Contents

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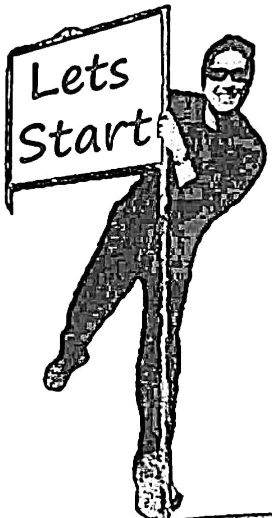


1 Units and Measurements

तो चलिए हमारी इस beautifull journey को start करते है पहले chapter unit and dimensions से.... ये chapter Advance/Mains/NEET तीनो के लिए important है..... मुझे पता है पहले chapter मे आपका excitation level बहुत high होता है.... from basic to advance का content हम इस chapter मे cover करेंगे मै बस इतना बोलुंगा हो सकता है शुरू मे ये chapter आपको थोड़ा मुश्किल लगे but at the end of physics ये chapter आपको सबसे आसान लगेगा.... from basic to Advance all relevent theory and question i will cover in this book.... तो चलिए शुरू करते है are you ready

PHYSICAL QUANTITIES

All quantities that can be measured are called physical quantities.e.g.time,length,mass,force,workdone,etc.

Classification of Physical Quantity

①

Based on Direction
- → Scaler
  - → Magnitude
  - → No Direction Ex. Distance, speed
- → Vector
  - 1 → magnitude
  - 2 → direction
  - 3 → Follow vector's law
  - → Example force, momentum velocity

क्या होता है vector क्या होता है vector's law अभी हमे इन पर बिल्कुल ध्यान नही देना है ये चीजे हम vector वाले chapter मे detail मे पढ़ेंगे।

② Based on dependency
- → Fundamental physical quantity
- → Derived physical quantity

Based on their Dependency

① Fundamental or base quantities: A set of physical quantities which are completely independent of each other and all other physical quantities can be expressed in terms of these physical quantities is called Set of Fundamental Quantities.

② Derived quantities: The quantities which can be expressed in terms of the fundamental quantities are known as derived quantities. Ex. Speed (= distance/time), volume, acceleration, force, pressure, etc.

Fundamental or base quantities

S. No.	Physical quantity	SI unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	Kelvin	K
5.	Electric current	ampere	A
6.	Luminous intensity	candela	cd
7.	Amount of substance	mole	mol

A quantity con be expressed as numerical value and unit.

Quantity (Q) = $n_1U_1 = n_2U_2$

↳ Physical quantity to measure.

$n_1 \rightarrow$ numerical value in 1st system of measurement

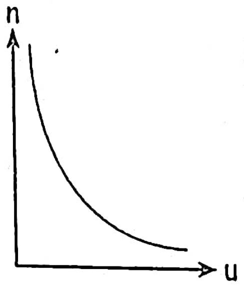
$U_1 \rightarrow$ Units value in 1st system of measurement

$n_2 \rightarrow$ numerical value in 2nd system of measurement

$U_2 \rightarrow$ Units value in 2nd system of measurement

If $U_1 > U_2 \Rightarrow n_1 < n_2$

If unit is bigger then numerical value is smaller and vice versa.



Classification of Units

meter, kilogram, sec. → MKS System	CGS System	FPS System
mass → kg	mass → gram	mass → pound
length → m	length → cm	length → foot
time → sec	time → sec	time → sec

Q. Convert Density = 50 kg/m³ into CGS.

Sol. $\text{Density} = 50\,\text{kg/m}^3 = \dfrac{50 \times 1000\,\text{gm}}{(100\,\text{cm})^3} = .05\,\text{gm/(cm)}^3$

Q. Convert Density = 0.6 gm/cc into MKS.

Sol. $\text{Density} = 0.6\ \text{gm/cc} = \dfrac{0.6 \times \dfrac{\text{kg}}{1000}}{\left(\dfrac{1}{100}\text{m}\right)^3} = \dfrac{600\,\text{kg}}{\text{m}^3}$

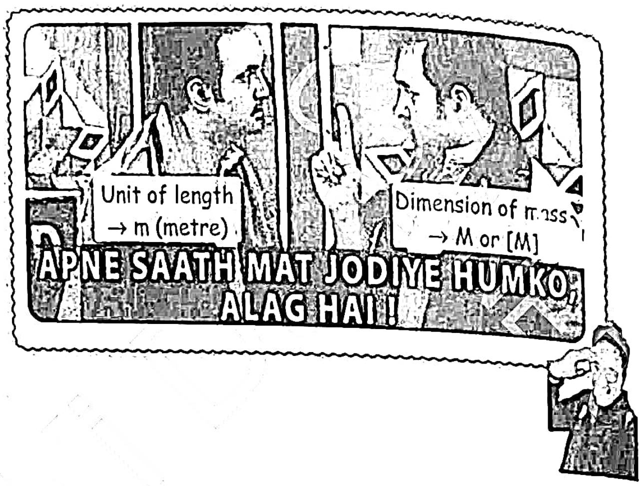
DIMENSIONS

Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

The physical quantity that is express in terms of base quantities is enclosed in square brackets to remind that the equation is among the dimensions and not among the magnitude, such expression for physical quantity is called dimension formula.

meter वाला 'm' नहीं है

Mass has dimension	[M]
Length has dimension	[L]
Time has dimension	[T]
Temp. has dimension	[K]
Electric current	[A]



Q. Write dimensional formula of speed.

Sol. $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

Dimensionally we can write

$$[\text{Speed}] = \left[\frac{\text{Distance}}{\text{Time}}\right] = \frac{[L]}{[T]} = [LT^{-1}]$$

So, dimensional formula of speed can be written as $LT^{-1} = [M^0 L^1 T^{-1}]$

Here

Dimension of mass → 0

Dimension of length → 1

Dimension of time → -1

सर speed का Dimensional Formula (DF) क्या हम only LT^{-1} लिख सकते है क्या इसे box मे बंद करना जरूरी है

वैसे तो speed का DF हम LT^{-1}, $[LT^{-1}]$, $[M^0LT^{-1}]$ तीनो लिख सकते है लेकिन box लगाना अच्छी बात है, हम अकसर physical quantity को M, L, T मे represent करते है अगर तुम school मे exam दे रहे हो तो box जरूर लगाओ ताकी school वाले master sahab number ना काटे लेकिन अगर तुम JEE का test/mock test/question practice कर रहे हो तो box लगाने मे time waste मत करना।

Q. Find the dimensions of mass length & time in density.

Sol. Formula of density is

$$[\text{Density}] = \frac{[\text{mass}]}{[\text{volume}]}$$

$$[\text{Density}] = \frac{M}{L \cdot L \cdot L} = \frac{M}{L^3} \Rightarrow [M^1L^{-3}T^0]$$

Hence in density we can say
Dimensional of mass → 1

Dimension of length → –3

Dimension of time → 0

① **Fundamental or base units:**
The units of fundamental quantities are called *base units*. In SI there are seven base units.

② **Derived units:**

The units of derived quantities or the units that can be expressed in terms of the base units are called <u>*derived*</u> units

Ex. Unit of speed can be derived from units of distance & unit of time.

$$\text{Unit of speed} = \frac{\text{unit of distance}}{\text{unit of time}} = \frac{\text{metre}}{\text{second}} = ms^{-1}$$

Dimensions of Quantities Related to Mechanics

S.N.	Quantity	Formula	Unit	Dimension
1.	Velocity or speed (v)	$v = \frac{d}{t} = \frac{\text{Displacement or Distance}}{\text{Time}}$	m/s	$[M^0L^1T^{-1}]$
2.	Acceleration (a)	$a = \frac{\Delta v}{\Delta t} = \frac{\text{Change in velocity}}{\text{Change in time}}$	m/s^2	$[M^0LT^{-2}]$
3.	Momentum (P)	P = mv = Mass × Velocity	kg – m/s	$[M^1L^1T^{-1}]$
4.	Impulse (I)	I = F × Δt Force × Time	Newton-second or kg – m/s	$[M^1L^1T^{-1}]$
5.	Force (F)	F = ma = Mass × Acceleration	Newton	$[M^1L^1T^{-2}]$
6.	Pressure (P)	$P = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}$	Pascal	$[M^1L^{-1}T^{-2}]$
7.	Kinetic energy (K_E)	$K = \frac{1}{2}mv^2 = \frac{1}{2}\ \text{Mass} \times \text{Velocity}^2$	Joule	$[M^1L^2T^{-2}]$
8.	Power (P)	$P = \frac{W}{t} = \frac{\text{Work}}{\text{Time}}$	Watt or Joule-sec	$[M^1L^2T^{-3}]$
9.	Density (d)	$\rho = \frac{m}{V} = \frac{\text{Mass}}{\text{Volume}}$	kg/m^3	$[M^1L^{-3}T^0]$
10.	Angular displacement (θ)	$\theta = \frac{S}{r} = \frac{\text{Arc}}{\text{Length}}$	Radian (rad)	$[M^0L^0T^0]$
11.	Angular velocity (ω)	$\omega = \frac{\Delta 0}{\Delta t} = \frac{\text{Angular displacement}}{\text{Time}}$	Radian/sec	$[M^0L^0T^{-1}]$
12.	Angular acceleration (α)	$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\text{Angular velocity}}{\text{Time}}$	Radian/sec^2	$[M^0L^0T^{-2}]$
13.	Moment of inertia (I)	$I = mx^2 = \text{Mass} \times \text{Distance}^2$	$kg\text{-}m^2$	$[M^1L^2T^0]$

S.N	Quantity	Formula	Unit	Dimension
14.	Torque (τ)	$\tau = F \times r_{\perp}$ Force × Perpendicular distance	Newton-meter	$[M^1L^2T^{-2}]$
15.	Angular momentum (L)	L = mvr = Mass × Velocity × Radius	Joule-sec	$[M^1L^2T^{-1}]$
16.	Force constant or spring constant (k)	F = -kx = Force constant × Displacement	Newton/m	$[M^1L^0T^{-2}]$
17.	Gravitational constant (G)	$F = \frac{Gm_1m_2}{r^2} = \frac{\text{Gravitational constant} \times \text{Mass}^2}{\text{Distance}^2}$	$N\text{-}m^2/kg^2$	$[M^{-1}L^3T^{-2}]$
18.	Gas constant (R)	PV = nrT Pressure = Velocity × Gas constant × Temperature	Joule/mol-K	$[M^1L^2T^{-2}\theta^{-1}]$
19.	Planck's constant (h)	E = hv Energy = Planck's constant × Frequency	Joule-s	$[M^1L^2T^{-1}]$
20.	Surface tension (S)	$S = \frac{F}{L} \Rightarrow \text{Surface Tension} = \frac{\text{Force}}{\text{Length}}$	N/m or Joule/ m^2	$[M^1L^0T^{-2}]$
21.	Coefficient of viscosity (η)	$\eta = \frac{F}{6\pi ru} = \frac{\text{Force}}{\text{Radius} \times \text{Velocity}}$	kg/m-s	$[M^1L^{-1}T^{-1}]$
22.	Time period (T)	$T = \frac{1}{n} = \frac{1}{\text{Frequency}}$	Second	$[M^0L^0T^1]$
23.	Frequency (n)	$n = \frac{1}{T} = \frac{1}{\text{Time}}$	Hz	$[M^0L^0T^{-1}]$

Dimensions of Quantities Related to Mechanics

S.N.	Quantity	Formula	Unit	Dimension
1.	Heat (Q)	Energy	Joule	$[ML^2T^{-2}]$
2.	Specific Heat (c)	$c = \frac{Q}{m \times \Delta\theta} = \frac{\text{Heat}}{\text{Mass} \times \text{Temperature}}$	Joule/kg-K	$[M^0L^2T^{-2}K^{-1}]$
3.	Thermal capacity (K)	$K = \frac{Q}{\Delta t} = \frac{\text{Heat}}{\text{Time}}$	Joule/K	$[M^0L^2T^{-2}K^{-1}]$
4.	Latent heat (L)	$L = \frac{Q}{m} = \frac{\text{Heat}}{\text{Mass}}$	Joule/kg	$[M^0L^2T^{-2}]$
5.	Boltzmann constant (k)	$k = \frac{2\,E}{3\,T} = \frac{\text{Energy}}{\text{Temperature}}$	Joule/K	$[M^1L^1T^{-2}K^{-1}]$

S.N.	Quantity	Formula	Unit	Dimension
6.	Coefficient of thermal conductivity (k)	$k = \frac{Qd}{A \times \Delta\theta \times t} = \frac{\text{Heat} \times \text{Distance}}{\text{Area} \times \text{Temp. difference} \times \text{Time}}$	Joule/m-s-K	$[M^1L^1T^{-3}K^{-1}]$
7.	Stefan's constant (σ)	$\sigma = \frac{\Delta E}{A \times \Delta t \times \theta^4} = \frac{\text{Energy}}{\text{Area} \times \text{Time} \times \text{Temperature}^4}$	Watt/m^2-K^4	$[M^1L^0T^{-3}K^{-4}]$
8.	Wien's constant (b)	$b = \lambda_{max} \times T$ = Wavelength × Temperature	Meter-K	$[M^0L^1T^0K^1]$
9.	Coefficient of linear expansion (α)	$\alpha = \frac{\Delta L}{L}\frac{1}{T} = \frac{\text{Change in length}}{\text{Length} \times \text{Temperature}}$	Kelvin^{-1}	$[M^0L^0T^0K^{-1}]$
10.	Mechanical eq. of Heat (J)	$J = \frac{W}{Q} = \frac{\text{Work}}{\text{Heat}}$	Joule/Calorie	$[M^0L^0T^0]$
11.	Vander wall's constant (a)	$a = \frac{RTV^2}{V-b} - PV^2$	Newton-m^4	$[M^1L^5T^{-2}]$
12.	Vander wall's constant (b)	Same as Volume (V)	m^3	$[M^0L^3T^0]$
13.	Temperature (T)	$T = \frac{Q}{M\Delta C}$	Kelvin (K)	$[M^0L^0T^0K^1]$

Electricity & Magnetism

S.N.	Quantity	Formula	Unit	Dimension
1.	Electric charge (q)	$q = I \times t$ = Electric current × Time	Coulomb	$[M^0L^0T^1A^1]$
2.	Electric current (i)	$I = \frac{q}{t} = \frac{\text{Charge}}{\text{Time}}$	Ampere	$[M^0L^0T^0A^1]$
3.	Capacitance (C)	$C = \frac{q}{V} = \frac{\text{Charge}}{\text{Voltage difference}}$	Coulomb/volt or Farad	$[M^{-1}L^{-2}T^4A^2]$
4.	Electric polential (V)	$V = \frac{q}{C} = \frac{\text{Charge}}{\text{Capacitance}}$	Joule/coulamb	$[M^1L^2T^{-3}A^{-1}]$
5.	Permittivity of free space (ε)	$\frac{\text{Charge}^2}{4\pi \times \text{Electric force} \times \text{Distance}^2}$	$\frac{\text{Coulomb}^2}{\text{Newton}-\text{meter}^2}$	$[M^{-1}L^{-3}T^4A^2]$
6.	Dielectric constant (K)	$K = \frac{\varepsilon}{\varepsilon_0} = \frac{\text{Permittivity in medium}}{\text{Permittivity in free space}}$	Unitless	$[M^0L^0T^0]$

S.N.	Quantity	Formula	Unit	Dimension
7.	Resistance (R)	$R = \frac{V}{I} = \frac{\text{Voltage difference}}{\text{Electric current}}$	Volt/Ampere or Ohm	$[M^1L^2T^{-3}A^{-2}]$
8.	Resistivity or Specific resistance (ρ)	$\rho = \frac{RA}{L} = \frac{\text{Resistance} \times \text{Area}}{\text{Length}}$	Ohm-meter	$[M^1L^3T^{-3}A^{-2}]$
9.	Coefficient of self-induction (L)	$L = \frac{\mu_0 N^2 A}{I}$	$\frac{\text{Volt - Second}}{\text{Ampere}}$ or Henery Ohm-second	$[M^1L^2T^{-2}A^{-2}]$
10.	Magnetic flux (ϕ)	$\phi = B \times A$ = Magnetic field × Area	Volt-second or Weber	$[M^1L^2T^{-2}A^{-1}]$
11.	Magnetic induction (B)	$B = \frac{F}{q \times v} = \frac{\text{Magnetic force}}{\text{Charge} \times \text{Velocity}}$	$\frac{\text{Newton}}{\text{Ampere - meter}}$ or Tesla	$[M^1L^0T^{-2}A^{-1}]$
12.	Magnetic Intensity (H)	$H = \frac{B}{\mu} = \frac{\text{Magnetic field}}{\text{Permeability}}$	Ampere/meter	$[M^0L^{-1}T^0A^1]$
13.	Magnetic dipole moment (M)	$M = I \times A$ = Current × Area	Ampere-meter^2	$[M^0L^2T^0A^1]$
14.	Permeability of free space (μ_0)	$\mu_0 = \frac{B \cdot l}{I} = \frac{\text{Magnetic field} \times \text{Length}}{\text{Current}}$	$\frac{\text{Newton}}{\text{Ampere}^2}$	$[M^1L^1T^{-2}A^{-2}]$
15.	Surface charge density (σ)	$\sigma = \frac{q}{A} = \frac{\text{Charge}}{\text{Area}}$	Coulomb-meter^2	$[M^0L^{-2}T^0A^1]$
16.	Electric dipole moment (p)	$p = q \times d$ = Charge × Distance	Coulomb-meter	$[M^0L^1T^1A^1]$
17.	Conductance (G)	$G = \frac{1}{R} = \frac{1}{\text{Resistance}}$	Ohm^{-1}	$[M^{-1}L^{-2}T^3A^1]$
18.	Conductivity (σ)	$\sigma = \frac{1}{\rho} = \frac{1}{\text{Resistivity}}$	Ohm^{-1} meter^1	$[M^{-1}L^{-3}T^3A^2]$
19.	Current density (J)	$J = \frac{I}{A} = \frac{\text{Current}}{\text{Area}}$	Ampere/m^2	$[M^0L^{-2}T^0A^1]$
20	Intensity of electric field (E)	$E = \frac{F}{q} = \frac{\text{Electric force}}{\text{Electric charge}}$	Volt/meter, Neuton/coulomb	$[M^1L^1T^{-3}A^{-1}]$
21.	Rydberg constant (R)	$R_H = \frac{me^4}{8h^3c\varepsilon_0^2}$	m^{-1}	$[M^0L^{-1}T^0]$

Quantities Having Same Dimensions

S.N.	Dimension	Quantity
1.	$[M^0L^0T^{-1}]$	Frequency, Angular frequency, Angular velocity and Velocity gradient
2.	$[M^1L^2T^{-2}]$	Work, Internal energy, Potential energy, Kinetic energy. Torque
3.	$[M^1L^{-1}T^{-2}]$	Pressure, Stress, Young's modulus, Bulk modulus, Modulus of rigidity, Energy density
4.	$[M^1L^1T^{-1}]$	Momentum, Impulse
5.	$[M^1L^1T^{-2}]$	Thrust, Force, Weight
6.	$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
7.	$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area), Force constant and Spring constant
8.	$[M^0L^2T^{-2}]$	Latent heat and Gravitational potential
9.	$[M^1L^2T^{-2}\theta^{-1}]$	Thermal capacity, Gas constant and Entropy
10.	$[M^0L^0T^1]$	L/R, $\sqrt{LC}$,RC where L = Inductance, R = Resistance, C = Capacitance
11.	$[M^0L^1T^0]$	Distance, Displacement, Radius, Wavelength, radius of gyration.
12.	$[M^0L^1T^{-1}]$	Speed, Velocity, Velocity of light.
13.	$[M^0L^1T^{-2}]$	Acceleration, Acceleration due to gravity, Centripetal acceleration.
14.	$[M^0L^0T^1]$	Decay constant, Rate of disintegration.

Units of Some Physical Quantities in Different Systems

Type of Physical Quantity	Physical Quantity	CGS (Originated in France)	MKS (Originated in France)	FPS (Originated in Britain)
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s
Derived	Force	dyne	newton(N)	poundal
	Work or Energy	erg	joule(J)	ft-poundal
	Power	erg/s	watt(W)	ft-poundal/s

Units and Dimensions of Physical Quantities

Quantity	Common Symbol	SI unit	Dimension
Displacement	s	METRE (m)	L
Mass	m, M	KILOGRAM (kg)	M
Time	t	SECOND (s)	T
Area	A	m^2	L^2
Volume	V	m^3	L^3
Density	ρ	kg/m^3	M/L^3
Velocity	v, u	m/s	L/T
Acceleration	a	m/s^2	L/T^2
Force	F	newton (N)	ML/T^2
Work	W	joule (J) (= N-m)	ML^2/T^2
Energy	E, U, K	joule (J)	ML^2/T^2
Power	P	watt (W) (= J/s)	ML^2/T^3
Momentum	p	kg-m/s	ML/T
Gravitational constant	G	$N\text{-}m^2/kg^2$	L^3/MT^2
Angle	θ, φ	radian	
Angular velocity	ω	radian/s	T^1
Angular acceleration	α	$radian/s^2$	T^{-2}
Angular momentum	L	$Kg\text{-}m^2/s$	ML^2/T
Moment of inertia	I	$Kg\text{-}m^2$	ML^2
Torque	τ	N-m	ML^2/T^2
Angular frequency	ω	radian/s	T^{-1}
Frequency	υ	hertz (Hz)	T^{-1}
Period	T	s	T
Young's modulus	Y	N/m^2	M/LT^2
Bulk modulus	B	N/m^2	M/LT^2
Shear modulus	η	N/m^2	ML/T^2
Surface tension	S	N/m	M/T^2
Coefficient of viscosity	η	$N\text{-}s/m^2$	M/LT

Pressure	P, p	N/m^2, Pa	M/LT^2
Wavelength	λ	m	L
Intensity of wave	I	W/m^2	M/T^3
Temperature	T	KELVIN (K)	K
Specific heat capacity	c	J/kg-K	L^2/T^2K
Stefan's constant	σ	W/m^2- K^4	M/T^3K^4
Heat	Q	J	ML^2/T^2
Thermal conductivity	K	W/m-K	ML/T^3K

सच-सच बताओ क्या तुम हर physical quantity का unit और dimensional formula रट रहे हो ना

Haa humne RATT liye

Nhi RATTNA tha

पूरे दो साल यही पढ़ना है कि कौन-सी physical quantity क्या है So, इस पर time बिलकुल waste ना करें और मस्त रहें अगर याद करना ही है तो velocity momentum, acc, force, work, power, kinetic energy याद कर सकते हो

PRINCIPLE OF HOMOGENEITY

In a physical equation, all terms which have been subtracted or added or equal to must have same dimension. In following equation A, B and C should have same dimension.

☆ $\left.\begin{array}{l}A = B + C \\ A = B - C\end{array}\right\}$ A, B, C are of same DF

☆ P = QR + S ⇒ DF of P, QR, S will be same

☆ Change in velocity = Final velocity – Initial velocity

$$\Delta\vec{V} = \vec{V}_f - \vec{V}_i$$

$$\downarrow \quad \downarrow \quad \downarrow$$

Same D.F

☆ Similarly, in the equation A = B + C + D all terms have same dimensions.

$$A = B + C + D$$

D.F. same

Points to be Remember for Dimension Analysis

☆ Quantity of different dimension cannot be added or subtracted but they can be multiplied or divided.

☆ Each additive component on the LHS and RHS must have the same dimensional formula this is known as principle of homogenity of equation.

☆ All trigonometric ratio, exponential and logarithm terms are dimensionless.

