18/12/2020

Unit - IV

Discrete Probability

Random Experiment : If an experiment is conducted any number of times under essentially identical conditions, there is a set of all possible outcomes associated with it. If the result is not certain and is any one of the several possible outcomes. The experiment is called Random Experiment.

Equally likely events :

Events are said to be equally likely events when there is no reason to expect rather than any one of the other.

Exhaustive events :

All possible events in any trial are known as exhaustive events.

Mutually exclusive events :

events are said to be mutually exclusive if the happening of any one of the events in a trial excludes the happening of any one of the others i.e., If no two or more of the events can happen simultaneously in the same trial.

Probability : Let E be the any event the P(E) is the ratio of no of favourable cases to the Total no. of outputs.

$$P(E) = \frac{n}{m}$$

$P(\bar{E}) = 1 - P(E)$ (complement of P(E))

Axioms of probability:

1. Axiom of certainity: The total probability is always equal to 1
$P(E) = 1$.

2. Axiom of positivity: The probability is always greater than or equal to 0
$P(E) \geq 0$.

3. Axiom of Union:
$P(E_1 \cup E_2) = P(E_1) + P(E_2)$ where E_1, E_2 are mutually exclusive events.

What is the probability that a card drawn at random from the pack of playing cards may be either be a queen or a king

Let S be the sample space associated with the drawing of card $n(S) = {}^{52}C_1 = 52$.

Let E_1 be event of the card drawn being a Queen
$n(E_1) = {}^4C_1 = 4$.

Let E_2 be the event of the card drawn being a king
$n(E_2) = {}^4C_1 = 4$.

But E_1, E_2 are mutually exclusive

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$
$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

Addition theorm on probability:

If S is a sample space. Let E_1, E_2 are any events in S.

1. Let $P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$
$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

ii. If E_1, E_2 are mutually exclusive

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

* A card is drawn from a well shuffled pack of cards. what is the probability that either a spade or aces.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

Multiplication theorem:

In a random experiment. if E_1, E_2 are two events such that $P(E_1) \neq 0$, $P(E_2) \neq 0$ then $P(E_1 \cap E_2) = P(E_2/E_1) \cdot P(E_1)$

$$P(E_1 \cap E_2) = P(E_1/E_2) \cdot P(E_2)$$

Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue, 15 orange marbles with replacement being made after each drop. Find the probability that

i. both are white.

ii. 1st is red and next is white.

Let E_1 be event of drawing a white marble.

$$P(E_1) = \frac{30}{75}.$$

Total no. of marbles $- n(S) = 75$

Let E_2 be the event of second drawn marble is also white

$$P(E_2/E_1) = \frac{30}{75}$$

The probability that both marbles are white (with replacement)

$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$

$= \frac{30}{75} \cdot \frac{30}{75} = \frac{4}{25}$

ii. Let E_1 be the event that the 1st drawn marbel is red then

$P(E_1) = \frac{10}{75}$

E_2 be the event that the 2nd drawn marbel is white then

$P(E_2/E_1) = \frac{30}{75}$

$\therefore$ The probability that the 1st marbel is red and 2nd marbel is white $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$

$= \frac{10}{75} \cdot \frac{30}{75} = \frac{4}{75}$

Three students A, B, C are in running race. A and B have the same probability of winning. And each is twice likely to win as C. Find the probability that B or C wins.

$S = A \cup B \cup C$

$P(A) = P(B) = 2P(C)$

$P(A) + P(B) + P(C) = 1$

$5P(C) = 1$

$P(C) = 1/5$

$P(B) = 2\left(\frac{1}{5}\right) = \frac{2}{5}$

$P(A) = \frac{2}{5}$

$P(B \cup C) = P(B) + P(C)$

$= \frac{2}{5} + \frac{1}{5} = 3/5$

Conditional probability :

If E_1, E_2 are two events in a sample space S. And $P(E_1) \neq 0$, then the probability of E_2 after the event E_1 has occurred is called the Conditional probability of the event of E_2 given E_1 and is denoted by $P\left(\frac{E_2}{E_1}\right) = \frac{P(E_2 \cap E_1)}{P(E_1)}$

11/2/2020

Baye's Theorem :

Let $E_1, E_2 \ldots\ldots E_n$ are n mutually exclusive events such that $P(E_i) > 0$ where $(i = 1, 2, \ldots, n)$ in a sample space S and A is any other event in S intersecting with every E_i (i.e. A can only occur in combination with any one of the events $E_1, E_2, \ldots, E_n$) Such that $P(A) > 0$. If E_i is any one events of $E_1, E_2, \ldots, E_n$ where $P(E_1), P(E_2), \ldots, P(E_n)$ and

$P(A/E_1), P(A/E_2) \ldots\ldots, P(A/E_n)$ are known then

$$\cancel{P(E_k/A) = \frac{P(E_k/A)\,P(A)}{P(A/E_1)\,P(E_1)}}$$

$$P(E_k/A) = \frac{P(A/E_k)P(E_k)}{P(A/E_1)P(E_1) + P(A/E_2)P(E_2) + \ldots\ldots + P(A/E_n)P(E_n)}$$

Proof :

$E_1, E_2, \ldots\ldots E_n$ are n events of S ∋ $P(E_i) > 0$ and $E_i \cap E_j = \phi$ (for $i \neq j$) where $i, j = 1, 2, 3, \ldots\ldots n$. also $E_1, E_2, \ldots\ldots E_n$ are mutually exclusive events of S and A is any other event of S where $P(A) > 0$ then.

$S = E_1 \cup E_2 \cup \ldots\ldots \cup E_n$.

$A \cap S = (A\cap E_1) \cup (A \cap E_2) \cup \dots \cup (A\cap E_n)$

Here $(A\cap E_1), (A \cap E_2)$ are mutually exclusive events. then

$$P(E_k/A) = \frac{P(E_k \cap A)}{P(A)}$$

$$= \frac{P(A/E_k)\,P(E_k)}{P(A\cap E_1) \cup P(A\cap E_2) \cup \dots \cup P(A\cap E_k)}$$

$$= \frac{P(A/E_k)\,P(E_k)}{P(A/E_1)\,P(E_1) + P(A/E_2)\,P(E_2) + \dots + P(A/E_k)\,P(E_k)}$$

In bolt factory mechanices manufacting 20%, 30%, 50% of the total of their o/p and 6%, 3%, 2% are defective. A bolt is drawn at random and found to be defective. Find the probabilities that it is manufactured from i. machine A ii machine B iii machine C.

let $P(A) = 20\% = 0.2$ $\qquad P(C) = 0.5$

$P(B) = 0.3$

let D be the defective bolt

$P(D/A) = 0.06 \qquad P(D/B) = 0.03 \qquad P(D/C) = 0.02$.

i. Defective bolt from machine A

$$P(A/D) = \frac{P(D/A)\,P(A)}{P(D/A)\,P(A) + P(D/B)\,P(B) + P(D/C)\,P(C)}$$

$$= \frac{0.06(0.2)}{(0.06)(0.2) + (0.03)(0.3) + (0.02)(0.5)}$$

$$= \frac{12}{31} \approx 0.38$$

ii. Defective from machine B

$$P(B|D) = \frac{(0.03)(0.3)}{0.031} = \frac{9}{31}$$

iii. Defective from machine C

$$= \frac{0.02(0.5)}{0.031} = \frac{0.01}{0.031} = \frac{10}{31}$$

20/02/20

2. In a certain college 25% of boys & 10% of girls are studying mathematics. The girls constitute 60% of the student body.

i. What is the probability that mathematics is being studied.

ii. If a student is selected at random and is found to be studying mathematics, find the probability that the student is girl

iii. student is boy

Given $P(M|B) = 25\% = 0.25$

$P(M|G) = 10\% = 0.10$

$P(G) = 60\% = 0.6$ $\quad P(B) = 40\% = 0.4$

i. $P(M) = P(M|B)P(B) + P(M|G)P(G) = 0.16$

ii. $P(G|M) = \dfrac{P(M|G)\,P(G)}{0.16} = 0.375$

iii. $P(B|M) = \dfrac{P(M|B)\,P(B)}{0.16} = \dfrac{(0.25)(0.4)}{0.16} = 0.625$

Suppose 5 men out of 100 and 25 women out of 10,000 are colourblind. Suppose a colorblind person is choosen at random what is the probability of being a person is female

$P(M) = \frac{1}{2}$ $\quad P(W) = \frac{1}{2}$

Let C is the colorblind

$P(C|m) = 5/100$, $P(C|w) = 25/10000$

i. $P(m|C) = \dfrac{P(C|m)\,P(m)}{P(C|m)\,P(m) + P(C|w)\,P(w)} = \dfrac{\frac{5}{100} \times \frac{1}{2}}{\frac{5}{100} \times \frac{1}{2} + \frac{25}{10000} \times \frac{1}{2}} = \dfrac{500}{525} = 0.952$

$P(w|C) = \dfrac{P(C|w) \cdot P(w)}{P(C|m)P(m) + P(C|w)P(w)} = \dfrac{1}{21} = 0.0476$

Random Variables:

A real variable x whose value is determined by the outcome of a random experiment is called a random variable

Random variables are of 2 types

i. Discrete random variable ii. Continuous random variable

22/02

Variance :

Variance characterises the variability in the distributions. Since two distributions with the same name ~~will~~ can still have different dispersion of data of about their means.

