FUNCTION

BASIC STRUCTURES, SETS, RELATIONS AND

UNIT- II

Sets: A well defined collection of objects is called a set the objects are called the elements or members.

The sets are denoted by A, B, C --- and the elements are denoted by a, b, c ----.

If the numbers of elements in a set is finite, then the set is called finite set otherwise ~~in finite~~ infinite set.

A set having only one element is called a Singleton set. If x is an element in a set A. then we write $x \in A$. If x is not an element in A then we write $x \notin A$.

The set is represented by two methods. They are: i, tabulated method

ii, set builder form or rule method

In the tabulation method, all elements of a set are written down within flower brackets.

In the set builder form, write the rule with all the elements satisfied.

Ex: S in the set of all +ve integer.

$S = \{1, 2, 3 \cdots\}$ in tabulated method.

$S = \{x / x$ is a +ve integer$\}$ in set builder form.

- A set contains of only one element is called the singleton set.

- A set not contain any element is called the null set or empty set. It is denoted by ϕ, $\{ \}$

- Two sets A and B are said to be equal if they have precisely the same elements. then we write $A = B$

Ex: $A = \{1, 2, 3, 4, 5\}$

$B = \{x / x$ is a +ve integer less than 6$\}$

Subsets:-

Consider two sets A and B, we say that A is contain set in B if a subset of B or that A is contained in B if every element of A is an element of B. It is denoted by $A \subset B$.

Ex:- $A = \{1, 2, 3\}$; $B = \{1, 2, 3, 4, 5\}$; $C = \{2, 3, 5, 6\}$

we observe that every element in A is an element in B

$\therefore A \subseteq B$.

Properties:-

1. Every set is a subset of itself.
2. Two sets A and B are equal if and only if $A \subseteq B$ and $B \subseteq A$.
3. The null set is a subset of every set A.
4. For any sets A, B and C if $A \subseteq B$: $B \subseteq C \Rightarrow A \subseteq C$

universal Set:-

All sets that we consider are subsets of a certain set 'U'.

- This set U is called the universal set or universe of discourse.

Power set:- Let A is a set. The set of all subsets of the set A is called the powerset of A and is denoted by P(A).

- If a finite set of A has 'n' elements, then P(A), the power set of A has 2^n elements.

Venm diagrams:-

Graphical representation of a set is called venm diagram.

- universal set is represented by rectangular shape and any set represented by circle shape.

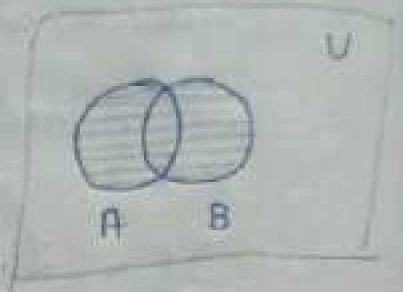
A B

$A \cap B$

$A \cup B$

Union of sets:-

Suppose A and B are any two sets. The union of A and B is the set of all those elements which belongs to either A or B or both. It is denoted by $A \cup B$.



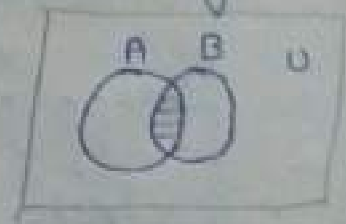
$$A \cup B = \{x / x \in A \text{ or } x \in B\}$$

we observe that

$$A \subseteq A \cup B : B \subseteq A \cup B.$$

Intersection of sets:-

Suppose A and B are any two sets. The intersection of A and B is the set of common elements in A and B. It is denoted by $A \cap B$



$$A \cap B = \{x / x \in A \text{ and } x \in B\}$$

we observe that

$$A \cap B \subseteq A = A \cap B \subseteq B.$$

Disjoint sets:-

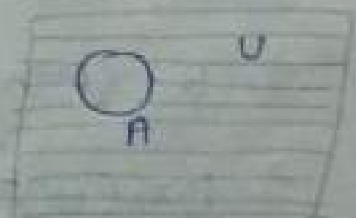
If A and B are any two sets. The sets A and B has no common element. Then the two sets are called disjoint sets.

i.e $A \cap B = \{\ \} = \phi$

Complement of a set:-

Let A be any set and U be an Universal set, the set of all elements that belongs to U but not belongs to A is called the complement of A, and is denoted by $\bar{A}$ or A'

$$\bar{A} = A' = \{x / x \in U \text{ and } x \notin A\}$$

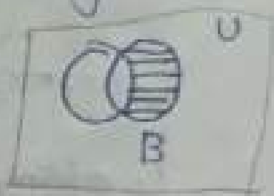


Relative Complement:-

Let A and B are two sets, the set of all elements that belongs to A but not belongs to B is called complement of B relative to A and is denoted by ~~B~~ $A-B$.

$\therefore A-B = \{x / x \in A \text{ and } x \notin B\}$

The set of all elements that belongs to B but not belongs to A is called the Complement of A relative to B and is denoted by $B-A$.



$\therefore B-A = \{x / x \in B \text{ and } x \in A\}$

→ $A-B$ and $B-A$ are disjoint sets.

→ If A and B are disjoint ⇒ $A-B = A$; $B-A = B$

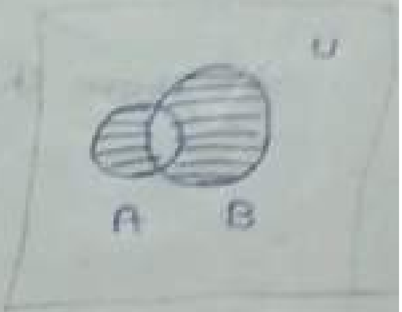
→ $A = (A \cup B) - (B-A)$; $B = (A \cup B) - (A-B)$

→ $A = (A \cap B) \cup (A-B)$; $B = (A \cap B) \cup (B-A)$.

Symmetr

Symmetric Difference:-

For twosets A and B, the relative complement of $A \cap B$ in $A \cup B$ is called the (Symmet.) Symmetric difference of A and B and is denoted by $A \Delta B$



$\therefore A \Delta r$

$\therefore A \Delta B = (A \cup B) - (A \cap B)$

$= (A - B) \cup (B - A)$

the laws of set theory:-

Commulative law: $A \cup B = B \cup A$; $A \cap B = B \cap A$

Assocative law: $A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

Distributive law: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Idempotent law: $A \cup A = A$; $A \cap A = A$

Identity law: $A \cup \phi = A$; $A \cap U = A$

Law of double Complement: $\bar{\bar{A}} = A$

Domination law:- $A \cup U = U$: $A \cap \phi = \phi$

Inverse law:- $A \cup \bar{A} = U$: $A \cap \bar{A} = \phi$

Demorgan laws:- $\overline{A \cup B} = \bar{A} \cap \bar{B}$: $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Absorption laws: $A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

→ prove that i, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii, $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$

iii, $\overline{A \cup B} = \bar{A} \cap \bar{B}$

iv, $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Sol: i, $A \cap (B \cup C) = \{x / x \in A \cap (B \cup C)\}$

$= \{x / x \in A \text{ and } (x \in B \cup C)\}$

$= \{x / x \in A \text{ and } (x \in B \text{ or } x \in C)\}$

$= \{x / x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)\}$

$= \{x / x \in (A \cap B) \text{ or } x \in (A \cap C)\}$

$= \{x / x \in (A \cap B) \cup (A \cap C)\}$

$= (A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

iv) $\overline{A \cap B} = \{x \mid x \in \overline{A \cap B}\}$

$= \{x \mid x \notin A \cap B\}$

$= \{x \mid x \notin A \text{ and } x \notin B\}$

$= \{x \mid x \in \bar{A} \text{ or } x \in \bar{B}\}$

$= \{x \mid x \in \bar{A} \cup \bar{B}\}$

$= \bar{A} \cup \bar{B}$

$\therefore \overline{A \cap B} = \bar{A} \cup \bar{B}$

cartesian product of sets:-

Let A and B two sets. then the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$ is called cartesian product or cross product or product set of A and B. It is denoted by $A \times B$.

$\therefore$ ~~AA~~ $\therefore A \times B = \{(a, b / a \in A \text{ and } b \in B\}$

$B \times A = \{(b, a / b \in B \text{ and } a \in A\}$

$A \times B \Rightarrow B \times A$

Ex: If $A = \{0,1,2\}$: $B = \{3,4,5\}$ Then find $A \times B$: $B \times A$

Sol: $A = \{0,1,2\}$: $B = \{3,4,5\}$

$A \times B = \{(a,b) / a \in A : b \in B\}$

$= \{(0,3), (0,4), (0,5), (1,3), (1,4), (1,5) (2,3), (2,4), (2,5)\}$

$B \times A = \{\cancel{(b \times} (b,a) / b \in B : a \in A\}$

$= \{(3,0), (3,1), (3,2), (4,0), (4,1), (4,2), (5,0), (5,1), (5,2)\}.$

→ $A \times B$ are finite sets with $n(A) = m$: $n(B) = n$

$\Rightarrow n(A \times B) = mn$: $n(B \times A) = mn$

$\Rightarrow n(A \times B) = n(A) \cdot n(B)$

The idea of cartesian product of sets ~~are~~ can be extended to any finite number of sets. For and non empty such $A_1, A_2, \dots A_k$ the k fold product $A_1 \times A_2 \times \dots \times A_k$ is defined as the set of all ordered k. triples $(a_1, a_2, \dots a_k)$ where $a_i \in A_i$, $i = 1,2 \dots k)$

$\therefore A_1 \times A_2 \times \cdots \times A_k = \{(a_1, a_2, \cdots a_k) /$

$a_i \in A_i, i = 1, 2 \cdots k\}$

$n(A_1 \times A_2 \times \cdots \times A_k) = n(A_1) \cdot n(A_2) \cdot n(A_3) \cdots n(A_k)$

Problem:

(P1) If $A = \{1, 3, 5\}$ $B = \{2, 3\}$ $C = \{4, 6\}$ Find the following

(1) $A \times B$ (2) $B \times A$ (3) $B \times C$ (4) $A \times C$ (5) $(A \cup B) \times C$

(6) $A \cup (B \times C)$ (7) $(A \times B) \cup C$ (8) $A \cap (B \times C)$

(9) $(A \times B) \cup (B \times C)$ (10) $(A \times B) \cap (B \times A)$

(11) $(A \times B) \cap (B \times C)$

Sol: Given
$A = \{1, 3, 5\}$ $B = \{2, 3\}$ $C = \{4, 6\}$

$A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$

$B \times A = \{(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$

$A \times C = \{(1,4), (1,6), (3,4), (3,6), (5,4), (5,6)\}$

$A \cup B = \{1, 2, 3, 5\}$

$(A \cup B) \times C = \{(1,4), (1,6), (2,4), (2,6), (3,4), (3,6), (5,4), (5,6)\}$

$B \times C = \{(2,4), (2,6), (3,4), (3,6)\}$

$A \cup (B \times C) = \{1, 3, 5, (2,4), (2,6), (3,4), (3,6)\}$

$(A \times B) \cup C = \{(1,2) (1,3) (3,2) (3,3) (5,2), (5,3), 4, 6\}$

$A \cap (B \times C) = \phi$

$(A \times B) \cup (B \times C) = \{(1,2) (1,3) \cancel{(3,2)}^{(3,2)} (3,3) (5,2)$
$(5,3) (2,4) (2,6) (3,4) (3,6)\}$

$(A \times B) \cap (B \times A) = \{(3,3)\}$

$(A \times B) \cap (B \times C) = \phi$.