☆ If a physical quantity is multiplied by a number then dimensional formula of that physical quantity remains same.

Q. Sir अगर दो physical quantity की dimension Same हैं तो क्या हम add या Subst. कर सकते है ?

Sol. जरूरी नही for example work and torque have same dimension but different unit but they cann't be added or subtracted

Work, torque → D.F. same

→ Add नही कर सकते

☆ Strain $= \dfrac{\Delta l}{l} = \dfrac{\text{Change in Length}}{\text{Original length}}$

☆ $\left[\dfrac{\Delta l}{l}\right] = \left[\dfrac{\text{Change in Length}}{\text{Original length}}\right] \Rightarrow \dfrac{L}{L} = 1$

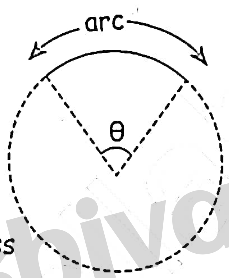
Strain is both unitless and dimensionless

$\Rightarrow [M^\circ L^\circ T^\circ]$

Q. What is the unit and dimension of plane angle?

Sol. Angle $\theta = \dfrac{\text{Arc length}}{\text{Radius}}$

$\boxed{\theta = \dfrac{Arc}{R}}$



Dimension → $\dfrac{L}{L} = 1 = [M^\circ L^\circ T^\circ]$

↓

Dimensionless

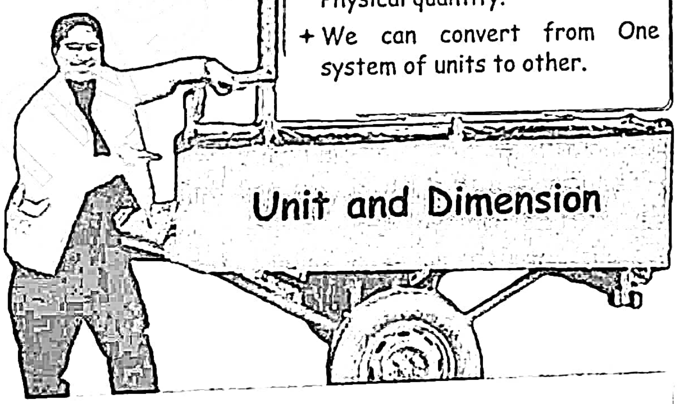
Unit → radian, rad

Q. If a physical quantity is dimension-less then it must be unitless. Is true or false?

Sol. False Ex. angle, Solid angle

Application of Dimensional Analysis

- To find the D.F of unknown Physical Quantity.
- We can check the correctness of any formula dimensionally.
- We can derive the relation b/w Physical quantity.
- We can convert from One system of units to other.



इस chapter से majority of the question ऊपर 4 type of application से पूछे जाते है।

TO FIND THE D.F OF UNKNOWN PHYSICAL QUANTITY

Q. If Gravitational force is given by $F = \dfrac{GM_1M_2}{r^2}$ then find dimensional formula (D.F) of Gravitational constant G = ?

Sol. $[G] = \left[\dfrac{Fr^2}{m_1m_2}\right] \Rightarrow \left[\dfrac{MLT^{-2}L^2}{MM}\right] \equiv [M^{-1}L^3T^{-2}]$

Q. Find dimensional formula of coefficient of viscosity viscous force is given by $F = 6\pi r\eta v$

r → radius, v → velocity

Sol. $\eta = \dfrac{F}{6\pi rv} \Rightarrow [\eta] = \left[\dfrac{F}{6\pi rv}\right] = \left[\dfrac{MLT^{-2}}{LLT^{-1}}\right] = [ML^{-1}T^{-1}]$

6π → Const dimentionless

Q. Dimensional formula of specific heat capacity (S).

Sol. $\underbrace{\Delta Q}_{\text{Heat given}} = \underbrace{M}\underbrace{S}_{\text{Specific heat capacity}}\underbrace{\Delta T}_{\text{Rise in temp.}}$

$S = \dfrac{\Delta Q}{M\Delta T} \Rightarrow [S] = \left[\dfrac{\Delta Q}{M\Delta T}\right] \equiv \left[\dfrac{ML^2T^{-2}}{MK}\right] \equiv [M^\circ L^2T^{-2}K^{-1}]$

Q. Find dimensional formula (D.F) of Boltzman Constant (k). If formula of internal energy is given by

$\underbrace{U}_{\text{Kinetic Energy (Internal Energy)}} = \dfrac{3}{2}K\underbrace{T}_{\text{temp}}$

Sol. From above formula Boltzman Constant (k)

$[K] = \left[\dfrac{2}{3}\dfrac{U}{T}\right]$ → temp

$\underbrace{[K]}_{\text{Boltzman Constant}} \Rightarrow \dfrac{[ML^2T^{-2}]}{\underbrace{[K]}_{\text{D.F. of temp}}} \equiv [ML^2T^{-2}K^{-1}]$

Q. Find dimensional formula of charge if formula of charge is equal to current × time

Sol. $q = i.\Delta t$

DF of charge $= [AT] \equiv [M^\circ L^\circ TA]$

Q. Find the dimensional formula of electric potential or voltage

Electric potential = $\dfrac{\text{Electric Potential Energy}}{\text{Charge}}$

Sol. Dimension of electric potential is = $\dfrac{ML^2T^{-2}}{AT}$

$=[ML^2T^{-3}A^{-1}]$

Q. Find the dimensional formula of resistance

We know $\underbrace{V}_{\text{Pot. Difference}} = i\underbrace{R}_{\text{Resistance}}$

Sol. The formula of resistance is

$$= [R] = \left[\frac{V}{i}\right] = \frac{[ML^2T^{-3}A^{-1}]}{[A]}$$

$$= \frac{[ML^2T^{-3}A^{-1}]}{[A]} = ML^2T^{-3}A^{-2}$$

Q. If $R = \rho\dfrac{\ell}{(\text{Area})}$ where R is resistance, ℓ is length and ρ is resistivity. Find DF of resistivity.

Sol. $\rho = \dfrac{R \times (\text{Area})}{\ell}$

$$\rho = \frac{ML^2T^{-3}A^{-2} \cdot L^2}{L} \Rightarrow [ML^3T^{-3}A^{-2}]$$

Q. Kinetic energy of a particle moving along elliptical trajectory is given by $K = \alpha s^2$ where s is the distance travelled by the particle. Determine dimensions of α.

Sol. $K = \alpha s^2 \Rightarrow \alpha = \dfrac{K}{s^2}$

$$[\alpha] = \left[\frac{(ML^2T^{-2})}{(L^2)}\right]$$

$$[\alpha] = [M^1 L^0 T^{-2}]$$

We can Check the Correctness of any Formula Dimensionally

Q. Check dimensionally if this formula $F = \dfrac{Mv^2}{r^3}$ is correct ? (Where v is speed r is radius, F is force, and M is mass)

Sol. DF of LHS = Force $\Rightarrow [ML^1T^{-2}]$

DF of RHS $\Rightarrow \dfrac{mv^2}{r^3} \Rightarrow \dfrac{M(LT^{-1})^2}{L^3} = [ML^{-1}T^{-2}]$

Since D.F. of LHS $\neq$ D.F. of RHS so given formula is wrong

Q. Is this formula $v = \sqrt{\dfrac{GM}{r}}$ correct or wrong where V is orbital velocity, M is mass of earth, R is radius of orbit and G is gravitational constant

Sol. In this formula DF of LHS $\Rightarrow [v] \Rightarrow [LT^{-1}]$

$$\underline{\text{RHS}} \Rightarrow \sqrt{\frac{GM}{r}} = \sqrt{\frac{M^{-1}L^3T^{-2}M}{L}} = \sqrt{L^2T^{-2}} = LT^{-1}$$

Since D.F. of LHS = D.F. of RHS so given formula is dimensionally correct.

Q. Check whether the given relation $\dfrac{Fv^2}{t} = KE$ (Kinetic Energy) is dimensionally correct? Here F = force, v = velocity and t = time?

Sol. $[\text{LHS}] = \left[\dfrac{Fv^2}{t}\right] = \dfrac{[MLT^{-2}]\times[L^2T^{-2}]}{[T]} = [ML^3T^{-5}]$

$[\text{RHS}] = [\text{KE}] = \left[\dfrac{1}{2}mv^2\right] = [M \times L^2T^{-2}] = [ML^2T^{-2}]$

$\therefore$ [LHS] $\neq$ [RHS], so the given relation is incorrect dimensionally.

☆ If a formula is dimensionally incorrect $\Rightarrow$ it must be incorrect overall.

☆ If a formula is dimensionally correct $\Rightarrow$ Then formula may be correct may be incorrect but अभी हम only dimensionally correctness पर बात करेंगे।

☆ If a formula is correct then it must be both dimensionally correct and numerically correct.

Q. The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$; find the dimensions of a, b, c and d.

Sol. The equation contains five terms. All of them should have the same dimensions. Since $[x]$ = length, each of the remaining four must have the dimension of length.

Thus, $[a]$ = length = L

$[bt] = L$, or $[b] = [LT^{-1}]$

$[ct^2] = L$, or $[c] = [LT^{-2}]$

and $[dt^3] = L$ or $[d] = [LT^{-3}]$

Q. If x is displacement t is time in the equation $x = \frac{A}{t} + Bx^2 + Cx$. Find the dimensional formula of A, B and C.

Sol. $[x] = \left[\frac{A}{t}\right] + \left[Bx^2\right] + \left[Cx\right]$

Dimension formula of $\frac{A}{t}$ = Dimensional formula of x.

$\left[\frac{A}{T}\right] = [L] \Rightarrow A = LT$

$[BL^2] = [L] \Rightarrow [L] \Rightarrow [B] = [L^{-1}]$

$[CL] = [L]$ So, C = dimensionless

Q. If x is displacement t is time in the equation $x = Ax^2 + \frac{B}{t^2} + \frac{C}{x}$. Find the dimensional formula of A, B and C.

If D.F. of $\frac{AB}{C}$ is $M^{\alpha}L^{\beta}T^{\gamma}$ find $\alpha + \beta + \gamma$

Sol. $x = Ax^2 + \frac{B}{t^2} + \frac{C}{x}$

From principle of homogeneity

$[x] = [Ax^2] = \left[\frac{B}{t^2}\right] = \left[\frac{C}{x}\right]$

(i) $[Ax^2] = [x]$

$\therefore AL^2 = L, \qquad [A] = [L^{-1}]$

(ii) $\left[\frac{B}{T^2}\right] = [x]$

$\therefore \frac{B}{T^2} = L, \qquad [B] = [LT^2]$

(iii) $\left[\frac{C}{x}\right] = [x]$

$\frac{C}{L} = L \qquad [C] = [L^2]$

$\frac{AB}{C} = \frac{L^{-1}LT^2}{L^2} = M^0L^{-2}T^2$

So, $\alpha + \beta + \gamma = 0 + (-2) + 2 = 0$

Q. If x is displacement t is time and v is velocity in the equation $v = At + \frac{B}{x^2} + Ct^2$.

Find the dimensional formula of ABC.

Sol. $[v] = [At] + \left[\frac{B}{x^2}\right] + [Ct^2]$

(i) $[v] = [At]$

$LT^{-1} = AT$

$\boxed{A = LT^{-2}}$

(ii) $\left[\frac{B}{x^2}\right] = [V]$

$\frac{B}{L^2} = LT^{-1}$

$\boxed{B = L^3T^{-1}}$

(iii) $[CT^2] = [v]$

$CT^2 = LT^{-1}$

$\boxed{C = LT^{-3}}$

So, dimensional formula of $ABC \equiv LT^{-2}L^3T^{-1}LT^{-3}$

$\equiv M^0L^5T^{-6}$

Q. If x is displacement, v is velocity and t is time in the equation $v = \frac{Ax}{B+t}$. Find the dimensional formula of A and B.

Sol. $[v] = \left[\frac{Ax}{B+t}\right]$

$\therefore B = T$

SKC: कद्दू मे कद्दू ही जुड़ सकता है और यहाँ B मे t जुड़ रहा है, so dimension of B and t are equal

Dimensionally

$\Rightarrow [v] = \left[\frac{Ax}{\text{time}}\right] \Rightarrow LT^{-1} = \frac{AL}{T}$

A = any number Dimensionless

Q. If x is displacement, v is velocity in the equation $v = \frac{A}{B - x^2}$. Find the dimensional formula of A and B.

Sol. From homogeneity principle D.F. of B = D.F. of x^2

$\boxed{\therefore [B] = [L^2]}$

$[v] = \left[\frac{A}{L^2}\right] \Rightarrow LT^{-1} = \frac{A}{L^2}$

$\boxed{[A] = [L^3T^{-1}]}$

Q. Potential energy (U) is given by $U = \dfrac{A\sqrt{x}}{x^2 + B}$ x is distance. Find the dimensional formula of A and B. Also find [AB].

Sol. From principle of homogeneity dimension of L.H.S = dimension of R.H.S

$$[U] = \left[\frac{A\sqrt{x}}{x^2 + B}\right]$$

$$\Rightarrow ML^2T^{-2} = \frac{AL^{\frac{1}{2}}}{L^2} = \frac{A}{L^{\frac{3}{2}}}$$

$$\Rightarrow [A] = [M^1L^{7/2}T^{-2}]$$

Since B is added to x^2

$[B] \equiv [L^2] \Rightarrow B = L^2$

D.F. of <u>A.B</u> is

$$[AB] \Rightarrow ML^{\frac{7}{2}}T^{-2}L^2 = ML^{\frac{11}{2}}T^{-2}$$

Q. If $\dfrac{v + A\sqrt{s}}{Bt^2} = F$; find the dimension of A and B where F = force, v = velocity, s = displacement and t = time

Sol. Using principle of homogeneity: $[A\sqrt{S}] = [v]$

$$\left[AL^{\frac{1}{2}}\right] = \left[LT^{-1}\right]$$

$$\therefore [A] = \left[L^{\frac{1}{2}}T^{-1}\right]$$

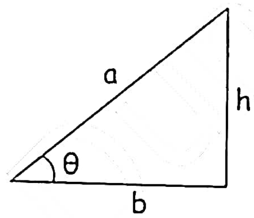
Also, we can write, $B = \dfrac{v + A\sqrt{S}}{Ft^2}$

$$[B] = \frac{[LT^{-1}]}{[(MLT^{-2})(T^2)]} \Rightarrow [B] = [M^{-1}L^0T^{-1}]$$

Important Points

- ☆ Angle is dimensionless $\Rightarrow$ $[\theta]$ is dimensionless because angle θ = arc/radius
- ☆ Sin (कद्दू) here sin के अंदर angle होगा so (कद्दू) is Dimensionless
- ☆ Sin (कद्दू) is also Dimensionless because

$$\sin\theta = \frac{h}{a} \equiv \text{dimensionless}$$



- ☆ All the trigonometry ratio $\sin\theta$, $\cos\theta$, $\tan\theta$ etc. are dimensionless.
- ☆ Log (कद्दू) and ln (कद्दू)

  Dimensionless
- ☆ Inverse function is also dimensionless
- ☆ In $x = \sin^{-1}(y)$ here x and y both are dimensionless.
- ☆ $y = \log x \longrightarrow$ here x and y are dimensionless
- ☆ $e^x = y \longrightarrow$ here x and y are dimensionless
- ☆ $a^x = y \longrightarrow$ here x and y are dimensionless
- ☆ Power of exponential function is dimensionless

देख भाई समझ रहा है ना मुद्दे की बात ये है की sin के अंदर, cos के अंदर, tan के अंदर, sin inverse के अंदर, log के अंदर, e की power मे जो भी है वो dimensionless होगा

Ex. In $y = \sin x \rightarrow$ Displacement मत समझना

$\rightarrow$ Dimensionless

Here y is number between [–1, 1] because range of sin is between [–1, 1]

Q. In equation $y = \sin(At^2)$. Find the dimension of A.

Sol. At^2 is dimensionless

$$\Rightarrow At^2 = M^oL^oT^o = 1$$

$$\Rightarrow A = T^{-2}$$

Q. In equation $y = \sin(At^3 + Bx^2)$. Find the dimension of A and B.

Sol. $[At^3] + [Bx^2]$ this will be dimensionless

$\Rightarrow AT^3 = 1$ and $BL^2 = 1$

$\boxed{A = T^{-3}}$ $\quad$ $\boxed{B = \frac{1}{L^2}}$

Q. In equation $y = \sin\left(\frac{A}{t^2} + Bx^2\right)$. Find the dimension of A or B.

Sol. Argument of sine are dimensionless

$\left[\frac{A}{t^2}\right] \Rightarrow \frac{A}{T^2} = 1 \Rightarrow A = T^2$

and $[Bx^2] = 1 \Rightarrow BL^2 = 1 \Rightarrow B = L^{-2}$

अव मै क्या वोलू अव तक तू खुद समझ गया होगा कि sin के अंदर जितने individual term है उनको 1 के वरावर करना है same thing for cos, log, e
अवे समझ तो गया ना

Q. In equation $y = \log\left(\frac{A}{t^2} + Bx^2\right)$ Find the dimension of A and B.

Sol. Argument of log is dimensionless so

$\left(\frac{A}{t^2} + Bx^2\right)$ is dimensionless

$\frac{A}{T^2} = 1$ and $[Bx^2] = 1 \Rightarrow BL^2 = [1]$

Q. In equation $y = e^{\frac{A}{t^2} + Bx^2}$. Find the dimension of A and B.

Sol. Same as above ques.

Q. In equation $y = A\sin(Bt + C)$, given A is amplitude in meter. Find the dimension of B and C.

Sol. Argument of trigonometric function are dimensionless

$\Rightarrow C = 1$

and $BT = 1$

$\boxed{B = T^{-1}}$

$\sin(Bt + C)$ is also dimensionless with Value $[-1, 1]$

$\boxed{\begin{aligned} &\text{D.F. of } y = L^1 \\ &\quad y = m^0 L^1 T^0 \end{aligned}}$

Q. In equation $y = A\sin\left(\frac{B}{t^2} + cv\right)$, given A is amplitude in meter v is velocity. Find the dimension of B and C.

Sol. $\left[\frac{B}{t^2} + cv\right] = 1$

$\Rightarrow \frac{B}{T^2} = 1 \Rightarrow B = T^2$

and $c \cdot v = 1$

$c \cdot LT^{-1} = 1$ $\qquad$ $[c] = [L^{-1}T]$

$[y] = [m^0 L^1 T^0]$

Q. In equation $q = Q_0 e^{-t/\tau}$ find D.F. of τ.

Sol. $\frac{t}{\tau}$ will be dimensionless because power of exponential function is dimensionless.

$\frac{t}{\tau} = 1 \Rightarrow [\tau] = [T]$

Q. In equation $y = A\, e^{Bx^2 + Cx + Dt^3}$ Find the dimensional formula of B, C and D.

Sol. Dimension of y will be same as that of A

$\boxed{y = m^0 L^1 T^0}$

power of e will be dimensionless so

$[Bx^2] = [Cx] = [Dt^3] = [1]$

$[B] = [L^{-2}]$

$[C] = [L^{-1}]$

$[D] = [T^{-3}]$

Q. In the equation pressure (P) is given by formula $P = \frac{\alpha}{\beta} e^{\frac{\alpha Z}{k\theta}}$ where Z is distance P is pressure k is Boltzmann constant θ is temperature. Find the dimension formula of α and β.

Sol. Internal energy U is given by $U = 3/2kT$ where T is temperature $\Rightarrow [kT] = [U]$

$[k\theta] = [ML^2T^{-2}]$

$[\alpha z]\ [\alpha L] = [ML^2T^{-2}]$

$\boxed{\alpha = MLT^{-2}}$

D.F. of P = D.F. of $\frac{\alpha}{\beta}$

$$\frac{MLT^{-2}}{L^2} = \frac{MLT^{-2}}{\beta}$$

$$\boxed{\beta = L^2}$$

Q. The position of a particle at time t, is given by the equation, $x(t) = \frac{v_0}{\alpha}(1 - e^{-\alpha t})$, where v_0 is a constant and $\alpha > 0$. The dimensions of v_0 and α are respectively.

Sol. $[a][t] = [M^0L^0T^0]$ and $[v_0] = [x][a]$

$[a] = [M^0L^0T^{-1}] \quad = [M^0L^1T^{-1}]$

WE CAN DERIVE THE RELATION BETWEEN PHYSICAL QUANTITY

भाई इस प्रकार के question की physics दो लाईन की but maths 10 line की भी हो सकती है..... all question are of same pattern..... let's try to understand.

Q. Time period of simple pendulum depends on mass (m) of block, length (l) of simple pendulum, acc. due to gravity (g) Obtain the formula of time period (T) dimensionally.

Sol. Suppose $T \propto m^x$

$T \propto l^y$ and k is dimensionless constant

$T \propto g^z$

$T \propto K m^x l^y g^z$

Using principle homogeneity dimension of L.H.S must be equal to dimension R.H.S

$T = K\ m^x l^y g^z$

$\underline{[LHS]} \rightarrow [M^0L^0T^1]$

$\underline{[RHS]} = [M^xL^yg^z]$

$[L.H.S] = [R.H.S]$

$M^0L^0T^1 = M^x L^y (LT^{-2})^z$

$M^0L^0T^1 = M^x L^{y+z} T^{-2z}$

Compareing powers of M, L, T in L.H.S and R.H.S

$x = 0, \qquad y + z = 0$

$-2z = 1$

$\boxed{z = -\frac{1}{2}} \qquad \boxed{y = +\frac{1}{2}}$

Putting values of x, y, z in the formula of time period T we get

$T = k m^0 l^{\frac{1}{2}} g^{-\frac{1}{2}}$

$T = k\sqrt{\frac{l}{g}}$ $\underline{\text{Actual}} \rightarrow T = 2\pi\sqrt{\frac{l}{g}}$

Ques की Language के मतलब पर नही जाना है ना ही Physics को समझने की कोशिश करनी है। बस ये देखना है की इस तरह के सवाल mathematically कैसे solve हो रहे है और eqn solve करते वक्त calculation mistake नही होनी चाहिए। इसी वात पे next ques खुद से solve करो।

Q. A gas bubble oscillate with time period T proportional to P (pressure), d (density), and E (energy) find relation between them using dimension analysis.

$$[\text{Pressure}(P)] \equiv \left[\frac{MLT^{-2}}{L^2}\right]$$

Sol.

