$f : A \longrightarrow B$.

$A \longleftarrow B : g$

$fog = gof = i$ identity

$g = f^{-1}$

$f : \mathbb{Z} \longrightarrow \mathbb{Z}$

$f(x) = 2x+1$

$y = 2x+1$

$f^{-1}(x) = \frac{x-1}{2}$

$\frac{y-1}{2} = x$

Computer Representation of Sets

$\{1,2,3\}$

$\Downarrow$

$\{1,3,2\}$

Set
unordered

Union
intersection
[illegible]

in Computer Science

Let $a_1, a_2, \ldots a_n$ be the arbitrary ordering of the elements of U.

$$f : N \longrightarrow U \quad \text{where } N = \{1, \ldots n\}$$

$$f(i) = a_i$$

Now, to represent a subset A of U with the bit string of length n, we follow:

→ i^{th} bit in this string is 1, if $a_i \in A$ and 0 if $a_i \notin A$.

example $U = \{1, 2, 3, 4, \ldots 10\}$

and give arbitrary ordering is in <u>increasing order</u>.

$f(i) = a_i = i$:

$f(1) = 1$
$f(2) = 2$
$f(3) = 3$
- - -
etc
$f(10) = 10$

$U = \{1, 2, \ldots, 10\}$ ✓
$A = \{1, 3, 5, 7, 9\}$ ✓
$B = \{2, 4, 6, 8, 10\}$ ✓
$C = \{1, 2, 3, 4, 5\}$ ✓

Q: What bit string represents the subset of all odd integers in U, the subset of all even integers in U, and the subset of integers not exceeding 5 in U.

A: The bit string that represents the set of odd integers in U ($U = \{1, 3, 5, 7, 9\}$), has a one bit in first, 3rd, 5th, 7th, 9th positions and zero bit elsewhere

It is: 1 0 1 0 1 0 1 0 1 0

Similarly: the bit string ~~represent~~ representation of set of even integer in U.

is:

0101010101 ✓ → {2,4,6,8,10}

And

1111100000 ← {1,2,3,4,5}

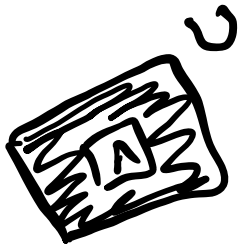
$U = \{1, 2, \ldots 10\} \leftrightarrow 1111111111$

$A = \{1,3,5,7,9\} \leftrightarrow 1010101010$

$B = \{2,4,6,8,10\} \leftrightarrow 0101010101$

$C = \{1,2,3,4,5\} \leftrightarrow 1111100000$

$A^c = 0101010101 = B$



A ∪ B: $\{1,3,5,7,9\} \cup \{2,4,6,8,10\}$

1010101010 ∨ 0101010101

(Join/wedge)