Variance of the probability distribution of a random variable is the mathematical ~~exception~~ expectation of $[x - E(x)]^2$

$$Var(x) = E[x - E(x)]^2$$
$$= E(x-\mu)^2$$
$$= \Sigma(x-\mu)^2 P_i$$
$$= \Sigma\left(x^2 + \mu^2 - 2x\mu\right)P_i$$
$$= \Sigma\left(x_i^2 P_i + \mu^2 P_i - 2x\mu P_i\right)$$
$$= \Sigma x_i^2 P_i + \mu^2 \Sigma P_i - 2\mu \Sigma x_i P_i$$
$$= \Sigma x_i^2 P_i + \mu^2 - 2\mu\mu$$
$$= x_i^2 P_i - \mu^2$$
$$= \Sigma E(x^2) - \mu^2$$

① $V(k) = 0$

② $V(kx) = k^2 V(x)$

③ $V(x+k) = V(x)$

Two dices are thrown. Let x assigned to each point (a, b) in S. The maximum of its number i.e. $X(a,b) = \max(a,b)$ find the probability distribution x a random variable with $X(S) = \{1,2,3,4,5,6\}$ also find mean, variance, S.D of the distribution

while throwing two dices
The total no of cases/chances are $6^2 = 36$

The maximum number could be $X(S) = \{1,2,3,4,5,6\}$

$$X(S) = X(a,b) = \max(a,b)$$

The no 1 will appear only in one case (1,1), $P(1) = P(X=1) = P(1,1) = \frac{1}{36}$

the favourable cases are (1,2)(2,1)(2,2)

$P(2) = P(X=2) = \frac{3}{36}$

$P(3) = P(X=3) = \frac{5}{36}$

$P(4) = P(X=4) = \frac{7}{36}$

$P(5) = P(X=5) = \frac{9}{36}$

$P(6) = P(X=6) = \frac{11}{36}$

x	1	2	3	4	5	6
p(x)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

i. mean $\mu = \Sigma x_i P_i$

$= 1\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right)$

$= \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36}$

$= \frac{161}{36} = 4.47$ //

ii. variance $V(x) = \Sigma x_i^2 P_i - \mu^2$

$= \frac{1}{36} + 4\left(\frac{3}{36}\right) + 9\left(\frac{5}{36}\right) + 16\left(\frac{7}{36}\right) + 25\left(\frac{9}{36}\right) + 36\left(\frac{11}{36}\right) - 4.47^2$

$= 21.97 - 20.0001$

$= 1.97$ //

iii. SD $= \sqrt{1.97}$

$= 1.40.$

iv. $P(1<x<6) = P(2)+P(3)+P(4)+P(5)$

$= \frac{3}{36}+\frac{5}{36}+\frac{7}{36}+\frac{9}{36} = \frac{14}{36}$

A random Variable x as the following probability function

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i. $k=?$ ii. $P(x<6)$, $P(x\geq 6)$, $P(0<x<5)$, $P(0\leq x\leq 4)$

iii. If $P(X\leq k) > \frac{1}{2}$

iv. Determine distribution f^n of x.

mean

Variance

sd

$k+2k+2k+3k+k^2+2k^2+7k^2+k = 1$

$10k^2+9k-1=0$

$k=\frac{1}{10}$, $k=-1$.

$P(x<6) = k+2k+2k+3k+k^2$

$= 8k+k^2 = \frac{8}{10}+\frac{1}{100} = 0.8+0.01 = 0.81$

$P(x\geq 6) = 2k^2+7k^2+k$

$= 9k^2+k = \frac{9}{100}+\frac{1}{10} = 0.09+0.1 = 0.19.$

$P(0<x<5) = k+2k+2k+3k$

$= 8k = \frac{8}{10} = 0.8$

$P(0 \le x \le 4)$
$8k = 0.5$

iii. $P(x \le k) > \frac{1}{2}$ is $k = 4$.

$P(x \le 1) = P(x=0) + P(x=1)$
$= 0 + k = k = \frac{1}{10}$

$P(x \le 2) = P(x=0) + P(x=1) + P(x=2) = 0 + k + 2k = 3k = \frac{3}{10} = 0.3$

$P(x \le 3) = 5k = 0.5$

$P(x \le 4) = 8k = 0.8 > 0.5$

iv.

x	$F(x) = P(x \le x)$
0	0
1	$k = \frac{1}{10}$
2	$3k$
3	$5k$
4	$8k$
5	$8k + k^2$
6	$8k + 3k^2$
7	$9k + 10k^2 = 0.9 + 10(\frac{1}{100}) = 1$.

v. $\mu = \Sigma x_i p_i$

$= 0 + 1(k) + 2(2k) + 3(2k) + 4(3k) + 5(k^2) + 6(2k^2) + 7(7k^2 + k)$

$= k + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k$.

$= 30k + 66k^2$

$= 3.66$

vi. $\sigma^2 = \Sigma x_i^2 p_i - \mu^2$

$= k + 4(2k) + 9(2k) + 16(3k) + 25(k^2) + 36(2k^2) + 49(7k^2 + k) - (3$

$= k + 8k + 18k + 48k + 25k^2 + 72k^2 + 343k^2 + 49k - 13.$

$= 16.8 - 13.395 = 3.41$.

$\sigma^2 = \sqrt{3.41}$

$\sigma = 1.84$ //.

24/02/2020.

A Sample of 4 items is selected at random from a Box containing 12 items of which 5 are defective. Find the expected no. of E of defective items

Let X denotes the no of defective items among 4 items drawn from 12 items. X can take the values of 0 to 4

$$X = 0, 1, 2, 3, 4$$

No. of defective items = 5.

No. of good items = 7

$$P(X=0) = P(\text{No defective item}) = \frac{{}^7C_4 \times {}^5C_0}{{}^{12}C_4} = \frac{35 \times 1}{495}$$

$$= 0.07.$$

$$P(X=1) = P(\text{only one defective item}) = \frac{{}^7C_3 \times {}^5C_1}{{}^{12}C_4} = \frac{210 \times 5}{495}$$

$$= 0.35$$

$$P(X=2) = P(\text{two defective items}) = \frac{{}^7C_2 \times {}^5C_2}{{}^{12}C_4} = \frac{21 \times 10}{495} = 0.42$$

$$P(X=3) = \frac{{}^7C_1 \times {}^5C_3}{{}^{12}C_4} = \frac{7 \times 10}{495} = 0.1414.$$

$$P(x=4) = \frac{{}^7C_0 \times {}^5C_4}{{}^{12}C_4} = \frac{1 \times \frac{5\times4\times3\times2}{4\times3\times2}}{495} = \frac{5}{495} = 0.01$$

x	0	1	2	3	4
P	0.07	0.35	0.42	0.14	0.01.

$$E = \sum_{i=0}^{4} x_i P_i$$

$$= 0(0.07) + 1(0.35) + 2(0.42) + 3(0.14) + 4(0.01)$$

$$= 0.35 + 0.84 + 0.42 + 0.04$$

$$= 1.65.$$

* A variate X has the following distribution on

x	-3	6	9
$P(x)$	1/6	1/2	1/3

Find i. $E(x)$ ii. $E(x^2)$ iii. $E[(2x+1)^2]$

i. $E(x) = \sum x_i P_i$

$$= -\cancel{3}^{1}\left(\frac{1}{\cancel{6}_2}\right) + \cancel{6}^{3}\left(\frac{1}{\cancel{2}}\right) + \cancel{9}^{3}\left(\frac{1}{\cancel{3}}\right)$$

$$= -\frac{1}{2} + 3 + 3 = 5.5$$

ii. $E(x^2) = 9\left(\frac{1}{6}\right) + 36\left(\frac{1}{2}\right) + 81\left(\frac{1}{3}\right)$

$$= 1.5 + 18 + 27$$

$$= 46.5$$

iii. $E(4x^2 + 1 + 4x) = 4E(x^2) + 1 + 4E(x)$

$$= 4(46.5) + 4(5.5) + 1 = 186 + 22 + 1 = 209$$

A player tosses two fair coins. He wins 100 if head appears, 200 if two heads appears. On the other hand he looses 500 if no head appears. Determine the Expected value E of the game and the game is favourable to the player.

Let x denotes the no. of heads occuring in tosses of two fair coins then the sample space S = {HH, TT, HT, TH}.

probability of ~~all~~ two heads occurs

$P(x=2) = \frac{1}{4}$

$P(x=1) = \frac{1}{2}$

$P(x=0) = \frac{1}{4}$

x	0	1	2
p	1/4	$\frac{1}{2}$	1/2

$E(x) =$

$\cancel{-100(0 \times \frac{1}{4} + (\frac{1}{2}) + 2(\frac{1}{4}))}$ $\quad = 200(\frac{1}{4}) + 100(\frac{1}{2}) - 500(\frac{1}{4})$

$\cancel{= 100(\frac{1}{2} + 1) \to 0(1.5) 150}$ $\quad = -25$

$\therefore$ The game is not favourable to player since $E<0$

25/02/2020

Recurrance Relations:

Generating function of a sequence:

Let us take a sequence of real numbers $(a_0, a_1, a_2, ---, a_n)$ and denoted this sequence with $\{a_r\}_{r=0}^{\infty}$ where $r = 0$ to ∞.

For this sequence, suppose there exist a function whose domain is a set of non-negative integers and whose range is a set of

-ral numbers.

Let the sequence is denoted as $A = \{a_r\}_{r=0}^{\infty}$ then the function on sequence is expressed as $A(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_r x^r$

$$= \sum_{r=0}^{\infty} a_r x^r$$

then for the sequence $a_0, a_1, a_2, \dots, a_r$ where $r = 0, 1, \dots$

$A(x)$ is called a generating function

Ex: Let us take the sequence $(1, 3, 9, 27, \dots)$ and is denoted by

$$A(x) = \sum_{r=0}^{\infty} 3^r x^r$$

$$= 1 + 3x + 9x^2 + 27x^3 + \dots$$

Suppose let the sequence $B = \{b_r\}_{r=0}^{\infty}$ where $b_r = \begin{cases} 0 & \text{if } 0 \le r \le 3 \\ 1 & \text{if } 4 \le r \le 6 \\ 2 & \text{if } r = 7 \\ 3 & \text{if } 8 \le r \end{cases}$

$b_0 = b_1 = b_2 = b_3 = 0$.

$b_4 = b_5 = b_6 = 1$

$b_7 = 2$

$b_8, b_9, \dots = 3$.

