

Analytical Learnability① Introduction (Inductive-Analytic approach to learnability)

- Inductive learnability requires a certain no. of training problems to achieve a given level of accuracy.
- In this learnability method, the generalization is done by observing training examples and identifying their features.
- On the basis of the features they are classified as +ve and -ve training examples.

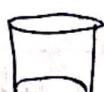


→ learnability algorithm → hypothesis

dataset (must be large)

Analytical Learnability:

- Analytical Learnability uses prior knowledge and deductive reasoning to increase the training examples.
- It contains domain theory which explains the feasibility example. So, it's called as explanation based learning.
- By using these explanations, analytical learnability is more accurate than inductive.



→ learnability algorithm → hypothesis

(dataset)

prior knowledge

Inductive-Analytic Learning

- In this learning method, we combine both analytical learning and inductive learning in order to gain the benefits of both approaches.
- In this method, the domain theory contains the explanations of training examples by the induction method.
- These explanations are used to perform predictions.

Learning problem:

Given: A set of training examples 'D' possibly containing an error.

. A domain theory 'B' containing errors.

. A space of candidate hypothesis 'H'

Determine: A hypothesis that best fits the training example and domain theory.

→ A best fit hypothesis is a hypothesis which contains zero errors.

→ There are 2 types of errors:

① Errors due to training examples

② Errors due to domain theory

→ Total errors can be given as

$$\underset{h \in H}{\operatorname{argmin}} K_D \text{error}_D(h) + K_B \text{error}_B(h)$$

Here, K_D , K_B are weights that are present for training example and domain theory
they can be calculated by using Bayesian method.

	Inductive Learning	Analytical Learning
Goal:	Hypothesis fits data	Hypothesis fits domain theory
Justification:	Statistical inference	Deductive inference
Advantage:	Requires little prior knowledge	Learns from scarce data

Comparison (Inductive & Analytical):

- Purely analytical learning methods offer the advantage of generalizing more accurately from less data by using prior knowledge to guide learning. However, they can be misled when given incorrect or insufficient prior knowledge.
- Purely inductive methods offer the advantage that they require no explicit prior knowledge & learn regularities based solely on training data. However, they can fail when given insufficient training data, & can be misled by the implicit inductive bias they must adopt in order to generalize beyond the observed data.

Hypothesis Space Search: we can characterize most learning methods or search algorithms by describing the hypothesis space H they search, the initial hypothesis ' h_0 ' of which they begin their search, the set of search operators ' O ' that define individual search steps, and the goal criterion ' G ' that specifies the search objective.

② Learning with perfect domain theories:

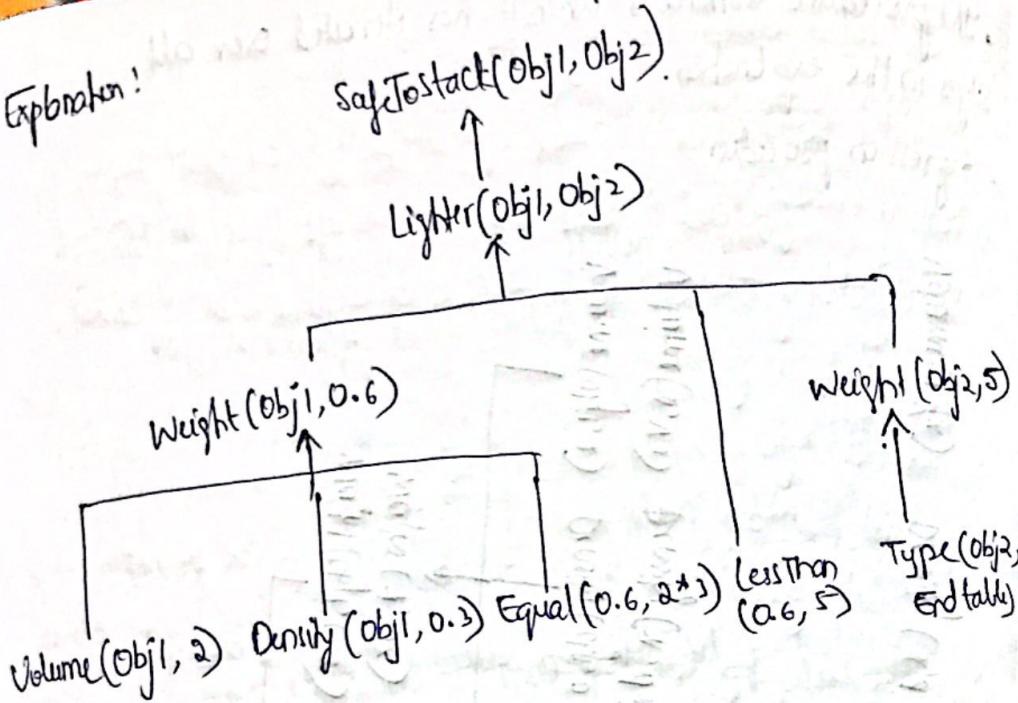
③ PROLOG-EBG:

- A domain theory is said to be correct, if each of its assertions is a truthful statement about the world.
- A domain theory is said to be complete with respect to a given target concept and instance space, if the domain theory covers every positive example in the instance space.
- PROLOG-EBG algorithm is a sequentially covering algorithm that considers the training data incrementally.
- For each new positive training example that is not yet covered by a learned Horn clause, it forms a new Horn clause by:

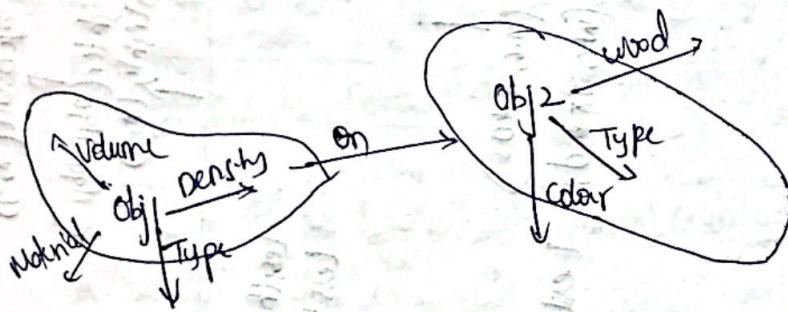
- (1) Explain the training example.
- (2) Analyse this explanation to determine whether it's an appropriate generalisation or not.
- (3) Refine the current hypothesis by adding a new Horn clause rule to cover this positive example, as well as other similar instances.

- PROLOG-EBG computes the most general rule by computing the weakest preimage (initial assertion) of the explanation.
- PROLOG-EBG computes the weakest preimage of the target concept with respect to the explanation, using a general procedure called regression.

Explanation:



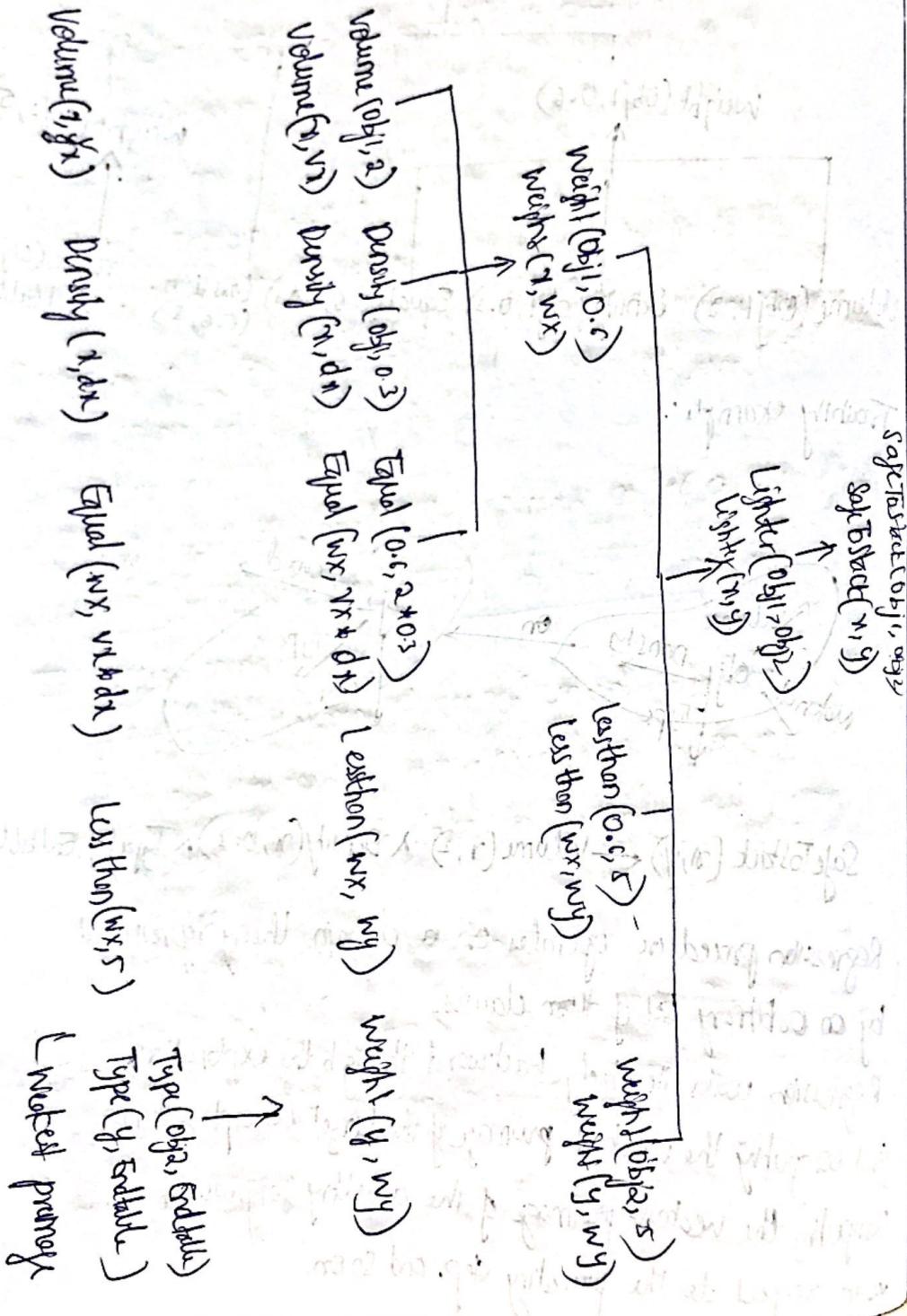
Traceability example:



$\text{SafeToStack}(x, y) \leftarrow \text{Volume}(x, 2) \wedge \text{Density}(x, 0.3) \wedge \text{Type}(y, \text{EndTable})$

- Regression procedure operates on a domain theory represented by an arbitrary set of Horn clauses.
- Regression works iteratively backward through the explanation, first computing the weakest preimage of the target concept, then computing the weakest preimage of the resulting expression with respect to the preceding step, and so on.

- The procedure terminates when it has iterated over all steps in the expression.
 - Regression procedure!



PROLOG-EBG:

PROLOG-EBG (Target Concept, Training Examples, Domain Theory)

- Learned Rules $\leftarrow \{ \}$
- Pos \leftarrow the positive examples from Training Examples
- foreach Positive Example in Pos that is not covered by Learned Rules do

1. Explain:

- Explanation \leftarrow an explanation (spec) in terms of the Domain theory that Positive Example satisfies the Target Concept.

2. Analyze:

- Sufficient Conditions \leftarrow the most general set of features of Positive Example sufficient to satisfy the Target Concept according to the Explanation.

3. Refine:

- Learned Rules \leftarrow LearnedRules + NewHornclauses where NewHornClause is of the form

Target Concept \leftarrow Sufficient Conditions

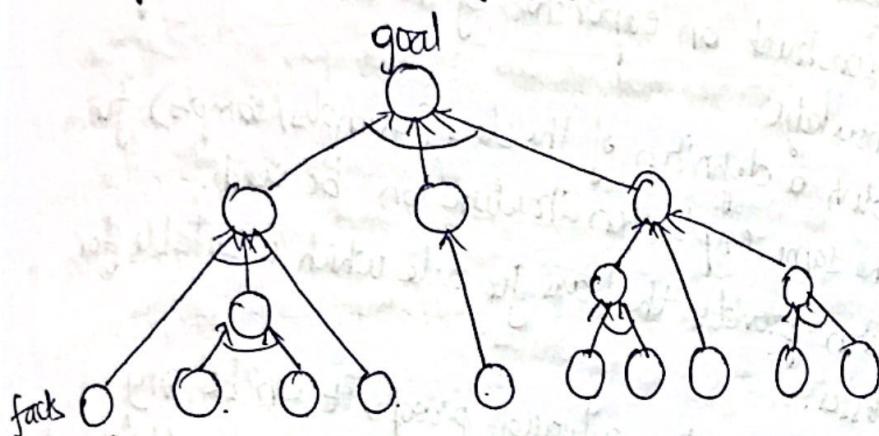
- Return Learned rules.

