MATHS

SET THEORY

- Set: A set is a well defined collection of distinct objects.
  A set is collection of objects of any sort

  Eg: A = {2, d, pencil, brick} ✗ (not well defined)

  A = {c, f, l, p} ✓ (well defined → alphabets)

- The objects of a set are called elements or members of that set.

  Eg: 1. Set of even numbers
  2. Set of prime numbers
  3. Set of indian rivers

Set Representation

→ ROASTER FORM:

- Tabulation form
- In this, the elements of a set are lised within a pair of branches {} and are seperated by commas.

- Eg: Let N denote set of first three natural no.s
  N = {1, 2, 3}
- Let V denote set of all vowels of English alphabet, V = {a, e, i, o, u}
- Let A is set of all letters in BOOK.

$A = \{B, O, K\}$

NOTE → The order within which elements are listed is immaterial but elements must not be repeated.

→ SET BUILDER FORM :

A rule or a formula or a statement is written within a pair of brackets so that the set is well defined. All the element of set must possess a single property to become member of that set.

$P = \{1, 2, 3, 4, 5\}$

$P = \{x \mid x \in N \text{ and } x \leq 5\}$

such that

Q. Represent the following sets in set builder form

1) $A = \{x \mid x = 3p$, where $1 \leq p \leq 5$ and p is natural number$\}$

2) $B = \{x \mid x^2 - 3x + 2 = 0\}$

i) Elements of set A are in the form of

$x = 3p$ where $1 \leq p \leq 5$

$p = 1, 2, 3, 4, 5$

$\therefore x = 3, 6, 9, 12, 15$

Hence the given set in tabular form is :

$A = \{3, 6, 9, 12, 15\}$

ii) $x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 1, 2$

Hence the given set in tabular form is:-
$B = \{1, 2\}$

CARDINALITY

- Cardinal Number
- No. of distinct elements of a set.
- denoted by $n(A)$ or $|A|$

Eg: $A = \{a, d, g, e, p\} \Rightarrow n(A) = 5$
$B = \{a, a, b, d, d\} \Rightarrow n(B) = 3$
$C = \{a, \{c, d\}, k\} \Rightarrow n(C) = 3$

TYPES OF SET

1) Null Set : Set that contains no element
- Empty Set   Eg: $P = \{x \mid x^2 = 4 ;\ x \text{ is odd}\}$, $P = \{\}$
- No elements
- denoted by ϕ or $\{\}$
- Cardinal no. $= 0$

Note → $\phi \neq \{\phi\}$

2) Singleton Set
- Only one element is present in the set
- $A = \{a\}$ or $B = \{5\}$
- Cardinal No. $= 1$

3) Finite Set
- $n(A) = N$

POPU
Page No.
Date: / /

4) Infinite set: Set that contain infinite elements.

- $n(A) = \infty$

Eg: $A = \{x : x \text{ is multiple of } 2, x \in N\} \Rightarrow A = \{2, 4, 6 \text{ ------}\}$

5) Equal Sets: When sets have exactly same elements

- $A = \{a, b, c\}$ $B = \{c, b, a\}$
- $n(A) = n(B)$
- Elements are also same

6) Equivalent sets: Sets have same number of elements

$A = \{a, b, c\}$ $B = \{1, 2, 2, 3\}$

- $n(A) = n(B)$

SUBSET:

- If all the elements of A is contained in B then

$$\underset{\text{Subset}}{A} \subset \underset{\text{superset}}{B}$$

Eg: $A = \{1, 5, 6\}$
$B = \{1, 2, 3, 5, 6\}$

Note:

ϕ is a subset of every set

i.e, $\phi \subset A$, $\phi \subset C$

- $A \subseteq A$
- $A \subset B$

all $x \in A \Rightarrow x \in B$

- $A \not\subset B$

atleast one $x \in A$ such that $x \notin B$

POWER SET:

- The family consisting of all the subsets of ~~the~~ a set is called power set of that set
- denoted by $P(A)$

Eg: $A = \{1, 2, 3\}$

$P(A) = \{ \phi, \{1\}, \{2\}, \{3\} + \{1,2\}, \{1,3\},$

$\{2,3\}, \{1,2,3\}\}$

$P(A) = \{x \mid x \subseteq A\}$

$n(A) = 3$

$n[P(A)] = 8 = 2^3$

$\therefore \boxed{n[P(A)] = 2^{|A|}}$

No. of proper subset $= 2^{n-1}$

(in proper subset $\{1,2,3\}$ would be excluded)

