22/10/2021

UNIT - 1

THE FOUNDATIONS

Symbol	Name
$\sim$	Negation
$\wedge$	AND (or) Meet
$\vee$	OR (or) Join
$\rightarrow$ (or) $\Rightarrow$	Implies
$\Leftrightarrow$ (or) $\Leftrightarrow$	if and only if (or) double implise (or) if condition
$\forall$	for all
$\in$	belongs to (or) Contained in
$\ni$ (or) /	such that
$\mathbb{N}$	Natural Numbers
$\mathbb{R}$	Real Numbers
$\mathbb{W}$	Whole Numbers
$\mathbb{Z}$	integers
$\mathbb{C}$	Complex Numbers
$\mathbb{Q}$	Rational Numbers
$\mathbb{Q}'$	Irrational Numbers
$\exists$	there exists
$\subseteq$	Sub set (contained in)
$\supseteq$	Super Set (Containes)
$\cup$	Union
$\cap$	Intersection
1 (or) T	Tautology
0 (or) F	contriduction

Sub set and Super Set: Read as

$\cong$	equalence
$\equiv$	concorence

} II unit

$\lvert x \rvert$	modules
$\lVert x \rVert$	Norm

Power Set :- It is set of all subsets of that main set

power set of A is define denoted with $P(n)$

Ex:- $A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

Truth tables :-

1) AND (or) Meet : (∧) :-

It is a one of the compound statement which is used to link two statements. p & q and resulting statement is denoted with p∧q

If both the statements are true then the result will be true in all the remainings cases the result will be false.

Truth table :-

1)

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

2)

p	q	r
T	T	T
F	T	T
T	F	T
T	T	F
F	F	T
T	F	F
F	T	F
F	F	F

2) OR (or) Join : (V) :-

If both the statements are false the result will be false in all other remainings cases the result will be true.

This compound statements is used to divide the two statements p & q the result will be denoted with p∨q

Truth table :-

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

3) Negation (~) :-

Negation indicates the opposite statement of the original statement

Negation of negation will be same as P

Ex: $\sim(\sim P) = P$

Truth table:

P	~P
T	F
F	T

Ex:

4) Implies (→) (or) (⇒)

This is the compound statement used to indicate the relation b/w two statements. (that is both the statements related to each other

Truth table :-

P	q	P→q	q→P
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

5) Double Implies ($\Leftrightarrow$) (or) $\Leftrightarrow$

This double implies is used to explains if only if conductions. It indicates with $p \Leftrightarrow q$ (or) p

Truth table :-

p	q	$p \Leftrightarrow q$	$q \Leftrightarrow p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

if either both the statements are true or false the result will be true in the remaing cases it will be false

* Find $p \wedge (\sim q \vee p)$ by truth table ?

Sol

p	q	$\sim q$	$(\sim q) \vee p$	$p \wedge ((\sim q) \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	F
F	F	T	T	F

* i) $[\sim p \wedge (q \vee p)] \Leftrightarrow p$

ii) $[(\sim p) \vee (\sim q)] \wedge [(\sim p) \vee r]$

iii) $[p \rightarrow (q \rightarrow r)] \Leftrightarrow [(p \wedge q) \vee r]$

iv) $(p \wedge q) \vee [(\sim p) \rightarrow q]$

(i) $[\sim p \wedge (q \vee p)] \leftrightarrow p$

Sol

p	q	$\sim p$	$q \vee p$	$(\sim p) \wedge (q \vee p)$	$[\sim p \wedge (q \vee p)] \leftrightarrow p$
T	T	F	T	F	F
T	F	F	T	F	F
F	T	T	T	T	F
F	F	T	F	F	T

(ii) $[\sim p \wedge (q \vee p)] \rightarrow$

$[(\sim p) \vee (\sim q)] \wedge ((\sim p) \vee r]$

Sol

p	q	r	$\sim p$	$\sim q$	$(\sim p) \vee (\sim q)$	$((\sim p) \vee r)$	$\sim p \vee (\sim q) \wedge ((\sim p) \vee r)$
T	T	T	F	F	F	T	F
F	T	T	T	F	T	T	T
T	F	T	F	T	T	T	T
T	T	F	F	F	F	F	F
F	F	T	T	T	T	T	T
T	F	F	F	T	T	F	F
F	T	F	T	F	T	T	T
F	F	F	T	T	T	T	T

iii) $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \wedge q) \vee r] = x$ (say)

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$[(p \wedge q) \vee r]$	$[p \rightarrow (q \rightarrow r)] \rightarrow [(p \wedge q) \vee r]$
T	T	T	T	T	T	T	T
F	T	T	T	T	F	T	T
T	F	T	T	T	F	T	T
T	T	F	F	F	T	T	T
F	F	T	T	T	F	T	T
T	F	F	T	T	F	F	F
F	T	F	F	T	F	F	F
F	F	F	T	T	F	F	F

iv) $(p \wedge q) \vee [(\sim p) \rightarrow q]$

p	q	$\sim p$	$p \wedge q$	$(\sim p) \rightarrow q$	$(p \wedge q) \vee ((\sim p) \rightarrow q)$
T	T	F	T	T	T
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	F	F	F

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Q) Prove that $(p \vee q) \vee r = p \vee (q \vee r)$ → Associative property

Sol 2

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$(q \vee r)$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
F	T	T	T	T	T	T
T	F	T	T	T	T	T
T	T	F	T	T	T	T
F	F	T	F	T	F	T
T	F	F	T	T	T	T
F	T	F	T	T	F	T
F	F	F	F	F	F	F

Both are equal.

from the above truth table both are equal.

Prove that $(p \wedge q)$

Prove that $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

Sol 2

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
F	T	T	F	T	F	F
T	F	T	F	F	F	F
T	T	F	T	F	F	F
F	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	F	F	F	F	F
F	F	F	F	F	F	F

from the above truth table both are equall

Mathematical logic expressions (or) propositions :-

Identity

*	+
1	0

	Identity	Name of Property
1	$p \wedge T = p$ $p \vee F = p$	Identity property
2	$p \wedge F = F$ $p \vee T = T$	Domination laws
3	$p \vee p = p$ $p \wedge p = p$	Idempotent laws
4	$\sim(\sim p) = p$	Double negation laws
5	$p \vee q = q \vee p$ $p \wedge q = q \wedge p$	commutative laws

6) $p \vee (q \vee r) = (p \vee q) \vee r$ } — Associative law
$p \wedge (q \wedge r) = (p \wedge q) \wedge r$

7) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$ } — Distributive law
$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

8) Demorgan laws } — Demorgan laws (or) Negation laws
$\sim(p \wedge q) = (\sim p) \vee (\sim q)$
$\sim(p \vee q) = (\sim p) \wedge (\sim q)$

9) $p \vee (p \wedge q) = p$ } — Absorption law
$p \wedge (p \vee q) = p$

} H.W

* Note points

$p \vee (\sim p) = T$

$p \wedge (\sim p) = F$

* Logical equivalances :-

$\cancel{p \to q \equiv (\sim p) \vee q}$ ✗

1) $p \to q \equiv (\sim p) \vee q$ Implication Equivalence.

2) $p \to q \equiv (\sim q) \to (\sim p)$

3) $p \vee q \equiv (\sim p) \to q$

4) $p \wedge q \equiv \sim[q \to (\sim p)]$

5) $\sim(p \to q) \equiv p \wedge (\sim q)$

6) $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

7) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$ } H.W

Extra (8) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

H.W

6) $(p \to r) \wedge (q \to r) = (p \vee q) \to r$

p	q	r	$p \to r$	$q \to r$	$p \vee q$	$(p \vee q) \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	T	T	T	T	T	T
T	T	F	F	F	T	F	F
F	F	T	T	T	F	T	T
T	F	F	F	T	T	F	F
F	T	F	T	F	T	F	F
F	F	F	T	T	F	T	T

given identity has proved

7) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

p	q	r	$p \to r$	$q \to r$	$p \wedge q$	$p \wedge q \to r$	$(p \to r) \vee (q \to r)$
T	T	T	T	T	T	T	T
F	T	T	T	T	F	T	T
T	F	T	T	T	F	T	T
T	T	F	F	F	T	F	F
F	F	T	T	T	F	T	T
T	F	F	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	F	T	T	F	F	F

given identity has proved

8) i) $\sim(p \wedge q) = (\sim p) \vee (\sim q)$

p	q	$p \wedge q$	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$(\sim p) \vee (\sim q)$
T	T	T	F	F	F	F
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

(i) $\sim(p \vee q) = (\sim p) \wedge (\sim q)$

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$(\sim p) \wedge (\sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

given identities is proved

(7) (i) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

p	q	r	$p \vee q$	$p \vee r$	$q \wedge r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	T	T	T	F	T	T
T	T	F	T	T	F	T	T
F	F	T	F	T	F	F	F
F	T	F	T	F	F	F	F
T	F	F	T	T	F	T	T
F	F	F	F	F	F	F	F

given identities is proved.

