10.1856043
120	−3	2.50%	2.58%	0.6576248	10.5186267
250	−3	0.50%	0.60%	0.9591163	9.2761988
250	−3	0.75%	0.80%	0.9274163	9.4190907
250	−3	1.00%	1.20%	0.8418314	9.5653098
250	−3	1.50%	1.57%	0.7378239	9.8681505
250	−3	2.00%	2.30%	0.5099090	10.1856043
250	−3	2.50%	2.58%	0.4176263	10.5186267

−3X; Low Volatility; Negative Single-Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	−3	−0.50%	0.60%	0.9992141	8.7359557
5	−3	−0.75%	0.80%	0.9986236	8.6082699
5	−3	−1.00%	1.20%	0.9969506	8.4833632
5	−3	−1.50%	1.57%	0.9949342	8.2415696
5	−3	−2.00%	2.30%	0.9894627	8.0099680
5	−3	−2.50%	2.58%	0.9871534	7.7879935
20	−3	−0.50%	0.60%	0.9968600	8.7359557
20	−3	−0.75%	0.80%	0.9945059	8.6082699
20	−3	−1.00%	1.20%	0.9878583	8.4833632
20	−3	−1.50%	1.57%	0.9798903	8.2415696
20	−3	−2.00%	2.30%	0.9585124	8.0099680
20	−3	−2.50%	2.58%	0.9495952	7.7879935
120	−3	−0.50%	0.60%	0.9813073	8.7359557
120	−3	−0.75%	0.80%	0.9674846	8.6082699
120	−3	−1.00%	1.20%	0.9293255	8.4833632
120	−3	−1.50%	1.57%	0.8852476	8.2415696
120	−3	−2.00%	2.30%	0.7755085	8.0099680
120	−3	−2.50%	2.58%	0.7332144	7.7879935
250	−3	−0.50%	0.60%	0.9614509	8.7359557
250	−3	−0.75%	0.80%	0.9334516	8.6082699
250	−3	−1.00%	1.20%	0.8583867	8.4833632
250	−3	−1.50%	1.57%	0.7757437	8.2415696
250	−3	−2.00%	2.30%	0.5888056	8.0099680
250	−3	−2.50%	2.58%	0.5238793	7.7879935

3X; High Volatility; Single-Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	3	0.50%	1.05%	0.9975950	8.7359557
5	3	0.75%	1.64%	0.9942285	8.6082699
5	3	1.00%	2.30%	0.9888435	8.4833632
5	3	1.50%	3.75%	0.9714415	8.2415696
5	3	2.00%	4.50%	0.9602607	8.0099680
5	3	2.50%	6.25%	0.9267656	7.7879935
20	3	0.50%	1.05%	0.9904148	8.7359557
20	3	0.75%	1.64%	0.9771132	8.6082699
20	3	1.00%	2.30%	0.9561151	8.4833632
20	3	1.50%	3.75%	0.8905669	8.2415696
20	3	2.00%	4.50%	0.8502696	8.0099680
20	3	2.50%	6.25%	0.7376997	7.7879935
120	3	0.50%	1.05%	0.9438497	8.7359557
120	3	0.75%	1.64%	0.8703004	8.6082699
120	3	1.00%	2.30%	0.7639431	8.4833632
120	3	1.50%	3.75%	0.4988836	8.2415696
120	3	2.00%	4.50%	0.3778677	8.0099680
120	3	2.50%	6.25%	0.1611675	7.7879935
250	3	0.50%	1.05%	0.8865725	8.7359557
250	3	0.75%	1.64%	0.7487051	8.6082699
250	3	1.00%	2.30%	0.5706596	8.4833632
250	3	1.50%	3.75%	0.2348723	8.2415696
250	3	2.00%	4.50%	0.1316612	8.0099680
250	3	2.50%	6.25%	0.0223098	7.7879935