$B(x) = x^4 + x^5 + x^6 + 2x^7 + 3x^8 + 3x^9 + \dots$

Find the generating f'n for a_r = the no. of non-negative integral solutions of $e_1 + e_2 + e_3 = r$ where $0 \le e_1 \le 3$, $2 \le e_2 \le 6$, e_3 is odd and $1 \le e_3 \le 9$

$e_1 + e_2 + e_3 = r$

$0 \le e_1 \le 3$

$2 \le e_2 \le 6$

e_3 odd and $1 \le e_3 \le 9$

$A(x) = 1 + x^1 + x^2 + x^3$

$B(x) = x^2 + x^3 + x^4 + x^5 + x^6$

$C(x) = x^1 + x^3 + x^5 + x^7 + x^9$

$G(x) = A(x) \cdot B(x) \cdot C(x)$

26/02/2020.

Calculating coefficients of generating functions $A(x) = \sum_{x=0}^{\infty} a_r x^r$

$B(x) = \sum_{k=0}^{\infty} b_k x^k$ be two formal power series.

Then $A(x)$ is said to be multiplicative inverse of $B(x)$. If

i) $A(x) \cdot B(x) = 1$ then $B(x) = \frac{1}{A(x)}$.

ii) Let $A(x)$, $C(x)$ and $D(x)$ are to be three formal power series then $A(x)$ divides $C(x)$ if $A(x) \cdot D(x) = C(x)$ then $D(x) = \frac{C(x)}{A(x)}$

iii) Geometric series, let $A(x) = 1-x$ be a multiplicative inverse and $B(x) = \frac{1}{A(x)} = \sum_{r=0}^{\infty} a_r x^r$. Then the following results hold

1. $\frac{1}{1-x} = \sum_{r=0}^{\infty} x^r$

2. $\frac{1}{1+x} = \sum_{r=0}^{\infty} (-1)^r x^r$

3. $\frac{1}{1-ax} = \sum_{r=0}^{\infty} a^r x^r$

4. $\frac{1}{(1-x)^n} = \frac{1}{[(1-x)^1]^n} = \sum_{r=0}^{\infty} [x^r]^{-n} = {}^{(n+r-1)}C_r x^r$

$= \binom{n+r-1}{r} x^r$

Find the Coefficients of the given functions -

1) x^9 in $(1+x^3+x^8)^{10}$ ii. x^{25} in $(1+x^3+x^8)^{10}$

i) x^9 in $(1+x^3+x^8)^{10}$

To know the coefficient of x^9

$$\boxed{e_1+e_2+\cdots\cdots e_n = r}$$

$e_1+e_2+e_3+\cdots\cdots+e_{10}=9$

Where $e_i = 0,3,8$ are those solutions where 3 values are equal to 3 and remaining 7 values are equal to 0.

$$7(0)+3(3)+8(0) = 9$$

$$\frac{10!}{7!\,3!\,0!} = 120.$$

ii) Let $e_1+e_2+e_3+\cdots\cdots+e_{10} = 25$

$e_i = 0,3,8$ are those solutions where 3 values are equal to 3 and 2 values are equal to 8 and 5 values are equal to 0

$$5(0)+3(3)+2(8) = 25$$

$$\frac{10!}{5!\,3!\,2!} = 2520.$$

H.W

1. $(1+x^5+x^9)^{10}$ in x^{23}, x^{32}.

$e_i = 0,5,9$

$$(9)2+(5)1+7(0)$$

$$\frac{10!}{2!\,7!} = \frac{10\times 9\times \cancel{8}^{4}\times \cancel{7!}}{\cancel{2}\times \cancel{7!}} = 360$$

$6(2) + 1(5) + 3(9) = 32$ $\qquad \frac{10!}{6!\,1!\,3!} = \frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{6!} \times \cancel{3} \times \cancel{2}}$

$= 840$

27/02/2020

Find the Sequences of generated by the following functions:

1. $(3+x)^3$

$= 27\left(1+\frac{x}{3}\right)^3$

$= 27\left[{}^3C_0 (1)^0 (x/3)^3 + {}^3C_1 (1)^1 (x/3)^{3-1} + {}^3C_2 (1)^2 (x/3)^{3-2} + {}^3C_3 (1)^3 (x/3)^0\right]$

$= 27\left[1(1)\frac{x^3}{27} + \cancel{3}(1)\frac{x^2}{\cancel{9}_3} + \cancel{3}(1)\frac{x}{\cancel{3}} + 1(1)(1)\right]$

$= x^3 + 9x^2 + 27x + 27$

$\therefore$ required Sequence : 27, 27, 9, 1, 0, 0, - - - - -

2. $2x^2(1-x)^{-1}$

$2x^2(1+x+x^2+x^3+ \dots)$

$2x^2 + 2x^3 + 2x^4 + 2x^5 + \dots$

$\therefore$ required Sequence : 0, 0, 2, 2, 2, - - - -

3. $\frac{1}{1-x} + 2x^3$

$(1-x)^{-1} + 2x^3$

$(1+x+x^2+x^3+ \dots) + 2x^3$

$1 + x + x^2 + 3x^3 + x^4 + \dots$

$\therefore$ required Sequence : 1, 1, 1, 3, 1, 1, - - - - -

4. $3x^3 + e^{2x}$

$= 3x^3 + \left[1 + \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \cdots\right]$

$= 3x^3 + \left[1 + 2x + \frac{4x^2}{2} + \frac{8x^3}{6} + \frac{16x^4}{4\times 6} + \cdots\right]$

$= 3x^3 + \left[1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \cdots\right]$

$= 1 + 2x + 2x^2 + \frac{13}{3}x^3 + \frac{2}{3}x^4 + \cdots$

$\therefore$ required sequence is $1, 2, 2, \frac{13}{3}, \frac{2}{3}, \cdots$

Find the Generating function for the following sequences.

1. $1, 2, 3, 4, \cdots$

$f(x) = (1-x)^{-2}$

$= 1 + 2x + 3x^2 + 4x^3 + \cdots$

$\therefore$ The required GF for given sequence is

$f(x) = (1-x)^{-2}$

2. $0, 1, -2, 3, -4, 5, \cdots$

$= 0 + x - 2x^2 + 3x^3 - 4x^4 + \cdots$

$= x(1 - 2x + 3x^2 - 4x^3 + \cdots)$

$= x(1+x)^{-2}$

3. $1^2, 2^2, 3^2, \cdots$

wkt $0 + 1x + 2x^2 + 3x^3 + 4x^4 + \cdots$

$x(1 + 2x + 3x^2 + 4x^3 + \cdots) = x(1-x)^{-2}$ — (1)

eqn (1) diff wrt to x

$1+4x+9x^2+16x^3 = \frac{d}{dx}\left[x(1-x)^{-2}\right]$

$1+2^2x+3^2x^2+4^2x^3 = \frac{d}{dx}\left[\frac{x}{(1-x)^2}\right]$

$= (x)(-2(1-x)^{-3}) + (1-x)^{-2}$

$= 2x(1-x)^{-3} + (1-x)^{-2} = \frac{2x+(1-x)}{(1+x)^3}$

$= \frac{1+x}{(1+x)^3}$

1. $0, 1^2, 2^2, 3^2, \ldots$

$x(1^2+2^2x+3^2x^2+4^2x^3+\ldots)$

$\frac{x(1+x)}{(1-x)^3}$

$\therefore f(x) = \frac{x(1+x)}{(1-x)^3}$ //

29/02/2020

$1^3, 2^3, 3^3, \ldots\ldots$

We know that $0 + 1x + 2^2x^2 + 3^2x^3 + \ldots\ldots$

$$x(1^2 + 2^2x + 3^2x^2 + \ldots\ldots) = \frac{x(x+1)}{(1-x)^3}$$

eqn ① diff wrt x

$$1^3 + 2^3x + 3^3x^2 + \ldots\ldots = \frac{d}{dx}\frac{(x^2+x)}{(1-x)^3}$$

$$= \frac{(2x+1)(1-x)^{\cancel{3}} - (x^2+x)3(1-x)^{\cancel{2}}(-1)}{(1-x)^{\cancel{6}}}$$

$$= \frac{(2x+1)(1-x) + 3(x^2+x)}{(1-x)^4}$$

$$= \frac{(2x+1)(1-x) + 3x^2 + 3x}{(1-x)^4}$$

$$= \frac{2x + 1 - 2x^2 - x + 3x^2 + 3x}{(1-x)^4}$$

$$= \frac{x^2 + 4x + 1}{(1-x)^4}$$

6. $0^3, 1^3, 2^3, 3^3, \ldots$

$0 + 1x + 2^3x^2 + 3^3x^3 + \ldots$

$x(1 + 2^3x + 3^3x^2 + 4^3x^2 + \ldots)$

$$= x\left[\frac{x^2+4x+1}{(1-x)^4}\right]$$

$$\therefore f(x) = x\left(\frac{x^2+4x+1}{(1-x)^4}\right)$$

Find the Coefficient of x^n in the following function

1. $(x^2+x^3+x^4+\ldots)^4$.
2. $(1+x^2+x^4+\ldots)^7$

1. $(x^2+x^3+x^4+\ldots)^4$

$= x^8\left[(1-x)^{-1}\right]^4$

$$\left[=(n+r-1)_{C_r}x^r\right]$$

$= x^8(1-x)^{-4}$

$= x^8 \sum_{r=0}^{\infty} (4+r-1)_{C_r} x^r$

$= \sum (3+r)_{C_r} x^{8+r}$

$x^{8+r} = x^n$

$8+r = n$

$r = n-8$.

$$\sum_{r=0}^{\infty} (3+n-8)\, C_{n-8}$$

$\therefore$ Coefficient of x^n

$$= (n-5)\, C_{n-8}$$

$$= (n-5)\, C_3$$

2. $(1+x^2+x^4+\ldots\ldots)^7$

$(1-x^2)^{-7}$

$$\sum_{r=0}^{\infty} (7+r-1)\, C_r\, (x^2)^r$$

$$\sum_{r=0}^{\infty} (6+r)\, C_r\, x^{2r}$$

The coefficient of x^n is

$(6+n/2)\, C_6 = (6+a)\, C_6$

The coefficient of x^n is $2a$ when n is even and it is 0 when n is odd.

