श्री BalaJi

Vikas Gupta
Pankaj Joshi

Solution

Advanced Problems in

Mathematics

for JEE

Main & Advanced

7th edition

श्री BalaJi

SOLUTION *to*

Advanced Problems

in

MATHEMATICS

for

JEE (MAIN & ADVANCED)

by :

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CALCULUS

ALGEBRA

CO-ORDINATE GEOMETRY

TRIGONOMETRY

VECTOR & 3DIMENSIONAL GEOMETRY

1

FUNCTION

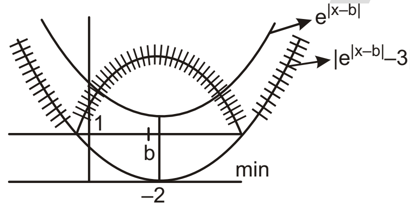
Exercise-1 : Single Choice Problems

1. $f(x) \quad \log_2(2 \quad 2\log_{\sqrt{2}}(16\sin^2 x \quad 1))$

$0 \quad \log_{\sqrt{2}}(16\sin^2 x \quad 1) \quad \log_2 17 \qquad 2 \quad 2\log_2 17 \quad 2 \quad 2\log_{\sqrt{2}}(16\sin^2 x \quad 1) \quad 2$

$0 \quad 2 \quad 2\log_{\sqrt{2}}(16\sin^2 x \quad 1) \quad 2 \qquad f(x) \quad 1$

2. For any $b \quad R\ e^{|x \quad b|}$ is



$|e^{|x \quad b|} \quad a|$ has four distinct solutions $a \quad 3$ so $a \quad (3, \quad)$

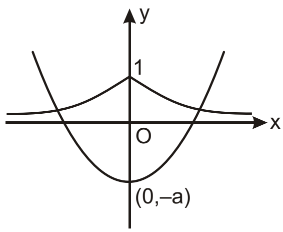
3. Domain $[\ 1, 1]$ and both are increasing functions.

$x \quad 1$, we get minimum value & $x \quad 1$, we get maximum value.

$\frac{}{4} \quad \frac{}{4}, \frac{}{4} \quad \frac{}{4} \qquad \frac{}{2}, \frac{}{2}$

4. $2^{2x^2 \quad 2y} \quad 2^{2x \quad 2y^2} \quad ^2 \quad 1 \quad 2^{2x^2 \quad 2y^2 \quad 2x \quad 2y \quad 1} \quad 0$

$x \quad \frac{1}{2} \quad ^2 \quad y \quad \frac{1}{2} \quad ^2 \quad 0$

5. 

7. $\sec^{1} \frac{5}{2} \quad \frac{2}{2(x^2 \quad 2)} \qquad x^2 \quad 2 \quad 2$

$\sec^{1} \frac{5}{2} \quad \frac{1}{(x^2 \quad 2)} \qquad \frac{1}{x^2 \quad 2} \quad \frac{1}{2}$

$\sec^{1}(\quad 2) \qquad \sec^{1}(2) \qquad \frac{5}{2} \quad \frac{1}{x^2 \quad 2} \qquad \frac{5}{2} \quad \frac{1}{2} \qquad 2$

$\frac{2}{3}$

8. $f(x) \quad x^2 \quad ax \quad b$ is injective if $D \quad 0$

$a^2 \quad 4b \quad 0$

If $\quad a \quad 1, b \quad 1, 2, 3, 4, 5$ Number of pair $\quad 5$

$a \quad 2, b \quad 1, 2, 3, 4, 5$ Number of pair $\quad 5$

$a \quad 3, b \quad 3, 4, 5$ Number of pair $\quad 3$

$a \quad 4, b \quad 4, 5$ Number of pair $\quad 2$

$a \quad 5$ $\quad b$ has no value

9. $f(x) \quad \log_x [x] \qquad f(x) \quad [0, 1]$

$g(x) \quad |\sin x| \quad |\cos x|$

$g(x) \quad [1, \sqrt{2}]$

10. $f(x) \quad 2x^3 \quad 3x^2 \quad 6$

$f(x) \quad 6x^2 \quad 6x \quad 0 \qquad x \quad [1, \quad)$

and $f(x) \quad [5, \quad)$

11. $0 \quad \{x\} \quad 1$

$\{x\}(\{x\} \quad 1)(\{x\} \quad 2) \quad 0$

$\{x\} \quad 0 \qquad x \quad z$

14. $1 \quad \sin^2 x \quad [1, 2]$

$\frac{1}{1 \quad \sin^2 x} \quad \frac{1}{2}, 1$

$\sin^{1} \frac{1}{1 \quad \sin^2 x} \qquad \overline{6}, \overline{2}$

$\frac{K}{6} \quad \frac{}{6}, \frac{}{2} \qquad K \quad [1, 3]$

15. $f(x \quad y) \quad f(x)f(y) \quad f(a \quad x)f(a \quad y)$

Put $x \quad y \quad 0$

$f(0) \quad [f(0)]^2 \quad f(a)f(a)$

$f(a) \quad 0 \qquad\qquad [\because f(0) \quad 1]$

Put $x \quad a$ and $y \quad x$

$f(a \quad x) \quad f(a)f(x) \quad f(0)f(a \quad x)$

$f(a \quad x) \quad f(a \quad x) \qquad f(2a \quad x) \qquad f(x)$

18. $f(x) \quad 4x \quad x^2 \quad y$

$x^2 \quad 4x \quad y \quad 0$

$f^{\quad 1}(x) \quad 2 \quad \sqrt{4 \quad x}$

19. $[5\sin x] \quad [\cos x] \qquad 6$

$1 \quad \cos x \quad 0$ and $\quad 5 \quad 5\sin x \quad 4$

$1 \quad \sin x \quad \frac{4}{5}$

20. $f(x) \quad ax \quad \cos x$

$f\,(x) \quad a \quad \sin x$

if $f(x)$ is invertible, then

$f\,(x) \quad 0$ or $f\,(x) \quad 0$

$a \quad 1$ or $a \quad 1$

21. $f(x) \quad [1 \quad \sin x] \quad 2 \quad \sin\frac{x}{2} \qquad 3 \quad \sin\frac{x}{3} \qquad\qquad n \quad \sin\frac{x}{n}$

$(1 \quad 2 \quad 3 \qquad n) \quad [\sin x] \quad \sin\frac{x}{2} \qquad \sin\frac{x}{3} \qquad\qquad \sin\frac{x}{n}$

22. $y \quad \frac{x^2 \quad ax \quad 1}{x^2 \quad x \quad 1}$

$(y \quad 1)x^2 \quad (y \quad a)x \quad (y \quad 1) \quad 0$

$D \quad 0$

$(y \quad a)^2 \quad 4(y \quad 1)^2 \quad 0$

$3y^2 \quad y(8 \quad 2a) \quad a^2 \quad 4 \quad 0 \quad y \quad R$

Not possible

23. $f(x) \quad [x] \quad [\ \ x]$

$f(x) \quad \begin{matrix} 0 & x & I \\ 1 & x & I \end{matrix}$

$g(x) \quad \{x\}$

$h(x) \quad f[g(x)] \quad f(\{x\})$ $\qquad \begin{matrix} \{x\} & 0 & x & I \\ \{x\} & \{x\} & x & I \end{matrix}$

$h(x) \quad \begin{matrix} f(0) & x & I \\ f(\{x\}) & x & I \end{matrix}$ $\qquad h(x) \quad \begin{matrix} 0 & x & I \\ 1 & x & I \end{matrix}$

Hence, the option (b).

24. $f(x) \quad \dfrac{x}{15} \quad \dfrac{15}{x} \qquad x \quad (0, 90)$

$0 \quad x \quad 15$ $\qquad f(x) \quad 0$

$15 \quad x \quad 30$ $\qquad f(x) \quad 1$

$30 \quad x \quad 45$ $\qquad f(x) \quad 2$

$45 \quad x \quad 60$ $\qquad f(x) \quad 3$

$60 \quad x \quad 75$ $\qquad f(x) \quad 4$

$75 \quad x \quad 90$ $\qquad f(x) \quad 5$

Total integers in range $f(x) \quad \{0, \quad 1, \quad 2, \quad 3, \quad 4, \quad 5\}$

25. $g(x) \quad \dfrac{1}{f(|x|)}$

$g(x)$ even functions symmetric about y-axis

$x \qquad f(x) \quad 0$

at $x \quad x_1 \qquad f(x) \quad 0 \qquad g(x_1)$

26. Homogeneous function $\quad f(tx, ty) \quad t^n f(x, y)$

27. $f(x) \quad \begin{matrix} 2x & 3 & x & 1 \\ a^2 x & 1 & x & 1 \end{matrix}$

For $x \quad 1 \qquad f(x) \quad 5$

So for range of $f(x)$ to be R.

$a^2 \quad 1 \quad 5$ and $a \quad 0$

$a \quad [\ 2, 2]$

Hence, $a \quad \{\ 2, \quad 1, 1, 2\}$

28. $\log_{1/3}(\log_4(x \quad 5)) \quad 0$

$0 \quad \log_4(x \quad 5) \quad 1$

$1 \quad x \quad 5 \quad 4$

$6 \quad x \quad 9$

29. $f(x) \quad \log_2 \dfrac{4}{\sqrt{2 \quad x} \quad \sqrt{2 \quad x}} \ ; \quad 2 \quad x \quad 2$

$\sqrt{2 \quad x} \quad \sqrt{2 \quad x} \quad y$

$4 \quad 2\sqrt{4 \quad x^2} \quad y^2$

$y \quad [2, 2\sqrt{2}]$

Range $f(x) \quad \log_2 \dfrac{4}{2\sqrt{2}}, \log_2 \dfrac{4}{2}$

$f(x)$ lies between $\dfrac{1}{2}, 1$

30. $|x^2 \quad 5x| \quad |x \quad x^2| \quad |6x| \qquad |x^2 \quad 5x| \quad |x \quad x^2| \quad |(x^2 \quad 5x) \quad (x \quad x^2)|$

$|a| \quad |b| \quad |a \quad b| \qquad ab \quad 0$

$(x^2 \quad 5x)(x \quad x^2) \quad 0$

$x(x \quad 5) \; x(x \quad 1) \quad 0 \qquad 5 \quad x \quad 1$

31. $f(x) \quad f\dfrac{1}{x} \quad f(x)f\dfrac{1}{x}$

$f(x) \quad 1 \quad x^n$

$f(2) \quad 33 \quad n \quad 5$

Hence, $f(x) \quad 1 \quad x^5$

Here, $f(x) \quad f(\quad x) \quad 0.$

Hence not an odd function.

32. $g(x) \quad \dfrac{\sin x \quad \sin 7x}{\cos x \quad \cos 7x} \quad |\sin x| \quad \dfrac{2\sin 4x\cos 3x}{2\cos 4x\cos 3x} \quad |\sin x|$

$\tan 4x \quad |\sin x|$

$g(x)$ period

33. $f(x) \quad \begin{matrix} \dfrac{x \quad 1}{2} & x \quad \text{odd} \\ \dfrac{x}{2} & x \quad \text{even} \end{matrix} \qquad f(x) : N \quad Z$

Let x odd $(2n \quad 1); n \quad 0$

$f(x) \quad \dfrac{2n \quad 1 \quad 1}{2} \quad n \qquad$ +ve integer

Let x even $2m; m \quad 0$

$f(x) \quad \dfrac{2m}{2} \quad m \qquad$ –ve integer

Range = codomains onto and clearly $f(x)$ is one-one function.

Hence, bijective.

34. $y \quad \dfrac{2^{x \quad 1} \quad 2^{1 \quad x}}{2^x \quad 2^{\quad x}} \quad \dfrac{2^{2x \quad 1} \quad 2}{2^{2x} \quad 1}$

$$y(2^{2x} \quad 1) \quad (2^{2x} \quad 1)2$$

$$2^{2x} \quad y \quad y \quad 2^{2x} \quad 2 \quad 2$$

$$2^{2x}(y \quad 2) \quad (2 \quad y)$$

$$2^{2x} \quad \frac{(y \quad 2)}{(2 \quad y)}$$

$$2x \quad \log_2 \frac{(2 \quad y)}{(2 \quad y)}$$

$$x \quad \frac{1}{2}\log_2 \frac{(2 \quad y)}{(2 \quad y)}$$

$$f^{\ \ 1}(x) \quad \frac{1}{2}\log_2 \frac{(2 \quad x)}{(2 \quad x)}$$

35. $\sqrt{|y|} \quad x$ $\qquad\qquad$ $(\because x \quad 0)$

$|y| \quad x^2$

$y \quad x^2 \qquad y \quad 0$

$y \quad\; x^2 \qquad y \quad 0$

36. $f(x) \quad \log_{[x]}(9 \quad x^2)$

Domains $\begin{matrix} 9 \quad x^2 \quad 0 \\ [x] \quad 0 \text{ and } [x] \quad 1 \end{matrix}$ $\qquad x \quad [2,3) \qquad f(x) \quad \log_2(9 \quad x^2)$

Range $(\quad ,\log_2 5]$

37. Gives $\quad e^x \quad e^{f(x)} \quad e$

$e^{f(x)} \quad e \quad e^x$

$f(x) \quad \log_e(e \quad e^x)$

Domain $e \quad e^x \quad 0$

$x \quad 1 \qquad x \quad (\quad ,1)$

Range $(\quad ,1)$

38. Gives $\quad y \quad |y| \quad x \quad |x|$

If $\quad x \quad 0, y \quad 0 \qquad 2y \quad 2x \qquad y \quad x$

$x \quad 0, y \quad 0 \qquad 2y \quad 0 \qquad y \quad 0$

$x \quad 0, y \quad 0 \qquad 0 \quad 2x \qquad x \quad 0$

$x \quad 0, y \quad 0 \qquad 0 \quad 0$ $\qquad$ whole region of III quadrant.

For person to be safe there should not be point common to the given curves and the voltage field graph. Only $y \quad m \quad |x|$ does not have any point of intersection with the curve.

39. Gives $\quad |f(x) \quad 6 \quad x^2| \quad |f(x)| \quad |4 \quad x^2| \quad 2$

$|f(x) \quad 2 \quad (4 \quad x^2)| \quad |f(x)| \quad |4 \quad x^2| \quad 2$

Since $|a \quad b \quad c| \quad |a| \quad |b| \quad |c|$

If $\quad a \quad 0, b \quad 0, c \quad 0$ or $a \quad 0, b \quad 0, c \quad 0$

$f(x) \quad 0$ and $4 \quad x^2 \quad 0 \qquad 2 \quad x \quad 2$ and $f(x) \quad 0$

40. $f(x) \quad \cos px \quad \sin x$

Period : L.C.M. of $\dfrac{2}{p}, \dfrac{2}{1}$

For period to exist p should be a rational number.

41. $y \quad f(e^x) \quad f(\ln|x|)$

Domain $f(x) \quad (0, 1)$

$0 \quad e^x \quad 1 \qquad x \quad 0$ ...(1)

and $\quad 0 \quad \ln|x| \quad 1 \qquad 1 \quad |x| \quad e \qquad x \quad (\quad e, \quad 1) \quad (1, e)$ ...(2)

Taking intersection $x \quad (\quad e, \quad 1)$

42. Givens $\quad f(1) \quad 2, \ f(2) \quad 3, \ f(3) \quad 4, \ f(4) \quad 1, \ g(1) \quad 3$ and $f[g(x)] \quad g[f(x)]$

at $x \quad 1 \quad f[g(1)] \quad g[f(1)] \qquad f(3) \quad g(2) \qquad g(2) \quad 4$

at $x \quad 2 \quad f[g(2)] \quad g[f(2)] \qquad f(4) \quad g(3) \qquad g(3) \quad 1$

at $x \quad 3 \quad f[g(3)] \quad g[f(3)] \qquad f(1) \quad g(4) \qquad g(4) \quad 2$

43. Gives $\quad [y \quad [y]] \quad 2\cos x \qquad [y] \quad [y] \quad 2\cos x \qquad 2[y] \quad 2\cos x; \ [y] \quad \cos x$ ...(1)

where
$$y \quad \frac{1}{3}[\sin x \quad [\sin x \quad [\sin x]]]$$
$$y \quad \frac{1}{3}[\sin x \quad [\sin x] \quad [\sin x]]$$
$$y \quad \frac{1}{3}(3[\sin x])$$
$$y \quad [\sin x]$$ ...(2)

From eqn. (1) & (2),

$$[\sin x] \quad \cos x$$
$$\cos x \quad 0, 1, \quad 1$$

Hence, no solution.

44. $$f(x) \quad \frac{x^{2n}}{(x^{2n} \ \operatorname{sgn} x)^{2n \quad 1}} \quad \frac{e^{\frac{1}{x}} \quad e^{\quad \frac{1}{x}}}{e^{\frac{1}{x}} \quad e^{\quad \frac{1}{x}}} \quad x \quad 0 \text{ and } f(0) \quad 1$$

when $$f(x) \quad \frac{(x^{2n})}{(x^{2n})^{2n \quad 1}} \quad \frac{e^{\frac{1}{x}} \quad e^{\quad \frac{1}{x}}}{e^{\frac{1}{x}} \quad e^{\quad \frac{1}{x}}} \ ; \ x \quad 0$$

$$f(x) \quad \frac{x^{2n}}{(x^{2n})^{2n \quad 1}} \quad \frac{e^{\frac{1}{x}} \quad e^{\quad \frac{1}{x}}}{e^{\frac{1}{x}} \quad e^{\quad \frac{1}{x}}} \; ; \; x \quad 0$$

Clearly, $f(x) \quad f(\quad x)$. Hence, $f(x)$ is even function.

45. $f(n) \quad 2(f(1) \quad f(2) \qquad f(n \quad 1))$

$f(2) \quad 2f(1)$

$f(3) \quad 2[f(1) \quad f(2)] \quad 2 \quad \frac{f(2)}{2} \quad f(2) \quad 3f(2)$

$f(4) \quad 2[f(1) \quad f(2) \quad f(3)] \quad 2 \quad \frac{f(3)}{2} \quad f(3) \quad 3f(3) \quad 3^2 f(2)$

$\vdots$

$$\sum_{r \quad 1}^{m} f(r) \quad f(1) \quad f(2) \qquad f(m) \quad f(1) \quad f(2) \quad 3f(2)$$

$$f(1) \quad f(2)[1 \quad 3 \quad 3^2 \qquad 3^{m \quad 2}]$$

$$f(1) \quad 2 \quad \frac{(3^{m \quad 1} \quad 1)}{(3 \quad 1)} \quad 3^{m \quad 1}$$

46. Gives $f(x) \quad \frac{x}{\sqrt{1 \quad x^2}}$

$$f(f(x)) \quad \frac{x}{\sqrt{1 \quad 2x^2}}$$

$$f(f(f(x))) \quad \frac{x}{\sqrt{1 \quad 3x^2}}$$

$\vdots$

$$\underbrace{fofo \qquad fof(x)}_{n \text{ times}} \quad \frac{x}{\sqrt{1 \quad nx^2}} \quad \frac{x}{\sqrt{1 \quad \sum_{r \quad 1}^{n} 1 \quad x^2}}$$

47. $f(x) \quad 2x \quad |\cos x|$

Range $f(x) \quad R$ codomain onto.

Clearly, $f(x)$ is increasing function one-one function.

48. Gives $f(x) \quad x^3 \quad x^2 \quad 3x \quad \sin x$

Since, $f(x)$ is continuous function.

and $f(x)$ as x

$f(x)$ as x

Range $f(x) \quad R$ codomains onto function

and $f(x) \quad 3x^2 \quad 2x \quad 3 \quad \cos x \quad 3 \quad x \quad \frac{1}{3}^{2} \quad \frac{8}{3} \quad \cos x \qquad f(x) \quad 0$

Hence, $f(x)$ is one-one.

49. $f(x) \quad \{x\} \quad \{x \quad 1\} \qquad \{x \quad 99\}$

Since $\quad \{x\} \quad \{x \quad I\}$ where I integer

$$f(x) \quad \underbrace{\{x\} \quad \{x\} \quad \{x\}}_{100 \text{ times}}$$

$$f(x) \quad 100\{x\} \qquad f(\sqrt{2}) \quad 100\{\sqrt{2}\} \quad 100 \quad 0.414 \quad 41.4$$

$[f(\sqrt{2})] \quad 41$

50. $|\cot x \quad \operatorname{cosec} x| \quad |\cot x| \quad |\operatorname{cosec} x|; x \quad [0, 2 \quad]$

$\cot x \quad 0$ and $\operatorname{cosec} x \quad 0 \qquad 1^{st}$ quadrant

or $\cot x \quad 0$ and $\operatorname{cosec} x \quad 0 \qquad 4^{th}$ quadrant

Hence, $\quad x \quad 0, \frac{}{2} \qquad \frac{3}{2}, 2$

51. If $f(4 \quad x) \quad f(4 \quad x)$

$f(x)$ is symmetric about $x \quad 4$.

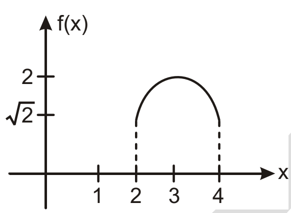
Roots of $f(x) \quad 0$ are of the form

$4 \quad , 4 \quad , 4 \quad , 4 \quad , 4 \quad , 4 \quad , 4 \quad , 4$

52. $f(x) \quad x \quad 6 \quad (x \quad 1)(x \quad 2)(x \quad 3)(x \quad 4)(x \quad 5)$

$f(6) \quad 120$

53. $f(x) \quad \sqrt{x \quad 2} \quad \sqrt{4 \quad x}$



54. $\frac{x}{9} \qquad \frac{x}{11}$

$x \quad [1, 9) \quad [11, 18) \quad [22, 27) \quad [33, 36) \quad [44, 45)$

55. $\log_{x \frac{1}{2}} (2x^2 \quad x \quad 1)$

$x \quad \frac{1}{2} \quad 0, \quad x \quad \frac{1}{2} \quad 1 \;\&\; 2x^2 \quad x \quad 1 \quad 0$

$$x \quad \frac{1}{2} \quad 2 \;\&\; (2x \quad 1)(x \quad 1) \quad 0$$

$$x \quad \frac{3}{2} \;\&\; x(\quad , \quad 1) \quad \frac{1}{2},$$

$$x \quad \frac{3}{2},$$

56. $[x^2] \quad [x] \quad 2 \quad 0$

Let $[x] \quad t$

$t^2 \quad t \quad 2 \quad 0$

$(t \quad 2)(t \quad 1) \quad 0$

$t \quad 2$ or $t \quad 1$

$[x] \quad 2$ or $[x] \quad 1$

$x \quad [\ 2, \quad 1) \quad [1, 2)$

58. $f(x)$ is many one function.

59. $f(f(x)) \quad 2 \quad f(x) \qquad f(x) \quad 0$

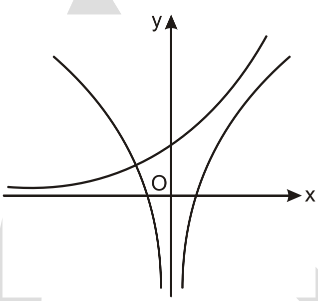
$\qquad 2 \quad f(x) \qquad f(x) \quad 0$

$f(f(x)) \quad 4 \quad x \qquad x \quad 0$

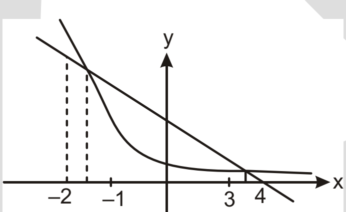
$\qquad 4 \quad (x) \qquad x \quad 0$

60. $f\,(x) \quad \dfrac{7(3x^2 \quad 2x \quad 3)}{(3 \quad 3x \quad 4x^2)^2} \quad 0 \qquad f(x)$

61.



62.



$[\ ] \quad 2$

$[\ ] \quad 3$

63. $f(x) \quad \sqrt{\sin(\log_7(\cos(\sin x)))}$

$\cos(\sin x) \quad 1 \qquad \cos(\sin x) \quad 1 \qquad f(x) \quad 0$

64. $3 \quad |[x]| \quad 2 \qquad 2 \quad [x] \quad 2 \qquad 2 \quad x \quad 3$

65. $f(x) \quad \frac{}{2} \quad \cot^{\ 1}\{\ x\}$

$0 \quad \{\ x\} \quad 1 \qquad \frac{}{4} \quad \cot^{\ 1}\{\ x\} \quad \frac{}{2}$

66. $f(f(x)) \quad x$

$f_{2008}(x) \quad f_{2009}(x) \quad x \quad f(x) \quad x \quad \frac{3x \quad 5}{2x \quad 3} \quad \frac{2x^2 \quad 5}{2x \quad 3}$

67. $f(x) \quad x \quad \frac{1}{x} \quad 1 \quad x^2 \quad \frac{1}{x^2}\ ;\ x^2 \quad \frac{1}{x^2} \quad 2\ ;\ x \quad \frac{1}{x} \quad 1 \quad 3 \qquad f(x) \quad 6$

68. $f(x) \quad e^{x^3 \quad 3x^2 \quad 9x \quad 2}$

$f\ (x) \quad e^{(x^3 \quad 3x^2 \quad 9x \quad 2)} 3(x \quad 3)(x \quad 1)$

$f(x)$ is many one.

at $x \quad 1, f(x) \quad e^7$

at $x \qquad , f(x) \quad 0$

Range of $f(x)$ is $(0, e^7]$.

69. $D_f : (\ 2, 1)$

$$\log \frac{\sqrt{4 \quad x^2}}{1 \quad x}$$

$$1 \quad \sin \ \log \frac{\sqrt{4 \quad x^2}}{1 \quad x} \qquad 1$$

70. $f\ (x) \quad 0 \quad x \quad R \qquad 3x^2 \quad 2(a \quad 2)x \quad 3a \quad 0 \quad x \quad R$

$D \quad 0$

$4(a \quad 2)^2 \quad 4 \ 9a \quad 0$

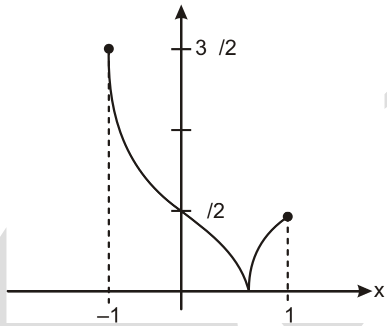
$a^2 \quad 5a \quad 4 \quad 0 \qquad (a \quad 1)(a \quad 4) \quad 0$

$a \quad [1, 4]$

71. Min. value of $3x^2 \quad bx \quad c \quad 0$

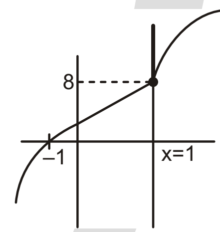
$D \quad 0$

72. $f(x) \quad \sin^{\ 1} x \quad \cos^{\ 1} x \quad 2\sin^{\ 1} x \quad \frac{}{2}$



73. is one-one when

$$2^3 \quad \ln 1 \quad b^2 \quad 3b \quad 10$$
$$b^2 \quad 3b \quad 2 \quad 0$$
$$b \quad 1, 2$$



80. We have, $[x]^2 \quad 7[x] \quad 10 \quad 0$

$$([x] \quad 5)([x] \quad 2) \quad 0$$
$$2 \quad [x] \quad 5$$
$$[x] \quad 3 \text{ or } 4$$
$$x \quad [3, 5)$$

and $\quad 4[y]^2 \quad 16[y] \quad 7 \quad 0$

$$(2[y] \quad 7)(2[y] \quad 1) \quad 0$$
$$\frac{1}{2} \quad [y] \quad \frac{7}{2}$$
$$[y] \quad 1 \text{ or } 2 \text{ or } 3$$
$$y \quad [1, 4)$$

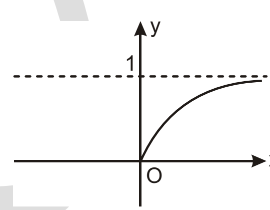
Therefore, $\quad x \quad y \quad [4, 9)$

$$[x \quad y] \quad \{4, 5, 6, 7, 8\}$$

Hence, $[x \quad y]$ cannot be 9.

81. $f : R \quad R \qquad f(x) \quad \dfrac{e^{|x|} \quad e^{\ x}}{e^x \quad e^{\ x}}$

$$f(x) \quad \begin{cases} \dfrac{e^x \quad e^{\ x}}{e^x \quad e^{\ x}} & \text{if } x \quad 0 \\ \dfrac{e^{\ x} \quad e^{\ x}}{e^x \quad e^{\ x}} & \text{if } x \quad 0 \end{cases}$$



Many one into function.

82. $f(x)$ such $f(1 \quad x) \quad 2f(x) \quad 3x \quad x \quad R$

$$x \quad \frac{1}{2} \quad x$$

$$f \quad \frac{1}{2} \quad x \quad 2f \quad \frac{1}{2} \quad x \quad 3 \quad \frac{1}{2} \quad x \qquad \ldots(1)$$

$$x \quad \frac{1}{2} \quad x$$

$$f \quad \frac{1}{2} \quad x \quad 2f \quad \frac{1}{2} \quad x \quad 3 \quad \frac{1}{2} \quad x \qquad \ldots(2)$$

(1) + (2)

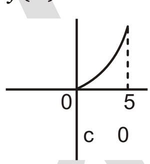
$$3 \quad f \quad \frac{1}{2} \quad x \quad f \quad \frac{1}{2} \quad x \quad 3; \quad f \quad \frac{1}{2} \quad x \quad 1 \quad f \quad \frac{1}{2} \quad x$$

$$1 \quad f \quad \frac{1}{2} \quad x \quad \frac{3}{2} \quad 3x; \quad f \quad \frac{1}{2} \quad x \quad \frac{1}{2} \quad 3x$$

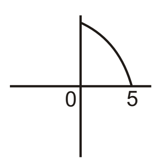
$$x \quad \frac{1}{2} \quad f(0) \quad \frac{1}{2} \quad \frac{3}{2} \quad 1$$

83. $f:[0,5] \quad [0,5]$

$f(x) \quad ax^2 \quad bx \quad c \qquad a,b,c \quad R, abc \quad 0$



or



$$25a \quad 5b \quad c \quad 0$$

$$f(5) \quad 0$$

$ax^2 \quad bx \quad c \quad 0(\quad)$ $\qquad \frac{c}{a} \quad 5$

$cx^2 \quad bx \quad a \quad 0 \quad \frac{1}{}$ $\qquad \frac{1}{a}$

So, roots are $a, \frac{1}{5}$.

84. $f(x) \quad x^2 \quad x \quad \cos x$

$f(x) \quad x$

85. $f(k)$ odd

$f(k \quad 1)$ even $\quad k \quad 1,2,3$

$f(1)$ odd
$f(2)$ even
$f(3)$ odd
$f(4)$ even

Hence, 4 functions.

$f(1)$ even
$f(2)$ odd
$f(3)$ even
$f(4)$ odd

Hence, 4 functions.

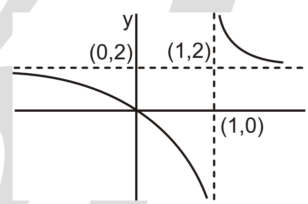
$f(1)$ odd
$f(2)$ even
$f(3)$ even
$f(4)$ odd

Hence, 4 functions.

86. $y \quad \tan(\sin x)$.

Here function is continuous and differentiable and $y_{\max} \quad \tan(1); y_{\min} \quad \tan 1$.

87. $f(x) \quad \dfrac{2x}{x \quad 1}$



$y \quad 2 \quad \dfrac{2}{(x \quad 1)}$

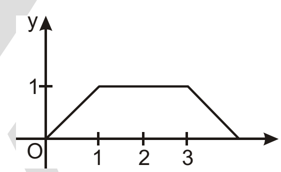
$(y \quad 2)(x \quad 1) \quad 2$

88. $R_f \quad [\ 2, 4]$

$R_g \quad [\ 1, 2]$

89. $f(x) \quad (x^4 \quad 1) \quad \dfrac{1}{x^2 \quad x \quad 1}$

90. $0 \quad f(x) \quad 1$



$0 \quad 7f(x) \quad 7$

$1 \quad \sin(7f(x)) \quad 1$

91. $\ln|\ln|x|| \quad 0$

$|\ln|x|| \quad 1$

$\ln|x| \quad (\quad , \ 1) \quad [1, \ )$

$|x| \quad 0, \dfrac{1}{e} \quad [e, \ )$

$x \quad (\quad , \ e] \quad \dfrac{1}{e}, 0 \quad 0, \dfrac{1}{e} \quad [e, \ )$

$|x|^2 \quad 7|x| \quad 10 \quad 0$

$(|x| \quad 2)(|x| \quad 5) \quad 0$

$2 \quad |x| \quad 5$

$x \quad (\ 5, \ 2] \quad [2, 5]$

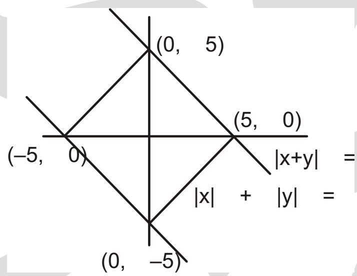
92. $\log_{[x]\quad 3\{x\}}\quad [x]\quad \frac{5}{2}^{\,2}\quad \frac{3}{4}\quad 0\qquad [x]\quad 3\{x\}\quad 1$

93. $x\quad 3\quad X\qquad\qquad |X|\quad |Y|\quad 5$

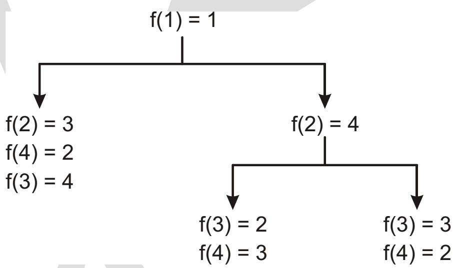
$y\quad 1\quad Y$

$x\quad y\quad 4\quad X\quad Y\qquad |X\quad Y|\quad 5$

number of pairs of (x, y) $\quad 12$



95.



96. $x^2\quad x\quad 0\qquad x\quad 0, 1$

97. Total one-one function – (at least one get right place) + (at least two get right place)

– (at least three get right place) + (at least four get right place)

$${}^{6}C_4\quad 4!\quad {}^{4}C_1\quad {}^{5}C_3\quad 3!\quad {}^{4}C_2\quad {}^{4}C_2\quad 2!\quad {}^{4}C_3\quad {}^{3}C_1\quad {}^{4}C_4\quad 181$$

98. $f(x)\quad x^2\quad 2x\quad 3$

$g(x)\quad f^{\ 1}(x)\quad 1\quad \sqrt{x\quad 4}\qquad x\qquad 4$

$f(x)\quad g(x)\quad f^{\ 1}(x)\qquad f(x)\quad x$

$$x^2\quad 3x\quad 3\quad 0\qquad x\quad \frac{3\quad \sqrt{21}}{2}$$

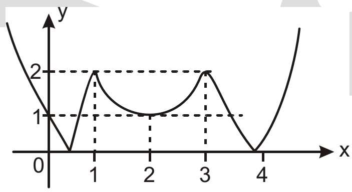
Exercise-2 : One or More than One Answer is/are Correct

1. $f(\ 4)\quad f(4)\quad 40$

$f(\ 13)\quad f(13)\quad f(3)\quad 19$

$f(\ 11)\quad f(11)\quad f(1)\quad 2$

2.



3. $f(x)\quad \cos^{\ 1}\dfrac{1\quad \tan^2(x/2)}{1\quad \tan^2(x/2)}$ is defined when

$\dfrac{x}{2}\quad (2n\quad 1)\dfrac{\ }{2}$

$x\quad (2n\quad 1)$

Domain $\ R\quad \{(2n\quad 1)\ :\ n\quad I\}$

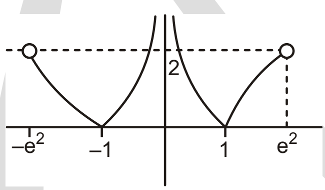
Range $\ [0,\ )$

$f(x)\quad \cos^{\ 1}(\cos x)$

$f(x)$ is even function.

when $x\quad (\ ,2\ )$, then $f(x)\quad 2\qquad x$ is differentiable.

4.



$0\quad |k\quad 1|\quad 3\quad 2$

$k\quad (\ 4,\ \ 2)\quad (4,6)$

5. (a) $D_f\quad R$

(b) $D_f\quad R$

(c) $f(x) \quad \sqrt{2\cos^2 x \quad \cos x \quad \frac{1}{8}}$

$D_f \quad R$

(d) $\ln(1 \quad |x|) \quad 0$

$D_f \quad \frac{(2n \quad 1)}{2}$

6. $f \frac{3}{2} \quad \frac{9}{4}$

$f \quad f \frac{3}{2} \quad \frac{3}{2}$

$f \quad f \quad f \frac{3}{2} \quad \frac{9}{4}$

$f \frac{5}{2} \quad 2$

$f \quad f \frac{5}{2} \quad 1$

$f \quad f \quad f \frac{5}{2} \quad 1$

8. $f^{\ 1}(f(x)) \quad f(f^{\ 1}(x)) \quad x$

if $f(f^{\ 1}(x)) \quad f^{\ 1}(x) \quad x \quad f^{\ 1}(x)$

if $f(f^{\ 1}(x)) \quad f^{\ 1}(x) \quad f(f^{\ 1}(f(x))) \quad f^{\ 1}(f(x)) \quad f(x) \quad f^{\ 1}(f(x)) \quad x$

9. $f(x) \quad \cos^{\ 1} x \quad \cos^{\ 1} \frac{x}{2} \quad \frac{\sqrt{3}\ \sqrt{1 \quad x^2}}{2}$

Let $x \quad \cos$

$f(x) \quad \cos^{\ 1}(\cos \quad) \quad \cos^{\ 1} \frac{1}{2}\cos \quad \frac{\sqrt{3}}{2}\sin$

$\cos^{\ 1}(\cos \quad) \quad \cos^{\ 1} \quad \cos \quad \overline{\ \ }\,3$

$\overline{\ \ }\,3 \quad 0 \quad \overline{\ \ }\,3$

$2 \quad \overline{\ \ }\,3 \quad \overline{\ \ }\,3$

10. $f(x) \quad \cos^{\ 1}(\ \{ \ x\})$

$\{ \ x\} \quad (\ 1, 0] \qquad \cos^{\ 1}(\ \{ \ x\}) \qquad \frac{}{2},$

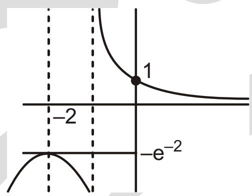
12. $h(x) \quad [\ln x \quad 1] \quad [1 \quad \ln x]$

$$h(x) \quad \begin{matrix} 1, & \ln x \quad 1 \quad I \\ 0, & \ln x \quad 1 \quad I \end{matrix}$$

14. $f(x) \quad \frac{1}{2}$, $f(x)$ is periodic & constant function.

16. $f(x) \quad \frac{e^{\ x}}{1 \quad x}$

$f\ (x) \quad \frac{e^{\ x}(x \quad 2)}{(1 \quad x)^2}$



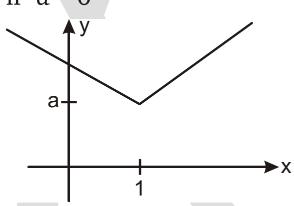
17. $[x] \quad \frac{2x\{x\}}{x \quad \{x\}} \quad \frac{2\{x\}([x] \quad \{x\})}{[x] \quad 2\{x\}}$

$[x]^2 \quad 2\{x\}^2$

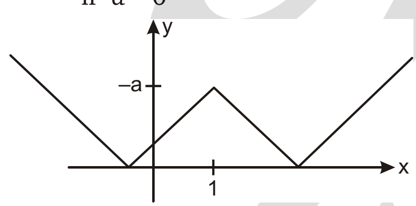
$x \quad 1 \quad \frac{1}{\sqrt{2}} \qquad\qquad (\because \ x \quad R \ )$

18. $||x \quad 1| \quad a| \quad 4$

if $a \quad 0$



if $a \quad 0$



(a) if eq. has three distinct real root then $a \quad 0$ and $a \qquad 4$

(b) 4 distinct roots for $a \quad (\qquad , \quad 4)$

(c) if $\ 4 \quad a \quad 4$, there are two distinct real roots

(d) if $a \quad 4$, no real root.

19. (a) $f_2(x) \quad (\sin x)^{1/2} \quad (\cos x)^{1/2}$

$\sqrt{\sin x} \quad \sin^2 x; \ \sqrt{\cos x} \quad \cos^2 x \qquad \sqrt{\sin x} \quad \sqrt{\cos x} \quad 1$

(b) $f_2(x) \quad (\sin x)^{1/2} \quad (\cos x)^{1/2} \qquad f_2(x) \quad 1$ at $x \quad 2k$

(c) $f_2(x) \quad (\sin x)^{1/2} \quad (\cos x)^{1/2}; \ f_3(x) \quad (\sin x)^{1/3} \quad (\cos x)^{1/3}$

if $x \quad (2k \ \ , 2k \qquad /2) \qquad 0 \quad \sin x \quad 1$ and $0 \quad \cos x \quad 1$

As power increases, value of function decreases.

$f_2(x) \quad f_3(x)$

(d) $f_3(x) \quad (\sin x)^{1/3} \quad (\cos x)^{1/3}$

$f_5(x) \quad (\sin x)^{1/5} \quad (\cos x)^{1/5}$

$f_3(x) \quad f_5(x)$

20. $1 \quad \log_3 \frac{x^2}{3} \quad 1 \qquad \frac{1}{3} \quad \frac{x^2}{3} \quad 3$

Range is $[0, 1]$.

21. $\frac{3x \quad 1}{2} \quad n$

$\frac{4n \quad 5}{9} \qquad \frac{4n \quad 5}{9} \quad \frac{1}{2} \qquad n$

$n \quad 2$

$\vdots$

$\vdots$

$n \quad 10$

22. $\sin^6 \qquad \cos^6 \qquad 1 \quad 3\sin^2 \quad \cos^2$

$1 \quad \frac{3}{4} \; \frac{1 \quad \cos 4}{2} \qquad \frac{5}{8} \quad \frac{3}{8}\cos(4\ )$

$\frac{5}{8} \quad \frac{3}{8}\cos(x)$

23. (a) $g(f(x)) \quad \ln(\sin x)$

(b) $x^2 \quad (a \quad 1)x \quad 9 \quad 0 \quad x \quad R$

$(a \quad 1)^2 \quad 36 \quad 0 \qquad 5 \quad a \quad 7$

(c) $f(f(x)) \quad (2011 \quad (2011 \quad x^{2012}))^{1/2012} \qquad x$

24. $\frac{1}{4} \quad \frac{150}{200} \qquad \frac{1}{4} \quad \frac{151}{200} \qquad\qquad \frac{1}{4} \quad \frac{199}{200} \qquad 50$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 4 to 6

Sol. $f(x) \quad \sqrt{x^2 \quad 2(\ ^2 \quad 3)x \quad 12}$

$g(x) \quad \ln(x^2 \quad 49)$

if domain of $f \quad g$ is same as domain of g. Then

$x^2 \quad 2(\ ^2 \quad 3)x \quad 12 \quad 0 \quad x \quad (\quad, \ 7) \quad (7, \ )$

$\frac{6}{7}, \frac{7}{2}$

$h(\ ) \quad \ln \quad 4\cos^2 t\,dt \quad {}^2 \quad \ln[2 \quad \sin 2 \quad {}^2]$

0

Paragraph for Question Nos. 7 to 8

7. For $x \quad [5^4, 5^5]$

$f(x) \quad {}^4 \; 2 \; \left|\frac{x}{5^4} \quad 3\right|$

2

$f(x)_{\max} \quad 32$

8. 5

$f(x) \quad 5^4 \; 2 \; \left|\frac{x}{5^4} \quad 3\right|$

$f(2007) \quad 5^4 \; 2 \; \frac{2007}{625} \quad 3 \quad 1118$

Paragraph for Question Nos. 9 to 10

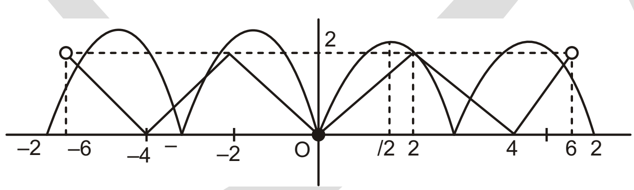
9. $f(x)$ → $x \quad 0 \quad x \quad 2$; → $-x \quad -2 \quad x < 0$

$f(x) \quad f(x \quad 4)$

$\{f(5.12)\} \quad \{f(1.12)\} \quad 0.12$

$\{f(7.88)\} \quad \{f(3.88)\} \quad \{f(\ 0.12)\} \quad 0.12$

10.



Paragraph for Question Nos. 13 to 14

13. $f(x) \quad 3$

$3 \quad \ln b_1, 3 \quad \ln b_2, 3 \quad \ln b_3$ are in A.P.

14. $y \quad 3x^2$

Let slope of tangent be m.

$$y \quad m(x \quad 2)$$
$$m(x_1 \quad 2) \quad 3x_1^2$$
Also, $$m \quad 6x_1$$
$$6x_1(x_1 \quad 2) \quad 3x_1^2$$
$$x_1 \quad 4$$
$$m \quad 24$$

Paragraph for Question Nos. 15 to 16

15. $y \quad 2^{x^4 \quad 4x^2} \qquad x^4 \quad 4x^2 \quad \log_2 y$

$$x^2 \quad \frac{4 \quad \sqrt{16 \quad 4\log_2 y}}{2} \qquad x \quad \sqrt{2 \quad \sqrt{4 \quad \log_2 y}}$$

16. $g(x) \quad 1 \quad \frac{6}{\sin x \quad 2}$ Range $[\ 5, \ \ 2]$

Exercise-4 : Matching Type Problems

1.
$$[x] \quad \{x\} \quad [y] \quad \{z\} \quad 12.7 \qquad \text{...(i)}$$
$$[x] \quad \{y\} \quad [z] \quad \{z\} \quad 4.1 \qquad \text{...(ii)}$$
$$\{x\} \quad [y] \quad \{y\} \quad [z] \quad 2 \qquad \text{...(iii)}$$

Adding (i), (ii) & (iii),
$$[x] \quad \{x\} \quad [y] \quad \{y\} \quad [z] \quad \{z\} \quad 9.4$$
$$\{y\} \quad [z] \quad 3.3, \{x\} \quad [y] \quad 5.3, [x] \quad \{z\} \quad 7.4$$
$$\{y\} \quad 0.7, [z] \quad 4, \{x\} \quad 0.3, [y] \quad 5$$
$$[x] \quad 7, \{z\} \quad 0.4$$

4. (A) $f(x) \quad \sin^2 2x \quad 2\sin^2 x \quad 2\sin^2 x\cos 2x$

Function is even, hence many one, function is also periodic.

$$f(x) \quad (1 \quad \cos 2x)\cos 2x \quad \frac{1}{4} \quad \cos 2x \quad \frac{1}{2}^{\,2}$$

Range of function is $\ 2, \frac{1}{4}$.

(B) $f(x) \quad 4x$

(C) $f(x) \quad \sqrt{\ln(\cos(\sin x))}$

$$\ln(\cos(\sin x)) \quad 0$$
$$\cos(\sin x) \quad 1$$
$$f(x) \quad 0$$

(D) $f(x) \quad \tan^{\ 1} \dfrac{x^2 \quad 1}{x^2 \quad \sqrt{3}}$

$f(x)$ is even & hence many one.

Range is $\dfrac{}{6}, \dfrac{}{4}$.

7. (A) Domain of $g(x)$ is [0, 3].

(B) Range of $g(x)$ is [0, 3].

(C) $f(f(f(2))) \quad 1$

$f(f(f(3))) \quad 2$

(D) $m \quad 3$

Exercise-5 : Subjective Type Problems

1. $f(x) \quad 2x \quad 1 \quad (x \quad 1)(x \quad 2)(x \quad 3)(x \quad 4)(x \quad 5)(2009x \quad)$

2. $f(x) \quad x^3 \quad 3x \quad 1$

$f(f(x)) \quad 0$

Let $f(x) \quad t$

$f(t) \quad 0$

$t \quad , \ ,$

$f(x) \quad , \quad (\ 2, \ 1)$

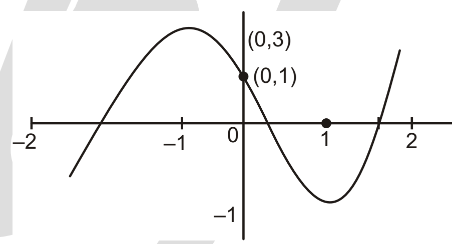
No. of solution 1

$f(x) \quad , \quad (0, 1)$

No. of solution 3

$f(x) \quad , \quad (1, 2)$

No. of solution 3



3. Put $x \quad y \quad 0 \qquad f(1) \quad 4$

Put $x \quad 0, y \quad 1 \qquad f(2) \quad 9$

4. $1 \quad \dfrac{2x}{3} \quad 1 \qquad \dfrac{3}{2} \quad x \quad \dfrac{3}{2}$

$12 \quad 3^x \quad \dfrac{27}{3^x} \quad 0 \qquad (3^x \quad 3)(3^x \quad 9) \quad 0 \qquad 1 \quad x \quad 2$

5. $\sin^{\ 1}(0) \quad \cos^{\ 1}(\ 1) \qquad 0 \quad x^2 \quad \dfrac{4}{9}$

$\sin^{\ 1}(1) \quad \cos^{\ 1}(0) \qquad \dfrac{4}{9} \quad x^2 \quad \dfrac{13}{9}$

8. Let $P(x) \quad ax^4 \quad bx^3 \quad cx^2 \quad dx \quad 2$

$P(1) \quad a \quad b \quad c \quad d \quad 2 \quad 5$ ...(1)

$P(\quad 1) \quad a \quad b \quad c \quad d \quad 2 \quad 5$ ...(2)

$b \quad d \quad 0$ and $a \quad c \quad 3$

$P(2) \quad 16a \quad 8b \quad 4c \quad 2d \quad 2 \quad 2$ ...(3)

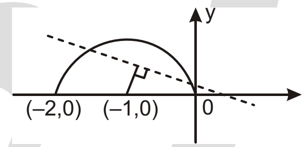
$P(\quad 2) \quad 16a \quad 8b \quad 4c \quad 2d \quad 2 \quad 2$ ...(4)

$4a \quad c \quad 0$ and $4b \quad d \quad 0$

$b \quad d \quad 0$ and $a \quad 1, c \quad 4$

$P(x) \quad x^4 \quad 4x^2 \quad 2$

9. $(x \quad 1)^2 \quad y^2 \quad 1 \qquad (\because\ y \quad 0)$



$x \quad y \quad k$

$\left|\frac{k \quad 1}{\sqrt{2}}\right| \quad 1$

$\sqrt{2} \quad 1 \quad k \quad \sqrt{2} \quad 1$

$0 \quad k \quad \sqrt{2} \quad 1 \qquad (\because\ k \quad 0)$

10. $\sqrt{[x] \quad \frac{x}{2}} \quad \frac{x}{3} \quad 3$

when $\frac{x}{3}$ is an integer then definitely $\sqrt{[x] \quad \frac{x}{2}}$ is also an integer.

So, $\sqrt{[x] \quad \frac{x}{2}} \quad 2$ and $\frac{x}{3} \quad 1$ (and check like this)

$[x] \quad \frac{x}{2} \quad 4, \quad \frac{x}{3} \quad 1 \qquad x \quad [3, 6)$

when $x \quad [3, 4)$

$[x] \quad 3, \frac{x}{2} \quad 1$

So, $x \quad [3, 4)$ satisfies.

when $x \quad [4, 5)\ [x] \quad 4\ \frac{x}{2} \quad 2 \qquad [x] \quad \frac{x}{2} \quad 6 \quad 4$ not satisfies, similarly on checking all possibilities we have only $x \quad [3, 4)$.

$a \quad 3, b \quad 4$

11. $f(f(x)) \quad \frac{1}{\sqrt[2011]{1 \quad \frac{1}{1 \quad x^{2011}}}} \quad \frac{\sqrt[2011]{1 \quad x^{2011}}}{x}$

$$f(f(f(x))) \quad \frac{\sqrt[2011]{1 \quad \frac{1}{1 \quad x^{2011}}}}{\frac{1}{\sqrt[2011]{1 \quad x^{2011}}}} \quad \frac{\frac{x}{\sqrt[2011]{1 \quad x^{2011}}}}{\frac{1}{\sqrt[2011]{1 \quad x^{2011}}}} \quad x$$

$$f_{2013}(x) \quad x \quad \{ \quad x\}$$

12. $f(x) \quad 0 \qquad 0 \quad x \quad 6$

$\qquad 1 \qquad 6 \quad x \quad 12$

$\qquad 2 \qquad 12 \quad x \quad 18$

$\qquad 3 \qquad 18 \quad x \quad 24$

$\qquad 4 \qquad 24 \quad x \quad 30$

$\qquad 5 \qquad x \quad 30$

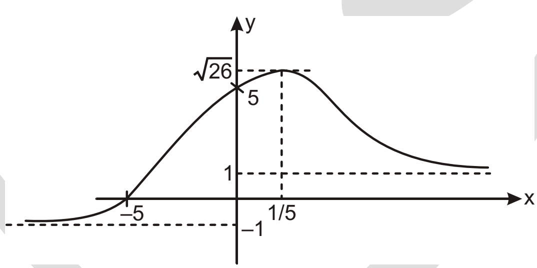
13. $(f(x, y))^2 \quad (g(x, y))^2 \quad \frac{1}{2}$

$$f(x, y) \;\; g(x, y) \quad \frac{\sqrt{3}}{4}$$

$$f(x, y) \quad x^2 \quad y^2 \qquad \frac{\sqrt{3}}{2}$$

$$g(x, y) \quad 2xy \qquad \frac{1}{2}$$

14. $f(x) \quad \frac{x \quad 5}{\sqrt{x^2 \quad 1}}$



15. $f(x)$ is injective for $x \qquad , \frac{1}{5}$

$[\;\;] \quad \frac{1}{5} \quad 0$

16. $f : R \quad R \; f(x) \quad \frac{x^3}{3} \quad (m \quad 1)x^2 \quad (m \quad 5)x \quad n$

$f\,(x) \quad x^2 \quad 2(m \quad 1)x \quad (m \quad 5) \quad 0$

$\qquad\qquad\qquad\qquad\qquad\qquad 0$

$$4(m \quad 1)^2 \quad 4(m \quad 5) \quad 0$$

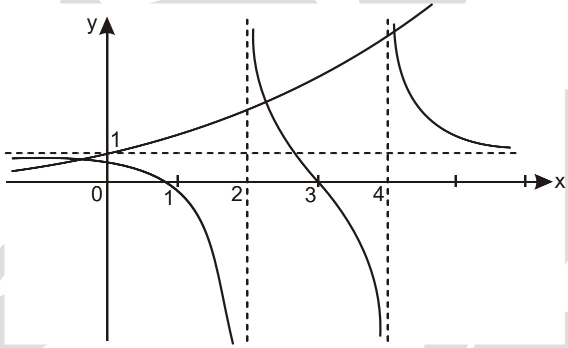
$$m^2 \quad 3m \quad 4 \quad 0$$

$$(m \quad 4)(m \quad 1) \quad 0$$

$$1 \quad m \quad 4$$

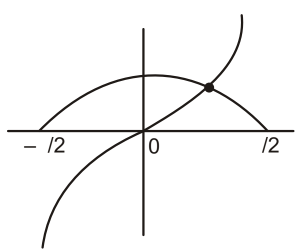
17. $f(x) \quad \dfrac{(x \quad 1)(x \quad 3)}{(x \quad 2)(x \quad 4)} \quad e^x$

$f(x) \quad 0$ has three solutions.



$f(\quad x) \quad \dfrac{(x \quad 1)(x \quad 3)}{(x \quad 2)(x \quad 4)} \quad e^{\quad x} \quad 0$ has three solutions.

$x^3 \quad \cos x$

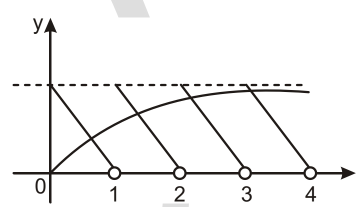


one solution

there are total 7 solutions.

18. $\cos^{\quad 1} \dfrac{2}{(1 \quad x)^2} \quad 1 \qquad (1 \quad \{x\})$

there are total 76 solutions.



19. $f(x) \quad x^2 \quad bx \quad c \quad 0 < \begin{matrix} p_1 \\ p_2 \end{matrix}$

$p_1 \quad p_2 \quad b$ (odd no.)

$p_1 \quad 2$

$p_1 p_2 \quad c$

$b \quad c \quad (p_2 \quad 2) \quad 2p_2 \quad 35$

$p_2 \quad 11$

$f(x) \quad x^2 \quad 13x \quad 22$

$$f(x)_{\min} \quad \frac{81}{4}$$

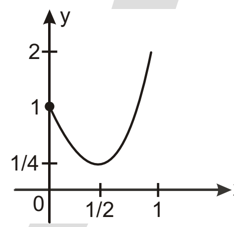
20. $f(x) \quad \lim_{x \quad 0} \dfrac{f(x) \quad f\left(\dfrac{x}{7}\right)}{x \quad \dfrac{x}{7}} \quad \dfrac{1}{6}$

$f(x) \quad \dfrac{x}{6} \quad 1 \qquad f(42) \quad 8$

21. $g(x) \quad f(x) \qquad 0 \quad x \quad \dfrac{1}{2}$

$\dfrac{1}{4} \qquad \dfrac{1}{2} \quad x \quad 1$

$3 \quad x \qquad 1 \quad x \quad 2$



22. $x \quad \left(\dfrac{10}{4}\right)^{100}_{r \quad 3} \dfrac{1}{r \quad 2} \quad \dfrac{1}{r \quad 2} \quad \dfrac{10}{4} \quad 1 \quad \dfrac{1}{2} \quad \dfrac{1}{3} \quad \dfrac{1}{4} \quad \dfrac{1}{102} \quad \dfrac{1}{101} \quad \dfrac{1}{100} \quad \dfrac{1}{99}$

$5 \quad 49 \quad \dfrac{1}{99} \quad \dfrac{1}{200} \quad \dfrac{1}{303} \quad \dfrac{1}{408}$

$[x] \quad 5$

23. $f(x) \quad x$ has two real roots.

$cx^2 \quad (d \quad a)x \quad b \quad 0 \begin{array}{l} \nearrow 7 \\ \searrow 11 \end{array}$

$\dfrac{a \quad d}{c} \quad 18$ and $\dfrac{b}{c} \quad 77$

if $f(f(x)) \quad x \quad x \quad R \qquad (ac \quad cd)x^2 \quad (d^2 \quad a^2)x \quad (a \quad d)b \quad 0$

$a \quad d \quad 0 \qquad a \quad d$

$f(x)$ will not attain the value $\dfrac{a}{c} \quad 9.$

24. $A \quad (1, 3)$

$p \quad 2^{1 \quad 1}, p \quad 2^{1 \quad 3}$

$1 \quad 2(p \quad 7) \quad 5 \quad 0$ and $9 \quad 6(p \quad 7) \quad 5 \quad 0 \qquad p \quad [\ 4, \quad 1]$

25. $y \quad \dfrac{x \quad \frac{1}{x}}{x^3 \quad \frac{1}{x^3} \quad 2}$ Let $t \quad x \quad \dfrac{1}{x} \quad 0$ for $x \quad 1$

$y \quad \dfrac{t}{t(t^2 \quad 3) \quad 2}$ $x^3 \quad \dfrac{1}{x^3} \quad t(t^2 \quad 3)$

$\dfrac{t}{t^3 \quad 3t \quad 2}$

$t^2 \quad \dfrac{2}{t} \quad t^2 \quad \dfrac{1}{t} \quad \dfrac{1}{t} \quad 3$

$\dfrac{1}{t^2 \quad \frac{2}{t} \quad 3}$ $t^2 \quad \dfrac{2}{t} \quad 3 \quad 6$ (AM GM)

$y_{\max} \quad \dfrac{1}{\left(t^2 \quad \frac{2}{t} \quad 3\right)_{\min}} \quad \dfrac{1}{6}$

$p \quad 1, q \quad 6$

28. $a \quad ar \quad ar^2 \quad 1$

$a^2r \quad a^2r^2 \quad a^2r^3 \qquad ar(a \quad ar \quad ar^2) \quad ar$

a^3r^3

29. $m \quad {}^6C_4 \quad 1 \quad 15$

$n \quad \dfrac{6!}{3!1!1!1!3!} \quad 4! \quad \dfrac{6!}{(2!)^4} \quad 4! \quad 1560$

30. $\displaystyle\sum_{r \quad 1}^{n} [\log_2 r] \quad 0 \quad 1 \quad 1 \quad (2 \quad 2 \quad 2 \quad 2) \quad \underbrace{(3 \quad 3 \qquad 3)}_{8 \text{ times}}$

$2 \quad 1 \quad 4 \quad 2 \quad 8 \quad 3$

32. 3

$|(x \quad 2y)(y \quad x)(x \quad 3y)| \quad f(x, y)$

No rain, then $f(x, y) \quad 0$ hence 3 lines.

33. Cubic $(x^2 \quad 5x \quad 6)(x \qquad) \quad 2(Bx \quad 100 \quad 4 \quad)$

$(x^2 \quad 5x \quad 4)(x \qquad) \quad Bx \quad 100 \quad 4$

Both identical $B \quad 2$

50

Cubic $(x^2 \quad 5x \quad 6)(x \quad 50) \quad 4x \quad 200$

34. $f(\ )\quad 0 \qquad\qquad 5\quad \sqrt{5}$

$f(f(f(x)))\quad 5\quad \sqrt{5}$

Since $f(x)\quad (x\quad 5)^2\quad 5$

$f(f(f(x)))\quad 5\quad \sqrt{5}$

$((f(f(f(x))))\quad 5)^2\quad 5\quad \sqrt{5}$

$(f(f)\quad 5)^2\quad \sqrt{5}$

$f(f)\quad 5\quad 5^{1/4}$

$f(f)\quad 5\quad 5^{1/4}$

$(f\quad 5)^2\quad 5\quad 5\quad 5^{1/4}$

$(f\quad 5)^2\quad 5^{1/4}$

$f\quad 5\quad 5^{1/8}$

35. Let $\ln x\quad t$

$$y\quad \frac{2t^2\quad 3t\quad 3}{t^2\quad 2t\quad 2}\qquad (y\quad 2)t^2\quad (2y\quad 3)t\quad (2y\quad 3)\quad 0$$

$$D\quad 0\qquad (2y\quad 3)(2y\quad 5)\quad 0\qquad \frac{3}{2}\quad y\quad \frac{5}{2}$$

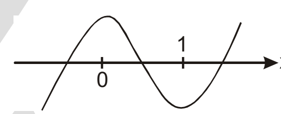
36. $P(x)\quad (x\quad 3)Q_1(x)\quad 6 \qquad P(3)\quad 6$ ...(1)

$P(x)\quad (x^2\quad 9)Q(x)\quad (ax\quad b)$

$P(3)\quad 3a\quad b\quad 6$

If equation of odd degree polynomial, then $b\quad 0, a\quad 2.$

37. $f(x)\quad 2x^3\quad 3x^2\quad P$



$f\,(x)\quad 6x^2\quad 6x\quad 6x(x\quad 1)$

$f(0)\quad 0\qquad f(1)\quad 0$

$P\quad 0\quad P\quad 1\quad 0$

38. $f(x)\quad \dfrac{1}{\sqrt{\ln(\cos^{\ 1} x)}}$

$\ln(\cos^{\ 1} x)\quad 0 \qquad \cos^{\ 1} x\quad 1$

❑❑❑

2

LIMIT

Exercise-1 : Single Choice Problems

1. $$\lim_{x \quad 0} \frac{2\sin\frac{x \quad \tan x}{2} \sin\frac{\tan x \quad x}{2}}{\frac{x \quad \tan x}{2} \quad \frac{\tan x \quad x}{2}} \qquad \frac{x \quad \tan x}{x^3} \qquad \frac{x \quad \tan x}{x} \qquad \frac{1}{4}$$

$$\frac{1}{2} \qquad \frac{1}{3} \quad 2 \quad \frac{1}{3}$$ (use expansions)

3. $a \quad \lim_{x \quad 0} \dfrac{\ln(1 \quad \cos 2x \quad 1)}{\cos 2x \quad 1} \quad \dfrac{(\cos 2x \quad 1)}{3x^2} \qquad \dfrac{2}{3}$

$b \quad \lim_{x \quad 0} \dfrac{\sin^2 2x}{4x^2} \quad \dfrac{4x^2}{x^2 \quad \dfrac{1 \quad e^x}{x}} \qquad 4$

$c \quad \lim_{x \quad 1} \dfrac{\sqrt{x}(1 \quad x)}{\dfrac{\ln(1 \quad x \quad 1)}{x \quad 1} \quad (x \quad 1)(\sqrt{x} \quad 1)} \qquad \dfrac{1}{2}$

4. $f(x) \quad \dfrac{}{2} \quad 3\tan^{1} x$

$g(x) \quad 2\tan^{1} x$

$\lim_{x \quad 0} \dfrac{f(x) \quad f(a)}{g(x) \quad g(a)} \qquad \dfrac{f\,(a)}{g\,(a)} \qquad \dfrac{3}{2}$

5. $\lim_{x \quad 0} e^{\frac{2}{x}\ln(1 \quad x) \quad 2 \quad \frac{4}{\sin x}} \quad e^{\lim_{x \quad 0}\frac{4}{\sin x} \quad e^{2\frac{\ln(1 \quad x)}{x} \quad 1} \quad 1}$

$$e^{\lim_{x \quad 0}\frac{4}{\sin x}\frac{e^{2\frac{\ln(1 \quad x)}{x} \quad 1} \quad 1}{2\frac{\ln(1 \quad x)}{x} \quad 1} \quad 2\frac{\ln(1 \quad x)}{x} \quad 1}$$

$$e^{\lim_{x \quad 0} 8 \frac{x \quad \frac{x^2}{2}}{x} \quad 1 \quad \frac{1}{\sin x}}$$

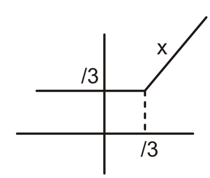
$$e^{\frac{8}{2}} \quad e^{4}$$

6. $\lim_{x} \frac{3}{x} \frac{x}{4} \quad \frac{x}{4} \quad \frac{3}{4} \quad 0 \quad \frac{3}{4}$

$p \quad q \quad 7$

7. $f(x) \quad \lim_{n} \frac{x \quad 1 \quad \overline{3x}^{\,n}}{1 \quad \overline{3x}^{\,n \quad 1}} \quad x\,; x \quad \overline{3}$

$$\lim_{n} \frac{\overline{3}\ \frac{3x}{}^{\,n} \quad 1}{\frac{3x}{}^{\,n \quad 1} \quad 1} \quad \overline{3}\,; x \quad \overline{3}$$



$\overline{3} \qquad x \quad \overline{3}$

$f(x) \quad x\,; \ x \quad \overline{3}$

$\overline{3}\,; \ x \quad \overline{3}$

Option (d) is wrong.

8. $\lim_{x \quad 0} \frac{\sin(\qquad \cos^2(\tan(\sin x))}{x^2} \quad \lim_{x \quad 0} \frac{\sin[\quad \sin^2(\tan(\sin x))]}{\sin^2(\tan(\sin x))} \quad \frac{\sin(\tan(\sin x))}{x}^{2}$

9. $\lim_{x \quad 3} f(x) \quad \lim_{x \quad 3} f(x) \quad \lim_{x \quad 3} \frac{(27)^{\frac{(x \quad 3)x}{27}} \quad 9}{3^x \quad 27} \quad \lim_{x \quad 3} \frac{1 \quad \cos(x \quad 3)}{(x \quad 3)^2}$

$$\lim_{x\quad 3}\frac{3^{2}\ 3^{\frac{x^{2}\ \ 3x}{9}\ \ 2}\ \ 1}{3^{3}(3^{x\ \ 3}\ \ 1)}\quad \frac{}{2}$$

$$\lim_{x\quad 3}\frac{1}{3}\frac{x^{2}\quad 3x\quad 18}{9(x\quad 3)}\quad \frac{}{2}\qquad \frac{1}{27}\ 9\quad \frac{}{2}\qquad \frac{2}{3}$$

10. $$\lim_{x\quad \frac{}{3}}\frac{2\sin\frac{\frac{}{3}\quad x}{2}\ \cos\frac{\frac{}{3}\quad x}{2}}{2\quad \cos x\quad \cos\frac{}{3}}\qquad \lim_{x\quad \frac{}{3}}\frac{\sin\frac{\frac{}{3}\quad x}{2}\ \cos\frac{\frac{}{3}\quad x}{2}}{2\sin\frac{\frac{}{3}\quad x}{2}\ \sin\frac{\frac{}{3}\quad x}{2}}$$

$$\frac{1}{2}\ \frac{1}{\frac{\sqrt{3}}{2}}\qquad \frac{1}{\sqrt{3}}$$

11. $$\lim_{x\quad \frac{}{2}}\frac{\sin x}{\cos^{1}[\sin^{3}x]}\qquad \frac{\sin\frac{}{2}}{\cos^{1}(0)}\quad \frac{1}{/2}\quad \frac{2}{}$$

13. $$\lim_{x\quad I}\{x\}\qquad \lim_{x\quad I}x\quad [x]\quad 1;\quad \lim_{x\quad I}\frac{e^{\{x\}}\quad \{x\}\quad 1}{\{x\}^{2}}\quad e\quad 2$$

16. $$\lim_{x} x\quad x^{5c\ \ 1}\ 1\quad \frac{7}{x}\quad \frac{2}{x^{5}}^{\ c}\quad 1\quad l$$

Case-I: $5c\quad 1\quad 0,$ then l

Case-II: $5c\quad 1\quad 0,$ then l

Since limit is finite and non-zero so $5c\quad 1\quad 0\qquad c\ \frac{1}{5}$

$$\lim_{x} x\quad 1\quad \frac{7}{x}\quad \frac{2}{x^{5}}^{\ 1/5}\qquad 1$$

$$\lim_{x} x\ \ 1\quad \frac{1}{5}\quad \frac{7}{x}\quad \frac{2}{x^{5}}\qquad 1$$ (by binomial approximation)

$$\frac{7}{5}$$

17. $\lim_{x \quad 0} \frac{\cos x \quad 1}{x^2} \quad \frac{\cos x \quad 1}{x^{n \quad 2}} \quad \frac{(e^x \quad 1)}{x^{n \quad 2}} \quad 0 \quad n \quad 1, 2, 3$

18. $1 \quad$ (form) $\quad e^{\lim_{x \quad 0} \frac{1}{1 \quad \cos x} \quad \frac{\sin x \quad x}{x}} \quad e^{2 \quad 1/6} \quad e^{\quad 1/3}$

19. $\lim_{x} [\sqrt{x^2 \quad x \quad 1} \quad (ax \quad b)] \quad 0$

So $a \quad 0$, on rationalizing

$$\lim_{x} \frac{(x^2 \quad x \quad 1) \quad [a^2x^2 \quad b^2 \quad (2ab)x]}{\sqrt{x^2 \quad x \quad 1} \quad ax \quad b} \quad 0$$

So, $1 \quad a^2 \quad 0 \qquad 1 \quad 2ab \quad 0$

$a \quad 1$

$\lim_{n} \sec^2[k! \ (\quad 1/2)] \quad 1 \quad a$

20. $f(x \quad T) \quad f(x \quad 2T) \qquad f(x \quad nT) \quad f(x)$

$$\lim_{n} \frac{nf(x)(1 \quad 2 \quad 3 \qquad n)}{f(x)(1 \quad 2^2 \quad 3^2 \qquad n^2)} \quad \lim_{n} \frac{n \quad \frac{n(n \quad 1)}{2}}{\frac{n(n \quad 1)(2n \quad 1)}{6}} \quad \frac{3}{2}$$

21. $265 \lim_{h \quad 0} \frac{h^2 \quad 3}{\frac{f(1 \quad h) \quad f(1)}{h} \quad \frac{\sin 5h}{h}} \qquad 265 \quad \frac{3}{f\,(1) \quad 5} \quad \frac{53 \quad 3}{f\,(1)}$

$\frac{53 \quad 3}{53}$ $\qquad [\because f\,(1) \quad 53]$

3

22. $\lim_{x \quad 0} \frac{\cos^2 x \quad 1}{\cos x \quad x^2 \quad (x \quad 1)}$

$$\lim_{x \quad 0} \frac{\sin^2 x}{x^2} \quad \frac{1}{\cos x(x \quad 1)} \quad 1$$

23. $f(x \quad y) \quad f(x) \ f(y)$

$f\,(x) \quad \lim_{h \quad 0} \frac{f(x \quad h) \quad f(x)}{h} \quad \lim_{h \quad 0} \frac{f(x) f(h) \quad f(x)}{h} \quad f(x) \lim_{h \quad 0} \frac{f(h) \quad 1}{h}$

If $f(h) \quad 1 \quad hP(h) \quad h^2Q(h) \qquad f\,(x) \quad f(x) \lim_{h \quad 0} \frac{hP(h) \quad h^2Q(h)}{h} \quad P(0) f(x)$

24. $$\lim_{x \quad \frac{}{2}} \frac{1 \quad \tan\frac{x}{2} \quad (1 \quad \sin x)}{1 \quad \tan\frac{x}{2} \quad (\quad 2x)^3}$$

$$\lim_{x \quad \frac{}{2}} \frac{\tan \quad \frac{}{4} \quad \frac{x}{2} \quad 1 \quad \cos \quad \frac{}{2} \quad x}{(\quad 2x)^3}$$

Let $x \quad \frac{}{2} \quad h$

$$\lim_{x \quad 0} \frac{\tan \quad \frac{h}{2} \quad (1 \quad \cos h)}{(\quad 2h)^3} \quad \frac{1}{32}$$

25. $$\lim_{x} \left(\frac{x \quad 3}{x \quad 2}\right)^{x} \quad e^{\lim\limits_{x} x \quad \frac{5}{x \quad 2}} \quad e^{5}$$

27. $\ln c \quad I,\quad (I \quad \text{integer})$

$c \quad e^{I}$

c is rational when $I \quad 0$

28. $$\lim_{x \quad 0} \left(1 \quad \frac{a \sin bx}{\cos x}\right)^{1/x} \quad e^{\lim\limits_{x \quad 0} \frac{1}{x} \quad 1 \quad \frac{a \sin bx}{\cos x} \quad 1} \quad e^{ab}$$

30. $a \quad \lim\limits_{x \quad 1} \dfrac{x}{\ln x} \quad \dfrac{1}{x \ln x} \quad \lim\limits_{x \quad 1} \dfrac{x^2 \quad 1}{x \ln x} \quad \lim\limits_{x \quad 1} \dfrac{x \quad 1}{x} \quad \dfrac{x \quad 1}{\ln x} \quad 2$

$b \quad 4, c \quad 1, d \quad 2$

32. $f(x) \quad x^2 \qquad 1 \quad x \quad 0$

$\qquad 1 \qquad x \quad 0$

$\qquad \dfrac{1}{x^2} \qquad 0 \quad x \quad 1$

$$\lim_{x \quad 0} \{f(x)\} \quad \lim_{x \quad 1} \{f(x)\} \quad \lim_{x \quad 1} \{f(x)\} \quad 0$$

33. Let $\sin^{1} x$

$$\lim_{\frac{}{4}} \frac{\cos^{1} \sin 2}{\sin \quad \sin\frac{}{4}} \quad \lim_{\frac{}{4}} \frac{2 \quad \frac{}{2}}{2 \sin \dfrac{\quad \frac{}{4}}{2} \cos \dfrac{\quad \frac{}{4}}{2}} \quad 2\sqrt{2}$$

$$\lim_{\frac{}{4}} \frac{\cos^{1} \sin 2}{\sin \quad \sin\frac{}{4}} \quad 2\sqrt{2}$$

34. $\lim_{n} \sum_{k\ 1}^{n} \sin\frac{}{2k} \quad \sin\frac{}{2(k \quad 2)} \qquad \lim_{n} \sum_{k\ 1}^{n} \cos\frac{}{2(k \quad 2)} \quad \cos\frac{}{2k}$

$$\lim_{n} \sin\frac{}{2} \quad \sin\frac{}{6} \quad \sin\frac{}{4} \quad \sin\frac{}{8} \quad \sin\frac{}{6} \quad \sin\frac{}{10} \qquad \sin\frac{}{2n} \quad \sin\frac{}{2(n \quad 2)}$$

$$\lim_{n} \cos\frac{}{6} \quad \cos\frac{}{2} \quad \cos\frac{}{8} \quad \cos\frac{}{4} \quad \cos\frac{}{10} \quad \cos\frac{}{6} \qquad \cos\frac{}{2(n \quad 2)} \quad \cos\frac{}{2n}$$

$$1 \quad \frac{1}{\sqrt{2}} \quad 2 \quad \frac{1}{\sqrt{2}} \quad 3$$

36. $\lim_{x \quad 0} \frac{(\cos x)^{\frac{1}{m} \ \frac{1}{n}} \quad 1}{x^2} \qquad \lim_{x \quad 0} \frac{\left(1 \quad 2\sin^2 \frac{x}{2}\right)^{\frac{1}{m} \ \frac{1}{n}} \quad 1}{x^2}$

$$\lim_{x \quad 0} 2 \quad \frac{1}{m} \quad \frac{1}{n} \quad \frac{\sin^2 \frac{x}{2}}{x^2} \qquad \frac{m \quad n}{2mn}$$

37. $\lim_{x \quad 0} \frac{x \quad xa\cos x \quad b\sin x}{x^3} \quad 1$

Using expansion,

$$\lim_{x \quad 0} \frac{x \quad xa\ 1 \quad \frac{x^2}{2!} \quad b\ x \quad \frac{x^3}{3!}}{x^3} \qquad \lim_{x \quad 0} \frac{x \quad ax \quad \frac{ax^3}{2!} \quad bx \quad \frac{bx^3}{3!}}{x^3}$$

Clearly, $1 \quad a \quad b \quad 0$ for limit to be finite ...(1)

$$\lim_{x \quad 0} \frac{b}{3!} \quad \frac{a}{2!} \quad \frac{x^3}{x^3} \quad 1 \qquad \frac{b}{6} \quad \frac{a}{2} \quad 1 \qquad b \quad 3a \quad 6 \qquad \text{...(2)}$$

From (1) and (2), $a \quad \frac{5}{2}, b \quad \frac{3}{2}$

38. $\lim_{x \quad 0} \frac{a\cos ax \quad \frac{e^x(\cos x \quad \sin x)}{e^x \quad \cos x}}{\sin bx \quad bx\ \cos bx} \quad \frac{1}{2}$

$$\lim_{x \quad 0} \frac{\cos^2 x \quad \cos x \quad \sin x}{\cos x(\sin bx \quad bx\cos bx)} \quad \frac{1}{2} \qquad (\because\ a \quad 1)$$

39. $\lim_{n} \frac{(1^3 \quad 2^3 \quad 3^3 \qquad n^3) \quad (1^2 \quad 2^2 \qquad n^2)}{n^4} \qquad \lim_{n} \frac{\left(\frac{n(n \quad 1)}{2}\right)^2 \quad \frac{n(2n \quad 1)(n \quad 1)}{6}}{n^4}$

$$\lim_{n} \frac{1}{4} \quad 1 \quad \frac{1}{n}^{2} \quad \frac{(2n \quad 1)(n \quad 1)}{6n^3} \quad \frac{1}{4}$$

40. $\lim_{x \quad 0} \frac{\cos(\sin x) \quad \cos x}{x^4} \qquad \lim_{x \quad 0} \frac{2\sin \frac{\sin x \quad x}{2} \quad \sin \frac{x \quad \sin x}{2}}{x^4}$

$$\lim_{x \quad 0} \frac{2}{x^4} \frac{(\sin x \quad x)}{2} \frac{(x \quad \sin x)}{2} \qquad \because \frac{\sin x \quad x}{2} \quad 0; \frac{x \quad \sin x}{2} \quad 0$$

$$\lim_{x \quad 0} \frac{1}{2} \quad 1 \quad \frac{\sin x}{x} \quad \frac{x \quad \sin x}{x^3} \quad \frac{1}{6}$$

42. $u_n \quad \frac{1}{2} \quad \frac{2}{2^2} \quad \frac{3}{2^3} \quad \frac{n}{2^n}$ ...(1)

$\frac{1}{2} u_n \quad \frac{1}{2^2} \quad \frac{2}{2^3} \quad \frac{n \quad 1}{2^n} \quad \frac{n}{2^{n \quad 1}}$ ...(2)

Substracting equation (1) and (2),

$$\frac{u_n}{2} \quad \frac{1}{2} \quad \frac{1}{2^2} \quad \frac{1}{2^3} \quad \frac{1}{2^n} \quad \frac{n}{2^{n \quad 1}} \qquad \frac{u_n}{2} \quad \frac{\frac{1}{2} \quad 1 \quad \frac{1}{2^n}}{1 \quad \frac{1}{2}} \quad \frac{n}{2^{n \quad 1}}$$

$$u_n \quad 2 \quad 1 \quad \frac{1}{2^n} \quad \frac{n}{2^{n \quad 1}}; \ \lim_{n \quad 0} u_n \quad 2$$

43. $e^{\lim_{x \quad 0} \frac{(\cos x \quad 1)}{\sin^2 x}} \qquad \lim_{x \quad 0} \frac{\frac{\sin 2x}{2x} \, 2x \quad 6x \, \frac{\tan^{\ 1} 3x}{3x} \quad 3x^2}{\frac{\ln(1 \quad 3x \quad \sin^2 x)}{3x \quad \sin^2 x} (3x \quad \sin^2 x) \quad xe^x} \qquad \frac{1}{\sqrt{e}} \quad 2$

44. $\tan \frac{x}{2} (1 \quad \sec x) \quad \tan x$

$$f_n(x) \quad \tan \frac{x}{2} (1 \quad \sec x)(1 \quad \sec 2x) \quad (1 \quad \sec 2^n x) \quad \tan 2^n x$$

45. $\lim_{x \quad \frac{}{4}} (1 \quad [x])^{\frac{1}{\ln(\tan x)}} \qquad \lim_{x \quad \frac{}{4}} (1)^{\frac{1}{\ln(\tan x)}} \quad 1$

46. $\lim_{x \quad 0} \frac{\{(a \quad n)nx \quad \tan x\}}{x^2} \frac{\sin nx}{nx} xn \quad 0$

$$\{(a \quad n)n \quad 1\}n \quad 0 \qquad a \quad n \quad \frac{1}{n}$$

47. $y \quad \lim_{n} \quad \frac{n!}{n^n} \quad \frac{3n^3 \quad 4}{4n^4 \quad 1}$

$$\ln y \quad \lim_{n} \frac{3n^3 \quad 4}{4n^4 \quad 1} \sum_{r \quad 1}^{n} \ln \frac{r}{n} \quad \frac{3}{4} \int_0^1 \ln x \, dx \quad \frac{3}{4} \qquad y \quad e^{3/4}$$

48. $\lim_{x} \frac{ax^2 \quad bx \quad c}{dx \quad e} \quad \lim_{x} \frac{ax \quad b \quad (c/x)}{d \quad (e/x)} \quad \lim_{x} \frac{a}{d} x \quad \frac{b}{d}$

if $\frac{a}{d}$ is positive.

if $\frac{a}{d}$ is negative.

Alternate solution :

$$\lim_{x} \frac{ax^2 \quad bx \quad c}{dx \quad e} \quad \lim_{x} \frac{a \quad (b/x) \quad (c/x^2)}{(d/x) \quad (e/x^2)}$$

Here $\frac{e}{x^2} \quad \frac{d}{x}$. Therefore,

$$\lim_{x} \frac{ax^2 \quad bx \quad c}{dx \quad e} \quad \lim_{x} \frac{a}{d/x}$$

$$\frac{a}{0} \text{ if } d \quad 0 \qquad \begin{matrix} \text{if } a \quad 0 \text{ and } d \quad 0 \\ \text{if } a \quad 0 \text{ and } d \quad 0 \end{matrix}$$

$$\frac{a}{0} \text{ if } d \quad 0 \qquad \begin{matrix} \text{if } a \quad 0 \text{ and } d \quad 0 \\ \text{if } a \quad 0 \text{ and } d \quad 0 \end{matrix}$$

49. $f(x) \quad \lim_{n} \tan^{1} \quad 4n^2 \quad 2\sin^2 \frac{x}{2n} \quad \lim_{n} \tan^{1} \quad 8n^2 \quad \left(\frac{\sin \frac{x}{2n}}{\frac{x}{2n}}\right)^2 \quad \frac{x^2}{4n^2} \quad \tan^{1}(2x^2)$

$$g(x) \quad \lim_{n} \frac{n^2}{2} \quad \frac{\ln \quad 1 \quad \cos^2 \frac{2x}{n} \quad 1}{\cos^2 \frac{2x}{n} \quad 1} \quad \cos \frac{2x}{n} \quad 1 \quad x^2$$

50. $\lim_{x \quad 0} \frac{\sin^2 x}{f(x)} \quad \frac{1}{3} \qquad f(x) \quad x^2(ax \quad 3) ; \quad a \quad 0$

51. $\lim_{x \quad 0} \frac{(2e^{2\sin x} \quad e^{\sin x} \quad 1)}{(x^2 \quad 2x)\, e^{\sin x}} \quad \lim_{x \quad 0} \frac{(2e^{\sin x} \quad 1)(e^{\sin x} \quad 1)}{x(x \quad 2)\, e^{\sin x}} \quad \frac{3}{2}$

52. $x^n \quad ax \quad b \quad (x \quad x_1)(x \quad x_2)(x \quad x_3) \quad (x \quad x_n)$

$$\lim_{x \quad x_1} \frac{x^n \quad ax \quad b}{x \quad x_1} \quad (x_1 \quad x_2)(x_1 \quad x_3) \quad (x_1 \quad x_n)$$

53. $$\lim_{x \quad 0} \frac{1 \quad \frac{1}{3}\sin^2 x \qquad 1 \quad \frac{1}{4}(2\tan x)}{\sin x \quad \tan^2 x} \quad \frac{1}{2}$$

54. $$\lim_{x \quad 0} \frac{f(x)}{x^2} \quad \lim_{x \quad 0} \begin{vmatrix} \cos x & \frac{2\sin x}{x} & \tan x \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} \quad 1$$

Exercise-2 : One or More than One Answer is/are Correct

1. $e^{\lim_{x \quad 0} \frac{1}{3x^2}(p\tan qx^2 \quad 3\cos^2 x \quad 3)}$

$e^{\lim_{x \quad 0} \frac{pq}{3} \quad \frac{3(1 \quad \cos^2 x)}{3x^2}}$

$\frac{pq}{3} \quad 1 \quad \frac{5}{3}; \qquad pq \quad 2$

3. $a \quad e \quad 2$

(a) $L \quad a \lim_{x} \quad 1 \quad \frac{2}{a}^{x} \quad \frac{e}{a}^{x} \quad {}^{1/x}$

$x \quad a, \frac{2}{a} \quad 0, \frac{e}{a} \quad 0, \frac{1}{x} \quad 0$

So, $L \quad a$

(b) If $a \quad 2e \quad 2$

$L \quad \lim_{x} (2^x \quad (2e)^x \quad e^x)^{1/x} \quad 2e \lim_{x} \quad \frac{1}{e}^{x} \quad 1 \quad \frac{1}{2}^{x} \quad {}^{1/x} \quad 2e(1) \quad 2e$

(c) If $0 \quad a \quad e$

$L \quad e \lim_{x} \quad \frac{2}{e}^{x} \quad \frac{a}{e}^{x} \quad 1 \quad {}^{1/x} \quad e$

(d) $a \quad \frac{e}{2} \quad 1$

$$L \quad \lim_{x} \quad 2^{x} \quad \frac{2a}{2}^{x} \quad e^{x} \quad ^{1/x} \quad 2a \lim_{x} \quad \frac{1}{a}^{x} \quad \frac{1}{2}^{x} \quad \frac{e}{2a}^{x} \quad ^{1/x} \quad 0$$

5. $f(x) \quad \cos(\sin x)$

Range is $[\cos 1, 1]$.

8. $f(x) \quad x \quad \frac{3}{2} \quad \frac{3}{2}[\cos x]$

9. If $x \quad \frac{1}{2^{2^n}}$ then $f(x) \quad 0$ but if $x \quad \frac{1}{2^{2^n}}$ then $\lim_{x \quad 0} f(x) \quad \lim_{n} (\quad 1)^n$, hence does not exist.

Also, if $x \quad \frac{1}{2^{2^n}}$ then $2x \quad \frac{1}{2^{2^n}} \quad f(2x) \quad 0$

11. $$\lim_{x \quad 0} \frac{\cos^{1}(1 \quad x)\sin^{1}(1 \quad x)}{\sqrt{2x}(1 \quad x)} \quad \lim_{x \quad 0} \frac{\frac{\sin^{1}\sqrt{2x \quad x^2}}{\sqrt{2x \quad x^2}} \sqrt{2x \quad x^2} \sin^{1}(1 \quad x)}{\sqrt{2x}(1 \quad x)} \quad \overline{2}$$

$$\lim_{x \quad 0} \frac{\cos^{1}(\quad x)\sin^{1}(\quad x)}{\sqrt{2(x \quad 1)}(\quad x)} \quad \overline{2\sqrt{2}}$$

12. $$\lim_{x \quad 0} \frac{2\sin\frac{\sin x \quad x}{2} \cos\frac{\sin x \quad x}{2}}{ax^3 \quad bx^5 \quad c} \quad \frac{1}{12}$$

$$\lim_{x \quad 0} \frac{2 \frac{\sin\frac{\sin x \quad x}{2}}{\frac{\sin x \quad x}{2}} \frac{\sin x \quad x}{2} \cos\frac{\sin x \quad x}{2}}{ax^3 \quad bx^5 \quad c} \quad \frac{1}{12}$$

14. $\cos^2 n \quad \overline{3}$

15. $\sin \quad \sin \quad \frac{\sin}{\sin} \quad \sin \quad \sin \quad \frac{1}{2}$

16. $\lim_{x \quad 2} [5 \quad 2x] \quad 0$

$\lim_{x \quad 2} [|x \quad 2| \quad a^2 \quad 6a \quad 9] \quad 0 \quad (a \quad 3)^2 \quad 1$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

1. $S_1 \quad 1, S_2 \quad 7, S_3 \quad 19$

$\qquad S_n \quad 1 \quad 3n(n \quad 1)$

$$\lim_{n} \frac{S_n}{n^2} \quad 3$$

2. $r_1 \quad 1, r_2 \quad \frac{1}{3}, r_3 \quad \frac{1}{5}$

or $\quad r_n \quad \frac{1}{2n \quad 1}$

$$\lim_{n} n \quad \frac{1}{2n \quad 1} \quad \frac{1}{2}$$

Paragraph for Question Nos. 3 to 4

3. $x \quad 0, \ x \quad \tan x$

$x \quad 0, \ x \quad \tan x \qquad x \quad \tan x \quad 0$

$\qquad [x \quad \tan x] \quad 0$

$$\lim_{x \quad 0} f([x \quad \tan x]) \quad f(0) \quad 4$$

4. $x \quad 0 \qquad x \quad \tan x$

$\frac{x}{\tan x} \quad 1$

$$\lim_{x \quad 0} \frac{x}{\tan x} \qquad \frac{x}{\tan x} \qquad 1$$

$$\lim_{x \quad 0} f \frac{x}{\tan x} \qquad \lim_{x \quad 0} f \frac{x}{\tan x} \qquad f(1 \quad) \quad 2 \quad 5 \quad 7$$

Paragraph for Question Nos. 5 to 6

5. $f(x) \quad 1 \quad |x \quad 2|$

$x \qquad 2 \quad , f(x) \qquad 1$ and $x \qquad 2 \quad , f(x) \qquad 1$

$$\text{R.H.L.} \quad \lim_{x \quad 2} (f(x))^{\frac{1}{\sin \frac{x}{2}}} \qquad e^{\lim\limits_{x \quad 2} \frac{f(x) \; 1}{\sin \frac{x}{2}}}$$

$$e^{\lim\limits_{x\ 2}\frac{1\ (x\ 2)\ 1}{\sin\ 1\ \frac{x}{2}}}\quad e^{\lim\limits_{x\ 2}\frac{(x\ 2)}{\frac{\sin\frac{}{2}(2\ x)}{\frac{}{2}(2\ x)}}\ \frac{}{2}(2\ x)}$$

$$e^{2/}$$

$$\text{L.H.L.}\quad \lim_{x\ 2}(f(x))^{\frac{1}{\sin x\ \frac{x}{2}}}\quad e^{\lim\limits_{x\ 2}\frac{f(x)\ 1}{\sin\ \frac{x}{2}}}$$

$$e^{\lim\limits_{x\ 2}\frac{1\ (x\ 2)\ 1}{\sin\frac{}{2}(2\ x)}}\quad e^{\lim\limits_{x\ 2}\frac{x\ 2}{\frac{}{2}(2\ x)}}$$

$$e^{\ 2/}$$

Limit does not exist.

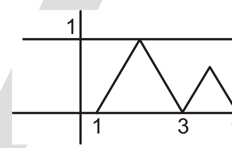
6. $[1,3]$

as $f(3x)\quad f(x)$

$x\quad [1,3]\quad ;\quad f(x)\quad [0,1]$

$3x\quad [3,9]\quad ;\quad f(3x)\quad f(x)\quad [0,\ ]$

$9x\quad [9,27]\ ;\quad f(9x)\quad f(3x)\quad [0,\ ^2]$



area between $[1,3]$ is $_1\ \frac{1}{2}\ 2\ 1\ 1$

area between $[3,9]$ is $_2\ \frac{1}{2}\ 6\quad 3$

area between $[9,27]$ is $_3\ \frac{1}{2}\ 18\ ^2\ 9\ ^2$

$1, 3\ , 9\ ^2,$ is converges when (g.p.) $|3\ |\ 1\quad \frac{1}{3},\frac{1}{3}$

Paragraph for Question Nos. 7 to 9

7. $$\lim_{x\ 0}\frac{[(1\ bx)\ (1\ ax)\sqrt{1\ x}]}{x^3}\quad \lim_{x\ 0}\frac{(1\ bx)\ (1\ ax)\ 1\ \frac{x}{2}\ \frac{x^2}{8}\ \frac{x^3}{16}}{x^3}$$

$$\lim_{x\ 0}\frac{bx\ \frac{x}{2}\ \frac{x^2}{8}\ \frac{x^3}{16}\ ax\ \frac{ax^2}{2}\ \frac{ax^3}{8}}{x^3}$$

coefficient of x and x^2 0 b a $\frac{1}{2}$ and $\frac{a}{2}$ $\frac{1}{8}$

a $\frac{1}{4}, b$ $\frac{3}{4}$

8. a b 1

9. l $\frac{1}{32}$; b $\frac{3}{4}$

Paragraph for Question Nos. 10 to 11

Sol. $\sin x$ $\sin y$ 1

$$y \quad \frac{\cos x}{\sqrt{2\sin x \quad \sin^2 x}}$$

$$y \quad \frac{\sin^2 x \quad \sin x \quad 1}{(2\sin x \quad \sin^2 x)^{3/2}}$$

Exercise-5 : Subjective Type Problems

1. $\lim\limits_{x \quad 0} \frac{\ln\tan \frac{}{4} \quad x}{\tan \quad x} \quad \lim\limits_{x \quad 0} \frac{\ln \frac{1 \quad \tan \quad x}{1 \quad \tan \quad x} \quad 1 \quad 1}{\tan \quad x}$

$1 \quad 2 \frac{}{} \quad 1$

$\frac{}{} \quad 2$

3. $a(x^3 \quad 1) \quad (x \quad 1) \quad 0$

$(x \quad 1)(ax^2 \quad ax \quad a \quad 1) \quad 0$

, $\quad 1$ so, , are roots of $ax^2 \quad ax \quad a \quad 1 \quad 0$

$1, \quad \frac{a \quad 1}{a}$

$$\lim_{x \quad \frac{1}{}} \frac{(1 \quad a)x^3 \quad x^2 \quad a}{(e^{1 \quad x} \quad 1)(x \quad 1)} \quad \lim_{x \quad \frac{1}{}} \frac{(x^3 \quad x^2) \quad a(x^3 \quad 1)}{(e^{1 \quad x} \quad 1)(x \quad 1)}$$

$$\lim_{x \quad \frac{1}{}} \frac{[x^2 \quad a(x^2 \quad x \quad 1)]}{(e^{1 \quad x} \quad 1)} \quad \lim_{x \quad \frac{1}{}} \frac{(1 \quad a)x^2 \quad ax \quad a}{\frac{e^{1 \quad x} \quad 1}{1 \quad x} \quad (1 \quad x)}$$

$$\lim_{x \quad 1} a \frac{\frac{1 \quad a}{a} x^2 \quad (1)x \quad 1}{(1 \quad x)} \quad \lim_{x \quad 1} a \frac{(\quad x^2 \quad (\qquad)x \quad 1)}{(1 \quad x)}$$

$$\lim_{x \quad 1} a \frac{(1 \quad (\quad)x)(1 \quad (\quad)x)}{(1 \quad x)} \quad \frac{a(\qquad)}{}$$

4. $\lim_{x \quad 0} \frac{(4^x \quad 1)(5^x \quad 1)(7^x \quad 1)}{x \sin^2 x} \quad 2 \ln 2 \ln 5 \ln 7$

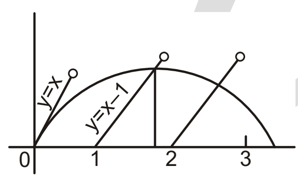
5. $\lim_{x \quad 0} \frac{ax \cos x \quad b \sin x}{x^2 \sin x} \quad \frac{1}{3}$

$$\lim_{x \quad 0} \frac{ax \; 1 \quad \frac{x^2}{2!} \quad \frac{x^4}{4!} \qquad b \quad x \quad \frac{x^3}{3!} \quad \frac{x^5}{5!}}{x^2 \sin x} \quad \frac{1}{3}$$

$a \quad b \quad 0$ and $\frac{a}{2} \quad \frac{b}{6} \quad \frac{1}{3}$

$b \quad 1, \; a \qquad 1$

7. $\lim_{x} \frac{\sin x}{x \quad 1} \qquad 0$



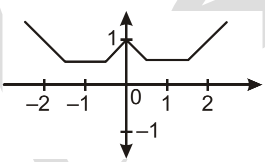
❑❑❑

3 CONTINUITY, DIFFERENTIABILITY AND DIFFERENTIATION

Exercise-1 : Single Choice Problems

1. $f\ (x) \quad \lim_{h \quad 0} \dfrac{f(x \quad h) \quad f(x)}{h} \quad \lim_{h \quad 0} \dfrac{f(x) \quad f(h) \quad 3hx(h \quad x) \quad f(x)}{h}$

$f\ (x) \quad 3x^2 \quad f\ (0) \qquad f\ (x) \quad 6x$

2. 

$f(x)$ is non-differentiable at five points.

3. $\dfrac{x}{5}$ is integer at 21 points in $[0, 100]$

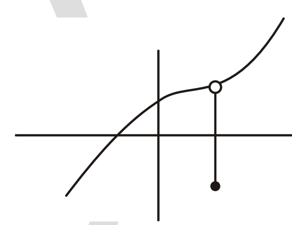
$\dfrac{x}{2}$ is integer at 51 points in $[0, 100]$

But when x is a multiple of 10 then $f(x)$ is continuous.

So that respective points should be subtract from both *i.e.*, multiple of 10 are 11 points in $[0, 100]$.

$$21 \quad 51 \quad 11 \quad 11 \quad 72 \quad 22 \quad 50$$

4. $f(x)$ has isolated point of discontinuity but $|f(x)|$ is continuous at $x \quad a$



So, $\lim_{x \quad a} f(x)$ and $f(a)$ has opposite sign, with same magnitude.

So, $\lim_{x \quad a} f(x) \qquad f(a)$

$\lim_{x \quad a} f(x) \quad f(a) \quad 0$

5. $\lim_{x \quad 0} \dfrac{f(4x) \quad 3f(3x) \quad 3f(2x) \quad f(x)}{x^3} \quad 12$

$$\lim_{x \quad 0} \frac{4f(4x) \quad 9f(3x) \quad 6f(2x) \quad f(x)}{3x^2} \quad 12$$

$$\lim_{x \quad 0} \frac{4^2 f\ (4x) \quad 27f\ (3x) \quad 12f\ (2x) \quad f\ (x)}{6x} \quad 12$$

$$\lim_{x \quad 0} \frac{4^3 f\ (4x) \quad 81f\ (3x) \quad 24f\ (2x) \quad f\ (x)}{6} \quad 12$$

$$(4^3 \quad 81 \quad 24 \quad 1) f\ (0) \quad 12 \quad 6$$

$$6f\ (0) \quad 12 \quad 6$$

$$f\ (0) \quad 12$$

6. $$y \quad \frac{1}{1 \quad (\tan\ )^{\sin \quad \cos} \quad (\tan\ )^{\cot \quad \cos}} \quad \frac{1}{1 \quad (\tan\ )^{\cos \quad \sin} \quad (\tan\ )^{\cot \quad \sin}} \quad \frac{1}{1 \quad (\tan\ )^{\cos \quad \cot} \quad (\tan\ )^{\sin \quad \cot}}$$

$$y \quad \frac{(\tan\ )^{\cos}}{(\tan\ )^{\cos} \quad (\tan\ )^{\sin} \quad (\tan\ )^{\cot}} \quad \frac{(\tan\ )^{\sin}}{(\tan\ )^{\cos} \quad (\tan\ )^{\sin} \quad (\tan\ )^{\cot}} \quad \frac{(\tan\ )^{\cot}}{(\tan\ )^{\cos} \quad (\tan\ )^{\sin} \quad (\tan\ )^{\cot}}$$

$$y \quad 1$$

$$\left.\frac{dy}{dx}\right|_{0 \quad /3} \quad 0$$

7. $f\ (x) \quad \sin(x^2)$

$$y \quad f(x^2 \quad 1)$$

$$\frac{dy}{dx} \quad f\ (x^2 \quad 1)2x$$

$$\left.\frac{dy}{dx}\right|_{x \quad 1} \quad 2 \quad f\ (2) \quad 2\sin 4$$

8. Clearly $\sin x, \cos x$ are negative at $x \quad \frac{7}{6}$

So, $f(x) \quad (\sin x \quad \cos x)$

$$f\ (x) \quad (\sin x \quad \cos x)$$

9. $2\sin x \cos y \quad 1$

$$\cos x \cos y \quad \sin x \sin y \ \ y \quad 0 \qquad y_{(\ /4,\ \ /4)} \quad 1$$

$$y \quad \cot x \cot y$$

$$y \quad \cot x \operatorname{cosec}^2 y \quad y \quad \cot y \operatorname{cosec}^2 x$$

$$y_{(\ /4,\ \ /4)} \quad (1 \quad 2 \quad 1) \quad (1 \quad 2) \quad 0$$

10. $\dfrac{dx}{dt} \quad 2t\, f\,(t^2), \quad \dfrac{dy}{dt} \quad 3t^2 f\,(t^3)$

$\dfrac{dy}{dx} \quad \dfrac{3}{2}\dfrac{tf\,(t^3)}{f\,(t^2)}$

$\dfrac{d^2y}{dx^2} \quad \dfrac{3}{2} \quad \dfrac{f\,(t^2)(f\,(t^3) \quad 3t^3 f\,(t^3)) \quad 2t^2 f\,(t^3) \quad f\,(t^2)}{(f\,(t^2))^2} \quad \dfrac{dt}{dx}$

$\left.\dfrac{d^2y}{dx^2}\right|_{t \quad 1} \quad \dfrac{3}{2} \quad \dfrac{f\,(1)(f\,(1) \quad 3f\,(1)) \quad 2f\,(1) \quad f\,(1)}{(f\,(1))^2} \quad \dfrac{1}{2f\,(1)} \quad \dfrac{3}{4} \quad \dfrac{f\,(1) \quad f\,(1)}{(f\,(1))^2}$

11. L.H.L. $a \quad 1$

R.H.L. $b \quad 1$

they are continuous L.H.L. R.H.L.

12. $y \quad \dfrac{\frac{1}{x}}{\frac{1}{x}} \quad \dfrac{\overline{x}}{\frac{1}{x} \quad \frac{1}{x}} \quad \dfrac{\overline{x^2}}{\frac{1}{x} \quad \frac{1}{x} \quad \frac{1}{x}}$

$\dfrac{\frac{1}{x^2}}{\frac{1}{x} \quad \frac{1}{x}} \quad \dfrac{\overline{x^2}}{\frac{1}{x} \quad \frac{1}{x} \quad \frac{1}{x}} \quad \dfrac{\frac{1}{x^3}}{\frac{1}{x} \quad \frac{1}{x} \quad \frac{1}{x}}$

$\log y \quad 3\ln x \quad \ln\dfrac{1}{x} \quad \ln\dfrac{1}{x} \quad \ln\dfrac{1}{x}$

$\dfrac{1}{y}y \quad \dfrac{3}{x} \quad \dfrac{\frac{1}{x^2}}{\frac{1}{x}} \quad \dfrac{\frac{1}{x^2}}{\frac{1}{x}} \quad \dfrac{\frac{1}{x^2}}{\frac{1}{x}}$

$y \quad \dfrac{y}{x} \quad 3 \quad \dfrac{\frac{1}{x}}{\frac{1}{x}} \quad \dfrac{\frac{1}{x}}{\frac{1}{x}} \quad \dfrac{\frac{1}{x}}{\frac{1}{x}}$

$y \quad \dfrac{y}{x} \quad \dfrac{}{1/x} \quad \dfrac{}{1/x} \quad \dfrac{}{1/x}$

13. $f(x) \quad \sqrt{\dfrac{1 \quad \sin^{1} x}{1 \quad \tan^{1} x}}$

$\ln f(x) \quad \dfrac{1}{2}[\ln(1 \quad \sin^{1} x) \quad \ln(1 \quad \tan^{1} x)]$

$$\frac{f(x)}{f(x)} \quad \frac{1}{2} \quad \frac{1}{(1 \quad \sin^{\ 1} x)\sqrt{1 \quad x^2}} \quad \frac{1}{(1 \quad \tan^{\ 1} x)(1 \quad x^2)}$$

$$f(0) \quad 1$$

14. $\sin^2 x \quad \sin^2 x \quad 2\sin^2 x \quad 0 \quad x \quad n$

15. $f(x)$ → $\tan x \quad \tan x \quad \cot x$

→ $\cot x \quad \tan x \quad \cot x$

Points of non-derivability $\frac{}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}$

16. $g(x) \quad |||x \quad 1| \quad 1| \quad 1|$

$x \quad 3 \qquad x \quad 3$

$(x \quad 3) \qquad 2 \quad x \quad 3$

18. $$\frac{d^2x}{dy^2} \quad \frac{1}{\frac{dy}{dx}^{\ 3}} \frac{d^2y}{dx^2}$$

$$\frac{dy}{dx} \quad 1 \quad e^x, \frac{d^2y}{dx^2} \quad e^x$$

at $x \quad \ln 2, \frac{dy}{dx} \quad 3, \frac{d^2y}{dx^2} \quad 2$

$$\frac{d^2x}{dy^2} \quad \frac{2}{27}$$

19. $g(f(x)) \quad \frac{1}{f(x)}$

$f(x) \quad 4$ at $x \quad 2$

$$g(\ 4) \quad \frac{1}{f(\ 2)} \quad \frac{1}{2}$$

20. $f(x) \quad 2 \quad x \qquad x \quad 1$

$x \qquad 0 \quad x \quad 1$

$x \qquad 1 \quad x \quad 0$

$x \quad 2 \qquad x \quad 1$

21. $f(x) \quad \cos x^2$

$f(x) \quad 2x\sin x^2$

22. $f(g(x)) \quad x \qquad f(g(x))g(x) \quad 1 \qquad g(x) \quad \frac{1}{f(g(x))} \quad 1 \quad (g(x))^5$

$g(x) \quad 5(g(x))^4 g(x)$

23. $f(x) \quad x^2 \qquad x \quad 1$

$\qquad x \qquad 0 \quad x \quad 1$

$\qquad 2x \qquad 1 \quad x \quad 0$

$\qquad x \quad 1 \qquad x \quad 1$

Clearly it is non-differentiable at $x \quad 0, \quad 1$ and 1.

24. $f(x) \quad \lim\limits_{n} \quad \cos\dfrac{x}{2} \cos\dfrac{x}{2^2} \cos\dfrac{x}{2^3} \qquad \cos\dfrac{x}{2^n} \qquad \lim\limits_{n} \dfrac{\sin x}{2^n \sin \dfrac{x}{2^n}} \qquad \dfrac{\sin x}{x}$

25. $f\,\dfrac{}{4} \qquad f\,\dfrac{}{4} \qquad f\,\dfrac{}{4}$

$$\lim_{x \quad \frac{}{4}} \frac{1 \quad \tan x}{4x} \qquad \lim_{x \quad \frac{}{4}} \frac{\tan \frac{}{4} \quad x \quad (1 \quad \tan x)}{4 \quad x \quad \frac{}{4}} \qquad \frac{1}{2}$$

26. $f\,(0) \quad \lim\limits_{h \quad 0} \dfrac{f(0 \quad h) \quad f(0)}{h} \qquad \lim\limits_{h \quad 0} \dfrac{e^{\frac{1}{h^2}} \sin \frac{1}{h}}{h}$

27. $\dfrac{dy}{dx} \quad 2y \quad 10$

$\dfrac{dy}{y \quad 5} \quad 2 \quad dx$

$\ln(y \quad 5) \quad 2x \quad c$

$\qquad y \quad 5(e^{2x} \quad 1) \qquad (\because c \quad \ln 5)$

$f(x) \quad 5\sec^2 x \quad 0 \qquad e^{2x} \quad \tan^2 x \quad 0$

28. $f\,\dfrac{}{2} \qquad \lim\limits_{x \quad \frac{}{2}} \dfrac{\sin\{\cos x\}}{x \quad \frac{}{2}} \qquad \lim\limits_{x \quad \frac{}{2}} \dfrac{\sin(\cos x)}{x \quad \frac{}{2}} \qquad 1$

$f\,\dfrac{}{2} \qquad \lim\limits_{x \quad \frac{}{2}} \dfrac{\sin\{\cos x\}}{x \quad \frac{}{2}} \qquad \lim\limits_{x \quad \frac{}{2}} \dfrac{\sin(\cos x \quad 1)}{x \quad \frac{}{2}}$

29. Let $g(x) \quad f(e^x)$

$g\,(x) \quad f\,(e^x)\;e^x$

$g\,(x) \quad f\,(e^x)\;e^{2x} \quad f\,(e^x)e^x$

30. $e^{f(x)} \quad \ln x \qquad f(x) \quad \ln(\ln x) \qquad g(x) \quad f^{\;1}(x) \quad e^{e^x}$

$g\,(x) \quad e^{e^x} \quad e^x \quad e^{e^x} \quad x$

32. $\ln f(x) \quad 4\ln(x \quad 1) \quad 3\ln(x \quad 2) \quad 2\ln(x \quad 3)$

$$\frac{f(x)}{f(x)} \quad \frac{4}{x \quad 1} \quad \frac{3}{x \quad 2} \quad \frac{2}{x \quad 3}$$

$$f(x) \quad f(x) \frac{4}{x \quad 1} \quad \frac{3}{x \quad 2} \quad \frac{2}{x \quad 3}$$

34. $f(2 \quad) \quad 0 \qquad c \quad 0$

$$f(2 \quad) \quad \frac{b\sin\{\ x\}}{\{\ x\}} \qquad f(2 \quad) \quad 0 \qquad b \quad 0$$

35. $f(0) \quad \lim_{x \quad 0} \frac{e^{\tan x} \quad e^x \quad \ln(\sec x \quad \tan x) \quad x}{\tan x \quad x}$

$$\lim_{x \quad 0} e^x \frac{(e^{\tan x \quad x} \quad 1)}{\tan x \quad x} \quad \lim_{x \quad 0} \frac{\ln(\sec x \quad \tan x) \quad x}{\tan x \quad x} \quad 1 \quad \lim_{x \quad 0} \frac{\sec x \quad 1}{\sec^2 x \quad 1} \quad 1 \quad \frac{1}{2} \quad \frac{3}{2}$$

36. $f(0 \quad) \quad e^a$

$f(0) \quad b$

$c \quad 1$

$$f(0 \quad) \quad \frac{2}{3} \qquad b \quad e^a \quad \frac{2}{3}$$

37. $\sqrt{x \quad y} \quad \sqrt{y \quad x} \quad 5$

$\sqrt{x \quad y} \quad 5 \quad \sqrt{y \quad x}$

Sq. both sides,

$x \quad y \quad 25 \quad y \quad x \quad 10\sqrt{y \quad x}$

$25 \quad 2x \quad 10\sqrt{y \quad x}$

$$2 \quad \frac{10(y \quad 1)}{2\sqrt{y \quad x}}$$

$2\sqrt{y \quad x} \quad 5(y \quad 1)$

$$5 \quad \frac{2x}{5} \quad 5(y \quad 1)$$

$$5 \quad \frac{2x}{5} \quad 5(y \quad 1)$$

$$y \quad \frac{2}{25}$$

38. $g(x) \quad f^{\ 1}(x)$

$f(g(x)) \quad x$

$f(g(x))g(x) \quad 1$

$$g\ (2)\quad \frac{1}{f\ (g(2))}$$

$$f(1)\quad 2$$

$$g(2)\quad 1$$

$$g\ (2)\quad \frac{1}{f\ (1)}$$

$$f\ (x)\quad 3x^2\quad 4x^3\quad \frac{1}{x}$$

$$f\ (1)\quad 8$$

$$g\ (2)\quad \frac{1}{8}$$

39. $f(x)\quad |x| \qquad x\quad (\quad ,\ 1)$

$\qquad x^2 \qquad x\quad [\ 1, 1)$

$\qquad 2x\quad 1 \qquad x\quad [1,\ \ )$

Function is not differentiable at $x\quad 1$.