−3X; High Volatility; Single-Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	−3	0.50%	1.05%	0.9974465	9.2761988
5	−3	0.75%	1.64%	0.9936866	9.4190907
5	−3	1.00%	2.30%	0.9874296	9.5653098
5	−3	1.50%	3.75%	0.9659022	9.8681505
5	−3	2.00%	4.50%	0.9497423	10.1856043
5	−3	2.50%	6.25%	0.9023787	10.5186267
20	−3	0.50%	1.05%	0.9898251	9.2761988
20	−3	0.75%	1.64%	0.9749846	9.4190907
20	−3	1.00%	2.30%	0.9506584	9.5653098
20	−3	1.50%	3.75%	0.8704274	9.8681505
20	−3	2.00%	4.50%	0.8136227	10.1856043
20	−3	2.50%	6.25%	0.6630639	10.5186267
120	−3	0.50%	1.05%	0.9404827	9.2761988
120	−3	0.75%	1.64%	0.8589870	9.4190907
120	−3	1.00%	2.30%	0.7381539	9.5653098
120	−3	1.50%	3.75%	0.4349061	9.8681505
120	−3	2.00%	4.50%	0.2900938	10.1856043
120	−3	2.50%	6.25%	0.0849831	10.5186267
250	−3	0.50%	1.05%	0.8799963	9.2761988
250	−3	0.75%	1.64%	0.7285713	9.4190907
250	−3	1.00%	2.30%	0.5312587	9.5653098
250	−3	1.50%	3.75%	0.1764645	9.8681505
250	−3	2.00%	4.50%	0.0759082	10.1856043
250	−3	2.50%	6.25%	0.0058809	10.5186267

−3X; High Volatility; Negative Single-Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	−3	−0.50%	1.05%	0.9975950	8.7359557
5	−3	−0.75%	1.64%	0.9942285	8.6082699
5	−3	−1.00%	2.30%	0.9888435	8.4833632
5	−3	−1.50%	3.75%	0.9714415	8.2415696
5	−3	−2.00%	4.50%	0.9602607	8.0099680
5	−3	−2.50%	6.25%	0.9267656	7.7879935
20	−3	−0.50%	1.05%	0.9904148	8.7359557
20	−3	−0.75%	1.64%	0.9771132	8.6082699
20	−3	−1.00%	2.30%	0.9561151	8.4833632
20	−3	−1.50%	3.75%	0.8905669	8.2415696
20	−3	−2.00%	4.50%	0.8502696	8.0099680
20	−3	−2.50%	6.25%	0.7376997	7.7879935
120	−3	−0.50%	1.05%	0.9438497	8.7359557
120	−3	−0.75%	1.64%	0.8703004	8.6082699
120	−3	−1.00%	2.30%	0.7639431	8.4833632
120	−3	−1.50%	3.75%	0.4988836	8.2415696
120	−3	−2.00%	4.50%	0.3778677	8.0099680
120	−3	−2.50%	6.25%	0.1611675	7.7879935
250	−3	−0.50%	1.05%	0.8865725	8.7359557
250	−3	−0.75%	1.64%	0.7487051	8.6082699
250	−3	−1.00%	2.30%	0.5706596	8.4833632
250	−3	−1.50%	3.75%	0.2348723	8.2415696
250	−3	−2.00%	4.50%	0.1316612	8.0099680
250	−3	−2.50%	6.25%	0.0223098	7.7879935

Table-3

2X; Low Volatility; Double & Triple Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	2	10.25%	11.50%	0.91294495	2.7547735
5	2	25.00%	26.00%	0.74048904	1.7777778
5	2	56.00%	60.00%	0.44888130	0.8899964
5	2	110.00%	121.00%	0.23936034	0.3906250
5	2	157.00%	160.00%	0.22455893	0.2333777
5	2	177.00%	183.00%	0.19695801	0.1940655
20	2	10.25%	11.50%	0.69466970	2.7547735
20	2	25.00%	26.00%	0.30065923	1.7777778
20	2	56.00%	60.00%	0.04060000	0.8899964
20	2	110.00%	121.00%	0.00328253	0.3906250
20	2	157.00%	160.00%	0.00254285	0.2333777
20	2	177.00%	183.00%	0.00150485	0.1940655
120	2	10.25%	11.50%	0.11237511	2.7547735
120	2	25.00%	26.00%	0.00073866	1.7777778
120	2	56.00%	60.00%	0.00000000	0.8899964
120	2	110.00%	121.00%	0.00000000	0.3906250
120	2	157.00%	160.00%	0.00000000	0.2333777
120	2	177.00%	183.00%	0.00000000	0.1940655
250	2	10.25%	11.50%	0.01052518	2.7547735
250	2	25.00%	26.00%	0.00000030	1.7777778
250	2	56.00%	60.00%	0.00000000	0.8899964
250	2	110.00%	121.00%	0.00000000	0.3906250
250	2	157.00%	160.00%	0.00000000	0.2333777
250	2	177.00%	183.00%	0.00000000	0.1940655