Find the coefficient of x^{18} in the following products

1. $(x+x^2+x^3+x^4+x^5)\,(x^2+x^3+x^4+\ldots\ldots)^5$

$x(1+x+x^2+x^3+x^4)\,x^{10}(1+x+x^2+x^3+\ldots\ldots)^5$

$x^{11}(1+x+x^2+x^3+x^4)\,\left[(1-x)^{-1}\right]^5$

$x^{11}(1+x+x^2+x^3+x^4)\,(1-x)^{-5}$

$x^{11}(1+x+x^2+x^3+x^4)\sum_{r=0}^{\infty}(5+r-1)_{C_r}x^r$

$= (1+x+x^2+x^3+x^4)\sum_{r=0}^{\infty}(4+r)\,C_r\,x^{11+r}$

$= (1+x+x^2+x^3+x^4)\sum_{r=0}^{\infty}(4+r)_{C_r}\,x^{11+r}$

$x^{11+r} = x^{18}$

$11+r=18$

$r=7$

The coefficient of x^{18} is

$= {}^{11}C_7 + {}^{10}C_6 + {}^{9}C_5 + {}^{8}C_4 + {}^{7}C_3$

$= 771.$

$(x+x^3+x^5+x^7+x^9)(x^3+2x^4+3x^5+----)^3$

$x(1+x^2+x^4+x^8)\,x^9(1+2x+3x^2+----)^3$

$x^{10}(1+x^2+x^4+x^8)\left[(1-x)^{-2}\right]^3$

$x^{10}(1+x^2+x^4+x^8)(1-x)^{-6}$

$(1+x^2+x^4+x^8)\sum_{r=0}^{\infty}(5+r)_{C_r}\,x^{10+r}$

$(1+x^2+x^4+x^8)\sum_{r=0}^{\infty}(5+r)_{C_r}\,x^{10+r}$

The coefficient of x^{18} is

${}^{13}C_8 + {}^{11}C_6 + {}^{9}C_4 + {}^{7}C_2 + {}^{5}C_0$

$1897.$

Find the no. of integral solutions of the equation

1. $x_1 + x_2 + x_3 + x_4 + x_5 = 30$. where under the constraints $x_i >= 0$ where $i = 1, 2, 3, 4, 5$ and also x_2 is even and x_3 is odd

First Order Recurrance Relations :

Let a Recurrance Relation of the form

$a_n = (a_{n-1} + f(n)) \rightarrow ①$ where $n \geq 1$. Here c is a constant and $f(n)$ is the known function. This is called a linear recurrance relation of 1st order with constant coefficient.

The relation ① can be solved in a trivial way. Let the relation be changed $n \rightarrow n+1$ then

$a_{n+1} = (a_n + f(n+1))$ for $n \geq 0$ where $n = 0, 1, 2, \ldots$

when $n = 0$.

$a_1 = ca_0 + f(1)$

$a_2 = ca_1 + f(2)$

$= c[ca_0 + f(1)] + f(2)$

$= c^2a_0 + cf(1) + f(2)$

$a_n = c^na_0 + c^{n-1}f(n-1) + \ldots\ldots + c^{n-k}f(k) + f(n)$

$\boxed{a_n = c^na_0 + \sum_{k=1}^{n} c^{n-k} f(k)}$ for $n \geq 1$

If $f(n) = 0$.

$\boxed{a_n = c^na_0}$

Solve the recurrance relation for $a_{n+1} = 8a_n$ for $n \geq 0$ $a_0 = 4$.

$a_{n+1} = 8a_n$

$n = n-1$

$a_n = 8a_{n-1}$ for $n \geq 1$ for $n = 1, 2, \ldots$

$a_1 = 8a_0$

$a_2 = 8a_1 = 8[8a_0] = 8^2 a_0$

$\vdots$

$a_n = 8^n a_0$ [General sol^n]

$a_n = 8^n (4)$

2. Solve the reccurance relation $a_n = na_{n-1}$ for $n \geq 1$ where $a_0 = 1$

$a_n = na_{n-1}$.

$a_1 = 1a_0$

$a_2 = 2a_1 = 2[1a_0]$

$a_3 = 3a_2 = 3[2.1.a_0]$

$\vdots$

$a_n = n! a_0$

$a_n = n! (1)$

$a_n = n!$

Find the reccurance relation and initial condition for the sequence 2, 10, 50, 250 and also find the General term of the sequence.

Sequence $A = \{a_n\}_{n=0}^{\infty}$ $a_0 = 2$, $a_1 = 10$, $a_2 = 50$, $a_3 = 250$.

$a_1 = 5(a_0) = 10$

$a_2 = 5(a_1) = 5^2(a_0)$

$a_3 = 5a_2 = 5^3(a_0)$

$- - - - - -$

$a_n = 5^n a_0$ (General sol^n)

$a_n = 5^n(2)$ (Required sol^n)

Solve the following Reccurence relations by substitutions $a_n = a_{n-1} + 3^n$ for $n \geq 1$ where $a_0 = 1$.

4b3b0

Sol Find a_{12} if $a_{n+1}^2 = 5a_n^2$ where $a_n > 0$ and $a_0 = 2$

n let $a_n^2 = b_n^2$

$b_{n+1} = 5b_n \qquad n \geq 0$

$n=0 \quad b_1 = 5b_0$

$n=1 \quad b_2 = 5b_1 = 5^2 b_0$

$b_n = 5^n . b_0$

$a_n^2 = 5^n . a_0^2$

$a_n^2 = 5^n . 4$

$a_n = \sqrt{5^n (4)}$

$a_{12} = \sqrt{5^{12}(4)}$

$= 31250$

2 Solve the recurrence relation for $a_n = 7a_{n-1}$ for $n \geq 1$ given that $a_2 = 98$ find $a_0 = ?$ and General sol^n.

$a_n = 7^n a_0$

$a_2 = 7^2 a_0$

$98 = 49a_0$

$a_0 = 2$

$\boxed{a_n = 7^n . 2}$

Second and Higher order linear Homogenous recurrance relation.

The recurrance relation is in the form

$C_n a_n + C_{n-1} a_{n-1} + C_{n-2} a_{n-2} = 0$ for $n \geq 2$ — ①

where C_n, C_{n-1}, C_{n-2} are real constants with $C_n \neq 0$ Such type of relation is called 2^{nd} order linear homogenous recurrance relation with constant coefficient.

In eq^n ① substitute $a_n = \alpha^n$ where $\alpha \neq 0$ then

$C_n \alpha^n + C_{n-1} \alpha^{n-1} + C_{n-2} \alpha^{n-2} = 0$ — ②

then eq^n ② α satisfies the quadratic eq^n. This eq^n is called characteristic eq^n or Auxilary eq^n

Similarly for higher order linear homogenous recurrance relation. Consider a recurrance relation

$C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} + \cdots + C_k a_{n-k} = 0$ ③

The characteristic eq^n of this homogenous eq^n is

$C_0 \alpha^n + C_1 \alpha^{n-1} + C_2 \alpha^{n-2} + \cdots + C_k \alpha^{n-k} = 0.$

This characteristic eq^n have k roots say $\alpha_1, \alpha_2, \cdots, \alpha_k$ then the following cases arises.

i. if the roots are not repeated (real & distinct)

i.e., $\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_k$

$a_n^h = C_1 \alpha_1^n + C_2 \alpha_2^n + C_3 \alpha_3^n + \cdots + k \alpha_k^n.$

ii. when the roots are repeated

i.e., $\alpha_1, \alpha_1, \alpha_3, \alpha_4, \dots \alpha_k$

$$a_n^H = (A + Bn)\alpha_1^n + C\alpha_3^n + D\alpha_4^n + \dots + K\alpha_k^n$$

iii. If the roots are Complex

i.e., $k = \alpha \pm i\beta$

$$a_n^H = r^n (A\cos n\theta + B\sin n\theta)$$

$$r = |k_1| = |k_2| = \sqrt{\alpha^2 + \beta^2}$$

$$\theta = \tan^{-1}(\beta/\alpha)$$

$$k = \alpha \pm \beta$$

$$a_n^H = A(\alpha + \sqrt{\beta})^n + B(\alpha - \sqrt{\beta})^n$$

Solve the recurrance relation. $a_n - 7a_{n-1} + 10a_{n-2} = 0$

where $a_0 = 10$, $a_1 = 4$.

A. R $\quad n = n+2$

$$a_{n+2} - 7a_{n+1} + 10a_n = 0.$$

$$\alpha^{n+2} - 7\alpha^{n+1} + 10\alpha^n = 0$$

$$(\alpha^2 - 7\alpha + 10)\alpha^n = 0$$

$$\alpha^2 - 7\alpha + 10 = 0$$

$$(\alpha - 5)(\alpha - 2) = 0$$

$\alpha = 2, 5$ roots are real and distinct.

$$a_n^H = A5^n + B2^n$$

$n=0$

$a_0 = A+B = 10$ —— ①

$n=1$

$a_1 = 5A + 2B = 41$ —— ②

$A = 7$, $B = 3$

$\therefore$ $a_n^H = 7(5)^n + 3(2)^n$

Solve the recurrance relation $a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0$

where $a_0 = 0$ $a_1 = 1$ $a_2 = 10$

$AE \Rightarrow n = n+3$

$a_{n+3} - 9a_{n+2} + 26a_{n+1} - 24a_n = 0$

$\alpha^{n+3} - 9\alpha^{n+2} + 26\alpha^{n+1} - 24\alpha^n = 0$

$(\alpha^3 - 9\alpha^2 + 26\alpha - 24)\alpha^n = 0$

$\alpha^3 - 9\alpha^2 + 26\alpha - 24 = 0$.