(ii) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
F	T	T	T	F	F	F	F
T	F	T	T	F	T	T	T
T	T	F	T	T	F	F	F
F	F	T	T	F	F	F	F
F	T	F	T	F	F	F	F
T	F	F	F	F	F	F	F
F	F	F	F	F	F	F	F

given identities is proved

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Q) Show that $(\sim P) \to (P \to q)$ is tautology by logic expressions

Sol: Consider $(\sim P) \to (P \to q)$

$\equiv P \vee (P \to q)$ $\quad [\because (\sim P) \to q \equiv P \vee q]$

$\equiv P \vee [(\sim P) \vee q)]$ $\quad [\because P \to q = (\sim P) \vee q]$

$\equiv [P \vee (\sim P)] \vee [P \vee q]$

$\equiv T \vee (P \vee q)$

$\equiv T$ (Tautology)

$$\boxed{\therefore (\sim P) \to (P \to q) \equiv T.}$$

Q) Show that negation of $(P) \vee [(\sim P) \wedge q]$ are logically equalent by $P \vee (\sim q)$.

Sol: Consider $\sim [(P) \vee [(\sim P) \wedge q]]$

$\sim (P \vee [(\sim P) \wedge q])$

$\equiv [(\sim P) \wedge (P \vee (\sim q))]$

$\equiv (\sim P \wedge P) \vee ((\sim P) \wedge (\sim q))$

$\equiv F \vee [(\sim P) \wedge (\sim q)]$

$\equiv \sim P \wedge \sim q$

Q) Find $\sim(p \vee q) \Leftrightarrow ((\sim p) \wedge (\sim q))$ by using truth table

p	q	$p \vee q$	$\sim(p \vee q)$	$(\sim q)$	$\sim(p \vee q)$	$((\sim p) \wedge (\sim q))$	$\Leftrightarrow$
T	T	T	F	F	F	F	T
T	F	T	F	T	F	F	T
F	T	T	T	F	F	F	T
T	T	T	F	F	F	F	T

Q

$\sim(p \vee q) \Leftrightarrow ((\sim p) \wedge (\sim q))$

$\equiv (\sim p) \wedge (\sim q) \Leftrightarrow [(\sim p) \wedge (\sim q)]$

$\equiv T$ [Tautology]

Q) Show that $[(p \vee q) \vee (q \vee (\sim r)) \wedge (p \vee r)] \equiv \sim[(\sim p) \wedge (\sim q)]$

by using truth table

Sol

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \vee q$	$p \vee r$	$q \vee (\sim r)$	$(\sim p) \wedge (\sim q)$	$(p \vee q)$	$(q \vee (\sim r$
T	T	T	F	F	F	T	T	T	F	T	
F	T	T	T	F	F	T	T	T	F	T	
T	F	T	F	T	F	T	T	F	F	T	
T	T	F	F	F	T	T	T	T	F	T	
F	F	T	T	T	F	F	T	F	T	F	
F	T	F	T	F	T	T	F	T	F	T	
T	F	F	F	T	T	T	T	T	F	T	
F	F	F	T	T	T	F	F	T	T	T	

truth table

$(p)\wedge(\sim q))$	x
F	T
	T
	T
	T

$\wedge(\sim q)]$

$(\sim q)$	$(p\vee q)\vee$	$(q\vee(\sim r))$	$[(p\vee q)\vee(q\vee(\sim r))]\wedge(p\vee r)$	$\sim((\sim p)\wedge(\sim q))]$
	T		T	T
	T		T	T
	T		T	T
	T		T	T
	F		F	F
	T		F	T
	T		T	T
	T		F	F

Logical Equivalanes

1) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

2) $p \leftrightarrow q \equiv (\sim p) \leftrightarrow (\sim q)$

3) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

4) $\sim(p \leftrightarrow q) \equiv p \leftrightarrow (\sim q)$

5) $p \to q \equiv (\sim p) \vee q$

H.W

Q) show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology by using logical equalance

Sol

$(p \wedge q) \rightarrow (p \vee q)$ $\quad$ $[\because p \rightarrow q \equiv (\sim p) \vee q]$

$\equiv \sim (p \wedge q) \vee (p \vee q)$

$\equiv [(\sim p) \vee (\sim q)] \vee (p \vee q)$

$\equiv (\sim p \vee p) \vee (\sim q \vee q)$

$\equiv T \vee T$

$\equiv T$ (Tautology)

Q) logical Equalalances :-

i) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q$
T	T	T	T	T	T	T
T	F	F	F	F	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T

given identities are proved

(ii) $p \leftrightarrow q \equiv (\sim p) \leftrightarrow (\sim q)$

p	q	$\sim p$	$\sim q$	$(\sim p) \leftrightarrow (\sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

given identities are proved

(iii) $p \leftrightarrow q$

p
T
T
F
F

(iv) $\sim (p \leftrightarrow$

p
T
T
F
F

(v) $p \rightarrow$

p
T
T
F
F

Q)

(i)

(iii) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

p	q	$p \wedge q$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \wedge q \vee (\sim p \wedge \sim p)$	$p \leftrightarrow q$
T	T	T	F	F	F	T	T
T	F	F	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	F	T	T	T	T	T

given identities are proved

(iv) $\sim(p \leftrightarrow q) \equiv p \leftrightarrow (\sim q)$

p	q	$\sim q$	$p \leftrightarrow (\sim q)$	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$	$p \leftrightarrow (\sim q)$
T	T	F	F	T	F	F
T	F	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	F	T	F	F

given identities are proved

(v) $p \rightarrow q \equiv (\sim p) \vee q$

p	q	$\sim p$	$(\sim p) \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

given identities are proved.

Q) Show the following by Truth tables.

(i) $p \wedge T \equiv p$

p	T	$p \wedge T$
T	T	T
F	T	F

We get p

given identities are proved.

ii) $p \vee F \equiv p$

p	F	p∨F
T	F	T
F	F	F

we get p

given identities are proved

iii) $p \wedge F \equiv F$

Sol

p	F	p∧F
T	F	F
F	F	F

we get F

given identities are proved

iv) $p \vee T \equiv T$

p	T	p∨T
T	T	T
F	T	T

we get T

given identities are proved

v) $p \vee p \equiv p$

p	p	p∨p
T	T	T
F	F	F

we get p.

given identities are proved

vi) $p \wedge p \equiv p$

p	p	p∧p
T	T	T
F	F	F

we get p

given identities are proved

vii) $p \vee q \equiv q \vee p$

p	q	p∨q	q∨p
T	T	T	T
F	F	T	T
F	T	T	T
F	F	F	F

given identities are proved
Both are equal

viii) $p \wedge q \equiv q \wedge p$

p	q	p∧q	q∧p
T	T	T	T
F	T	F	F
T	F	F	F
F	F	F	F

given identities are proved
Both are equal

(ix) $p \to q \equiv \sim p \vee q$

p	q	~p	p→q	(~p)∨q
T	T	F	T	T
F	T	T	T	T
T	F	F	F	F
F	F	T	T	T

given identities are proved
Both are equal

Find (p

p	o
T	
T	
F	
F	

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Q)

ex

Sol

Find $(p \wedge q) \rightarrow (p \vee q)$?

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

Both are equal given identities are proved.

$\therefore (p \wedge q) \rightarrow (p \vee q)$ is T (Tautology)

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Q) Show that $(p \wedge q) \vee ((\sim p) \rightarrow q)$ is a toutology in logic expressions ?