Q. Determine the power set of the following sets:

i) $\{a\}$ ii) $\{\phi, \{\phi\}\}$ iii) $\{a, b\}$

i) Let $X = \{a\}$

then power set of X,

$P(X) = \{\phi, \{a\}\}$ $n[P(X)] = 2^1 = 2$

ii) Let $Y = \{\phi, \{\phi\}\}$

then power set of Y,

$P(Y) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

$n[P(Y)] = 2^2 = 4$

iii) Let $Z = \{a, b\}$
Then power set of Z,
$P(Z) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$
$n[P(Z)] = 2^2 = 4$

UNIVERSAL SET

- All the elements under discussion

Eg: $U = \{x \mid x \text{ is an integer}, x \leq 8\}$
$A = \{1, 2, 4\}$
(U has the elements of 4 as well as some extra elements)

DISJOINT SETS

- Two sets A are B are called disjoint if $A \cap B = \phi$

Eg: $A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$
$\therefore A \cap B = \phi$

ORDERED PAIR

- An element of the form (a, b) is called an Ordered pair.
  $a \rightarrow$ 1st element
  $b \rightarrow$ 2nd element

POPU
Page No.
Date: / /

Equality Of Order pair:
Let (a,b) and (c,d) be any two ordered pairs
$(a,b) = (c,d)$
When $a=c$ & $b=d$

PRODUCT OF TWO SETS

Let $A=\{1,2,3\}$ and $B=\{a,b\}$

$A\times B = \{(1,a),(2,a),(3,a),(1,b),(2,b),(3,b)\}$

$B\times B = \{a,b\}\times\{a,b\}$
$\Rightarrow \{(a,a),(a,b),(b,a),(b,b)\}$

Q. Find the values of
1) $A\times B\times C$
2) B^2
3) $B^2\times 4$
where $A=\{1\}$, $B=\{a,b\}$, $C=\{2,3\}$

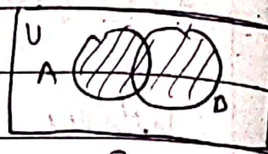
Ans ii) $B^2 = \{a,b\}\times\{a,b\}$
$= \{(a,a)(a,b),(b,a),(b,b)\}$

iii) $B^2\times 4 = \{(a,a,4),(a,b,4),(b,a,4),(b,b,4)\}$

1) $A\times B\times C$
$A\times B = \{1\}\times\{a,b\}$
$= \{(1,a),(1,b)\}$
$A\times B\times C = \{(1,a),(1,b)\}\times\{2,3\}$
$= \{(1,a,2),(1,a,3),(1,b,2),(1,b,3)\}$

OPERATION ON SETS :

→ **UNION:**



$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

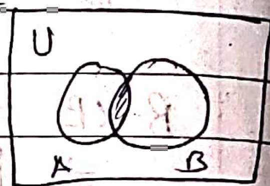
$A = \{1, 2, 3, 4\}$, $B = \{2, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\} = B \cup A$

→ **INTERSECTION:**

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$A \cap B = \{2, 4\} = B \cap A$



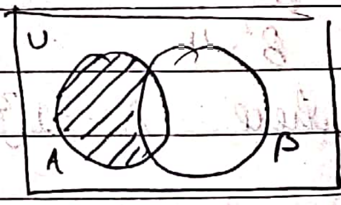
→ **Difference of two sets:**

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

= Difference of B from A

$A - B = \{1, 3\}$

$\therefore A - B = A - (A \cap B)$



Note : $A - B \neq B - A$

→ **Complement of a set:**

$A \Rightarrow$ — $U \Rightarrow$ —

- denoted by A^c or $\bar{A}$ or A'

$A' = \{x \mid x \in U \text{ and } x \notin A$

$A' = U - A$

→ $(A')' = A$

→ $U' = \phi$

→ $A' \cup A = U$

→ $\phi' = U$

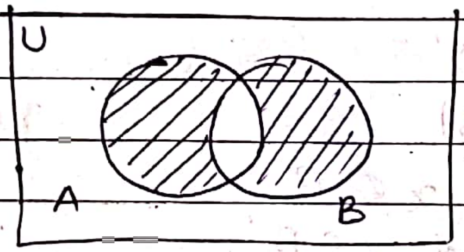
- $A' \cap A = \phi$

SYMMETRIC DIFFERENCE:

- denoted by $A \oplus B$ or $A \Delta B$
- $A \oplus B = (A-B) \cup (B-A)$

Eg: $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6\}$

$A \oplus B = \{1, 4, 5, 6\}$



Key points

or
and
Conjunction

1. $x \in (A \cup B)$
   $\Rightarrow x \in A$ or $x \in B$
2. $x \in (A \cap B)$
   $\Rightarrow x \in A$ and $x \in B$
3. $x \notin (A \cup B)$
   $\Rightarrow x \notin A$ and $x \notin B$
4. $x \notin (A \cap B)$
   $\Rightarrow x \notin A$ and $x \notin B$
5. $x \in A \Rightarrow x \notin A'$
   $x \in A' \Rightarrow x \notin A$
6. $x \in (A-B)$
   $\Rightarrow x \in A$ and $x \notin B$

DEMORGAN'S LAW

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$

Q. A and B are any two sets: Prove that
i) $(A \cup B)' = A' \cap B'$
ii) $(A \cap B)' = A' \cup B'$

Soln i) $\Rightarrow x \in (A \cup B)'$
$\Rightarrow x \notin (A \cup B)$
$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \in A'$ and $x \in B'$
$\Rightarrow x \in A' \cap B'$
$\therefore$ Proved
$\therefore (A \cup B)' \subset A' \cap B'$ —— 1

~~ii) $\Rightarrow x \in$~~

Again, $y \in A' \cap B'$
$y \in A'$ and $y \in B'$
$y \notin A$ and $y \notin B$
$y \in (A \cup B)'$
$\therefore A' \cap B' \subset (A \cup B)'$ —— 2

From 1 and 2
$(A \cup B)' = A' \cap B'$

ii) Suppose, $x \in (A \cap B)'$
$\Rightarrow x \notin A \cap B$
$\Rightarrow x \notin A$ or $x \notin B$
$\Rightarrow x \in A'$ or $x \in B'$
$\Rightarrow x \in A' \cup B'$

$\therefore \quad (A \cap B)' \subset A' \cup B'$ —— (1)

Again, $y \in A' \cup B'$

$\Rightarrow y \in A'$ or $y \in B'$

$\Rightarrow y \notin A$, or $y \notin B$

$\Rightarrow y \notin (A \cap B)$

$\Rightarrow y \in (A \cap B)'$

$\therefore \quad A' \cup B' \subset (A \cap B)'$ —— (2)

From (1) and (2)

$(A \cap B)' = A' \cup B'$

Q. If A, B and C are any three non-empty sets, prove that

$(A - B) \times C = (A \times C) - (B \times C)$

Sol) $(x, y) \in (A - B) \times C$

$(x \in A$ and $x \notin B)$ and $(y \in C)$

$(x \in A$ and $y \in C)$ and $(x \notin B$ and $y \in C)$

$(x, y) \in (A \times C)$ and $(x, y) \notin (B \times C)$

$(x, y) \in (A \times C) - (B \times C)$

$(A - B) \times C \subset (A \times C) - (B \times C)$ —— (1)

Again, $(x, y) \in (A \times C) - (B \times C)$

$(x, y) \in (A \times C)$ and $(x, y) \notin (B \times C)$

$(x \in A$ and $y \in C)$ and $(x \notin B$ and $y \in C)$

$(x \in A$ and $x \notin B)$ and $(y \in C)$

$x \in (A - B)$ and $(y \in C)$

$(x, y) \in (A - B) \times C$

$\therefore (A \times C) - (B \times C) \subset (A - B) \times C$ —— ②

From ① and ②
$(A-B) \times C = (A \times C) - (B \times C)$
$\therefore$ Hence proved

→ **Overlapping Set:** Two sets are said to be overlapping set if they have atleast one element in common
$A = \{1, 2, 3, 4\}$ $B = \{4, 5, 6\}$

→ **Family of sets:** If a set A contains elements which are itself sets then it is called family of sets or set of sets.