–2X; Low Volatility; Double & Triple Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	–2	10.25%	11.50%	0.81119399	6.3288636
5	–2	25.00%	26.00%	0.06693723	16.0000000
5	–2	56.00%	60.00%	0.00000000	277.7777778
5	–2	110.00%	121.00%	0.00003840	2.7777778
5	–2	157.00%	160.00%	0.00373500	0.8734387
5	–2	177.00%	183.00%	0.00556752	0.6200012
20	–2	10.25%	11.50%	0.43301097	6.3288636
20	–2	25.00%	26.00%	0.00002008	16.0000000
20	–2	56.00%	60.00%	0.00000000	277.7777778
20	–2	110.00%	121.00%	0.00000000	2.7777778
20	–2	157.00%	160.00%	0.00000000	0.8734387
20	–2	177.00%	183.00%	0.00000000	0.6200012
120	–2	10.25%	11.50%	0.00659164	6.3288636
120	–2	25.00%	26.00%	0.00000000	16.0000000
120	–2	56.00%	60.00%	0.00000000	277.7777778
120	–2	110.00%	121.00%	0.00000000	2.7777778
120	–2	157.00%	160.00%	0.00000000	0.8734387
120	–2	177.00%	183.00%	0.00000000	0.6200012
250	–2	10.25%	11.50%	0.00002859	6.3288636
250	–2	25.00%	26.00%	0.00000000	16.0000000
250	–2	56.00%	60.00%	0.00000000	277.7777778
250	–2	110.00%	121.00%	0.00000000	2.7777778
250	–2	157.00%	160.00%	0.00000000	0.8734387
250	–2	177.00%	183.00%	0.00000000	0.6200012

–2X; Low Volatility; Negative Double & Triple Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	–2	–10.25%	11.50%	0.91294495	2.7547735
5	–2	–25.00%	26.00%	0.74048904	1.7777778
5	–2	–56.00%	60.00%	0.44888130	0.8899964
5	–2	–110.00%	121.00%	0.23936034	0.3906250
5	–2	–157.00%	160.00%	0.22455893	0.2333777
5	–2	–177.00%	183.00%	0.19695801	0.1940655
20	–2	–10.25%	11.50%	0.69466970	2.7547735
20	–2	–25.00%	26.00%	0.30065923	1.7777778
20	–2	–56.00%	60.00%	0.04060000	0.8899964
20	–2	–110.00%	121.00%	0.00328253	0.3906250
20	–2	–157.00%	160.00%	0.00254285	0.2333777
20	–2	–177.00%	183.00%	0.00150485	0.1940655
120	–2	–10.25%	11.50%	0.11237511	2.7547735
120	–2	–25.00%	26.00%	0.00073866	1.7777778
120	–2	–56.00%	60.00%	0.00000000	0.8899964
120	–2	–110.00%	121.00%	0.00000000	0.3906250
120	–2	–157.00%	160.00%	0.00000000	0.2333777
120	–2	–177.00%	183.00%	0.00000000	0.1940655
250	–2	–10.25%	11.50%	0.01052518	2.7547735
250	–2	–25.00%	26.00%	0.00000030	1.7777778
250	–2	–56.00%	60.00%	0.00000000	0.8899964
250	–2	–110.00%	121.00%	0.00000000	0.3906250
250	–2	–157.00%	160.00%	0.00000000	0.2333777
250	–2	–177.00%	183.00%	0.00000000	0.1940655