$= 1111111111$

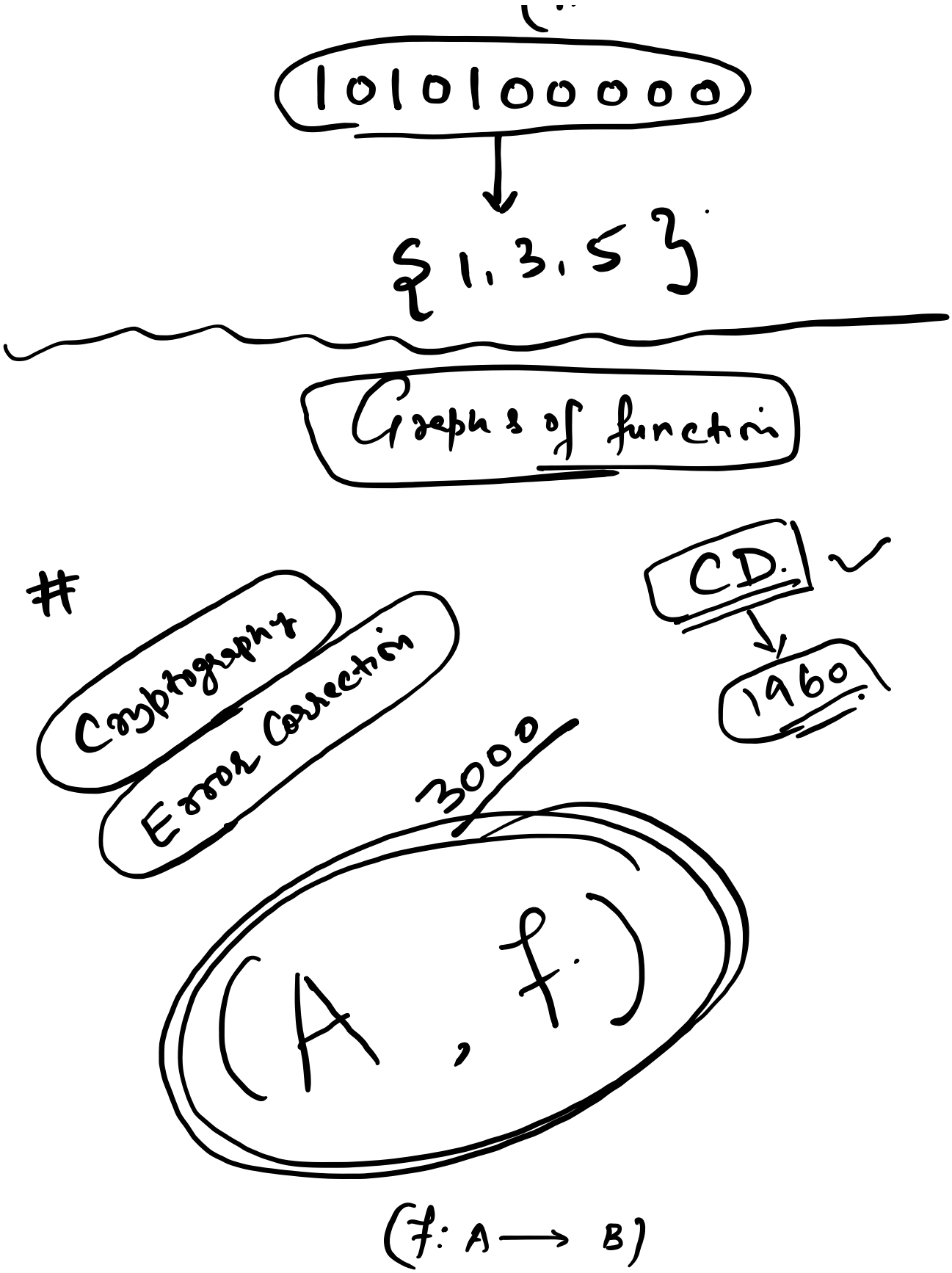
$\Rightarrow$ ∪

$1 \vee 0 = 1$
$1 \vee 1 = 1$
$0 \vee 0 = 0$
$0 \wedge 0 = 0$
$1 \wedge 0 = 0$
$1 \wedge 1 = 1$

A ∩ C: $\{1,3,5,7,9\} \cap \{1,2,3,4,5\}$

1010101010 ∧ 1111100000

(meet)


1010100000
{1, 3, 5}
Graphs of functions
#
Cryptography
Error Correction
CD.
1960
3000
(A, f)
(f: A → B)

A function f from A to B can be represented as a subset of $A \times B$.

if $f: A \to B$, then

$$f = \{(a, b): a \in A, b = f(a)\}$$

$$\{(a,b): a \in A, b \in B\}$$

Ex:

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, \ldots, 20\}$

$f: A \to B$

$(x \to x^2)$

$f(x) = x^2$

$f(1) = 1$

$f(2) = 4$

$f(3) = 9$

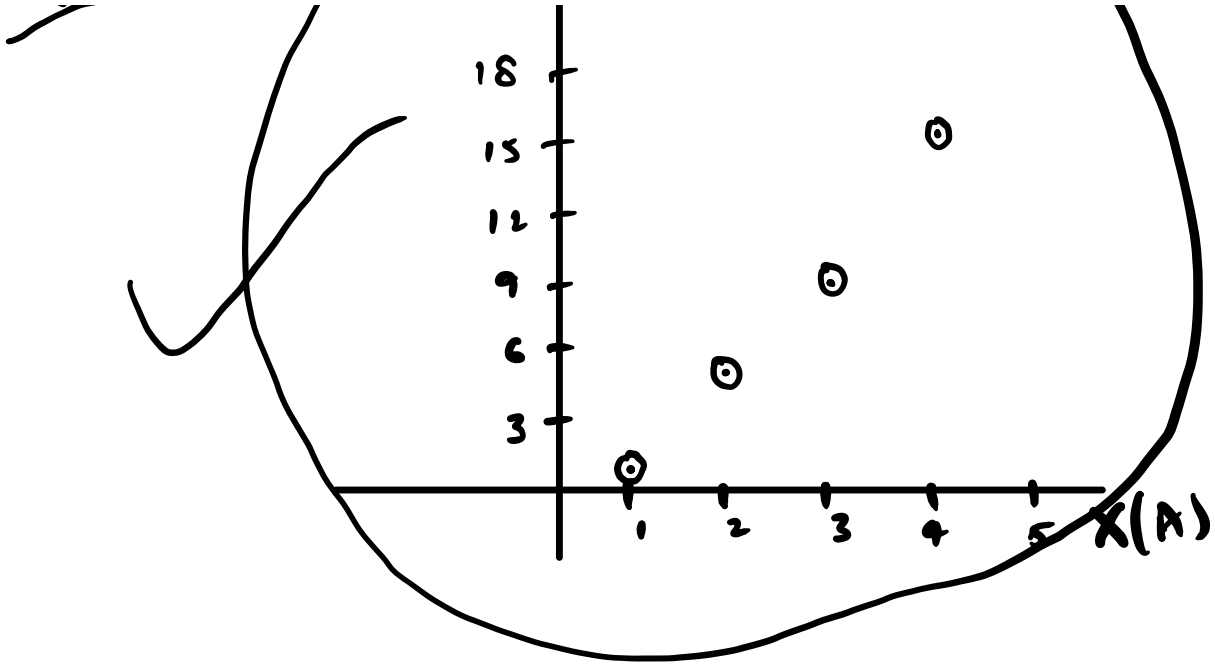
$f(4) = 16$

$\{\ \}$

$(\ )$

$f = \{(1,1), (2,4), (3,9), (4,16)\}$

Graph



Ceiling and Floor functions.

Floor function: assigns to the real no. x, the **largest integer** that is **less than or equal to** x. It is denoted by $\lfloor x \rfloor$

$$\lfloor 3.5 \rfloor = 3$$

$x : 3.5$

3

2

Ceiling function:- assign

the smallest integer that is greater than or equal to x

$\lceil x \rceil$

$\lceil 3.5 \rceil = 4$

$\lceil 4 \rceil = 4$

$3.5 \leq 4, 5, 6, 7$