Sir ये ocsillation क्या होता है।

ये सव हम mechanics मे पढ़ेंगे अभी इसका load मत लो

Suppose time period varies with p, d and E by the following relation

$T = K\, p^x d^y E^z$

Using principle of homogeneity

$[T] = [K\, p^x d^y E^z]$ K is dimensionless

$M^0L^0T^1 = (ML^{-1}T^{-2})^x . (ML^{-3})^y (ML^2T^{-2})^z$

$M^0L^0T^1 = M^{x+y+z} L^{-x-3y+2z} T^{-2x-2z}$

$$\boxed{\begin{array}{l} -2x - 2z = 1 \\ x + y + z = 0 \\ -x - 3y + 2z = 0 \end{array}}$$

Solve yourself

Q. A satellite is orbiting around a planet, its orbiting velocity depends on R (radius of orbit), m (mass of planet) and G (gravitational constant). Using dimensional analysis find an expression relating orbital velocity of satellite

Sol. Suppose orbital velocity (v) depends on R, M and G by the following relation

$v = KR^x m^y G^z$ K is dimensionless constant

Using principle homogeneity

$[v] = [KR^x m^y G^z]$

$LT^{-1} = L^x M^y (M^{-1}L^3T^{-2})^z$

$M^0L^1T^{-1} = M^{y-z} L^{x+3z} T^{-2z}$

Comparing powers of M, L, T on both sides of the equation we get

$-2z = -1$ ....(i) $\Rightarrow z = \frac{1}{2}$

$y - z = 0$, ...(ii) $y = z$

$\Rightarrow y = \frac{1}{2}$, $z = \frac{1}{2}$

$x + 3z = 1$ (iii)

$x + \frac{3}{2} = 1$

$x = -\frac{1}{2}$

$$\left.\begin{aligned} x &= -\frac{1}{2} \\ y &= \frac{1}{2} \\ z &= \frac{1}{2} \end{aligned}\right\}$$

putting values of x, y, z in $v = KR^x M^y G^z$ we get

$v = kR^{-\frac{1}{2}} M^{\frac{1}{2}} G^{\frac{1}{2}}$

$$\boxed{v = k\sqrt{\frac{Gm}{R}}}$$

Sir, क्या इन सवालो मे proportionality constant K हमेशा dimensionless होगा
हाँ भाई ऐसे सवालो मे K को dimensionless ही मानना है अगर K की dimension हुई तो वो ques इस method से solve नही होगा और ये एक dimensional analysis की limitation भी है।

Q. Energy of a particle oscillating depends on mass (m) frequency 'f', and amplitude 'A' such that

$E = \underbrace{K}_{\text{dimensionless}} m^x f^y A^z$ find value of x + y + z

$ML^2T^{-2} = M^x(T^{-1})^y L^z$

$ML^2T^{-2} = M^x L^z T^{-y}$

$x = 1$

$z = 2$

$y = 2$

$\boxed{1 + 2 + 2 = 5 \text{ Ans.}}$

Q. When a solid sphere moves through a liquid, the liquid opposes the motion with a force *F*. The magnitude of *F* depends on the coefficient of viscosity η of the liquid, the radius *r* of the sphere and the speed *v* of the sphere. Assuming that *F* is proportional to different powers of these quantities, guess a formula for *F* using the method of dimensions.

Sol. Suppose the formula is $F = k\eta^a r^b v^c$

Then, $[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L]^b \left[\frac{L}{T}\right]^c$

$= [M^a L^{-a+b+c} T^{-a-c}]$

Equating the exponents of M, L and T from both sides,

$a = 1$

$-a + b + c = 1$

$-a - c = -2$

Solving these, $a = 1$, $b = 1$ and $c = 1$

Thus, the formula for *F* is $F = k\eta rv$.

UNIT CONVERSION FROM DIMENSION ANALYSIS

Q. Convert 1 Newton into dyne

dyne → It is unit of force in CGS system

Sol. The dimensional equation of force is $[F] = [M^1 L^1 T^{-2}]$

Therefore if n_1, u_1 and n_2, u_2 corresponds to SI and CGS numerical value and unit respectively, then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 1\left[\frac{kg}{g}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2} = 1\times(1000)(100)(1) = 10^5$$

Method-2:

$$1\text{ N} = 1\text{ kgm/sec}^2 \xrightarrow{CGS} 1.\frac{1000\,gm\cdot 100\,cm}{sec^2}$$

$$= 10^5\ \frac{gm\cdot cm}{sec^2} = 10^5\text{ (dyne)}$$

Q. Convert 1Joule into erg

erg → Unit of energy in CGS system

Sol. $1\text{ Joule} = \frac{1\,kg\,m^2}{sec^2} \Rightarrow \frac{1\times 1000\,gm\times(100\,cm)^2}{sec^2}$

$$= 10^7\ \underbrace{\boxed{\frac{gm\ cm^2}{sec^2}}}_{erg}$$

$= 10^7$ erg

Q. The dimensional formula for viscosity of fluids is $[\eta] = [M^1L^{-1}T^{-1}]$. Find how many poise (CGS unit of viscosity) is equal to 1 poiseuille (SI unit of viscosity).

Sol. $[\eta] = [M^1\,L^{-1}\,T^{-1}]$

1 CGS units = $g\ cm^{-1}\ s^{-1}$

1 SI units = $kg\ m^{-1}\ s^{-1}$

$= (1000\ g)(100\ cm)^{-1}\ s^{-1} = 10\ g\ cm^{-1}\ s^{-1}$

Thus, 1 Poiseuilli = 10 poise

Q. A calorie is a unit of heat or energy and it equals about 4.2 J, where 1 J = 1 kg m^2/s^2. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β metre, the unit of time is γ second. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\beta^{-2}\gamma^2$ in terms of the new units.

Sol. 1 cal = 4.2 kg m^2s^{-2}

SI New system

$n_1 = 4.2 \quad n_2 = ?$

$M_1 = 1$ kg $M_2 = \alpha$ kg

$L_1 = 1$ m $\quad L_2 = \beta$ metre

$T_1 = 1$ s $\quad T_2 = \gamma$ second

Dimensional formula of energy is $[ML^2T^{-2}]$

Comparing with $[M^aL^bT^c]$,

We find that $a = 1, b = 2, c = -2$

$$\text{Now, } n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 4.2\left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1\left[\frac{1\text{ m}}{\beta\text{ m}}\right]^2\left[\frac{1\text{ s}}{\gamma\text{ s}}\right]^{-2} = 4.2\ \alpha^{-1}\beta^{-2}\gamma^2$$

अब मै आपके लिए नीचे बहुत सारे attach कर रहा हुँ इन्हे sincerely solve करे खुद से bcz Mains/NEET/Advance सभी exam मे ऐसे ही question आने की probability सबसे highest है ये लो pen और start हो जाओ शुरू।

Q. In a given system of Units. 1 Unit of mass = 2kg 1 Unit of length = 5m 1 Unit of time = 5sec

Then in this system, 1N represent

Sol. Unit of force in hypothetical system

$$= \frac{2kg5m}{(5sec)^2} = \frac{10}{25}\boxed{\frac{kgm}{sec^2}}$$

$1N' = \frac{2}{5}N$ हमारा वाला

$$\boxed{1N = \frac{5}{2}N',}$$

Q. Unit of force in given system is represented by N'. In this system 1 Unit of mass = 20kg 1 Unit of length = 4m 1 Unit of time = 2sec. Answer following parts of questions.

Sol. (i) 1 Unit of force in this system

$$= \frac{20kg\cdot 4m}{(2sec)^2} = \frac{20kg\,m}{sec^2} = 20N$$

(ii) Numerical value of 1N in this system = $\frac{1}{20}N'$

(iii) Numerical value of 50N in this system

$$= \frac{50}{20}\ N'$$

Few Important Question You Must Solve Because ये exam मे repeatedly पुछे गए है

Q. If P is the pressure of a gas and ρ is its density, then find the dimension of velocity in terms of P and ρ.

Sol. **Method - I**

$[P] = [ML^{-1}T^{-2}]$ ...(i)

$[\rho] = [ML^{-3}]$ ...(ii)

Dividing eq. (i) by (ii)

$[P\rho^{-1}] = [L^2T^{-2}]$

$\Rightarrow [LT^{-1}] = [P^{1/2}\rho^{-1/2}]$

$\Rightarrow [v] = [P^{1/2}\rho^{-1/2}]$

Method - II

$v \propto P^a \rho^b$

$v = kP^a \rho^b$

$[LT^{-1}] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b$

$\Rightarrow a = \frac{1}{2}, b = -\frac{1}{2}$ (Equating dimensions)

$\Rightarrow [v] = [P^{1/2}\rho^{-1/2}]$

Q. Find relationship between speed (v) of sound in a medium, the elastic constant (E) and the density of the medium (ρ).

Sol. Let the speed depends upon elastic constant and density according to the relation

$v \propto E^a \rho^b$

or $v = KE^a\rho^b$ ...(i)

Where K is a dimensionless constant of proportionality.

Considering dimensions of the quantities

$[v] = M^0 L T^{-1}$

$[E] = \frac{[\text{stress}]}{[\text{strain}]} = \frac{[\text{force}]/[\text{area}]}{[\Delta l]/[l]} = \frac{[M^1L^1T^{-2}]/[L^2]}{[L^1]/[L^1]} = [M^1L^{-1}T^{-2}]$

$\therefore [E^a] = [M^a L^{-a} T^{-2a}]$

$[\rho] = [\text{mass}]/[\text{volume}] = [M]/[L^3] = [M^1L^{-3}T^0]$

$\therefore [\rho^b] = [M^b L^{-3b} T^0]$

Equating the dimensions of the LHS and RHS quantities of equation (i), we get

$[M^0 L^1 T^{-1}] = [M^a L^{-a} T^{-2a}] \times [M^b L^{-3b} T^0]$

or $[M^0 L^1 T^{-1}] = [M^{a+b} L^{-a-3b} T^{-2a}]$

Comparing the individual dimensions of M, L and T

$a + b = 0,$ ...(ii)

$-a - 3b = 1,$ and ...(iii)

$-2a = -1$ ...(iv)

Solving we get

$a = \frac{1}{2}, b = -\frac{1}{2}$

Therefore the required relation is

$v = K\sqrt{\frac{E}{\rho}}$.

Q. Pressure (P) acting due to a fluid kept in a container depends on, weight of liquid (w), Area of cross-section of container (A) and density of fluid (ρ). Establish a formula of pressure (P).

Sol. According to the question

$P \propto w^x A^y \rho^z$

$P = K[w^x A^y \rho^z]$

Now writing the dimension of each quantity on either side.

$[ML^{-1}T^{-2}] = K[MLT^{-2}]^x [L^2]^y [ML^{-3}]^z$

$[ML^{-1}T^{-2}] = K[M]^{x+z} [L]^{x+2y-3z} [T]^{-2x}$

Now comparing the powers

For M, $1 = x + z$ ...(i)

L, $-1 = x + 2y - 3z$ ...(ii)

T, $-2 = -2x$ ...(iii)

$\Rightarrow$ Solving we get, $x = 1, y = -1, z = 0$

$\therefore P = KwA^{-1}\rho^0$

Q: If ε_0 is the permittivity of free space and E is the electric field, then $\varepsilon_0 E^2$ has the dimensions:

Ans. $ML^{-1}T^{-2}$

Hint: Energy density $= \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$

Q. Match List-I with List-II.

	List-I		List-II
A.	Coefficient of viscosity	I.	$[ML^2T^{-2}]$
B.	Surface Tension	II.	$[ML^2T^{-1}]$
C.	Angular momentum	III.	$[ML^{-1}T^{-1}]$
D.	Rotational kinetic energy	IV.	$[ML^0T^{-2}]$

Sol. As we know,

$$F = \eta A\frac{dv}{dy},\ L = mvr,\ S.T = \frac{F}{\ell},\ K.E = \frac{1}{2}I\omega^2$$

Hence,

$$F = \eta A\frac{dv}{dy},$$

$$[MLT^{-2}] = \eta\,[L^2]\,[T^{1}]$$

$$\eta = [ML^{-1}T^{1}]$$

$$S.T = \frac{F}{\ell} = \frac{[MLT^{-2}]}{[L]} = [ML^0T^{-2}]$$

$$L = mvr = [ML^2T^{-1}]$$

$$K.E = \frac{1}{2}I\omega^2 = [ML^2T^{-2}]$$

Q. The equation of state of a real gas is given by $\left(P+\frac{a}{V^2}\right)(V-b) = RT$, where P, V and T are pressure. volume and temperature respectively and R is the universal gas constant. The dimensions of $\frac{a}{b^2}$ is similar to that of:

Sol. $[P] = \left[\frac{a}{V^2}\right] \Rightarrow [a] = [PV^2]$

And dimension of V = dimension of b

$$\therefore\ \frac{[a]}{[b^2]} = \frac{[PV^2]}{[V^2]} = [P]$$

Q. Young's modulus of elasticity Y is expressed in terms of three derived quantities, namely, the gravitational constant G, Planck's constant h and the speed of light c, as $Y = c^\alpha h^\beta G^\gamma$. Which of the following is the correct option?

Sol. $Y = c^\alpha h^\beta G^\gamma$

$$[ML^{-1}T^{-2}] = [LT^{-1}]^\alpha[ML^2T^{-1}]^\beta[M^{-1}L^3T^{-2}]^\gamma$$

$$[ML^{-1}T^{2}] = [M^{\beta-\gamma}]\,[L^{\alpha+2\beta+3\gamma}]\,[T^{-\alpha-\beta-2\gamma}]$$

$1 = \beta - \gamma$ ...(i)

$-1 = \alpha + 2\beta + 3\gamma$ ...(ii)

$-2 = -\alpha - \beta - 2\gamma$ ...(iii)

After solving above equations

$\alpha = 7,\ \beta = -1,\ \gamma = -2$

Q. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e, electron mass m_e, Planck's constant h, and coulomb's constant $k = \frac{1}{4\pi\varepsilon_0}$, where ε_0 is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^\alpha\,[m_e]^\beta\,[h]^\gamma\,[k]^\delta$. The value of $\alpha + \beta + \gamma + \delta$ is ______________.

Sol. $MT^{-2}I^{-1} = [IT]^\alpha[M]^\beta[ML^2T^{-1}]^\gamma\,[ML^3T^{-4}I^{-2}]^\delta$

$\therefore\ \beta + \gamma + \delta = 1$ ...(i)

$2\gamma + 3\delta = 0$ ...(ii)

$\alpha - \gamma - 4\delta = -2$ ...(iii)

$\alpha - 2\delta = -1$ ...(iv)

On solving these equations, we get

$\alpha + \beta + \gamma + \delta = 4$

Q. If time (t), velocity (v), and angular momentum (ℓ) are taken as the fundamental units. Then the dimension of mass (m) in terms of t, v,and ℓ is:

Sol. Let us suppose,

Mass $\alpha\ [t^a\,v^b\,\ell^c] \Rightarrow [M^1L^0T^0] \equiv T^a \times [LT^{-1}]^b \times [ML^2T^{-1}]^c$

On equating both sides, we get

$\Rightarrow a - b - c = 0;\ c = 1$, and $b + 2c = 0 \Rightarrow b = -2c = -2$

$\Rightarrow a = b + c = 1 - 2 = -1$

Hence, dimension of mass $= [t^a\,v^b\,\ell^c] = [t^{-1}\,v^{-2}\,\ell^1]$

Q. In a hypothetical system if 1 Unit of mass = 10 kg 1 Unit of length = 5 m 1 Unit of sec = 2 sec.

Find

(1) 1 unit of force in this hypothetical system in terms of newton.

(2) 1 Unit of Energy in this hypothetical system

1 Unit of force in this system

(3) Find numerical value of 500 Joule in terms of new system

Sol. SKC Method

(i) Value of 1 unit of force in hypothetical system = 1 N' = $\frac{10\text{kg}\times 5\text{m}}{(2\text{sec})^2} = \frac{50}{4}\frac{\text{kgm}}{(\text{sec})^2}$

1 N' = 12.5 Newton = 12.5N

(ii) Value of 1 unit of energy in hypothetical system $= 1\,J' = \frac{10\,kg \times (5m)^2}{(2\,sec)^2} = 62.5\,\frac{kgm^2}{sec^2}$

$= 62.5$ Joule

(iii) $1\,J' = 62.5\,J$

$\Rightarrow 1\,J = \frac{1}{62.5} J'$

$\Rightarrow 500\,J = 500 \times \frac{J'}{62.5} = 8\,J'$

Ans. 8

Method-2:

$n_1 u_1 = n_2 u_2$

$n_1 M_1 L_1^2 T_1^{-2} = n_2 M_2 L_2^2 T_2^{-2}$

$\Rightarrow 500 \times \frac{1kg \times 1m^2}{1 sec^2} = n_2 \frac{(10kg)(5m)^2}{(2 sec)^2}$

$\Rightarrow 500 = n_2 \times \frac{250}{4}$

$n_2 = 4$

Q. In a typical combustion engine the work done by a gas molecule is given by $W = \alpha^2 \beta e^{-\frac{\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be:

Sol. We have, $W = \alpha^2 \beta e^{-\frac{\beta x^2}{kT}}$

$\therefore [\beta x^2] = [kT]$

$= [ML^2T^{-2}] \left[\because E = \frac{3}{2}kT\right]$

$\Rightarrow [\beta] = [MT^{-2}]$

And, $[W] = [\alpha^2 \beta] \Rightarrow [ML^2\ T^{2}] = [\alpha^2][MT^{-2}]$

$\Rightarrow [\alpha] = [L]$

Q. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct?

Sol. Mass $= M^0L^0T^0$

$Mvr = M^0L^0T^0$

$M^0 \frac{L^1}{T^1} . L^1 = M^0L^0T^0$

$L^2 = T^1$ ...(i)

Force $= M^1L^1T^{-2}$ (in SI)

$= M^0L^1L^{-4}$ [In new system from equation (i)]

$= L^{-3}$

Energy $= M^1L^2T^{-2}$ (In SI)

$= M^0L^2L^{-4}$ [In new system from equation (i)]

$= L^{-2}$

Power $= \frac{\text{Energy}}{\text{Time}}$

$= M^1L^2T^{-3}$ (in SI)

$= M^0L^2L^{-6}$ [In new system from equation (i)]

$= L^{-4}$

Linear momentum $= M^1L^1T^{-1}$ (in SI)

$= M^0L^1L^{-2}$ [In new system from equation(i)]

$= L^{-1}$

Vector

सुनो भाई ये chapter वैसे तो mathematics का है जो maths वाले बच्चे 12th class मे extreme detail मे पढ़ोगे यहाँ हमे उतना ही पढ़ना है जितना physics मे use होना है जरूरत से ज्यादा PHD करने की कोशिश ना करे bcz वो हम 12th मे करेंगे कुछ बच्चे जो इसे पहली बार पढ़ रहे है उन्हे starting मे यह chapter मुश्किल लग सकता है उनसे मै यही कहुँगा की जितना content मै यहाँ दे रहा हुँ उसको 3-4 बार अच्छे से rough copy पर practice कर ले इतने से physics मे अच्छे से काम चल जाएगा

Scalar Quantity

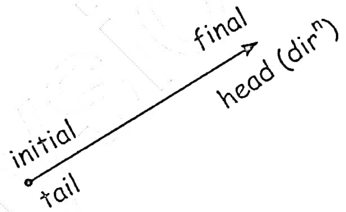
Those physical quantities which can be completely described by its magnitude only are called Scalar.

Vector Quantity

Those physical quantity which have magnitude & direction and they follow law of vector algebra.
Eg. Force, Acceleration, Momentum.

REPRESENTATION OF VECTOR

Diagram वाला तरीका



Length of arrow represent magnitude of vector.

SKC

कौनसी Physical quantity vector है या scalar है अभी तुम इस बात से कोई लेना देना नहीं है। क्योंकि अभी तुम्हारी physics start नहीं हुई है so take it lightly Vector का सफर चालू हो चुका।

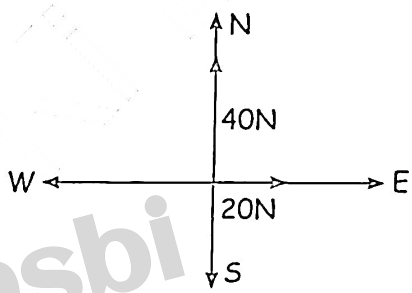
Mathematically/Analytical

☆ A vector P is represented by $\vec{P}$

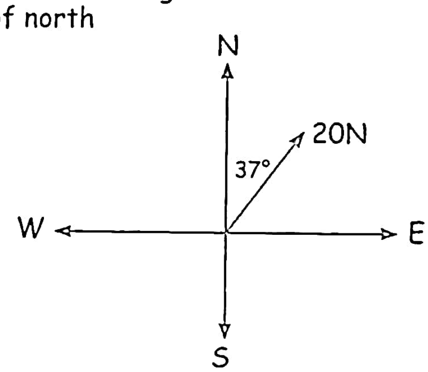
☆ Force = $\vec{F}$

Q. Represent vector $\vec{P}$ & vector $\vec{Q}$
$\vec{P}$ = 20N along East
$\vec{Q}$ = 40N along north

Sol.



☆ Represent $\vec{P}$ of magnitude 20N in direction of 37° east of north

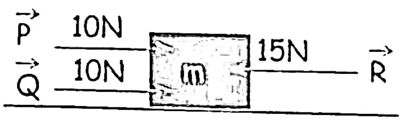


TYPES OF VECTOR

अबे अच्छे से इनकी reading ले लेना।

Equal Vector

Two vector are said to be equal vector, if they have same magnitude, same direction and same physical quantity.



$\vec{R}$ and $\vec{P}$ are equal vector : ✗

$\vec{P}$ and $\vec{Q}$ are equal vector : ✓

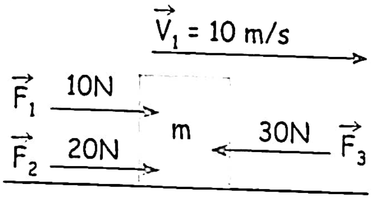
Parallel Vector →

Two vectors are said to be parallel if they have same direction.

> Here, Physical quantity, अलग - अलग हो सकती है। All equal vectors are parallel vectors

Anti-Parallel Vector = Direction Opposite

A Block is moving with constant velocity 10 m/s along east direction



$\vec{F}_1$ & $\vec{F}_2$ are parallel vector.

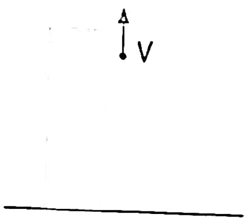
$\vec{F}_1$ & $\vec{F}_2$ are not equal vector.

$\vec{F}_1$ & $\vec{V}_1$ are parallel vector

$\vec{F}_2$ & $\vec{V}_1$ are parallel vector

$\vec{F}_1$ & $\vec{F}_3$ are anti-parallel vector

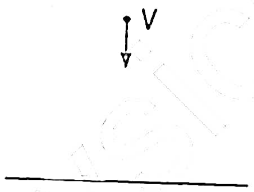
$\vec{V}_1$ & $\vec{F}_3$ are anti-parallel vector.



$\vec{V}$ → ऊपर

$\vec{a}$ → नीचे

$\vec{V}$, $\vec{a}$ are antiparallel.



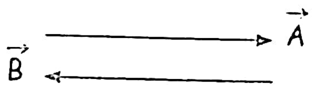
$\vec{V}$ → नीचे

$\vec{a}$ → नीचे

$\vec{V}$, $\vec{a}$ are parallel.

Negative Vector

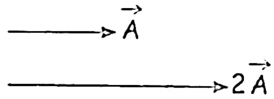
Two vectors $\vec{A}$, & $\vec{B}$, are said to be negative of each other if they have same magnitude & opposite direction.



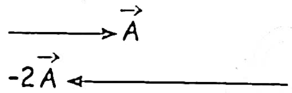
$\vec{A} = -\vec{B}$ are negative vector to each other.

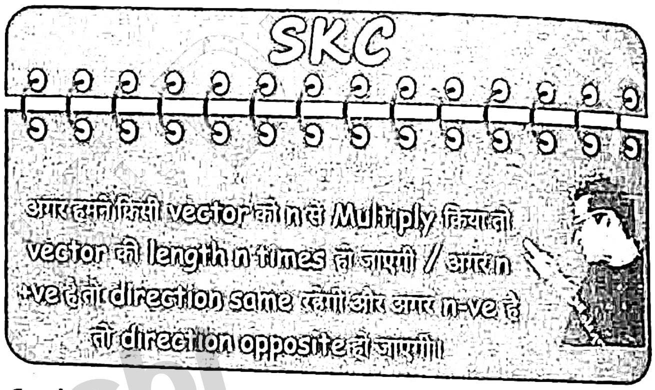
Multiplication of a Vector with a Number

☆ If a vector $\vec{A}$ is multiply by a number n > 0. Then magnitude of vector becomes n times & direction remains same.