40. $g(x)\quad (f(x))^2\quad (f\ (x))^2 \qquad g\ (x)\quad 2f(x)f\ (x)\quad 2f\ (x)f\ (x)$

or $g\ (x)\quad 2f(x)f\ (x)\quad 2f(x)f\ (x)\quad 0 \qquad g(x)\quad c \qquad g(8)\quad 8$

41. $l\quad \lim_{x} f\ \frac{a}{\sqrt{x}}^{\ x} \qquad e^{\lim_{x} \frac{f\ \frac{a}{\sqrt{x}}\quad 1}{\frac{1}{x}}}$

Using L' Hospital's rule, we get

$$l\quad e^{\frac{a^2}{2}f\ (0)}\quad e^{\frac{a^2}{2}}$$

42. $\frac{d}{dx}f_n(x)\quad e^{f_{n\ 1}(x)}\frac{d}{dx}f_{n\ \ 1}(x)\quad f_n(x)\frac{d}{dx}f_{n\ \ 1}(x)$

$$f_n(x)f_{n\ \ 1}(x) \qquad f_2(x)f_1(x)$$

43. $y\quad \tan^{\ 1}(x^{1/3})\quad \tan^{\ 1}(a^{1/3})$

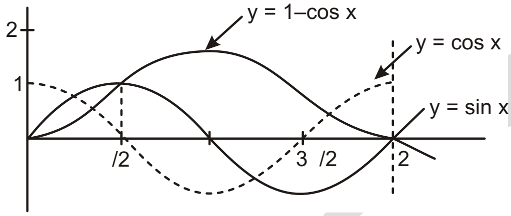
44. $f(x)$ is continuous at $x\quad 0$ then $\frac{4k\quad 1}{3}\quad \frac{4k\quad 1}{5}$

45. Put $x\quad \sin\quad$ then $y\quad \tan^{\ 1}\tan\frac{}{2}$

46. $\lim_{x\quad 0}\frac{e^x\cos x\quad \ln(1\quad x)\quad 1}{x}$

$$\lim_{x\quad 0}\frac{(e^x\quad 1)}{x}\cos x\quad \frac{\ln(1\quad x)}{x}\quad \frac{\cos x\quad 1}{x}\qquad 0$$

47.



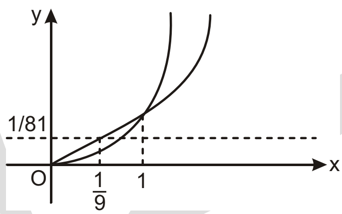
Clearly 3 sharp points.

48. $g(x) \quad f^{\ 1}(x)$ $\qquad f(4) \quad 2 \qquad g(2) \quad 4$

$G(x) \quad \dfrac{1}{g(x)}$ $\qquad f\,(4) \quad \dfrac{1}{16} \qquad g\,(2) \quad 16$

$G\,(x) \quad \dfrac{1}{(g(x))^2} \quad g\,(x) \qquad G\,(2) \quad \dfrac{1}{(g(x))^2} \quad g\,(2) \quad \dfrac{1}{16} \; 16 \qquad 1$

49.



$f(x) \quad \text{maximum} \quad x^4, x^2, \dfrac{1}{81} \qquad \dfrac{1}{81} \qquad\qquad x \quad \dfrac{1}{9}$

$x^2 \qquad \dfrac{1}{9} \quad x \quad 1$

$x^4 \qquad x \quad 1$

$f(x)$ is non-differentiable at $x \quad \dfrac{1}{9}, 1$

50. $\lim\limits_{h \quad 0} \dfrac{\ln(f(2 \quad h^2)) \quad \ln(f(2 \quad h^2))}{h^2}$

Apply L' Hospital rule,

$$\lim_{h \quad 0} \frac{\dfrac{2hf\,(2 \quad h^2)}{f(2 \quad h^2)} \quad \dfrac{2hf\,(2 \quad h^2)}{f(2 \quad h^2)}}{2h} \quad 4$$

51. $f(x) \quad (x^2 \quad 3x \quad 2)|(x \quad 1)(x \quad 2)(x \quad 3)| \quad \left|\sin \quad x \quad \dfrac{}{4}\right|$

Not differentiable at $x \quad 3, \dfrac{3}{4}, \dfrac{7}{4}$

52. $h(x) \quad f(2x\,g(x) \quad \cos \quad x \quad 3)$

$h\,(x) \quad f\,(2x\,g(x) \quad \cos \quad x \quad 3)[2g(x) \quad 2xg\,(x) \qquad \sin \quad x]$

$h\,(1) \quad f\,(2g(1) \quad 4)[2g(1) \quad 2g\,(1)] \quad 32$

53. $f(x) \quad \dfrac{(x \quad 1)^7\sqrt{1 \quad x^2}}{(x^2 \quad x \quad 1)^6} \qquad (f(0) \quad 1)$

$$\ln f(x) \quad 7\ln(1 \quad x) \quad \frac{1}{2}\ln(1 \quad x^2) \quad 6\ln(x^2 \quad x \quad 1)$$

$$\frac{f\ (x)}{f(x)} \quad \frac{7}{1 \quad x} \quad \frac{x}{1 \quad x^2} \quad \frac{6(2x \quad 1)}{x^2 \quad x \quad 1}$$

$f\ (0) \quad 13$

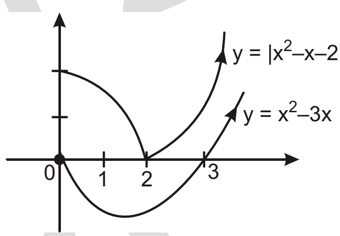
54. $f(x) \quad \begin{matrix} \sin 2x; & x \quad 1 \\ \ln(1 \quad x); & x \quad 1 \\ \dfrac{\ln 2 \quad \sin 2}{2}; & x \quad 1 \end{matrix}; \qquad f(1 \quad) \quad f(1 \quad) \quad f(1)$

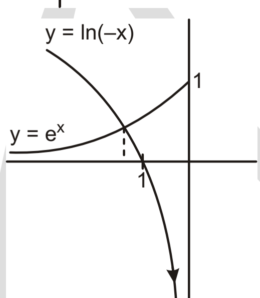
55. $f(f(x)) \quad \begin{matrix} f(x); & \text{if } f(x) \text{ is rational} \\ 1 \quad f(x); & \text{if } f(x) \text{ is irrational} \end{matrix}$

$f(f(x)) \quad \begin{matrix} x; & \text{if } x \text{ is rational} \\ 1 \quad (1 \quad x); & \text{if } x \text{ is irrational} \end{matrix}$

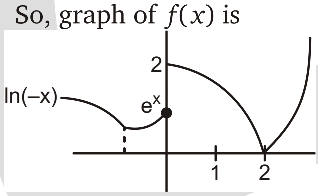
$f(f(x)) \quad \begin{matrix} x; & \text{if } x \text{ is rational} \\ x; & \text{if } x \text{ is irrational} \end{matrix}$

56.





So, graph of $f(x)$ is



Clearly, 3 non-differentiability points.

57. $g(f(x)) \quad x$

58. $\lim_{x \quad 0} \frac{\ln(2 \quad \cos 2x)}{\ln^2(1 \quad \sin 3x)} \quad K \quad \lim_{x \quad 0} \frac{e^{\sin 2x} \quad 1}{\ln(1 \quad \tan 9x)}$

$\lim_{x \quad 0} \frac{1 \quad \cos 2x}{\sin^2 3x} \quad K \quad \lim_{x \quad 0} \frac{\sin 2x}{\tan 9x}$

59. $\frac{dx}{dt} \quad \frac{3}{t^4} \quad \frac{2}{t^3} \quad \frac{3 \quad 2t}{t^4}$

$\frac{dy}{dt} \quad \frac{3}{t^3} \quad \frac{2}{t^2} \quad \frac{3 \quad 2t}{t^3}$

$\frac{dy}{dx} \quad t$

$\frac{dy}{dx} \quad x \quad \frac{dy}{dx}^{3} \quad t \quad \frac{1 \quad t}{t^3} \quad t^3 \quad 1$

60. $\frac{2}{y^3} y \quad 2\sqrt{2}(\quad 2\sin 2x)$

$\frac{(y)^2}{y^6} \quad 8 \quad (2\sqrt{2}\cos x)^2 \quad 8 \quad \frac{1}{y^2} \quad 1^{\;2}$

$\frac{(y)^2}{y^6} \quad \frac{8y^2 \quad (1 \quad y^2)^2}{y^4}$

$(y)^2 \quad 8y^4 \quad y^2(1 \quad y^2)^2$ then diff.

61. $f(x) \quad x$ satisfy the equation.

$f(5) \quad 5$

62. $f(x) \quad \begin{matrix} x & & x \quad 0 \\ x^2 & 0 & x \quad 1 \\ 2x \quad 1 & & x \quad 1 \end{matrix}$

$f(x) \quad \begin{matrix} 1 & & x \quad 0 \\ 2x & 0 & x \quad 1 \\ 2 & & x \quad 1 \end{matrix}$

$f(x)$ is not derivable at $x \quad 0$.

63. $y \quad (x \quad \sqrt{1 \quad x^2})^n$

$\frac{dy}{dx} \quad \frac{ny}{\sqrt{1 \quad x^2}}$

$$\frac{d^2y}{dx^2} \quad n \quad \frac{\sqrt{1 \quad x^2}\,y \quad \dfrac{yx}{\sqrt{1 \quad x^2}}}{1 \quad x^2} \qquad (1 \quad x^2)\frac{d^2y}{dx^2} \quad x\frac{dy}{dx} \quad n^2 y$$

64. $g(x) \quad f(x \quad \sqrt{1 \quad x^2}) \quad 1 \quad \dfrac{x}{\sqrt{1 \quad x^2}} \qquad 1 \quad x \quad \sqrt{1 \quad x^2}\;^{2} \qquad \dfrac{x \quad \sqrt{1 \quad x^2}}{\sqrt{1 \quad x^2}}$

$$2x \quad x \quad \sqrt{1 \quad x^2}$$

66. $f(x) \quad \lim\limits_{h \quad 0} \dfrac{f(x \quad h) \quad f(x)}{h} \quad \lim\limits_{h \quad 0} f(x) \; \dfrac{f(h) \quad 1}{h} \qquad f(x) \;\; f(0) \quad 3f(x) \qquad (\because \; f(0) \quad 3)$

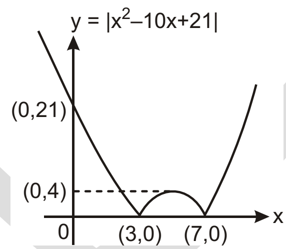
67. $f(x) \quad \lim\limits_{n} \dfrac{\log_e(2 \quad x) \quad x^{2n}\sin x}{1 \quad x^{2n}}$

$$f(x) \quad \begin{array}{ll} \ln(2 \quad x) & |x| \quad 1 \\ \sin x & |x| \quad 1 \\ \dfrac{\ln 3 \quad \sin 1}{2} & x \quad 1 \\ \dfrac{\sin 1}{2} & x \quad 1 \end{array}$$

68. $\lim\limits_{x \quad 0} \dfrac{x \quad e^x \quad 1 \quad \{1 \quad \cos 2x\}}{x^2} \qquad \lim\limits_{x \quad 0} \dfrac{x \quad e^x \quad 1 \quad 1 \quad \cos 2x}{x^2}$

$$\lim_{x \quad 0} \frac{x \quad e^x \quad \cos 2x}{x^2} \quad \lim_{x \quad 0} \frac{1 \quad x \quad e^x}{x^2} \quad \frac{(\cos 2x \quad 1)}{x^2} \quad \frac{5}{2}$$

69.



71. $xy \quad$ const.

$$y \quad xy \quad 0 \qquad y \quad \frac{y}{x}$$

72. $f(x) \quad 1 \quad |x \quad 2|$ is a continuous function.

$g(x) \quad 1 \quad |x|$ is a continuous function.

$f(g(x))$ is a continuous function.

73. $f\,(K\ \ ) \quad \lim\limits_{h \quad 0} \dfrac{f(k \quad h) \quad f(k)}{h}$

$$\lim_{h \quad 0} \frac{K \tan(\ k \quad h) \quad k \tan k}{h}$$

$$\lim_{h \quad 0} k \ \frac{\tan \ h}{h} \qquad k$$

74.
$$\lim_{x \quad 0} \frac{ae^{\sin x} \quad be^{\ \sin x} \quad c}{x^2} \quad 2$$

$$a \quad b \quad c \quad 0 \qquad \ldots(1)$$

Applying L Hospital Rule,

$$\lim_{x \quad 0} \frac{ae^{\sin x} \ \cos x \quad be^{\ \sin x} \quad \cos x}{2x} \quad 2 \qquad a \quad b$$

75. $\tan x \quad \sec \quad \ \tan y$

$\sec^2 x \quad \sec \quad \sec^2 y \ \ y$

$y \quad 1 \text{ at } \ \dfrac{}{4}, \dfrac{}{4}$

$2\sec^2 x \tan x \quad \sec \quad (\sec^2 y \ \ y \quad 2\sec^2 y \ \ \tan y \ \ (y\ )^2) \qquad y \quad 0$

76. We gave,
$$y \quad (x^2 \quad 9)(x^2 \quad 4)(x^2 \quad 1)\,x$$
$$\{x^6 \quad 14x^4 \quad x^2(49) \quad 36\}\,x$$
$$x^7 \quad 14x^5 \quad 49x^3 \quad 36x$$

Therefore, $\dfrac{dy}{dx} \quad 7x^5 \quad 70x^4 \quad 147x^2 \quad 36$

Thus, $\dfrac{d^2y}{dx^2} \quad 42x^5 \quad 280x^3 \quad 294x$

$$\left.\frac{d^2y}{dx^2}\right|_{x \quad 1} \quad 42 \quad 280 \quad 294 \quad 56$$

77. $f\,(x) \quad \lim\limits_{h \quad 0} \dfrac{f(x \quad h) \quad f(x)}{h} \quad \lim\limits_{h \quad 0} \dfrac{f(x)\,f(h) \quad f(x)}{h} \quad \lim\limits_{h \quad 0} f(x) \ \dfrac{f(h) \quad 1}{h}$

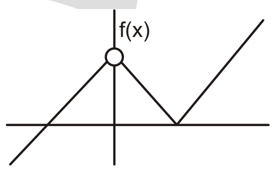
$f\,(x) \quad f\,(0),\ f(x) \qquad f(x) \quad e^{kx} \qquad$ (where $k \quad f\,(0)$)

78. $f(g(x)) \quad x; \qquad f(g(x))g(x) \quad 1 \qquad g(6) \quad \dfrac{1}{f(g(6))} \quad \dfrac{1}{f(0)}$

79. $\dfrac{dy}{dz} \quad \dfrac{dy/dx}{dz/dx} \quad \dfrac{f(x)}{g(x)}$

$$\frac{d^2y}{dz^2} \quad \frac{d}{dz}\ \frac{dy}{dz} \quad \frac{\frac{d}{dx}\ \frac{dy}{dz}}{\frac{dz}{dx}} \quad \frac{gf \quad fg}{(g)^3}$$

80.

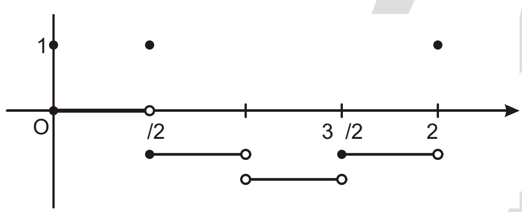


$f(x) \quad 1 \quad x \quad 2, x \quad (\quad , \quad 1)\ x \quad 1, 1$ non differentiable

$g(f(x)) \quad (f(x) \quad 1)^2 \quad (x \quad 1 \quad 1)^2 \quad x^2, x \quad (\ 1, 0)$

$(|x \quad 1| \quad 1)^2, x \quad 0$

81. $f(x) \quad [\sin x] \quad [\cos x]$



82. $g(x) \quad \begin{matrix} \cos x & , x \quad [0, \quad] \\ \sin x \quad 1 & , x \end{matrix}$

$g(\quad) \quad g(\ ) \quad g(\quad) \quad 1$

but not differentiable at x .

83. $\underset{r\ 0}{} \dfrac{f^r(0)}{r!} \quad \dfrac{f(0)}{0!} \quad \dfrac{f(0)}{1!} \quad \dfrac{f(0)}{2!}$

$$4^n \quad \frac{n \ 4^{n \ 1}}{1!} \quad \frac{n(n \quad 1) \ 4^{n \ 2}}{2!}$$

$${}^nC_0 4^n \quad {}^nC_1 4^{n \ 1} \quad {}^nC_2 4^{n \ 2}$$

$$(4 \quad 1)^n \quad 5^n$$

84. $f(x) \quad \dfrac{x}{1 \quad x} \qquad x \quad 1$

$\qquad \dfrac{x}{1 \quad x} \qquad 1 \quad x \quad 0$

$\qquad \dfrac{x}{1 \quad x} \qquad 0 \quad x \quad 1$

$\qquad \dfrac{x}{1 \quad x} \qquad x \quad 1$

Function is discontinuous at $x \quad 1, 1$

$f(x)$ is not differentiable at $x \quad 1, 1$

85. $f(g(x)) \quad x$

$f\,(g(x))\,g\,(x) \quad 1 \qquad g \;\dfrac{7}{6} \quad \dfrac{1}{f \quad g\;\dfrac{7}{6}} \quad \dfrac{1}{f\,(1)}$

86. $f(x) \quad 0 \qquad\qquad x \quad 0$

$\qquad 4x^2(1 \quad 2x)^2 \qquad x \quad 0$

Differentiable everywhere.

88. $f(x)$ is discontinuous at $x \quad 1, 2$

$g(x) \quad x^2 \quad ax \quad b \quad 0 \begin{cases} x=1 \\ x=2 \end{cases}$

89. $f \;^{1}(f(x)) \quad x$

$(f \;^{1}(f(x)))\; f\,(x) \quad 1$

$(f \;^{1}(f(9)))\; f\,(g) \quad 1$

$(f \;^{1}(3)) \quad \dfrac{1}{f\,(9)} \quad \dfrac{1}{5}$

90. $f(0 \quad) \quad \lim\limits_{h \quad 0} h^n \sin\dfrac{1}{h} \quad 0 \qquad n \quad 0$

$f(0 \quad) \quad \lim\limits_{h \quad 0} (\quad h)^n \sin \quad \dfrac{1}{h} \quad 0 \qquad n \quad 0$

$f\,(x) \quad n \quad x^{n \quad 1} \sin\dfrac{1}{x} \quad x^{n \quad 2} \cos\dfrac{1}{x} \quad \text{finite} \qquad n \quad 2$

Exercise-2 : One or More than One Answer is/are Correct

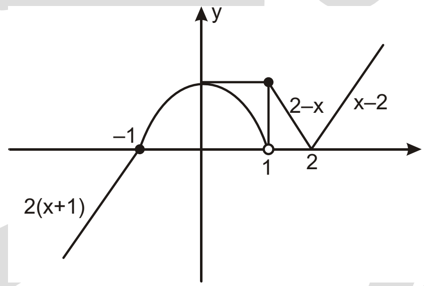
1. $f(x)$ has exactly one point of discontinuity so that sgn $(x^2 \quad x \quad 1)$ is equal to zero for some values of .

$D \quad 0$

$^2 \quad 4 \quad 0$

2

2. Answer from the graph.



3. (a) L.H.L. $\lim\limits_{x \quad 0} \dfrac{x(3e^{1/x} \quad 4)}{2 \quad e^{1/x}} \quad 0 \quad \dfrac{4}{2} \quad 0$

R.H.L. $\lim\limits_{x \quad 0} \dfrac{x(3e^{1/x} \quad 4)}{2 \quad e^{1/x}} \quad \lim\limits_{x \quad 0} x \dfrac{3 \quad 4e^{\quad 1/x}}{2e^{\quad 1/x} \quad 1} \quad 0 \quad \dfrac{3}{1} \quad 0$

$f(0) \quad 0$

$f(x)$ is continuous at $x \quad 0$.

(b) $f\,(0 \quad) \quad \lim\limits_{x \quad 0} \dfrac{f(x) \quad f(0)}{x \quad 0} \quad \lim\limits_{x \quad 0} x \dfrac{\dfrac{3e^{1/x} \quad 4}{2 \quad e^{1/x}}}{x}$

$\lim\limits_{x \quad 0} \dfrac{3 \quad 4e^{\quad 1/x}}{2e^{\quad 1/x} \quad 1} \quad 3$

$f\,(0 \quad) \quad \lim\limits_{x \quad 0} \dfrac{f(x) \quad f(0)}{x \quad 0} \quad \lim\limits_{x \quad 0} \dfrac{3e^{1/x} \quad 4}{2 \quad e^{1/x}} \quad \dfrac{4}{2} \quad 2$

$f\,(0 \quad) \quad f\,(0 \quad)$

(c) $f\,(0 \quad) \quad 3$

(d) $f\,(0 \quad) \quad 2$ exist

4. Given $|\,f(x)| \quad \sin^2 x$

Clearly $|\,f(0)| \quad 0 \qquad f(0) \quad 0$

$$\lim_{x \quad 0} |f(x)| \quad \left|\lim_{x \quad 0} f(x)\right| \quad 0$$

$$|f(0)| \quad \left|\lim_{x \quad 0} \frac{f(x) \quad f(0)}{x}\right| \quad \left|\lim_{x \quad 0} \frac{f(x)}{x}\right| \quad 0$$

5. $$f(0 \quad) \quad \lim_{x \quad 0} \frac{a \; 1 \quad x \quad x \quad \dfrac{x^3}{3!} \qquad b \; 1 \quad \dfrac{x^2}{2!} \quad \dfrac{x^4}{4!} \qquad 5}{x^2} \qquad f(0)$$

$$\lim_{x \quad 0} \frac{(a \quad b \quad 5) \quad a \quad \dfrac{b}{2} \; x^2}{x^2} \quad 3$$

$$a \quad b \quad 5 \quad 0 \qquad \ldots(1)$$

$$a \quad \frac{b}{2} \quad 3$$

$$2a \quad b \quad 6 \qquad \ldots(2)$$

On solving (1) and (2),

$$\begin{array}{rrrr} a & b & 5 & 0 \\ 2a & b & 6 & 0 \\ \hline & a & 1 & 0 \end{array}$$

$$a \quad 1$$

$$b \quad 4$$

$$a \quad b \quad 5$$

$f(0 \quad)$ is exist when $c \quad 0$

$$\lim_{x \quad 0} (1 \quad dx)^{1/x} \quad 3$$

$$e^{\lim\limits_{x \quad 0} \frac{1}{x}(dx)} \quad 3$$

$$e^d \quad 3$$

$$d \quad \ln 3$$

7. (a) $f(x) \quad \sqrt[3]{x^2|x|} \quad 1 \quad |x|$

But $x^2|x| \quad |x|^3$

So, $f(x) \quad |x| \quad 1 \quad |x| \qquad 1$ is every where differentiable.

So, no where non-differentiable.

(b) $\lim\limits_{x} [x(\tan^{1}(x \quad 1) \quad x\tan^{1}(x \quad 1))] \quad [5\tan^{1}(x \quad 1) \quad \tan^{1}(x \quad 1)]$

$$\lim_{x} 4\tan^{1}(x \quad 1) \quad 4 \frac{}{2} \quad 2$$

(c) $f(\ x) \quad \sin \ \ln \quad x \quad \sqrt{x^2 \quad 1} \qquad \sin \ \ln \frac{1}{x \quad \sqrt{x^2 \quad 1}}$

$\sin \quad \ln \ x \quad \sqrt{x^2 \quad 1} \qquad \sin \ \ln \ x \quad \sqrt{x^2 \quad 1}$

$f(x)$

So, $f(x)$ is an odd function.

(d) $f(x) \quad \dfrac{4 \quad x^2}{4x \quad x^3}$ is discontinuous at where denominator is zero, $4x \quad x^3 \quad 0$

$x \quad 0, x \quad 2$

a, b, c only correct.

8. $g(x) \quad ae^{ax} \quad f(x) \qquad g(0) \quad a \quad 5$

$g(x) \quad a^2e^{ax} \quad f(x)$

$g(0) \quad a^2 \quad 3$

$a^2 \quad a \quad 2 \quad 0; \quad a \quad 2, 1$

10. $f(0 \quad) \quad f(0) \quad f(0 \quad) \quad 0$

11. $f(x)\,dx \qquad f(\ x)\,dx$

$f(x) \quad f(\ x) \quad c$

12. $|f(x)| \quad x^{4n}$

$f(0) \quad 0$

$\lim\limits_{h \quad 0}(\ h^{4n}) \quad \lim\limits_{h \quad 0} f(0 \quad h) \quad \lim\limits_{h \quad 0}(h)^{4n} \qquad f(0 \quad h) \quad 0$

$\lim\limits_{h \quad 0}(\ (\ h)^{4n}) \quad \lim\limits_{h \quad 0} f(0 \quad h) \quad \lim\limits_{h \quad 0}(\ h)^{4n} \qquad f(0 \quad h) \quad 0$

$f(x)$ is continuous at $x \quad 0$.

$f(0 \quad) \quad \lim\limits_{h \quad 0} \dfrac{f(0 \quad h) \quad f(0)}{h} \quad 0 \qquad \lim\limits_{h \quad 0} \dfrac{h^{4n}}{h} \quad \lim\limits_{h \quad 0} \dfrac{f(0 \quad h) \quad f(0)}{h} \quad \lim\limits_{h \quad 0} \dfrac{h^{4n}}{h}$

$f(x)$ is differentiable at $x \quad 0$.

13. $g(x) \quad 0 \qquad x \quad I$

$x^2 \qquad x \quad I$

$gof(x) \quad 0$ for $x \quad R$

14. If $f(x)$ is continuous at $x \quad 2$ then $3p \quad 10q \quad 4$

$f(x)$ is differentiable at $x \quad 2$ then $2p \quad 11q \quad 4$

16. $$f(x) \quad \begin{array}{ll} x^2 & 2 \quad x \quad 0 \\ x & 0 \quad x \quad 1 \\ x^3 & 1 \quad x \quad 2 \end{array}$$

17. $$f\,(x) \quad \lim_{h \quad 0} \frac{f(x \quad h) \quad f(x)}{h} \qquad f(x) \lim_{h \quad 0} \frac{\dfrac{f(x \quad h)}{f(x)} \quad 1}{h}$$

$$f(x) \lim_{h \quad 0} \frac{f\ 1 \quad \dfrac{h}{x} \quad 1}{h}$$

So, $f\,(x) \quad \dfrac{f(x)}{x} \quad f\,(1)$

$\ln(f(x)) \quad 3\ln x \quad \ln c$

$f(x) \quad cx^3$

$f(1) \quad 1$ so $c \quad 1$

$f(x) \quad x^3$

So, we can check options.

18. $f(x) \quad (x \quad 1)(x \quad 2)(x \quad 1)(x \quad 2) \quad (x^2 \quad 1)(x^2 \quad 4)$

$f\,(x) \quad (x^2 \quad 1)\,2x \quad (x^2 \quad 4)(2x) \quad 2x(2x^2 \quad 5) \quad 0$

$x \quad 0, \quad \sqrt{\dfrac{5}{2}}$

19. If $f(x)$ is continuous at $x \quad 2$ then $3p \quad 10q \quad 4$

$f(x)$ is differentiable at $x \quad 2$ then $2p \quad 11q \quad 4$

20. $y \quad e^{x\sin x^3} \quad e^{x\ln(\tan x)}$

$$\frac{dy}{dx} \quad e^{x\sin(x^3)}[x\cos(x^3)3x^2 \quad \sin(x^3)] \quad e^{x\ln(\tan x)} \quad \ln(\tan x) \quad x\frac{1}{\tan x}\sec^2 x$$

$$y \quad e^{x\sin(x^3)}[3x^3\cos(x^3) \quad \sin(x^3)] \quad (\tan x)^x(\ln(\tan x) \quad 2x\,\mathrm{cosec}\,2x)$$

21. $f(x) \quad 1 \quad (1 \quad x) \quad (1 \quad x)x^2 \quad (1 \quad x)(1 \quad x^2)x^3 \qquad (1 \quad x)(1 \quad x^2) \quad (1 \quad x^{n \quad 1})\,x^n$

$$1 \quad (1 \quad x)(1 \quad x^2)(1 \quad x^3) \quad (1 \quad x^n) \quad 1 \quad \mathop{}_{r \ 1}^{n} (1 \quad x^r)$$

$$(f(x) \quad 1) \quad \mathop{}_{r \ 1}^{n} (1 \quad x^r)$$

22. f and g must be continuous.

$1 \quad a \quad 2 \quad b$

$a \quad 1 \quad b$

$3 \quad b \quad 1 \qquad b \quad 2$

$a \quad 1$

23. $f(x) \begin{cases} ax^3+b\,; & 0 \quad x \quad 1 \\ 2\cos \quad x + \tan^{-1}x\,; & 1 < x \quad 2 \end{cases}$

is must be continuous and differentiable at $x \quad 1$.

$$a \quad b \quad 2 \quad \frac{}{4} \qquad \ldots(1) \qquad \text{(continuity)}$$

$$3a \quad 0 \quad \frac{1}{2} \qquad \ldots(2) \qquad \text{(By differentiable)}$$

We get, $\quad a$ and b

24. $f(f(x)) \quad 2 \quad x \qquad 0 \quad x \quad 1$

$2 \quad x \qquad 1 \quad x \quad 2$

$4 \quad x \qquad 2 \quad x \quad 3$

25. $\ln(f(x)) \quad \ln(x \quad 1) \quad \ln(x \quad 2) \qquad \ln(x \quad 100)$

$$\frac{f\ (x)}{f(x)} \quad \frac{1}{x \quad 1} \quad \frac{1}{x \quad 2} \qquad \frac{1}{x \quad 100}$$

$$\frac{f(x)f\ (x) \quad (f\ (x))^2}{(f(x))^2} \qquad \frac{1}{(x \quad 1)^2} \quad \frac{1}{(x \quad 2)^2} \quad \frac{1}{(x \quad 3)^2} \qquad \frac{1}{(x \quad 100)^2}$$

if $\quad g(x) \quad f(x)f\ (x) \quad (f\ (x))^2 \quad 0$

$$\frac{1}{(x \quad 1)^2} \quad \frac{1}{(x \quad 2)^2} \qquad \frac{1}{(x \quad 100)^2} \quad 0$$

$g(x) \quad 0$ has no solution.

26. $h(x) \quad 1 \qquad\qquad x \quad 1$

$|x \quad 2| \quad a \quad 2 \quad |x| \qquad 1 \quad x \quad 2$

$|x \quad 2| \quad a \quad 1 \quad b \qquad x \quad 2$

if $h(x)$ is continuous at $x \quad 1$, then $a \quad 3$

if $h(x)$ is continuous at $x \quad 2$, then $b \quad 1$

27. $\lim\limits_{x \quad 0} f(x) \quad 1 \quad \lim\limits_{x \quad 0} f(x)$

Clearly, $C \quad 1$ and use L' Hospital's rule.

28. Differentiable w.r.t. 'x'

$$2f(x)f\ (x) \quad 2y \quad 2f(x \quad y)f\ (x \quad y)$$

put $x \quad 0$

$$k \quad y \quad f'(y) f(y)$$

integrate on both sides,

$$ky \quad \frac{y^2}{2} \quad \frac{f^2(y)}{2} \quad c \qquad \ldots(1)$$

put $x \quad y \quad 0$ in given equation, we get

$$f^2(0) \quad 2$$

$f(0) \quad \sqrt{2}$ as $(f(x) \quad 0)$

put $y \quad 0$ in (1)

$$1 \quad c \quad 0 \qquad c \quad 1$$

also put $y \quad \sqrt{2}$

$$k\sqrt{2} \quad 1 \quad \frac{4}{2} \quad 1$$

$$k\sqrt{2} \quad 0$$

$$k \quad 0$$

$$\frac{y^2}{2} \quad \frac{f^2(y)}{2} \quad 1$$

$$f^2(y) \quad y^2 \quad 2$$

$$f(y) \quad \sqrt{y^2 \quad 2}$$

$$f(x) \quad \sqrt{x^2 \quad 2}$$

Hence, we can answer.

30. $f'(x) \quad \lim\limits_{h \quad 0} \dfrac{f(x \quad h) \quad f(x)}{h} \quad \lim\limits_{h \quad 0} \dfrac{f(x \quad h) \quad f(x)}{h}$

$\lim\limits_{h \quad 0} \dfrac{f(h)}{h} \quad x^2 \quad x^2 \quad f'(0)\,;\ f'(x) \quad x^2 \quad 1$

32. $f(1 \quad) \quad f(1 \quad) \quad f(1) \quad \dfrac{1}{2}$

$f'(x) \quad x \qquad 0 \quad x \quad 1$

$\qquad 4x \quad 3 \qquad 1 \quad x \quad 2$

$f''(x) \quad 1 \qquad 0 \quad x \quad 1$

$\qquad 4 \qquad 1 \quad x \quad 2$

34. $gof(x) \quad 0$

$fog(x) \quad 0 \qquad x \quad I$

$\qquad [x^2] \qquad x \quad I$

35. $f(g(x)) \quad x$

$f\ (g(x))g\ (x) \quad 1$

$g\ (x) \quad \dfrac{1}{f\ (g(x))}$

$g\ (e) \quad \dfrac{1}{f\ (g(e))} \quad \dfrac{1}{f\ (1)} \quad \dfrac{1}{e \quad 1}$

$g\ (x) \quad \dfrac{1}{(f\ (g(x)))^2} f\ \ (g(x)) \ \ g\ (x)$

$g\ \ (e) \quad \dfrac{1}{(f\ (1))^2} f\ \ (1) \ \ g\ (e)$

36. $f(2 \quad) \quad \lim\limits_{x \quad 2} [x \quad 1] \quad 1$

$f(2 \quad) \quad \lim\limits_{x \quad 2} \dfrac{3x \quad x^2}{2} \quad 1$

$f(3 \quad) \quad \lim\limits_{x \quad 3} [x \quad 1] \quad 1$

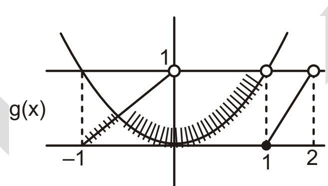
$f(3 \quad) \quad \lim\limits_{x \quad 3} (x^2 \quad 8x \quad 17) \quad 2$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

2. $f(x) \quad \lim\limits_{n} n^2 \dfrac{\tan(\ln(\sec(x/n)))}{\ln(\sec(x/n))} \dfrac{(\ln(\sec(x/n) \quad 1) \quad 1)}{\sec(x/n) \quad 1} \dfrac{\sec(x/n) \quad 1}{(x/n)^2} \quad \left(\dfrac{x}{n}\right)^2$

$f(x) \quad \dfrac{x^2}{2}$



Paragraph for Question Nos. 3 to 4

4. $f\ (x) \quad 2x \quad g\ (1)$

$f\ \ (x) \quad 2 \qquad\qquad g\ (1) \quad 2f(1) \quad 2 \quad f\ (1)$

$f\ (1) \quad 2 \quad 3 \quad 1 \qquad\qquad f(1) \quad 2$

$f(x) \quad x^2 \quad 3x$ $\qquad\qquad$ $g\ (2) \quad 2(\ \ 2) \quad 2(2) \quad 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2 \quad 1 \quad g\ (1)$

$g(x) \quad 2x^2 \quad x(2x \quad 3) \quad 2$ $\qquad g\ (1) \quad 3$

$\qquad 3x \quad 2$

$f(1) \quad g(\ \ 1) \quad 2 \quad (3 \quad 2) \quad 3$

Paragraph for Question Nos. 5 to 6

5. Clearly, 3 is non-repeated root where as 1 is repeats and also $(x \quad 2)^{1/3}$ is not diff. at $x \quad 2.$

at 3, 2 is non-diff. and sum is 5.

6. $h(x)$ is continuous.

$x \quad 1 \quad x^2 \quad x \quad 2$

$x^2 \quad 2x \quad 1 \quad 0$

$(x \quad 1)^2 \quad 2$

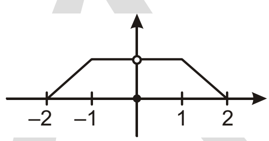
$x \quad 1 \quad \sqrt{2}$

$\tan\frac{3}{8} \quad 1 \quad \sqrt{2}, \tan\ \frac{}{8} \quad \sqrt{2} \quad 1$

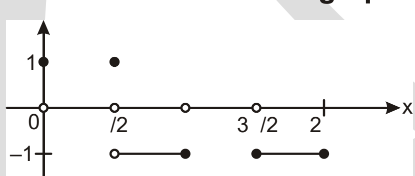
$\tan\frac{7}{8} \quad 1 \quad \sqrt{2}$

$\sqrt{2} \quad 1$ is not differentiable.

Paragraph for Question Nos. 7 to 8

Sol. 

Paragraph for Question Nos. 9 to 10

10. 

Paragraph for Question Nos. 11 to 13

11. $f(x) \quad \begin{array}{ll} 0 & 0 \quad x \quad 1 \\ x & 1 \quad x \quad 2 \\ 2(x \quad 1) & 2 \quad x \quad 3 \\ 3(x \quad 1) & \quad x \quad 3 \end{array}$

No. of values where $f(x)$ is discontinuous 2

12. $f(x)$ is non-differentiable at $x \quad 1, 2, 3.$

13. No. of integers in the range of $f(x) \quad 5$

Paragraph for Question Nos. 14 to 16

Sol. $f\ (x) \quad \lim_{h \quad 0} \dfrac{f(x) f(h) \quad f(x)}{h} \quad f\ (0) f(x)$

$f(x) \quad e^{2x} \qquad (f\ (0) \quad 2)$

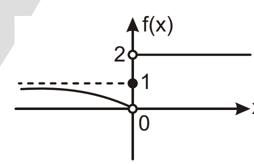
$g(x) \quad x^2$

Paragraph for Question Nos. 17 to 18

Sol. $g\ (x) \quad \sec^2 x \quad (1 \quad) \cos x \quad 1 \quad \dfrac{(1 \quad \cos x)(\quad f(x))}{f(x)}$

Paragraph for Question Nos. 19 to 21

Sol. $f(x) \quad \begin{array}{ll} \dfrac{x^2}{x^2 \quad 1} & x \quad 0 \\ 1 & x \quad 0 \\ 2 & x \quad 0 \end{array}$



Paragraph for Question Nos. 22 to 24

Sol. $f(x) \quad g\ (1) \sin x \quad (g\ (2) \quad 1)x$

$f\ (x) \quad g\ (1) \cos x \quad g\ (2) \quad 1 \qquad f \quad \overline{\ \ }\!\!\!\!\!\!\! \dfrac{}{2} \quad g\ (2) \quad 1$

$f\ \ (x) \quad g\ (1) \sin x \qquad f \quad \dfrac{}{2} \quad g\ (1)$

$g(x) \quad x^2 \quad f \quad \dfrac{}{2} \quad x \quad f \quad \dfrac{}{2} \qquad g\ (x) \quad 2x \quad f \quad \dfrac{}{2}$

$g\ \ (x) \quad 2 \qquad g\ \ (2) \quad 2$

$f(x) \quad \sin x \quad x$ and $g(x) \quad x^2 \quad x \quad 1$

Paragraph for Question Nos. 25 to 26

Sol. $f\,(x) \quad \lim_{h \quad 0} \dfrac{f(x \quad h) \quad f(x)}{h} \quad \lim_{h \quad 0} \dfrac{\dfrac{f(x) \quad f(h)}{1 \quad f(x)\,f(h)} \quad f(x)}{h}$

$f\,(x) \quad \lim_{h \quad 0} f(h) \dfrac{(1 \quad (f(x))^2)}{h}$

$f\,(x) \quad \lim_{h \quad 0} f\,(0)(1 \quad f(x)^2) \qquad\qquad (f(0) \quad 0)$

$f(x) \quad \dfrac{e^{2x} \quad 1}{e^{2x} \quad 1}$

$f\,(x) \quad 0 \quad x \quad R$

$\lim_{x \quad 0} (f(x))^x \quad e^{\lim_{x \quad 0} \frac{2x}{(e^{2x} \quad 1)}} \quad 1$

Paragraph for Question Nos. 27 to 28

Sol. $f(x) \quad 3(x \quad 6)(x \quad 1)(x \quad 2)(x \quad 3) \quad x^2 \quad 1$ then k why!

27. $\lim_{x \quad 6} \dfrac{3(x \quad 1)(x \quad 2)(x \quad 3)(x \quad 6)}{x \quad 6} \qquad \dfrac{6!}{2}$

28. $g(x) \quad \dfrac{1}{3(x \quad 6)(x \quad 1)(x \quad 2)(x \quad 3)}$

Paragraph for Question Nos. 29 to 30

Sol. $f(x) \quad g(x)$

$x^{\ln x} \quad e^2 x$

$(\ln x)^2 \quad 2 \quad \ln x$

$x \qquad \dfrac{1}{e}, \qquad e^2$

29. $\lim_{x \quad e^2} \dfrac{f(x) \quad c}{g(x) \qquad 2} \quad \dfrac{f\,(x)}{g\,(x)} \quad 4$

$c \quad e^2$

30. $h\,(\ ) \quad \dfrac{g(\ )\,f\,(\ ) \quad g\,(\ )\,f(\ )}{g^2(\ )} \quad \dfrac{e(\ 2e^2) \quad e^2 \quad e}{(e)^2} \quad 3e$

Exercise-4 : Matching Type Problems

1. (A) Let $I \quad \int_{0} \frac{\log(\sin x)}{\cos^2 x}\,dx$

$$2 \int_0^{/2} \frac{\log(\sin x)}{\cos^2 x}\,dx \quad 2 \int_0^{/2} \log(\sin x)\sec^2 x\,dx \quad 2\ \log(\sin x)\tan x\Big|_0^{/2} \quad \int_0^{/2} \frac{\cos x}{\sin x}\tan x\,dx$$

$$2(0 \quad 0) \quad 2 \int_0^{/2} dx \quad 0 \quad 2\ \frac{}{2} \qquad k$$

$$k$$

$$\frac{3k}{} \quad 3 \quad 0, 1, 2$$

(B) $e^{x \ \ y} \quad e^{y \ \ x} \quad 1$

$e^x \quad e^{\ x} \quad e^{\ y}$

$e^x \quad e^{\ x} \quad e^{\ y}(\ \ y\ )$

$e^x \quad e^{\ x} \quad e^{\ y}(\ \ y\ ) \quad e^{\ y}(y\ )^2$

$e^{\ y} \quad e^{\ y}(\ \ y\ ) \quad e^{\ y}(y\ )^2 \qquad y \quad (y\ )^2 \quad 1 \quad 0$

$k \quad 1$

(C) Let $f^{\ 1} \quad g$

$g\{f(x)\} \quad x \qquad (g\,f(x))\,f\,(x) \quad 1$

$g\,(2\ln 2)\,f\,(2) \quad 1$

$g\ 2(\ln 2) \quad \frac{1}{1 \quad \ln 2}$

$2(f^{\ 1})\ (\ln 4) \quad \frac{2}{1 \quad \ln 2} \quad 0, 1$

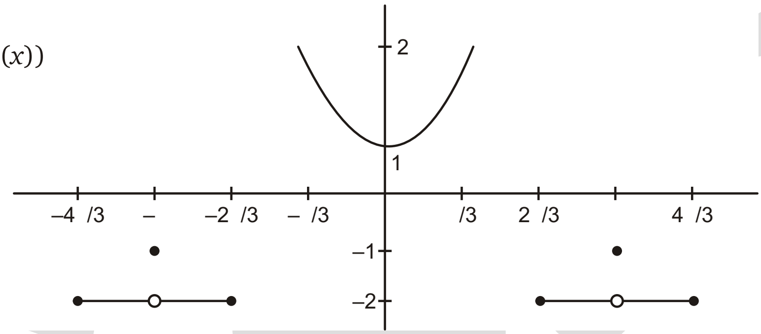
(D) $l \quad \lim_{x} (x \ln x)^{\frac{1}{x^2 \ \ 1}}$

$$\ln l \quad \lim_{x} \frac{\ln x \quad \ln(\ln x)}{x^2 \quad 1} \quad \lim_{x} \frac{\frac{1}{x} \quad \frac{1}{\ln x}\ \frac{1}{x}}{2x} \quad 0$$

$\ln(l) \quad 0 \qquad l \quad 1$

2. $g(f(x))$ →
- sec 2, $\quad -2 \quad x \quad -1$
- sec 1, $\quad -1 \quad x \quad 0$
- sec x, $\quad 0 \quad x \quad 2$

$f(g(x))$



3. (A) $f(1\ \ )\quad f(1\ \ )\quad 1$

(B) $\int_2^3 ([x]\ \{x\}\ \ |x|)\ dx \quad \int_2^3 (2(x\quad 2)\quad x)\ dx \quad \frac{x^2}{2}\quad 4x\Big|_2^3 \quad \frac{3}{2}$

(C) $[x]\ \{x\}\quad 1 \qquad x\quad 0$

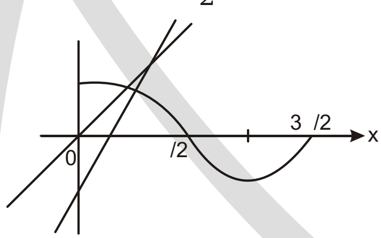
$x\quad 3\quad \frac{1}{3},\quad 2\quad \frac{1}{2}$

(D) $l\quad \lim_{x\quad 4} f(x)\quad \lim_{x\quad 4}([x]\{x\}\quad |x|)\quad 4$

4. (A) $\lim_{x} \left(\frac{x^2\quad 2x\quad 1}{2x^2\quad 3x\quad 2}\right)^{\frac{2x\ \ 1}{2x\ \ 1}}\quad \frac{1}{2}$

(B) $\lim_{x\quad 0} \frac{\log_{\sec x/2}\cos x}{\log_{\sec x}\cos\dfrac{x}{2}}\quad \lim_{x\quad 0}(\log_{\sec\frac{x}{2}}\cos x)^2\quad \lim_{x\quad 0}\left(\frac{\ln\cos x}{\ln\sec x/2}\right)^2\quad 2$

(C)



(D) $\sin x\quad \frac{1}{3},\frac{2}{3},\frac{3}{3}$

5. $f(1\ \ )\quad f(1\ \ )\quad f(1)\qquad b\quad 0$

$f(3\ \ )\quad f(3\ \ )\quad f(3)$

$3\quad 9p\quad 3q\quad 2\qquad 3p\quad q\quad 0$

$f\ (x)\quad 2ax\quad a\qquad x\quad 1$

$\qquad 1 \qquad\qquad 1\quad x\quad 3$

$\qquad 2px\quad q\qquad x\quad 3$

$f(3\ \ )\quad f(3\ \ )\quad f(3)$

$6p\quad q\quad 1\qquad p\quad \dfrac{1}{3}, q\qquad 1$

$f(1\ \ )\quad f(1\ \ )$

$a\quad 1$

Exercise-5 : Subjective Type Problems

1. $f(x)$ is discontinuous at $x\quad 1, |f(x)|$ is diff. every where

$f(1)\qquad f(1\ \ )\qquad f(1\ \ )$

$3\qquad (0\quad b)$

$b\qquad 3$

$f(1\ \ )\qquad f(1\ \ )$ (as $|f(x)|$ is differentiable every where)

$1\qquad (2a\quad a)\qquad a\qquad 1$

Continuous at $x\quad 3$,

So, $5\quad 9p\quad 3q\quad 2$

$3p\quad q\quad 1$ ...(1)

$f(x)$ is continuous at $x\quad 3$

So, $f(3\ \ )\quad f(3\ \ )$

$6p\quad q\quad 1$ ...(2)

On solving (1) & (2) we get, $p\quad 0, q\quad 1$

So, $|a\quad b\quad p\quad q|\quad |\ 1\quad 3\quad 0\quad 1|\quad 3$

2. $\sin^{\ 1} y\quad 8\sin^{\ 1} x$

$\dfrac{y}{\sqrt{1\quad y^2}}\quad \dfrac{8}{\sqrt{1\quad x^2}}$

$(1\quad x^2)(y\ )^2\quad 64(1\quad y^2)$

$(1\quad x^2)y\quad xy\qquad 64y$

3. $yy\quad 4a$

$(y\ )^2\quad yy\quad 0$

4. $f(x)$ is discontinuous at $x\qquad \sqrt{3},\quad \sqrt{2}, \sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}$

$\sin\ \ x\quad 0$ at $x\qquad 2,\quad 1, 0, 1, 2, 3$

So, continuous at these points.

5. Let $f(x)\quad K$

$f(x)\quad Kx\quad c$

$f(9) \quad f(\ \ 3) \quad 12K$

Maximum value of $f(9) \quad f(\ \ 3) \quad 96$

6. $g(x) \quad \sin x^3 \quad x^3 \quad 1 \qquad\qquad x \quad 1$

$\sin x^3 \quad x^3 \quad 1 \qquad\qquad 0 \quad x \quad 1$

$\sin x^3 \quad x^3 \quad 1 \qquad\qquad 1 \quad x \quad 0$

$\sin x^3 \quad x^3 \quad 1 \qquad\qquad x \quad\ 1$

Function is not differentiable at $x \quad\ 1, 1$

7. $F(x) \quad g(x) \qquad\qquad x \quad 1$

$\dfrac{f(x) \quad g(x)}{2} \qquad\qquad x \quad 1$

$f(x) \qquad\qquad 1 \quad x \quad 1$

$\dfrac{f(x) \quad g(x)}{2} \qquad\qquad x \quad\ 1$

$g(x) \qquad\qquad x \quad\ 1$

If $F(x)$ is continuous at $x \quad 1$

$F(1\ \ ) \quad F(1) \quad F(1\ \ )$

$b \quad a \quad 3$

If $F(x)$ is continuous at $x \quad\ 1$

$F(\ \ 1\ \ ) \quad F(\ \ 1) \quad F(\ \ 1\ \ )$

$a \quad b \quad 5$

8. $f^{\ \ 1}(x) \quad 2 \quad x \quad\ 2 \quad x \quad 5$

$2 \quad x \quad\ 2 \quad x \quad 2$

9. $f(x) \quad 2f(1 \quad x) \quad x^2 \quad 2$

$f(1 \quad x) \quad 2f(x) \quad (1 \quad x)^2 \quad 2 \qquad f(x) \quad \dfrac{(x \quad 2)^2}{3}$

10. $g(x) \quad x(x \quad 3)(x \quad 7)$

$f(g(x)) \quad \text{sgn}(x(x \quad 3)(x \quad 7))$

11. $\dfrac{d^2}{dx^2}(\sin^2 x \quad \sin x \quad 1) \qquad 4\sin^2 x \quad \sin x \quad 2$

12. $f(x) \quad a\cos(\ \ x) \quad b$

$f\ (x) \quad\ a \quad \sin(\ \ x)$

$\int_{1/2}^{3/2} f(x)\,dx \qquad \dfrac{2a}{\ } \quad b\ \dfrac{2}{\ } \quad 1 \qquad a \quad\ 1,\ b \quad 1$

13. $\ (x) \quad f\ (x) \quad 2f\ (2x)\ ; \qquad (x) \quad f\ (x) \quad 4f\ (4x)$

$\ (1) \quad f\ (1) \quad 2f\ (2) \quad 5$

(2) $f(2)$ $2f(4)$ 7

(1) $f(1)$ $4f(4)$ (1) 2 (2) 5 (2 7) 19

(1) 10 19 10 9

14. $g(f(x))$ x

$g(f(x))f(x)$ 1

$f(1)$ 7/6

x 1

$g \quad \frac{7}{6} \quad f(1) \quad 1$

$g \quad \frac{7}{6} \quad \frac{1}{f(1)}$

$f(x) \quad 4 \; e^{\frac{1 \quad x}{2}} \quad \frac{1}{2} \quad x^2 \quad x \quad 1$

$f(1)$ 2 1 1 1 5

$h(x) \quad ax^{\frac{5}{4}} \quad bx^{\frac{1}{4}}$

$h(x) \quad \frac{5a}{4} x^{\frac{9}{4}} \quad \frac{b}{4} x^{\frac{3}{4}}$

$h(5) \quad 0 \qquad \frac{5a}{4} \; 5^{\;9/4} \quad \frac{b}{4} \; 5^{\;3/4} \quad 0$

$5a \;\; 5^{\;3/2} \quad b$

$\frac{a}{b} \quad 5^{1/2}$

$\frac{a}{b}^{2} \quad 5$

$\frac{a^2}{5b^2 g \quad \frac{7}{6}} \quad \frac{5}{5 \quad \frac{1}{5}} \quad 5$

16. Let $\lim_{x} \quad f(x) \quad \int_0^x f(t)\,dt \quad l$ ...(1)

$$\lim_{x} \frac{e^x \int_0^x f(t)\,dt}{(e^x)} \quad l$$

$$\lim_{x\to\infty}\frac{e^x\int_0^x f(t)\,dt}{e^x}=l$$

$$\Rightarrow \lim_{x\to\infty}\int_0^x f(t)\,dt = l \qquad \ldots(2)$$

From (1) and (2) we get, $\lim_{x\to\infty} f(x) = 0$

17. $f(0)=0,\ f'(0)=1,\ f''(0)=1,\ f'''(0)=2$

$g(f(x)) = x \Rightarrow g'(f(x))f'(x) = 1$

$$\Rightarrow g''(f(x)) = -\frac{f''(x)}{(f'(x))^3}$$

$$\Rightarrow g'''(f(x))f'(x) = -\frac{(f'(x))^3 f'''(x) - 3(f'(x))^2(f''(x))^2}{(f'(x))^6}$$

Put $x = 0$

$$g'''(0)1 = -\frac{1\cdot 2 - 3\cdot 1}{1} = 1$$

19. $$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{f\left(x\left(1+\frac{h}{x}\right)\right)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{f(x)}{1+h/x}+\frac{f\left(1+\frac{h}{x}\right)}{x}-f(x)}{h} = \lim_{h\to 0}\frac{f(x)\left(-\frac{h}{x}\right)}{h\left(1+\frac{h}{x}\right)}+\frac{f\left(1+\frac{h}{x}\right)}{hx}$$

$$= -\frac{f(x)}{x}+\lim_{h\to 0}\frac{f\left(1+\frac{h}{x}\right)-f(1)}{x^2\,\frac{h}{x}} \qquad (\text{as } f(1)=0)$$

$$f'(x) = -\frac{f(x)}{x}+\frac{f'(1)}{x^2}$$

$$xf'(x)+f(x)=\frac{1}{x}$$

$$\frac{d}{dx}(xf(x))=\frac{1}{x}$$

$$xf(x)=\int\frac{1}{x}\,dx$$

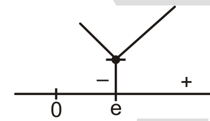
$$xf(x)=\ln x+k$$

Put $x \quad 1$, we get $k \quad 0$

$$f(x) \quad \frac{\ln x}{x}$$

$$H(x) \quad \frac{1}{f(x)} \quad \frac{x}{\ln x}$$

$$H\ (x) \quad \frac{\ln x \ \ 1 \ \ 1}{(\ln x)^2} \qquad\qquad H(x) \quad e$$



$$H(e) \quad e$$

$$\lim_{x \quad e} \frac{1}{f(x)} \quad 2$$

21. $f\ (x) \quad \tan\ ^1(x^2) \quad \frac{2x^2}{1 \quad x^4} \quad 4x^3$

$$f\ \ (x) \quad \frac{2x}{1 \quad x^4} \quad 2\ \frac{(1 \quad x^4)\ 2x \quad x^2(4x^3)}{(1 \quad x^4)^2} \quad 12x^2$$

22. $\frac{dy}{dx} \quad \frac{dy/d}{dx/d} \quad 3\sin \quad \cos \quad \frac{3}{2}\sin 2$

$$\frac{d^2y}{dx^2} \quad \frac{3\cos 2}{\sin}$$

23. Let $8x \quad 16 \quad t^2 \quad \sqrt{\frac{t^2 \quad 16 \quad 8t}{8}} \quad \sqrt{\frac{t^2 \quad 16 \quad 8t}{8}} \quad \frac{|t \quad 4| \quad |t \quad 4|}{2\sqrt{2}}$

24. $f(x) \quad [x] \qquad 0 \quad x \quad 1$

$\{x\} \qquad 1 \quad x \quad \frac{5}{4}$

$\left|x \quad \frac{3}{2}\right| \qquad \frac{5}{4} \quad x \quad 2$

No. of points where $f(x)$ is non-differentiable are three.

$$x \quad 1, \frac{5}{4}, \frac{3}{2}$$

❑❑❑

4

APPLICATION OF DERIVATIVES

Exercise-1 : Single Choice Problems

1. Maximum value of $f(x) \quad 3$

Minimum value of $f(x) \quad 1$

2. $f\ (x) \quad 6x \quad 6$

$f\ (x) \quad 3x^2 \quad 6x \quad 3 \qquad\qquad (\because f\ (2) \quad 3)$

$f(x) \quad x^3 \quad 3x^2 \quad 3x \quad 1 \qquad\qquad (\because f(2) \quad 1)$

5. $V \quad \frac{4}{3} \ (10 \quad T)^3 \quad \frac{4}{3} \ (10)^3$

$$\frac{dV}{dt} \quad 4\ (10 \quad T)^2 \frac{dT}{dt}$$

$$\frac{dT}{dt} \quad \frac{1}{18} \qquad (\because T \quad 5\text{ cm})$$

6. $g(x) \quad \frac{(|x| \quad 1)(|x| \quad 2)}{(|x| \quad 3)(|x| \quad 4)}$

$g(x)$ is an even function so there is an extrema at $x \quad 0$.

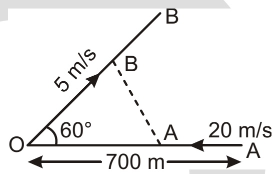
Also number of extrema for $x \quad 0$ will be equal to number of extrema for $x \quad 0$

for $x \quad 0$

$$g(x) \quad \frac{(x \quad 1)(x \quad 2)}{(x \quad 3)(x \quad 4)}$$

Number of extrema $\quad 2$

Total extrema $\quad 5$



7. $AB \quad \sqrt{700 \quad \frac{45}{2}t \quad ^2 \quad \frac{75}{4}t^2}$

$(AB)_{min}$ at t 30 sec

8. $f(0 \quad) \quad f(0) \quad a \quad 3$

9. $f(x) \begin{cases} 3+k-x, & x \quad k \\ a^2-2+\frac{\sin(x-k)}{x-k}, & x > k \end{cases}$ $\quad f(k \quad) \quad f(k), f(k \quad) \quad f(k)$

So, $\lim_{x \quad k} (a^2 \quad 2) \quad \frac{\sin(x \quad k)}{(x \quad k)} \quad a^2 \quad 1 \quad 3$

$a^2 \quad 4$

$|a| \quad 2$

10. $\frac{dy}{dx} \quad 3x^2 \quad 4x \quad C_1$

$y \quad x^3 \quad 2x^2 \quad C_1 x \quad C_2$

Also, $\left.\frac{dy}{dx}\right|_{at\ x \quad 1} \quad 0$ and $y]_{at\ x \quad 1} \quad 5$

11. $m_1 \quad \left.\frac{dy}{dx}\right|_{at\ (1,\ 2)} \quad 2a \quad b$

$m_2 \quad g(x) \quad \left.\frac{dy}{dx}\right|_{at\ (\quad 2,\ 2)} \quad 2 \quad 2a \quad b \quad \frac{1}{2}$

Also, $2 \quad a \quad b \quad \frac{7}{2}$

12. $18y\frac{dy}{dx} \quad 3x^2$

$\frac{dy}{dx} \quad \frac{3x^2}{18y}$

$\frac{a^2}{6b} \quad 1 \quad a^2 \quad 6b$

Also, $9b^2 \quad a^2$

13. $\frac{dy}{dx} \quad 3x^2 \quad 4x \quad c$

at $x \quad 1, \frac{dy}{dx} \quad 0 \quad c \quad 1$

$$\frac{dy}{dx} \quad 3x^2 \quad 4x \quad 1$$

$$y \quad x^3 \quad 2x^2 \quad x \quad d$$

at $x \quad 1, y \quad 5 \qquad 5 \quad 1 \quad 2 \quad 1 \quad d$

$$d \quad 5$$

14. $A(0, 2)$

$$5 \quad ^2(3x^2) \quad 10 \quad (2x) \quad 1 \quad 2\frac{dy}{dx} \quad 0 \qquad \frac{dy}{dx} \quad \frac{15 \quad ^2x^2 \quad 20 \quad x \quad 1}{2}$$

$$\frac{dy}{dx} \text{ at } A \quad \frac{1}{2}$$

Equation of normal at A is $y \quad 2x \quad 2$

Let normal meets the curve at B

$$5 \quad ^2x^3 \quad 10 \quad x^2 \quad x \quad 4x \quad 4 \quad 4 \quad 0$$

$$5x(\quad x \quad 1)^2 \quad 0$$

$$x \quad \frac{1}{}$$

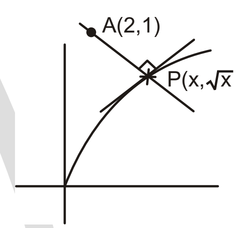
So, $B \quad \frac{1}{}, \frac{2}{} \quad 2$

Slope of tangent at $B \quad \dfrac{15 \quad 20 \quad 1}{2} \quad 2$

15. $f(x) \quad \cos x \quad \frac{1}{2}\cos 2x \quad \frac{1}{3}\cos 3x$

$$f\ (x) \quad \sin x \quad \sin 2x \quad \sin 3x \quad 2\sin x(2\cos x \quad 1)(\cos x \quad 1) \quad 0$$

16. Closest distance exist always alone the normal



$$\frac{1 \quad \sqrt{x}}{2 \quad x} \quad \frac{dy}{dx} \quad 1$$

$$\frac{1 \quad \sqrt{x}}{2 \quad x} \quad \frac{1}{2\sqrt{x}} \quad 1$$

Let $\sqrt{x} \quad t$

$$x \quad \frac{2 \quad \sqrt{3}}{2}$$

17. Let $x \quad 2\sin$

$$y \quad \ln \frac{2 \quad 2\cos}{2 \quad 2\cos} \quad 2\cos \quad \ln \frac{2\cos^2 \quad /2}{2\sin^2 \quad /2} \quad 2\cos$$

$$2\ln \quad \cot\frac{}{2} \quad 2\cos$$

$$\frac{dy}{d} \quad \frac{1}{\cot \ /2} \quad \text{cosec}^2 \frac{}{2} \quad 2\sin$$

$$\frac{dy}{d} \quad \frac{2}{\sin} \quad 2\sin \quad \frac{2\cos^2}{\sin}$$

$$\frac{dx}{d} \quad 2\cos \ ; \ \frac{dy}{dx} \quad \cot$$

$$y \quad 2\ln \ \cot\frac{}{2} \quad 2\cos \qquad \cot \ (x \quad 2\sin \ )$$

$$T \quad 0, 2\ln \ \cot\frac{}{2} \quad 2\cos \quad 2\cos$$

$$P \quad 2\sin \ , 2\ln\cot\frac{}{2} \quad 2\cos$$

$$PT^2 \quad (\sqrt{4\sin^2 \quad 4\cos^2} \ ) \quad 4$$

18. $g(x) \quad (2x^2 \quad \ln x) f(x)$

$$f(x) \quad \frac{1}{\ln x^3} 3x^2 \quad \frac{1}{\ln x^2} 2x$$

$$f(x) \quad \frac{x^2 \quad x}{\ln x}$$

$$f(x) \quad \frac{x(x \quad 1)}{\ln x} \quad 0 \quad x \quad 1; \quad f(x) \quad f(1) \qquad f(x) \quad 0 \quad x \quad 1$$

For $g(x)$ is increasing

$g(x) \quad 0 \qquad 2x^2 \quad \ln x \quad 0$ as $(f(x) \quad 0)$

Let $H(x) \quad 2x^2 \quad \ln x$

$H(x) \quad 4x \quad \frac{1}{x} \quad \frac{4x^2 \quad 1}{x} \quad 0$ when $x \quad 1$

$H(x) \quad H(1) \qquad H(x) \quad 2$

$g(x) \quad 0 \quad x \quad (1, \quad)$

$g(x)$ is increasing on $(1, \quad)$.

19. $f(x) \quad 3x^2 \quad 12x \quad a$

$f(x) \quad 0$ in $(\ 3, \ \ 1)$

Product of the roots $\frac{a}{3} \quad 3 \qquad a \quad 9$

20. $f(x) \quad \tan^{1} \frac{1 \quad x}{1 \quad x}$

$$f(x) \quad \frac{1}{1 \quad \left(\frac{1 \quad x}{1 \quad x}\right)^2} \quad \frac{2}{(1 \quad x)^2} \quad \frac{2}{2(1 \quad x^2)} \quad \frac{1}{1 \quad x^2} \quad 0$$

$f(x)$ is decreasing $\quad x \quad R$

So, in $[0,1] \quad f(0) \quad \tan^{\ \ 1}(1) \quad \frac{\ }{4}$ (max)

$f(1) \quad 0$ (min)

21. $f(x) \quad 3x^2 \quad 2(a \quad 2)x \quad 3a$

$D \quad 0$

$a^2 \quad 5a \quad 4 \quad 0$

$a \quad [1, 4]$

22. $f(x) \quad 0$

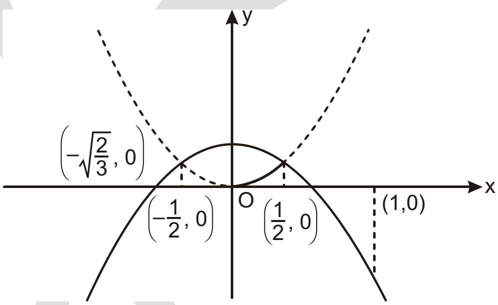
$\cos^2 x \quad \sqrt[3]{x} \quad x^{1/3} \quad \frac{1}{2} \quad 0$

$\cos^2 x \quad \frac{1}{2}$

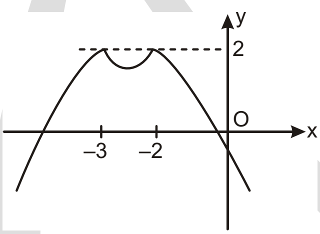
$\cos x \quad \frac{1}{\sqrt{2}}$

total number is 12.

23.



24.



$b^2 \quad 1 \quad 2$

25. $f(x)$ is continuous and differentiable in $[\ \ 1, 1]$.

26. $\frac{\cos x_1}{x_1} \quad \sin x_1 \quad x_1 \quad \cot x_1$

Point $(x_1, \cos x_1)$ always lie on $\frac{1}{y^2} \quad \frac{1}{x^2} \quad 1$

27. $x \quad \frac{a}{x^2} \quad 2 \quad x \quad (0, \quad)$

$f(x) \quad x^3 \quad 2x^2 \quad a \quad 0 \quad x \quad (0, \quad)$

$f\ (x) \quad 3x^2 \quad 4x \quad 3x \quad x \quad \frac{4}{3}$

Minimum value at $x \quad \frac{4}{3}$

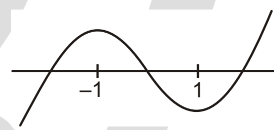
$\frac{64}{27} \quad 2 \quad \frac{16}{9} \qquad a \quad 0 \qquad a \quad \frac{32}{27}$

29. $f\ (x) \quad \cos^2 x \quad \cos x \quad 2 \quad 0$

$f(x)_{\text{min}} \quad f(0) \quad 0$

$f(x)_{\text{max}} \quad f(2 \quad) \quad 5$

31. $f(x) \quad x^3 \quad 3x \quad c \quad 0$



$f\ (x) \quad 3\,(x^2 \quad 1)$

$f(1)\,f(\quad 1) \quad 0$

$(c \quad 2)(2 \quad c) \quad 0$

32. $f\ (x) \quad e^x(x \quad 1)(x \quad 2) \quad 0$

33. $\frac{dy}{dx} \quad 3ax^2 \quad 2bx \quad c \quad 0$ has one root $\quad D \quad b^2 \quad 3ac \quad 0 \quad b^2 \quad 6$

34. Let $x \quad \tan$ then $y \quad \cos^2$

$\left|\frac{dy}{dx}\right| \quad |2\sin \quad \cos^3 \quad |$

$\left|\frac{dy}{dx}\right|_{\text{max}}$ at $\quad \overline{6}$

35. $h(x) \quad f(x) \quad g(x) \quad 2x \quad 3\sin x \quad x\cos x$

$h(0) \quad 0$

$h\ (x) \quad 2 \quad 2\cos x \quad x\sin x$

$h\ (0) \quad 0$

$h\ \ (x) \quad \sin x \quad x\cos x$

$h\ \ (0) \quad 0$

$h\ \ (x) \quad x\sin x \quad 0 \quad x \quad 0, \overline{2}$

36. $f(x) \quad 2\tan^{\ 1}(g(x)) \qquad |g(x)| \quad 1$

$2\tan^{\ 1} g(x) \qquad g(x) \quad 1$

$2\tan^{\ 1} g(x) \qquad g(x) \quad 1$

$$f\,(x) \quad \frac{2g\,(x)}{1 \quad (g(x))^2} \qquad\qquad |g(x)| \quad 1$$

$$\frac{2g\,(x)}{1 \quad (g(x))^2} \qquad\qquad |g(x)| \quad 1$$

37. $\lim\limits_{x \quad e^a} \dfrac{7}{3} \dfrac{\ln(1 \quad 7f(x))}{7f(x)} \quad \dfrac{1}{3} \dfrac{\sin f(x)}{f(x)} \qquad 2$

38. If $f(x)$ is strictly decreasing for all x,

$f\,(x) \quad \log_{1/3}(\log_3(\sin x \quad a)) \quad 0$

$\sin x \quad a \quad 3 \quad x \quad R$

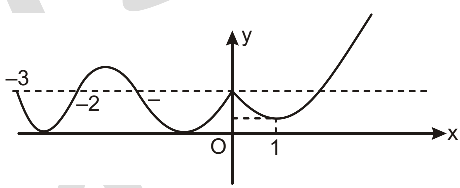
$a \quad 4$

39. $f(x) \quad a\ln|x| \quad bx^2 \quad x$

$f\,(x) \quad \dfrac{a}{x} \quad 2bx \quad 1 \quad \dfrac{2bx^2 \quad x \quad a}{x}$

if $x \quad 1$ and $x \quad 3$ are point of extrema.

$\dfrac{1}{2b} \quad 4$ and $\dfrac{a}{2b} \quad 3$

40. 

$f(x)$ has local maximum at $x \quad 0$.

41. $f(x) \quad \int_1^x (t \quad a)^{2n}(t \quad b)^{2m \quad 1}\,dt$

$f\,(x) \quad (x \quad a)^{2n}(x \quad b)^{2m \quad 1}$

No sign change of $f\,(x)$ about $x \quad a$.

$f\,(x)$ will change sign from negative to positive at $x \quad b$ Point of minima.

43. Let point P on the curve $y^2 \quad x^3$ is $P(t_1^2, t_1^3)$.

Equation of tangent at $P(t_1^2, t_1^3)$ is

$$y \quad t_1^3 \quad \frac{3}{2}t_1(x \quad t_1^2)$$

If this intersect the curve again at $Q(t_2^2, t_2^3)$

$$t_2 \quad \frac{t_1}{2}$$

$$\frac{\tan}{\tan} \quad \frac{(3t_1/2)}{(3t_2/2)} \quad 2$$

44. $y^2 \quad x^3$

if $(2, 3)$ is lie on the curve

$$8 \qquad 9 \qquad \ldots(1)$$

Slope of normal at $(2, 3)$

$$\frac{1}{4} \quad \frac{1}{2} \qquad 2$$

45. Equation of tangent at $(0, 1)$ to the curve $y \quad 1 \quad kx$ meet x-axis at $(a, 0)$ then

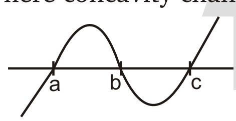
$$2 \quad \frac{1}{k} \quad 1 \qquad k \quad \frac{1}{2}, 1$$

46. $f(x) \quad \int_0^{\sqrt{x}} e^{\frac{u^2}{x}} du \quad \sqrt{x} \int_0^1 e^{t^2} dt$

where $t \quad \dfrac{u}{\sqrt{x}}$

$f(x) \quad K\sqrt{x}, \quad K \quad 0$

47. $f\,(\;)\quad 0 \qquad x$ is the point where concavity changes.



48. $f(x) \quad x^6 \quad x \quad 1$

$f\,(x) \quad 6x^5 \quad 1 \quad 0 \quad x \quad [1, 2]$

If $f(1) \quad 1 \quad 0$ and $f(2) \quad 2^6 \quad 3 \quad 0$ then $f(x)$ has one root in [1, 2].

49. Every line passing from (a, b) is normal to the circle $(x \quad a)^2 \quad (y \quad b)^2 \quad k$

50. $f\,(x) \quad \cos x(3\sin^2 x \quad m) \quad 0$

$\sin^2 x \quad \dfrac{m}{3} \qquad 0 \quad \dfrac{m}{3} \quad 1$

$0 \quad m \quad 3$

51. Let $y \quad x^{1/x}$

$y \quad x^{(1/x)\;\;2}(1 \quad \ln x)$

$f(x)$ is increasing $\qquad (0, e)$

and $f(x)$ is decreasing $\qquad (e, \;\;)$

52. Let $y \quad mx$

Point of tangency be (x_1, y_1)

$$mx_1 \quad x_1^3 \quad x_1 \quad 16 \text{ \& } m \quad 3x_1^2 \quad 1$$

$$x_1(3x_1^2 \quad 1) \quad x_1^3 \quad x_1 \quad 16$$

$$x_1 \quad 2$$

$$m \quad 13$$

53. $y \quad 3x^2 \quad 6x \quad 6$

$y \quad 6x \quad 6 \quad 0$

$x \quad 1$

$y \quad 3$

54. Let $H(x) \quad \ln(f(x) \quad f(x) \qquad f^n(x)) \quad x$

$$H(a) \quad H(b)$$

$$H(c) \quad 0 \qquad \text{(by L.M.V.T.)}$$

$$\frac{f(c) \quad f(c) \qquad f^{n \quad 1}(c)}{f(c) \quad f(c) \qquad f^n(c)} \quad 1 \quad 0$$

$$f^{n \quad 1}(c) \quad f(c)$$

55. $h(x) \quad g(x) \quad x$

$h(x) \quad g(x) \quad 1$

$g(x) \quad h(x) \quad 1$

$g(x) \quad h(x)$

$h(x) \quad 3(h(x) \quad 1) \quad 3$

$h(x) \quad 3h(x) \quad 0$

$\dfrac{d}{dx}(e^{\quad 3x} h(x)) \quad 0$

Let $P(x) \quad e^{\quad 3x} h(x)$

$P(x) \quad 0$

$P(x)$ is an increasing function.

$P(0) \quad h(0) \quad 0$

$P(x) \quad 0 \quad x \quad 0$

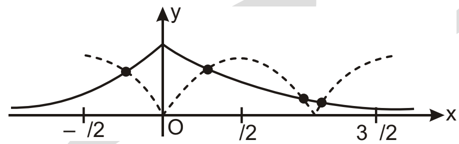
$h(x) \quad 0 \quad x \quad 0$

$h(x)$ is an increasing function $\quad x \quad 0$

56. $\dfrac{dy}{dx} \qquad \dfrac{c}{(x \quad 1)^2} \quad 1 \quad (x \quad 1)^2 \quad c$

Point $(\sqrt{c} \quad 1, \sqrt{c})$ lie on the line $x \quad y \quad 3 \qquad \sqrt{c} \quad 2$

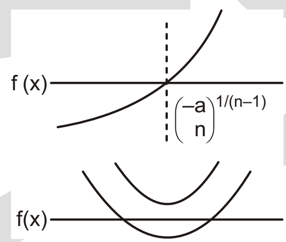
57. $|\sin x| \quad e^{\;x^2}$



60. $x^n \quad ax \quad b \quad 0$

x is even.

$nx^{n\;\;1} \quad a \quad f\,(x)$



61. $f(b) \quad \left|\sin x \quad \frac{2}{3 \quad \sin x} \quad b\right|_{\max} \quad x \quad R$

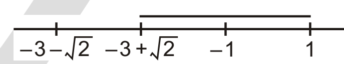
$\sin x \quad t$

$g(t) \quad t \quad \frac{2}{3 \quad t} \quad t \quad [\;\;1, 1]$

$g\,(t) \quad 1 \quad \frac{2}{(3 \quad t)^2} \quad 0$

$(3 \quad t)^2 \quad 2 \quad 0$

$(3 \quad t \quad \sqrt{2})(3 \quad t \quad \sqrt{2}) \quad 0$

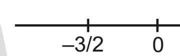


$g(t) \quad t \quad \frac{2}{3 \quad 1}$ increasing $\quad [\;\;1, 1]$

$g(t)_{\max} \quad \frac{3}{2}$

$g(t)_{\min} \quad 0$

$f(b) \quad \left|\frac{3}{2} \quad b \atop b\right. \qquad \text{if } b \quad \frac{3}{4}$



$\qquad\qquad b \quad \frac{3}{4}$

min. of $f(b) \quad \frac{3}{4} \quad \frac{3}{4}$

62. $y \quad \frac{x}{1 \quad x^2} \qquad \frac{dy}{dx} \quad \frac{1 \quad x^2}{(1 \quad x^2)^2}$

63. $f^{\ 1}(x) \quad 2\cos^{\ 1}\dfrac{x}{3}$

$$\frac{d}{dx}f^{\ 1}(x) \quad \frac{2}{\sqrt{1 \quad \dfrac{x^2}{9}}} \quad \frac{1}{3}$$

64. $f(x) \quad \sin x \quad \tan x \quad 2x$

$f(x) \quad \cos x \quad \sec^2 x \quad 2 \quad 0$

$\cos^3 x \quad 2\cos^2 x \quad 1 \quad 0 \qquad (\cos x \quad 1)(\cos^2 x \quad \cos x \quad 1) \quad 0$

$\cos x \quad 1, \dfrac{1 \quad \sqrt{5}}{2}$

65. $\dfrac{a}{b} \quad \dfrac{2c}{3b} \quad \dfrac{4}{3} \quad 0 \qquad 3a \quad 4b \quad 6c \quad 12b \quad 0$

$$\frac{1}{4}a \quad \frac{b}{3} \quad d \quad 0$$

Consider $f(x) \quad \dfrac{ax^4}{4} \quad \dfrac{bx^3}{3} \quad \dfrac{cx^2}{2} \quad dx$

then $\quad f(0) \quad 0 \quad f(1)$

$f(x)$ satisfies the conditions of Rolle's theorem in [0, 1].

Hence, $f(x) \quad 0$ has at least one solution in (0, 1).

66. $f(x) \quad (x) \ (x \quad 2)^2$

$(2) \quad 0 \quad f(x) \quad 0 \quad f(x)$

$(2) \quad 0 \quad f(x) \quad 0 \quad f(x)$

67. $f(1) \quad f(3) \quad a \quad b \quad 5 \quad 3a \quad b \quad 27 \quad a \quad 11$

$f(c) \quad 3c^2 \quad 12c \quad a \quad 0 \quad b \quad R$

70. Let $x \quad \dfrac{3at}{2^{2/3}}, y \quad at^{3/2}$

$$\frac{dy}{dx} \quad \frac{2 \quad \dfrac{9a^2t^2}{2^{4/3}}}{9a^2t^{3/2}} \quad \cot \quad \sqrt{t} \quad 2^{1/3}\cot$$

and $P \quad \cos \quad \dfrac{3at}{2^{2/3}} \quad \dfrac{at^{3/2}}{\cot}$

$$\frac{P}{a} \quad \cos \quad \cot^2$$

Exercise-2 : One or More than One Answer is/are Correct

1. Equation of tangent to $y \quad x^3$

$$y \quad x_1^3 \quad 3x_1^2(x \quad x_1)$$

Equation of tangent to $y \quad x^{1/3}$ is

$$y \quad x_2^{1/3} \quad \frac{1}{3x_2^{2/3}}(x \quad x_2)$$

If these tangents represent same line

$$\frac{1}{1} \quad \frac{9x_1^2 x_2^{2/3}}{1} \quad \frac{2x_1^3}{\frac{2}{3}x_2^{1/3}} \qquad x_1 \qquad \frac{1}{\sqrt{3}}$$

2. (a) $f(C_1) \quad \dfrac{f(4) \quad f(0)}{4 \quad 0} \quad \dfrac{1}{4};\ C_1 \quad (0, 4)$

(c) $f(C_1) \quad \dfrac{f(8) \quad f(0)}{8 \quad 0} \quad \dfrac{1}{8};\ C_1 \quad (0, 8)$

$f(C_2) \quad f(8) \quad 1$

(d) Let $g(x) \quad \int_0^{x^3} f(t)\, dt$

$$g(0) \quad 0, g(2) \quad \int_0^8 f(t)\, dt$$

$$g(\ ) \quad 3 \quad {}^2 f(\ {}^3) \quad \frac{g(2) \quad g(0)}{2} \qquad (0, 2)$$

and $\quad g(\ ) \quad 3 \quad {}^2 f(\ {}^3) \quad \dfrac{g(2) \quad g(0)}{2} \qquad (0, 2)$

$$g(\ ) \quad g(\ ) \quad g(2) \quad g(0) \quad \int_0^8 f(t)\, dt$$

4. $f(x) \quad 2x^4 \quad x^4 \sin\dfrac{1}{x} \qquad x \quad 0$

$\qquad 0 \qquad\qquad x \quad 0$

$f(x) \quad 8x^3 \quad 4x^3 \sin\dfrac{1}{x} \quad x^2 \cos\dfrac{1}{x}$

5. $1 \quad f(x) \quad 1$

$x \quad f(x) \quad x \qquad (\because f(0) \quad 0)$

$\dfrac{x^2}{2} \quad f(x) \quad \dfrac{x^2}{2} \qquad (\because f(0) \quad 0)$

6. $f\ (x)\quad 0\quad x\quad [\ 3, 4]$

$\quad f\ (x)$ is increasing for $x\quad [\ 3, 4]$

7. $f\ (x)\quad 0\quad x\quad [0, 2]$

$\quad f\ (x)$

$$f\ (C_1)\quad \frac{f(1)\quad f(0)}{1\quad 0}, C_1\quad (0, 1) \text{ and } f\ (C_2)\quad \frac{f(2)\quad f(1)}{2\quad 1}, C_2\quad (1, 2)$$

$$f\ (C_1)\quad f\ (C_2)\qquad f(0)\quad f(2)\quad 2 f(1)$$

Similarly applying LMVT between $0, \frac{2}{3}$ and $\frac{2}{3}, 2$

$$\frac{f(2)\quad f\ \frac{2}{3}}{\frac{4}{3}}\quad \frac{f\ \frac{2}{3}\qquad f(0)}{\frac{2}{3}}\qquad 2 f(0)\quad f(2)\quad 3 f\ \frac{2}{3}$$

8. Let $g\ (x)\quad a(x\quad 1)$

$$g\ (x)\quad \frac{ax^2}{2}\quad ax\quad b$$

$$g\ (\ 1)\quad 0\qquad b\quad \frac{3a}{2}$$

$$g(x)\quad \frac{ax^3}{6}\quad \frac{ax^2}{2}\quad bx\quad c\qquad g(x)\quad x^3\quad 3x^2\quad 9x\quad 5\qquad (\because g(\ 1)\quad 10, g(3)\qquad 22)$$

9. $f(x)\quad 2x^3\quad 3(\qquad 2)\,x^2\quad 2\ x\quad 5$

$f\ (x)\quad 6x^2\quad 6(\qquad 2)\,x\quad 2\qquad 0$ has two real roots, then

$$D\quad 0\qquad 3\ ^2\quad 8\qquad 12\quad 0$$

$$R$$

10. $f(x)\quad 1\quad x \ln(x\quad \sqrt{1\quad x^2})\quad \sqrt{1\quad x^2}$

$f\ (x)\quad \ln(x\quad \sqrt{1\quad x^2})$

$f\ (x)\quad 0$ for $\quad x\quad [0,\quad)$

$f\ (x)\quad 0$ for $\quad x\quad (\qquad, 0]$

11. $f(x, y)\quad x^m (k\quad x)^n$

$$f\ (x, y)\quad mx^{m\quad 1}(k\quad x)^n\quad x^m n\ (k\quad x)^{n\quad 1}\quad 0$$

$$x\quad \frac{mk}{m\quad n}$$

Maximum value $\dfrac{k^{m\quad n}\quad m^m\quad n^n}{(m\quad n)^{m\quad n}}$

12. Let line is tangent at $(3t_1^2, 2t_1^3)$ and normal at $(3t_2^2, 2t_2^3)$

$$\left.\frac{dy}{dx}\right|_{3t_1^2,\ 2t_1^3} \quad t_1$$

So, slope of normal at $(3t_2^2, 2t_2^3)$ $\quad \dfrac{1}{t_2}$

$$t_1 \quad \frac{1}{t_2}$$

$$t_2 \quad \frac{1}{t_1}$$

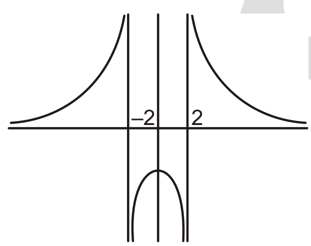
$$t_1 \quad \frac{2t_1^3 \quad \dfrac{2}{t_1^3}}{3 \quad t_1^2 \quad \dfrac{1}{t_1^2}}$$

$t_1^2(t_1^4 \quad 3) \quad 2$ or $t_1 \qquad \sqrt{2}$

13. $m \quad \dfrac{dy}{dx}$

$\dfrac{|my|}{x \quad y} \quad \dfrac{y}{x}$ then solve it.

14. First draw the graph $f(x) \quad \dfrac{1}{x^2 \quad 4}$



While drawing diff. possibilities of $y \quad ax^2 \quad bx \quad c$

We get possible intersections.

15. $y \quad 3x_1^2$

$3x_1^2 \quad \dfrac{x_1^3 \quad 8}{x_1 \quad 2} \qquad x_1 \quad 1$ or 2

$y \quad 3$ or $y \quad 12$

16. Let $f(x) \quad x \quad \cos x \quad a$

$f(x) \quad 1 \quad \sin x \quad 0 \quad x \quad R$

$f(x)$ is increasing.

$x \quad \cos x \quad a \quad 0$ for one positive value of $x, a \quad (1, \quad)$

17. Let $y \quad ax \quad b \quad f(x) \quad a \quad 0$

$f \ ^{1}(x) \quad \dfrac{x \quad b}{a}$

(1) $m_1 \quad a, m_2 \quad \dfrac{1}{a} \qquad\qquad m_1 m_2 \quad 1$

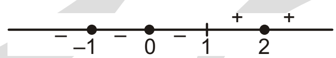
(2) $m_1 \quad a, m_2 \quad \dfrac{1}{a} \qquad\qquad m_1 m_2 \quad 1$

(3) $m_1 \quad a, m_2 \quad \dfrac{1}{a} \qquad\qquad m_1 m_2 \quad 1$

(4) $m_1 \quad a, m_2 \quad \dfrac{1}{a} \qquad\qquad m_1 m_2 \quad 1$

18. $f\ (x) \quad e^{x}(x^{2} \quad 1)x^{2}(x \quad 1)^{2011}(x \quad 2)^{2012}$

$\qquad e^{x}x^{2}(x \quad 1)^{2012}(x \quad 1)(x \quad 2)^{2012}$



$x \quad 1, 0, 2$ are points of inflections and might be more points in (1, 2).

$x \quad 1$ is point of minima (Answer can be given either d, ad, bd or abd)

19. $f(x) \quad \sin x \quad ax \quad b$

$f\ (x) \quad \cos x \quad a$

if $a \quad 1$ then $f(x)$ is increasing.

So, only one real root, which is positive if $b \quad 0$ and negative if $b \quad 0$

if $a \quad 1$

$f(x)$ is decreasing so only one real root, which is negative if $b \quad 0$.

20. $f\ (c) \quad 0 \quad \text{for} \quad c \quad (0, 1)$

$f\ (x) \quad 0 \quad \text{for} \quad x \quad (0, c)$

$f\ (x) \quad 0 \quad \text{for} \quad x \quad (c, 1)$

21. $f\ (x) \quad 5\sin x \cos x(\sin x \quad \cos x)(1 \quad \sin x \cos x)$

Clearly, $\quad f\ (x) \quad 0 \quad x \quad \dfrac{}{4}, \dfrac{}{2}$

$\qquad f\ (x) \quad 0 \quad x \quad 0, \dfrac{}{4}$

$\qquad f(0) \quad 0, f \ \dfrac{}{2} \quad 0$

By Rolle's theorem $\quad c \quad 0, \dfrac{}{2} \qquad f\ (c) \quad 0$

Clearly, $\quad f(x) \quad f \ \dfrac{}{4}$

$$f(x) \quad 2 \quad \frac{1}{\sqrt{2}}^{5} \quad 1 \quad 2 \quad \frac{1}{4\sqrt{2}} \quad 1 \quad \frac{1}{2\sqrt{2}} \quad 1 \text{ and } f(x) \quad 0 \quad x \quad 0, \frac{}{2}$$

22. $f(x) \quad x^{2} \quad {}^{1} \ln x \qquad x \quad 0$

$\qquad 0 \qquad\qquad x \quad 0$

$f(x)$ is not continuous at $\quad \frac{1}{2}, \quad 1$

23. $f(x) \quad \frac{\cos x}{x}$

Clearly, $f(x) \quad 0 \quad x \quad 0, \frac{}{2} \quad (4n \quad 1)\frac{}{2}, (4n \quad 1)\frac{}{2} \quad n \quad N$

and $f(x) \quad 0 \quad x \quad (4n \quad 1)\frac{}{2}, (4n \quad 3)\frac{}{2} \quad n \quad N$

$f(x)$ has a local minima at $x \quad (4n \quad 1)\frac{}{2} \quad n \quad N$

and $f(x)$ has a local maxima at $x \quad \frac{}{2}$ and $(4n \quad 1)\frac{}{2} \quad n \quad N$

Also, $f(x) \quad \frac{x(\quad \sin x) \quad \cos x}{x^{2}} \quad 0 \qquad x \tan x \quad 1 \quad 0$

All the points of inflection of $f(x)$ lie on the curve $x \tan x \quad 1 \quad 0$

Also, $f(x) \quad 0 \quad x \quad (2n \quad 1)\frac{}{2} \quad n \quad N$

Number of values of x in $(0, 10 \quad)$ in which $f(x) \quad 0$ are 20.

24. $|f(x)| \quad 1$

Applying L.M.V.T. in $x \quad (0, 1)$

$|f(x)| \quad |f(1) \quad f(0)|$

$|f(1) \quad f(0)| \quad 2$

$|f(x)| \quad 2$ for atleast one x in $(0, 1)$

Similarly $|f(x)| \quad 2$ for atleast one x is $(\quad 1, 0)$

$F(x) \quad (f(x))^{2} \quad (f(x))^{2}$

For atleast one x in $(0, 1)$ & $(\quad 1, 0)$

$|f(x)| \quad 2$ & $|f(x)| \quad 1$

$(f(x))^{2} \quad 4$ & $(f(x))^{2} \quad 1$

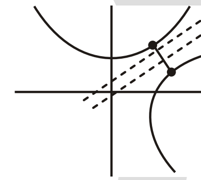
$(f(x))^{2} \quad (f(x))^{2} \quad 5$

$F(x) \quad 5$, for atleast one x in $(\quad 1, 0)$ & $(0, 1)$

25. $f \frac{}{2} \quad f \frac{}{2}$ and $f \frac{}{2} \quad f \frac{}{2}$

Also, absolute maximum occurs at $x \quad 1$

26. Symmetric about $y \quad x$



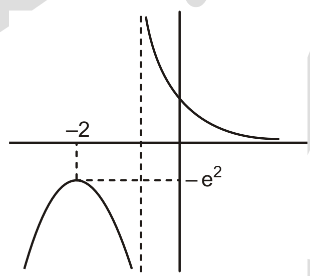
$$\frac{dy}{dx} \quad 1$$

$$2x \quad 1 \qquad x \quad \frac{1}{2}$$

$$\text{Point} \quad \frac{1}{2}, \frac{5}{4}$$

27. $f(x) \quad 0 \qquad \frac{(1 \quad x) e^{\ x} \quad e^{\ x}}{(1 \quad x)^2} \quad 0$

for $x \quad 2$ increasing.



28. Point (1, 2) lies on $y \quad mx \quad 5 \qquad m \quad 3$ ...(1)

Point (1, 2) lies on $x^3 y^3 \quad ax^3 \quad by^3 \qquad 8b \quad a \quad 8$ ...(2)

$3x^2 y^3 \quad x^3 \ 3y^2 y^1 \quad 3ax^2 \quad 3by^2 y^1 \qquad a \quad 12b \quad 4$ ...(3)

$$a \quad \frac{16}{5}, \ b \quad \frac{3}{5}$$

29. $\frac{f(x) \quad 1}{f(x) \quad 1} \quad \frac{x^4 \quad x^2 \quad 1}{(x^2 \quad x \quad 1)^2} \quad \frac{x^2 \quad x \quad 1}{x^2 \quad x \quad 1}$

$$f(x) \quad \frac{x^2 \quad 1}{x}$$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

1. $f(x) \quad \frac{x \quad 1}{x \quad 1}$

$g(x) \quad \frac{x(x \quad 1)}{x \quad 1} \quad x \quad 2 \quad \frac{2}{x \quad 1}$

$g\,(x) \quad 1 \quad \frac{2}{(x \quad 1)^2} \quad 0$

$x \quad 1 \quad \sqrt{2}, 1 \quad \sqrt{2}$

$g\ (x) \quad \frac{4}{(x \quad 1)^3}$

$g\ (1 \quad \sqrt{2}) \quad 0$

Minimum value of $g(x)$ is $3 \quad 2\sqrt{2}$.

2. $g\,(x) \quad \frac{1}{2} \quad 1 \quad \frac{2}{(x \quad 1)^2} \quad \frac{1}{2} \quad x \quad 3, \quad 1$

Paragraph for Question Nos. 3 to 5

3. $g(1) \quad \int_0^1 f(t)\,dt \quad \int_0^1 (1 \quad t)\,dt \quad t \quad \left.\frac{t^2}{2}\right|_0^1 \quad \frac{1}{2}$

4. For $x \quad [2,3]$

$g(x) \quad \int_0^1 f(t)\,dt \quad \int_1^2 f(t)\,dt \quad \int_2^x f(t)\,dt$

$g(x) \quad \frac{1}{2} \quad \frac{(x \quad 2)^3}{3}$

at $x \quad \frac{5}{2}, g \quad \frac{5}{2} \quad \frac{13}{24}$

$g\,(x) \quad f(x)$

$g \quad \frac{5}{2} \quad \frac{1}{4}$

$y \quad \frac{13}{24} \quad \frac{1}{4} \quad x \quad \frac{5}{2}$

$12y \quad 3x \quad 1$

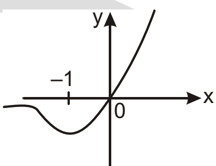
5. Slope of tangent at $P \quad \frac{1}{4}$

Slope of tangent at $R \quad \frac{2}{3}$

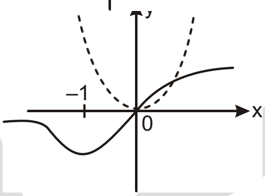
$$\tan \quad \frac{5}{14}$$

Paragraph for Question Nos. 6 to 8

6.



7.



Paragraph for Question Nos. 9 to 11

9. By putting $x \quad 1, 2, \quad 1, 0$ we get a, b, c, d clearly other roots product is 1.

10. $P(x) \quad k \quad 0$ has 4 distinct real roots.

$P(x) \quad k$, where $\quad k \quad (1, 2) \qquad k \quad (\ 2, \quad 1)$

pull the graph more than 1 and less than 2, now the graph intersect the x-axis in $(\ 2, \quad 1), (\ 1, 0), (0, 1), (2, 3)$

$$2 \quad (\ 1) \quad 0 \quad 2 \qquad 1$$

11. $P(x) \quad 0$ has two roots.

$P \ (x) \quad 0$ has three root

$P(x) \quad 0$ has atleast 5 roots.

$(P \ (x))^2 \quad P(x)P \ (x) \quad 0 \qquad (P \ (x)P(x)) \quad 0$ has atleast four roots.

Paragraph for Question Nos. 12 to 14

Sol. The equation of chord AB will be $y \quad f(x_1) \quad \frac{f(x_2) \quad f(x_1)}{x_2 \quad x_1}(x \quad x_1)$

This line passes through $(0, 2x_1x_2)$

$$2x_1x_2 \quad f(x_1) \quad \frac{f(x_2) \quad f(x_1)}{x_2 \quad x_1}(\ x_1)$$

$$2x_1x_2 \quad \frac{(x_2 \quad x_1)f(x_1) \quad x_1f(x_2) \quad x_1f(x_1)}{x_2 \quad x_1}$$

$$2x_1x_2(x_2 \quad x_1) \quad x_2f(x_1) \quad x_1f(x_2)$$

$$\frac{f(x_1)}{x_1} \quad \frac{f(x_2)}{x_2} \quad 2(x_2 \quad x_1) \qquad \frac{f(x_1)}{x_1} \quad 2x_1 \quad \frac{f(x_2)}{x_2} \quad 2x_2 \quad k$$

$$\frac{f(x)}{x} \quad 2x \quad k$$

$$f(x) \quad kx \quad 2x^2$$

Given that $f(1) \quad 1$

$$1 \quad k \quad 2 \qquad k \quad 1$$

$$f(x) \quad x \quad 2x^2$$

12. $\displaystyle {}_0^{1/2} f(x)\,dx \quad \frac{x^2}{2} \quad \frac{2x^3}{3} \Big|_0^{1/2} \quad \frac{1}{8} \quad \frac{2}{3}\,\frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{12} \quad \frac{1}{24}$

13. $f(x) \quad 1 \quad 4x \quad 0 \qquad x \quad \dfrac{1}{4}$

14. $F(x) \quad f(x) \quad x \quad 2x \quad 2x^2$

Clearly, $F(0) \quad F(1) \quad 0$

Rolle's theorem is applicable in $[0,1]$.

Paragraph for Question Nos. 15 to 16

15. $\displaystyle f(x) \quad 1 \quad x \;{}_0^1\, e^y f(y)\,dy \quad e^x \;{}_0^1\, y\,f(y)\,dy$

Let $\displaystyle A \quad {}_0^1\, e^y f(y)\,dy, \qquad B \quad {}_0^1\, y\,f(y)\,dy$

$$f(x) \quad 1 \quad Ax \quad Be^x \qquad A \quad {}_0^1\, e^x(1 \quad Ay \quad Be^y)\,dy$$

$$B \quad \frac{2}{e \quad 1}, A \quad \frac{3}{2}$$

$$f(x) \quad 3 \quad 0$$

$$\frac{3}{2} \quad \frac{2e^x}{(e \quad 1)} \quad 3 \quad 0 \qquad e^x \quad \frac{3(e \quad 1)}{4} \qquad \frac{4}{3}e^x \qquad [e \quad 1] \quad 3$$

16. $Ax_1 \quad Be^{x_1} \qquad \dfrac{3x_1}{2} \quad 2 \qquad Be^{x_1} \qquad 2$

$$f(x_1) \quad A \quad Be^{x_1} \quad m_1$$

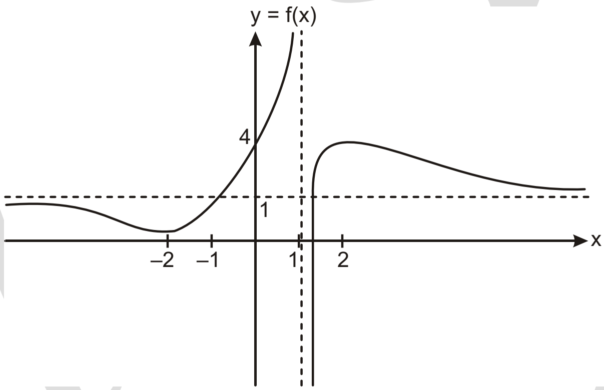
$$m_1 \quad \frac{3}{2} \quad 2 \quad \frac{7}{2}$$

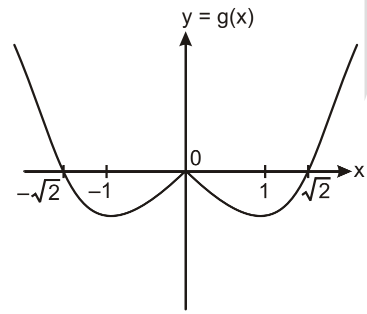
$$m_2 \quad \frac{3}{2}$$

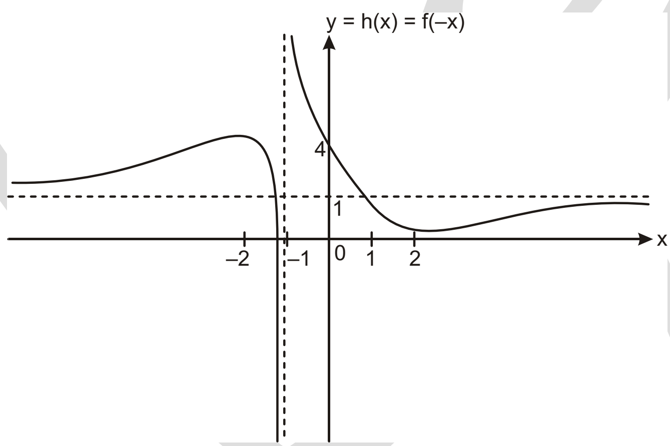
$$\tan \quad \frac{8}{25}$$

Exercise-4 : Matching Type Problems

3.







4. (A) $y \quad \dfrac{x^3}{(x \quad\ )(x \quad\ )(x \quad\ )}$

$$y \quad \frac{8}{(2 \quad\ )(2 \quad\ )(2 \quad\ )}$$

$$x^3 \quad 3x^2 \quad 2x \quad 4 \quad (x \quad\ )(x \quad\ )(x \quad\ )$$

Put $x \quad 2$

$8 \quad 12 \quad 4 \quad 4 \quad (2 \quad)(2 \quad)(2 \quad) \quad 4$

$$y|_{\text{at } x \quad 2} \quad \frac{8}{4} \quad 2$$

(B) $x^3 \quad ax \quad 1 \quad 0, \qquad x^4 \quad ax \quad 1 \quad 0$ ...(1)

$x^4 \quad ax^2 \quad x \quad 0$ ...(2)

(2) – (1) gives $\quad ax^2 \quad ax \quad x \quad 1 \quad 0$

$ax(x \quad 1) \quad (x \quad 1) \quad 0$

$(x \quad 1)(ax \quad 1) \quad 0$

$x \quad 1 \text{ or } x \quad \frac{1}{a}$

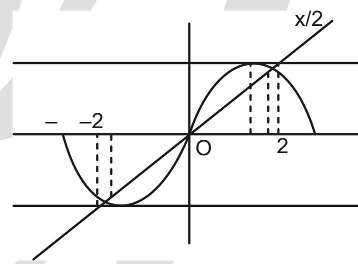
put $x \quad 1$ in (1) we get, $\quad 1 \quad a \quad 1 \quad 0 \qquad a \quad 2$

$|a| \quad 2$

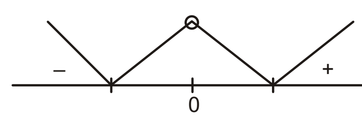
(C) $f(x) \quad x^2 \quad 4\cos x \quad 5$

$f(x) \quad 2x \quad 4\sin x \quad 2(x \quad 2\sin x) \quad 0$

$\sin x \quad \frac{x}{2}$



$f(x)$ — (>0) when x; <0 when $0 \quad x$; >0 when $\quad x \quad 0$; <0 when x



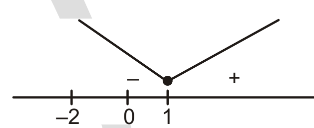
$x \quad 0$ only a point of maxima

So, number of local maxima is 1.

(D) Let $|x| \quad t \quad [0, 2]$

$f(x) \quad 2t^3 \quad 3t^2 \quad 12t \quad 1 \quad g(t)$

$g(t) \quad 6t^2 \quad 6t \quad 12t \quad 6(t^2 \quad t \quad 2) \quad 6(t \quad 2)(t \quad 1)$



$g(1)$ is min. of $f(x)$ *i.e.*, $f_{\min} \quad 2 \quad 3 \quad 12 \quad 1 \quad 6$

$g(0) \quad 1, g(2) \quad 16 \quad 12 \quad 24 \quad 1 \quad 5$

max. of $f(x)$ is $5 \quad 4, 3, 2, 0$

6. $f \quad \frac{1}{8x} \quad a \quad 2x \quad 0$ since $x \quad 0$

$1 \quad 8ax \quad 16x^2 \quad 0$

See $D \quad 0$

$D \quad 0$

$D \quad 0$

7. Let $g(x) \quad ax^3 \quad bx^2 \quad cx \quad d$

$f(x) \quad \sqrt{g(x)}$

$f(x)$ has local maxima and local maxima at $x \quad 2$ and $x \quad 2$.

$g(x)$ has same local minima and maxima and $x \quad 2$ and 2.

$a \quad 0\,; a \quad 2$

$$f\ (x) \quad \frac{3ax^2 \quad 2bx \quad c}{2\sqrt{ax^3 \quad bx^2 \quad cx \quad d}} \quad 0$$

$f\ (\ \ 2) \quad 0$ and $f\ (2) \quad 0$

$b \quad 0, c \quad 24$

Also, $\quad g(2) \quad 0$ and $g(\ \ 2) \quad 0$

$16 \quad 48 \quad d \quad 0$ and $16 \quad 48 \quad d \quad 0$

$d \quad 32$ and $d \quad 32$

$d \quad 32$

8. (A) $V \quad r^2h \quad h \quad R^2 \quad \dfrac{h^2}{4}$ $\qquad \because R^2 \quad r^2 \quad \dfrac{h^2}{4}$

$$\frac{dV}{dh} \quad R^2 \quad \frac{3h^2}{4} \quad 0 \quad h \quad \frac{2R}{\sqrt{3}} \text{ and } r \quad \sqrt{\frac{2}{3}}R$$

(B) $V \quad \dfrac{r^2h}{3} \quad \dfrac{h}{3}\{R^2 \quad (h \quad R)^2\}$ $\qquad \{\because R^2 \quad r^2 \quad (h \quad R)^2\}$

$$\frac{dV}{dh} \quad \frac{}{3}(4hR \quad 3h^2) \quad 0 \quad h \quad \frac{4R}{3} \text{ and } r \quad \frac{2\sqrt{2}R}{3}$$

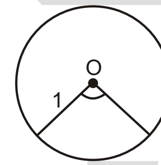
(C) $\sin \quad \dfrac{r}{h \quad r} \quad \dfrac{R}{\sqrt{R^2 \quad h^2}} \quad R^2 \quad \dfrac{h^2r^2}{h^2 \quad 2hr}$

Volume of cone $\quad \dfrac{}{3}R^2h \quad \dfrac{}{3}\ \dfrac{h^2r^2}{h \quad 2r}$

$$\frac{dV}{dh} \quad \frac{}{3}r^2 \quad \frac{(h \quad 2r)\,2h \quad h^2}{(h \quad 2r)^2} \quad 0 \quad h \quad 4r$$

(D) $$\frac{\dfrac{2x}{3} \quad \dfrac{2x}{3} \quad \dfrac{2x}{3} \quad \dfrac{3y}{4} \quad \dfrac{3y}{4} \quad \dfrac{3y}{4} \quad \dfrac{3y}{4}}{7} \quad \left(\frac{8x^3}{27} \quad \frac{81y^4}{256}\right)^{1/7} \quad x^3y^4 \quad \frac{32}{3}$$

Exercise-5 : Subjective Type Problems



1. $\dfrac{A_2}{A_1} \quad \dfrac{r^2}{2 \quad \dfrac{}{2} \quad r^2} \quad \dfrac{2}{2}$

$V \quad \dfrac{}{3} \; \dfrac{}{2} \quad {}^2\sqrt{1^2 \qquad \dfrac{}{2}^{\,2}}$

$\dfrac{dV}{d} \quad 0 \qquad \sqrt{\dfrac{8}{3}}$

$\dfrac{A_2}{A_1} \quad \dfrac{\sqrt{3}}{\sqrt{3} \quad \sqrt{2}} \quad 3 \quad \sqrt{6}$

2. $f(x) \quad x^2 \ln x$

$f\,(x) \quad x(1 \quad 2\ln x)$

and $f\,(x) \quad 0$ for $\quad [1, e]$

$f(x)$ is continuously increasing on $[1, e]$ with the least value zero at $x \quad 1$ and the greatest value e^2 at $x \quad e$.

