Module 1

$A = \{\ ,\ ,\ ,\ \}$ a b c

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$\{1, 1, 2\}$

$= \{1, 2\}, \quad \{2, 1\}.$

What is a Set?

A set is an unordered collection of objects, called elements or members of the set.
We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

$a \in A$

Example

The set O of odd positive integers less than 10 can be expressed by $O = \{1, 3, 5, 7, 9\}$.

$x^2 + 1 = 0$

$\boxed{\sqrt{-1} = i}$

ϕ

$\sqrt{2}$

x_1 , , x_2 1.312153121531215

$\boxed{(0,1)}$

Some existing notations of sets

$\mathbb{N} = \{1, 2, 3, \cdots\}$, the set of natural numbers

$\mathbb{Z} = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$ = the set of integers

π $\frac{22}{7}$

$\mathbb{Z}^+ = \{1, 2, 3, \cdots\}$, the set of positive integers

$\frac{10}{4} = 2.500$ $\mathbb{N}$ $\mathbb{N}$

$\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z}$ and $q \neq 0\}$= the set of rational numbers

$\mathbb{R}$ = the set of real numbers

$\mathbb{R}^+$ = the set of positive real numbers

1, 2

$\mathbb{C}$ = the set of complex numbers.

5 6 A 9 1 7

Some definitions

- A set $\mathcal{A}$ is a subset of another set $\mathcal{B}$ if every element of $\mathcal{A}$ is also an element of $\mathcal{B}$. We use the notation $\mathcal{A} \subseteq \mathcal{B}$ to indicate that $\mathcal{A}$ is a subset of the set $\mathcal{B}$.
- Two sets A and B are equal $\iff$ $A \subseteq B$ and $B \subseteq A$.
- Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is a finite set and that n is the cardinality of S. The cardinality of S is denoted by $|S|$.

$A \subsetneq B$

Cartesian Product

Let A and B be sets. The Cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence,

$$A \times B = \{(a, b) \mid a \in A, b \in B\}.$$

$\{\ \}$

$(\ )$

For example, if $A = \{1, 2\}$ and $B = \{a, b\}$, then

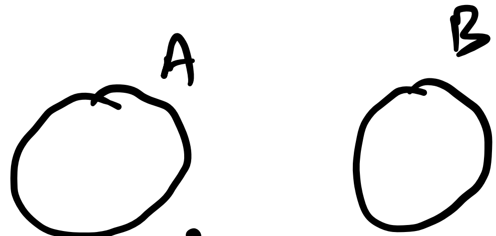
$$A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$$

Note that $A \times B \neq B \times A$.

$\{1,2\} = \{2,1\}$

$(1,2) \neq (2,1)$

Operations



Let A and B be sets.

$A = \{a, 1, 2\}$, $B = \{1, a, b\}$

$A \cup B = \{1, 2, a, b\}$

Union

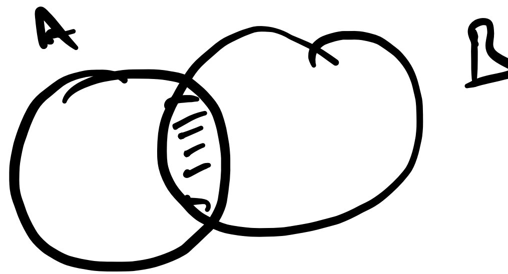
The union of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