☆ If a vector $\vec{A}$ is multiply by a number n < 0, then magnitude of vector becomes n times & direction become opposite.





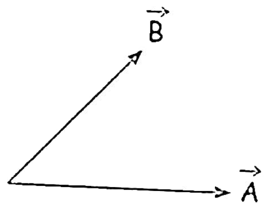
Coplanar Vector

If all vectors lie in same plane then vectors are known as co-planar.

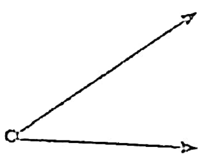
NOTE:- **TWO VECTORS are always COPLANAR**

Coinitial Vector

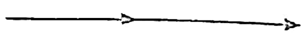
[Tail to Tail]: Two vectors are said to be coinitial if they have same initial point.



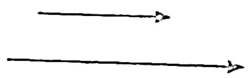
Co-Initial vector



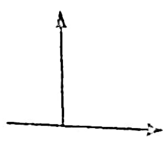
Collinear vector



Parallel vector



Orthogonal vector



कौन-सी physical quantity scalar है कौन-सी vector है ये तू याद कर रहा है ????

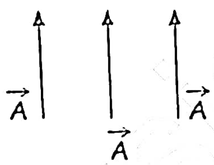
Haa humne RATT liye

Nhi RATTNA tha

पूरे दो साल यही पढ़ना है कि कौन-सी physical quantity क्या है So, इस पर time बिलकुल waste ना करें और मस्त रहे।

Parallel Shifting of Vector

Whenever we need, we can parallely shift any vector without changing direction and magnitude we can can say that vector will not change.



SKC

✧ देख भाई अभी हम अपनी जरूरत के अनुसार vector को parallelly shift कर सकते है जैसे computer के mouse का cursor sign होता है।

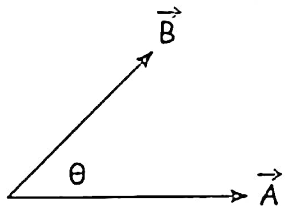
✧ अगर किसी vector की dirxn बदली मतलब vector बदल जाएगा।

✧ अगर किसी vector का Magnitude बदला मतलब vector बदल जाएगा।

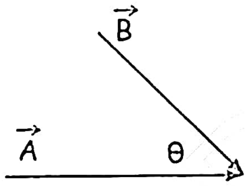
✧ अगर Magnitude, direction same = vector same

Angle Between Two Vectors

☆ It is the angle b/w their tail.



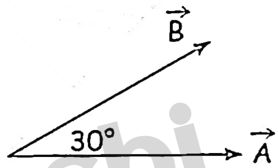
☆ It is the angle b/w their head.



[$0 < \theta < 180$] [$\theta \rightarrow$ angle b/w $\vec{A}$ & $\vec{B}$]

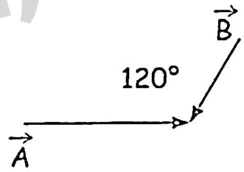
Q. Find angle b/w $\vec{A}$ & $\vec{B}$

(a)



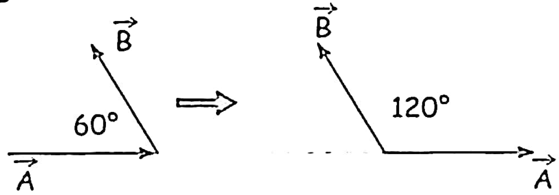
Angle b/w vectors = 30°

(b)



Angle b/w vectors = 120°

(c)

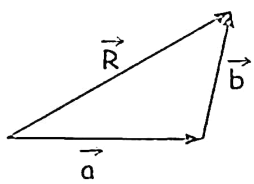


Angle b/w vectors = 180 − 60° = 120°

ADDITION OF 2 VECTORS

Triangle Rule of Vector Addition

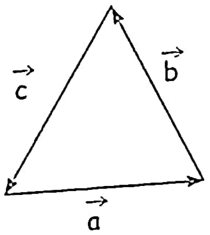
If 2 vectors represent two sides of a triangle in same sense (head of first vector coincide with tail of other vector) resultant is 3rd side of triangle in opposite sense.



$\vec{a} + \vec{b} = \vec{R}$

Q. Write the eqn for following fig. using triangle of vector addition.

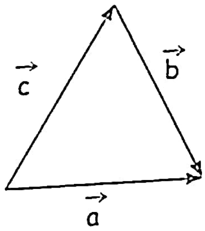
(a)



Closed loop same sense

$\vec{a}+\vec{c}+\vec{b}=0$ इसका मतलब तीनो vector का sum/ addition/resultant 0 आगया।

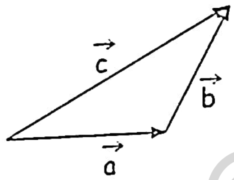
(b)



$\vec{a}=\vec{b}+\vec{c} \Rightarrow \vec{a}-\vec{c}=\vec{b}$

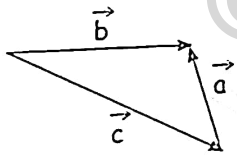
$\vec{a}$ is resultant of $\vec{c}$ and $\vec{b}$

(c)



$\vec{a}+\vec{b}=\vec{c}$

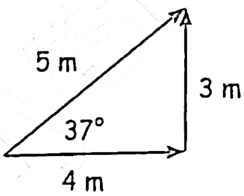
(d)



$\vec{c}+\vec{a}=\vec{b}$

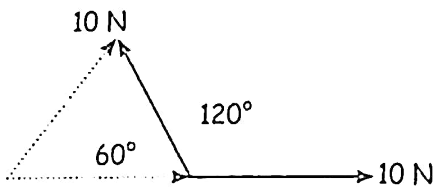
Q. A man moves 4m in east direction and then 3m in north direction. Find the resultant displacement of man.

Sol.



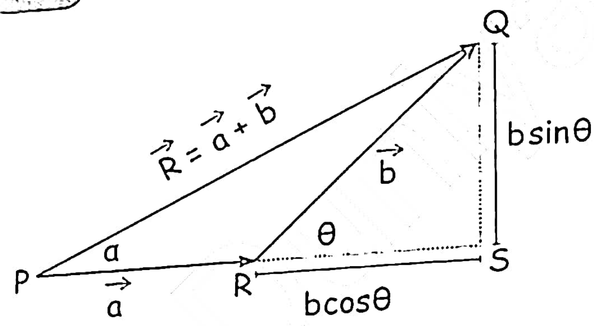
5 m in 37° north of east OR 5 m in 53° East of north.

Q.



Sol. Resultant 10 in 60° north of east.

ANALYTICAL METHOD TO FIND RESULTANT AND ITS DIRECTION (OF 2 VECTORS)



$|\vec{a}|=a,\ |\vec{b}|=b,\ |\vec{R}|=|\vec{a}+\vec{b}|=R$

In ΔRQS

$\sin\theta = QS/b \Rightarrow QS = b\sin\theta$

$\cos\theta = RS/b \Rightarrow RS = b\cos\theta$

In ΔPQS

$(PQ)^2 = (PR+RS)^2 + QS^2$

$R^2 = (a+b\cos\theta)^2 + (b\sin\theta)^2$

$R^2 = a^2 + b^2(\cos^2\theta + \sin^2\theta)^2 + 2ab\cos\theta$

$\boxed{R=\sqrt{a^2+b^2+2ab\cos\theta}}$ $\boxed{\tan\alpha = \dfrac{b\sin\theta}{a+b\cos\theta}}$

R = Magnitude of resultant.

α = Angle b/w $\vec{R}$ & $\vec{a}$

देख भाई काम की बात यह है कि दो vector $\vec{a}$ and $\vec{b}$ के resultant का magnitude $R=\sqrt{a^2+b^2+2ab\cos\theta}$ होता है जो $\vec{a}$ के साथ angle α बनाता है

Where $\tan\alpha = \dfrac{b\sin\theta}{a+b\cos\theta}$ ये formula गलती से भी मत भूलना वरना पिटाई होगी अगर resultant R ने $\vec{b}$ के साथ angle β बनाया तो $\tan\beta = \dfrac{a\sin\theta}{b+a\cos\theta}$

Q. Two vector of magnitude 10N each gives a resultant of magnitude $10\sqrt{3}$ N. Find angle b/w both vector.

Sol. $R = 10\sqrt{3}$ N

$R = \sqrt{10^2 + 10^2 + 200\cos\theta}$

$300 = 200 + 200\cos\theta \Rightarrow \theta = 60°$

Q. Two forces $\vec{A}$ and $\vec{B}$ of magnitude 10 N and 20 N are acting on a block having 60° angle between them. Find the magnitude of resultant of them.

Sol. $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{100 + 400 + 200} = \sqrt{700}$

Angle made by resultant of $\vec{A}$ & $\vec{B}$ with $\vec{A}$.

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} = \frac{20 \times \frac{\sqrt{3}}{2}}{10 + 20 \times \frac{1}{2}} = \frac{\sqrt{3}}{2}$$

SPECIAL CASES

① θ = 0° (Parallel Vectors)

$$\xrightarrow{\vec{a}}\ \xrightarrow{\vec{b}}$$

$\vec{R} = \vec{a} + \vec{b}$

$R = \sqrt{a^2 + b^2 + 2ab} = a + b$

$R_{max} = a + b = |\vec{a}| + |\vec{b}|$

② θ = 180° (Antiparallel Vectors)

$$\xrightarrow{\vec{a}}\ \xleftarrow{\vec{b}}$$

$\vec{R} = \vec{a} + \vec{b}$

$R = \sqrt{a^2 + b^2 - 2ab} = |a - b|$

$R_{min} = |a - b| = \left||\vec{a}| - |\vec{b}|\right|$

$\boxed{|a - b| \le R \le (a + b)}$

$R_{max} = a + b \qquad R_{min} = |a - b|$

③ θ = π/2 or 90°

$R = \sqrt{a^2 + b^2}$

NOTE: Value of θ increases from 0° to 180° then magnitude of resultant will decrease.

Vector की understanding बढ़ाने के लिए नीचे कुछ important ques. attach कर रहा हुँ भले ही आपको कुछ ques. easy लगे फिर भी आपको सारे ques. in the last solve करने है।

Q. Max and min magnitude of resultant of two forces are 7 N and 1 N respectively. Find the resultant of 2 forces when it act orthogonally.

Sol. $F_1 + F_2 = 7$ N;

$F_1 - F_2 = 1$ N

$\Rightarrow F_1 = 4$ N; $F_2 = 3$ N

When F_1 and F_2 are perpendicular to each other

then $R = \sqrt{3^2 + 4^2} = 5$

Q. Two vector A & B with same magnitude x. Find magnitude of resultant of A and B. θ = 60°.

Sol. $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{2x^2 + 2x^2\cos 60°}$

$= \sqrt{2x^2 + 2x^2\frac{1}{2}} = \sqrt{3x^2} = x\sqrt{3}$

Q. In above question if A = x, B = x, θ = 120°, R = ?

Sol. $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{2x^2 + 2x^2\left(-\frac{1}{2}\right)}$

$= \sqrt{x^2} = x$

Q. If two vector $\vec{A}$ and $\vec{B}$ of magnitude 10 N and 6 N are at an angle 60°. If $\vec{B}$ become twice to its initial value & added to A. Find magnitude of resultant of A & B after $\vec{B}$ change.

Sol. B = 2 × 6 = 12 N

A = 10 N

अबे θ तो 60° ही रहेगा

$C = \sqrt{100 + 144 + 2 \times 10 \times 12 \times \cos 60} = \sqrt{364}$

Q. Magnitude of $\vec{A}$ is 8 N.

Magnitude of $\vec{B}$ is 6 N.

Which of the following can be magnitude of $\vec{A} + \vec{B}$

(a) 10 N (b) 22 N (c) 48 N
(d) 2 N (e) 2.001 N (f) 1.99 N
(g) 14.1 N (h) 13.999 N

Sol. $C_{max} = A + B = 14$

$C_{min} = A - B = 2$

$2 \le C \le 14$

Ans. (a), (d), (e), (h)

Q. Magnitude of resultant of $\vec{A}$ and $\vec{B}$ is 5 unit where magnitude of $\vec{A}$ is $5\sqrt{3}$ unit and magnitude of $\vec{B}$ = 5 unit. Find angle between A and B.

Sol. $\vec{C} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$5^2 = 75 + 25 + 50\sqrt{3}\cos\theta$

$\frac{-\sqrt{3}}{2} = \cos\theta \Rightarrow \theta = 150°$

Q. Sum of the magnitude of $\vec{A}$ and $\vec{B}$ is 16 N. Magnitude of resultant of vector $\vec{A}$ and $\vec{B}$ is 8 N. When resultant is perpendicular to the $\vec{A}$. Find magnitude of $\vec{A}$ and $\vec{B}$.

Sol. $A + B = 16$ N

Magnitude of $\vec{R} = 8$ N

$\alpha = 90°$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$, $\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

$\tan 90° = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow A + B\cos\theta = 0$

$B\cos\theta = -A$

$64 = A^2 + B^2 + 2AB\cos\theta$

$64 = A^2 + B^2 - 2A^2$

$64 = B^2 - A^2 = (A + B)(B - A)$

$64 = 16(B - A)$

$4 = (B - A)$

$B - A = 4$

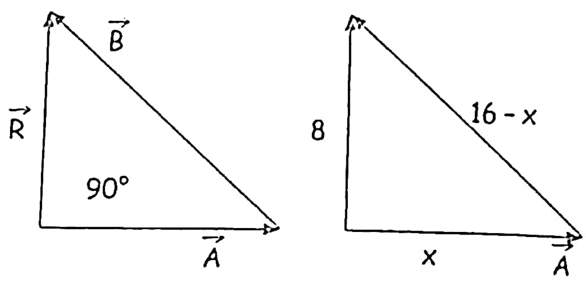
$B + A = 16$

$2B = 20$

$B = 10$

$A = 6$

Method-2:



By using Pythagoras theorem.

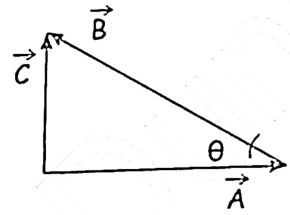
$(16 - x)^2 = 8^2 + x^2$

$256 + x^2 - 32x = 64 + x^2$

$256 - 32x = 64$

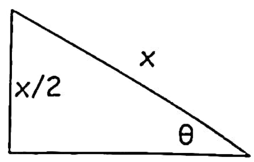
$\Rightarrow 6 = x$ & $16 - x = 10$

Q. Resultant of $\vec{A}$ and $\vec{B}$ is $\perp$ to $\vec{A}$ and its magnitude is equal to half of magnitude of $\vec{B}$. Find the angle between $\vec{A}$ and $\vec{B}$.



Sol. $\vec{A} + \vec{B} = \vec{C}$

$\sin\theta = \frac{\frac{x}{2}}{x} = \frac{1}{2}$



$\theta = 30°$

Angle between $\vec{A}$ & $\vec{B}$ = 180° – 30° = 150°

Angle between $\vec{B}$ and $\vec{C}$ = 60°

Angle between $\vec{A}$ and $\vec{C}$ = 90°

3. Two vector $\vec{A}$ and $\vec{B}$ have same magnitude 'a' and resultant has magnitude R. Now $\vec{B}$ is doubled and added to $\vec{A}$ and now new resultant become $a\sqrt{3}$. Find angle b/w $\vec{A}$ and $\vec{B}$.

Sol. $\vec{A} + \vec{B} = \vec{R}$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$R_{old} = \sqrt{2a^2 + 2a^2\cos\theta}$

$|2\vec{B} + \vec{A}| = a\sqrt{3}$

$R_{new} = \sqrt{a^2 + (2a)^2 + 2(a)(2a)\cos\theta}$

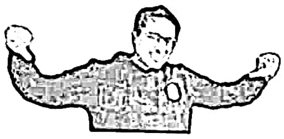
$a\sqrt{3} = \sqrt{a^2 + 4a^2 + 4a^2\cos\theta}$

$3a^2 = 5a^2 + 4a^2\cos\theta$

$-2\cancel{a^2} = 4\cancel{a^2}\cos\theta$

$\frac{-2}{4} = \frac{-1}{2} = \cos\theta$

$\theta = 120°$



काम का डव्वा

☆ If $\vec{A}+\vec{B}=\vec{R}$

Magnitude of $\vec{R} = R = \sqrt{A^2+B^2+2AB\cos\theta}$

θ = angle between A & B

☆ $\tan\alpha = \dfrac{b\sin\theta}{a+b\cos\theta}$ (कैसे आया Not Important)

(α is the angle made by $\vec{R}$ with $\vec{A}$)

☆ $R = |\vec{A}+\vec{B}|$ = magnitude of resultant of $\vec{A}$ and $\vec{B}$.

☆ $|A-B| \le R \le (A+B)$

☆ दो vector का Resultant max तव होगा जव उनके बीच angle 0° होगा। और Min तव होगा जव उनके बीच angle 180° होगा।

If $\theta = 0 \Rightarrow R = A + B = R_{max}$

If $\theta = 180 \Rightarrow R = |A-B| = R_{min}$ = [वडा-छोटा]

If $\theta = 90 \Rightarrow R = \sqrt{A^2+B^2}$

☆ If $|\vec{A}| = |\vec{B}| = A$(let)

$\theta = 0 \longrightarrow R = 2A$

$\theta = 60° \longrightarrow R = A\sqrt{3}$

$\theta = 90° \longrightarrow R = A\sqrt{2}$

$\theta = 120° \longrightarrow R = A$

$\theta = 180° \longrightarrow R = 0$

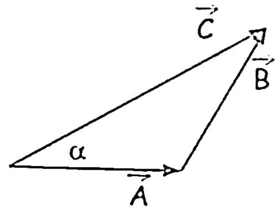
$R = 2A\cos\theta/2$

दो equal magnitude के vector अगर 60° पर है तो उनका resultant $\sqrt{3}$ times i.e $A\sqrt{3}$ होगा और वो 120° पर है तो उनका resultant A होगा।

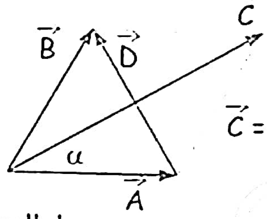
☆ दो equal magnitude वाले vector का resultant उनके बीचो-बीच (along angle bi-sector) निकलेगा।

PARALLELOGRAM LAW OF VECTOR ADDITION

If two coinitial vectors are given then resultant of these two vectors are given by diagonal of parallelogram (II gm) made from 2 given vector by shifting them parallel to their cointial vector & other diagonal of the II gm gives difference of vectors.



$\vec{C} = \vec{A} + \vec{B}$

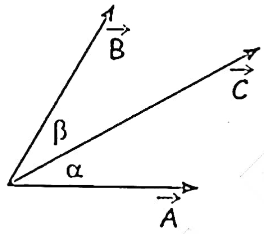


$\vec{C} = \vec{A} + \vec{B}$

Parallelogram

$\vec{A} + \vec{D} = \vec{B}$

$\vec{D} = \vec{B} - \vec{A}$



If A > B, α < β

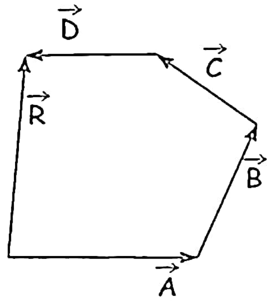
If A < B, α > β

[जो Vector भारी होगा Resultant उस तरफ झुकेगा]

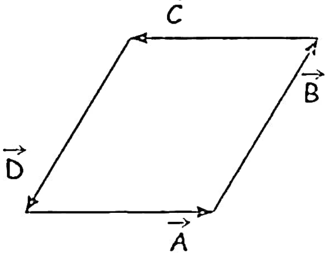
POLYGON-LAW

SKC

Resultant vector को draw करने के लिए पहले की Tail को Last के Head se Join Krdo



$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{R}$

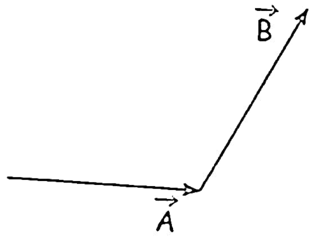


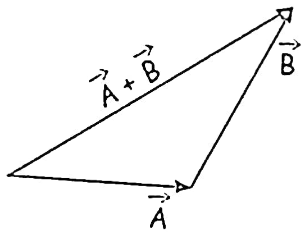
$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{O}$ [zero vector]

Dirxn of this zero vector is arbitary. [Jo Man Kre]

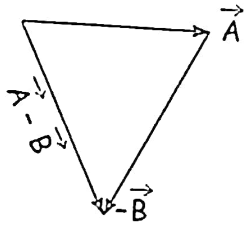
It is a vector where magnitude is zero.

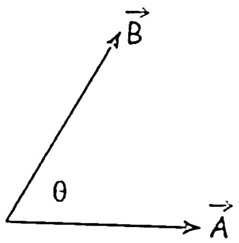
SUBTRACTION OF TWO VECTOR

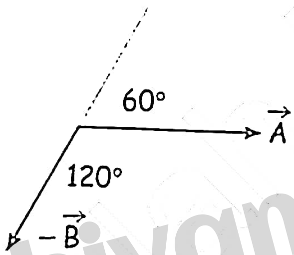




$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$







$\vec{A}$ और $\vec{B}$ के बीच अगर angle θ है तो $\vec{A}$ में और $-\vec{B}$ के बीच 180-θ angle होगा।

If angle between $\vec{A}$ & $\vec{B}$ is θ,

Then, $\vec{C} = \vec{A} + \vec{B}$

$$C = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

& $\vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

↳ Magnitude = ?

angle between $\vec{A}$ & $-\vec{B}$ will be 180 − θ.

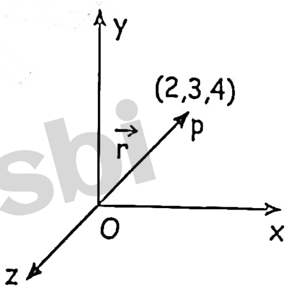
$$D = \sqrt{A^2 + B^2 + 2AB\cos(180-\theta)}$$

$$= \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin(180-\theta)}{A + B\cos(180-\theta)} = \frac{B\sin\theta}{A - B\cos\theta}$$

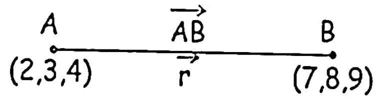
POSITION VECTOR

A vector representing location or position of a point in space w.r.t origin is called position vector.



Position vector of P w.r.t origin = $2\hat{i} + 3\hat{j} + 4\hat{k}$

☆

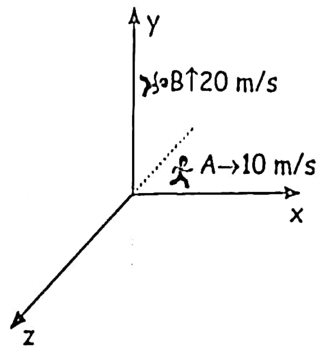


$\vec{AB}$ = P.V of B − P.V of A

$= (7-2)\hat{i} + (8-3)\hat{j} + (9-4)\hat{k} = 5\hat{i} + 5\hat{j} + 5\hat{k}$

Q. A particle A is moving with speed 10 m/s along + x-axis and another particle B is moving along y-axis with speed 20 m/s. Find their velocity.

Sol.