2X; High Volatility; Double & Triple Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	2	10.25%	22.00%	0.7165354	2.7547735
5	2	25.00%	56.00%	0.2481361	1.7777778
5	2	56.00%	120.00%	0.0406000	0.8899964
5	2	110.00%	235.00%	0.0045480	0.3906250
5	2	157.00%	340.00%	0.0011772	0.2333777
5	2	177.00%	365.00%	0.0015592	0.1940655
20	2	10.25%	22.00%	0.2636032	2.7547735
20	2	25.00%	56.00%	0.0037911	1.7777778
20	2	56.00%	120.00%	0.0000027	0.8899964
20	2	110.00%	235.00%	0.0000000	0.3906250
20	2	157.00%	340.00%	0.0000000	0.2333777
20	2	177.00%	365.00%	0.0000000	0.1940655
120	2	10.25%	22.00%	0.0003355	2.7547735
120	2	25.00%	56.00%	0.0000000	1.7777778
120	2	56.00%	120.00%	0.0000000	0.8899964
120	2	110.00%	235.00%	0.0000000	0.3906250
120	2	157.00%	340.00%	0.0000000	0.2333777
120	2	177.00%	365.00%	0.0000000	0.1940655
250	2	10.25%	22.00%	0.0000001	2.7547735
250	2	25.00%	56.00%	0.0000000	1.7777778
250	2	56.00%	120.00%	0.0000000	0.8899964
250	2	110.00%	235.00%	0.0000000	0.3906250
250	2	157.00%	340.00%	0.0000000	0.2333777
250	2	177.00%	365.00%	0.0000000	0.1940655

–2X; High Volatility; Double & Triple Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	–2	10.25%	22.00%	0.4649653	6.3288636
5	–2	25.00%	56.00%	0.0000036	16.0000000
5	–2	56.00%	120.00%	0.0000000	277.7777778
5	–2	110.00%	235.00%	0.0000000	2.7777778
5	–2	157.00%	340.00%	0.0000000	0.8734387
5	–2	177.00%	365.00%	0.0000000	0.6200012
20	–2	10.25%	22.00%	0.0467393	6.3288636
20	–2	25.00%	56.00%	0.0000000	16.0000000
20	–2	56.00%	120.00%	0.0000000	277.7777778
20	–2	110.00%	235.00%	0.0000000	2.7777778
20	–2	157.00%	340.00%	0.0000000	0.8734387
20	–2	177.00%	365.00%	0.0000000	0.6200012
120	–2	10.25%	22.00%	0.0000000	6.3288636
120	–2	25.00%	56.00%	0.0000000	16.0000000
120	–2	56.00%	120.00%	0.0000000	277.7777778
120	–2	110.00%	235.00%	0.0000000	2.7777778
120	–2	157.00%	340.00%	0.0000000	0.8734387
120	–2	177.00%	365.00%	0.0000000	0.6200012
250	–2	10.25%	22.00%	0.0000000	6.3288636
250	–2	25.00%	56.00%	0.0000000	16.0000000
250	–2	56.00%	120.00%	0.0000000	277.7777778
250	–2	110.00%	235.00%	0.0000000	2.7777778
250	–2	157.00%	340.00%	0.0000000	0.8734387
250	–2	177.00%	365.00%	0.0000000	0.6200012