Sol: $(p \wedge q) \vee ((\sim p) \rightarrow q)$

$\equiv (p \wedge q) \vee (\sim(\sim p) \vee q)$ $[\because p \rightarrow q \equiv (\sim p) \vee q]$

$\equiv (p \wedge q) \vee (p \vee q)$

$\equiv (p \vee q) \vee (p \wedge q)$ [$\because$ commutative law]

$\equiv [(p \vee q) \vee p] \wedge [(p \vee q) \vee q]$ [$\because$ Distributive law]

$\equiv [(p \vee q) \vee q] \wedge [p \vee (q \vee q)]$ [$\because$ Associative law]

$\equiv [p \vee q] \wedge [p \vee q]$

$\equiv p \vee q$

$(p \wedge q) \vee ((\sim p) \rightarrow q)$ is not a tautology

Q) Show that it is a tautology

$[(\sim p \wedge (p \vee q)] \rightarrow q$

(8) show that $[\sim p \wedge (p \vee q)] \to q$ is a tautology by using logical equivalences

Sol: Given $[\sim p \wedge (p \vee q)] \to q$

$\equiv [(\sim p \wedge p) \vee (\sim p \wedge q)] \to q$ [∵ Distributive law]

$\equiv [F \vee (\sim p \wedge q)] \to q$ [∵ Negation law]

$\equiv [\sim p \wedge q] \to q$ [∵ Identity law]

$\equiv [\sim(\sim p \wedge q)] \vee q$ [∵ $p \to q \equiv \sim p \vee q$]

$\equiv [p \vee (\sim q)] \vee q$

$\equiv p \vee (\sim q \vee q)$ [∵ Associative law]

$\equiv p \vee T$ [Negation law]

$\equiv T$ [∵ Domination law]

∴ $[\sim p \wedge (p \vee q)] \to q$ is a Tautology

(1) show that the following are logically equivalent in logical Expressions.

i) $\sim(p \to q)$ and $p \wedge (\sim q)$

(2) ii) $\sim[p \vee (\sim p \wedge q)]$ and $\sim p \wedge (\sim q)$

(3) iii) $(p \wedge q) \to (p \vee q)$ is Tautology

iv) $(p \wedge q) \to p$ is Tautology.

(i) $\sim(p \to q)$ and $p \wedge (\sim q)$

Sol: Given $\sim(p \to q)$

$\equiv p \wedge (\sim q)$ $[\because \sim(p \to q) = p \wedge (\sim q)]$

$\boxed{\therefore \sim(p \to q) \equiv p \wedge (\sim q)}$

(ii) $\sim[p \vee (\sim p \wedge q)]$ and $\sim p \wedge (\sim q)$

Sol: Given $\sim[p \vee (\sim p \wedge q)]$

$\equiv [\sim p \wedge (p \vee (\sim q))]$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge p)$

$= (\sim p \wedge \sim q)$

(iii) $(p \wedge q) \to (p \vee q)$

Sol: Given $(p \wedge q) \to (p \vee q)$

$\equiv ((\sim p) \vee (\sim q)) \vee (p \vee q)$ $(p \to q \equiv \sim p \vee q)$

$\equiv (\sim p \vee p) \vee (\sim q \vee q)$

$= T \vee T$

$= T$

$\therefore$ The given $(p \wedge q) \to (p \vee q)$ is a tautology

(iv) $(p \wedge q) \to p$

Sol: $(\sim(p \wedge q)$

Given $(p \wedge q) \to p$

$\equiv \sim(p \wedge q) \vee q$

$= (\sim p) \vee (\sim q) \vee q$ [∵ Associative

$= \sim p \vee (\sim q \vee q)$

$= \sim p \vee T$

$= T$

∴ $(p \wedge q) \to p$ is a tautology

Predicate

A

Called

A d

Examples

i)

ii)

iii)

⇒ The

values of

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02/11/21 Problems

Q) What i

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Sol: G

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Q) Le

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Sol

Predicates

A property (or) a relationship between objects is called a predicate. Indicates by $P(x)$

A discription of predicate in logic is called formula.

Examples of Predicates :-

i) $x > 3$

ii) $x+3 = y$

iii) $x+y = z$

⇒ The above predicates are neither true nor false when the values of variables are not specified.

⇒ The statement "$x > 3$" has two parts the first part is the variable 'x' and the subject of statement where the second part is greater than 3 (predicate)

02/11/21 Problems

Q) What is the truth value of the predicate $P(x)$ denotes the statement "$x > 3$", for $x = 4$ and $x = 2$ [2 or 3] Marks

Sol: Given statement is "$x > 3$"

$P(x) : x > 3$

for $x = 4 \Rightarrow P(4) : 4 > 3$ is True

for $x = 2 \Rightarrow P(2) : 2 > 3$ is False

Q) Let $Q(x, y)$ denotes the statement $x = y+3$. What are the truth values of prepositions $Q(1, 2)$ and $Q(3, 0)$

Sol: Given statement is $x = y+3$

Where $Q(x, y) = Q(1, 2)$

$x = 1, \quad y = 2$

for $Q(1, 2) \Rightarrow$ for $x = 1, y = 2 \Rightarrow Q(1, 2)$ $1 = 2+3$ is False

for $x = 3, y = 0 \Rightarrow Q(3, 0)$ $3 = 0+3$ is True

$\therefore$ $Q(1,2)$ is false, $Q(3,0)$ is true

Q) What are the truth values of prepositions $R(1,2,3)$ & $R(0,0,1)$ where $R(x,y,z)$ denotes the statement $x+y=z$

Sol: Given statement is $x+y=z$

for $x=1, y=2, z=3 \Rightarrow R(1,2,3): 1+2=3$ is true

for $x=0, y=0, z=1 \Rightarrow R(0,0,1): 0+0=1$ is false

~~for~~ $\therefore$ $R(1,2,3)$ is true.

$R(0,0,1)$ is False.

Quantifiers :-

1) Universal Quantifier :- The quantifier $\forall$ (for all) is called Universal Quantifier. The Universal Quantification of the statement "$P(x)$ for all x" in the given domain. The Universal Quantification is indicated by $\forall x\, P(x)$

We can read it as [for all of x the statement $P(x)$]

Ex:- Let $P(x)$ be the statement "$x+1 > x$". What is the truth value of the quantification $\forall x\, P(x)$, where the domain consists for all the real values. ($\mathbb{R}$) 3M (or 2M)

Sol: Given statement is "$x+1 > x$"

Given Quantification is $\forall x\, P(x)$

Given domain $\mathbb{R}$

for all the values of $x \in \mathbb{R}$ the given quantification $\forall x\, P(x)$ is true.

2) Existantial Quantifier :- The quantifier "$\exists$" [there exist] is called Existantial. The existantial quantification of $P(x)$ is the preposition there exist at least one value for x from the given domain. Such that $P(x)$ is true. We can say

that the quantification is true. The existantial quantification is denoted with $\exists x\, P(x)$. we can read it as there exist of x for the statement $P(x)$.

Ex:- Let $P(x)$ denotes the statement "$x > 3$". what is the truth value of the Quantification $\exists x\, P(x)$. where the domain is the set of all real numbers. (2 or 3M)

Sol: Given statement is $P(x)$: "$x > 3$"

Given Quantification is $\exists x\, P(x)$

Given domain is set of all $\mathbb{R}$ $\Rightarrow x \in \mathbb{R}$

for $x = 4$

$4 > 3$

($\therefore P(x) : 4 > 3$ is true)

Note 1) When all the elements ~~and~~ in domain say $x_1, x_2, \ldots, x_n$ follows the universal Quantification $\forall x\, P(x)$ is same as the conjuction (meet or AND($\wedge$)). $P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$ is true if and only if the universal Quantification $\forall x\, P(x)$ is true. (or) $P(x_1), P(x_2), \ldots, P(x_n)$.

finite ~~[illegible]~~ Note pa
infinite ~~[illegible]~~ definit

Ex) What is the truth value of $\forall x P(x)$ where $P(x)$ is the statement "$x^2 > 10$". where the domain consists positive integers not exceeding 4

Sol: Given statement is $x^2 > 10$

Given Quantification is $\forall x\, P(x)$

Given domain the positive integers not exceeding 4 (1,2,3,4)

$P(1) : 1^2 > 10$ (false)

$P(2) : 2^2 > 10$ is False

$P(3) : 3^2 > 10$ is False

$P(4) : 4^2 > 10$ is true

$\therefore P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge P(x_4)$

$= F \wedge F \wedge F \wedge T = F$

$= F$

Hence $\forall x\, P(x)$ in given domain is false

(Q) $P(x) : 2x > x$

Sol: Given statement is $P(x) : 2x > x$.