3. $f(x) \quad px\,e^{\quad x} \quad \dfrac{x^2}{2} \quad x$

$f\,(x) \quad (1 \quad x)[pe^{\quad x} \quad 1] \quad 0$

$p \quad 1$

4. $f\,(x) \quad \begin{matrix} ax\,e^{ax} \quad e^{ax}; & x \quad 0 \\ 1 \quad 2ax \quad 3x^2; & x \quad 0 \end{matrix}$

Clearly, $f\,(x)$ is continuous at $x \quad 0$

$f\,\,(x) \quad \begin{matrix} a^2 x\,e^{ax} \quad 2ae^{ax}; & x \quad 0 \\ 2a \quad 6x; & x \quad 0 \end{matrix}$; $f\,(x)$ increasing if $(ax \quad 2)\,ae^{ax} \quad 0$ and $2a \quad 6x \quad 0$

5. $f(x) \quad x^2 \quad 2bx \quad 1$

Case I : $b \quad 1$

$f(0) \quad f(1) \quad 4$

$1 \quad (2 \quad 2b) \quad 4$

$b \quad \dfrac{5}{2}$

Case II : $0 \quad b \quad \dfrac{1}{2}$

$f(1) \quad f(b) \quad 4$

$b \quad 3, \quad 1$ (Not possible)

Case III : $\frac{1}{2} \quad b \quad 1$

$$f(0) \quad f(b) \quad 4$$

$$b \quad 2 \qquad \text{(Not possible)}$$

Case IV : $b \quad 0$

$$f(1) \quad f(0) \quad 4$$

$$b \quad \frac{3}{2}$$

6. $x^2 \quad 9y^2 \quad 36 \qquad\qquad x^2 \quad y^2 \quad 12$

$12 \quad y^2 \quad 9y^2 \quad 36$

$8y^2 \quad 24 \qquad y^2 \quad 3 \qquad y \quad \sqrt{3}, \quad \sqrt{3}$

when $y \quad \sqrt{3}, \; x^2 \quad 12 \quad 3 \quad 9$

$$x \quad 3$$

point of intersections are $(\quad 3, \quad \sqrt{3})$

Let one of point of intersect is $(3, \sqrt{3})$

Now, $\frac{2x}{36} \quad \frac{2y}{4} y \quad 0 \qquad y \quad \frac{2x}{36} \quad \frac{4}{2y}$

$$(y\ )_{(3,\sqrt{3})} \quad \frac{1}{3\sqrt{3}} \quad (m_1)$$

$$2x \quad 2y\,y \quad 0 \qquad y \quad \frac{x}{y} \qquad (y\ )_{(3,\sqrt{3})} \quad \frac{3}{\sqrt{3}} \quad \sqrt{3} \quad m_2$$

$$\tan \quad \left|\frac{m_1 \quad m_2}{1 \quad m_2 m_1}\right| \quad \left|\frac{\frac{1}{3\sqrt{3}} \quad \sqrt{3}}{1 \quad \frac{1}{3}}\right| \quad \left|\frac{8}{4\sqrt{3}}\right| \quad \left|\frac{2}{\sqrt{3}}\right|$$

$$\tan \quad \frac{2}{\sqrt{3}} \qquad\qquad \tan^{\ 1} \frac{2}{\sqrt{3}}$$

7. $f(x) \quad 2e^{2x} \quad (\quad 1)\,e^{x} \quad 2 \quad 0 \quad x \quad R$

$i.e., 2e^{2x} \quad 2 \quad (\quad 1)\,e^{x}$

$1 \quad 2(e^{x} \quad e^{\ x})$

$\frac{1}{2} \quad (e^{x} \quad e^{\ x}) \quad x \quad R$

$\frac{1}{2} \quad (e^{x} \quad e^{\ x})_{\min} \quad x \quad R$

So, $\frac{1}{2} \quad 2$

$1 \quad 4$

$(\quad ,3]$

$k \quad 3$

9. $f(x) \quad x^2,$ $\qquad g(x) \quad \frac{8}{x}$

$q \quad p^2$ $\qquad s \quad \frac{8}{r}$

Also, $\frac{s \quad q}{r \quad p} \quad 2p \quad \& \quad 2p \quad \frac{8}{r^2}$

$pr^2 \quad 4$ ...(1)

$\frac{s \quad q}{r \quad p} \quad 2p$

$\frac{\frac{8}{r} \quad p^2}{r \quad p} \quad 2p \qquad \frac{8}{r} \quad p^2 \quad 2pr \quad 2p^2$

$p^2 \quad \frac{8}{r} \quad 2pr$

$p^2 r \quad 16$ ...(2)

Multiply (1) & (2),

$pr \quad 4$

$r \quad 1, \quad p \quad 4$

10. $f(x) \quad \begin{matrix} |x \quad 2|, & x \quad 0 \\ |x \quad 2|, & x \quad 0 \end{matrix}$

Minimum value of $f(x)$ is 2.

11. $f(x) \quad {}_0^x [(a \quad 1)(t^2 \quad t \quad 1)^2 \quad (a \quad 1)(t^4 \quad t^2 \quad 1)]\, dt$

$2 \quad {}_0^x (t^2 \quad t \quad 1)(at \quad t^2 \quad 1)\, dt$

$f\,(x) \quad 2(x^2 \quad x \quad 1)(ax \quad x^2 \quad 1) \quad 0$

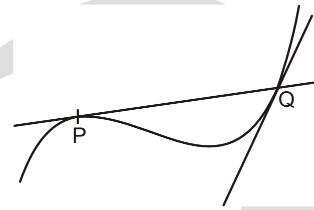
$D \quad 0 \qquad a^2 \quad 4 \quad 0$

12. $f(x) \quad x^{2013} \qquad e^{2014x}$

$f\,(x) \quad 2013\,x^{2012} \qquad 2014\,e^{2014x} \qquad 0$

$f(x)$ is increasing function.

14. $P \quad (x_1, x_1^3 \quad ax_1)$



$Q \quad (x_2, x_2^3 \quad ax_2)$

$y \quad x^3 \quad ax$

$\frac{dy}{dx} \quad 3x^2 \quad a$

Slope at P slope of PQ

$$(3x_1^2 \quad a) \quad \frac{x_2^3 \quad ax_2 \quad x_1^3 \quad ax_1}{x_2 \quad x_1} \qquad (\because x_1 \quad x_2)$$

$$(x_2 \quad x_1)(x_2 \quad 2x_1) \quad 0$$

$$x_2 \quad 2x_1 \qquad \ldots(1)$$

Slope at P Slope at Q 1

$$(3x_1^2 \quad a)(3x_2^2 \quad a) \quad 1 \qquad \ldots(2)$$

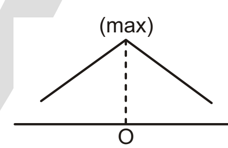
Put (1) in (2),

$$36x_1^4 \quad 15ax_1^2 \quad (a^2 \quad 1) \quad 0$$

$$D \quad 0 \qquad (\because x_1 \quad R)$$

$$9a^2 \quad 16 \qquad a \quad \frac{4}{3}$$

15. $I(t) \quad (x^2 \quad 2x \quad t^2)\,dx \quad \frac{x^3}{3} \quad x^2 \quad t^2x|$



$I(t) \quad \frac{\;^3 \qquad ^3}{3} \quad (\;^2 \qquad ^2) \quad t^2(\qquad)$

$I\,(t) \quad 2t(\qquad) \quad 0$

$I(t) \quad I(0)$

$$I(0) \quad {}_{2}^{0}\,(x^2 \quad 2x)\,dx \quad \frac{4}{3} \qquad \frac{p}{q} \quad |I(0)| \quad \frac{4}{3}$$

16. $\frac{dy}{dt} \quad 1 \quad \frac{3y}{100 \quad 2t}$ when $t \quad 0, y \quad 0$

$\frac{dy}{dt} \quad \frac{3}{100 \quad 2t} \quad y \quad 1$

$y(100 \quad 2t)^{3/2} \qquad (100 \quad 2t)^{1/2} \qquad c$

as when $t \quad 0, y \quad 0$

$c \quad \frac{1}{10}$

$y \quad (100 \quad 2t) \quad \frac{1}{10}(100 \quad 2t)^{3/2}$

$$\frac{dy}{dt} \quad 2 \quad \frac{1}{10} \ \frac{3}{2} \ (100 \quad 2t)^{1/2} (\ 2) \quad 0$$

$$t \quad \frac{250}{9} \quad 27 \ \frac{7}{9}$$

17. Let $f\ (x) \quad K$

$f(x) \quad Kx \quad c$

$f(9) \quad f(\ 3) \quad 12\,K$

Maximum value of $f(9) \quad f(\ 3) \quad 96$

19. Equation of normal at $P \ \frac{3}{4} y_1^3, y_1$ is $y \quad y_1 \quad \frac{9 y_1^2}{4} \quad x \quad \frac{3}{4} y_1^3$

If it passes from (0, 1) then $27 y_1^5 \quad 16 y_1 \quad 16 \quad 0$ has only one real root.

20. $e^{\ x} \ \frac{x^2}{2} \quad x \quad 1 \quad a$

Let $f(x) \quad e^{\ x} \ \frac{x^2}{2} \quad x \quad 1$

$f\ (x) \quad e^{\ x} \quad \frac{x^2}{2} \quad 0$

21. $f\ (x) \quad a \quad 2 \sin 2x \quad \cos x \quad \sin x$

Let $g(x) \quad 2 \sin 2x \quad \cos x \quad \sin x$

$2\,\{(\cos x \quad \sin x)^2 \quad 1\} \quad \cos x \quad \sin x$

where $\cos x \quad \sin x \quad t$

$2t^2 \quad t \quad 2 \quad t \quad [\ \sqrt{2}, \sqrt{2}]$

$2 \quad \sqrt{2} \quad g(x) \quad \frac{17}{8} \quad a \quad \frac{17}{8}$

22. Let $x \quad 6 \cos^3 \quad , \quad y \quad 6 \sin^3$

$$\frac{dy}{dx} \quad \frac{6\,(3 \sin^2 \quad \cos \ )}{6\,(3 \cos^2 \quad \sin \ )} \quad \tan$$

Equ. of tangent

$y \quad 6 \sin^3 \quad \tan \ (x \quad 6 \cos^3 \ ) \quad p_1 \quad 6 \sin \quad \cos$

Equ. of normal

$y \quad 6 \sin^3 \quad \cot \ (x \quad 6 \cos^3 \ ) \quad p_2 \quad 6\,(\cos^2 \quad \sin^2 \ )$

$\sqrt{4 p_1^2 \quad p_2^2} \quad 6 \sqrt{4 \sin^2 \quad \cos^2 \quad \cos^4 \quad \sin^4 \quad 2 \sin^2 \quad \cos^2} \quad 6$

❑❑❑

5

Indefinite and Definite Integration

Exercise-1 : Single Choice Problems

1. $a^x \ln x \quad \underbrace{a^x \ln a}_{\text{II}} \; \underbrace{x(\ln x \quad 1)}_{\text{I}} \quad dx$

$$a^x \quad \ln x\, dx \qquad x(\ln x \quad 1)\, a^x \qquad x \;\frac{1}{x} \quad (\ln x \quad 1) \;\; a^x \quad dx$$

$$a^x \quad \ln x\, dx \quad [x(\ln x/e)]\, a^x \qquad (\ln x)\, a^x dx$$

2. $\lim\limits_{n} \dfrac{1}{n} \displaystyle\sum_{r\;1}^{n} \dfrac{1}{\sqrt{1 \quad \dfrac{r}{n}}} \qquad \int_0^1 \dfrac{dx}{\sqrt{x} \quad 1} \qquad 2(\sqrt{2} \quad 1)$

3. $\dfrac{\sin x}{\sin(x \qquad)}\, dx$

Let $x \qquad t \qquad dx \quad dt$

$\dfrac{\sin(t \qquad)}{\sin t}\, dt \quad t\cos \qquad \sin \quad \log \sin t \quad C \quad x\cos \qquad \sin \quad \log \sin(x \qquad) \quad C$

4. $\displaystyle\int_0^2 \frac{\log(x^2 \quad 2)}{(x \quad 2)^2}\, dx \qquad \frac{\log(x^2 \quad 2)}{x \quad 2}\Bigg|_0^2 \quad \int_0^2 \frac{2x\, dx}{(x \quad 2)(x^2 \quad 2)}$

$$\frac{\sqrt{2}}{3} \tan^{\;1} \sqrt{2} \qquad \frac{5}{12} \log 2 \qquad \frac{1}{12} \log 3$$

5. For $0 \quad x \quad 1$

$1 \quad x^9 \quad 1 \quad x^8 \quad 1 \quad x^4 \quad 1 \quad x^3$

6. Let $g(x) \qquad \displaystyle\int_0^x \sqrt{1 \quad (f(s))^2}\, ds$

$\lim\limits_{t \quad x} \dfrac{g(t) \quad g(x)}{f(t) \quad f(x)} \quad f(x) \qquad g(x) \quad f(x) f(x) \quad \sqrt{1 \quad (f(x))^2}$

$\dfrac{y\,dy}{\sqrt{1 \quad y^2}} \quad dx \qquad \sqrt{1 \quad y^2} \quad 1 \quad x \qquad \because f \; \dfrac{1}{2} \quad \dfrac{\sqrt{3}}{2}$

7. $\lim\limits_{n} \dfrac{1}{n} \sum\limits_{r \; 1}^{n} \sqrt{f \; \dfrac{r}{n}} \quad {}_{0}^{1} \sqrt{f(x)}\,dx \quad \dfrac{2}{\sqrt{3}} \, {}_{0}^{1} \sqrt{1 \quad \cos^6 x \quad \sin^6 x}\,dx$

${}_{0}^{1} \sin 2x\,dx \quad \dfrac{1 \quad \cos 2}{2}$

8. ${}_{0}^{1} \dfrac{(x^6 \quad x^3)}{(2x^3 \quad 1)^3}\,dx \quad \dfrac{1}{2} \, {}_{0}^{1} \dfrac{2 \quad 1 \quad \dfrac{1}{x^3}}{2x \quad \dfrac{1}{x^2}}^{3}\,dx \quad \dfrac{1}{36} \qquad$ Put $2x \quad \dfrac{1}{x^2} \quad t$

9. $2 \, {}_{0}^{1/\sqrt{2}} \dfrac{\sin^{\;1} x}{x}\,dx \quad {}_{0}^{1} \dfrac{\tan^{\;1} x}{x}\,dx$

$2 \, {}_{0}^{/4} \dfrac{\cos}{\sin}\,d \quad {}_{0}^{/4} \dfrac{\sec^2}{\tan}\,d \quad \dfrac{}{4}\ln 2 \quad 2 \, {}_{0}^{/4} \ln\sin \; d \quad {}_{0}^{/4} \ln\tan \; d$

${}_{0}^{/4} \ln\sin 2 \; d \quad \dfrac{}{4}\ln 2$

10. $f(x) \quad x^2 \quad {}_{0}^{x} e^{\;t} f(x \quad t)\,dt \quad x^2 \quad e^{\;x} \, {}_{x}^{0} e^{u} f(u)\,du \qquad$ (Let $x \quad t \quad u$)

$f(x) \quad 2x \quad e^{\;x} \, {}_{x}^{0} e^{u} f(u)\,du \quad f(x) \qquad f(x) \quad x^2 \quad 2x$

$f(x) \quad \dfrac{x^3}{3} \quad x^2$

11. $f(x) \quad f(x) \quad k_1 \qquad k_1 \, {}_{0}^{1} f(x)\,dx$

$y \quad ke^x \quad k_1$

if $f(0) \quad 1 \qquad k \quad k_1 \quad 1$

$k_1 \, {}_{0}^{1} (ke^x \quad k_1)\,dx \qquad 2k_1 \quad k(e \quad 1) \qquad k \; \dfrac{2}{3 \quad e}$ and $k_1 \; \dfrac{e \quad 1}{3 \quad e}$

12. $I_1 \quad {}_{\sin^2 x}^{1 \quad \cos^2 x} \quad tf(t(2 \quad t))\,dt \quad 2 \quad {}_{\sin^2 x}^{1 \quad \cos^2 x} \quad f(t(2 \quad t))\,dt \quad {}_{\sin^2 x}^{1 \quad \cos^2 x} \quad tf(t(2 \quad t))\,dt$

$I_1 \quad 2I_2 \quad I_1 \qquad I_1 \quad I_2$

13. $\dfrac{5\sin x\,dx}{\sin x \quad 2\cos x} \qquad dx \quad 2 \quad \dfrac{\cos x \quad 2\sin x}{\sin x \quad 2\cos x}\,dx \quad x \quad 2\ln|\sin x \quad 2\cos x| \quad C$

14. $\dfrac{(2 \quad \sqrt{x})\,dx}{(x \quad 1 \quad \sqrt{x})^2} \qquad \dfrac{\dfrac{2}{x^2} \quad \dfrac{1}{x^{3/2}}}{\dfrac{1}{x} \quad \dfrac{1}{\sqrt{x}} \quad 1^{\;2}}\,dx$

Let $\dfrac{1}{x} \quad \dfrac{1}{\sqrt{x}} \quad 1 \quad t \qquad \dfrac{1}{x^2} \quad \dfrac{1}{2x^{3/2}} \quad dx \quad dt$

15. $\dfrac{\sqrt[3]{x \quad \sqrt{2 \quad x^2}} \quad \sqrt[6]{1 \quad x\sqrt{2 \quad x^2}} \quad dx}{\sqrt[3]{1 \quad x^2}} \qquad \dfrac{\sqrt[3]{x \quad \sqrt{(2 \quad x^2)}}\sqrt[3]{\sqrt{\dfrac{\sqrt{2 \quad x^2} \quad x}{2}}}}{\sqrt[3]{1 \quad x^2}}\,dx$

$2^{1/6} \quad dx \quad 2^{1/6}x \quad C$

16. $\dfrac{dx}{\sqrt{1 \quad \tan^2 x}}$

$\dfrac{\cos x\,dx}{\sqrt{1 \quad 2\sin^2 x}} \qquad \dfrac{1}{\sqrt{2}}\sin^{\;1}(\sqrt{2}\sin x) \quad C$

17. $I \qquad \dfrac{dx}{x^{5/6} \quad (x \quad 1)^{7/6}}$

$\dfrac{x}{x \quad 1} \quad t \qquad \dfrac{dx}{(x \quad 1)^2} \quad dt$

$I \qquad \dfrac{(x \quad 1)^2\,dt}{[t(x \quad 1)]^{5/6}(x \quad 1)^{7/6}} \qquad t^{\;5/6}dt \quad 6t^{1/6} \quad C$

18. $I_n \qquad \sin^n x\,dx \qquad \sin^{n \;\; 2} x(1 \quad \cos^2 x)\,dx$

$I_n \quad I_{n \;\; 2} \qquad \underbrace{\sin^{n \;\; 2} x \;\; \cos x}_{\mathrm{II}} \;\; \underbrace{\cos x}_{\mathrm{I}}\,dx$

$I_n \quad I_{n \;\; 2} \qquad \dfrac{\cos x \;\; \sin^{n \;\; 1} x}{n \quad 1} \qquad \sin x \;\; \dfrac{\sin^{n \;\; 1} x}{n \quad 1}\,dx$

$I_n \quad I_{n \;\; 2} \qquad \dfrac{\cos x \;\; \sin^{n \;\; 1} x}{n \quad 1} \qquad \dfrac{1}{n \quad 1} I_n$

$nI_n \quad (n \quad 1)I_{n \;\; 2} \qquad \cos x \;\; \sin^{n \;\; 1} x$

19. $x^2 \frac{1}{(a \quad bx)^2} dx$ Let $a \quad bx \quad t$ then $dx \quad \frac{dt}{b}$

$$x^2 \frac{1}{(a \quad bx)^2} dx \qquad \frac{t \quad a}{b}^2 \quad \frac{1}{t^2} \quad \frac{dt}{b} \quad \frac{1}{b^3} \quad \frac{t^2 \quad 2at \quad a^2}{t^2} \quad dt$$

$$\frac{1}{b^3} \quad 1 \quad \frac{2a}{t} \quad \frac{a^2}{t^2} \quad dt \quad \frac{1}{b^3} \quad t \quad 2a \ln|t| \quad \frac{a^2}{t} \qquad C$$

$$\frac{1}{b^3} \quad a \quad bx \quad 2a \ln|a \quad bx| \quad \frac{a^2}{a \quad bx} \qquad C$$

20. $\frac{8x^{43} \quad 13x^{38}}{(x^{13} \quad x^5 \quad 1)^4} dx$

$$\frac{8x^{\ \ 9} \quad 13x^{\ \ 14}}{(1 \quad x^{\ \ 8} \quad x^{\ \ 13})^4} dx$$

Let $1 \quad x^{\ \ 8} \quad x^{\ \ 13} \quad t$

$$(\ 8x^{\ \ 9} \quad 13x^{\ \ 14}) \, dx \quad dt$$

$$\frac{dt}{t^4} \quad \frac{1}{3t^3} \quad C \quad \frac{1}{3(1 \quad x^{\ \ 8} \quad x^{\ \ 13})^3} \quad C \quad \frac{x^{39}}{3(x^{13} \quad x^5 \quad 1)^{13}} \quad C$$

21. $\frac{(\cos 6x \quad \cos 4x) \quad 5(\cos 4x \quad \cos 2x) \quad 10(\cos 2x \quad 1)}{10\cos^2 x \quad 5\cos x \cos 3x \quad \cos x \cos 5x} dx$

$$\frac{2\cos 5x \cos x \quad 5(2\cos 3x \cos x) \quad 10(2\cos^2 x)}{10\cos^2 x \quad 5\cos x \cos 3x \quad \cos x \cos 5x} dx$$

$$2\,dx \quad 2x \quad C$$

$f(x) \quad 2x \qquad f(10) \quad 20$

22. $(1 \quad x \quad x^{\ \ 1}) e^{x \ \ x^{\ 1}} dx \qquad e^{x \ \ x^{\ 1}} \ 1\,dx \qquad (x \quad x^{\ \ 1}) e^{x \ \ x^{\ 1}} dx$

$$e^{x \ \ x^{\ 1}} \quad x \qquad e^{x \ \ x^{\ 1}} \quad 1 \quad \frac{1}{x^2} \quad x\,dx \qquad (x \quad x^{\ \ 1}) e^{x \ \ x^{\ 1}} dx \quad C$$

$$xe^{x \ \ x^{\ 1}} \qquad C$$

23. $e^x \frac{2\tan x}{1 \quad \tan x} \quad \frac{1}{\frac{1 \quad \cos(\ \ /2 \quad 2x)}{2}} \quad dx \qquad e^x \frac{2\tan x}{1 \quad \tan x} \quad \frac{2}{(1 \quad \sin 2x)} \quad dx$

$$2 \quad e^x \frac{\sin x}{\sin x \quad \cos x} \quad \frac{1}{(\sin x \quad \cos x)^2} \quad dt$$

Let $f(x) \quad \frac{\sin x}{\sin x \quad \cos x}, \quad f\ (x) \quad \frac{1}{(\sin x \quad \cos x)^2}$

$$2 \quad e^x \quad \frac{\sin x}{\sin x \quad \cos x} \qquad C$$

$$g \ \frac{5}{4} \qquad 1$$

24. $\frac{d}{dx}(x \sin x \quad \cos x) \quad x \cos x \qquad f\ (x) \quad x \cos x$

Let $f(x) \quad x \sin x \quad \cos x \qquad f\ (x) \quad x \sin x \quad \cos x$

$$e^{f(x)} \quad xf\ (x) \quad \frac{f\ (x)}{(f\ (x)^2)} \quad dx \qquad xe^{f(x)} f\ (x)\, dx \qquad e^{f(x)} \quad \frac{f\ (x)}{(f\ (x)^2)}\, dx$$

$$xe^{f(x)} \qquad e^{f(x)} dx \quad e^{f(x)} \quad \frac{1}{f\ (x)} \qquad e^{f(x)} \quad f\ (x) \quad \frac{1}{f\ (x)} \quad dx$$

$$xe^{f(x)} \quad \frac{e^{f(x)}}{f\ (x)} \quad C \quad e^{f(x)} \quad x \quad \frac{1}{f\ (x)} \qquad C$$

$$e^{x \sin x \quad \cos x} \quad x \quad \frac{1}{x \cos x} \qquad C$$

25. $\int_0^1 \sqrt{x} \quad \frac{1}{\sqrt{x} \quad \sqrt{1 \quad x}} \quad dx$

$$\int_0^1 (\sqrt{x} \quad (\sqrt{1 \quad x} \quad \sqrt{x})\, dx \qquad \int_0^1 (\sqrt{1 \quad x}\, dx \quad \frac{2}{3}(1 \quad x)^{3/2}\Big|_0^1 \quad \frac{2}{3}(2^{3/2} \quad 1)$$

26. $x^{x^2} x(2 \ln x \quad 1)\, dx$

$x^{x^2} \quad t$

$x^2 \ln x \quad \ln t$

$$x^2 \ \frac{1}{x} \quad (\ln x)\, 2x \quad dx \quad \frac{1}{t}\, dt$$

$$t \ \frac{dt}{t} \qquad dt \quad t \quad C \quad x^{x^2} \quad C \quad (x^x)^x \quad C$$

27. $\sec^{2010} x \operatorname{cosec}^2 x\, dx \qquad 2010 \sec^{2010} x\, dx$

$$\sec^{2010} x(\quad \cot x) \qquad 2010 \sec^{2010} x \quad \tan x \ (\quad \cot x) \qquad 2010 \sec^{2010} x\, dx$$

$$\frac{\cot x}{(\cos x)^{2010}} \quad 2010 \quad \sec^{2010} x\, dx \quad 2010 \quad \sec^{2010} x\, dx \quad C \quad \frac{\cot x}{(\cos x)^{2010}} \quad C$$

$\frac{f(x)}{g(x)} \quad \frac{1}{\sin x} \quad \{x\}$ no solution.

28. Let $x^x \ln x \quad t$

$$x^x \ln x(1 \quad \ln x) \quad \frac{x^x}{x} \quad dx \quad dt$$

$$x^x \quad \ln x \quad (\ln x)^2 \quad \frac{1}{x} \quad dx \quad dt$$

$$dt \quad t \quad C \quad x^x \ln x \quad C$$

29. $$I \quad \frac{\frac{1}{x^3} \quad \frac{1}{x^5} \quad dx}{\sqrt{2 \quad \frac{2}{x^2} \quad \frac{1}{x^4}}}$$

Let $2 \quad \frac{2}{x^2} \quad \frac{1}{x^4} \quad t \qquad I \quad \frac{1}{4} \quad \frac{dt}{\sqrt{t}}$

30. Put $\ln x \quad t$

$$I \quad e^t \quad \frac{t \quad 1}{t^2 \quad 1}^2 \quad dt \qquad e^t \quad \frac{1}{t^2 \quad 1} \quad \frac{2t}{(t^2 \quad 1)^2} \quad dt$$

31. $$I \quad \frac{dx}{(x \quad 1)^{3/4}(x \quad 2)^{5/4}} \qquad \frac{dx}{\frac{x \quad 1}{x \quad 2}^{3/4} (x \quad 2)^2}$$

Let $\frac{x \quad 1}{x \quad 2} \quad t \qquad dt \quad \frac{3\,dx}{(x \quad 2)^2}$

32. $$\frac{2 \quad (1 \quad x^7)}{x(1 \quad x^7)} dx \qquad \frac{dx}{x} \quad \frac{2}{7} \quad \frac{7x^6}{x^7(1 \quad x^7)} dx \qquad \ln|x| \quad \frac{2}{7}\ln|1 \quad x^7| \quad C$$

33. $$I \quad \frac{(\sin^4 x \quad \cos^4 x)(\sin^4 x \quad \cos^4 x)}{1 \quad 2\sin^2 x \cos^2 x} dx \qquad (\sin^2 x \quad \cos^2 x)\, dx \qquad \cos 2x\, dx$$

34. $$I \quad 2^{1/3} \quad \frac{(\tan x)^{1/3}\, d((\tan x)^{1/3})}{(\tan x)^{2/3} \quad 1}$$

Let $(\tan x)^{1/3} \quad t \qquad d((\tan x)^{1/3}) \quad dt$

$$I \quad \frac{2^{1/3}}{2} \quad \frac{2t}{t^2 \quad 1} dt$$

35. $$\frac{(2012)^x}{\sqrt{1 \quad (2012)^{2x}}} \quad (2012)^{\sin^{\ 1}(2012)^x} \quad dx$$

Let $\sin^{\ 1}(2012)^x \quad t \qquad \frac{1}{\ln 2012} \quad (2012)^t\, dt \quad \frac{(2012)^{\sin^{\ 1}(2012)^x}}{\ln^2(2012)} \quad C$

36. Let $x \quad 1 \quad t^2 \qquad dx \quad 2t\,dt$

$$2 \quad \frac{(t^2 \quad 1)\,dt}{t^4 \quad t^2 \quad 1} \quad 2 \quad \frac{1 \quad \frac{1}{t^2} \quad dt}{t \quad \frac{1}{t} \quad ^2 \quad 3}$$

37. $\dfrac{f(x)g\ (x) \quad f\ (x)g(x)}{f(x)g(x)} \ \ln \ \dfrac{g(x)}{f(x)} \ dx$

Let $\dfrac{g(x)}{f(x)} \quad t \qquad \dfrac{f(x)g\ (x) \quad g(x)f\ (x)}{(f(x))^2}dx \quad dt; \qquad \dfrac{\ln t}{t}dt \quad \dfrac{(\ln t)^2}{2} \quad C$

38. $e^x \ \ln x \ \ \dfrac{2}{x} \ \ \dfrac{1}{x^2} \ \ dx \ \ dx$

$$e^x \ \ln x \ \ \frac{1}{x} \ dx \qquad e^x \ \frac{1}{x} \ \ \frac{1}{x^2} \ \ dx \ \ dx \qquad e^x \ \ln x \ \ \frac{1}{x} \quad C_1 \ \ dx \quad e^x \ln x \quad C_1 x \quad C_2$$

39. $f(x) \qquad 2 \left| \dfrac{t\cos(x \qquad t)}{} \right|_0^1 \quad \int_0^1 \dfrac{1 \ \cos(x \qquad t)}{} dt \qquad \cos x \quad 2\sin x$

40. $\dfrac{2}{x} \quad \sqrt{5} \qquad x \qquad \dfrac{2}{\sqrt{5}}, 1$

$$\int_0^1 f(x)\,dx \qquad \int_0^{2/\sqrt{5}} f(x)\,dx \qquad \int_{2/\sqrt{5}}^1 f(x)\,dx \quad \sqrt{5} \ \ \frac{2}{\sqrt{5}} \quad 0 \qquad \int_{2/\sqrt{5}}^1 \frac{2}{x}dx$$

$$\int_0^1 f(x)\,dx \quad 2 \quad 2\,[\ln x]_{2/\sqrt{5}}^1$$

$$a \quad 2 \quad 2\ln \ \frac{\sqrt{5}}{2}$$

42. $f(0) \quad 0, f(2\ \ ) \quad 2$

$$\int_0^2 f(x)\,dx \qquad \int_0^2 f^{\ 1}(x)\,dx \qquad \int_0^2 2 \ \ dx \quad 4^{\ 2}$$

$$\frac{x^2}{2} \quad \cos x \ \Big|_0^2 \qquad I \quad 4^{\ 2} \qquad I \qquad \int_0^2 f^{\ 1}(x)\,dx \quad 2^{\ 2}$$

43. $2 \ \ 2 \int_0^1 e^{\ x^4} dx \qquad \int_0^1 8x^4 e^{\ x^4} dx \qquad 2 \ \ 2 \ \ (xe^{\ x^4})_0^1 \qquad \int_0^1 4x^4 e^{\ x^4} dx \qquad \int_0^1 8x^4 e^{\ x^4} dx$

$$\frac{4}{e}$$

46. Put $y \quad 2 \quad z$

$$I \quad {}_{2}^{2} \frac{z^2 \quad 1}{2z^2 \quad 3}\sin(z)\,dz \quad 0$$

47. $${}_1^4 \frac{3}{x} e^{\sin x^3} dx$$

Let $x^3 \quad t \qquad 3x^2 dx \quad dt$

$${}_1^{64} \frac{e^{\sin t}}{t} dt \quad F(64) \quad F(1)$$

51. $$\lim_{x} x \; {}_0^x e^{t^2 \; x^2} dt \quad \lim_{x} \frac{x \quad {}_0^x e^{t^2} dt}{e^{x^2}}$$

Apply L' Hospital's rule,

$$\lim_{x} \frac{x\;\;(e^{x^2}) \quad {}_0^x e^{t^2} dt \;\; 1}{e^{x^2} \quad 2x} \quad \lim_{x} \frac{1}{2} \; \frac{{}_0^x e^{t^2} dt}{2x\, e^{x^2}} \quad \frac{1}{2}$$

52. $$L \quad \sum_{r\;1}^{n} \frac{2\;\;r\;\;\;n}{r^2\quad n\;\;r\quad n^2} \quad {}_0^1 \frac{(2x\quad 1)\,dx}{x^2\quad x\;\;1} \quad \ln(x^2\quad x\quad 1)\big|_0^1$$

$L \quad \ln 3$

53. Let $\sqrt[3]{x^2 \quad 2x} \quad y \quad f(x)$

$x \quad 1 \quad (y^3 \quad 1)^{1/2}$

$$I \quad {}_0^2 (f^{\;1}(x) \quad f(x) \quad 1)\,dx$$

Consider ${}_0^2 f^{\;1}(x) \quad {}_0^2 tf\,(t)\,dt$ Let $f^{\;1}(x) \quad t;\, x \quad f(t);\, dx \quad f\,(t)\,dt$

$$tf(t)\big|_0^2 \quad {}_0^2 dx \quad 6$$

54. Put $x \quad 2\tan$ then $I \quad {}_0^{/2} \frac{\ln 2 \quad \ln\tan}{4\sec^2} \; 2\sec^2 \quad d$ then solve it.

55. Put $x \quad 5 \quad t$

$x \quad 0, t \quad 5$

$x \quad 10, t \quad 5$

$$\int_{5}^{5} (t \quad t^2 \quad t^3)\, dt \quad \left.\frac{t^3}{3}\right|_{5}^{5} \quad \frac{250}{3}$$

56. Let $\quad I \quad \int_0 \frac{dx}{(1 \quad x^9)(1 \quad x^2)}$ ...(1)

Put $x \quad \frac{1}{t} \qquad dx \quad \frac{1}{t^2}\, dt$

$$I \quad \int^{0} \frac{\frac{dt}{t^2}}{\frac{t^9 \quad 1}{t^9}\ \frac{1 \quad t^2}{t^2}} \quad \int_0 \frac{t^9\, dt}{(t^9 \quad 1)(1 \quad t^2)}$$ ...(2)

On adding (1) & (2),

$$2I \quad \int_0 \frac{dt}{(1 \quad t^2)} \quad \tan^{1} t \Big|_0$$

$$2I \quad \frac{}{2} \qquad I \quad \frac{}{4}$$

57. $I \quad \int_0^{/2} \frac{1 \quad \sin 3x}{1 \quad 2\sin x}\, dx \quad \int_0^{/2} \frac{1 \quad 3\sin x \quad 4\sin^3 x}{1 \quad 2\sin x}\, dx$

$$\int_0^{/2} \frac{(1 \quad 2\sin x)(\quad 2\sin^2 x \quad \sin x \quad 1)}{(1 \quad 2\sin x)} \quad 2\ \frac{1}{2}\frac{}{2} \quad 1 \quad \frac{}{2} \quad 1$$

58. $\lim_{x} \frac{(\tan^{1} x)^2}{\frac{1}{2\sqrt{x^2 \quad 1}}\, 2x}$

$$\lim_{x} (\tan^{1} x)^2 \frac{\sqrt{1 \quad x^2}}{x} \quad \frac{^2}{4}$$

59. Let $t \quad \prod_{r \quad 1}^{2013} (x^2 \quad r^2)$

$$\ln t \quad \sum_{r \quad 1}^{2013} \ln(x^2 \quad r^2)$$

$$\frac{1}{t}\, dt \quad \sum_{r \quad 1}^{2013} \frac{2x}{x^2 \quad r^2}\, dx \qquad dt \quad 2 \sum_{r \quad 1}^{2013} \frac{x}{x^2 \quad r^2} \quad t\, dx$$

$$\frac{dt}{2} \quad \frac{t}{2} \quad \frac{1}{2} \sum_{r \ 1}^{2013} (x^2 \quad r^2) \Big|_0^1 \quad \frac{1}{2} \sum_{r \ 1}^{2013} (1 \quad r^2) \quad \sum_{r \ 1}^{2013} r^2$$

60. $f(x) \quad 2 \quad \frac{1}{1 \quad x^2} \quad \frac{1}{x \quad \sqrt{1 \quad x^2}} \quad 1 \quad \frac{2x}{2\sqrt{1 \quad x^2}} \quad 2 \quad \frac{1}{1 \quad x^2} \quad \frac{1}{\sqrt{1 \quad x^2}}$

$$\frac{2(1 \quad x^2) \quad 1 \quad \sqrt{1 \quad x^2}}{1 \quad x^2} \quad \frac{(1 \quad x^2) \quad \sqrt{1 \quad x^2} \quad x^2}{1 \quad x^2} \quad 0 \quad x \quad R$$

61. $I \quad \int_0^{/2} \frac{\sin x \cos x \, dx}{1 \quad \sin x \quad \cos x} \quad \int_0^{/2} \sin \frac{x}{2} \quad \cos \frac{x}{2} \quad \sin \frac{x}{2} \quad dx$

$$\frac{1}{2} \int_0^{/2} [\sin x \quad \cos x \quad 1] \, dx \quad 1 \quad \frac{}{4}$$

62. $I \quad \int_3^7 \frac{\cos x^2 dx}{\cos x^2 \quad \cos(10 \quad x)^2}$

$I \quad \int_3^7 \frac{\cos(10 \quad x)^2 dx}{\cos(10 \quad x)^2 \quad \cos x^2}$

$2I \quad 4 \qquad I \quad 2$

63. $\int_{e \ 1}^{1} \frac{\ln x}{x} dx \quad \int_1^{e^2} \frac{\ln x}{x} dx \quad \frac{(\ln x)^2}{2} \Big|_{e \ 1}^{1} \quad \frac{(\ln x)^2}{2} \Big|_1^{e^2} \quad 2 \quad \frac{1}{2} \quad \frac{5}{2}$

64. $\lim_{x \quad \frac{}{4}} \frac{\int_2^{\mathrm{cosec}^2 x} t g(t) \, dt}{x^2 \quad \frac{2}{16}} \quad \lim_{x \quad \frac{}{4}} \frac{\mathrm{cosec}^2 x \, g(\mathrm{cosec}^2 x) \, 2\mathrm{cosec} \, x(\quad \mathrm{cosec} \, x) \cot x}{2x} \quad \frac{16}{} g(2)$

65. $\lim_{n} \sum_{k \ 1}^{n} \frac{n \quad k}{n^2} \cos \frac{4k}{n}$

$$\lim_{n} \sum_{k \ 1}^{n} \frac{1 \quad \frac{k}{n}}{n} \cos \frac{4k}{n} \quad \int_0^1 (1 \quad x) \cos 4x \, dx$$

$$(1 \quad x) \frac{\sin 4x}{4} \Big|_0^1 \quad \int_0^1 \frac{\sin 4x}{4} dx \quad \frac{1}{16} \cos 4x \Big|_0^1$$

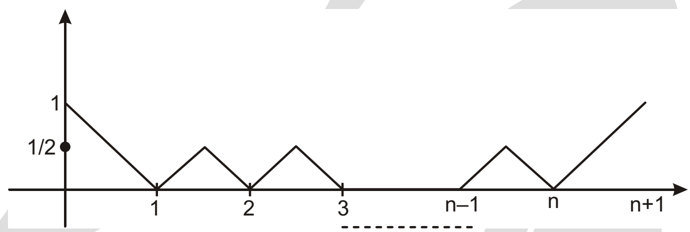
$$\frac{1}{16} (\cos 4 \quad 1) \quad \frac{1}{16} (1 \quad \cos 4)$$

66. $\lim_{n} {}_{0}^{1/n} \sin\frac{}{2n}\,dx \quad {}_{1/n}^{2/n} \sin\frac{2}{2n}\,dx \qquad {}_{1\ 1/n}^{1} \sin\frac{n}{2n}\,dx$

$\lim_{n} \frac{1}{n}\ \sin\frac{}{2n} \quad \sin\frac{2}{2n} \qquad \sin\frac{n}{2n}$

$\lim_{n} \frac{\sin\frac{n}{4n}}{n\sin\frac{}{4n}} \sin\frac{(n\quad 1)}{4n} \quad \frac{2}{}$

67. ${}_{0}^{n\ 1} \min.\{|x\quad 1|,|x\quad 2|,|x\quad 3|, \quad |x\quad n|\}\,dx \quad \frac{1}{2}(1) \quad \frac{1}{2}\ \frac{1}{2} \quad (n\quad 1)\ \frac{1}{2}\quad (1)\quad \frac{n\quad 3}{4}$



68. $S_k \quad \frac{1}{2}k\sin\frac{k}{2n}$

$\lim_{n} \frac{1}{n^2} {}_{k\ 1}^{n} \frac{1}{2}k\sin\frac{k}{2n} \quad {}_{0}^{1} \frac{1}{2}x\sin\ \frac{x}{2}\ \ dx \quad \frac{2}{{}^{2}}$

71. $f(x) \quad {}_{0}^{g(x)} \frac{dt}{\sqrt{1\quad t^3}} \qquad g(x) \quad \sin x\ \ (1\quad \sin(\cos x))^2$

$f(x)\quad g(x)\ \frac{1}{\sqrt{1\quad (g(x))^3}}$

$f\ \ \frac{}{2} \quad g\ \ \frac{}{2} \qquad \sin\frac{}{2} \qquad 1$

72. $x^2 f(x) \quad {}_{4}^{x} (4t^2\quad 2f(t))\,dt$

$x^2 f(x)\quad 2x\,f(x)\quad 4x^2\quad 2f(x)$

$16f(4)\quad 8f(4)\quad 64\quad 2f(4)$

$18f(4)\quad 64$

$9f(4)\quad 32$

73. $\lim_{n} {}_{r\;1}^{2n} \frac{r^2}{n^3 \quad r^3} \quad \lim_{n} \frac{1}{n} {}_{r\;1}^{2n} \frac{\left(\frac{r}{n}\right)^2}{1 \quad \left(\frac{r}{n}\right)^3} \quad {}_0^2 \frac{x^2 dx}{1 \quad x^3} \quad \left|\frac{1}{3}\ln|1 \quad x^3|\right|_0^2 \quad \frac{1}{3}\ln 9$

74. ${}_0^2 \cos^{1}(\cos x)\, dx \quad 2 \; {}_0 \cos^{1}(\cos x)\, dx \quad 2 \left|\frac{x^2}{2}\right|_0 \quad 2$

75. $2f(x) \quad {}_0^x (x^2 \quad 2xt \quad t^2)\, g(t)\, dt$

$2f(x) \quad x^2 {}_0^x g(t)\, dt \quad 2x {}_0^x t \;\; g(t)\, dt \quad {}_0^x t^2 g(t)\, dt$

$2f'(x) \quad x^2 \;\; g(x) \quad {}_0^x g(t)\, dt \;\; 2x \quad 2x(x\, g(x)) \quad {}_0^x t \;\; g(t)\, dt \;\; 2 \quad x^2 g(x)$

$2f''(x) \quad 2x {}_0^x g(t)\, dt \quad 2 {}_0^x tg(t)\, dt$

$f'''(x) \quad x \;\; g(x) \quad {}_0^x g(t)\, dt \quad xg(x)$

$f'''(x) \quad {}_0^x g(t)\, dt$

$f''''(x) \quad g(x)$

76. $I \quad {}_0 \frac{x^3 \cos^4 x \sin^2 x}{{}^2 \quad 3 \;\; x \quad 3x^2} dx \quad {}_0^{/2} \sin^2 x\, dx$

$I \quad {}_0 \frac{(\quad x)^3 \cos^4 x \sin^2 x}{{}^2 \quad 3 \;\; x \quad 3x^2} dx \quad 2I \quad {}_0 \cos^4 x \sin^2 x\, dx$

77. $\frac{1}{2}\left|\tan^{1} \frac{2x}{1 \quad x^2}\right|_0^{\sqrt{3}} \quad \left|\tan^{1} x\right|_0^{\sqrt{3}} \quad \overline{3} \quad 0 \quad \overline{3}$

78. ${}_0^3 \{x\}^{[x]} dx \quad {}_0^3 (x \quad [x])^{[x]} dx \quad {}_0^1 1 \;\; dx \quad {}_1^2 (x \quad 1)\, dx \quad {}_2^3 (x \quad 2)^2 dx$

79. $I \quad {}_0^1 \frac{\tan^{1} x}{x} dx$

$x \quad \tan$

$$I \quad {}_{0}^{/4} \quad \frac{}{\tan} \quad \sec^2 \quad d \qquad {}_{0}^{/4} \frac{2}{\sin 2} d \qquad \frac{1}{2} \quad {}_{0}^{/2} \frac{t}{\sin t} dt$$

80. $${}_{0}^{4/} \quad \underbrace{3x^2}_{\text{II}} \underbrace{\sin\frac{1}{x}}_{\text{I}} dx \qquad {}_{0}^{4/} \quad x\cos\frac{1}{x} dx \qquad \left|\sin\frac{1}{x} \quad x^3\right|_{0}^{4/} \qquad {}_{0}^{4/} \quad \cos\frac{1}{x} \quad \frac{1}{x^2} \quad x^3 dx \qquad {}_{0}^{4/} \quad x\cos\frac{1}{x} dx$$

$$\frac{64}{3} \quad \frac{1}{\sqrt{2}} \qquad \lim_{x \quad 0} \quad x^3 \sin\frac{1}{x} \qquad \frac{32\sqrt{2}}{3}$$

81. $${}_{1}^{x} \quad 8t^2 \quad \frac{28t}{3} \quad 4 \quad dt \qquad \frac{\frac{3x}{2} \quad 1}{\log_{(x \quad 1)} \sqrt{x \quad 1}}$$

$$\left|\frac{8t^3}{3} \quad \frac{14t^2}{3} \quad 4t\right|_{1}^{x} \qquad \frac{\frac{3x}{2} \quad 1}{\frac{1}{2}}$$

$$\frac{8x^3}{3} \quad \frac{14x^2}{3} \quad 4x \qquad \frac{8}{3} \quad \frac{14}{3} \quad 4 \qquad 3x \quad 2$$

$$8x^3 \quad 14x^2 \quad 12x \quad 8 \quad 14 \quad 12 \quad 9x \quad 6$$

$$8x^3 \quad 14x^2 \quad 3x \quad 0$$

$$x(8x^2 \quad 14x \quad 3) \quad 0$$

$$x(2x \quad 3)(4x \quad 1) \quad 0$$

$$x \quad 0, \quad \frac{3}{2}, \quad \frac{1}{4}$$

But $x \quad 1$ & $x \quad 0$

So, $x \quad \frac{1}{4}$

85. $$f(x) \quad {}_{0}^{4} \quad e^{|x \quad t|} dt \qquad {}_{0}^{x} \quad e^{(x \quad t)} dt \qquad {}_{x}^{4} \quad e^{(t \quad x)} dt \qquad e^{x} \quad e^{4 \quad x} \quad 2 \quad 2e^{2} \quad 2$$

86. $$\frac{1}{4} \quad {}_{0} \quad \frac{4\cos^3 x}{x} dx \qquad \frac{1}{4} \quad {}_{0} \quad \frac{\cos 3x \quad 3\cos x}{x} dx \qquad \frac{1}{4} \quad {}_{0} \quad \frac{\cos 3x}{x} dx \qquad \frac{3}{4} \quad {}_{0} \quad \frac{\cos x}{x} dx$$

87. $$\sin\frac{x}{2} \quad \cos\frac{x}{2} \quad \cos\frac{x}{2} \quad \sin\frac{x}{2} \quad dx \qquad \cos^2\frac{x}{2} \quad \sin^2\frac{x}{2} \quad dx \qquad \cos x \, dx$$

88. $I_{n}\ \frac{1}{2}\ \ _{0}\ \frac{\sin(2nx\quad x)}{\sin 2x}dx\quad _{0}\ \frac{\sin 2nx\ \ \cos x}{\sin 2x}dx\quad _{0}\ \frac{\cos 2nx\ \ \sin x}{\sin 2x}dx$

$$\frac{1}{2}\ _{0}\ \frac{\sin 2nx}{\sin x}dx\quad \frac{1}{2}\ _{0}\ \frac{\cos 2nx}{\cos x}dx$$

89. $f(x)\quad 1\quad \ln^2 x\quad 2\ln x\quad 0\qquad x\quad \frac{1}{e}$

$f\ \frac{1}{e}\quad 1\ \frac{1}{e}\ \ _{1}^{1/e}(\ln^2 t\quad 2\ln t)\,dt$

Let $I\ \ _{1}^{1/e}(\ln^2 t\quad 2\ln t)\,dt$

$\ln t\quad x\qquad t\quad e^x;\ dt\quad e^x dx\quad _{0}^{1}(x^2\quad 2x)\,e^x dx\quad [e^x\quad x^2]_0^1\quad \frac{1}{e}$

90. $f(x)\quad x^2\quad _{0}^{x} e^{\ t} f(x\quad t)\,dt$

$x^2\quad _{0}^{x} e^{t\quad x} f(t)\,dt\quad x^2\quad e^{\ x}\ _{0}^{x} e^{t} f(t)\,dt$

$f\,(x)\quad 2x\quad e^{\ x}\ _{0}^{x} e^{t} f(t)\,dt\quad f(x)$

$f\,(x)\quad 2x\quad x^2\qquad f(x)\quad \frac{x^3}{3}\quad x^2$

$y\quad \frac{1}{4}(\ \ 2x^2\quad 6x\quad 1)$

91. $I\quad _{/2}^{/2}\frac{\cos^2 x}{1\quad 5^x}dx\qquad _{/2}^{/2}\frac{\cos^2 x}{1\quad 5^{\ x}}dx$

$2I\quad _{/2}^{/2}\cos^2 x\,dx$

$I\quad _{0}^{/2}\cos^2 x\,dx\quad \frac{}{4}$

92. $\frac{x^2 \quad x \quad 1}{x^2 \quad 1} e^{\cot^{1} x} dx$

Let $\cot^{1} x \quad 1 \quad \frac{1}{1 \quad x^2} dx \quad dt$

$e^t (\cot t \quad \operatorname{cosec}^2 t)\, dt \quad e^t \quad \cot t \quad c$

$x \quad e^{\cot^{1} x} \quad c$

93. $\lim_{x} \frac{1}{n} {}_{r\,1}^{n} \frac{r\sqrt{n^2 \quad r^2}}{n^2} \quad \lim_{x} \frac{1}{n} {}_{r\,1}^{n} \frac{r}{n}\sqrt{1 \quad \left(\frac{r}{n}\right)^2} \quad {}_0^1 x\sqrt{1 \quad x^2}\,dx \quad \left.\frac{(1 \quad x^2)^{3/2}}{3}\right|_0^1$

94. $\frac{(x^3 \quad 1)}{(x^4 \quad 1)(x \quad 1)}dx \quad \frac{x^3}{x^4 \quad 1}dx \quad \frac{1}{x \quad 1}dx \quad \frac{1}{4}\ln(1 \quad x^4) \quad \ln(1 \quad x) \quad c$

95. $\lim_{x \quad 0} \frac{(\cos^{1} \cos x)(\quad \sin x)}{2 \quad 2\cos 2x} \quad \lim_{x \quad 0} \frac{x \sin x}{4\sin^2 x} \quad \frac{1}{4}$

96. $f(x) \begin{cases} 0 & x > \tan 1 \\ \cos x & 0 < x < \tan 1 \\ \frac{\cos x}{2} & x = \tan 1 \end{cases}$

${}_0 f(x)\,dx \quad {}_0^{\tan 1} \cos x \quad {}_{\tan 1} 0\,dx \quad \sin(\tan 1)$

97. $\lim_{n} {}_{k\,1}^{n} \frac{k}{n^2 \quad n \quad 2k} \quad \lim_{n} \frac{1}{n} {}_{k\,1}^{n} \frac{\frac{k}{n}}{1 \quad \frac{1}{n} \quad \frac{2k}{n^2}} \quad {}_0^1 x\,dx$

98. $\lim_{y \quad 1} \frac{{}_1^y |t \quad 1|\,dt}{\tan(y \quad 1)} \quad \lim_{y \quad 1} \frac{y \quad 1}{\sec^2(y \quad 1)} \quad 0$ (Applying L' Hospital Rule)

99. ${}_0^1 \frac{dx}{(1 \quad x^2)^4} \quad \left.\frac{x}{2(4 \quad 1)(1 \quad x^2)^{4\,1}}\right|_0^1 \quad \frac{5}{6} {}_0^1 \frac{dx}{(1 \quad x^2)^3}$

$\frac{1}{6(2)^3} \quad 0 \quad \frac{5}{6} \left.\frac{x}{2(2)(1 \quad x^2)^2}\right|_0^1 \quad \frac{5}{6} \frac{3}{4} {}_0^1 \frac{dx}{(1 \quad x^2)^2}$

$$= \frac{1}{48}\left(\frac{5}{6}+\frac{1}{16}-0\right)+\frac{5}{8}\left[\frac{x}{2(1)(1+x^2)}\right]_0^1+\frac{5}{8}\cdot\frac{1}{2}\int_0^1\frac{dx}{1+x^2}$$

$$= \frac{1}{48}\left(\frac{5}{6\cdot 16}\right)+\frac{5}{8}\left(\frac{1}{4}-0\right)+\frac{5}{16}[\tan^{-1}x]_0^1$$

$$= \frac{7}{6\cdot 16}+\frac{5}{8\cdot 4}+\frac{5}{16}\left(\frac{\pi}{4}-0\right)$$

$$= \frac{22}{6\cdot 16}+\frac{5\pi}{64}$$

$$= \frac{11}{48}+\frac{5\pi}{64}$$

Alternate solution :

$$I=\int_0^1\frac{dx}{(1+x^2)^4}$$

Put $x=\tan\theta$; therefore, $dx=\sec^2\theta\, d\theta$.

$$I=\int_0^{\pi/4}\frac{\sec^2\theta\, d\theta}{(\sec\theta)^8}$$

That is,

$$I=\int_0^{\pi/4}(\cos\theta)^6 d\theta$$

$$=\int_0^{\pi/4}\left(\frac{3\cos\theta+\cos 3\theta}{4}\right)^2 d\theta$$

$$=\frac{9}{16}\int_0^{\pi/4}\cos^2\theta\, d\theta+\frac{1}{16}\int_0^{\pi/4}(\cos 3\theta)^2 d\theta+\frac{3}{8}\int_0^{\pi/4}\cos\theta\cos 3\theta\, d\theta$$

$$=\frac{9}{16}\int_0^{\pi/4}\frac{1+\cos 2\theta}{2}d\theta+\frac{1}{16}\int_0^{\pi/4}\frac{1+\cos 6\theta}{2}d\theta+\frac{3}{8}\int_0^{\pi/4}\frac{\cos 4\theta+\cos 2\theta}{2}d\theta$$

$$=\frac{9}{32}\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\pi/4}+\frac{1}{16\cdot 2}\left[\theta+\frac{\sin 6\theta}{6}\right]_0^{\pi/4}+\frac{3}{8\cdot 2}\left[\frac{\sin 4\theta}{4}+\frac{\sin 2\theta}{2}\right]_0^{\pi/4}$$

$$=\frac{9}{32}\left(\frac{\pi}{4}+\frac{1}{2}\right)+\frac{1}{16\cdot 2}\left(\frac{\pi}{4}-\frac{1}{6}\right)+\frac{3}{8\cdot 2}\left(0+\frac{1}{2}-0\right)$$

$$=\frac{5\pi}{64}+\frac{11}{48}$$

100. We have, $I=\int_0^{\pi/4}(\sin x)^4 dx$ ...(1)

We know that, $\sin^2 x=\frac{1-\cos 2x}{2}$

Therefore,

$$\sin^4 x \quad (\sin x)^4 \quad \frac{1 \quad \cos 2x}{2}^{\,2}$$

$$\frac{1}{4}[1 \quad 2\cos 2x \quad (\cos 2x)^2]$$

$$\frac{1}{4} \ 1 \quad 2\cos 2x \quad \frac{1 \quad \cos 4x}{2}$$

$$\frac{1}{4} \ \frac{3}{2} \quad 2\cos 2x \quad \frac{\cos 4x}{2}$$

Substituting this value of $\sin^4 x$ in Eq. (1), we get

$$I \quad \overset{/4}{\underset{0}{}} \ \frac{3}{8} \quad \frac{1}{2}\cos 2x \quad \frac{1}{8}\cos 4x \quad dx$$

$$\frac{3}{8}x \ \overset{/4}{\underset{0}{}} \quad \frac{1}{4}[\sin 2x]_0^{/4} \quad \frac{1}{32}[\sin 4x]_0^{/4}$$

$$\frac{3}{8} \ \frac{}{4} \quad \frac{1}{4}(1 \quad 0) \quad \frac{1}{32}(0 \quad 0)$$

$$\frac{3}{32} \quad \frac{1}{4}$$

Alternate solution : We have,

$$I \quad \overset{/4}{\underset{0}{}} (\sin x)^4 \, dx$$

which can be written as

$$J \quad (\sin^2 x)(1 \quad \cos^2 x)\, dx$$

$$\sin^2 x\, dx \quad \frac{1}{4} \quad 4\sin^2 x \cos^2 x\, dx$$

$$\frac{1 \quad \cos 2x}{2} dx \quad \frac{1}{4} \quad (\sin 2x)^2 dx$$

$$\frac{1}{2}x \quad \frac{1}{4}\sin 2x \quad \frac{1}{4} \quad \frac{1 \quad \cos 4x}{2} dx$$

$$\frac{x}{2} \quad \frac{\sin 2x}{4} \quad \frac{1}{8}x \quad \frac{1}{32}\sin 4x \quad c$$

$$\frac{3}{8}x \quad \frac{\sin 2x}{4} \quad \frac{\sin 4x}{32} \quad c$$

Using the given limits, the above equation becomes

$$I \quad [J]_0^{/4} \quad \frac{3}{8}x \Big|_0^{/4} \quad \frac{\sin 2x}{4}\Big|_0^{/4} \quad \frac{\sin 4x}{32}\Big|_0^{/4}$$

$$\frac{3}{32} \quad \frac{1}{4}$$

101. $\dfrac{(\cos 9x \quad \cos 6x)\sin 5x}{\sin 10x \quad \sin 5x}\,dx \quad 2\cos\dfrac{5x}{2}\cos\dfrac{3x}{2} \quad (\cos 4x \quad \cos x)$

$$\frac{\sin 4x}{4} \quad \sin x \quad C$$

$A \quad \dfrac{1}{4}, B \quad 1$

102. $\dfrac{\dfrac{dx}{x^{2014}}}{1 \quad \dfrac{1}{x^{2013}}} \quad \dfrac{1}{2013}\ln \quad \dfrac{x^{2013}}{1 \quad x^{\ 013}} \quad C$

103. $\dfrac{1}{2}\ {}_0^1 x\ (2x \quad e^{\ x^2})\,dx \quad \dfrac{1}{2} \quad xe^{\ x^2} \quad {}_0^1 e^{\ x^2}dx$

$$\frac{1}{2} \quad \frac{1}{e} \quad a$$

104. $2 \quad {}_0^1 f(x)\,dx \quad {}_1^2 f(x)\,dx \quad {}_2^3 f(x)\,dx \quad {}_3^4 f(x)\,dx \quad {}_4^5 f(x)\,dx$

$$2\ \frac{0^2}{2} \quad \frac{1^2}{2} \quad \frac{2^2}{2} \quad \frac{3^2}{2} \quad \frac{4^2}{2} \quad \frac{35}{2}$$

105. $\dfrac{1}{3} \quad \dfrac{3x^2}{x^6(1 \quad x^3)^2}\,dx$

Let $1 \quad x^3 \quad t \quad 3x^2dx \quad dt$

$$\frac{1}{3} \quad \frac{dt}{t^2(t \quad 1)^2} \quad \frac{1}{3} \quad \frac{2}{t} \quad \frac{1}{t^2} \quad \frac{2}{t \quad 1} \quad \frac{1}{(t \quad 1)^2} \quad dt$$

106. $\lim\limits_{n} \sum_{r \ 1}^{3n} \dfrac{1}{n\sqrt{1 \quad \dfrac{r}{n}}} \quad {}_0^3 \dfrac{1}{\sqrt{1 \quad x}}\,dx \quad (2\sqrt{1 \quad x})_0^3 \quad 2$

107. ${}_0^2\ x\,f(x)\,dx \quad \dfrac{x^2}{2}\ f(x)\Big|_0^2 \quad {}_0^2 \dfrac{x^2}{2}\ f\,(x)\,dx \quad 0 \quad {}_0^2 \dfrac{x^2}{2\sqrt{1 \quad x^3}}\,dx$

108. $${}_{0}^{/3}(\ln(\cos x \quad \sqrt{3}\sin x) \quad \ln\cos x)\,dx$$

$${}_{0}^{/3} \ln\ 2\cos\ x\ \ \frac{}{3}\ \ \ln\cos x\ \ dx\ \ \frac{}{3}\ln 2$$

109. $${}_{r\ 1}^{100}\ {}_{0}^{1} f(r\quad 1\quad x)\,dx \quad {}_{0}^{1} f(x)\,dx \quad {}_{0}^{1} f(x\quad 1)\,dx \quad {}_{0}^{1} f(x\quad 2)\,dx \quad\quad {}_{0}^{1} f(x\quad 99)\,dx$$

$${}_{0}^{1} f(x)\,dx \quad {}_{1}^{2} f(x)\,dx \quad {}_{2}^{3} f(x)\,dx \quad\quad {}_{99}^{100} f(x)\,dx \quad {}_{0}^{100} f(x)\,dx \quad a$$

110. $$\lim_{n}\ {}_{k\ 0}^{n}\ x^2\,\frac{(2x)^k}{k!} \quad x^2\ \ e^{2x} \quad {}_{0}^{1} x^2e^{2x}dx \quad \frac{e^2\quad 1}{4}$$

111. $$x^5\sqrt{1\quad x^3}\,dx$$

Let $1\quad x^3\quad t^2$

$3x^2dx\quad 2t\,dt$

$$\frac{2}{3}\quad t^2(t^2\quad 1)\,dt\quad \frac{2}{3}\ \frac{t^5}{5}\quad \frac{t^3}{3}\quad\quad c$$

112. $f(x)\quad \dfrac{\sin x}{x}$

$f(x)\quad 0\quad x\quad (0,\quad)$

$f(x)\quad 0\quad x\quad (\quad ,2\quad)$

113. $$\frac{x(x^2\quad 1)\quad 3(x^2\quad 3)}{(x^2\quad 1)(x^2\quad 3)}\,dx$$

$$\frac{x}{x^2\quad 3}\quad \frac{3}{x^2\quad 1}\quad dx$$

$$\frac{1}{2}\ln|x^2\quad 3|\quad 3\tan^{\ 1}x\quad c$$

114. $$\frac{\sqrt{\sec^5 x}}{\sqrt{\sin^3 x}}\,dx \quad\quad \frac{\sec^4 x}{\sqrt{\tan^3 x}}\,dx$$

Let $\tan x\quad t^2$

$\sec^2 x\,dx\quad 2t\,dt$

$$\frac{(1\quad t^4)\ 2t\ \ dt}{t^3}\quad 2\quad \frac{1}{t^2}\quad t^2\quad dt$$

115. Let $tx \quad y \quad x\,dt \quad dy$

$$\lim_{x \quad 0} \frac{\overset{x^2}{\underset{0}{}} e^{\sin y}\, dy}{x^2} \quad \lim_{x \quad 0} \frac{e^{\sin x^2} 2x}{2x} \quad 1$$

116. $$\overset{/2}{\underset{0}{}} \frac{\cos 2x}{x} dx \quad \frac{\sin 2x}{2x} \overset{/2}{\underset{0}{}} \quad 2 \overset{/2}{\underset{0}{}} \frac{\sin 2x}{(2x)^2} dx \quad 1 \quad \underset{0}{} \frac{\sin}{2} d \qquad (\because \text{ Let } 2x \quad)$$

Exercise-2 : One or More than One Answer is/are Correct

1. $$\frac{dx}{(1 \quad \sqrt{x})^8} \qquad \text{Let } x \quad t^2 \quad dx \quad 2t\,dt$$

$$\frac{2t\,dt}{(1 \quad t)^8} \quad 2 \quad \frac{dt}{(t \quad 1)^7} \quad \frac{dt}{(t \quad 1)^8} \quad 2 \quad \frac{1}{6(1 \quad t)^6} \quad \frac{1}{7(1 \quad t)^7} \quad C$$

2. $$e^x \quad \cos x \ln(x \quad \sqrt{1 \quad x^2}) \ dx \quad 2 \underset{0}{} e^x dx \quad 2(e \quad 1) \quad e \quad \frac{7}{4}$$

3. $$I \quad \sqrt{\frac{x}{a^3 \quad x^3}}\, dx \qquad \text{Let } x^{3/2} \quad a^{3/2} \cos$$

$$\frac{\sqrt{x}}{\sqrt{(a^{3/2})^2 \quad (x^{3/2})^2}} dx \quad \frac{2}{3} \quad \frac{a^{3/2} \cos}{\sqrt{a^3 \quad a^3 \sin^2}} d$$

$$\frac{2}{3} \quad d \quad \frac{2}{3} \quad C \quad \frac{2}{3} \sin^{\ 1} \frac{x^{3/2}}{a^{3/2}} \quad C$$

4. $$x \sin x \sec^3 x\, dx \quad \underbrace{x}_{\mathrm{I}} \quad \underbrace{\tan x \sec^2 x}_{\mathrm{II}} \quad dx \quad x \frac{\tan^2 x}{2} \quad \frac{\tan^2 x}{2} dx$$

$$x \frac{\tan^2 x}{2} \quad \frac{(\sec^2 x \quad 1)}{2} dx \quad x \frac{\tan^2 x}{2} \quad \frac{1}{2}(\tan x \quad x) \quad C$$

$$\frac{1}{2}(x \sec^2 x \quad \tan x) \quad C$$

$f(x) \quad \sec^2 x, \quad g(x) \quad \tan x$

(a) Clear $f(x) \quad (\quad 1, 1)$

(b) $\tan x \quad \sin x$

$\cos x \quad 1 \qquad \tan x$ is not defined.

no solution

(c) $g(x) \quad f(x) \quad x \quad R$ except $(2n \quad 1)\frac{}{2}$

(d) $\sec^2 x \quad \tan x$

$1 \quad \tan^2 x \quad \tan x \quad 0$ has no solution.

5. $(\sin 3 \quad \sin \;)\cos \quad e^{\sin} \quad d \qquad (4\sin \quad 4\sin^3 \;)\, e^{\sin} \quad \cos \quad d$

Let $t \quad \sin$

$dt \quad \cos \quad d \qquad 4[\; t^3 \quad 3t^2 \quad 5t \quad 5]\, e^t \quad C$

Compare it

$A \quad 4, B \quad 12, C \quad 20$

7. $I \quad \frac{2x\, dx}{{}_0\sqrt{(3 \quad 2x)(\quad 2x)}} \qquad \frac{2(\quad x)\, dx}{{}_0\sqrt{(3 \quad 2x)(\quad 2x)}}$

$I \quad \frac{}{2} \; \frac{dx}{{}_0\sqrt{\;^2 \quad (x \quad /2)^2}}$

8. Let $f(x) \quad a^x, F(x) \quad F(\quad x)$

9. $J \quad {}^0_1 \quad \cot^{1}\frac{1}{x} \quad \cot^{1}(x) \quad dx \qquad {}^2_0 \quad \cot^{1}\frac{1}{x} \quad \cot^{1} x \quad dx$

${}^0_1 \quad \frac{}{2}\, dx \qquad {}^2_0 \, \frac{}{2}\, dx$

$K \quad {}_0 \quad dx$ (As 2 is period)

11. $l_1 \quad \lim_{x} \sqrt{\frac{x \quad \cos^2 x}{x \quad \sin x}} \quad 1$

$l_2 \quad \lim_{h \quad 0} \; {}^1_1 \frac{h\, dx}{h^2 \quad x^2} \quad \lim_{h \quad 0} 2\tan^{1}\frac{1}{h}$

13. $\frac{dx}{(1 \quad \sin^2 x)\cos^2 x} \qquad \frac{\sec^4 x}{1 \quad 2\tan^2 x}\, dx$

$$\frac{(1 \quad \tan^2 x)\sec^2 x}{(1 \quad 2\tan^2 x)}dx \quad \frac{1}{2} \quad \sec^2 x\,dx \quad \frac{1}{2} \quad \frac{\sec^2 x\,dx}{1 \quad 2\tan^2 x}$$

$$\frac{1}{2}\tan x \quad \frac{1}{2\sqrt{2}}\tan^{\ 1}(\sqrt{2}\tan x) \quad c$$

14. $\dfrac{(1 \quad \sin^{2015} x) \quad \sqrt{1 \quad \sin^{4030} x}}{2\sin^{2015} x}$ (Rationalise)

$$\int_{2014}^{2014} \frac{1}{2}dx \qquad\qquad \text{odd} \quad 0$$

15. $\tan^{\ 1}(nx)|_a \quad \overline{2} \quad \tan^{\ 1}(na)$

$a \quad 0, a \quad 0, a \quad 0$

16. Let $\sqrt{x} \quad \cos 2$

$dx \quad \sin 4 \quad d$

$I \quad \int_0^{/4} \cot \quad \sin 4 \quad d$ and $J \quad \int_0^{/4} \tan \quad \sin 4 \quad d$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

2. $f(x) \quad (2x^3\cos^2 x \quad 6x^2\sin x\cos x \quad 2x^3\sin^2 x)\,dx$

$$\underbrace{2x^3}_{\text{I}}\ \underbrace{\cos 2x}_{\text{II}} \quad 3x^2\sin 2x \quad dx$$

$f(x) \quad x^3\sin 2x \quad c$

$f(\ \ ) \quad 0 \quad c \quad 0 \qquad c \quad 0$

$f(x) \quad x^3\sin 2x$

Paragraph for Question Nos. 6 to 8

6. $g(x) \quad x \quad A$

$A \quad \int_0^1 f(t)\,dt$

$$f(x) \quad \frac{x^3}{2} \quad 1 \quad x \;{}_{0}^{x}(x \quad A)\,dx \quad \frac{x^3}{2} \quad 1 \quad \frac{x^3}{2} \quad Ax^2$$

$$f(x) \quad Ax^2 \quad 1$$

$$A \quad {}_{0}^{1} Ax^2 \quad 1 \qquad A \quad \frac{3}{2}$$

$$f(x) \quad \frac{3x^2}{2} \quad 1; \quad \text{min. } f(x) \quad 1$$

7. $\frac{3}{2}x^2 \quad 1 \quad x \quad \frac{3}{2}$

$$3x^2 \quad 2x \quad 5 \quad 0$$

0, no solution

8. $g(x) \quad x \quad \frac{3}{2}$

$$A \quad \frac{1}{2} \quad \frac{3}{2} \quad \frac{3}{2} \quad \frac{9}{8}$$

Paragraph for Question Nos. 9 to 11

9. $${}_{0}^{a} f(x)\,dx \quad {}_{a}^{1} f(x)\,dx \quad 2f(a) \quad 3a \quad b \qquad \ldots(1)$$

Diff. w.r.t. 'a' on both sides,

$$(f(a) \quad 0) \quad (0 \quad f(a)) \quad 2f\,(a) \quad 3$$

$$2f(a) \quad 2f\,(a) \quad 3$$

$$(2f(a) \quad 3) \quad 2f\,(a)$$

$$\frac{2f\,(a)}{2f(a) \quad 3} \quad 1$$

$$\frac{2f\,(a)}{2f(a) \quad 3}da \qquad da$$

$$\ln|2f(a) \quad 3| \quad a \quad c$$

$$2f(a) \quad 3 \quad e^{a \quad c}$$

$$2f(a) \quad 3 \quad ke^{a}$$

$$2f(a) \quad ke^{a} \quad 3$$

Put $a \quad 1 \quad 0 \quad ke \quad 3 \quad k \quad \dfrac{3}{e}$

$$2f(a) \quad \frac{3}{e}e^{a} \quad 3$$

$$f(a) \quad \frac{3}{2} \quad \frac{3}{2e}e^{a}$$

$$f(x) \quad \frac{3}{2} \quad \frac{3}{2e}e^{x}$$

Put $f(x)$ in (1) (By taking limiting case)

$$_{0}^{a} \quad \frac{3}{2} \quad \frac{3}{2e}e^{x} \quad dx \quad _{a}^{1} \quad \frac{3}{2} \quad \frac{3}{2e}e^{x} \quad dx \quad 3 \quad \frac{3}{e}e^{a} \quad 3a \quad b$$

$$\frac{3a}{2} \quad \frac{3}{2e}e^{a} \quad 0 \quad \frac{3}{2e} \qquad \frac{3}{2} \quad \frac{3}{2} \qquad \frac{3a}{2} \quad \frac{3e^{a}}{2e} \qquad 3 \quad \frac{3}{e}e^{a} \quad 3a \quad b$$

$$\frac{3}{2e} \quad 3 \quad b$$

10. Length of subtangent $\left|\dfrac{y}{(dy/dx)}\right|$

$$y \quad f(x) \quad \frac{3}{2} \quad \frac{3}{2e}e^{x}$$

$$\frac{dy}{dx} \quad f\,(x) \quad 0 \quad \frac{3}{2e}e^{x}$$

$$\left.\frac{dy}{dx}\right|_{x \quad 1/2} \qquad \frac{3}{2\sqrt{e}}$$

when $x \quad \dfrac{1}{2},\ y \quad f \quad \dfrac{1}{2} \quad \dfrac{3}{2} \quad \dfrac{3}{2e}e^{1/2} \quad \dfrac{3}{2} \quad 1 \quad \dfrac{1}{\sqrt{e}}$

Length of subtangent $\left|\dfrac{\dfrac{3}{2} \quad 1 \quad \dfrac{1}{\sqrt{e}}}{\dfrac{3}{2\sqrt{e}}}\right| \quad \left|\sqrt{e} \quad 1\right| \quad \sqrt{e} \quad 1$

11. $_{0}^{1} \; f(x)\,dx \quad _{0}^{1} \quad \dfrac{3}{2} \quad \dfrac{3}{2e}e^{x} \quad dx \quad \left.\dfrac{3x}{2} \quad \dfrac{3}{2e}e^{x}\right|_{0}^{1} \quad \dfrac{3}{2} \quad \dfrac{3e}{2e} \qquad 0 \quad \dfrac{3}{2e}$

$$\frac{3}{2} \quad \frac{3}{2} \qquad \frac{3}{2e} \quad \frac{3}{2e}$$

Paragraph for Question Nos. 12 to 13

12. $f_3(x) \quad f_0(x)$ see options or 3 times by parts.

13. $f_n(x) \quad \frac{x^n}{\lfloor n} \quad \ln x \quad 1 \quad \frac{1}{2} \quad \frac{1}{3} \qquad \frac{1}{n}$

Paragraph for Question Nos. 14 to 15

$$f(x) \quad a \quad x \quad \frac{1}{2}^{\;2} \quad \frac{3}{4}$$

$$f(1) \quad 1 \qquad\qquad a \quad 1$$

$$f(x) \quad x^2 \quad x \quad 1$$

14. $g(x) \quad 1 \quad x^2$ now integrate

15. $\frac{e^x}{e^{2x} \quad e^x \quad 1} \quad e^x \quad t$

Paragraph for Question Nos. 16 to 17

17. $$L \quad \lim_{x} \frac{xe^{2x}(1 \quad 3x^2)^{1/2}}{C \quad (xe^x)^{C \quad 1} \quad (e^x \quad xe^x)} \quad \lim_{x} \frac{\left((xe^x)^2 \quad \frac{1}{x^2} \quad 3\right)^{1/2}}{C \quad (xe^x)^C \quad \frac{1}{x} \quad 1}$$

Exercise-4 : Matching Type Problems

2. (A) $$\frac{dx}{(x^2 \quad 1)\sqrt{x^2 \quad 2}} \qquad \frac{\frac{1}{x^3}\,dx}{1 \quad \frac{1}{x^2} \quad \sqrt{1 \quad \frac{2}{x^2}}}$$

Let $1 \quad \frac{2}{x^2} \quad t^2$

(C) $$\frac{x^4 \quad x^8}{(1 \quad x^4)^{7/2}} \qquad \frac{x \quad \frac{1}{x^3} \quad dx}{\frac{1}{x^2} \quad x^2 \;^{7/2}}$$

Let $\frac{1}{x^2} \quad x^2 \quad t^2$

(D) Let $\sqrt{x} \quad \cos 2 \qquad dx \quad 2\sin 4 \quad d$

$$\sqrt{\frac{1 \quad \sqrt{x}}{1 \quad \sqrt{x}}}\,dx \quad 2 \quad \tan \quad \sin 4 \quad d$$

3. (A) Let $\sin x \quad t \qquad \cos x\,dx \quad dt$

$$\int_0^1 \frac{dt}{(1 \quad t)(2 \quad t)} \qquad \int_0^1 \frac{1}{t \quad 1} \quad \frac{1}{t \quad 2}\,dt \quad [\ln(1 \quad t) \quad \ln(t \quad 2)]_0^1$$

(B) $\displaystyle\int_0^{41\ /4} |\cos x|\,dx \quad 10 \int_0 |\cos x|\,dx \qquad \int_0^{/4} \cos x\,dx$

$$10 \int_0^{/2} \cos x\,dx \qquad \int_{/2} \cos x\,dx \qquad \int_0^{/4} \cos x\,dx$$

(C) $\displaystyle\int_{1/2}^{0} [x]\,dx \qquad \int_0^{1/2} [x]\,dx \qquad \int_{1/2}^{1/2} \ln \frac{1 \quad x}{1 \quad x}\,dx$

$$\int_{1/2}^{0} 1\,dx \qquad \int_0^{1/2} 0\,dx \qquad \frac{1}{2}$$

(D) $\displaystyle I \qquad \int_0^{/2} \frac{2\sqrt{\cos}}{3(\sqrt{\cos} \qquad \sqrt{\sin}\ )}\,d \qquad \int_0^{/2} \frac{2\sqrt{\sin}}{3(\sqrt{\sin} \qquad \sqrt{\cos}\ )}\,d$

$$2I \qquad \int_0^{/2} \frac{2}{3}\,d \qquad \frac{}{3}$$

$$I \quad \frac{}{6}$$

4. (A) Common root $\quad b \quad a \quad 3(b \quad a)^2 \quad a(b \quad a) \quad 1 \quad 0 \quad 2a^2 \quad 3b^2 \quad 5ab \quad 1 \quad 0$

(B) $\displaystyle \frac{x^4 \quad 1}{2x^2} \quad \sin^2 \frac{x}{2} \qquad \frac{x^2 \quad \frac{1}{x^2}}{2} \quad \sin^2 \frac{x}{2} \qquad x \quad 1$

(C) $\displaystyle y \quad \frac{1}{\frac{1}{(x \quad 1)^2} \qquad \frac{1}{x \quad 1} \quad 2} \qquad x \quad 1 \;; \quad \frac{1}{x \quad 1} \qquad 2\,,\,1$

(D) $\displaystyle \frac{x}{1 \quad x}^{7/6} \frac{dx}{x^2}.$

Let $\dfrac{x \quad 1}{x} \quad t^6 \qquad \dfrac{1}{x^2}\,dx \quad 6t^5\,dt$

$$I \quad 6 \quad t^{\;7}(\;\; t^5)\,dt \quad \frac{6}{t} \quad C \quad 6\left(\frac{x}{x \quad 1}\right)^{1/6} \quad C$$

5. (A) We have,

$$\overset{1.5}{\underset{0}{}}[x^2]\,dx \qquad \overset{1}{\underset{0}{}}[x^2]\,dx \qquad \overset{\sqrt{2}}{\underset{1}{}}[x^2]\,dx \qquad \overset{1.5}{\underset{\sqrt{2}}{}}[x^2]\,dx$$

$$\overset{1}{\underset{0}{}}0\,dx \qquad \overset{\sqrt{2}}{\underset{1}{}}1\;dx \qquad \overset{1.5}{\underset{\sqrt{2}}{}}2\;dx \quad 0 \quad (\sqrt{2} \quad 1) \quad 2(1.5 \quad \sqrt{2}) \quad 2 \quad \sqrt{2}$$

(B) We have,

$$\overset{4}{\underset{0}{}}\{\sqrt{x}\}\,dx \qquad \overset{1}{\underset{0}{}}\sqrt{x}\,dx \qquad \overset{4}{\underset{1}{}}(\sqrt{x} \quad 1)\,dx \quad \frac{2}{3} \quad \frac{2}{3}(8 \quad 1) \quad 3 \quad \frac{7}{3}$$

Aliter : $\overset{4}{\underset{0}{}}\{\sqrt{x}\}\,dx \qquad \overset{4}{\underset{0}{}}\sqrt{x}\,dx \qquad \overset{4}{\underset{0}{}}[\sqrt{x}\,]\,dx$

(C) We have,

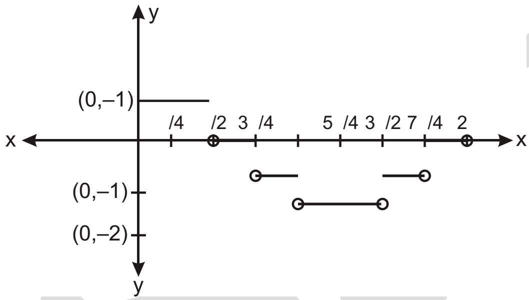
$$\sin x \quad \cos x \quad \sqrt{2}\sin\left(\frac{}{4} \quad x\right)$$

$$[\sin x \quad \cos x] \quad \left[\sqrt{2}\sin\left(\frac{}{4} \quad x\right)\right]$$

The graph of $y \quad \left[\sqrt{2}\sin\left(\frac{}{4} \quad x\right)\right]$ is obtained from the graph of $y \quad [\sqrt{2}\sin x]$ by translating it by $\frac{}{4}$ units in the direction of OX . The graph so obtained is shown in figure.

It is evident from the graph of $y \quad [\sqrt{2}\sin(x \quad\quad /4)]$ that

$$f(x) \quad [\sin x \quad \cos x] \quad \begin{cases} 1, & 0 \quad x \quad\quad /2 \\ 0, & \quad /2 \quad x \quad 3\;/4 \\ 1, & 3\;/4 \quad x \\ 2, & \quad\quad x \quad 3\;/2 \\ 1, & 3\;/2 \quad x \quad 7\;/4 \\ 0, & 7\;/4 \quad x \quad 2 \end{cases}$$



$$\int_0^{2\pi}[\sin x + \cos x]\,dx$$

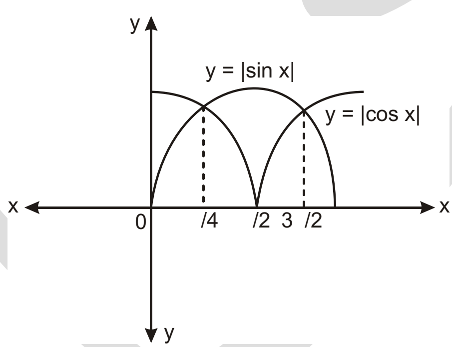
$$= \int_0^{\pi/2} 1\,dx + \int_{\pi/2}^{3\pi/4} 0\,dx + \int_{3\pi/4}^{\pi}(-1)\,dx + \int_{\pi}^{3\pi/2}(-2)\,dx + \int_{3\pi/2}^{7\pi/4}(-1)\,dx + \int_{7\pi/4}^{2\pi} 0\,dx$$

$$= \frac{\pi}{2} + 0 - \frac{\pi}{4} - 2\cdot\frac{\pi}{2} + (-1)\frac{\pi}{4} + 0$$

(D) We have,

$$\int_0^{\pi}\big||\sin x| - |\cos x|\big|\,dx$$

$$= -\int_0^{\pi/4}(|\sin x| - |\cos x|)\,dx + \int_{\pi/4}^{3\pi/4}(|\sin x| - |\cos x|)\,dx - \int_{3\pi/4}^{\pi}(|\sin x| - |\cos x|)\,dx$$



$$= -\int_0^{\pi/4}(\sin x - \cos x)\,dx + \int_{\pi/4}^{\pi/2}(\sin x - \cos x)\,dx + \int_{\pi/2}^{3\pi/4}(\sin x + \cos x)\,dx$$

$$- \int_{3\pi/4}^{\pi}(\sin x + \cos x)\,dx$$

$$[\ \cos x \quad \sin x]_0^{/4} \quad [\ \cos x \quad \sin x]_{/4}^{/2} \quad [\ \cos x \quad \sin x]_{/2}^{3\ /4} \quad [\ \cos x \quad \sin x]_{3\ /4}$$

$$[\ \sqrt{2} \quad 1] \quad [\ 1 \quad \sqrt{2}] \quad [\sqrt{2} \quad 1] \quad [1 \quad \sqrt{2}] \quad 4\sqrt{2} \quad 4 \quad 4(\sqrt{2} \quad 1)$$

Exercise-5 : Subjective Type Problems

1. $$\frac{x\,dx}{\sqrt{1 \quad 9x^2}} \qquad \frac{(\cos^{\ 1} 3x)^2}{\sqrt{1 \quad 9x^2}} dx \quad I_1 \quad I_2$$

$$I_1 \qquad \frac{x\,dx}{\sqrt{1 \quad 9x^2}} \qquad\qquad I_2 \qquad \frac{(\cos^{\ 1} 3x)^2}{\sqrt{1 \quad 9x^2}} dx$$

Let $1 \quad 9x^2 \quad t^2$ Let $\cos^{\ 1} 3x \quad k$

2. $$I \qquad {}_0 \frac{x^3 dx}{(a^2 \quad x^2)^5}$$

$$I \quad \frac{1}{a^6} \ {}_0^{/2} \sin^3 \quad \cos^5 \quad d \qquad \frac{1}{a^6} \ {}_0^{/2} \cos^3 \quad \sin^5 \quad d \qquad (\text{Let } x \quad a \tan \ )$$

$$2I \quad \frac{1}{8a^6} \ {}_0^{/2} \sin^3 2 \quad d \qquad \frac{1}{32a^6} \ {}_0^{/2} (3 \sin 2 \qquad \sin 6 \ )\, d$$

$$I \quad \frac{1}{24a^6}$$

3. $${}_0^{2} \ g(x)\, dx$$

$${}_{3\ /2}^{2} \ t\, f(t)\, dt \qquad (\because x \quad f(t))$$

$${}_{3\ /2}^{2} \ (t \cos t \quad t^2 \sin t)\, dt \quad 4^{\ 2} \quad \frac{3}{2} \quad 1$$

4. $$(x^5 \quad x^3 \quad x)\sqrt{2x^6 \quad 3x^4 \quad 6x^2}\, dx$$

Let $2x^6 \quad 3x^4 \quad 6x^2 \quad t^2 \qquad 12(x^5 \quad x^3 \quad x)\, dx \quad 2t\, dt$

$$\frac{1}{12} \quad 2t^2 dt \quad \frac{1}{18}(2x^6 \quad 3x^4 \quad 6x^2)^{3/2} \quad C$$

5. Put $x \quad \sin$

6. $\dfrac{\tan x}{\tan^2 x \quad \tan x \quad 1}\,dx$

Let $\tan x \quad t, \sec^2 x\,dx \quad dt, dx \quad \dfrac{dt}{1 \quad t^2}$

$$\frac{t}{(1 \quad t \quad t^2)}\frac{dt}{1 \quad t^2} \qquad \frac{1}{1 \quad t^2}\quad\frac{1}{1 \quad t \quad t^2}\,dt \qquad \frac{dt}{1 \quad t^2} \qquad \frac{dt}{t \quad \frac{1}{2}{}^2 \quad \frac{\sqrt3}{2}{}^2}$$

$$\tan^{\ 1}(t) \quad \frac{1}{\frac{\sqrt3}{2}}\tan^{\ 1}\frac{2t \quad 1}{\sqrt3} \quad C$$

$$\tan^{\ 1}(\tan x) \quad \frac{2}{\sqrt3}\tan^{\ 1}\frac{2\tan x \quad 1}{\sqrt3} \quad C$$

7. Let $x^4 \quad t$

$4x^3 dx \quad dt$

$${}_0^1 \frac{1 \quad t^{2010}}{(1 \quad t)^{2012}}\,dt \quad {}_0^1\frac{1}{(1\quad t)^{2012}}\,dt \quad {}_0^1 \frac{1}{t^2 \quad 1 \quad \frac1t {}^{2012}}\,dt \quad \left.\frac{(1\quad t)^{\ 2011}}{2011}\right|_0^1 \quad \left.\frac{1 \quad \frac1t {}^{2011}}{2011}\right|_0^1$$

$$\frac{1}{2011}\frac{1}{2^{2011}} \quad 1 \quad \frac{1}{2011}\frac{1}{2^{2011}} \quad 0 \quad \frac{1}{2011}\frac{1}{2^{2011}} \quad 1 \quad \frac{1}{2^{2011}} \quad \frac{1}{2011} \quad -$$

8. $${}_1^{\sqrt3}(x^{2x^2}x \quad 2x^{2x^2}\quad x\ln x)\,dx \quad {}_1^{\sqrt3} x^{2x^2}(x \quad 2x\ln x)\,dx$$

$${}_1^{\sqrt3} x^{x^2}{}^{2} (x \quad 2x\ln x)\,dx \qquad \text{Let } x^{x^2} \quad t \quad x^2\ln x \quad \ln t;\ (2x\ln x \quad x)\,dx \quad \frac{dt}{t}$$

$${}_1^{(\sqrt3)^3} t^2\frac{dt}{t} \quad {}_1^{(\sqrt3)^3} t\,dt \quad \left.\frac{t^2}{2}\right|_1^{3^{3/2}} \quad \frac{3^3 \quad 1}{2} \quad 13$$

9. $$\frac{dx}{(\cos x \quad \sin x)(1 \quad \sin x\cos x)} \quad 2\,\frac{(\cos x \quad \sin x)\,dx}{(\cos x \quad \sin x)^2(2 \quad (\sin x \quad \cos x)^2 \quad 1)}$$

$$2\,\frac{(\cos x \quad \sin x)\,dx}{((\sin^2 x \quad \cos^2 x) \quad 2\sin x\cos x)(1 \quad (\sin x \quad \cos x)^2)}$$

$$2 \quad \frac{(\cos x \quad \sin x)\, dx}{(2 \quad (\sin x \quad \cos x)^2)(1 \quad (\sin x \quad \cos x)^2)}$$

$$2 \quad \frac{dt}{(2 \quad t^2)(1 \quad t^2)} \text{ where } t \quad \sin x \quad \cos x$$

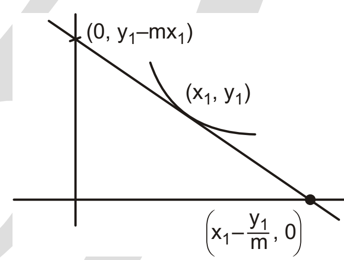
$$2 \quad \frac{dt}{(2 \quad t^2)(1 \quad t^2)} \quad \frac{2}{3} \quad \frac{1}{1 \quad t^2} dt \qquad \frac{dt}{2 \quad t^2}$$

$$\frac{2}{3} \tan^{\ 1}(t) \quad \frac{1}{2\sqrt{2}} \ln \frac{\sqrt{2} \quad t}{\sqrt{2} \quad t} \qquad C$$

$$A \quad \frac{2}{3},\ B \quad \frac{1}{3\sqrt{2}}$$

$$12A \quad 9\sqrt{2}B \quad 3 \quad 12 \quad \frac{2}{3} \quad 9\sqrt{2}\, \frac{1}{3\sqrt{2}} \quad 3 \quad 8$$

10. $x^a \ \ y \quad {}^a; \ \ y \quad \frac{a}{x^a}$



$$\frac{dy}{dx} \quad a \ {}^a x^{\ a \ 1} \qquad a \frac{x^a \quad y}{x^{a \ 1}}$$

$$m \quad \frac{ay_1}{x_1}$$

$$A \quad \frac{1}{2}|y_1 \quad mx_1|\left|x_1 \quad \frac{y_1}{m}\right| \quad \frac{1}{2} y_1 x_1 (1 \quad a)^2$$

$$\frac{1}{2} \quad {}^a \quad x_1^{1 \ a}(1 \quad a)^2$$

For A to be constant $1 \quad a \quad 0$

11. $I_{(6,8)} \quad \int_0 x^6(\quad x)^8 dx \qquad \frac{x^6(\quad x)^9}{9} \Big|_0 \quad \int_0 6x^5 \frac{(\quad x)^9}{9} dx$

$$I_{(6,8)} \quad \frac{6}{9} I_{(5,9)} \quad \frac{6}{9} \quad \frac{5}{10} \quad \frac{4}{11} \quad \frac{3}{12} \quad \frac{2}{13} \quad \frac{1}{14} \int_0 (\quad x)^{14} dx \quad \frac{6! \quad 8!}{15!} \quad 15$$

14. $I \quad \int_0^{100} \sqrt{x}\, dx \qquad \int_0^{1^2} 0 \ \ dx \qquad \int_{1^2}^{2^2} dx \qquad \int_{2^2}^{3^2} 2dx \qquad \int_{9^2}^{10^2} 9\, dx$

$$I \quad \frac{155}{3}$$

17. $f(\ ) \quad \frac{\sin \quad dx}{{}_{1}(x \quad \cos \quad)^2 \quad \sin^2}^{1} \quad \tan^{1} \frac{x \quad \cos}{\sin}\Big|_{1}^{1}$

Clearly, $f(\ )$ is not defined when $\sin \quad 0$

$0, \quad ,2$

20. $f(x) \quad \frac{1}{2} \int_0^x (x \quad t)^2 g(t)\, dt \quad \frac{1}{2} \quad x^2 \int_0^x g(t)\, dt \quad 2x \int_0^x tg(t)\, dt \quad \int_0^x t^2 g(t)\, dt$

$f(x) \quad x \int_0^x g(t)\, dt \quad \int_0^x t\, g(t)\, dt$

$f(x) \quad \int_0^x g(t)\, dt$

$f(x) \quad g(x)$

22. $f(2 \quad x) \quad f(2 \quad x)$, it means it symmetric about $x \quad 2 \quad \int_0^2 f(x)\, dx \quad \int_2^4 f(x)\, dx \quad 5$

Let $2 \quad x \quad t;\ f(t) \quad f(4 \quad t) \quad i.e.,\ f(x) \quad f(4 \quad x) \quad f(4 \quad x)$

$\int_0^{50} f(x)\, dx \quad 25 \int_0^{2} f(x)\, dx \quad 25 \quad 5 \quad 125$

23. $I_n \quad \int_1^1 |x| \quad 1 \quad x \quad \frac{x^2}{2} \quad \frac{x^3}{3} \quad \frac{x^{2n}}{2n} \quad dx \quad 2 \int_0^1 \quad x \quad \frac{x^3}{2} \quad \frac{x^5}{4} \quad \frac{x^{2n \quad 1}}{2n} \quad dx$

$$2 \quad \frac{x^2}{1 \quad 2} \quad \frac{x^4}{2 \quad 4} \quad \frac{x^6}{4 \quad 6} \quad \frac{x^{2n \quad 1}}{2n \ (2n \quad 2)}\Bigg|_0^1$$

$$2 \quad \frac{1}{1 \quad 2} \quad \frac{1}{2 \quad 4} \quad \frac{1}{4 \quad 6} \quad \frac{1}{2n \ (2n \quad 2)}$$

$I_n \quad 1 \quad \frac{1}{2} \quad \frac{1}{1 \quad 2} \quad \frac{1}{2 \quad 3} \quad \frac{1}{n(n \quad 1)} \quad 1 \quad \frac{1}{2} \quad \frac{1}{1} \quad \frac{1}{n \quad 1}$

$\lim_{n} I_n \quad 1 \quad \frac{1}{2} \quad \frac{3}{2} \quad \frac{p}{q}$

$pq(p \quad q) \quad 3 \quad 2(5) \quad 30$

25. $\int_a^b |\sin x|\, dx \quad 8 \quad b \quad a \quad 4$

$$\int_0^{a\quad b} |\cos x|\, dx \quad 9 \qquad a \quad b \quad \frac{9}{2} \qquad a \quad \frac{}{4};\, b \quad \frac{17}{4}$$

$$\frac{1}{\sqrt{2}}\left|\int_a^b x \sin x \quad dx\right| \quad \frac{1}{\sqrt{2}}\left|\int_{/4}^{17\ \ /4} x \sin x\, dx\right| \quad 2$$

28. $f(x) \quad 0 \qquad \int_0^x e^{\ \ y} f(y)\, dy \quad x^2 \quad x \quad 1$

$$e^{\ \ x} f(x) \quad 2x \quad 1 \qquad f(x) \quad (2x \quad 3)e^x$$

29. $I_n \quad 2 \int_0 \frac{}{2} \quad |x| \ \cos nx\, dx \quad 2\ \frac{1 \quad \cos n}{n^2}$

$$I_1 \quad I_2 \quad I_3 \quad I_4 \quad 4\ 1 \ \ \frac{1}{9} \qquad \frac{40}{9}$$

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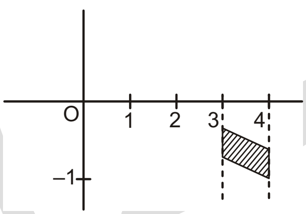
6

AREA UNDER CURVES

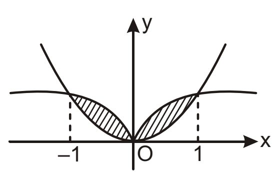
Exercise-1 : Single Choice Problems

1. 

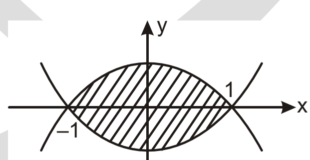
$1 \quad x \quad 3y \quad 2$

Area of shaded region $\frac{1}{3}$

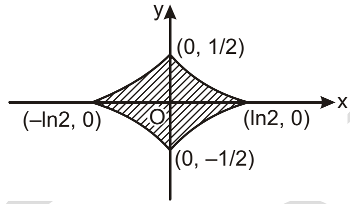
2. Area $2 \int_0^1 (\sqrt{x} \quad x^2)\, dx$



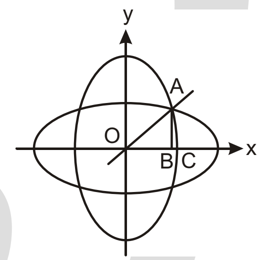
3. $y^2 \quad (x^2 \quad 1)^2 \quad 4 \int_0^1 (1 \quad x^2)\, dx \quad \frac{8}{3}$



4. Area $4 \int_0^{\ln 2} e^{\ x} \quad \frac{1}{2}\ dx \quad 2 \quad 2\ln 2$



5. As given relations are inverse of each other so A lies on $y \quad x$

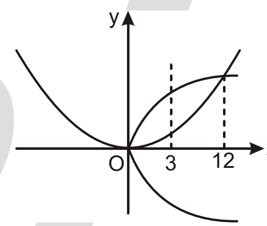


i.e., $\dfrac{n}{\sqrt{n^2 \quad 1}}, \dfrac{n}{\sqrt{n^2 \quad 1}}$

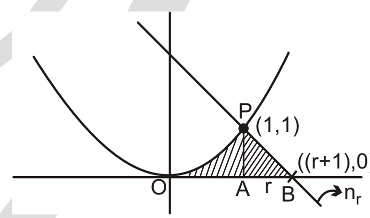
So, required area $\quad$ 8 area ($OACBO$) $\quad$ 8($\quad OAB \quad$ area $BACB$)

$$8 \quad \frac{1}{2} \quad \frac{n}{\sqrt{n^2 \quad 1}}^{2} \qquad \int_{n/\sqrt{n^2 \quad 1}}^{1} n \sqrt{1 \quad x^2}\, dx$$

6. $$\frac{\int_0^3 \left(12x \quad \frac{x^2}{12}\right) dx}{\int_3^{12} \left(12x \quad \frac{x^2}{12}\right) dx} \quad \frac{15}{49}$$



8. $\left.\dfrac{dy}{dx}\right|_{(1,\,1)} \quad r$

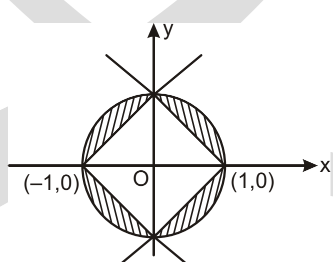


Normal at (1, 1) is $\quad y \quad \dfrac{1}{r}(x \quad 1) \quad 1$

Required area $\quad \int_0^1 x^r\, dx \quad$ ar($\quad PAB$) $\quad \dfrac{1}{r \quad 1} \quad \dfrac{1}{2}r \quad f(r)$

$$f\,(r) \quad \frac{1}{(r \quad 1)^2} \quad \frac{1}{2} \quad 0$$

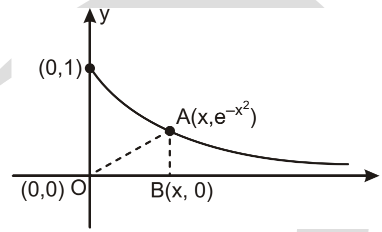
9. Area $\quad 4 \int_0^1 \sqrt{1 \quad x}\, dx \quad \dfrac{1}{2} \quad \dfrac{2}{3}$



10. Area of $AOB \quad \frac{1}{2} xe^{\quad x^2}$

$\frac{dA}{dx} \quad (1 \quad 2x^2)\, e^{\quad x^2}$

Area is maximum at $x \quad \frac{1}{\sqrt{2}}$



12. $\overset{2}{\underset{1}{}} \; g(x)\, dx$

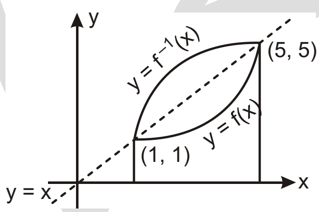
Let $x \quad f(t) \qquad dx \quad f\,(t)\, dt$

$\overset{1}{\underset{0}{}} \; t(3t^2 \quad 6t \quad 3)\, dt \quad \frac{1}{4}$

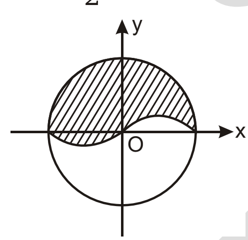
13. $x^2 y^2 \quad y^4 \quad x^2 \quad 5y^2 \quad 4 \quad 0$

$(x^2 \quad y^2 \quad 4)(y^2 \quad 1) \quad 0$

14.



15. Ar. of shaded region $\frac{1}{2}$ Ar. of circle $\frac{3}{2}$.



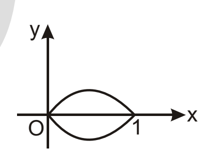
16. We have,

$$y^2 \quad x(1 \quad x^3) \qquad \ldots(1)$$

For $x \quad 1$, y^2 is negative. Since the square of a real number cannot be negative, y does not exist at $x \quad 0$ or at $x \quad 1$; $y \quad 0$. Let $x \quad \frac{1}{2}$. Therefore, from Eq. (1), we get

$$y^2 \quad \frac{1}{2} \; 1 \quad \frac{1}{8} \quad \frac{7}{16}$$

$$y \quad \frac{\sqrt{7}}{4}$$

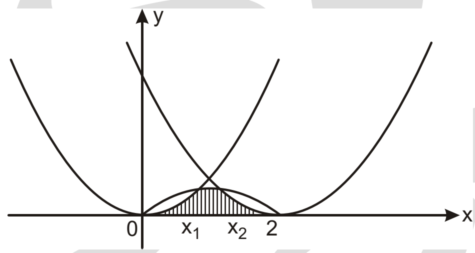


Also, for $x \quad 0, y^2$ is negative. Therefore, the required area is

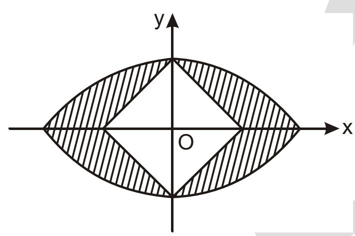
$$2\int_0^1 y\,dx \quad 2\int_0^1 (\quad)\sqrt{x}\sqrt{1 \quad x^3}\,dx$$

$$2\int_0^1 \sqrt{x \quad x^4}\,dx$$

17.



18. $|x| \quad |y| \quad 2$ and $\dfrac{x^2}{9} \quad \dfrac{y^2}{4} \quad 1$



Ar. of ellipse Ar. of square $(2)(3) \quad 8 \quad 6 \quad 8$

Exercise-2 : One or More than One Answer is/are Correct

1. (a) $f(x) \quad (x \quad a)(x \quad b)(x \quad c) \quad (x \quad a)(x)(x \quad c)$ [$\because b \quad 0$]

Clearly option (a) is correct.

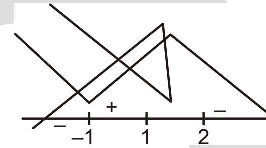
(b) $\int_a^c f(x)\,dx \quad \int_a^b f(x)\,dx \quad \int_b^c f(x)\,dx \quad 0$ (from graph)

which incorrect

(c) $\int_a^b f(x)\,dx \quad 0$ & $\int_c^b f(x)\,dx \quad 0$ but second term is large negative value so option (c) is incorrect.

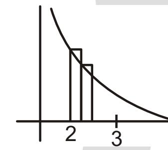
(d) Clearly, (d) is incorrect.

2. $T_n \quad \frac{1}{n} \; {}^{3n\;\;1}_{r\;\;2n} \; \frac{(r/n)}{1 \quad (r/n)^2}, \quad S_n \quad \frac{1}{n} \; {}^{3n}_{r\;\;2n\;\;1} \; \frac{(r/n)}{1 \quad (r/n)^2}$



Let $\quad f(x) \quad \frac{x}{1 \quad x^2}$

$f\ (x) \quad \frac{(1 \quad x^2) \quad 2x^2}{(1 \quad x^2)^2} \quad \frac{1 \quad x^2}{(1 \quad x^2)^2}$



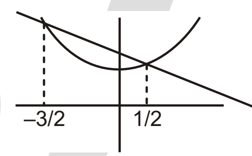
$f(x)$ is decreasing in (2, 3).

$T_n \quad {}^{3}_{2} \; f(x)\,dx, \quad S_n \quad {}^{3}_{2} \; f(x)\,dx$

3. $a \quad b \quad 2$ ...(1)

${}^{4}_{0} \; (a\sqrt{x} \quad bx)\,dx \quad 8 \qquad \frac{2a}{3} \quad b \quad 1$ ...(2)

4. Normal $y \quad x \quad \frac{7}{4}$



${}^{1/2}_{3/2} \; \frac{7}{4} \quad x \qquad (x^2 \quad 1)\,dx$

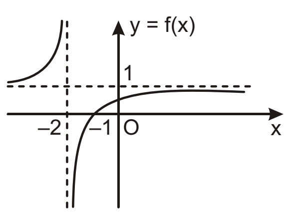
Exercise-3 : Comprehension Type Problems

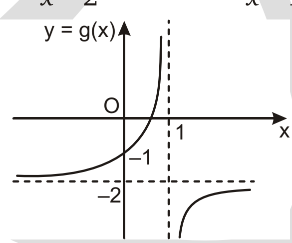
Paragraph for Question Nos. 1 to 3

Sol. $f(x) \quad \frac{x \quad a}{bx^2 \quad cx \quad 2}$

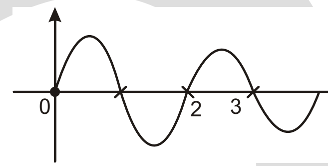
$f(\;\;1) \quad 0 \qquad a \quad 1$

If $y \quad 1$ is asymptotes, then $b \quad 0$ and $c \quad 1 \qquad f(x) \quad \frac{x \quad 1}{x \quad 2}$ and $g(x) \quad \frac{1 \quad 2x}{x \quad 1}$





Paragraph for Question Nos. 4 to 6



Sol. $y \quad e^{\ x} \sin x$

$$\frac{dy}{dx} \quad e^{\ x}[\cos x \quad \sin x] \quad 0$$

$$\tan x \quad 1 \qquad x \quad n \qquad \overline{4}$$

So, $(i.e., 0 \quad x \qquad)$

$$I \qquad e^{\ x} \sin x\, dx \quad \frac{e^{\ x}}{2}[\sin x \quad \cos x] \quad c$$

$$S_j \quad \left| \ {}_{j}^{(j\ 1)} \ e^{\ x} \sin x\, dx \right| \quad \left| \frac{e^{\ x}}{2}[\sin x \quad \cos x] \right|_{j}^{(j\ 1)} \quad \frac{e^{\ j}}{2}(e \qquad 1)$$

4. Put $j \quad 0, S_0 \quad \frac{1 \quad e}{2}$

5. $\frac{S_{2009}}{S_{2010}} \quad \frac{e^{\ 2009}}{e^{\ 2010}} \quad e$

6. $\frac{S_{j\ 1}}{S_j} \quad e$

$$\because \qquad {}_{j\ 0} S_j \quad \frac{S_0}{1 \quad e} \quad \frac{\frac{1 \quad e}{2}}{1 \quad e} \quad \frac{1 \quad e}{2(e \qquad 1)}$$

Exercise-5 : Subjective Type Problems

1. $f(x) \quad x^2$

$$A \quad 2 \ {}_0^1 (\sqrt{2 \quad x^2} \quad x^2)\, dx \quad \overline{2} \quad \frac{1}{3}$$

2. $f(x) \quad 2 \ln x$

$$A \quad {}_0^1 (\ x^3 \quad 6x^2 \quad 11x \quad 6 \quad 2 \ln x)\, dx \quad \frac{17}{4}$$

4. At $x \quad 0, y \quad 0$

$x \quad 5y \quad y^5 \quad 0 \qquad 1 \quad 5y \quad 5y^4 y \quad 0$

at $x \quad 0, y \quad 0$

$$y \qquad \frac{1}{5}$$

Equation of tangent : $y \quad \frac{x}{5}$, equation of normal : $y \quad 5x$

Area $\frac{1}{2} \quad 5 \quad 26 \quad 65$

5. $[x]^2 \quad [y]^2$

$[y] \quad [x]$

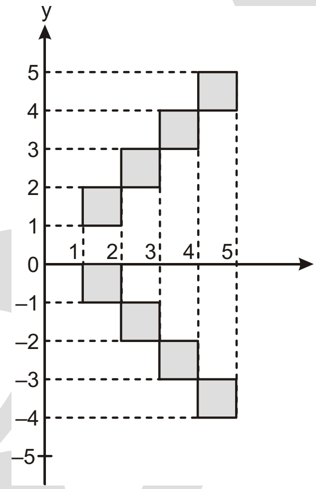
$[y] \quad 1, \quad 1 \quad x \quad 2$

$2, \quad 2 \quad x \quad 3$

$3, \quad 3 \quad x \quad 4$

$4, \quad 4 \quad x \quad 5$

$5, \quad x \quad 5$



Now, when, $x \quad [1,2)$

then $y \quad [\ 1,0) \quad [1,2)$

when $x \quad [2,3]$

then $y \quad [\ 2, \quad 1) \quad [2,3)$

when $x \quad [3,4)$

then $y \quad [\ 3, \quad 2) \quad [3,4)$

when $x \quad [4,5)$

then $y \quad [\ 4, \quad 3) \quad [4,5)$

when $x \quad 5$

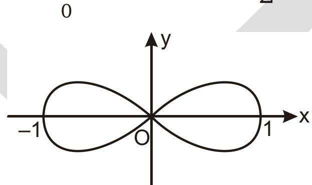
then $y \quad [\ 5, \quad 4) \quad [5,6)$

Hence, required area $2(4) \quad 8$ sq. unit

6. $f(x) \quad f(z) \quad f(x \quad z)$ and $f(0) \quad 0$ and $f\,(0) \quad 4 \qquad f(x) \quad 4x$

So, area bounded $\int_0^4 (4x \quad x^2)\,dx$

7. Required area $4\int_0^1 \sqrt{x^2 \quad x^6}\,dx \quad \int_0^1 4x\sqrt{1 \quad x^4}\,dx \quad \frac{}{2}$



Put $x^2 \quad \sin$

8. Ar $4\int_0^1 (1 \quad x^{2/5})\,dx \quad 4\left[x \quad \frac{5}{7}x^{7/5}\right]_0^1 \quad \frac{8}{7}$

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DIFFERENTIAL EQUATIONS

Exercise-1 : Single Choice Problems

4. $\frac{dy}{dx} \quad \frac{xy}{x^2 \quad y^2}$

Let $\frac{y}{x} \quad t \qquad \frac{dy}{dx} \quad t \quad x\frac{dt}{dx}$

$\frac{dx}{x} \qquad \frac{1 \quad t^2}{t^3} dt$; $\quad \ln y \quad \frac{x^2}{2y^2} \quad \frac{1}{2} \qquad (\because y(1) \quad 1)$

5. $\frac{y}{\sqrt{1 \quad y^2}} dy \qquad dx$

$\sqrt{1 \quad y^2} \quad x \quad c$

$(x \quad c)^2 \quad y^2 \quad 1$

6. $\frac{dy}{y} \qquad \frac{dx}{(x \quad 3)^2}$

$\ln y \quad \frac{1}{x \quad 3} \quad c$

7. Let $f(x) \quad y$

$\frac{dy}{dx} \quad 2xy \quad \frac{e^{x^2}}{(x \quad 1)^2}$

8. Let $x^2 y^2 \quad t$

$2xy^2 \quad 2x^2 y\frac{dy}{dx} \quad \frac{dt}{dx}$

$\frac{dt}{dx} \quad \tan t$

9. $y \quad (C_1 \cos C_2) \cos x \quad (C_5 \quad C_1 \sin C_2) \sin x \quad C_3 e^{C_4} e^{\quad x}$

$y \quad A \cos x \quad B \sin x \quad Ce^{\quad x}$

$\because \quad C_1, C_2, C_3, C_4$ are arbitrary constants.