③ Explanation based learning (EBL):

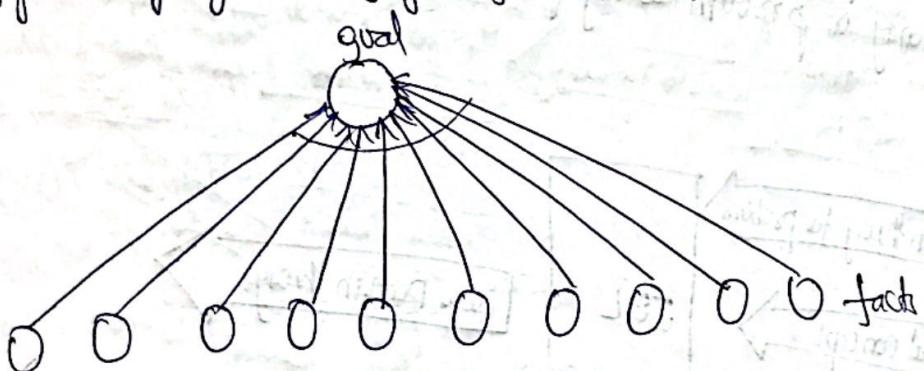
- Explanation-Based Learning (EBL) system accepts an example (i.e. training example) and explains what it learns from the example.
- The objective of EBL is to formulate a generalized explanation of the goal concept.
- It has two steps:
 - first, explain method
 - second, generalize method
- During the first step, the domain theory is used to prune away all the unimportant aspects of training examples with respect to the goal concept.
- The second step is to generalize the explanation as far as possible while still describing the goal concept.
- EBL requirements:
 - (1) A formal statement of the goal concept to be learned
 - (2) At least one positive training example of the concept
 - (3) Domain theory, which relates to the concept and training examples.
 - (4) Criteria for concept operation.
- Explanation Based Learning (EBL) is a principled method for explicitly available domain knowledge to improve supervised learning.

standard approach to EBL:

An exploration (detailed proof of goal)



After learning (go directly from facts to solution)



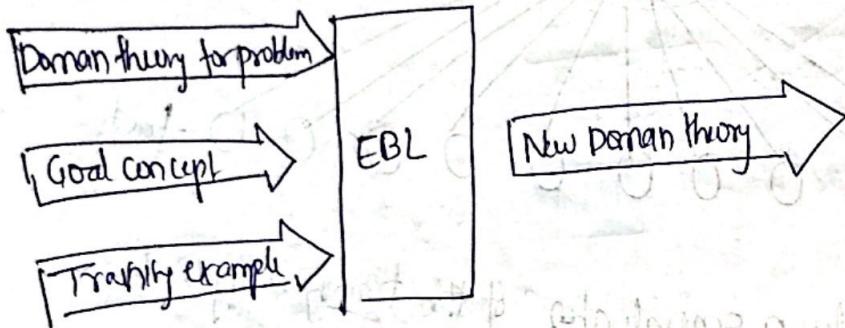
- . EBL computes a generalization of the training example, to describe the goal concept.
- . It also satisfies the operating criteria

EBL Memorisation:

- . Once the rule is identified then that should be memorized.
- . whenever it finds an example, immediately it will apply the same rule (taken from cache memory) on the example.

Extracting general rules from examples:

- The basic idea behind EBL is
 - first to construct an explanation of the observation using prior knowledge.
 - to establish a definition of the class (examples/samples) for which the same explanation structure can be used.
- This definition provides the basis for rule which is suitable for all the classes.
- The "explanation" can be a logical proof, it can be any reasoning or problem solving process, whose steps are well defined.



Domain theory:

Set of rules and facts that are used to explain how the examples fit a case of the goal concept.

Goal concept:

A high level description of what program is supposed to know.

Triviality Example:

What the learner sees in the world.

④ Use your knowledge to ~~offer the search space~~ initialize the hypothesis

KBANN Algorithm: (Combining Inductive & Analytical (Covert))
One approach to perfectly fit the domain theory using prior knowledge is to initialize the hypothesis to perfectly fit the domain theory, then inductively refine this initial hypothesis as needed to fit the training data. This approach is used by the KBANN (Knowledge Based Artificial Neural Network) to learn artificial neural network.

In KBANN an initial network is constructed so that for every possible instance, the classification assigned by the network is identical to that assigned by the domain theory. The BACKPROPAGATION Algorithm is then employed to adjust the weights of this initial network as needed to fit the training examples.

The KBANN algorithm ~~exploits~~ ^{classifies} the initialise-the-hypothesis approach using domain theories. It assumes a domain theory represented by a set of propositional non-recursive Horn clauses.

Given:

- A set of training examples.
- A domain theory consisting of non-recursive propositional Horn clauses.