$\alpha = 2, 4, 3$. roots are real & distinct

$a_n^H = A2^n + B3^n + C4^n$

$n = 0$

$A + B + C = 0$ —— ①

$n=1$

$2A + 3B + 4C = 1$ —— ②

$n=2$

$4A + 9B + 16C = 10$. —— ③

Solving eq^{ns} we get ① + ②

$$\cancel{2A} + 2B + 2C = 0$$
$$\cancel{2A} + 3B + 4C = 1$$

$$-B - 2C = -1$$
$$-2C = B - 1$$
$$2C = 1 - B$$
$$C = \frac{1-B}{2}$$

$$0 = A + B + C$$
$$1 = 2A + 3B + 4C$$

$$2A + 4B + 3C = 1$$
$$10 = 4A + 16B + 9C$$
$$A = -(B+C)$$
$$-2B - 2C + 4B + 3C = 1$$
$$2B + C = 1$$
$$C = 1 - 2B$$

$$C = 1 - 2\left(\frac{5}{2}\right)$$
$$C = -4.$$

$$0 = A + B + 1 - 2B$$
$$0 = A - B + 1$$
$$A - B = -1 \qquad B = A + 1$$

$$B = \frac{3}{2} + 1$$
$$B = \frac{5}{2}$$

$$10 = 4A + 16B + 9(1 - 2B)$$
$$10 = 4A + 16B + 9 - 18B$$
$$10 = 4A - 2B + 9$$
$$1 = 4A - 2B$$
$$1 = 4A - 2(A+1)$$
$$1 = 4A - 2A - 2$$
$$3 = 2A$$
$$A = 3/2$$

$$a_n^H = \frac{3}{2} 2^n + \frac{5}{2} 4^n + (-4) 3^n$$

5/3/20.

1. Solve the recurrance relation $a_n - 6a_{n-1} + 9a_{n-2} = 0$ for $n \geq 2$, where $a_0 = 5$ $a_1 = 12$.

AE : $\alpha^2 - 6\alpha + 9 = 0$

$(\alpha - 3)^2 = 0$

$\alpha = 3, 3$

roots are real and repetated.

$a_n^H = (A + Bn)3^n$ — (1)

$a_0 = A = 15$

$a_1 = (A+B)3 = 12$

$A + B = 4$,

$B = -1$

$a_n^H = (5-n)3^n$.

2. Solve the recurrance relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$ where $F_0 = 0$, $F_1 = 1$,

$F_{n+2} - F_{n+1} - F_n = 0$

AE : $\alpha^2 - \alpha - 1 = 0$

$\alpha = \frac{1 \pm \sqrt{5}}{2}$

$F_n^H = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$

$0 = A + B$ — (1)

$1 = A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right)$ — (2)

$$A = \frac{1}{\sqrt{5}} \qquad B = -\frac{1}{\sqrt{5}}$$

$$F_n^H = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right) - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)$$

Find the General Sol^n of the recurrance relation.

$a_n - 7a_{n-2} + 10a_{n-4} = 0$ for $n \geq 4$.

AE : $\alpha^4 - 7\alpha^2 + 10 = 0$.

$\alpha^2 = 2, 5$

$\alpha = \pm\sqrt{2}, \pm\sqrt{5}$

$$a_n^H = A\sqrt{2}^n + B(-\sqrt{2})^n + C\sqrt{5}^n + D(-\sqrt{5})^n$$

Solve the recurrance relation

1. $a_n - 7a_{n-1} + 16a_{n-2} - 12a_{n-3} = 0$ for $a_0 = 1$
$a_1 = 4$
$a_2 = 8$

2. $a^2_{n+2} - 5a^2_{n+1} + 4a^2_n = 0$ for $a_0 = 4$, $a_1 = 13$

Let $a_n^2 = b_n$

$b_{n+2} - 5b_{n+1} + 4b_n = 0$

AE : $\alpha^2 - 5\alpha + 4 = 0$

Linear Non-Homogenous recurrance relations of 2nd and Higher Order.

Consider a recurrance relation $c_0a_n + c_1a_{n-1} + \dots + c_k a_{n-k} = f(n)$ — ①
for $n \geq k \geq 2$.

where $c_0, c_1, \dots, c_k$ are constants. The GS of the recurrance relation ① is given by $a_n = a_n^H + a_n^P$ — ②
where a_n^H is Homogenous part, a_n^P is Particular sol^n. To find particular sol^n of the relation we have to follow the following cases.

i. Let $f(x)$ is a polynomial of degree t and β^n is not a root of the characteristic eq^n. then ~~f(n)=~~

$f(x) =$ (polynomial of degree t). β^n in this case a_n^P is taken in the form

$a_n^P = (P_0 n^t + P_1 n^{t-1} + \dots + P_t)\beta^n$ — ③ where
$P_0, P_1, P_2, \dots, P_t$ are constants to be evaluated by using the fact $a_n = a_n^P$ satisfy the relation/eq^n ①.

ii. Suppose $f(x)$ is a polynomial of degree t and β^n is a root of multiplicity not the characteristic eq^n then $a_n^P = n^m(P_0 n^t + P_1 n^{t-1} + \dots + P_t)\beta^n$ — ④
where $P_0, P_1, \dots, P_t$ are constants to be evaluated by using the fact $a_n = a_n^P$ satisfy the

relation 1. where n is the multiplicity.

Solve the recurrance relation $a_n - 6a_{n-1} + 8a_{n-2} = 9$

where $a_0 = 10$, $a_1 = 25$

AE : $\alpha^2 - 6\alpha + 8 = 0$

$\alpha = 4, 2$

$a_n^H = A4^n + B2^n$ —— ①

To find particular soln a_n^P of the relation.

$f(n) = 9 = 9.1^n$

where 9 is a polynomial of degree 0, $\beta = 1$

1 is not a characteristic root

Here $\alpha = 2, 4$ any of them not equal to $\beta = 1$ then

$a_n^P = P_0 . 1^n$

$a_{n-1} = a_{n-2} = P_0$

$P_0 - 6P_0 + 8P_0 = 9$

$3P_0 = 9$

$P_0 = 3$

$\therefore a_n^P = 3.1^n$ is y^P

$\therefore a_n = a_n^H + a_n^P$

$a_n = A4^n + B2^n + 3.1^n$

$n=0$

$10 = A + B + 3$ $\qquad A + B = 7$. —— ①

$n = 1$

$25 = 4A + 2B + 3$

$22 = 4A + 2B$ — ②

$$\begin{array}{r} 2A + \cancel{2B} = 14 \\ 4A + \cancel{2B} = 22 \\ \hline -2A = -8 \end{array}$$

$A = 4 \quad B = 3$

$\therefore a_n = 4.4^n + 3.2^n + 3.1^n$

07/03/20.

2. Solve the recurrance relation $a_n - 6a_{n-1} + 8a_{n-2} = 3^n$ where $a_0 = 3$, $a_1 = 7$.

AE : $x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = 4, 2$

$a_n^H = A4^n + B.2^n$

$f(n) = 3^n = 1.3^n$ is the polynomial of degree 0, $B = 3$ which is not a characteristic root.

Then $a_n^P = (P_0) 3^n$

$a_{n-1} = P_0 . 3^{n-1}$

$a_{n-2} = P_0 . 3^{n-2}$

$P_0 3^n - 6P_0 3^{n-1} + 8P_0 . 3^{n-2}$

$P_0(3^n - 6.3^{n-1} + 8.3^{n-2}) = 3^n$

$3^{n-2} P_0(3^2 - 6.3 + 8) = 3^n$

$3^{n-2} P_0(9 - 18 + 8) = 3^n$

$3^{n-2} P_0(-1) = 3^n$

$P_0 . 3^{n-2}(-1) = 3^n$

$-P_0 = \frac{3^n}{3^{n-2}} = 9$

$P_0 = 9$

$a_n^P = (-9) 3^n$

$a_{n-1} = -9.3^{n-1}$

$a_{n-2} = -9.3^{n-2}$

$\therefore a_n^P = -9.3^n$

$a_n = a_n^H + a_n^P$

$a_n = A.4^n + B.2^n - 9.3^n$ — (*)

$n=0$

$3 = A. + B - 9$

$A + B = 12$

when $n=1$

$7 = 4A + 2B - 27$

$4A + 2B = 34$

$2A + B = 17$

$$\begin{aligned} A + \cancel{B} &= 12 \\ -2A \cancel{+B} &= -17 \end{aligned}$$

$$-A = -5$$

$$A = 5, \quad B = 7$$

$$\therefore a_n = 5 \cdot 4^n + 7 \cdot 2^n - 9 \cdot 3^n$$

Solve the recurrance relation $a_n - 6a_{n-1} + 8a_{n-2} = n \cdot 4^n$ where $a_0 = 3$, $a_1 = 7$.

$$AE = x^2 - 6x + 8 = 0.$$

$$(x-4)(x-2) = 0$$

$$x = 4, 2$$

roots are real and distinct

$$a_n^H = A 4^n + B \cdot 2^n$$

$f(n) = n \cdot 4^n$ is a polynomial of degree 1, and $B = 4$ which is a characteristic eq^n of multiplicity 1.