Eg: $A = \{\{1, 2\}, \{3, 4\}, \phi\}$

POPU

Page No.
Date: / /

RELATIONS :

CARTESIAN PRODUCT (X)

Let $A = \{1, 2\}$
$B = \{x, y, z\}$

$A \times B = \{(a,b) \mid a \in A, b \in B\}$
$A \times B = \{(1,x), (1,y), (1,z), (2,x), (2,y), (2,z)\}$
$B \times A = \{(x,1) (y,1) (z,1), (x,2), (y,2), (z,2)\}$

$\Rightarrow \boxed{A \times B \neq B \times A}$
(Not commutative)

$|A| = 2$, $|B| = 3$ (Cardinal No.)
$|A \times B| = 2 \times 3 = 6$, $|B \times A| = 3 \times 2 = 6$

$\boxed{|A \times B| = |B \times A|}$

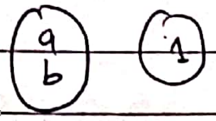
In general, if $|A| = m$, $|B| = n$
then $|A \times B| = |B \times A| = m \times n$

$R = \{(1,x), (2,y), (2,z)\}$ (R is relation b/w two sets A & B)
$A \times B = \{(1,x), (1,y), (1,z), (2,x), (2,y), (2,z)\}$

$\Rightarrow R \subset A \times B$
$\Rightarrow A \times B \Rightarrow$ max. possible relations

$\boxed{\text{No. of relations} = 2^{mn}}$

Eg. A B



$|A| = 2, \ |B| = 1 \Rightarrow |A \times B| = 2$

$R = \phi, (a,1), (b,1), \{(a,1), (b,1)\}$

No. of relations $= 2^2 = 4$

OPERATIONS ON RELATIONS

1) Union of two relations R_1 and R_2

$R_1 \cup R_2 = \{(a,b) \mid (a,b) \in R_1 \text{ or } (a,b) \in R_2\}$

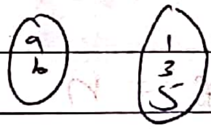
2) Intersection of two relations R_1 and R_2

$R_1 \cap R_2 = \{(a,b) \mid (a,b) \in R_1 \text{ and } (a,b) \in R_2\}$

3) Complement of Relation:

denoted by $R^c, \bar{R}, R'$

Let A B



$A \times B = \{(a,1), (a,3), (a,5), (b,1), (b,3), (b,5)\}$

and $R = \{(a,3), (b,1), (b,3)\}$

Then $R' = \{(a,1)(a,5), (b,5)\}$

$$\boxed{R' = (A \times B) - R}$$

Hence, $R' = \{(a,b) \mid (a,b) \in A \times B, (a,b) \notin R\}$

Note: 1) $A \cup A' = U$

$R \cup R' = A \times B$

2) $A \cap A' = \phi$

$R \cap R' = \phi$

Scanned with CamScanner

4) Inverse of a Relation:

Let $R = \{(a,3), (b,1), (b,3)\}$
then $R^{-1} = \{(3,a), (1,b), (3,b)\}$

$\therefore R^{-1} = \{(b,a) \mid (a,b) \in R\}$

Note: $|R^{-1}| = |R|$

MULTISET

Multisets are sets where an element can occur as a member more than once.

Eg: $A = \{a, a, a, b, b, c\}$, $B = \{a, a, a, a, b, b, d, d\}$ are multisets
They can also be written as,
$A = \{3 \cdot a, 2 \cdot b, 1 \cdot c\}$, $B = \{4 \cdot a, 2 \cdot b, 2 \cdot d\}$

Multiplicity:
The multiplicity of an element in a multiset is defined to be number of times the element occurs in a multiset.

Q) Let $P = \{3 \cdot a, 2 \cdot b, 1 \cdot c\}$ and $Q = \{4 \cdot a, 3 \cdot b, 2 \cdot d\}$ be two multisets.
a) $P \cup Q$ b) $P \cap Q$ c) $P - Q$ d) $Q - P$

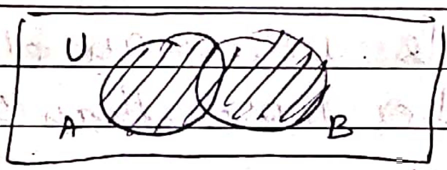
a) $P \cup Q = \{4 \cdot a, 3 \cdot b, 1 \cdot c, 2 \cdot d\}$
b) $P \cap Q = \{3 \cdot a, 2 \cdot b\}$
c) $P - Q = \{1 \cdot c\}$
d) $Q - P = \{1 \cdot a, 1 \cdot b, 2 \cdot d\}$

VENN DIAGRAM:

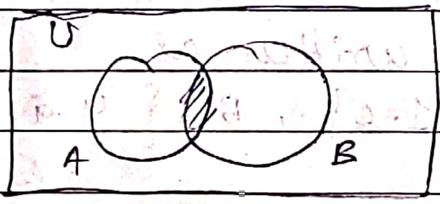
A venn diagram is a pictorial representation of sets which are used to show representation between sets.