Relations:-

Let A and B be two sets. then a subset of $A \times B$ is called a relation from A to B. so, if R is a relation from A to B, then R is a set of ordered pairs (a,b) where $a \in A : b \in B$.

Conversely if R is a set of order pair (a,b) where $a \in A$ and $b \in B$. then R is a relation from A to B.

If $(a,b) \in R \longleftrightarrow aRb$.

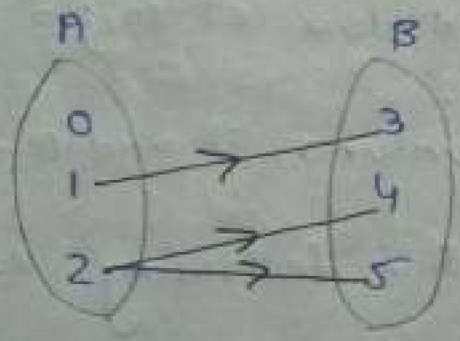
If R is a relation from A to A, that is R is a subset of $A \times A$ then R is called binary relation on A.

Ex: $A = \{0,1,2\}$ $B = \{3,4,5\}$

$R = \{(1,3)(2,4)(2,5)\}$

R is a subset of $A \times B$

The relation can be depicted in a diagram as shown below called the arrow diagram.



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Matrix of a Relation :-

Let $A = \{a_1, a_2, a_3, \ldots a_m\}$,
$B = \{b_1, b_2, b_3, \ldots b_n\}$ be any finite
sets of order m & n respectively.

- then $A \times B$ consists of mn ordered pairs
of the form (a_i, b_j)

Here $1 \leq i \leq m$ and $1 \leq j \leq n$.

- let R be any relation with the condition
$R \subseteq A \times B$. and also let $m_{ij} = (a_i, b_j)$; and
assigns the values 1 and 0 two m_{ij}
according to the following rule.

$$m_{ij} = \begin{cases} 1, & \text{if } (a_i, b_j) \in R \\ 0, & \text{if } (a_i, b_j) \notin R \end{cases}$$

- the main matrix followed by these m_{ij}'s
is called the matrix of a relation r.
which is denoted with M_R or $M(R)$

NOTE:- The rows of ~~M~~ M_R are corresponding to the elements of A and the columns are corresponding to the elements of B

when B = A Then the order of the matrix M_R is ~~nxn~~ m×m

Ex:-

(Pb) ~~Let~~ $A = \{0,1,2\}$, $B = \{p,q\}$

If $R = \{(0,p), (1,q), (2,p)\}$ then find M_R

Sol: Given Data

$A = \{0,1,2\}$ $B = \{p,q\}$

$R = \{(0,p), (1,q), (2,p)\}$.

$A \times B = \{(0,p), (0,q), (1,p), (1,q), (2,p), (2,q)\}$

we have

$$\cancel{m_{ij} = \begin{cases} 1, & \text{for } (a,b) \in R \\ 0, & \text{for } (a,b) \end{cases}}$$

$$m_{ij} = \begin{cases} 1, & \text{for } (a_i, b_j) \in R \\ 0, & \text{for } (a_i, b_j) \notin R \end{cases}$$

$m_{11} = 1$, Since $(0, p) \in R$

$m_{12} = 0$, Since $(0, q) \notin R$

$m_{21} = 0$, Since $(1, p) \notin A$

$m_{22} = 1$, Since $(1, q) \in R$

$m_{31} = 1$, Since $(2, p) \in R$

$m_{32} = 0$, Since $(2, q) \notin R$

matrix of a given relation is

$$M_R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}_{3\times 2}$$

H.W

Pb $A = \{1, 2, 3, 4\}$, $B = \{(1,2); (1,3), (2,4), (3,2)\}$

Find the M_R (Relation is A to A)

Hint:

Sol: Given data

$A \times A = \{(1,1) (1,2) (1,3) (1,4) (2,1) (2,2)$
$(2,3) (2,4) (3,1) (3,2) (3,3)$
$, (3,4) (4,1) (4,2) (4,3) (4,4)\}$

we have

$$m_{ij} = \begin{cases} 1, & \text{for } (a_i, b_j) \in R \quad \cancel{(a,b) \in R} \\ 0, & \text{for } (a_i, b_j) \notin R \end{cases}$$

$m_{11} = 0$ since $(1,1) \notin R$

$m_{12} = 1$ since $(1,2) \in R$

$m_{13} = 1$ since $(1,3) \in R$

$m_{14} = 0$ since $(1,4) \notin R$

$m_{21} = 0$ since $(2,1) \notin R$

$m_{22} = 0$ since $(2,2) \notin R$

$m_{23} = 0$ since $(2,3) \notin R$

$m_{24} = 1$ since $(2,4) \in R$

$m_{31} = 0$ since $(3,1) \notin R$

$m_{32} = 1$ since $(3,2) \overset{\in}{\cancel{\notin}} R$

$m_{33} = 0$ since $(3,3) \notin R$

$m_{34} = 0$ since $(3,4) \notin R$

$m_{41} = 0$ since $(4,1) \notin R$

$m_{42} = 0$ since $(4,2) \notin R$

$m_{43} = 0$ since $(4,3) \notin R$

$m_{44} = 0$ since $(4,4) \notin R$

matrix of a given relation is

$$m_R \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{4\times4}$$

26/11/21

Digraph:-

- Let R be a binary relation on a set of A.

Then R can be represented petriotically as

describe as follows.

~~Document~~ Draw the ~~[illegible]~~

- Draw a small circle each element of A

with corresponding a is circle are called

~~[illegible]~~ ~~varies~~ or nodes.

vertices

Draw an arrow edges from a verties 'x'

to 'y', if and only if the ordered pair $(x,y) \in R$

the pictorial represenation of R is called digraph or directed graph. ~~or digraph~~

- If a relation pictorially represented by digraph, a verties From which an edge leaves is called sources or horizon.

- A vertices where the edge ends is called terminal for that edge. a vertices which is neither source nor ~~source not~~ terminal is called An Isolated Vertex

- An Edge for which the Source and terminal are same is called a loop. the No. of Edges terminating at a vertex is called the indegree of that vertex and the no. of Edges leaving a vertex is called the outdegree of that vector.

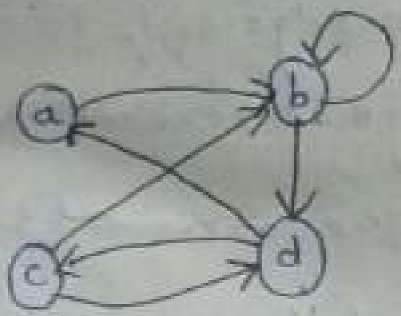
$A \times A = \{(a,b)\ (b,c)\ (a,c)\ (a,d)\ (b,d)\ (b,a), (c,a)$
$(c,b)\ (c,d)\ (d,a)\ (d,b)\ (d,c)\ (a,a)\ (b,b)$
$(c,c)\ (d,d)\}$

(Pb) If $A = \{a,b,c,d\}$ and $R = \{(a,b),(b,b),(b,d),$
$(c,b),(c,d),(d,a)$
$(d,c)\}$

then draw the digraph 'R'?

Sol: Given data $A = \{a,b,c,d\}$ and

$R = \{(a,b),(b,b),(b,d),(c,b),(c,d),(d,a),$
$(d,c)\}$



vertex	Indegree	outdegree
a	1	1
b	3	2
c	1	2
d	2	2

NOTE:- If $n(A) = m$ & $n(B) = n$ then the number of relation from A to B is 2^{mn} (or) $2^{n(A \times B)}$

(Pb) Let A and B are two finite sets with $n(B)=3$. if there are 4096 relations from A to B. what is $n(A)=$?

Sol: Let $n(A)=m$ Given $n(B)=n=3$

The no. of relations from A to B $=2^{mn}=4096$

$\Rightarrow 2^{3m}=4096$

$\Rightarrow 3m\log 2=\log 4096$

$\Rightarrow 3m\log 2=\log 2^{12}$

$\Rightarrow 3m\log 2=12\log 2$

$\Rightarrow 3m=12$

$m=4$

$n(A)=4$

(Pb) Let $A=\{1,2\}$, $B=\{p,q,r,s\}$ and

$R=\{(1,q),(1,r),(2,p),(2,q),(2,s)\}$

write down M_R ?