Determine:

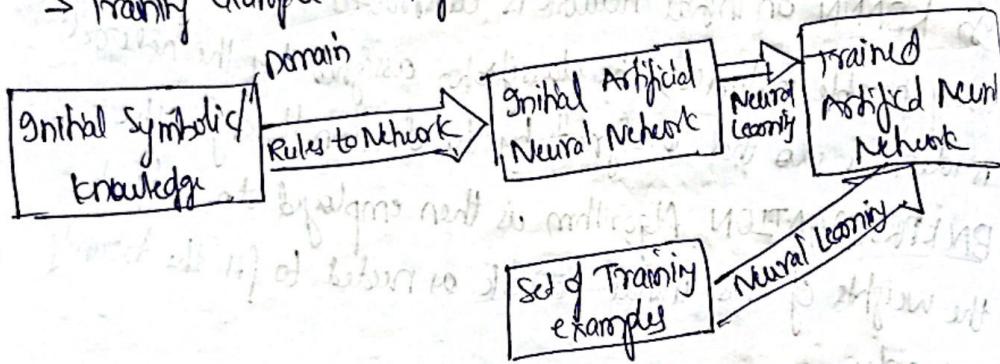
- An artificial neural network that fits the training examples.

- The two stages of KBANN algorithm are first to create an artificial neural network that perfectly fits the domain theory and second, to use the BACKPROPAGATION algorithm to refine this initial network to fit the training example.

- KBANN is a hybrid learning system (combine both domain knowledge + training example)

→ Domain knowledge - set of rules

→ Training example - train set of neural networks



- KBANN algorithm Combination:
 - 1) rules-to-network algorithm.
 - 2) Refiner algorithm

Ex: Learning Task! Identify Cup

Domain theory:

Cup \leftarrow Stable, Liftable, OpenVessel

Stable \leftarrow BottomIsFlat

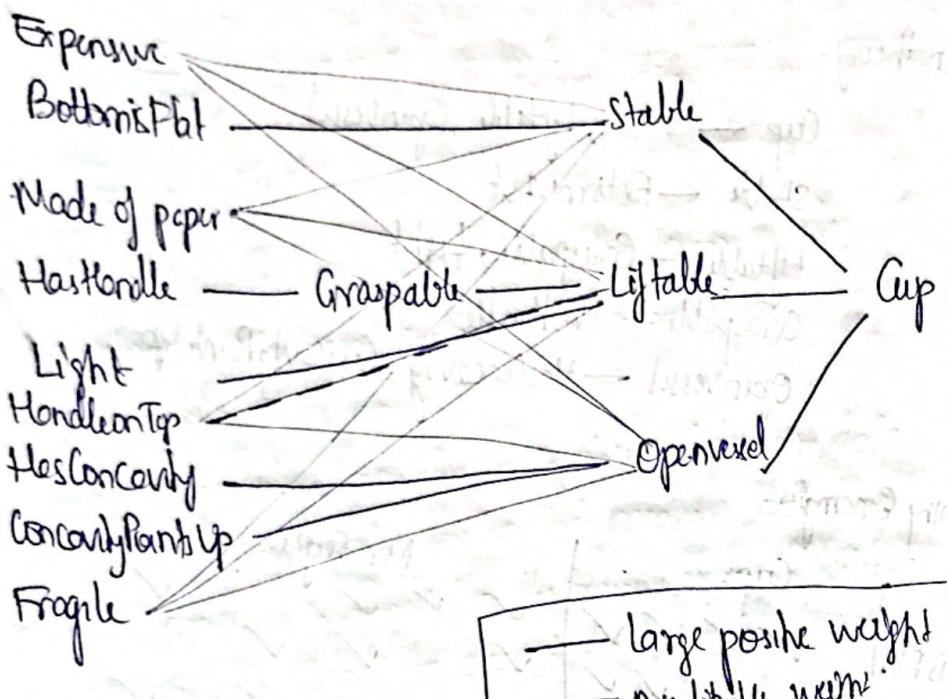
Liftable \leftarrow Graspable, Light

Graspable \leftarrow HasHandle

OpenVessel \leftarrow HasConcavity, ConcavityPointsUp

Training Examples:

	Cups	Non-Cups
BottomIsFlat	✓ ✓ ✓ ✓ ✓	✓ ✓ ✓ ✓ ✓
ConcavityPointsUp	✓ ✓ ✓ ✓ ✓	✓ ✓ ✓ ✓ ✓
Expensive	✓ ✓ ✓	✓
Fragile	✓ ✓ ✓	✓
Light	✓ ✓ ✓ ✓	✓
BottomIsFlat HandleOnTop	✓ ✓ ✓ ✓	✓ ✓ ✓ ✓
HasHandle	✓ ✓ ✓ ✓	✓ ✓ ✓ ✓
HasConcavity	✓ ✓ ✓ ✓	✓ ✓ ✓ ✓
ModelOfPaper	✓ ✓ ✓ ✓	✓ ✓ ✓ ✓



⑤ Using prior knowledge to alter the search objective!