Intersection

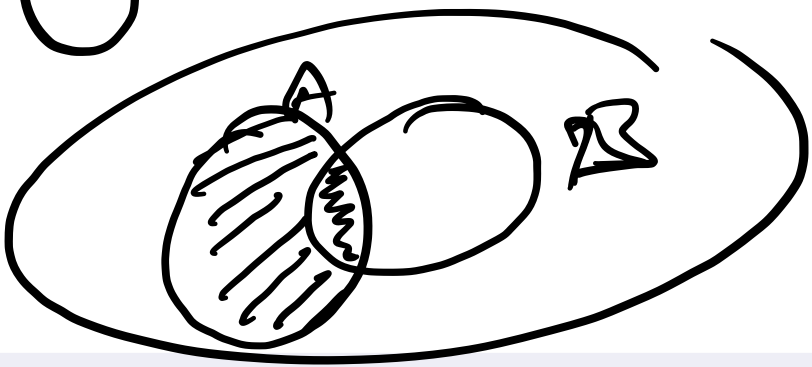
The intersection of the sets A and B, denoted by $A \cap B$, is the set that contains those elements that are in both A and B.

$\{1, a\}$

- Two sets are called disjoint if their intersection is the empty set.
- $|A \cup B| = |A| + |B| - |A \cap B|$



Operations



Difference of sets

The difference of A and B, denoted by $A - B$, is the set containing those elements that are in A but not in B. The difference of A and B is also called the complement of B with respect to A.

Complement

Let U be the universal set. The complement of the set A, denoted by $\bar{A}$ or A^c, is the complement of A with respect to U. Therefore, the complement of the set A is $U - A$.

Multiset

It is a modification of the concept of a set that, unlike a set, allows for multiple instances for each of its elements. In the multiset $\{a, a, b\}$, the element a has multiplicity 2, and b has multiplicity 1.

Set identities

TABLE 1 Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

$(U \cap A)$

$A \cap A^c$

$A \cup A^c = U$

$\bar{A}$ or A^c

$\text{Range} := \underbrace{\{ f(x) : x \in A \}} \neq B$ in general

What is function?

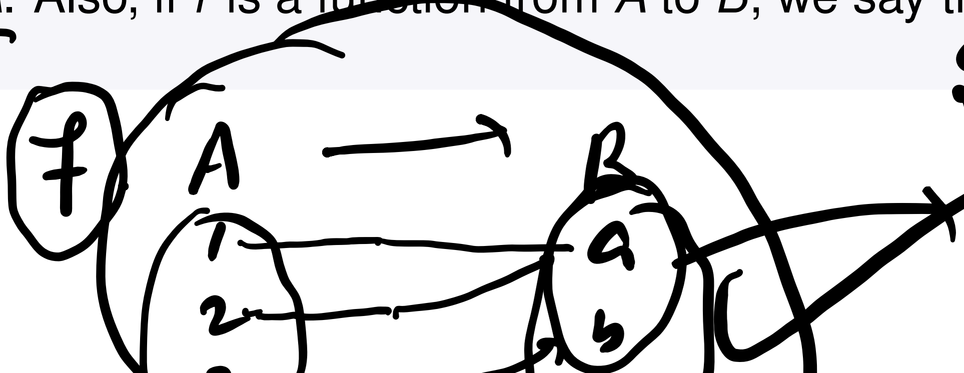
Let A and B be nonempty sets. A function f from A to B is an assignment of exactly one element of B to each element of A. We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A. If f is a function from A to B, we write $f : A \to B$.

Domain and Co-domain, $\boxed{\text{Range} \mid \text{Image}}$

If f is a function from A to B, we say that A is the $\boxed{\text{domain of } f}$ and B is the codomain of f. If $f(a) = b$, we say that b is the image of a and a is a preimage of b. The range, or image, of f is the set of all images of elements of A. Also, if f is a function from A to B, we say that f maps A to B.

Range | Image

$f(a) = b$



$\{a, b\}$

$f(1) = a$

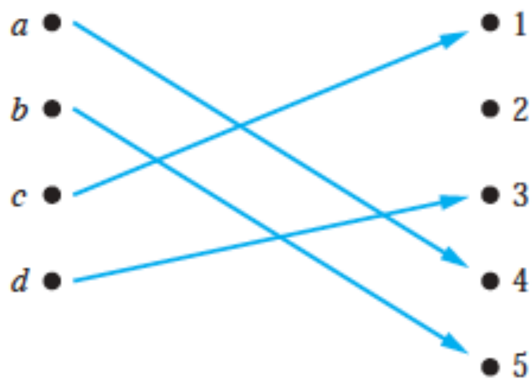
$f(2) = a$

$f(3) = b.$

One-one function

A function f is said to be one-to-one, or an injunction, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f. A function is said to be injective if it is one-to-one.