Sol: $A \times B = \{(1,p)\ (1,q)\ (1,r)\ (1,s)\ (2,p),$
$(2,q)\ (2,r)\ (2,s)\}$

$m_{11} = 0$ since $(1,p) \notin R$

$m_{12} = 1$ since $(1,q) \in R$

$m_{13} = 1$ since $(1,r) \in R$

$m_{14} = 0$ since $(1,s) \notin R$

$m_{21} = 1$ since $(2,p) \in R$

$m_{22} = 1$ since $(2,q) \in R$

$m_{23} = 0$ since $(2,r) \notin R$

$m_{24} = 1$ since $(2,s) \in R$

matrix of given relation is

$$M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}_{2\times 4}$$

(Pb) Let $A = \{1, 2, 3, 4\}$ and R be a relation on A, defined by xRy if and only if 'x divides y'

i.e, $\frac{x}{y}$

i, write R as set of ordered pairs

ii, Draw the digraph of R

iii, Determine the indegree and outdegree in ~~all vertex of R?~~ of all vertices in graph

Sol: Let $A = \{1, 2, 3, 4\}$

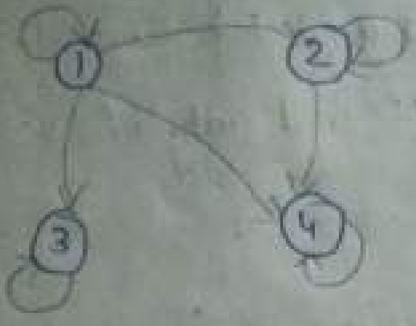
Given Relation xRy iff 'x divides y'

$$A \times A = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (3,1)\ (3,2)\ (3,3)\ (3,4)\ (4,1)\ (4,2)\ (4,3)\ (4,4)\}$$

i, Then the relation is

$$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$$

ii,



iii,

vertex	Indegree	outdegree
1	1	4
2	2	2
3	2	1
4	3	1

(Pb) Determine the relation R from the set A to set B as described by the following matrix $M_R = \left[\begin{matrix}1&0&1\\1&1&0\\0&0&1\\1&0&0\end{matrix}\right]$

Sol: Given

$$M_R = \begin{bmatrix}1&0&1\\1&1&0\\0&0&1\\1&0&0\end{bmatrix}_{4\times3}$$

let $A = \{a_1, a_2, a_3, a_4\}$

$B = \{b_1, b_2, b_3\}$

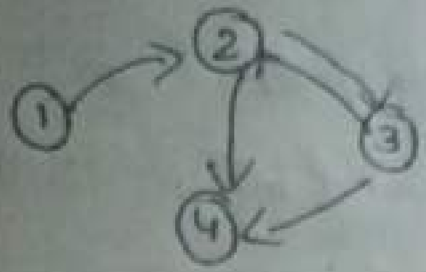
$$M_R = \begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline a_1 & 1 & 0 & 1 \\ a_2 & 1 & 1 & 0 \\ a_3 & 0 & 0 & 1 \\ a_4 & 1 & 0 & 0 \end{array}$$

~~$R = \{(a_1,b_1), (a_2,b_2), (a_3.$~~

$R = \{(a_1,b_1), (a_1,b_3), (a_2,b_1), (a_2,b_2),$

$(a_3,b_3), (a_4,b_1)\}$

(Pb) If the relation R determine by each of the digraph given below, also write down the matrix of the relation



Sol. $R = \{(1,2)\ (2,3)\ (2,4)\ ,(3,2)\ (3,4)\}$

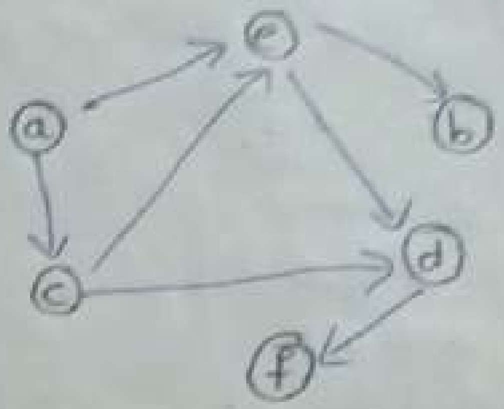
$M_R =$

$A \times A = \{(1,1)\ (1,2)\ (1,3)\ (1,4)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\}$

$m_{11}=0$	$m_{21}=0$	$m_{31}=0$	$m_{41}=0$
$m_{12}=1$	$m_{22}=0$	$m_{32}=1$	$m_{42}=0$
$m_{13}=0$	$m_{23}=1$	$m_{33}=0$	$m_{43}=0$
$m_{14}=0$	$m_{24}=1$	$m_{34}=1$	$m_{44}=0$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{4 \times 4}$$

(Pb)



Sol: $R = \{(a,e)\ (e,b)\ (a,c)\ (c,e)\ (e,d)\ (d,f)\ (c,d)\}$

$A = \{a, b, c, d, e, f\}$

$A \times A = \{(a,a)\ (a,b)\ (a,c)\ (a,d)\ (a,e)\ (a,f)$
$(b,a)\ (b,b)\ (b,c)\ (b,d)\ (b,e)\ (b,f)$
$(c,a)\ (c,b)\ (c,c)\ (c,d)\ (c,e)\ (c,f)$
$(d,a)\ (d,b)\ (d,c)\ (d,d)\ (d,e)\ (d,f)$
$(e,a)\ (e,b)\ (e,c)\ (e,d)\ (e,e)\ (e,f)$
$(f,a)\ (f,b)\ (f,c)\ (f,d)\ (f,e)\ (f,f)\}$

$m_{11}=0$	$m_{23}=0$	$m_{35}=1$	$m_{51}=0$	$m_{63}=0$
$m_{12}=0$	$m_{24}=0$	$m_{36}=0$	$m_{52}=1$	$m_{64}=0$
$m_{13}=1$	$m_{25}=0$	$m_{41}=0$	$m_{53}=0$	$m_{65}=0$
$m_{14}=0$	$m_{26}=0$	$m_{42}=0$	$m_{54}=1$	$m_{66}=0$
$m_{15}=1$	$m_{31}=0$	$m_{43}=0$	$m_{55}=0$	
$m_{16}=0$	$m_{32}=0$	$m_{44}=0$	$m_{56}=0$	
$m_{21}=0$	$m_{33}=0$	$m_{45}=0$	$m_{61}=0$	
$m_{22}=0$	$m_{34}=1$	$m_{46}=1$	$m_{62}=0$	

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$$M_R = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{6 \times 6}$$

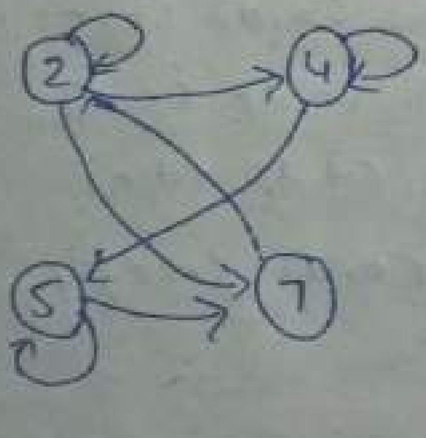
(Pb) Let $A = \{2,4,5,7\}$ and R be a relation on A

having the matrix $M_R = \begin{array}{c|c} & \begin{matrix} 2 & 4 & 5 & 7 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$.

construct the digraph.

Sol: $A \times A = \{(2,4)\ (2,5)\ (2,2)\ (2,7)$
$(4,2)\ (4,4)\ (4,5)\ (4,7)$
$(5,2)\ (5,4)\ (5,5)\ (5,7)$
$(7,2)\ (7,4)\ (7,5)\ (7,7)\}$

$R = \{(2,2)\ (2,4)\ (2,7),\ (4,4)(4,5)\ (5,5)$
$(5,7)\ (7,2)\}$



vertices	Indegree	outdegree
2	2	3
4	2	2
5	2	2
7	2	1

(P) Find the Relation R on a set A and write down its digraph.

Given $A = \begin{bmatrix} a & b & c & d & e \end{bmatrix}$

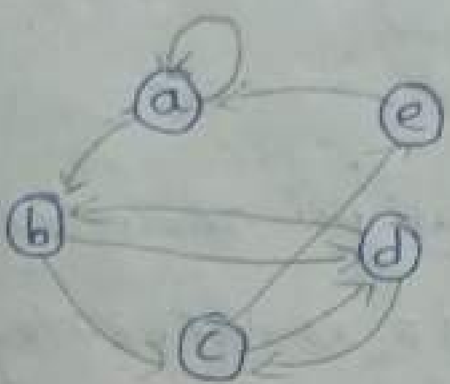
Sol: Given $A = \{a, b, c, d, e\}$ and the matrix of R is

$$M_R = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 1 & 1 & 0 & 0 & 0 \\ b & 0 & 0 & 1 & 1 & 0 \\ c & 0 & 0 & 0 & 1 & 1 \\ d & 0 & 1 & 1 & 0 & 0 \\ e & 1 & 0 & 0 & 0 & 0 \end{array}$$

$A \times A = \{ (a,a)\ (a,b)\ (a,c)\ (a,d)\ (a,e)$
$(b,a)\ (b,b)\ (b,c)\ (b,d)\ (b,e)$
$(c,a)\ (c,b)\ (c,c)\ (c,d)\ (c,e)$
$(d,a)\ (d,b)\ (d,c)\ (d,d)\ (d,e)$
$(e,a)\ (e,b)\ (e,c)\ (e,d)\ (e,e) \}$

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$R = \{ (a,a)\ (a,b)\ (b,c)\ (b,d)\ (c,d)\ (c,e)$
$(d,b)\ (d,c)\ (e,a) \}$



vertex	indegree	outdegree
a	2	2
b	2	2
c	2	2
d	2	2
e	1	1

Operations on Relations:

1. Union of Relations:- Let R_1, R_2 be two relations from a set A to a set B.

- The union of R_1 and R_2 is a relation from A to B with the property $(a,b) \in R_1 \cup R_2$ iff $(a,b) \in R_1$ (or) $(a,b) \in R_2$

2. Intersection of Relations:- The intersection of the Relations R_1 and R_2 is defined by a relation from ~~a to b~~ A to B with the property $(a,b) \in R_1 \cap R_2$ iff $(a,b) \in R_1$ and $(a,b) \in R_2$.

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3. Complement of a Relation:

Given a Relation R from a set A to a set B then the compliment of R is denoted with $\bar{R}$. is defined a relation from A to B with the property $(a,b) \in \bar{R}$ iff $(a,b) \notin R$

4. Inverse of a Relation: Given a Relation R from a set A to a set B then the inverse of R is denoted with R^{-1}. is a relation from B to A with the property $(a,b) \in R^{-1}$ iff $(b,a) \in R$

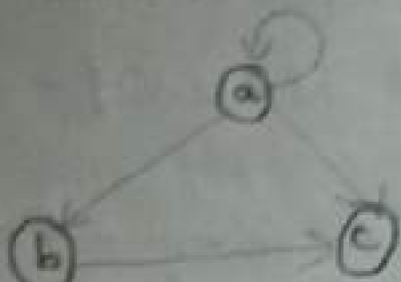
~~(or) $(b,a) \in R^{-1}$ iff $(a,b) \in R$.~~

other name: Inverse of a Relation is also called Converse of a Relation.

