Tutorial -1
Aug 8, 2021

Last Date: Within 10 days | Platform: Google Classroom/MSTeams

Do all the questions.

1. Let $A = \{a, b, c\}$. Then find the power set ($\mathcal{P}(A)$) of A, i.e., find the collection of all the subsets of A.

2. Which of the following subset(s) of $A \times B$, where $A = \{1, 2, 3, 4, 5\}$ and $B = \{a, b, c, d, e\}$, is/are functions? If it is a function, mention the type of the function also, like 1-1, onto, 1-1 and onto , or neither 1-1 nor onto.
   - $\{(1, a), (1, b), (2, d), (3, e), (4, c)\}$
   - $\{(1, a), (2, b), (3, d), (4, c), (5, c)\}$
   - $\{(1, a), (2, b), (3, d), (4, c)\}$
   - $\{(1, a), (2, a), (3, b), (4, c), (5, e)\}$
   - $\{(1, a), (2, b), (3, e), (4, c), (5, d)\}$

   Do you think, every one-one function is also an onto function and vice-versa in the above case? If this is so, what is the reason you have observed ?

3. Which of the following subsets are subgroups of the group $G = (\mathbb{R}, \cdot)$, where $\cdot$ represents the usual multiplication of real numbers?
   - $(\mathbb{Z}, \cdot)$, where $\mathbb{Z}$ is set of all integers.
   - $(\mathbb{Z}, +)$, where $+$ is the usual addition of integers.
   - $(\mathbb{Q}, \cdot)$, where $\mathbb{Q}$ is set of all rational numbers.
   - $(\mathbb{Q}^+, \cdot)$, where $\mathbb{Q}^+$ is set of all positive rational numbers.
   - $(\mathbb{Q}^+, \cdot)$, where $\mathbb{Q}^-$ is set of all negative rational numbers.
   - $(\mathbb{R} - \mathbb{Q}, \cdot)$.

4. Prove that for all $n \geq 2$, $\frac{\mathbb{Z}}{n\mathbb{Z}}(\ or\ \mathbb{Z}_n)$ is not a group under multiplication of residue classes. What about the set $\{1, 2, 3, \ldots, n-1\}$ under multiplication modulo n, when n is a prime?

5. Find the order of each element of additive group $\mathbb{Z}_{12}$ and multiplicative group $U(12)$.

6. Prove or disprove that the set of positive irrational numbers together with 1 under multiplication is a group.

7. Prove or disprove that the set of 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ with real entries a, b, c such that $ac \neq 0$ is a group.

8. A set of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with determinant 1 with entries from Z_5 is denoted by $SL(2, Z_5)$, a special linear group. Find the inverse of $\begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix}$ in $SL(2, Z_5)$.

9. Prove that set of all rational numbers of the form $3^m 6^n$ for integers m, n, is a group under multiplication.

10. Answer the following:

    - the order of $\bar{4}$ in the group $Z_5 - \{\bar{0}\}$ under multiplication?
    - the order of i in the multiplicative group $\{1, -1, i, -i\}$?
    - the generators of the group $\{w^{15} = 1, w, w^2, \ldots, w^{14}\}$?
    - If $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, then find $f^2 = f \circ f$.
    - Check whether $U(16)$ and $\mathbb{Z}_8$ are isomorphic?