$f(x) = x + 1$



$f(x) = |x|$

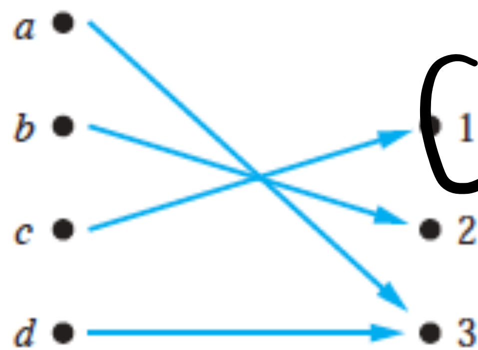
$f(a) = f(b) \Rightarrow a = b$

$a \Rightarrow b$ ---> $f(a) = f(b)$ but $a \neq b$,

Onto function

A function f from A to B is called onto, or a surjection, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$. A function f is called surjective if it is onto.

1 → 1
2, 3, 4, 5, 6 → 2, 3, 4, 5, 6, 7, 8

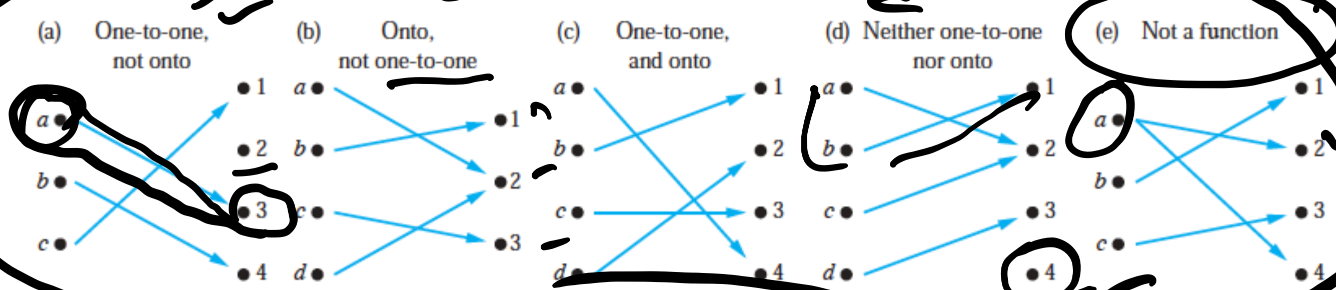


$(a)^{1/3} \in R$ → a

	Rule	1-1	onto
1) $f: Z' \to Z'$	$x \to x^3$	✓	✗
2) $f: R \to R$	$x \to x^3$	✓	✓
3) $f: Z \to N$	$x \to \lvert x \rvert$	✗	✓

4)

$f: \mathbb{Z} \to \mathbb{Z}, \quad x \to x^2$ X | X



• The function f is a one-to-one correspondence, or a bijection, if it is both one-to-one and onto. We also say that such a function is bijective.

$f: A \longrightarrow B$

$A \longleftarrow B : g$

Composition of functions

Let g be a function from the set A to the set B and let f be a function from the set B to the set C. The composition of the functions f and g, denoted by $f \circ g$, is defined by $(f \circ g)(a) = f(g(a))$ for all $a \in A$

Inverse of a function

The inverse of a function f is another function g if $f \circ g = g \circ f =$ *identity*, In this case we write $g = f^{-1}$.

$(g \circ f)(x) = g(f(x))$

$f : A \to B$ $\quad g : B \to C$.

$(g \circ f) : A \to C$