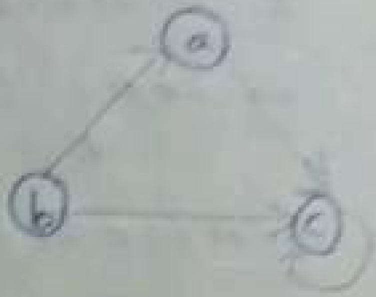
Imp NOTE:- 1. If R is a Relation from A to B then the set of universe is $A \times B$.

2. If M_R is a matrix of a Relation R then $(M_R)^T$ is a matrix of the Relation of R^{-1}.

(16) The digraph of two relations R and S on the set $A = \{a,b,c\}$ are given below. Draw the digraphs of $\bar{R}$, $R \cup S$, $R \cap S$, R^{-1}, S^{-1}!



R



S

Sol: Given $A = \{a,b,c\}$

$A \times A = \{(a,a)(a,b)(a,c),(b,a)(b,b),(b,c)(c,a)(c,b)(c,c)\}$

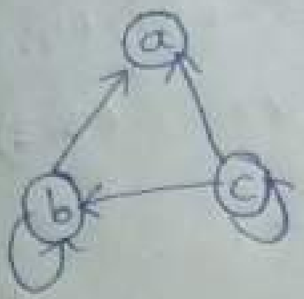
From the given diagraphs

$R = \{(a,a),(a,b),(a,c),(b,c)\}$

$S = \{(a,c),(b,a),(b,c),(c,c)\}$

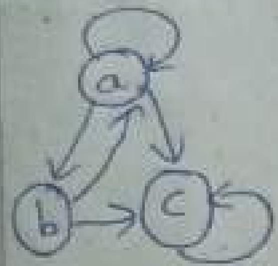
$\bar{R} = (A \times A) - R$

$\bar{R} = \{(b,a)(b,b)(c,a)(c,b).(c,c)\}$



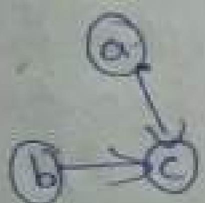
vertex	Indegree	outdegree
a	2	0
b	2	2
c	1	3

$R \cup S = \{(a,a), (a,b), (a,c) (b,a) (b,c) (c,c)\}$



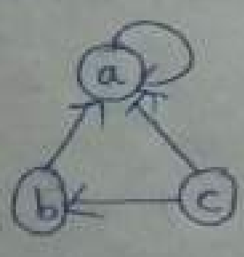
vertex	Indegree	outdegree
a	2	3
b	1	2
c	3	1

$R \cap S = \{(a,c) (b,c)\}$



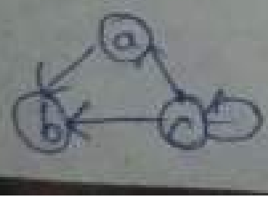
vertex	Indegree	outdegree
a	0	1
b	0	1
c	2	0

$R^{-1} = \{(a,a) (b,a) (c,a) (c,b)\}$



vertex	Indegree	outdegree
a	3	1
b	1	1
c	0	2

$S^{-1} = \{(c,a) (a,b) \cdot (c,b) (c,c)\}$



vertex	Indegree	outdegree
a	1	1
b	2	0
c	1	3

(Pb) Let $A = B = \{1,2,3\}$ and $R = \{(1,1)\ (1,2)\ (2,3)\ (3,1)\}$ and $S = \{(2,1)\ (3,1)\ (3,2)\ (3,3)\}$

Then compute $\bar{R}$, $R \cup S$, S^{-1}

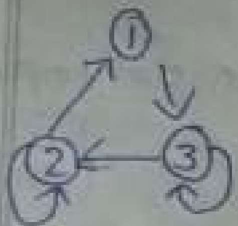
Sol: Given $A = \{1,2,3\}$ $B = \{1,2,3\}$

$A \times B = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$

$R = \{(1,1), (1,2), (2,3), (3,1)\}$

$S = \{(2,1), (3,1), (3,2), (3,3)\}$

$\bar{R} = \{(1,3), (2,1), (2,2), (3,2), (3,3)\}$



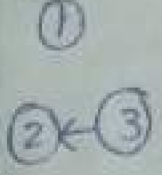
vertex	Indegree	outdegree
1	1	1
2	2	2
3	2	1

$R \cup S = \{(1,1), (1,2), (2,1), (2,3), (3,1), (3,2), (3,3)\}$

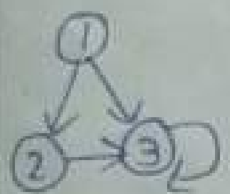
vertex	Indegree	outdegree
1	3	2
2	2	2
3	2	2

$R \cap S = \{(3,2)\}$

vertex	Indegree	outdegree
1	0	0
2	1	0
3	0	1



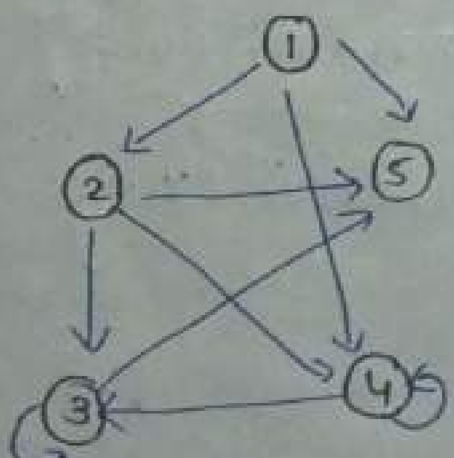
$\rho^{-1} = \{(1,2), (1,3), (2,3), (3,3)\}$



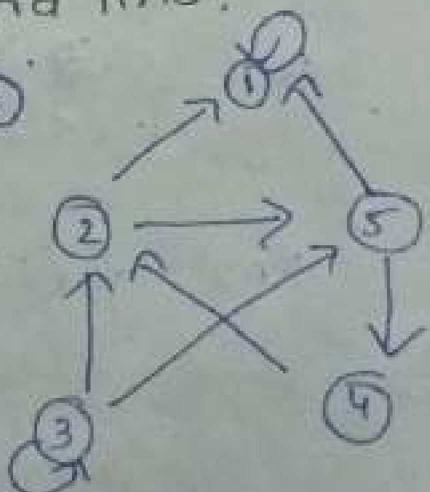
vertex	Indegree	outdegree
1	0	2
2	1	1
3	3	1

(16) Let $A = \{1,2,3,4,5\}$ and R, S are relations on A whos corresponding digraphs are given below. Find $\bar{R}$, R^{-1}, and $R \cap S$.

R



S



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Sol: Given data = $\{1,2,3,4,5\}$

$$A \times A = \left\{ \begin{array}{l} (1,1)(1,2)(1,3)(1,4)(1,5) \\ (2,1)(2,2)(2,3)(2,4)(2,5) \\ (3,1)(3,2)(3,3)(3,4)(3,5) \\ (4,1)(4,2)(4,3)(4,4)(4,5) \\ (5,1)(5,2)(5,3)(5,4)(5,5) \end{array} \right\}$$

from given digraph

$$R = \{(1,2)(1,4)(1,5)(2,3)(2,4)(2,5),(3,3)$$
$$(3,5)(4,3)(4,4)\}$$

$$S = \{(4,1)(1,2)(2,5),(3,2),(3,3),(3,5),(4,2),$$
$$(5,1),(5,4)\}$$

$$\bar{R} = \left\{ \begin{array}{l} (1,1)(1,3)(2,1)(2,2)(3,1)(3,2)(3,4)(4,1) \\ (4,2)(4,5)(5,1)(5,2)(5,3)(5,4)(5,5) \end{array} \right\}$$

$$R^{-1} = \left\{ \begin{array}{l} (2,1)(4,1)(5,1)(3,2)(4,2) \\ (5,2)(3,3)(5,3)(3,4)(4,4) \end{array} \right\}$$

$$R \cap S = \{(1,2),(2,5),(3,3),(3,5)\}$$

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Composition of relations:-

Consider a relation R from a set A to a set B and S be a relation from the set B to set C. Define a new relation called the product (or) composition of R and S, from the set A to set C. This new relation is denoted with ROS and it defines as follow:

$$ROS = \{(a,c) \mid \exists b \in B \Rightarrow (a,b) \in R, (b,c) \in S\}$$

NOTE: 1: If R is a relation on a set A then we can define the composition of R within itself is a relation from A to A and it can be written as ROR (or) R^2. Similarly Ro(ROR) (or) R^3.

2. Let R be a relation from a set $A = \{a_1, a_2, a_3, \ldots, a_m\}$ to a set $B = \{b_1, b_2, b_3, \ldots, b_n\}$ and S be relation from the set B to a set $C = \{c_1, c_2, c_3, \ldots c_p\}$ then the matrices of the relations R, S & ROS satisfies the following identity

$$M(R) \times M(S) = M(ROS)$$

(or)

$$M_R \times M_S = M_{ROS}$$

3. Let A, B, C, D be 4 sets. R be a relation from A to B, S be a relation from B to C, T be a relation from C to D then $R \circ (S \circ T) = (R \circ S) \circ T$.