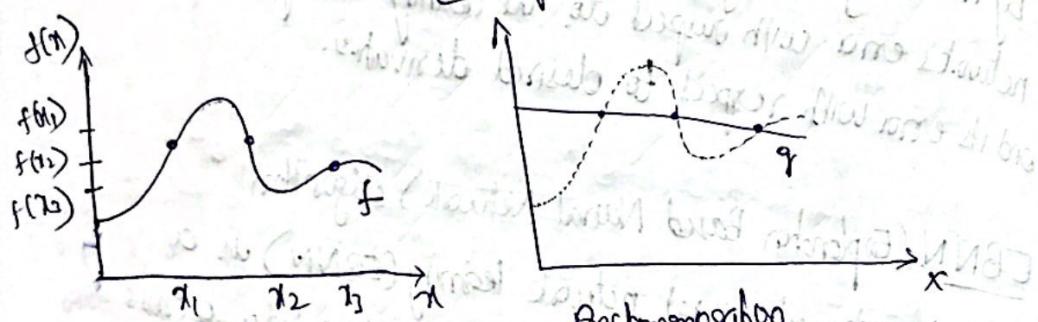
TangentProp algorithm:

- TangentProp accommodates domain knowledge expressed as derivatives of the target function with respect to transformations of its inputs.
- Prior knowledge is incorporated into the error criterion, minimized by gradient descent, so that the network must fit a combined function of the training data and domain theory.

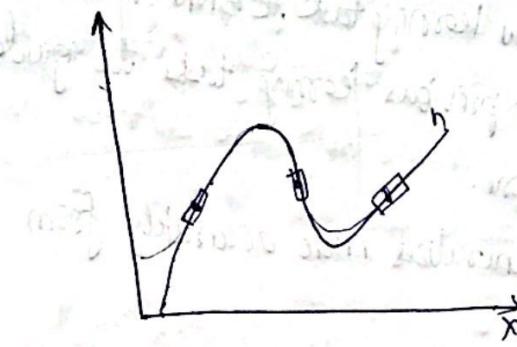
Ex: Handwritten character recognition.

- Tangent Prop algorithm trains a neural network to fit both training values and training derivatives
- Each training example consists of a pair $(x_i, f(x_i))$
- The tangentprop algorithm assumes various training derivatives of the target function.
- If each instance x_i is described by a single real value, then each training example may be of the form

$$(x_i, f(x_i), \frac{\partial f(x)}{\partial x}|_{x_i}) \rightarrow [3\text{tuples}]$$



Backpropagation
(it considers data in the form of
2 tuples only)



Graph for TangentProp algorithm
(Considers 3 tuple training data)

(Due to consideration of slope, it gives more accurate results)

than Backpropagation algo.

- The modified error function in Tangent Prop is

$$E_i \leq \left[(f(x_i) - f(\hat{x}_i))^2 + \mu \sum_j \left(\frac{\partial f(s_j(x, x_i))}{\partial x} - \frac{\partial \hat{f}(s_j(x, \hat{x}_i))}{\partial x} \right)^2 \right]$$

μ = constant value.

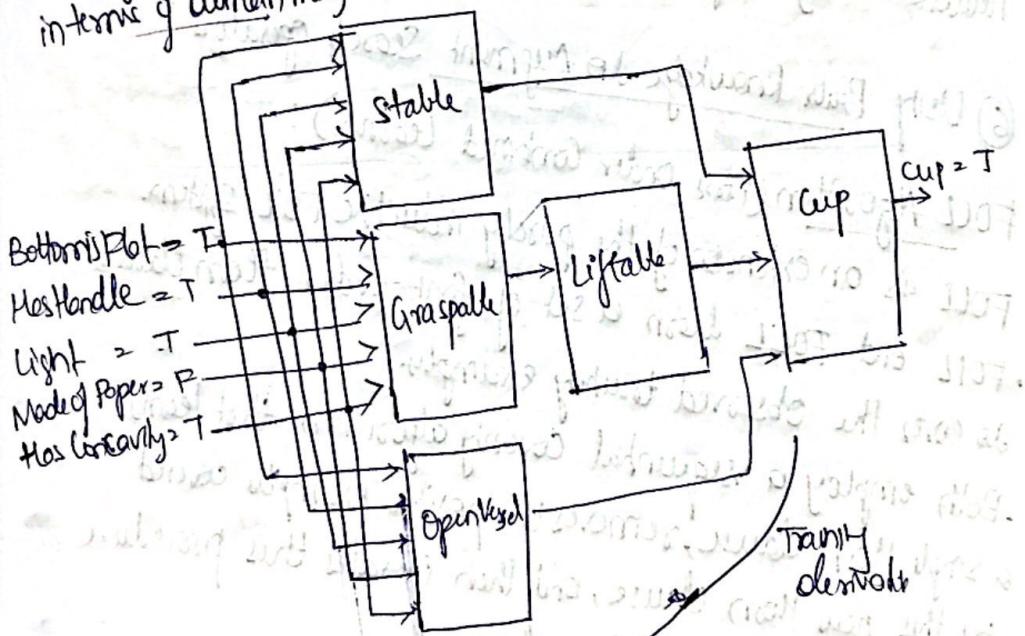
- Tangent Prop uses prior knowledge in the form of desired derivatives of the target function with respect to transformations of its inputs.
- Combines this prior knowledge with observed training data by minimizing the objective function that measures both the network's error with respect to the training example values and its error with respect to desired derivatives.