10. $y \quad e^{(\quad 1)x}$

$y \quad e^{(\quad 1)x}(\quad 1)$

$y \quad e^{(\quad 1)x}(\quad 1)^2$

12. $\frac{dy}{dx} \quad 1 \quad \frac{f\,(x)}{f(x)} \quad y \quad f(x)$

I.F. $e^{\quad 1 \quad \frac{f\,(x)}{f(x)} \quad dx} \quad \frac{e^{\quad x}}{f(x)}$

$\frac{ye^{\quad x}}{f(x)} \quad e^{\quad x} dx \quad C \qquad \frac{ye^{\quad x}}{f(x)} \quad e^{\quad x} \quad C$

13. Equation of tangent at $t, \frac{t^2}{2}$ is

$$y \quad tx \quad \frac{t^2}{2} \qquad t \quad \frac{dy}{dx}$$

Differential equation is

$$\frac{dy}{dx}^{\quad 2} \quad 2x\frac{dy}{dx} \quad 2y \quad 0$$

14. Let $x \quad y \quad t \quad 1 \quad \frac{dy}{dx} \quad \frac{dt}{dx}; \quad \frac{1}{t^3}\frac{dt}{dx} \quad \frac{x}{t^2} \quad x \qquad \frac{e^{\quad x^2}}{(x \quad y)^2} \quad e^{\quad x^2} \quad C$

15. $\frac{dy}{dx} \quad 2y \tan x \quad \tan^2 x$

I.F. $e^{\quad 2 \quad \tan x\, dx} \quad \cos^2 x$

$y \cos^2 x \quad \sin^2 x\, dx \quad \frac{1}{2} \quad (1 \quad \cos 2x)\, dx$

16. $f(x) \quad 2e^x \quad 1$

17. Let $\frac{dy}{dx} \quad t$ then $\frac{d^2y}{dx^2} \quad \frac{dt}{dx}$

$\frac{dt}{dx} \quad \frac{2tx}{x^2 \quad 1}$

$t \quad \frac{dy}{dx} \quad 3(x^2 \quad 1) \qquad (\because y\,(0) \quad 3)$

$$y \quad x^3 \quad 3x \quad 1 \qquad\qquad (\because y(0) \quad 1)$$

18. $cy^2 \quad 2x \quad c$

$$2cyy \quad 2 \qquad c \quad \frac{1}{yy}$$

$$y^2 \quad 2x\,yy \quad 1$$

19. Let $\operatorname{cosec} y \quad t$

$$\operatorname{cosec} y \cot y \, dy \quad dt$$

$$\frac{dt}{dx} \quad \frac{t}{x} \quad \frac{1}{x^2}$$

$$\frac{t}{x} \quad \frac{1}{2x^2} \quad c \qquad \frac{1}{x \sin y} \quad \frac{1}{2x^2} \quad c$$

20. $\dfrac{xdy \quad ydx}{x^2} \quad \dfrac{\sqrt{x^2 \quad y^2}}{x^2} dx$

$$\frac{d\left(\frac{y}{x}\right)}{\sqrt{1 \quad \left(\frac{y}{x}\right)^2}} \qquad \frac{dx}{x}$$

21. $\lim\limits_{t \quad x} \dfrac{t^3 f(x) \quad x^3 f(t)}{t^2 \quad x^2} \quad \dfrac{1}{2} \qquad 3x\, f(x) \quad x^2 f\,(x) \quad 1$

$$\frac{dy}{dx} \quad \frac{3y}{x} \quad \frac{1}{x^2} \qquad y \quad \frac{1}{4x} \quad \frac{3}{4}x^2 \qquad (\because f(1) \quad 1)$$

22. $\dfrac{2dp(t)}{dt} \quad p(t) \quad 900$

$$2 \quad \frac{dp(t)}{p(t) \quad 900} \qquad dt$$

$$2\ln|900 \quad p(t)| \quad t \quad c$$

$$p(t) \quad 900 \quad 50e^{t/2} \qquad\qquad (\because p(0) \quad 850)$$

$$p(t) \quad 0 \qquad t \quad 2\ln 18$$

23. $\dfrac{\sin y}{\cos^2 y}\dfrac{dy}{dx} \quad \dfrac{\tan x}{\cos y} \quad \sec x$

Let $\dfrac{1}{\cos y} \quad t \qquad \dfrac{\sin y}{\cos^2 y} dy \quad dt$

$$\frac{dt}{dx} \quad t \ \tan x \quad \sec x$$

$$t \quad \sec x \qquad \sec^2 x \; dx \quad C$$

$$\sec y \quad \sec x \quad \tan x \quad C$$

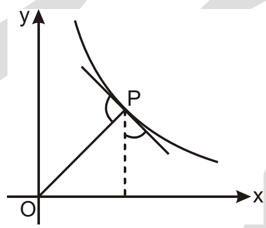
24. $\dfrac{dy}{dx} \quad (4x \quad y \quad 1)^2$

Let $4x \quad y \quad 1 \quad t$

$$4 \quad \frac{dy}{dx} \quad \frac{dt}{dx} \qquad \frac{dt}{dx} \quad t^2 \quad 4$$

$$\frac{1}{2}\tan^{1}\frac{t}{2} \qquad x \quad C$$

25. $\tan \left| \dfrac{\dfrac{dy}{dx} \quad \dfrac{y}{x}}{1 \quad \dfrac{y}{x}\dfrac{dy}{dx}} \right| \quad \dfrac{dx}{dy}$



$$\frac{dy}{dx}^2 \qquad \frac{2y}{x}\frac{dy}{dx} \quad 1$$

26. I.F. $e^{\frac{1}{x}dx} \quad e^{\ln x} \quad x$

$$y \;\; x \qquad x^3 dx \quad \frac{x^4}{4} \quad c$$

27. $x^3 dy \quad 3x^2 y\, dx \quad y^2 dx \quad 2xy\, dy$

$$d(x^3 y) \quad d(xy^2)$$

$$d(x^3 y) \qquad d(xy^2) \qquad x^3 y \quad xy^2$$

Exercise-2 : One or More than One Answer is/are Correct

1. $\dfrac{xdy \quad ydx}{x^2} \quad \dfrac{x^2 \quad 2}{x^2}dx$

$$d\frac{y}{x} \qquad 1 \quad \frac{2}{x^2} \quad dx$$

$$y \quad x^2 \quad 2x \quad 2 \qquad\qquad (\because f(1) \quad 1)$$

2. I.F. $x \sec x$; $yx \sec x \quad \tan x \quad c$

3. Put $y \quad h$

$$x[f(x \quad h) \quad f(x \quad h)] \quad h[f(x \quad h) \quad f(x \quad h)] \quad 2(x^2 h \quad h^3)$$

or $\lim\limits_{h \quad 0} x \dfrac{[f(x \quad h) \quad f(x \quad h)]}{h} \quad [f(x \quad h) \quad f(x \quad h)] \quad \lim\limits_{h \quad 0} 2(x^2 \quad h^2)$

$$xf(x) \quad f(x) \quad x^2 \qquad f(x) \quad x^2 \quad x$$

4. L.D.E., I.F. $1 \quad \sin^2 x$; $(1 \quad \sin^2 x) f \quad \sin x \quad C, C \quad 0$

5. $2ydx \quad 2xdy \quad (2x^2 y^{3/2} dx \quad x^3 y^{1/2} dy) \quad 0$

$$2d(xy) \quad \frac{2}{3} d(x^3 \quad y^{3/2}) \quad 0$$

6. I.F. $\dfrac{1}{\sin^3 x}$; $\dfrac{y}{\sin^3 x} \quad \dfrac{\sin 2x}{\sin^3 x} dx$; $\dfrac{y}{\sin^3 x} \quad 2 \quad \cot x \ \operatorname{cosec} x \, dx \quad 2 \operatorname{cosec} x \quad c$

$y \quad 2\sin^2 x \quad 4\sin^3 x \quad \because y \ \dfrac{}{2} \quad 2$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Sol. $x \int_0^x g(t)\,dt \quad \int_0^x (1 \quad t) g(t)\,dt \quad x^4 \quad x^2$

differentiate w.r.t. 'x'

$$x\,g(x) \quad \int_0^x g(t)\,dt \quad (1 \quad x)\,g(x) \quad 4x^3 \quad 2x \qquad \ldots(1)$$

1. From (1)

$$\int_0^x g(t)\,dt \quad g(x) \quad 4x^3 \quad 2x$$

Let $g(x) \quad g(x) \quad 12x^2 \quad 2 \qquad \dfrac{dy}{dx} \quad y \quad 12x^2 \quad 2$ $\qquad (\because y \quad g(x))$

2. Put $x \quad 0$ in (1) we get $g(0) \quad 0$

Paragraph for Question Nos. 3 to 5

3. $f(g(x)) \quad e^{2x}$

$\dfrac{x \ [f(g(x))]}{f(g(x))} \quad \dfrac{[g(f(x))]}{g[f(x)]}$

$$g(f(x)) \quad e^{\;x^2}$$

$$H(x) \quad e^{\;(x \quad 1)^2 \quad 1}$$

4. $f(g(0)) \quad g(f(0)) \quad 2$

5. $H(x)_{\max} \quad e$

Paragraph for Question Nos. 6 to 8

Sol. $g(x) \quad \lim_{h \quad 0} \dfrac{g(x \quad h) \quad g(x)}{h} \quad \lim_{h \quad 0} \dfrac{e^x g(h)}{h} \quad \lim_{h \quad 0} g(x) \quad \dfrac{e^h \quad 1}{h}$ $\quad (\because g(0) \quad 2)$

$$2e^x \quad g(x)$$

$$\frac{dy}{dx} \quad y \quad 2e^x \qquad y \quad 2x\,e^x \quad ce^x$$

$$y \quad 2x\,e^x \qquad\qquad (\because g(0) \quad 0)$$

Exercise-4 : Matching Type Problems

1. (A) $y\dfrac{dx}{dy} \quad x \quad y^2\dfrac{dx}{dy} \quad 1 \quad (y^2 \quad y)\dfrac{dx}{dy} \quad (1 \quad x) \quad \dfrac{dx}{1 \quad x} \quad \dfrac{dy}{y(y \quad 1)}$

(B) $y\dfrac{dx}{dy} \quad 2x \quad 10y^3 \qquad \dfrac{dx}{dy} \quad \dfrac{2}{y} \quad x \quad 10y^2$

I.E. $e^{\;\frac{2}{y}dy} \quad e^{2\ln y} \quad y^2$

$d(xy^2) \quad 10y^4$

(C) $\dfrac{dy}{dx} \quad y$

$y \;\; y \quad (3y\;)^2$

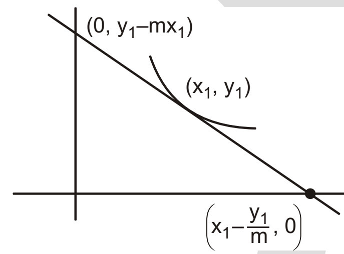
$\dfrac{y}{y} \quad \dfrac{3y}{y}$ then integrate it.

(D) Put $x^2 \quad t$

$\dfrac{dt}{dy} \quad \dfrac{t}{y} \quad \dfrac{1}{y^3}$

then solve it.

Exercise-5 : Subjective Type Problems



1. $x^{a} \quad y \quad\quad {}^{a}; \quad\quad y \quad \frac{a}{x^{a}}$

$$\frac{dy}{dx} \quad a \quad {}^{a} x \quad {}^{a \quad 1} \quad\quad a\frac{x^{a} \quad y}{x^{a \quad 1}}$$

$$m \quad \frac{ay_1}{x_1}$$

$$A \quad \frac{1}{2}|y_1 \quad mx_1|\left|x_1 \quad \frac{y_1}{m}\right| \quad \frac{1}{2}y_1x_1(1 \quad a)^2$$

$$\frac{1}{2} \quad {}^{a} \quad x_1^{1 \quad a}(1 \quad a)^2$$

For A to be constant $1 \quad a \quad 0$.

2. $\frac{dy}{dx} \quad xy(1 \quad y)$

$$\frac{dy}{(1 \quad y)\,y} \quad\quad x\,dx$$

$$\frac{2y}{1 \quad y} \quad e^{\frac{x^2}{2}} \qquad\qquad (\because f(0) \quad 1)$$

$$f(2) \quad \frac{e^2}{2 \quad e^2}$$

3. $y^2 \quad \cos^2 x \quad 2$

$$2y\frac{dy}{dx} \quad\quad \sin 2x$$

$$y\frac{d^2y}{dx^2} \quad \frac{dy}{dx}^{\;2} \quad\quad \cos 2x$$

$$y^4 \quad y^3\frac{d^2y}{dx^2} \quad (\cos^2 x \quad 2)^2 \quad (\cos^2 x \quad 2) \quad \frac{dy}{dx}^{\;2} \quad \cos 2x \quad 6$$

4. $\lim\limits_{t \quad x \quad 1} \frac{t^2 f(x \quad 1) \quad (x \quad 1)^2 f(t)}{f(t) \quad f(x \quad 1)} \quad 1$

$$\lim_{t \quad x \quad 1} \frac{2tf(x \quad 1) \quad (x \quad 1)^2 f\,(t)}{f\,(t)} \quad 1$$

$$[x \quad 1][2f(x \quad 1) \quad (x \quad 1)f\,(x \quad 1)] \quad f\,(x \quad 1)$$

$$f\ (x) \quad \frac{2xf(x)}{x^2 \quad 1}$$

$$f(x) \quad x^2 \quad 1$$

$$\lim_{x \quad 1} \frac{\ln(f(x)) \quad \ln 2}{x \quad 1} \qquad \lim_{x \quad 1} \frac{f\ (x)}{f(x)} \quad 1$$

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QUADRATIC EQUATIONS

Exercise-1 : Single Choice Problems

1. Let $3^{x/2} \quad a, 2^{y} \quad b$

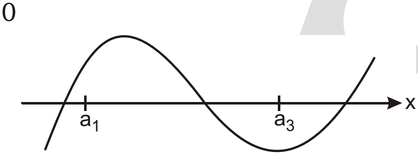
$a^2 \quad b^2 \quad 77, a \quad b \quad 7 \quad a \quad 3^{x/2} \quad 9 \quad x \quad 4$

$b \quad 2^{y} \quad 2 \quad y \quad 1$

2. $f(x) \quad \sum_{i\ 1}^{3} (x \quad a_i) \quad \sum_{i\ 1}^{3} a_i \quad 3x \quad (x \quad a_1)(x \quad a_2)(x \quad a_3) \quad (a_1 \quad a_2 \quad a_3) \quad 3x$

$f(a_1) \quad a_2 \quad a_3 \quad 2a_1 \quad 0 \qquad (a_1 \quad a_2 \quad a_3)$

$f(a_3) \quad a_1 \quad a_2 \quad 2a_3 \quad 0$



3. $x^4 \quad 2ax^2 \quad x \quad a^2 \quad a \quad 0$

$a^2 \quad a(2x^2 \quad 1) \quad x^4 \quad x \quad 0$

$$a \quad \frac{2x^2 \quad 1 \quad (2x \quad 1)}{2}$$

$$a \quad x^2 \quad x, \quad a \quad x^2 \quad x \quad 1$$

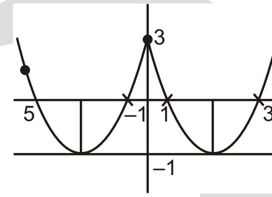
$$a \quad \frac{1}{4}, \qquad a \quad \frac{3}{4} \qquad (\because x \quad R)$$

4. $x^3 \quad 3x^2 \quad 4x \quad 12 \quad 0$

Equation whose roots are $\quad 3, \quad 3, \quad 3$ is

$(x \quad 3)^3 \quad 3(x \quad 3)^2 \quad 4(x \quad 3) \quad 12 \quad 0$

$$f(x) \quad x^3 \quad 6x^2 \quad 5x \quad 0 \begin{matrix} -3 \\ -3 \\ -3 \end{matrix}$$

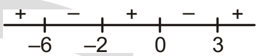


5. $|K \quad 1| \quad 3$

$3 \quad K \quad 1 \quad 3$

$2 \quad K \quad 4$

6. $\frac{x}{x \quad 6} \quad \frac{1}{x} \quad 0 \qquad \frac{x^2 \quad x \quad 6}{x(x \quad 6)} \quad 0 \qquad \frac{(x \quad 3)(x \quad 2)}{x(x \quad 6)} \quad 0$



$x \quad (\quad 6, \quad 2] \quad (0, 3]$

7. $P(x) \quad x^4 \quad 8x^2 \quad 15 \quad 2x^3 \quad 6x \quad (x^2 \quad 3)(x^2 \quad 5) \quad 2x(x^2 \quad 3)$

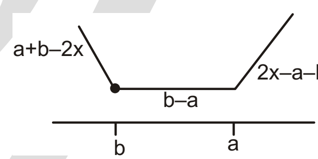
$(x^2 \quad 3)(x^2 \quad 2x \quad 5)$

$Q(x) \quad (x \quad 2)(x^2 \quad 2x \quad 5)$

8. $a \quad 1, h \quad \frac{}{2}, b \quad 1, g \quad \frac{5}{2}, f \quad \frac{7}{2}, c \quad 6$

$\begin{vmatrix} 1 & /2 & 5/2 \\ /2 & 1 & 7/2 \\ 5/2 & 7/2 & 6 \end{vmatrix} \quad 0 \qquad \frac{5}{2}, \frac{10}{3}$

10. Let $f(x) \quad |x \quad a| \quad |x \quad b|$



Suppose $a \quad b$

$f(0) \quad f(1) \quad f(\quad 1)$

$f(x)$ const. in $[b, a]$

So, $b \quad 1 \quad a \quad 1$

$a \quad b \quad 2$

Minimum $|a \quad b| \quad 2$

12. $y \quad \frac{x^2 \quad 2x \quad c}{x^2 \quad 4x \quad 3c} \qquad (y \quad 1)x^2 \quad 2(2y \quad 1)x \quad (3cy \quad c) \quad 0 \qquad (D \quad 0)$

$D \quad 0 \quad y \quad R$ and $D \quad 0$

But at $c \quad 0$ and 1 there will be common factors among numerator and denominator.

$c(c \quad 1) \quad 0$

13. $f(t) \quad t^2 \quad mt \quad 2 \quad 0$

$f(2) \quad 0$

$4 \quad 2m \quad 2 \quad 0 \qquad m \quad 3$

But $\quad \dfrac{3|x|}{9 \quad |x|^2} \quad \dfrac{3}{\dfrac{9}{|x|} \quad |x|} \quad \dfrac{3}{6}$ (by A.M. G.M. in equality)

$$\left(\frac{3|x|}{9 \quad x^2}\right)^m \quad \frac{1}{2^m} \quad 1 \qquad [\because m \quad 3]$$

So, $\quad \left(\dfrac{3|x|}{9 \quad x^2}\right)^m \quad\quad 0$

14. $x^2(x^6 \quad 24x^5 \quad 18x^3 \quad 39) \quad 3 \quad 5 \quad 7 \quad 11$

If 'x' is integer, then there is no value of 'x'.

15. $m^4 \quad \dfrac{1}{m^4} \quad 119$

$$m^2 \quad \frac{1}{m^2} \quad 11$$

$$\left(m \quad \frac{1}{m}\right)^2 \quad 9$$

$$\left|m^3 \quad \frac{1}{m^3}\right| \quad \left|m \quad \frac{1}{m}\right|\left|m^2 \quad \frac{1}{m^2} \quad 1\right| \quad |3 \quad 12| \quad 36$$

16. $ax^2 \quad 2bx \quad c \quad 0$

$ax^2 \quad 2cx \quad b \quad 0$

By condition of common root

Common root $\quad \dfrac{1}{2}$ and $\dfrac{a}{4} \quad b \quad c \quad 0$

$\dfrac{2c}{a}$ and $\quad \dfrac{2b}{a}$

Equation whose roots are and is

$$x^2 \quad \frac{2c}{a} \quad \frac{2b}{a} \quad x \quad \frac{4bc}{a^2} \quad 0$$

$$2a^2x^2 \quad a^2x \quad 8bc \quad 0 \qquad\qquad \frac{a}{4} \quad b \quad c \quad 0$$

17. $9x^2(x \quad 1) \quad 1(x \quad 1) \quad 0$

$$x \quad 1, x \quad \frac{1}{3}, \quad \frac{1}{3}$$

$$\cos \quad 1, \cos \quad \frac{1}{3}, \cos \quad \frac{1}{3}$$

$$0,$$

$$(\quad , \cos \quad) \quad (\quad , 1) \quad \text{centre}$$

$$2\sin^{1} \tan\frac{}{4}, 4 \quad 2\frac{}{2}, 4 \quad (\quad , 4)$$ point lies on the circle.

Radius is 3.

18. $y \quad \dfrac{11x^2 \quad 12x \quad 6}{x^2 \quad 4x \quad 2}$

$(y \quad 11)x^2 \quad (4y \quad 12)x \quad (2y \quad 6) \quad 0 \quad x \quad R$

$D \quad 0$

$(4y \quad 12)^2 \quad 4(y \quad 11)(2y \quad 6) \quad 0$

$y^2 \quad 20y \quad 51 \quad 0$

$(y \quad 17)(y \quad 3) \quad 0$

$y \quad (\quad , \quad 17] \quad [\ 3, \quad)$

19. $\dfrac{x \quad 3}{x^2 \quad x \quad 2} \quad \dfrac{1}{x \quad 4} \quad 0$

$\dfrac{(x^2 \quad x \quad 12) \quad (x^2 \quad x \quad 2)}{(x^2 \quad x \quad 2)(x \quad 4)} \quad 0$

$\dfrac{10}{(x \quad 2)(x \quad 1)(x \quad 4)} \quad 0$

$(x \quad 1)(x \quad 2)(x \quad 4) \quad 0$

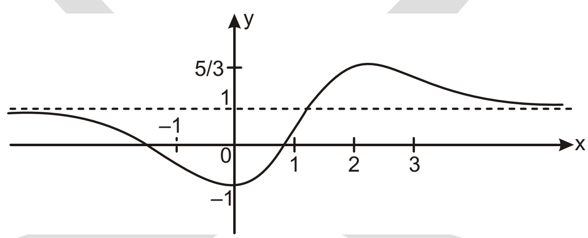
$x \quad (\quad , \quad 1) \quad (2, 4)$

20. $x \quad 4 \quad 3i$

$(x \quad 4)^2 \quad 9 \quad x^2 \quad 8x \quad 25 \quad 0$

$x^3 \quad 4x^2 \quad 7x \quad 12 \quad (x^2 \quad 8x \quad 25)(x \quad 4) \quad 88 \quad 88$

21. $f(x) \quad \dfrac{x^2 \quad x \quad 1}{x^2 \quad x \quad 1}$



By graph : Min. $f(0)$; Max. $f(2)$

If $x \quad [\ 1, 3]$; $y_{\max.} \quad \dfrac{5}{3}$

22. By graph min. $f(0)$; max. $f(1)$

if $x \quad [\ 1, 1]$; $y \quad [\ 1, 1]$

23.
$$\frac{1}{x \quad p} \quad \frac{1}{x \quad q} \quad \frac{1}{r}$$

$x^2 \quad x(p \quad q \quad 2r) \quad pq \quad r(p \quad q) \quad 0$

If one root is . Then other root must be .

$$p \quad q \quad 2r \quad 0 \qquad r \quad \frac{p \quad q}{2}$$

Product of the roots $pq \quad r(p \quad q) \quad pq \quad \frac{(p \quad q)^2}{2} \qquad \frac{(p^2 \quad q^2)}{2}$

24. If $a_1 x^2 \quad b_1 x \quad c_1 \quad 0$ has one root .

$a_2 x^2 \quad b_2 x \quad c_2 \quad 0$ has one root $\frac{1}{}$.

$c_2 x^2 \quad b_2 x \quad a_2 \quad 0$ has one root

Condition of common root is

$$(a_1 a_2 \quad c_1 c_2)^2 \quad (a_1 b_2 \quad b_1 c_2)(a_2 b_1 \quad b_2 c_1)$$

25. If $^2 \quad 5 \quad 3 \quad 0$ and $^2 \quad 5 \quad 3 \quad 0$

$x^2 \quad 5x \quad 3 \quad 0$ has two roots and .

5, 3

Sum of the roots $\quad - \quad - \quad \frac{(\qquad)^2 \quad 2}{} \quad \frac{19}{3}$

Product of roots $\quad - \quad - \quad 1$

Equation whose roots are — and — is $3x^2 \quad 19x \quad 3 \quad 0$

26. $| \qquad | \ | \ _1 \qquad _1|$

$a^2 \quad 4b \quad b^2 \quad 4a$

$a^2 \quad b^2 \quad 4(b \quad a)$

$(a \quad b)(a \quad b \quad 4) \quad 0$

$a \quad b \qquad a \quad b \quad 4 \quad 0$

27. $\tan(\ _1 \qquad _2 \qquad _3 \qquad _4) \quad \frac{S_1 \quad S_3}{1 \quad S_2 \quad S_4} \quad \frac{\sin 2 \qquad \cos}{1 \quad \cos 2 \qquad \sin} \quad \frac{\cos \ (2\sin \qquad 1)}{\sin \ (2\sin \qquad 1)} \quad \cot$

28. $(a^2 \quad b^2)x^2 \quad 2x(bd \quad ac) \quad (c^2 \quad d^2) \quad 0$

$(a^2 x^2 \quad 2acx \quad c^2) \quad (b^2 x^2 \quad 2bdx \quad d^2) \quad 0$

$(ax \quad c)^2 \quad (bx \quad d)^2 \quad 0$

This equation has imaginary roots.

29. If , are roots of $ax^2 \quad bx \quad c \quad 0$

$2, \quad 2$ are roots of $a(x \quad 2)^2 \quad b(x \quad 2) \quad c \quad 0$

$ax^2 \quad x(b \quad 4a) \quad 4a \quad 2b \quad c \quad 0$

30. 1

2

3

$(\quad 1)(\quad 1) \quad 2$

atleast one root is positive.

31. $D \quad 0 \quad 3k^2 \quad 8k \quad 16 \quad 0 \quad 4 \quad k \quad \frac{4}{3}$

$^2 \quad ^2 \quad (\quad)^2 \quad 2 \quad k^2 \quad 2(k^2 \quad 2k \quad 4) \quad k^2 \quad 4k \quad 8 \quad 12 \quad (k \quad 2)^2$

32. $P(x) \quad (x \quad 2)Q_1(x) \quad R(x)$

$Q(x) \quad (x \quad 2)Q_2(x) \quad R(x)$

$P(2) \quad Q(2)$

33. $a \quad b \quad a$ and $ab \quad b$

if $b \quad 0, a \quad 1$ and $b \quad 2$

$x^2 \quad ax \quad b \quad x^2 \quad x \quad 2 \quad x \quad \frac{1}{2}^2 \quad \frac{9}{4}$

34. $x^2 \quad \frac{b}{a} \quad \frac{c}{a} \quad x \quad \frac{c}{a}^3 \quad 0$

$x^2 \quad (\quad) \quad x \quad ^3 \quad ^3 \quad 0 \begin{cases} ^2 \\ ^2 \end{cases}$

35. $x^2 \quad 2(a \quad b \quad c)x \quad 6k(ab \quad bc \quad ca) \quad 0$

$D \quad 0$

$4(a \quad b \quad c)^2 \quad 24k(ab \quad bc \quad ca) \quad 0$

$k \quad \frac{1}{6} \quad \frac{a^2 \quad b^2 \quad c^2}{ab \quad bc \quad ca} \quad 2$

also, $|a \quad b| \quad c, |b \quad c| \quad a, |c \quad a| \quad b$

$a^2 \quad b^2 \quad c^2 \quad 2(ab \quad bc \quad ca) \quad 0$

$\frac{a^2 \quad b^2 \quad c^2}{ab \quad bc \quad ca} \quad 2 \quad k \quad \frac{2}{3}$

36. $9|x|^2 \quad 18|x| \quad 5 \quad 0$

$(3|x| \quad 1)(3|x| \quad 5) \quad 0$

$x \quad \frac{1}{3}, \quad \frac{5}{3}$

and $x^2 \quad x \quad 2 \quad 0 \qquad (x \quad 2)(x \quad 1) \quad 0 \qquad x \quad 1$ or $x \quad 2$

37. Difference of roots is same in both equation

$b^2 \quad c \quad B^2 \quad C$

38. $|x \quad p| \quad |x \quad 15| \quad |x \quad p \quad 15| \quad (x \quad p) \quad (x \quad 15) \quad (x \quad P \quad 15) \quad 30 \quad x$

min. 15

39. $4p(q \quad r)x^2 \quad 2q(r \quad p)x \quad r(p \quad q) \quad 0$ $= -1/2$

$= -1/2$

If $x \quad \frac{1}{2}$ is also the root of $4x^2 \quad 2x \quad m \quad 0$

$m \quad 2$

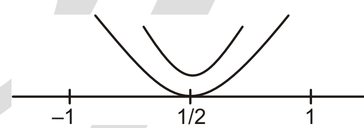
40. Let $\cos x \quad t$

$t \quad [\ 1, 1]$

$kt^2 \quad kt \quad 1 \quad 0 \quad t \quad [\ 1, 1]$

Case I : $k \quad 0$

x coordinate of vertex is $\frac{1}{2}$.



$f \quad \frac{1}{2} \qquad 0$

$\frac{k}{4} \quad \frac{k}{2} \quad 1 \quad 0$

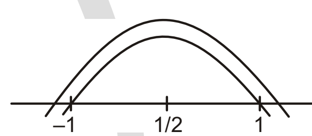
$k \quad 4$

Also, $k \quad 0$

$k \quad [0, 4]$

Case II : $k \quad 0$

$f(1) \quad 0$ and $f(\ 1) \quad 0$



$k \quad k \quad 1 \quad 0$ and $k \quad k \quad 1 \quad 0$

$1 \quad 0$ and $k \quad \frac{1}{2}$

Also, $k \quad 0$

$k \quad \frac{1}{2}, 0 \qquad k \quad \frac{1}{2}, 4$

41. $\frac{1 \quad x}{1 \quad x} \quad y \qquad x \quad \frac{y \quad 1}{y \quad 1}$

$H(y) \quad 3\left(\frac{y \quad 1}{y \quad 1}\right)^3 \quad 2\left(\frac{y \quad 1}{y \quad 1}\right) \quad 5 \quad 0$

$H(y) \quad 3(y \quad 1)^3 \quad 2(y \quad 1)(y \quad 1)^2 \quad 5(y \quad 1)^3 \quad 0$

$H(y) \quad 3(y^3 \quad 3y^2 \quad 3y \quad 1) \quad 2(y \quad 1)(y^2 \quad 2y \quad 1) \quad 5(y^3 \quad 3y^2 \quad 3y \quad 1) \quad 0$

$H(y) \quad 3(y^3 \quad 3y^2 \quad 3y \quad 1) \quad 2(y^3 \quad y^2 \quad y \quad 1) \quad 5(y^3 \quad 3y^2 \quad 3y \quad 1) \quad 0$

$H(y) \quad 3y^3 \quad 2y^2 \quad 13y \quad 2 \quad 0$

$H\,(x) \quad 9x^2 \quad 4x \quad 13 \qquad D \quad 0$

$H(x) \quad 0 \quad x \quad 0$

Hence, it has one –ve real root.

42. $(\ ^2 \qquad 2)\,x^2 \quad (\qquad 2)\,x \quad 1 \quad 0 \quad x \quad R$

$^2 \qquad 2 \quad 0 \quad (\qquad 2)^2 \quad 4(\ ^2 \qquad 2) \quad 0$

$(\qquad 2)(\qquad 1) \quad 0 \quad 5\ ^2 \quad 8 \quad 4 \quad 0$

$(\ \ 2, 1) \qquad 2, \frac{2}{5}$

$2, \frac{2}{5}$

2 is also the solution of this equation.

43. $1, \quad 1, \quad 1, \quad 1\,(\text{as})\,(\qquad 1)^2 \quad (\qquad 1)^2 \quad (\qquad 1)^2 \quad (\qquad 1)^2 \quad 0$

The roots of given equation is equal to 1.

$S_2 \quad \frac{a_2}{a_0} \quad 6$

44. $|x \quad 1| \quad |x \quad 2| \quad |x \quad 3| \quad 6$

Case I : $x \quad 3$

$3x \quad 6 \quad 6 \qquad x \quad 4$

Case II : $2 \quad x \quad 3$

$x \quad 6$ (Not possible)

Case III : $1 \quad x \quad 2$

$4 \quad x \quad 6$

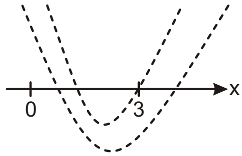
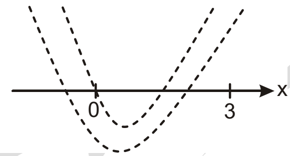
$x \qquad 2$ (Not possible)

Case IV : $x \quad 1$

$6 \quad 3x \quad 6$

$x \quad 0$

$x \quad (\qquad, 0] \quad [4, \quad)$

45.  

Case-I : $f(0) \quad 0 \quad f(3) \quad 0$ **Case-II :** $f(3) \quad 0 \quad f(0) \quad 0$

46. $x^3 \quad 3px^2 \quad 3qx \quad r \quad 0$

$\frac{2}{} \quad \frac{1}{} \quad \frac{1}{}$ (∵ , , are in H.P.)

$\frac{3}{} \quad \frac{1}{} \quad \frac{1}{} \quad \frac{1}{} \quad \underline{\qquad\qquad}$

$\frac{r}{q}$ which satisfy the given equation.

47. $4y^2 \quad 4xy \quad (x \quad 6) \quad 0 \quad y \quad R$

$D \quad 0 \qquad x^2 \quad x \quad 6 \quad 0$

48. $\log_{\cos x^2}(3 \quad 2x) \quad \log_{\cos x^2}(2x \quad 1)$

$0 \quad \cos x^2 \quad 1 \quad 3 \quad 2x \quad 2x \quad 1 \quad 3 \quad 2x \quad 0 \quad 2x \quad 1 \quad 0$

$x \quad 1 \qquad x \quad 3/2 \qquad x \quad 1/2$

49. $px^2 \quad qx \quad r \quad 0$

0

$(x \quad)^2 \quad (x \quad)^2 \quad (\quad)x^2 \quad 4 \quad x \quad (\quad) \quad 0$

Product of roots $\quad 0$

$D \quad 16 \quad {}^2 \quad {}^2 \quad 4 \quad (\quad)^2 \quad 4 \quad (\quad)^2 \quad 0$

50. $x^3 \quad 2x^2 \quad 4x \quad 4 \quad 0 \rightarrow a, b, c$

$4x^3 \quad 4x^2 \quad 2x \quad 1 \quad 0 \rightarrow 1/a, 1/b, 1/c$

$q \quad 1, r \quad \frac{1}{2}, s \quad \frac{1}{4}$

51. $\log_2(x^2 \quad 3x) \quad 2$

$0 \quad x^2 \quad 3x \quad 4$

52. $k \quad 2 \quad 0 \quad D \quad 0$

$k \quad 2 \quad (k \quad 6)(k \quad 4) \quad 0$

$k \quad 4$

53. 0

$\frac{3m \quad 8}{m \quad 2} \quad 0 \qquad 2 \quad m \quad \frac{8}{3}$

54. $\log_6 \frac{x^2 \quad x}{x \quad 4} \quad 1$

$\frac{x^2 \quad x}{x \quad 4} \quad 6 \qquad \frac{x^2 \quad 5x \quad 24}{x \quad 4} \quad 0 \qquad \frac{(x \quad 8)(x \quad 3)}{x \quad 4} \quad 0$

55. $ax^2 \quad c \quad 0$

$0, \qquad \frac{c}{a}$

$^3 \quad ^3 \quad (\quad)(\quad ^2 \quad ^2 \quad) \quad 0$

56. $(k \quad 1)x^2 \quad (k \quad 1)x \quad (k \quad 1) \quad 0 \quad x \quad R$

$k \quad 1 \quad 0 \qquad (k \quad 1)^2 \quad 4(k \quad 1)(k \quad 1) \quad 0$

$k \quad 1 \qquad (k \quad 1)(3k \quad 5) \quad 0$

$k \quad \frac{5}{3}$

57. $y \quad 2x^2 \quad 4ax \quad k$; abscissa corresponding to the vertex is $\frac{b}{2a}$ *i.e.*, $\frac{4a}{4} \quad 2 \quad a \quad 2$

now, $y(\quad 2) \quad 7$

$7 \quad 8 \quad 16 \quad k \quad k \quad 1$

58. If $a \quad b \quad c \quad 0$

Sum of coefficient $(b \quad c \quad a) \quad (c \quad a \quad b) \quad (a \quad b \quad c) \quad a \quad b \quad c \quad 0$

$x \quad 1$ is one root of the equation.

other root $\frac{a \quad b \quad c}{b \quad c \quad a}$

59. $x^3 \quad ax^2 \quad bx \quad c \quad 0$

Sum of roots $a \quad a$

If is root of the equation, then $ab \quad c$.

60. $2q^2 \quad r \quad 2 \quad ^2 \quad ^2 \quad (\quad ^4 \quad ^4) \quad (\quad ^2 \quad ^2)^2 \quad 0$

62. In ABC,

$$\cot\frac{A}{2} \quad \cot\frac{B}{2} \quad \cot\frac{C}{2} \quad \cot\frac{A}{2} \; \cot\frac{B}{2} \; \cot\frac{C}{2}$$

If $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A.P. then $\cot\frac{A}{2} \quad \cot\frac{C}{2} \quad 2\cot\frac{B}{2}$

$$\cot\frac{A}{2}\cot\frac{C}{2} \quad 3$$

63. $f(x) \quad x \quad x \quad [\;\; 9, 9]$

64. $$(3|x| \quad 3)^2 \quad |x| \quad 7$$

$$(|x| \quad 2)(9|x| \quad 1) \quad 0$$

$$|x| \quad 2, \frac{1}{9} \qquad x \qquad 2, \quad \frac{1}{9}$$

$$y \quad \sqrt{x(x \quad 4)}$$

$D_f : (\quad ,0] \quad [4, \quad)$

65. $x^2 \quad 3|x| \quad 2 \quad 0 \qquad (|x| \quad 2)(|x| \quad 1) \quad 0$

66. $x^2 \quad bx \quad c \quad 0$ $+1$

Sum of roots $2 \qquad b \quad 1 \qquad \dfrac{b \quad 1}{2}$

If is the root of equation, then $\left(\dfrac{b \quad 1}{2}\right)^2 \quad b\,\dfrac{b \quad 1}{2} \quad c \quad 0 \qquad b^2 \quad 4c \quad 1$

67. $y \quad \dfrac{3x^2 \quad 9x \quad 17}{3x^2 \quad 9x \quad 7}$

$$3(y \quad 1)x^2 \quad 9(y \quad 1)x \quad (7y \quad 17) \quad 0$$

$y \quad 1 \quad 0$ then $D \quad 0$

$$81(y \quad 1)^2 \quad 12(y \quad 1)(7y \quad 17) \quad 0$$

$$(y \quad 1)(y \quad 41) \quad 0$$

$$1 \quad y \quad 41$$

68. $\dfrac{x^2 \quad 2x \quad 7}{2x \quad 3} \quad 6 \quad 0 \quad x \quad R$

$\dfrac{x^2 \quad 10x \quad 11}{(2x \quad 3)} \quad 0$

$\dfrac{(x \quad 11)(x \quad 1)}{2x \quad 3} \quad 0$

$x \qquad , \quad \dfrac{3}{2} \qquad (\quad 1, 11)$

69. $y \quad \dfrac{3x \quad 2}{7x \quad 5} \qquad x \quad \dfrac{5y \quad 2}{3 \quad 7y} \qquad y \quad R \quad \dfrac{3}{7}$

70. $\dfrac{x \quad 2}{x \quad 4} \quad 0 \qquad x \quad [\ 2, 4)$

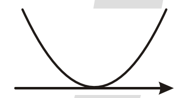
$x^2 \quad ax \quad 4 \quad 0$

$f(\ 2) \quad 0 \qquad f(4) \quad 0$

$a \quad 0 \qquad a \quad 3$

$a \quad [0, 3)$

71. $P(x) \quad (P \quad 3)x^2 \quad 2Px \quad (3P \quad 6) \quad x \quad R$



$P \quad 3 \quad 0 \qquad\qquad D \quad 0$

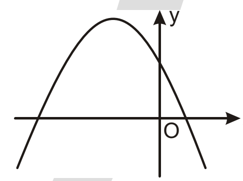
$P \quad 3 \qquad\qquad P^2 \quad (P \quad 3)(3P \quad 6) \quad 0$

$P \quad 6$

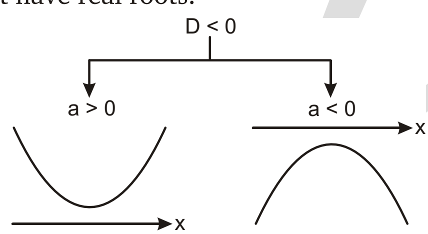
72. Graph is downward $\quad a \quad 0$

Graph cut y-axis $\quad c \quad 0$

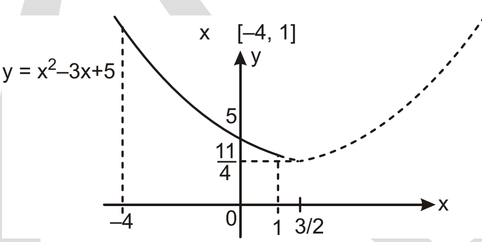
x-coordinate of vertex $\dfrac{b}{2a} \quad 0 \qquad b \quad 0$



73. $ax^2 \quad bx \quad c \quad 0$ does not have real roots.



74.



$y \quad [3, 33]$

75. $3x^2 \quad 17x \quad 10 \quad 0 \qquad (x \quad 5)(3x \quad 2) \quad 0$

If $x \quad 5$ is common root, then $m \quad 0$

If $x \quad \dfrac{2}{3}$ is common root, then $m \quad \dfrac{26}{9}$

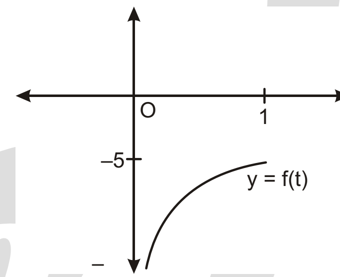
76. $x^2 \quad (y \quad 2)x \quad (y^2 \quad y \quad 1) \quad 0$

$D \quad 0 \quad (y \quad 2)^2 \quad 4(y^2 \quad y \quad 1) \quad 0 \quad y \quad\quad , \frac{8}{5} \quad [0, \quad)$

77. If $x \quad 3$ is root of this equation, then $k \quad 5$

$3x^4 \quad 6x^3 \quad 5x^2 \quad 8x \quad 12 \quad (x \quad 3)(3x^2 \quad 4)(x \quad 1)$

78. $a \quad \frac{(4 \quad \sin^4 x)}{\sin^2 x}$ put $\sin^2 x \quad t \quad t \quad [0, 1]$



$a \quad \frac{4}{t} \quad t \quad f(t)$

Here, $f(t) \quad \frac{4}{t^2} \quad 1 \quad 0$

For atleast one real root, $a \quad (\quad , \quad 5]$

79. $(rs)^2 \quad (st)^2 \quad (tr)^2 \quad (rs \quad st \quad tr)^2 \quad 2rst(r \quad s \quad t) \quad b^2 \quad 2(\quad c)(\quad a)$

80. Let the roots be $t, t \quad 1$ and $t \quad 2$.

$t \quad (t \quad 1) \quad (t \quad 2) \quad a \quad\quad 3(t \quad 1) \quad a$

$t(t \quad 1) \quad b \quad\quad b \quad 1 \quad 3(t \quad 1)^2$

$$\frac{a^2}{b \quad 1} \quad \frac{[3(t \quad 1)]^2}{3(t \quad 1)^2} \quad 3$$

81. $(3x^2 \quad kx \quad 3)(x^2 \quad kx \quad 1) \quad 0$

$D_1 \quad k^2 \quad 36$ and $D_2 \quad k^2 \quad 4 \quad 0$

82. $\frac{1}{r \quad s} \quad \frac{1}{r} \quad \frac{1}{s} \quad\quad \frac{r}{s}^{2} \quad \frac{r}{s} \quad 1 \quad 0 \quad\quad \frac{r}{s}^{3} \quad 1$

84. If $\quad x \quad (\quad , \quad 2] \quad [3, \quad)$

$x^2 \quad 2x \quad 8 \quad 0 \quad\quad x \quad 2, 4$

if $\quad x \quad (\ 2, 3)$

$x^2 \quad 4 \quad\quad x \quad 2$

85. $5x^2 \quad 12x \quad 3 \quad 0$ has $D \quad 0$

Both roots common.

86.

6

5

1

$^2 \quad ^2 \quad ^2 \quad 26$

$^2\ ^2 \quad ^2\ ^2 \quad ^2\ ^2 \quad 13$

87. $2x^2 \quad 6x \quad k \quad 0 \begin{array}{l} \nearrow \dfrac{+5i}{2} \\ \searrow \dfrac{-5i}{2} \end{array}$

Sum of roots $\quad 3$

Product of roots $\dfrac{\ ^2 \quad 25}{4} \quad \dfrac{k}{2} \quad k \quad 17$

88. $x_1^2 \quad x_2^2 \quad (k \quad 2)^2 \quad 2(k^2 \quad 3k \quad 5) \quad (k^2 \quad 10k \quad 6) \quad 18$

89. $a(x^2 \quad x \quad 1) \quad (x^2 \quad x \quad 1) \quad 0$

$$a \quad \frac{x^2 \quad x \quad 1}{x^2 \quad x \quad 1}$$

90. $f(1) \quad 13 \quad 0 \quad 13$

$f(2) \quad 18 \quad 0 \quad 18$

$f(3) \quad 15 \quad 0 \quad 15$

$15 \quad 18$

94. $D \quad (b \quad c)^2 \quad 4a(2b \quad a \quad c) \quad (b \quad c)^2 \quad (4ac \quad 4b^2) \quad (2a \quad 2b)^2 \quad 0$

95. $x^3 \quad x \quad 1 \quad 0 \begin{array}{l} \nearrow a \\ \rightarrow b \\ \searrow c \end{array}$

$$(1 \quad x)^3 \quad x^2(1 \quad x) \quad x^3 \quad 0 \begin{array}{l} \nearrow \dfrac{1}{a+1} \\ \rightarrow \dfrac{1}{b+1} \\ \searrow \dfrac{1}{c+1} \end{array}$$

$x^3 \quad 2x^2 \quad 3x \quad 1 \quad 0$

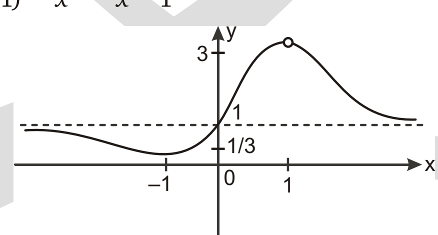
Sum of roots $\dfrac{1}{1 \quad a} \quad \dfrac{1}{1 \quad b} \quad \dfrac{1}{1 \quad c} \quad 2$

96. $x^2 \quad 2(4k \quad 1)x \quad 15k^2 \quad 2k \quad 7 \quad 0 \quad x \quad R$

$D \quad 0$

$k^2 \quad 6k \quad 8 \quad 0 \qquad 2 \quad k \quad 4$

97. $f(x) \quad \dfrac{x^3 \quad 1}{(x \quad 1)(x^2 \quad x \quad 1)} \quad \dfrac{x^2 \quad x \quad 1}{x^2 \quad x \quad 1} \qquad (\because x \quad 1)$



98. $\dfrac{2x^2 \quad 2}{x^2 \quad mx \quad 4} \quad 0 \quad x \quad R \qquad x^2 \quad mx \quad 4 \quad 0 \quad x \quad R$

$D \quad 0 \qquad m^2 \quad 16 \quad 0$

99. $x^2 \quad 2|a \quad 1|x \quad 1 \quad 0$

$D \quad 0 \qquad 4(a \quad 1)^2 \quad 4 \quad 0 \qquad a \quad (\quad , \quad 2] \quad [0, \quad)$

100. $P(x) \quad a_1x^2 \quad 2b_1x \quad c_1 \quad 0; \; D_1 \quad 4(b_1^2 \quad a_1c_1) \quad 0, \; a_1 \quad 0, c_1 \quad 0$

$Q(x) \quad a_2x^2 \quad 2b_2x \quad c_2 \quad 0; \; D_2 \quad 4(b_2^2 \quad a_2c_2) \quad 0, \; a_2 \quad 0, c_2 \quad 0$

$f(x) \quad a_1a_2x^2 \quad b_1b_2x \quad c_1c_2$

$D \quad b_1^2b_2^2 \quad 4a_1a_2c_1c_2 \quad 0$

101. $x^2 \quad 2x \quad 4 \qquad 3\cos(ax \quad b)$

$(x \quad 1)^2 \quad 3 \qquad 3\cos(ax \quad b)$

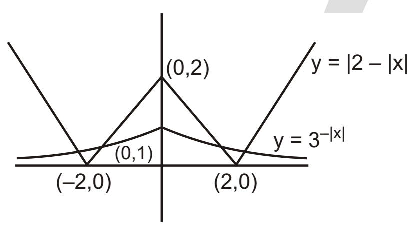
$x \quad 1$ and $ax \quad b$

102. $r \quad 4$

$r^2 \qquad r^3 \quad 36 \qquad r \quad 3, \qquad 1$

$A \quad 3, B \quad 243 \quad 3^5$

103.



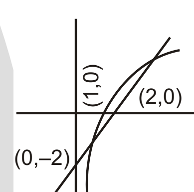
104. We have $4x^2 \quad 16x \quad 15 \quad 0 \qquad \frac{3}{2} \quad x \quad \frac{5}{2} \qquad \cot \qquad 2$, the integral solution of the given inequality and $\sin \qquad \tan 45 \quad 1$

$\sin(\qquad)\sin(\qquad) \quad \sin^2 \qquad \sin^2 \qquad \dfrac{1}{1 \quad \cot^2} \quad 1 \quad \dfrac{1}{1 \quad 4} \quad 1 \quad \dfrac{4}{5}$

105. $f_1(x) \quad f_2(x)$

$2 \quad \log_e x \quad x$

$\log_e x \quad (x \quad 2)$



Clearly graphs intersect once in (0,1).

Now check

$g(x) \quad 2 \quad \ln x \quad x$

$g(e) \quad 0$

$g(e^2) \quad 0$

one root between (e, e^2)

106. $x^4 \quad 3x^3 \quad 2x^2 \quad 3x \quad 1 \quad 0 \qquad x^2 \quad 3x \quad 2 \quad \frac{3}{x} \quad \frac{1}{x^2} \quad 0$

$$x^2 \quad \frac{1}{x^2} \quad 3 \quad x \quad \frac{1}{x} \quad 2 \quad 0 \qquad x \quad \frac{1}{x}^{\;2} \quad 3 \quad x \quad \frac{1}{x} \quad 4 \quad 0$$

$t^2 \quad 3t \quad 4 \quad 0$ (where $x \quad \frac{1}{x} \quad t$)

$(t \quad 4)(t \quad 1) \quad 0 \qquad t \quad 4$ or $t \quad 1$

$x \quad \frac{1}{x} \quad 4$ or $x \quad \frac{1}{x} \quad 1$

Real solutions are from $x \quad \frac{1}{x} \quad 4 \qquad x^2 \quad 1 \quad 4x \qquad x^2 \quad 4x \quad 1 \quad 0$

Hence, sum of roots 4.

107. $f(x) \quad x^2 \quad (k \quad 4) \quad k^2 \quad 12$

$f(4) \quad 16 \quad 4(k \quad 4) \quad k^2 \quad 12 \quad 0$

$2 \quad k \quad 6$

108. $^2 \quad ^2 \quad (\quad)^2 \quad 2 \qquad k^2 \quad 2(k^2 \quad 2k \quad 4) \qquad k^2 \quad 4k \quad 8$

Maximum value 12

109. $f(x) \quad a^x \quad x \ln a$

$f\ (x) \quad (a^x \quad 1) \ \ln a$

110. As a, b and c are the roots of $x^3 \quad 2x^2 \quad 1 \quad 0$, we have

$$a \quad b \quad c \quad 2$$

$$ab \quad bc \quad ca \quad 0$$

Now, for finding the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$, evaluating using first row, we get

$$a(bc \quad a^2) \quad b(b^2 \quad ac) \quad c(ab \quad c^2) \quad abc \quad a^3 \quad b^3 \quad abc \quad abc \quad c^3$$

$$3abc \quad a^3 \quad b^3 \quad c^3$$

$$(a^3 \quad b^3 \quad c^3 \quad 3abc)$$

$$(a \quad b \quad c)(a^2 \quad b^2 \quad c^2 \quad ab \quad bc \quad ca)$$

$$(\quad 2)[(\quad 2)^2 \quad 3(0)] \quad 8$$

111. $x^2 \quad px \quad q \quad 0, \ p, q \quad R, q \quad 0 \qquad ,$ real roots.

$g(x) \quad 0 \qquad \frac{1}{\ }, \quad \frac{1}{\ }$

$$\frac{1}{} \quad \frac{1}{} \qquad \frac{1}{} \qquad \frac{1}{}$$

$$p \quad \frac{p}{q} \qquad \_ \quad \_ \quad \frac{1}{}$$

$$p \quad \frac{p}{q} \quad q \quad \frac{p^2 \quad 2q}{q} \quad \frac{1}{q}$$

$$pq \quad p \quad q^2 \quad p^2 \quad 2q \quad 1$$

$$p^2 \quad p(p \quad 1) \quad q^2 \quad 2q \quad 1 \quad 0$$

$$(q \quad 1)^2 \quad 4(q^2 \quad 2q \quad 1) \quad 0$$

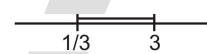
$$q^2 \quad 2q \quad 1 \quad 4q^2 \quad 8q \quad 4 \quad 0$$

$$3q^2 \quad 10q \quad 3 \quad 0$$

$$3q^2 \quad 2q \quad q \quad 3 \quad 0$$

$$3q(q \quad 3) \quad (q \quad 3) \quad 0$$

$$\frac{1}{3}, 3$$



112. $\ln(x^2 \quad 5x) \quad \ln(x \quad a \quad 3) \qquad x^2 \quad 5x \quad x \quad a \quad 3 \quad 0$

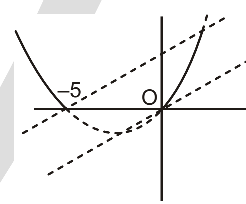
$$a \quad 3 \quad 0$$

$$a \qquad 3$$

$y \quad x \quad a \quad 3 \qquad 5 \quad a \quad 3 \quad 0$

$$a \quad 2$$

$$3 \quad a \quad 2$$



113. $f(x) \quad x^2 \quad \frac{1}{x^2} \quad 6x \quad \frac{6}{x} \quad 2 \qquad x \quad \frac{1}{x}^{\;2} \quad 6 \;\; x \quad \frac{1}{x}$

Let $x \quad \frac{1}{x} \quad t$

$f(x) \quad t^2 \quad 6t \qquad t \quad (\qquad , \quad 2] \quad [2, \quad)$

min. value $\quad 9$ at $t \quad 3$

114. $x^3 \quad 2x^2 \quad 2x \quad c \quad (x^2 \quad bx \quad b) \quad x \quad \frac{c}{b}$

$b \quad \frac{c}{b} \quad 2$ and $b \quad c \quad 2 \qquad b \quad c \quad 1$

115. $\qquad 0 \qquad (\quad)^3 \quad (\quad)^3 \quad (\quad)^3 \quad 3(\quad)(\quad)(\quad)$

118. $(\ ^{r}\quad ^{r})\ (\quad ^{2}\quad ^{3}\quad)\ (\quad ^{2}\quad ^{3}\quad)$

$r\ 1$

$\overline{\quad 1}\quad \overline{\quad 1}$

$4x^2 \quad 2x \quad 1 \quad 0$

$4\ \dfrac{x}{1\quad x}^{2} \quad 2\ \dfrac{x}{1\quad x} \quad 1\ \ 0 \qquad 5x^2 \quad 1 \quad 0$ $\overline{1-}$ $\overline{1-}$

119. $\dfrac{2011}{2014}^{x} \quad \dfrac{2012}{2014}^{x} \quad \dfrac{2013}{2014}^{x} \quad 1$

Let $f(x) \quad \dfrac{2011}{2014}^{x} \quad \dfrac{2012}{2014}^{x} \quad \dfrac{2013}{2014}^{x}$ $\quad f(x)$ is a decreasing function for $x \quad R$.

123.

$x^2 \quad ax \quad 12 \quad 0 \qquad \ldots(1)$

$x^2 \quad bx \quad 15 \quad 0 \qquad \ldots(2) \qquad$ Common roots

$x^2 \quad (a \quad b)\,x \quad 36 \quad 0 \quad \ldots(3)$

(1) + (2) – (3)

$^{2}\quad 9 \qquad 3$

positive root $\quad 3$

124. $e^{\sin x} \quad t$

$t^2 \quad 4t \quad 1 \quad 0 \qquad t \quad 2 \quad \sqrt{5} \qquad e^{\sin x} \quad 2 \quad \sqrt{5}$ (Not possible)

125. Maximum value of $f(x) \quad 3$

Minimum value of $f(x) \quad 1$

126. $f(1) \qquad 2 \quad 0$

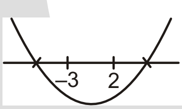
127. $2x^2 \quad 5x \quad 7 \quad 0$ has non-real roots $\quad \dfrac{a}{2} \quad \dfrac{b}{5} \quad \dfrac{c}{7}$

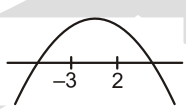
Min. value of $a \quad b \quad c \quad 2 \quad 5 \quad 7 \quad 14$

Max. value of $a \quad b \quad c \quad 28 \quad 70 \quad 98 \quad 196$

128. Distance $\quad \sqrt{(x_B \quad x_A)^2 \quad (y_B \quad y_A)^2} \quad \sqrt{(1 \quad 2t)^2 \quad t^2} \quad \sqrt{5t^2 \quad 4t \quad 1}$

Min. distance $\quad \dfrac{1}{\sqrt{5}}$ at $t \quad \dfrac{2}{5}$

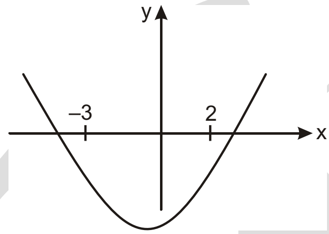
129. 



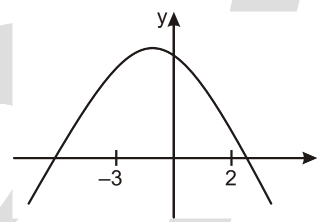
$af(1) \quad 0$ and $af(\quad 1) \quad 0$; $f(\quad 3)\,f(2) \quad 0$

We have the equation $ax^2 \quad bx \quad c \quad 0$ has two roots and such that $\quad 3$ and $\quad 2$.

If $a \quad 0$, then we have the following graphical representation :



Then, for all $x \quad [\ 3, 2]$, $f(x) \quad 0$, we have the following graphical representation :



This implies that

$$f(\ 1) \quad 0 \text{ and } f(1) \quad 0$$

$$a \quad b \quad c \quad 0 \text{ and } a \quad b \quad c \quad 0$$

$$a(a \quad |b| \quad c) \quad 0$$

If $a \quad 0$, then for all $x \quad [\ 3, 2]$, $f(x) \quad 0$. This imply that

$$f(\ 1) \quad 0 \text{ and } f(1) \quad 0$$

$$a \quad b \quad c \quad 0 \text{ and } a \quad b \quad c \quad 0$$

$$a(a \quad |b| \quad c) \quad 0$$

130. Let $x^2 \quad 5x \quad t$

$$t^2 \quad 2t \quad 24 \quad 0 \quad (t \quad 6)(t \quad 4)$$

$$x^2 \quad 5x \quad 6 \quad 0 \quad (x \quad 6)(x \quad 1)$$

$$x^2 \quad 5x \quad 4 \quad 0 \quad (x \quad 4)(x \quad 1)$$

131. Case-1: $x \quad 2$

$$3(x \quad 2) \quad (1 \quad 5x) \quad 4(3x \quad 1) \quad 13 \qquad x \quad \frac{4}{5} \quad \text{(Not possible)}$$

Case-2: $\frac{1}{5} \quad x \quad 2$

$$3(x \quad 2) \quad (1 \quad 5x) \quad 4(3x \quad 1) \quad 13 \qquad x \quad \frac{2}{7} \quad \text{(Possible)}$$

Case-3: $\frac{1}{3} \quad x \quad \frac{1}{5}$

$$3(x \quad 2) \quad (1 \quad 5x) \quad 4(3x \quad 1) \quad 13 \qquad x \quad \frac{1}{2} \quad \text{(Not possible)}$$

Case-4: $x \quad \frac{1}{3}$

$$3(x \quad 2) \quad (1 \quad 5x) \quad 4(3x \quad 1) \quad 13 \qquad x \quad \frac{1}{2} \quad \text{(Possible)}$$

132. $\log_{\cos x} \sin x \quad 2 \qquad \sin x \quad \cos^2 x$

$\sin^2 x \quad \sin x \quad 1 \quad 0$

$$0 \quad \sin x \quad \frac{\sqrt{5} \quad 1}{2} \qquad (\sin x \quad 0)$$

133. Minimum value $\frac{D}{4} \qquad 5 \qquad D \quad 20$

$$| \qquad | \quad \frac{\sqrt{D}}{1} \quad \sqrt{20}$$

134. $|x \quad 3| \quad |x \quad 5| \quad 7x$

$2x \quad 2 \quad 7x \qquad\qquad x \quad 3$

$(x \quad 3) \quad (x \quad 5) \quad 7x \qquad\qquad 5 \quad x \quad 3$

$(x \quad 3) \quad (x \quad 5) \quad 7x \qquad\qquad x \qquad 5$

136. $(a \quad b \quad c)^2 \quad a^2 \quad b^2 \quad c^2 \quad 2(ab \quad bc \quad ac) \qquad ab \quad bc \quad ac \qquad 4$

$a^3 \quad b^3 \quad c^3 \quad 3abc \quad (a \quad b \quad c)(a^2 \quad b^2 \quad c^2 \quad ab \quad bc \quad ac) \qquad abc \qquad 4$

140. $x^2 \quad 3x \quad 4 \quad x^2 \quad 3x \quad 4$

$x \quad 0$

142. $x^2 \quad 4x \quad 3 \quad 0$

$3, \qquad 1$

143. $a^3 \quad b^3 \quad c^3 \quad 3abc$

$a \quad b \quad c \quad 0$

$ax^2 \quad bx \quad c \quad 0$ has one root $x \quad 1$

145. $x_1 \quad x_2 \quad x_1 x_2 \quad a$

$x_1 x_2 \quad x_1 x_2 (x_1 \quad x_2) \quad b$

$x_1^2 x_2^2 \quad c \qquad b \quad c \quad x_1 x_2 (a \quad 1)$

147. $(|x| \quad 2)(|x| \quad 1) \quad 0 \qquad x \qquad 1, \quad 2$

149. $^2 \qquad ^2 \quad (\qquad)^2 \quad 2 \qquad 4(1 \quad \sin 2 \ )^2 \quad 4\cos^2 2$

$4(2 \quad 2\sin 2 \ )$

150. $\sin^2 x \quad \sin x \qquad b \quad x \quad [0, \ ]$

$0 \quad b \quad 2$

$2 \quad b \quad 0$

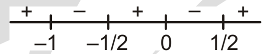
152. $x^2 \quad px \quad r \quad 0 \quad (x \qquad)(x \qquad)$

$^2 \quad p \qquad r \quad (\qquad)(\qquad) \qquad q \quad r$

153. $2^{x \ 2} \quad 4^x \quad 9 \quad 2^{x \ 2} \quad 4^x \quad 0; \ 2^x(4 \quad 2^x) \quad 0$

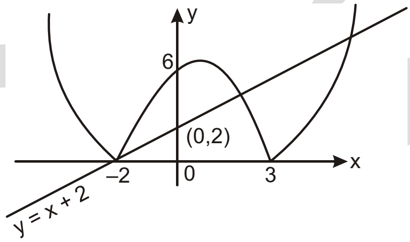
Exercise-2 : One or More than One Answer is/are Correct

1. $\dfrac{2x \quad 1}{x(x \quad 1)(2x \quad 1)} \quad 0$



$x \quad (\quad , \quad 1) \quad \dfrac{1}{2}, 0 \quad \dfrac{1}{2},$

3.



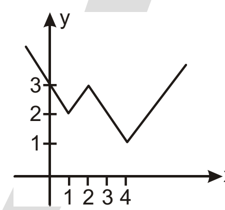
4. Apply $D \quad 0 \quad f(2) \quad 0 \quad f(\ 2) \quad 0 \quad 2 \quad \dfrac{a}{2} \quad 2$

5. $f(x) \quad x \quad 3 \qquad x \quad 4$

$5 \quad x \qquad 2 \quad x \quad 4$

$x \quad 1 \qquad 1 \quad x \quad 2$

$3 \quad x \qquad x \quad 1$



6. $a \quad b \quad \dfrac{1}{b} \quad \dfrac{1}{a} \qquad ab \quad 1 \qquad (\because a \quad b)$

$a \quad b \quad \dfrac{a}{b} \qquad a \quad \dfrac{1}{a} \quad a^2 \qquad a^3 \quad a^2 \quad 1 \quad 0$

7. If $f(2 \quad x) \quad f(2 \quad x)$ and $D \quad 0$

Vertex of parabola is $2, \dfrac{D}{4a}$ lies in IV$^{\text{th}}$ quadrant.

$f(0) \quad f(1) \quad f(2)$

8. If $f(2 \quad x) \quad f(2 \quad x)$ and $D \quad 0$

$f(\ 2) \quad 4a \quad 2b \quad c \quad 0$

If $\log_{f(2)} f(3)$ is not defined then $f(2) \quad 1$

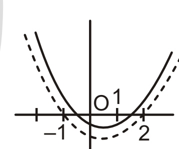
$f(x) \quad 1$

If $\dfrac{b}{2a} \quad 2 \qquad a$ and b are opposite sign.

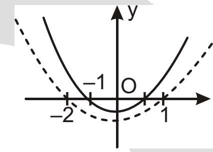
9. **Case-I :** $f(\ 1) \quad 0 \quad f(1) \quad 0 \quad f(2) \quad 0$

$a \quad 0 \quad a \quad 0 \quad a \quad \dfrac{3}{2}$

$a \quad \dfrac{3}{2}, 0$



Case-II: $f(1) \quad 0 \quad f(\ 1) \quad 0 \quad f(\ 2) \quad 0$



$$a \quad 0 \quad a \quad 0 \quad a \quad \frac{3}{2}$$

$$a \quad\quad 0, \frac{3}{2}$$

10. As expression taking minimum value

So, $a \quad 0$

$$\frac{b}{2a} \quad 0; \quad \frac{D}{4a} \quad 0$$

$$a \quad 0, b \quad 0, D \quad 0$$

11. $ax^2 \quad bx \quad c \quad 0 \quad x \quad R$

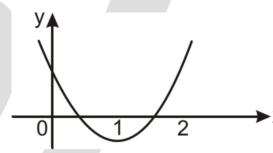
$a \quad 0, D \quad 0$

$f(0) \quad c \quad 0$

$f(\ 3) \quad f(\ 2) \quad 13a \quad 5b \quad 2c \quad 0$

$f(\ 3) \quad f(2) \quad 13a \quad b \quad 2c \quad 0$

12. $D \quad 0 \qquad\quad k \quad \sqrt{5}$



$f(1) \quad 0 \qquad\quad k \quad 3$

$f(2) \quad 0 \qquad\quad k \quad \frac{21}{4}$

$$3 \quad k \quad \frac{21}{4}$$

13. $x^2 \quad px \quad q \quad 0$

Sum of the roots $\quad 13$

Product of the roots $\quad 30$

$$x^2 \quad 13x \quad 30 \quad 0 \quad (x \quad 10)(x \quad 3)$$

Correct roots are $x \quad 10, \quad 3$

14. $x^2 \quad 3x \quad 2 \quad 0$

$(x \quad 2)(x \quad 1) \quad 0 \qquad x \quad (\quad ,1) \quad (2, \quad)$

$x^2 \quad 3x \quad 4 \quad 0$

$(x \quad 4)(x \quad 1) \quad 0 \qquad x \quad [\ 1, 4]$

then $x \quad [\ 1, 1) \quad (2, 4]$

15. $5^x \quad (2\sqrt{3})^{2x} \quad 169 \quad 0$

$5^x \quad 12^x \quad 169 \quad 0$

if $x \quad 2 \qquad 5^2 \quad 12^2 \quad 169$

$x \quad 2 \qquad 5^x \quad 12^x \quad 169$

$x \quad 2 \qquad 5^x \quad 12^x \quad 169$

$x \quad (\quad ,2]$

16. $f(x) \quad x^2 \quad ax \quad b$

$D_1 : a^2 \quad 4b$

$g(x) \quad x^2 \quad cx \quad d$

$D_2 : c^2 \quad 4d$

$D_1 \quad D_2 \quad a^2 \quad c^2 \quad 4(b \quad d) \quad (a \quad c)^2 \quad 0$ atleast one of them is positive.

17. Let $x \quad 1 \quad t^2$

$$\frac{1}{\sqrt{x \quad 2\sqrt{x \quad 1}}} \quad \frac{1}{\sqrt{x \quad 2\sqrt{x \quad 1}}} \quad \frac{1}{\sqrt{t^2 \quad 2t \quad 1}} \quad \frac{1}{\sqrt{t^2 \quad 2t \quad 1}}$$

$$\frac{1}{|t \quad 1|} \quad \frac{1}{|t \quad 1|} \quad \frac{1}{|1 \quad \sqrt{x \quad 1}|} \quad \frac{1}{|\sqrt{x \quad 1} \quad 1|}$$

If $1 \quad x \quad 2$, then $0 \quad \sqrt{x \quad 1} \quad 1$

$$\frac{1}{|1 \quad \sqrt{x \quad 1}|} \quad \frac{1}{|\sqrt{x \quad 1} \quad 1|} \quad \frac{1}{1 \quad \sqrt{x \quad 1}} \quad \frac{1}{1 \quad \sqrt{x \quad 1}} \quad \frac{2}{2 \quad x}$$

If $x \quad 2$, then $\sqrt{x \quad 1} \quad 1$

$$\frac{1}{|1 \quad \sqrt{x \quad 1}|} \quad \frac{1}{|\sqrt{x \quad 1} \quad 1|} \quad \frac{1}{\sqrt{x \quad 1} \quad 1} \quad \frac{1}{\sqrt{x \quad 1} \quad 1} \quad \frac{2\sqrt{x \quad 1}}{x \quad 2}$$

18. $\log_{1/3}(x^2 \quad 2px \quad p^2 \quad 1) \quad 0$

$(x \quad p)^2 \quad 1 \quad 1 \qquad (x \quad p)^2 \quad 0 \qquad x \qquad p$

$kp^2 \quad kp \quad k^2 \quad 0 \quad k \quad R$

$k^2 \quad (p \quad p^2) \quad 0 \quad k \quad R$

$D \quad 0$

19. (a) $\qquad ^2 \qquad ^2$

and $\qquad ^{2\ 2} \qquad (\qquad 1) \quad 0 \qquad 0$ or $\quad 0$ or $\quad 1$

(b) $\tan 2 \qquad \tan 3 \qquad \dfrac{\sin 5}{\cos 2 \quad \cos 3} \quad 0 \qquad \sin 5 \quad 0 \qquad \dfrac{n}{5}$

(c) $\dfrac{\dfrac{2x_1}{x_2} \quad \dfrac{128x_3^2}{x_2^2} \quad \dfrac{x_2^3}{4x_1x_3^2}}{3} \quad 4 \qquad (\because \text{ AM} \quad \text{GM})$

(d) Equation of chord with mid-point (h, k) is $T \quad S_1$

$$(h \quad 1)\,x \quad (k \quad 3)\,y \quad (h \quad 3k \quad h^2 \quad k^2) \quad 0$$

If it is passes from $(0, 0)$.

Then, $h^2 \quad k^2 \quad h \quad 3k \quad 0$

20. $2 \quad a \quad 2 \qquad\qquad a^2 \quad [0, 4)$

$x^2 \quad 4x \quad a^2 \quad 0 \qquad x \quad 2 \quad \sqrt{4 \quad a^2}$

21. If $\quad 2 \quad 0$

$0 \qquad 2 \;{}^2 \quad 0 \quad q \quad 0$

p

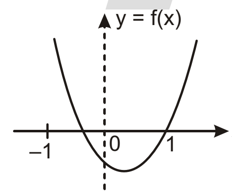
$2 \;{}^2 \quad q \quad 2p^2 \quad q \quad 0$

22. $f(x) \quad (a \quad b \quad 2c)\,x^2 \quad (b \quad c \quad 2a)\,x \quad (c \quad a \quad 2b) \quad 0 \qquad f(1) \quad 0$

(a) if $a \quad b \quad c \quad 0$

$a \quad b \quad 2c$

$f(0) \quad a \quad c \quad 2b \quad 0$

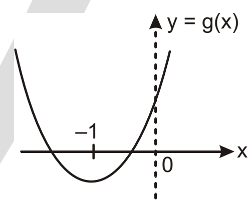


(c) $g(x) \quad ax^2 \quad 2bx \quad c \quad 0$

$g(0) \quad c \quad 0$

$g(\;\;1) \quad a \quad 2b \quad c \quad 0$

(d) $cx^2 \quad 2bx \quad a \quad 0$ $1/$ $1/$



23. $f(x) \quad 4x^2 \quad 8ax \quad a$

$D \quad 48a^2 \quad 0$

(a) If $f(x)$ is non-negative $\quad x \quad R$, then $a \quad 0$

(b) If $a \quad 0$, then $f(0) \quad 0$

(c) If $f(x) \quad 0$ has two distinct solutions in $(0, 1)$, then

$$f(0) \quad 0 \qquad a \quad 0$$

$$f(1) \quad 0 \qquad a \quad \frac{4}{7}$$

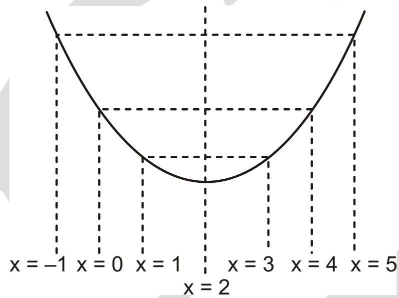
$$0 \quad \frac{b}{2a} \quad 1 \qquad 0 \quad a \quad 1$$

24. $ax^2 \quad bx \quad c \quad 0$ has no real roots, then $D \quad 0$

$$f \quad \frac{1}{2} \quad a \quad 2b \quad 4c \quad 0 \qquad a \quad 0$$

$$\frac{4a \quad 2b \quad c}{a \quad 3b \quad 9c} \quad \frac{f(2)}{f\ \frac{1}{3}} \quad 0$$

25.



26. $ax^2 \quad bx \quad c \quad 0$

$$x \quad \frac{b \quad \sqrt{b^2 \quad 4ac}}{2a} \quad \frac{b \quad i\sqrt{4ac \quad b^2}}{2a}$$

$$| \ | \ | \ | \quad \sqrt{\frac{b}{2a}^2 \quad \frac{4ac \quad b^2}{4a^2}} \quad \sqrt{\frac{c}{a}}$$

27. $3x \quad 6 \quad 6 \qquad x \quad 3$

$x \quad 6 \qquad 2 \quad x \quad 3$

$4 \quad x \quad 6 \qquad 1 \quad x \quad 2$

$6 \quad 3x \quad 6 \qquad x \quad 1$

28. $f(x) \quad ax^2 \quad x \quad b \quad a$

$D \quad 0$ and $f(1) \quad b \quad 1 \quad 0$

$f(0) \quad b \quad a \quad 0$

$f(1/2) \quad 4b \quad 2 \quad 3a \quad 0$

29. $a^2 \quad b^2 \quad (a \quad b)^2 \quad 2ab \quad 7$ ...(1)

$a^3 \quad b^3 \quad (a \quad b)(7 \quad ab) \quad 10$ ...(2)

$(a \quad b)[21 \quad (a \quad b)^2] \quad 20$

Let $(a \quad b) \quad x$

$x^3 \quad 21x \quad 20 \quad 0$

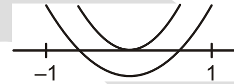
$(x \quad 1)(x \quad 5)(x \quad 4) \quad 0$

31. 0

Root $\frac{1}{\ }, \frac{1}{\ }, \frac{1}{\ }, \frac{1}{\ }$

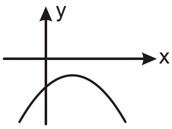
Put $x \quad \frac{1}{x}$

32. $D \quad 0 \qquad f(\ 1) \quad 0 \qquad f(1) \quad 0$



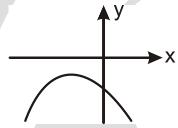
$2 \quad K \quad \frac{1}{4}$

33. (a)



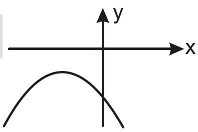
$a < 0$
$c < 0$
$b < 0$

(b)



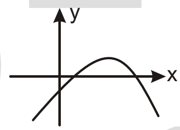
$a < 0$
$c < 0$
$b > 0$

(c)



$a < 0$
$c > 0$
$b > 0$

(d)



$a < 0$
$c < 0$
$b < 0$

34. (a) $f(1)f(\ 1) \quad 0$ (b) $f(1)f \quad \frac{1}{2} \quad 0$

(c) $f(\ 1)f(\ 2) \quad 0$ (d) $b^2 \quad 4ac \quad 0$

but a can be +ve or –ve.

35. $\frac{b}{a}, \qquad \frac{c}{a}$

$$^2 \quad ^2 \quad (\qquad)^2 \quad 2 \qquad \frac{b^2}{a^2} \quad \frac{2c}{a} \quad \frac{b^2 \quad 2ac}{a^2}$$

$$\frac{1}{^2} \quad \frac{1}{^2} \quad \frac{(\qquad)^2 \quad 2}{^2 \ ^2} \quad \frac{b^2 \quad 2ac}{a^2}$$

$$\frac{1}{^3} \quad \frac{1}{^3} \quad \frac{(\qquad)(\ ^2 \qquad ^2 \qquad)}{(\quad)^3} \quad \frac{b(b^2 \quad 3ac)}{c^3}$$

36. $\sin^2 x \quad \sin x \quad 1$

$1 \qquad 1$

37. $x^2 \quad 5x \quad x \quad a \quad 3 \quad x \quad (\ 5, 0)$

$x^2 \quad 4x \quad 3 \quad a \quad x \quad (\ 5, 0)$

$a \quad (\ 3, 2]$

39. $x^2 \quad 2ax \quad a^2 \quad 0 \qquad\qquad x \quad a$

$x \quad a(1 \quad \sqrt{2})$

$x^2 \quad 2ax \quad 5a^2 \quad 0 \qquad\qquad x \quad a$

43. $(\qquad) \quad (\qquad) \quad 12 \qquad\qquad 6$ ...(1)

$(\qquad\qquad (\qquad) \quad 54 \qquad\qquad 9$ ...(2)

$(\quad)(\quad) \quad 14$ ...(3)

$7, \quad 2$

44. $l \quad \frac{K^3}{K \quad 1} \quad m \quad \frac{K^2 \quad 3}{K \quad 1} \quad n \quad 0 \begin{cases} K = a \\ K = b \\ K = c \end{cases}$

$lK^3 \quad mK^2 \quad nK \quad (3m \quad n) \quad 0$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

1. $f(\ 1 \quad x) \quad f(\ 1 \quad x) \quad x \quad R$

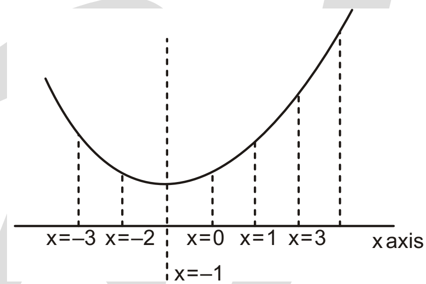
graph of $f(x)$ is symmetric about $x \quad 1.$

$\frac{b}{2a} \quad 1 \quad b \quad 2a$

$f(\ 2) \quad 4a \quad 2b \quad c$

$f(3) \quad 9a \quad 3b \quad c$

$f(\ 3) \quad 9a \quad 3b \quad c$



Using graph $f(3) \quad f(\ 3) \quad f(\ 2)$

2. $p \quad b \quad 4a \quad 2a \quad 0$

$q \quad 2a \quad b \quad 4a \quad 0$

$p \quad q \quad 0$

Paragraph for Question Nos. 3 to 4

Sol. $(k \quad 1)x^2 \quad (20k \quad 14)x \quad 91k \quad 40 \quad 0$

$f(4) \quad 27k \quad 0$

$f(7) \quad 9 \quad 0$

One root is lie (4, 7)

$f(10) \quad 9k \quad 0$

$f(13) \quad 27 \quad 0$

Other root is lie (10, 13)

Paragraph for Question Nos. 5 to 7

5. $f(x) \quad x^2 \quad bx \quad c \quad x \quad R$

Least value at $\frac{b}{2} \quad 1 \quad b \quad 2$

Graph of $f(x)$ cuts y-axis, when $x \quad 0$

$c \quad 2$

$$f(x) \quad x^2 \quad 2x \quad 2$$

Least value of $f(x) \quad 1$

6. $f(\quad 2) \quad f(0) \quad f(1) \quad 9$

7. $a \quad (1, \quad)$

Paragraph for Question Nos. 8 to 9

Sol. $(\log_2 x)^2 \quad 4(\log_2 x) \quad m^2 \quad 2m \quad 13 \quad 0$

8. $D \quad 0 \quad m^2 \quad 2m \quad 17 \quad 0 \quad m \quad R$

9. $m^2 \quad 2m \quad (\log_2 x)^2 \quad 4(\log_2 x) \quad 13 \quad 0$

$D \quad 0$

$$(\log_2 x \quad 6)(\log_2 x \quad 2) \quad 0 \qquad x \quad 0, \frac{1}{4} \qquad [64, \quad)$$

Paragraph for Question Nos. 10 to 11

Sol. $x^4 \quad 2x^3 \quad 3x^2 \quad 4x \quad 1 \quad 0$ has four roots $a, \frac{1}{a}, b, \frac{1}{b}$

$$a \quad \frac{1}{a} \qquad b \quad \frac{1}{b} \qquad 2 \qquad \ldots(1)$$

$$b \quad \frac{1}{b} \qquad a \quad \frac{1}{a} \qquad 4 \qquad \ldots(2)$$

Paragraph for Question Nos. 12 to 14

Sol. $f(x) \quad (6 \quad x) \quad 0 \quad (x \quad 1)(x \quad 2)(x \quad 3)(x \quad 4)(x \quad 5)$

$f(x) \quad (x \quad 1)(x \quad 2)(x \quad 3)(x \quad 4)(x \quad 5) \quad (6 \quad x)$

Paragraph for Question Nos. 15 to 16

Sol. $x^3 \quad x^2(1 \quad \sin \quad \cos \quad) \quad x(\sin \quad \cos \quad \sin \quad \cos \quad) \quad \sin \quad \cos \quad 0$

Roots are $1, \sin \quad , \cos \quad .$

Paragraph for Question Nos. 17 to 18

Sol. $2[1 \quad P(x)] \quad P(x \quad 1) \quad P(x \quad 1)$

$2 \quad 2[ax^2 \quad bx \quad c] \quad a(x \quad 1)^2 \quad b(x \quad 1) \quad c \quad a(x \quad 1)^2 \quad b(x \quad 1) \quad c$

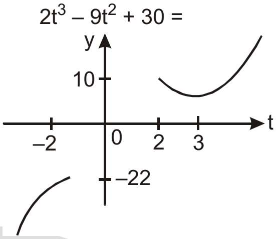
$a \quad 1$

$P(0) \quad c \quad 8$

$P(2) \quad 4a \quad 2b \quad c \quad 32 \qquad b \quad 10$

Paragraph for Question Nos. 19 to 21

Sol.



Paragraph for Question Nos. 22 to 23

22. $D \quad 0$

$(2t \quad 1)^2 \quad 4t(5t \quad 1) \quad 0$

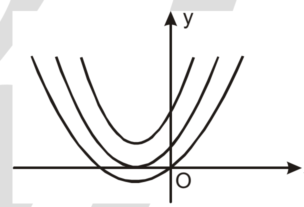
$16t^2 \quad 1 \quad 0 \qquad \frac{1}{4} \quad t \quad \frac{1}{4} \qquad (t \quad 0)$

23. $t \quad 0$

Case-I: $D \quad 0$

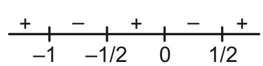
$(2t \quad 1)^2 \quad 4t(5t \quad 1) \quad 0 \qquad t \quad \frac{1}{4} \quad (\because t \quad 0)$

Case-II: $D \quad 0 \quad f(0) \quad 0 \quad \frac{b}{2a} \quad 0 \qquad \frac{1}{5} \quad t \quad \frac{1}{4}$



Exercise-4 : Matching Type Problems

1. (A) $\frac{(2x \quad 1)}{x(2x \quad 1)(x \quad 1)} \quad 0$



(B) $3x^2 \quad 2(a^2 \quad 1)x \quad (a^2 \quad 3a \quad 2) \quad 0$

0

$a^2 \quad 3a \quad 2 \quad 0 \qquad (a \quad 1)(a \quad 2) \quad 0$

(C) $\sqrt{x \quad 3 \quad 4\sqrt{x \quad 1}} \quad \sqrt{x \quad 8 \quad 6\sqrt{x \quad 1}} \quad 1$

Let $x \quad 1 \quad t^2$; $|t \quad 2| \quad |t \quad 3| \quad 1$

(D) A.M. $\frac{}{4} \quad 2$

G.M. $(\qquad)^{1/4} \quad 2$

A.M. G.M. $\qquad 2$

3. (A) $x^4 \quad 8x^2 \quad 9 \quad 0$

$(x^2 \quad 9)(x^2 \quad 1) \quad 0 \qquad x \quad 3, \quad 3$

(B) $x^{2/3} \quad x^{1/3} \quad 2 \quad 0$

$(x^{1/3} \quad 2)(x^{1/3} \quad 1) \quad 0 \qquad x \quad 8, 1$

(C) $(\sqrt{3x \quad 1})^2 \quad (\sqrt{x} \quad 1)^2$

$3x \quad 1 \quad x \quad 1 \quad 2\sqrt{x} \qquad 2x \quad 2\sqrt{x}$ (Not possible)

(D) $(3^x \quad 9)(3^x \quad 1) \quad 0 \qquad x \quad 0, 2$

4. (A) $(a \quad b) \quad a$ & $ab \quad b \quad (1, \quad 2)$ and $(0, 0)$

(B) $P \quad \overline{O}, \quad Q \quad 8\cos\frac{}{9}\cos\frac{2}{9}\cos\frac{4}{9} \quad 1$

(C) $ar^6 \quad \sqrt{2}$

Product $(\sqrt{2})^{11} \quad 2^{11/2}$

$m \quad 11$

$n \quad 4$

(D) $x \quad y \quad 3$

$\because \quad (x \quad y)^2 \quad (y \quad 3)^2 \quad 0$

$5x \quad 4y \quad 3$

Exercise-5 : Subjective Type Problems

1. $f \quad \cos\frac{}{7} \quad \sin\frac{}{7}\sin\frac{3}{7} \quad \sin\frac{3}{7}\sin\frac{5}{7} \quad \sin\frac{}{7}\sin\frac{5}{7}$

$2\cos^2\frac{}{7} \quad \cos\frac{}{7} \quad 1$

$f(x) \quad 2x^2 \quad x \quad 1$

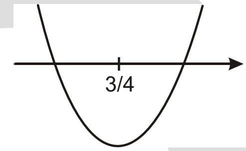
2. $(r \quad a)(r \quad b)(r \quad c)(r \quad d) \quad (\quad 1) \quad (\quad 3) \quad (1) \quad (3)$

$(r \quad a) \quad (r \quad b) \quad (r \quad c) \quad (r \quad d) \quad 0$

3. Let $x^2 \quad x \quad 1 \quad t \qquad t \quad \frac{3}{4},$

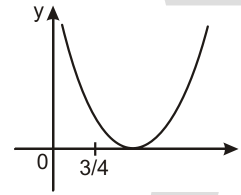
$t^2 \quad (m \quad 3)t \quad m \quad 0$

Case-I : $f \frac{3}{4} \quad 0$



$\frac{9}{16} \quad \frac{3}{4}(m \quad 3) \quad m \quad 0$

$m \quad \frac{45}{4}$

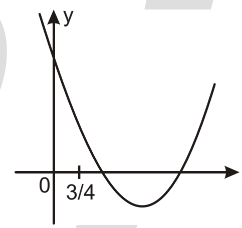


Case-II : $D \quad 0 \qquad m \quad 1, 9$

$\frac{b}{2a} \quad \frac{3}{4} \qquad m \quad \frac{9}{2}$

There is one positive integral value of $m \quad 9$.

4. $t^2 \quad (m \quad 3)t \quad m \quad 0$



$t \quad [3/4, \quad)$ has four distinct real roots, then

$D \quad 0$

$m^2 \quad 10m \quad 9 \quad 0$

$m \quad (\quad , 1) \quad (9, \quad)$

$\frac{b}{2a} \quad \frac{3}{4} \qquad m \quad \frac{9}{2}$

$f \frac{3}{4} \quad 0 \qquad m \quad \frac{45}{4} \qquad m \quad (9, \quad)$

5. $f(t) \quad (m^2 \quad 12)t^2 \quad 8t \quad 4 \quad 0 \qquad\qquad (t \quad 0)$

$f(0) \quad 4 \quad 0$

$m^2 \quad 12 \quad 0 \qquad m \quad [\ 2\sqrt{3}, 2\sqrt{3}]$

Case-I : $D \quad 0$

$m^2 \quad 8 \quad 0 \qquad m \quad (\ 2\sqrt{2}, 2\sqrt{2})$

Case-II : $D \quad 0 \qquad m \quad (\quad , \quad 2\sqrt{2}] \quad [2\sqrt{2}, \quad)$

$\frac{b}{2a} \quad \frac{4}{m^2 \quad 12} \quad 0 \qquad m \quad (\ 2\sqrt{3}, 2\sqrt{3})$

$m \quad [\ 2\sqrt{3}, 2\sqrt{3}]$

6. $(e^x \quad 2) \ \sin \ x \quad \frac{}{4} \quad (x \quad \ln 2)(\sin x \quad \cos x) \quad 0$

$\frac{1}{\sqrt{2}}(x \quad \ln 2) \ (\sin x \quad \cos x)(x \quad \ln 2)(\sin x \quad \cos x) \quad 0 \quad \frac{1}{\sqrt{2}}(x \quad \ln 2)^2(\sin^2 x \quad \cos^2 x) \quad 0$

$\cos 2x \quad 0, \quad x \quad \ln 2$

$$x \quad 0, \frac{}{4} \qquad \frac{3}{4}, \qquad \{\ln 2\}$$

Least positive integral value is 3.

7. $x^2 \quad 17x \quad 71 \quad {}^2 \qquad\qquad Z$

$x^2 \quad 17x \quad (71 \quad {}^2) \quad 0$

D perfect square m^2 (say)

$m^2 \quad 289 \quad 4(71 \quad {}^2)$

$(m \quad 2 \;)(m \quad 2 \;) \quad 1 \quad 5$

$m \quad 2 \quad 1$

$m \quad 2 \quad 5$

8. $P(x) \quad (x^4 \quad x^3 \quad x^2 \quad 1)(x^2 \quad 1) \quad (x^2 \quad x \quad 1)$

$P(\;) \quad P(\;) \quad P(\;) \quad P(\;) \quad (\;{}^2 \qquad 1) \quad (\;{}^2 \qquad 1) \quad (\;{}^2 \qquad 1) \quad (\;{}^2 \qquad 1) \quad 6$

9. If $\frac{a}{2} \quad 1$

$f(x)_{\max} \quad f(4) \qquad 4a \quad 18 \quad 6 \qquad a \qquad 3$ (Not possible)

if $\frac{a}{2} \quad 1$

$f(x)_{\max} \quad f(\;2) \qquad a \quad 0$ (Not possible)

There is no real value of 'a'.

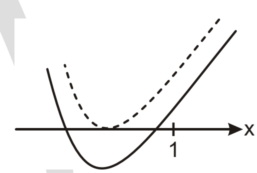
10. $x^2 \quad 8x \quad (n^2 \quad 10n) \quad 0$

$D \quad 0 \qquad n^2 \quad 10n \quad 16 \quad 0$

$(n \quad 8)(n \quad 2) \quad 0$

$2 \quad n \quad 8$ and $n \quad 10$

11. $x^2 \quad 2(m \quad 1)x \quad (m \quad 5) \quad 0 \quad (x \quad 1)$



Case-I : $D \quad 0$

$m^2 \quad 3m \quad 4 \quad 0 \qquad 1 \quad m \quad 4$

Case-II : $D \quad 0$

$m \quad (\quad , \quad 1] \quad [4, \quad)$

$f(1) \quad 0 \qquad m \qquad \frac{4}{3}$

$\frac{b}{2a} \quad 1 \qquad m \quad 0$

$m \quad (\;1, \quad)$

12. $ax^4 \quad bx^3 \quad x^2 \quad 2x \quad 3 \quad (x \quad 2)(x \quad 1)\,Q(x) \quad (4x \quad 3)$

Put $x \quad 1 \qquad a \quad b \quad 3$

$x \quad 2 \qquad b \quad 2a$

13. $D \quad 0 \qquad \dfrac{b}{2a} \quad 4 \qquad f(4) \quad 0$

$k \quad 1 \quad 0 \qquad 4k \quad 4 \qquad k^2 \quad 3k \quad 2 \quad 0$

$k \quad 0 \qquad k \quad 1 \qquad (k \quad 2)(k \quad 1) \quad 0$

$k \quad 2$

14. $x^2 \quad 3x \quad 2 \quad (x \quad 1)(x \quad 2)$

If $(x \quad 1)$ is a factor of $x^4 \quad px^2 \quad q \quad 0$. Then

$$p \quad q \quad 1 \qquad \ldots(1)$$

If $(x \quad 2)$ is a factor of $x^4 \quad px^2 \quad q \quad 0$. Then

$$4p \quad q \quad 16 \qquad \ldots(2)$$

$p \quad 5, q \quad 4$

$p \quad q \quad 9$

15. $x^2 \quad 2xy \quad ky^2 \quad 2x \quad k \quad 0$

if it can be resolved into two linear factors, then

$$abc \quad 2fgh \quad bg^2 \quad af^2 \quad ch^2 \quad 0$$

$$k^2 \quad k \quad k \quad 0$$

$$k \quad 0, 2$$

16. $(a \quad 1)\,x^2 \quad 2 \quad ax \quad 3$ has exactly one solution.

$D \quad 0$

$a^2 \quad 4(a \quad 1) \quad 0$

$(a \quad 2)^2 \quad 0 \qquad a \quad 2 \qquad a^2 \quad 4$

17. $y \quad \dfrac{x^2 \quad ax \quad 1}{x^2 \quad 3x \quad 2}$

$x^2(y \quad 1) \quad x(3y \quad a) \quad 2y \quad 1 \quad 0 \quad x \quad R$

$D \quad 0$

$(3y \quad a)^2 \quad 4(y \quad 1)(2y \quad 1) \quad 0 \quad y \quad R$

$y^2 \quad 6y(a \quad 2) \quad a^2 \quad 4 \quad 0 \quad y \quad R$

$D \quad 0$

$36(a \quad 2)^2 \quad 4(a^2 \quad 4) \quad 0$

$(a \quad 2)(2a \quad 5) \quad 0$

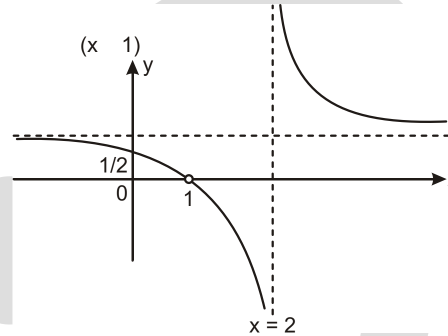
$$2 \quad a \quad \frac{5}{2}$$

Integral value of $a \quad 2$

At $a \quad 2$

$$f(x) \quad \frac{x^2 \quad 2x \quad 1}{x^2 \quad 3x \quad 2} \quad \frac{(x \quad 1)^2}{(x \quad 2)(x \quad 1)}$$

$$f(x) \quad \frac{x \quad 1}{x \quad 2}(x \quad 1)$$



Range $R \quad \{0, 1\}$

No integral values of 'a' for which range is R.

18. $x^{100} \quad (x^2 \quad 3x \quad 2)\ Q(x) \quad (ax \quad b)$

at $\quad x \quad 1 \qquad\qquad a \quad b \quad 1$ ...(1)

at $\quad x \quad 2 \qquad\qquad 2a \quad b \quad 2^{100}$ ...(2)

$a \quad 2^{100} \quad 1,\ b \quad 2 \quad 2^{100}$

Remainder $\quad (2^{100} \quad 1)\,x \quad 2(1 \quad 2^{99})$

$k \quad 99$

19. $x \quad 7^{1/3} \quad 7^{2/3}$

$x^3 \quad 7 \quad 49 \quad 3 \quad 7(x) \qquad x^3 \quad 21x \quad 56 \quad 0$

Product of all roots $\quad 56$

21. Clearly $P(x)$ is a second degree polynomial.

$$P(x) \quad ax^2 \quad bx \quad c$$

$$P\ (x) \quad 2ax \quad b$$

$$P(x) \quad P\ (x) \quad ax^2 \quad (b \quad 2a)\,x \quad c \quad b \quad x^2 \quad 2x \quad 1$$

$$a \quad 1, b \quad 2a \quad 2, c \quad b \quad 1$$

$$a \quad 1, b \quad 4, c \quad 5$$

$$P(x) \quad x^2 \quad 4x \quad 5$$

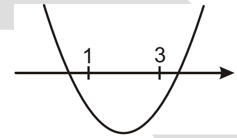
$$P(\ 1) \quad 1 \quad 4 \quad 5 \quad 6 \quad 4 \quad 2$$

23. Let $x^2 \quad t$

$t^2 \quad kt \quad k \quad 0$

$D \quad 0 \qquad k \quad (\quad , 0) \quad (4, \quad)$

$f(0) \quad 0 \qquad k \quad 0$



25. $f(1) \quad 0 \qquad a \quad \frac{4}{3}$

$f(3) \quad 0 \qquad a \quad \frac{8}{7}$

Integral values of 'a' are 5, 4, 3, 2.

26. $f(0) f \quad \frac{}{2} \qquad 0$

$(n \quad 1)(2n \quad 1)(n \quad 3) \quad 0 \qquad n \quad [3, \quad)$

27. $f(x) \quad ax^2 \quad bx \quad c \qquad\qquad a, b, c \quad I$

$ax^2 \quad bx \quad c \quad a(x \quad)(x \quad) \quad p \qquad , \quad I$

$ax^2 \quad bx \quad c \quad 2p \quad a(x \quad)(x \quad) \quad p \quad 0$

Not possible for integral values of x.

28. $9x^2 \quad 2x(y \quad 46) \quad y^2 \quad 20y \quad 244 \quad 0$

$D \quad 0 \qquad y^2 \quad 11y \quad 10 \quad 0$

$(y \quad 1)(y \quad 10) \quad 0 \qquad 1 \quad y \quad 10$

$y^2 \quad 2y(x \quad 10) \quad 9x^2 \quad 92x \quad 244 \quad 0$

$D \quad 0 \qquad x^2 \quad 9x \quad 18 \quad 0$

$(x \quad 3)(x \quad 6) \quad 0 \qquad 3 \quad x \quad 6$

29. $a \quad b \quad 3$ and $a^3 \quad b^3 \quad 7 \qquad a^3 \quad (3 \quad a)^3 \quad 7 \qquad 9a^2 \quad 27a \quad 20 \quad 0$

Sum of distinct values of 'a' is 3.