Note: ① The universal set is represented by rectangle
② Its subsets are represented by circle.

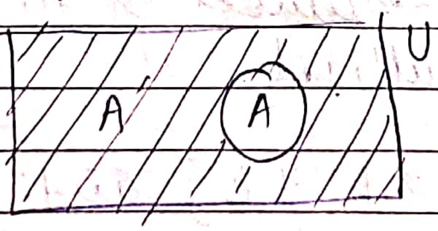
→ Union : $A \cup B$



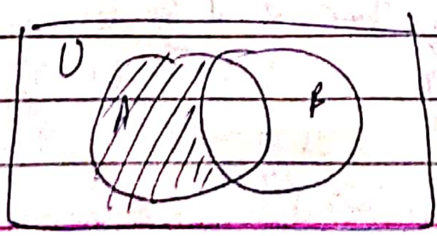
→ Intersection : $A \cap B$



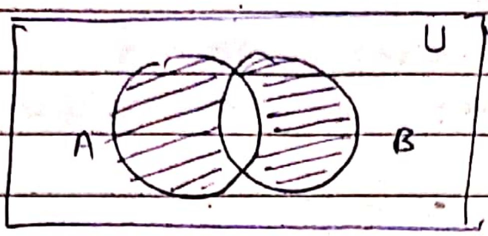
→ Complement : (A^c, A', $\bar{A}$)



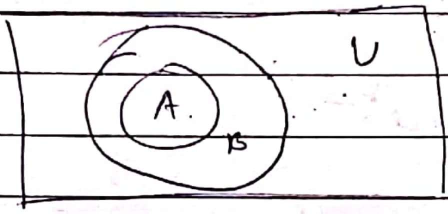
→ Difference : $A - B$



→ Symmetric difference : $A \oplus B$



→ Proper Subset : $A \subset B$



ALGEBRA OF SET

1) Idempotent laws :-

ⓐ $A \cup A = A$

ⓑ $A \cap A = A$

2) Associative laws :-

ⓐ $(A \cup B) \cup C = A \cup (B \cup C)$

ⓑ $(A \cap B) \cap C = A \cap (B \cap C)$

3) Commutative laws :-

ⓐ $A \cup B = B \cup A$

ⓑ $A \cap B = B \cap A$

4) Distributive laws :-

ⓐ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ⓑ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5) Demorgan's Laws :-

a) $(A \cup B)' = A' \cap B'$

b) $(A \cap B)' = A' \cup B'$

6) Identity laws:-

a) $A \cup \phi = A$

b) $A \cap U = A$

c) $A \cup U = U$

d) $A \cap \phi = \phi$

7) Involution laws:-

a) $(A')' = A$

8) Complement laws:-

a) $A \cup A' = U$

b) $A \cap A' = \phi$

c) $U' = \phi$

d) $\phi' = U$

INCLUSION - EXCLUSION PRINCIPLE:

(Addition principle)

1) If A and B are finite sets, then

$$|A \cup B| = |A| + |B| - |A \cap B|$$

2) If A, B, C are finite sets then

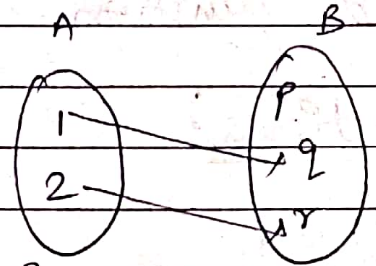
$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

FUNCTIONS

→ Special type of relation.
$F : A \to B$

Eg. $\underbrace{A = \{1, 2\}}_{\text{Domain}}$ $\underbrace{B = \{p, q, r\}}_{\text{Co-domain}}$

$F = \{(1, q), (2, r)\}$



$\underbrace{\{1, 2\}}_{\text{Domain}}$ (Pre-image) $\underbrace{\{q, r\}}_{\text{Range}}$ (Image)

→ Every element of domain must have a mapping
→ Every preimage has a unique image
→ Every function is one kind of relation.