(Pb) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,1), (1,3), (3,2), (3,4), (4,2)\}$ and $S = \{(2,1), (3,3), (3,4), (4,1)\}$ be relations on set A. Find ROS, SOR, ROR, SOS

Sol: Given, $R = \{(1,1), (1,3), (3,2), (3,4), (4,2)\}$

$A = \{1, 2, 3, 4\}$

$S = \{(2,1), (3,3), (3,4), (4,1)\}$

$R \circ S = \{(1,3), (1,4), (3,1), (4,1)\}$

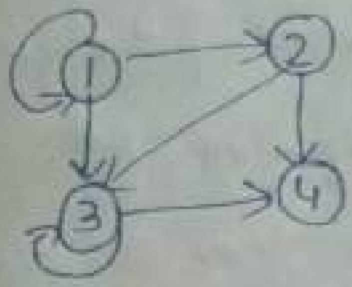
$S \circ R = \{(2,1), (2,3), (3,2), (3,4), (4,1), (4,3)\}$

$R \circ R = \{(1,1), (1,3), (1,2), (1,4), (3,2)\}$

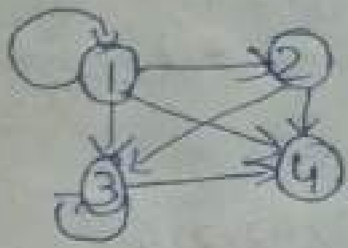
$S \circ S = \{(3,4), (3,1), (3,3)\}$

(Pb) For $A = \{1, 2, 3, 4\}$ and the relation $R = \{(1,1), (1,2), (2,3), (3,3), (3,4)\}$ is a relation on A. Find R^3, R^4, R^2 and draw the digraphs of above relation

Sol $R^2 = RoR = \{(1,2), (1,1), (1,3), (2,3), (2,4)$
$(3,3)\ (3,4)\}$

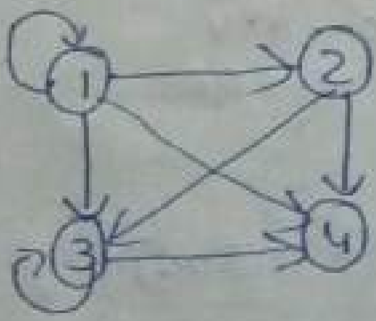


$R^3 = Ro(RoR) = \{(1,1), (1,2), (1,3), (1,4), (2,3),$
$(2,4), (3,3)\ (3,4)\}$



$R^4 = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4),$

$(3,3), (3,4)\}$



3) Let R be a relation on set $A = \{1,2,3,4\}$. Given $R = \{(1,2),(1,3),(2,4),(3,2)\}$. write down the relation matrix M_R Compute $M(R)^2$ and hence draw the diagraph of R^2

Sol: Given $A = \{1,2,3,4\}$

$A \times A = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$

$R = \{(1,2),(1,3),(2,4),(3,2)\}$

$$M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{4 \times 4}$$

By known result,

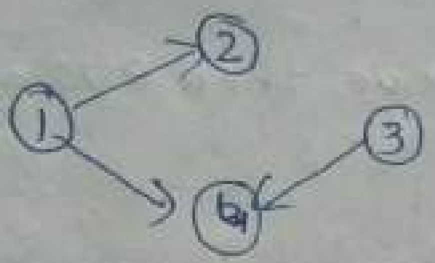
$$\boxed{\therefore\ M(R) \times M(S) = M(R \circ S)}$$

$$M(R) \times M(R) = M(R^2)$$

$$M_{R^2} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$R^2 = \{(1,2)\ (1,4)\ (3,4)\}$



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Properties of Relations.

1. Reflexive Relation: A binary Relation R on a set A is called reflexive Relation. If ~~$(x,x) \in$~~ $(a,a) \in R$ for all $a \in A$.

NOTE:- If R is reflexive then R^{-1} is also reflexive.

- If R is reflexive then $\bar{R}$ is not reflexive.
- If R & S reflexive then $R \cup S$ and $R \cap S$ are also reflexive.

2. IrReflexive Relation: A binary Relation R on a set A is called Irreflexive Relation. If $(a,a) \notin R$ for some $a \in A$.

3. Symmetric Relation: - A binary Relation R on a set A is called Symmetric. If $(a,b) \in R \Rightarrow (b,a) \in R$, for every ordered pai[r] of R

NOTE: If R is Symmetric then R^{-1} is also Symmetric.

If R and S are Symmetric then $R \cap S$ is Symmetric.

4. Anti-Symmetric Relation: A binary Relation R on a set A is Called Anti-Symmetric Relation. If $(a,b) \in R, (b,a) \in R$ $\Rightarrow a = b$, for every ordered pair of R.

5. Compatibility Relation: A binary Relation R on a set A is called compatibility Relation. If it satisfies reflexive and Symmetric Conditions.

6. Transitive Relation:- A binary Relation R on set A is called Transitive Relation. If $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$ for every ordered pair in R.

(Pb) Let $A = \{1,2,3\}$. Determine the nature of the following relations on A.

Sol: i, $R_1 = \{(1,2), (2,1), (1,3), (3,1)\}$

H.W
ii, $R_2 = \{(1,1), (2,2), (3,3), (2,3)\}$

H.W
iii, $R_3 = \{(1,1), (2,2), (3,3)\}$

H.W
iv, $R_4 = \{(1,1), (2,2), (3,3), (2,3), (3,2)\}$

H.W
v, $R_5 = \{(1,1), (2,3), (3,3)\}$

Sol: i, Let $A = \{1,2,3\}$

$R_1 = \{(1,2), (2,1), (1,3), (3,1)\}$

$(1,1) \notin R_1$

$(2,2) \notin R_1$ $\quad (a,a) \notin R$ for some $a \in A$

$(3,3) \notin R_1$

$\therefore$ R_1 is not reflexive

ii, $(1,2) \in R \Rightarrow (2,1) \in R$

$(1,3) \in R \Rightarrow (3,1) \in R$

For every ordered Pair of R_1

If $(a,b) \in R_1 \Rightarrow (b,a) \in R$

$\therefore R_1$ is Symmetric relation

$(1,2), (2,1) \in R_1$ but $(1,1) \notin R_1$

$(2,1), (1,3) \in R_1$ but $(2,3) \notin R_1$

for some ordered pair in R_1

If $(a,b), (b,c) \in R_1 \Rightarrow (a,c) \notin R_1$

$\therefore R_1$ is not transtitive relation

Hence R_1 is not reflexive and Not transitive But R_1 is Symmetric.

(ii) $R_2 = \{(1,1)\ (2,2)\ (3,3)\ (2,3)\}$

(i) $(1,1) \in R$

$(2,2) \in R$

$(3,3) \in R$

$(a,a) \in R$ for all $a \in A$

$\boxed{\therefore R_2 \text{ is reflexive}}$

(ii) $(1,1) \in R \Rightarrow (1,1) \in R$

$(2,2) \in R \Rightarrow (2,2) \in R$

$(2,3) \in R \Rightarrow (3,2) \notin R$

for some ordered pair of R_2

If $(a,b) \in R_2 \Rightarrow (b,a) \notin R_2$

$\therefore R_2$ is not symmetric relation

(ii) $(1,1)\ (1,1) \in R_2 \Rightarrow (1,1) \in R_2$

$(2,2)\ (2,2) \in R_2 \Rightarrow (2,2) \in R_2$

$(2,2)\ (2,3) \in R_2 \Rightarrow (2,3) \in R_2$

$(3,3)\ (3,3) \in R_2 \Rightarrow (3,3) \in R_2$

$(2,3)\ (3,3) \in R_2 \Rightarrow (2,3) \in R_2$

for every ordered pair in R_2

If $(a,b)\ (b,c) \in R_2 \Rightarrow (a,c) \in R_2$

$\boxed{\therefore R_2 \text{ is transitive relation}}$

Hence R_2 is reflexive and transitive but ~~not~~

R_2 is not Symmetric

(iii) $R_3 = \{(1,1)\ (2,2)\ (3,3)\}$

Sol: $\Rightarrow (1,1) \in R$

$(2,2) \in R$

$(3,3) \in R$

$(a,a) \in R$ for all $a \in A$

$\boxed{\therefore R_3 \text{ is reflexive}}$

$\Rightarrow (1,1) \in R \Rightarrow (1,1) \in R$

$(2,2) \in R \Rightarrow (2,2) \in R$

$(3,3) \in R \Rightarrow (3,3) \in R$

for every ordered pair of R_3

If $(a,b) \in R_3 \Rightarrow (b,a) \in R_3$

$\boxed{\therefore R_3 \text{ is Symmetric relation}}$

$\Rightarrow (1,1)(1,1) \in R_3 \Rightarrow (1,1) \in R_3$
$(2,2)(2,2) \in R_3 \Rightarrow (2,2) \in R_3$
$(3,3)(3,3) \in R_3 \Rightarrow (3,3) \in R_3$

for every ordered pair in R_3
If $(a,b)(b,c) \in R_3 \Rightarrow (a,c) \in R_3$

$\therefore R_3$ is transitive relation

$\therefore$ Here R is reflexive, symmetric and transitive

iv) $R_4 = \{(1,1)(2,2)(3,3)(2,3)(3,2)\}$

$\Rightarrow (1,1) \in R_4$
$(2,2) \in R_4$
$(3,3) \in R_4$
$(a,a) \in R$ for all $a \in A$

$\therefore R_1$ is reflexive relation

$\Rightarrow (1,1) \in R_4 \Rightarrow (1,1) \in R_4$
$(2,2) \in R_4 \Rightarrow (2,2) \in R_4$
$(3,3) \in R_4 \Rightarrow (3,3) \in R_4$
$(2,3) \in R_4 \Rightarrow (3,2) \in R_4$
$(3,2) \in R_4 \Rightarrow (2,3) \in R_4$

for every ordered pair of R_4
If $(a,b) \in R_4 \Rightarrow (b,a) \in R_4$

$\therefore R_4$ is Symmetric relation

$\Rightarrow (2,2)(2,3) \in R_4 \Rightarrow (2,3) \in R_4$
$(2,2)(2,2) \in R_4 \Rightarrow (2,2) \in R_4$
$(3,3)(3,2) \in R_4 \Rightarrow (3,2) \in R_4$
$(2,3)(3,3) \in R_4 \Rightarrow (2,3) \in R_4$
$(2,3)(3,2) \in R_4 \Rightarrow (2,2) \in R_4$
$(3,2)(2,3) \in R_4 \Rightarrow (3,3) \in R_4$
$(1,1)(1,1) \in R_4 \Rightarrow (1,1) \in R_4$

for every orderd pair
If $(a,b)(b,c) \in R_4$
$\Rightarrow (a,c) \in R_4$

$\therefore R_4$ is transitive relation

Here R_4 is reflexive, Symmetric, transitive

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(v) $R_5 = \{(1,1), (2,3), (3,3)\}$

$\exists$ $(2,2) \notin R_5$

$(a,a) \notin R$ for some $a \in A$

$\boxed{\therefore R_5 \text{ is not reflexive}}$

$\Rightarrow (1,1) \in R_5 \Rightarrow (1,1) \in R_5$

$(2,3) \in R_5 \Rightarrow (3,2) \in R_5$

for some ordered pair of R_5

If $(a,b) \in R_5 \Rightarrow (b,a) \in R_5$

$\boxed{\therefore R_5 \text{ is Symmetric relation}}$

$\Rightarrow (1,1)(1,1) \in R_5 \ (1,1) \in R_5$

$\Rightarrow (2,3)(3,3) \in R_5 \Rightarrow (2,3) \in R_5$

$\Rightarrow (3,3)(3,3) \in R_5 \Rightarrow (3,3) \in R_5$

For every ordered pair in R_5

If $(a,b)\ (b,c) \in R_5 \Rightarrow (a,c) \in R_5$

$\boxed{\therefore R_5 \text{ is transitive Relation}}$

Hence R_5 is not reflexive and not Symmetric

$\Rightarrow R_5$ is transitive relation.