30. $(y^2 \quad 3)^2 \quad (x \quad 4)^2 \quad 1$

$x \quad 4 \quad \cos \quad , \quad y^2 \quad 3 \quad \sin$

$M \quad 36, m \quad 1$

31. $x_1 \quad x_2 \quad x_1 x_2 \quad a$

$x_1 x_2 \quad x_1 x_2 (x_1 \quad x_2) \quad b$

$x_1^2 x_2^2 \quad c$

If $\quad b \quad c \quad 2(a \quad 1) \qquad x_1 x_2 \quad 2$

32. $x^3 \quad 3x^2 \quad 4x \quad 5 \quad 0 \qquad x \qquad$ is root

$x^3 \quad 3x^2 \quad 4x \quad 5 \quad 0 \qquad x \qquad$ is root

0

33. 5

$(1) \quad (2) \qquad\qquad (2) \quad (3)$

$(x \quad z)(x \quad y \quad z) \quad 1$ and $(y \quad x)(x \quad y \quad z) \quad 2$

Divide $z \quad \dfrac{x \quad y}{2}$

$$y^2 \quad y\,\frac{x \quad y}{2} \qquad \left(\frac{x \quad y}{2}\right)^2 \quad 1 \text{ and } x^2 \quad 2xy \quad 5y^2 \quad 0$$

$$x \quad (1 \quad \sqrt{6})y$$

$y^2 \quad \dfrac{2}{9 \quad 3\sqrt{6}}$ put values

34. $\dfrac{4(1 \quad a \quad b) \quad (a \quad b)^2}{4} \qquad \dfrac{4(1 \quad a \quad b) \quad (a \quad b)^2}{4}$

$$8 \quad (a \quad b)^2 \quad (a \quad b)^2 \qquad a^2 \quad b^2 \quad 4$$

35. $\sqrt[3]{20x \quad \sqrt[3]{20x \quad 13}} \quad 13$

$$\sqrt[3]{20x \quad \sqrt[3]{20x \quad \sqrt[3]{20x \quad \sqrt[3]{20x \quad \sqrt[3]{20x}}}}} \qquad 13$$

$$\sqrt[3]{20x \quad 13} \quad 13 \qquad 20x \quad 2197 \quad 13$$

$$x \quad \frac{2184}{20} \quad \frac{546}{5}$$

36. Let $f(x) \quad x^2 \quad 2(a \quad 1)x \quad a(a \quad 1)$

$f(1 \quad a) \quad 0 \qquad\qquad f(1 \quad a) \quad 0$

$4a^2 \quad 3a \quad 1 \quad 0 \qquad\qquad 3a \quad 1 \quad 0$

$\dfrac{1}{4} \quad a \quad 1 \qquad\qquad a \quad \dfrac{1}{3}$

$$a \qquad \frac{1}{4}, 1$$

37. $(x \quad 8)(x \quad 2) \quad 0$

$$2 \quad x \quad 8$$

38. $\sin \qquad \cos \qquad \dfrac{b}{a}$

$\sin \qquad \cos \qquad \dfrac{c}{a}$

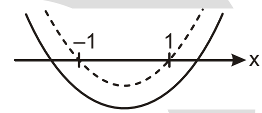
$$(\sin \qquad \cos \;)^2 \quad 1 \quad 2\sin \quad \cos \qquad \frac{b^2}{a^2} \quad 1 \quad \frac{2c}{a}$$

$$\frac{b^2 \quad a^2}{a^2} \quad \frac{2c}{a}$$

39. $\cos^2 x \quad (1 \quad a)\cos x \quad a^2 \quad 0 \quad x \quad R$

Let $\cos \quad t \quad t \quad [\ 1, 1]$

$t^2 \quad (1 \quad a)t \quad a^2 \quad 0 \quad t \quad [\ 1, 1]$



$f(\ 1) \quad 0$

$a^2 \quad a \quad 0$

$a \quad (\quad , 0] \quad [1, \quad)$

$f(1) \quad 0$

$a^2 \quad a \quad 2 \quad 0$

$(a \quad 2)(a \quad 1) \quad 0 \qquad a \quad (\quad , \quad 2] \quad [1, \quad)$

40. $2x^2 \quad 35x \quad 2 \quad 0$

$2 \quad 35 \quad \frac{2}{} \quad$ and $\quad 2 \quad 35 \quad \frac{2}{}$

42. $xF(x) \quad 1 \quad k(x \quad 1)(x \quad 2)(x \quad 3) \quad (x \quad 9)$

$$F(x) \quad \frac{k(x \quad 1)(x \quad 2)(x \quad 3) \quad (x \quad 9) \quad 1}{x}$$

Constant term $\quad k(\ 9!) \quad 1 \quad 0$

$$k \quad \frac{1}{9!}$$

44. $\cos A \quad \cos B \quad \cos C \quad a$

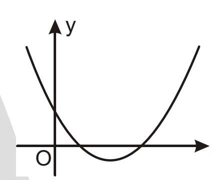
$\cos A \cos B \quad \cos B \cos C \quad \cos A \cos C \quad b$

$\cos A \cos B \cos C \quad c$

$a^2 \quad 2b \quad 2c \quad \cos^2 A \quad \cos^2 B \quad \cos^2 C \quad 2\cos A \cos B \cos C$

1

45. $k \quad 0$



$D \quad 0 \quad \frac{b}{2a} \quad 0$

$k^2 \quad 10k \quad 9 \quad 0 \quad \frac{k \quad 3}{k} \quad 0$

$k \quad (\quad , 1) \quad (9, \quad) \quad k \quad (0, 3) \qquad k \quad (0, 1)$

❑❑❑

9 SEQUENCE AND SERIES

Exercise-1 : Single Choice Problems

1. AM GM

3. $2\sec \quad \sec(\quad 2\ ) \quad \sec(\quad 2\ )$

$$\frac{2}{\cos} \quad \frac{\cos(\quad 2\ ) \quad \cos(\quad 2\ )}{\cos(\quad 2\ )\cos(\quad 2\ )}$$

$$\cos 2 \quad \cos 4 \quad \cos \quad (2\cos \quad \cos 2\ )$$

$$2\cos^2 \quad 1 \quad 2\cos^2 2 \quad 1 \quad 2\cos^2 \quad \cos 2$$

$$\cos^2 \quad (1 \quad \cos 2\ ) \quad (\cos 2 \quad 1)(\cos 2 \quad 1) \quad 0$$

$$\cos^2 \quad 1 \quad \cos 2$$

4. If a, b, c A.P. $\qquad b \quad \dfrac{a \quad c}{2}$

if c, d, e H.P. $\qquad d \quad \dfrac{2ec}{e \quad c}$

if b, c, d G.P. $\qquad c^2 \quad bd$

$$c^2 \quad \frac{a \quad c}{2} \quad \frac{2ec}{e \quad c}$$

$$c^2 \quad ae$$

5. $(a \quad nd)^2 \quad (a \quad md)(a \quad rd)$

$$\frac{a}{d} \quad \frac{mr \quad n^2}{2n \quad m \quad r}$$

if m, n, r in H.P., then $n \quad \dfrac{2mr}{m \quad r} \qquad \dfrac{a}{d} \quad \dfrac{n}{2}$

6. A.M. (, , ,) $\dfrac{4}{4} \quad 1$

G.M. (, , ,) $1 \qquad\qquad 1$

So, equation is $(x \quad 1)^4 \quad 0$

7. $S_3 \quad S_1^2 \qquad \dfrac{S_1^4 S_2^2 \quad S_2^2 S_3^2}{S_1^2 \quad S_3^2} \quad \dfrac{S_2^2(S_1^4 \quad S_3^2)}{S_1^2 \quad S_3^2} \quad 0$

8. $T_r \quad \dfrac{r \;\; 2^r}{(r \quad 2)!}$

$T_r \quad \dfrac{(r \quad 2 \quad 2)\, 2^r}{(r \quad 2)!} \quad \dfrac{1}{(r \quad 1)!} 2^r \quad \dfrac{1}{(r \quad 2)!} 2^{r \quad 1}$

$S_n \quad \dfrac{2!}{2!} \quad \dfrac{2^{n \quad 1}}{(n \quad 2)!}$

$\lim\limits_{n} \; S_n \quad S \qquad 1 \qquad\qquad \text{as} \lim\limits_{n} \dfrac{2^{n \quad 1}}{(n \quad 2)!} \quad 0$

9. $\tan^2 \dfrac{}{12} \quad \tan \dfrac{}{12} \quad x \;\tan \dfrac{}{12} \quad x$

$\tan^2 \dfrac{}{12} \quad \dfrac{\tan^2 \frac{}{12} \quad \tan^2 x}{1 \quad \tan^2 \frac{}{12} \tan^2 x} \qquad \tan^2 x \;\tan^4 \dfrac{}{12} \quad 1 \quad 0 \qquad \tan x \quad 0$

$x \quad 0, \quad ,2 \quad ,3 \quad , \qquad 99$

10. $\dfrac{S_n}{S_n \quad 1} \quad \dfrac{n}{n \quad 1} \dfrac{n \quad 1}{n \quad 2}$

$Q_n \quad \dfrac{2}{1} \quad \dfrac{3}{2} \quad \dfrac{4}{3} \quad \dfrac{5}{4} \qquad \dfrac{n}{n \quad 1} \qquad \dfrac{3}{4} \quad \dfrac{4}{5} \quad \dfrac{5}{6} \qquad \dfrac{n}{n \quad 1} \quad \dfrac{n \quad 1}{n \quad 2}$

$Q_n \quad \dfrac{n}{1} \quad \dfrac{3}{n \quad 2} \quad \dfrac{3n}{n \quad 2}$

$\lim\limits_{n} Q_n \quad 3$

11. $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} \quad \begin{vmatrix} \log A & (p \quad 1)\log R & p & 1 \\ \log A & (q \quad 1)\log R & q & 1 \\ \log A & (r \quad 1)\log R & r & 1 \end{vmatrix}$

$\begin{vmatrix} \log A & p & 1 \\ \log A & q & 1 \\ \log A & r & 1 \end{vmatrix} \quad \begin{vmatrix} (p \quad 1)\log R & p & 1 \\ (q \quad 1)\log R & q & 1 \\ (r \quad 1)\log R & r & 1 \end{vmatrix} \quad 0$

12. Numbers divisible by 6   49

Numbers divisible by 18   16

13. $\dfrac{y \quad z}{2} \quad \sqrt{yz} \qquad\qquad 1 \quad x \quad 2\sqrt{yz}$

Thus, $(1 \quad x)(1 \quad y)(1 \quad z) \quad 2\sqrt{yz} \;\; 2\sqrt{zx} \;\; 2\sqrt{xy} \quad 8xyz$

$$\frac{xyz}{(1 \quad x)(1 \quad y)(1 \quad z)} \quad \frac{1}{8}$$

17. Clearly, both roots are lies in between 1 and 1.

$$\lim_{n} \sum_{r\,1}^{n} (\quad ^{r} \quad ^{r}) \quad \lim_{n} \sum_{r\,1}^{n} \quad ^{r} \quad \lim_{n} \sum_{r\,1}^{n} \quad ^{r}$$

$$\frac{}{1} \quad \frac{}{1} \quad \frac{1}{12}$$

18. $\dfrac{a_i}{a_j} \quad \dfrac{a_1}{a_2} \quad \dfrac{a_1}{a_3} \quad \dfrac{a_1}{a_4} \quad \dfrac{a_2}{a_1} \quad \dfrac{a_2}{a_3} \quad \dfrac{a_2}{a_4} \quad \dfrac{a_3}{a_1} \quad \dfrac{a_3}{a_2} \quad \dfrac{a_3}{a_4} \quad \dfrac{a_4}{a_1} \quad \dfrac{a_4}{a_2} \quad \dfrac{a_4}{a_3}$

$$12 \qquad \qquad \because x \quad \frac{1}{x} \quad 2$$

19. $\dfrac{x^2 \quad 2xy \quad 2xy \quad 4y^2 \quad z^2 \quad z^2}{6} \quad \sqrt[6]{2^2 \quad 4 \quad x^4 \quad y^4 \quad z^4}$

20. Let first term be 'a' and difference be d.

$$5(a \quad 4d) \quad 8(a \quad 7d)$$

$$a \quad 12d \quad 0$$

$$S_{25} \quad \frac{25}{2}[2a \quad 24d]$$

$$S_{25} \quad 25(a \quad 12d) \quad 0$$

21. $10\sin x \quad \sqrt{5}(4\sin^2 x \quad 1) \qquad \sin x \quad 0$

$$\sin x \quad \frac{\sqrt{5} \quad 1}{4}$$

22. Let first term of G.P. be a and ratio be r.

$$a \quad ar \quad ar^2 \quad 70 \quad \text{and} \quad 10ar \quad 4a \quad 4ar^2$$

$$a \quad 40 \qquad r \quad \frac{1}{2}$$

$$S \quad \frac{a}{1 \quad r} \quad \frac{40}{1 \quad \frac{1}{2}} \quad 80$$

23. $\displaystyle \sum_{n\,1} \frac{k}{2^{n \quad k}} \quad \frac{k}{2^k} \sum_{n\,1} \frac{1}{2^n} \quad \frac{k}{2^k}$

$$\sum_{k\,1} \frac{k}{2^k} \quad \frac{1}{2} \quad \frac{2}{2^2} \quad \frac{3}{2^3} \quad \frac{4}{2^4} \qquad 2$$

24. $(pqr)^{1/3} \quad \dfrac{p \quad q \quad r}{3} \qquad p \quad q \quad r$

if $3p \quad 4q \quad 5r \quad 12 \qquad p \quad q \quad r \quad 1$

25. $\frac{1}{3} \; \frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{10} \quad \frac{1}{15} \quad \frac{1}{21} \qquad \frac{1}{3} \; \frac{1}{3} \; 1 \quad \frac{1}{2} \quad \frac{1}{5} \; \frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{7} \; \frac{1}{3} \quad \frac{1}{4}$

$$\frac{1}{3} \quad \frac{1}{1\;2} \quad \frac{1}{2\;3} \quad \frac{1}{3\;4} \qquad \frac{1}{3}$$

26. $\dfrac{\frac{a}{2}[2A \quad (a \quad 1)D]}{a^2} \quad \dfrac{\frac{b}{2}[2A \quad (b \quad 1)D]}{b^2} \quad c \qquad D \quad 2c, A \quad c$

27. $\dfrac{x/r}{1 \quad r} \quad 4 \qquad \dfrac{x}{4} \quad r \quad r^2$

if $1 \quad r \quad 1$ then $2 \quad r \quad r^2 \quad \dfrac{1}{4}$

$$2 \quad \frac{x}{4} \quad \frac{1}{4} \qquad 8 \quad x \quad 1$$

28. $t_1 \quad t_3 \quad t_5 \qquad t_{2n \quad 1} \quad \dfrac{n \quad 1}{2}[2a \quad n(2d)] \quad 248$

$t_2 \quad t_4 \quad t_6 \qquad t_{2n} \quad \dfrac{n}{2}[2(a \quad d) \quad (n \quad 1)2d] \quad 217$

$$t_{2n \quad 1} \quad t_1 \quad 2n \;\; d \quad 56$$

$$\frac{n \quad 1}{2}[2a \quad 56] \quad 248 \text{ and } \frac{n}{2}[2a \quad 56] \quad 217$$

$$n \quad 7, \; a \quad 3$$

29. length of side $A_1 \quad 20$

length of side $A_2 \quad \dfrac{20}{\sqrt{2}}$

length of side $A_3 \quad \dfrac{20}{(\sqrt{2})^2}$

length of side $A_n \quad \dfrac{20}{(\sqrt{2})^{n \quad 1}}$

Area of $A_n \quad \dfrac{400}{2^{n \quad 1}} \quad 1$

30. $S_k \quad \underset{i \;\; 0}{} \dfrac{1}{(k \quad 1)^i} \quad 1 \quad \dfrac{1}{k \quad 1} \quad \dfrac{1}{(k \quad 1)^2} \qquad \dfrac{k \quad 1}{k}$

$$\overset{n}{\underset{k \;\; 1}{}} kS_k \quad \overset{n}{\underset{k \;\; 1}{}} (k \quad 1) \quad \overset{n}{\underset{k \;\; 1}{}} k \quad \overset{n}{\underset{k \;\; 1}{}} 1 \quad \frac{n(n \quad 1)}{2} \quad n \quad \frac{n(n \quad 3)}{2}$$

31. $T_r \quad \dfrac{(r^2 \quad 1)}{r(r \quad 1)} \, 2^{r \quad 1} \quad 1 \quad \dfrac{1}{r} \quad \dfrac{2}{r \quad 1} \quad 2^{r \quad 1}$

$$S_n \quad {}_{r\ 1}^{\ n} T_r \quad {}_{r\ 1}^{\ n} 2^{r\ 1} \quad {}_{r\ 1}^{\ n} \frac{2^{r\ 1}}{r} \quad \frac{2^r}{r \quad 1} \quad (2^{n\ 1}) \quad 1 \quad \frac{2^n}{n \quad 1} \quad \frac{n}{n \quad 1} \ 2^n$$

32. $${}_{n\ 2}^{29} (1.5)^n \quad (1.5)^2 \quad (1.5)^3 \qquad (1.5)^{29}$$

$$(1.5)^2 \ \frac{(1.5)^{28} \quad 1}{0.5} \quad 2k \quad 2(1.5)^2$$

33. $7, A_1, A_2, A_3, \qquad , A_n$, 49 are in A.P.

$$A_1 \quad A_2 \quad A_3 \qquad A_n \quad \frac{n \quad 2}{2} (7 \quad 49) \quad (7 \quad 49)$$

$$\frac{n}{2} \ 56 \quad 364 \qquad n \quad 13$$

34. $\frac{2}{r^2}, \frac{2}{r}, 2, 2r, 2r^2$

35. $S_n \quad 5n^2 \quad 4n$

$t_n \quad S_n \quad S_{n\ 1} \quad 10n \quad 1$

36. $$x^3 \quad y^3 \quad (x \quad y)(x^2 \quad y^2 \quad xy) \quad a \ b \ \frac{a^2 \quad b}{2} \qquad \because xy \ \frac{(x \quad y)^2 \quad (x^2 \quad y^2)}{2}$$

$$\frac{3ab}{2} \quad \frac{a^3}{2}$$

37. $$S_1 \ \frac{1}{1 \ \frac{2}{3}} \quad 3$$

$$S_2 \ \frac{3}{1 \ \frac{2}{5}} \quad 5$$

$\vdots$

$$S_n \ \frac{2n \quad 1}{1 \ \frac{2}{2n \quad 1}} \quad 2n \quad 1$$

$$\frac{1}{S_1 S_2 S_3} \quad \frac{1}{S_2 S_3 S_4} \quad \frac{1}{S_3 S_4 S_5} \qquad \frac{1}{3 \ 5 \ 7} \quad \frac{1}{5 \ 7 \ 9} \quad \frac{1}{7 \ 9 \ 11}$$

$$S \quad {}_{r\ 1} t_r \quad {}_{r\ 1} \frac{1}{(2r \quad 1)(2r \quad 3)(2r \quad 5)} \quad {}_{r\ 1} \frac{1}{(2r \quad 1)(2r \quad 3)} \quad \frac{1}{(2r \quad 3)(2r \quad 5)}$$

38. $ar^5, 2, 5, ar^{13}$ are in G.P.

$(ar^9)^2 \quad 10$

$t_1 t_2 t_3 \qquad t_{19} \quad a^{19} r^{9\ 19} \quad (ar^9)^{19} \quad 10^{19/2}$

39. A.M. G.M.

$$A \quad \frac{1}{A} \quad 1 \quad 3; \qquad B \quad \frac{1}{B} \quad 1 \quad 3; \qquad C \quad \frac{1}{C} \quad 1 \quad 3; \qquad D \quad \frac{1}{D} \quad 1 \quad 3$$

$$A \quad \frac{1}{A} \quad 1 \quad B \quad \frac{1}{B} \quad 1 \quad C \quad \frac{1}{C} \quad 1 \quad D \quad \frac{1}{D} \quad 1 \qquad 3^4$$

40. $(\ r)^2 \qquad r^2 \quad 2 \ r_1 r_2$

$$r_1 r_2 \quad \frac{a \quad b}{2}$$

41. $\frac{2n}{2}[2a \quad (2n \quad 1)d] \quad x$ and $\frac{n}{2}[2(a \quad 2nd) \quad (n \quad 1)d] \quad y$

$$\frac{2y}{n} \quad \frac{x}{n} \quad 3nd \qquad d \quad \frac{2y \quad x}{3n^2}$$

44. $2, 6, 2(k \quad 1)$ are in G.P.

$6^2 \quad 2 \quad 2(k \quad 1)$

$k \quad 10$

$x^2 \quad x \quad 6 \quad 0$ and $|x| \quad 100$

$x \quad (\quad 100, \quad 2) \quad (3, 100)$

Number of integers 193

45. $$\mathop{}_{r\ 1}^{n} \sqrt{1 \quad T_r T_{r\ 1} T_{r\ 2} T_{r\ 3}} \qquad \mathop{}_{r\ 1}^{n} \sqrt{1 \quad r \quad \frac{3}{2} \quad r \quad \frac{1}{2} \quad r \quad \frac{1}{2} \quad r \quad \frac{3}{2}}$$

$$\mathop{}_{r\ 1}^{n} \sqrt{r^2 \quad \frac{5}{4}^{\ 2}} \qquad \mathop{}_{r\ 1}^{n} \left| r^2 \quad \frac{5}{4} \right|$$

$$\mathop{}_{r\ 1}^{2} \left| r^2 \quad \frac{5}{4} \right| \quad \mathop{}_{r\ 2}^{n} \left| r^2 \quad \frac{5}{4} \right| \quad \frac{1}{4} \qquad \mathop{}_{r\ 2}^{n} r^2 \qquad \mathop{}_{r\ 2}^{n} \frac{5}{4}$$

46. $T_r \qquad T_r \qquad T_{r\ 1} \quad r^2 \quad r$

$$\mathop{}_{r\ 1}^{n} \frac{2008}{T_r} \quad 2008 \mathop{}_{r\ 1}^{n} \frac{1}{r(r \quad 1)} \quad (2008) \mathop{}_{r\ 1}^{n} \frac{1}{r} \quad \frac{1}{r \quad 1} \qquad (2008)\frac{n}{n \quad 1}$$

$$\lim_{h} \frac{(2008)n}{n \quad 1} \quad 2008$$

48. $$P(x) \quad \mathop{}_{r\ 1}^{n} \quad x \quad \frac{1}{r} \quad x \quad \frac{1}{r \quad 1} \quad x \quad \frac{1}{r \quad 2}$$

Absolute term $$\mathop{}_{r\ 1}^{n} \frac{1}{r(r \quad 1)(r \quad 2)} \quad \frac{1}{2} \mathop{}_{r\ 1}^{n} \frac{1}{r(r \quad 1)} \quad \frac{1}{(r \quad 1)(r \quad 2)}$$

$$\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{(n \quad 1)(n \quad 2)}$$

$$\lim_{n} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{(n \quad 1)(n \quad 2)} \qquad \frac{1}{4}$$

50. $\frac{1}{T_1}, \frac{1}{T_2}, \frac{1}{T_3}, \quad , \frac{1}{T_k}$ are in A.P.

$$\frac{T_2}{T_6} \quad \frac{\frac{1}{a} \quad 5d}{\frac{1}{a} \quad d} \quad 9 \qquad d \quad \frac{2}{a}$$

$$\frac{T_{10}}{T_4} \quad \frac{\frac{1}{a} \quad 3d}{\frac{1}{a} \quad 9d} \quad \frac{5}{17}$$

52. $1 \quad \frac{1}{3^2} \quad \frac{1}{3^4} \qquad \frac{2}{3} \; 1 \quad \frac{1}{3^2} \quad \frac{1}{3^4} \qquad \frac{15}{8}$

53. $(x \quad 1)(x \quad 2)(x \quad 3)(x \quad 4) \qquad (x \quad 10)$

Coefficient of x^8 sum of terms taken two at a time

$$\frac{1}{2}[(1 \quad 2 \quad 3 \qquad 10)^2 \quad (1^2 \quad 2^2 \qquad 10^2)]$$

55. AM GM

$$\frac{\qquad}{4} \quad (\qquad)^{1/4} \quad \frac{1}{2}$$

$$\frac{1}{2}$$

56. Use AM GM

57. $\sum_{r \; 1} (\;^{r} \quad ^{r}) \quad (\qquad ^2 \quad ^3 \qquad) \quad (\qquad ^2 \quad ^3 \qquad)$

$$\frac{\qquad}{1} \qquad \frac{\qquad}{1}$$

$$4x^2 \quad 2x \quad 1 \quad 0$$

$$4 \quad \frac{x}{1 \quad x}^2 \quad 2 \quad \frac{x}{1 \quad x} \quad 1 \quad 0 \qquad 5x^2 \quad 1 \quad 0 \quad \frac{1-}{\quad} \quad \frac{\quad}{1-}$$

58. $2^2[1 \quad 2^3 \quad 3^3 \quad 4^3 \qquad 10^3] \quad 4 \left(\frac{10 \quad 11}{2}\right)^2 \quad 12100$

59. AM HM

$$\frac{b \quad \frac{a}{2} \quad \frac{a}{2}}{3} \qquad \frac{3}{\frac{4}{a} \quad \frac{1}{b}}$$

60. $4^x \quad 15 \quad 4^{2 \quad x} \qquad 4^x \quad 16 \qquad x \quad 2$

Common ratio $\cos \frac{2011}{3} \qquad \cos\ 670 \quad \overline{3} \qquad \frac{1}{2}$

61. AM GM

$$\frac{a^4 \quad b^4 \quad \frac{c^2}{2} \quad \frac{c^2}{2}}{4} \qquad \left(\frac{a^4b^4c^4}{4}\right)^{1/4}$$

62. $x^2 \quad y^2 \quad x^2 \quad \frac{1}{x^2} \quad 2$

63. $\frac{2}{1^3} \quad \frac{6}{1^3 \quad 2^3} \quad \frac{12}{1^3 \quad 2^3 \quad 3^3} \quad \frac{20}{1^3 \quad 2^3 \quad 3^3 \quad 4^3}$

$$\frac{1 \quad 2}{1^3} \quad \frac{2 \quad 3}{1^3 \quad 2^3} \quad \frac{3 \quad 4}{1^3 \quad 2^3 \quad 3^3}$$

$$\lim_{n} \quad {}_{1}^{n} \frac{n(n \quad 1)}{1^3 \quad 2^3 \qquad n^3} \qquad \lim_{n} \quad {}_{1}^{n} \frac{n(n \quad 1)}{\left(\frac{n(n \quad 1)}{2}\right)^2}$$

$$\lim_{n} \; 4 \quad {}_{1}^{n} \frac{1}{n(n \quad 1)} \qquad 4 \lim_{n} \quad {}_{1}^{n} \frac{1}{n} \quad \frac{1}{n \quad 1}$$

$$4 \lim_{n} \quad \frac{1}{1} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{3} \qquad \frac{1}{n} \quad \frac{1}{n \quad 1}$$

$$4 \lim_{n} \frac{n}{n \quad 1} \quad 4$$

64. ${}_{n \; 1} \frac{1}{(k \quad 1)} \quad \frac{1}{(n \quad 1)(n \quad 2) \quad (n \quad k \quad 1)} \quad \frac{1}{(n \quad 2)(n \quad 3) \quad (n \quad k)} \qquad \frac{1}{(k \quad 1)} \; \frac{1}{2 \; 3 \; 4 \quad k}$

65. $A \quad G \quad \frac{3}{2}$ and $G \quad H \quad \frac{6}{5}$

As we know,

$$G^2 \quad AH \qquad G^2 \quad \frac{3}{2} \quad G \quad G \quad \frac{6}{5} \qquad G \quad 6 \text{ and } A \quad \frac{15}{2}$$

$ab \quad 36$ and $a \quad b \quad 15 \qquad a \quad 12$ and $b \quad 3$

66. $S \quad \frac{2 \quad 5}{2^2 \; 5^2} \quad \frac{5 \quad 8}{5^2 \; 8^2} \quad \frac{8 \quad 11}{8^2 \; 11^2} \qquad \frac{1}{3}\; \frac{5^2 \quad 2^2}{2^2 \; 5^2} \quad \frac{8^2 \quad 5^2}{5^2 \; 8^2} \quad \frac{11^2 \quad 8^2}{8^2 \; 11^2}$

$$\frac{1}{3}\; \frac{1}{2^2} \quad \frac{1}{5^2} \quad \frac{1}{5^2} \quad \frac{1}{8^2} \quad \frac{1}{8^2} \quad \frac{1}{11^2} \qquad \frac{1}{29^2} \quad \frac{1}{32^2}$$

$$\frac{1}{3}\; \frac{1}{4} \quad \frac{1}{32^2} \quad \frac{85}{1024}$$

67. $\overset{10}{\underset{r\;1}{}} \frac{r}{(r^2 \quad 1)^2 \quad r^2} \quad \overset{10}{\underset{r\;1}{}} \frac{r}{(r^2 \quad r \quad 1)(r^2 \quad r \quad 1)}$

$$\frac{1}{2}\overset{10}{\underset{r\;1}{}} \frac{1}{r^2 \quad r \quad 1} \quad \frac{1}{r^2 \quad r \quad 1}$$

68. $\underset{r\;1}{} t_r \quad \underset{r\;1}{} \frac{r}{r^4 \quad r^2 \quad 1}$

$$\underset{r\;1}{} \frac{r}{(r^2 \quad 1)^2 \quad r^2} \quad \underset{r\;1}{} \frac{r}{(r^2 \quad r \quad 1)(r^2 \quad r \quad 1)}$$

$$\frac{1}{2}\underset{r\;1}{} \frac{1}{r^2 \quad r \quad 1} \quad \frac{1}{r^2 \quad r \quad 1}$$

69. $S \quad 1 \quad \frac{4}{5} \quad \frac{7}{5^2} \quad \frac{10}{5^3}$

$$\frac{1}{5} S \quad \frac{1}{5} \quad \frac{4}{5^2} \quad \frac{7}{5^3}$$

$$\frac{4}{5} S \quad 1 \quad \frac{3}{5} \quad \frac{3}{5^2} \quad \frac{3}{5^3} \qquad \frac{7}{4}$$

$$S \quad \frac{35}{16}$$

71. $x_1, x_2, x_3 \quad x_{2n}$

$$\overset{2n}{\underset{r\;1}{}} (\;1)^{r\;\;1} x_r^2$$

$$x_1^2 \quad x_2^2 \quad x_3^2 \qquad x_{2n}^2$$

$$(x_1 \quad x_2)(x_1 \quad x_2 \quad x_3 \qquad x_{2n})$$

$$(x_2 \quad x_1)(x_1 \quad x_2 \quad x_3 \qquad x_{2n})$$

$$\frac{(x_{2n} \quad x_1)}{2n \quad 1} \frac{2x}{2}[x_1 \quad x_{2n}]$$

$$\frac{x}{2x \quad 1}(x_1^2 \quad x_{2n}^2)$$

72. $\frac{\quad}{2} \quad 9; \quad \sqrt{\quad} \quad 4$

73. rms AM

$$\sqrt{\frac{p^2 \quad q^2}{2}} \quad \frac{p \quad q}{2}$$

74. $150 \quad 9 \quad \frac{n}{2}[300 \quad (n \quad 1)(\ 2)] \quad 4500 \qquad n \quad 25$

Total term $n \quad 9 \quad 34$

75. $S_{20} \quad \frac{20}{2}[2(1 \quad ad) \quad 19d] \quad 20$

$$19d \quad 2ad \quad 0$$

76. $\sum_{n \ 3} \frac{1}{(n \quad 2)(n \quad 1)n(n \quad 1)(n \quad 2)} \quad \frac{1}{4} \sum_{n \ 3} \frac{1}{(n \quad 2)(n \quad 1)n(n \quad 1)} \quad \frac{1}{(n \quad 1)n(n \quad 1)(n \quad 2)}$

78. $2^x \quad 2^{2x \ 1} \quad \frac{5}{2^x} \quad 2^x \quad 2^{2x} \quad 2^{2x} \quad \frac{1}{2^x} \quad \frac{1}{2^x} \quad \frac{1}{2^x} \quad \frac{1}{2^x} \quad \frac{1}{2^x}$

$$\frac{2^x \quad 2^{2x \ 1} \quad (5/2^x)}{8} \quad 2^x \quad (2^{2x})^2 \quad \frac{1}{(2^x)^5}^{1/8} \quad 1$$

$$2^x \quad 2^{2x \ 1} \quad \frac{5}{2^x} \quad 8$$

79. $\sum_{r \ 1} \frac{(4r \quad 5)}{r(5r \quad 5)} \quad \frac{1}{5^r} \quad \sum_{r \ 1} \frac{1}{r} \quad \frac{1}{5r \quad 5} \quad \frac{1}{5^r} \quad \sum_{r \ 1} \frac{1}{r \ 5^r} \quad \frac{1}{(r \quad 1) \ 5^{r \ 1}} \quad \frac{1}{5}$

Exercise-2 : One or More than One Answer is/are Correct

1. $a \quad \frac{a_1 \quad a_n}{2}, \quad b \quad \sqrt{a_1 a_n}, \quad c \quad \frac{2a_1 a_n}{a_1 \quad a_n}$

$a \quad b \quad c$ and $b^2 \quad ac$

2. $D_1 : b^2 \quad 4ac \quad 0$

$D_2 : c^2 \quad 4ab \quad 0$

$D_3 : a^2 \quad 4bc \quad 0$

$D_1 \quad D_2 \quad D_3 : a^2 \quad b^2 \quad c^2 \quad 4(ab \quad bc \quad ac)$

$$1 \quad \frac{a^2 \quad b^2 \quad c^2}{ab \quad bc \quad ac} \quad 4$$

3. If a, b, c are in H.P.

A.M. H.M.

$$\frac{a \quad c}{2} \quad b \qquad a \quad c \quad 2b$$

$$a \quad b \quad b \quad c$$

or $$\frac{1}{a \quad b} \quad \frac{1}{b \quad c} \quad 0$$

G.M. > H.M.

also $\sqrt{ac} \quad b$ or $ac \quad b^2$

4. $$T_p \quad a \quad (p \quad 1)\, d \quad \frac{1}{q(p \quad q)}$$

$$T_q \quad a \quad (q \quad 1)\, d \quad \frac{1}{p(p \quad q)} \qquad a \quad d \quad \frac{1}{pq(p \quad q)}$$

5. (a) $a, H_1, H_2, H_3, \quad , H_n, b$ are in H.P.

$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \quad , \frac{1}{H_n}, \frac{1}{b}$ are in A.P.

$$\frac{1}{H_1} \quad \frac{1}{H_2} \quad \frac{1}{H_3} \qquad \frac{1}{H_n} \quad \frac{n}{2} \quad \frac{1}{a} \quad \frac{1}{b}$$

(c) $a, A_1, A_2, A_3, \quad , A_{2n}, b$ are in A.P.

$A_1 \quad A_{2n} \quad A_2 \quad A_{2n \quad 1} \quad A_3 \quad A_{2n \quad 2} \qquad a \quad b$

(d) $4g_2 \quad 5g_3 \quad 4r \quad 5r^2$

This is minimum at $r \quad \frac{2}{5}$

6. a, b, c are in H.P.

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(a) $\frac{a \quad b \quad c}{a} \quad 2, \frac{a \quad b \quad c}{b} \quad 2, \frac{a \quad b \quad c}{c} \quad 2$ are in A.P.

(b) $\frac{a \quad b \quad c}{c} \quad 1, \frac{a \quad b \quad c}{b} \quad 1, \frac{a \quad b \quad c}{c} \quad 1$ are in A.P.

(c) $\frac{2}{b} \quad \frac{1}{a} \quad \frac{1}{c} \quad \frac{2}{\sqrt{ac}} \qquad \sqrt{ac} \quad b$

$a^5 \quad c^5 \quad 2\ (ac)^{5/2} \quad b^5$

(d) $2ac \quad ab \quad bc$

7. Let the roots be a, ar, ar^2, ar^3 and ar^4.

$$\frac{a(r^5 \quad 1)}{r \quad 1} \quad 40 \qquad \ldots(1)$$

and $$\frac{1}{a}\frac{\frac{1}{r^5} \quad 1}{\frac{1}{r} \quad 1} \quad 10 \qquad \ldots(2)$$

put $\frac{r^5 \quad 1}{r \quad 1} \quad \frac{40}{a}$ in (2) we get $ar^2 \qquad 2$

Now, $\qquad (ar^2)^5 \quad (\quad 2)^5$

8. (a) $\because \quad 2a_{k \; 1} \quad a_k \quad a_{k \; 2}$

$f_k(\quad 1) \quad 0$

1 is a root.

Other is also real root.

(b) From (a) (1) is root for any 'k' so any pair of equation has a common root.

(c) If one root is 1, other roots are c/a (form)

$\frac{a_{k \; 2}}{a_k} \quad i.e., \quad \frac{a_3}{a_1}, \frac{a_4}{a_2}, \frac{a_5}{a_3}$ are not in A.P.

9. $b \quad \frac{a \quad c}{2}, d \quad \frac{2ce}{c \quad e}$

if $c^2 \quad bd$, then $c^2 \quad 36 \qquad (\because a \quad 2, e \quad 18)$

10. If a, b, c are in A.P. then

$a \quad b \quad d$ and $c \quad b \quad d$

$a \quad b \quad c \quad 60 \qquad b \quad 20$

If $(a \quad 2), b, (c \quad 3)$ are in G.P., then

$400 \quad (18 \quad d)(23 \quad d) \qquad d \quad 2, \quad 7$

12. $$\frac{81 \quad 144a^4 \quad 16b^4 \quad 9c^4}{4} \quad 36abc$$

A.M. G.M.

$81 \quad 144a^4 \quad 16b^4 \quad 9c^4$

13. $x, y, z \quad$ A.P.

Let $x \quad y \qquad$ and $z \quad y$

$$\cos(y \quad) \quad \cos y \quad \cos(y \quad) \quad 1 \quad \frac{\sin\frac{3}{2}}{\sin\frac{}{2}} \cos(y)$$

$$\sin(y\quad)\quad \sin y\quad \sin(y\quad)\quad \frac{1}{\sqrt{2}}\quad \frac{\sin\frac{3}{2}}{\sin\frac{}{2}}\quad \sin(y)\quad \cot y\quad \sqrt{2}$$

$$\frac{\sin\frac{3}{2}}{\sin\frac{}{2}}\quad \sqrt{\frac{3}{2}}\quad 3\quad 4\sin^2\frac{}{2}\quad \cos\quad \frac{\sqrt{3}\quad\sqrt{2}}{2\sqrt{2}}$$

15. $\dfrac{10^{n\ 1}\quad 1}{10^{n\ 2}\quad 1}\quad \dfrac{10^{m\ 1}\quad 1}{10^{m\ 2}\quad 1}$

$10^{n\ 1}\ 10^{m\ 2}\quad 10^{n\ 1}\quad 10^{m\ 2}\quad 1\quad 10^{n\ 2}\ 10^{m\ 1}\quad 10^{n\ 2}\quad 10^{m\ 1}\quad 1$

$10^{m\ 1}\quad 10^{n\ 1}$

16. $S_r\quad \sqrt{r\quad S_r}\qquad S_r^2\quad S_r\quad r$

17. 50, 48, 46, 44, ……… A.P.

$T_n\quad 50\quad (n\quad 1)(\ 2)\quad 0$

$n\quad 26$

18. $S_n\quad S_n\ \overset{n}{\underset{r\ 1}{}}\ t_r\quad \overset{n}{\underset{r\ 1}{}}\ \dfrac{2r\quad 1}{1^2\quad 2^2\quad 3^2\qquad r^2}\quad \overset{n}{\underset{r\ 1}{}}\ 6\ \dfrac{1}{r}\quad \dfrac{1}{r\quad 1}\quad 6\ 1\quad \dfrac{1}{n\quad 1}$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Sol. $T_1\quad A\quad B\quad 0\qquad\qquad A\quad B$

$T_2\quad A\quad B\quad 1\qquad\qquad A(\quad)\ 1$

$T_3\quad A\ ^2\quad B\ ^2\quad 1\qquad\qquad A(\ ^2\quad ^2)\ 1$

$T_4\quad A\ ^3\quad B\ ^3\quad 2\qquad\qquad A(\ ^3\quad ^3)\ 2$

1 and $\qquad 1$

Paragraph for Question Nos. 3 to 4

Sol. Set A: $\quad 5\quad D, 5, 5\quad D$ and

Set B: $\quad 5\quad d, 5, 5\quad d$

$$\frac{p}{q}\quad \frac{25\quad D^2}{25\quad d^2}\quad \frac{7}{8}$$

$25\quad 8D^2\quad 7d^2\quad d^2\quad 16d\quad 8 \qquad\qquad (\because D\quad 1\quad d)$

$d\quad 1$ and $D\quad 2$

Set $A\ \{3, 5, 7\}$ and set $B\ \{4, 5, 6\}$

Paragraph for Question Nos. 5 to 7

5. $\dfrac{(x \quad 3) \quad (y \quad 1) \quad (z \quad 5)}{3} \quad [(x \quad 3)(y \quad 1)(z \quad 5)]^{1/3}$

$(x \quad 3)(y \quad 1)(z \quad 5) \quad (21)^3$

6. Term is $6(x \quad 3) \quad y \quad \dfrac{1}{2} \quad z \quad \dfrac{5}{3} \qquad \dfrac{(x \quad 3) \quad y \quad \frac{1}{2} \quad z \quad \frac{5}{3}}{3} \quad (x \quad 3) \quad y \quad \dfrac{1}{2} \quad z \quad \dfrac{5}{3}^{\;1/3}$

$(x \quad 3) \quad y \quad \dfrac{1}{2} \quad z \quad \dfrac{5}{3} \quad \dfrac{(355)^3}{6^3 \quad 3^3}$

Maximum value $\dfrac{(355)^3}{6^2 \quad 3^3}$

7. $\dfrac{x \quad y \quad z}{3} \quad (xyz)^{1/3}; \quad xyz \quad (20)^3$

Paragraph for Question Nos. 8 to 10

Sol. Let removed number are A and $A \quad 1$.

$$\frac{n(n \quad 1)}{2} \quad 2A \quad 1 \quad (n \quad 2)\frac{105}{4}$$

$$2n^2 \quad 103n \quad 206 \quad 8A$$

$$n \quad 50, A \quad 7$$

Paragraph for Question Nos. 11 to 13

Sol. $a_{n \quad 1} \quad 1 \quad (a_n \quad 1)^2$

$a_n \quad 1 \quad (a_{n \quad 1} \quad 1)^2$

$a_{n \quad 1} \quad 1 \quad (a_{n \quad 2} \quad 1)^2$

$(a_2 \quad 1) \quad (a_1 \quad 1)^2$

$a_1 \quad 1 \quad (a_0 \quad 1)^2$

$(a_n \quad 1)(a_{n \quad 1} \quad 1)^2(a_{n \quad 2} \quad 1)^{2^2} \qquad (a_1 \quad 1)^{2^{n \quad 1}} \quad (a_{n \quad 1} \quad 1)^2(a_{n \quad 2} \quad 1)^2 \qquad (a_0 \quad 1)^{2^n}$

$(a_n \quad 1) \quad 3^{2^n}$

$$b_n \quad \frac{2 \;\; (3^{2^0} \quad 1)(3^2 \quad 1) \qquad (3^{2^{n \quad 1}} \quad 1)}{(3^{2n} \quad 1)}$$

$$b_n \quad \frac{3^{2^n} \quad 1}{3^{2^n} \quad 1}$$

Paragraph for Question Nos. 14 to 15

$$f(n) \quad {}_{r\ 2}^{\ n} \frac{4}{(r \quad 1)\, r(r \quad 1)} \quad 2 \ {}_{r\ 2}^{\ n} \frac{1}{(r \quad 1)\, r} \quad \frac{1}{r(r \quad 1)} \quad 2 \ \frac{1}{1\ 2} \quad \frac{1}{n\,(n \quad 1)} \ ; a \quad \lim_{n} f(n) \quad 1$$

14. $f(7) \quad f(8) \quad \dfrac{122}{63}$

15. $x^2 \quad \dfrac{3}{2} x \quad t \quad 0 <$

Paragraph for Question Nos. 16 to 17

Sol. $\dfrac{a_1}{a_1 \quad 1} \quad \dfrac{a_2}{a_2 \quad 3} \quad \dfrac{a_3}{a_3 \quad 5} \qquad \dfrac{a_{1005}}{a_{1005} \quad 2009} \quad \dfrac{1}{k}$

$a_1 \quad \dfrac{1}{k \quad 1}, a_2 \quad \dfrac{3}{k \quad 1}, a_3 \quad \dfrac{5}{k \quad 1}, \qquad a_{1005} \quad \dfrac{2009}{k \quad 1}$

$a_1 \quad a_2 \quad a_3 \qquad a_{1005} \quad \dfrac{(1005)^2}{k \quad 1} \quad 2010 \qquad k \quad 1 \quad \dfrac{1005}{2}$

Exercise-4 : Matching Type Problems

1. (A) a, b, c are in A.P.

$b \quad a \quad c \quad b$

$b \quad a, c \quad b, a$ are in G.P.

$\dfrac{c \quad b}{b \quad a} \quad \dfrac{a}{c \quad b} \qquad c \quad b \quad a \qquad (\because b \quad a \quad c \quad b)$

(B) a, x, b are in A.P.

$x \quad \dfrac{a \quad b}{2}$

a, y, z, b are in G.P.

$y \quad a^{2/3} b^{1/3}, \quad z \quad a^{1/3} b^{2/3}$

(C) $a, b \quad ar, c \quad ar^2$

If $c \quad 4b \quad 3a$

$r^2 \quad 4r \quad 3 \quad 0 \qquad (\because a \quad 0)$

$(r \quad 3)(r \quad 1) \quad 0$

(D) $7x^2 \quad 8x \quad 9 \quad 0$

$a \quad 7 \quad 0, \quad D \quad 64 \quad 252 \quad 0$

No solution

2. (A) $a \quad d \quad b \quad c \quad 20$

(B) $2, G_1, G_2, G_3, G_4, G_5, G_6, 5$ are in G.P.

$G_1G_6 \quad G_2G_5 \quad G_3G_4 \quad 10$

(C) $a_4h_7 \quad a_1h_{10} \quad a_{10}h_1 \quad 6$

(D) $(2^x \quad 5)^2 \quad 2 \;\; 2^x \quad \frac{7}{2} \qquad (2^x \quad 8)(2^x \quad 4) \quad 0 \qquad x \quad 3$

3. (A) $2 \;\; 2^{x^2} \quad 2^x \quad 2^{x^3}$

Exponential series can't be in A.P.

(B) If $a_1, a_2, a_3, \qquad , a_n$ are in A.P.

$$a_2 \quad a_1 \quad a_3 \quad a_2 \quad a_4 \quad a_3 \qquad a_n \quad a_{n \;\, 1} \quad d$$

$$S \quad d \;\; \frac{1}{\sqrt{a_1} \quad \sqrt{a_2}} \quad \frac{1}{\sqrt{a_2} \quad \sqrt{a_3}} \qquad \frac{1}{\sqrt{a_{n \;\, 1}} \quad \sqrt{a_n}}$$

$$d \;\; \frac{\sqrt{a_n} \quad \sqrt{a_1}}{d} \quad \sqrt{a_1} \quad \sqrt{a_n}$$

(C)
$$\frac{S_{2n}}{S_n} \quad \frac{\frac{2n}{2}[2a \quad (2n \quad 1)\,d]}{\frac{n}{2}[2a \quad (n \quad 1)\,d]} \quad 3$$

$$2a \quad (n \quad 1)\,d$$

$$\frac{S_{3n}}{2S_n} \quad \frac{\frac{3n}{2}[2a \quad (3n \quad 1)\,d]}{\frac{2n}{2}[2a \quad (n \quad 1)\,d]} \quad 3$$

(D) $t_1 \quad t_5 \quad t_2 \quad t_4 \quad 2t_3$

$$\frac{4(t_1 \quad t_2 \quad t_4) \quad 6t_3 \quad t_5}{3t_1} \quad \frac{3t_1 \quad (t_1 \quad t_5) \quad 4(t_2 \quad t_4) \quad 3(2t_3)}{3t_1} \quad 1$$

4. A Q; B P; C T; D S

5. (A) $\frac{1}{3}\log_2 x \quad \log_2 y \quad 5$ and $\frac{1}{3}\log_2 y \quad \log_2 x \quad 7$

$\log_2 x \quad 6$ and $\log_2 y \quad 3$

$x \quad 2^6$ and $y \quad 2^3$

(B) $B \quad 60$ and $b^2 \quad ac$

$$\cos B \quad \frac{a^2 \quad c^2 \quad b^2}{2ac} \quad \frac{1}{2} \qquad a^2 \quad c^2 \quad 2ac$$

$$a \quad c$$

(C) AM GM

$$\frac{\frac{b}{a} \quad \frac{c}{a} \quad \frac{a}{b} \quad \frac{c}{b} \quad \frac{a}{c} \quad \frac{b}{c}}{6} \quad 1$$

(D) $(b \quad c)^2 \quad a^2 \quad bc$

$$b^2 \quad c^2 \quad a^2 \quad (\quad 2)\,bc$$

$$\frac{b^2 \quad c^2 \quad a^2}{2bc} \quad \frac{\quad 2}{2}$$

$$1 \quad \frac{2}{2} \quad 1$$

$$0 \quad 4$$

6. $P(n) \quad (f(n \quad 2) \quad f(n)) \quad q(n)$

$$P(n) \quad \frac{1}{n \quad 1} \quad \frac{1}{n \quad 2} \quad q(n)$$

$$P(n) \quad (2n \quad 3) \quad (n^2 \quad 3n \quad 2) \quad q(n)$$

$P(n) \quad n^2 \quad 3n \quad 2$ and $q(n) \quad (2n \quad 3)$

Exercise-5 : Subjective Type Problems

1. If a, b, c, d are in A.P. with common difference 'k', then

$$9k^3 \quad (x \quad 4)\,k^2 \quad 4k \quad 0$$

$$k\{9k^2 \quad (x \quad 4)\,k \quad 4\} \quad 0$$

$$D \quad 0 \qquad (x \quad 4)^2 \quad 144 \quad 0$$

$$(x \quad 8)(x \quad 16) \quad 0$$

$$x \quad (\quad , \quad 8] \quad [16, \quad)$$

2. $S \quad 1\,2 \quad 2\,2^2 \quad 3\,2^3 \quad 4\,2^4 \qquad n\,2^n$

$$2\,S \quad 1\,2^2 \quad 2\,2^3 \quad 3\,2^4 \qquad (n \quad 1)\,2^n \quad n\,2^{n \quad 1}$$

$$S \quad (n \quad 1)\,2^{n \quad 1} \quad 2 \quad 2 \quad 2^{n \quad 10}$$

$$2(n \quad 1) \quad 2^{10}$$

$$n \quad 513$$

3. $$\lim_{n} \sum_{r \quad 1}^{n} \frac{r \quad 2}{2^{r \quad 1}\,r(r \quad 1)} \quad \sum_{r \quad 1} \frac{1}{r\,2^r} \quad \frac{1}{(r \quad 1)\,2^{r \quad 1}} \quad \frac{1}{2}$$

4. $$\sum_{r \quad 1} \frac{8r}{4r^4 \quad 1} \quad 2 \sum_{r \quad 1} \frac{1}{2r^2 \quad 2r \quad 1} \quad \frac{1}{2r^2 \quad 2r \quad 1} \quad 2$$

5. Let three terms in A.P. $a \quad d, a, a \quad d$

If $(a \quad d)^2, a^2, (a \quad d)^2$ are in G.P. $\quad d \quad \sqrt{2}\, a$

$$r \quad \frac{a^2}{(a \quad d)^2} \quad \frac{1}{(1 \quad \sqrt{2})^2}$$

6. $\sqrt{\frac{10^{2n} \quad 1}{9} \quad 2 \quad \frac{10^n \quad 1}{9}} \quad P \quad \frac{10^n \quad 1}{9} \qquad P \quad 3$

7. $a \quad d, a, a \quad d, a \quad d \quad 30$

If last three terms are in G.P.

$$(a \quad d)^2 \quad a(a \quad d \quad 30)$$

$$a \quad \frac{d^2}{30 \quad 3d}$$

8. $\frac{1}{8n^4} \quad {}^{n}_{k \;\; 1} \; [k(k \quad 2)(k \quad 4)(k \quad 6) \quad (k \quad 2)\,k(k \quad 2)(k \quad 4)]$

$\frac{1}{8} \quad \frac{(n \quad 1)(n \quad 1)(n \quad 3)(n \quad 5) \quad n(n \quad 2)(n \quad 4)(n \quad 6) \quad 15}{n^4} \quad \frac{1}{4}(n \qquad)$

9. Unit digit of $\frac{n(n \quad 1)}{2}^{\;2} \quad 1$

Then unit digit of $\frac{n(n \quad 1)}{2}$ is 1 because unit digit of $n(n \quad 1)$ can not be 8.

10. $2\log_b c \quad \log_c a \quad \log_a b$

$$2 \quad \frac{\log a \quad 2\log r}{\log a \quad \log r} \quad \frac{\log a}{\log a \quad 2\log r} \quad \frac{\log a \quad \log r}{\log a}$$

Let $A \quad \log a$ and $R \quad \log r \quad 3A^2 \quad 3Ar \quad 2R^2 \quad 0 \quad \frac{A}{R} \quad \frac{3 \quad \sqrt{33}}{6}$

$$d \quad \log_b c \quad \log_c a \quad \frac{A \quad 2R}{A \quad R} \quad \frac{A}{A \quad 2R} \quad \frac{3}{2}$$

11. $3, \frac{3}{r}, \frac{3r}{s}, 7s; \quad \frac{2}{r} \quad 1 \quad \frac{r}{s}$ and $\frac{6r}{s} \quad \frac{3}{r} \quad 7s$

$$7r^3 \quad 6r^2 \quad 21r \quad 18 \quad 0 \qquad (r^2 \quad 3)(7r \quad 6) \quad 0$$

$$r \quad \frac{6}{7} \text{ and } s \quad \frac{9}{14}$$

12. $S \quad \frac{1^2}{3^1} \quad \frac{2^2}{3^2} \quad \frac{3^2}{3^3} \quad \frac{4^2}{3^4}$

$$\frac{S}{3} \quad \frac{1^2}{3^2} \quad \frac{2^2}{3^3} \quad \frac{3^2}{3^4}$$

$$\frac{2S}{3} \quad S \quad \frac{S}{3} \quad \frac{1}{3} \quad \frac{3}{3^2} \quad \frac{5}{3^3} \quad \frac{7}{3^4}$$

$$\frac{2S}{9} \quad \frac{1}{3^2} \quad \frac{3}{3^3} \quad \frac{5}{3^4}$$

$$\frac{2S}{3} \quad \frac{2S}{9} \quad \frac{1}{3} \quad \frac{2}{3^2} \quad \frac{2}{3^3} \quad \frac{2}{3^4}$$

$$\frac{4S}{9} \quad \frac{1}{3} \quad \frac{2}{3^2} \quad 1 \quad \frac{1}{3} \quad \frac{1}{3^2}$$

$$\frac{4S}{9} \quad \frac{1}{3} \quad \frac{2}{9} \quad \frac{1}{2/3} \quad \frac{2}{3} \qquad S \quad \frac{3}{2} \quad \frac{p}{q}$$

13. $S \quad f(x)_{\max} \qquad x \quad [\ 4, 3]$

$a \quad ar \quad f(0) \quad 3$

$f(x) \quad 3x^2 \quad 3 \quad 0 \qquad\qquad f(x)_{\max} \quad f(3) \quad 27 \quad 9 \quad 9 \quad 27$

$$S \quad 27 \quad \frac{a}{1 \quad r}$$

$$a(1 \quad r) \quad 3 \qquad \frac{1}{1 \quad r} \quad \frac{a}{3}$$

$$27 \quad a \quad \frac{a}{3}$$

$$a^2 \quad 81 \qquad a \quad 9$$

If $a \quad 9 \qquad 1 \quad r \quad \dfrac{3}{9}$ $\qquad$ If $a \quad 9 \qquad 1 \quad r \quad \dfrac{1}{3}$

$r \quad \dfrac{2}{3}$ $\qquad\qquad$ $r \quad \dfrac{4}{3} \quad 1$ (rejected)

$$\frac{p}{q} \quad \frac{2}{3} \qquad p \quad q \quad 5$$

14. Total runs from 1 to 9 $\quad$ 1350

Let, number of terms in A.P. be n.

$$\frac{n}{2}[300 \quad (n \quad 1) \quad (\ 1)] \quad 4500 \quad 1350 \quad 3150$$

$n \quad 25$ or 126, $n \quad 126$ (not possible)

$n \quad 25$, total matches $\quad 34$

15. $x \quad \dfrac{10}{4} \displaystyle\sum_{n\ 3}^{100} \dfrac{1}{n \quad 2} \quad \dfrac{1}{n \quad 2} \quad \dfrac{10}{4} \quad 1 \quad \dfrac{1}{2} \quad \dfrac{1}{3} \quad \dfrac{1}{4} \quad \dfrac{1}{102} \quad \dfrac{1}{101} \quad \dfrac{1}{100} \quad \dfrac{1}{99}$

16. $$f(n) \quad \frac{(2n \quad 1) \quad (2n \quad 1) \quad \sqrt{(2n \quad 1)(2n \quad 1)}}{\sqrt{2n \quad 1} \quad \sqrt{2n \quad 1}}$$

Let $\sqrt{2n \quad 1} \quad a$ and $\sqrt{2n \quad 1} \quad b$

$$f(n) \quad \frac{(a^2 \quad b^2 \quad ab)}{(a \quad b)}\frac{(a \quad b)}{(a \quad b)} \quad \frac{a^3 \quad b^3}{a^2 \quad b^2}$$

$$f(n) \quad \frac{(2n \quad 1)^{3/2} \quad (2n \quad 1)^{3/2}}{2}$$

$$\mathop{}_{n \quad 1}^{60} f(n) \quad \mathop{}_{n \quad 1}^{60} \frac{(2n \quad 1)^{3/2} \quad (2n \quad 1)^{3/2}}{2} \quad \frac{(121)^{3/2} \quad 1}{2} \quad 665$$

17. $3^0\{2^0 \quad 2^{\quad 1} \quad 2^{\quad 2} \quad\quad \} \quad 1\{2\}$

$3^{\quad 1}\{2^0 \quad 2^{\quad 1} \quad 2^{\quad 2} \quad\quad \} \quad \frac{1}{3}\{2\}$

$3^{\quad 2}\{2^0 \quad 2^{\quad 1} \quad 2^{\quad 2} \quad\quad \} \quad \frac{1}{3}\{2\}$

$\vdots \qquad\qquad\qquad\qquad \vdots$

Hence, $\dfrac{2 \quad 1}{1 \quad \frac{1}{3}} \quad 3$

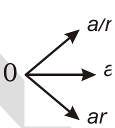
18. $15^2 \quad (15 \quad d)^2 \quad (15 \quad 2d)^2 \qquad\quad (15 \quad 9d)^2 \quad 1185$

$19d^2 \quad 90d \quad 71 \quad 0$

$d \quad 1$

$S_n \quad S_{n \quad 1}$

$\frac{n}{2}(31 \quad n) \quad \frac{n \quad 1}{2}(32 \quad n) \qquad n \quad 16$

19. $24x^3 \quad 14x^2 \quad kx \quad 3 \quad 0$ 

Product of roots $a^3 \quad \frac{1}{8} \qquad a \quad \frac{1}{2}$

$k \quad 7$

If $x \quad 7$ lies between the roots, then

$f(7) \quad 49 \quad 7^{\quad 2} \quad 112 \quad 0$

$^{2} \quad 9 \quad 0$

20. $9x^3 \quad 3y^3 \quad 1 \quad 9xy$

$(9^{1/3}x)^3 \quad (3^{1/3}y)^3 \quad 1^3 \quad 3(9^{1/3}x)(3^{1/3}y) \qquad 9^{1/3}x \quad 3^{1/3}y \quad 1$

21. If a, x, y, z, b A.P.

$x \quad \frac{3a \quad b}{4}, y \quad \frac{a \quad b}{2}$ and $z \quad \frac{a \quad 3b}{4}$

If a, x, y, z, b H.P.

$x \quad \frac{4ab}{3b \quad a}, y \quad \frac{2ab}{a \quad b}$ and $z \quad \frac{4ab}{3a \quad b}$

If $\frac{3a \quad b}{4} \quad \frac{a \quad b}{2} \quad \frac{a \quad 3b}{4} \quad 55$ and $\frac{4ab}{3b \quad a} \quad \frac{2ab}{a \quad b} \quad \frac{4ab}{3a \quad b} \quad \frac{343}{55} \quad ab \quad 7$

❑❑❑

10 DETERMINANTS

Exercise-1 : Single Choice Problems

1. Direct expansion.

2. $$D \quad \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} \quad 0 \qquad\qquad (k \quad 1)^2(k \quad 2) \quad 0$$

also $$D_1 \quad \begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ k^2 & 1 & k \end{vmatrix} \quad 0 \qquad\qquad k \quad 1 \qquad k \quad 2$$

3. $$\begin{vmatrix} a & a^2 & 1 & a^3 \\ b & b^2 & 1 & b^3 \\ c & c^2 & 1 & c^3 \end{vmatrix} \quad (1 \quad abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad 0$$

4. $$D \quad \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} \quad 0 \qquad \frac{2}{b} \quad \frac{1}{a} \quad \frac{1}{c}$$

6. $$2x \quad ay \quad 6z \quad 8$$

and $$4x \quad 2ay \quad 6z \quad 8 \qquad 2x \quad ay \quad 0$$

and $$6x \quad 12y \quad 6z \quad 30 \qquad 4x \quad (12 \quad a)\,y \quad 22$$

$$y \quad \frac{22}{12 \quad 3a} \qquad a \quad 4$$

7. $R_1 \quad R_1 \quad R_2 \quad R_3$

$$\begin{vmatrix} x^2 \quad 4 & x^2 \quad 4 & x^2 \quad 4 \\ 2 & x^2 \quad 13 & 2 \\ x^2 \quad 13 & 3 & 7 \end{vmatrix} \quad (x^2 \quad 4)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x^2 \quad 13 & 2 \\ x^2 \quad 13 & 3 & 7 \end{vmatrix} \quad 0$$

$$(x^2 \quad 4)(x^2 \quad 15)(20 \quad x^2) \quad 0$$

8. $D \quad \begin{vmatrix} k & k \quad 1 & k \quad 1 \\ k \quad 1 & k & k \quad 2 \\ k \quad 1 & k \quad 2 & k \end{vmatrix} \quad 0$

$R_1 \quad R_1 \quad R_2 \qquad\qquad R_2 \quad R_2 \quad R_3$

$D \quad \begin{vmatrix} 1 & 1 & 3 \\ 2 & 2 & 2 \\ k \quad 1 & k \quad 2 & k \end{vmatrix} \quad 0$

9. $\begin{vmatrix} \log a \quad (n \quad 1)\log r & \log a \quad (n \quad 1)\log r & \log a \quad (n \quad 3)\log r \\ \log a \quad (n \quad 5)\log r & \log a \quad (n \quad 7)\log r & \log a \quad (n \quad 9)\log r \\ \log a \quad (n \quad 11)\log r & \log a \quad (n \quad 13)\log r & \log a \quad (n \quad 15)\log r \end{vmatrix}$

$C_3 \quad C_3 \quad C_2 \qquad\qquad C_2 \quad C_2 \quad C_1$

$\begin{vmatrix} \log a \quad (n \quad 1)\log r & 2\log r & 2\log r \\ \log a \quad (n \quad 5)\log r & 2\log r & 2\log r \\ \log a \quad (n \quad 11)\log r & 2\log r & 2\log r \end{vmatrix} \quad 0$

10. $D_2 \quad \begin{vmatrix} a_1 & 2a_3 & 5a_2 \\ b_1 & 2b_3 & 5b_2 \\ c_1 & 2c_3 & 5c_2 \end{vmatrix} \quad 10 \begin{vmatrix} a_1 & a_3 & a_2 \\ b_1 & b_3 & b_2 \\ c_1 & c_3 & c_2 \end{vmatrix} \quad 10 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

11. $_2 \quad \begin{vmatrix} 1 & bc & a \\ 1 & ac & b \\ 1 & ab & c \end{vmatrix}$

$R_1 \quad aR_1 \quad R_2 \quad bR_2 \quad R_3 \quad cR_3$

$_2 \quad \frac{1}{abc} \begin{vmatrix} a & abc & a^2 \\ b & abc & b^2 \\ c & abc & c^2 \end{vmatrix} \quad \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} \quad \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad _1$

12. $C_1 \quad C_1 \quad C_2 \quad C_3 \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \quad a \\ 1 & a \quad 1 & a \quad b \end{vmatrix} \quad 1$

13. $\begin{vmatrix} 1 & 2 & x \\ 2 & 3 & x^2 \\ 3 & 5 & 2 \end{vmatrix} \quad 10 \qquad x^2 \quad x \quad 12 \quad 0$

Sum $\quad 1$

14. $R_1 \quad R_1 \quad R_2, \quad R_2 \quad R_2 \quad R_3$

$$D \begin{vmatrix} 1 & 1 & 1 \\ d \quad a \quad 1 & e \quad b \quad 1 & f \quad c \quad 1 \\ x \quad a & x \quad b & x \quad c \end{vmatrix}, \; C_1 \quad C_1 \quad C_2 \text{ and } C_2 \quad C_2 \quad C_3$$

On solving D does not depend on x.

15. $R_1 \quad R_1 \quad R_2 \quad R_3$

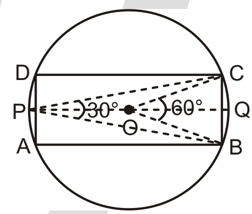
$$(x \quad y \quad z)\begin{vmatrix} 1 & 1 & 1 \\ 2y & y \quad z \quad x & 2y \\ 2z & 2z & z \quad x \quad y \end{vmatrix} \quad C_1 \quad C_1 \quad C_2 \text{ and } C_2 \quad C_2 \quad C_3$$

$$(x \quad y \quad z)^3\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 2y \\ 0 & 1 & z \quad x \quad y \end{vmatrix} \quad (x \quad y \quad z)^3$$

16. $BOC \quad 60$

$BC \quad OB \quad OC \quad r$

$AB \quad 2r\cos 30 \quad \sqrt{3}\, r$



$$\frac{\text{Area or rectangle}}{\text{Area of circle}} \quad \frac{\sqrt{3}\, r^2}{r^2} \quad \sqrt{3}$$

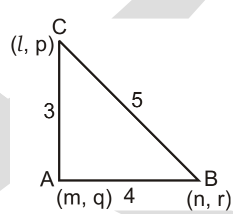
17. $C_1 \quad C_1 \quad bC_3, C_2 \quad C_2 \quad aC_3$

$$(1 \quad a^2 \quad b^2)^2\begin{vmatrix} 1 & 0 & 2b \\ 0 & 1 & 2a \\ b & a & 1 \quad a^2 \quad b^2 \end{vmatrix} \quad (1 \quad a^2 \quad b^2)^3$$

18.
$$\begin{vmatrix} 2 & a \quad b \quad c \quad d & ab \quad cd \\ a \quad b \quad c \quad d & 2(a \quad b)(c \quad d) & ab(c \quad d) \quad cd(a \quad b) \\ ab \quad cd & ab(c \quad d) \quad cd(a \quad b) & 2abcd \end{vmatrix} \begin{vmatrix} 1 & 1 & 0 \\ c \quad d & a \quad b & 0 \\ cd & ab & 0 \end{vmatrix}\begin{vmatrix} 1 & a \quad b & ab \\ 1 & c \quad d & cd \\ 0 & 0 & 0 \end{vmatrix}$$

$$0$$

19. $|B| \begin{vmatrix} l & m & n \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} \quad [2\text{Ar}(\ ABC)]^2$



20. $D \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 1 & 5 & 10 \end{vmatrix} \quad 0$

$$D_1 \begin{vmatrix} 1 & 2 & 1 \\ K & 3 & 4 \\ K^2 & 5 & 10 \end{vmatrix} \quad 5(K^2 \quad 3K \quad 2) \quad 5(K \quad 1)(K \quad 2)$$

$$D_2 \begin{vmatrix} 1 & 1 & 1 \\ 1 & K & 4 \\ 1 & K^2 & 10 \end{vmatrix} \quad 3(K^2 \quad 3K \quad 2) \quad 3(K \quad 2)(K \quad 1)$$

$$D_3 \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & K \\ 1 & 5 & K^2 \end{vmatrix} \quad K^2 \quad 3K \quad 2 \quad (K \quad 2)(K \quad 1)$$

21. $$(x \quad 1)(x \quad 2)(x \quad 3) \begin{vmatrix} 1 & x & 1 & (x \quad 1)^2 \\ 1 & x & 2 & (x \quad 2)^2 \\ 1 & x & 3 & (x \quad 3)^2 \end{vmatrix} \quad 2(x \quad 1)(x \quad 2)(x \quad 3)$$

22. $$\begin{vmatrix} 2 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix}$$

$$2(1 \quad \cos^2 A) \quad \cos C(\quad \cos C \quad \cos A \cos B) \quad \cos B(\cos C \cos A \quad \cos B)$$

$$2 \quad \cos 2A \quad \frac{1 \quad \cos 2C}{2} \quad \frac{1 \quad \cos 2B}{2} \quad 2\cos A \cos B \cos C$$

$$\cos 2A \quad \cos 2C \quad \cos 2B \quad 2\cos A \cos B \cos C$$

$$2\cos(A \quad B)\cos(A \quad B) \quad 2\cos^2 C \quad 1 \quad 2\cos A \cos B \cos C$$

$$2\cos C[\cos C \quad \cos(A \quad B)]$$

$$2\cos C \cos A \cos B \quad 1 \quad 2\cos A \cos B \cos C \quad 1$$

24. As a, b and c are the roots of $x^3 \quad 2x^2 \quad 1 \quad 0$, we have

$$a \quad b \quad c \quad 2$$

$$ab \quad bc \quad ca \quad 0$$

$$abc \quad 1$$

Now, for finding the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$, evaluating using first row, we get

$$a(bc \quad a^2) \quad b(b^2 \quad ac) \quad c(ab \quad c^2) \quad abc \quad a^3 \quad b^3 \quad abc \quad abc \quad c^3$$

$$3abc \quad a^3 \quad b^3 \quad c^3 \quad (a^3 \quad b^3 \quad c^3 \quad 3abc)$$

$$(a \quad b \quad c)(a^2 \quad b^2 \quad c^2 \quad ab \quad bc \quad ca)$$

$$(\quad 2)[(\quad 2)^2 \quad 3(0)] \quad 8$$

25. For non-trivial solution, $|A|$ or $D \quad 0$, That is, $\begin{vmatrix} & 1 & 1 \\ 1 & & 2 \\ 1 & 2 & \end{vmatrix} \quad 0$

Now, $R_2 \quad R_2 \quad R_1; R_3 \quad R_3 \quad R_1$ gives $\begin{vmatrix} & 1 & 1 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} \quad 0$

Also, $R_3 \quad R_3 \quad R_2$ gives $\begin{vmatrix} & 1 & 1 \\ 1 & 1 & 3 \\ 0 & 0 & 4 \end{vmatrix} \quad 0$

Evaluation using third row, we get

$$4(\qquad 1) \quad 0 \qquad \frac{1}{2}$$

which is exactly the real value of .

Exercise-2 : One or More than One Answer is/are Correct

1. $f(a, b) \quad a(a \quad b)(a \quad 2b)$

2. $R_1 \quad R_1 \quad R_2$ and $R_2 \quad R_2 \quad R_3$

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ \cos^2 & \sin^2 & 1 \quad 2\sqrt{3}\tan \end{vmatrix} \quad 0 \qquad \tan \qquad \frac{1}{\sqrt{3}}$$

3. $R_1 \quad R_1 \quad R_2, R_2 \quad R_2 \quad R_3$

$$d^2 \begin{vmatrix} 1 & 1 & 3 \\ 1 & 2 & 1 \\ a \quad 2d & a & a \quad d \end{vmatrix} \quad d^2(13d \quad 12a)$$

4. $\begin{vmatrix} 1 & 3 & 4 \\ 1 & (3 \quad) & 5 \\ 3 & 1 & \end{vmatrix} \quad 0$

5. $D(x) \begin{vmatrix} x^2 \quad 4x \quad 3 & 2x \quad 4 & 13 \\ 2x^2 \quad 5x \quad 9 & 4x \quad 5 & 26 \\ 8x^2 \quad 6x \quad 1 & 16x \quad 6 & 104 \end{vmatrix}$

$C_3 \quad C_3 \quad 4C_2, C_2 \quad C_2 \quad 2C_1$

$$D(x) \begin{vmatrix} 3x \quad 3 & 3 & 0 \\ 26x \quad 37 & 26 & 0 \\ 8x^2 \quad 6x \quad 1 & 16x \quad 6 & 104 \end{vmatrix}$$

7. $D \begin{vmatrix} a & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & a \end{vmatrix} \quad 0 \qquad a^2 \quad a \quad 2 \quad 0 \qquad (a \quad 2)(a \quad 1) \quad 0$

$D_1 \begin{vmatrix} 0 & 1 & 2 \\ b & 2 & 1 \\ 0 & 1 & a \end{vmatrix} \qquad D_2 \begin{vmatrix} a & 0 & 2 \\ 1 & b & 1 \\ 2 & 0 & a \end{vmatrix} \qquad D_3 \begin{vmatrix} a & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{vmatrix}$

$a \quad 2$ infinite solution

$a \quad 1, b \quad 0$ has no solution.

8. $D \begin{vmatrix} a & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & a \end{vmatrix} \quad 0 \qquad a^2 \quad a \quad 2 \quad 0$

$(a \quad 2)(a \quad 1) \quad 0$

$D \begin{vmatrix} 0 & 1 & 2 \\ b & 2 & 1 \\ 0 & 1 & a \end{vmatrix} \qquad D \begin{vmatrix} a & 0 & 2 \\ 1 & b & 1 \\ 2 & 0 & a \end{vmatrix} \qquad D \begin{vmatrix} a & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{vmatrix}$

$a \quad 2$ infinite solution

$a \quad 1, b \quad 0$ has no solution.

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

$D \begin{vmatrix} 2 & & 6 \\ 1 & 2 & \\ 1 & 1 & 3 \end{vmatrix} \quad (\quad 2)(\quad 3); \qquad D_1 \begin{vmatrix} 8 & & 6 \\ 5 & 2 & \\ 4 & 1 & 3 \end{vmatrix} \quad (\quad 2)(4 \quad 15)$

$D_2 \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & \\ 1 & 4 & 3 \end{vmatrix} \quad 0; \qquad D_3 \begin{vmatrix} 2 & & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{vmatrix} \quad (\quad 2)$

Exercise-4 : Subjective Type Problems

2. $R_1 \quad R_1 \quad R_2 \quad R_3$

$3 \begin{vmatrix} a_1 \quad b_1 \quad c_1 & a_2 \quad b_2 \quad c_2 & a_3 \quad b_3 \quad c_3 \\ 2b_1 \quad c_1 & 2b_2 \quad c_2 & 2b_3 \quad c_3 \\ 2c_1 \quad a_1 & 2c_2 \quad a_2 & 2c_3 \quad a_3 \end{vmatrix}$

$$R_2 \quad R_2 \quad R_1 \quad R_3 \quad 9\begin{vmatrix} a_1 \quad b_1 \quad c_1 & a_2 \quad b_2 \quad c_2 & a_3 \quad b_3 \quad c_3 \\ b_1 & b_2 & b_3 \\ 2c_1 \quad a_1 & 2c_2 \quad a_2 & 2c_3 \quad a_3 \end{vmatrix}$$

Now, operate as $R_3 \quad R_3 \quad R_1 \quad R_2$

then $R_1 \quad R_1 \quad R_2 \quad R_3$

3. Let $f(x) \begin{vmatrix} (1 \quad x)^2 & (1 \quad x)^4 & (1 \quad x)^6 \\ (1 \quad x)^3 & (1 \quad x)^6 & (1 \quad x)^9 \\ (1 \quad x)^4 & (1 \quad x)^8 & (1 \quad x)^{12} \end{vmatrix}$

Coefficient of 'x' is $f\,(0)$.

$$f\,(x) \begin{vmatrix} 2(1 \quad x)^2 & 4(1 \quad x)^3 & 6(1 \quad x)^5 \\ (1 \quad x)^3 & (1 \quad x)^6 & (1 \quad x)^9 \\ (1 \quad x)^4 & (1 \quad x)^8 & (1 \quad x)^{12} \end{vmatrix} \begin{vmatrix} (1 \quad x)^2 & (1 \quad x)^4 & (1 \quad x)^6 \\ 3(1 \quad x)^2 & 6(1 \quad x)^5 & 9(1 \quad x)^8 \\ (1 \quad x)^4 & (1 \quad x)^8 & (1 \quad x)^{12} \end{vmatrix}$$

$$\begin{vmatrix} (1 \quad x)^2 & (1 \quad x)^2 & (1 \quad x)^6 \\ (1 \quad x)^3 & (1 \quad x)^6 & (1 \quad x)^9 \\ 4(1 \quad x)^3 & 8(1 \quad x)^7 & 12(1 \quad x)^{11} \end{vmatrix}$$

Put $x \quad 0, \quad f\,(0) \quad 0$

5. For non-zero solution, $\quad 0$

$$\begin{vmatrix} 2 & 3 & 1 \\ 3 & 2 & k \\ 4 & 1 & 1 \end{vmatrix} \quad 0 \qquad k \quad 0$$

Now, let x

So, $\quad y \quad \dfrac{3}{2}, \quad z \quad \dfrac{5}{2}$

Minimum positive integer value of z is at $\quad 2\,;\, z \quad 5$

6. $\begin{vmatrix} 2a & 2 & 3 \\ 1 & a & 2 \\ 2 & 0 & a \end{vmatrix} \quad 0 \qquad a \quad 2$

7. Let three terms be $A \quad d, A, A \quad d$.

$$A^4 \quad (A \quad d)^2(A \quad d)^2 \quad A^4 \quad d^4 \quad 2A^2d^2$$

$$d \quad \sqrt{2}A, r \quad 3 \quad 2\sqrt{2} \text{ or } r \quad 3 \quad 2\sqrt{2}$$

8. $_3 \begin{vmatrix} 3a_1 \quad b_1 & 3a_2 \quad b_2 & 3a_3 \quad b_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix} \begin{vmatrix} 3a_1 & 3a_2 & 3a_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix} \quad 27 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$$2\begin{vmatrix} 6a_1 & 2a_2 & 2a_3 \\ 3b_1 & b_2 & b_3 \\ 12c_1 & 4c_2 & 4c_3 \end{vmatrix} \quad 24\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

9. $$\begin{vmatrix} 1 & \cos & 1 \\ \cos & 1 & \cos \\ 1 & \cos & 2 \end{vmatrix} \quad 3(1 \quad \cos^2 \quad)$$

Its minimum value 3

10. $$D \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & \end{vmatrix} \quad 8 \quad 0 \quad\quad 8$$

$$D_3 \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 14 \\ 2 & 5 & \end{vmatrix} \quad 36 \quad 0 \quad\quad 36$$

11. $$n\sin 2 \quad 1 \quad 1 \quad \frac{1}{\lfloor 2} \quad \frac{1}{\lfloor 3} \quad \frac{1}{\lfloor N} \quad \lfloor n$$

$$n\sin 2 \quad 1 \quad \frac{1}{n \quad 1} \quad \frac{1}{(n \quad 1)(n \quad 2)} \quad\quad \frac{1}{(n \quad 1)(n \quad 2) \quad (N)}$$

Using $\sin(2n \quad) \quad \sin$

$$n(2 \quad) \quad \frac{1}{n \quad 1} \quad \frac{1}{(n \quad 1)(n \quad 2)} \quad\quad \frac{1}{(n \quad 1)(n \quad 2) \quad N}$$

Using $\lim_{0} \frac{\sin}{} \quad 1$

$$2$$

12. $$\begin{vmatrix} \cos & \sin & \cos \\ \sin & \cos & \sin \\ \cos & \sin & \cos \end{vmatrix} \quad 0 \quad\quad 2\cos \quad \cos 2 \quad\quad 0$$

❑❑❑

11 COMPLEX NUMBERS

Exercise-1 : Single Choice Problems

2. $\arg(z \quad 2 \quad 7i) \quad \cot^{\;1}(2) \qquad \dfrac{y \quad 7}{x \quad 2} \quad \dfrac{1}{2}$

$\arg \dfrac{z \quad 5i}{z \quad 2 \quad i} \qquad \overline{}\!\!\!\!2 \qquad\qquad x(x \quad 2) \quad (y \quad 5)(y \quad 1) \quad 0$

4. $z_1^2 \quad z_2^2 \quad z_1 z_2$

5. Let $\quad re^{i} \;$ then $z \quad \dfrac{1}{r} e^{i(\;\;/2\;\;\;)}$

$\bar{z} \quad \dfrac{1}{r} e^{\;\;i(\;\;/2\;\;\;)} r \;\; e^{i} \qquad e^{\;\;i\;\;/2}$

6. $a \sum_{r\;1}^{n} \;\; r^{\;r\;1} \quad b \sum_{r\;1}^{n} \;\;^{r\;1} \quad a(1 \quad 2 \quad\;\; 3\;^{2} \qquad\qquad n\;\;^{n\;1}) \quad b(1 \qquad\qquad ^{2} \qquad\qquad\qquad ^{n\;1})$

$a \; \dfrac{1 \qquad\quad ^{2} \qquad\qquad ^{n\;1}}{1} \quad \dfrac{n\;\;^{n}}{1} \qquad b(0)$

$a \;\; 0 \quad \dfrac{n}{1} \qquad 0$

8. $z^4 \quad z^3 \quad 2 \quad 0$ has roots z_1, z_2, z_3 and z_4.

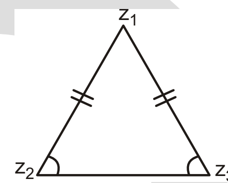
$(z \quad 1)^4 \quad 2(z \quad 1)^3 \quad 32 \quad 0$ has roots $(2z_1 \quad 1), (2z_2 \quad 1), (2z_3 \quad 1)$ and $(2z_4 \quad 1)$

9. $\qquad \arg \dfrac{z \quad 6 \quad 3i}{z \quad 3 \quad 6i} \quad \overline{}\!\!\!\!4$

$(x \quad 6)^2 \quad (y \quad 6)^2 \quad 9$

11. $|iz \quad z_1| \quad |i||z \quad iz_1| \quad |z \quad iz_1|$

Maximum distance of $iz_1(\;\;3 \quad 5i)$ from z is $2 \quad \sqrt{3^2 \quad (5 \quad 1)^2} \quad 7$



12. $$\frac{z_1 \quad z_2}{z_3 \quad z_2} \quad \frac{|z_1 \quad z_2|}{|z_3 \quad z_2|} e^{i}$$

$$\frac{z_1 \quad z_3}{z_2 \quad z_3} \quad \frac{|z_1 \quad z_3|}{|z_2 \quad z_3|} e^{\quad i}$$

$$\arg \frac{z_1 \quad z_2}{z_3 \quad z_2} \quad \frac{z_1 \quad z_3}{z_3 \quad z_2} \quad \arg \left|\frac{z_1 \quad z_2}{z_3 \quad z_2}\right| e^{i} \quad \left|\frac{z_1 \quad z_3}{z_2 \quad z_3}\right| e^{\quad i}$$

$$\overline{} \\ 2$$

13. $$\frac{z_2}{z_1} \quad \frac{3}{2} e^{i \ /3}$$

$$\left|\frac{z_1 \quad z_2}{z_1 \quad z_2}\right| \quad \left|\frac{1 \quad \frac{3}{2} e^{i \ /3}}{1 \quad \frac{3}{2} e^{i \ /3}}\right| \quad \left|\frac{2 \quad 3\cos\frac{}{3} \quad 3i\sin\frac{}{3}}{2 \quad 3\cos\frac{}{3} \quad 3i\sin\frac{}{3}}\right|$$

$$\sqrt{\frac{\frac{7}{2}^{\ 2} \quad \frac{3\sqrt{3}}{2}^{\ 2}}{\frac{1}{2}^{\ 2} \quad \frac{3\sqrt{3}}{2}^{\ 2}}} \quad \sqrt{\frac{49 \quad 27}{1 \quad 27}} \quad \frac{\sqrt{133}}{7}$$

14. $z_1 z_2 z_3 \qquad c$

$1 \ |c| \qquad |c| \ 1$

$|z_1 \quad z_2 \quad z_3| \ |z_1| \ |z_2| \ |z_3|$

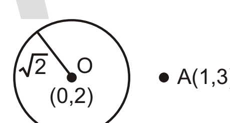
$|a| \ 3$

$|b| \ |z_1 z_2 \quad z_2 z_3 \quad z_3 z_1| \ |z_1 z_2| \ |z_2 z_3| \ |z_3 z_1|$

$|b| \ 3$

15. $\frac{1}{2} \quad |z| \quad 4$

$$\left|z \quad \frac{1}{z}\right| \quad \sqrt{\quad r \quad \frac{1}{r} \ \cos \qquad ^{2} \quad r \quad \frac{1}{r} \ \sin \qquad ^{2}} \quad \sqrt{r^2 \quad \frac{1}{r^2} \quad 2(\cos \quad \sin \ )}$$



16. $|3 \quad i(z \quad 1)| \ |z \quad 1 \quad 3i|$

Maximum distance of A from (z) $\quad OA \quad r$

$$\sqrt{1 \quad 1} \quad \sqrt{2} \quad 2\sqrt{2}$$

17. $x^2 \quad (\sqrt{2}\,i)\,x \quad 1 \quad 0$

$$x \quad \frac{\sqrt{2}\,i \quad \sqrt{\quad 2 \quad 4}}{2} \quad \frac{1}{\sqrt{2}}(\ 1 \quad i)$$

$x \quad cis \dfrac{}{4}, cis \dfrac{3}{4}$

$x^{2187} \quad cis \dfrac{3}{4}, \; cis \dfrac{}{4}$

$\dfrac{1}{x^{2187}} \quad cis \dfrac{3}{4} \;, \quad cis \quad \dfrac{}{4} \qquad x^{2187} \quad \dfrac{1}{x^{2187}} \quad 2i\sin\dfrac{3}{4}, 2i\sin\dfrac{}{4} \quad \sqrt{2}\,i$

18. $1 \dfrac{(1 \quad z^9)}{1 \quad z} \quad 0, z \quad 1$

$z^9 \quad 1$

$re^{i} \quad e^{\frac{i(2n \quad 1)}{9}}, n \quad 1, 2, \quad 8$

19. Let $P(re^{i} \quad)$ & $Q(re^{i} \quad)$

Point of intersection of tangents at ‘ ’,‘ ’ to circle $x^2 \quad y^2 \quad r^2$ is

$$r\frac{\cos\frac{}{2}}{\cos\frac{}{2}} \quad i\frac{r\sin\frac{}{2}}{\cos\frac{}{2}} \quad \frac{re^{i\frac{}{2}}}{\cos\frac{}{2}} \quad \frac{2_{\;1\;\;2}}{_1 \quad _2}$$

20. $|z_1 \quad z_2|^2 \quad |z_2 \quad z_3|^2 \quad |z_3 \quad z_1|^2 \quad 2(4 \quad 9 \quad 16) \quad 2(a \quad b \quad b \quad c \quad c \quad a)$

where a, b, c are position vectors of points z_1, z_2, z_3

Maximum value $58 \quad 2(6 \quad 12 \quad 8) \quad \dfrac{1}{2} \quad 84$

21. We have $\qquad Z \quad \dfrac{7 \quad i}{3 \quad 4i}$

Simplifying ($i.e.$, rationalizing the denominator), we get

$$\frac{7 \quad i}{3 \quad 4i} \; \frac{3 \quad 4i}{3 \quad 4i} \quad \frac{21 \quad 4 \quad 28i \quad 3i}{9 \quad 16}$$

$$\frac{25 \quad 25i}{25} \quad 1 \quad i$$

Therefore, $\qquad \left(\dfrac{7 \quad i}{3 \quad 4i}\right)^{14} \quad (1 \quad i)^{14}$

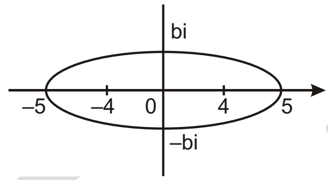
$[(1 \quad i)^2]^7 \quad (1 \quad i^2 \quad 2i)^7$

$(\quad 2^7)i$

22. $|Z \quad 4| \quad |Z \quad 4| \quad 10$

$PS \quad PS \quad 2a$

which implies that foci at 4 and 4 and a 5 as shown in the following figure.



Now, $b^2 \quad 25(1 \quad e^2) \quad 25 \quad (5e)^2$

$$25 \quad 16 \quad 9$$

$$b \quad 3$$

Z lies on the ellipse circumference $|Z|$ denotes the distance from the origin. Therefore,

$$|Z|_{\max} \quad 5$$

$$|Z|_{\min} \quad 3$$

Thus, the difference between the maximum and the minimum values of $|Z|$ is

$$|Z|_{\max} \quad |Z|_{\min} \quad 5 \quad 3 \quad 2$$

Exercise-2 : One or More than One Answer is/are Correct

1. Let $z_1 \quad re^{i}$ and $z_2 \quad re^{i}$

$|z_1 \quad z_2| \; |z_1|$

$|e^{i} \quad e^{i}| \; |e^{i}| \quad 1$

$(\cos \quad \cos\ )^2 \quad (\sin \quad \sin\ )^2 \quad 1$

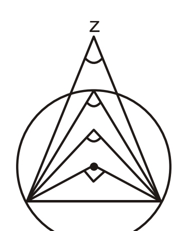
$\cos(\quad) \quad \dfrac{1}{2} \qquad\qquad \dfrac{2}{3}$ or $\dfrac{2}{3}$

$\dfrac{z_1}{z_2} \quad e^{i(\quad)} \quad e^{i2\ /3}$ or $e^{\ i2\ /3}$

2. (a) If arg $\dfrac{z_1}{z_2} \quad \dfrac{\ }{2}$ then z_1 and z_2 subtend right-angle at circumcentre origin.

the chord joining z_1 and z_2 will subtend an angle at 'z' such that

$$/4 \quad \text{if} \quad |z| \quad 1$$
$$/4 \quad \text{if} \quad |z| \quad 1$$
$$/4 \quad \text{if} \quad |z| \quad 1$$



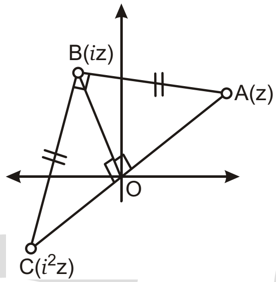
(b) $|z_1z_2 \quad z_2z_3 \quad z_3z_1| \quad |z_1|\,|z_2|\,|z_3|\left|\dfrac{1}{z_1} \quad \dfrac{1}{z_2} \quad \dfrac{1}{z_3}\right|$

$$|\overline{z_1 \quad z_2 \quad z_3}|$$

(c) $\dfrac{(z_1 \quad z_2)(z_2 \quad z_3)(z_3 \quad z_1)}{z_1z_2z_3} \qquad \dfrac{\overline{(z_1 \quad z_2)(z_2 \quad z_3)(z_3 \quad z_1)}}{z_1z_2z_3}$

(d) The triangle formed by joining z_1, z_3 and z_2 is isosceles and right angled at z_3.

3.



Method I : Multiplying a complex number by i rotates a vector for z in the anticlockwise direction by an angle of 90°.

$$AOB \quad BOC \quad 90$$

As shown in figure, the ABC is a right angled isosceles triangle.

Method II : Let z, iz, i^2z are vertices A, B and C of the triangle ABC.

$$|AB| \quad |BC| \text{ also } |AB|^2 \quad |BC|^2 \quad |AC|^2$$

Since, $\quad |AB| \quad |BC| \text{ also } |AB|^2 \quad |BC|^2 \quad |AC|^2$

the ABC is a right angled isosceles triangle.

7. $(z \quad i)^4 \quad 1 \quad i$

$$z \quad i \quad 2^{1/8} \cos \frac{}{8} \quad \frac{2m}{4}$$

Square side length $\quad \dfrac{2^{1/8} \quad 2}{\sqrt{2}}$

8. $z \quad 4 \quad \dfrac{1}{2} \quad i\dfrac{\sqrt{3}}{2}$

Roots $\quad 4^{1/4} \cos \dfrac{2m}{4} \quad \dfrac{60}{4}$

$m \quad 0, 1, 2, 3 \qquad m \quad 1$

9. $a \quad b \quad c^{\ 2}$

$a \quad b^{\ 2} \quad c \quad \bar{}$

$$| \ | \quad 1 \qquad \left| a \quad b \quad \frac{1}{2} \quad \frac{\sqrt{3}\,i}{2} \qquad c \quad \frac{1}{2} \quad \frac{\sqrt{3}\,i}{2} \right| \quad 1$$

10. Check option for z

$^{62} \quad 1 \quad 0 \qquad ^{2} \quad 1 \quad 0$

$^{155} \quad 1 \quad 0 \qquad ^{2} \quad 1 \quad 0$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

Sol. $f(z) \quad \overline{f(z)} \quad f(\bar{z}) \quad \overline{f(\bar{z})}$

$(\; z \quad) \quad (\bar{\ } \bar{z} \quad \bar{\ }) \quad \bar{z} \quad \bar{\ } z \quad \bar{\ }$

$(\quad \bar{\ })(z \quad \bar{z}) \quad 0$

$\text{Im}(\;) \quad 0 \qquad (\text{Im}(z) \quad 0)$

$f(z) \quad \overline{f(z)} \quad 0$

$(z \quad \bar{z}) \quad (\quad \bar{\ }) \quad 0 \qquad (\because \quad \bar{\ })$

$\text{Re}(\;) \quad 0 \qquad (\text{Re}(z) \quad 0)$

$|f(z)|^2 \quad (z \quad 1)^2$

$^2 z^2 \quad ^2 \quad z^2 \quad 2z \quad 1$

$(\; ^2 \quad 1) z^2 \quad 2z \quad (\; ^2 \quad 1) \quad 0 \quad z \quad R$

Paragraph for Question Nos. 3 to 5

Sol. $| \qquad | \quad 2\sqrt{7}$

$|(\qquad)^2 \quad 4 \quad | \quad 28$

$|z_1^2 \quad 4(z_2 \quad m)| \quad 28$

$|m \quad (4 \quad 5i)| \quad 7$

$\text{greatest}\,(|m|) \quad \sqrt{16 \quad 25} \quad 7$

$\text{least}\,|m| \quad 7 \quad \sqrt{16 \quad 25}$

Paragraph for Question Nos. 6 to 7

Sol. $C_1 : |z \quad z_1|^2 \quad |z \quad z_2|^2 \quad 10 \qquad C_1 : (x \quad 5)^2 \quad y^2 \quad 1$

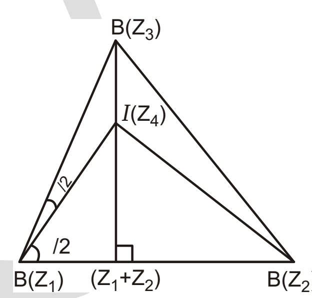
$C_2 : |z \quad z_1|^2 \quad |z \quad z_2|^2 \quad 16 \qquad C_2 : (x \quad 5)^2 \quad y^2 \quad 4$

Paragraph for Question Nos. 8 to 9

Sol. $\frac{Z_2 \quad Z_1}{|Z_2 \quad Z_1|} \quad \frac{Z_4 \quad Z_1}{|Z_4 \quad Z_1|} e^{i \quad /2}, \quad \frac{Z_3 \quad Z_1}{|Z_3 \quad Z_1|} \quad \frac{Z_4 \quad Z_1}{|Z_4 \quad Z_1|} e^{i \quad /2}$

$\frac{(Z_2 \quad Z_1)(Z_3 \quad Z_1)}{|Z_2 \quad Z_1||Z_3 \quad Z_1|} \quad \frac{(Z_4 \quad Z_1)^2}{|Z_4 \quad Z_1|^2} e^{\;\; i}$

$\frac{(Z_2 \quad Z_1)(Z_3 \quad Z_1)}{(Z_4 \quad Z_1)^2} \quad \frac{AB \quad AC}{(IA)^2}$



Similarly, others.

Exercise-4 : Matching Type Problems

1. Let $BC \quad n, CA \quad n \quad 1, AB \quad n \quad 2$

(A) $\left|\arg \dfrac{z_1 \quad z_3}{z_2 \quad z_3}\right| \quad \left|2\arg \dfrac{z_3 \quad z_1}{z_2 \quad z_1}\right| \qquad C \quad 2 \quad A$

$\dfrac{\sin C}{c} \quad \dfrac{\sin A}{a} \qquad \dfrac{\sin 2A}{n \quad 2} \quad \dfrac{\sin A}{n}$

$\cos A \quad \dfrac{n \quad 2}{2n} \qquad \dfrac{(n \quad 2)^2 \quad (n \quad 1)^2 \quad n^2}{2(n \quad 2)(n \quad 1)} \quad \dfrac{n \quad 2}{2n}$

$n(n^2 \quad n \quad 5) \quad (n^2 \quad 3n \quad 2)(n \quad 2) \qquad n^2 \quad 3n \quad 4 \quad 0 \qquad n \quad 4$

biggest side $n \quad 2 \quad 6$

(B) $(c \quad a)\ (b \quad c) \quad 0 \qquad C \quad 90 \qquad a^2 \quad b^2 \quad c^2$

$n^2 \quad (n \quad 1)^2 \quad (n \quad 2)^2 \qquad n \quad 3$

Area $\dfrac{1}{2} \ 3 \ 4 \quad 6$

$|a \quad b \quad b \quad c \quad c \quad a| \quad 2 \qquad 12$

(C) $\left|\dfrac{a_1 b_2 \quad a_2 b_1}{a_1 a_2 \quad b_1 b_2}\right| \quad \tan A \quad \dfrac{4}{3} \qquad \cos A \quad \dfrac{3}{5}$

$\dfrac{(n \quad 2)^2 \quad (n \quad 1)^2 \quad n^2}{2(n \quad 2)(n \quad 1)} \quad \dfrac{3}{5}$

$5(n^2 \quad 6n \quad 5) \quad 6(n^2 \quad 3n \quad 2)$

$n^2 \quad 12n \quad 13 \quad 0 \qquad n \quad 13$

$S \quad c \quad \dfrac{1}{2}(a \quad b \quad c) \quad \dfrac{1}{2}(13 \quad 14 \quad 15) \quad 6$

(D) Altitudes are in H.P. sides are in A.P.

Also, $b \quad a \quad c, a \quad b \quad c, c \quad a \quad b$ least value of $a \quad 2$

least value of $b \quad 3$

3. (A) $\{0, 1, \quad 1\}^m \quad \{0, 1, \quad ^2\}^m$

$0, 1, \quad 1, \quad ^2, \quad ,$

(B) $2 \ , (x^2 \quad x \quad 10) \quad 0$ roots are $2 \quad 3 \ , \ 2 \quad 3 \quad ^2$

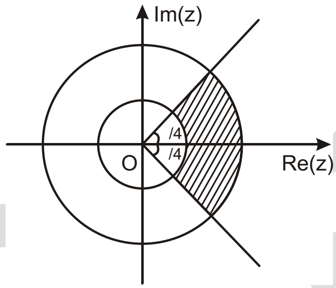
Last number is 3.

(C) Central angle 60 Equilateral

(D) Put $z \quad 1 \qquad z_1 \quad 1, z_2 \quad , z_3 \quad ^2$

Exercise-5 : Subjective Type Problems

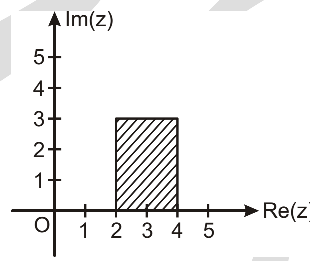
1.



$2 \quad |z| \quad 4$

Probability $\quad \dfrac{1}{4}$

2.



3. $z \quad \bar{z} \quad 2|z \quad 1| \qquad y^2 \quad 2x \quad 1$

$\arg(z_1 \quad z_2) \quad \dfrac{}{4} \qquad y_1 \quad y_2 \quad x_1 \quad x_2$

$y_1^2 \quad y_2^2 \quad 2(x_1 \quad x_2) \quad 2(y_1 \quad y_2) \qquad y_1 \quad y_2 \quad 2 \quad (y_1 \quad y_2)$

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12 MATRICES

Exercise-1 : Single Choice Problems

1. $A \begin{bmatrix} \cos \\ \sin \end{bmatrix} [\cos \quad \sin\ ] \quad \begin{bmatrix} \sin \\ \cos \end{bmatrix} [\sin \quad \cos\ ]$

$\begin{bmatrix} \cos^2 & \cos\ \sin \\ \sin\ \cos & \sin^2 \end{bmatrix} \quad \begin{bmatrix} \sin^2 & \sin\ \cos \\ \sin\ \cos & \cos^2 \end{bmatrix} \quad I$

2. $|A| \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} \quad 1$

$A^2 \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. $A \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\det(\text{adj}(\text{adj}(A))) \quad |A|^4 \quad 27^4$

$\frac{27^4}{5} \quad \frac{1}{5}$

4. $A^{1}B^{1} \quad B^{1}A^{1} \qquad C \quad (A^{1} \quad B^{1})^5 \quad (I)^5$

5. $A^4 \quad I \qquad A(A^3) \quad I$

7. $(\text{adj } A)A \quad |A|I$

$|A| \quad xyz \quad 8x \quad 4y \quad 3z \quad 28 \quad 2$

8. $(x \quad 2) \quad (x^2 \quad x \quad 3) \quad (x \quad 7) \quad 0$

$x^2 \quad x \quad 6 \quad 0 \qquad (x \quad 3)(x \quad 2) \quad 0$

9. $A \quad \begin{matrix} 1 & 3 \\ 3 & 0 \end{matrix} \qquad A^{\ 1} \quad \frac{1}{9}\begin{matrix} 0 & 3 \\ 3 & 1 \end{matrix}$

10. $\begin{matrix} 1 & \tan \\ \tan & 1 \end{matrix} \quad \begin{matrix} \cos^2 & \sin\ \cos \\ \sin\ \cos & \cos^2 \end{matrix} \quad \begin{matrix} a & b \\ b & a \end{matrix}$

$a \quad \cos 2 \;,\; b \quad \sin 2$

11. $P^2 \quad I \quad P$

or $P^3 \quad P \quad P^2 \quad 2P \quad I$

or $P^4 \quad 2I \quad 3P$

or $P^5 \quad 3I \quad 5P$

or $P^6 \quad 5I \quad 8P$

12. $|\text{adj}(\text{adj}(A))| \quad |A|^{(n\ \ 1)^2}$

$|A| \quad x \quad y \quad z \quad 12$

$x \quad 1, y \quad 1, z \quad 1$

${}^{11}C_2 \quad 55$

13. Let $A \quad \begin{matrix} a & b \\ c & d \end{matrix}$; $\text{adj}(A) \quad \begin{matrix} d & c \\ b & a \end{matrix}^{T} \quad \begin{matrix} d & b \\ c & a \end{matrix}$; $\text{adj}(\text{adj}(A)) \quad \begin{matrix} a & b \\ c & d \end{matrix}$

14. $M \quad A^{2m} \quad A^{\ 1}$

$M \quad \frac{A^{2m\ \ 1}}{a^2 \quad b^2}$

If $A^2 \quad (a^2 \quad b^2)\ I \qquad A^{2m} \quad (a^2 \quad b^2)^m \quad I$

$A^{2m\ \ 1} \quad (a^2 \quad b^2)^m \quad A$

15. $A^2 \quad 5A \quad 6I \quad I$

$(A \quad 2I)(A \quad 3I) \quad I$

$A \quad 2I$ and $A \quad 3I$ are inverse of each other.

16. $AB \quad \begin{matrix} 3 & 5 \\ 7 & 12 \end{matrix} \quad \begin{matrix} 12 & 5 \\ 7 & 3 \end{matrix} \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \quad I$

17. $\text{adj}(A) \quad \begin{matrix} 3 & 2 \\ 2 & 2 \end{matrix}$

18. $AA^1 \quad I$

$\begin{matrix} \cos & 2\sin \\ \sin & \cos \end{matrix} \quad \begin{matrix} \cos & \sin \\ 2\sin & \cos \end{matrix} \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$

$\begin{matrix} \cos^2 \quad 4\sin^2 & 3\sin\ \cos \\ 3\sin\ \cos & \sin^2 \quad \cos^2 \end{matrix} \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \quad \sin \quad 0$

20. If $A \quad \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix}$, $P \quad \begin{matrix} \cos & \sin \\ \sin & \cos \end{matrix}$, $Q \quad P^T AP$, we have

$$PQ^{2014}P^T \quad \underbrace{P(P^T AP)(P^T AP) \quad (P^T AP)P^T}_{2014 \text{ times}}$$

$$(PP^T)A(PP^T)A(PP^T) \quad (PP^T)A(PP^T)$$

Matrix multiplication is associative.

$$PP^T \quad \begin{matrix} \cos & \sin \\ \sin & \cos \end{matrix} \quad \begin{matrix} \cos & \sin \\ \sin & \cos \end{matrix}$$

$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \quad I_2$$

Hence, $\quad PQ^{2014}P^T \quad A^{2014}$

$$A \quad \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix} \qquad A^2 \quad \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix} \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix} \quad \begin{matrix} 1 & 4 \\ 0 & 1 \end{matrix}$$

$$A^3 \quad \begin{matrix} 1 & 4 \\ 0 & 1 \end{matrix} \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix} \quad \begin{matrix} 1 & 6 \\ 0 & 1 \end{matrix}$$

$$A^4 \quad \begin{matrix} 1 & 6 \\ 0 & 1 \end{matrix} \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix} \quad \begin{matrix} 1 & 8 \\ 0 & 1 \end{matrix}$$

$$A^n \quad \begin{matrix} 1 & 2n \\ 0 & 1 \end{matrix} \text{ and } A^{2014} \quad \begin{matrix} 1 & 4028 \\ 0 & 1 \end{matrix}$$

21. $\left| adj \; \frac{M}{2} \right| \quad \left|\frac{M}{2}\right|^2 \quad \left(\frac{1}{8}|M|\right)^2$

22. $|A^{\;1}| \quad \frac{1}{|A|} \quad \frac{1}{5}$

$|(AB)^T| \quad |AB| \quad |A \; (adj\; A)| \quad |A|\,|adj\,(A)| \quad 5 \quad 5^2 \quad 5^3$

$||A^{\;1}|(AB)^T| \quad |\frac{1}{5}(AB)^T| \quad \frac{1}{5^3}|AB| \quad 1$

Exercise-2 : One or More than One Answer is/are Correct

3. $\quad A \; A \quad A$

Also, $\quad A_0 \quad \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \quad I$

and $\quad A \; A \quad A \quad A_0 \quad I$

we get $A^{\;1} \quad A$

However, $A^{\;1} \quad A$ and $A^2 \quad I$ do not hold.

4. $A(A^2 \quad I) \quad 2(A^2 \quad I) \quad 0$

$(A^2 \quad I)(A \quad 2I) \quad 0$

Exercise-3 : Matching Type Problems

1. (A) Possible non-negative value of $|A| \quad 2, 4, 8$

(B) Sum is 0.

(C) $|\text{adj}(\text{adj}(\text{adj } A)))| \quad |A|$

least absolute value of $|A| \quad 2$

$|A| \quad 2$

(D) least $|A| \quad 8$

$|4A^{\;1}| \quad \dfrac{16}{|A|} \quad 2$

2. (A) Since A is idempotent, $A^2 \quad A^3 \quad A^4 \qquad A$. Now,

$$(A \quad I)^n \quad I \quad {}^nC_1A \quad {}^nC_2A^2 \qquad {}^nC_nA^n$$

$$I \quad {}^nC_1A \quad {}^nC_2A \qquad {}^nC_nA$$

$$I \quad ({}^nC_1 \quad {}^nC_2 \qquad {}^nC_n)A$$

$$I \quad (2^n \quad 1)A$$

$2^n \quad 1 \quad 127 \qquad n \quad 7$

(B) We have,

$$(I \quad A)(I \quad A \quad A^2 \qquad A^7)$$

$$I \quad A \quad A^2 \qquad A^7 \quad (\; A \quad A^2 \quad A^3 \quad A^4 \qquad A^8)$$

$$I \quad A^8$$

$$I \qquad (\text{if } A^8 \quad 0)$$

(C) Here matrix A is skew-symmetric and since $|A| \quad |A^T| \quad (\;1)^n|A|$, so $|A|(1 \quad (\;1)^n) \quad 0$.

As n is odd, hence $|A| \quad 0$. Hence A is singular.

(D) If A is symmetric, $A^{\;1}$ is also symmetric for matrix of any order.

5. (A) $\dfrac{1}{n} \sum_{r\;1}^{n} \dfrac{1}{\sqrt{\dfrac{r}{n}}} \quad \int_0^1 \dfrac{1}{\sqrt{x}}\,dx$

(B) $D \quad 4\cos t \cos 2t$

(C) $3x^2 \quad 2px \quad g \quad 0$

$f \quad \frac{5}{3} \quad 0 \quad f(\quad 1) \quad 0$

(D) $(2^x \quad 2)^2 \quad 1 \quad ||b \quad 1| \quad 3| \quad |\sin y|$

$b \quad 1 \quad 3$

$|\sin y| \quad 1$

Exercise-4 : Subjective Type Problems

1. $(AB)^2 \quad AB \; AB \quad A^3B^2$

$(AB)^3 \quad (AB)^2 \; AB \quad A^3B^2 \; AB \quad A^7B^3$

$(AB)^4 \quad (AB)^3 \; AB \quad A^7B^3 \; AB \quad A^{15}B^4 \qquad (AB)^{10} \quad A^{1023}B^{10}$

2. $l \quad \lim_{n} 18 \; \frac{3}{3^2} \quad \frac{3^2}{3^4} \quad \frac{3^3}{3^6} \qquad 18 \; \frac{1}{3 \; 1 \quad \frac{1}{3}} \quad 9$

$m \quad \lim_{n} 12 \; \frac{2}{2^2} \quad \frac{2^2}{2^4} \qquad 12 \; \frac{1}{2 \; 1 \quad \frac{1}{2}} \quad 12$

4. $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \quad a_1b_2c_3 \quad a_1c_2b_3 \quad \ldots$six elements

All cannot be simultaneously 1.

5. First element of matrix $A_{10} \quad 286 \quad (10^{\text{th}}$ of sequence 1, 2, 6, 15, …)

Trace of $A_{10} \quad 286 \quad 297 \quad 308 \quad 319 \qquad 385 \quad 3055$

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13 PERMUTATION AND COMBINATIONS

Exercise-1 : Single Choice Problems

1. $\underline{7}\ \underset{9}{\underline{\ \uparrow\ }}\ \underset{9}{\underline{\ \uparrow\ }} = 81$; $\underset{8}{\underline{\ \uparrow\ }}\ \underline{7}\ \underset{9}{\underline{\ \uparrow\ }} = 72$; $\underset{8}{\underline{\ \uparrow\ }}\ \underset{9}{\underline{\ \uparrow\ }}\ \underline{7} = 72$

2. $\frac{8!}{3!3!2!2!}\quad {}^2C_1\quad 3!\qquad \frac{8!}{3!2!2!2!}\quad 3!\quad 8400$

3. Number of ways $\quad 6\quad \frac{3!}{2!}\quad 3!\quad 108$

4. ${}^4C_1\quad \frac{5!}{2!}\quad 240$

5. ${}^6C_2\quad 1\quad 4!\quad 360$

6. $x^2\quad 5x\quad 3\quad 0$

$5,\qquad 3$

Sum of roots $\quad \frac{}{}\quad \frac{}{}\quad \frac{\ ^2\qquad ^2}{}\quad \frac{19}{3}$

7. ${}^5C_4\quad {}^8C_6\quad {}^5C_5\quad {}^8C_5\quad 196$

8. $(1\quad 2\quad 3\qquad\qquad 22)\ {}^{21}C_{10}$

9. $x\quad \frac{2009\quad 2008\quad 2007\quad 1}{2008\quad 2007\quad 2007}\quad 2008\quad \frac{1}{2009\quad 2007}$

$[x]\quad 2008$

10. $N\quad p_1^n p_2 p_3\qquad p_{m\ \ 1}$

No. of factors $\quad (n\quad 1)\,2^m$

11. Number of ways $\quad (11)!\quad 2^{12}$

12. $\underset{5}{\uparrow}\ \underset{5}{\uparrow}\ \_\ \_$

$m \quad 5 \quad 5 \quad {}^8C_2 \quad 2!$

$\underset{4}{\uparrow}\ \underset{5}{\uparrow}\ \_\ \_$

$n \quad 4 \quad 5 \quad {}^8C_2 \quad 2!$

13. Three different digits (not including zero)

$${}^9C_3 \quad 2!$$

Two digits (not including zero)

$${}^9C_2 \quad 2$$

Three digits (including zero)

$${}^9C_2 \quad 1$$

14. Let no. of elements in $A \quad n$

No. of elements in $B \quad m$

$$2^n \quad 2^m \quad 1920 \quad 2^7 \quad 15$$

$$n \quad 11, \; m \quad 7$$

$$n(A \quad B) \quad n(A) \quad n(B) \quad n(A \quad B) \quad 15$$

15. $C \qquad 4! \quad 24$

$D \qquad 4! \quad 24$

$M \qquad 4! \quad 24$

S C ...... $= 3! = 6$

S D ...... $= 3! = 6$

S M C $D\ W = 1$

S M C $W\ D = 1$

16. P All A's together $\dfrac{5!}{3!}$; $\quad Q$ All B's together $\dfrac{6!}{4!}$

$$n(P \quad Q) \quad 3!; \qquad n(P \quad Q) \quad \frac{5!}{3!} \quad \frac{6!}{4!} \quad 3! \quad 50 \quad 6 \quad 44$$

17. $5^6 \quad 6^7 \quad 7^8 \quad 8^9 \quad 9^{10} \quad 10^{11} \qquad 30^{31}$

No. of zero's no. of 5's

$$6 \quad 11 \quad 16 \quad 21 \quad (2 \quad 26) \quad 31 \quad 137$$

18. $(x \quad y)(x \quad y) \quad 10 \quad 337$

$x \quad y \quad 10$ and $x \quad y \quad 337$

$x \quad \dfrac{347}{2}$ (not possible)

19. Total number of different things $n \quad 2$

20. Let the numbers are $10 \quad d, 10, 10 \quad d$.

$$d \quad \{\ 9, \quad 8, \quad 7, \qquad , 7, 8, 9\}$$

22. $m \quad 2 \quad 5! \quad 5!$

$n \quad 4! \quad 5!$

23. Total ways $4 \quad 4! \quad 96$

25. ${}^4C_2 \quad 5^2 \quad (21)^2 \quad 66150$

26. Total all letters are different.

$$10^5 \quad {}^{10}C_5 \quad 5! \quad 69760$$

29. $M \quad 1440$

$M \quad 2^5 \quad 3^2 \quad 5$

No. of divisions $6 \quad 3 \quad 2 \quad 36$

P Product of divisors $(1440)^{18}$

$P \quad 2^{90} \quad 3^{36} \quad 5^{18}$

Hence, $x \quad 30$

30. **Case-1 :** All digits same 9

Case-2 : Excluding zero :

(i) No's having 3 digits same : ${}^9C_2 \quad {}^2C_1 \quad \dfrac{4!}{3!} \quad 288$

(ii) No's having 2 digits same, 2 other same : ${}^9C_2 \quad \dfrac{4!}{2!2!} \quad 216$

Case-3 : Including zero :

(i) No's having 3 zero's : 9

(ii) No's having 2 zero's : ${}^9C_1 \quad \dfrac{3!}{2!} \quad 27$

(iii) No's having 1 zero $\quad {}^9C_1 \quad \dfrac{3!}{2!} \quad 27$

Hence, total no's 576

31. **Case-I :** When two T's contain exactly one vowel between them,

$$5! \quad (\ {}^5C_1 \quad {}^5C_4 \quad 4!) \quad 15 \quad 5! \quad 5!$$

Case-II : When two T's also contain consonant between them,

$$4! \quad ({}^5C_2) \quad ({}^7C_5 \quad 5!) \quad 42 \quad 5! \quad 5!$$

32. $6\ 6\ 6\ 6\ 6\ 0 \quad 6$

$6\ 6\ 6\ 6\ 3\ 3 \quad \dfrac{6!}{4!2!}$

$6\ 6\ 6\ 6\ 4\ 2 \quad \dfrac{6!}{4!}$

6 6 6 4 4 4 $\quad \frac{6!}{3!3!}$

33. Five 4 runs + one 0 run $\quad \frac{6!}{5!}$

Four 4 runs + two 2 runs $\quad \frac{6!}{4!2!}$

Three 4 runs + two 3 runs + one 2 runs $\quad \frac{6!}{3!2!}$

Two 4 runs + four 3 runs $\quad \frac{6!}{2!4!}$

$N \quad 96$

34. ${}^7C_2 \quad 21$

35. $x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad 101$

Let $x_1 \quad 2k_1 \quad 1, x_2 \quad 2k_2 \quad 1, x_3 \quad 2k_3 \quad 1, x_4 \quad 2k_4 \quad 1, x_5 \quad 2k_5 \quad 1$

$k_1 \quad k_2 \quad k_3 \quad k_4 \quad k_5 \quad 48; \quad {}^{48\ 5\ 1}C_{5\ 1}$

36. Total ways (largest number is 4)

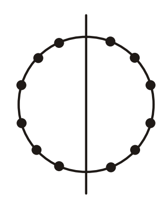
$6^4 \quad (4^4 \quad 3^4) \quad 1121$

37. ${}^6C_3 \quad 4!$

38. If two points are selected from one side of main diagonal ${}^6C_2$.

Then other two points are selected on other side of main diagonal 1.

Total ways ${}^6C_2 \quad 1 \quad 15$



39. $(9 \quad x_1) \quad (9 \quad x_2) \quad (9 \quad x_3) \quad (9 \quad x_4) \quad (9 \quad x_5) \quad 43$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad 2$

Number of ways ${}^{2\ 5\ 1}C_{5\ 1} \quad {}^6C_4 \quad 15$

Exercise-2 : One or More than One Answer is/are Correct

1. **Case-I :** All five letters are different.

$5!$

Case-II : Two letters are same and remaining are different.

$${}^3C_1 \quad {}^4C_3 \quad \frac{5!}{2!} \quad 720$$

Case-III : Two alike, two other alike and remaining different.

$${}^3C_2 \quad {}^3C_1 \quad \frac{5!}{2!2!} \quad 270$$

Total number of words 1110

2. $\overset{100}{\underset{k\ \ 0}{}}\ {}^{100}C_k(x \quad 2)^{100 \quad k}\ 3^k \quad (x \quad 1)^{100}$

$$\text{Coeff. of } x^{50} \quad {}^{100}C_{50}$$

3. $\dfrac{\text{Total} \quad \text{Row 1} \quad \text{Row 2}}{\underline{|2}} \quad \{\underline{|2} \ \text{ for } N\}$

$\dfrac{{}^8C_5 \underline{|6} \quad \underline{|6} \quad \underline{|6}}{\underline{|2}}$

4. (four odd) + (4 even) + (3 even + 1 odd) + (2 even + 2 odd)

$^5C_4 \quad 4! \quad {}^4C_4 \quad 4! \quad {}^4C_3 \quad {}^5C_1 \quad 4! \quad {}^4C_2 \quad {}^5C_2 \quad 4 \quad 4$

1584

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

1. 0

2. Digit 6 always come at last three place digit 5 always come at last four place and digit 4 always come at last five place.

$$^3C_1 \quad {}^3C_1 \quad {}^3C_1 \quad 3! \quad 162$$

Exercise-4 : Matching Type Problems

1. (A) $\dfrac{6!}{2!} \quad {}^7C_2 \quad 7560$ (B) $5! \quad {}^6C_2 \quad 1800$

(C) $7560 \quad 1800 \quad 5760$ (D) $4! \quad {}^5C_4 \quad \dfrac{4!}{2!2!} \quad 720$

2. (A) Total ways (No repeating letter is at odd position)

$$\frac{11!}{2!2!2!} \quad 0 \quad \frac{11!}{(2!)^3}$$

(B) $\dfrac{7!}{2!2!} \quad {}^8C_4 \quad \dfrac{4!}{2!} \quad 210 \quad 7!$

(C) $MM \quad TT \quad H\,E\,I\,C\,S$

$7! \quad {}^8C_2 \quad 1 \quad 28 \quad 7!$

(D) $\dfrac{4!}{2!} \quad \dfrac{7!}{2!2!} \quad \dfrac{4!\,7!}{(2!)^3}$

Exercise-5 : Subjective Type Problems

1. $^{9}C_4 \quad {}^{5}C_4 \quad 630$

2. $^{9}C_2 \quad \dfrac{7!}{2!2!} \quad \dfrac{9!}{8}$

4. $^{10}C_3 \quad {}^{8}C_3 \quad 64$

5. **Case-I:** If Ravi is include.

$$^{7}C_5 \quad {}^{9}C_8 \quad 189$$

Case-II: If Ravi is not include.

$$^{7}C_6 \quad [{}^{8}C_7 \quad {}^{9}C_8] \quad 119$$

Total number of ways 308

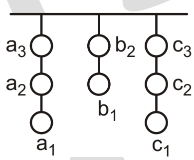
6. $^{6}C_4 \quad {}^{4}C_2 \quad 9$

7. $5! \quad (1 \quad {}^{5}C_2 \quad 1) \quad 109$

8. Let other two sides are a and b.

$$a \quad b \quad 11 \qquad 0 \quad a \quad 11, \quad 0 \quad b \quad 11$$

9.