Eg:

1) $f(x) = x^2$, $\underbrace{R}_{\text{Domain}} \longrightarrow \underbrace{R}_{\text{co-domain}}$

→ Every element will have its square
– $x \in R \to x^2$ (unique)
∴ Valid function

2) $f(x) = \sqrt{x}$, $R \to R$
$x = -1 \longrightarrow \sqrt{-1} \notin R$

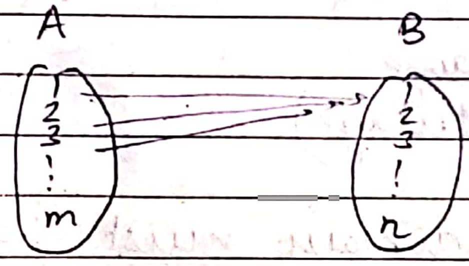
- Square root of (-ve) numbers is imaginary which is not in the co-domain

- $4 \to 2$, $4 \to -2$ ✗

- A pre-image cannot have multiple images
$\therefore$ Not a valid function.

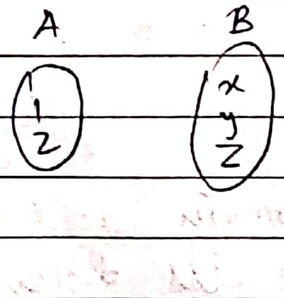
Number of Functions:

$|A| = m \qquad |B| = n$



$\Rightarrow$ Total functions $= n \times n \times n$ - - - - - m times
$= n^m$

Eg:



$f : A \to B$

$\boxed{\begin{matrix} (1,x) & (2,x) \\ (1,y) & (2,y) \\ (1,z) & (2,z) \end{matrix}} \Rightarrow$

$\{(1,x), (2,x)\}$
$\{(1,x), (2,z)\}$
$\{(1,x), (2,y)\}$

$\{(1,z), (2,x)\}$
$\{(1,z), (2,y)\}$
$\{(1,z), (2,z)\}$

$\{(1,y), (2,x)\}$
$\{(1,y), (2,y)\}$
$\{(1,y), (2,y)\}$

Total functions $\Rightarrow 3^2 = 9$
$\therefore$ Verified

TYPES OF FUNCTIONS

1) One-One (Injective) : $f: X \to Y$

→ distinct elements of X must map with distinct elements of Y.

$x_1, x_2 \in X \quad , \quad y_1, y_2 \in Y$

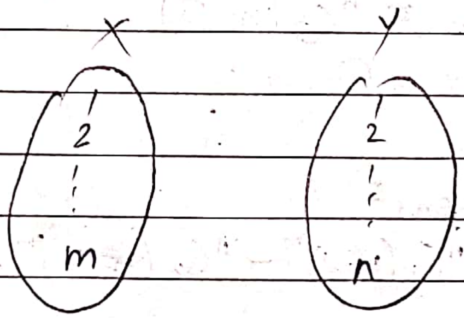
$x_1 \longrightarrow y_1$

$x_2 \longrightarrow y_2$

⇒ To check if function is one-one, we have to prove that for $f(a) = f(a) \Rightarrow a = b$
If $a \neq b$, then its not one-one function

• $|X| = m \qquad |Y| = n$

For a function to be one-one, $|X| \leq |Y|$



Total one-one functions:

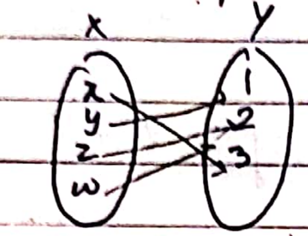
$\Rightarrow n(n-1)(n-2)(n-3) \text{-----} (n-m+1)$

$\Rightarrow {}^nP_m$

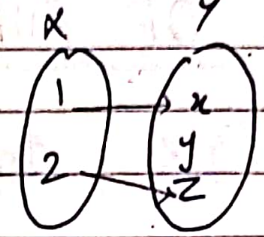
If $n = m$: $n!$

2) ONTO FUNCTION (Surjective)

for a $f: X \to Y$, every image in Y must have a pre image in X. (Codomain == Range)



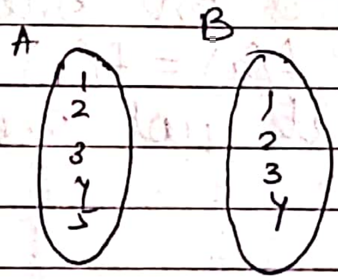
✓ Onto
✗ One-One



✗ Onto
✓ One-One

Opposite of Onto $\Rightarrow$ Into function.