–2X; High Volatility; Negative Double & Triple Digit Daily Returns

N	β	Rm	σ	VolEff	BetaCoeff
5	–2	–10.25%	22.00%	0.7165354	2.7547735
5	–2	–25.00%	56.00%	0.2481361	1.7777778
5	–2	–56.00%	120.00%	0.0406000	0.8899964
5	–2	–110.00%	235.00%	0.0045480	0.3906250
5	–2	–157.00%	340.00%	0.0011772	0.2333777
5	–2	–177.00%	365.00%	0.0015592	0.1940655
20	–2	–10.25%	22.00%	0.2636032	2.7547735
20	–2	–25.00%	56.00%	0.0037911	1.7777778
20	–2	–56.00%	120.00%	0.0000027	0.8899964
20	–2	–110.00%	235.00%	0.0000000	0.3906250
20	–2	–157.00%	340.00%	0.0000000	0.2333777
20	–2	–177.00%	365.00%	0.0000000	0.1940655
120	–2	–10.25%	22.00%	0.0003355	2.7547735
120	–2	–25.00%	56.00%	0.0000000	1.7777778
120	–2	–56.00%	120.00%	0.0000000	0.8899964
120	–2	–110.00%	235.00%	0.0000000	0.3906250
120	–2	–157.00%	340.00%	0.0000000	0.2333777
120	–2	–177.00%	365.00%	0.0000000	0.1940655
250	–2	–10.25%	22.00%	0.0000001	2.7547735
250	–2	–25.00%	56.00%	0.0000000	1.7777778
250	–2	–56.00%	120.00%	0.0000000	0.8899964
250	–2	–110.00%	235.00%	0.0000000	0.3906250
250	–2	–157.00%	340.00%	0.0000000	0.2333777
250	–2	–177.00%	365.00%	0.0000000	0.1940655

Bibliography

Ahn, H., Cai, J., & Cheung, Y. (2005). Price clustering on the limit-order book: Evidence from the stock exchange of Hong Kong. *Journal of Financial Markets, 8*(4), 421–451.

Ait-Sahalia, Y., Fan, J., & Li, Y. (2013). The leverage effect puzzle: Disentangling sources of bias at high frequency. *Journal of Financial Economics, 109*(1), 224–249.

Akhtar, S., Faff, R., Oliver, B., & Subrahmanyam, A. (2011). The power of bad: The negativity bias in Australian consumer sentiment announcements on stock returns. *Journal of Banking & Finance, 35*(5), 1239–1249.

Asparouhova, E., Bessembinder, H., & Kalcheva, I. (2010). Liquidity biases in asset pricing tests. *Journal of Financial Economics, 96*(2), 215–237.

Bakken, H., Lindset, S., & Olson, L. H. (2006). The Pricing of multi-period rate of return guarantees: The Monte Carlo approach original. *Insurance: Mathematics and Economics, 39*(1), 135–149.

Benos, E., & Jochec, M. (2012). Patriotic name bias and stock returns. *Journal of Financial Markets, 16*(3), 550–570.

Bickel, S. (1969). Minimum variance and optimal asymptotic portfolios. *Management Science, 16*(3), 221–226.

Blume, M. (1974). Unbiased estimators of long-run expected rates of return. *Journal of the American Statistical Association, 69*(347), 634–638.

Blume, M. E., & Stambaugh, R. F. (1983). Biases in computed returns: An application to the size effect. *Journal of Financial Economics, 12*, 387–404.

Booth, L. (2002). Discounting expected values with parameter uncertainty. *Journal of Corporate Finance, 9*, 505–519.

Branch, B., & Echevarria, D. P. (1998). The bid-ask bias and the size effect: A test of the Blume-Stambaugh bid-ask bias effect hypothesis. *Quarterly Review of Economics & Finance, 38*(1), 129–148.

Cao, G. & Shan, D. (2013). The effect of exit strategy on optimal portfolio selection with bi-random returns. *Journal of Applied Mathematics*, *2013*(6), Article ID 236579.

Chae, J., & Yang, C. (2013). Commonality in individuals' trading: A systematic path between behavioral bias and expected returns. *Pacific-Basin Finance Journal, 21*(1), 1008–1023.

Chari, V., Kehoe, P. J, & McGrattan, E. (Feb. 2005). *A critique of structural VARs using real business cycle theory* (Working paper #631). Minneapolis: Federal Reserve Bank of Minneapolis: Research Department.