$$\therefore a_n^P = n^1 (P_0 + P_1 n) 4^n = (P_0 n + P_1 n^2) 4^n$$

$$a_{n-1} = (n-1)(P_0 + P_1(n-1)) 4^{n-1}$$

$$= (P_0(n-1) + P_1(n-1)^2) 4^{n-1}$$

$$a_{n-2} = (P_0(n-2) + P_1(n-2)^2) 4^{n-2}$$

$$(P_0 n + P_1 n^2)4^n - 6((P_0(n-1) + P_1(n-1)^2)4^{n-1})$$

$$+8([P_0(n-2) + P_1(n-2)^2]4^{n-2}] = n.4^n$$

$$(P_0 n + P_1 n^2)4^n - 6[P_0(n-1) + P_1(n^2+1-2n)]4^{n-1}$$

$$+8[P_0(n-2) + P_1(n^2-4n+4)]4^{n-2} = n.4^n$$

$$4^{n-2}\Big[(P_0 n + P_1 n^2)16 - 24[P_0(n-1) + P_1(n^2+1-2n)]$$

$$+8[P_0(n-2) + P_1(n^2-4n+4)]\Big] = n.4^n$$

$$n^2[16P_1 - 24P_1 + 8P_1] + n[16P_0 - 24P_0 + 48P_1 + 8P_0 - 32P_1]$$

$$+ 24P_0 - 24P_1 - 16P_0 + 32P_1 = \frac{n.4^n}{4^{n-2}} = 16n$$

$$n^2(0) + n(0 + 16P_1) + 8P_1 + 8P_0 = 16n$$

Compare coefficients

$16P_1 = 16$

$\boxed{P_1 = 1}$

$8P_0 + 8P_1 = 0$

$P_0 = P_1$ $\boxed{P_0 = -1}$

$2A + B = 1+$

$a_n^p = n(-1+n)\,4^n$

$= (-n+n^2)\,4^n$

$= (n^2-n)\,4^n$

$\therefore$ GS: $a_n = a_n^h + a_n^p$

$= A4^n + B.2^n + (n^2-n)4^n$ — (x)

$a_0 = 3$

$n = 0$

$3 = A+B$

$n = 1$

$7 = 4A + 2B$

$$\begin{array}{r} 2A+2B = 6 \\ 4A+2B = 7 \\ \hline -2A = -1 \end{array}$$

$A = \frac{1}{2}$

$B = 3 - \frac{1}{2}$

$B = \frac{5}{2}$

$\therefore a_n = \frac{1}{2}4^n + \frac{5}{2}2^n + (n^2-n)4^n$

Solve the recurance relation $a_n + 3a_{n-1} - 10a_{n-2} = n^2+n+1$

AE: $\alpha^2 + 3\alpha - 10 = 0$

$(\alpha-2)(\alpha+5) = 0$

$\alpha = -5, 2$. roots are real and distinct.

$a_n^h = A(2)^n + B(-5)^n$

$f(n) = (n^2 + n + 1) 1^n$

is a ponimoial of degree 2 and $B = 1$ which is not a characteristic root / eq^n.

$\therefore \; a_n^p = (P_0 + P_1 n + P_2 n^2) 1^n$

$a_{n-1} = (P_0 + P_1(n-1) + P_2(n-1)^2)$

$= P_0 + P_1(n-1) + P_2[n^2 + 1 - 2n)$

$a_{n-2} = P_0 + P_1(n-2) + P_2(n^2 - 4n + 4)$

$(P_0 + P_1 n + P_2 n^2) + 3(P_0 + P_1(n-1) + P_2(n^2+1-2n))$

$-10(P_0 + P_1(n-2) + P_2(n^2 - 4n + 4)) = n^2 + n + 1.$

$n^2(P_2 + 3P_2 - 10P_2) + n(P_1 + 3P_1 - 6P_2 - 10P_1 + 40P_2)$

$+ P_0 + 3P_0 - 3P_1 + 3P_2 - 10P_0 + 20P_1 - 40P_2 = n^2 + n + 1$

$n^2(-6P_2) + n(-6P_1 + 34P_2) + -6P_0 + 17P_1 - 37P_2$

$= n^2 + n + 1$

Comparing the Coefficients:

$-6P_2 = 1$

$\boxed{P_2 = -1/6}$

$$-6P_1 + 34P_2 = 1$$

$$-6P_1 + 34\left(\frac{-1}{6}\right) = 1$$

$$-6P_1 - \frac{17}{3} = 1$$

$$-6P_1 = 1 + \frac{17}{3}$$

$$-6P_1 = \frac{20}{3}$$

$$\boxed{P_1 = -\frac{10}{9}}$$

$$-37P_2 + 17P_1 - 6P_0 = 1$$

$$-37\left(\frac{-1}{6}\right) + 17\left(\frac{-10}{9}\right) - 6P_0 = 1.$$

$$\frac{37}{6} - \frac{170}{9} - 6P_0 = 1.$$

$$\frac{37}{6} - \frac{170}{9} - 1 = 6P_0$$

$$6P_0 = \frac{-247}{18}$$

$$P_0 = \frac{-247}{108}$$

$$\therefore \text{Go} : a_n^P = \frac{-247}{108} + \left(-\frac{10}{9}\right)n + \left(\frac{-1}{6}\right)n^2$$

$$\therefore a_n = A(2)^n + B(-1)^n 5^n + \left(\frac{-247}{108} - \frac{10}{9}n - \frac{1}{6}n^2\right) //$$

Solve the recurrance relation $a_n - 7a_{n-1} + 10a_{n-2} = 7.3^n + 4^n$

$\rightarrow a_n - 5a_{n-1} + 6a_{n-2} = n^2 . 4^n$

Method of generating functions for 1 order Recurance relations.

Let the recurrance relation is in the form

$a_n = Ca_{n-1} + f(n)$ for $n \geq 1$ which is equivalent to

$a_{n+1} = Ca_n + \overset{\phi(n)}{f(n+1)}$ for $n \geq 0$ — ①

where C is a constant and $\phi(n) = f(n+1)$ is a given function. multiple both side relation ① by x^{n+1} to get

$a_{n+1} . x^{n+1} = Ca_n . x^{n+1} + \phi(n) x^{n+1}$ This can be written

as $\sum\limits_{n=0}^{\infty} a_{n+1} x^{n+1} = \sum\limits_{n=0}^{\infty} Ca_n . x^{n+1} + \sum\limits_{n=0}^{\infty} \phi(n) . x^{n+1}$ for $n = 0, 1, 2$

suppose $\sum\limits_{n=1}^{\infty} a_n x^n - Cx \sum\limits_{n=0}^{\infty} a_n x^n = x \sum\limits_{n=0}^{\infty} \phi(n) x^n$

$$\left(\sum_{n=0}^{\infty} (a_n x^n - a_0)\right) - Cx \sum_{n=0}^{\infty} a_n x^n = x \sum_{n=0}^{\infty} \phi(n) x^n$$

$$\Rightarrow \sum_{n=0}^{\infty} a_n x^n (1 - Cx) = a_0 + x \sum_{n=0}^{\infty} \phi(n) x^n \quad \text{— ②}$$

$$f(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$g(x) = \sum_{n=0}^{\infty} \phi(n) x^n$$

$$(1 - Cx) f(x) = a_0 + x g(x)$$

$$\boxed{f(x) = \frac{a_0 + xg(x)}{1 - cx}}$$

10/08/20

Find a generating f'n for the recurrence relation $a_{n+1} - a_n = 3^n$, $n \geq 0$ where $a_0 = 1$ and also solve the relation.

$a_{n+1} = Ca_n + \phi(n) \qquad a_{n+1} - a_n = 3^n$

$C = 1 \,,\ \phi(n) = 3^n \qquad a_{n+1} = a_n + 3^n$

$$f(x) = \frac{a_0 + xg(x)}{1 - Cx}$$

$$= \frac{1 + xg(x)}{1 - x}$$

$$g(x) = \sum_{n=0}^{\infty} \phi(n) x^n$$

$$= \sum_{n=0}^{\infty} 3^n x^n$$

$$= 1 + 3x + 9x^2 + \cdots$$

$$g(x) = (1 - 3x)^{-1}$$

$$f(x) = \frac{1 + x(1-3x)^{-1}}{(1-x)} = \frac{1}{(1-x)} + \frac{x}{(1-x)(1-3x)}$$

$$= \frac{1 - 3x + x}{(1-x)(1-3x)} = \frac{1 - 2x}{(1-x)(1-3x)}$$

$$f(x) = \frac{1 - 2x}{(1-x)(1-3x)} = \frac{A}{(1-x)} + \frac{B}{(1-3x)}$$

$1-2x = A(1-3x) + B(1-x)$

Put $x = \frac{1}{3}$

$1-2\left(\frac{1}{3}\right) = B\left(1-\frac{1}{3}\right)$

$\frac{1}{3} = B\left(\frac{2}{3}\right)$

$B = \frac{1}{2}$

$x = 1$

$1-2 = A(1-3)$

$-1 = -2A$

$A = \frac{1}{2}$

$$f(x) = \frac{1}{2(1-x)} + \frac{1}{2(1-3x)}$$

$$= \frac{1}{2}\left[(1-x)^{-1} + (1-3x)^{-1}\right]$$

$$f(x) = \frac{1}{2}\left[\sum_{n=0}^{\infty}(x^n + 3^n x^n)\right]$$

$$= \sum_{n=0}^{\infty}\left(\frac{1+3^n}{2}\right)x^n$$

$$\therefore a_n = \frac{1+3^n}{2}$$

2. $a_n - 3a_{n-1} = n \qquad n \geq 1, \; a_0 = 1$

$n = n+1$

$a_{n+1} - 3a_n = n+1 \qquad n \geq 0,$

$C = 3 \qquad \phi(n) = n+1$

$$f(x) = \frac{a_0 + x\,g(x)}{1-Cx} = \frac{1+x\,g(x)}{1-3x}$$

$$g(x) = \sum_{n=0}^{\infty} \phi(n) x^n$$

$$= \sum_{n=0}^{\infty} (n+1) x^n$$

$$g(x) = (1-x)^{-2}.$$

$$\therefore f(x) = \frac{1 + x(1-x)^{-2}}{1-3x}$$

$$= \frac{1 + \frac{x}{(1-x)^2}}{1-3x} = \frac{(1-x)^2 + x}{(1-3x)(1-x)^2}$$

$$= \frac{(1-2x+x^2)+x}{(1-3x)(1-x)^2} = \frac{x^2-x+1}{(1-3x)(1-x)^2}.$$

$$f(x) = \frac{A}{(1-3x)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2}.$$