$(a_1, a_2, a_3), (b_1, b_2)$ and (c_1, c_2, c_3) are alike things so these can be arranged is

$$\frac{8!}{2!3!3!} \quad \frac{4 \;\; 5 \;\; 6 \;\; 7 \;\; 8}{2 \quad 6} \quad 560$$

10. $^{n}C_2 \quad n \quad 14 \qquad n \quad 7$

11. $x_1 \quad x_2 \quad x_3 \qquad\qquad x_7 \quad x_8 \quad 93$

$x_1 \quad 0, \; x_2 \quad 6, \; x_3 \quad 6, \qquad x_7 \quad 6, \; x_8 \quad 0$

$x_1 \quad x_2 \quad x_3 \qquad\qquad x_7 \quad x_8 \quad 57$

No. of ways $\quad {}^{64}C_7$

12. $^{4}C_4({}^{2}C_1)^4 \quad 16$

13. Let $\quad x_1$ objects of one type

x_2 objects of second type

x_3 objects of third type

$x_1 \quad x_2 \quad x_3 \quad 3n$

$0 \quad x_1 \quad 2n, 0 \quad x_2 \quad 2n, 0 \quad x_3 \quad 2n$

Number of ways $\;{}^{3n\;\;2}C_2 \quad 3 \quad {}^{n\;\;1}C_2 \quad 3n^2 \quad 3n \quad 1$

14. $x \quad y \quad z \quad w \quad 15$

$x \quad 0, y \quad 6, z \quad 2, w \quad 1$

$x \quad y \quad z \quad w \quad 6$

Number of ways $\;{}^9C_3 \quad 84$

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14

BINOMIAL THEOREM

Exercise-1 : Single Choice Problems

1. Let $x \quad 2^{\frac{153}{2}}$ $\qquad$ $N \quad x^{16} \quad 1$

$x^2 \quad \sqrt{2}\,x \quad 1$ $\qquad$ $N \quad (x^4 \quad 1)(x^4 \quad 1)(x^8 \quad 1)$

$N \quad (x^4 \quad 1)(x^2 \quad \sqrt{2}x \quad 1)(x^2 \quad \sqrt{2}x \quad 1)(x^8 \quad 1)$

Let $y \quad 2^{204}$ $\qquad$ $N \quad y^6 \quad 1 \quad (y^3 \quad 1)(y^3 \quad 1)$

$y^2 \quad y \quad 1$ $\qquad$ $(y^3 \quad 1)(y \quad 1)(y^2 \quad y \quad 1)$

3. ${}^4C_2 \quad {}^2 \quad {}^6C_3 \quad {}^3$

$\frac{3}{10}$

5. ${}_n \quad (2 \quad \sqrt{3})^n$

Let ${}_n \quad (2 \quad \sqrt{3})^n \quad {}_n \quad {}_n$ integer

$[\ {}_n] \quad \{\ {}_n\} \quad {}_n$ integer $\quad \{\ {}_n\} \quad 1 \quad {}_n$

So, $\lim_{n} (\ {}_n \quad [\ {}_n]) \quad \lim_{n} [1 \quad (2 \quad \sqrt{3})^n] \quad 1 \quad 0 \quad 0$ $\qquad (\because 0 \quad \{\ {}_n\}, \quad {}_n \quad 1)$

6. $N \quad {}^{20}C_7 \quad {}^{20}C_8 \quad {}^{20}C_9 \quad {}^{20}C_{10} \quad \quad {}^{20}C_{20}$

$({}^{20}C_7 \quad {}^{20}C_9 \quad {}^{20}C_{11} \quad \quad {}^{20}C_{19}) \quad ({}^{20}C_8 \quad {}^{20}C_{10} \quad \quad {}^{20}C_{20})$

$({}^{20}C_0 \quad {}^{20}C_2 \quad {}^{20}C_4 \quad {}^{20}C_6) \quad ({}^{20}C_1 \quad {}^{20}C_3 \quad {}^{20}C_5)$

$(1 \quad 190 \quad 4845 \quad 38760) \quad (20 \quad 1140 \quad 15504)$

$43796 \quad 16664 \quad 27132 \quad 3 \quad 4 \quad 7 \quad 19 \quad 17$

7. $\log_2 1 \quad \frac{1}{2}(2^{12} \quad 2) \quad \log_2 2^{11} \quad 11 \qquad [\because \quad {}^nC_r \quad 2^n]$

8. $T_{r \quad 1} \quad {}^nC_r \quad x^{n \quad r} \quad y^r \quad {}^{12}C_r \quad x^{12 \quad r} \quad {\frac{1}{x^3}}^{r}$

$12 \quad 4r \quad 0 \qquad r \quad 3$

$$T_4 \quad {}^{12}C_3 \quad \frac{12 \quad 11 \quad 10}{3 \;\; 2 \;\; 1} \quad 220$$

9. $\frac{3}{4!} \quad \frac{4}{5!} \quad \frac{5}{6!} \qquad \frac{1}{3!} \quad \frac{1}{(k \quad 3)!}$

$$\overset{52}{\underset{r\ 3}{}}\frac{r}{(r \quad 1)!} \quad \overset{52}{\underset{r\ 3}{}}\frac{1}{r!} \quad \frac{1}{(r \quad 1)!} \qquad \frac{1}{3!} \quad \frac{1}{53!}$$

$k \quad 50$

10. $f(x) \quad \overset{n}{\underset{r\ 1}{}} [(r \quad 1)^2 \; {}^nC_r \quad r^2 \; {}^nC_{r\ 1}]$

$f(n) \quad (n \quad 1)^2 \quad 1$

$f(30) \quad 960$

12. ${}^nC_1 \qquad {}^nC_2 \quad {}^2 \quad {}^nC_3 \quad {}^3 \qquad {}^nC_n \quad {}^n \quad (1 \qquad)^n \quad 1$

where $\quad e^{\frac{2\ i}{n}} \qquad \frac{2}{1}$

13. $2^{30} \quad 3^{20} \quad 2^{10} \quad (6)^{20} \quad 1024(7 \quad 1)^{20} \quad 1024(7K \quad 1) \quad 7k \quad 1024 \quad 7k \quad 1022 \quad 2$

14. ${}^{26}C_0 \quad {}^{26}C_1 \quad {}^{26}C_2 \qquad {}^{26}C_{26} \quad 2^{26}$

$2({}^{26}C_0 \quad {}^{26}C_1 \qquad {}^{26}C_{13}) \quad 2^{26} \quad {}^{26}C_{13}$

15. $(1 \quad x \quad x^2)^n \quad a_0 \quad a_1x \quad a_2x^2 \quad a_3x^3 \qquad a_{2n}x^{2n}$

differentiate w.r.t. x

$n(1 \quad x \quad x^2)^{n\ 1}(1 \quad 2x) \quad a_1 \quad 2a_2x \quad 3a_3x^2 \qquad 2n \; a_{2n}x^{2n\ 1}$

Put $x \quad 1 \qquad n \;\; 3^n \quad a_1 \quad 2a_2 \quad 3a_3 \qquad 2n\,a_{2n}$ ...(1)

Put $x \qquad 0 \quad a_1 \quad 2a_2 \quad 3a_3 \;\; {}^2 \qquad 2n\,a_{2n} \;\; {}^{2n\ 1}$ ...(2)

Put $x \;\; {}^2 \qquad 0 \quad a_1 \quad 2a_2 \;\; {}^2 \quad 3a_3 \;\; {}^4 \qquad 2n\,a_{2n} \;\; {}^{4n\ 2}$ ...(3)

(1) + (2) + (3)

$n \;\; 3^{n\ 1} \quad a_1 \quad 4a_4 \quad 7a_7 \quad 10a_{10}$

16. ${}^nC_r \quad {}^nC_{r\ 1} \quad {}^{n\ 1}C_r$

${}^3C_0 \quad {}^3C_1 \quad {}^4C_2 \quad {}^5C_3 \qquad {}^{99}C_{97} \quad {}^{100}C_{97}$

17. Last digit of $9! \quad 0$

Last digit of $3^{9966} \qquad 9$

Hence last digit 9.

18. $x \quad T_7 \quad {}^nC_6(3^{1/3})^{n\ 6} \quad (4 \;\; {}^{1/3})^6$

$y \quad T_{n\ 5} \quad {}^nC_{n\ 6}(3^{1/3})^6 \quad (4 \quad ^{1/3})^{n\ 6}$

$y \quad 12x$

${}^nC_{n\ 6}(3^{1/3})^6(4 \quad ^{1/3})^{n\ 6} \quad 12 \quad {}^nC_6(3^{1/3})^{n\ 6}(4 \quad ^{1/3})^6$

$12 \quad (12^{1/3})^{12\ n} \qquad n \quad 9$

20. $t_{r\ 1} \quad {}^{15}C_r(x^2)^{15\ r} \quad \dfrac{2}{x}^{\ r}$

Coeff. of $x^{15} \quad {}^{15}C_5 \quad 2^5$

Coeff. of $x^0 \quad {}^{15}C_{10} \quad 2^{10}$

21. $(1 \quad x)^2(1 \quad y)^3(1 \quad z)^4(1 \quad w)^5$

General term $\quad {}^2C_a \ {}^3C_b \ {}^4C_d \ {}^5C_e x^{a\ b\ d\ e}$

$\underset{a\ b\ d\ e\ 12}{\quad} {}^2C_a \quad {}^3C_b \quad {}^4C_d \quad {}^5C_e \quad {}^{14}C_{12}$ or ${}^{14}C_{12} \quad \dfrac{14 \quad 13}{2} \quad 91$

22. $\overset{n}{\underset{r\ 0}{}} \ r\ {}^nC_r \quad 2 \ \overset{n}{\underset{r\ 0}{}} \dfrac{1}{r \quad 1}\ {}^nC_r; \quad n \ \overset{n}{\underset{r\ 0}{}} \ {}^{n\ 1}C_{r\ 1} \quad \dfrac{2}{n \quad 1} \ \overset{n}{\underset{r\ 0}{}} \ {}^{n\ 1}C_{r\ 1}$

$n \ 2^{n\ 1} \quad \dfrac{2}{n \quad 1}(2^{n\ 1} \quad 1)$

Exercise-2 : One or More than One Answer is/are Correct

1. $N \quad {}^{20}C_7 \quad {}^{20}C_8 \quad {}^{20}C_9 \quad {}^{20}C_{10} \qquad {}^{20}C_{20}$

$({}^{20}C_7 \quad {}^{20}C_9 \quad {}^{20}C_{11} \qquad {}^{20}C_{19}) \ ({}^{20}C_8 \quad {}^{20}C_{10} \qquad {}^{20}C_{20})$

$({}^{20}C_{20} \quad {}^{20}C_2 \quad {}^{20}C_4 \quad {}^{20}C_6) \ ({}^{20}C_1 \quad {}^{20}C_3 \quad {}^{20}C_5)$

2. For B and D put $x \quad 1, \quad 1$

For A differentiate with respect to x then put $x \quad 0$

For C replace x with $\dfrac{1}{x}$

3. $\overset{4}{\underset{r\ 0}{}} (\quad 1)^r \ {}^{16}C_r \quad {}^{16}C_0 \quad {}^{16}C_1 \quad {}^{16}C_2 \quad {}^{16}C_3 \quad {}^{16}C_4 \quad 1365$

4. $2 \quad \dfrac{1}{2} \quad {}^nC_1 \quad 1 \quad \dfrac{1}{2^2} \quad {}^nC_2 \qquad n \quad 8, 1$

$T_{r\ 1} \quad {}^8C_r \ \dfrac{1}{2}^{\ r} \ x^{\frac{16\ 3r}{4}} \qquad r \quad 0, 4, 8$

5. LHS $\quad (1 \quad 2x^2 \quad x^4)(1 \quad C_1x \quad C_2x^2 \quad C_3x^3 \qquad)$

RHS $a_0 \quad a_1 x \quad a_2 x^2 \quad a_3 x^3$

Comparing the coefficients of x, x^2, x^3,

Now, $2a_2 \quad a_1 \quad a_3$

$2({}^nC_2 \quad 2) \quad {}^nC_1 \quad ({}^nC_3 \quad 2\,{}^nC_1)$

$2\dfrac{n(n \quad 1)}{2} \quad 4 \quad 3n \quad \dfrac{n(n \quad 1)(n \quad 2)}{6}$

or $n^3 \quad 9n^2 \quad 26n \quad 24 \quad 0$

$(n \quad 2)(n^2 \quad 7n \quad 12) \quad 0 \qquad (\because 8 \quad 52 \quad 36 \quad 24)$

or $(n \quad 2)(n \quad 3)(n \quad 4) \quad 0$

$n \quad 2, 3, 4$

6. $\displaystyle \sum_{i\,0}^{n}\sum_{j\,0}^{n}\sum_{k\,0}^{n} {}^nC_i \quad {}^nC_j \quad {}^nC_k \quad \sum_{i\,0}^{n} {}^nC_i \quad \sum_{j\,0}^{n} {}^nC_j \quad \sum_{k\,0}^{n} {}^nC_k \quad 2^{3n}$

7. $({}^{100}C_6 \quad {}^{100}C_7) \quad 3({}^{100}C_7 \quad {}^{100}C_8) \quad 3({}^{100}C_8 \quad {}^{100}C_9) \quad ({}^{100}C_9 \quad {}^{100}C_{10})$

$\quad {}^{101}C_7 \quad 3\,{}^{101}C_8 \quad 3\,{}^{101}C_9 \quad {}^{101}C_{10}$

$({}^{101}C_7 \quad {}^{101}C_8) \quad 2({}^{101}C_8 \quad {}^{101}C_9) \quad ({}^{101}C_9 \quad {}^{101}C_{10}) \quad {}^{102}C_8 \quad 2\,{}^{102}C_9 \quad {}^{102}C_{10}$

$\quad ({}^{102}C_8 \quad {}^{102}C_9) \quad ({}^{102}C_9 \quad {}^{102}C_{10}) \quad {}^{103}C_9 \quad {}^{103}C_{10} \quad {}^{104}C_{10}$

8. $\dfrac{{}^{15}C_{2r}}{{}^{15}C_{2r \quad 1}} \quad \dfrac{1}{2} \quad \dfrac{2r \quad 1}{15 \quad 2r} \quad \dfrac{1}{2} \quad \dfrac{6r \quad 13}{2r \quad 15} \quad 0 \quad \dfrac{13}{6} \quad r \quad \dfrac{15}{2}$

9. $f(x) \quad 1 \quad x^{111} \quad x^{222} \qquad x^{999}$

if $f(x)$ is divided by $x \quad 1$, then remainder $f(\quad 1) \quad 0$

if $f(x)$ is divided by $x \quad 1$, then remainder $f(1) \quad 10$

$f(x) \quad (1 \quad x^{222} \quad x^{444} \quad x^{666} \quad x^{888}) \quad x^{111}(1 \quad x^{222} \quad x^{444} \quad x^{666} \quad x^{888})$

$\quad (1 \quad x^{111})(1 \quad x^{222} \quad x^{444} \quad x^{666} \quad x^{888})$

Exercise-3 : Matching Type Problems

2. (B) $\displaystyle P \quad \sum_{r\,0}^{n} {}^nC_r \quad 2^n$

$\displaystyle Q \quad \sum_{r\,0}^{m} {}^mC_r (15)^r \quad (1 \quad 15)^m \quad 16^m$

$n \quad 4m$

(C) $1 \quad 6 \quad 120 \quad 56K$

Reminder $\quad 15$

3. (A) $\frac{a^2 \quad b^2 \quad ab}{a \quad b} \quad \frac{(a \quad b)(a^2 \quad b^2 \quad ab)}{(a \quad b)(a \quad b)} \quad \frac{a^3 \quad b^3}{a^2 \quad b^2}$

$\frac{4 \quad \sqrt{3}}{\sqrt{3} \quad 1} \quad \frac{8 \quad \sqrt{15}}{\sqrt{5} \quad \sqrt{3}} \quad \frac{12 \quad \sqrt{35}}{\sqrt{7} \quad \sqrt{5}} \qquad \frac{1}{2}((\sqrt{169})^3 \quad 1^3) \quad 1098$

(B) $\frac{8}{5}(2\cos^2 \quad 3\sin \ ) \quad \frac{8}{5}(\ 2\sin^2 \quad 3\sin \quad 2)$

Greatest value 5 at sin $\quad \frac{3}{4} \qquad (\because 4 \qquad 6)$

(C) Let $(\sqrt{3} \quad 1)^6 \quad I \quad f$

and $(\sqrt{3} \quad 1)^6 \quad f \quad (\sqrt{3} \quad 1)^6 \quad (\sqrt{3} \quad 1)^6 \quad 416 \quad I \quad 1$

$I \quad 415 \quad 1 \quad 5 \quad 83$

(D) $(1 \quad x)(1 \quad x^2) \quad (1 \quad x^{128}) \quad \frac{1 \quad x^{256}}{1 \quad x} \quad \frac{1 \quad x^{n \quad 1}}{1 \quad x}$

$n \quad 1 \quad 256$

Exercise-4 : Subjective Type Problems

2. Coefficient of $x^{60} \quad 6 \quad 5 \quad 8 \quad 6 \quad 1$

7. $(1 \quad x)^{3n} \quad {}^{3n}C_0 \quad {}^{3n}C_1 x \quad {}^{3n}C_2 x^2 \qquad {}^{3n}C_{3n} x^{3n}$

Put $x \quad 1 \qquad 2^{3n} \quad {}^{3n}C_0 \quad {}^{3n}C_1 \quad {}^{3n}C_2 \qquad {}^{3n}C_{3n}$

Put $x \qquad (\quad ^2)^{3n} \quad {}^{3n}C_0 \quad {}^{3n}C_1 \quad {}^{3n}C_2 \quad ^2 \qquad {}^{3n}C_{3n}$

Put $x \quad ^2 \qquad (\quad)^{3n} \quad {}^{3n}C_0 \quad {}^{3n}C_1 \quad ^2 \quad {}^{3n}C_2 \quad ^4 \qquad {}^{3n}C_{3n}$

$2^{3n} \quad (\quad ^2)^{3n} \quad (\quad)^{3n} \quad 3[{}^{3n}C_0 \quad {}^{3n}C_3 \qquad {}^{3n}C_{3n}]$

10. $\overset{5}{\underset{K \quad 1}{}} \ {}^{20}C_{2K \quad 1} \quad 2^{18} \qquad 2^{108} \quad 2^3(2^5)^{21} \quad 8(33 \quad 1)^{21}$

Remainder $\quad 8$ or 3

11. $f(n) \quad {}^nC_0 a^{n \quad 1} \quad {}^nC_1 a^{n \quad 2}$

$f(n) \quad \frac{(a \quad 1)^n}{a}$

$f(2007) \quad f(2008) \quad 3^7 K$

$\frac{3^9 \quad (a \quad 1)3^9}{a} \quad 3^7 K \qquad K \quad 9$

13. $(360 \quad 1)^{44} \quad 1 \quad {}^{44}C_0 \ (360)^{44} \quad {}^{44}C_1 \ (360)^{43} \qquad {}^{44}C_{43} \ (360)^1$

$360[{}^{44}C_0 \ (360)^{43} \quad {}^{44}C_1 \ (360)^{42} \qquad {}^{44}C_{43}]$

14. $(3^{|x \;\; 2|} \quad (3^{|x \;\; 2| \;\; 9})^{1/5})^7$

$T_6 \quad {}^7C_5 \;\; (3^{|x \;\; 2|})^2 \;\; 3^{|x \;\; 2| \;\; 9} \quad 567$

$3^{3|x \;\; 2| \;\; 9} \quad 27 \quad |x \quad 2| \quad 4 \quad x \quad 6, \quad 2$

15. $1 \quad \sum_{r \;\; 1}^{10} 3^r \;\; {}^{10}C_r \quad \sum_{r \;\; 1}^{10} r \;\; {}^{10}C_r$

$1 \quad ((1 \quad 3)^{10} \quad {}^{10}C_0) \quad 10 \;\; 2^9 \quad 4^{10} \quad 5 \;\; 2^{10} \quad 2^{10}(4^5 \quad 5)$

$1, \quad 5$

if $\quad ,$ lies between the roots of $f(x) \quad 0$

$f(1) \quad 0 \quad f(5) \quad 0$

$k^2 \quad 0 \quad 16 \quad k^2 \quad 0$

16. $S_n \quad {}^nC_0 \;\; {}^nC_1 \quad {}^nC_1 \;\; {}^nC_2 \qquad {}^nC_{n \;\; 1} \;\; {}^nC_n \quad {}^{2n}C_{n \;\; 1}$

$S_{n \;\; 1} \quad {}^{2n \;\; 2}C_n$

$$\frac{S_{n \;\; 1}}{S_n} \quad \frac{{}^{2n \;\; 2}C_n}{{}^{2n}C_{n \;\; 1}} \quad \frac{15}{4}$$

$$\frac{(2n \quad 2)(2n \quad 1)}{n(n \quad 2)} \quad \frac{15}{4}$$

$$n^2 \quad 6n \quad 8 \quad 0$$

❑❑❑

15

PROBABILITY

Exercise-1 : Single Choice Problems

2. $$f(x)\quad 3\sqrt{x}\quad 4\sqrt{1\quad x}$$ [where $x\quad P(A)$]

$$f(x)_{\text{max.}}\quad 5 \text{ at } x\quad \frac{9}{25}$$

3. $$P(A\quad B)\quad 1\quad P(\overline{A\quad B})\quad \frac{5}{6}$$

$$P(A\quad B)\quad \frac{1}{4}, P(A)\quad \frac{3}{4}$$

$$P(B)\quad P(A\quad B)\quad P(A\quad B)\quad P(A)\quad \frac{1}{3}$$

4. $1\quad \frac{1}{2}^{\,n}\quad \frac{31}{32}\qquad n\quad 5$

5. Required probability $\dfrac{3!\quad 2}{\dfrac{9!}{3!3!3!}}\quad \dfrac{1}{140}$

6. Required probability $\dfrac{\dfrac{3!(3n\quad 3)!}{[(n\quad 1)!]^3}}{\dfrac{(3n)!}{(n!)^3}}$

7. If product of two numbers equal to third number, then possibilities are $(2, 3, 6)$, $(2, 4, 8)$, $(2, 5, 10)$.

Probability $\dfrac{3}{{}^{10}C_3}\quad \dfrac{1}{40}$

8. $P\quad \dfrac{3}{3}\quad \dfrac{2}{3}\quad \dfrac{2}{3}\quad \dfrac{1}{3}\quad \dfrac{4}{27}$

9. Total word $n \quad \dfrac{7!}{2!2!}$

T $\boxed{\text{ITANIC}}$ I $\boxed{\text{TTANIC}}$ A $\boxed{\text{TTINIC}}$

Favourable word $m \quad \dfrac{6!}{2!} \quad \dfrac{6!}{2!} \quad \dfrac{6!}{2!2!} \qquad P \quad \dfrac{m}{n} \quad \dfrac{5}{7}$

10. Probability $\dfrac{n!}{n^n} \quad \dfrac{3}{32} \quad \dfrac{6}{64}$

$\dfrac{(n \quad 1)!}{n^{n \quad 1}} \quad \dfrac{1 \;\; 2 \;\; 3}{4^3} \qquad n \quad 4$

11. Total case $n \quad 9 \quad 10^3$

Favourable case $m \quad (9 \quad 10^3) \quad 6^4$

$$P \quad 1 \quad \frac{6^4}{9 \quad 10^3} \quad \frac{107}{125}$$

12. Total case $n \quad 6!$

Favourable case $m \quad (3! \quad 2!) \quad (2! \quad 2!) \quad 16$

Probability $\dfrac{16}{6!} \quad \dfrac{1}{45}$

13. E_1 "No card is king from removed cards"

E_2 "1 card is king from removed cards"

E_3 "2 card is king from removed cards"

E_4 "3 card is king from removed cards"

E_5 "4 card is king from removed cards"

F 3 cards are drawn from pack those are kings.

$$P(F) \quad \sum_{i \quad 1}^{S} P(E_i) \; P \; \frac{F}{E_i} \quad \frac{^{48}C_{26}}{^{52}C_{26}} \quad \frac{^{4}C_{3}}{^{26}C_{3}} \quad \frac{^{48}C_{25} \quad ^{4}C_{1}}{^{52}C_{26}} \quad \frac{^{3}C_{3}}{^{26}C_{3}} \quad 0 \quad 0 \quad 0$$

$$\frac{4}{^{52}C_{26} \quad ^{26}C_{3}} (^{48}C_{26} \quad ^{48}C_{25}) \quad \frac{4 \quad ^{49}C_{26}}{^{52}C_{26} \quad ^{26}C_{3}}$$

$$\frac{1}{(13)(17)(25)}$$

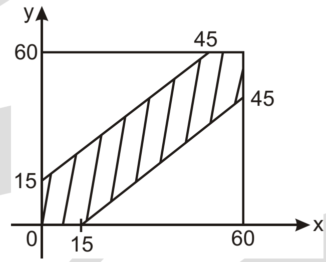
14. $\dfrac{^{3}C_{2} \quad ^{10}C_{4}}{^{13}C_{6}} \quad \dfrac{1}{7} \quad \dfrac{15}{286}$

15. Let f be function from $\{1, 2, \quad, 10\}$ to itself total functions possible is 10^{10}. The number of one-one onto functions possible is 10!.

Hence, the probability of selected function to be one-one onto is $\dfrac{10!}{10^{10}} \quad \dfrac{9!}{10^9}$.

16. Let the friends come to the restaurant at 5 h x min and 5 h y min, respectively, where $x, y \quad [0, 60]$.

Hence, the sample space consists of all points (x, y) lying in $60 \quad 60$ square as shown above and for favourable cases, $|x \quad y| \quad 15$, that is $15 \quad x \quad y \quad 15$ which is shown by shaded region in the graph shown below :



Hence, the probability that they will meet is given by :

$$1 \quad \frac{2 \quad \frac{1}{2} \quad 45 \quad 45}{60 \quad 60} \quad 1 \quad \frac{3}{4}^{2} \quad \frac{7}{16}$$

17. Total ways $\quad {}^{91}C_3$

Favourable ways (Common ratio is 2) + (Common ratio is 3) $\quad 16 \quad 2 \quad 18$

Exercise-2 : One or More than One Answer is/are Correct

1. Probability $\dfrac{{}^{4}C_3 \quad {}^{11}C_5}{{}^{15}C_8} \quad \dfrac{1}{7} \quad \dfrac{8}{195}$

2. Probability $\quad 1 \quad \dfrac{1}{2} \quad 1 \quad \dfrac{1}{4} \quad 1 \quad \dfrac{1}{6} \quad 1 \quad \dfrac{1}{8} \qquad 1 \quad \dfrac{1}{2012}$

$$\frac{1}{2} \quad \frac{3}{4} \quad \frac{5}{6} \quad \frac{7}{8} \qquad \frac{2011}{2012} \quad \frac{2012!}{2^{2012}(1006!)^2}$$

3. We have $P(E_i) \quad \dfrac{2}{4} \quad \dfrac{1}{2}$ or $i \quad 1, 2, 3.$

Also for $i \quad j, P(E_i \quad E_j) \quad \dfrac{1}{4} \quad P(E_j)P(E_i)$. Therefore, E_i and E_j are independent for $i \quad j$.

Also, $\quad P(E_1 \quad E_2 \quad E_3) \quad \dfrac{1}{4} \quad P(E_1)P(E_2)P(E_3)$

E_1, E_2, E_3 are not independent.

4. Max. $(P(A \quad B)) \quad P(A) \quad \dfrac{3}{5}$

$$\text{Min.}\,(P(A \quad B)) \quad P(A) \quad P(B) \quad 1 \quad \frac{4}{15}$$

$$P(A \quad B) \quad P(A) \quad P(B) \quad P(A \quad B) \quad \frac{19}{15} \quad P(A \quad B)$$

$$P(A \quad \overline{B}) \quad P(A) \quad P(A \quad B) \quad \frac{3}{5} \quad P(A \quad B)$$

$$P \quad \frac{\overline{A}}{B} \quad \frac{P(\overline{A} \quad B)}{P(B)} \quad \frac{P(B) \quad P(A \quad B)}{P(B)}$$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

1. $P(E_1) \quad \frac{1}{10} \quad 1 \quad \frac{2}{10} \quad \frac{1}{2} \quad \frac{3}{10} \quad \frac{1}{3} \quad \frac{4}{10} \quad \frac{1}{4} \quad \frac{4}{10} \quad \frac{2}{5}$

2. $P \quad \frac{B_3}{E_2} \quad \frac{P(B_3 \quad E_2)}{P(E_2)} \quad \frac{\frac{3}{10} \quad \frac{1}{3}}{\frac{2}{10} \quad \frac{1}{2} \quad \frac{3}{10} \quad \frac{1}{3} \quad \frac{4}{10} \quad \frac{1}{4}} \quad \frac{1}{3}$

Paragraph for Question Nos. 3 to 5

3. Mr. A's 3 digit number is always greater than Mr. B's 3 digit numbers then A should always pick digit 9.

$$\text{Probability} \quad \frac{{}^8C_3 \quad {}^8C_2}{{}^8C_3 \quad {}^9C_3} \quad \frac{1}{3}$$

4. Probability $\quad \frac{{}^8C_3 \quad 1}{{}^9C_3 \quad {}^8C_3} \quad \frac{1}{{}^9C_3} \quad \frac{1}{84}$

5. $P(E)$ $\quad A$ picks 9 or A does not pick 9 and his number is greater than B

$$\frac{1}{3} \quad \frac{2}{3} \quad \frac{1}{2} \quad 1 \quad \frac{{}^8C_3}{{}^8C_3} \quad \frac{1}{{}^8C_3} \quad \frac{37}{56}$$

Paragraph for Question Nos. 6 to 7

6. Let a_n number of ways of outcomes of n tosses when no 2 consecutive heads occur

$a_n \quad a_{n \quad 2} \quad a_{n \quad 1}$

Also, $a_1 \quad 2$ (H or T)

$a_2 \quad 3$ (TT or HT or TH)

$a_3 \quad 5, a_4 \quad 8$ ……

$a_{10} \quad 144$

Probability $\frac{144}{2^{10}}$

7. [HT HT HTH] T, T, T

Number of ways of arranging $\frac{4!}{3!} \quad 4$

Probability $\frac{4}{2^{10}}$

Paragraph for Question Nos. 8 to 10

8. $6n \quad 2^n, n \quad N$

$n \quad 1, 2, 3, 4$

9. $\frac{4}{6} \quad \frac{\text{Number of solutions of } x \quad y \quad 4, 1 \quad x, y \quad 6}{36}$

$\frac{\text{Number of solutions of } x \quad y \quad z \quad 8, 1 \quad x, y, z \quad 6}{6^3}$

$\frac{4}{6} \quad \frac{30}{36} \quad \frac{160}{216} \quad \frac{100}{243}$

10. Probability $\frac{4}{6} \quad \frac{30}{36} \quad 1 \quad \frac{160}{216} \quad \frac{4}{6} \quad \frac{30}{36} \quad \frac{56}{216} \quad \frac{35}{243}$

Paragraph for Question Nos. 11 to 12

11. Let p_1 be the probability of being an answer correct from section 1. Then $p_1 \quad 1/5$. Let p_2 be the probability of being an answer correct from section 2. Then $p_2 \quad 1/15$.

Hence, the required probability is $\frac{1}{5} \quad \frac{1}{15} \quad \frac{1}{75}$

12. Scoring 10 marks from four questions can be done in $3 \quad 3 \quad 3 \quad 1 \quad 10$ ways so as to answer 3 questions from section 2 and 1 question from section 1 correctly.

Hence, the required probability is $\frac{{}^{10}C_3 \; {}^{10}C_1}{{}^{20}C_4} \frac{1}{5} \quad \frac{1}{15}^{3}$.

Exercise-5 : Subjective Type Problems

1. $\frac{2}{3} \quad \frac{1}{3}\frac{2}{3} \quad \frac{1}{3}\frac{1}{3}\frac{2}{3} \quad \frac{2}{3} \quad \frac{1}{3}\frac{2}{3}\frac{2}{3} \quad \frac{416}{729}$

5. Probability $\frac{{}^6C_5 \quad {}^7C_4 \quad {}^8C_3 \quad {}^9C_2 \quad {}^{10}C_1 \quad 1}{2^{10}} \quad \frac{9}{64}$

6. $p \quad \dfrac{{}^3C_1}{{}^7C_1} \quad \dfrac{3}{7}$

7. Total ways $\quad \dfrac{6!}{2!2!2!3!} \quad 3! \quad 90$

Favourable cases $\quad 90 \quad [3! \quad {}^3C_1 \quad {}^3C_1 \quad 2 \quad 2] \quad 48$

$$p \quad \frac{48}{90} \quad \frac{8}{15}$$

9. E_1 be the event of both getting the correct answer

E_2 both getting wrong answers.

E both obtaining same answer.

$$P(E_1) \quad \frac{1}{8}\frac{1}{12} \quad \frac{1}{96}, \quad P(E_2) \quad 1 \quad \frac{1}{8} \quad 1 \quad \frac{1}{12} \quad \frac{77}{96}$$

$$P \quad \frac{E}{E_1} \quad 1; \quad P \quad \frac{E}{E_2} \quad \frac{1}{1001}$$

$$P \quad \frac{E_1}{E} \quad \frac{1 \quad \frac{1}{96}}{1 \quad \frac{1}{96} \quad \frac{1}{1001} \quad \frac{77}{96}} \quad \frac{13}{14}$$

10. Total ways $\quad {}^9C_7 \quad 7!$

Favourable ways $\quad {}^9C_7 \quad 7! \quad ({}^7C_3 \quad 3!) \quad ({}^6C_4 \quad 4!)$

$$P(E) \quad 1 \quad \frac{({}^7C_3 \quad 3!) \quad ({}^6C_4 \quad 4!)}{{}^9C_7 \quad 7!} \quad 1 \quad \frac{15}{36} \quad \frac{7}{12}$$

11. $\dfrac{1}{2} \; \dfrac{1}{2} \quad \dfrac{1}{2} \quad 2 \quad \dfrac{1}{4} \quad \dfrac{1}{4} \quad \dfrac{1}{4} \quad \dfrac{1}{4} \quad \dfrac{1}{4} \quad \dfrac{1}{2}^{\;3} \quad \dfrac{27}{128}$

12. $1^{st}2^{nd} \quad \dfrac{1}{4} \quad \dfrac{1}{6} \quad a$

$2^{nd}1^{st} \quad \dfrac{1}{4} \quad \dfrac{1}{6} \quad b$

$1^{st}1^{st} \quad \dfrac{1}{4} \quad \dfrac{1}{36} \quad c$

$2^{nd}2^{nd} \quad \dfrac{1}{4} \quad \dfrac{1}{36} \quad d$

$$\frac{c \quad d}{a \quad b \quad c \quad d} \quad \frac{2}{5}$$

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16

LOGARITHMS

Exercise-1 : Single Choice Problems

1. $\log_{10} x \quad A \qquad x \quad 0$

$\log_{10}(x \quad 2) \quad B, \; x \quad 2 \quad 0 \qquad x \quad 2$

$$A^2 \quad 3AB \quad 2B^2 \quad 0$$

$$(A \quad 2B)(A \quad B) \quad 0$$

$$(\log x \quad 2\log(x \quad 2))(\log x \quad \log(x \quad 2)) \quad 0$$

Case-I : $\log x \quad 2\log(x \quad 2) \quad 0$

and $\log x \quad \log(x \quad 2) \quad 0$ ...(1)

Case-II : $\log x \quad 2\log(x \quad 2) \quad 0$

and $\log x \quad \log(x \quad 2) \quad 0$ ...(2)

From (1) & (2), $x \quad (4, \quad)$

2. $(\log_e x)^2 \quad (\log_e x) \quad (\log_e x)^2 \quad 1$ $\qquad (\log_e x \quad 0)$

$$2(\log_e x)^2 \quad \log_e x \quad 1 \quad 0$$

$$(2\log_e x \quad 1)(\log_e x \quad 1) \quad 0$$

$$\log_e x \qquad \frac{1}{2} \text{ (not possible)}$$

$$\log_e x \quad 1$$

3. $S \quad (a^{\log_3 7})^{\log_3 7} \quad (b^{\log_7 11})^{\log_7 11} \quad (c^{\log_{11} 25})^{\log_{11} 25}$

$$27^{\log_3 7} \quad 49^{\log_7 11} \quad \sqrt{11}^{\log_{11} 25} \qquad 469$$

4. $a^2 \quad 3a \quad 3 \qquad x \quad \frac{1}{x}^{\;0}$ and $a^2 \quad 3a \quad 3 \quad 0$

$a^2 \quad 3a \quad 2 \quad 0$

$(a \quad 1)(a \quad 2) \quad 0 \qquad a \quad (\quad ,1) \quad (2, \quad)$

5. $P \quad \dfrac{5}{\log_x 120} \quad \log_{120} x^5; \quad (120)^P \quad x^5 \quad 32 \qquad x \quad 2$

6. $x \quad \dfrac{z^{1/3}}{2}, \; y \quad \dfrac{z^{1/6}}{5}$

If $xy \quad z^{3/2}; \quad \dfrac{z^{1/3}}{2} \; \dfrac{z^{1/6}}{5} \quad z^{3/2} \qquad z \quad \dfrac{1}{10}$

7. $\log_x(\log_3(\log_x y)) \quad 0 \qquad y \quad x^3, \log_y 27 \quad 1 \qquad y \quad 27$

8. $\log_{10^{\;2}} 10^3 \quad \log_{10^{\;1}} 10^{\;4} \quad \dfrac{3}{2} \quad 4 \quad \dfrac{5}{2}$

9. $a \quad \dfrac{3}{1 \quad 2\log_3 2} \qquad \log_3 2 \quad \dfrac{3 \quad a}{2a}; \quad \log_6 16 \quad \dfrac{4\log_3 2}{1 \quad \log_3 2}$

10. $\log_2(\log_2(\log_3 x)) \quad 0 \qquad x \quad 9$

$\log_2(\log_3(\log_2 y)) \quad 0 \qquad y \quad 8$

11. Let $\log_3 a \quad x, \; \log_3 b \quad y; \; \dfrac{x}{3} \quad \dfrac{y}{2} \quad \dfrac{7}{2}$ and $\dfrac{x}{2} \quad \dfrac{y}{3} \quad \dfrac{2}{3}$

12. $a \quad \log_2 5; \; b \quad \log_5 8; \; c \quad \log_8 11; \; d \quad \log_{11} 14$

$2^{abcd} \quad 2^{\log_2 14} \quad 14$

14. $\dfrac{\log_8 17}{\log_9 23} \quad \dfrac{\log_{2\sqrt{2}} 17}{\log_3 23}$

15. **Case-I :** $2x \quad 3 \quad 1$

$3x \quad 4 \quad 1$

$x \quad \dfrac{5}{3} \qquad x \quad 2$

Case-II : $0 \quad 2x \quad 3 \quad 1$

$0 \quad 3x \quad 4 \quad 1$

$x \quad \dfrac{5}{3} \quad \dfrac{3}{2} \quad x \quad \dfrac{5}{3}$

16. $p \quad \log_{10} N \quad p \quad 1 \qquad P \quad 10^{p \;\; 1} \quad 10^p$

$q \quad \log_{10} 1/N \qquad q \quad 1 \qquad Q \quad 10^q \quad 10^{q \;\; 1}$

17. $n \quad 1$ number of digits $\quad 1$ characteristic

18. $\log_{10}(0.15)^{20} \quad 20(\log_{10} 15 \quad 2) \qquad 16.478$

19. $\log_2(\log_4(\log_{10} 10^{16})) \quad \log_2(\log_4 16) \quad 1$

20. $2\log x \quad \log(2x \quad 75) \quad 2$

$\dfrac{x^2}{2x \quad 75} \quad 100 \qquad x^2 \quad 200x \quad 7500 \quad 0$

21. $x^{\log_x a \log_a y \log_y z} \quad x^{\log_x z} \quad z$

22. $x^{x\sqrt{x}} \quad x^{3x/2}$

$x \quad 0, 1 \qquad x\sqrt{x} \quad \dfrac{3}{2}x \qquad x \quad \dfrac{9}{4}$

If $\; x \quad 1$, then it also satisfy.

23. $(\log_3 x)^2 \quad 2\log_3 x$

$\log_3 x \quad 0 \qquad \text{or} \qquad \log_3 x \quad 2$

$x \quad 1 \qquad \text{or} \qquad x \quad 9$

24. $\log_{10} x \quad \log_{10} y \quad 2 \qquad xy \quad 100$ ...(1)

$x \quad y \quad 15$ ...(2)

$x \quad 20, \quad y \quad 5$

25. $2^{\left(x \;\frac{1}{3}\; 2x \;\frac{3}{x}\right)^{1/2}} \qquad 2^{\frac{7}{3}}$

$x \quad \frac{2}{3}x \quad \frac{1}{x} \quad \frac{14}{3} \qquad 5x^2 \quad 14x \quad 3 \quad 0$

26. $25^{(2x \;\; x^2 \;\; 1)} \quad 9^{(2x \;\; x^2 \;\; 1)} \quad 34\,\frac{3^{2x \;\; x^2 \;\; 1}}{3} \quad \frac{5^{2x \;\; x^2 \;\; 1}}{5}$

Let $3^{2x \;\; x^2 \;\; 1} \quad a$ and $5^{2x \;\; x^2 \;\; 1} \quad b$

$a^2 \quad b^2 \quad \frac{34}{15}ab$

$15a^2 \quad 34ab \quad 15b^2 \quad 0 \qquad (3a \quad 5b)(5a \quad 3b) \quad 0$

Case-1 : if $\frac{a}{b} \quad \frac{5}{3}$

$\frac{3}{5}^{\,2x \;\; x^2 \;\; 1} \qquad \frac{5}{3}$

$2x \quad x^2 \quad 1 \quad 1 \qquad x^2 \quad 2x \quad 2 \quad 0$

Sum of two values of $x \quad 2$

Case-2 : if $\frac{a}{b} \quad \frac{3}{5}$

$\frac{3}{5}^{\,2x \;\; x^2 \;\; 1} \qquad \frac{3}{5}$

$2x \quad x^2 \quad 1 \quad 1 \qquad x \quad 0$ and 2

Sum of all values of x is 4.

27. $a^x \quad b^y \quad c^z \quad d^w$

$b \quad a^{x/y}, \; c \quad a^{x/z}, \; d \quad a^{x/w}$

$\log_a(bcd) \quad \log_a a^{\frac{x}{y} \;\frac{x}{z}\; \frac{x}{w}} \qquad \frac{x}{y} \quad \frac{x}{z} \quad \frac{x}{w} \qquad x \;\; \frac{1}{y} \quad \frac{1}{z} \quad \frac{1}{w}$

28. $x \quad \dfrac{4}{(\sqrt{5} \quad 1)(\sqrt[4]{5} \quad 1)(\sqrt[8]{5} \quad 1)(\sqrt[16]{5} \quad 1)}$

Multiply and divide by $(1 \quad \sqrt[16]{5})$ then

$$x \quad 1 \quad \sqrt[16]{5}$$

$$(x \quad 1)^{48} \quad 5^3 \quad 125$$

29. $$\log_x \log_{18}(\sqrt{2} \quad \sqrt{8}) \quad \frac{1}{3}$$

$$\log_x \log_{(3\sqrt{2})^2} 3\sqrt{2} \quad \frac{1}{3}$$

$$\log_x \frac{1}{2} \quad \frac{1}{3} \qquad x \quad \frac{1}{8}$$

30. $$f(n) \quad \frac{1}{3}\log_2 n \qquad \text{if } \log_8 n \text{ is integer}$$

$$0 \qquad \text{otherwise}$$

$$\sum_{n \quad 1}^{2011} f(n) \quad \log_8 2^3 \quad \log_8 2^6 \quad \log_8 2^9 \quad 1 \quad 2 \quad 3 \quad 6$$

32. $$\log_{0.3}(x \quad 1) \quad \log_{(0.3)^2}(x \quad 1) \qquad (x \quad 1)^2 \quad x \quad 1$$

$$(x \quad 1)(x \quad 2) \quad 0$$

Also, for log to be defined $(x \quad 1) \quad 0$

$$x \quad (2, \quad)$$

33. $$\sqrt{7^{2x^2 \quad 5x \quad 6}} \quad (49)^{3\log_2\sqrt{2}} \quad 7^3$$

$$2x^2 \quad 5x \quad 6 \quad 6$$

$$2x^2 \quad 5x \quad 12 \quad 0$$

$$(2x \quad 3)(x \quad 4) \quad 0$$

34. $(\log_2 x)^4 \quad 16(\log_2 x)^2 \log_2 \dfrac{16}{x}$

$$t^4 \quad 16t^2(4 \quad t) \qquad (\text{where } \log_2 x \quad t)$$

$$t^2(t^2 \quad 64 \quad 16t)$$

$$t^2(t \quad 8)^2$$

Since $1 \quad x \quad 256 \qquad 0 \quad t \quad 8$

Maximum of $(t \quad 8)^2 t^2$ lies at $t \quad 4$.

Hence, maximum $(4 \quad 8)^2 \quad 4^2 \quad 256$

37. $\because \quad 0$

$$\log_{16} x \quad \frac{1 \quad \sqrt{(1 \quad 4\log_{16} \quad)}}{2}$$

$\because$ The given equation will have exactly one solution, if

$$1 \quad 4\log_{16} \qquad 0 \quad \text{or} \quad \log_{16} \qquad \frac{1}{4} \quad 4^{\quad 1}$$

$$(16)^{4^{\quad 1}} \quad (2^4)^{1/4} \quad 2, \quad 2, 2i, \quad 2i, \text{where } i \quad \sqrt{\quad 1}$$

But is real and positive.

$\quad 2$

Number of real values $\quad 1$

38. Let x be the rational number, then according to question,

$$x \quad 50 \quad \log_{10} x$$

By trial $x \quad 100$

39. $\because \quad x \quad \log_5(1000) \quad \log_5(5^3 \quad 8) \quad 3 \quad \log_5 8$

and $y \quad \log_7(2058) \quad \log_7(7^3 \quad 6) \quad 3 \quad \log_7 6$

$$x \quad y \quad \log_5 8 \quad \log_7 6 \quad 0$$

$\because \log_5 8 \quad 1, \log_7 6 \quad 1$
$\log_5 8 \quad \log_7 6 \quad 0$

$$x \quad y$$

40. $7\log \dfrac{2^4}{5 \quad 3} \quad 5\log \dfrac{5^2}{2^3 \quad 3} \quad 3\log \dfrac{3^4}{2^4 \quad 5}$

$7\{4\log 2 \quad \log 5 \quad \log 3\} \quad 5\{2\log 5 \quad 3\log 2 \quad \log 4\} \quad 3\{4\log 3 \quad 4\log 2 \quad \log 5\}$
$\log 2$

41. $\log_{10}\{\tan 1 \ \tan 2 \ \tan 3 \quad \tan 45 \quad \tan 87 \ \tan 88 \ \tan 89 \ \}$

$\log_{10}\{\tan 1 \ \tan 2 \ \tan 3 \quad \tan 45 \quad \cot 3 \ \cot 2 \ \cot 1 \ \}$

$\log_{10} 1 \quad 0$

42. $\log_7 \log_7 7^{\frac{1}{2} \ \frac{1}{4} \ \frac{1}{8}} \quad \log_7 \dfrac{1}{2} \quad \dfrac{1}{4} \quad \dfrac{1}{8} \quad \log_7 \dfrac{7}{8}$

$$1 \quad \log_7 8 \quad 1 \quad 3\log_7 2$$

43. $(4)^{\log_{3^2} 3} \quad (9)^{\log_2 2^2} \quad (10)^{\log_x 83}$

$$(4)^{1/2} \quad 9^2 \quad (10)^{\log_x 83}$$

$$(83)^1 \quad (83)^{\log_x 10}$$

$$1 \quad \log_x 10 \qquad x \quad 10$$

44. $(10^{\log_{10} x})^{\log_{10} \frac{y}{z}} \ (10^{\log_{10} y})^{\log_{10} \frac{z}{x}} \ (10^{\log_{10} z})^{\log_{10} \frac{x}{y}}$

45. $\because \quad \log_x 2 \log_{2x} 2 \quad \log_{4x} 2$

$x \quad 0, 2x \quad 0$ and $4x \quad 0$ and $x \quad 1, 2x \quad 1, 4x \quad 1$

$x \quad 0$ and $x \quad 1, \frac{1}{2}, \frac{1}{4}$

Then, $\quad \frac{1}{\log_2 x} \quad \frac{1}{\log_2 2x} \quad \frac{1}{\log_2 4x}$

$$\log_2 x \quad \log_2 2x \quad \log_2 4x$$
$$\log_2 x \ (1 \quad \log_2 x) \quad (2 \quad \log_2 x)$$
$$(\log_2 x)^2 \quad 2$$
$$\log_2 x \quad \sqrt{2}$$
$$x \quad 2^{\quad \sqrt{2}}$$
$$x \quad \{2^{\quad \sqrt{2}}, 2^{\sqrt{2}}\}$$

46. $\because \quad 2\log_{10} x \quad \log_x (0.01) \quad 2\log_{10} x \quad \log_x (10^{\quad 2})$

$2(\log_{10} x \quad \log_x 10)$ $\qquad (\because x \quad 0$ and $x \quad 1)$

$2 \quad \frac{\log_e x}{\log_e 10} \quad \frac{\log_e 10}{\log_e x} \quad\quad 2 \ 2$ $\qquad (\because \text{AM} \quad \text{GM})$

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47. Let $\sqrt{\log_2 x} \quad a$

$a^2 \quad 2a \quad 1 \qquad a \quad 1$

if $\sqrt{\log_2 x} \quad 1 \qquad x \quad 2$

48. $\log_e (e^2 x^{\ln x}) \quad \log_e x^3$

$2 \quad (\ln x)^2 \quad 3 \ln x$

Let $\ln x \quad a$

$a^2 \quad 3a \quad 2 \quad 0 \qquad (a \quad 2)(a \quad 1) \quad 0$

$x_1 \quad e^2, x_2 \quad e$

49. $M \quad \text{antilog}_{32} 0.6 \quad (32)^{0.6} \quad 2^3 \quad 8$

$N \quad 49^1 \quad 49^{\ \log_7 2} \quad 5^{\ \log_5 4}$

$\frac{49}{4} \quad \frac{1}{4} \quad \frac{25}{2}$

50. $\log_2(\log_2(\log_3 x)) \quad 0 \qquad x \quad 9$

$\log_3(\log_3(\log_2 y)) \quad 0 \qquad y \quad 8$

51. $|\log_{1/2} 10 \quad |\log_4 625 \quad \log_2 5| \quad |\log_{1/2} 10 \quad \log_2 5| \quad 1$

52. $\log_3 2 \quad \dfrac{\log_5 2}{\log_5 3} \quad \dfrac{\dfrac{1}{2a}}{b \quad \dfrac{1}{2a}} \quad \dfrac{1}{2ab \quad 1}$

55. $(x \quad 3)^2 \quad 9 \qquad x \quad 6$

57. $\log_a \left(\dfrac{16}{15}\right)^7 \left(\dfrac{25}{24}\right)^5 \left(\dfrac{81}{80}\right)^3 \qquad 8$

$$\log_a 2 \quad 8 \qquad\qquad a \quad 2^{1/8}$$

58. $\log_{2^3}(2^7) \quad \log_{3^2}(3 \quad ^{1/2}) \quad \dfrac{7}{3} \quad \dfrac{1}{4} \quad \dfrac{31}{12}$

59. $\left(\dfrac{1}{\sqrt{27}}\right)^2 \quad \left(\dfrac{1}{\sqrt{27}}\right)^{\frac{\log_5 16}{2\log_5 9}} \quad \dfrac{1}{27} \quad \left(\dfrac{1}{\sqrt{27}}\right)^{\log_3 2}$

$$\frac{1}{27} \quad 2^{\log_3 \frac{1}{\sqrt{27}}} \quad \frac{2\sqrt{2}}{27}$$

60. $\log_2 \dfrac{(x \quad 1)(x \quad 2)}{3x \quad 1} \quad \log_2 4 \qquad \dfrac{(x \quad 1)(x \quad 2)}{3x \quad 1} \quad 4$

$$x^2 \quad 11x \quad 2 \quad 0$$

61. $\log_{100} 10 \quad \dfrac{1}{2}$

$\log_2(\log_4 2) \quad \log_2 1/2 \quad\quad 1$

$\log_4[\log_2(256)^2]^2 \quad \log_4 16^2 \quad 4$

$\log_4 8 \quad \log_{2^2} 2^3 \quad \dfrac{3}{2}$

62. $\quad \log_5(\log_5 3) \quad\quad 5 \quad\quad \log_5 3$

$3^{k \quad 5} \quad\quad 3^k \quad 3^5 \quad\quad 3^k \quad 3^{\log_3 5} \quad 5 \quad 3^k$

63. $\log_{10} b^4 \quad 2 \quad \log_{10} a^2$

$\dfrac{\log_{10} b}{\log_{10} a} \quad \log_a b$

64. $2^x \quad 3^y \quad 6^{\quad z} \quad k$ (let)

$x \quad \log_2 k, \; y \quad \log_3 k, \; z \quad\quad \log_6 k$

$\dfrac{1}{x} \quad \dfrac{1}{y} \quad \dfrac{1}{z} \quad \log_k 2 \quad \log_k 3 \quad \log_k 6 \quad 0$

65. $(\sqrt{2} \quad 1)^3 \quad 5\sqrt{2} \quad 7$

66. $1 \quad \log_a b \quad \frac{1}{4} \qquad \log_a b \qquad \frac{3}{4} \qquad \frac{\frac{1}{3} \quad \frac{1}{2}\log_a b}{1 \quad \log_a b} \quad \frac{17}{6}$

68. Let $\log_y x \quad t$

$5t^2 \quad 26t \quad 5 \quad 0 \qquad (5t \quad 1)(t \quad 5) \quad 0$

Either $x \quad y^5$ or $y \quad x^5$

69. $1 \quad \frac{1}{\log_3 x} \quad \frac{1}{\log_3 x \quad 1} \qquad (\log_3 x \quad 1)^2 \quad \log_3 x \qquad (\log_3 x)^2 \quad 3\log_3 x \quad 1 \quad 0$

70. $\log_2 x \quad \frac{1}{2}\log_2 y \quad \frac{1}{2}\log_2 z \quad 2 \qquad x\sqrt{y}\sqrt{z} \quad 4$

$\log_3 y \quad \frac{1}{2}\log_3 x \quad \frac{1}{2}\log_3 z \quad 2 \qquad \sqrt{x} \quad y \quad \sqrt{z} \quad 9$

$\log_4 z \quad \frac{1}{2}\log_4 x \quad \frac{1}{2}\log_4 y \quad 2 \qquad \sqrt{x} \quad \sqrt{y} \quad z \quad 16 \qquad xyz \quad 24$

71. $\frac{1}{49} \quad 2^{\log_7 {}^{1/49}} \qquad 7^{\log_{1/5} {}^5}$

$\frac{1}{49} \quad \frac{1}{4} \quad 7$

72. $\log_2(3 \quad x) \quad \log_2 \frac{1}{\sqrt{2}} \quad \log_2(5 \quad x) \quad \frac{1}{2} \quad \log_2(x \quad 7)$

$\log_2(3 \quad x)(5 \quad x) \quad \log_2(x \quad 7)$

$x^2 \quad 9x \quad 8 \quad 0 \qquad x \quad 8$

73. $\log_5 x \quad \log_x 5 \qquad x \quad 5, \frac{1}{5}$

74. $|x \quad 1|^{\log_3 x^2 \quad 2\log_x 9} \quad (x \quad 7)^7$

either $x \quad 2$ or $\log_3 x^2 \quad 2\log_x 9 \quad 7$

$(\log_3 x \quad 4)(2\log_3 x \quad 1) \quad 0$

75. $9^{x \quad 1} \quad 7 \quad 4(3^{x \quad 1} \quad 1)$

Let $3^x \quad t$

$\frac{t^2}{9} \quad 7 \quad 4 \; \frac{t}{3} \quad 1 \qquad t^2 \quad 12t \quad 27 \quad 0$

$(t \quad 3)(t \quad 9) \quad 0$

76. If $\quad 1$

$\log \quad 10 \quad \log \quad 3 \quad \log \quad e \quad \log \quad 2$

$\log_{10} \qquad \log_3 \qquad \log_e \qquad \log_2$

78. $\overset{4}{\underset{r\ 1}{}}\ \log_4 2^r \quad \overset{4}{\underset{r\ 1}{}}\ \frac{r}{2} \quad 5$

79. $\log_3 2 \quad \log_3 5 \quad \log_3 10$

$\log_3 9 \quad \log_3 10 \quad \log_3 27$

80. $\sin 3 \quad 3\sin \quad 4\sin^3$

$$\frac{k}{2}\ a^3 \quad \frac{1}{a^3} \quad \frac{3}{2}\ a \quad \frac{1}{a} \quad \frac{4}{8}\ a \quad \frac{1}{a}^{\ 3}$$

$$\frac{1}{2}\ a^3 \quad \frac{1}{a^3}$$

$$k \quad 1$$

81. $2x \quad 3 \quad 0 \quad x^2 \quad 5x \quad 6 \quad 0 \quad 2x \quad 3 \quad 1$

$x \quad \frac{3}{2} \quad (x \quad 6)(x \quad 1) \quad 0 \quad x \quad 2$

$(6, \quad)$

Exercise-2 : One or More than One Answer is/are Correct

1. $$6(\log x)^2 \quad \log x \quad 1 \quad 0$$

$$(3\log x \quad 1)(2\log x \quad 1) \quad 0$$

$$x \quad 10^{1/3} \quad \text{or} \quad x \quad 10^{\quad 1/2}$$

3. $3(\log_{10} 2)x^2 \quad (1 \quad \log_{10} 2)x \quad 2\log_{10} 2 \quad x$

$$\log_{10} 2(3x^2 \quad x \quad 2) \quad 0$$

$$\log_{10} 2(x \quad 1)(3x \quad 2) \quad 0$$

Roots of this eq. are $x \quad 1, \frac{2}{3}$

Sum of coeff. $\quad 2\log_{10} 2$ (irrational)

Discriminant $\quad b^2 \quad 4ac \quad 25(\log_{10} 2)^2$ (irrational)

4. $A \quad \min.(x^2 \quad 2x \quad 7) \quad x \quad R \qquad A \quad 6$

$B \quad \min.(x^2 \quad 2x \quad 7) \quad x \quad [2, \quad) \qquad B \quad 7$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

1. If $\quad _1 \quad 4,$ then $3^4 \quad N \quad 3^5$

If $\quad _2 \quad 2,$ then $5^2 \quad N \quad 5^3$

$$81 \quad N \quad 125$$

3. If $\quad _1 \quad 5,$ then $3^5 \quad N \quad 3^6$

If $\quad _2 \quad 3,$ then $5^3 \quad N \quad 5^4$

If $\quad _3 \quad 2,$ then $7^2 \quad N \quad 7^3$

$$243 \quad N \quad 343$$

Paragraph for Question Nos. 4 to 5

Sol. $|x^2 \quad y^2| \quad 221$

Paragraph for Question Nos. 6 to 7

Sol. $(1 \quad 4\log_{p^2}(2p))^2 \quad (1 \quad \log_2 p)^2 \quad (1 \quad \log_2 4p)^2$

Let $\log_2 p \quad t$

$$1 \quad 2 \quad \frac{1 \quad t}{t}^{2} \quad (1 \quad t)^2 \quad (3 \quad t)^2 \qquad t \quad \log_2 p \quad 2$$

Exercise-4 : Matching Type Problems

1. (A) $\quad a \quad 3((\sqrt{7} \quad 1) \quad (\sqrt{7} \quad 1)) \quad 6$

$\quad b \quad \sqrt{1296} \quad 36$

(B) $\quad a \quad (\sqrt{3} \quad 1) \quad (\sqrt{3} \quad 1) \quad 2$

$\quad b \quad (3 \quad \sqrt{2}) \quad (3 \quad \sqrt{2}) \quad 2\sqrt{2}$

(C) $\quad a \quad (\sqrt{2} \quad 1), \quad b \quad (\sqrt{2} \quad 1)$

(D) $\quad a \quad 2 \quad \sqrt{3}, \quad b \quad 2 \quad \sqrt{3}$

Exercise-5 : Subjective Type Problems

1. $N \quad 6^{\log_{10} 40} \quad 6^{2\log_{10} 5} \quad 6^{\log_{10} 1000} \quad 6^3 \quad 216$

2. $\log_b(a^{\log_2 b}) \quad \log_a(b^{\log_2 b})$

$$\log_b a \quad \log_a b \qquad a \quad b \text{ or } a \quad \frac{1}{b} \text{ (not possible)}$$

$$\log_a(c \quad (b \quad a)^2) \quad 3 \qquad c \quad a^3$$

Minimum value of c 8 at a 2

3. $\log_b 729 \quad 6\log_b 3$

if this is an integer, then $b \quad 3, 3^2, 3^3, 3^6$

4. **Case-1 :** If $\quad x \quad \frac{5}{2} \quad 1 \qquad x \quad \frac{3}{2}$

then $\quad (x \quad 5)^2 \quad (2x \quad 3)^2 \qquad 3x^2 \quad 2x \quad 16 \quad 0 \qquad x \quad \frac{8}{3},$

Case-2 : If $\quad 0 \quad x \quad \frac{5}{2} \quad 1 \qquad \frac{5}{2} \quad x \quad \frac{3}{2}$

then $\quad (x \quad 5)^2 \quad (2x \quad 3)^2 \qquad x \qquad 2, \frac{3}{2}$

there is no negative integral value of x.

5. $\frac{6}{5} a^{(\log_a x)(\log_{10} a)(\log_a 5)} \quad 3^{(\log_{10} x \quad 1)} \quad 9^{\log_{100} x \quad \frac{1}{2}}$

$$6 \quad 5^{(\log_{10} x \quad 1)} \quad 3^{(\log_{10} x \quad 1)} \quad 3^{(\log_{10} x \quad 1)}$$

$$6 \quad 5^{(\log_{10} x \quad 1)} \quad \frac{3^{\log_{10} x}}{3} \quad 3 \quad 3^{\log_{10} x}$$

$$6 \quad 5^{(\log_{10} x \quad 1)} \quad \frac{10}{3} \quad 3^{\log_{10} x}$$

$$\left(\frac{5}{3}\right)^{\log_{10} x \quad 2} \qquad 1$$

$$\log_{10} x \quad 2 \quad 0$$

$$x \quad 100$$

Integer part of $\log_3 100$ is 4.

6.
$$\log_5 \frac{a \quad b}{3} \quad \frac{\log_5 a \quad \log_5 b}{2}$$

$$\log_5 \left(\frac{a \quad b}{3}\right)^2 \quad \log_5(ab)$$

$$(a \quad b)^2 \quad 9ab \qquad a^2 \quad 7ab \quad b^2 \quad 0$$

$$a^4 \quad b^4 \quad 2a^2b^2 \quad 49a^2b^2$$

$$\frac{a^4 \quad b^4}{a^2b^2} \quad 47$$

8. $\log_{10}\sqrt{1 \quad x} \quad 3\log_{10}\sqrt{1 \quad x} \quad 2 \quad \log_{10}\sqrt{1 \quad x} \quad \log_{10}\sqrt{1 \quad x}$

$$\log_{10}\sqrt{1 \quad x} \quad 1$$

$$\sqrt{1 \quad x} \quad 10 \qquad x \quad 99 \quad \text{(not possible)}$$

9. $$x^2 \quad 1 \quad 6\log_4 y$$

$$y^2 \quad 2^x y \quad 2^{2x \quad 1} \quad 0$$

$$y \quad 2^{x \quad 1} \text{ and } y \qquad 2^x$$

if $y \qquad 2^x$ (not possible, because $y \quad 0$)

if $y \quad 2^{x \quad 1}$

$$\log_2 y \quad x \quad 1$$

$$x^2 \quad 1 \quad 3\log_2 y$$

$$x^2 \quad 1 \quad 3(x \quad 1)$$

$$x^2 \quad 3x \quad 4 \quad 0$$

$$(x \quad 4)(x \quad 1) \quad 0$$

$$x_1 \quad 4$$

$$y_1 \quad 2^5 \quad 32$$

$$x_2 \qquad 1$$

$$y_2 \quad 2 \quad 1$$

$$\log_2|x_1x_2y_1y_2| \quad \log_2 128 \quad 7$$

10. $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}} \quad \log_7 \log_7 (7^{7/8}) \quad \log_7 (7/8) \quad 1 \quad 3\log_7 2$

$$a \quad 3$$

$$\log_{15}\log_{15}\sqrt{15\sqrt{15\sqrt{15\sqrt{15}}}} \quad \log_{15}\log_{15}(15^{15/16})$$

$$\log_{15}\frac{15}{16} \quad 1 \quad 4\log_{15} 2$$

$$b \quad 4$$

Then $$a \quad b \quad 7$$

11. $\log_{1 \quad x}(1 \quad y) \quad \log_{1 \quad y}(1 \quad x) \quad 2 \qquad\qquad t \quad \frac{1}{t} \quad 2 \qquad t \quad 1$

$$1 \quad x \quad 1 \quad y$$

$$x \qquad y$$

$$\log_{1 \quad y}(1 \quad 2y) \quad \log_{1 \quad y}(1 \quad 2y) \quad 2$$

$$\log_{1\quad y}(1\quad 4y^2)\quad 2$$

$$1\quad 4y^2\quad 1\quad y^2\quad 2y$$

$$5y^2\quad 2y\quad 0$$

$$y\quad 0, y\quad \frac{2}{5}$$

But $y\quad 0$ rejected.

12. $$\log_b n\quad 2$$

$$\log_n(2b)\quad \log_n 2\quad \log_n b\quad 2$$

$$\log_n 2\quad \frac{1}{2}\quad 2$$

$$\log_n 2\quad \frac{3}{2}\qquad n\quad 2^{2/3}$$

if $\log_b n\quad 2 \qquad b\quad n^{1/2}\quad 2^{1/3}$

$$n\quad b\quad 2^{2/3}\quad 2^{1/3}\quad 2$$

13. $$\log_y x\quad \frac{1}{\log_y x}\quad 2$$

$$\log_y x\quad 1\qquad x\quad y$$

$$x^2\quad y\quad 12$$

$$x^2\quad x\quad 12\quad 0$$

$$(x\quad 4)(x\quad 3)\quad 0$$

$$x\quad 4 \text{ or } x\quad 3$$

but $x\quad 0$, then $x\quad 3$

$$xy\quad 9$$

14. $$y^x\quad x^y$$

if $x\quad 2y$ then $y^{2y}\quad (2y)^y$

$$2y\log y\quad y\log(2y)$$

if $y\quad 0$ then $\log y^2\quad \log(2y)$

$$y^2\quad 2y\qquad y\quad 2$$

$$x^2\quad y^2\quad 5y^2\quad 20$$

15. $(\log_2 4\quad \log_2(4^x\quad 1))\log_2(4^x\quad 1)\quad 3$

Let $\log_2(4^x\quad 1)\quad t$

$$t^2 \quad 2t \quad 3 \quad 0 \qquad t \quad 3 \text{ or } 1$$

$$\log_2(4^x \quad 1) \quad 1 \qquad 4^x \quad 1 \qquad x \quad 0$$

17. $x^2 \quad 4x \quad 3 \quad 0 \quad (x \quad 0)$

18. $\log_{3^{1/4}}(\log_{3\sqrt{5}} x) \quad 4 \qquad \log_3^{(\log_{3\sqrt{5}} x)} \quad 1$

$$\log_5 x \quad 1 \qquad x \quad 5$$

❑❑❑

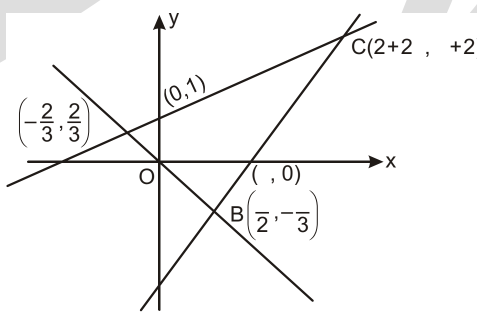
17

STRAIGHT LINES

Exercise-1 : Single Choice Problems

1. Let ratio be $\quad : 1 \qquad \frac{6 \quad 3}{\quad 1} \quad 0, \qquad \frac{1}{2}$

3.



if $(a, \sin a)$ lie inside the triangle, then $a \quad (0, \quad)$

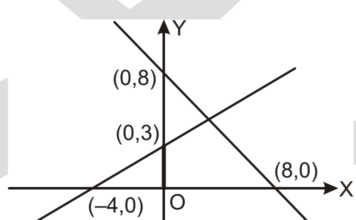
4. $x \quad \frac{711}{13 \quad 11m} \quad \frac{9 \quad 79}{13 \quad 11m}$

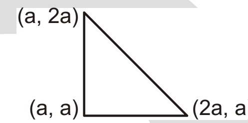
if x is an integer, then $m \quad 6$

6. $7 \quad \frac{y}{x}^{2} \quad 2c \quad \frac{y}{x} \quad 1 \quad 0$

$m_1 \quad m_2 \quad 4m_1m_2 \qquad c \quad 2$

10.





13. $\frac{1}{2}a^2 \quad 72$

$a \quad 12$

Centroid $\quad (16, 16)$ or $(\quad 16, \quad 16)$

14. $g(x) \quad ax \quad b$

$g(1) \quad 2$

$a \quad b \quad 2$

$g(3) \quad 0$

$2a \quad 2$

$a \quad 1$

$b \quad 3$

$g(x) \quad x \quad 3$

$\cot[\cos^{\;1}(|\sin x| \quad |\cos x|) \quad \sin^{\;1}(|\sin x|) \quad |\cos x|]$

$|\sin x| \quad |\cos x| \quad [1, \sqrt{2}]$

$\cot[\cos^{\;1} 1 \quad \sin^{\;1} 1] \quad 0 \quad g(3)$

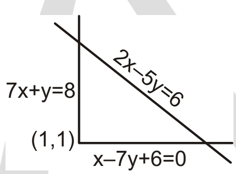
15. Points A and B are mirror images about $y \quad x$.

Point P will lie on the bisector of line joining A and B $\quad P$ lie on $y \quad x$.

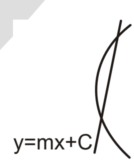
16. $4m^3 \quad 3am^2 \quad 8a^2m \quad 8 \quad 0$ $\begin{cases} \to m_1 \\ \to m_2 \\ \to m_3 \end{cases}$

$m_1m_2m_3 \quad 2$

$m_3 \quad 2 \qquad (\because m_1m_2 \quad 1)$

17. 

18. $2x^2 \quad 3y^2 \quad 5x \dfrac{y \quad mx}{C} \quad 0$



Coefficient of x^2 coefficient of $y^2 \quad 0$

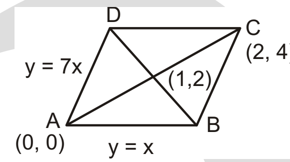
$5 \quad \dfrac{5m}{C} \quad 0 \qquad m \quad C \quad 0$

Then the equation of family of line is $y \quad m(x \quad 1)$

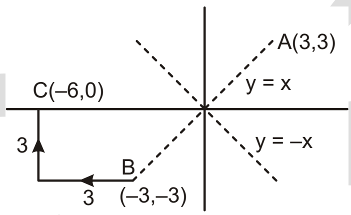
20. Equation of line BC is $y \quad 7x \quad 10$

Equation of line CD is $y \quad x \quad 2$

Area of rhombus $\left|\dfrac{(2 \quad 0)(10 \quad 0)}{(7 \quad 1)}\right| \quad \dfrac{10}{3}$



21.

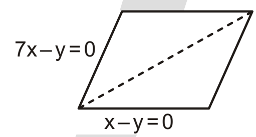


22. $y \quad \dfrac{3}{4}(x \quad 9) \quad 6$

23. Acute angle bisector is

$$\frac{7x \quad y}{\sqrt{50}} \qquad \frac{x \quad y}{\sqrt{2}}$$

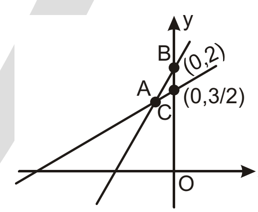
$$y \quad 2x$$



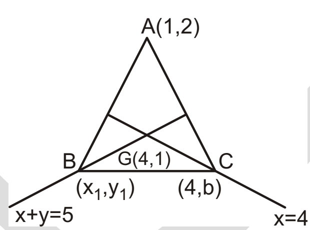
24. Either $x \quad y$ or $x \quad \left|\dfrac{3x \quad 4y \quad 12}{5}\right|$ or $y \quad \left|\dfrac{3x \quad 4y \quad 12}{5}\right| \qquad (1, 1)$

25. Co-ordinate of point $A \quad \dfrac{1}{7}, \dfrac{10}{7}$

Ar (ABC) $\dfrac{1}{2} \quad \dfrac{1}{2} \quad \dfrac{1}{7} \quad \dfrac{1}{28}$



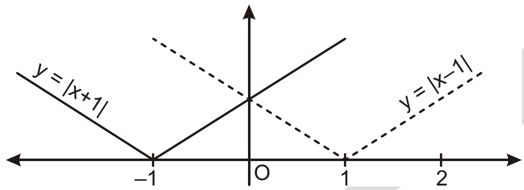
26.



Co-ordinate of centroid $G(4, 1) \quad \dfrac{x_1 \quad 4 \quad 1}{3} \quad 4$

$$x_1 \quad 7 \text{ and } y_1 \quad 2$$

27.



The image of $y \quad |x \quad 1|$ w.r.t. y-axis is $y \quad |x \quad 1| \qquad y \qquad (x \quad 1)$

Required solution $\quad (y \quad (x \quad 1))(y \quad (x \quad 1)) \quad 0$

28.

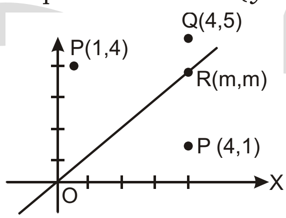
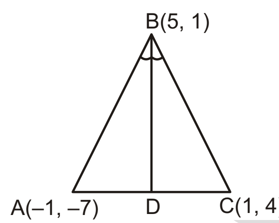


Image of $(1, 4)$ about the line $y \quad x$ is $(4, 1) \qquad P\ (4, 1)\ Q(4, 5)$ and $R(m, m)$ are collinear.

$$m \quad 4$$

29. $\dfrac{AD}{CD} \quad \dfrac{AB}{BC} \quad \dfrac{10}{5} \quad \dfrac{2}{1}$



30. $4c\ \dfrac{y}{x}^{2} \qquad \dfrac{y}{x} \quad 6 \quad 0$ has one root is $\dfrac{3}{4} \qquad c \quad 3$

33.
$$\frac{x}{a} \quad \frac{y(a \quad c)}{2ac} \quad \frac{1}{c} \quad 0$$

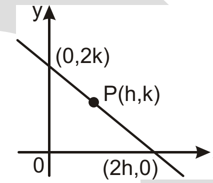
$$a(y \quad 2) \quad c(2x \quad y) \quad 0$$

Passes through a fixed point $(1, \quad 2)$

34.
$$\frac{1}{b}\ \frac{y}{x}^{2} \quad \frac{2}{h}\ \frac{y}{x} \quad \frac{1}{a} \quad 0 \begin{matrix} \nearrow m \\ \searrow 2m \end{matrix}$$

$$3m \quad \frac{2b}{h} \text{ and } 2m^2 \quad \frac{b}{a} \qquad \frac{ab}{h^2} \quad \frac{9}{8}$$

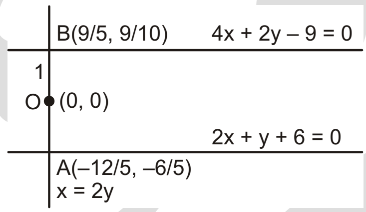
35. Equation of line is $\quad \frac{x}{2h} \quad \frac{y}{2k} \quad 1$



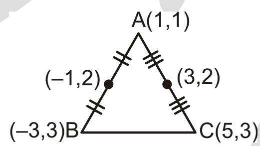
if it passes through fixed point (x_1, y_1)

$$\frac{x_1}{2h} \quad \frac{y_1}{2k} \quad 1$$

36. $OA : OB \qquad : 1 \qquad \frac{4}{3}$



37. $G \quad 1, \frac{7}{3}$



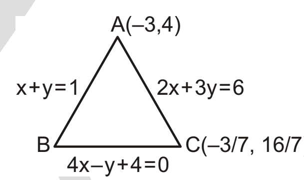
38. Diagonals are perpendicular.

39. Let point on the line $x \quad y \quad 4$ is $(a, 4 \quad a)$.

$$\left|\frac{4(a) \quad 3(4 \quad a) \quad 10}{5}\right| \quad 1 \qquad a^2 \quad 4a \quad 21 \quad 0 \begin{matrix} \nearrow a_1 \\ \searrow a_2 \end{matrix}$$

$$a_1 \quad a_2 \qquad 4 \qquad b_1 \quad b_2 \quad 12$$

40. Equation of altitude on BC



$$x \quad 4y \quad 13$$

Equation of altitude on AB

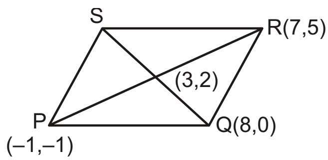
$$7x \quad 7y \quad 19 \quad 0$$

$$H \quad \frac{3}{7}, \frac{22}{7}$$

41. Equation of line is $(3x \quad 4y \quad 5) \qquad (4x \quad 6y \quad 6) \quad 0$

$$\frac{(3 \quad 4\ )}{4 \quad 6} \quad \frac{7}{5} \qquad 1 \qquad\qquad \frac{1}{2}$$

42. $\frac{5 \quad 1}{8 \quad 2} \quad \frac{7 \quad 5}{x \quad 8} \qquad x \quad 11$

43. 

$$S(\ \ 2, 4)$$

44. Area $\frac{1}{2}\begin{vmatrix} a & & a & 1 \\ a & 1 & a \ \ 1 & 1 \\ a & 2 & a & 1 \end{vmatrix} \ \ 1$

45. $(x \quad y)^2 \quad 1$

$$x \quad y \quad 1 \text{ and } x \quad y \quad 1 \quad 0$$

46. AB subtend an acute angle at point C, then

$$a^2 \quad (a \quad 1)^2 \quad 4$$

$$a \qquad , \frac{\sqrt{7} \quad 1}{2} \qquad \frac{\sqrt{7} \quad 1}{2},$$

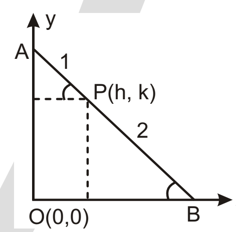
48.

$$h \quad \cos$$

$$k \quad 2\sin$$

$$h^2 \quad \frac{k^2}{4} \quad 1$$

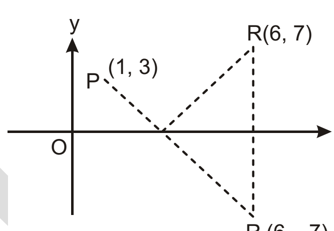
$$4x^2 \quad y^2 \quad 4$$



50. Let the point of reflection is (h, k).

$$\frac{h \quad a}{1} \quad \frac{k \quad 0}{t} \quad \frac{2(a \quad at^2)}{1 \quad t^2} \qquad x \qquad a$$

51.



52. Let (x, y) and (X, Y) be the old and the new coordinates, respectively. Since the axes are rotated in the anticlockwise direction, $\quad 60$. Therefore,

$$\begin{matrix} x & \cos 60 & \sin 60 & X \\ y & \sin 60 & \cos 60 & Y \end{matrix}$$

$$\begin{matrix} x & 1/2 & \sqrt{3}/2 & X \\ y & \sqrt{3}/2 & 1/2 & Y \end{matrix}$$

$$\begin{aligned} x &\quad \frac{X}{2} \quad \frac{\sqrt{3}}{2}Y \\ y &\quad \frac{\sqrt{3}}{2}X \quad \frac{Y}{2} \end{aligned}$$

$$x \quad \frac{X}{2} \quad \frac{\sqrt{3}}{2}Y \text{ and } y \quad \frac{\sqrt{3}}{2}X \quad \frac{Y}{2}$$

$$\frac{X}{2} \quad \frac{\sqrt{3}}{2}Y \quad ^2 \qquad \frac{\sqrt{3}}{2}X \quad \frac{Y}{2} \quad ^2 \qquad a^2$$

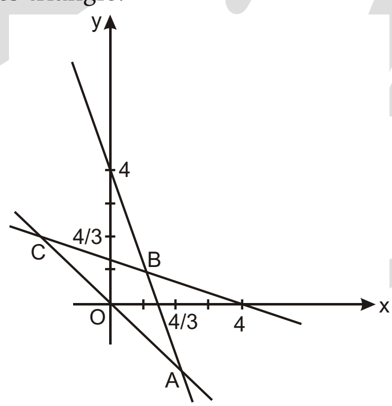
$$(X^2 \quad 3Y^2 \quad 2\sqrt{3}XY) \quad (3X^2 \quad Y^2 \quad 2\sqrt{3}XY) \quad 4a^2$$

$$2X^2 \quad 2Y^2 \quad 4\sqrt{3}XY \quad 4a^2$$

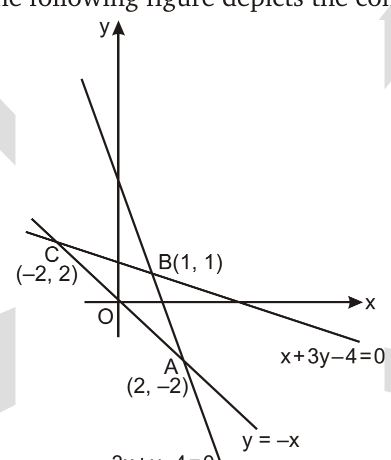
$$Y^2 \quad X^2 \quad 2\sqrt{3}XY \quad 2a^2$$

which is the required equation.

53. The following figure depicts the condition. By observation from the figure, ABC is clearly an obtuse angled and isosceles triangle.



Alternate solution : The following figure depicts the condition.



From the figure, we get

$A : 3x \quad y \quad 4$ and $y \quad x \quad x \quad 2 ; y \quad 2$

$B : (1, 1)$ by solving the equations.

$C : x \quad 3y \quad 4 \quad 0$ and $y \quad x \quad x \quad 2 ; y \quad 2$

Thus, $$AB \quad BC \quad \sqrt{1 \quad 9} \quad \sqrt{10}$$

$$AC \quad \sqrt{4^2 \quad 4^2} \quad 4\sqrt{2}$$

$$\cos B \quad \frac{10 \quad 10 \quad 16(2)}{2(\sqrt{10})(\sqrt{10})} \quad 0$$

Therefore, the given triangle is isosceles and obtuse angled triangle.

56. $\dfrac{y_2 \quad y_1}{x_2 \quad x_1} \quad \dfrac{y_3 \quad y_2}{x_3 \quad x_2}$ Points are collinear.

57. $3h \quad a\cos t \quad b\sin t \quad 1$

$3k \quad a\sin t \quad b\cos t$

$$(3h \quad 1)^2 \quad (3k)^2 \quad (a\cos t \quad b\sin t)^2 \quad (a\sin t \quad b\cos t)^2 \quad a^2 \quad b^2$$

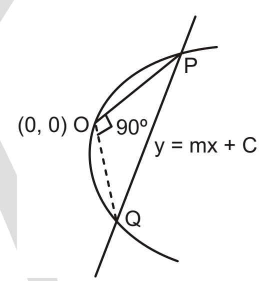
58. Equation of line $\dfrac{x}{a} \quad \dfrac{y}{1 \quad a} \quad 1.$

Lines passes from (4, 3).

62. The given triangle is equilateral. Therefore, the orthocentre of the triangle is same as centroid of the triangle. Thus, the orthocentre, that is, the centroid is given by

$$\frac{5 \quad 0 \quad (5/2)}{3}, \frac{0 \quad 0 \quad (5\sqrt{3}/2)}{3} \quad \frac{5}{2}, \frac{5}{2\sqrt{3}}$$

63. Using homogenization,



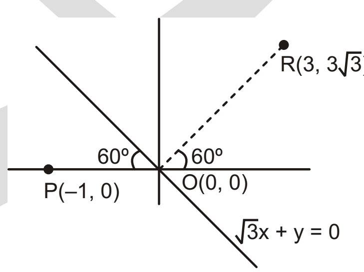
$$3x^2 \quad y^2 \quad 2x \frac{y \quad mx}{C} \quad 4y \frac{y \quad mx}{C} \quad 0$$

$$\text{Coefficient of } x^2 \quad \text{Coefficient of } y^2 \quad 0$$

$$3 \quad \frac{2m}{C} \quad 1 \quad \frac{4}{C} \quad 0$$

$$C \quad m \quad 2$$

64.



Exercise-2 : One or More than One Answer is/are Correct

1. Let line be $\frac{x}{a} \quad \frac{y}{b} \quad 1$

$$a \quad b \quad 9 \quad \text{and} \quad ab \quad 20$$

$$a \quad 5,\, b \quad 4 \quad \text{or} \quad a \quad 4,\, b \quad 5$$

2. Centroid is $\;4, \frac{4}{3}\;$.

3. $\begin{vmatrix} 2 & 3 & 5 \\ t^2 & t & 6 \\ 3 & 2 & 1 \end{vmatrix} \quad 0 \qquad t^2 \quad t \quad 6 \quad 0$

4.
$$b \quad \frac{y}{x}^{2} \quad 6 \quad \frac{y}{x} \quad a \quad 0$$

$$bm^2 \quad 6m \quad a \quad 0$$

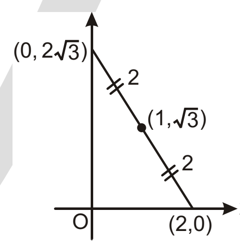
if $\; m \quad 1$ is root of the equation

$$a \quad b \quad 6$$

if $\; m \quad 1$ is root of the equation

$$a \quad b \quad 6$$

6. Co-ordinate of other two points

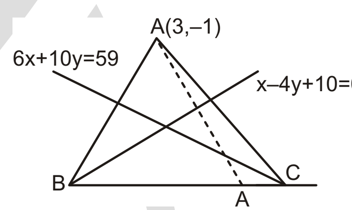


$$(1 \quad 2\cos \quad ,\, \sqrt{3} \quad 2\sin \quad)$$

$$1 \quad 2\,\frac{\sqrt{3}}{2},\, \sqrt{3} \quad 2\,\frac{1}{2}$$

$$(1 \quad \sqrt{3}, \sqrt{3} \quad 1) \text{ and } (1 \quad \sqrt{3}, \sqrt{3} \quad 1)$$

8. Image of $A(3, \quad 1)$ about angle bisector $x \quad 4y \quad 10 \quad 0$ is $A\ (a, b)$.



$$\frac{a \quad 3}{1} \quad \frac{b \quad 1}{4} \quad \frac{2(3 \quad 4 \quad 10)}{17}$$

$$A\ (1, 7)$$

Let point $B \quad x_1, \frac{x_1 \quad 10}{4}$ on the line $x \quad 4y \quad 10 \quad 0$

If mid-point of AB lie on the line $6x \quad 10y \quad 59$

$$6\ \frac{x_1 \quad 3}{2} \quad 10\ \frac{x_1 \quad 10 \quad 4}{8} \quad 59$$

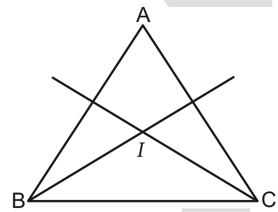
$$B(10, 5)$$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 3 to 4

3. $x \quad y \quad 2$ and $x \quad 3y \quad 6$

Meet at $(3, \quad 1)$



4. Image of $A(2, \quad 4)$ about $x \quad y \quad 2$ lie on BC.

$$\frac{x_2 \quad 2}{1} \quad \frac{y_2 \quad 4}{1} \quad 2 \; \frac{4}{2}$$

$$x_2 \quad 6, y_2 \quad 0$$

Image of $A(2, \quad 4)$ about $x \quad 3y \quad 6$ lie on BC.

$$\frac{x_3 \quad 2}{1} \quad \frac{y_3 \quad 4}{3} \quad 2\,\frac{8}{10}$$

$$x_3 \quad \frac{2}{5}, y_3 \quad \frac{4}{5}$$

Equ. of line $BC, \quad x \quad 7y \quad 6$

$$B \quad \frac{4}{3}, \frac{2}{3} \quad \text{and } C(6, 0)$$

Exercise-4 : Matching Type Problems

2. (A) $\overset{n \; 1}{\underset{r \; 1}{}} \; ({}^1C_{r \; 1} \quad {}^2C_{r \; 1} \quad {}^3C_{r \; 1} \qquad {}^nC_{r \; 1})$

$$\overset{n \; 1}{\underset{r \; 1}{}} \; {}^1C_{r \; 1} \qquad \overset{n \; 1}{\underset{r \; 1}{}} \; {}^2C_{r \; 1} \qquad \overset{n \; 1}{\underset{r \; 1}{}} \; {}^3C_{r \; 1} \qquad\qquad \overset{n \; 1}{\underset{r \; 1}{}} \; {}^nC_{r \; 1}$$

$$2^1 \quad 2^2 \quad 2^3 \qquad 2^n \quad 2(2^n \quad 1)$$

(B) Family of line $(x \quad y \quad 2) \quad (2x \quad y \quad 4) \quad 0$ always passes from $(\; 2, 0)$.

If almost one tangent can be drawn from $(\; 2, 0)$ then

$$S_1 \quad 4 \quad 8g \quad 36 \quad 4g^2 \quad 0$$

$$g^2 \quad 2g \quad 8 \quad 0$$

(C) $2\sin 7x \; \cos 2x \quad \cos 2x$

$$\cos 2x \quad 0 \quad \text{or} \quad \sin 7x \quad \frac{1}{2}$$

$$x \quad \frac{\;}{4}, \frac{3}{4} \qquad 7x \quad \frac{\;}{6}, \frac{5}{6}, \frac{13}{6}, \frac{17}{6}, \frac{25}{6}, \frac{29}{6}, \frac{37}{6}, \frac{41}{6}$$

(D) $a \quad b \quad \tan 65 \;\; \tan 70 \quad \tan 65 \quad \tan 70$

$$\tan 135 \quad \frac{\tan 65 \quad \tan 70}{1 \quad \tan 65 \quad \tan 70} \quad 1$$

$$\tan 65 \ \tan 70 \quad \tan 65 \quad \tan 70 \quad 1$$

3. (A) $\cos 40 \quad 2\cos 40 \ \sin 10 \quad \cos 40 \quad (\sin 50 \quad \sin 30\ )$

(B) $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 4 \\ 1 & 1 & 3 \end{vmatrix} \quad 0 \qquad (3 \quad 2\ )(1 \quad 3\ ) \quad 0$

(C) $\begin{vmatrix} k & 2 \quad 2k & 1 \\ k \quad 1 & 2k & 1 \\ 4 \quad k & 6 \quad 2k & 1 \end{vmatrix} \quad 0 \qquad 2k^2 \quad k \quad 1 \quad 0$

$$k \quad 1, \frac{1}{2}$$

(D) $\sum_{k \ 3} \frac{1}{2}^{k} \quad \frac{1}{2}^{3} \quad \frac{1}{2}^{4} \quad \frac{1}{2}^{5} \qquad \frac{1}{4}$

Exercise-5 : Subjective Type Problems

1. 132

2. $ax \quad by \quad c \quad 3x \quad 4y \quad c$

$$a \quad 3, b \quad 4$$

Distance of $3x \quad 4y \quad c$ from $A(3,1)$ is 1.

$$\frac{|9 \quad 4 \quad c|}{5} \quad 1$$

$$|c \quad 5| \quad 5$$

Also, $3x \quad 4y \quad c \quad 0$ and $3x \quad 4y \quad 5 \quad 0$ lie on same side of A

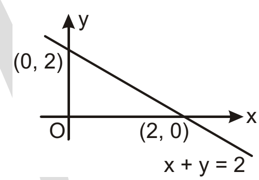
$$c \quad 5 \quad 0$$

$$c \quad 5 \quad 5 \qquad c \quad 0$$

3. $xy(x \quad y \quad 2) \quad 0$

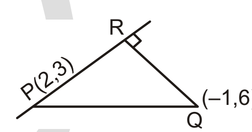
$$^{4} \quad 2 \quad 0 \qquad (\quad 0)$$

$$1$$



5. $PQ \quad 3\sqrt{2}$

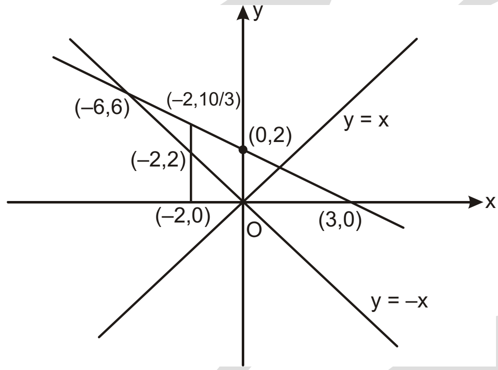
$QR \quad PQ$



6. $x^2(y^2 \quad x^2) \quad 0$

has 3 different lines $x \quad 0, y \quad x$ and $y \quad x$.

8. $2 \quad a \quad \frac{10}{3}$



9. Describe a circle whose diameter is *AB*.

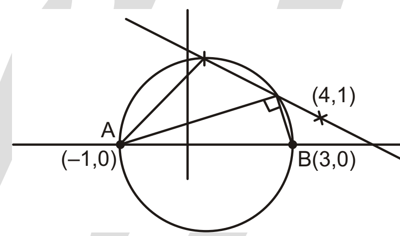
centre $\quad (1,0)$

Radius $\quad 2$

Let '*m*' the slope of the line passing through $(4,1)$.

$(y \quad 1) \quad m(x \quad 4)$ intersect the circle

distance from centre $\quad$ radius of circle.



$$\left|\frac{3m \quad 1}{\sqrt{m^2 \quad 1}}\right| \quad 2$$

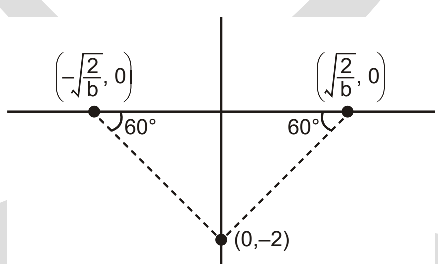
$$9m^2 \quad 6m \quad 1 \quad 4m^2 \quad 4$$

$$m \quad \frac{6 \quad \sqrt{96}}{10}, \frac{6 \quad \sqrt{96}}{10} \quad \frac{1}{5}, 1$$

$$_1 \quad _2 \quad \frac{12}{10} \quad \frac{6}{5}$$

$$5(_1 \quad _2) \quad 6$$

10. $\sqrt{\frac{2}{b}} \quad \frac{2}{\sqrt{3}} \qquad b \quad \frac{3}{2}$

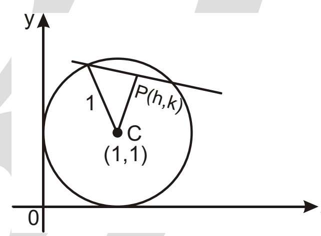


18

CIRCLE

Exercise-1 : Single Choice Problems

1. $CP \quad \frac{\sqrt{3}}{2} \quad \sqrt{(h \quad 1)^2 \quad (k \quad 1)^2}$



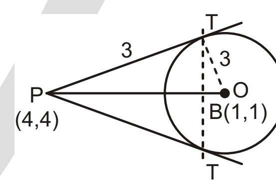
Locus of point $P(h, k)$ is

$$(x \quad 1)^2 \quad (y \quad 1)^2 \quad \frac{3}{4}$$

2. $\sqrt{d^2 \quad (r_1 \quad r_2)^2} \quad 15; \quad \sqrt{d^2 \quad (r_1 \quad r_2)^2} \quad 5 \qquad r_1 r_2 \quad 50$

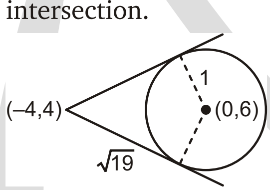
4. $PT \quad \sqrt{16 \quad 16 \quad 8 \quad 8 \quad 7} \quad 3$

$$TT \quad 2BT \quad 2 \;\; 3\cos 45 \quad 3\sqrt{2}$$



5. It will be circle with diametric ends as $(1, 1)$ and $(4, 2)$ *i.e.*, point of intersection.

6.



8. Let centroid be (h, k).

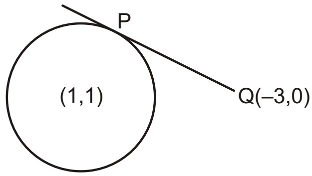
$$h \quad \frac{\cos \qquad \sin \qquad 1}{3}, \quad k \quad \frac{\sin \qquad \cos \qquad 2}{3}$$

$$3h \quad 1 \quad \cos \qquad \sin \quad , \quad 3k \quad 2 \quad \sin \qquad \cos$$

$$(3h \quad 1)^2 \quad (3k \quad 2)^2 \quad 2$$

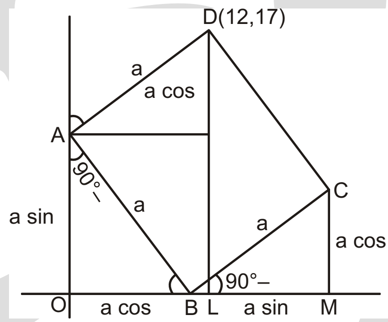
$$x \quad \frac{1}{3}^{2} \qquad y \quad \frac{2}{3}^{2} \quad \frac{2}{9}$$

9.



Length of tangent $PQ \quad \sqrt{4^2 \quad 1^2 \quad 5} \quad \sqrt{12}$

10.

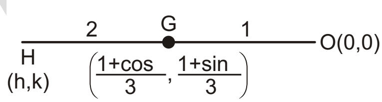


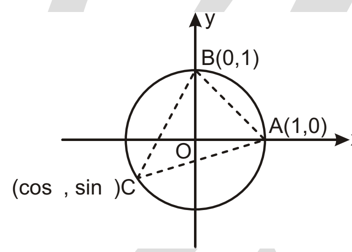
$OA \quad a\sin \quad 12, \quad DL \quad a\sin \quad a\cos \quad 17$

$$a\cos \quad 5$$

$$C \quad (a\cos \quad a\sin \ , a\cos\ ) \quad (17, 5)$$

12. Centroid divide the line joining orthocentre and circumcentre in 2 : 1.





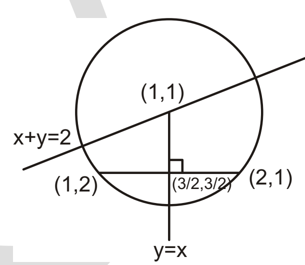
$$h \quad 1 \quad \cos \ , \quad k \quad 1 \quad \sin$$

$$(x \quad 1)^2 \quad (y \quad 1)^2 \quad 1$$

13. Co-ordinate of centre is $C(1, 1)$.

$$(x \quad 1)^2 \quad (y \quad 1)^2 \quad 1$$

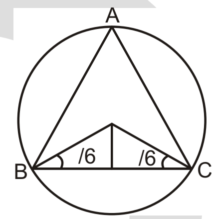
$$x^2 \quad y^2 \quad 2x \quad 2y \quad 1 \quad 0$$



14. $a \quad 2R\cos\frac{}{6}$

$a \quad 4\sqrt{3}$ cm

Area of $ABC \quad \frac{\sqrt{3}}{4}a^2 \quad 12\sqrt{3}$ cm^2



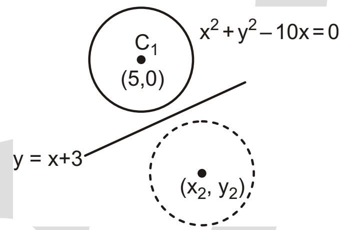
15. Image of centre $C_1(5,0)$ about the line $y \quad x \quad 3$ is

$$\frac{x_2 \quad 5}{1} \quad \frac{y_2 \quad 0}{1} \quad \frac{2(5 \quad 3)}{1^2 \quad 1^2}$$

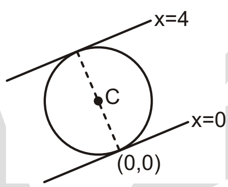
$C_2(\quad 3, 8)$

Equation of reflected circle is

$$(x \quad 3)^2 \quad (y \quad 8)^2 \quad 25$$



19.



20. Let the equation of line is $3x \quad 4y \quad C$

$$\left|\frac{C}{5}\right| \quad 3 \qquad C \quad 15 \text{ (in first quadrant)}$$

21. $C_1(5,0), C_2(3, \quad 1), C_3(3/2, 2)$ do not lie on straight line.

22. Let equation of diameter is $3x \quad 5y \quad C$

$C \quad 7$

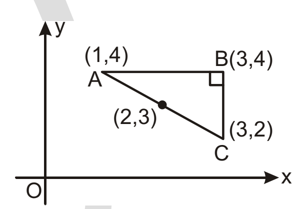
23. Equation of circle is

$$(x \quad 1)(x \quad 2) \quad (y \quad 2)(y \quad 3) \quad (x \quad 3y \quad 7) \quad 0$$

If its radius is $\sqrt{5}$.

1

25. Equation of circle is $(x \quad 1)(x \quad 3) \quad (y \quad 4)(y \quad 2) \quad 0$

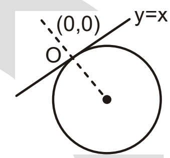


26. Equation of tangent at $O(0,0)$.

$$x(0) \quad y(0) \quad g(x \quad 0) \quad f(y \quad 0) \quad 0$$

$$gx \quad fy \quad 0$$

27. Equation of normal at $O(0,0)$



$$y \quad x$$

$$\text{Centre} \quad 0 \qquad \frac{1}{\sqrt{2}}, 0 \qquad \frac{1}{\sqrt{2}}$$

$$\text{Either} \quad \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \quad \text{or} \quad \frac{1}{\sqrt{2}}, \quad \frac{1}{\sqrt{2}}$$

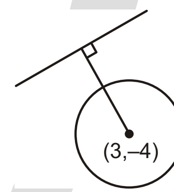
28. Here, $\quad C_1C_2 \quad r_1 \quad r_2$ (Condition for external touch)

30. The triangle is right angled and the radical centre will be the orthocentre of the triangle.

32. Equation of common chord is $6x \quad 14y \quad (l \quad m) \quad 0$

If it passes through $(1, \quad 4)$. Then, $l \quad m \quad 50$

33. $\qquad x^2 \quad y^2 \quad 6x \quad 8y \quad 0$

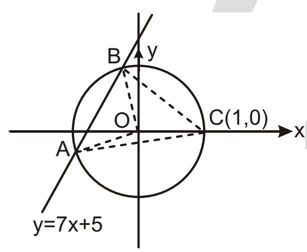


Distance of line from centre

$$\left|\frac{9 \quad 16 \quad 25}{5}\right| \quad \frac{32}{5}$$

$$\text{Shortest distance} \quad \frac{32}{5} \quad 5 \quad \frac{7}{5}$$

34. $\quad AOB \quad \frac{}{2} \qquad ACB \quad \frac{}{4}$



37. Equation of required circle :

$S:(x \quad 1)^2 \quad (y \quad 1)^2 \quad (x \quad y) \quad 0$

$S \; : x^2 \quad y^2 \quad 2y \quad 3 \quad 0$

Common chord of $S \quad 0$ and $S \quad 0$ is $S \quad S \quad 0$

$$(\quad 2)x \quad (\quad 4)y \quad 5 \quad 0$$

Centre of $S \; : (0, \quad 1)$ lies on common chord $\qquad 9$

$S:(x \quad 1)^2 \quad (y \quad 1)^2 \quad 9(x \quad y) \quad 0$

$$r \quad \frac{9}{\sqrt{2}}$$

40. Point lie inside the circle $k^2 \quad (k \quad 2)^2 \quad 4 \quad 2k^2 \quad 4k \quad 0; \; 2 \quad k \quad 0$

41. The length of the normal is

$$\left| y\sqrt{1 \quad \left(\frac{dy}{dx}\right)^2} \right|$$

The length of radius vector of a point (x, y) on the curve is $|xi \quad yj|$, that is $\sqrt{x^2 \quad y^2}$, it is given that

$$\sqrt{x^2 \quad y^2} \quad |y|\sqrt{1 \quad (y')^2}$$

Squaring on both sides of this equation, we get

$$x^2 \quad y^2 \quad y^2[1 \quad (y')^2]$$

$$x^2 \quad y^2 \quad y^2 \quad y^2\left(\frac{dy}{dx}\right)^2$$

$$x^2 \quad \left(y\frac{dy}{dx}\right)^2$$

$$y\frac{dy}{dx} \quad x \text{ or } y\frac{dy}{dx} \quad x$$

Now, $$y\frac{dy}{dx} \quad x$$

$$y\,dy \quad x\,dx$$

Integrating on both sides, we get

$$\frac{y^2}{2} \quad \frac{x^2}{2} \quad c$$

$$x^2 \quad y^2 \quad 2c \text{ or } x^2 \quad y^2 \quad \text{constant}$$

This answer does not exist in the given options. So, consider the other alternative.

$$y\,dy \quad x\,dx$$

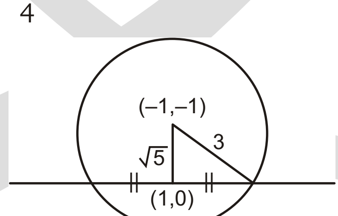
Integrating on both sides, we get

$$\frac{y^2}{2} \quad \frac{x^2}{2} \quad c$$

$$x^2 \quad y^2 \quad \text{constant}$$

and this constant is 0 in practical sense.

44. Length of chord $2\sqrt{3^2 \quad 5} \quad 4$



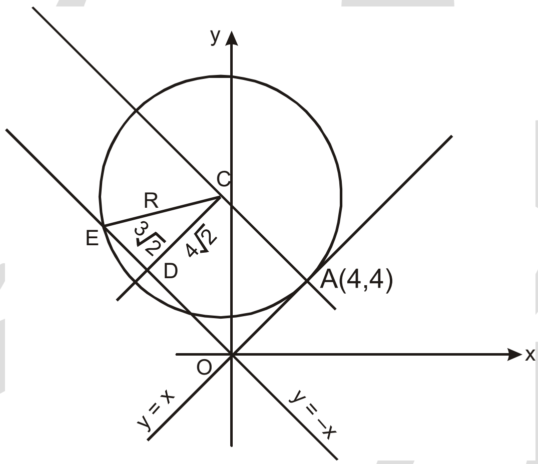
47. Family of circles touching the line $y \quad x$ at the point $(4, 4)$ is

$$(x \quad 4)^2 \quad (y \quad 4)^2 \quad (y \quad x) \quad 0$$

We need to find the member of this family which has length of chord $\quad 6\sqrt{2}$ on $x \quad y \quad 0$. For different $\quad$'s, we get different circles.

$$x^2 \quad y^2 \quad 8x \quad 8y \quad 32 \quad y \quad x \quad 0$$

$$x^2 \quad y^2 \quad x(\ 8 \quad) \quad y(\ 8 \quad) \quad 32 \quad 0 \qquad \ldots(1)$$



Now,

$$OA \quad DC \quad 4\sqrt{2}$$

$$DE \quad 3\sqrt{2} \quad \frac{6\sqrt{2}}{2} \text{ (given)}$$

Therefore, $\quad R^2 \quad (3\sqrt{2})^2 \quad (4\sqrt{2})^2$

$$\frac{\ ^2}{2} \quad 50 \qquad ^2 \quad 100 \qquad\qquad 10$$

Substituting $\quad 10$ in eq. (1), we get

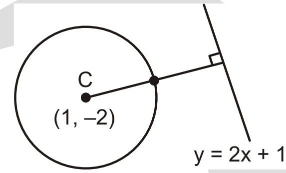
$$x^2 \quad y^2 \quad 2x \quad 18y \quad 32 \quad 0$$

[Substituting $\quad 10$; in eq. (1); we get $x^2 \quad y^2 \quad 18x \quad 2y \quad 32 \quad 0$, which does not exist in the given options]

Note : From eq. (1), we get

$$R^2 \quad (\text{Radius})^2 \quad g^2 \quad f^2 \quad c \quad \frac{(\quad 8)^2}{4} \quad \frac{(\quad 8)^2}{4} \quad 32 \quad \frac{\ ^2}{2}$$

48. Slope of line normal to circle and perpendicular to line



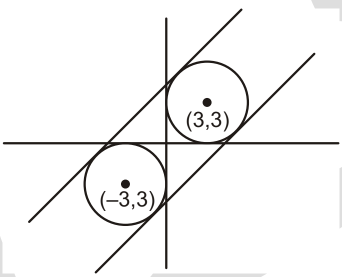
$$m \quad \frac{1}{2} \quad \tan$$

Co-ordinate of point lie on normal at a dist. of 3 from centre

$$1 \quad 3 \quad \frac{2}{\sqrt{5}}, \quad 2 \quad 3 \quad \frac{1}{\sqrt{5}}$$

Exercise-2 : One or More than One Answer is/are Correct

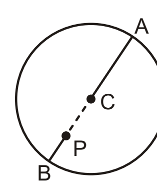
2.



3. $x^2 \quad y^2 \quad x \quad \overline{2} \quad y \quad \overline{2} \quad 2\sin^{1} \quad 0$

$$\text{Length of chord} \quad 2\sqrt{\frac{}{4}^{2} \quad \frac{}{4} \quad \sin^{1}{}^{2}}$$

7. $(x \quad 2)^2 \quad (y \quad 3)^2$ is nothing but square of distance between (x, y) and $(\quad 2, 3)$ where (x, y) is point lies on the circle.



Centre $(\quad 4, 5)$, $\qquad r \quad \sqrt{16 \quad 25 \quad 40} \quad 9$

Clearly, $(\quad 2, 3)$ is lies inside the circle.

