

# Heat Transfer Analysis with Critical Radius

## Key Analysis

### Heat Transfer Mechanism:

- Conduction through insulation (radial direction).
- Convection from the insulation's outer surface to the surroundings.

### Thermal Resistances:

#### Conduction Resistance (insulation):

$$R_{\text{cond}} = \ln(r_2 / r_1) / (2\pi k L)$$

#### Convection Resistance:

$$R_{\text{conv}} = 1 / (h \cdot 2\pi r_2 L)$$

#### Total Resistance:

$$R_{\text{total}} = R_{\text{cond}} + R_{\text{conv}}$$

#### Heat Transfer Rate:

$$Q = (T_1 - T) / R_{\text{total}}$$

For simplicity, assume  $L = 1\text{m}$ :

$$Q = (2\pi (T_1 - T)) / [\ln(r_2 / r_1) / k + 1 / (h \cdot r_2)]$$

#### Critical Radius of Insulation:

For cylinders, critical radius  $r_c = k / h$

- If  $r_2 < r_c$ : Adding insulation increases heat loss.
- If  $r_2 > r_c$ : Adding insulation decreases heat loss.

#### Plotting $Q$ vs $r$ (for $r_2$ ):

##### Behavior:

- $Q$  initially decreases until  $r_2 = r_c$ .
- $Q$  then increases for  $r_2 > r_c$ .

##### Conclusion:

The plot shows a U-shaped curve due to the critical radius effect. Insulation is only beneficial when  $r_2 > r_c$ .

Heat Transfer vs Radius for Cylindrical Pipe