Given Quantification is $\forall x\, P(x)$

Given domain is not exceeding 4 $(1, 2, 3, 4)$

$P(1) : 2 > 1$ is true

$P(2) : 4 > 2$. is true

$P(3) : 6 > 3$ is true

$P(4) : 8 > 4$ is true.

$\therefore P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge P(x_4)$

$= T \wedge T \wedge T \wedge T$

$= T$

$\therefore$ Hence $\forall x\, P(x)$ in given domain is true.

09/11/2021 Note :-

Where all the elements in domain $x_1, x_2, \ldots x_n$ follows the existancial quantification $\exists x\, P(x)$ is same as the disjunction $P(x_1) \vee P(x_2) \vee P(x_3) \vee \ldots \vee P(x_n)$ because the disjunction is true $(\Leftrightarrow)$ if and only if atleast one of the $P(x_1), P(x_2), P(x_3) \ldots P(x_n)$ is true.

Example :- What is the truth value of exisistancial quantification $\exists x\, P(x)$. where $P(x)$ is the statement $x^2 > 10$. When the domain consist all the positive integers not exceeding 4.

Sol Given statement is $P(x) : x^2 > 10$

Given Quantification is $\exists x\, P(x)$

Given domain is not exceeding 4 $\{1, 2, 3, 4\}$

$P(1) : 1 > 10$ is False

$P(2) : 4 > 10$ is False

$P(3) : 9 > 10$ is False

$P(4) : 16 > 10$ is True

Now $P(x_1) \vee P(x_2) \vee P(x_3) \vee P(x_4)$

$\equiv F \vee F \vee F \vee T$

$\equiv T$

The Quantification $\exists x\, (x^2 > 10)$ is true in the given domain.

Q) What is the truth value of universal quantification $\forall x\, P(x)$ where $P(x)$ is the statement $x^2 - x < 9$ and domain consists all positive integers not exceeding 5.

Sol Given statement is $P(x) : x^2 - x < 9$

Given Quantification is $\forall x\, P(x)$

Given domain is not exceeding 5 for all positive integers $\{1, 2, 3, 4, 5\}$.

$P(1) : 1 - 1 < 9$ is true

$P(2) : 2^2 - 1 < 9$
$4 - 1 < 9$
$3 < 9$ is true

$P(3) : 3^2 - 1 < 9$
$9 - 1 < 9$
$8 < 9$ is true

$P(4) : 4^2 - 1 < 9$
$16 - 1 < 9$
$15 < 9$ is False

$P(5) : 5^2 - 1 < 9$
$25 - 1 < 9$
$24 < 9$ is False.

Now $P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge P(x_4) \wedge P(x_5)$

$\equiv T \wedge T \wedge T \wedge F \wedge F$

$\equiv F$

The Quantification $\forall x\, P(x)$ is $\forall x\, (x^2 - x < 9)$ is false in the given domain.

Q) What is the truth value of Exisistancial quantification $\exists x P(x)$ where $P(x)$ is the statement "$x^2+x-9 < 25$". and domain the set of all positive integers not exceeding 5

Sol: Given statement is $x^2+x-9 < 25$

Given Quantification $\exists x P(x)$

Given domain set of all + integers not exceeding 5. $\{1,2,3,4,5\}$

$P(1) = 1^2+1-9 < 25$
$= 2-9 < 25$
$P(1) = -7 < 25$ is True

$P(2) = 2^2+1-9 < 25$
$= 5-9 < 25$
$= -4 < 25$ is True

$P(3) = 3^2+1-9 < 25$
$= 10-9 < 25$
$= 1 < 25$ is True

$P(4) = 4^2+1-9 < 25$
$= 17-9 < 25$
$= 8 < 25$ is True

$P(5) = 5^2+1-9 < 25$
$= 26-9 < 25$
$= 17 < 25$ is True

Now $P(x_1) \vee P(x_2) \vee P(x_3) \vee P(x_4) \vee P(x_5)$
$\equiv T \vee T \vee T \vee T \vee T$
$\equiv T$

The Quantification $\exists x (x^2+x-9 < 25)$ is true in the given domain

Q) Let $p(x)$ be the statement "$x = x^2$". If the domain consists of integers. what are the truth values of $P(0)$, $P(1)$, $P(2)$, $P(-1)$. (Exisitancial quantification) $\exists x P(x)$, $\forall x P(x)$.

Sol: Given statement is $x = x^2$

Given Quantification is $\exists x P(x)$ & $\forall x P(x)$

Given domain are integers. $(0, 1, 2, -1)$

$P(-1) \Rightarrow -1 = (-1)^2$
$\Rightarrow -1 \neq 1$ is false

$P(1) \Rightarrow 1 = 1$ is True

$P(0) \Rightarrow 0 = 0$ is True

$P(2) \Rightarrow 2 = 4$ is False

$\exists_x P(x)$

Now $P(x_1) \vee P(x_2) \vee P(x_3) \vee P(x_4)$

$\equiv F \vee T \vee T \vee F$

$\equiv T$

$\forall_x P(x)$

Now $P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge P(x_4)$

$\equiv F \wedge T \wedge T \wedge F$

$\equiv F$

The Quantification $\exists_x P(x)$ is $\exists_x (x = x^2)$ is true in the given domain but

the Quantification $\forall_x (x = x^2)$ is false in the given domain.

Q) Determine the truth values of the following statement If the domain consists all the real numbers.

i) $\exists_x (x^3 = -1)$ True ($x=-1$)

ii) $\exists_x (x^4 < x^2)$ True (0.1, 0.2)

iii) $\forall_x [(-x)^2 = x^2]$ True no need example

iv) $\forall_x [2x > x]$ False (-1) (-2) Eg

Sol i) $\exists_x (x^3 = -1)$

Statement $x^3 = -1$

Quantification $\exists_x P(x)$

domain is real numbers

Ex: $P(-1) \Rightarrow (-1)^3 = -1$

$\Rightarrow -1 = -1$ is True

$\therefore$ The Quantification $\exists_x P(x)$ is true in the given domain

(iii) $\forall_x [(-x)^2 = x^2]$

Statement $\rightarrow (-x)^2 = x^2$

Quantification $\rightarrow \forall_x P(x)$

domain is real numbers

The Quantification $\forall_x P(x)$ denotes the propositional for every x, for all real numbers x, such that $(-x)^2 = x^2$ is true.

$\therefore$ The given Quantification $\forall_x ((-x)^2 = x^2)$ is true. //

(ii) $\exists_x (x^4 < x^2)$

Statement $x^4 < x^2$

Quantification $\exists_x P(x)$

domain is real numbers

Ex: $P(0.1) = (0.1)^4 < (0.1)^2$

$= 0.0001 < 0.01$ is True

$\therefore$ The Quantification $\exists_x P(x)$ is true in the given domain.

(iv) $\forall_x [2x > x]$

Statement $\rightarrow 2x > x$

Quantification $\rightarrow \forall_x P(x)$

domain is real numbers

Ex: $P(-2) \Rightarrow 2(-2) > -2$

$-4 > -2$ is False

$\therefore$ The give Quantification $\forall_x P(x)$ does not denote the proposition for every x, for all real number x, such that $(2x) > x$ is False

$\therefore$ The given Quantification $\forall_x (2x > x)$ is false //

Nested Quantifiers :- The quantifiers are said to be nested quantifiers if one is with the scope of other.

Example $\forall x \exists y (x+y=0)$ is same as $\forall x\, Q(x,y)$ where $Q(x,y)$ is the statement $\exists y\, P(x,y)$, Here $P(x,y): x+y=0$

order is important

Order of quantifiers

Many mathematical statements involve multiple number of quantifiers of propositional functions involving more than one variables. It is important to note that the order of the quantifiers is important unless all the quantifiers are same quantifiers. (i.e unless all the quantifiers are universal (or) all the quantifiers are exisitancial).

Example Let $p(x,y)$ be the statement "$x+y=y+x$" What are the truth values of quantifications $\forall x \forall y\, p(x,y)$ and $\forall y \forall x\, P(x,y)$. where the domain consists all the real numbers.