Cheng, P., & Deets, M. K. (1971). Statistical biases and security rates of return. *The Journal of Financial and Quantitative Analysis, 6*(3), 977–994.
Choe, H., & Shin, H. S. (1993). An analysis of interday and intraday return volatility—Evidence from the Korea stock exchange. *Pacific-Basin Finance Journal, 1*(2), 175–188.
Co, R. & Labuszewski, J. (July 2012). Leveraged ETFs: Where is the missing performance? *Stock Market Landscape*, pp. 1–4; July 26, 2012 |CME Group, USA.
Comerton-Forde, C., & Putniņš, T. (2010). Measuring closing price manipulation. *Journal of Financial Intermediation, 20*(2), 135–158.
Conrad, J., & Kaul, G. (1993). Long-term market overreaction or biases in computed returns. *Journal of Finance, 48*(1), 39–63.
Cooper, I. (2006). Arithmetic versus geometric mean estimators: Setting discount rates for capital budgeting. *European Financial Management, 2*(2), 157–167.
Cowan, A., & Sergeant, A. M. (2001). Interacting biases, non-normal return distributions and the performance of tests for long-horizon event studies. *Journal of Banking & Finance, 25*(4), 741–765.
Dorfleitner, G. (2003). Why the return notion matters. *International Journal of Theoretical & Applied Finance, 6*, 73–86.
Fenn, D. J., et al. (2011). Temporal evolution of financial-market correlations. *Physics Review-E, 84*, 61–65.
Fisher, L., Weaver, D., & Webb, G. (2009). Removing biases in computed returns: An analysis of bias in equally-weighted return indexes of REITs. *International Real Estate Review, 15*, 43–71.
Fisher, L., Weaver, D., & Webb, G. (2010). Removing biases in computed returns. *Review of Quantitative Finance and Accounting, 21*, 137–151.
Gerrard, R., & Haberman, S. (1996). Stability of pension systems when gains/losses are amortized and rates of return are autoregressive. *Insurance: Mathematics and Economics, 18*(1), 59–71.
Goetzmann, W., Ingersoll, J., Spiegel, M., & Welch, I. (2007). Portfolio performance manipulation and manipulation-proof performance measures. *Review Of Financial Studies, 20*, 1503–1546.
Graff, C. (2014). Expressing relative differences (in percent) by the difference of natural logarithms. *Journal of Mathematical Psychology, 60*, 82–85.
Guillen, M., Jorgensen, P., & Nielsen, J. (2006). Return smoothing mechanisms in life and pension insurance: Path-dependent contingent claims. *Insurance: Mathematics and Economics, 38*(2), 229–252.

Haberman, S., & Wong, L. (1997). Moving average rates of return and the variability of pension contributions and fund levels for a defined benefit pension scheme. *Insurance: Mathematics and Economics, 20*(2), 115–135.

Hillion, P., & Suominen, M. (2004). The manipulation of closing prices. *Journal of Financial Markets, 7*(4), 351–375.

Hommes, C. H. (2002). Modeling the stylized facts in finance through simple nonlinear adaptive systems. *PNAS, 99*, 7221–7228.

Hsieh, D. A. (1993). Implications of nonlinear dynamics for financial risk management. *Journal of Financial and Quantitative Analysis, 28*(1), 41–64.

Indro, D., & Lee, W. Y. (1997). Biases in arithmetic and geometric averages as estimates of long-run expected returns and risk premia. *Financial Management, 26*(4), 81–90.

Jacquier, E., Kane, A., & Marcus, A. (2003). Geometric or arithmetic mean: A re-consideration. *Financial Analysts Journal, 59*(6), 46–53.

Jacquier, E., Kane, A., & Marcus, A. (2005). Optimal estimation of the risk premium for the long run and asset allocation: A case of compounded estimation risk. *Journal Of Financial Econometrics, 3*, 37–55.

Jean, W., & Helms, W. P. (1983). Geometric mean approximations. *Journal of Financial and Quantitative Analysis, 18*, 287–293.

Kallunki, J., & Martikainen, T. (1997). The covariance-factor structure of daily returns in a thinly traded stock market. *Journal of Multinational Financial Management, 7*(2), 113–125.

Keim, D. (1989). Trading patterns, bid-ask spreads, and estimated security returns: The case of common stocks at calendar turning points. *Journal of Financial Economics, 25*(1), 75–97.