No. of Onto Functions: $|A| = m$, $|B| = n$



$U = 4^5 = n^m$
(Total relations)

$F_{-1} = 3^5 \times {}^4C_1 = {}^nC_1 (n-1)^m$
(Removing any 1 element from co-domain)

$F_{-2} = 2^5 \times {}^4C_2 = {}^nC_2 (n-2)^m$

$F_{-3} = 1^5 \times {}^4C_3 = {}^nC_3 (n-3)^m$

$F_{-4} = 0^5 \times {}^4C_4 = 0$

Total onto functions:

$$n(f) = n^m - \left[{}^nC_1 (n-1)^m - {}^nC_2 (n-2)^m + {}^nC_3 (n-3)^m \cdots \right]$$

$$= \sum_{r=1}^{n} (-1)^{n-r} \, {}^nC_r \, r^m$$

Q. Consider the following functions on set of all integers $(Z \to Z)$, $f(x) = x^3$ and $g(x) = \lceil \frac{x}{2} \rceil$ which of the following are true.

POPU
Page No.
Date: / /

$$f(a) = f(b)$$
$$a^3 = b^3$$
$$a^3 - b^3 = 0$$
$$(a-b)(a^2+ab+b^2) = 0$$
$$a = b \quad [a^2+ab+b^2 \neq 0]$$

$f(x)$ is one-one

For funtion to be onto, every image in co-domain should have a preimage in domain.

But eg: $(2)^{1/3} \notin Z$

$\therefore$ Range $\subset$ Co-domain

Range $\neq$ Co-domain

$\therefore$ $f(x) = x^3$ is not an onto function

ii) $g(x) = \left[\frac{x}{2}\right]$

$x=1$, $g(1) = \left[\frac{1}{2}\right] = [0.5] = 1$

$x=2$, $g(2) = \left[\frac{2}{2}\right] = [1] = 1$

Both 1 & 2 point to 1, so clearly $g(x)$ is not one-one

For every image in co-domain, there is an image in domain

$\therefore$ It is onto

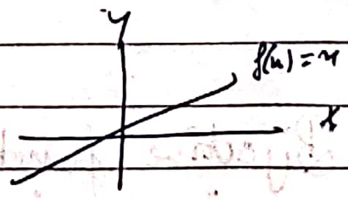
TYPES OF FUNCTIONS

i) $f(x) = C \in R$ → Constant function

Eg: $f(x) = 1$

ii) Identity function:

$f(x) = x$



iii) Equal functions:

Scanned with CamScanner

$f: A \to B$, $g: C \to D$

$f(x) = g(x)$

i) Domain of f & $g \to A = C$

ii) $f(x) = g(x) \quad \forall\, x \in A, C$

Q. $Z \to Z$, $f(x) = x$ and $g(x) = \sqrt{x^2}$ are identical?

$f(x) = x$ | $g(x) = \sqrt{x^2}$

$x = 1 \quad f(x) = 1$ | $x = -1 \quad g(x) = \sqrt{(-1)^2} = 1$

$x = -1 \quad f(x) = -1$ | $x = 1 \quad g(x) = \sqrt{1^2} = 1$

$(1, 1), (-1, -1)$ | $(1, 1), (-1, 1)$

$f(x)$ and $g(x)$ are not identical.

Q. How many onto functions are there from n-element to 2-elements?

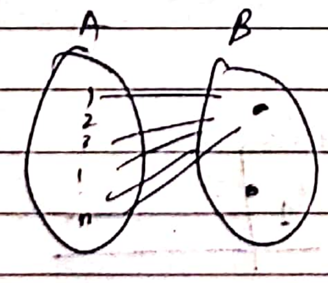
$|A| = n \quad |B| = 2$

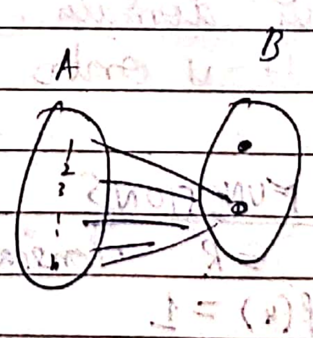
$f: A \to B$

$n(f) \Rightarrow n^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m$

$\Rightarrow 2^n - {}^2C_1 (2-1)^n + {}^2C_2$

$\Rightarrow 2^n - 2$





[Total − (these 2 cases)]

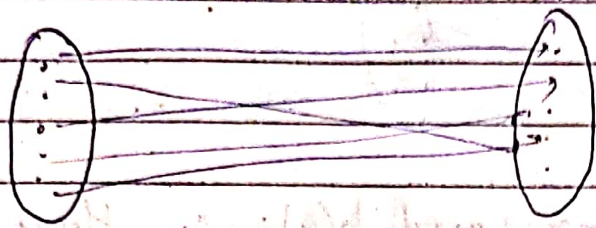
Bijective function / Bijection:
A function is said to be bijective iff it is both one-one & onto