(16) Find the Nature of the relation represented by the following matrices

H.W i, $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ H.W ii, $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

H.W iii, $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ iv, $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Sol iv, $M_R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Assume $A = \{a_1, a_2, a_3, a_4\}$

$$M_R = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} \begin{array}{c} \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$R = \{(a_1, a_3), (a_1, a_4), (a_2, a_3), (a_3, a_4),$

$(a_4, a_1)\}$

$(a_1, a_1) \notin R$

for some $a_1 \in A$

$(a_1, a_1) \notin R$

$\therefore$ a is not reflexive//

$(a_1, a_3) \in R$

But $(a_3, a_1) \notin R$

for some ordered pair of R

if $(a, b) \in R$ but $(b, a) \notin R$.

$\therefore$ R is not symmetric relation//

~~Ca~~ $(a_2, a_3), (a_3, a_4) \in R$ but $(a_2, a_4) \notin R$

(a_3, a_4) $(a_4, a_1) \in R$ but $(a_3, a_1) \notin R$

For some ordered pair in R

If $(a, b), (b, c) \in R \Rightarrow (a, c) \notin R$

$\therefore$ R is not transitive relation//

i,
$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Given $M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Assume $A = \{1, 2, 3, 4\}$

$$M_R = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$\Rightarrow R = \{(1,2)\ (1,3)\ (2,1)\ (2,2)\ (3,1)\ (3,3)\ (3,4)\ (4,3)\ (4,4)\}$

eflexive $\Rightarrow (1,1) \notin R$

If $(a,a) \notin R$, for some $a \in A$

$\boxed{\therefore \text{R is not reflexive}}$

ymmetric: $(1,2) \in R \Rightarrow (2,1) \in R$

$(1,3) \in R \Rightarrow (3,1) \in R$

$(2,2) \in R \Rightarrow (2,2) \in R$

$(3,3) \in R \Rightarrow (3,3) \in R$

$(3,4) \in R \Rightarrow (4,3) \in R$

for every ordered pairs in R

$\Rightarrow$ If $(a,b) \in R \Rightarrow (b,a) \in R$

$\boxed{\therefore \text{R is Symmetric}}$

Transitive: $(1,2)(2,1) \in R$ but $(1,1) \notin R$

for some ordered pair in R

If $(a,b)(b,c) \in R$ but $(a,c) \notin R$

$\boxed{\therefore \text{R is not transitive}}$

Hence R is not reflexive and transitive but R is Symmetric

(iii) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Assume $A = \{a_1, a_2, a_3\}$

$$M_R = \begin{array}{c|ccc} & a_1 & a_2 & a_3 \\ \hline a_1 & 1 & 1 & 1 \\ a_2 & 0 & 0 & 1 \\ a_3 & 0 & 0 & 1 \end{array}$$

$R = \{(a_1,a_1)\ (a_1,a_2)\ (a_1,a_3)\ (a_2,a_3)\ (a_3,a_3)\}$

$(a_2,a_2) \notin R$

for some $a_2 \in A$

$(a_2,a_2) \notin R$

$\therefore$ R is not reflexive

$(a_1,a_3) \in R$ but $(a_3,a_1) \notin R$

for some ordered pair of R

If $(a,b) \in R$ but $(b,a) \notin R$

$\therefore$ R is not Symmetric

$(a_1,a_1)\ (a_1,a_2) \in R \Rightarrow (a_1\ a_2) \in R$

$(a_1,a_1)\ (a_1,a_3) \in R \Rightarrow (a_1, a_3) \in R$

$(a_1,a_2)\ (a_2 a_3) \in R \Rightarrow (a_1, a_3) \in R$

$(a_3\ a_3)\ (a_3\ a_3) \in R \Rightarrow (a_3, a_3) \in R$

$(a_2\ a_3)(a_3\ a_3) \in R \Rightarrow (a_2\ a_3) \in R$

for every ordered pair in R

If $(a,b)\ (b,c) \in R \Rightarrow (a,c) \in A$

$\therefore$ R is transitive relation.

ii. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Assume $A = \{1, 2, 3, 4\}$

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 0 & 1 \end{array}$$

$R = \{(1,1)\ (1,3)\ (2,2)\ (2,4)\ (3,1)\ (3,3)\ (4,2)\ (4,4)\}$

$(1,1) \in R$

$(2,2) \in R$

$(3,3) \in R$

$(4,4) \in R$ for all $(a,a) \in R$

$\therefore$ R is reflexive

$(2,4) \in R \Rightarrow (4,2) \in R$

$(1,3) \in R \Rightarrow (3,1) \in R$

for all $(a,b) \in R \Rightarrow (b,a) \in R$

$\therefore$ R is Symmetric

$(4,2)\ (4,4) \in R \Rightarrow (4,4) \in R$

$(3,1)\ (3,3) \in R \Rightarrow (3,3) \in R$

$(1,1)\ (1,3) \in R \Rightarrow (1,3) \in R$

$(2,2)\ (2,4) \in R \Rightarrow (2,4) \in R$

$(2,4)\ (2,2) \in R \Rightarrow (2,2) \in R$

for all ordered pair in R

$(a,b)\ (b,c) \in R\ \ (a,c) \in R$

$\therefore$ R is transitive relation

(Pb) All the set positive integers Z^+, a relation R is defined by $aRb \Leftrightarrow$ a divides b (Exactly)

Find the nature of relation R.

Sol: Given that the set $A = Z^+$

and $aRb \Leftrightarrow$ a divides b

reflexive:

let $a \in Z^+$

$\Rightarrow a|a$ (or) a divides a

$\Rightarrow (a,a) \in R$

$\therefore$ for every $a \in Z^+, (a,a) \in R$

Hence R is reflexive

Symmetric:

Let $(a,b) \in R$ $\quad a,b \in Z^+$

$\Rightarrow a|b$ $\quad a,b \in Z^+$, $a \neq b$

$\Rightarrow$ b is multiple of a

$\Rightarrow b \nmid a$ $\quad a,b \in Z^+, a \neq b$

$\Rightarrow (b,a) \notin R$

$\therefore$ for some $(a,b) \in R \Rightarrow (b,a) \notin R$

Hence R is not Symmetric

Transitive:

let $(a,b) \in R$

& $(b,c) \in R$

$\Rightarrow a|b,\ b|c,\ a,b,c \in \mathbb{Z}^+$

$\Rightarrow$ b is multiple of a

and c is multiple of b

$\Rightarrow$ c is multiple of a

$\Rightarrow a|c$

$\Rightarrow (a,c) \in R$

for every ordered pair in R

If $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$

$\therefore$ R is transitive

Hence R is Reflexive, transitive but not Symmetric Relation.

Equivalence Relation:- A binary Relation R on a set A is said to be Equivalence Relation.

If (i) R is Reflexive
(ii) R is Symmetric
(iii) R is transitive

H.W (Pb) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)\}$ be a relation on set A. Verify R is equivalence Relation or not.

Sol: Given $R = \{(1,1)(1,2)(2,1)(2,2)(3,4)(4,3)(3,3)(4,4)\}$

$A = \{1,2,3,4\}$

$(1,1) \in R$

$(1,2) \in R$

$(3,3) \in R$

$(4,4) \in R$

for all ordered pairs $(a,a) \in R$

$\therefore$ R is Reflexive relation

$(2,1) \in R \Rightarrow (1,2) \in R$

$(3,4) \in R \Rightarrow (4,3) \in R$

for all ordered pairs

$(a,b) \in R$ $(b,a) \in R$

$\therefore$ R is a Symmetric relation

$(1,1)(1,2)\in R \Rightarrow (1,2)\in R$

$(2,1)(2,2)\in R \Rightarrow (2,2)\in R$

$(4,3)(3,3)\in R \Rightarrow (4,3)\in R$

$(4,4)(4,4)\in R \Rightarrow (4,4)\in R$

$(3,4)(4,3)\in R \Rightarrow (3,3)\in R$

$(2,2)(2,1)\in R \Rightarrow (2,1)\in R$

for all ordered pairs

$(a,b)(b,c)\in R \quad (a,c)\in R$

$\therefore$ R is Transitive relation

$\therefore$ R is a equivalence Relation

(Pb) A relation R on a set $A = \{a, b, c\}$ is represented by the following matrix. Determine R is Equivalence Relation or not

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sol: Given that $A = \{a, b, c\}$

$$M_R = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{(a,a), (a,c), (b,b), (c,c)\}$

Reflexive:

$(a,a) \in R$, $(b,b) \in R$

$(c,c) \in R$

for every $a \in A$, $(a,a) \in R$

$\therefore$ R is reflexive

Symmetric:

$(a,a) \in R \Rightarrow (a,a) \in R$

$(a,c) \in R$ but $(c,a) \notin R$

for some ordered pair in R

If $(a,b) \in R$ but $(b,a) \notin R$

$\boxed{\therefore R \text{ is not Symmetric}}$

<u>transitive</u>

$(a,a)\,(a,a) \in R \Rightarrow (a,a) \in R$

$(c,c)\,(c,c) \in R \Rightarrow (c,c) \in R$

$(b,b)\,(b,b) \in R \Rightarrow (b,b) \in R$

$(a,a)\,(a,c) \in R \Rightarrow (a,c) \in R$

$(a,c)\,(a,c) \in R \Rightarrow (a,c) \in R$

for every ordered pair in R

If $(a,b)\,(b,c) \in R \Rightarrow (a,c) \in R$

$\boxed{\therefore R \text{ is transitive relation}}$

Hence R is reflexive and transitive but R is not symmetric so

so the given relation is not equivalent because the symmetric not executed.