Sol Given statement $p(x,y)$: $x+y=y+x$

Given quantifier

Given domain

The quantification $\forall x \forall y\, P(x,y)$ denotes all the propositions for every "x, for all real numbers y, such that $x+y=y+x$" is true. [$\because$ Comitative property w.r.t addition]

$\therefore$ Given quantification $\forall x \forall y\, (x+y=y+x)$ is true

The given quantification $\forall x \forall y\, P(x,y)$ denotes the proposition "for every y all the real numbers x such that $x+y=y+x$" is true [$\because$ Comitative property

$\therefore$ for all y, for all x, $(x+y = y+x)$ is true.

Hence both the quantifications $\forall x \forall y\, P(x,y)$ and $\forall y \forall x\, P(x,y)$ are same, so the order is not important in the nested universal quantification.

(8) *** Let $Q(x,y)$ be the statement "$x+y=0$". what are the truth values of the quantifications $\exists y \forall x\, Q(x,y)$ and $\forall x \exists y\, Q(x,y)$ when the domain consists all the real numbers.

$\exists y \forall x\, Q(x,y)$	$\forall x \exists y\, Q(x,y)$
1 0	$x=1$ -1
	$x=-2$ $+2$
-1	
1	
2	

Sol Given Statement $Q(x,y)$: $x+y=0$

Given domain is set of all real numbers

The quantification $\exists y \forall x\, Q(x,y)$ denotes the proposition "there is a real number y, for every real number x, such that $x+y=0$" is False. [$\because$ $y=1$ $x=2$ $1+2 \neq 0$]

$\therefore$ There is no real number y such that $x+y=0$ for all x.

The quantification $\forall x \exists y\, Q(x,y)$ denotes the proposition "for every real number x, there is some real number y, such that $x+y=0$" is True [$\because$ Identity property w.r.t addition]

$\therefore$ The quantification $(\forall x \exists y)$ for all x there exisit some real number y, such that $x+y=0$.

11/01/2021

Quantifiers of variables

S.No	Statement	When is true	When is False
1)	$\forall x\, P(x)$	$P(x)$ is true for every 'x'	There is an 'x' for which $P(x)$ is false
2)	$\exists x\, P(x)$	There is an 'x' for which $P(x)$ is true	$P(x)$ is false for every 'x'
3)	$\forall x \forall y\, P(x,y)$ $\forall y \forall x\, P(x,y)$	$P(x,y)$ is true for every pair (x,y)	There is a pair (x,y) for which $P(x,y)$ is false
4)	$\forall x \exists y\, P(x,y)$	For every 'x' there is a 'y' for which $P(x,y)$ is true	For One 'x' There is an 'x' such that $P(x,y)$ is false for every 'y'
5)	$\exists x \forall y\, P(x,y)$	There is an 'x' for which $P(x,y)$ is true for every 'y'	For every 'x' there is a 'y' for which $P(x,y)$ is false.
6)	$\exists x \exists y\, P(x,y)$	There is a pair (x,y) for which $P(x,y)$ is true	$P(x,y)$ is False for every pair (x,y)

Rules of Inferences :-

Consider the prepositions $P_1, P_2, P_3 \ldots P_n$ and also the preposition 'C'. The compound statement (preposition) $[P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n] \rightarrow C$ is called an argument

⇒ In this argument the conjuction $[P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n]$ is said to 'Premises'.

⇒ $P_1, P_2, P_3, \ldots P_n$ are also called Premises of the argument and C is called conclusion of the argument.

⇒ An argument in a propositional logic is a sequence of propositions but the first proposition is called premises the final preposition called conclusion.

⇒ An argument is <u>valied</u> if the truth values are of all its possible situations are true.

Consistency of a premises :-

The premises $P_1, P_2, P_3 \ldots P_n$ of an argument are said to be <u>inconsistent</u> if there conjunction $P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n$ is false for every possible situations.

The premises $P_1, P_2, P_3 \ldots P_n$ of an argument are said to be <u>consistent</u> if there conjunction $P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n$ is true for over atleast one of its possible situations.

Q) Test the consistency of premises $P, (\sim P \wedge q)$

Sol) Given premises $P, (\sim P \wedge q)$

P	q	$\sim P$	$(\sim P \wedge q)$	$[P \wedge (\sim P \wedge q)]$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

Given premises is inconsistent

8) Test the consistency of premises $(p \vee q)$ and $\sim p$

Sol

p	q	$\sim p$	$p \vee q$	$[(p \vee q) \wedge (\sim p)]$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

The given consistency of premises is consistant

∵ One of its possible situation is true.

Rules of Inferences :-

S.No	Rule	Tautology	Name of the rule
1)	p $p \to q$ $\therefore q$ (Conclusion)	$[p \wedge (p \to q)] \to q$	Modus Ponens
2)	$\sim q$ $p \to q$ $\therefore \sim p$	$[(\sim q) \wedge (p \to q)] \to (\sim p)$	Modus tollens
3)	$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical Syllogism
4)	$p \vee q$ $\sim p$ $\therefore q$	$[(p \vee q) \wedge (\sim p)] \to (q)$	Disjective Syllogism (or) Disjunction Syllogism
5)	p $\therefore p \vee q$	$p \to (p \vee q)$	Addition rule

S.No	Rule	Tautology	Name of the rule
6)	$\frac{p \wedge q}{\therefore p}$	$[p \wedge q] \rightarrow p$	Simplification rule
7)	$\frac{\begin{array}{c} p \\ q \end{array}}{\therefore p \wedge q}$	$[(p) \wedge (q)] \rightarrow (p \wedge q)$	Conjuction rule
8)	$\frac{\begin{array}{c} p \vee q \\ \sim p \vee r \end{array}}{\therefore q \vee r}$	$[(p \vee q) \wedge (\sim p \vee r)] \rightarrow (q \vee r)$	Resolution rule

Ex :- State which rule of inference is the basis of the following arguments

i) " It is below freezing now. Therefore, it is either below freezing or raining now".

Sol Let 'p' be the proposition " It is below freezing now".

Let 'q' be the proposition "It is raining now".

Given argument can be written as

$$\frac{p}{\therefore p \vee q}$$

$\therefore$ The given argument is addition rule.

ii) " It is below freezing and raining now". Therefore It is below freezing now"

Sol Let 'p' be the preposition " It is below freezing now

let 'q' be the preposition " It is raining now

Given argument can be written as $\frac{p \wedge q}{\therefore p}$

$\therefore$ The given argument is "simplification rule."

8) Test wether the following arguments are valied or not

i) If Sachin hits a century then he gets a free car

Sachin hits a century.

Sachin gets a free car

Sol Let 'p' be the preposition "Sachin hits a century"

Let 'q' be the preposition "Sachin gets a free car"

the given argument can be written as

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

By Modus Ponens rule the given argument is valid

ii) If I work to drive

If I drive to work then I will arrive tired

I am not tired

I do not to drive

Sol Let 'p' be the preposition "If I drive to work"

Let 'q' be the preposition "I will arrive tired"

Let '$\sim p$' be the preposition "I do not to drive"

Let '$\sim q$' be the preposition "I am not tired"

Given argument can be written as

$$\begin{array}{l} \sim q \\ p \to q \\ \hline \therefore \sim p \end{array}$$

By Modus tollens rule the given argument is valid.

(iii) If I study, then I do not fail in the exam
If I do not fail in the exam, then my father gifts a two wheeler to me

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∴ If I study then my father gifts a two wheeler to me

Sol: Let 'p' be the preposition " I study "
Let 'q' be the preposition " I do not fail in the exam
Let 'r' be the preposition " My father gifts a two wheeler to me

Given argument can be written as

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

By Modus Hypothetical syllogism rule the given argument is valid.

(iv)

$$\begin{array}{l} p \to q \\ q \to s \\ p \vee r \\ \hline \therefore q \vee s \end{array}$$

Sol: Consider the conjuction

$(p \to q) \wedge (r \to s) \wedge (p \vee r)$

$\equiv (p \to q) \wedge (r \to s) \wedge (\sim p \to r)$ $(\because p \to q \equiv \sim p \vee q)$

$\equiv (p \to q) \wedge (\sim p \to r) \wedge (r \to s)$ [∵ Commutative law]

$\equiv (p \to q) \sim (\sim p \to s)$ [∵ Hypothetical syllogism]

$\equiv (\sim q \to \sim p) \wedge (\sim p \to s)$ [∵ By contrapositive]

$\equiv (\sim q \to s)$ [∵ Hypothetical syllogism]

$\equiv \sim(\sim q) \vee s$ $[\because p \to q \equiv \sim p \vee q]$

$\equiv q \vee s$

Hence the given argument is valid.