$$x^2-x+1 = A(1-x)^2 + B(1-3x)(1-x) + C(1-3x)$$

Put $x = \frac{1}{3}$

$$\frac{1}{9} - \frac{1}{3} + 1 = A\left(1-\frac{1}{3}\right)^2 + B(0) + C(0)$$

$$\frac{7}{9} = A\left(\frac{4}{9}\right) \qquad A+B+C = 1.$$

$$A = \frac{7}{9}\left(\frac{9}{4}\right) = \frac{7}{4}. \qquad \frac{7}{4} - \frac{1}{2} + B = 1$$

$$B = -\frac{1}{4}$$

Put $x = 1$

$$1-1+1 = A(0) + B(0) + C(-2)$$

$$C = -1/2$$

$$= \frac{7}{4(1-3x)} - \frac{1}{4(1-x)} - \frac{1}{2(1-x)^2}$$

$$= \frac{1}{4}\left[\frac{7}{(1-3x)} - \frac{1}{(1-x)} - \frac{2}{(1-x)^2}\right]$$

$$= \frac{1}{4}\left[7(1-3x)^{-1} - (1-x)^{-1} - 2(1-x)^{-2}\right]$$

$$= \frac{1}{4}\left[7\sum_{n=0}^{\infty} 3^n x^n - \sum_{n=0}^{\infty} x^n - 2\sum_{n=0}^{\infty}(n+1)\right]$$

$$= \sum_{n=0}^{\infty}\left[\frac{7}{4}3^n - \frac{1}{4} - \frac{1}{2}(n+1)\right]x^n$$

$$a_n = \frac{7}{4}3^n - \frac{1}{4} - \frac{(n+1)}{2}$$

Method of Generating f'n for 2nd Order recurrance relations.

Let the recurrance relation is in the form

$a_n + A_1 a_{n-1} + A_2 a_{n-2} = f(n)$ for $n \geq 2$ which is equivalent to $a_{n+2} + A a_{n+1} + A_2 a_n = f(n+2)$ $n \geq 0$ — (1)

where $\phi(n) = f(n+2)$.

$$f(x) = \frac{a_0 + (a_1 + a_0 A_1)x + x^2 g(x)}{1 + A_1 x + A_2 x^2}$$

$$g(x) = \sum_{n=0}^{\infty} \phi(n) x^n \qquad f(x) = \sum_{n=0}^{\infty} a_n x^n$$

solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n$ $n \geq 0$ where $a_0 = 1$, $a_1 = 2$. By the method of generating f'ns.

$a_{n+2} - 2a_{n+1} + a_n = 2^n$.

$A_1 = -2 \quad A_2 = 1 \quad \phi(n) = 2^n$

$$f(x) = \frac{1 + (2 + 1(-2))x + x^2 g(x)}{1 + (-2)x + 1(x^2)}$$

$$= \frac{1 + x^2 g(x)}{1 - 2x + x^2} = \frac{1 + x^2 g(x)}{x^2 - 2x + 1}$$

$$g(x) = \sum_{n=0}^{\infty} \phi(n) x^n = \sum_{n=0}^{\infty} 2^n x^n$$

$$g(x) = (1-2x)^{-1}$$

$$f(x) = \frac{1 + x^2 (1-2x)^{-1}}{(1-x)^2} = \frac{1 - 2x + x^2}{(x^2 - 2x + 1)(1-2x)}$$

$$= \frac{\cancel{(x^2 - 2x + 1)}}{\cancel{(x^2 - 2x + 1)}(1-2x)} = \frac{1}{(1-2x)}$$

$$= (1-2x)^{-1}$$

$$= \sum_{n=0}^{\infty} x^n 2^n$$

$\therefore$ $a_n = 2^n$ // is the required solution.

$a_{n+2} - 3a_{n+1} + 2a_n = 0$, $n \geq 0$ $\quad a_0 = 1$, $a_1 = 6$.

$A_1 = -3 \quad A_2 = 2 . \quad \phi(n) = 0$

$$f(x) = \frac{1 + (6 + 1(-3))x + x^2 g(x)}{1 + (-3)x + 2x^2}$$

$$= \frac{1 + 3x + x^2 g(x)}{1 - 3x + 2x^2}$$

$$g(x) = \sum_{n=0}^{\infty} \phi(n) x^n$$

$$= 0.$$

$$f(x) = \frac{1 + 3x}{1 - 3x + 2x^2} = \frac{1 + 3x}{(1 - x)(1 - 2x)}$$

$$\frac{1 + 3x}{(1 - x)(1 - 2x)} = \frac{A}{(1 - x)} + \frac{B}{(1 - 2x)}$$

$$1 + 3x = A(1 - 2x) + B(1 - x)$$

Put $x = 1$ $\qquad x = \frac{1}{2}$

$1 + 3 = A(-1)$ $\qquad 1 + \frac{3}{2} = B\left(1 - \frac{1}{2}\right)$

$\boxed{A = -4}$ $\qquad \frac{5}{2} = B\left(\frac{1}{2}\right)$

$\boxed{B = 5}$

$$f(x) = \frac{-4}{(1-x)} + \frac{5}{(1-2x)}$$

$$= -4(1-x)^{-1} + 5(1-2x)^{-1}$$

$$= -4\sum_{n=0}^{\infty} x^n + 5\sum_{n=0}^{\infty} 2^n x^n$$

$$= \sum_{n=0}^{\infty} \left[-4 + 5.2^n\right] x^n.$$

$a_n = 5.2^n - 4$ //

11/3/20.

$a_{n+2} - 5a_{n+1} + 6a_n = 2 \quad n \geq 0. \quad a_0 = 3$
$a_1 = 7.$

$A_1 = -5 \quad A_2 = 6 \quad \phi(n) = 2.$

~~$f(x) = \frac{a_0 + x g(x)}{1 - Cx} = \frac{3 + x g(x)}{1 -}$~~

$$f(x) = \frac{a_0 + (a_1 + a_0 A_1)x + x^2 g(x)}{1 + A_1 x + A_2 x^2}$$

$$= \frac{3 + (7 + 3(-5))x + x^2 g(x)}{1 - 5x + 6x^2}$$

$$= \frac{3 + (-8)x + x^2 g(x)}{1 - 5x + 6x^2}$$

$$= \frac{3 - 8x + x^2 g(x)}{1 - 5x + 6x^2}$$

$g(x) = \sum_{n=0}^{\infty} \phi(n) x^n$

$= \sum_{n=0}^{\infty} 2x^n$

$= 2\sum_{n=0}^{\infty} x^n$

$g(x) = 2(1-x)^{-1}$

$$f(x) = \frac{3-8x+x^2\,2(1-x)^{-1}}{6x^2-5x+1} = \frac{(3-8x)(1-x)+2x^2}{6x^2-5x+1\,(1-x)}$$

$$= \frac{3-8x-3x+8x^2+2x^2}{(6x^2-5x+1)(1-x)} = \frac{10x^2-11x+3}{(3x-1)(2x-1)(1-x)}$$

$$\frac{10x^2-11x+3}{(3x-1)(2x-1)(1-x)} = \frac{A}{3x-1} + \frac{B}{2x-1} + \frac{C}{1-x}$$

$$10x^2-11x+3 = A(2x-1)(1-x) + B(3x-1)(1-x) + C(3x-1)(2x-1)$$

Put $x=1$

$10-11+3 = C(2)(1)$

$\boxed{C=1}$

Put $x = \frac{1}{2}$

$\frac{10}{4} - \frac{11}{2} + 3 = B\left(\frac{3}{2}-1\right)\left(\frac{1}{2}\right) \quad \cancel{x = \frac{10}{4}}$

$\boxed{B=0}$

Put $x = \frac{1}{3}$

$10\left(\frac{1}{9}\right) - \frac{11}{3} + 3 = A\left(\frac{2}{3}-1\right)\left(1-\frac{1}{3}\right)$

$\frac{4}{9} = A\left(-\frac{2}{9}\right) \quad \boxed{A=-2}$

$$f(x) = \frac{-2}{3x-1} + \frac{1}{1-x} = -2(3x-1)^{-1} + (1-x)^{-1}$$

$$= +2(1-3x)^{-1} + (1-x)^{-1} = 2\sum_{n=0}^{\infty} 3^n x^n + \sum_{n=0}^{\infty} x^n$$

$$\sum_{n=0}^{\infty}\left[2.3^n+1\right]x^n$$

$\therefore\ a_n = 2.3^n+1$

Principal of inclusion and exclusion.

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

2. $n(A \cup B \cup C) = n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cap B) - n(A \cap C) - n(B \cap C)$

The sum rule which is applied to the non-disjoint sets is called principal of inclusion and exclusion.

Find the number of integers b/w 1 to 1000 that are not divisible by 2, 3, 5, 7.