4/12/21

M-I

(Pb) For a fixed integer $n \geq 1$. prove that the relation "congruent modulo n" Is an equivalence relation on the set of all positive integers $\mathbb{Z}$.

Given that

Sol: The set $A = \mathbb{Z}$

fixed $n \geq 1$ is an integer.

Given Relation

$aRb \Leftrightarrow a$ congruent b modulo n

If $(a,b) \in R \Rightarrow a \equiv b \pmod{n}$

$\Rightarrow n | a-b$

$\Rightarrow a-b$ is multiple n

$\Rightarrow a-b = nk$, for some $k \in \mathbb{Z}$

Reflexive:

let $a \in A$ $(=\mathbb{Z})$

$a \in \mathbb{Z}$

$\Rightarrow a-a=0$

$\Rightarrow a-a = 0 \cdot n$ for $n \geq 1$

$\Rightarrow a-a = n \cdot 0$

$\Rightarrow a-a =$ multiple of n.

$\Rightarrow n | a-a$

$\Rightarrow a \equiv a \pmod{n}$

$\Rightarrow (a,a) \in R$

for all '$a \in A$', $(a,a) \in R$

$\therefore$ R is reflexive

Symmetric:

let $(a,b) \in R$

$\Rightarrow a \equiv b \pmod{n}$

$\Rightarrow n | a-b$

$\Rightarrow$ ~~a-b+~~ $a-b = nk$ for some $k \in \mathbb{Z}$

$\Rightarrow b-a = n(-k)$ $\left(\because \text{If } k \in \mathbb{Z} \Rightarrow -k \in \mathbb{Z}\right)$

$\Rightarrow b \equiv a \pmod{n}$

$\Rightarrow (b,a) \in R$

for every ordered pair in R

If $(a,b) \in R \Rightarrow (b,a) \in R$

$\therefore$ R is Symmetric

Transitive:-

let $(a,b), (b,c) \in R$

$\Rightarrow a \equiv b \pmod{n}$ & $b \equiv c \pmod{n}$

$\Rightarrow n | a-b$ & $n | b-c$

$\Rightarrow a-b = nk_1$ & $b-c = nk_2$

for some $k_1, k_2 \in \mathbb{Z}$

$\Rightarrow a-b+b-c = nk_1 + nk_2$

$\Rightarrow a-c = n(k_1+k_2)$

$\Rightarrow a-c$ = multiple of n

$\Rightarrow n | a-c$

$\Rightarrow a \equiv c \pmod{n}$

$\Rightarrow (a,c) \in R$

$\therefore$ for every ordered pair in R

If $(a,b)\ (b,c) \in R \Rightarrow (a,c) \in R$

$\therefore$ R is transitive//

Hence congruent module n is equivalence relation.

(16) R is a Symmetric Relation on A. If and only if $R=R^{-1}$.

Sol: Let A be a set and R be a binary relation on set A

i) Necessary Part:

Assume that R is Symmetric

To prove $R=R^{-1}$

we have prove $R \subseteq R^{-1}$ & $R^{-1} \subseteq R$

let $(a,b) \in R \Rightarrow (b,a) \in R^{-1}$

$\Rightarrow (b,a) \in R$ ($\because R$ is Symmetric)

$\Rightarrow (a,b) \in R^{-1}$

$\Rightarrow R \subseteq R^{-1}$

Let $(a,b) \in R^{-1} \Rightarrow (b,a) \in R$

w.r.t R is Symmetric

$\Rightarrow R^{-1}$ is Symmetric

$(a,b) \in R^{-1}$

$\Rightarrow (b,a) \in R^{-1}$

$\Rightarrow (a,b) \in R$

$R^{-1} \subseteq R$

$\therefore R \subseteq R^{-1}$ & $R^{-1} \subseteq R \Rightarrow R = R^{-1}$

ii) Converse Part:-

Assume that $R = R^{-1}$

let $(a,b) \in R$

$\Rightarrow (b,a) \in R^{-1}$

But $R^{-1} = R$

$\Rightarrow (b,a) \in R$

For $(a,b) \in R \Rightarrow (b,a) \in R$

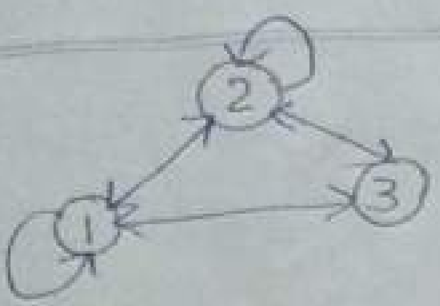
$\therefore$ R is Symmetric

Hence R is Symmetric

$\therefore$ R is a Symmetric Relation on A. $\Leftrightarrow$

$R = R^{-1}$.

(Pb) The digraph of a Relation R on a set $A = \{1,2,3\}$ is as given below. Determine whether the R is equivalence or not.



Sol: G.T $A = \{1,2,3\}$

$R = \{(1,1), (1,2) (1,3), (2,1) (2,2) (2,3) (3,1) (3,2)\}$

Reflexive: $(1,1) \in R$
$(2,2) \in R$
$(3,3) \notin R$

If $(a,a) \notin R$ for some $a \in A$

$\therefore$ R is reflexive

Symmetric: $(1,1) \in R \Rightarrow (1,1) \in R$
$(1,2) \in R \Rightarrow (2,1) \in R$
$(1,3) \in R \Rightarrow (3,1) \in R$
$(2,1) \in R \Rightarrow (1,2) \in R$
$(2,2) \in R \Rightarrow (2,2) \in R$
$(2,3) \in R \Rightarrow (1,3) \in R$
$(3,1) \in R \Rightarrow (2,3) \in R$
$(3,2) \in R \Rightarrow (2,3) \in R$

for all order pair in R

If $(a,b) \in R \Rightarrow (b,a) \in R$

$\therefore$ R is Symmetric

G.T $A = \{1,2,3\}$

$R = \{(1,1), (1,2)\ (1,3), (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\}$

Reflexive: $(1,1) \in R$

Sy

transitive: $(1,1)(1,2) \in R \Rightarrow (1,2) \in R$

$(1,1)(1,3) \in R \Rightarrow (1,3) \in R$

$(1,2)(2,1) \in R \Rightarrow (1,1) \in R$

$(1,2)(2,2) \in R \Rightarrow (1,2) \in R$

$(1,3)(3,1) \in R \Rightarrow (1,1) \in R$

$(1,3)(3,2) \in R \Rightarrow (1,2) \in R$

$(1,2)(2,3) \in R \Rightarrow (1,3) \in R$

$(2,1)(2,2) \in R \Rightarrow (2,2) \in R$

$(2,1)(2,3) \in R \Rightarrow (2,3) \in R$

$(3,1)(3,2) \in R \Rightarrow (3,2) \in R$

$(3,1)(4,3) \in R \Rightarrow (3,3) \notin R$

for some order pair in R

if $(a,b)\ (b,c) \in R$ but $(a,c) \notin R$

$\therefore$ R is not transitive

Hence the given relation is not ~~too~~ equivilence

6/12/21

POSET

→ poset is a p

A relation R on a set A is said to be partial order relation. If it is reflexive, Anti Symmetric & Transitive. so it is POSET

Ex:-) $A = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\ (3,3)\}$

(Pb) $A = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (1,3)\ (2,3)\}$

Sol Given $A = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (1,3)\ (2,3)\}$

R is a transitive, reflexive & Anti Symmetric.

∴ R is a POSET

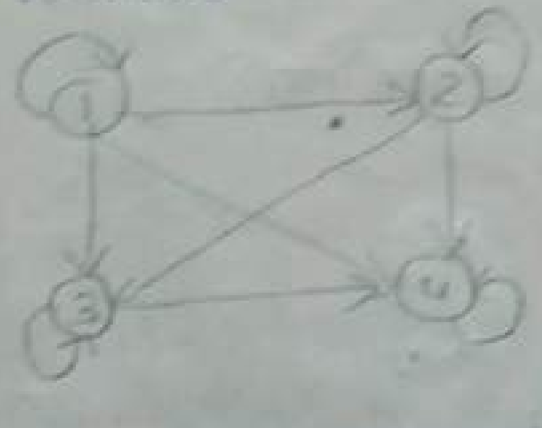
Hasse Diagram :- A pictorial representation of partial order relation (Poset) is called hasse Diagram.

(b) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,2)\ (2,3)\ (2,4)\ (3,3)\ (3,4)\ (4,4)\}$. Draw the Hasse Diagram of R

Sol: STEP-1: check the given Relation is POSET or not. If yes proceed to the next step

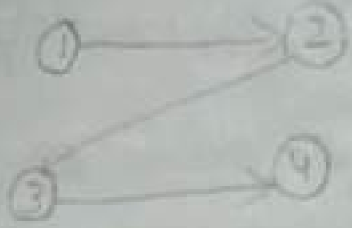
the given Relation R is poset.

~~STEP-2:- Remove~~



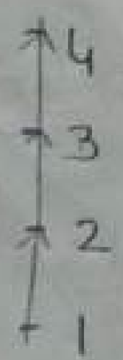
STEP-2:- Remove all reflexive edges (all self loops)

STEP-3:- Remove all transitive edges



∵ (2,3), (3,4) → (2,4) | (1,3) (3,4) → (1,4) | (1,2) (2,3) → (1,3)

STEP-4:- Represent the resulting diagram in upward direction.