13/01/2021 $p \to r$
$q \to r$
$\therefore (p \vee q) \to r$

Sol:

Consider the conjunction

$= (p \to r) \wedge (q \to r)$

$\equiv (p \vee q) \to r$ $\quad (\because (p \to r) \wedge (q \to r) = (p \vee q) \to r)$

$\therefore$ $p \to r$
$q \to r$
$\therefore (p \vee q) \to r$

Hence the given argument is valid.

Note :- Statement : $p \to q$
Converse : $q \to p$
Inverse : $\sim p \to \sim q$
Contropositive : $\sim q \to \sim p$

Negation of a quantified statement :-

In a quantified statement change the quantifier from universal to existancial and existancial to universal also replace open statement by it's negation then we will get the negation of a quantified statement.

8) Consider the following preposition with the set of all real numbers as the universe. With the statements $p(x): |x| > 3$, $q(x): x > 3$ Find the truth value of the quantified preposition $\forall x [p(x) \to q(x)]$. And also write down Converse, Inverse, Contropositive of the given quantified statements and find their truth values ?

Given that,

Sol) Given statements $p(x) : |x| > 3$, $q(x) : x > 3$

Given domain is Real number set

Given Quantified preposition is $\forall x\,[p(x) \to q(x)]$

for $x = -4$ then $p(-4) : |-4| > 3$ is true

$q(-4) : -4 > 3$ is false

T	T	T
T	F	F
F	T	T
F	F	T

$\therefore$ for $x = -4$, $p(x) \to q(x)$ is false.

Hence $\forall x[p(x) \to q(x)]$ is false

Converse :- $\forall x\,[q(x) \to p(x)]$

for every x if $q(x) : x > 3$ is true then $p(x) : |x| > 3$ is also true.

T F F

X

for all Hence $\forall x\,[q(x) \to p(x)]$ is true.

Inverse :- $\forall x\,[\sim(p(x)) \to \sim q(x)]$

$x = -3$

$p(-3) = |-3| \leq 3$

$3 \leq 3$

$q(-3) : -3 \leq 3$

F F T

F T T

T F F

T T T

for every 'x' if

$\sim q(x) : x \leq 3$

$\sim p(x) : |x| \leq 3$

for every 'x' if $\sim p(x) : x \leq 3$ is true then $\sim p(x) \leq 3$ is also true.

Hence $\forall x\,[\sim p(x) \to \sim q(x)]$

Contrapositive :- $\forall x\,[\sim q(x) \to \sim p(x)]$

$\sim q(x) : x \leq 3$; $\sim p(x) : |x| \leq 3$

for $x = -4$ then $\sim q(-4) : -4 \leq 3$ is true

$\sim p(-4) : |-4| \leq 3$ is false

$4 \leq 3$

$\therefore$ for $x = -4$, $\sim q(x) \to \sim p(x)$ is false

Hence $\forall x\,[\sim q(x) \to \sim p(x)]$ is false "

Q) Consider the following quantified statements with the universe of real numbers as the universe. Determine the truth values of the following quantified prepositions

$$\left\{\begin{array}{l} p(x) : x \geq 0 \\ q(x) : x^2 \geq 0 \\ r(x) : x^2-3x-4=0 \\ s(x) : x^2-3 > 0 \end{array}\right\}$$

(i) $\exists x\, p(x) \wedge q(x)$

(ii) $\forall x\, [p(x) \to q(x)]$

(iii) $\forall x\, [q(x) \to s(x)]$

(iv) $\forall x\, [r(x) \vee s(x)]$

(v) $\exists x\, [p(x) \wedge r(x)]$

(vi) $\forall x\, [r(x) \to p(x)]$

(vii) $\forall x\, [q(x) \to s(x)]$

Sol Given domain is $\mathbb{R}$

$q(x) = x^2 \geq 0$

$s(x) = x^2-3 > 0$

for $x=1$ $q(1): 1 \geq 0$ is true

$s(1): 1^2-3 > 0$ is false

for $x=1$ $q(x) \to s(x)$ is false

Hence $\forall x\, [q(x) \to s(x)]$ is false

i) $\exists x\, p(x) \wedge q(x)$

Sol Given domain is $\mathbb{R}$

$p(x) = x \geq 0$

$q(x) = x^2 \geq 0$

for $x=1$, $p(1): 1 \geq 0$ is true

$q(1): 1^2 \geq 0$ is true

for $x=1$, $p(x) \wedge q(x)$ is true

Hence $\exists x\, p(x) \wedge q(x)$ is true

ii) $\forall x\, [p(x) \to q(x)]$

Sol Given domain is $\mathbb{R}$

$p(x) = x \geq 0$

$q(x) = x^2 \geq 0$

for $x=2$, $p(2): 2 \geq 0$ is true

$q(2): 2^2 \geq 0$ is true

for $x=2$, $p(x) \to q(x)$ is true

Hence $\forall x\, [p(x) \to q(x)]$ is true

iv) $\forall x\, [r(x) \vee s(x)]$

Given domain is $\mathbb{R}$

$r(x) = x^2-3x-4=0$

$s(x) = x^2-3 > 0$

for $x=4$, $r(x) = 4^2-3(4)-4=0$ is true

$s(4) = 4^2-3 > 0$ is false

for $x=4$ $[r(x) \vee s(x)]$ is false

Hence $\forall x\, [r(x) \vee s(x)]$ is false

(v) $\exists x\, [p(x) \wedge r(x)]$

Sol Given domain

$p(x) = x \geq 0$

$r(x) = x^2-3x-4$

for $x=4$ $p(4): 4$

$r(4): 4^2$

for $x=4$ $p(x) \wedge s$

Hence $\exists x\, [p(x)$

is true

Q) Prove

Sol Given

Let s

$\forall x$ P

16/11/2021 Q)

Sol Consider

with the set
values of the
repositions
is $\mathbb{R}$
0
0 is true
0 is true
is true
is true
$\mathbb{R}$
true
true
true
true
true
true
se.

(v) $\exists x [p(x) \wedge q(x)]$

Sol Given domain is $\mathbb{R}$

$p(x) = x \geq 0$

$q(x) = x^2 - 3x - 4 = 0$

for $x = 4$ $p(4) : 4 \geq 0$

$q(4) : 4^2 - 3(4) - 4 = 0$

for $x = 4$ $p(x) \wedge q(x)$ is true

Hence $\exists x [p(x) \wedge q(x)]$ is true

(vi) $\forall x [q(x) \rightarrow p(x)]$

Sol Given domain is $\mathbb{R}$

$q(x) = x^2 - 3x - 4 = 0$

$4 \geq 0$

$p(x) = x \geq 0$

for $x = 4$, $p(4) : 4 \geq 0$

$q(x) : 4^2 - 3(4) - 4 = 0$

for $x = 4$ $q(x) \rightarrow p(x)$ is true

Hence $\forall x [q(x) \rightarrow p(x)]$ is true

(5 Marks) (or) 7 Mar

Q) Prove that $\forall x\, P(x) \Rightarrow \exists x\, P(x)$

Sol Given statement is $P(x) \rightarrow \exists x\, P(x)$

Let S be the set of universe

$\forall x\, P(x) = \{$for every $x \in S$, $P(x)$ is true$\}$ for all $\forall x$

$\Rightarrow \{$for some $a \in S$, $P(a)$ is true$\}$ for some $\exists x$

$\Rightarrow \exists a \in S \ni P(a)$ is true

$\Rightarrow \exists x\, P(a)$

$\Rightarrow \exists x\, P(x)$

16/11/2021 Q) Prove that $\forall x [P(x) \vee q(x)] \Rightarrow \forall x\, P(x) \vee \exists x\, q(x)$

Sol Let S be the set of universe

Consider $\forall x [P(x) \vee q(x)] = \{$for all $x \in S$, $P(x)$ OR $q(x)$ is true$\}$

$\Rightarrow \{$for all $x \in S$, $P(x)$ is true or $q(x)$ is true$\}$

$\Rightarrow \{$for all $x \in S$, $P(x)$ is true$\} \vee \{$for all $x \in S$, $q(x)$ is true$\}$

$\Rightarrow \{$for all $x \in S$, $P(x)$ is true$\} \vee \{$for some $x \in S$, $q(x)$ is true$\}$

$\Rightarrow \forall x\, P(x) \vee \exists x\, q(x)$ //

16/11/2021 (iii) Prove that $\exists x[p(x) \wedge q(x)] \Rightarrow \exists x\, p(x) \wedge \exists x\, q(x)$

Sol: Let s be the set of universe.

