SECTIONS 14.6 - 15.4 / materials in previous sections could be indirectly used

14.6 14.7 14.7 14.8 15.1 15.2 15.2 15.3 15.4

{CHAPTER 14}

$\nabla f(x,y,z) = \langle f_x, f_y, f_z \rangle$

$D_{\vec{u}} f(a,b,c) = \lim_{h \to 0} \dfrac{f(a+hu_1, b+hu_2, c+hu_3) - f(a,b,c)}{h}$ is the directional derivative along unit vector $\vec{u} = \langle u_1, u_2, u_3 \rangle$.

- ∇f is vector ; $D_{\vec{u}} f$ is a scalar
- $D_{\vec{i}} f(x,y,z) = f_x$; $D_{\vec{j}} f(x,y,z) = f_y$; $D_{\vec{k}} f(x,y,z) = f_z$
- $D_{\vec{u}} f(a,b,c) = \nabla f(a,b,c) \cdot \vec{u}$
- $\vec{\nabla} f(a,b,c)$ is direction of fastest increase @ (a,b,c)
- $-\vec{\nabla} f(a,b,c)$ is direction of fastest decrease @ (a,b,c)
- $|\nabla f(a,b,c)|$ = max directional derivative @ (a,b,c)
- $-|\nabla f(a,b,c)|$ = min directional derivative @ (a,b,c)
- $\nabla f(a,b,c)$ is normal vector of tangent plane to level curve $f(x,y,z) = k$ @ (a,b,c)
- $\dfrac{df}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt} + \dfrac{\partial f}{\partial z}\dfrac{dz}{dt}$
  $= \nabla f \cdot r'(t)$

- $f(x,y)$ has critical pt at (a,b) if:
  $\nabla f(a,b) = \vec{0}$ or undefined

$H = \det \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = f_{xx} f_{yy} - f_{xy}^2$

(If f_{xx}, f_{yy} have diff signs, then it is saddle).

(1) If $H > 0$ and $f_{xx} > 0$, (a,b) is a local min
(2) If $H > 0$ and $f_{xx} < 0$, (a,b) is a local max
(3) If $H < 0$, (a,b) is a saddle point
(4) If $H = 0$, the test is inconclusive.

Saddle: 2 intersecting level curves of same value
Max/Min: Concentric circles

LAGRANGE MULTIPLIERS

- $f(x,y,z)$ with constraint $g(x,y,z) = 0$

Solve $\nabla f(x,y,z) = \lambda \nabla g(x,y,z)$;
$g(x,y,z) = 0$

Evaluate $f(x,y,z)$ @ all solutions

Compare values from step 2

Types of domains:

- OPEN: None of boundary included
- CLOSED: Contains all points on boundary
- BOUNDED: If domain finite (contained in some large enough circle)

$D = \mathbb{R}^2 \to$ closed AND open, not bounded

$D = \{(x,y) \mid x, y \geq 0\}$ closed, not bounded

- EXTREME VALUES
  - $f(x,y)$ @ critical points
  - Find min/max on boundary
  - Compare values from S1/S2

hyperbolas on level curves $\to$ saddle

Can use to find extreme value of functions on boundary of domain for global extrema

To show a func has no max/min, fix variables and show single-variable limit as $x \to \infty$ is $\infty / -\infty$.

CHAPTER 15

$$\iint_R f(x,y)\,dA = \lim_{m,n\to 0} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i^*, y_j^*)\,\Delta A$$

(Divide rectangular domain into $m \times n$ squares)

(Multiply sample point in each subregion w/ area of subregion)

Midpoint Rule: Use midpoint of each subregion as sample points.

$$\iint_R f(x,y)\,dA = \int_a^b\int_c^d f(x,y)\,dy\,dx = \int_c^d\int_a^b f(x,y)\,dx\,dy$$

Type I Domain: dy dx

Type II Domain: dx dy

- Bounds for outer integral must be const.
- Cannot randomly flip – must draw out to convert

(1) $\iint_D f+g\,dA = \iint_D f\,dA + \iint_D g\,dA$

(2) $c\iint_D f(x,y)\,dA = c\iint_D f(x,y)\,dA$

$$\iint_D f(x,y)\,dA = \iint_D f(r\cos\theta, r\sin\theta)\cdot r\,dr\,d\theta$$

bounds in r, θ

JACOBIAN

$x = r\cos\theta$

$y = r\sin\theta$ $\quad r^2 = x^2 + y^2$

For lamina w/ density $\rho(x,y)$ over domain D

- Mass: $\iint_D \rho(x,y)\,dA = m$
- COM $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{1}{m}\iint_D x\,\rho(x,y)\,dA$$

$$\bar{y} = \frac{1}{m}\iint_D y\,\rho(x,y)\,dA$$

pay attention to symmetry

(3) If $D = D_1 + D_2$:

$$\iint_D f(x,y)\,dA = \iint_{D_1} f(x,y)\,dA + \iint_{D_2} f(x,y)\,dA$$

(4) $\iint_D 1\,dA = \text{Area}(D)$

(5) Average Value of $f(x,y)$ on D $\quad \frac{1}{\text{Area}(D)}\iint_D f(x,y)\,dA$

(6) $\iint_D f(x,y)\,dA$ = signed volume of solid between $z = f(x,y)$ and D on xy-plane.

If $f(x,y)$ is odd in x and is symmetric about the y-axis, then $\iint_D f(x,y)\,dA = 0$.

Similarly, if $f(x,y)$ is odd in y and symmetric about x-axis, then $\iint_D f(x,y)\,dA = 0$.

If $f(x,y)$ even, and domain symmetric, can be split up & multiplied.