$PC \quad 2\sqrt{2} \qquad a \quad PA^2 \quad (9 \quad 2\sqrt{2})^2$

$b \quad PB^2 \quad (9 \quad 2\sqrt{2})^2$

$a \quad b \quad 178, \quad a \quad b \quad 72\sqrt{2}$

8. Let point of intersection $P(h, k)$

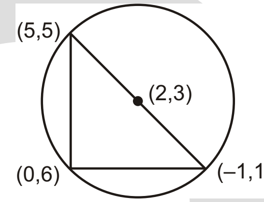
Equation of chord of contact is

$$hx \quad ky \quad a^2$$

If it is tangent to $x^2 \quad y^2 \quad 2ax \quad 0$

$$\left|\frac{ha \quad a^2}{\sqrt{h^2 \quad k^2}}\right| \quad a$$

9. Equation of tangent to circle



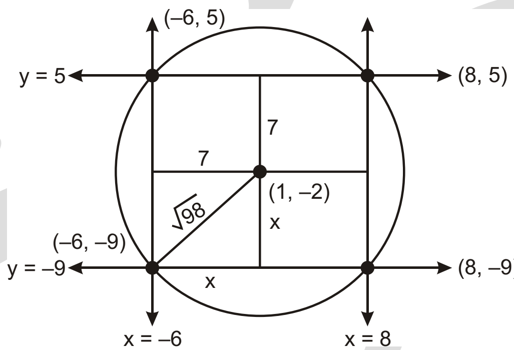
$$y \quad 3 \quad \frac{3}{2}(x \quad 2) \quad \sqrt{13}\sqrt{1 \quad \frac{9}{4}}$$

$$2y \quad 3x \quad 13, \; 2y \quad 3x \quad 13$$

$$\frac{x_2 \quad 2}{3} \quad \frac{y_2 \quad 3}{2} \quad \frac{13}{13} \qquad (\quad 1, 5)$$

$$\frac{x_3 \quad 2}{3} \quad \frac{y_3 \quad 3}{2} \quad \frac{13}{13} \qquad (5, 1)$$

10. $2x^2 \quad 98 \quad x^2 \quad 49 \quad x \quad 7$



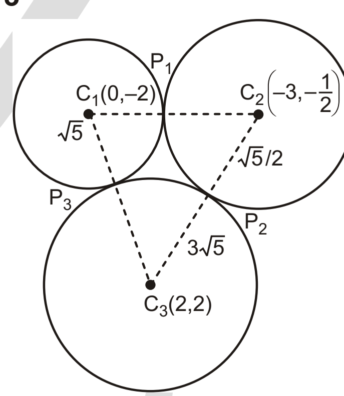
Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

Sol. $P_1(\quad 2, \quad 1)$

$P_2 \quad \frac{16}{7}, \quad \frac{1}{7}$

$P_3 \quad \frac{1}{2}, \quad 1$



Paragraph for Question Nos. 4 to 6

4. $S : x^2 \quad y^2 \quad x(2 \quad 9) \quad y(3 \quad 12) \quad 53 \quad 27 \quad 0$

$C : x^2 \quad y^2 \quad 4x \quad 6y \quad 3 \quad 0$

Equation of line : $S \quad C \quad 0$

or $x(2 \quad 5) \quad y(3 \quad 6) \quad 56 \quad 27 \quad 0$

or $5x \quad 6y \quad 56 \quad 0$ or $2x \quad 3y \quad 27 \quad 0$

$$x \quad 2, \quad y \quad \frac{23}{3}$$

5. Centre of C lies on common chord of S and C.

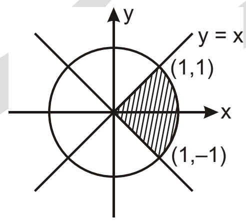
$(2, 3)$ lies on $x(2 \quad 5) \quad y(3 \quad 6) \quad 56 \quad 27 \quad 0$

$S : x^2 \quad y^2 \quad 5x \quad 6y \quad 1 \quad 0$

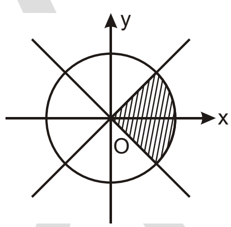
6. Difference of squares of lengths of tangents from A and B is 3, which is equal to $|AP^2 \quad BP^2|$.

Paragraph for Question Nos. 7 to 8

7. Max. dist. between any two arbitrary points $\quad 2$



8.



Paragraph for Question Nos. 9 to 10

Sol. Let $P(h, k)$

$L_1 \quad \sqrt{h^2 \quad k^2 \quad 4}$

$L_2 \quad \sqrt{h^2 \quad k^2 \quad 4h}$

$L_3 \quad \sqrt{h^2 \quad k^2 \quad 4k}$

If $\quad L_1^4 \quad L_2^2 L_3^2 \quad 16$

$(h^2 \quad k^2 \quad 4)^2 \quad (h^2 \quad k^2 \quad 4h)(h^2 \quad k^2 \quad 4k) \quad 16$

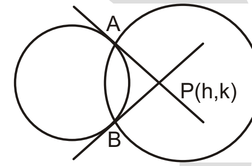
$(h \quad k)(h^2 \quad k^2 \quad 2h \quad 2k) \quad 0$

$C_1 : x \quad y \quad 0$

$C_2 : x^2 \quad y^2 \quad 2x \quad 2y \quad 0$

Exercise-5 : Subjective Type Problems

1. Equation of chord of contact w.r.t. P



$$hx \quad ky \quad 1$$

Equation of common chord is

$$(\quad 3)x \quad (2 \quad 2)y \quad 3 \quad 0$$

$$\frac{3}{h} \quad \frac{2 \quad 2}{k} \quad 3$$

Equation of locus is $6x \quad 3y \quad 8 \quad 0$

2. By using system of circles any circle passing through $(1, 1)$ and $(\quad 2, 1)$ is

$$(x \quad 1)(x \quad 2) \quad (y \quad 1)^2 \quad (y \quad 1) \quad 0 \qquad \ldots(1)$$

Given circles $\qquad x^2 \quad y^2 \quad 1 \quad 0 \qquad \ldots(2)$

Now radical axis of (1) and (2) is

$$(x \quad 2y) \quad (y \quad 1) \quad 0 \qquad \ldots(3)$$

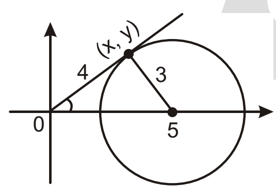
$\because$ Radical centre of given circles is $(0,0)$.

So, eq. (3) is passing through $(0,0)$.

$$0$$

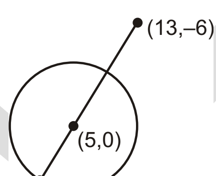
Put $\quad 0$ in eq. (1) we get required circle.

3. $\frac{y}{x} \quad \tan \quad \frac{3}{4}$



4. $x^2 \quad y^2 \quad 26x \quad 12y \quad 210$

$(x \quad 13)^2 \quad (y \quad 6)^2 \quad 5$



5. $\qquad S \quad x^2 \quad y^2 \quad 2gx \quad 2fy \quad c \quad 0$

$$2g \quad 2f \quad c \quad 2 \qquad \ldots(1)$$

$(1, 1)$ satisfy circle.

$$2g \quad 2f \quad c \quad 2$$

$$c \quad 0$$

and $g \quad f \quad 1$

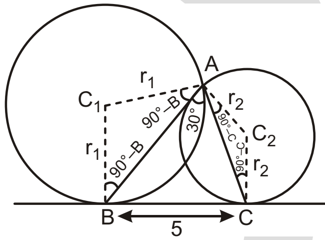
Length of tangent $\sqrt{8 \quad 4g \quad 4f \quad c} \quad 2$

6. Length of common external tangent

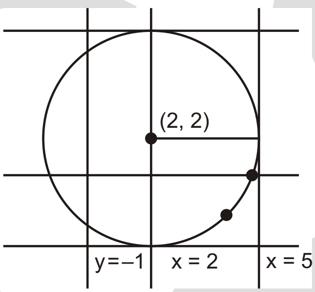
$$\sqrt{d^2 \quad (r_1 \quad r_2)^2} \quad 5 \qquad \ldots(1)$$

$$\cos(90 \quad B \quad 90 \quad C \quad 30\ ) \quad \cos 60$$

$$\frac{r_1^2 \quad r_2^2 \quad d^2}{2r_1 r_2} \qquad \ldots(2)$$



9.

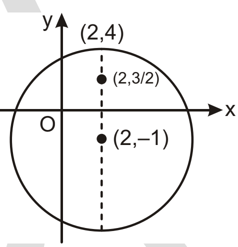


From diagram common points are 3.

10. $(C_1C_2)^2 \quad r_1^2 \quad r_2^2$

$$18 \quad 2r^2 \qquad r^2 \quad 9$$

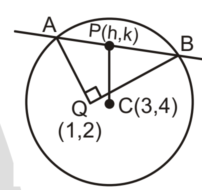
11.



12. $PQ \quad PA \quad PB$

$$\sqrt{(h \quad 1)^2 \quad (k \quad 2)^2} \quad \sqrt{6^2 \quad (h \quad 3)^2 \quad (k \quad 4)^2}$$

$$h^2 \quad k^2 \quad 4h \quad 6k \quad 3 \quad 0$$



13. $c \quad 3, a^2 \quad b^2 \quad 36$

Length of chord $AB \quad 2\sqrt{r^2 \quad p^2}$

$$2\sqrt{c \qquad \frac{2c}{\sqrt{a^2 \quad b^2}}^{\,2}} \qquad 2\sqrt{2}$$

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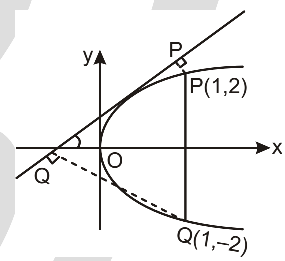
PARABOLA

Exercise-1 : Single Choice Problems

1. $PQ \quad PQ\cos(90 \quad)$

$$\frac{4}{\sqrt{t^2 \quad 1}}(t^2 \quad 1)$$

$(PQ)_{\min} \quad 2\sqrt{2}$



2. Equation of circle with SP as diameter

$$(x \quad 4)\ x \quad \frac{9}{4} \qquad y(y \quad 6) \quad 0$$

$$\text{Centre} \quad \frac{25}{8}, 3 \quad \text{and radius} \quad \frac{25}{8}$$

Equation of normal at $P(4, 6)$ is

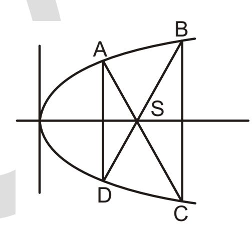
$$4x \quad 3y \quad 34$$

$$\text{Length of chord} \quad 2\sqrt{\frac{25}{8}^2 \quad \frac{\frac{25}{8} \quad 9 \quad 34}{5}^2} \qquad \frac{15}{4}$$

3. The diagonals are the focal chord.

$$AS \quad 1 \quad t^2 \quad c \text{ (say)}$$

$$\frac{1}{c} \quad \frac{1}{\frac{25}{4} \quad c} \quad 1 \qquad \because \frac{1}{AS} \quad \frac{1}{CS} \quad \frac{1}{a}$$



$$c \quad \frac{5}{4}, 5$$

$A\ \frac{1}{4}, 1\ , B(4,4), C(4,\quad 4)$ and $D\ \frac{1}{4},\quad 1$

Area of trapzium $\ \frac{1}{2}(2\quad 8)\ \frac{15}{4}$

4. For normal chord $t_2 \qquad t_1 \ \frac{2}{t_1}$

Also chord substends an angle of 90° at the vertex

$$t_1 t_2 \qquad 4 \qquad\qquad t_2^2 \quad 8$$

9. $(y \quad x \quad 2) \quad (y \quad x \quad 2) \quad 0$

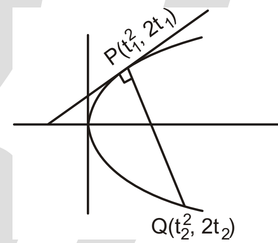
The family of lines passes through $(2, 0)$.

The chord is $x \quad 2$ and end points are $(2,\quad 4)$.

10.

$$t_2 \qquad t_1 \ \frac{2}{t_1}$$

$$h \ \frac{t_1^2 \quad t_2^2}{2} \text{ and } k \ \frac{2t_1 \quad 2t_2}{2}$$



Put the value of t_2 and eliminate t_1 we get

$$h \quad 2 \ \frac{4}{k^2} \ \frac{k^2}{2} \qquad a \quad 2, b \quad 4, c \quad 2$$

11. The parabola is $(y \quad 1)^2 \quad 4(x \quad 1)$. The coordinates of $P(1 \quad t_1^2, 1 \quad 2t_1)$ and $Q(1 \quad t_2^2, 1 \quad 2t_2)$.

Here $S(2, 1)$ is the focus. The coordinates of T are G.M. of abscissa and A.M. of ordinates of P and Q.

$$ST^2 \quad 16 \qquad\qquad SP \ SQ \quad ST^2$$

12. Let $P(t_1)$ and $Q(t_2)$ are point of $y^2 \quad 8x$

$$2t_1^2 \quad 2t_2^2 \quad 17 \text{ and } (2t_1^2)(2t_2^2) \quad 11$$

$$ST^2 \quad SP \ SQ \quad 2(1 \quad t_1^2)\, 2(1 \quad t_2^2) \quad 34 \quad 4 \quad 11$$

$$ST \quad \sqrt{49}$$

13. $ay \quad x^2 \qquad \frac{1}{\frac{dy}{dx}} \ \frac{a}{2x_1} \ \frac{A}{B}$ (slope of normal)

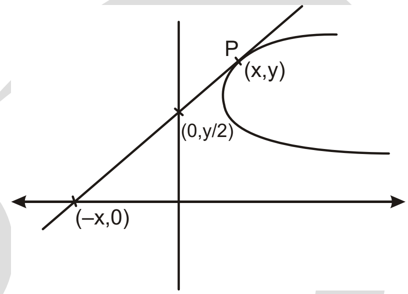
$x_1 \ \frac{aB}{2A}$ and $y_1 \ \frac{1}{B} \ \frac{a}{2}$ put (x_1, y_1) in $ay \quad x^2$

14. $$\frac{dy}{dx} \quad \frac{y}{2x}$$

$$\frac{2dy}{y} \quad \frac{1}{x}dx$$

$$2\log y \quad \log x \quad \log c$$

$$y^2 \quad cx \quad \text{put } (3,1)$$



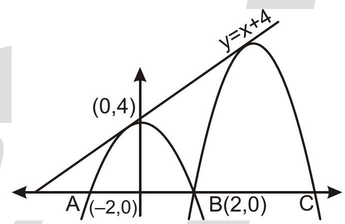
15. $$(x \quad)^2 \quad (y \quad (\quad 4))$$

The curve passes through $(2,0)$

$$(2 \quad)^2 \quad (0 \quad (\quad 4))$$

$$^2 \quad 5 \quad 0 \qquad 0 \text{ or } \quad 5$$

$$(x \quad 5)^2 \quad (y \quad 9) \quad \text{put } y \quad 0 \qquad x \quad 2, 8$$

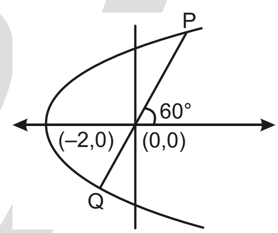


16. $y \quad (\tan 60 \quad) x$ is the focal chord.

Coordinates of P and Q are intersection of $y \quad \sqrt{3}\,x$ with parabola

$$P(4, 4\sqrt{3}),\ Q \quad \frac{4}{3}, \frac{4}{\sqrt{3}}$$

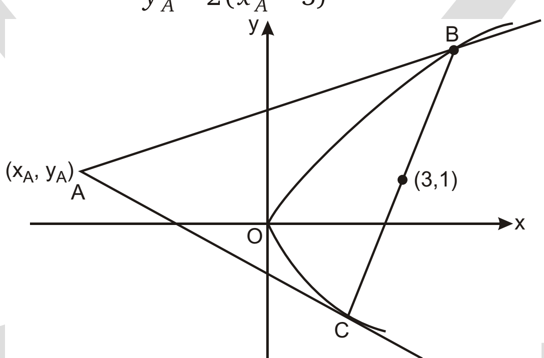
Find bisector of PQ.



17. The director circle of the parabola is its directrix $(x \quad 11 \quad 0)$. Now apply condition of tangency.

18. The following figure depicts the condition. Chord of contact of a point $A(x_A, y_A)$ with respect to $y^2 \quad 4x$ is $y_A y \quad 2(x \quad x_A)$. Since this chord passes through the point (3, 1), we have

$$y_A \quad 2(x_A \quad 3)$$



AB and AC are tangents to the parabola.

BC is chord of contact of point A with respect to the parabola $y^2 \quad 4ax$.

Given that point A lies on $x^2 \quad y^2 \quad 25$, we have

$$x_A^2 \quad y_A^2 \quad 25$$

$$x_A^2 \quad 4(x_A \quad 3)^2 \quad 25$$

$$x_A^2 \quad 4(x_A^2 \quad 9 \quad 6x_A) \quad 25$$

$$5x_A^2 \quad 24x_A \quad 36 \quad 25 \quad 0$$

$$5x_A^2 \quad 24x_A \quad 11 \quad 0$$

Exercise-2 : One or More than One Answer is/are Correct

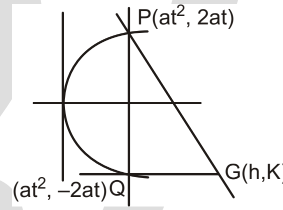
1. Equation of normal at $P(at^2, 2at)$ is

$$y \quad tx \quad 2at \quad at^3$$

$$G(4a \quad at^2, \quad 2at)$$

Locus of point $G(h, K)$ is

$$y^2 \quad 4a(x \quad 4a)$$



Exercise-3 : Comprehension Type Problems

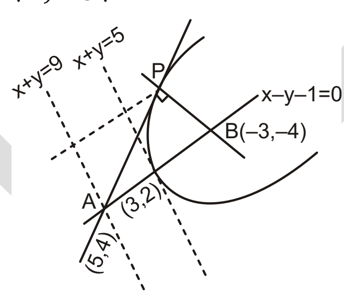
Paragraph for Question Nos. 1 to 3

Sol. Tangent and normal are angle bisectors of focal radius and perpendicular to directrix.

Circle 'C' circumscribing $\quad ABP$ is

$$(x \quad 5)(x \quad 3) \quad (y \quad 4)(y \quad 4) \quad 0$$

Length of latus rectum $\quad 4(2\sqrt{2}) \quad 8\sqrt{2}$

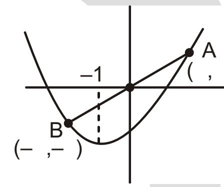


Exercise-5 : Subjective Type Problems

1.

$$2 \quad ^2 \quad 4 \quad 2 \qquad \ldots(1)$$

$$2 \quad ^2 \quad 4 \quad 2 \qquad \ldots(2)$$



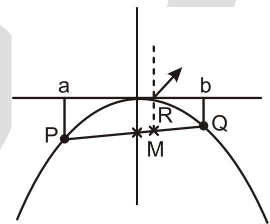
(1) & (2) $\quad 4 \quad ^2 \quad 4 \quad 0 \qquad 1$

Put $\quad 1, \quad 2 \quad 4 \quad 2 \quad 4$

$A(1,\ 4), B(\quad 1, \quad 4)$

$$AB^2 \quad l^2 \quad (\sqrt{4 \quad 64})^2 \quad 68$$

2. $R \quad \dfrac{a \quad b}{2}, \quad \dfrac{a \quad b}{2}^2 \quad , M \quad \dfrac{a \quad b}{2}, \dfrac{a^2 \quad b^2}{2}$



$$PQ \quad y \quad b^2 \quad \frac{b^2 \quad a^2}{b \quad a}(x \quad b)$$

$$y \quad b^2 \quad (b \quad a)(x \quad b)$$

$$y \quad (b \quad a)(x \quad b) \quad b^2$$

$$_1 \quad \int_a^b [[\ (a \quad b)(x \quad b) \quad b^2] \quad x^2]\,dx$$

$$(a \quad b)\frac{(x \quad b)^2}{2} \quad b^2 x \quad \frac{x^3}{3}\Bigg|_a^b \quad \frac{(a \quad b)^3}{6}$$

Area of $\quad PQR \quad {}_2 \quad \dfrac{1}{2}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ \dfrac{a \quad b}{2} & \dfrac{a \quad b}{2}^2 & 1 \end{vmatrix}$

$R_1 \quad R_1 \quad R_2, R_2 \quad R_2 \quad R_3$, we get $\quad {}_2 \quad \dfrac{(a \quad b)^3}{8}$

3.

$$m_{AB} \quad m_{BC} \quad 1$$

$$\frac{2}{(t_1 \quad t_2)} \quad \frac{2}{(t_2 \quad t_3)} \quad 1$$

$$(t_1 \quad t_2)(t_2 \quad t_3) \quad 4$$

Similarly,

$$m_{AD} \quad m_{CD} \quad 1$$

$$(t_1 \quad t_4)(t_3 \quad t_4) \quad 4$$

$$(t_1 \quad t_2)(t_2 \quad t_3) \quad (t_1 \quad t_4)(t_3 \quad t_4)$$

Solving this

$$\frac{t_2 \quad t_4}{t_1 \quad t_3} \quad 1$$

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ELLIPSE

Exercise-1 : Single Choice Problems

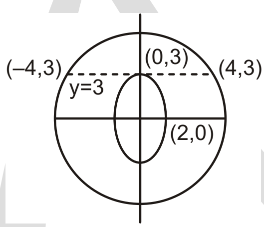
1. Length of perpendicular from $C(0,0)$ to the tangent at $P(2\sqrt{3}\cos\quad, 2\sqrt{2}\cos\quad)$ is

$$CF \quad \left| \frac{1}{\sqrt{\frac{\cos^2}{12} \quad \frac{\sin^2}{8}}} \right|$$

Equation of normal at P is $\frac{2\sqrt{3}x}{\cos} \quad \frac{2\sqrt{2}\,y}{\sin} \quad 12 \quad 8$ which meets the major axis at $G \quad \frac{2}{\sqrt{3}}\cos\quad, 0$

$$CF \quad PG \quad 8$$

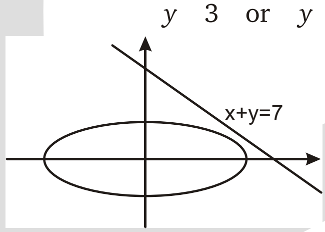
2.



The minimum length of intercept will be possible when

$$y \quad 3 \quad \text{or} \quad y \quad\quad 3 \qquad AB \quad 8$$

3.

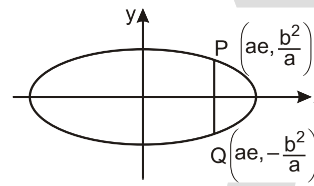


$$\frac{dy}{dx} \quad \frac{x}{2y} \quad 1$$

Put $x \quad 2y$ in the equation of ellipse

The point lies in I quad $\quad (2, 1)$

4. Equation of tangent at P is



$$ex \quad y \quad a$$

$$e \quad \frac{2}{3}, \quad a \quad \frac{10}{3}$$

and $\quad b \quad \frac{10\sqrt{5}}{9}$

Length of latus rectum $\quad \frac{2b^2}{a} \quad \frac{100}{27}$

5. Area bounded by circle & ellipse $\quad a^2 \quad ab \quad a(a \quad b)$

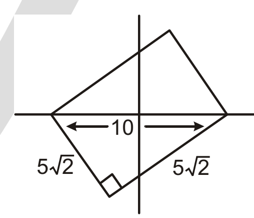
6. $\frac{S_1F_1 \quad S_2F_2}{2} \quad \sqrt{(S_1F_1)(S_2F_2)} \quad \sqrt{16}$

$\because$ Product of perpendiculars from two foci of an ellipse upon any tangent is equal to the square of semi-minor axis.

7. $\quad f(k^2 \quad 2k \quad 5) \quad f(k \quad 11)$

$$k^2 \quad 2k \quad 5 \quad k \quad 11 \qquad k \quad (\quad 3, 2)$$

8. Since sides of the square are tangent and perpendicular to each other, so the vertices lie on director circle



$$x^2 \quad y^2 \quad 16 \quad b^2 \quad \frac{10}{2}^{\;2}$$

$$b \quad 3$$

$$\frac{A}{} \quad \frac{(4)(3)}{} \quad 12$$

9. $T \quad S_1 \qquad px \quad qy \quad \frac{xq \quad py}{2} \quad 1 \quad p^2 \quad q^2 \quad pq \quad 1$

$$p^2 \quad q^2 \quad pq \qquad p \quad 0, q \quad 0$$

10. The combined equation of pair of tangents drawn from a point (x_1, y_1) to the ellipse $S \quad \frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1 \quad 0$ is $T^2 \quad SS_1$. Therefore,

$$\frac{xx_1}{a^2} \quad \frac{yy_1}{b^2} \quad 1^{\;2} \quad \frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1 \quad \frac{x_1^2}{a^2} \quad \frac{y_1^2}{b^2} \quad 1$$

$$\frac{4x}{9} \quad 2y \quad 1^{\quad 2} \qquad \frac{x^2}{9} \quad y^2 \quad 1 \quad \frac{4^2}{9} \quad 2^2 \quad 1$$

$$3x^2 \quad 7y^2 \quad 16xy \quad 8x \quad 36y \quad 52 \quad 0$$

$$\tan \quad \frac{2\sqrt{h^2 \quad ab}}{a \quad b}$$

where, $a \quad 3, b \quad 7$ and $h \qquad 8.$ Therefore,

$$\tan \quad \frac{2\sqrt{64 \quad 21}}{10} \quad \frac{\sqrt{43}}{5}$$

Note : is acute angle between the pair of tangents. Therefore,

$$(a \quad b \quad c)^2 \quad a^2 \quad b^2 \quad c^2 \quad 2ab \quad 2ac \quad 2bc$$

Alternate solution : Any line passing through the point (4, 2) is given by

$$y \quad 2 \quad m(x \quad 4)$$

$$y \quad mx \quad 4m \quad 2$$

For this line to be tangent to the given ellipse, put this y into the equation of the ellipse and make

$$D \quad 0$$

That is,

$$\frac{x^2}{9} \quad (mx \quad 4m \quad 2)^2 \quad 1$$

$$(1 \quad 9m^2)x^2 \quad x(36m \quad 72m^2) \quad 16(9)m^2 \quad 16(9)m \quad 27 \quad 0$$

Now,

$$D \quad 0 \qquad B^2 \quad 4AC \quad 0$$

$$(36m \quad 72m^2)^2 \quad 4(1 \quad 9m^2)(16 \;\; 9m^2 \quad 16 \;\; 9m \quad 27) \quad 0$$

$$(36m)^2(1 \quad 2m)^2 \quad 36(1 \quad 9m^2)(16m^2 \quad 16m \quad 3) \quad 0$$

$$m^2(1 \quad 4m^2 \quad 4m) \quad 36(16m^2 \quad 16m \quad 3 \quad 9 \;\; 16m^4 \quad 9 \;\; 16m \quad 27m^2) \quad 0$$

$$7m^2 \quad 16m \quad 3 \quad 0$$

Now,

$$\tan \quad \left|\frac{m_1 \quad m_2}{1 \quad m_1 m_2}\right| \quad \left|\frac{\sqrt{(m_1 \quad m_2)^2 \quad 4m_1 m_2}}{1 \quad m_1 m_2}\right|$$

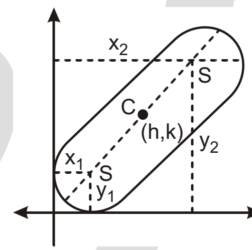
$$\tan \quad \frac{\sqrt{\frac{16}{7}^{\;2} \quad 4 \;\; \frac{3}{2}}}{1 \quad \frac{3}{7}} \quad \frac{7}{10} \quad \frac{\sqrt{16^2 \quad 4 \;\; 3 \;\; 7}}{7}$$

$$\frac{1}{10} \quad \sqrt{4(43)} \quad \frac{\sqrt{43}}{5}$$

where   is the acute angle between the tangents.

Exercise-2 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2



1. $SS \quad 2ae$

$$(x_1 \quad x_2)^2 \quad (y_1 \quad y_2)^2 \quad 4(2)^2 \quad \frac{\sqrt{3}}{2}^{\ 2}$$

$$(x_1 \quad x_2)^2 \quad (y_1 \quad y_2)^2 \quad 4(x_1 x_2 \quad y_1 y_2) \quad 12$$

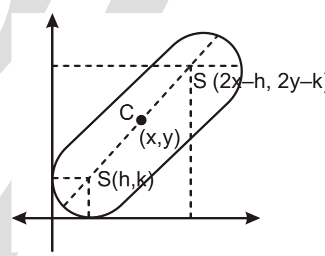
$$(2h)^2 \quad (2k)^2 \quad 4(1 \quad 1) \quad 12$$

($\because$ $x_1 x_2$ and $y_1 y_2$ are   distance of the foci from their tangents $b^2 \quad 1^2$)

$$h^2 \quad k^2 \quad 5$$

2. $(2x \quad h)(h) \quad 1 \qquad x \quad \dfrac{1 \quad h^2}{2h}$

$(2y \quad k)(k) \quad 1 \qquad y \quad \dfrac{1 \quad k^2}{2k}$



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HYPERBOLA

Exercise-1 : Single Choice Problems

1. The normal is $y \quad 4 \quad \frac{1}{4}(x \quad 1)$. Put the value of y in $xy \quad 4$ we get co-ordinates.

3. $c^2 \quad a^2m^2 \quad b^2 \qquad c^2 \qquad {}^2m^2 \quad (\;{}^3 \quad {}^2 \quad)^2$

$c^2 \quad 0 \qquad m^2 \quad (\;{}^2 \qquad 1)^2$

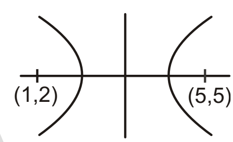
${}^2 \qquad 1$ has minimum value $\frac{3}{4} \quad m^2 \quad \frac{9}{16}$

4. The asymptotes are $y \quad \frac{\sqrt{3}}{2}x$ and the double ordinate be

$P \quad h, \frac{\sqrt{3}}{2}\sqrt{h^2 \quad 4}$ and $P \quad h, \quad \frac{\sqrt{3}}{2}\sqrt{h^2 \quad 4}$

$(PQ)(PQ\;) \quad 3$

5. $2ae \quad 5$ and $2a \quad 3$



$e \quad \frac{5}{3}$

$\frac{1}{e^2} \quad \frac{1}{(e\,)^2} \quad 1 \qquad e \quad \frac{5}{4}$

6. The equation of normal $at\,(2\sec \quad, \tan \quad)$ is $2x\cos \qquad y\cot \qquad 5$

Equal intercepts $\qquad \sin \quad \frac{1}{2}$

Also touches ellipse $\qquad a^2 \quad b^2 \quad \frac{25}{3} \qquad \because \; c^2 \quad a^2m^2 \quad b^2$

7. Let locus of point be (h, k).

Equation of chord of contact is $hx \quad ky \quad 4$

For tangent, $x \quad \frac{4 \quad hx}{k} \qquad 1$ has two equal roots.

$$hk \quad 4 \qquad xy \quad 4$$

8. $$\frac{x^2}{16} \quad \frac{y^2}{18} \quad \frac{x\cos \quad y\sin}{p}^{2} \quad 0$$

$$\text{Coeff. of } x^2 \quad \text{coeff. of } y^2 \quad 0 \qquad P \quad 12$$

The chord $x\cos \quad y\sin \quad 12 \quad 0$ is tangent to the circle $x^2 \quad y^2 \quad \frac{d}{2}^{2} \quad \frac{d}{4} \quad 6$

9. Let the rectangular hyperbola be $x^2 \quad y^2 \quad a^2$ and the point be $(a\sec \quad , a\tan \quad)$.

$$a_1a_2 \quad b_1b_2 \quad (a\cos \quad) \frac{2a}{\cos} \qquad \frac{a\cos}{\sin} \quad \frac{2a\sin}{\cos}$$

Exercise-2 : One or More than One Answer is/are Correct

3. Let $t, \frac{1}{t}$ be any point on $xy \quad 1$

$$xy \quad 1$$

$$xy \quad y \quad 0$$

$$y \quad \frac{y}{x}$$

$$y \quad \frac{1}{t^2}$$

$$\frac{b}{a} \quad t^2$$

a and b are of opp. sign.

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COMPOUND ANGLES

Exercise-1 : Single Choice Problems

2. $a \sin x \quad b(2 \cos c \cos x)$

$$\cos c \quad \frac{a \sin x}{2b \cos x}$$

$$\frac{1}{2b}(\quad \sec x \quad a \tan x) \text{ differentiate w.r.t. } x$$

$$\sec x \tan x \quad a \sec^2 x \quad 0$$

$$\sin x \quad \frac{a}{}$$

3. $$\tan x \quad \frac{3 \tan x \quad \tan^3 x}{1 \quad 3 \tan^2 x} \quad 1$$

$$t \quad \frac{3t \quad t^3}{1 \quad 3t^2} \quad 1 \quad 0 \qquad (\text{Let } \tan x \quad t\ )$$

$$\frac{1 \quad t^4}{1 \quad 3t^2} \quad 0 \qquad \frac{(t \quad 1)(t \quad 1)}{(3t^2 \quad 1)} \quad 0$$

$$t \quad \frac{1}{\sqrt{3}}, 1$$

4. $$\sum_{r\ 1}^{8} \tan(rA) \tan\{(r \quad 1)A\} \quad \sum_{r\ 1}^{8} \frac{\tan(r \quad 1)A \quad \tan(rA) \quad \tan A}{\tan A} \quad \frac{\tan 9A \quad 9 \tan A}{\tan A} \quad 10$$

5. $$f(x) \quad 2 \text{ cosec } 2x \quad \sec x \quad \text{cosec } x$$

$$\frac{1 \quad \sin x \quad \cos x}{\sin x \cos x}$$

$$f\ (x) \quad \frac{\sin^3 x \quad \sin^2 x \quad \cos^3 x \quad \cos^2 x}{\sin^2 x \cos^2 x} \quad 0 \qquad x \quad \frac{}{4}$$

$$f(x)_{\min} \quad \frac{2}{\sqrt{2} \quad 1} \qquad \text{at } x \quad \frac{}{4}$$

6. $\text{cosec} \quad \text{cosec}\,(60 - \;) \quad \text{cosec}\,(60 \quad)$

where $\quad 10$

10. $\frac{1}{2}(2\sin x\cos x \quad 2\cos^2 x) \quad \frac{1}{2}(\sin 2x \quad \cos 2x \quad 1)$

11. $\frac{\tan A}{\sqrt{3}} \quad \frac{\tan B}{\sqrt{5}} \quad k \qquad (k \quad 0)\text{, if } 2\sin A \quad \sqrt{3}\sin B$

$$\frac{2\tan A}{\sqrt{1 \quad \tan^2 A}} \quad \frac{\sqrt{3}\tan B}{\sqrt{1 \quad \tan^2 B}} \qquad \frac{2\sqrt{3}k}{\sqrt{1 \quad 3k^2}} \quad \frac{\sqrt{3} \quad \sqrt{5}k}{\sqrt{1 \quad 5k^2}} \qquad k \quad \frac{1}{\sqrt{5}}$$

12. Gives equations can be written as

$$2\cos \quad 9\cos \quad 6\cos \quad 7\cos \qquad \ldots(1)$$

$$2\sin \quad 9\sin \quad 6\sin \quad 7\sin \qquad \ldots(2)$$

Square and add equation (1) and (2),

$$4 \quad 36 \quad 36[\cos \quad \cos \quad \sin \quad \sin \;] \quad 36 \quad 49 \quad 84[\cos \quad \cos \quad \sin \quad \sin \;]$$

$$36[\cos(\quad)] \quad 84[\cos(\quad)]$$

$$\frac{\cos(\quad)}{\cos(\quad)} \quad \frac{84}{36} \quad \frac{7}{3} \quad \frac{m}{n}; \qquad m \quad n \quad 10$$

13. $\left|\frac{1 \quad \sin \quad 1 \quad \sin}{\sqrt{1 \quad \sin^2}}\right| \quad \left|\frac{2}{\cos}\right| \quad 2\sec$

14. $A \quad \sum_{r\;1}^{3} \cos\frac{2r}{7} \quad \cos\frac{2}{7} \quad \cos\frac{4}{7} \quad \cos\frac{6}{7} \quad \cos\frac{2}{7} \quad \cos\frac{4}{7} \quad \cos\frac{8}{7} \quad B$

15. $\tan \quad \frac{x}{z} \quad \frac{1}{3}$

$$\tan \quad \frac{y}{z} \quad \frac{}{2}$$

$$\tan(\quad) \quad \frac{\tan \quad \tan}{1 \quad \tan \quad \tan} \quad 1 \qquad \frac{}{4}$$

17. $f(x) \quad 2\sin^2 x \quad \sin x \quad 2 \quad x \quad \frac{}{6}, \frac{2}{3}$

Let $\sin x \quad t$

$$f(t) \quad 2t^2 \quad t \quad 2 \quad t \quad \frac{1}{2}, 1$$

18. $1 \quad (\cos^2 A \quad \sin^2 B) \quad \cos A \cos B \quad 1 \quad \cos(A \quad B)\cos(A \quad B) \quad \frac{1}{2}[\cos(A \quad B) \quad \cos(A \quad B)] \quad \frac{3}{4}$

19. $(2\sin x \quad \operatorname{cosec} x)^2 \quad (\tan x \quad \cot x)^2 \quad 0$

$$\sin^2 x \quad \frac{1}{2} \qquad \tan^2 x \quad 1$$

20. $\cos^2 A \quad \sin A \;\; \tan A \qquad\qquad \cos^3 A \quad \sin^2 A$

21. $f(x) \quad \frac{\sqrt{3} \quad 1}{2} \sin x \quad \frac{\sqrt{3} \quad 1}{2} \cos x \quad \frac{\sqrt{3} \quad 1}{2} (\sin x \quad \cos x)$

22. $A \quad B \quad C$

$$\tan A \tan B \tan C \quad \tan A \quad \tan B \quad \tan C$$

23.

$$E \quad \sin A \quad \sin 2B \quad \sin 3C$$

$$E \quad \frac{3}{5} \quad 2 \;\; \frac{4}{5} \; \frac{3}{5} \quad 1$$

$$\frac{15}{25} \quad \frac{24}{25} \quad 1 \quad \frac{39 \quad 25}{25} \quad \frac{14}{25}$$

24. $\frac{\cos A \cos C \quad \cos A \cos C}{\cos A \sin C \quad \cos A \sin C} \quad \cot C \qquad\qquad (\because \; A \quad B \quad C \qquad)$

25. $\frac{\sin \qquad \sin}{\cos \qquad \cos} \quad \frac{2\cos \;\frac{\qquad}{} \quad \sin \;\frac{\qquad}{2}}{2\sin \;\frac{\qquad}{2} \quad \sin \;\frac{\qquad}{2}} \quad \cot \frac{\qquad}{2} \quad \cot$

26. $\cos \frac{x}{256} \;\; \cos \frac{x}{128} \cos \frac{x}{64} \cdots\cdots \cos \frac{x}{4} \;\; \cos \frac{x}{2} \quad \frac{\sin x}{256 \sin \;\frac{x}{256}}$

27. $\frac{(\sin 7 \qquad \sin 5 \;\;) \quad 5(\sin 5 \qquad \sin 3 \;\;) \quad 12(\sin 3 \qquad \sin \;\;)}{\sin 6 \qquad 5\sin 4 \qquad 12\sin 2}$

$$\frac{2\sin 6 \quad \cos \qquad 5(2\sin 4 \quad \cos \;\;) \quad 12(2\sin 2 \quad \cos \;\;)}{\sin 6 \qquad 5\sin 4 \qquad 12\sin 2} \quad 2\cos$$

28. $\tan^2 A \quad \tan^2 B \quad \tan^2 C \quad \tan A \tan B \quad \tan B \tan C \quad \tan A \tan C$

$$\tan A \quad \tan B \quad \tan C$$

$$A \quad B \quad C \quad \frac{}{3}$$

29. $\log_{|\sin x|} |\cos x| \;\; \log_{|\cos x|} |\sin x| \quad 2 \qquad\qquad \log_{|\sin x|} |\cos x| \quad 1 \qquad\qquad |\cos x| \quad |\sin x|$

30. $f(x) \quad \sin^6 x \quad \cos^6 x \quad 1 \quad 3\sin^2 x \cos^2 x \quad 1 \quad \frac{3}{4} \sin^2 2x$

31. $y \quad \dfrac{2\sin}{1 \quad \cos \qquad \sin} \qquad \dfrac{(1 \quad \sin \quad) \quad \cos}{(1 \quad \sin \quad) \quad \cos} \qquad \dfrac{2\sin \quad [(1 \quad \sin \quad) \quad \cos \quad]}{(1 \quad \sin \quad)^2 \quad \cos^2}$

$\dfrac{1 \quad \sin \qquad \cos}{1 \quad \sin}$

32. $\dfrac{\tan^3 A}{1 \quad \tan^2 A} \qquad \dfrac{\cot^3 A}{1 \quad \cot^2 A} \quad \dfrac{\sin^3 A}{\cos A} \quad \dfrac{\cos^3 A}{\sin A}$

$\dfrac{\sin^4 A \quad \cos^4 A}{\sin A \cos A} \quad \dfrac{1 \quad 2\sin^2 A\cos^2 A}{\sin A\cos A}$

$\sec A \operatorname{cosec} A \quad 2\sin A\cos A$

33. $\sqrt{\dfrac{1 \quad \sin}{1 \quad \sin}} \qquad \sqrt{\dfrac{1 \quad \sin}{1 \quad \sin}} \qquad \dfrac{2}{\sqrt{1 \quad \sin^2}} \qquad \dfrac{2}{|\cos \quad |}$

34. $y \quad (\sin^2 \qquad \operatorname{cosec}^2 \qquad 2) \quad (\cos^2 \qquad \sec^2 \qquad 2) \quad 7 \quad (\tan^2 \qquad \cot^2 \quad) \quad 9$

35. $\log_3 \sin x \quad \log_3 \cos x \quad \log_3 (1 \quad \tan x) \quad \log_3 (1 \quad \tan x) \qquad 1$

$\log_3 \dfrac{\tan x}{1 \quad \tan^2 x} \qquad 1 \qquad\qquad \dfrac{\tan x}{1 \quad \tan^2 x} \quad \dfrac{1}{3} \qquad\qquad \tan 2x \quad \dfrac{2}{3}$

36. $\sin \qquad \operatorname{cosec} \qquad 2 \qquad\qquad \sin \qquad \operatorname{cosec} \qquad 1; \qquad x \quad \dfrac{1}{x} \quad 2$

37. $(\tan \qquad \cot \quad)(\tan^2 \qquad \cot^2 \qquad 1) \quad 52$

$(\tan \qquad \cot \quad)\{(\tan \qquad \cot \quad)^2 \quad 3\} \quad 52$

Let $\qquad\qquad\qquad \tan \qquad \cot \qquad t$

$t^3 \quad 3t \quad 52 \quad 0 \qquad\qquad t \quad 4$

$\tan^2 \qquad \cot^2 \qquad (\tan \qquad \cot \quad)^2 \quad 2 \quad 14$

38. $5 \quad 3\sin x \quad 4\cos x \quad 5$

$10 \quad 3\sin x \quad 4\cos x \quad 15 \quad 20$

$\log_{20} 10 \quad \log_{20}(3\sin x \quad 4\cos x \quad 15) \quad \log_{20} 20$

39. $x^2 \quad y^2 \quad 9$

Let $x \quad 3\cos \quad , y \quad 3\sin$

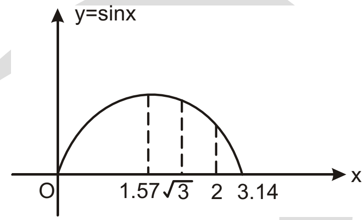
$4a^2 \quad 9b^2 \quad 16$

Let $a \quad 2\cos \quad , \; b \quad \dfrac{4}{3}\sin$

$4a^2x^2 \quad 9b^2y^2 \quad 12abxy \quad (2ax \quad 3by)^2$

$(12\cos \quad \cos \qquad 12\sin \quad \sin \quad)^2 \quad 144\cos^2(\qquad)$

40. $A^2 \quad \sin 2 \quad \sin\sqrt{3}$



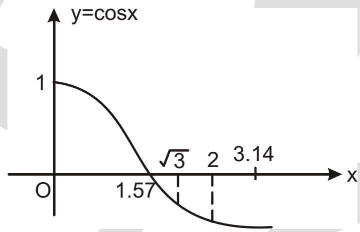
$$\sin\sqrt{3} \quad \sin 2$$

$$A^2 \quad 0$$

$$B^2 \quad \cos 2 \quad \cos\sqrt{3}$$

$$\cos\sqrt{3} \quad \cos 2$$

$$B^2 \quad 0$$



Both A and B are not real numbers.

41. $(2^x \quad 2^{\;\;x} \quad 2\cos x)(3^x \quad 3^{\;\;x} \quad 2\cos x)(5^{\;\;x} \quad 5^x \quad 2\cos x) \quad 0$

If $\dfrac{2^x \quad 2^{\;\;x}}{2} \quad \cos x \qquad x \quad 0$

If $\dfrac{3^x \quad 3^{\;\;x}}{2} \quad \cos x \qquad x$

If $\dfrac{5^{\;\;x} \quad 5^x}{2} \quad \cos x \qquad$ (Not possible)

There are two real values of x.

42.

$$e^{\sin x} \quad e^{\;\;\sin x} \quad 4 \quad 0$$

$$e^{2\sin x} \quad 4e^{\sin x} \quad 1 \quad 0$$

$$e^{\sin x} \quad 2 \quad \sqrt{5}$$

If $\qquad e^{\sin x} \quad 2 \quad \sqrt{5}$

$\qquad \sin x \quad \ln(2 \quad \sqrt{5}) \qquad [\ln(2 \quad \sqrt{5}) \quad 1$, Not possible]

If $\qquad e^{\sin x} \quad 2 \quad \sqrt{5} \qquad (2 \quad \sqrt{5} \quad 0)$ Not possible

There is no solution.

43. $\sqrt{4\sin^4 \quad 4\sin^2 \quad \cos^2} \quad 4\cos^2(\quad /4 \quad /2)$

$$\sqrt{4\sin^2} \quad 2[1 \quad \cos(\quad /2 \quad)]$$

$$2|\sin \quad | \quad 2 \quad 2\sin$$

$$2\sin \quad 2 \quad 2\sin \quad 2 \qquad \left(\text{If} \quad \frac{3}{2} \text{ then } \sin \quad 0\right)$$

44. $\cos\frac{}{12} \quad \sin\frac{}{12} \quad \dfrac{\sin\frac{}{12}}{\cos\frac{}{12}} \quad \dfrac{\cos\frac{}{12}}{\sin\frac{}{12}}$

$$\frac{\cos\frac{}{12} \quad \sin\frac{}{12}}{\sin\frac{}{12} \quad \cos\frac{}{12}} \quad \frac{2\sqrt{1 \quad \sin \quad /6}}{\sin \quad /6} \quad 2\sqrt{2}$$

45. $\tan(100 \quad 125\ ) \quad \dfrac{\tan 100 \quad \tan 125}{1 \quad \tan 100\ \tan 125} \quad 1$

$\tan 100 \quad \tan 125 \quad \tan 100\ \tan 125 \quad 1$

46. If $\sin x \quad \sin^2 x \quad 1 \qquad \sin x \quad 1 \quad \sin^2 x \qquad \sin x \quad \cos^2 x$

$\cos^8 x \quad 2\cos^6 x \quad \cos^4 x \quad \sin^4 x \quad 2\sin^3 x \quad \sin^2 x$

$\sin^2 x(\sin^2 x \quad 2\sin x \quad 1)$

$(1 \quad \sin x)\,(2 \quad \sin x)$

$2 \quad \sin x \quad \sin^2 x \quad 1$

47. Let $x \quad 5\cos \quad , y \quad 5\sin$

$0 \quad 3x \quad 4y \quad 25 \qquad (\because\ 3x \quad 4y \quad 0)$

48. $5\cos 2 \quad 2\cos^2 \frac{}{2} \quad 1 \quad 0$

$10\cos^2 \qquad \cos \qquad 3 \quad 0 \qquad\qquad \cos \quad \frac{1}{2}, \quad \frac{3}{5}$

49. $\sin \quad \frac{4}{5}$ where $0 \qquad$ and $\tan \quad 0$

then $\qquad \cos \quad \frac{3}{5}$

$5 \quad \frac{3}{5}\sin(\qquad) \quad \frac{4}{5}\cos(\qquad) \quad \text{cosec} \quad 5$

50. $\sin \quad x \quad \frac{}{6} \quad \cos \quad x \quad \frac{}{6} \quad \sqrt{2} \quad \cos\frac{}{4}\sin \quad x \quad \frac{}{6} \quad \sin\frac{}{4} \quad \cos \quad x \quad \frac{}{6} \quad \sqrt{2}\sin \quad x \quad \frac{5}{12}$

This attained maximum value when $x \quad \frac{5}{12} \quad \frac{}{2} \qquad x \quad \frac{}{12}$

51. $\sin 2x \quad \cos 2x \quad 2a \quad 1$

$\sqrt{2} \quad 2a \quad 1 \quad \sqrt{2}$

$\frac{1 \quad \sqrt{2}}{2} \quad a \quad \frac{1 \quad \sqrt{2}}{2}$

52. $(\cos 12 \quad \cos 24 \quad \cos 48 \quad \cos 84\ )(\cos 36 \quad \cos 72\ ) \quad \cos 60$

$(\quad \cos 12 \quad \cos 24 \quad \cos 48 \quad \cos 96\ )(\cos 36 \quad \cos 72\ ) \quad \cos 60$

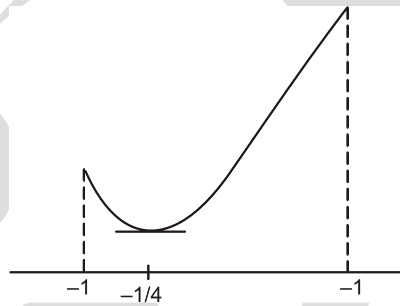
$$\frac{\sin(2^4 \quad 12\ )}{2^4 \sin 12} \quad \frac{\sqrt{5} \quad 1}{4} \quad \frac{\sqrt{5} \quad 1}{4} \quad \frac{1}{2} \quad \frac{1}{128}$$

53. $2\cos^2 \quad \cos \quad 1$

$y_{\min} \quad \dfrac{7}{8}$ at $\cos \quad \dfrac{1}{4}$

$y_{\max} \quad 4$ at $\cos \quad 1$

$$\frac{y_{\max}}{y_{\min}} \quad \frac{32}{7}$$

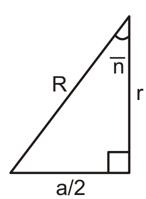


54. $\tan x \quad \dfrac{3\tan x \quad \tan^3 x}{1 \quad 3\tan^2 x} \quad 1 \quad 0\,;\ \dfrac{\tan^4 x \quad 1}{3\tan^2 x \quad 1} \quad 0$

$$\frac{(\tan^2 x \quad 1)(\tan x \quad 1)(\tan x \quad 1)}{(\sqrt{3}\tan x \quad 1)(\sqrt{3}\tan x \quad 1)} \quad 0$$

$$\frac{}{6} \quad x \quad \frac{}{4}$$

55. $a \quad 2R\sin\dfrac{}{n} \quad 2r\tan\dfrac{}{n}$



56. $(\cos 12 \quad \cos 132\ ) \quad (\cos 84 \quad \cos 156\ )$

$$2\cos\frac{12 \quad 132}{2}\cos\frac{12 \quad 132}{2} \quad 2\cos\frac{84 \quad 156}{2}\cos\frac{84 \quad 156}{2}$$

$$2\cos 72 \ \ \cos 60 \quad 2\cos 120 \ \ \cos 36$$

$$2 \quad \frac{\sqrt{5} \quad 1}{4} \quad \frac{1}{2} \quad 2 \quad \frac{1}{2} \quad \frac{\sqrt{5} \quad 1}{4} \quad \frac{1}{2}$$

57. $\dfrac{1}{2} \quad \dfrac{2\sin \quad \cos}{\cos \quad \cos 3} \quad \dfrac{2\sin 3 \quad \cos 3}{\cos 9 \quad \cos 3} \quad \dfrac{2\sin 9 \quad \cos 9}{\cos 9 \quad \cos 27} \quad \dfrac{2\sin 27 \quad \cos 27}{\cos 27 \quad \cos 81}$

$$\frac{1}{2} \quad \frac{\sin(3 \qquad)}{\cos \quad \cos 3} \quad \frac{\sin(9 \quad 3\ )}{\cos 3 \quad \cos 9} \quad \frac{\sin(27 \quad 9\ )}{\cos 9 \quad \cos 27} \quad \frac{\sin(81 \quad 27\ )}{\cos 27 \quad \cos 81}$$

$$\frac{1}{2}[\tan 81 \quad \tan\ ] \quad \frac{1}{2} \quad \frac{\sin 80}{\cos \quad \cos 81}$$

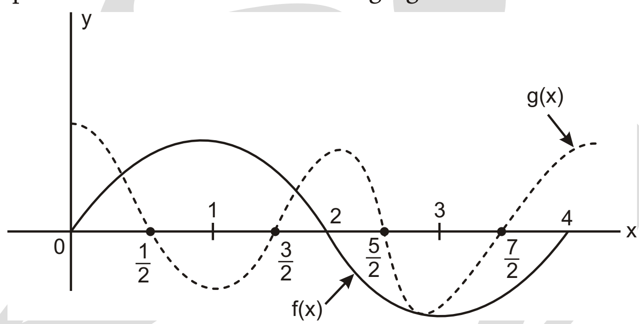
58. $\sin 20 \quad \dfrac{4\cos 20 \quad 1}{\cos 20} \quad \dfrac{2\sin 40 \quad \sin 20}{\cos 20} \quad \dfrac{2\sin(60 \quad 20\ ) \quad \sin 20}{\cos 20} \quad \sqrt{3}$

59. Let us draw the graph of

$$f(x) \quad \sin\ \frac{x}{2}$$

and $\qquad g(x) \quad \cos(x\ \ )$

On the same *xy*-plane as shown in the following figure.



From this graphical representation, it is clear that y is strictly increasing in $\dfrac{5}{2}, \dfrac{7}{2}$

Because for all values of x,

$$\frac{5}{2} \quad x \quad \frac{7}{2}$$

That is, $\qquad \sin\ \dfrac{x}{2} \quad 0$

and $\qquad \cos(x\ \ ) \quad 0$

which imply that $\qquad \dfrac{dy}{dx} \quad 0$

which means that y is strictly increasing.

60. $8\sin \quad \sin 3 \quad \dfrac{\sin 8}{4\sin 2} \quad \cos 6$

$$\sin 3 \ \ \sin 8 \quad \cos 6 \ \ \cos$$

$$\cos 5 \quad \cos 11 \quad \cos 7 \quad \cos 5$$

$$\cos 7 \quad \cos 11 \quad 0$$

$$2\cos 9 \quad \cos 2 \quad 0$$

61. $\tan A \quad \dfrac{1}{3} \quad \sin A \quad \dfrac{1}{\sqrt{10}}; \quad \cos A \quad \dfrac{3}{\sqrt{10}}$

63. $(2\cos\ \ )^2 \quad (1 \quad \sin\ \ )^2 \quad \sin \quad 1 \text{ or } \sin \quad \dfrac{3}{5}$

64. $\sin \quad \dfrac{1}{\sin} \quad 2 \quad \sin \quad 1$

65. $\tan^2 \quad \cot^2 \quad a \quad \tan^3 \quad \cot^3 \quad \sqrt{a \quad 2}\,(a \quad 1) \quad 52$

66. $\tan A \quad \tan C \quad \dfrac{5}{12}$

$\cos B \quad \cos D \quad \dfrac{3}{5} \qquad \tan D \quad \dfrac{4}{3}$

67. $\sqrt{\tan^2 \quad \sin^2} \quad \sqrt{\tan^2 \quad \sin^2} \quad |\tan \quad \sin \quad|$

68. $\dfrac{\sin 10 \quad \sin 20}{\cos 10 \quad \cos 20} \quad \tan 15 \quad 2 \quad \sqrt{3}$

69. $(\sin^2 \quad)^3 \quad (\cos^2 \quad)^3 \quad (\sin^2 \quad \cos^2 \quad)(\sin^4 \quad \cos^4 \quad \sin^2 \quad \cos^2 \quad)$

$1 \quad 3\sin^2 \quad \cos^2$

70. $\dfrac{\tan x \quad 1}{\tan x \quad 1} \quad \dfrac{\sec^2 x \quad 2}{\tan^2 x \quad 1} \qquad \dfrac{(\tan x \quad 1)^2 \quad (\sec^2 x \quad 2)}{\tan^2 x \quad 1}$

$\dfrac{2\tan x \quad 2}{\tan^2 x \quad 1} \qquad \dfrac{2}{\tan x \quad 1}$

71. $\dfrac{\cot \quad \tan}{\cot \quad \tan} \quad [\cos 450 \quad \cos(2 \quad 180 \quad)]$

$(\cos^2 \quad \sin^2 \quad) \quad \cos 2 \quad 2\cos 2$

72. $\dfrac{1 \quad \tan}{1 \quad \tan} \quad \dfrac{1 \quad \tan}{1 \quad \tan} \quad 1$

$1 \quad \tan^2 \quad \dfrac{}{4} \qquad \sec^2 \quad \dfrac{}{4} \qquad \mathrm{cosec}^2 \quad \dfrac{}{4}$

73. $\dfrac{\tan \quad \sin}{1 \quad \cos} \quad \tan$

74. $(\cos 2 \quad \cos 5 \quad) \quad (\cos 3 \quad \cos 4 \quad)$

$2\cos\dfrac{7}{2} \quad \cos\dfrac{3}{2} \quad 2\cos\dfrac{7}{2} \quad \cos\dfrac{}{2}$

$2\cos\dfrac{7}{2} \quad \cos\dfrac{3}{2} \quad \cos\dfrac{}{2} \qquad 4\sin\dfrac{}{2}\sin \quad \cos\dfrac{7}{2}$

75. $\cos 2 \quad \dfrac{1 \quad \tan^2}{1 \quad \tan^2} \quad \dfrac{1 \quad \left(\dfrac{1 \quad \sin \quad \sin}{\cos \quad \cos}\right)^2}{1 \quad \left(\dfrac{1 \quad \sin \quad \sin}{\cos \quad \cos}\right)^2}$

$\dfrac{(\cos \quad \cos \quad)^2 \quad (1 \quad \sin \quad \cos \quad)^2}{(\cos \quad \cos \quad)^2 \quad (1 \quad \sin \quad \sin \quad)^2} \quad \dfrac{[1 \quad \cos(\qquad)][\cos(\qquad) \quad 1]}{(\cos \quad \cos \quad)^2 \quad (1 \quad \sin \quad \sin \quad)^2} \quad 0$

76. $x \quad \frac{2}{3}$ (IInd quadrant)

$\cos x \quad \cos 2x \quad \cos 3x \qquad \cos 100x \; \frac{\sin 50x}{\sin \frac{x}{2}} \cos \frac{101x}{2} \qquad \frac{1}{2}$

77. $\cos^3 0 \quad \cos^3 \frac{}{3} \quad \cos^3 \frac{2}{3} \quad \cos^3 \qquad \cos^3 \frac{10}{3} \quad \frac{1}{8}$

78. $\frac{1 \quad 2(\cos 60 \quad \cos 80\ )}{2 \sin 10} \quad \frac{2\cos 80}{2\sin 10} \quad 1$

79. $(x \quad 5)^2 \quad (y \quad 12)^2 \quad 14^2$

Let $x \quad 5 \quad 14\cos \ , \ y \quad 12 \quad 14\sin$

$x^2 \quad y^2 \quad 365 \quad 336\sin \quad 140\cos$

80. tan has three distinct solution in $[0, 2\ ]$ 0 and 0, , 2 .

81. $\sqrt{\frac{1 \quad \tan}{1 \quad \tan}} \quad \sqrt{\frac{1 \quad \tan}{1 \quad \tan}} \quad \frac{2}{\sqrt{1 \quad \tan^2}}$

82. $3\sin \quad 4\cos \quad 5\ \frac{3}{5}\sin \quad \frac{4}{5}\cos \quad 5\sin(\quad 53\ )$

83. $f(n) \quad \sum_{r\ 1}^{n} \cos r$

$f(4) \quad \cos 1 \ \cos 2 \ \cos 3 \ \cos 4 \quad 0$

$f(5) \quad \cos 1 \ \cos 2 \ \cos 3 \ \cos 4 \ \cos 5 \quad 0$

84. $\frac{(p^2 \quad q^2)^2}{pq} \quad \frac{(4\tan A \sin A)^2}{\tan^2 A \quad \sin^2 A} \quad 16$

85. $0 \quad \sin \quad \cos \quad 1 \qquad 0, \frac{}{4}$

$(\sin\ )^{\cos} \quad (\sin\ )^{\sin}$

$(\cos\ )^{\cos} \quad (\cos\ )^{\sin}$

86. $32\sin\frac{A}{2}\sin\frac{5A}{2} \quad 16(\cos 2A \quad \cos 3A)$

$16[(2\cos^2 A \quad 1) \quad (4\cos^3 A \quad 3\cos A)]$

87. $\cos \quad \cos \quad \sin \quad \sin \quad \sin \quad \cos \quad \cos \quad \sin \quad 0$

$\cos \quad (\cos \quad \sin\ ) \quad \sin \quad (\cos \quad \sin\ ) \quad 0$

$(\cos \quad \sin\ )(\cos \quad \sin\ ) \quad 0$

$\cos \quad \sin \qquad (\because \cos \quad \sin\ )$

$\tan \quad 1$

88. $2^{x} \quad 3^{y} \quad 6^{\ z} \quad k$

$x \quad \log_2 k, y \quad \log_3 k, z \quad \log_6 k$

$\frac{1}{x} \quad \frac{1}{y} \quad \frac{1}{z} \quad 0$

89. $(\sin \quad \sin \ )^2 \quad (\cos \quad \cos \ )^2 \quad \frac{21}{65}^{\ 2} \quad \frac{27}{65}^{\ 2}$

$2 \quad 2\cos(\quad) \quad \frac{1170}{(65)^2} \quad 4\cos^2 \frac{\quad}{2}$

90. $^{2} \quad a^2 \quad b^2 \quad 2\sqrt{(a^2\cos^2 \quad b^2\sin^2 \ )(a^2\sin^2 \quad b^2\cos^2 \ )}$

$a^2 \quad b^2 \quad 2\sqrt{a^2b^2 \quad (a^4 \quad b^4 \quad 2a^2b^2)\sin^2 \quad \cos^2}$

91. $Q \quad \sum_{r\ 0}^{n} \frac{\sin(3^r \ )\cos(3^r \ )}{\cos(3^r \ )\cos(3^{r\ 1} \ )} \quad \frac{1}{2}\sum_{r\ 0}^{n} \tan(3^{r\ 1} \ ) \quad \tan(3^r \ ) \quad \frac{1}{2}P$

92. When 270 $\quad$ 360 , we have

$$\sqrt{2(1 \quad \cos \ )} \quad \sqrt{2\cos^2 \frac{\quad}{2}}$$

which is non-negative. Now, the above equation can be written as

$$\sqrt{2(1 \quad \cos \ )} \quad 2\left|\cos\frac{\quad}{2}\right|$$

$$2\cos\frac{\quad}{2} \qquad \because \cos\frac{\quad}{2} \quad 0 \text{ when } 135 \quad \frac{\quad}{2} \quad 180$$

Now, let us consider that $\sqrt{2 \quad \sqrt{2(1 \quad \cos \ )}}$

which is not-negative. That is,

$$\sqrt{2 \quad \sqrt{2(1 \quad \cos \ )}} \quad \sqrt{2 \quad 2\cos\frac{\quad}{2}}$$

$$\sqrt{2}\sqrt{1 \quad \cos\frac{\quad}{2}} \quad \sqrt{2}\sqrt{2\sin^2\frac{\quad}{4}}$$

$$2\left|\sin\frac{\quad}{4}\right|$$

$$2\sin\frac{\quad}{4} \qquad \because \sin\frac{\quad}{4} \quad 0 \text{ when } \frac{135}{2} \quad \frac{\quad}{4} \quad 90$$

93. We know that $\sqrt{2} \quad \sin x \quad \cos x \quad \sqrt{2}$

When $x \quad \frac{3}{4}$, we have $\sin x \quad \cos x \quad \sqrt{2}$

when $x \quad \frac{3}{4}$, we have $y \quad \sqrt{2} \quad 1 \quad 0$

which implies that options (1) and (2) are incorrect.

Now, at $x \quad \frac{}{4}$, we have $\sin x \quad \cos x \quad \sqrt{2}$

That is, $(\sin 4x \quad \cos 4x)^2 \quad 2$. Therefore, $y \quad \sqrt{2} \quad 2$ for any $x \quad R$.

which implies that option (4) is incorrect.

Note : The maximum value of $\sin x \quad \cos x$ is $\sqrt{2}$, for $x \quad \frac{}{4}$ and the maximum value of $(\sin 4x \quad \cos 4x)^2$ is 2, for $x \quad \frac{}{16}$.

94. $(\cos x \quad \cos y)^2 \quad (\sin x \quad \sin y)^2 \quad (\quad \cos z)^2 \quad (\quad \sin z)^2$

$$2 \quad 2\cos(x \quad y) \quad 1$$

95. $\frac{1}{\sin 10} \quad \frac{1}{\sin 50} \quad \frac{1}{\sin 70} \quad \frac{\sin 50 \ \sin 70 \quad \sin 10 \ \sin 70 \quad \sin 10 \ \sin 50}{\sin 10 \ \sin 50 \ \sin 70}$

$$\frac{\frac{1}{2}(\cos 20 \quad \cos 120 \quad \cos 60 \quad \cos 80 \quad \cos 40 \quad \cos 60 \ )}{\frac{1}{4}\sin 30}$$

$$\frac{\frac{1}{2} \ \frac{3}{2} \quad \cos 20 \quad 2\cos 60 \ \cos 20}{\frac{1}{4}\sin 30} \quad 6$$

Exercise-2 : One or More than One Answer is/are Correct

1. $\cot 12 \ \cot 24 \ \cot 48 \ [\cot 28 \ \cot(60 \quad 28 \ )\cot(60 \quad 28 \ )] \quad (\cot 12 \ \cot 48 \ )(\cot 24 \ \cot 84 \ )$

$$\frac{\cot 36}{\cot 72} \quad \frac{\cot 72}{\cot 36} \quad 1$$

2. $$\cot^4 x \quad 2(1 \quad \cot^2 x) \quad a^2 \quad 0$$

$$\cot^4 x \quad 2\cot^2 x \quad a^2 \quad 2 \quad 0$$

$$(\cot^2 x \quad 1)^2 \quad 3 \quad a^2$$

to have atleast one solution

$$3 \quad a^2 \quad 0$$

$$a^2 \quad 3 \quad 0$$

$$a \quad [\ \sqrt{3}, \sqrt{3}]$$

Integral values –1, 0, 1

$$\text{Sum} = 0$$

3. (A) $\tan 1 \quad \tan^{\ 1} 1 \qquad \tan 1 \quad \frac{}{4}$

(B) $\sin 1 \quad \cos 1$

$\sin 57.3 \qquad \cos 57.3$

(C) $\tan 1 \quad \sin 1$ (not possible)

Because $\tan 57.3 \quad 1 \quad \sin 57.3$

(D) $\cos 1 \quad \frac{}{4}$

$$\cos(\cos 1) \quad \cos \ \frac{}{4}$$

4. (A) $\tan 1 \quad 1$ and $\sin 1 \quad 1$, then $\log_{\sin 1} \tan 1 \quad 0$

(B) $1 \quad \tan 3 \quad 1$ and $\cos 1 \quad 1$, then $\log_{\cos 1}(1 \quad \tan 3) \quad 0$

(C) $\cos \qquad \sec \qquad 2$ and $\log_{10} 5 \quad 1$, then $\log_{\log_{10} 5}(\cos \qquad \sec \ ) \quad 0$

(D) $2 \sin 18 \quad 1$ and $\tan 15 \quad 1$, then $\log_{\tan 15} 2 \sin 18 \quad 0$

5. Put $\sin \ \dfrac{2 \tan \frac{}{2}}{1 \quad \tan^2 \frac{}{2}}$, $\cos \ \dfrac{1 \quad \tan^2 \frac{}{2}}{1 \quad \tan^2 \frac{}{2}}$

6. Given $\dfrac{\sin(2 \qquad)}{\sin} \quad \dfrac{3}{1}$

Option (C) $\dfrac{\sin(2 \qquad) \quad \sin}{\sin(2 \qquad) \quad \sin} \quad \dfrac{3 \quad 1}{3 \quad 1}$ (Use C and D method)

$$\tan(\qquad) \quad 2 \tan$$

Option (B) $3 \sin \qquad \sin(2 \qquad)$

$2 \sin \qquad \sin(2 \qquad) \quad \sin$

$2 \sin \qquad 2 \cos(\qquad) \sin$

Option (D) $3 \sin \qquad \sin\{ \quad (\qquad)\}$

$3 \sin \qquad \sin \quad \cos(\qquad) \quad \cos \quad \sin(\qquad)$

Subtract from (B) option

$$2 \sin \qquad \cos \quad \sin(\qquad)$$

Option (A) $\cot \qquad 3 \cot(2 \qquad) \quad \dfrac{\cos}{\sin} \quad 3 \dfrac{\cos(2 \qquad)}{\sin(2 \qquad)}$

$$\frac{\cos}{\sin} \quad 3 \frac{\cos(2 \qquad)}{3 \sin} \quad \frac{2 \sin(\qquad) \sin}{\sin} \quad 4 \tan \qquad \text{(from D)}$$

Also $\cot \qquad \cot(\qquad) \quad \dfrac{3}{2} \cot$ (from C)

Now multiply the two relations.

7. $\sin(x \quad 20\ ) \quad \sin(x \quad 40\ ) \quad \sin(x \quad 40\ )$

$\sin(x \quad 20\ ) \quad \sin(x \quad 40\ ) \quad \sin(x \quad 40\ )$

$\cos(x \quad 10\ ) \quad \sin(x \quad 40\ ) \quad \cos[90 \quad (x \quad 40\ )]$

$x \quad 30$ now check the option, only (a) and (b) satisfy

8. $2\ (\cos x \cos y) \quad 2\ (\sin x \sin y) \quad 3 \quad 0$

$(\ \cos x)^2 \quad (\ \sin x)^2 \quad 0$

$\cos x \quad 0$ and $\sin x \quad 0$

$\cos 3x \quad \cos 3y \quad \cos 3z \quad 4(\cos^3 x \quad \cos^3 y \quad \cos^3 z) \quad 3(\cos x \quad \cos y \quad \cos z)$

$12 \cos x \cos y \cos z$

9. $0 \quad \sin x \quad 1, \ 0 \quad \cos x \quad 1$

If $\sin^n x \quad \cos^n x \quad 1 \qquad n \quad 2$

$\sin^n x \quad \cos^n x \quad 1 \qquad n \quad 2$

$\sin^n x \quad \cos^n x \quad 1 \qquad n \quad 2$

10. If $x \quad \sin(\qquad)\sin(\qquad)$

$2x \quad \cos(\qquad\qquad) \quad \cos(\qquad\qquad)$

$y \quad \sin(\qquad)\sin(\qquad)$

$2y \quad \cos(\qquad\qquad) \quad \cos(\qquad\qquad)$

$z \quad \sin(\qquad)\sin(\qquad)$

$2z \quad \cos(\qquad\qquad) \quad \cos(\qquad\qquad)$

$2x \quad 2y \quad 2z \quad 0 \qquad x \quad y \quad z \quad 0$

If $x \quad y \quad z \quad 0$ then $x^3 \quad y^3 \quad z^3 \quad 3xyz$

11. $X^2 \quad 4XY \quad Y^2 \quad (x\cos \quad y\sin\ )^2 \quad (x\sin \quad y\cos\ )^2$

$4(x\cos \quad y\sin\ )(x\sin \quad y\cos\ )$

$x^2 \quad y^2 \quad 4\{x^2 \sin\ \cos \quad y^2 \sin\ \cos \quad xy(\cos^2 \quad \sin^2\ )\}$

$x^2(1 \quad 4\sin\ \cos\ ) \quad y^2(1 \quad 4\sin\ \cos\ ) \quad 4xy(\cos^2 \quad \sin^2\ )$

$\cos^2 \quad \sin^2 \quad 0$

$\overline{}$ $\qquad (0 \qquad / 2)$
4

$x^2 \quad 4XY \quad Y^2 \quad 3x^2 \quad y^2$

$A \quad 3$ and $B \quad 1$

12. (A) $2(a \quad d) \quad 2(b \quad c)$

(B) $\tan 50 \quad \dfrac{1}{\tan 40} \quad \dfrac{1 \quad \tan^2 20}{2\tan 20}$

$\tan 20 \quad 2\tan 50 \quad \tan 70$

$$2a \quad 2b \quad 2c$$

(D) $\quad \tan 20 \quad 2\tan 10 \quad \tan 20 \;\; \tan^2 10 \quad 0$

$\quad \tan 20 \quad 2\tan 10$

$\quad b \quad a$ and $d \quad c$

13. (A) $\frac{1}{2}(2\sin 75 \;\; \cos 75 \;) \quad \frac{1}{2}\sin 150 \quad \frac{1}{4}$

(B) $\log_2^{28} \quad 2 \quad \log_2^7$ (irrational)

(C) $\log_3^5 \; \log_5^6 \quad \log_3^6 \quad 1 \quad \log_3^2$ (irrational)

(D) $8^{\;\log_{27}^3} \quad 8^{\;1/3} \quad \frac{1}{2}$

14. $\quad \sin x\cos x(\cos^2 x \quad \sin^2 x) \quad \frac{1}{2}\sin 2x\cos 2x \quad \frac{1}{4}\sin 4x$

$\quad \sin x\cos x \quad \frac{1}{2}\sin 2x$

15. $\quad \sqrt{2 \quad \sqrt{2 \quad 2\cos 4}} \quad \sqrt{2 \quad \sqrt{2(1 \quad \cos 4\;)}}$

$\quad \sqrt{2 \quad \sqrt{4\cos^2 2}}$

$\quad \sqrt{2 \quad 2|\cos 2\;|}$

If $\quad 2 \quad 3 \;/2$ then $\frac{}{2} \quad \frac{3}{4}$

$\quad \sqrt{2 \quad 2|\cos 2\;|} \quad \sqrt{2 \quad 2\cos 2} \quad 2|\sin \;| \quad 2\sin$

If $\frac{3}{2} \quad 2 \quad 2$ then $\frac{3}{4}$

$\quad \sqrt{2 \quad 2|\cos 2\;|} \quad \sqrt{2 \quad 2\cos 2} \quad 2|\cos \;| \quad 2\cos$

16. $1 \quad \tan \quad \tan^2 \quad \tan^3$

$\quad 1 \quad \tan^2 \quad \tan \quad (\tan^2 \quad 1)$

18. $\quad \frac{1}{\sin^6 x \quad \cos^6 x} \quad \frac{1}{1 \quad 3\sin^2 x\cos^2 x}; \quad 1 \quad \frac{1}{1 \quad 3\sin^2 x\cos^2 x} \quad 4$

19. $\log_{10}\sin x \quad \log_{10}\cos x \quad 2\log_{10}\cot x \quad \log_{10}\tan x \quad 1$

$\log_{10}(\sin x \;\; \cos x \;\; \cot x) \quad k \quad \log_{10}\cos^2 x \quad 1$

20. $\tan A \quad \tan B \quad \tan C \quad \tan A\tan B\tan C$

$\frac{3}{\tan C} \quad \frac{6}{\tan C} \quad \tan C \quad \frac{3}{\tan C}\;\frac{6}{\tan C} \quad \tan C$

$\quad \tan^2 C \quad 9 \quad \tan C \quad 3$

21. $\dfrac{(1 \quad \cot x)}{\sin^2 x} \quad (1 \quad \cot x) \;\; \operatorname{cosec}^2 x$

$$(1 \quad \cot x)(1 \quad \cot^2 x)$$

22. $f(x) \quad \dfrac{1}{2} \;\; 2\sin^2 x \quad 2\sin^2 \quad x \quad \dfrac{2}{3} \quad\quad 2\sin^2 \quad x \quad \dfrac{4}{3}$

23. $y \quad \dfrac{\tan x}{\tan 3x} \quad \dfrac{1 \quad 3\tan^2 x}{3 \quad \tan^2 x}$

$$\tan^2 x \quad \frac{1 \quad 3y}{3 \quad y} \quad 0$$

24. $\sqrt{2}\sin(A \quad B) \quad \cos B(\sin B \quad \sin^3 B) \quad \sin B(\cos B \quad \cos^3 B)$

$$\sin B \cos B$$

$$\frac{1}{2}\sin 2B \qquad \sin(A \quad B) \qquad \frac{\sin 2B}{2\sqrt{2}}$$

25. $\dfrac{1}{\sin^6 x \quad \cos^6 x} \qquad \dfrac{1}{1 \quad 3\sin^2 x\cos^2 x}; \;\; 1 \quad \dfrac{1}{1 \quad 3\sin^2 x\cos^2 x} \quad 4$

26. $1 \quad \tan \qquad \tan^2 \qquad \tan^3$

$$1 \quad \tan^2 \qquad \tan \quad (\tan^2 \qquad 1)$$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

1. $286.5 \quad$ (IV quadrant) $l \quad 0, m \quad 0$

2. $\tan(\;\; 1042 \;\;) \qquad \tan(1080 \qquad 38 \;\;) \qquad \tan 38 \qquad \tan 45$

3. $401.1 \quad$ (I quadrant) $\quad l \quad 0, m \quad 0$

Paragraph for Question Nos. 4 to 6

$a \quad \sin \qquad b \quad \sin \qquad \dfrac{2}{3} \qquad c \quad \sin \qquad \dfrac{4}{3}$

$p \quad \cos \qquad q \quad \cos \qquad \dfrac{2}{3} \qquad r \quad \cos \qquad \dfrac{4}{3}$

4. $a \quad b \quad c \quad \sin \quad \sin \quad \frac{2}{3} \quad \sin \quad \frac{4}{3}$

$$\sin \quad 2\sin(\quad)\cos \frac{}{3} \quad 0$$

5. $ab \quad bc \quad ac \quad \sin \quad \sin \quad \frac{2}{3} \quad \sin \quad \frac{2}{3} \quad \sin \quad \frac{4}{3} \quad \sin \quad \sin \quad \frac{4}{3}$

$$\frac{1}{2} \quad \cos\frac{2}{3} \quad \cos \ 2 \quad \frac{2}{3} \quad \cos\frac{2}{3} \quad \cos(2 \quad 2\ ) \quad \cos\frac{4}{3} \quad \cos \ 2 \quad \frac{4}{3} \quad \frac{3}{4}$$

6. $qc \quad rb \quad \cos \quad \frac{2}{3} \quad \sin \quad \frac{4}{3} \quad \cos \quad \frac{4}{3} \quad \sin \quad \frac{2}{3} \quad \sin\frac{2}{3} \quad \frac{\sqrt{3}}{2}$

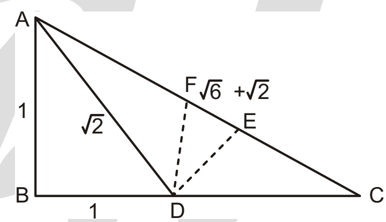
Paragraph for Question Nos. 7 to 8

7. $\tan A \quad \sqrt{7 \quad 4\sqrt{3}} \quad \cot C$

$$\sqrt{\tan A \quad \cot C} \quad \sqrt{2\sqrt{7 \quad 4\sqrt{3}}}$$

$$\sqrt{2(2 \quad \sqrt{3})} \quad \sqrt{4 \quad 2\sqrt{3}}$$

$$\sqrt{3} \quad 1$$



8. $\log_{AE} \frac{AC}{CD} \quad \log_{\sqrt{2}} \frac{\sqrt{2} \quad \sqrt{6}}{1 \quad \sqrt{3}} \quad \log_{\sqrt{2}}^{\sqrt{2}} \quad 1$

Paragraph for Question Nos. 9 to 10

9. In a ABC, $\cot A \quad \cot B \quad \cot C \quad \sqrt{3} \quad \cot \quad \sqrt{3}$

10. $\cot \quad \cot A \quad \cot B \quad \cot C \quad \sin(A \quad) \quad \frac{\sin^2 A \sin}{\sin B \sin C}$

$\sin(B \quad) \quad \frac{\sin^2 B \ \sin}{\sin A \sin C}$ and $\sin(C \quad) \quad \frac{\sin^2 C \sin}{\sin A \sin B}$

Paragraph for Question Nos. 11 to 12

11. $f(x) \quad \dfrac{\left|\cos\frac{x}{2}\right| \quad \left|\sin\frac{x}{2}\right|}{\left|\cos\frac{x}{2}\right| \quad \left|\sin\frac{x}{2}\right|}$

12. If $\frac{}{2} \quad \frac{x}{2} \qquad f(x) \quad \dfrac{\cos\frac{x}{2} \quad \sin\frac{x}{2}}{\cos\frac{x}{2} \quad \sin\frac{x}{2}} \quad \dfrac{1 \quad \tan\frac{x}{2}}{1 \quad \tan\frac{x}{2}}$

Exercise-4 : Matching Type Problems

1. (A) If $A \quad B \quad 45$ then $(1 \quad \tan A)(1 \quad \tan B) \quad 2$

(B) $a^2 \quad 5a \quad 6 \sin x \qquad x \quad R$

$$a^2 \quad 5a \quad 6$$

$$a^2 \quad 5a \quad 6 \quad 0 \qquad (a \quad 3)(a \quad 2) \quad 0$$

(C) $$\frac{a \quad \frac{1}{a}^{4} \quad a^4 \quad \frac{1}{a^4} \quad 2}{a \quad \frac{1}{a}^{2} \quad a^2 \quad \frac{1}{a^2}} \quad \frac{a \quad \frac{1}{a}^{4} \quad a^2 \quad \frac{1}{a^2}^{2}}{a \quad \frac{1}{a}^{2} \quad a^2 \quad \frac{1}{a^2}} \quad a \quad \frac{1}{a}^{2} \quad a^2 \quad \frac{1}{a^2} \quad 2$$

(D) $\sum_{k \; 1}^{3} (x \quad k)^2 \quad (x \quad 1)^2 \quad (x \quad 2)^2 \quad (x \quad 3)^2 \quad 0$ No real root

2. (A) $y \quad \dfrac{1 \quad \tan^2(\quad / 4 \quad x)}{1 \quad \tan^2(\quad / 4 \quad x)} \quad \cos(\quad / 2 \quad 2x) \quad \sin 2x$

(B) $0 \quad \log_3 \dfrac{5 \sin x \quad 12 \cos x \quad 26}{13} \quad 1$

(C) $y \quad 2 \sin^2 x \quad \cos x \quad 3 \quad 2 \cos^2 x \quad \cos x \quad 1 \quad 2 \quad \cos x \quad \frac{1}{4}^{2} \quad \frac{7}{8}$

(D) $y \quad 4 \sin^2 \quad 4 \sin \quad \cos \quad \cos^2 \quad (2 \sin \quad \cos \quad)^2$

4. (A) $\cos^2 x \quad \frac{1}{5} \quad \sin x^{2}$

$$(5 \sin x \quad 4)(5 \sin x \quad 3) \quad 0$$

$$\sin x \quad \frac{4}{5} \quad \text{or} \quad \frac{3}{5}$$

(B) $\cot \frac{}{2} \quad 1 \quad \cot$

$$2 \cos^2 \frac{}{2} \quad \cos \quad \sin$$

$$\sin \quad 1 \qquad \frac{3}{2}, \frac{}{2}$$

(C) $f(x) \quad \sin^4 x \quad 8 \sin^2 x \quad 2$

$$f(x) \quad [2, 9]$$

(D) $\log_2 \dfrac{(2x^2 \quad 5x \quad 27)}{(2x \quad 1)^2} \quad 0 \qquad x \quad \frac{1}{2}$

$$2x^2 \quad 9x \quad 26 \quad 0$$

$$2 \quad x \quad \frac{13}{2}$$

5. (A) $f(x) \quad 2\sin^2 x \quad \sin x \quad 6$

$y_{\min} \quad 9 \quad \text{at} \quad \sin x \quad 1$

$y_{\max} \quad \frac{47}{8} \quad \text{at} \quad \sin x \quad \frac{1}{4}$

(B) $f(x) \quad 2\cos^2 x \quad 6$

$y_{\min} \quad 6; \quad y_{\max} \quad 8$

(C) $f(x) \quad \frac{1}{2}[4\sin 2x \quad 1 \quad \cos 2x \quad 3(1 \quad \cos 2x)]$

$\frac{1}{2}[2 \quad 4\sin 2x \quad 4\cos 2x]$

$1 \quad 2(\sin 2x \quad \cos 2x)$

$y_{\max} \quad 1 \quad 2\sqrt{2}; \quad y_{\min} \quad 1 \quad 2\sqrt{2}$

(D) $f(x) \quad \sqrt{2}\sin \quad \frac{}{4} \quad \sin x$

Exercise-5 : Subjective Type Problems

1. $\dfrac{\sin 80 \quad \sin 65 \quad \sin 35}{2\sin 35 \quad \cos 15 \quad 2\sin 35 \quad \cos 35} \quad \dfrac{\sin 80 \quad \sin 65}{2(\cos 15 \quad \cos 35\ )} \quad \dfrac{\sin 80 \quad \sin 65}{4\cos 25 \quad \cos 10} \quad \dfrac{1}{4}$

2. If $A \quad B \quad 45$

$$(1 \quad \cot A)(1 \quad \cot B) \quad 2$$

$$(1 \quad \cot 23\ )(1 \quad \cot 22\ ) \quad 2$$

3.

$$4x^2 \quad 7x \quad 1 \quad 0$$

$$\tan A \quad \tan B \quad \frac{7}{4}$$

$$\tan A \quad \tan B \quad \frac{1}{4}$$

$$\tan(A \quad B) \quad \frac{\tan A \quad \tan B}{1 \quad \tan A \quad \tan B} \quad \frac{7}{3}$$

$$4\sin^2(A \quad B) \quad 7\sin(A \quad B)\cos(A \quad B) \quad \cos^2(A \quad B)$$

$$\frac{4\tan^2(A \quad B) \quad 7\tan(A \quad B) \quad 1}{1 \quad \tan^2(A \quad B)} \quad 1$$

4. $\dfrac{(18 \quad 2) \quad 180}{18} \qquad \dfrac{(n \quad 2) \quad 180}{n} \qquad 60 \quad 360 \qquad n \quad 9$

5. $10\,(1 \quad \cos 2 \quad)^2 \quad 15\,(1 \quad \cos 2 \quad)^2 \quad 24$

$$(5\cos 2 \qquad 1)^2 \quad 0 \qquad\qquad \cos 2 \quad \frac{1}{5} \qquad\qquad \tan^2 \quad \frac{3}{2}$$

6. $\tan \quad \dfrac{3}{8} \quad \dfrac{}{8} \quad \tan \dfrac{3}{8} \quad \tan \dfrac{}{8} \qquad \tan \quad \dfrac{5}{8} \quad \dfrac{3}{8} \quad \tan \dfrac{5}{8} \quad \tan \dfrac{3}{8}$

$$\tan \quad \frac{7}{8} \quad \frac{5}{8} \quad \tan \frac{7}{8} \quad \tan \frac{3}{8} \qquad \tan \quad \frac{9}{8} \quad \frac{7}{8} \quad \tan \frac{9}{8} \quad \tan \frac{7}{8} \qquad \tan \frac{9}{8} \quad \tan \frac{}{8} \quad 0$$

7. $\dfrac{\cos \dfrac{2}{7} \quad 2\cos^2 \dfrac{}{7}}{\cos \dfrac{}{7} \cos \dfrac{2}{7}} \qquad \dfrac{4 \quad \cos \dfrac{2}{7} \quad 2\cos^2 \dfrac{}{7} \quad \sin \dfrac{}{7}}{\sin \dfrac{4}{7}} \qquad \dfrac{4 \quad 1 \quad 2\cos \dfrac{2}{7} \quad \sin \dfrac{}{7}}{\sin \dfrac{3}{7}}$

$$\frac{4 \quad 1 \quad 2\cos \dfrac{2}{7} \quad \sin \dfrac{}{7}}{\sin \dfrac{}{7} \quad 3 \quad 4\sin^2 \dfrac{}{7}} \qquad 4$$

8.
$$a^2 \sec^2 200 \qquad c^2 \tan^2 200 \qquad d^2 \qquad 2cd \tan 200$$
$$b^2 \sec^2 200 \qquad c^2 \qquad d^2 \tan^2 200 \qquad 2cd \tan 200$$
$$a^2 \quad b^2 \quad c^2 \quad d^2$$
$$(a \sec 200 \qquad c \tan 200 \quad)^2 \quad d^2$$
$$(b \sec 200 \qquad d \tan 200 \quad)^2 \quad c^2$$
$$(c^2 \quad d^2)\,(\sec^2 200 \qquad \tan^2 200 \quad) \quad (2bd \quad 2ac) \sec 200 \;\tan 200 \qquad c^2 \quad d^2$$
$$(c^2 \quad d^2)\,(2\tan^2 200 \quad) \quad (2ac \quad 2bd) \sec 200 \;\tan 200$$
$$\frac{2(c^2 \quad d^2)}{ac \quad bd} \quad \frac{2\sec 200}{\tan 200} \quad \frac{2}{\sin 200} \quad \frac{2}{\sin 20}$$

9. $2\cos \dfrac{}{17} \cos \dfrac{9}{17} \quad \cos \dfrac{7}{17} \quad \cos \dfrac{9}{17} \quad \cos \dfrac{10}{17} \quad \cos \dfrac{8}{17} \quad \cos \dfrac{7}{17} \quad \cos \dfrac{9}{17} \quad 0$

10. $\dfrac{\cos \quad\quad \cos 3 \quad\quad \cos 3 \quad\quad \cos 9 \quad\quad \cos 9 \quad\quad \cos 17}{\sin 3 \quad\quad \sin \quad\quad \sin 9 \quad\quad \sin 3 \quad\quad \sin 17 \quad\quad \sin 9} \quad \dfrac{\cos \qquad \cos 17}{\sin 17 \qquad \sin} \quad \tan 9$

11. $8abc \quad 8 \sin 10 \;\sin 50 \;\sin 70 \quad 1$

$$\frac{a \quad b}{c} \quad \frac{\sin 10 \quad \sin 50}{\sin 70} \quad \frac{2\sin 30 \;\cos 20}{\sin 70} \quad 1$$

$$\frac{1}{a} \quad \frac{1}{b} \quad \frac{1}{c} \quad \frac{1}{\sin 10} \quad \frac{1}{\sin 50} \quad \frac{1}{\sin 70} \quad \frac{\sin 50 \;\sin 70 \quad \sin 10 \;\sin 70 \quad \sin 10 \;\sin 50}{\sin 10 \;\sin 50 \;\sin 70} \quad 6$$

12. $\frac{1}{4}\ 4\sin^3 \quad 4\sin^3 \quad \frac{2}{3} \quad 4\sin^3 \quad \frac{4}{3}$

$$\frac{1}{4}\ 3\sin \quad \sin 3 \quad 3\sin \quad \frac{2}{3} \quad \sin(3 \quad 2\ ) \quad 3\sin \quad \frac{4}{3} \quad \sin(3 \quad 4\ )$$

$$\frac{1}{4}\ 3\ \ \sin \quad \sin \quad \frac{2}{3} \quad \sin \quad \frac{4}{3} \quad 3\sin 3 \quad \frac{3}{4}\sin 3$$

13. $\sum_{r\ 1}^{n} \frac{\sin(2^r \quad 2^{r\ 1})}{\cos 2^r \cos 2^{r\ 1}} \quad \sum_{r\ 1}^{n} (\tan 2^r \quad \tan 2^{r\ 1}) \quad \tan 2^n \quad \tan 1$

14. $x \quad \sec \quad \tan\ , \ y = \text{cosec} \quad \cot$

$$y \quad x \quad xy \quad \frac{1 \quad \cos}{\sin} \quad \frac{1 \quad \sin}{\cos} \quad \frac{(1 \quad \sin\ )(1 \quad \cos\ )}{\sin \quad \cos} \quad 1$$

15. $\cos 18 \quad \cos 72 \quad 2\sin 45\ \sin 27$

$$\sqrt{2}\sin 27$$

16. $3(\sin 1 \quad \cos 1)^4 \quad 6(\sin 1 \quad \cos 1)^2 \quad 4(\sin^6 1 \quad \cos^6 1)$

$$3(1 \quad 2\sin 1\cos 1)^2 \quad 6(1 \quad 2\sin 1\cos 1) \quad 4(1 \quad 3\sin^2 1\cos^2 1)$$

$$3(1 \quad 4\sin^2 1\cos^2 1 \quad 4\sin 1\cos 1) \quad 10 \quad 12\sin 1\cos 1 \quad 12\sin^2 1\cos^2 1$$

$$13$$

17. $3^{\sin 2x \quad 2\cos^2 x} \quad \frac{3^3}{3^{\sin 2x \quad 2\cos^2 x}} \quad 28$

Let $3^{\sin 2x \quad 2\cos^2 x} \quad t, \quad t^2 \quad 28t \quad 27 \quad 0 \quad\quad t \quad 1, 27$

If $t \quad 1 \quad\quad \sin 2x \quad 2\cos^2 x \quad 0$

$$2\cos x(\sin x \quad \cos x) \quad 0 \quad\quad x \quad \frac{}{2}, \frac{3}{4}, \frac{7}{4}$$

If $t \quad 27$

$\sin 2x \quad 2\cos^2 x \quad 3$ (Not possible)

$(\sin 2 \quad \cos 2\ )^2 \quad 8\sin 4 \quad 1 \quad 7\sin 4 \quad 1$ (at $\frac{}{2}, \frac{3}{4}, \frac{7}{4}$)

18. $(\sin \quad \text{cosec}\ )^2 \quad (\cos \quad \sec\ )^2$

$$5 \quad \text{cosec}^2 \quad \sec^2$$

$$7 \quad \tan^2 \quad \cot^2$$

$$9$$

19. $\tan 20 \quad \tan 40 \quad \tan 80 \quad \tan 60$

$$\frac{\sin 20\ \cos 80 \quad \sin 80\ \cos 20}{\cos 20\ \cos 80} \quad \frac{\sin 40\ \cos 60 - \sin 60\ \cos 40}{\cos 40\ \cos 60}$$

$$\frac{\sin 100}{\cos 20\ \cos 80} \quad \frac{\sin 20}{\cos 40\ \cos 60}$$

$$\frac{\sin 80}{\cos 20\ \cos 80} \quad \frac{2\sin 20}{\cos 40}$$

$$\frac{\sin 80\ \cos 40 \quad \sin 40\ \cos 80}{\cos 20\ \cos 40\ \cos 80} \quad \frac{\sin 40}{\cos 20\ \cos 40\ \cos 80}$$

$$\frac{8\sin 40\ \sin 20}{\sin(8 \quad 20\ )} \quad 8\sin 40$$

20. $1 \quad \cos 10x \cos 6x \quad 2\cos^2 8x \quad \sin^2 8x$

$2 \quad \cos 16x \quad \cos 4x \quad 2(1 \quad \cos 16x) \quad 1 \quad \cos 16x$

$\cos 4x \quad 1$

$$x \quad \frac{n}{2} \quad (n \quad 0, \quad 1, \quad 2, \quad 3 \ldots\ldots)$$

If $360 \quad k \quad 540$

$k \quad 450 \quad (n \quad 5)$

21. $\cos 20 \quad 2\sin^2 55 \quad 1 \quad \sqrt{2}\sin k$

$\cos 20 \quad 1 \quad \cos 110$

$1 \quad \cos 20 \quad \sin 20$

$1 \quad \sqrt{2}\sin(45 \quad 20\ )$

$k \quad 65$

23. $\tan 19x \quad \dfrac{\cos 96 \quad \cos 6}{\cos 96 \quad \cos 6} \quad \dfrac{2\cos 51\ \cos 45}{2\sin 51\ \sin 45} \quad \cot 51 \quad \tan 141$

$19x \quad 180\ n \quad 141$

24. $\dfrac{2\sin 40 \quad \sin 20}{\cos 20\ \cos 30} \quad \dfrac{2\sin(60 \quad 20\ ) \quad \sin 20}{\cos 20\ \cos 30}$

25. $\cos\dfrac{2}{7} \quad \cos\dfrac{4}{7} \quad \cos\dfrac{6}{7} \quad 1 \quad \dfrac{\sin\dfrac{3}{7}}{\sin\dfrac{}{7}} \quad \cos\dfrac{4}{7} \quad 1 \quad \dfrac{3}{2}$

26. $\dfrac{k}{2}(\cos 2A \quad \cos 3A) \quad \dfrac{11}{8}$

$\dfrac{k}{2}[2\cos^2 A \quad 1 \quad 4\cos^3 A \quad 3\cos A] \quad \dfrac{11}{8}$

$k \quad 4$

27. $3\sin^2 x \quad 4\cos^2 x \quad 3 \quad \cos^2 x$

28. $\tan \quad \tan \quad 12$

$\tan \quad \tan \quad 3$

$$\tan(\quad) \quad \frac{\tan \quad \tan}{1 \quad \tan \quad \tan} \quad 3$$

29. $$\frac{\cos 24 \quad \cos 33}{2\sin 33 \quad \sin^2 57} \qquad \frac{\sin 18 \quad \cos 9}{\sin 9} \quad \cos 18$$

$$\frac{\cos 24 \quad \cos 33}{\sin 57 \quad \cos 24} \quad \frac{\sin 9}{\sin 9} \quad 2$$

30. $$\tan \quad \frac{1 \quad \cos 2}{\cos 2} \quad \frac{1 \quad \cos 4}{\cos 4} \quad \frac{1 \quad \cos 8}{\cos 8}$$

$$\frac{\sin}{\cos} \quad \frac{2\cos^2}{\cos 2} \quad \frac{2\cos^2 2}{\cos 4} \quad \frac{2\cos^2 4}{\cos 8} \qquad \frac{8\sin \quad \cos \quad \cos 2 \quad \cos 4}{\cos 8} \qquad \frac{\sin 8}{\cos 8} \quad \tan 8$$

31. $$y \quad \sin^2 x \quad \cos^2 x \quad \tan^2 x \quad \cot^2 x \quad \text{cosec}^2 x \quad \sec^2 x \quad 6$$

$$y \quad 9 \quad 2(\tan^2 x \quad \cot^2 x) \quad 13$$

❑❑❑

23 TRIGONOMETRIC EQUATIONS

Exercise-1 : Single Choice Problems

1. $\tan^2 x \quad \sec^2 y \quad \frac{5a}{6} \quad 3 \quad 2 \quad a^2 \qquad 6a^2 \quad 5a \quad 6 \quad 0$

2. $[\tan(x \quad y) \quad \cot(x \quad y)]^2 \quad (x \quad 1)^2 \quad 0$

$x \quad 1$ and $\tan^2(x \quad y) \quad 1$

$$x \quad y \quad n \quad \frac{}{4}$$

3. $\sin x \quad \cos x \quad 1$

$$\sin \quad x \quad \frac{}{4} \quad \frac{1}{\sqrt{2}}$$

$$x \quad n \quad (\quad 1)^n \frac{}{4} \quad \frac{}{4}$$

4. $\sin^2(\sin x) \quad 3\sin(\sin x) \quad 2 \quad 0$

$\{\sin(\sin x) \quad 2\}\{\sin(\sin x) \quad 1\} \quad 0$

Equation has no solution.

5. $\tan 2x \quad \tan 6x \qquad \sin 4x \quad 0$

$$4x \quad ,2 \quad ,3 \quad , \qquad ,11$$

$$x \quad \frac{}{4}, \frac{2}{4}, \frac{3}{4}, \qquad , \frac{11}{4}$$

But $\frac{}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}$ are rejected. So number of solutions 5.

6. $3\sin^2 x \quad 6\sin x \quad \sin x \quad 2 \quad 0$

$(3\sin x \quad 1)(\sin x \quad 2) \quad 0$

$\sin x \quad 2$, then

$$\sin x \quad \frac{1}{3}$$

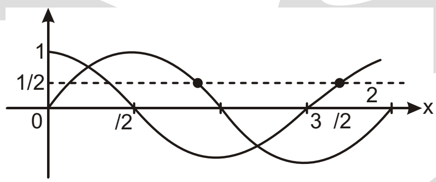
$\sin x \quad \frac{1}{3}$ has 6 solutions for $x \quad [0, 5\ \ ]$

7. $\cos \quad \cos 2 \quad 1$

$2\cos^2 \quad \cos \quad 0$

$\cos \quad 0$ or $\cos \quad \frac{1}{2}$

$\frac{}{2}, \frac{2}{3}, \frac{4}{3}, \frac{3}{2}$

8. 

in $[0, 2\ \ )$ max. $(\sin x, \cos x) \quad \frac{1}{2}$ has two solutions.

9. $(\cot^2 x \quad 2\sqrt{3}\cot x \quad 3) \quad (\cot^2 x \quad 1) \quad (4\operatorname{cosec} x \quad 4) \quad 0$

$(\cot x \quad \sqrt{3})^2 \quad (\operatorname{cosec} x \quad 2)^2 \quad 0$

$\cot x \quad \sqrt{3}$ and $\operatorname{cosec} x \quad 2$

10. $\sin^2 x \quad \sin^2 3x$

$3x \quad n \quad x$

$x \quad \frac{n}{4}, \ x \quad \frac{n}{2}$, hence general solution is $\frac{n}{4}$.

11. $\sin x \quad 0$

$8\sin^2 x \cos^2 x \quad 1$

$2\sin^2 2x \quad 1$

$\cos 4x \quad 0$

$x \quad (2n \quad 1)\frac{}{8} \quad (n \quad I)$

12. $\cos x \quad \cos 2x \quad \cos 3x \quad \cos 4x \quad \cos 5x \quad 5$

$\cos x \quad 1 \quad \cos 2x \quad 1 \quad \cos 3x \quad 1 \quad \cos 4x \quad 1 \quad \cos 5x \quad 1$

$x \quad 2n \quad x \quad n \quad x \quad \frac{2n}{3} \quad x \quad \frac{2n}{4} \quad x \quad \frac{2n}{5}$

$x \quad 2n$

13. $(2\sin x \quad \operatorname{cosec} x)^2 \quad (\tan x \quad \cot x)^2 \quad 0$

$\sin^2 x \quad \frac{1}{2} \quad \tan^2 x \quad 1 \quad x \quad n \quad \frac{}{4}$

14. $$\cos^3 3x \quad \cos^3 5x \quad (2\cos 4x \cos x)^3$$
$$\cos^3 3x \quad \cos^3 5x \quad (\cos 5x \quad \cos 3x)^3$$
$$3\cos 5x \cos 3x(\cos 5x \quad \cos 3x) \quad 0$$
$$\cos 5x \cos 3x \;\; \cos 4x \cos x \quad 0$$

15. $\sin^{100} x \quad 1 \quad \cos^{100} x \qquad \sin^{100} x \quad 1$ and $\cos^{300} x \quad 0$

16. $\sin \qquad 1$ and $\sec^2 4 \quad 1 \quad \sin \qquad \sec 4 \quad 1; \qquad \overline{2}$

17. $(4\sin^2 x \quad \mathrm{cosec}^2 x) \quad (\tan^2 x \quad \cot^2 x) \quad 6$
$$(2\sin x \quad \mathrm{cosec}\, x)^2 \quad (\tan x \quad \cot x)^2 \quad 0$$
$$2\sin x \quad \mathrm{cosec}\, x \quad \text{and} \quad \tan x \quad \cot x$$

18. $\sin^4 \qquad 2\sin^2 \qquad 1 \quad 2$

$(\sin^2 \qquad 1)^2 \quad 2 \quad \cos^4$ (not possible)

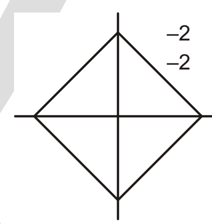
19. $\cos(P\sin x) \quad \sin(P\cos x)$

$$\cos(P\sin x) \quad \cos \quad \overline{2} \quad P\cos x$$

$$P\sin x \quad P\cos x \quad 2n \qquad \overline{2}$$

$$P\sin x \quad P\cos x \quad 2n \qquad \overline{2}$$

20. $|x| \quad |y| \quad 2$



$-2 \quad x \quad 2$
$-2 \quad y \quad 2$

$$\sin \frac{x^2}{3} \qquad 1$$

$$\frac{x^2}{3} \quad \overline{2}$$

$$x \qquad \sqrt{\frac{3}{2}}$$

21. $x \qquad \overline{2},$

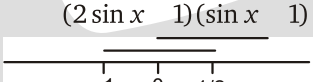
$$\cos 2x \quad |\sin x|$$

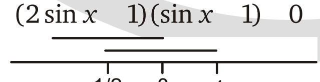
$\sin x \quad 0 \qquad\qquad \sin x \quad 0$

$1 \quad 2\sin^2 x \quad \sin x \quad 0$

$2\sin^2 x \quad 2\sin x \quad \sin x \quad 1 \quad 0 \qquad\qquad 2\sin^2 x \quad 2\sin x \quad \sin x \quad 1 \quad 0$

$(2\sin x \quad 1)(\sin x \quad 1) \quad 0 \qquad\qquad (2\sin x \quad 1)(\sin x \quad 1) \quad 0$





$$\frac{1}{2} \quad \sin x \quad \frac{1}{2}$$

$$\overline{6}, \overline{6} \qquad \frac{5}{6},$$

22. $\sin^4 x \quad \cos^4 x \quad \sin x \cos x$

$1 \quad 2\sin^2 x \cos^1 x \quad \sin x \cos x$

$$2y^2 \quad y \quad 1 \quad 0$$

$$(2y \quad 1)(y \quad 1) \quad 0$$

$$y \quad \frac{1}{2} \qquad\qquad y \quad 1$$

$$2\sin x \cos x \quad 1 \qquad\qquad \sin x \cos x \quad 1$$

$$\sin 2x \quad 1$$

$$2x \quad \overline{2}, \frac{5}{2}$$

$$x \quad \overline{4}, \frac{5}{4}$$

23. $\sin\frac{5x}{2} \quad 1 \quad \sin\frac{x}{2} \quad 1$

24. $\cos 2 \quad \sin^2 \quad \sin^2 \quad \frac{1}{3} \quad \sin \quad \frac{1}{\sqrt{3}}$

25. $b\sin \quad c \quad a\cos$

$b^2(1 \quad \cos 2\ ) \quad c^2 \quad a^2\cos 2 \quad 2ac\cos$

$$(a^2 \quad b^2)\cos 2 \quad 2ac\cos \quad (c^2 \quad b^2) \quad 0$$

$$\cos \quad \cos \quad \frac{c^2 \quad b^2}{a^2 \quad b^2} \qquad \ldots(1)$$

$$a^2(1 \quad \sin 2\ ) \quad c^2 \quad b^2\sin 2 \quad 2bc\sin$$

$$(a^2 \quad b^2)\sin 2 \quad 2bc\sin \quad (c^2 \quad a^2) \quad 0$$

$$\sin \quad \sin \quad \frac{c^2 \quad a^2}{a^2 \quad b^2} \qquad \ldots(2)$$

$$\cos(\qquad) \quad \cos \quad \cos \quad \sin \quad \sin \quad \frac{a^2 \quad b^2}{a^2 \quad b^2}$$

Exercise-2 : One or More than One Answer is/are Correct

1. $2\cos^2 \quad 2\sqrt{2}\cos \quad 3 \quad 0$

$$(\sqrt{2}\cos \quad 1)^2 \quad 4 \qquad \cos \quad \frac{1}{\sqrt{2}} \text{ or } \frac{3}{\sqrt{2}} \text{ (Not possible)}$$

3. $4\sin 3x \quad 5 \quad 4\cos 2x \quad 5\sin x$

$(\sin x \quad 1)(4\sin x \quad 1)^2 \quad 0 \quad x \quad R$

4. $4\cos x(2 \quad 3\sin^2 x) \quad \cos 2x \quad 1 \quad 0$

$\cos x(3\cos x \quad 2)(2\cos x \quad 1) \quad 0$

Least difference $\frac{}{6}$

5. $\cos x \cos 6x \quad 1$

Case-1 : $\cos x \quad 1$ and $\cos 6x \quad 1$

Not possible

Case-2 : $\cos 6x \quad 1$ and $\cos x \quad 1$

$x \quad (2n \quad 1) \quad , (n \quad I)$

7. $2k \quad \sin^2 2x \quad 2\sin 2x \quad 2$

Let $\sin 2x \quad t \qquad t \quad [\ 1, 1]$

$2k \quad t^2 \quad 2t \quad 2 \qquad k \quad \frac{3}{2}, \frac{1}{2}$

8. $f(\ ) \quad \cos \quad \cos\frac{}{8} \quad \cos \quad \cos\frac{3}{8} \quad \cos \quad \cos\frac{5}{8} \quad \cos \quad \cos\frac{7}{8} \quad \cos^4 \quad \cos^2 \quad \frac{1}{8}$

9. $\dfrac{4\sin^2 x\cos^2 x \quad 4\sin^4 x \quad 4\sin^2 x\cos^2 x}{4\cos^2 x \quad 4\sin^2 x\cos^2 x} \quad \tan^4 x \quad \frac{1}{9}$

$\tan x \quad \frac{1}{\sqrt{3}}$

10. $\tan \ (1 \quad \sin^2 \ ) \quad \cot \ (1 \quad \cos^2 \ ) \quad 1 \quad \sin 2 \quad 0 \quad \sin 2 \quad \frac{1}{2}$

11. $2 \ \frac{1}{2}\sin \quad \frac{\sqrt{3}}{2}\cos \quad (x \quad 3)^2 \quad 2$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3

1. $h(x) \quad f^2(x) \quad g^2(x) \quad 2 \quad 2\sin 4x$

$$8\cos 4x \quad 0$$

$$\cos 4x \quad 0$$

Longest interval $\frac{}{4}$

2. $2 \quad 2\sin 4x \quad 4$

$$\sin 4x \quad 1$$

$$x \quad (4n \quad 1)\frac{}{8}$$

3.

$$\sin 3x \quad \cos x \quad \cos 3x \quad \sin x$$

$$\sin 3x \quad \sin x \quad \cos 3x \quad \cos x$$

$$2\sin x\cos 2x \quad 2\sin 2x\sin x$$

$$\sin x \quad 0 \qquad \text{or} \quad \tan 2x \quad 1$$

$$x \quad 0, \qquad \text{or} \qquad x \quad \frac{3}{8}, \frac{7}{8}$$

Exercise-4 : Matching Type Problems

1. (A)

$$\cos^2 x \quad \frac{1}{5} \quad \sin x^{\;2}$$

$$(5\sin x \quad 4)(5\sin x \quad 3) \quad 0$$

$$\sin x \quad \frac{4}{5} \quad \text{or} \quad \frac{3}{5}$$

(B)

$$\cot\frac{}{2} \quad 1 \quad \cot$$

$$2\cos^2\frac{}{2} \quad \cos \quad \sin$$

$$\sin \quad 1 \qquad \frac{3}{2}, \frac{}{2}$$

(C) $f(x) \quad \sin^4 x \quad 8\sin^2 x \quad 2$

$$f(x) \quad [2, 9]$$

(D) $\log_2 \dfrac{(2x^2 \quad 5x \quad 27)}{(2x \quad 1)^2} \quad 0 \qquad x \quad \dfrac{1}{2}$

$$2x^2 \quad 9x \quad 26 \quad 0$$

$$2 \quad x \quad \frac{13}{2}$$

2. (A) $\sin x \quad 1, \cos y \quad 1$ or $\sin x \quad 1, \cos y \quad 1$

(B) $f(x) \quad \cos x \quad \sin x \quad K$

$$k \quad \sqrt{2}$$

(C) $|x^2 \quad 1| \quad 1$ and $|2x^2 \quad 5| \quad 1$

$$x^2 \quad 2$$

(D) $\sin x \quad \sin y \quad \sin(x \quad y)$

$$2\sin\frac{x \quad y}{2}\cos\frac{x \quad y}{2} \quad 2\sin\frac{x \quad y}{2}\cos\frac{x \quad y}{2}$$

$$\sin\frac{x \quad y}{2} \quad 0 \text{ or } \cos\frac{x \quad y}{2} \quad \cos\frac{x \quad y}{2}$$

$$x \quad y \quad 2n \qquad \text{or} \quad x \quad \frac{n}{2}, y \quad \frac{n}{2}$$

if $x \quad 0, y \quad 1$

if $x \quad \frac{1}{2}, y \quad \frac{1}{2}$

if $x \quad \frac{1}{2}, y \quad \frac{1}{2}$

if $y \quad 0, x \quad 1$

Exercise-5 : Subjective Type Problems

1. Let $\sin x \quad 1 \quad a, \cos x \quad 1 \quad b, \sin x \quad c$

$$a^3 \quad b^3 \quad c^3 \quad (a \quad b \quad c)^3$$

$$a \quad b \quad 0 \text{ or } b \quad c \quad 0 \text{ or } c \quad a \quad 0$$

$$\sin x \quad \cos x \quad 2 \text{ or } \sin x \quad \cos x \quad 1 \text{ or } \sin x \quad \frac{1}{2}$$

Total solution $\quad 5$

2. $\sin y \quad 2014\cos y \quad 1$

$$y \quad \frac{}{2}$$

3. $\dfrac{2\sin 6x}{\sin x \quad 1} \quad 0$

$$\sin 6x \quad 0$$

$$x \quad 0, \frac{}{6} \qquad \frac{}{3}, \frac{}{2}$$

$$1 \quad \tan^2 x \quad 2\sqrt{2}\tan x \quad 0$$

$$x \quad \frac{}{8}, \frac{3}{8} \qquad x \quad \frac{}{8}, \frac{}{6} \qquad \frac{}{8}, \frac{3}{8}$$

4. $\sin^4 x \quad 4\sin^2 x \quad (2 \quad k) \quad 0$

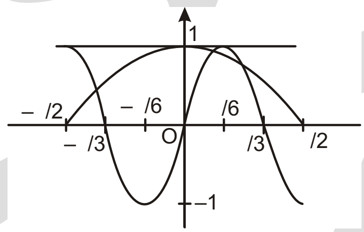
Let $\qquad \sin^2 x \quad t \qquad t \quad [0, 1]$

$$t^2 \quad 4t \quad (2 \quad k) \quad 0$$

$$f(0)\, f(1) \quad 0$$

$$(k \quad 2)(k \quad 1) \quad 0 \qquad 2 \quad k \quad 1$$

5.