Consider $\exists x[p(x) \wedge q(x)] = \{$for some $x \in s$, $p(x)$ AND $q(x)$ is true$\}$

$= \{$for some $x \in s$, $p(x)$ is true and $q(x)$ is true$\}$

$= \exists x\, p(x) \wedge \exists x\, q(x)$.

$\therefore \exists x[p(x) \wedge q(x)] \Rightarrow \exists x\, p(x) \wedge \exists x\, q(x)$ is proved.

(iv) Prove that $\forall x\, p(x) \vee \forall x\, q(x) \Rightarrow \forall x[p(x) \vee q(x)]$

Sol: Let s be the set of Universe

Cosider $\forall x\, p(x) \vee \forall x\, q(x) = \{$for all $x \in s$, $p(x)$ is true$\}$ or $\{$for all $x \in s$, $q(x)$ is true$\}$

$= \{$for all $x \in s$, $p(x)$ is true or $q(x)$ is true$\}$

$= \{$for all $x \in s$, $p(x)$ or $q(x)$ is true$\}$

$= \forall x[p(x) \vee q(x)]$

(v) Prove that the following argument is valid

all men are mortal

Sachin is a man

$\therefore$ Sachin is mortal

Sol: Let s be the set of all men

Let $p(x)$: x is mortal, a is sachin

a = Sachin

Given argument can be written as

$\forall x\ p(x)$ — for all $x \in S$, $p(x)$ is true

$a \in S$ — and $a \in S$

$\therefore p(a)$ — so $p(a)$ is true

$\therefore$ The given argument is valid.

8) Find whether the following argument is valid or not for which the universe is the set of all students

No Engineering student is bad in studies → compound stmt

Anil is not bad in studies

$\therefore$ Anil is an engineering student

Sol Let S be the set of all students

let $p(x)$: x is an engineering students

let $q(x)$: x is not bad in studies

a = Anil

Given argument can be written as

$\forall x\,[p(x) \to q(x)]$

$q(a)$

$\therefore p(a)$

To prove validity

Consider $\forall x\,[p(x) \to q(x)] \wedge q(a)$

$\Rightarrow \forall x\,[\sim p(x) \vee q(x)] \wedge q(a)$

for $a \in S$ $\Rightarrow [\sim p(a) \vee q(a)] \wedge q(a)$ $(\because p \to q \equiv \sim p \vee q)$

$\Rightarrow [\sim p(a) \wedge q(a)] \vee [q(a) \wedge q(a)]$

$\Rightarrow [\sim p(a) \wedge q(a)] \vee q(a) \not\Rightarrow p(a)$

$\therefore$ The given argument is invalid.

Q) Test whether the following argument is valid or not

No graduate of commerce or literature studies physics

Anil is a graduate student who studies physics

$\therefore$ Anil is not a graduate student of literature.

Sol

Let $p(x)$: x studies commerce

Let $q(x)$: x studies literature

Let $r(x)$: x studies physics

a = Anil

Given argument can be written as

$$\forall x\,[p(x) \vee q(x)] \rightarrow (\sim r(x))$$

$$r(a)$$

premises

$$\therefore \sim q(a)$$

prove avathundhi

To prove validity

Consider $[\forall x\,[p(x) \vee q(x)] \rightarrow (\sim r(x))] \wedge r(a)$

for $a \in S$ $\Rightarrow [[p(a) \vee q(a)] \rightarrow (\sim r(a))] \wedge r(a)$ $(\because p \rightarrow q \equiv \sim p \vee q)$

$\Rightarrow [\sim(p(a) \vee q(a)) \vee (\sim r(a))] \wedge r(a)$

$\Rightarrow [(\sim p(a) \wedge \sim q(a)) \vee (\sim r(a))] \wedge r(a)$ $(\because$ Demorgan law$)$

$\Rightarrow [(\sim p(a) \wedge \sim q(a)) \wedge r(a)] \vee (\sim r(a) \wedge r(a))$

$(\because$ distributive law$)$

$\Rightarrow [(\sim p(a) \wedge \sim q(a)) \wedge r(a)] \vee F$

$\Rightarrow [\sim p(a) \wedge (\sim q(a))] \wedge r(a)$ $(\because$ Identity law$)$

$\Rightarrow [\sim p(a) \wedge \sim q(a)]$ $(\because$ Simplification rule$)$

$\Rightarrow \sim q(a) \wedge \sim p(a)$ (∵ commutative rule)

$\Rightarrow \sim q(a)$ (∵ simplification rule)

Hence the given argument is valid //

8) Prove that the following argument is valid

$$\begin{array}{l} \forall x\,[p(x) \to q(x)] \\ \forall x\,[q(x) \to r(x)] \\ \hline \therefore \forall x\,[p(x) \to r(x)] \end{array}$$

Hypothetical syllogism

To prove validity

Sol:

Consider $[\forall x[p(x) \to q(x)]] \wedge [\forall x[q(x) \to r(x)]]$

$\Rightarrow [\sim p(x) \vee q(x)] \wedge [\sim q(x) \vee r(x)]$

$\Rightarrow$

9) $$\begin{array}{l} \forall x\,[p(x) \to (q(x) \wedge r(x))] \\ \forall x\,[p(x) \wedge s(x)] \\ \hline \forall x\,[p(x) \wedge s(x)] \end{array}$$

Sol: To prove validity

Consider $[\forall x[p(x) \to (q(x) \wedge r(x))]] \wedge [\forall x[p(x) \wedge s(x)]]$

Q) Determine the truth values of each of the following quantified statements, Here the universe set is set of all non zero integers ? (i) $\exists x \exists y [xy = 1]$

Sol Let S be the set of all non zero integers

$p(x, y) : [xy = 1]$

for $x = 1$, $y = 1$

$P(1, 1) : [(1)(1) = 1]$ is true

$\exists x \exists y [xy = 1]$ is true for the given domain (S)

(ii) $\exists x \forall y [xy = 1]$ (iii) $\forall x \exists y (xy = 1)$ (iv) $\exists x \exists y [(2x+y=5) \wedge (x-3y=-8)]$

(v) $\exists x \exists y [(3x-y=17) \wedge (2x+4y=3)]$

(iv) $\exists x \exists y [(2x+y=5) \wedge (x-3y=-8)]$

Sol Let S be the set of all non zero integers as the universe

$P(x,y) : 2x+y=5$

$Q(x,y) : x-3y=-8$

by solving for x, y

$x = 1$ $y = 3$

$(x, y) = (1, 3) \in S$

for $x=1$, $y=3$

$P(1, 3) : 2(1) + 3 = 5$ is true

$Q(1, 3) : 1 - 3(3) = -8$ is true

$P(x, y) \wedge Q(x, y)$ is true for $x=1$, $y=3$

Hence $\exists x \exists y [(2x+y=5) \wedge (x-3y=-8)]$ is true

(v) $\exists x \exists y [(3x-y=17) \wedge (2x+4y=3)]$

Sol Let S be the set of all non zero integers as the universe

$P(x, y) : 3x-y=17$

$Q(x, y) : 2x+4y=3$

by solving for x, y for $x = 5.07$, $y = -1.78$

for $x = \frac{71}{14}$, $y = -\frac{25}{14}$

$P(5.07, -1.78)$: $3(5.07) - (-1.78) = 17$ is true

$Q(5.07, -1.78)$: $2(5.07) + 4(-1.78) = 3$ is true

$P(x,y) \wedge Q(x,y)$ is true for $x = 5.07$, $y = -1.78$

but (x, y) does not belongs to S (or) But $(x,y) = \left(\frac{71}{14}, \frac{-25}{14}\right) \notin S$

Hence $\exists x \exists y \left[(3x - y = 17) \wedge (2x + 4y = 3) \right]$ is false.