Kenett, D. Y., Preis, T., et al. (2012). Dependency network and node influence: Application to the study of financial markets. *International Journal Of Bifurcation & Chaos, 22*, 1250181 (2012).

Kriener, B., Helias, M., Rotter, S., et. al. (2014). How pattern formation in ring networks of excitatory and inhibitory spiking neurons depends on the input current regime. *Frontiers Of Computational Neuroscience*, 7:187. doi: 10.3389/fncom.2013.00187.

Lakonishok, J., & Smidt, S. (1984). Volume and turn-of-the-year behavior. *Journal of Financial Economics, 13*(3), 435–455.

Liu, W., & Strong, N. (2008). Biases in decomposing holding-period portfolio returns. *The Review Of Financial Studies, 21*(5), 2243–2274.

Llerbach, W. (2003). Cross- and auto-correlation effects arising from averaging: The case of US interest rates and equity duration. *Applied Financial Economics, 13*, 4), 287–4), 294.

MacLean, L., Ziemba, W., & Blazenko, G. (1992). Growth versus security in dynamic investment analysis. *Management Science, 38*(11), 1562–1585.

Menna, M., Rotundo, G., & Tirozzi, B. (2002). Distinguishing between chaotic and stochastic systems in financial time series. *International Journal of Modern Physics-C, 13*(1), 31–39.

Nwogugu, M. (2013). Decision-making, sub-additive recursive "matching" noise and biases in risk-weighted index calculation methods in in-complete markets with partially observable multi-attribute preferences. *Discrete Mathematics, Algorithms & Applications, 5*, 1350020. doi:10.1142/S1793830913500201.

Ortoleva, P. (2010). Status quo bias, multiple priors and uncertainty aversion. *Games and Economic Behavior, 69*(2), 411–424.

Perocco, M., & Borgonovo, E. (2012). A note on the sensitivity analysis of the internal rate of return. *International Journal of Production Economics, 135*(1), 526–529.

Podobnik, B., Horvatic, D., Petersen, A. M., & Stanley, H. E. (2009). Cross-correlations between volume change and price change. *PNAS, 106*, 22079–22084.

Preis, T. (2011). Econophysics—Complex correlations and trend switchings in financial time series. *European Physical Journal, 194*, 5–86.

Preis, T, Kenett, D., et. al. (2012). Quantifying the behavior of stock correlations under market stress. *Scientific Reports 2, 752*(2012). doi:10.1038/srep00752

Roll, R. (1983). On computing mean returns and the small firm premium. *Journal of Financial Economics, 12*(3), 371–386.

Scherer, B. (2011). A note on the returns from minimum variance investing. *Journal of Empirical finance, 18*(4), 652–660.

South, D. B. (1995). Relative growth rates: A critique. *South African Forestry Journal, 173*, 43–48.

Strong, N. (2006). Modeling abnormal returns. *Journal of Business Finance & Accounting, 19*(4), 533–553.

Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *Journal of Mathematical Behavior, 31*(3), 344–355.

Wen, F., Huang, D., Lan, Q., & Yang, X. (2007). Numerical simulation for influence of overconfidence and regret aversion on return distribution. *Systems Engineering - Theory & Practice, 27*(7), 10–18.

Wilson, J., Jones, C. P., & Lundstrum, L. (2005). Stochastic properties of unaveraged financial data: Explanation and empirical demonstration using monthly stock prices. *Financial Review, 36*(3), 175–190.

Wong, B. (2009). On modelling long term stock returns with ergodic diffusion processes: Arbitrage and arbitrage-free specifications. *Journal of Applied Mathematics and Stochastic Analysis*, *2009* (2009), Article ID 215817.

Zhang, G. (2012). Optimal geometric mean returns of stocks and their options. *International Journal of Stochastic Analysis*. http://dx.doi.org/10.1155/2012/498050

Zhao, D., & Fang, Y. (2013). Can representation bias help the returns forecast and portfolio selection? *Procedia Computer Science, 17*, 603–610.

Zhu, Z. (2002). Time-varying forward bias and the expected excess return. *Journal of International Financial Markets, Institutions and Money, 12*(2), 119–137.