6. $2\sin^2 x \quad \sin^2 2x \quad 2$

$$2\sin^4 x \quad 3\sin^2 x \quad 1 \quad 0 \qquad (2\sin^2 x \quad 1)(\sin^2 x \quad 1) \quad 0$$

$$\sin 2x \quad \cos 2x \quad \tan x$$

$$2\tan x \quad 1 \quad \tan^2 x \quad \tan x(1 \quad \tan^2 x)$$

$$\tan^3 x \quad \tan^2 x \quad \tan x \quad 1 \quad 0 \qquad (1 \quad \tan x)(\tan^2 x \quad 1) \quad 0$$

$$2\cos^2 x \quad \sin x \quad 2$$

$$2\sin^2 x \quad \sin x \quad 0$$

$$\sin x(2\sin x \quad 1) \quad 0$$

7. $(3\cot \qquad 1)(\cot \qquad 3) \quad 0$

$\cot \qquad \frac{1}{3}$ and $\cot \qquad 3$

$$, \qquad \frac{}{2} \quad , \quad \frac{}{2}$$

8. $(8\cos 4 \qquad 3)(\cot \qquad \tan \quad)^2 \quad 12$

$$8(2\cos^2 2 \qquad 1) \quad 3 \quad \frac{4\cos^2 2}{\sin^2 2} \qquad 12$$

$$16\cos^4 2 \qquad 8\cos^2 2 \qquad 3 \quad 0$$

$$(4\cos^2 2 \qquad 3)(4\cos^2 2 \qquad 1) \quad 0$$

$$\cos 2 \qquad \frac{\sqrt{3}}{2}$$

9. $2\sin^2 x \quad 4\sin^2 x\cos^2 x \quad 2$

$2\sin^4 x \quad 3\sin^2 x \quad 1 \quad 0 \qquad (\sin^2 x \quad 1)(2\sin^2 x \quad 1) \quad 0$

$$\sin x \qquad \frac{1}{\sqrt{2}}, \quad 1$$

$$\sin 2x \quad \cos 2x \quad \tan x$$

$$\frac{2\tan x}{1 \quad \tan^2 x} \quad \frac{1 \quad \tan^2 x}{1 \quad \tan^2 x} \quad \tan x$$

$$\tan^3 x \quad \tan^2 x \quad \tan x \quad 1 \quad 0$$

$$(\tan x \quad 1)^2(\tan x \quad 1) \quad 0$$

$$2\cos^2 x \quad \sin x \quad 2$$

$$2\sin^2 x \quad \sin x \quad 0$$

$$\sin x(2\sin x \quad 1) \quad 0$$

❑❑❑

24 SOLUTION OF TRIANGLES

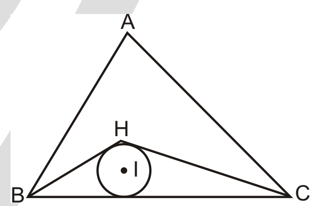
Exercise-1 : Single Choice Problems

1. $\dfrac{\cot A \quad \cot B}{\cot C} \quad \dfrac{\cos A \sin B \quad \cos B \sin A}{(\sin A \sin B)\dfrac{\cos C}{\sin C}} \quad \dfrac{\sin^2 C}{(\sin A \sin B)\cos C} \quad \dfrac{c^2}{ab\ \dfrac{a^2 \quad b^2 \quad c^2}{2ab}}$

$\dfrac{2c^2}{\dfrac{17}{9} \quad 1 \quad c^2} \quad \dfrac{18}{8}$

2. $BIC \quad \overline{2} \quad \dfrac{\quad A}{2}$

$\overline{2} \quad \dfrac{B \quad C}{2}$



3. $\dfrac{1}{64}[(2R\cos A)^2 \quad a^2][(2R\cos B)^2 \quad b^2][(2R\cos C)^2 \quad c^2]$

$\dfrac{1}{64}[(2R\cos A)^2 \quad (2R\sin A)^2][(2R\cos B)^2 \quad (2R\sin B)^2][(2R\cos C)^2 \quad (2R\sin C)^2]$

R^6

4. $B \quad 60$

$2\sin^2 B \quad 3\sin^2 C \qquad \sin C \quad \dfrac{1}{\sqrt{2}} \qquad C \quad 45$

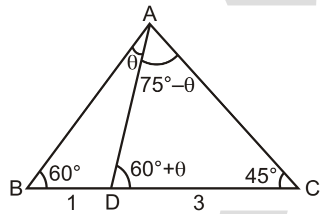
5. $\tan\dfrac{A}{2}\tan\dfrac{C}{2} \quad \dfrac{1}{3}$

$\dfrac{s \quad b}{s} \quad \dfrac{1}{3} \qquad b \quad \dfrac{2}{3}s \qquad \dfrac{a \quad c}{2} \qquad b \qquad b \quad 2 \qquad$ (A.M. G.M.)

6. $\cos A \cos B \cos C \qquad \dfrac{a}{\cos A} \quad 2R\cos A\cos B\cos C \qquad \tan A \quad 2R\cos A \quad \cos B \quad \cos C \qquad (\tan A)$

$$2R\cos A \quad \cos B \quad \cos C \quad \frac{\sin A\sin B\sin C}{\cos A\cos B\cos C} \quad 2R\sin A\sin B\sin C$$

8. In BAD, $\dfrac{BD}{\sin} \quad \dfrac{AD}{\sin 60}$



In CAD, $\dfrac{CD}{\sin(75 \quad)} \quad \dfrac{AD}{\sin 45}$

$$\frac{BD}{\sin}\sin 60 \quad \frac{CD\sin 45}{\sin(75 \quad)}$$

$$\frac{\sin}{\sin(75 \quad)} \quad \frac{BD}{CD}\,\frac{\sin 60}{\sin 45} \quad \frac{1}{\sqrt{6}}$$

9. Length of angle bisector AD $\quad \dfrac{2bc}{b \quad c}\cos\dfrac{A}{2}$

Length of angle bisector BE $\quad \dfrac{2ac}{a \quad c}\cos\dfrac{B}{2}$

Length of angle bisector CF $\quad \dfrac{2ab}{a \quad b}\cos\dfrac{C}{2}$

$$\text{H.M.} \quad \frac{3}{\dfrac{b \quad c}{2bc} \quad \dfrac{a \quad c}{2ac} \quad \dfrac{a \quad b}{2ab}} \quad \frac{3}{\dfrac{1}{a} \quad \dfrac{1}{b} \quad \dfrac{1}{c}}$$

10. $2b \quad a \quad c$

$2\sin B \quad \sin A \quad \sin C$

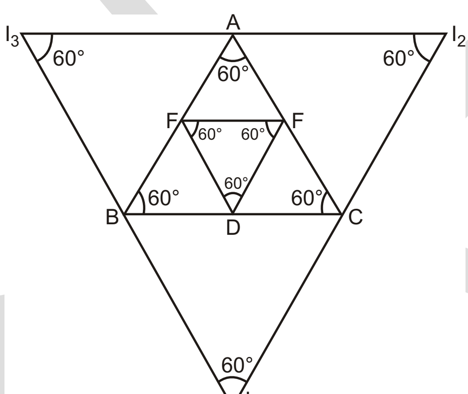
$$2 \quad 2\sin\frac{B}{2}\cos\frac{B}{2} \quad 2\sin\frac{A \quad C}{2} \quad \cos\frac{A \quad C}{2} \quad \sin\frac{B}{2} \quad \frac{1}{2\sqrt{2}}$$

11. $2\cos\dfrac{B \quad C}{2} \quad \dfrac{b \quad c}{a} \quad \dfrac{\sin B \quad \sin C}{\sin A}$

$$\sin\frac{A}{2} \quad \frac{1}{2} \qquad\qquad A \quad 60$$

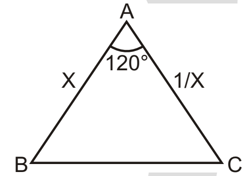
12. $\cos A \quad \dfrac{4 \quad c^2 \quad 1}{4c} \quad \dfrac{1}{4} \quad c \quad \dfrac{3}{c} \quad \dfrac{\sqrt{3}}{2}$

13.



If ABC is an equilateral triangle then DEF and $I_1 I_2 I_3$ are also equilateral triangle

Side of DEF 1 unit $Ar(\ DEF)\ \frac{\sqrt{3}}{4}$



14. $$AD \quad \frac{2x \quad \frac{1}{x}}{x \quad \frac{1}{x}} \cos\frac{}{3} \quad \frac{1}{x \quad \frac{1}{x}}$$

$$AD_{\max} \quad \frac{1}{2}$$

15. $r \quad \frac{\sqrt{3}a}{6}, R \quad \frac{\sqrt{3}a}{3}, r_1 \quad \frac{\sqrt{3}a}{2} \qquad r, R, r_1$ are in A.P.

16. $\sin(B \quad C)\sin(B \quad C) \quad \sin(A \quad B)\sin(A \quad B)$

$$\sin^2 B \quad \sin^2 C \quad \sin^2 A \quad \sin^2 B$$

$$2\sin^2 B \quad \sin^2 A \quad \sin^2 C$$

$$2b^2 \quad a^2 \quad c^2$$ (Using sine rule)

17. $$\tan(A \quad B) \quad \frac{\tan A \quad \tan B}{1 \quad \tan A \tan B} \quad \tan(\quad C)$$

$$\tan C \quad \frac{7}{4} \qquad \sin C \quad \frac{7}{\sqrt{65}}$$

Using sine rule

$$R \quad \frac{c}{2\sin C} \quad \frac{65}{14}$$

18. $\frac{\cos A}{a} \quad \frac{\cos B}{b} \quad \frac{\cos C}{c}$

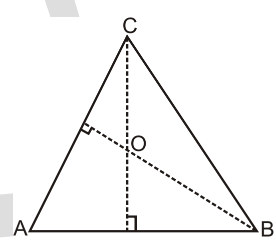
$$\frac{b^2 \quad c^2 \quad a^2}{2abc} \quad \frac{a^2 \quad c^2 \quad b^2}{2abc} \quad \frac{a^2 \quad b^2 \quad c^2}{2abc} \quad \frac{a^2 \quad b^2 \quad c^2}{2abc} \quad \frac{(a \quad b \quad c)^2 \quad 2(ab \quad bc \quad ac)}{2abc}$$

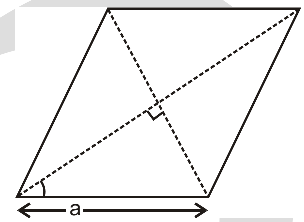
19. $\frac{a \quad c}{b} \quad \frac{b \quad c}{a} \quad \frac{a^2 \quad b^2 \quad ac \quad bc}{ab} \quad \frac{c^2(a \quad b \quad c)}{abc} \quad \frac{c^2(2s)}{4R} \quad \frac{2R}{r} \quad \frac{c}{r}$

20. $a^2(\sin B \quad 1) \quad b^2 \quad c^2 \quad a^2 \quad 2bc\cos A \qquad \cos A \quad 0$

21. $2R \quad \frac{a}{\sin(\quad A)} \quad \frac{a}{\sin A} \quad 2R$

$$R \quad R$$



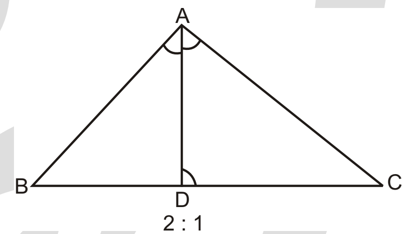


22. $a \quad \sqrt{d_1 d_2}$

$$\frac{d_1}{2} \quad a\cos \quad , \frac{d_2}{2} \quad a\sin$$

$$1 \quad 4\sin \quad \cos$$

$$\sin 2 \quad \frac{1}{2} \quad 2 \quad 30$$



23. $\because \quad (m \quad n)\cot \quad m\cot \quad n\cot$

$$(2 \quad 1)\cot \quad 2\cot \quad \cot$$

Put $\cot \quad \frac{1}{3}, \cot \quad \cot \quad \overline{2} \quad \tan$

We have $1 \quad \frac{2}{\tan} \quad \tan \quad \tan^2 \quad \tan \quad 2 \quad 0$

$$\tan \quad 1 \quad 45$$

24. Circumradius of equilateral $, R \quad \frac{l}{2\sin 60} \quad \frac{l}{\sqrt{3}}$

Diagonal of square $2R \quad a\sqrt{2} \quad 2R \quad a \quad R\sqrt{2} \quad \frac{l\sqrt{2}}{\sqrt{3}}$ Area of square $\frac{2l^2}{3}$

25. $\cos \quad \frac{2^2 \quad (\sqrt{6})^2 \quad (\sqrt{3} \quad 1)^2}{4\sqrt{6}} \quad \frac{\sqrt{3} \quad 1}{2\sqrt{2}}$

26. If a, b, c are in A.P.

$$2\sin B \quad \sin A \quad \sin C \quad \sin\frac{B}{2} \quad \frac{1}{4}$$

$$\frac{s}{r} \quad \frac{6\cos\frac{B}{2}}{1 \quad 2\sin\frac{B}{2}} \quad 3\sqrt{15}$$

27. $\cos(A \quad B) \quad \frac{1 \quad \tan^2\frac{A \quad B}{2}}{1 \quad \tan^2\frac{A \quad B}{2}} \quad \frac{31}{32} \quad \tan\frac{A \quad B}{2} \quad \frac{1}{3\sqrt{7}}$

$$\tan\frac{A \quad B}{2} \quad \frac{a \quad b}{a \quad b}\cot\frac{C}{2} \quad \cos C \quad \frac{1}{8}$$

$$\cos C \quad \frac{a^2 \quad b^2 \quad c^2}{2ab} \quad \frac{1}{8} \quad c \quad 6$$

28. $(b \quad c)\cos(B \quad C) \quad (c \quad a)\cos(C \quad A) \quad (a \quad b)\cos(A \quad B)$

$$[(b\cos A \quad a\cos B) \quad (c\cos A \quad a\cos C) \quad (c\cos B \quad b\cos C)] \quad [a \quad b \quad c] \quad 30$$

30. $A \quad \frac{\ }{7}, \quad B \quad \frac{2}{7}, \quad C \quad \frac{4}{7}$

$$(a^2 \quad b^2)(b^2 \quad c^2)(c^2 \quad a^2) \quad a^2b^2c^2 \quad 1 \quad \frac{b^2}{a^2} \quad 1 \quad \frac{c^2}{b^2} \quad 1 \quad \frac{a^2}{c^2}$$

$$a^2b^2c^2 \quad 1 \quad \frac{\sin^2 \frac{2}{7}}{\sin^2 \frac{\ }{7}} \quad 1 \quad \frac{\sin^2 \frac{4}{7}}{\sin^2 \frac{2}{7}} \quad 1 \quad \frac{\sin^2 \frac{\ }{7}}{\sin^2 \frac{4}{7}} \quad a^2b^2c^2$$

36. $h \quad \frac{a}{2\sqrt{3}}$

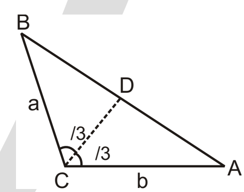
$$Ar(\ ABC) \quad Ar(\ APB) \quad Ar(\ BPC) \quad Ar(\ APC)$$

$$\frac{\sqrt{3}}{4}a^2 \quad \frac{1}{2}a(h \quad h_1 \quad h_2) \qquad h_1 \quad h_2 \quad \frac{a}{\sqrt{3}}$$

37. $\cos 60 \quad \frac{6^2 \quad 7^2 \quad x^2}{2 \quad 6 \quad 7} \qquad x \quad \sqrt{43}$

39. $CD \quad \frac{2ab}{a \quad b}\cos\frac{\ }{3}$

$$\frac{ab}{a \quad b}$$



42. $a \quad b \quad c \quad 48$

$a \quad 20$

$b \quad c \quad 28$

$a \quad b \quad c, a \quad c \quad b$

$20 \quad b \quad 28 \quad b, 20 \quad c \quad 28 \quad c$

$b \quad 4, c \quad 4$

43. In an equilateral triangle

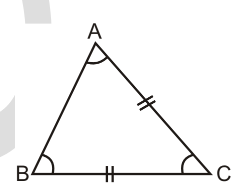
$a \quad b \quad c$

44. $\frac{(a \quad b \quad c)(b \quad c \quad a)(c \quad a \quad b)(a \quad b \quad c)}{4b^2c^2}$

$$\frac{2s(2s \quad 2a)(2s \quad 2b)(2s \quad 2c)}{4b^2c^2} \quad 4 \quad \frac{2(s \quad a)}{bc} \quad \frac{(s \quad b)(s \quad c)}{bc} \quad 4\sin^2\frac{A}{2}\cos^2\frac{A}{2} \quad \sin^2 A$$

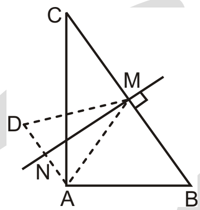
45. $R \quad 4r$

$$R \quad 4 \quad 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$



$$1 \quad 16\sin^2\frac{\ }{2} \quad \cos \qquad 8(1 \quad \cos\ )\cos$$

46. $DMN \sim \quad AMN \qquad DM \quad AM$



47. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \quad (a^3 \quad b^2 \quad c^3 \quad 3abc)$

$$(a \quad b \quad c)(a^2 \quad b^2 \quad c^2 \quad ab \quad bc \quad ac)$$

48. $A \quad \frac{}{7}, B \quad \frac{2}{7}, C \quad \frac{4}{7}$

$$1 \quad \frac{b^2}{a^2} \quad 1 \quad \frac{c^2}{b^2} \quad 1 \quad \frac{a^2}{c^2}$$

$$1 \quad \frac{\sin^2 \frac{2}{7}}{\sin^2 \frac{}{7}} \quad 1 \quad \frac{\sin^2 \frac{4}{7}}{\sin^2 \frac{2}{7}} \quad 1 \quad \frac{\sin^2 \frac{}{7}}{\sin^2 \frac{4}{7}}$$

$$\frac{\sin^2 \frac{}{7} \quad \sin^2 \frac{2}{7}}{\sin^2 \frac{}{7}} \quad \frac{\sin^2 \frac{2}{7} \quad \sin^2 \frac{4}{7}}{\sin^2 \frac{2}{7}} \quad \frac{\sin^2 \frac{4}{7} \quad \sin^2 \frac{}{7}}{\sin^2 \frac{4}{7}}$$

$$1(\sin^2 A \quad \sin^2 B \quad \sin(A \quad B) \ \sin(A \quad B))$$

49. $r_1 \quad \frac{}{s \quad a}, \quad r_2 \quad \frac{}{s \quad b}, \quad r_3 \quad \frac{}{s \quad c}$

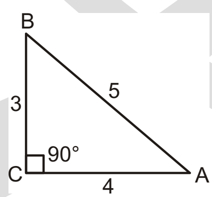
$$\frac{r_1 r_2 r_3}{r^3} \quad \frac{s^3}{(s \quad a)(s \quad b)(s \quad c)}$$

$$\frac{\frac{s \quad a}{s} \quad \frac{s \quad b}{s} \quad \frac{s \quad c}{s}}{3} \quad \frac{s \quad a}{s} \quad \frac{s \quad b}{s} \quad \frac{s \quad c}{s}$$

50. $\sin A \quad \frac{3}{5}$

$\sin B \quad \frac{4}{5}$

$\sin C \quad 1$



51. $\dfrac{r_1 \quad r_2}{1 \quad \cos C} \quad \dfrac{4R\sin\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2} \quad 4R\cos\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}}{2\cos^2\dfrac{C}{2}}$

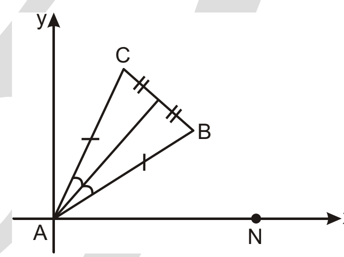
$$\frac{2R \quad \sin\dfrac{A}{2}\cos\dfrac{B}{2} \quad \cos\dfrac{A}{2}\sin\dfrac{B}{2}}{\cos\dfrac{C}{2}} \quad 2R$$

53. $\cos \quad \dfrac{\sin^2 \quad \cos^2 \quad (1 \quad \sin \quad \cos \;)}{2\sin \quad \cos} \quad \dfrac{1}{2}$

55. Since we need to compute the radius of an escribed circle, we would be needing the length of all the sides of the given triangle ABC.

From the question, we already know $AB \quad AC \quad 5$.

For finding the length of side BC, let us draw a line AD which is the bisector of angle BAC, as shown in the figure below.



$$BAD \quad DAC \quad 15$$

Therefore, $\quad \sin 15 \quad \dfrac{BD}{AB} \quad \dfrac{BD}{5}$ and $\sin 15 \quad \dfrac{\sqrt{3} \quad 1}{2\sqrt{2}}$

Therefore, $\quad BD \quad 5\sin 15 \quad \dfrac{5(\sqrt{3} \quad 1)}{2\sqrt{2}}$

We also know that $\quad BC \quad 2BD$

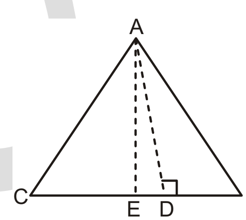
Therefore, $\quad BC \quad \dfrac{5(\sqrt{3} \quad 1)}{\sqrt{2}}$

Now, we know that the required radius

$$r_1 \quad s\tan\frac{A}{2} \quad \frac{AB \quad BC \quad CA}{2} \quad \tan\frac{A}{2}$$

$$\frac{5 \quad \dfrac{5(\sqrt{3} \quad 1)}{\sqrt{2}} \quad 5}{2} \quad (\tan 15\;) \quad \frac{10\sqrt{2} \quad 5\sqrt{3} \quad 5}{2\sqrt{2}} \quad (2 \quad \sqrt{3})$$

56. $ED \quad BE \quad BD \quad \dfrac{a}{2} \quad C\cos B$



$$\frac{a}{2} \quad C \quad \frac{a^2 \quad c^2 \quad b^2}{2ac}$$

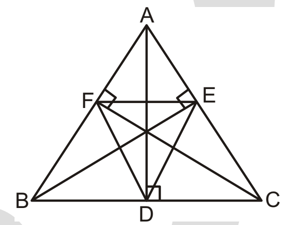
$$\frac{b^2 \quad c^2}{2a}$$

57. $2R(\sin A \cos B \cos C \quad \cos A \sin B \cos C \quad \cos A \cos B \sin C)$

$$2R(\sin(A \quad B)\cos C \quad \cos A \cos B \sin C)$$

$$R(2\sin A \sin B \sin C) \quad \frac{abc}{4R^2} \quad \frac{}{R} \quad \frac{rs}{R}$$

58. In AFE, $\dfrac{b\cos A}{\sin B} \quad 2R_1$



$R_1 \quad R\cos A$

Similarly, $R_2 \quad R\cos B$

and $R_3 \quad R\cos C$

$R_1 \quad R_2 \quad R_3 \quad R(\cos A \quad \cos B \quad \cos C) \quad \dfrac{3}{2}R$

59. Ar (ABC) Ar (OAB) Ar (OBC) Ar (OAC)

$$8 \quad \frac{1}{2}R^2(\sin \quad \sin \quad \sin \ )$$

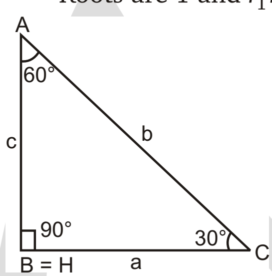
$$\sin \quad \sin \quad \sin \quad \frac{4}{5} \qquad \because R^2 \quad \frac{20}{}$$

Exercise-2 : One or More than One Answer is/are Correct

1. $x^2 \quad r(r_1r_2 \quad r_2r_3 \quad r_1r_3)x \quad (r_1r_2r_3 \quad 1) \quad 0$

$$x^2 \quad (r_1r_2r_3)x \quad (r_1r_2r_3 \quad 1) \quad 0$$

Roots are 1 and $r_1r_2r_3 \quad 1$

2. 

3. $R \quad 2r, r \quad (s \quad a)\tan 30 \quad \dfrac{s}{3}\tan 30 \qquad s$ is irrational is irrational

$$r_1 \quad s\tan 30 \quad 3r \text{ (rational)}$$

4. $D \quad E \quad F \quad \dfrac{}{2}$

5. $a \quad 4, b \quad 8, \quad C \quad 60$

$$\cos C \quad \frac{1}{2} \quad \frac{a^2 \quad b^2 \quad c^2}{2ab} \qquad c \quad 4\sqrt{3}$$

6. If $\frac{r}{r_1} \quad \frac{r_2}{r_3} \qquad \frac{s \quad a}{s} \quad \frac{s \quad c}{s \quad b}$

$a^2 \quad b^2 \quad c^2$

$C \quad 90$

7. $BOC \quad 2 \quad A$

$BIC \quad /2 \quad A/2$

$BHC \qquad A$

8. $\sqrt{3}x^2 \quad 4x \quad \sqrt{3} \quad 0$

$(\sqrt{3}x \quad 1)(x \quad \sqrt{3}) \quad 0 \qquad \frac{1}{\sqrt{3}} \quad x \quad \sqrt{3}$

$30 \quad A, B \quad 60 \qquad 60 \quad C \quad 120$

9. $\cos 2 \quad 2\cos^2 \quad 1$

$$\frac{1}{\sqrt{2}} \quad 2\cos^2 \frac{}{8} \quad 1$$

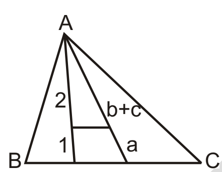
$$2\cos^2 \frac{}{8} \quad 1 \quad \frac{1}{\sqrt{2}}$$

$$\cos^2 \frac{}{8} \quad \frac{\sqrt{2} \quad 1}{2\sqrt{2}}$$

$$\cos C \quad \frac{a^2 \quad b^2 \quad c^2}{2ab}$$ then solve it

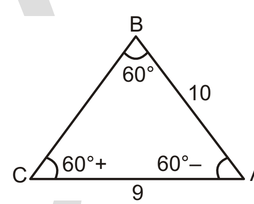
11. $(3\sin A \quad 4\cos B)^2 \quad (4\sin B \quad 3\cos A)^2 \quad 37\,; \quad 9 \quad 16 \quad 24\sin(A \quad B) \quad 37$

12. $\frac{b \quad c}{a} \quad \frac{2}{1}$



13. $A, \quad B, \quad C \quad$ A.P. $\quad B \quad 60$

$\cos 60 \quad \frac{a^2 \quad 10^2 \quad 9^2}{20a}$



14. $\frac{1}{2}ab\sin C$

$\frac{a \quad b}{2} \quad \sqrt{ab} \qquad \frac{\sin A \quad \sin B}{2} \quad \sqrt{\sin A \quad \sin B}$

15. $3\cos A \quad \cos(B \quad C) \quad \cos(B \quad C) \qquad 2\cos A \quad \cos B \quad C) \qquad \cos(A \quad 2C)$

$2 \quad (\tan A \sin 2C \quad \cos 2C)$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 2

2. $r \quad 4r\sin \frac{}{4} \quad \frac{A}{4} \sin \frac{}{4} \quad \frac{B}{4} \sin \frac{}{4} \quad \frac{C}{4}$

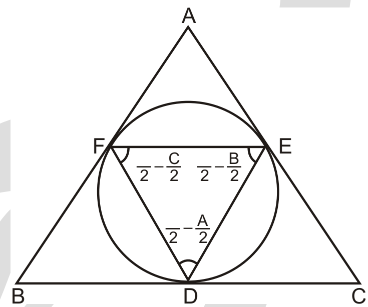
$\frac{r}{r} \quad 4\sin \frac{}{4} \quad \frac{A}{4} \sin \frac{}{4} \quad \frac{B}{4} \sin \frac{}{4} \quad \frac{C}{4}$

$\sin\frac{A}{2} \quad \sin\frac{B}{2} \quad \sin\frac{C}{2} \quad 1$

$r_1 \quad 4r\sin \frac{}{4} \quad \frac{A}{4} \cos \frac{}{4} \quad \frac{B}{4} \cos \frac{}{4} \quad \frac{C}{4}$

$\frac{r_1}{r} \quad 4\sin \frac{}{4} \quad \frac{A}{4} \cos \frac{}{4} \quad \frac{B}{4} \cos \frac{}{4} \quad \frac{C}{4}$

$1 \quad \sin\frac{A}{2} \quad \sin\frac{B}{2} \quad \sin\frac{C}{2}$

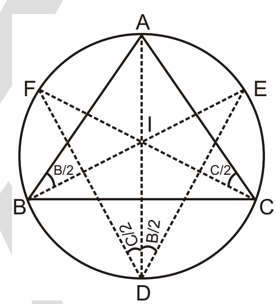


Paragraph for Question Nos. 3 to 4

3. $Ar(\quad DEF)$

$2R^2 \sin \frac{}{2} \quad \frac{A}{2} \sin \frac{}{2} \quad \frac{B}{2} \sin \frac{}{2} \quad \frac{C}{2}$

$2R^2 \cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$



4. $\frac{Ar(\quad ABC)}{Ar(\quad DEF)} \quad \frac{2R^2 \sin A \sin B \sin C}{2R^2 \cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} \quad 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \quad 1$

Paragraph for Question Nos. 5 to 6

Sol. $c/2 \quad R \qquad c \quad 82$

$2r \quad a \quad b \quad c$

$a \quad b \quad 98$ ...(1)

$a^2 \quad b^2 \quad c^2 \quad (82)^2$ ...(2)

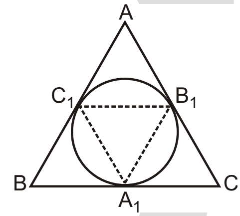
$a \quad 18, b \quad 80$

Paragraph for Question Nos. 7 to 8

Sol. $A_1 \quad \frac{\ }{2} \quad \frac{A}{2}$

$$A_2 \quad \frac{\ }{2} \quad \frac{1}{2}(\quad A_1) \quad \frac{\ }{2} \quad \frac{1}{2} \quad \frac{\ }{2} \quad \frac{A}{2}$$

$$\frac{\ }{4} \quad \frac{A}{4}$$



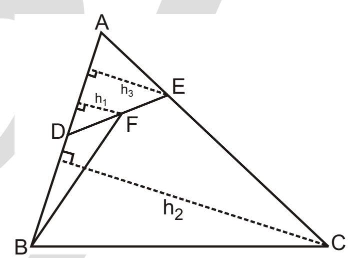
$$A_3 \quad \frac{\ }{2} \quad \frac{1}{2}(\quad A_2)$$

$$\frac{3}{8} \quad \frac{A}{8}$$

$$A_n \quad \frac{\ }{2} \quad 1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \ldots\ldots\ldots \quad \frac{(\quad 1)^n A}{2^n}$$

Paragraph for Question Nos. 9 to 10

Sol. $\frac{\ }{1} \quad \frac{\frac{1}{2} \quad BD \quad h_1}{\frac{1}{2} \quad AB \quad h_2} \quad (1 \quad x)\frac{h_1}{h_2} \quad \frac{h_3}{h_3} \quad (1 \quad x)\, yz$



$$\frac{\ }{2} \quad \frac{\frac{1}{2} \quad EC \quad h_4}{\frac{1}{2} \quad AC \quad h_5} \quad x(1 \quad y)(1 \quad z)$$

Paragraph for Question Nos. 11 to 13

Sol. $\log \quad 1 \quad \frac{c}{a} \quad \log a \quad \log b \quad \log 2$

$a \quad c \quad 2b$

$(c \quad a)\, x^2 \quad 2bx \quad (a \quad c) \quad 0$ has equal roots, then

$$a^2 \quad b^2 \quad c^2$$

Paragraph for Question Nos. 14 to 16

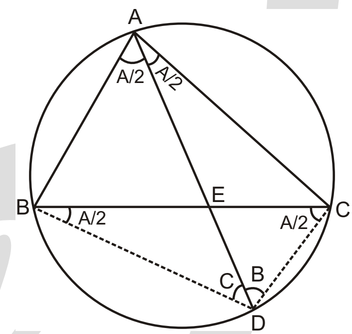
Sol. $\frac{BE}{\sin C} \quad \frac{ED}{\sin \frac{A}{2}}, \frac{EC}{\sin B} \quad \frac{ED}{\sin \frac{A}{2}}$

$$BE \quad EC \quad a \quad \frac{ED}{\sin\frac{A}{2}}(\sin B \quad \sin C)$$

$$ED \quad \frac{a\sin\frac{A}{2} \quad 2R}{b \quad c}$$

$$\frac{l_a \quad \frac{2bc}{b \quad c}\cos\frac{A}{2}}{\frac{2bc}{b \quad c}\cos\frac{A}{2} \qquad \frac{a\sin\frac{A}{2} \quad 2R}{b \quad c}}$$

$$l_a \quad \frac{2\sin B\sin C}{2\sin B\sin C \quad 2\sin^2\frac{A}{2}} \quad \frac{\sin B\sin C}{\sin^2 \quad B \quad \frac{A}{2}}$$



Exercise-4 : Matching Type Problems

2. (A) $3^0\{2^0 \quad 2^{\ 1} \quad 2^{\ 2} \qquad\qquad \} \quad 1\{2\}$

$3^{\ 1}\{2^0 \quad 2^{\ 1} \quad 2^{\ 2} \qquad\qquad \} \quad \frac{1}{3}\{2\}$

$3^{\ 2}\{2^0 \quad 2^{\ 1} \quad 2^{\ 2} \qquad\qquad \} \quad \frac{1}{3}\{2\}$

$\vdots \qquad\qquad\qquad\qquad \vdots$

Hence, $\quad \frac{2 \quad 1}{1 \quad \frac{1}{3}} \quad 3$

(B) $b^2 \quad c^2 \quad a^2 \quad 2bc\cos A \quad 54$

$bc\cos A \quad 27 \quad a^3 \quad\quad a \quad 3$

$$\frac{b^2 \quad c^2}{9} \quad \frac{63}{9} \quad 7$$

(C) Circumcentre of ABC is $(\ \ 1, 0)$.

Point A lie on the circle $(x \quad 1)^2 \quad y^2 \quad 4 \quad x^2 \quad y^2 \quad 2x \quad 3 \quad 0$

(D) $(\cos \quad \sin \quad\quad 6) \quad 6(\sin \quad\quad \cos\ ) \quad 36 \quad \sin^2 \quad \cos^2 \quad 12\sin \quad \cos \quad 36(1 \quad 2\sin \quad \cos\ )$

Let $\sin \quad \cos \quad\quad t$

$$t^2 \quad 84t \quad 0 \qquad t \quad 0$$

If $\sin \quad 0 \qquad \cos \quad 1$

If $\cos \quad 0 \qquad \sin \quad 1 \qquad \overline{\ \ 2}$

3. $r_1 r_2 \quad r_3 r_2 \quad r_1 r_3 \quad S^2 \qquad S \quad 42$

$$\frac{1}{r_1} \quad \frac{1}{r_2} \quad \frac{1}{r_3} \quad \frac{1}{r} \qquad r \quad 8$$

$$r \quad \frac{}{S} \qquad 336$$

4. Use $r \quad \frac{}{s}, \; r_1 \quad \frac{}{s \quad a}, \; r_2 \quad \frac{}{s \quad b}, \; r_3 \quad \frac{}{s \quad a}$

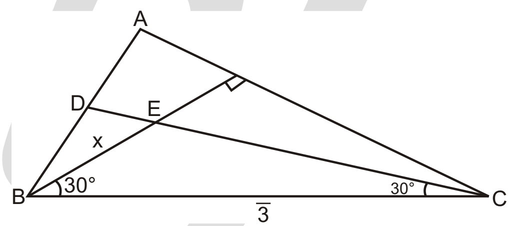
(C) $r \quad 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

and similarly r_1, r_2, r_3

Exercise-5 : Subjective Type Problems

2. $O_1 E O_2 \quad 90$, E is the orthocentre of $O_1 E O_2$

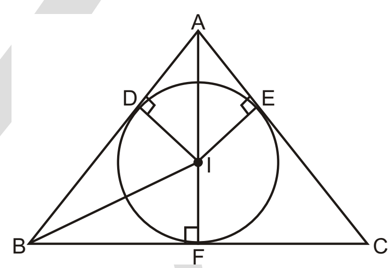
$$\frac{x}{\sin 30} \quad \frac{\sqrt{3}}{\sin 120}; x \quad 1$$



3. $\frac{1}{2} r(AD \quad AE) \quad 5$

$\frac{1}{2} r(BF \quad BD) \quad 10$

$$\frac{BF \quad BD}{AD \quad AE} \quad 2 \qquad \frac{r\cot\frac{B}{2} \quad r\cot\frac{B}{2}}{r\cot\frac{A}{2} \quad r\cot\frac{A}{2}} \quad 2$$



Applying C and D, $\dfrac{\cos\frac{C}{2}}{\sin\frac{A \quad B}{2}} \quad 3$

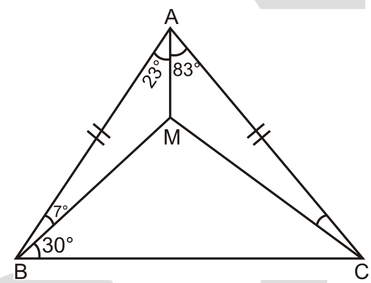
4. $$\frac{1 \quad 2 \quad 3}{3} \quad \frac{(r_1 r_2 r_3)^2}{r^6} \qquad \frac{s}{s \quad a} \quad \frac{s}{s \quad b} \quad \frac{s}{s \quad c}^{\;2}$$

$$\frac{(s \quad a) \quad (s \quad b) \quad (s \quad c)}{3} \qquad [(s \quad a)(s \quad b)(s \quad c)]^{1/3}$$

$$\frac{s^3}{(s \quad a)(s \quad b)(s \quad c)} \quad 27$$

Minimum value 1

5. In ABM, $\dfrac{AB}{\sin 150} \quad \dfrac{AM}{\sin 7}$



In ACM, $\dfrac{AC}{\sin(97 \quad)} \quad \dfrac{AM}{\sin}$

$$\sin \quad 2\sin 7 \ \sin(97 \quad)$$

$$\sin \quad \sin \quad \cos(104 \quad)$$

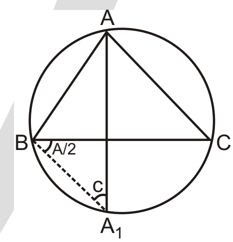
$$\cos(104 \quad) \quad 0$$

$$14$$

6. $\dfrac{AH}{AD} \quad \dfrac{BH}{BE} \quad \dfrac{CH}{CF} \quad \dfrac{R}{}(a\cos A \quad b\cos B \quad c\cos C) \quad \dfrac{R^2}{}(\sin 2A \quad \sin 2B \quad \sin 2C)$

$$\frac{4R^2}{}\sin A\sin B\sin C \quad \frac{bc\sin A}{} \quad 2$$

7. $\dfrac{c}{\sin C} \quad \dfrac{AA_1}{\sin \ B \quad \dfrac{A}{2}}$



$AA_1 \cos\dfrac{A}{2} \quad \sin B \quad \sin C \qquad (\because R \quad 1)$

$$\frac{AA_1\cos\frac{A}{2} \quad BB_1\cos\frac{B}{2} \quad CC_1\cos\frac{C}{2}}{\sin A \quad \sin B \quad \sin C} \quad 2$$

8. $ax^2 \quad bx \quad c \quad 0$ has equal roots, then

$$b^2 \quad 4ac \qquad \ldots(1)$$

$$\frac{\sin A}{\sin C} \quad \frac{\sin C}{\sin A} \quad \frac{a}{c} \quad \frac{c}{a} \quad \frac{a^2 \quad c^2}{ac} \quad \frac{b^2 \quad 2ac\cos B}{ac}$$

$$4 \quad 2\cos B$$

9. $\cot\dfrac{A}{2}, \cot\dfrac{B}{2}, \cot\dfrac{C}{2}$ is AP

In ABC,

$$\cot\frac{A}{2} \quad \cot\frac{B}{2} \quad \cot\frac{C}{2} \quad \cot\frac{A}{2} \ \cot\frac{B}{2} \ \cot\frac{C}{2}$$

$$\cot\frac{A}{2} \ \cot\frac{C}{2} \quad 3$$

AM GM

$$\frac{\cot\frac{A}{2} \quad \cot\frac{C}{2}}{2} \quad \sqrt{\cot\frac{A}{2} \quad \cot\frac{C}{2}} \qquad \cot\frac{B}{2} \quad \sqrt{3}$$

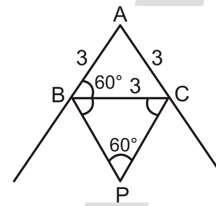
10. $(R^2 \quad 4Rr \quad 4r^2) \quad (4r^2 \quad 12r \quad 9) \quad 0$

$$(R \quad 2r)^2 \quad (2r \quad 3)^2 \quad 0$$

$$r \quad \frac{3}{2} \; ; \; R \quad 2r$$

ABC is an equilateral triangle.

11. In BCP,



$$\frac{3}{\sin 60} \quad \frac{PC}{\sin}$$

$$PC \quad 2\sqrt{3}\sin$$

12. $b \quad c \quad \dfrac{2ab\cos C \quad 2\sqrt{3}ab\sin C}{2b} \quad \dfrac{(a^2 \quad b^2 \quad c^2) \quad 12}{2b}$

13. $R \quad 3, \qquad 6$

$P_{DEF} \quad DE \quad EF \quad DF \quad R(\sin 2A \quad \sin 2B \quad \sin 2C)$

$$4R\sin A\sin B\sin C$$

$$4R \quad \frac{b}{2R}\frac{c}{2R}\sin A \qquad \frac{1}{R}(2 \quad) \quad 4$$

❑❑❑

25 INVERSE TRIGONOMETRIC FUNCTIONS

Exercise-1 : Single Choice Problems

2. $(\cot^{1}x)\ \frac{}{2}\quad \cot^{1}x \quad 2\cot^{1}x \quad \frac{}{2}\cot^{1}x \quad 3\ \frac{}{2}\quad \tan^{1}x \quad 6 \quad 0$

$$(\cot^{1}x)^2 \quad 5\cot^{1}x \quad 6 \quad 0$$

$$(\cot^{1}x)^2 \quad 5(\cot^{1}x) \quad 6 \quad 0$$

$$(\cot^{1}x \quad 3)(\cot^{1}x \quad 2) \quad 0$$

$$2 \quad \cot^{1}x \quad 3$$

$$\cot 3 \quad x \quad \cot 2 \qquad\qquad (\because\ \cot^{1}x \text{ is decreasing})$$

3. $1 \quad \tan^2(\tan^{1}2) \quad 1 \quad \cot^2(\cot^{1}3) \quad 1 \quad 2^2 \quad 1 \quad 3^2 \quad 15$

4. $\displaystyle \sum_{n\ 1} \tan^{1}\frac{(n \quad 1)^2 \quad (n \quad 1) \quad ((n \quad 1)^2 \quad (n \quad 1))}{1 \quad (n \quad 1)^4 \quad (n \quad 1)^2}$

5. $\cot^{1}(\sqrt{\cos\ }) \quad \tan^{1}(\sqrt{\cos\ }) \quad x$

$$\frac{}{2} \quad 2\tan^{1}\sqrt{\cos\ } \quad x$$

$$\frac{}{4} \quad \frac{x}{2} \quad \tan^{1}\sqrt{\cos\ }$$

$$\sqrt{\cos\ } \quad \frac{1 \quad \tan\frac{x}{2}}{1 \quad \tan\frac{x}{2}}$$

$$\tan\frac{x}{2} \quad \frac{1 \quad \sqrt{\cos\ }}{1 \quad \sqrt{\cos\ }} \qquad \sin x \quad \tan^2\frac{}{2}$$

6. $T_n \quad \tan^{\ 1} \dfrac{4}{4n^2 \quad 3} \quad \tan^{\ 1} \dfrac{1}{n^2 \quad (3/4)} \quad \tan^{\ 1} \dfrac{n \quad \frac{1}{2} \quad n \quad \frac{1}{2}}{1 \quad n \quad \frac{1}{2} \quad n \quad \frac{1}{2}}$

$T_n \quad \tan^{\ 1} n \quad \dfrac{1}{2} \quad \tan^{\ 1} n \quad \dfrac{1}{2}$

$S_n \quad \tan^{\ 1} n \quad \dfrac{1}{2} \quad \tan^{\ 1} \dfrac{1}{2} \qquad S \quad \dfrac{}{2} \quad \tan^{\ 1} \dfrac{1}{2}$

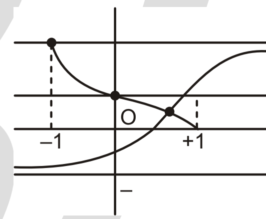
7. $\cos^{\ 1}(1 \quad x) \quad m\cos^{\ 1} x \quad \dfrac{n}{2}$

Domain $x \quad [0, 1]$

$\cos^{\ 1}(1 \quad x) \quad m\cos^{\ 1} x \quad 0 \qquad (\because m \quad 0)$

There is no solution.

8. $2\tan^{\ 1}(2x \quad 1) \quad \cos^{\ 1} x$



$2x \quad 1 \quad 0 \qquad\qquad 1 \quad x \quad 0$

$x \quad \dfrac{1}{2}$

Only one solution

9. Put $x \quad 2\sin \quad , y \quad 3\cos$

$\dfrac{x}{2\sqrt{2}} \quad \dfrac{y}{3\sqrt{2}} \quad 2 \quad \dfrac{\sin}{\sqrt{2}} \quad \dfrac{\cos}{\sqrt{2}} \quad 2 \quad [\ 3, \quad 1]$

$\dfrac{\sin}{\sqrt{2}} \quad \dfrac{\cos}{\sqrt{2}} \quad 2 \quad 1$ only

10. $(\cos^{\ 1} x)^2 \quad (\sin^{\ 1} x)^2 \quad 0 \qquad (\cos^{\ 1} x \quad \sin^{\ 1} x)(\cos^{\ 1} x \quad \sin^{\ 1} x) \quad 0$

$\cos^{\ 1} x \quad \sin^{\ 1} x \quad 0$

$\dfrac{}{2} \quad 2\sin^{\ 1} x \quad 0 \qquad \dfrac{}{2} \quad \sin^{\ 1} x \quad \dfrac{}{4} \qquad 1 \quad x \quad \dfrac{1}{\sqrt{2}}$

11. $f(x) \quad x^2 \quad 7x \quad k(k \quad 3) \quad 0$

$f(0) \quad 0 \quad (\because k \quad (0, 3))$

and are of opposite sign.

$\tan^{\ 1} \qquad \tan^{\ 1} \dfrac{1}{} \quad \tan^{\ 1} \qquad \tan^{\ 1} \dfrac{1}{} \quad 0$

12. $f(x) \quad a \quad 2b\cos^{\ 1} x$

$D_f : [\ 1, 1]$

$f(x)$ is decreasing function.

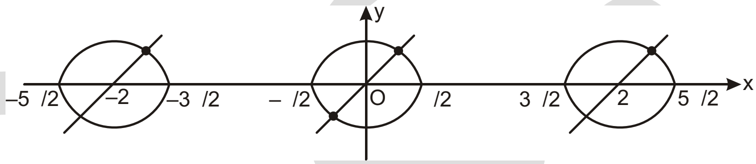
$f(\ 1) \quad 1 \qquad a \quad 2b \quad 1$

and $f(1) \quad 1 \qquad a \quad 1$

13. Let $\tan^{1} x \quad t \qquad t^{2} \quad \frac{}{2} \quad t \quad ^{2} \quad \frac{5 \quad ^{2}}{8}$

$$t \quad \frac{3}{4} \text{ or } \frac{}{4} \qquad \tan^{1} x \quad \frac{}{4} \qquad x \quad 1$$

14.



15. $1 \quad \sin^{1}(\cos^{1}(\sin^{1}(\tan^{1} x))) \quad \frac{}{2}$

$\sin 1 \quad \cos^{1}(\sin^{1}(\tan^{1} x)) \quad 1$

$\cos(\sin 1) \quad \sin^{1}(\tan^{1} x) \quad \cos 1$

$\sin(\cos(\sin 1)) \quad \tan^{1} x \quad \sin(\cos 1)$

$\tan(\sin(\cos(\sin 1))) \quad x \quad \tan(\sin(\cos 1))$

16. $x \quad \frac{1}{x} \quad 2\sin(\cos^{1} y) \qquad x \quad 1 \text{ and } y \quad 0$

17. $\tan^{1} 1 \quad \tan^{1} 2 \quad \tan^{1} 3$

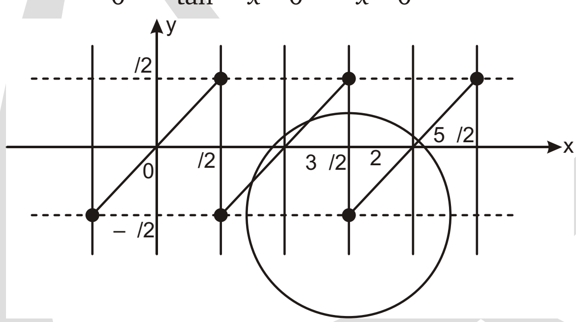
$\tan^{1} 1 \qquad \tan^{1} \frac{5}{1 \quad 6}$

18. Let $\tan^{1} x \quad , \qquad \frac{}{2}, \frac{}{2}$

$$2 \quad \cos^{1}\cos 2 \qquad 2 \quad 0$$

$$0 \quad \tan^{1} x \quad 0 \qquad x \quad 0$$

19.



$$16(x^{2} \quad y^{2}) \quad 48 \; x \quad 16 \; y \quad 31 \; ^{2} \quad 0$$

$$x^{2} \quad y^{2} \quad 3 \; x \quad y \quad \frac{31 \; ^{2}}{16} \quad 0$$

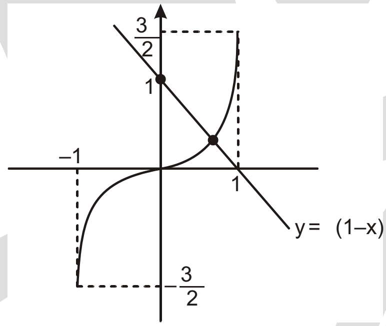
$$x \quad \frac{3}{2}^{2} \qquad y \quad \frac{}{2}^{2} \quad \frac{9}{16}^{2}$$

22. $\sin^{1}(\sin 8) \quad 3 \quad 8 \quad t$

$\tan^{1}(\tan 8) \quad 8 \quad 3 \qquad t$

$$f(t) \quad f(\;\; t)$$
$$2$$

23. Graphs of $y \quad 3\sin^{1} x$ and $y \quad (1 \quad x)$ are



Clearly one point of intersection

24. $D_f : [\;\; 1, 1]$

$f(x)_{\max} \quad \frac{}{2} \quad 6$ at $x \quad 1$

$f(x)_{\min} \quad \frac{}{2} \quad 2$ at $x \quad 1$

27. $\tan^{1}\frac{1}{3} \quad \tan^{1}\frac{1}{7} \quad \tan^{1}\frac{1}{13} \qquad \underset{r\;1}{} \tan^{1}\frac{1}{r^2 \quad r \quad 1}$

$$\underset{r\;1}{} (\tan^{1}(r \quad 1) \quad \tan^{1}(r))$$

28. $\frac{1}{2}\cos^{1} x \quad \tan^{1}\frac{(1/4) \quad (2/9)}{1 \quad (1/4) \quad (2/9)} \quad \tan^{1}\frac{1}{2} \quad \frac{}{2} \quad 2\tan^{1}\frac{1}{2} \quad \frac{1}{2}\cos^{1}\frac{1 \quad \frac{1}{4}}{1 \quad \frac{1}{4}}$

using $2\tan^{1} x \quad \cos^{1}\frac{1 \quad x^2}{1 \quad x^2}$ for $x \quad 0 \quad \frac{1}{2}\cos^{1}\frac{3}{5}$.

29. $\tan^{2}(\sin^{1} x) \quad 1 \qquad \frac{}{2} \quad \sin^{1} x \qquad \frac{}{4}$ or $\frac{}{4} \quad \sin^{1} x \quad \frac{}{2}$

30. $\cot^{1}\frac{1 \quad 2 \quad 4}{4 \quad 2} \quad \cot^{1}\frac{1 \quad 4 \quad 8}{8 \quad 4} \quad \cot^{1}\frac{1 \quad 8 \quad 16}{16 \quad 8}$

$$\cot^{\ 1}(2) \quad \cot^{\ 1}(4) \quad \cot^{\ 1}(4) \quad \cot^{\ 1}(8) \quad \cot^{\ 1}(8) \quad \cot^{\ 1}(16)$$
$$\cot^{\ 1}(2)$$

32. $\sin^{\ 1}(1 \quad x)$ is defined for $x \quad 0$ and $\sin^{\ 1} x \quad \frac{}{2} \quad \cos^{\ 1} x \qquad 1 \quad x \quad 1.$

The given equation is $\sin^{\ 1} x \quad \sin^{\ 1}(1 \quad x) \quad \cos^{\ 1} x$

which can be written as

$$\frac{}{2} \quad \cos^{\ 1} x \quad \frac{}{2} \quad \cos^{\ 1}(1 \quad x) \quad \cos^{\ 1} x$$
$$\cos^{\ 1}(1 \quad x) \quad 2\cos^{\ 1} x$$
$$\cos^{\ 1}(\ 1 \quad x) \quad 2 \qquad \cos^{\ 1}(2x^2 \quad 1)$$
$$\cos^{\ 1}(\ 1 \quad x) \quad \cos^{\ 1}(2x^2 \quad 1) \quad 2$$
$$\cos^{\ 1}(\ 1 \quad x) \quad \cos^{\ 1}(2x^2 \quad 1)$$
$$1 \quad x \quad 2x^2 \quad 1 \qquad 1$$
$$x \quad 0$$

which implies that the total number of solutions $\sin^{\ 1} x \quad \sin^{\ 1}(1 \quad x) \quad \cos^{\ 1} x$ is only one.

33. $(\sin^{\ 1} x)^3 \quad (\cos^{\ 1} x)^3 \quad (\sin^{\ 1} x)(\cos^{\ 1} x)(\sin^{\ 1} x \quad \cos^{\ 1} x) \quad \frac{3}{16}$

$$(\sin^{\ 1} x \quad \cos^{\ 1} x)\{(\sin^{\ 1} x)^2 \quad (\cos^{\ 1} x)^2 \quad (2\cos^{\ 1} x \sin^{\ 1} x)\} \quad \frac{3}{16}$$
$$(\sin^{\ 1} x \quad \cos^{\ 1} x)(\sin^{\ 1} x \quad \cos^{\ 1} x)^2 \quad \frac{3}{16}$$
$$(\sin^{\ 1} x \quad \cos^{\ 1} x)\frac{2}{4} \quad \frac{3}{16}$$
$$2\sin^{\ 1} x \quad \frac{}{2} \quad \frac{}{4}$$
$$2\sin^{\ 1} x \quad \frac{3}{4}$$
$$\sin^{\ 1} x \quad \frac{3}{8}$$
$$x \quad \sin\frac{3x}{8} \text{ or } \cos\frac{x}{8}$$

35. $f(x) \quad \tan^{\ 1} \dfrac{\sqrt{1 \quad x^2} \quad 1}{x}$

$$\frac{\sqrt{1 \quad x^2} \quad 1}{x} \quad y$$

$$y \quad \frac{x\frac{1}{2}\frac{2x}{\sqrt{1 \quad x^2}} \quad (\sqrt{1 \quad x^2} \quad 1)}{x^2}$$

$$\frac{\sqrt{1 \quad x^2} \quad 1}{x^2(\sqrt{1 \quad x^2})} \quad 0 \quad \text{always}$$

$x \qquad\qquad y \quad 1$

$x \qquad\qquad y \quad 1$

$\tan^{1}(\; 1 \quad 1)$

$$\overline{4}, \overline{4} \quad \{0\}$$

40. $\cos^{1} x \quad \cot^{1} x \qquad x \quad [\; 1, 1]$

$$\overline{4}, \frac{7}{4}$$

41. $x^3 \quad bx^2 \quad cx \quad 1 \quad 0$

$f(\; 1) \quad b \quad c \quad 0$

$f(0) \quad 1 \quad 0$

$1 \qquad 0$

B

$B \quad (0, 1)$

$$y \quad 2\tan^{1}(\text{cosec } B) \quad \tan^{1} \frac{2\sin B}{\cos^2 B}$$

$$\tan^{1} \frac{2\cos B}{1 \quad \text{cosec}^2 B} \quad \tan^{1} \frac{2\sin B}{\cos^2 B}$$

42. $f(x) \quad \overline{2} \quad \cot^{1}\{\; x\}$

$$\overline{4} \quad \cot^{1}\{\; x\} \quad \overline{2}$$

43. $\sin^{1}(\sin 3) \quad \tan^{1}(\tan 3) \quad \sec^{1}(\sec 3)$

$(\quad 3) \quad (3 \quad) \quad 3 \quad 3$

44. $(2n \quad, 0)\, n \quad I$

45. $f(x) \quad \sin^{1}([x] \quad 1) \quad 2\cos^{1}([x] \quad 2)$

$1 \quad [x] \quad 1 \quad 1 \qquad 0 \quad [x] \quad 2$

$1 \quad [x] \quad 2 \quad 1 \qquad 1 \quad [x] \quad 3 \qquad [x] \quad 1 \text{ or } 2$

Exercise-2 : One or More than One Answer is/are Correct

2. $\cos^{1} x \quad \tan^{1} x \qquad x \quad [0, 1]$

$\tan^{1} \dfrac{\sqrt{1 \quad x^2}}{x} \qquad \tan^{1} x$

$$x^2 \quad \sqrt{1 \quad x^2} \qquad x^4 \quad x^2 \quad 1 \quad 0$$

$$x^2 \quad \frac{\sqrt{5} \quad 1}{2}$$

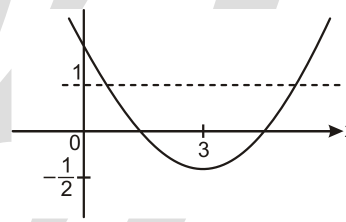
3. $\tan \quad \cos^{1}\dfrac{4}{5} \quad \tan^{1}\dfrac{2}{3} \qquad \tan \quad \tan^{1}\dfrac{17}{6}$

$a \quad 17,\ b \quad 6$

5. $\sin^{1} \quad x^2 \quad 6x \quad \dfrac{17}{2} \qquad \sin^{1} k$

where $1 \quad k \quad 1$

$$y \quad x^2 \quad 6x \quad \frac{17}{2}$$



6. $(\sin^{1} x \quad \cos^{1} x)((\sin^{1} x)^2 \quad (\cos^{1} x)^2 \quad 2\sin^{1} x\cos^{1} x) \quad \dfrac{3}{16}$

$$\sin^{1} x \quad \cos^{1} x \quad \frac{\ }{4} \qquad \cos^{1} x \quad \frac{\ }{8} \qquad x \quad \cos\frac{\ }{8}$$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 2

1. $a \quad 2$

$b \quad 3$

2. $a \quad 0$

$b \quad 3$

Exercise-4 : Matching Type Problems

3. (A) $33n \quad \frac{n}{2}[2 \quad (n \quad 1)^2] \quad\quad n \quad 9$

(B) $x \quad [\ 1, 1] \quad\quad \cos^{\ 1} x \quad \cot^{\ 1} x \quad\quad \frac{}{4}, \frac{7}{4}$

(C) $\cos \quad\quad |1 \quad \sin\ | \quad\quad \cos \quad\quad 0$

Sq. both sides,

$\cos^2 \quad\quad 1 \quad \sin^2 \quad\quad 2\sin$

$\sin \quad\quad 0$ or $\sin \quad\quad 1$

Number of solution $\quad 3$

(D) $a \quad x(x \quad 1)$

Possible values of a are 6, 12, 20, 30.

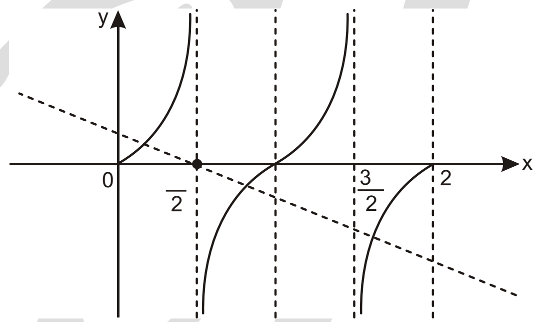
4. (A) $\tan^{\ 1}(3) \quad \tan^{\ 1}(\ 3) \quad 0$

(B) $1 \quad \sin x \quad 2\cos^2 x$

$2\sin^2 x \quad \sin x \quad 1 \quad 0$

$(2\sin x \quad 1)(\sin x \quad 1) \quad 0$

(C) $\tan x \quad \frac{}{4} \quad \frac{}{2}$



(D) $f(x) \quad x^3 \quad x^2 \quad 4x \quad 2\sin x$

$f\ (x) \quad 3x^2 \quad 2x \quad 4 \quad 2\cos x \quad 0$

and $f(0) \quad 0$

Exercise-5 : Subjective Type Problems

1. $5 \quad 2 \quad\quad x^2 \quad 4x$

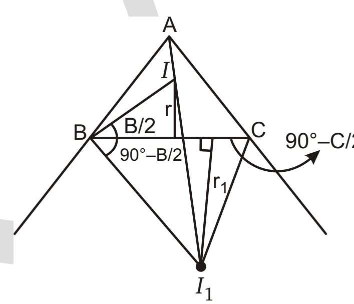
$x^2 \quad 4x \quad (2 \quad\quad 5) \quad 0$

$2 \quad \sqrt{9 \quad 2} \quad\quad x \quad 2 \quad \sqrt{9 \quad 2} \quad\quad\quad\quad 9$

2. $\sin\frac{B}{2} \quad \frac{r}{IB}$

$IB \quad 4R\sin\frac{A}{2}\sin\frac{C}{2}$

$\sin \quad 90 \quad\quad \frac{B}{2} \quad\quad \frac{r_1}{BI_1} \quad\quad BI_1 \quad 4R\sin\frac{A}{2}\cos\frac{C}{2}$



$(II_1)^2 \quad (BI)^2 \quad (BI_1)^2 \quad 16R^2\sin^2\frac{A}{2} \quad\quad\quad\quad \ldots(1)$

$$I_2 I_3 \cos \quad 90 \quad \frac{A}{2} \quad a \qquad \text{(by using pedal triangle)}$$

$$I_2 I_3 \quad 4R\cos\frac{A}{2}$$

$$(I_2 I_3)^2 \quad 16R^2\cos^2\frac{A}{2} \qquad \ldots(2)$$

From (1) & (2) we get $\quad 16$

3. $2\tan^{\ 1}\dfrac{1}{5} \quad \sin^{\ 1}\dfrac{3}{5}$

$\tan^{\ 1}\dfrac{5}{12} \quad \sin^{\ 1}\dfrac{3}{5}$

$\tan^{\ 1}\dfrac{5}{12} \quad \tan^{\ 1}\dfrac{3}{4} \qquad \tan^{\ 1}\dfrac{3}{4} \quad \tan^{\ 1}\dfrac{5}{12}$

$$\tan^{\ 1}\frac{16}{63} \qquad \cos^{\ 1}\frac{63}{65}$$

$$65$$

5. $\displaystyle \mathop{2\tan^{\ 1}}_{n\ \ 0} \frac{2}{n^2 \quad n \quad 4} \qquad \mathop{2\tan^{\ 1}}_{n\ \ 0} \frac{\frac{1}{2}}{\frac{n^2}{4} \quad \frac{n}{4} \quad 1}$

$$\mathop{2\tan^{\ 1}}_{n\ \ 0} \frac{\frac{n}{2} \quad \frac{1}{2} \quad \frac{n}{2}}{\frac{n}{2}\ \frac{n}{2} \quad \frac{1}{2} \quad 1}$$

$$\mathop{2}_{n\ \ 0} \quad \tan^{\ 1}\frac{n}{2} \quad \frac{1}{2} \qquad \tan^{\ 1}\frac{n}{2}$$

6. $\cos^{\ 1}(|3\log_6^2(\cos x) \quad 7|) \quad \cos^{\ 1}(|\log_6^2(\cos x) \quad 1|)$

$$|3\log_6^2(\cos x) \quad 7| \quad |\log_6^2(\cos x) \quad 1|$$

Let $\log_6^2(\cos x) \quad t$

$$|3t \quad 7| \quad |t \quad 1|$$

$$t \quad 3 \text{ and } t \quad 2$$

$$\cos x \quad 6^{\ \sqrt{3}} \text{ and } 6^{\ \sqrt{2}}$$

❑❑❑

26

VECTOR & 3DIMENSIONAL GEOMETRY

Exercise-1 : Single Choice Problems

1. Perpendicular distance from origin

$$d \quad \frac{p}{\sqrt{a^2 \quad b^2 \quad c^2}}$$

$$d^2 \quad \frac{p^2}{a^2 \quad b^2 \quad c^2}$$

2. Area of triangle $\frac{1}{2}|a \quad b| \quad 3$

$$|a||b|\sin\frac{}{3} \quad 6 \qquad |a||b| \quad \frac{12}{\sqrt{3}}$$

$$a \quad b \quad |a||b|\cos\frac{}{3} \quad 2\sqrt{3}$$

4. $|c \quad a|^2 \quad 8 \qquad |c|^2 \quad 2\, c \quad a \quad |a|^2 \quad 8 \qquad |c|^2 \quad 2|c| \quad 1 \quad 0 \qquad |c|^2 \quad 1$

Also, $\quad a \quad b \quad 2\hat{i} \quad 2\hat{j} \quad \hat{k} \qquad |a \quad b| \quad 3$

$$|(a \quad b) \quad c| \quad |a \quad b||c|\sin\frac{}{6} \quad 3 \;1\; \frac{1}{2} \quad \frac{3}{2}$$

5.

$$\cos \quad _1 \quad \frac{4}{5}$$

$$\cos \quad _2 \quad \frac{4}{5}$$

$$\cos^2 \quad _1 \quad \sin^2 \quad _2 \quad 1$$

7. $(a \quad b)\,(a \quad b) \quad 4\sqrt{3} \qquad ab\cos\frac{}{3} \quad 1 \qquad b \quad 1$

$$(a^2b^2 \quad (a \quad b)^2) \quad 4\sqrt{3}$$

$$(4 \quad 1 \quad (1)^2) \quad 4\sqrt{3}$$

$$\frac{4\sqrt{3}}{3}$$

8. $x(3\hat{i} \quad 2\hat{j} \quad 4\hat{k}) \quad y(2\hat{i} \quad 2\hat{k}) \quad z(4\hat{i} \quad 2\hat{j} \quad 3\hat{k}) \qquad (x\hat{i} \quad y\hat{j} \quad z\hat{k})$

$$(3 \qquad)x \quad 2y \quad 4z \quad 0$$

$$2x \qquad y \quad 2z \quad 0$$

$$4x \quad 2y \quad (3 \qquad)z \quad 0$$

For non-trivial solution

$$\begin{vmatrix} 3 & 2 & 4 \\ 2 & & 2 \\ 4 & 2 & 3 \end{vmatrix} \quad 0$$

9. $\begin{vmatrix} a \quad a & a \quad b & a \quad c \\ b \quad a & b \quad b & b \quad c \\ c \quad a & c \quad b & c \quad c \end{vmatrix} \quad [a \quad b \quad c]^2$

10. $|\,c|^2 \quad 4(a \quad b)^2 \quad 9b^2 \quad 4(a^2b^2 \quad (a \quad b)^2) \quad 9b^2 \quad 192$

$$c \quad 3\,b \quad 2\,a \quad b \qquad c^2 \quad 9b^2 \quad 6\,b \quad c \quad 4(a^2b^2 \quad (a \quad b)^2)$$

$$6 \;\; 4 \;\; \sqrt{192}\cos \qquad 288 \qquad \cos \quad \frac{\sqrt{3}}{2}$$

11. $|\,a \quad 2\,b|^2 \quad |\,b \quad 2\,c|^2 \quad |\,c \quad 2\,a|^2 \quad 5a^2 \quad 5b^2 \quad 5c^2 \quad 4(a \quad b \quad b \quad c \quad c \quad a)$

$$15 \quad 4(a \quad b \quad b \quad c \quad c \quad a) \quad 15 \quad 4 \quad \frac{3}{2} \qquad 21$$

$$\because \quad a \quad b \quad b \quad c \quad c \quad a \quad \frac{3}{2}$$

12. $16|\,a||\,b|\sin\frac{}{2} \quad 3|\,a|^2 \quad 3|\,b|^2 \quad 6|\,a||\,b|$

$$3a^2 \quad 10ab \quad 3b^2 \quad 0 \qquad (3a \quad b)(a \quad 3b) \quad 0$$

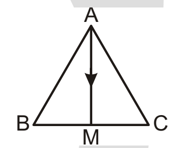
Now $\qquad \overline{OC} \quad \overline{AB} \quad (a \quad b)\;(b \quad a) \quad |\,\overline{OC}||\,\overline{AB}|\cos$

$$\frac{b^2 \quad a^2}{\sqrt{a^2 \quad b^2}\sqrt{a^2 \quad b^2}} \quad \cos \quad \frac{9a^2 \quad a^2}{9a^2 \quad a^2} \qquad \text{(using } b \quad 3a\text{)}$$

$$\cos \quad \frac{4}{5}$$

$$\tan\frac{}{2} \quad \sqrt{\frac{1 \quad \cos}{1 \quad \cos}} \quad \sqrt{\frac{1 \quad \frac{4}{5}}{1 \quad \frac{4}{5}}} \quad \frac{1}{3}$$

13. $\overline{AM} \quad \frac{1}{2}(\overline{AB} \quad \overline{AC})$



14. $\begin{vmatrix} 2 & & 3 \\ 3 & 3 & 5 \\ & 2 & 2 \end{vmatrix} \quad 0 \qquad {}^2 \quad 3 \quad 2 \quad 0$

15. $(a \quad b \quad c) \quad (b \quad c \quad a) \quad (c \quad a \quad b)$

$$(a \quad b \quad c)\,(b \quad c \quad b \quad a \quad c \quad a \quad c \quad b \quad a \quad c \quad a \quad b) \quad 2(a \quad b \quad c)\,(b \quad c \quad c \quad a)$$

$$2([a\ b\ c] \quad [b\ c\ a]) \quad 4[a\ b\ c]$$

16. $(\hat{a} \quad \hat{b}) \quad (\hat{a} \quad \hat{b}) \quad (\hat{a} \ (\hat{a} \quad \hat{b}))\,\hat{b} \quad (\hat{b} \ (\hat{a} \quad \hat{b}))\,\hat{a} \quad (1 \quad \hat{a} \ \hat{b})(\hat{b} \quad \hat{a})$

17. Angle between planes is angle between n_1 and n_2, where $n_1 \quad \overline{AB} \quad \overline{AC}$ and $n_2 \quad \overline{AD} \quad \overline{AC}$

$$n_1 \quad 2\hat{i} \quad 4\hat{j} \quad 3\hat{k}, \qquad n_1 \quad 6\hat{i} \quad 3\hat{j} \quad 6\hat{k}$$

18. $a_1 \quad x_1\hat{i} \quad y_1\hat{j} \quad z_1\hat{k}$, $a_2 \quad x_2\hat{i} \quad y_2\hat{j} \quad z_2\hat{k}$ and $a_3 \quad x_3\hat{i} \quad y_3\hat{j} \quad z_3\hat{k}$ are mutually perpendicular unit vectors, then

$$[a_1 \quad a_2 \quad a_3] \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \quad 1$$

22. On solving, $\quad Ax \quad C$ and $Bx \quad D$

$$\begin{matrix} & 1 & & 3 \\ x & 2 & x & 1 \\ & 3 & & 2 \end{matrix}$$

$$P \quad (1,2,3),\ Q \quad (3,1,2)$$

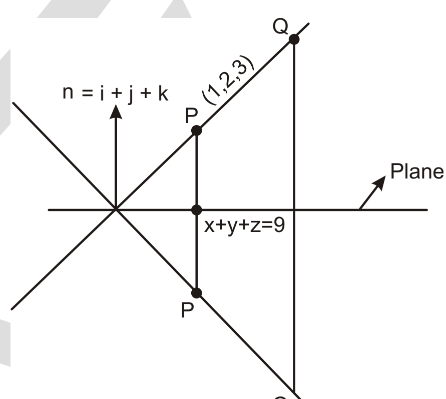
$$PP : \frac{x \quad 1}{1} \quad \frac{y \quad 2}{1} \quad \frac{z \quad 3}{1}$$

$(\quad 1, \qquad\qquad 3)$ lies on plane

$3 \quad 6 \quad 9$

$$P \quad (3,4,5)$$

Similarly $\quad Q \quad (5,3,4)$



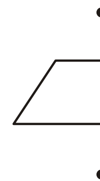
Now check the options.

23. $\overline{AM} \quad (\quad 1)\hat{i} \quad \hat{j}$

$\overline{BM} \quad (\quad 1)\hat{i}$

$\overline{CM} \quad (\quad 3)\hat{i} \quad 2\hat{j} \quad 2\hat{k}$ are coplanar, then $\begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 0 \\ 3 & 2 & 2 \end{vmatrix} \quad 0$

24. Normal vector is parallel to $\overline{PQ}$



$$\frac{x_1 \quad 1}{1} \quad \frac{y_1 \quad 2}{1} \quad \frac{z_1 \quad 3}{1}$$

$$x_1 \qquad 1, \; y_1 \quad 2 \qquad , z_1 \quad 3$$

Mid point of PQ is lie on the plane

$$\frac{2}{3}$$

$$Q \quad \frac{5}{3}, \frac{8}{3}, \frac{11}{3}$$

25.

$$|\hat{a} \quad \hat{b}| \quad 1$$

$$\cos \quad \frac{1}{2}$$

Volume of parallelopiped $[\hat{a} \quad \hat{b} \quad \hat{a} \quad \hat{b}] \quad \sin^2 \qquad \frac{3}{4}$

26. Equation of line PQ

$$\frac{x \quad 3}{1} \quad \frac{y \quad 7}{2} \quad \frac{z \quad 1}{6}$$

Point $Q(3 \qquad , 7 \quad 2 \quad , 1 \quad 6 \quad)$

If it lies on plane $3x \quad 2y \quad 11z \quad 9$, then

$$\frac{25}{59}$$

27. $V_1 \quad [a \quad b \quad c]$

$V_2 \quad [a \quad b \quad 2\,c \quad 3\,a \quad 2\,b \quad c \quad a \quad 4\,b \quad 2\,c] \quad 15[a \quad b \quad c]$

28. Line represented by $x \quad ay \quad b \quad 0, cy \quad z \quad d \quad 0$ is parallel to

$$(\hat{i} \quad a\hat{j}) \quad (c\hat{j} \quad \hat{k}) \quad a\hat{i} \quad \hat{j} \quad c\hat{k}$$

Line represented by $x \quad a\,y \quad b \quad 0, c\,y \quad z \quad d \quad 0$ is parallel to

$$(\hat{i} \quad a\,\hat{j}) \quad (c\,\hat{j} \quad \hat{k}) \quad a\,\hat{i} \quad \hat{j} \quad c\,\hat{k}$$

If these two lines are perpendicular, then

$$aa \quad cc \quad 1$$

29. Equation of line PQ

$$r \quad (2\hat{i} \quad 2\hat{j} \quad 3\hat{k}) \qquad (\hat{i} \quad 5\hat{j} \quad \hat{k})$$

Co-ordinate of $Q(2 \qquad ,5 \qquad 2,3 \qquad)$

If point Q lies on plane, then

$$\frac{10}{27}$$

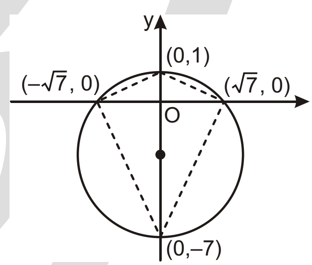
$$\overline{PQ} \qquad \hat{i} \quad 5 \quad \hat{j} \qquad \hat{k} \quad \frac{10}{27}\hat{i} \quad \frac{50}{27}\hat{j} \quad \frac{10}{27}\hat{k}$$

30. $$(a \quad b) \quad c \quad a \quad (b \quad c)$$

$$(a \quad c)\, b \quad (b \quad c)\, a \quad (a \quad c)\, b \quad (a \quad b)\, c$$

$$(a \quad b)\, c \quad (b \quad c)\, a$$

31. Let $\qquad r \quad x\hat{i} \quad y\hat{j}$

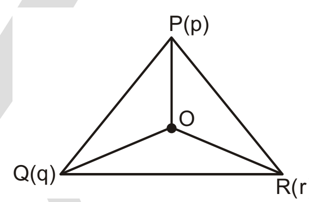


$$r\ (r \quad 6\hat{i}) \quad 7$$

$$x^2 \quad (y \quad 3)^2 \quad 16$$

Area of quadrilateral $\quad 8\sqrt{7}$

33. $$\frac{\frac{1}{2}|(p \quad q) \quad (r \quad q)|}{\frac{1}{2}|q \quad r|} \quad 4$$



Also, $\quad p \quad k_1\, q \quad k_2\, r \quad 0$

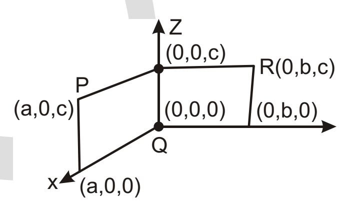
$$p \qquad k_1\, q \quad k_2\, r \quad 0$$

$$k_1 \quad k_2 \quad 1 \quad 4$$

$$k_1 \quad k_2 \quad 3$$

34. Let length, breadth and height of rectangular box be a, b, c respectively.

$$P \quad a\hat{i} \quad c\hat{k}$$



$$R \quad b\hat{j} \quad c\hat{k}$$

$$O \quad \frac{a}{2}\hat{i} \quad \frac{b}{2}\hat{j} \quad \frac{c}{2}\hat{k}$$

$$|\overline{OQ}||\overline{OR}|\cos \qquad \frac{a}{2}\hat{i} \quad \frac{b}{2}\hat{j} \quad \frac{c}{2}\hat{k} \qquad \frac{a}{2}\hat{i} \quad \frac{b}{2}\hat{j} \quad \frac{c}{2}\hat{k}$$

$$\cos \quad \frac{1}{3}$$

Similarly, $\cos \quad \frac{1}{3}$

36. $\mathbf{r} \quad a(\mathbf{m} \quad \mathbf{n}) \quad b(\mathbf{n} \quad \mathbf{l}) \quad c(\mathbf{l} \quad \mathbf{m})$

where $[\mathbf{l}\ \mathbf{m}\ \mathbf{n}] \quad 4, \quad \mathbf{r} \quad \mathbf{l} \quad 4a, \quad \mathbf{r} \quad \mathbf{m} \quad 4b, \quad \mathbf{r} \quad \mathbf{n} \quad 4c$

which imply that

$$\frac{a \quad b \quad c}{\mathbf{r} \quad (\mathbf{l} \quad \mathbf{m} \quad \mathbf{n})} \quad \frac{1}{4}$$

37. The volume tetrahedron is given by $k \quad \frac{1}{6}[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] \quad [\mathbf{a}\ \mathbf{b}\ \mathbf{c}] \quad 6k$

The volume of parallelepiped is given by

$$[\mathbf{a} \quad \mathbf{b}\ \mathbf{b} \quad 2\mathbf{c}\ 3\mathbf{a} \quad \mathbf{c}] \ [\mathbf{a}\ \mathbf{b} \quad 2\mathbf{c}\ 3\mathbf{a} \quad \mathbf{c}] \ [\ \mathbf{b}\ \mathbf{b} \quad 2\mathbf{c}\ 3\mathbf{a} \quad \mathbf{c}]$$

$$[\mathbf{a}\ \mathbf{b}\ 3\mathbf{a} \quad \mathbf{c}] \ [\mathbf{a}\ 2\mathbf{c}\ 3\mathbf{a} \quad \mathbf{c}] \ [\ \mathbf{b}\ \mathbf{b}\ 3\mathbf{a} \quad \mathbf{c}] \ [\ \mathbf{b}\ 2\mathbf{c}\ 3\mathbf{a} \quad \mathbf{c}]$$

$$[\mathbf{a}\ \mathbf{b} \quad \mathbf{c}] \ [\ \mathbf{b}\ 2\mathbf{c}\ 3\mathbf{a}] \quad [\mathbf{a}\ \mathbf{b}\ \mathbf{c}] \ 6[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$$

$$7[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$$

Volume is 42 k.

38. We know that the equation of the plane passing through the line of intersection of planes $p_1 \quad 0$ and $p_2 \quad 0$ is

$$p_1 \quad p_2 \quad 0$$

That is,

$$(x \quad 2y \quad z \quad 10) \quad (3x \quad y \quad z \quad 5) \quad 0 \qquad \ldots(1)$$

Since, this plane passes through the origin $(0, 0, 0)$ satisfies this equation. This implies that

$$(\ 10) \quad (\ 5) \quad 0$$

$$2$$

Substituting the value of in Eq. (1), we get

$$(x \quad 2y \quad z \quad 10) \ 2(3x \quad y \quad z \quad 5) \quad 0$$

That is, $\quad 5x \quad 3z \quad 0$

$$5x \quad 3z \quad 0$$

39. Let the point $P(x_p, y_p, z_p)$ be the required point.

The distance of the point from x-axis is $\sqrt{y_p^2 \quad z_p^2}$.

The distance from the point $(1, \quad 1, 2)$ is

$$\sqrt{(x_p \quad 1)^2 \quad (y_p \quad 1)^2 \quad (z_p \quad 2)^2}$$

$$y_p^2 \quad z_p^2 \quad (x_p \quad 1)^2 \quad (y_p \quad 1)^2 \quad (z_p \quad 2)^2$$

$$x_p^2 \quad 2x_p \quad 2y_p \quad 4z_p \quad 6 \quad 0$$

Therefore, the locus of point P is

$$x^2 \quad 2x \quad 2y \quad 4z \quad 6 \quad 0$$

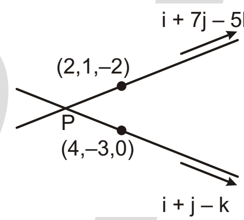
Exercise-2 : One or More than One Answer is/are Correct

3. Point P on line L_1

$$P(2 \quad ,1 \quad 7 \quad , \quad 2 \quad 5 \quad)$$

Point P on line L_2

$$P(4 \quad r, \quad 3 \quad r, \quad r) \qquad 1, r \quad 3$$



Acute angle between L_1 and L_2

$$\cos \quad \frac{13}{15}$$

Equation of plane containing L_1 and L_2 is $x \quad 2y \quad 3z \quad 2 \quad 0$

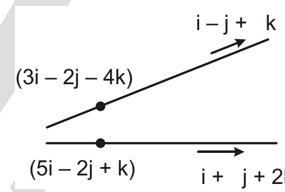
4. $\hat{a} \quad \hat{b} \quad (\hat{b} \quad \hat{c})$

$\hat{a} \quad \hat{b} \quad 1$ and $\hat{a} \quad \hat{c} \quad \hat{b} \quad \hat{c}$

$|\hat{a} \quad \hat{b}| \quad |\hat{b} \quad \hat{c}| \qquad \sin \quad 0 \qquad (\because \qquad \hat{b}\,\hat{c})$

$|\hat{a} \quad \hat{b} \quad \hat{c}|^2 \quad 3 \quad 2(\hat{a} \quad \hat{b} \quad \hat{b} \quad \hat{c} \quad \hat{a} \quad \hat{c}) \quad 5 \quad 4(\hat{b} \quad \hat{c})$

5. If these two lines are coplanar, then



$$\begin{vmatrix} 1 & 1 & \\ 1 & & 2 \\ 2 & 0 & 5 \end{vmatrix} \quad 0$$

$$2 \quad ^2 \quad 5 \quad 1 \quad 0$$

6. $\hat{i} \quad [(a \quad \hat{j}) \quad \hat{i}] \quad \hat{j} \quad [(a \quad \hat{k}) \quad \hat{j}] \quad \hat{k} \quad [(a \quad \hat{i}) \quad \hat{k}] \quad 0$

$$2\,a \quad (\hat{i} \quad \hat{j} \quad \hat{k}) \quad 0 \qquad (2x \quad 1)\hat{i} \quad (2y \quad 1)\hat{j} \quad (2z \quad 1)\hat{k} \quad 0$$

$$x \quad y \quad z \quad \frac{1}{2}$$

7. $[a \quad b \quad c \quad d \quad e \quad f] \quad (a \quad b)\,[(c \quad d) \quad (e \quad f)] \quad (c \quad d)\,[(e \quad f) \quad (a \quad b)]$

$$(e \quad f)\,[(a \quad b) \quad (c \quad d)]$$

$$(a \quad b)\,[(c \quad d) \quad f]\,e \quad [(c \quad d \quad e) \quad f]$$

$$[c \quad d \quad f][a \quad b \quad e] \quad [c \quad d \quad e][a \quad b \quad f]$$

Similarly, solve other 2.

8. $3(a \quad b) \quad (b \quad c) \quad 2(c \quad d) \quad 0$

$$\frac{\overline{BC} \quad 2\,\overline{CD}}{1 \quad 2} \quad \overline{BA}$$

10. $b \quad 2\hat{c} \quad \hat{a}$

$$|b|^2 \quad 4 \quad ^2 \quad 4 \quad \frac{1}{4} \quad 16 \qquad 4, 3$$

11. $L_1 : x \quad y \quad z$

$$L_2 : \frac{x \quad 1}{1} \quad \frac{y \quad 1}{1} \quad \frac{z}{1}$$

Shortest distance $\frac{1}{\sqrt{2}}$

Equation of plane containing line L_2 and parallel to L_1

$$y \quad z \quad 1 \quad 0$$

Distance of origin from this plane $\frac{1}{\sqrt{2}}$

12. $r\ (a \quad b \quad c) \quad 0$

$$[a\ b\ c](\sin x \quad \cos y \quad 2) \quad 0$$

$$\sin x \quad 1 \text{ and } \cos y \quad 1$$

13. $(a \quad b) \quad (c \quad d) \quad [(a \quad b) \quad d]\,c \quad (a \quad b \quad c)\,d \quad r\,c \quad s\,d$

where $r \quad [a \quad b \quad c]$ and $s \quad [a \quad b \quad c]$ as c and d are non-collinear.

Similarly, $h \quad [b \quad c \quad d]$ and $k \quad [a \quad c \quad d]$

14. Here, $\hat{i} \quad 2\hat{j}, \quad 2\hat{i} \quad a\hat{j} \quad 10\hat{k}$ and $12\hat{i} \quad 20\hat{j} \quad a\hat{k}$

$$[\qquad] \begin{vmatrix} 1 & 2 & 0 \\ 2 & a & 10 \\ 12 & 20 & a \end{vmatrix} \quad a^2 \quad 24a \quad 240 \quad 0, \text{for all } a$$

, and are non-coplanar or linearly independent for all a.

Hence, (a, b, c) is the correct answer.

19. Let $\mathbf{r} \quad x\hat{\mathbf{i}} \quad y\hat{\mathbf{j}} \quad z\hat{\mathbf{k}}$

If $\mathbf{r} \quad \hat{\mathbf{i}} \quad \hat{\mathbf{j}} \quad \hat{\mathbf{k}} \qquad\qquad y\hat{\mathbf{k}} + z\hat{\mathbf{j}} \quad \hat{\mathbf{j}} \quad \hat{\mathbf{k}}$

$\mathbf{r} \quad x\hat{\mathbf{i}} \quad \hat{\mathbf{j}} \quad \hat{\mathbf{k}}$

If $\mathbf{r} \quad \hat{\mathbf{j}} \quad \hat{\mathbf{i}} \quad \hat{\mathbf{k}} \qquad\qquad x\hat{\mathbf{k}} \quad z\hat{\mathbf{i}} \quad \hat{\mathbf{i}} \quad \hat{\mathbf{k}}$

$\mathbf{r} \quad \hat{\mathbf{i}} \quad y\hat{\mathbf{j}} \quad \hat{\mathbf{k}}$

20. (A) See dot product

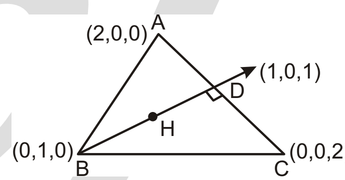
(C) $y \quad \ln(e^{\ 2} \quad e^x)$

$e^y \quad e^{\ 2} \quad e^x$

21. $(\ 3 \quad 4 \ ,6 \quad 3 \ ,2 \ ) \quad (\ 2 \quad 4 \ ,7 \qquad , \ )$

Exercise-3 : Comprehension Type Problems

Paragraph for Question Nos. 1 to 3



1. $AB \quad BC$

p.v. of $H \quad \hat{j} \quad r(\hat{i} \quad \hat{j} \quad \hat{k})$

Also, $\quad \overline{AH} \ \ \overline{BC} \quad 0$

$[(r \quad 2)\hat{i} \quad (1 \quad r)\hat{j} \quad r\hat{k}] \ (\ \hat{j} \quad 2\hat{k}) \quad 0$

$r \quad 1 \quad 2r \quad 0 \qquad\qquad r \quad \frac{1}{3}$

2. p.v. of $H \quad \frac{\hat{i}}{3} \quad \frac{2\hat{j}}{3} \quad \frac{\hat{k}}{3}$

p.v. of centroid $\quad \frac{2}{3}\hat{i} \quad \frac{\hat{j}}{3} \quad \frac{2\hat{k}}{3}$

p.v. of $S \quad \frac{3(\text{p.v.) of centroid} \quad \text{p.v. of } H}{2}$

y coordinate of $S \quad \frac{1}{6}$

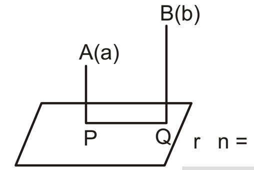
3. Let $P \quad (a, b, c)$

$(a \quad 2)^2 \quad b^2 \quad c^2 \quad a^2 \quad (b \quad 1)^2 \quad c^2 \quad a^2 \quad b^2 \quad (c \quad 2)^2 \quad a^2 \quad b^2 \quad c^2$

$P \quad 1, \frac{1}{2}, 1$

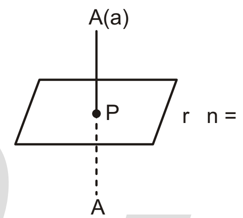
$PA \quad \frac{3}{2}$

Paragraph for Question Nos. 4 to 6



4. $PQ \quad (b \quad a)\cos$ (where angle between AB and plane)

$$\frac{|(b \quad a) \quad n|}{|n|}$$



5. Equation of line AP is $r \quad a \quad n$

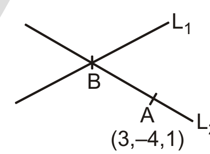
For point $P(a \quad n) \quad n \quad d$

$$\frac{d \quad a \quad n}{|n|^2}$$

$$P \quad a \quad \frac{d \quad a \quad n}{|n|^2} \qquad A \text{ is } a \quad 2\frac{d \quad a \quad n}{|n|^2} n$$

6. Distance $|BQ \quad AP|$ $\left|\frac{b \quad n \quad d}{|n|} \quad \frac{a \quad n \quad d}{|n|}\right| \left|\frac{(b \quad n) \quad n}{|n|}\right|$

Paragraph for Question Nos. 7 to 9



7. $B(3 \quad 2 \ , \quad 1 \quad 3 \ , 2 \quad)$

d_r of $L_2 \quad 2 \ , \quad 3 \qquad 1$

L_2 is parallel to plane $r \ (2\hat{i} \quad \hat{j} \quad \hat{k}) \quad 5$

$$4 \quad 3 \quad 3 \quad 1 \qquad 0$$

$$2 \qquad 2 \qquad\qquad 1$$

$B(1, 2, 3)$

So, equation of L_2 is $r \quad (3\hat{i} \quad 4\hat{j} \quad \hat{k}) \quad (\hat{i} \quad 3\hat{j} \quad \hat{k})$

8. Equation of plane contain L_1 & L_2 is

$$\begin{vmatrix} x \quad 3 & y \quad 1 & z \quad 2 \\ 0 & 3 & 1 \\ 1 & 6 & 2 \end{vmatrix} \quad 0$$

i.e., $(x \quad 3)(6 \quad 6) \quad (y \quad 1)(0 \quad 1) \quad (z \quad 2)(0 \quad 3) \quad 0$

$$y \quad 3z \quad 5 \quad 0$$

9. Any point of L_1 is $(3 \quad 2 \ , \quad 1 \quad 3 \ , 2 \quad)$

if on plane , then

$2(3 \quad 2\ ) \quad 1(\ 1 \quad 3\ ) \quad 1(2 \qquad) \quad 5$

$2 \quad 2 \qquad 1$

$Q(5, \quad 4, 1)$

if on xy plane, then $\quad 2 \qquad 0 \qquad 2$

$R(7, \quad 7, 0)$

Volume of tetrahedran $\quad \frac{1}{6}[\overline{OA} \quad \overline{OQ} \quad \overline{OR}\,] \quad \frac{1}{6}\begin{vmatrix} 3 & 4 & 1 \\ 5 & 4 & 1 \\ 7 & 7 & 0 \end{vmatrix} \quad \frac{7}{3}$

Paragraph for Question Nos. 10 to 11

Sol. Use crammer rule,

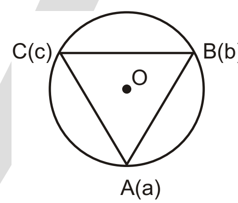
Intersect at a unique point $\quad D \quad 0$

Do not have any common point of intersection.

$D \quad 0$ and atleast any one of D_x, D_y, D_z is non-zero (condition of no solution)

Paragraph for Question Nos. 12 to 14

Sol. $|\mathbf{a}| \quad |\mathbf{b}| \quad |\mathbf{c}| \quad r$



PV of E : $\dfrac{\mathbf{a} \quad \dfrac{\mathbf{a} \quad \mathbf{b}}{2} \quad \mathbf{c}}{3}$

$\mathbf{e} \quad \dfrac{3\,\mathbf{a} \quad \mathbf{b} \quad 2\,\mathbf{c}}{6}$

PV of G : $\mathbf{g} \quad \dfrac{\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}}{3}$

12. $\overline{\mathbf{OE}} \ \overline{\mathbf{CD}} \quad 0 \qquad \dfrac{3\,\mathbf{a} \quad \mathbf{b} \quad 2\mathbf{c}}{6} \quad \dfrac{\mathbf{a} \quad \mathbf{b}}{2} \quad \mathbf{c} \quad 0$

$\mathbf{a}\ (\mathbf{b} \quad \mathbf{c}) \quad 0$

$\overline{\mathbf{OA}} \quad \overline{\mathbf{BC}}$

ABC must be isosceles with base BC.

$|\overline{\mathbf{AC}}| \quad |\overline{\mathbf{AB}}|$

13. $\overline{\mathbf{GE}}\ \overline{\mathbf{CD}}\quad 0 \qquad \frac{3\mathbf{a}\quad \mathbf{b}\quad 2\mathbf{c}}{6}\quad \frac{\mathbf{a}\quad \mathbf{b}\quad \mathbf{c}}{3}\qquad \frac{\mathbf{a}\quad \mathbf{b}}{2}\quad \mathbf{c}\quad 0$

$(\mathbf{a}\quad \mathbf{b})\ \mathbf{c}\quad 0 \qquad \overline{\mathbf{AB}}\quad \overline{\mathbf{OC}}$

ABC must be isosceles with base *AB*.

Circumcentre and centroid lie on median through *C*.

Orthocenter also lie on median through *C*.

14. $[\overline{\mathbf{AB}}\ \overline{\mathbf{AC}}\ \overline{\mathbf{AB}}\quad \overline{\mathbf{AC}}]\quad (\overline{\mathbf{AB}}\ \overline{\mathbf{AC}})^2$

$(\mathbf{a}\quad \mathbf{b}\quad \mathbf{b}\quad \mathbf{c}\quad \mathbf{c}\quad \mathbf{a})^2$

$[\overline{\mathbf{AE}}\ \overline{\mathbf{AG}}\ \overline{\mathbf{AE}}\quad \overline{\mathbf{AG}}]\quad (\overline{\mathbf{AE}}\quad \overline{\mathbf{AG}})^2 \qquad \frac{1}{18}(\mathbf{a}\quad \mathbf{b}\quad \mathbf{b}\quad \mathbf{c}\quad \mathbf{c}\quad \mathbf{a})^2$

$\frac{1}{324}(\overline{\mathbf{AB}}\quad \overline{\mathbf{AC}})^2$

Paragraph for Question Nos. 15 to 16

15. $D(3,\quad 1,2)$ $\qquad$ *AB* lies along $(0,1,2)$

CD lies along $(3,\quad 2,0)$

Equation of plane containing *AB* line

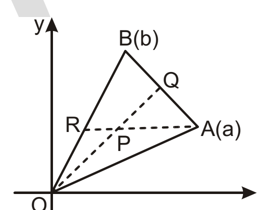
$$\begin{vmatrix} x\quad 1 & y\quad 1 & z\quad 1 \\ 0 & 1 & 2 \\ 2 & 2 & 0 \end{vmatrix}\quad 2(x\quad 1)\quad 2(y\quad 1)\quad (z\quad 1)\quad 0$$

Containing *CD* line $2(x\quad 1)\quad 2(y\quad 1)\quad (z\quad 2)\quad 0$

16. $r\quad (3,\quad 1,2)\quad d(1,0,0)$

Equation of *ABC* plane is $x\quad 1$.

Paragraph for Question Nos. 17 to 18



17. $R\ \frac{2\mathbf{b}}{5}$ and $Q\ \frac{3\mathbf{b}\quad 2\mathbf{a}}{5}$

$$\frac{\frac{3\mathbf{b}\quad 2\mathbf{a}}{5}}{1}\qquad \frac{\mathbf{a}\quad 1\ \frac{2\mathbf{b}}{5}}{1}$$

$$\frac{2}{5(\quad 1)}\quad \frac{}{1}\ \text{and}\ \frac{3}{5(\quad 1)}\quad \frac{2}{5(\quad 1)}\qquad \frac{10}{9}$$

18. $\text{Ar}(\ \ OPA)\ \ \frac{1}{2}\left|\overline{\mathbf{OP}}\ \ \ \overline{\mathbf{OA}}\right|\ \ \frac{1}{2}\ \frac{2}{19}(3\,\mathbf{b}\ \ \ 2\,\mathbf{a})\ \ \ \mathbf{a}\ \ \ \frac{3}{19}(\mathbf{b}\ \ \ \mathbf{a})$

$\text{Ar}(PQBR)\ \ \frac{1}{2}\left|\overline{\mathbf{OQ}}\ \ \ \overline{\mathbf{OB}}\ \ \ \overline{\mathbf{OP}}\ \ \ \overline{\mathbf{OR}}\right|\ \ \frac{1}{2}\ \ \frac{3\,\mathbf{b}\ \ \ 2\,\mathbf{a}}{5}\ \ \ \mathbf{b}\ \ \frac{2}{19}(3\,\mathbf{b}\ \ \ 2\,\mathbf{a})\ \ \frac{2\,\mathbf{b}}{5}$

$$\frac{3}{19}(\mathbf{a}\ \ \ \mathbf{b})$$

Exercise-4 : Matching Type Problems

1. (A) Line $\frac{x\ \ 1}{2}\ \frac{y\ \ 2}{3}\ \frac{z}{1}$ is along the vector $a\ \ \ 2\hat{i}\ \ \ 3\hat{j}\ \ \ \hat{k}$ and line $r\ \ \ (3\hat{i}\ \ \ \hat{j}\ \ \ \hat{k})\ \ \ t(\hat{i}\ \ \ \hat{j}\ \ \ \hat{k})$ is along the vector $b\ \ \ \hat{i}\ \ \ \hat{j}\ \ \ \hat{k}$. Here $a\ \ \ \ b$.

Also, $\begin{vmatrix} 3\ \ 1 & 1\ \ (\ \ 2) & 1\ \ 0 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix}\ \ 0$

(B) The direction ratios of the line $x\ \ \ y\ \ \ 3z\ \ \ 4\ \ \ 0\ \ \ 2x\ \ \ y\ \ \ 3z\ \ \ 5\ \ \ 0$ are

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{vmatrix}\ \ \hat{i}\ \ \ 7\hat{j}\ \ \ 3\hat{k}$$

Hence, the give two lines are parallel.

(C) The given lines are

$(x\ \ \ t\ \ \ 3, y\ \ \ 2t\ \ \ 1, z\ \ \ \ 3t\ \ \ 2)$ and $r\ \ \ (t\ \ \ 1)\hat{i}\ \ \ (2t\ \ \ 3)\hat{j}\ \ \ (\ \ t\ \ \ 9)\hat{k}$,

or $\frac{x\ \ 3}{1}\ \ \frac{y\ \ 1}{2}\ \ \frac{z\ \ 2}{3}$ and $\frac{x\ \ 1}{1}\ \ \frac{y\ \ 3}{2}\ \ \frac{z\ \ 9}{1}$

The lines are perpendicular as $(1)(1)\ \ \ (\ \ 2)(2)\ \ \ (\ \ 3)(\ \ 1)\ \ \ 0$

Also, $\begin{vmatrix} 3\ \ 1 & 1\ \ 3 & 2\ \ (\ \ 9) \\ 1 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix}\ \ 0$

Hence, the lines are intersecting.

(D) The given lines are $r\ \ \ (\hat{i}\ \ \ 3\hat{j}\ \ \ \hat{k})\ \ \ t(2\hat{i}\ \ \ \hat{j}\ \ \ \hat{k})$ and $r\ \ \ (\ \hat{i}\ \ \ 2\hat{j}\ \ \ 5\hat{k})\ \ \ s(\hat{i}\ \ \ 2\hat{j}\ \ \ \frac{3}{4}\hat{k})$.

$$\begin{vmatrix} 1\ \ (\ \ 1) & 3\ \ (\ \ 2) & 1\ \ 5 \\ 2 & 1 & 1 \\ 1 & 2 & 3/4 \end{vmatrix}\ \ 0$$

Hence, the lines are coplanar and hence intersecting (as the lines are not parallel).

2. (A) If a, b and c are mutually perpendicular, then $[a \quad b \;\; b \quad c \;\; c \quad a] \quad [a \quad b \quad c]^2$

$$(|a||b||c|)^2 \quad 16$$

(B) Given a and b are two unit vectors, $i.e.$, $|a| \quad |b| \quad 1$ and angle between them is $\frac{}{3}$.

$$\sin \quad \frac{|a \quad b|}{|a||b|} \qquad \sin\frac{}{3} \quad |a \quad b|; \quad \frac{\sqrt{3}}{2} \quad |a \quad b|$$

Now $[a \quad b \quad a \quad b \quad b] \quad [a\;b\;b] \quad [a\;a \quad b\;b] \quad 0 \quad [a\;a \quad b\;b]$

$$(a \quad b)\,(b \quad a) \qquad |a \quad b|^2 \quad \frac{3}{4}$$

(C) If b and c are orthogonal, $b \quad c \quad 0$

Also, it is given that $b \quad c \quad a$

Now $[a \quad b \quad c \quad a \quad b \quad b \quad c] \quad [a \quad a \quad b \quad b \quad c] \quad [b \quad c \quad a \quad b \quad b \quad c]$

$$[a \quad b \quad c] \quad a\;(b \quad c] \quad a \quad a \quad |a|^2 \quad 1$$

(because a is a unit vector)

(D) $[x \quad y \quad a] \quad 0$

Therefore, x, y and a are coplanar.

$$[x, y\; b] \quad 0$$

Therefore, x, y and b are coplanar.

Also, $\qquad [a\;b\;c] \quad 0$

Therefore, a, b and c are coplanar.

From (i), (ii) and (iii)

x, y and c are coplanar. Therefore, $[x, y\; c] \quad 0$

Exercise-5 : Subjective Type Problems

1. Line L is the shortest distance line of given lines.

2. $[\hat{a}\quad \hat{b}\quad \hat{c}]\quad [\hat{b}\quad \hat{c}\quad \hat{c}\quad \hat{a}\quad \hat{a}\quad \hat{b}]\quad [\hat{a}\quad \hat{b}\quad \hat{c}]^2$

$$[\hat{a}\quad \hat{b}\quad \hat{c}]\quad 1$$

Projection of $\hat{b}\quad \hat{c}$ on $\hat{a}\quad \hat{b}\quad \dfrac{(\hat{b}\quad \hat{c})\ (\hat{a}\quad \hat{b})}{|\hat{a}\quad \hat{b}|}\quad \dfrac{[\hat{a}\quad \hat{b}\quad \hat{c}]}{|\hat{a}\quad \hat{b}|}$

3. Let $l\quad m\quad n\quad \dfrac{1}{\sqrt{2}}$

4. $\overline{OC}\qquad {}^2(a\quad b)^2\qquad {}^2(a\quad b)^2\quad 2\quad [a\ (a\quad b)\quad b\ (a\quad b)]$

$$1\quad {}^2\ 1\quad 1\quad 2\ 1\ 1\frac{1}{2}\qquad {}^2\ 1\ 1\ \frac{\sqrt{3}}{2}^{\ 2}\qquad 0 \qquad \text{...(1)}$$

Also, $\qquad \overline{OB}\quad \overline{OC}\quad |\overline{OB}|\,|\overline{OC}|\cos\dfrac{}{3}$

$$1\ 1\ \frac{1}{2}\qquad 1\quad \frac{1}{2}\qquad\qquad \frac{1}{3} \qquad \text{...(2)}$$

From (1) and (2), $\qquad {}^2\quad \dfrac{8}{9}$

5. $v_{n\ 1}\quad v_n\quad \begin{matrix} 0 & 1/2 \\ 1/2 & 0 \end{matrix}^{\ n\ 1}\quad v_0$

$v_2\quad v_1\quad \begin{matrix} 0 & 1/2 \\ 1/2 & 0 \end{matrix}\quad v_0$

$v_3\quad v_2\quad \begin{matrix} 0 & 1/2 \\ 1/2 & 0 \end{matrix}^{\ 3}\quad v_0$

$v_n\quad v_{n\ 1}\quad \begin{matrix} 0 & 1/2 \\ 1/2 & 0 \end{matrix}^{\ n}\quad v_0$

Adding all the equations,

$v_n\quad v_0\quad (A\quad A^2\quad A^3\qquad A^n)\,v_0$

where $A\quad \begin{matrix} 0 & 1/2 \\ 1/2 & 0 \end{matrix}\qquad v_n\quad (I\quad A\quad A^2\qquad)\,v_0$

6. Let $B \quad A \quad \frac{1}{3}A^2 \quad \frac{1}{9}A^3 \quad \frac{1}{27}A^4$

$$\frac{AB}{3} \quad \frac{A^2}{3} \quad \frac{1}{9}A^3 \quad \frac{1}{27}A^4$$

$$I \quad \frac{A}{3} \quad B \quad A$$

$$B \quad \frac{1}{3}(3I \quad A)^{\ 1} A$$

7. $\det M_n \quad \sum_{k\ 0}^{n} \frac{1}{(2k \quad 1)!} \quad \frac{1}{(2k \quad 2)!} \quad \frac{1}{1!} \quad \frac{1}{(2n \quad 2)!}$

8. $|a \quad b| \quad \sqrt{3}$

Squaring both sides

$$a \quad b \quad \frac{1}{2}$$

$$c \quad a \quad 2\,b \quad 3\,a \quad b$$

$$a \quad c \quad 2 \ \& \ b \quad c \quad \frac{5}{2}$$

$$p \quad |(a \quad a)\,b \quad (b \quad c)\,a|$$

$$p \quad \sqrt{\left|2\,b \quad \frac{5}{2}\,a\right|^2}$$

$$p \quad \frac{\sqrt{21}}{2} \qquad [p] \quad 2$$

9. $r \quad (a \quad b)\sin x \quad (b \quad c)\cos y \quad 2(c \quad a)$

$$r \quad a \quad [b \ c \ a]\cos y$$

$$r \quad b \quad 2[c \ a \ b]$$

$$r \quad c \quad \sin x[a \ b \ c]$$

$\sin x \quad \cos y \quad 2 \quad 0$

$\sin x \quad 1$ and $\cos y \quad 1$

$$x \quad \frac{}{2} \qquad\qquad y$$

10. New equation of plane : $4x \quad 7y \quad 4z \quad 81 \qquad (5x \quad 3y \quad 10z \quad 25) \quad 0$

$$(4 \quad 5\ )4 \quad (7 \quad 3\ )7 \quad (4 \quad 10\ )4 \quad 0$$

$$1$$

Equation of plane : $x \quad 4y \quad 6z \quad 106 \quad 0$

$$\text{distance} \quad \frac{106}{\sqrt{53}} \quad \sqrt{212}$$

18.

$$(\qquad) \qquad 1$$

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