POPU

Page No.
Date: / /

$$\text{Bijection} = \underbrace{\text{Injection}}_{\text{one one}} + \underbrace{\text{Surjection}}_{\text{onto}}$$

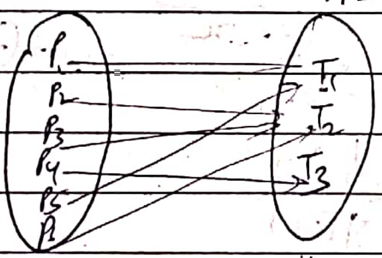
$f: A \longrightarrow B$



· Total bijective functions $= n\,(n-1)\,(n-2) \cdots 1$
$= n!$

Q. In how many ways we can asign 6 people 3 diff tasks so that every person is assigned to only one task and every task is assigned to atleast 1 person.

$n = 3$



Onto funtion

$m = 6$
Domain

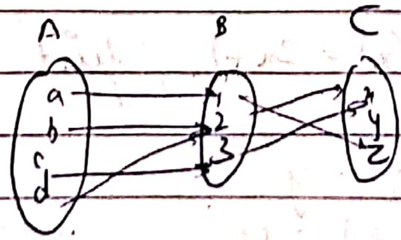
codomain = Range

$$n(f) = n^m - \left[{}^nC_1 (n-1)^m - {}^nC_2 (n-2)^m\right]$$
$$\Rightarrow 3^6 - {}^3C_1\, 2^6 + {}^3C_2 . 1^6$$
$$= 729 - 192 + 3 = \underline{540 \text{ ways}}$$

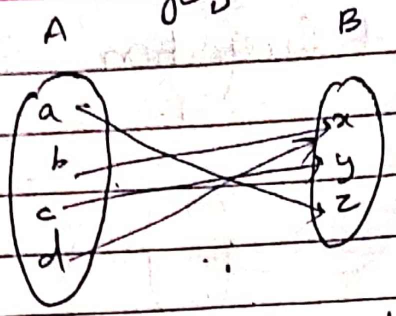
Composition of functions:

$f: A \rightarrow B$ $\qquad A \rightarrow B \rightarrow C : g \circ f = g(f(x))$

$g: B \rightarrow C$



gof



Q. If $g(x) = 1-x$ and $h(x) = \frac{x}{x-1}$ then $g[h(x)] = goh$
$h[g(x)]$ hog

$$goh = g(h(x)) \Rightarrow g\left(\frac{x}{1-x}\right) = 1 - \frac{x}{1-x}$$

$$= \frac{1-x-x}{1-x} = \frac{1-2x}{1-x}$$

$$hog \Rightarrow h(g(x)) \Rightarrow h(1-x) \Rightarrow \frac{1-x}{1-x-1} \Rightarrow \frac{1-x}{-x}$$

$$= \frac{x-1}{x}$$

$$\frac{goh}{hog} \Rightarrow \frac{1-2x}{1-x} \cdot \frac{x}{x-1} \Rightarrow \frac{-(x-2x^2)}{(x-1)^2}$$

$$\Rightarrow \frac{h(x)}{g(x)}$$

Inverse of function

Given $f: A \to B$, if the relation $f^{-1}: B \to A$ is a function, then its called inverse of a function.

Theorem: Inverse of a function f exists if f is bijective.

Q. $R \to R$ and $f(x) = 2x+1$, find if its bijective or not

Let $f(x) = y$

$2x+1 = y$

POPU

Page No.:
Date: / /

$$x = \frac{y-1}{2}$$
$$f^{-1}(x) = \frac{x-1}{2}$$

$\therefore$ It is bijective

Q. Let $A = R - \{3\}$ and $B = R - \{1\}$ where $f: A \to B$ is defined by $f(x) = \frac{x-2}{x-3}$.

$$f(x) = y \Rightarrow \frac{x-2}{x-3} = y$$

$$x-2 = y(x-3)$$
$$x-2 = xy - 3y$$
$$3y - 2 = x(y-1)$$
$$x = \frac{3y-2}{y-1}, \quad y \neq 1$$

$$f^{-1}(x) = \frac{3x-2}{x-1}, \quad x \neq 1$$

Scanned with CamScanner