(ii) $\exists x \forall y [xy = 1]$

Sol Let S be the set of all non zero integers as the universe

$P(x,y)$: $(xy = 1)$

for $x = 2$, $y = 2$

$P[2,2]$: $[(2)(2) = 1]$ is false

$\exists x \forall y [xy = 1]$ is false for the given domain(s)

(iii) $\forall x \exists y (xy = 1)$

Sol Let S be the set of all non zero integers as the universe

$P(x,y)$: $(xy = 1)$

for $x = 2$; $y = 2$

$P(2,2)$: $[(2)(2) = 1]$ is false

$\forall y \exists y [xy = 1]$ is false for the given domain(s)

Q) Find the negation of the following quantified statement

(i) $\forall x \exists y [\{p(x,y) \wedge q(x,y)\} \rightarrow r(x,y)]$

Sol The negation of the given quantified statement is

$$\sim[\forall x \exists y [\{p(x,y) \wedge q(x,y)\} \rightarrow r(x,y)]]$$

$$\equiv \exists x \forall y \sim[\{p(x,y) \wedge q(x,y)\} \rightarrow r(x,y)]$$

$$\equiv \exists x \forall y \sim[\sim\{p(x,y) \wedge q(x,y)\} \vee r(x,y)] \quad (\because p \rightarrow q \equiv \sim p \vee q)$$

$$\equiv \exists x \forall y [\{p(x,y) \wedge q(x,y)\} \wedge \sim r(x,y)]$$

$$\equiv \exists x \forall y [p(x,y) \wedge q(x,y) \wedge \sim r(x,y)]$$

(ii) $\forall x \forall y [(x>y) \rightarrow (x-y) > 0]$

Sol The negation of the given quantified statement is

$$\sim[\forall x \forall y [(x>y) \rightarrow (x-y) > 0]$$

$$\equiv \exists x \exists y \sim[(x>y) \rightarrow (x-y) > 0])$$

$$\equiv \exists x \exists y \sim[\sim(x>y) \vee (x>y) > 0)] \quad (\because p \rightarrow q \equiv \sim p \vee q)$$

$$\equiv \exists x \exists y [(x>y) \wedge (x>y) \geq 0])$$

$$\equiv \exists x \exists y [(x>y) \wedge (x>y) \geq 0)]$$

18/11/2021 Q) Find the negation of the following statement

$\forall x \forall y \left[(x < y) \rightarrow \exists z (x < z < y) \right]$

Sol The negation of the given statement

$\sim \left[\forall x \forall y \{ (x < y) \rightarrow \exists z (x < z < y) \right]$

$\equiv \exists x \exists y \left[\sim \{ (x < y) \rightarrow \exists z (x < z < y) \right]$

$\equiv \exists x \exists y \left[\sim (\sim (x < y) \vee \exists z (x < z < y)) \right]$. $\left(\because p \rightarrow q \equiv \sim p \vee q \right)$

$\equiv \exists x \exists y \left[(x < y) \wedge \forall z (x < z < y) \right]$

1) Proofs of Introduction :-

i) Direct Proofs :-

Stratagey of Direct proofs :-

The Direct method of proving a condition $P \to q$ has the following steps

i) Hypothesis :- First Assume that P is true.

ii) Analysis :- starting with hypothesis and emplaying the rules, laws, known logical equivalences, rules of inferences, known facts in for that q is true.

iii) Conclusion :- $P \to q$ is true

Q) Give a direct proof of the statement "The square of an odd integer is an odd integer".

Sol i) Hypothesis :- Assume that 'n' is an odd integer

ii) Analysis :- we have 'n' is an odd integer

$$n = 2k+1$$

$$n^2 = (2k+1)^2$$

$$= 4k^2 + 4k + 1 \text{ is not divisible by } 2$$

So n^2 is not even integer

$\therefore n^2$ is an odd integer.

iii) Conclusion :- The square of an odd integer is an odd integer

Q) Give a direct proof of the following statement

i) "The square of an even integer is an even integer".

ii) "The Sum of any five consequetive integer is divisible by 5.

$n, n+1, n+2, n+3, n+4$

i) Hypothesis :- Assume that 'n' is an even integer.

ii) Analysis :- we have 'n' is an even integer

$n = 2k + 2$

$n^2 = 4k^2 + 4k + 2$ is divisible by 2.

So n^2 is an even integer

iii) Conclusion :- The square of an even integer is an even integer

i) Hypothesis :- Assume that 'n' is sum of five consequ integer div by 5

ii) Analysis :- we have 'n' is an integer

$(n) + (n+1) + (n+2) + (n+3) + (n+4)$ is divisible by 5

$5n + 10 \,/\, 5$

If $n=1 \rightarrow \frac{15}{5} = 3$ is divisible by 5

So 'n' is an any integer.

iii) Conclusion :- The sum of any five consequetive integer is divisible by 5

2) Proofs of Contridiction(*) :-

Strategy of Contri :- The method of proof of a contridiction is having the following steps for $p \rightarrow q$ is True.

i) Hypothesis :- Assume that $p \rightarrow q$ is False. That is p is true and q is false

ii) Analysis :- Starting with the hypothesis that q is false and employing the rules, known laws, known logical equivalences known facts, it is in for that p is False, which is contridictes the assumption.

(iii) Conclusion :- Because of the contridiction in Analysis we can conclude that $p \to q$ is true.

8) Provide a proof of contridiction for the following statement. " For every integer 'n', if n^2 is odd then 'n' is odd ". Let 'n' be an integer,

$p: n^2$ is odd ; $q: n$ is odd

Sol (i) Hypothesis :-

Assume $p \to q$ is false

i.e, p is true and q is false

means n^2 is odd and n is not odd (or) then n is even

(ii) Analysis :- q is false i.e n is even then n can be written as $\Rightarrow n = 2k$

$\Rightarrow n^2 = 4k^2$ is divisible by 2

$\Rightarrow n^2$ is even

which contridicts the assumption.

(iii) Conclusion :- Because of the contridiction, assumption is corong.

Hence $p \to q$ is true

i.e If n^2 is odd then 'n' is odd.

9) The so Given

"The square of an even integer, then it is an even int"

Sol Let n be an integer

$p: n$ is even , $q: n^2$ is even

(i) Hypothesis :- Assume $p \to q$ is false

i.e p is true and q is false

means n is even and n^2 is odd

(ii) Analysis :- q is false

i.e n^2 is odd

$\Rightarrow n^2 = 2k+1$

$n = \sqrt{2k+1}$ is not divisible by 2.

$\Rightarrow n$ is odd

which contridicts the assumption.

(iii) Conclusion :- Because of the contridiction, assumption is wrong.

Hence $p \to q$ is true.

i.e If n is even then n^2 is even.

3) Indirect Proofs :-

$\Rightarrow$ We know that a condition $p \to q$ and it's contrapositive $\sim q \to \sim p$ both are logically equivalent

$\Rightarrow$ In some situations the proof of $p \to q$ is not possible are difficulty to prove. then use the contrapositive $\sim q \to \sim p$ to get easy proof. On the basis of this proof we infer that $p \to q$ is true. The method of proving a condition in this way is called "Indirect Proof"

Q) Provide an indirect proof of the following statement "For all positive real numbers x and y, if the product xy exceeds 25, then $x > 5$ or $y > 5$

Sol

Let $x, y \in \mathbb{R}^+$

$p : xy > 25$; $q : x > 5$; $y > 5$

proof of contrapositive $\sim q \to \sim p$

i) Hypothesis :- Assume $\sim q$.

Sol Let $x, y \in \mathbb{R}^+$

$p: xy > 25$, $q: x > 5$ or $y > 5$

proof of contrapositive $\sim q \to \sim p$

i) Hypothesis :- Assume $\sim q$ is true

i.e $x \leq 5$ and $y \leq 5$ is true

ii) Analysis :- $\sim q$ is true

$x \leq 5$ and $y \leq 5$

$(x)(y) \leq 5(5)$

$xy \leq 25$ $\left[\because \begin{array}{l} p: xy > 25 \\ \sim p: xy \leq 25 \end{array}\right]$

$\Rightarrow \sim p$ is true

$\therefore$ If $x \leq 5$ and $y \leq 5$ then $xy \leq 25$

$\sim q \to \sim p$ is true

(iii) Conclusion :- $P \to q$ and $\sim q \to \sim P$

are logically equivalent

$\therefore$ Hence $P \to q$ is true //

Q) Provide an indirect proof for the following statements:

i) "Let n be an integer prove that if n^2 is odd then n is odd."

ii) "The product of two even integers is an even integers"

(i) Sol ~~Hypothesis~~ :- Let $x, y \in \mathbb{Q}$