→ Velocity of A = $10\hat{i}$

→ Velocity of B = $20\hat{j}$

MAGNITUDE OF A VECTOR

Q. $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

Sol. Magnitude of $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Q. $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $|\vec{A}| = ?$

Sol. Magnitude of A

$= \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4+9+16} = \sqrt{29}$

Q. $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, $|\vec{A}| = ?$

Sol. Magnitude of A

$= \sqrt{2^2 + 3^2 + (-4)^2} = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$

Q. $\vec{A} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$. Find magnitude of $\vec{A}$.

Sol. $|\vec{A}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$

Q. $\vec{A} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$, $|\vec{A}| = ?$

Sol. $|\vec{A}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = 1$

UNIT VECTOR

है तो vector ही $\hat{A}$ vector whose magnitude is 1. It represent direx^n.

अंधे की लाठी

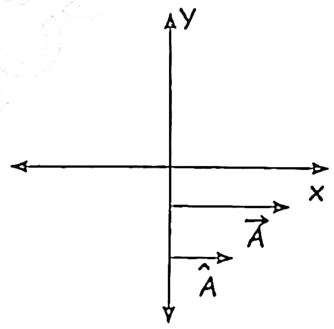
it is used to give direx^n and is unitless & dimensionless

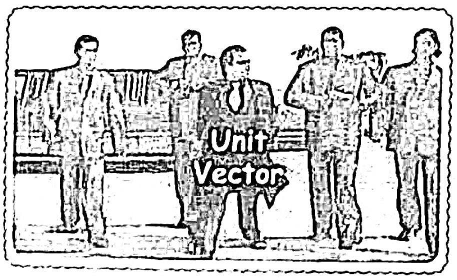
$\vec{A} = |\vec{A}|$ direx^n

$\vec{A} = |\vec{A}|\,\hat{A}$

└→ unit vector

$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$





$\hat{A} \rightarrow$ unit vector

└→ ये ऐसा vector है जिसकी Magnitude '1' और dirx^n $\vec{A}$ की तरफ है।

→ Unit vector along +x-axis = $\hat{i}$

→ Unit vector along +y-axis = $\hat{j}$

→ Unit vector along +z-axis = $\hat{k}$

Q. If $\vec{A}$ is unit vector find value of α where $A = 0.6\hat{i} + \alpha\hat{j}$

Sol. $\vec{A} = 0.6\hat{i} + a\hat{j}$

$\vec{A} = \sqrt{0.36 + \alpha^2}$

$1 = 0.36 + \alpha^2$

$0.64 = \alpha^2$

$\pm 0.8 = \alpha$

Q. If a vector is given by $\vec{A} = 3\hat{i} + 4\hat{j}$. Find unit vector along $\vec{A}$.

Sol. Magnitude of $\vec{A} = \sqrt{3^2 + 4^2} = 5$

$\vec{A} = |\vec{A}| \cdot \hat{A}$

- → dirx^n
- → unit vector
- → Magnitude

किसी भी vector को उसके magnitude से divide करदो तो उस vector की तरफ का unit vector आजाएगा।

Unit vector along $\vec{A}$ or $\hat{A}$

$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{3\hat{i} + 4\hat{j}}{5} = \frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}$

$\vec{A}$ and $\hat{A}$ are parallel vector.

Q. Find the unit vector along $\vec{A}$ where $\vec{A} = 2\hat{i} + 6\hat{j} - 3\hat{k}$

Sol. $\hat{A} = \frac{2\hat{i} + 6\hat{j} - 3\hat{k}}{\sqrt{49}} = \frac{2}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{3}{7}\hat{k}$

Q. A particle has momentum of magnitude 20 kg m/s. If momentum is in the dirx^n of $\vec{A}$. Find momentum in vector form, where $\vec{A} = \hat{i} + \hat{j}$

Sol. Momentum $= 20 \times \hat{A} = 20 \times \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) = \sqrt{2}\left(10\hat{i} + 10\hat{j}\right)$

Q. A bird is flying with speed 10 m/s in the dirx^n of a vector $\vec{A} = 3\hat{i} + 4\hat{j}$. Find velocity of bird.

Sol. $\vec{V}$ = Magnitude of dirx^n.

$\vec{V} = 10\hat{A} = 10\left[\frac{3\hat{i} + 4\hat{j}}{5}\right] = 2[3\hat{i} + 4\hat{j}] = 6\hat{i} + 8\hat{j}$

Q. Find force F in vector form if its magnitude is 21 N and dirx^n is

(a) parallel to $6\hat{i} - 3\hat{j} + 2\hat{k}$

(b) opposite to $6\hat{i} - 3\hat{j} + 2\hat{k}$

Sol. (a) [Parallel] $\vec{F}$

$$= 21\hat{A} = 21\left[\frac{6\hat{i} - 3\hat{j} + 2\hat{k}}{7}\right] = 18\hat{i} - 9\hat{j} + 6\hat{k}$$

(b) [Opposite] $\vec{F}$

$$= 21(-\hat{A}) = -21\hat{A} = -18\hat{i} + 9\hat{j} - 6\hat{k}$$

Q. A bird is flying with 10 m/s speed from point A to directly point $\vec{B}$. Find velocity of Bird.

A (3,4,6) ———— B (7,4,3)

Sol. $\overrightarrow{AB} = 4\hat{i} + 0\hat{j} - 3\hat{k}$

$$\vec{V} = 10 \times \hat{AB} = 10 \times \left(\frac{4\hat{i} - 3\hat{k}}{5}\right) = 8\hat{i} - 6\hat{k}$$

Q. $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$.

$\vec{B} = 2\hat{i} + 3\hat{j} - 2\hat{k}$. Find $\vec{A} + \vec{B}$

Sol. $\vec{A} + \vec{B} = 5\hat{i} + 7\hat{j} + 3\hat{k}$

Q. $\vec{A} = 3\hat{i} + 2\hat{j} + 5\hat{k}$.

$\vec{B} = 2\hat{i} + 3\hat{j} + 4\hat{k}$. Find following vectors.

Sol. $\vec{A} + \vec{B} = 5\hat{i} + 5\hat{j} + 9\hat{k}$

$\vec{A} - \vec{B} = \hat{i} - \hat{j} + \hat{k}$

$2\vec{A} + 3\vec{B} = 12\hat{i} + 13\hat{j} + 22\hat{k}$

$|\vec{A} + \vec{B}| = \sqrt{5^2 + 5^2 + 9^2}$

$|\vec{A} - \vec{B}| = \sqrt{3}$

$|2\vec{A} + 3\vec{B}| = \sqrt{12^2 + 13^2 + 22^2}$

$2\vec{A} - 3\vec{B} = 0\hat{i} + (4-9)\hat{j} + (10-12)\hat{k} = -5\hat{j} - 2\hat{k}$

Q. $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$.

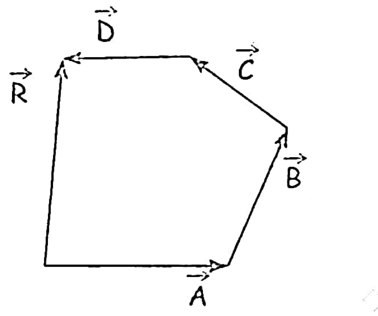
$\vec{B} = 4\hat{i} + \hat{j} + 4\hat{k}$.

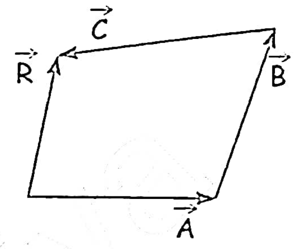
Find a vector $\vec{C}$ whose magnitude is 20 and dirx^n is opposite to $4\vec{A} + \vec{B}$.

Sol. $4\vec{A} + \vec{B} = 12\hat{i} + 5\hat{j} + 8\hat{k}$

$$\vec{C} = -20 \times \left[\frac{12\hat{i} + 5\hat{j} + 8\hat{k}}{\sqrt{12^2 + 5^2 + 8^2}}\right]$$

COMPONENT OF VECTOR





$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

Here, $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ are said to be four component of $\vec{R}$.

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

Here, $\vec{A}$, $\vec{B}$, $\vec{C}$ are three component of $\vec{R}$.

Q. A man move 5 m along east, then turn left and move 10 m along north, then turn right move 20 m east and then turn right to south & move 15 m. Find net displacement and distance travel

Sol. Distance = 5 + 10 + 20 + 15 = 50 m

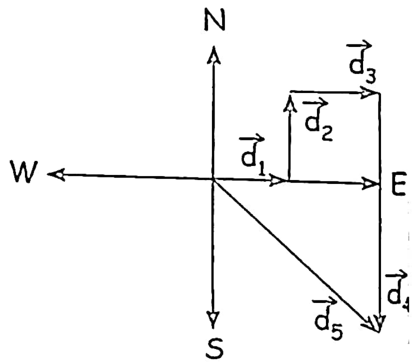
$\vec{d}_1 = 5\hat{i}$

$\vec{d}_2 = 10\hat{j}$

$\vec{d}_3 = 20\hat{i}$

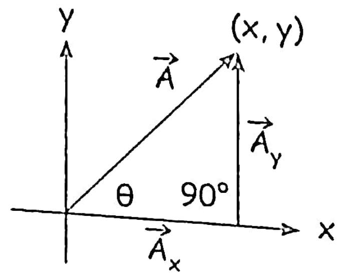
$\vec{d}_4 = -15\hat{j}$

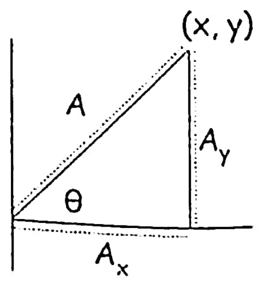
$d_{net} = 25\hat{i} - 5\hat{j}$



Magnitude = $\sqrt{25^2 + 5^2} = \sqrt{625 + 25} = \sqrt{650}$

RECTANGULAR COMPONENT/ CARTESIAN COMPONENT OF A VECTOR (2D)





$\vec{A} = x\hat{i} + y\hat{j}$,

$\vec{A} = \vec{A}_x + \vec{A}_y$,

*A_x & A_y are two rectangular component of $\vec{A}$.

$\vec{A} = A_x\hat{i} + A_y\hat{j},$

$\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j},$

$\cos\theta = \dfrac{A_x}{A} \Rightarrow A_x = A\cos\theta$

$\sin\theta = \dfrac{A_y}{A} \Rightarrow A_y = A\sin\theta$

$\vec{A}_x$ = Component of $\vec{A}$ along x-axis

$\vec{A}_y$ = Component of $\vec{A}$ along y-axis

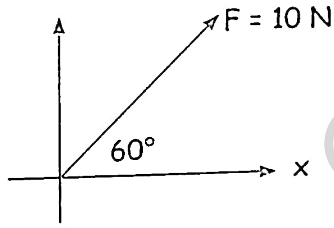
$\vec{A}_x = A\cos\theta\hat{i}$

$\vec{A}_y = A\sin\theta\hat{j}$

यह वहुत important है और कम से कम एक करोड़ वार use होगा so, अच्छे से समझ लेना कि कैसे एक vector के दो rectangular component ले रहे है या कैसे एक vector को तोड़ रहे है।

Q. Resolve the vector into $\hat{i}$ & $\hat{j}$

(a)

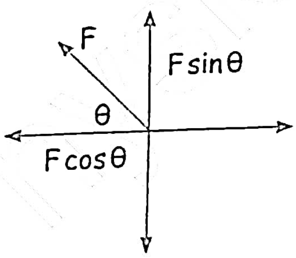


$\vec{F} = 10\cos 60\hat{i} + 10\sin 60\hat{j}$

$\vec{F} = 10 \times \dfrac{1}{2}\hat{i} + 10\,\dfrac{\sqrt{3}}{2}\,\hat{j}$

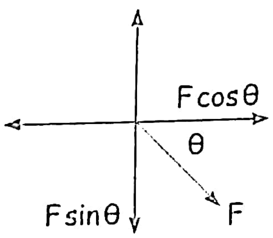
$= 5\hat{i} + 5\sqrt{3}\,\hat{j}$

(b)



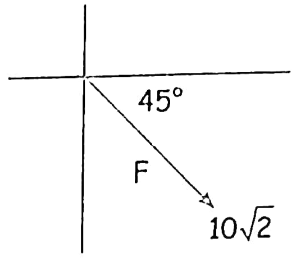
$\vec{F} = -F\cos\theta\hat{i} + F\sin\theta\hat{j}$

(c)



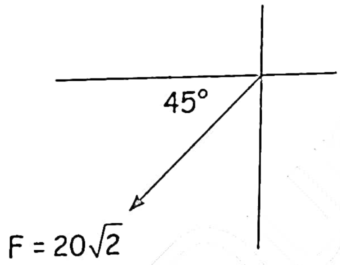
$\vec{F} = F\cos\theta\hat{i} - F\sin\theta\hat{j}$

(d)



$\vec{F} = 10\sqrt{2}\,\cos 45°\hat{i} - 10\sqrt{2}\,\sin 45°\hat{j} = 10\hat{i} - 10\hat{j}$

(e)

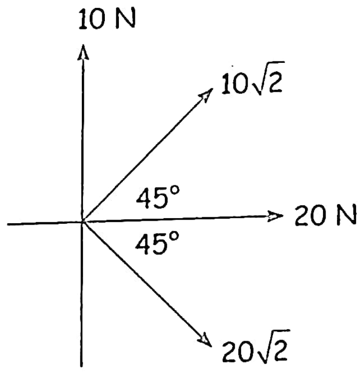


$\vec{F} = -20\hat{i} - 20\hat{j}$

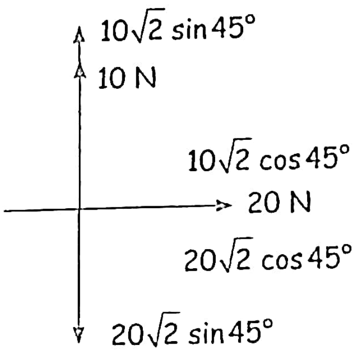
SKC

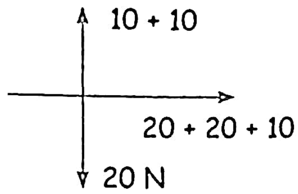
अब हम vector का सबसे ज्यादा important part पढ़ने जा रहे है जो electrostatic, mechanics मे बहुत use होगा यहाँ हमे बहुत सारे vector का net resultant निकालना होगा। कुछ नहीं करना बस सारे forces को x-y मे तोड़ लो और individually x और y मे collect करके answer लिख दो।

Q. Find net force.



Sol.

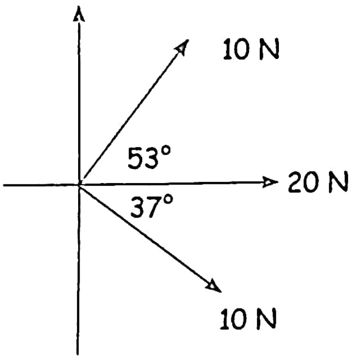


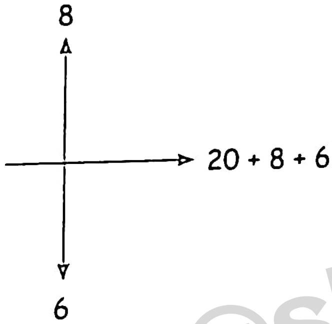


$F_{net} = 50\hat{i}$

Q. Find net force

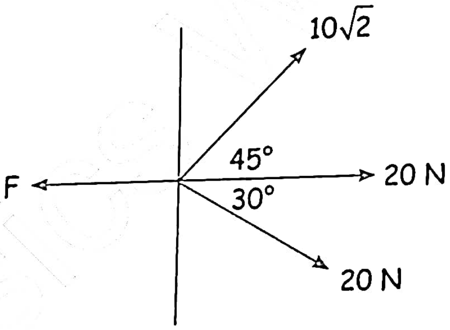
Sol.



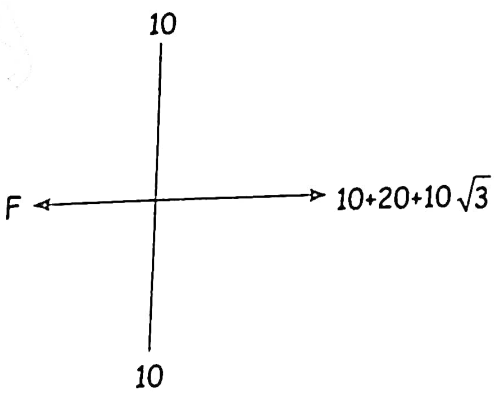


$F_{net} = 34\hat{i} + 2\hat{j}$

Q. Find the value of F so that the net force is zero

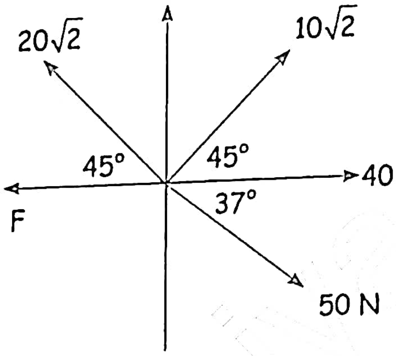


Sol.

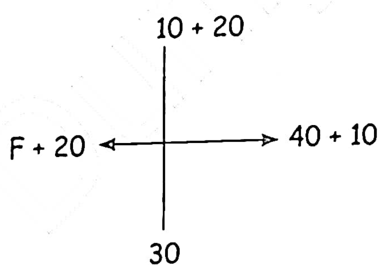


$30 + 10\sqrt{3} = F$

Q. Find value of F so that $\vec{F}_{net} = 0$.



Sol.



$F + 20 = 50$

$F = -30\hat{i}$

अब अगर मैं 50 vector भी दे दूँ तो सभी को resolve करके कर लोगे ना, मानता हुँ maths/calculation लंबी होगी but physics तो आसान है अगर ये चीज समझ गए हो तो मुझे insta पर confirmation दो saleem.nitt only if you have account.

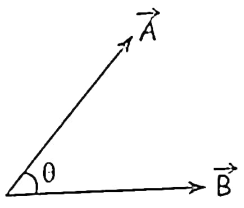
VECTOR PRODUCT

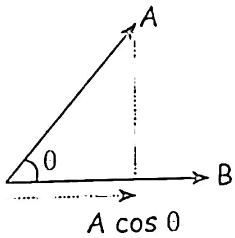
1. Dot Product
2. Cross Product

Dot Product

$\vec{A}\cdot\vec{B} = A\ B\ \cos\theta$

Here, A and B are the magnitude of $\vec{A}$ and $\vec{B}$ and θ is the angel between $\vec{A}$ and $\vec{B}$.





$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = A(B\cos\theta)$

$= A$ (magnitude of component $\vec{B}$ along $\vec{A}$)

★ $\vec{A}\cdot\vec{B} = (A\cos\theta)B$

= (magnitude of componentof $\vec{A}$ along $\vec{B}$)B

★ If $\vec{A}\perp\vec{B} \Rightarrow \vec{A}\cdot\vec{B}$ $A\cdot B\cos 90° = 0$

★ $\hat{i}\cdot\hat{j} = 1\times1\times\cos 90° = 0$

★ $\hat{i}\cdot\hat{k} = 1\times1\times\cos 90° = 0$

★ $\hat{j}\cdot\hat{k} = 1\times1\times\cos 90° = 0$

★ $\vec{A}\cdot\vec{B}$ = number ayega scalar.

★ $\hat{i}\cdot\hat{i} = 1\times1\times\cos 0° = 1$

★ $\hat{j}\cdot\hat{j} = 1\times1\times\cos 0° = 1$

★ $\hat{k}\cdot\hat{k} = 1\times1\times\cos 0° = 1$

★ $[\vec{A}\cdot\vec{A} = A^2]$

If $\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$

$\vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$

then $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

$\because \vec{A}\cdot\vec{B} = AB\cos\theta \Rightarrow \cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

★ Dot product of two vector is commutative

$\vec{A}.\vec{B} = \vec{B}.\vec{A}$

★ $\vec{A}.\vec{A} = A^2$

★ $(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B}) = A^2 + B^2 + \vec{A}.\vec{B} + \vec{B}.\vec{A}$

$= A^2 + B^2 + 2\vec{A}.\vec{B}$

Q. Find dot product of $\vec{A}$ and $\vec{B}$.

$\vec{A} = 2\hat{i} + 3\hat{j}$

$\vec{B} = 4\hat{i} + 5\hat{j}$

$\vec{A}\cdot\vec{B} = 8 + 15 = 23$

Q. $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{B} = a\hat{i} + 2\hat{j} + 5\hat{k}$

if $\vec{A}\perp\vec{B}$ find a

> VVVVVVVVVV
> **Important**
> अगर दो vector ⊥ है तो उनका Dot product zero hoga

Sol. $\vec{A}\cdot\vec{B} = 2a - 6 + 20 = 0 \Rightarrow a = -7$

Q. Find angle between $\vec{A}$ & $\vec{B}$ if $\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = 3\hat{i} + 4\hat{j}$

Sol. $\vec{A}\cdot\vec{B} = AB\cos\theta$

> $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

Magnitude of B = 5

Mgnitude of A = $\sqrt{2}$

$\therefore \cos\theta = \dfrac{7}{5\sqrt{2}}$

Q. $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ Find angle between $\vec{A}$ & $\vec{B}$

Sol. $\vec{A}\cdot\vec{B} = 6 + 12 + 30 = 48$, $A = 5\sqrt{2}$, $B = 7$

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$48 = 5\sqrt{2}\times 7\cos\theta$

$\dfrac{48}{35\sqrt{2}} = \cos\theta$

Q. Find angle between $\vec{A}$ & $\vec{B}$ if

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{B} = \hat{i} - \hat{j} + \hat{k}$

Sol. $\vec{A}\cdot\vec{B} = AB\cos\theta$

$1 = \sqrt{3}\times\sqrt{3}\cos\theta$

$\dfrac{1}{3} = \cos\theta$

$\theta = \cos^{-1}\left(\dfrac{1}{3}\right)$

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

IF-

$\dfrac{A_x}{B_x} = \dfrac{A_y}{B_y} = \dfrac{A_z}{B_z} = n$

- $n > 0 \Rightarrow \vec{A}$ is parallel to $\vec{B}$
- $n < 0 \Rightarrow \vec{A}$ is anti-parallel to $\vec{B}$

Q. $\vec{A} = 3\hat{i} - 4\hat{j} + 5\hat{k}$

$\vec{B} = -9\hat{i} + 12\hat{j} - 15\hat{k}$. Are these vectors parallel or anti parallel?