$U = \{x \mid 1 \leq x \leq 1000\}$

$A = \{x \mid 1 \leq x \leq 1000, x \text{ is divisible by } 2\}$

$B = \{x \mid 1 \leq x \leq 1000, x \text{ is divisibly by } 3\}$

$C = \{x \mid 1 \leq x \leq 1000, x \text{ is divisible by } 5\}$

$D = \{x \mid 1 \leq x \leq 1000, x \text{ is divisible by } 7\}$

$n(A) = |A| = \frac{1000}{2} = 500$

$n(B) = |B| = \frac{1000}{3} = 333$

$n(C) = |C| = \frac{1000}{5} = 200$

$n(D) = |D| = \frac{1000}{7} = 142$

$A \cap B = \{x \mid 1 \leq x \leq 1000, x \text{ is divisible by } 2, 3\}$

$|A \cap B| = \frac{1000}{2\times 3} = 166$ 2, 3, 5, 7

$|B \cap C| = \frac{1000}{3\times 5} = 66$

$|C \cap D| = \frac{1000}{5\times 7} = 28$

$|D \cap A| = \frac{1000}{7\times 2} = 71$

$|D \cap B| = \frac{1000}{7\times 3} = 47$

$|C \cap A| = \frac{1000}{5\times 3} = 100.$

$(A \cap B \cap C) = \{x \mid 1 \le x \le 1000,\ x \text{ is divisibly by } 2, 3, 5\}$

$|A \cap B \cap C| = \frac{1000}{30} = 33$

$|B \cap C \cap D| = \frac{1000}{3\times 5\times 7} = 9$

$|C \cap D \cap A| = \frac{1000}{5\times 7\times 2} = 14$

$|D \cap A \cap B| = \frac{1000}{2\times 3\times 7} = 23$

$(A \cap B \cap C \cap D) = \{x \mid 1 \le x \le 1000,\ x \text{ is div by } 2, 3, 5, 7\}$

$A \cap B C \cap D = \frac{1000}{2\times 3\times 5\times 7} = 4$

$$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |B \cap C| - |C \cap D| - |D \cap A| - |D \cap B| - |C \cap A| + |A \cap B \cap C| + |B \cap C \cap D| + |C \cap D \cap A| + |D \cap A \cap B| - |A \cap B \cap C \cap D|$$

$$= 500 + 333 + 200 + 142 - 166 - 66 - 28 - 71 - 47 - 100 + 33 + 9 + 14 + 23 - 4$$

$$= 1175 - 478 + 79 - 4$$

$$= 772$$

$$|\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}| = U - |A \cup B \cup C \cup D|$$

$$= 1000 - 772 = 228$$

How many integers from 1 to 10^6 inclusive are neither perfect squares nor perfect cubes nor perfect fourth powers.

$U = \{x \mid 1 \le x \le 10^6\}$

$A = \{x \mid 1 \le x \le 10^6, x \text{ is a perfect square}\}$

$B = \{x \mid 1 \le x \le 10^6, x \text{ is a perfect cube}\}$

$C = \{x \mid 1 \le x \le 10^6, x \text{ is a perfect fourth powers}\}$

$|A| = \sqrt{10^6} = 1000$

$|B| = (10^6)^{\frac{1}{3}} = 100$

$|C| = (10^6)^{\frac{1}{4}} = 31,$

$|A \cap B| = \{x \mid 1 \le x \le 10^6,\ x \text{ is a perfect square \& perfect cube}\}$

$= \underline{(10^6)^{\frac{1}{6}}} = 10$

$|B \cap C| = (10^6)^{\frac{1}{3} \times \frac{1}{4}} = 3$

$|C \cap A| = (10^6)^{\frac{1}{4} \times \frac{1}{2}} = \cancel{5}\ 31$

$|A \cap B \cap C| =$

$= (10^6)^{\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}} = 3$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A|$

$+ |A \cap B \cap C|$

$= 1000 + 100 + \cancel{31} - 10 - 3 - \overset{31}{\cancel{5}} + 3$

$= 1131 - 43$

$= 1090$

$|A \cap \bar{B} \cap \bar{C}| = U - |A \cup B \cup C|$

$= \cancel{10^5 - 1088} = 10^6 - 1090$

$= \cancel{998856} . = 998910$ //

12/03/20.

*

From a group of 10 professers how many ways can commite of 5 members we found so that atleast one of the professersA and profeser B will be included. solve this by using principal of inclusion and exclusion.

$^{10}C_5 = U.$

$|A| = {}^9C_4 = \frac{9 \times 8 \times 5 \times 4}{4 \times 3 \times 2 \times 1} = 126$ $= 252$ ways

$|B| = {}^9C_4 = 126$

$|A \cap B| = {}^8C_3 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 45 + 45 - 56 = 34$

A comitte of 5 members is formed out of 10members in $^{10}C_5 = 252$ ways

The comitte should include atleast one of the profesor A or professor B or the comittee can include both professors A and B.

Let A and B two comittes with include professor A and professor B respectively.

A comitte which include both the professors is 56 $|A \cap B|$

∴ the total no. of comittes that include atleast one of the professor is $|A \cup B| = |A| + |B| - |A \cap B|$

$= 126 + 126 - 56$

$= 196$

★

In howmany ways can the letters $\{4a, 3b, 2c\}$ be arranged so that all the letters of same kind are not in a single block.

Let S be any set with 9 objects that can be arranged

in $S = \frac{9!}{4!3!2!} = 1260$

Let A denote the set of arranged letters where 4a are in single block.

$|A| = \frac{6!}{1!3!2!} = 60$

Let B denote the set of arranged letters where 3b are in single block

$|B| = 105$

$|C| = \frac{8!}{4!3!} = 280$

$|A \cap B| = \frac{4!}{2!} = 12$

$|B \cap C| = \frac{6!}{4!} = 30$

$|C \cap A| = \frac{5!}{3!} = 20$

$|A \cap B \cap C| = \frac{3!}{1!} = 6$

$|\bar{A} \cap \bar{B} \cap \bar{C}| = U - |A \cup B \cup C|$

$= 1260 - [60 + 105 + 280 - 12 - 30 - 20 + 6]$

$= 1260 - 389$

14/03/20 $= 871$

Find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$. Under the constraints x_i is less than or equal to 7 $x_i \leq 7$ for $i = 1, 2, 3, 4$.

Let 'S' denote the eqn $x_1 + x_2 + x_3 + x_4 = 18$ — (1)

$n = 4, \quad r = 18$

no. of non-negative integer sol^{n}s

$^{(n+r-1)}C_r = {}^{(4+18-1)}C_{18} = C(21, 18) = {}^{21}C_{18} = 5$

Let A be the subset of S that contains the non-negative integer sol^{n}s of the given eq^n under the condition.

$x_1 > 7,\ x_2 \geq 0,\ x_3 \geq 6,\ x_4 \geq 6$

i.e., $A = \{(x_1, x_2, x_3, x_4) \in S \mid x_1 > 7\}$

$B = \{(x_1, x_2, x_3, x_4) \in S \mid x_2 > 7\}$

$C = \{(x_1, x_2, x_3, x_4) \in S \mid x_3 > 7\}$

$D = \{(x_1, x_2, x_3, x_4) \in S \mid x_4 > 7\}$

Then the required no. of solutions would be

$|\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}|$

Let us set $y_1 = x_1 - 8$

then $x > 7$ (i.e., $x \geq 8$) corresponds to

$y_1 \geq 0$. then write eqn ① in terms of y.

$$y_1 + x_2 + x_3 + x_4 = 10 \quad - ②$$

$\therefore$ no. of non-negative integer solutions of

$$^{(4+10-1)}C_{10} = C(13, 10)$$

$\therefore |A| = C(13, 10)$

$|||^{ly}$ $|B| = |C| = |D| = C(13, 10)$

Let us take $y_1 = x_1 - 8$, $y_2 = x_2 - 8$

$$y_1 + y_2 + x_3 + x_4 = 2 \quad — ③$$

no of non-ve integer soln for eqn ③

$$^{(4+2-1)}C_2 = C(5, 2)$$

$|A \cap B| = C(5,C) = |A\ C| = |A\ D| = |B\ C| = |B\ D| =$
$|C\ D| = |A\ B|$

In the given eq^n more than 2 x_i's cannot be greater than 7 simultaneously.

Hence

$|A \cap B \cap C| = 0 \quad |A \cap B \cap D| = |B \cap C \cap D| = |A \cap C \cap D|$

$\therefore |A \cap B \cap C \cap D| = 0$

$\therefore$ The no. of non-ve integer solutions is

$|\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}| = S - [|A \cup B \cup C \cup D|]$

$= 1330 - [{}^4C_1(286) - {}^4C_2(10) + 0 - 0]$

$= 1330 - [1144 - 60]$

$= 246$

Find the no of non negative integer sol^{n}s of the eq^n $x_1 + x_2 + x_3 = 20$ such that $2 \le x_1 \le 5$, $4 \le x_2 \le 7$, $-2 \le x_3 \le 9$

Let $y_1 = x_1 - 2$, $y_2 = x_2 - 4$, $y_3 = x_3 + 2$.

since $x_1 \ge 2$, $x_2 \ge 4$, $x_3 \ge -2$ we have $y_1 \ge 0$, $y_2 \ge 0$, $y_3 \ge 0$. Then write the given eq^n in terms of y's.

$y_1 + y_2 + y_3 = 16$ — ①

no. of non-negative integer sol^{n}s

$^{(3+16-1)}C_{16} = C(18,16)$

when $x_1 \le 5$ we have $y_1 \le 3$,

$x_2 \le 7$ $\quad y_2 \le 3$

$x_3 \le 9$ $\quad y_3 \le 11$

Now let $A = \{(y_1, y_2, y_3) \in S \mid y_1 > 3\}$

$B = \{(y_1, y_2, y_3) \in S \mid y_2 > 3)$

$C = \{(y_1, y_2, y_3) \in S \mid y_3 > 11\}$

we have to find $|\bar{A} \cap \bar{B} \cap \bar{C}|$

Let us set $z_1 = y_1 - 4$ then $(y_1 > 3)$ (ie., $y_1 \ge 4$)

correspondingly

$z_1 \ge 0$.

$z_1 + y_2 + y_3 = 12$

the no. of non-negtive integer sol^ns for

$^{(3+12-1)}C_{12} = C(14,12)$

$|A| = |B| = C(14,12)$

$z_3 = y_3 - 12$

$y_1 + y_2 + z_3 = 4$

$^{(3+4-1)}C_4 = C(6,4) = |C|$.

Let set $z_1 = y_1 - 4$, $z_2 = y_2 - 4$

$z_1 + z_2 + y_3 = 8$

$(3+8-1)C_8 = C(10,8) = |A \cap B|$

$|B \cap C| = |C \cap A| = C(2,0)$

In any soln of given eqn ① we cannot have

$y_1 > 3,\ y_2 > 3,\ y_3 > 11.$

$\therefore\ |A \cap B \cap C| = 0$

$|\bar{A} \cap \bar{B} \cap \bar{C}| = |S| - [|A \cup B \cup C|]$

$= 153 - [91 + 91 + 15 - 45 - 1 - 1 + 0]$

$= 153 - 150$

$= 3$