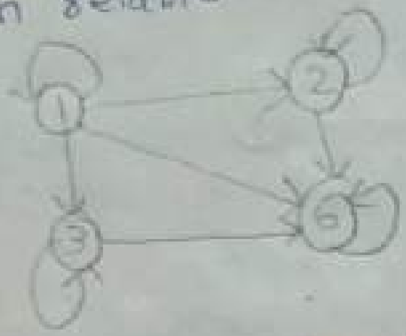
(8) Let $A = \{1, 2, 3, 6\}$. Draw the hasse Diagram of Relation $R = \{(1,1)\ (1,2)\ (1,3)\ (1,6)\ (2,2)\ (2,6)\ (3,3)\ (3,6)\ (6,6)\}$

Sol: the given relation is POSET

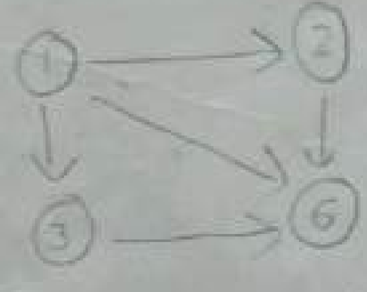
the given Relation is POSET

step-1:- check the given Relation is POSET or not. If yes proceed to the next step

the given relation is POSET.



step-2:- Remove all reflexive edges (all self loops)



step-3:- Remove all transitive edges

(1,2)(2,6) | (1,3)(3,6) |

(1,6) | (1,6)

$$\begin{tikzcd} 1 \arrow[r] \arrow[d] & 2 \arrow[d] \\ 3 \arrow[r] & 6 \end{tikzcd}$$

Represent the resulting diagram in upward direction.

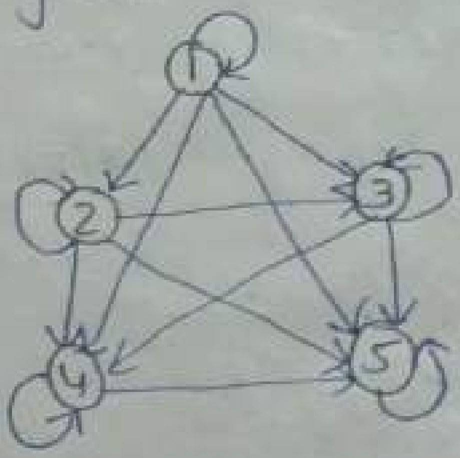
(14) POSET ({1,2,3,4,5}, ≤). Find Hasse Diagram if exists.

Sol: Given POSET ({1,2,3,4,5}, ≤)

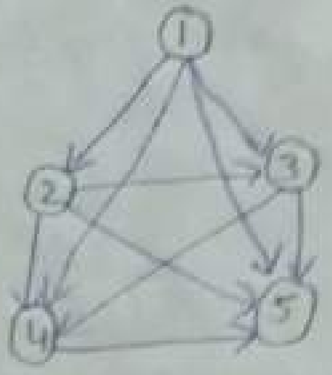
R = { (1,1), (1,2), (1,3), (1,4), (1,5), (2,2) (2,3) (2,4) (2,5) (3,3) (3,4) (3,5) (4,4) (4,5) (5,5) }

STEP 1: check the given Relation is POSET or not. If yes proceed to the next step

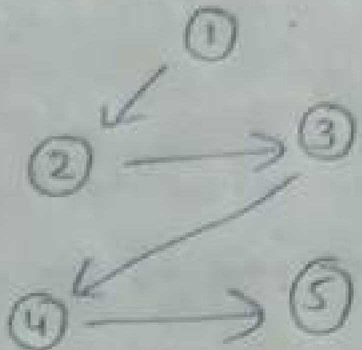
the given Relation R is POSET



STEP 2: Remove all reflexive edges (all self loops)



STEP 3:- Remove all transitive edges



(1,2) (2,4)	(2,3) (3,5)	(3,4) (4,5)	(2,4) (4,5)
(1,4)	(2,5)	(3,5)	(2,5)

(1,3) (3,5)	(1,3) (3,5)
(1,5)	(1,5)

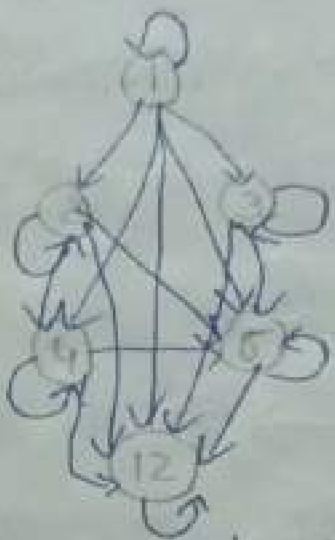
STEP 4:- Representing the resulting diagram in upward direction

(Pb) POSET ({1,2,3,4,6,12}, /). Find Hasse Diagram if exists

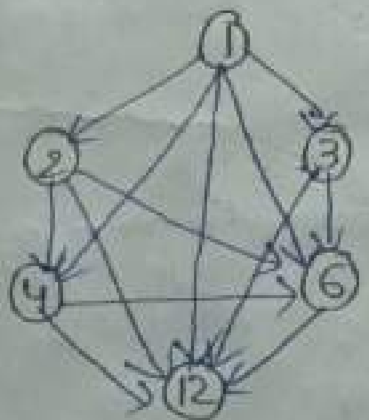
Sol: Given POSET ({1,2,3,4,6,12}, /)

∃R = {(1,1) (2,2) (3,3) (4,4) (6,6) (12,12) (1,2) (1,3) (1,4) (1,6) (1,12) (2,4) (2,6) (2,12) (3,6) (3,12) (4,6) (4,12) (6,12)}.

Step-1: R is POSET



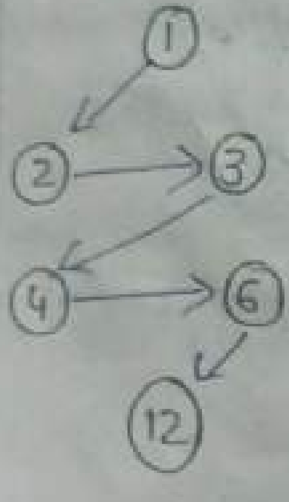
Step-2: remove the reflexive edges [all self loops]



Step-3: remove the transitive edges

(1,2) (2,3) → (1,3)	(1,2) (2,4) → (1,4)	(1,2) (2,6) → (1,6)	(1,2) (2,12) → (1,12)

(1,3)(3,4)	(1,3)(3,6)	(1,3)(3,12)	(1,4)(4,6)	
(1,4)	(1,6)	(1,12)	(1,6)	
(1,4)(4,12)	66	(2,3)(3,4)	(2,3)(3,6)	
(1,12)	(1,12)	(2,4)	(2,6)	
(2,3)(3,12)	(2,4)(4,6)	(2,4)(4,12)	(2,6)(6,12)	
(2,12)	(2,6)	(2,12)	(2,12)	
(3,4)(4,6)	(3,4)(4,12)	(3,6)(6,12)	(4,6)(6,12)	(6,12)(6,12)
(3,6)	(3,12)	(3,12)	(4,12)	(6,12)



-) represent the resulting diagram in upward directio

Functions :- (mappings or transformations):

Def:- Let A and B be non empty sets. A function from A to B is an assignment of exactly one element of B to each element of A. we write $f(a)=b$ if b is the unique element of B assigned by the function f to the element a of A. If f is a function from A to B, we write $f: A \to B$.

~~If f~~ :-

Note:-

(1) If $f: A \to B$ is a function then A - domain
B - codomain

The range of f is the set of all images of elements of A

(2) If f_1 and f_2 be functions ~~th~~ from A to B then f_1+f_2 and $f_1 f_2$ are also functions from A to $\mathbb{R}$ defined by

$$(f_1+f_2)(x) = f_1(x)+f_2(x)$$
$$(f_1 f_2)(x) = f_1(x) \cdot f_2(x)$$

one-one or injective function:-

A function f is said to be one-one or injective iff $f(a)=f(b)$ implies that $a=b$ for all a and b in the domain of f.

Note:- A function f is one-one iff $f(a)\neq f(b)$ wherever $a\neq b$.

onto or surjective:- A function f from A to B is called onto or surjective, iff for every element $b\in B$ there is an element $a\in A$ with $f(a)=b$.

Bijection:- The function f is a one-to-one ~~conver~~ correspondence or bijection, if it is both one-one and onto.

Inverse Function:- Let f be a one-to-one correspendence from the set A to the set B. The inverse function of f is the function that assigns to an element b belonging to B the unique element a in A. Such that

$f(a)=b$. The inverse function of f is denoted by f^{-1}.

Hence, $f^{-1}(b)=a$ when $f(a)=b$

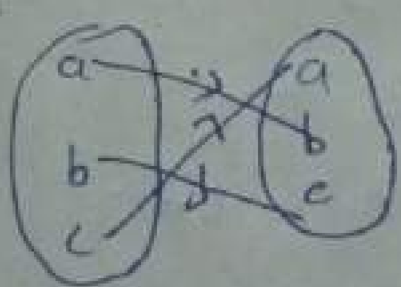
composition:- Let g be a function from the set A to set B and let f be a function from the set B to the set C. The composition of the functions f and g, denote by $f \circ g$, is defined by

$$(f \circ g)(a) = f(g(a)).$$

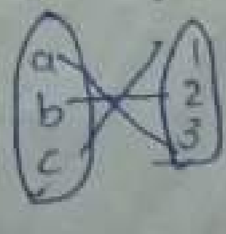
eg: ① let g be the function from the set $\{a,b,c\}$ to itself such that $g(a)=b$, $g(b)=c$, and $g(c)=a$. let f be the function from the set $\{a,b,c\}$ to the set $\{1,2,3\}$ such that $f(a)=3$, $f(b)=2$ and $f(c)=1$ find ~~fg~~ $f \circ g$ & $g \circ f$.

Sol. Given $g(a)=b$, $g(b)=c$, $g(c)=a$, $f(a)=3$, $f(b)=2$, $f(c)=1$.

g



f



$(f \circ g)(a) = f(g(a)) = f(b) = 2$

$(f \circ g)(b) = f(g(b)) = f(c) = 1$

$(f \circ g)(c) = f(g(c)) = f(a) = 3$

Now slly

$(g \circ f)(a) = g(f(a)) = g(3)$

gof is not defined, because the range of f is not a subset of the domain of g.

② Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x+3$ and $g(x) = 3x+2$. Find fog & gof?

Sol: Given $f(x) = 2x+3$

$g(x) = 3x+2$

$(f \circ g)(x) = f(g(x)) = f(3x+2)$

$= 2(3x+2)+3$

$= 6x+4+3$

$(f \circ g)(x) = 6x+7$

Now

$(gof)(x) = g(f(x)) = g(2x+3)$

$= 3(2x+3)+2$

$= 6x+9+2$

$= 6x+11$

(h) ~~If~~ R is reflexive if and only if $\bar{R}$ is irreflexive