Sol. $\dfrac{3}{-9} = \dfrac{-4}{12} = \dfrac{-5}{15}$

$\dfrac{-1}{3} = \dfrac{-1}{3} = \dfrac{-1}{3}$ [Antiparallel]

SKC

$\vec{A}, \vec{B}$ ||^r है या anti-||^r है या ⊥ है या कुछ नहीं, ये पता करने के लिए

$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$ निकाललो $\Rightarrow$ If $\theta = 0° =$ parallel

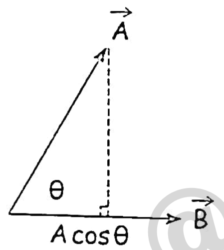
$\theta = 180° =$ Anti||^r If $\theta = 90° = \perp$

Q. $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{B} = 4\hat{i} + 6\hat{j} - 8\hat{k}$. Are they parallel?

Sol. $\frac{2}{4} = \frac{3}{6} \neq \frac{-4}{8}$

$\frac{1}{2} = \frac{1}{2} \neq \frac{-1}{2}$ [Nothing]



Component of $\vec{A}$ along $\vec{B} = A\cos\theta$ (magnitude)

$= A\cos\theta.\ \hat{B}$ (In vector form) $= \vec{A}_{||}$

$\because A\cos\theta = \frac{\vec{A}.\vec{B}}{B}$

Component of $\vec{A}$ along $\vec{B} = \frac{\vec{A}.\vec{B}}{B} = \vec{A}.\hat{B}$

Component of $\vec{B}$ along $\vec{A} = \frac{\vec{A}.\vec{B}}{A} = \vec{B}.\hat{A}$

Component of $\vec{A}$ along $\vec{B}$ in vector form $(\vec{A}_{||}) = \left(\frac{\vec{A}.\vec{B}}{B}\right)\hat{B}$

Component of $\vec{A}$ perpendicular to $\vec{B} = \vec{A} - \vec{A}_{||}$

$\vec{A}_{||} + \vec{A}_{\perp} = \vec{A}$

इसका use आगे mechanics में होगा इसलिए rough copy पर पाँच चार लिख-लिख कर practice करे (for 11th students)

Q. Find component of $\vec{A}$ along $\vec{B}$

$\vec{A} = 3\hat{i} + 4\hat{j}$ $\quad\quad \vec{B} = \hat{i} + \hat{j}$

Sol. Component of $\vec{A}$ along $\vec{B}$ (scalar) $= A\cos\theta$

$= \frac{\vec{A}\cdot\vec{B}}{B} = \frac{3+4}{\sqrt{2}} = \frac{7}{\sqrt{2}}$ (magnitude)

Component of $\vec{A}$ along $\vec{B}$ (Vector) $= \frac{\vec{A}\cdot\vec{B}}{B} \times \hat{B}$

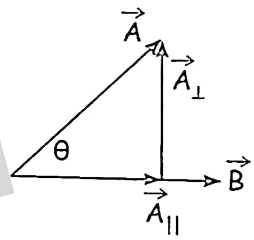
$= \frac{7}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \frac{7}{2}(\hat{i}+\hat{j})$

Q. If $\vec{A} = \hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$,

Find component of $\vec{A}$ perpendicular to $\vec{B}$.

Sol. Component of $\vec{A}$ parallel (along) to $\vec{B} = \frac{3}{5}(3\hat{i} + 4\hat{j})$

Component of $\vec{A}$ perpendicular to $\vec{B} = A\sin\theta$



$\vec{A}_{||} + \vec{A}_{\perp} = \vec{A} \Rightarrow \vec{A}_{\perp} = \vec{A} - \vec{A}_{||}$

$A_{\perp} = \hat{i} + 3\hat{j} - \left[\frac{3}{5}(3\hat{i} + 4\hat{j})\right] = \hat{i} + 3\hat{j} - \left(\frac{9\hat{i} + 12\hat{j}}{5}\right)$

$= \frac{-4\hat{i}}{5} + 3\hat{j}$

Q. $\vec{A} = 4\hat{i} - 2\hat{j}$, $\vec{B} = 3\hat{i} + 4\hat{j}$

Find component of $\vec{A}$ perpendicular to $\vec{B}$.

Sol. Component of $\vec{A}$ parallel to $\vec{B}$

$= \frac{\vec{A}\cdot\vec{B}}{B} = \frac{12-8}{5} = \frac{4}{5}$

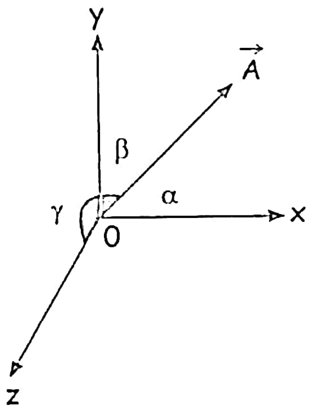
Vector form $= \frac{4}{5}\left(\frac{3\hat{i} + 4\hat{j}}{5}\right) = \frac{12\hat{i} + 16\hat{j}}{25}$

Component of $\vec{A}$ perpendicular to $\vec{B}$

$= A_{\perp} = \vec{A} - \vec{A}_{||}$

$= 4\hat{i} - 2\hat{j} - \left[\frac{12\hat{i} + 16\hat{j}}{25}\right] = \frac{88\hat{i} - 66\hat{j}}{25}$

DIRECTION COSINE



$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

If $\vec{A}$ makes angle α with + x-axis

If $\vec{A}$ makes angle β with + y-axis

If $\vec{A}$ makes angle γ with + z-axis

Component of $\vec{A}$ along x-axis = $A\cos\alpha = A_x$

Component of $\vec{B}$ along y-axis = $A\cos\beta = A_y$

Component of $\vec{C}$ along z-axis = $A\cos\gamma = A_z$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$\cos\alpha = \frac{A_x}{A}$ $\cos\beta = \frac{A_y}{A}$ $\cos\gamma = \frac{A_z}{A}$ here, $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosine of $\vec{A}$

Q. $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$. Find direction cosine

Sol. $A = |\vec{A}| = \sqrt{2^2 + 3^2 + 6^2} = 7$

$$\cos\alpha = \frac{A_x}{A} = \frac{2}{7} \quad \cos\beta = \frac{A_y}{A} = \frac{3}{7} \quad \cos\gamma = \frac{A_z}{A} = \frac{6}{7}$$

Q. $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$, Find direction cosine

Sol. $A = \sqrt{9 + 16 + 25} = 5\sqrt{2}$

(a) Direction cosine

$$\cos\alpha = \frac{3}{5\sqrt{2}} \quad \cos\beta = \frac{4}{5\sqrt{2}} \quad \cos\gamma = \frac{5}{5\sqrt{2}}$$

(b) Component of $\vec{A}$ along x-axis = $3\hat{i}$

(c) Component of $\vec{A}$ along y-axis = $4\hat{j}$

(d) Component of $\vec{A}$ along z-axis = $5\hat{k}$

(e) Component of $\vec{A}$ on x-y plane = $3\hat{i} + 4\hat{j}$

(f) Component of $\vec{A}$ on y-z plane = $4\hat{j} + 5\hat{k}$

Cross Product

भाई cross product की ज्यादा tension मत लेना बस ये सिख लेना कि कैसे cross product निकालते है rotation से पहले इसका वहुत ही कम use होगा, थोड़ा सा circular motion में use होगा।

If cross product of vector $\vec{A}$ and $\vec{B}$ is $\vec{C}$ means $\vec{A} \times \vec{B} = \vec{C}$ then $\vec{C}$ is a vector which is perpendicular to both $\vec{A}$ and $\vec{B}$ and magnitude of $\vec{C}$ is given by

$|\vec{C}| = AB\sin\theta$

☆ $\vec{C} = (AB\sin\theta)\hat{C}$

☆ $\vec{a} \times \vec{b} = \vec{c} = ab\sin\theta \cdot \hat{c}$

☆ $\vec{c} \cdot \vec{a} = 0$, $\vec{c} \cdot \vec{b} = 0$

☆ $\vec{c} \perp \vec{a}$, $\vec{c} \perp \vec{b}$

☆ $(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$, $(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$

☆ $\vec{a} \times \vec{a} = 0$

☆ $\hat{i} \times \hat{i} = 0$, $\hat{j} \times \hat{j} = 0$, $\hat{k} \times \hat{k} = 0$

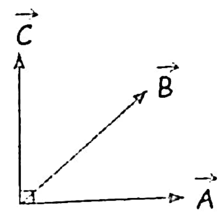
दो vector का cross product खुद मे एक vector होता है।

☆ In terms of components $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$= \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

अगर आपने determinant नही पढ़ा तो tension लेने की कोई जरूरत नही है जब पढ़लो तब ये method apply करके cross product निकाल लेना।

☆ $\vec{A} \times \vec{B} = \vec{C}$



SKC

★ $\hat{i} \times \hat{j} = \hat{k}$

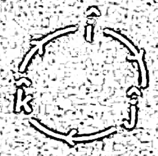
★ $\hat{j} \times \hat{k} = \hat{i}$

★ $\hat{k} \times \hat{i} = \hat{j}$

★ $\hat{j} \times \hat{i} = -\hat{k}$

★ $\hat{i} \times \hat{k} = -\hat{j}$

★ $\hat{k} \times \hat{j} = -\hat{i}$



अब याद करने का तरीका सुन, जबान से बोलो $\hat{i}\,\hat{j}\,\hat{k}\,\hat{i}\,\hat{j}\,\hat{k}\,\hat{i}\,\hat{j}\,\hat{k}\,\hat{i}\,\hat{j}$...... और ये sequence catch करो जैसे i और j के बाद k आया तो $\hat{i} \times \hat{j} = \hat{k}$ उसी sequence में j और k के बाद i आया है तो $\hat{j} \times \hat{k} = \hat{i}$ अगर sequence गड़बड़/उल्टा हुआ तो minus लगा दो जैसे $\hat{j} \times \hat{i} = -\hat{k}$ अब समझ गए ना।

Q. $\vec{a} = 3\hat{i} + 4\hat{j}$

$\vec{b} = 2\hat{i} + 5\hat{j}$ find $\vec{a} \times \vec{b}$

Sol. $\vec{a} \times \vec{b} = (3\hat{i} + 4\hat{j}) \times (2\hat{i} + 5\hat{j})$

$= 6\hat{i} \times \hat{i} + 15\hat{i} \times \hat{j} + 8\hat{j} \times \hat{i} + 20\hat{j} \times \hat{j}$

$= 0 + 15\hat{k} - 8\hat{k} + 0$

$= 7\hat{k} = \vec{C}$

$\vec{C} \perp \vec{A}$

$\vec{C} \perp \vec{B}$

Q. $\vec{a} = 4\hat{i} + 7\hat{j}$

$\vec{b} = 2\hat{i} + 3\hat{j}$ find $\vec{a} \times \vec{b}$

Sol. $\vec{a} \times \vec{b} = 8\hat{i} \times \hat{i} + 12\hat{i} \times \hat{j} + 14\hat{j} \times \hat{i} + 21\hat{j} \times \hat{j}$

$= 0 + 12\hat{k} - 14\hat{k} + 0 = -2\hat{k} = \vec{C}$

$\vec{C} \perp \vec{A}, \vec{C} \perp \vec{B}$

$\vec{b} \times \vec{a} = (2\hat{i} + 3\hat{j}) \times (4\hat{i} + 4\hat{j})$

$= 0 + 14\hat{k} - 12\hat{k} + 0 = 2\hat{k}$

$\therefore \vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

अब मै नीचे कुछ important question attach कर रहा हुँ इनको अच्छे से solve करो...... अबे करोगे ना......

कुछ PRACTICE करलो

Q. If the sum of two unit vectors is a unit vector, then find the magnitude of their difference.

Sol. Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum is $\bar{n}_s = \hat{n}_1 + \hat{n}_2$ or $n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta = 1 + 1 + 2\cos\theta$

Since it is given that n_s is also a unit vector, therefore $1 = 1 + 1 + 2\cos\theta$

or $\cos\theta = -\frac{1}{2}$ or $\theta = 120°$

Now the difference of the vectors is $\bar{n}_d = \hat{n}_1 - \hat{n}_2$

or $n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta = 1 + 1 - 2\cos(120°)$

$\therefore n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3 \Rightarrow n_d = \sqrt{3}$

Q. If vector $\bar{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\bar{B} = -4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are perpendicular to each other then value of λ will be?

Sol. If $\bar{A}$ and $\bar{B}$ are perpendicular to each other then $\bar{A}.\bar{B} = 0$

So, $2(-4) + 3(-6) + (-1)(-\lambda) = 0 \Rightarrow \lambda = +26$

Q. If $\bar{a}_1$ and $\bar{a}_2$ are two non collinear unit vectors and if $|\bar{a}_1 + \bar{a}_2| = \sqrt{3}$, then find the value of $(\bar{a}_1 - \bar{a}_2).(2\bar{a}_1 + \bar{a}_2)$.

Sol. $a_1 = a_2 = 1$ and $a_1^2 + a_2^2 + 2a_1a_2\cos\theta = 3$

Or $1 + 1 + 2\cos\theta = 3$ or $\cos\theta = \frac{1}{2}$

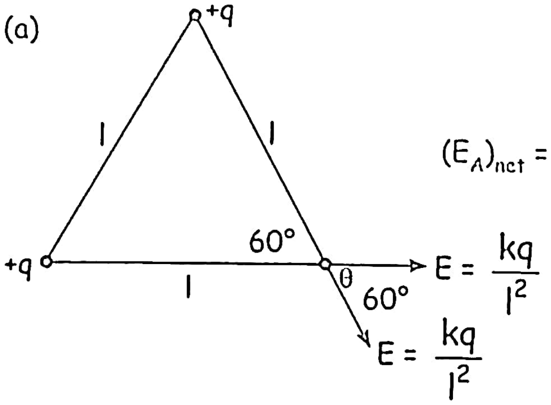
Now $(\bar{a}_1 - \bar{a}_2)\cdot(2\bar{a}_1 + \bar{a}_2) = 2a_1^2 - a_2^2 - a_1a_2\cos\theta$

$= 2 - 1 - \frac{1}{2} = \frac{1}{2}$

Q. In 12th class we will study in electrostatics that a point charge produce electric field which is a vector quantity and electric field due to a positive point charge at a distance r from charge has value $\frac{kq}{r^2}$ radially away from charge (along the line joining point to the charge) k is constant

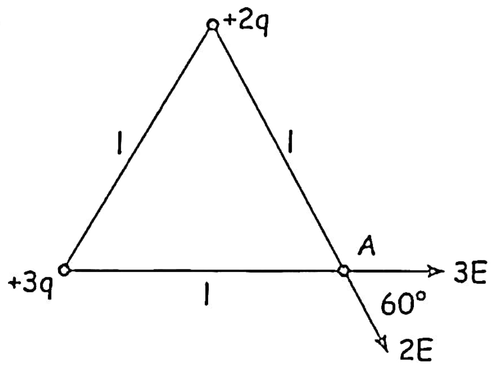
Using above data find the net electric field at point A in following case.

(a)



$(E_A)_{net} = E\sqrt{3}$

(b)

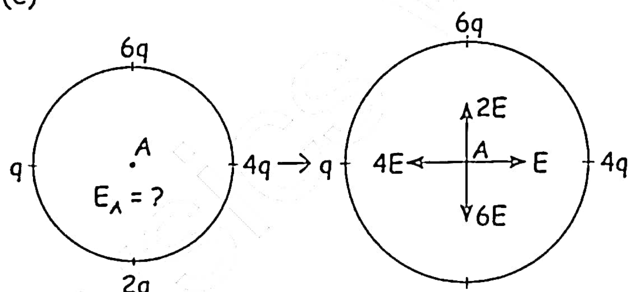


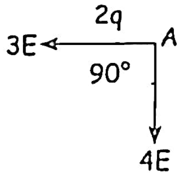
Let $E = \frac{kq}{l^2}$

$$E_{net} = \sqrt{(2E)^2 + (3E)^2 + 2 \times 2E \times 3E \times \cos 60^\circ}$$

$$= E\sqrt{19}$$

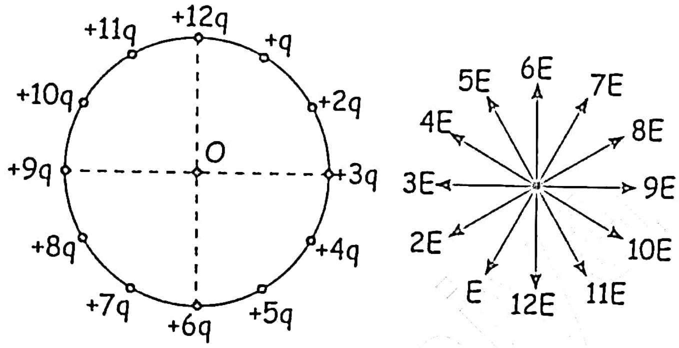
(c)

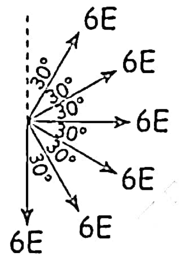




$(E_{net})_A = 5E = \frac{5kq}{r^2}$

(d) Find E_{net} at centre O.





अब component लेकर solve कर लोगे ना..... नही तो कोई बात नही 12th मे कर लेना

$$\vec{E}_{net} = (12E + 6E\sqrt{3})\hat{i} - 6E\hat{j}$$

IMPORTANT RESULT

If $|\vec{A}| = |\vec{B}| = x$

and angle between $\vec{A}$ and $\vec{B}$ is θ

then $|\vec{A} + \vec{B}| = 2x \cos \theta/2$

$|\vec{A} - \vec{B}| = 2x \cos \theta/2$

Q. Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P} + \vec{Q}$ is n times the magnitude of $\vec{P} - \vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is:

Ans. $\cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$

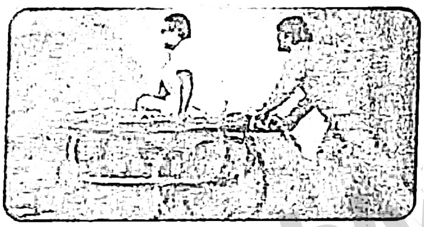
3 Kinematics

MOTION IN A STRAIGHT LINE

Motion: If a body changes its position with time body is called in motion

☆ Rest and motion are relative terms. They depend on observer.

जैसे की यहाँ बाबू भईया आपके respect में motion मे है but ठेले के respect मे rest पर है।



Distance: Actual path travelled by the body -

☆ Dependent upon path (we should know the path)

☆ It can't be decreasing

☆ It can't be negative

☆ It can be zero or positive

DISPLACEMENT

Change in position vector $\vec{r}$ or change in position

$\vec{d} = \Delta\vec{r} = \vec{r}_f - \vec{r}_i$

☆ Displacement = final position - Initial position

☆ It is the shortest distance between initial and final point.

☆ Vector term

☆ It can be positive, negative or zero.

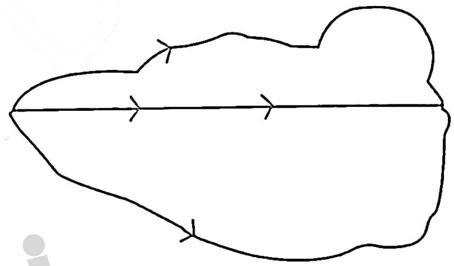
☆ It can be increasing or decreasing.

☆ Independent of path travelled.

Q. A particle move from point A to point B. Find it displacement in following case.

Sol. $\vec{d}$ = displacement $= 5\hat{i} + 2\hat{j} + 5\hat{j}$

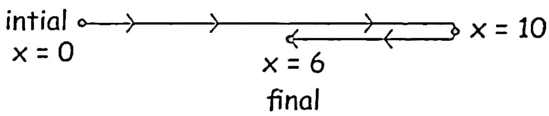
☆



Displacement - same

Distance - different (depend upon path)

☆

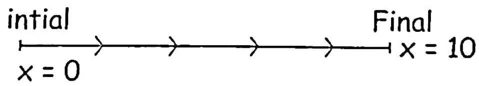


distance → 10 + 4 = 14

displacement → 6

(vector form) → $6\hat{i}$

☆



Distance = 10

Displacement = $10\hat{i}$

Here, Distance = |Displacement|

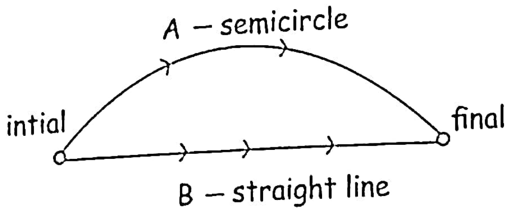
☆ |Displacement| = magnitude of displacement

☆ If a particle does not change direction then distance = |displacement|

☆ Distance ≥ |displacement|

☆ $\dfrac{\text{Distance}}{|\text{Displacement}|} \geq 1$

Q. A particle move from initial to final position in two different path A and B as shown in fig. Find distance and displacement for both.



Sol.

	A	B
Distance	πR	2R
Displacement	2R	2R

Q. A particle move 5m along east then 6m along north and 10m in upward direction. Find distance & displacement?

Sol. Distance 5m + 6m + 10m = 21

Displacement = $5\hat{i} + 6\hat{j} + 10\hat{k}$

Magnitude = $\sqrt{5^2 + 6^2 + 10^2}$

Q. A particle move

10m east = $10\hat{i}$

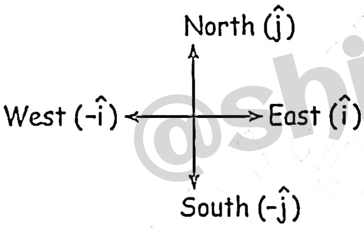
5m north = $5\hat{j}$

6m south = $-6\hat{j}$

8m west = $-8\hat{i}$

15m east = $15\hat{i}$

20m north = $20\hat{j}$

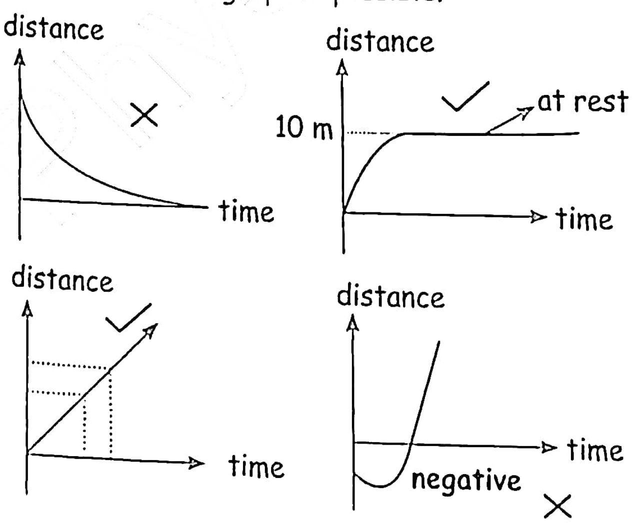


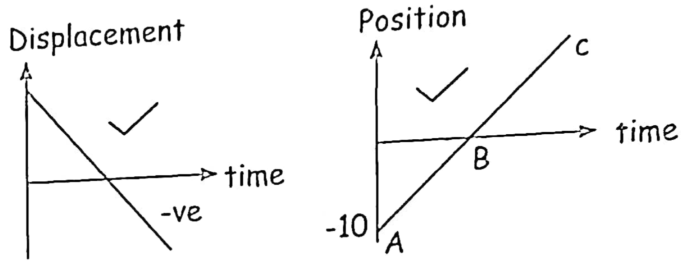
distance = 64 m

displacement = $(10 - 8 + 15)\hat{i} + (5 - 6 + 20)\hat{j}$

$= 17\hat{i} + 19\hat{j}$

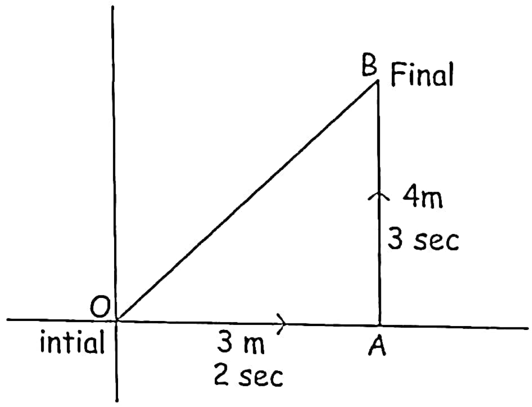
Q. Which of the graph is possible.





- displacement = change in position.
- Average velocity = $\dfrac{\text{total displacement}}{\text{total time}}$
- Instantaneous velocity = $\dfrac{d\bar{r}}{dt}$
- Average speed = $\dfrac{\text{total distance}}{\text{total time}}$
- Instantaneous speed = kisi inst. par speed
- |Instantaneous velocity| = Instantaneous speed

Q. A particle move from origin to point A and took 2 sec, then it move to point B and took 3 sec in following fig. Find displacement, average velocity and average speed.