EBNN (Exploration Based Neural Network) algorithm:

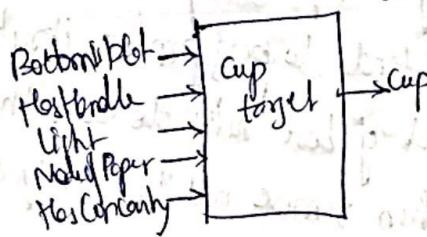
- Exploration-based neural network learning (EBNN) is a machine learning algorithm that transfers knowledge across multiple learning tasks.
- When faced with a new learning task, EBNN exploits domain knowledge accumulated in previous learning tasks to guide generalisation in the new one.
- As a result, EBNN generalizes more accurately from less data.

- EBNN builds on TangentProp algorithm in two significant ways.
- First, instead of relying on the user to provide training derivatives, EBNN computes training derivatives itself for each observed training example.
- Second, EBNN addresses the issue of how to weight the relative importance of the inductive (training data) and analytical (domain theory, prior knowledge) component of learning.
- The value of α is chosen independently for each training example.

Explanation of training examples in terms of domain theory.



Target network



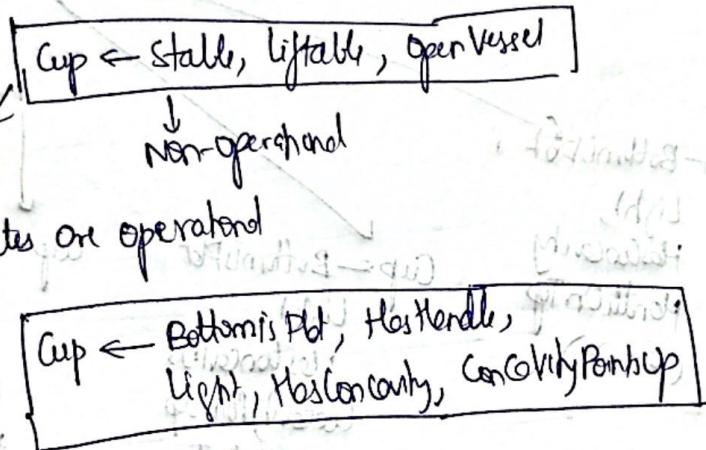
- The top portion of this figure depicts an EBNN domain theory for the target function cup, with each rectangular block representing a distinct neural network in the domain theory.
- Some networks take the outputs of other networks as their inputs (e.g. the rightmost network labeled cup takes its input from the outputs of the Stable, Liftable and OpenVessel networks).
- Thus, the networks can be chained together to infer the target function value for the input instance.
- EBNN makes use of these domain theory networks to learn the new target function. It does not alter the domain theory networks during the process.

⑥ Using Prior Knowledge to Augment Search Operators

FOCL Algorithm (First Order Combined Learner):