9

Conclusion

In financial markets in most developed and developing countries, investors' preferences diverge substantially. Furthermore, all existing asset pricing models are inaccurate because the underlying assumptions are not realistic. The focus on consumption and price as the definition of investors' preferences and constraints is misleading and the investors' decision problem is much broader. The elasticity of intertemporal substitution is also very inaccurate, especially where markets are incomplete and investors' preferences are both dynamic and multifaceted. Given the problems and inaccuracies inherent in the CSIP dichotomy, the UIWD and MRIJS are more likely to result in better policy decisions. The MRIJS is distribution free, does not require use of any specific utility functions, implicitly accounts for risk (by multifaceted wealth allocation) and provides a more unified and accurate indication/analysis of the average investor's wealth allocation decisions. Obviously, these have important ramifications for asset management and capital budgeting.

This book identifies and analyzes the many pervasive biases inherent in the NPV/IRR/MIRR models and related approaches (such as AIRR; NFV; Selective-IRR; etc.) which are used extensively in the automated

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financial systems of government agencies, ordinary companies and financial institutions around the world, and directly affect daily transactions that exceed €5 trillion (in the form of: swaps; money-market investments; bonds; futures; stocks; loans; capital budgeting in companies; etc.). The necessary and sufficient conditions for monotonic NPV ("well behaved" NPV) do not exist in many circumstances. NPV, MIRR and related approaches are deeply flawed and very sensitive to the time horizon, the signs of periodic cash flows, and discount rates that exceed 100% or are below −100%. The NPV-MIR-IRR model does not accommodate the differences between compounded interest rates and simple interest rates, and does not account for Real Options, Regret, or Rejoice in decision-making. The NPV-IRR-MIRR model (and related approaches, such as AIRR, NFV, SVA, etc.) can also be distorted by framing effects. Regret-based decision models are a viable alternative to the NPV-MIRR-IRR model. However, while the NPV-MIRR model (and related approaches) does not maximize any objective functions (it merely states the extent to which a project adds value, but cannot be used to choose among many positive-NPV projects where there are budget constraints), Regret can be minimized, and thus can be used to maximize objective functions in project evaluation/selection, given specific criteria. While Regret-based decision models can capture inflexibility, Real Options, framing effects and other factors, its main disadvantage is that there is no unified method for calculating Regret; or unified Regret-based decision model.

The substantial statistical biases that are caused by the compounding of periodic returns tend to increase over time, and also tend to increase if the growth rates of the associated periodic cash flows/benefits are serially correlated (as explained in Booth 2002). Where daily data are used, there are other biases caused by differences in the calculation of prices, and the possible manipulation of closing prices. These biases have critical implications for portfolio management, capital budgeting, insurance, and risk management. However, the *leverage effect* puzzle (discussed in Ait-Sahalia et al. 2013) can also be partly attributed to some of the biases introduced in this book—as the frequency increases, the periodic returns become smaller and more subject to these biases. Similarly, substantial portions of the leverage effect (which has been documented in the literature) can be attributed to some of the biases introduced in this book—such as the *downward-bias effect,* the *negative-returns effect*, the *proportionality effect*, and the *volume-loss effect*.

The inaccuracy of Descartes' Sign Rule, the Fourier-Boudan Theorem, Sturm's Theorem, Vincent's Theorem, the Binomial Theorem, the Fundamental Theorem of Algebra, traditional root-verification methods in algebra, and the power rule and inverse function rule (in differential calculus), all have significant implications for researchers and professionals in mathematics, finance, operations research and all areas of fundamental research that rely on algebra and calculus.

Overall, the biases and functional errors in Polynomials Theory are significant and some are, or can be, recursive and additive (sub-additive or super-additive).

Bibliography

Ait-Sahalia, Y., Fan, J. & Li, Y. (2013). The leverage effect puzzle: Disentangling sources of bias at high frequency. *Journal of Financial Economics*, *109*(1), 224–249.

Booth, L. (2002). Discounting expected values with parameter uncertainty. *Journal of Corporate Finance*, *9*, 505–519.