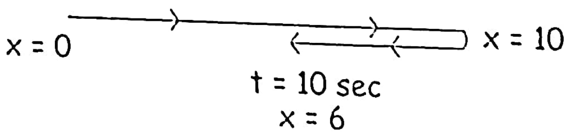


Sol. Displacement $\bar{d} = 3\hat{i} + 4\hat{j}$

Avrg. velocity = $\dfrac{3\hat{i} + 4\hat{j}}{5}$

Avrg. speed = $\dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{3+4}{2+3} = \dfrac{7}{5}$

Q.



Sol. Displacement = 6 (mag)

Average velocity = $\frac{6}{10}$

Distance = 10 + 4 = 14

Average speed = $\frac{14}{10}$

Q. x = 0, t = 0 ———— t = 5 sec, x = 10

Sol. Distance = 10

Average speed = $\frac{10}{5}$

Displacement = 10 (Magnitude)

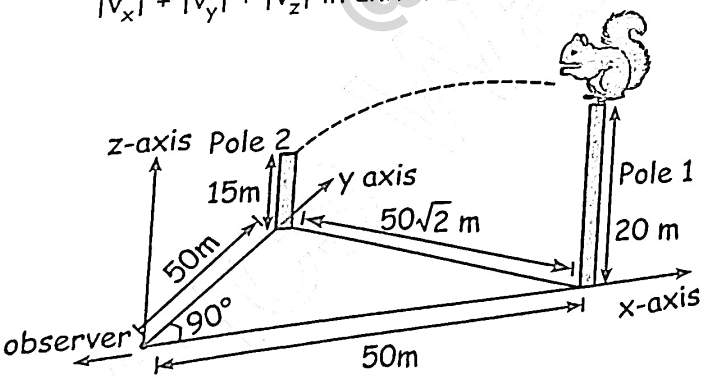
Average velocity = $\frac{10}{5}$ (Magnitude)

☆ Agar particle ne apni direction nahi badli

☆ distance = |displacement|

☆ Avrg speed = |Avrg velocity|

Q. A small squirrel jumps from pole 1 to pole 2 and took 3 sec. What is average velocity vector of squirrel? If average velocity vector is expressed as $v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, express your answer as sum of magnitudes of its components $|v_x| + |v_y| + |v_z|$ in unit m/s.



Sol. Intial coordinate is (50, 0, 20)

Final coordinate is (0, 50, 15)

Displacement = $-50\hat{i} + 50\hat{j} - 5\hat{k}$

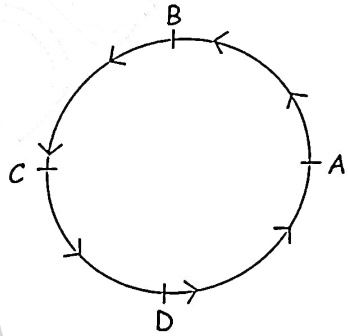
Average velocity = $\frac{-50\hat{i} + 50\hat{j} - 5\hat{k}}{3}$

$= -\frac{50}{3}\hat{i} + \frac{50}{3}\hat{j} - \frac{5}{3}\hat{k}$

$\frac{50}{3} + \frac{50}{3} + \frac{5}{3} = \frac{105}{3} = 35$ m/s

मुझे पता है कुछ google boys ने google पर search करके answer 105 निकाला होगा....... अबे सवाल ठीक से पढ़ लिया करो

Q. A particle is performing uniform circular motion with constant speed v_1 having time period T Anticlockwise. Find avg velocity and avg speed.



Sol.

	Avrg Speed	Arvg Velocity
$A \to B$	$\frac{2\pi R/4}{T/4} = \frac{2\pi R}{T}$	$\frac{R\sqrt{2}}{T/4}$
$A \to B \to C$	$\frac{\pi R}{T/2} = \frac{2\pi R}{T}$	$\frac{2R}{T/2}$
$A \to B \to C \to D$	$\frac{(3/4)2\pi R}{3T/4} = \frac{2\pi R}{T}$	$\frac{R\sqrt{2}}{3T/4}$
$A \to B \to C \to D \to A$	$\frac{2\pi R}{T}$	0

Q. A car is moving along x-axis, in 1st four hour it travel with speed 50 km/hr, in next 2 hours it move with 70 km/hr and in last part of journey it travel for 5 hour with 80 km/hr. Find avg speed.

Sol. Avrg speed = $\frac{\text{total distance}}{\text{Time}}$

$= \frac{d_1 + d_2 + d_3}{t_1 + t_2 + t_3} = \frac{50 \times 4 + 70 \times 2 + 80 \times 5}{4 + 2 + 5}$

$= \frac{200+140+400}{11}$

$$\boxed{\text{Averg speed} = \frac{740}{11}}$$

note: 1D motion (on x-axis)

Average velocity $= \frac{\vec{x}_f - \vec{x}_i}{\text{time}} = \frac{\Delta \vec{x}}{\Delta t}$

मैं average value को represent करने के कुछ symbol नीचे लिख रहा हुँ जो अक्सर use होते है और space बचाने के लिए हम भी use करेंगे

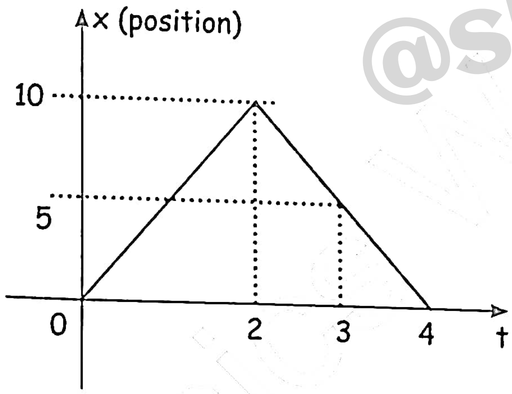
Average velocity = $<\vec{v}>$

Average speed = < speed > = $<|\vec{v}|>$

Average acc = $<\vec{a}>$

Average कद्दू = < कद्दू >

Q. A particle is moving on the x-axis as show its x-coordinate unit time. Find average velocity from t = 0 to t = 2 sec, from t = 0 to t = 3 sec.



Sol. ① $t = 0 \to t = 2$ sec

Average velocity $= \frac{x_f - x_i}{\text{time}} = \frac{10-0}{2} = 5$

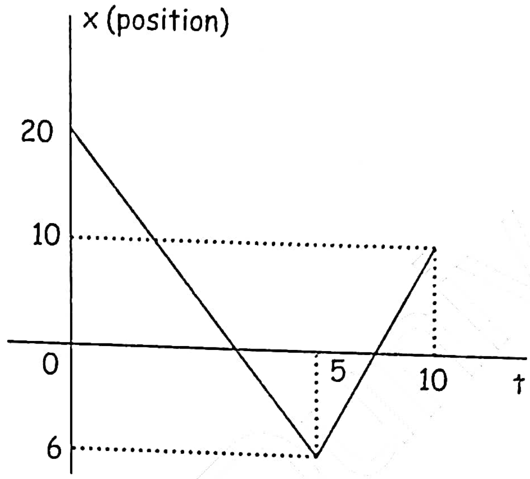
Average speed $= \frac{10}{2} = 5 \left| \frac{10_f - 0_i}{t} \right.$

② $t = 0 \to t = 3$

Average velocity $= \frac{x_f - x_i}{\text{time}} = \frac{5-0}{3}$

Average speed $= \frac{10+5}{3}$

Q. Find average velocity from t = 0 to t = 5s and t = 0 to t = 10s.



Sol. 1. $t = 0 \rightarrow t = 5$

Average velocity $= \frac{x_f - x_i}{\text{time}} = \frac{(-6)-(+20)}{5}$

Average speed $= \frac{20+6}{5}$

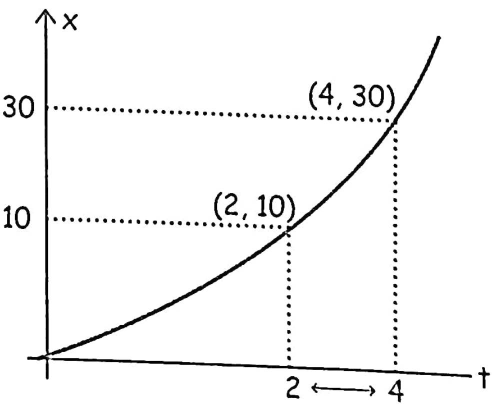
2. From $t = 0 \rightarrow t = 10$

Average velocity $= \frac{x_f - x_i}{10} = \frac{10-20}{10} = \frac{-10}{10}$

Average speed $= \frac{20+6+6+10}{10} = 4.2$ m/s

Q. Find average velocity from

$t = 2 \rightarrow t = 4$ sec



Sol. Average velocity (mag) $= \frac{x_f - x_i}{\Delta t}$

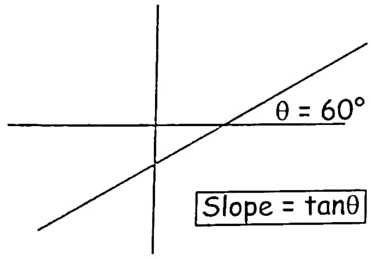
$= \frac{20}{2} = 10$

Slope of line $= \frac{x_2 - x_1}{t_2 - t_1} = \frac{30-10}{4-2}$

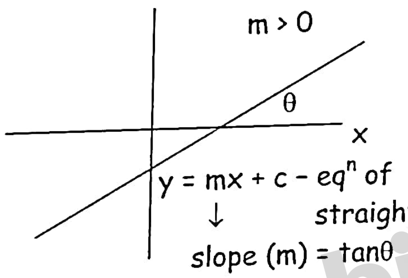
$= \frac{20}{2} = 10$

चलो अब थोड़ा सा काम का mathematics का revision हो जाए जिसकी हमे यहाँ बहुत जरूरत है।

SLOPE OF LINE

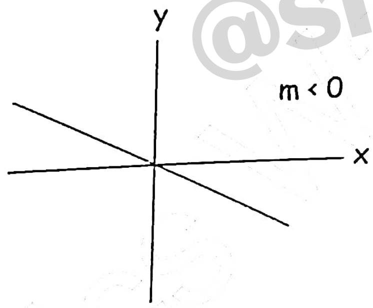


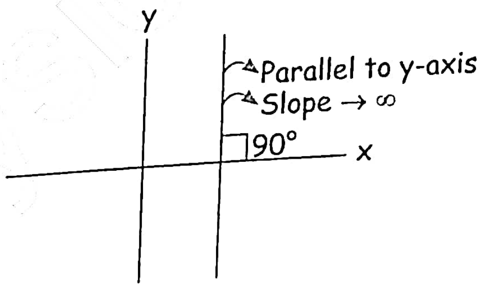
slope = $\sqrt{3} = \tan 60°$

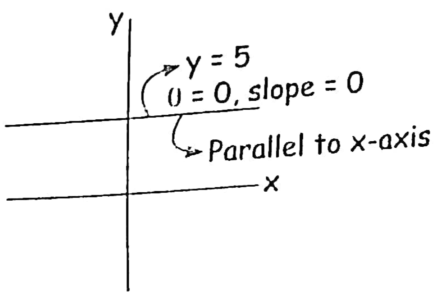


y = mx + c – eq^n of straight line
↓
slope (m) = tanθ

slope = tanθ





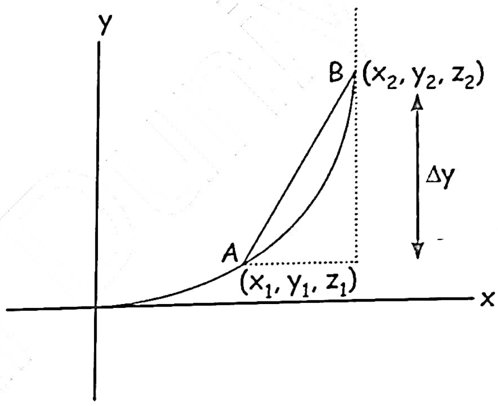


Slope of line join A to B

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \tan\theta$$

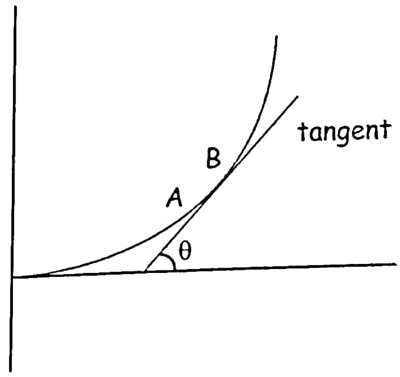
Δy = change in y

Δx = change in x



If Δy is very small $\Delta y = \to 0$ ($\Delta y = dy$)

If Δx is very small $\Delta x \to 0$ ($\Delta x = dx$)



$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

☆ $\frac{dy}{dx}$ मतलब differentiation of y wrt x

→ और rate of change of y wrt x

→ Slope of tangent at that point

→ Ratio of very-2 small change in y to very small change in x

→ 1st derivation of y wrt x

IMP FORMULA

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(\text{cosec}\,x) = -\text{cosec}\,x\cot x$
$\frac{d}{dx}\sin x = \cos x$	$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$
$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}e^x = e^x$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\ln x) = \frac{1}{x}$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	

Q. $y = x^3$

Sol. $\frac{dy}{dx} = 3x^{3-1} = 3x^2$

Q. $y = t^3$

Sol. $\frac{dy}{dt} = 3t^2$

Q. $y = 2x^5$

Sol. $\frac{dy}{dx} = 2 \times 5x^4 = 10x^4$

Q. $y = \frac{1}{x^2} = x^{-2}$

Sol. $\frac{dy}{dx} = -2x^{-2-1} = \frac{-2}{x^3}$

Q. $y = x^2 + x^5$

Sol. $\frac{dy}{dx} = 2x + 5x^4$

Q. $y = x^9 + \frac{1}{x^5}$

Sol. $\frac{dy}{dx} = 9x^8 - \frac{5}{x^6}$

$\frac{dy}{dx} = 9x^8 - \frac{5}{x^6}$

differentiation of y wrt x

Q. $y = x$

Sol. $\frac{dy}{dx} = 1$

Q. $y = 5$ constant

Sol. $\frac{dy}{dx} = 0$

Q. $y = x^3 + \sin x$

Sol. $\frac{dy}{dx} = 3x^2 + \cos x$

Q. $y = x^7 + \tan x + 10$

Sol. $\frac{dy}{dx} = 7x^6 + \sec^2 x + 0$

Q. $y = 3x^3 + \sin x + \tan x$

Sol. $\frac{dy}{dx} = 9x^2 + \cos x + \sec^2 x$

Q. $y = 2x^2 + \cos x + 5$

Sol. $\frac{dy}{dx} = 4x - \sin x + 0$

Q. $y = 3x^2 + \cos x + e^x - \sin x + 10$

Sol. $\frac{dx}{dy} = 6x - \sin x + e^x - \cos x + 0$

Q. If $y = x^2 + 4x^{-1/2} - 3x^{-2}$ find $\frac{dy}{dx}$

Sol. $\frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2})$

$= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$

$= \frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2})$

$= 2x - 2x^{-3/2} + 6x^{-3}$

Q. If $3y = 4x^2 - 5$ find $\frac{dy}{dx}$

Sol. $y = \frac{4}{3}x^2 - \frac{5}{3} \Rightarrow \frac{dy}{dx} = \frac{8x}{3}$

Q. $\sqrt{y} = x - 1$, find $\frac{dy}{dx}$

Sol. $y = x^2 - 2x + 1 \Rightarrow \frac{dy}{dx} = 2x - 2$

$\frac{dy}{dx}$ को shortcut मे y' भी लिखते है इसका मतलब है y का एक बार differentiation, उसी प्रकार अगर y का दो बार differentiation करेंगे तो उसे y'' या $\frac{d^2y}{dx^2}$ लिखते है जैसे

$y = x^3$

$y' = \frac{dy}{dx} = 3x^2$

$y'' = \frac{d^2y}{dx^2} = 6x$

Multiply

$y = u \cdot v$

$y' = uv' + vu'$

Q. $y = x^3 . \sin x$

Sol. $\frac{dy}{dx} = x^3\left(\frac{d}{dx}\sin x\right) + \sin x\left(\frac{d}{dx}x^3\right)$

Q. $y = x^5 \tan x$

Sol. $\frac{dy}{dx} = x^5.\sec^2 x + [\tan x](5x^4)$

Q. $y = x^3 . e^x$

Sol. $\frac{dy}{dx} = x^3e^x + e^x.3x^2$

Q. $y = e^x . \sin x$

Sol. $\frac{dx}{dy} = e^x.\cos x + \sin x.e^x$

Q. $y = x^3 e^x$

Sol. $\frac{dy}{dx} = x^3e^x + e^x.3x^2$

Q. $y = x^4 \ln x$

Sol. $\frac{dy}{dx} = x^4.\frac{1}{x} + (\ln x)(4x^3)$

CHAIN RULE

Q. $y = \sin(x^2 + x^7)$

Sol. $\frac{dx}{dy} = \cos(x^2 + x^7) \times (2x + 7x^6)$

ऊपर वाले chain rule के question मे यह pattern पहचानने की कोशिश करो कि पहले बाहर वाला function का differentiation करना है फिर multiply करते हुए अंदर वाले function का differentiation करते जाना है जैसे

$y = \sin$ (कददू)

$\frac{dy}{dx} = \cos$ (कददू) $\times \frac{d}{dx}$ (कददू)

Q. $y = \ln x^5$

Sol. $\frac{dy}{dx} = \frac{1}{x^5} \times 5x^4$

Q. $y = \ln x^4$

Sol. $\frac{dy}{dx} = \frac{1}{x^4} \times 4x^3$

Q. $y = \ln(\sin x)$

Sol. $\frac{dy}{dx} = \frac{1}{\sin x} \times \cos x$

Q. $y = \tan^2 x$

Sol. $y = (2 \tan x).\sec^2 x$

Q. $y = \sin^3 x = (\sin x)^3$

Sol. $\frac{dy}{dx} = 3(\sin x)^2.\cos x$

Q. $y = \sin^4 x = (\sin x)^4$

Sol. $\frac{dy}{dx} = 4(\sin x)^3.\cos x$

Q. $y = \sin(x^2 + x^3)$

Sol. $\frac{dy}{dx} = \cos(x^2 + x^3)(2x + 3x^2)$

Q. $y = \ln(\sin x^3)$

Sol. $\frac{dy}{dx} = \frac{1}{\sin x^3} \times \cos(x^3) \times [3x^2]$

Q. $y = \sin(e^x)$

Sol. $\frac{dy}{dx} = \cos(e^x) . e^x$

☆ $\frac{d}{dx}\sin(ax + b) = a\cos(ax + b)$

☆ $\frac{d}{dx}\sin(2x + 3) = \cos(2x + 3)[2 + 0]$

☆ $\frac{d}{dx}e^{2x+5} = e^{2x+5}(2 + 0)$

Q. $y = x^3$ find $\frac{dy}{dx}$ at $x = 2$

Sol. $y = x^3$

$\frac{dy}{dx} = 3x^2$

at $x = 2$, $\frac{dy}{dx} = 12$

Q. $x = t^2 - 4t + 20$ find value of $\frac{dx}{dt}, \frac{d^2x}{dt}$ at $t = 2$ sec

Sol. $\frac{dx}{dt} = 2t - 4 + 0 \Rightarrow$ at $t = 2$ sec, $\frac{dx}{dt} = 0$

$\frac{d^2x}{dt^2} = 2 \Rightarrow$ at $t = 2$ sec, $\frac{d^2x}{dt^2} = 2$

Chain Rule Power वाले questions

Q. $y = \sin^2 x$

$\frac{dy}{dx} = 2\sin x \frac{d}{dx}(\sin x) = 2\sin x \cos x$

Q. $y = \sin^2(3x + 4)$

$\frac{dy}{dx} = 2\sin(3x + 4)\cos(3x + 4) \times 3$

Q. $y = [\ln(3x^2)]^3$

$$\frac{dy}{dx} = 3\ln(3x^2) \times \frac{1}{3x^2} \times 6x$$

Q. $y = \left(x^3 + 4\right)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(x^2 + 4)^{-\frac{1}{2}} \times 2x$$

Very Important

1. $y = \pi x^3$

$$\frac{dy}{dx} = \pi \times 3x^2$$

2. $y = x^5$

$$\frac{dy}{dt} = 5x^4 \frac{dx}{dt}$$

3. $y = \sin x$

$$\frac{dy}{dt} = \cos x \frac{dx}{dt}$$

4. $A = \pi r^2$

$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$$

5. $y = 4/3\ \pi x^3$

$$\frac{dy}{dx} = \frac{4}{3}\pi 3x^2$$

Q. Radius of a circle changes wrt time with the rate of + 5 m/sec, find rate of change of area wrt time when radius is 10 m

Sol. $A = \pi r^2,\ \frac{dr}{dt} = 5$

$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt} \Rightarrow \frac{dA}{dt} = \pi \times 2 \times 10 \times 5$$

$\frac{dy}{dx} \rightarrow$ Rate of change of y wrt x

$\frac{dA}{dt} \rightarrow$ Rate of change of Area with time

$\frac{dr}{dt} \rightarrow$ Rate of change of radius wrt time

Q. If radius of a sphere is increasing at the rate of 10 m/sec at what rate volume of sphere will change, when radius is 3 m

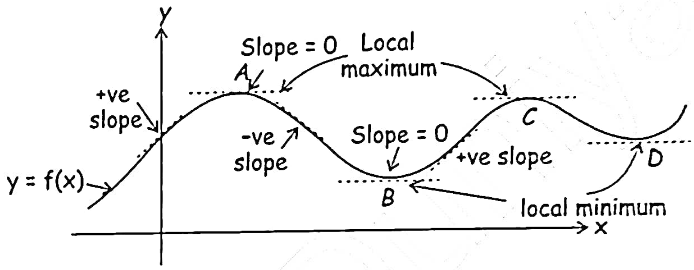
Sol. $V = 4/3\pi r^3 - \frac{dv}{dt} = \frac{4\pi}{3} 3r^2 \frac{dr}{dt}$

$$\frac{dv}{dt} = \frac{4}{3}\pi 3(3)^2 \times 10 = 360\pi$$

MAXIMA AND MINIMA

Finding maxima and minima of a function using derivatives:-

A maximum is a high point and minimum is low point of a function (see figure)



In a smoothly changing function a maximum or a minimum is always where function flattens out or where slope of tangent line is zero. We know slope = $\frac{dy}{dx}$. So a function reaches its maximum or minimum value when $\frac{dy}{dx} = 0$.

In the neighbourhood of maximum (point A), slope changes from positive to zero at point A and then becomes negative as x increases which means

$$\frac{d}{dx}(\text{slope}) < 0 \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} < 0$$

In the neigbourhood of minimum (point B), slope changes from negative to zero and then becomes positive as x increases which means

$$\frac{d}{dx}(\text{slope}) > 0 \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} > 0$$

SECOND DERIVATIVE TEST

When a function's slope $\left(\frac{dy}{dx}\right) = 0$ at a point and its second derivative at that point is

(*i*) less than zero, it is a local maximum.

(*ii*) greater than zero, it is a local minimum.

☆ Jab भी कभी किसी function की max या min value chahiye होगी हम उसे diff करके Zero कर देगे.

☆ For point of maxima $\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} < 0$

☆ For point of minima $\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} > 0$

Q. $y = x^2 - 4x + 20$

Find y_{min} = ?