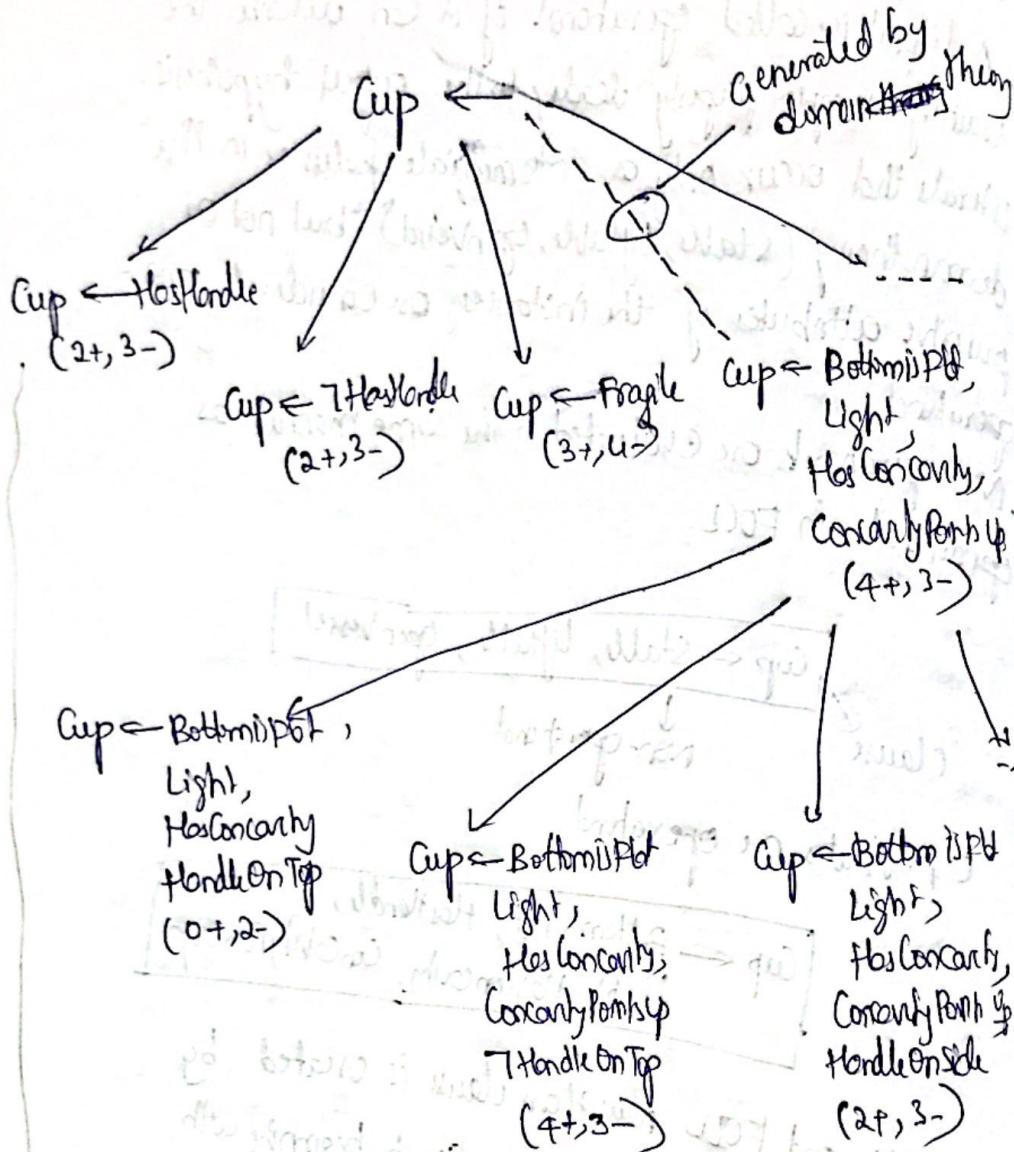
- FOCL is an extension of the purely inductive FOIL system.
- FOIL and FOCL learn a set of first-order Horn clauses to cover the observed training examples.
- Both employ a sequential covering algorithm that learns a single Horn clause, removes the positive examples covered by this new Horn clause, and then iterates this procedure over the remaining training examples.
- Like FOIL, FOCL also tends to perform an iterative process of learning a set of best rules to cover the training examples and then remove all the training examples

- covered by the best rule (using sequential covariant algorithm).
- A literal is called operational if it can describe the truth-value correctly leading to the output hypothesis.
 - Literals that occur only as intermediate features in the domain theory (stable, liftable, openVessel), but not as primitive attributes of the instances, are considered non-operational.
 - Non-operational literals are evaluated in the same manner as operational in FOIL.



- In FOIL and FOIL new *from clause* is created by performing a general-to-specific search, beginning with the most general possible *from clause* (*target function*).
- FOIL generates each candidate specialization by adding a single new literal to clause precondition.
- FOIL uses this same method for producing candidate specializations, but also generates additional specializations based on the domain theory.

FOL Example (domain theory + trinity example)



Here $(2+, 3-)$ -means, in the given data, when
2 times HasHandle is positive example
3 times HasHandle is a negative example