Sol. $\frac{dy}{dx} = 2x - 4 = 0 \Rightarrow x = 2$

$\frac{d^2y}{dx^2} = 2 - 0 = 2 > 0$

$\frac{d^2y}{dx^2} > 0$ hence at $x = 2$, y is min

So, $y_{min} = 2^2 - 4 \times 2 + 20 = 16$

Q. $y = \sqrt{3}\sin\theta + \cos\theta$ Find y max

Sol. $\frac{dy}{d\theta} = \sqrt{3}\cos\theta - \sin\theta = 0$

$\sqrt{3}\cos\theta = \sin\theta$

$\theta^\circ = 60^\circ$

$\frac{d^2y}{d\theta^2} = -\sqrt{3}\sin\theta - \cos\theta$

at $\theta = 60^\circ$

$\frac{d^2y}{d\theta^2} = \frac{-3}{2} - \frac{1}{2} < 0$

So y is max.

$y_{max} = \sqrt{3}\sin 60^\circ + \cos 60^\circ$

$\sqrt{3}(\sqrt{3}/2) + 1/2 = 2$

$y = a\sin\theta + b\cos\theta$

$y_{max} = \sqrt{a^2 + b^2}$

$y_{max} = \sqrt{3+1} = 2$

Q. What is the minimum value of y for the curve $y = -8x^2 + x^4$.

Sol. $y = -8x^2 + x^4$

$\frac{dy}{dx} = -16x + 4x^3 = -x(16 - 4x^2)$

The function will have a maximum or minimum value when $\frac{dy}{dx} = 0$

$\Rightarrow x(16 - 4x^2) = 0$

$\Rightarrow x = 0$ or $x = \pm 2$

Now

$\frac{d^2y}{dx^2} = -16 + 12x^2$

At $x = 0$, $\frac{d^2y}{dx^2} = -16$ (maximum)

At $x = \pm 2$, $\frac{d^2y}{dx^2} = -16 + 48 = +32$ (minimum)

So function has minimum value at $x = \pm 2$

$y_{min} = -8 \times 4 + 16 = -16$

Q. A ball is thrown vertically upward in the air. It height y at any time t is given by $y = 10t - 5t$ where y is in meters and t is in seconds. Wha is the maximum height attained by the ball?

Sol. $y = 10t - 5t^2$

$\frac{dy}{dt} = 10 - 10t = 0$

$\Rightarrow t = 1$ sec

$\frac{d^2y}{dt^2} = -10$ (maximum)

ball attains maximum height at t = 1s

$y_{max} = 10 \times 1 - 5 \times 1^2 = 5m.$

☆ Inst. velocity = $\bar{v} = \frac{d\bar{r}}{dt}$

☆ If particle moving on x-axis

Inst. velocity = $\overline{v_x} = \frac{dx}{dt}\hat{i}$

सीधी बात x को time के respect मे differentiate करेंगे तो velocity आएगी।

Q. A particle is moving on x-axis such that it x-coordinate w.r.t time change as $x = t^2 - 6t + 1$

(a) Find velocity at t = 0, t = 3, t = 6 sec, = Ins velocity

Sol. $v = \frac{dx}{dt} = 2t - 6$

(now put the value of t)

$t = 0, v = -6$

$t = 3, v = 0$

$t = 6, v = +6$

☆ किसी time पर v मतलब instantaneous velocity

☆ किसी time पर velocity का magnitude ही उस time पर instantaneous speed है

(b) Find avrg velocity between $t = 0 \rightarrow t = 3$

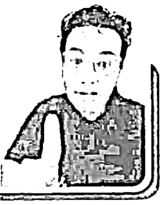
Sol. Averge velocity = $\frac{x_f - x_i}{t_f - t_i}$

$t = 0, x_i = 0 - 0 + 10 = 10$

$t = 3, x_f = 3^2 - 6 \times 3 + 10 = 1$

Average velocity = $\frac{1-10}{3} = -3\hat{i}$

Definition carefully याद रखनी हैं, avg velocity के लिए x को differentiate मत कर देना।
Avg velocity निकालने के लिए बस ये देखो x_i और x_f क्या है।



Q. $x = t^2 - 2t + 10$. Find avg velocity and avg speed from $t = 0$ to $t = 3$ sec

Sol. At $t = 0, x = 10$

At $t = 3, x = 9 - 6 + 10 = 13$

Avg velocity $= \frac{13-10}{3} = 1$

Avg speed = 1 (क्यों भाई तूने यही किया ना)

अबे गलत है Avg speed हमेशा avg velocity के बराबर नही होती definition याद करो क्या थी।

SKC

Avg speed के लिए तुम्हें रास्ता पता होना जरूरी है। So, सबसे पहले initial और final position निकालो और velocity zero करके turning point निकालो, फिर रास्ता बना कर देखो particle कितना चला है।

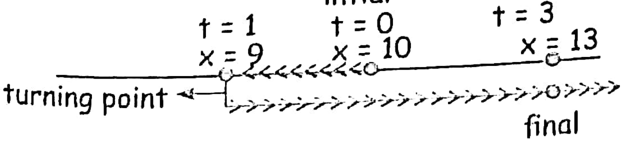
At $t = 0, x = 10$

At $t = 3, x = 13$ now find turning point

$v = 2t - 2 = 0$

$t = 1$ (देखो इस time पर particle मे अपनी direction change की है पता लगाओ $t = 1$ पर particle कहाँ है)

$t = 1, x = 1^2 - 2 \times 1 + 10 = 9$



Average speed $= \frac{1+4}{3} = \frac{5}{3}$

Better understanding के लिए नीचे वाले ques और solve करो

Q. $x = 2t^2 - 4t + 5$ from $t = 0 \to t = 2$ Find Average speed

Sol. $t = 0, x_i = 5$

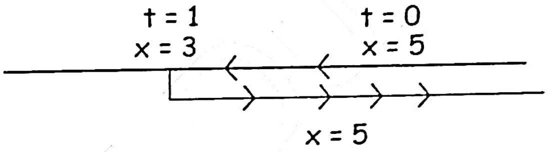
$t = 2, x_f = 5$

$v = 4t - 4 = 0$ (for turning point)

$4t = 4$

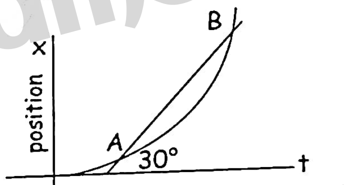
$\boxed{t = 1}$

At $t = 1$, $x = 2 - 4 + 5 = 3$

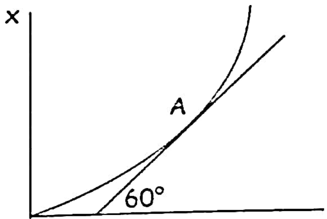


Average speed $= \frac{2+2}{4} = 1$

GRAPHS

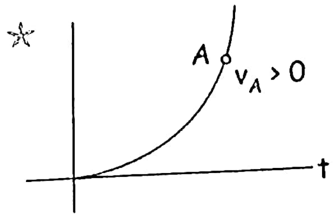


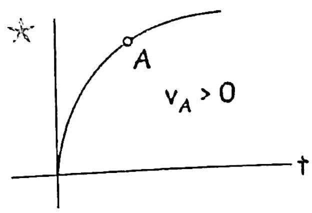
Average velocity = slope of line AB between A & B = tan 30°

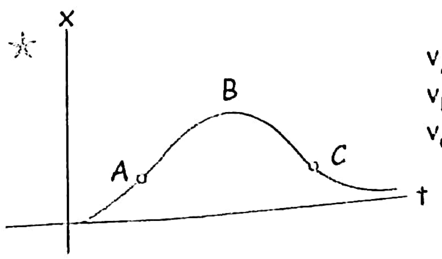


Instantaneous velocity at A = slope of tangent at A = tan 60°

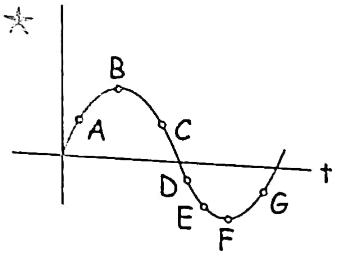
x-t graph मे किसी point पर tangent का slope उस point पर velocity देगा...... इसे मत भूलना







$v_A > 0$
$v_B = 0$ - turning point
$v_C < 0$



$v_A > 0$
$v_B = 0$
$v_C < 0$
$v_D < 0$
$v_E < 0$
$v_F = 0$
$v_G > 0$

ACCELERATION

☆ Inst acceln → Rate of change of velocity $\bar{a} = \frac{d\bar{v}}{dt}$

If $\vec{V} \rightarrow$ constant

$a = 0$

Agar velocity badli to acceleration hai.

☆ Average acceleration → $\frac{\text{change in velocity}}{\text{time}}$

$$= \frac{\Delta \bar{v}}{\Delta t} = \frac{\bar{v}_F - \bar{v}_i}{t_f - t_i}$$

Q. A particle is moving on x-axis such that it's velocity is given at $V = t^2 + 4t + 10$.
Find acc at t = 3 sec

avg acc from $t = 0 \rightarrow t = 3$

Sol. $\bar{a} = \frac{dv}{dt} = 2t + 4$

At t = 3 $a = 2 \times 3 + 4 = 10$

$$\text{Average acc} = \frac{V_f - V_i}{\text{time}}$$

$t = 3$, $V_f = 9 + 12 + 10 = 31$

$t = 0$, $v_i = 10$

$$\text{Average acc} = \frac{31 - 10}{3 - 0} = 7$$

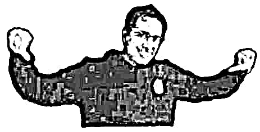
Q. $x = t^3 + 2t^2 + 5t$ Find velocity & acc at t = 2sec

Sol. $v = \frac{dx}{dt} = 3t^2 + 4t + 5$

$$a = \frac{dv}{at} = 6t + 4$$

$t = 2$, $a = 12 + 4 = 16$

$t = 2$, $v = 12 + 8 + 5 = 25$

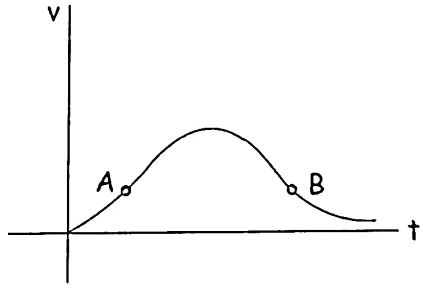


काम का डब्बा

अब तक का final result ये है की

+ x-t graph मे किसी point पर tangent का slope उस point पर velocity देगा।
+ v-t graph मे किसी point पर tangent का slope उस point पर acc देगा।
+ $\vec{v}, \vec{a}$ ⇒ same sign ⇒ speed up
+ $\vec{v}, \vec{a}$ ⇒ opposite sign ⇒ speed down.
+ Average velocity = $\frac{\text{total displacement}}{\text{total time}}$
+ Average speed = $\frac{\text{total distance}}{\text{total time}}$
+ Average acc = $\frac{\text{change in velocity}}{\text{total time}} = \frac{\bar{v}_f - \bar{v}_i}{t}$
+ (a - t) graph ka slope कददू देगा।

Q. Check if particle is slowing down or speed up points A & B?

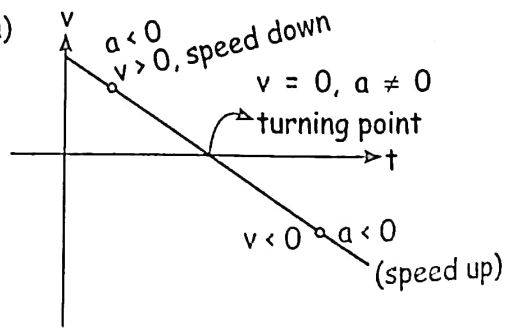


Sol. $v_A > 0, a_A > 0 \Rightarrow$ speed up

$v_B > 0,\ a_B < 0 \Rightarrow$ speed down

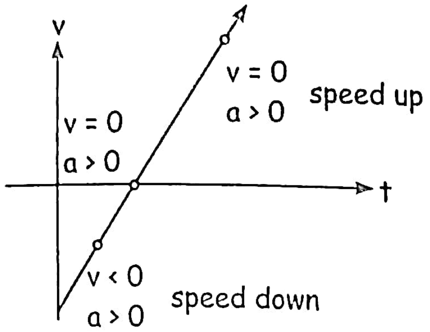
Q. Where is particle slowing down?

(a)

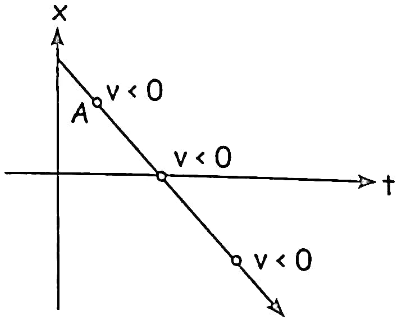


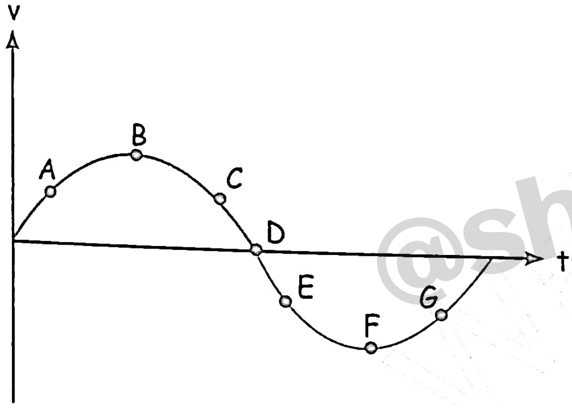
At turning point ⇒ v = 0 & v changes its sign this point.

(b)



Q. Is velocity +ve or –ve in the x-t graph?

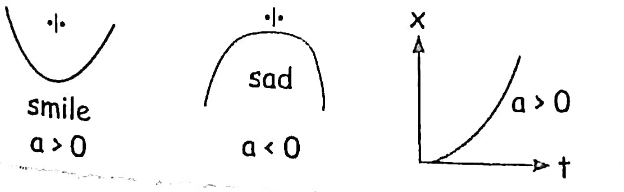




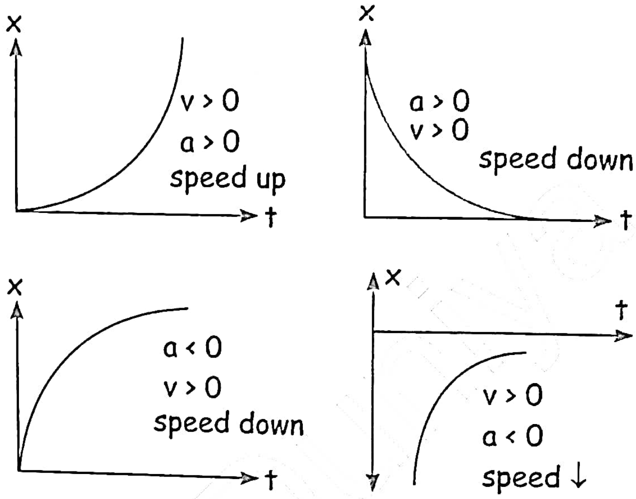
A ⇒ v > 0, a > 0, speed up
B ⇒ v > 0, a = 0
C ⇒ v > 0, a < 0, speed down
D ⇒ v = 0, a < 0
E ⇒ v < 0, a < 0, speed up
F ⇒ v < 0, a = 0
G ⇒ v < 0, a > 0, speed down

1. पहले ये देखो given क्या है पूछा क्या है ?
2. x - y axis mai kya given hai.

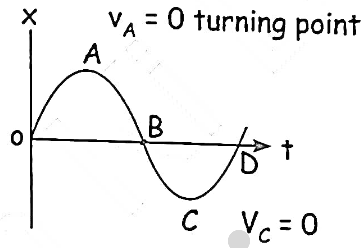
x - t graph se direct acc ka sign pata karna ho.



Only for x - t graph



Q.



	v	a	Speed ↓↑
0 → A	+	-	Speed ↓
A → B	-	-	Speed up
B → C	-	+	Speed ↓
C → D	+	+	Speed ↑

INTEGRATION (MATHS)

Reverse of differentiation

$\frac{d}{dx}f(x) \Rightarrow$ differentiation of f(x) wrt x

$\frac{dy}{dx} \Rightarrow$ differentiation of y wrt x

$\int y dx \Rightarrow$ integration of y wrt x

$\frac{d}{dx}f(x) = f'(x) \Rightarrow \int f'(x) = f(x) + c$, where c is constant of integration.

$$f(x) \xrightarrow{\text{differentiation}} g(x) \xrightarrow{\text{integration}} f(x) + c$$

Indefinite Integration

$\int x^n\,dx$ = Integration of x^n wrt x

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Physics में सबसे ज्यादा यही formula use होगा

Examples:

1. $\int x^4 dx = \frac{x^{4+1}}{4+1} + C = \frac{x^5}{5} + C$

2. $\int x^7 dx = \frac{x^{7+1}}{7+1} + C = \frac{x^8}{8} + C$

3. $\int (x^2 + x^3) dx = \frac{x^3}{3} + \frac{x^4}{4} + C$

4. $\int 5x^4\,dx = 5\int x^4\,dx$

   $= 5.\frac{x^5}{5} + C = x^5 + C$

5. $\int (3x^2 + 7x^6)\,dx$

   $\frac{3x^3}{3} + \frac{7x^7}{7} + C \Rightarrow = x^3 + x^7 + C$

6. $\int x^{-3} dx = \frac{x^{-3+1}}{-3+1} + C$

   $= \frac{-1}{2x^2} + C$

7. $\int \left(\frac{1}{x^9} + \frac{1}{x^{10}}\right) dx$

   $= \int (x^{-9} + x^{-10})\,dx$

   $= \frac{x^{-9+1}}{-9+1} + \frac{x^{-10+1}}{-10+1} + C = -\frac{x^{-8}}{8} - \frac{x^{-9}}{9} + C$

8. $\int dx = \int 1\,dx$

   $= \int x^0\,dx$

   $= \frac{x^{0+1}}{0+1} + C = x + C$

9. $\int 5\,dx = 5\int dx = 5x + C$

10. $\int \left(2t + 7t^6 + 10\right) dt = \frac{2t^2}{2} + \frac{7t^7}{7} + 10t + c$

    $= t^2 + t^7 + 10t + c$

FORMULA

- ☆ $f(x^n dx) = \frac{x^{n+1}}{n+1} + c$
- ☆ $\int e^x dx = e^x + c$
- ☆ $\int \frac{1}{x} dx = \ln x + c$
- ☆ $\int \sin x = -\cos x + c$
- ☆ $\int \cos x\,dx = \sin x + c$
- ☆ $\int \frac{1}{ax+b} dx = \frac{1}{a}\ln(ax+b) + c$
- ☆ $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + c$
- ☆ $\int \cos(ax+b)dx = +\frac{1}{a}\sin(ax+b) + c$
- ☆ $\int 2\cos(2x+3) = \sin(2x+3)$
- ☆ $\int \cos(2x+3) = 1/2\,\sin(2x+3) + c$
- ☆ $\int \frac{1}{4x+5} dx = \frac{1}{4}\ln(4x+5) + c$

Definite Integration

Upper limit of x

$$\int_{x_i = a}^{x_f = b} f(x)\,.\,dx = g(x)\Big|_{x_i = a}^{x_f = b} = g(b) - g(a)$$

Lower limit of x

Ex (1):

$$\int_0^3 x^2 dx = \frac{x^3}{3}\Big|_0^3 = \left(\frac{3^3}{3}\right) - \left(\frac{0^3}{3}\right) = 9 \text{ Ans}$$

Ex (2):

$$\int_2^3 4x^3 dx = 4\frac{x^4}{4}\Big|_2^3 = x^4\Big|_2^3 = 3^4 - 2^4 = 81 - 16 = 65 \text{ Ans}$$

Ex (3):

$$\int_0^{\pi/2} \cos x\,dx = \sin x\Big|_0^{\pi/2} = \left(\sin\frac{\pi}{2}\right) - \sin 0 = 1 - 0 = 1$$

Ex (4):

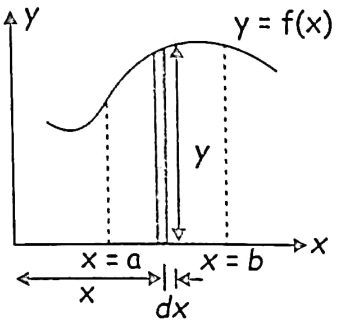
$$\int_2^3 e^x dx = e^x\Big|_2^3 = e^3 - e^2$$

Q. $\int_0^1 (3x^2 + 2x) = 3\frac{x^3}{3} + 2\frac{x^2}{2}\Big|_0^1$

$= (x^3 + x^2)\Big|_0^1 = (1^3 + 1^2) - (0^3 + 0^2) = 2$ Ans

APPLICATION OF INTEGRATION

☆ Area under the curve :



Area of small element = ydx = f(x)dx

If we sum up all areas between x = a and x = b then

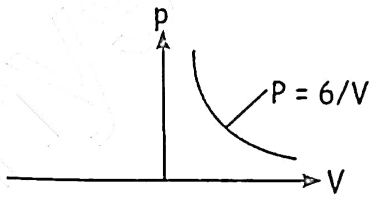
$\int_a^b f(x)dx$ = shaded area between curve and x-axis.

☆ Average value

if y = f(x) then $\bar{y}$ or $y_{average} = \frac{\int_{x=a}^{x=b} f(x)dx}{b-a}$

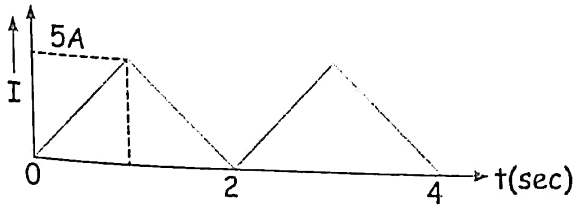
$= \frac{\text{Area under the curve}}{\text{interval}}$

Q. A gas expands its volume from V to 3V as shown in figure. Calculate the work in this process if $W = \int pdv$.



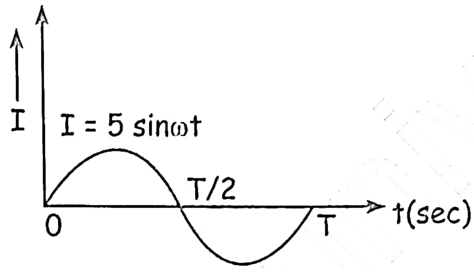
Sol. $W = \int_V^{3V} pdv \Rightarrow \int_V^{3V} \frac{6}{v}dv = [6\log_e v]_V^{3V} = 6\ln 3$

Q. Calculate average value of current from t = 0 to t = 4 seconds.



Sol. $I_{average} = \frac{\int_a^b Idt}{\int_a^b dt} = \frac{\text{total area}}{\text{total time}} = \frac{5 \times 2}{4} = 2.5$ amp

Q. Calculate average value of current from t = 0 to t = T seconds ($T = 2\pi/\omega$).



Sol. $I_{average} = \frac{\int_0^T 5\sin\omega t}{\int_0^T dt} = \frac{5}{\omega T}[-\cos\omega t]_0^T$

$= -\frac{5}{\omega T}[\cos\omega T - \cos 0] = 0 \qquad \left[\omega = \frac{2\pi}{T}\right]$

☆ For any moving object, the average speed can never be zero or negative, as total distance covered is always +ve.

☆ If a particle travels distances s_1, s_2, s_3 , ..., etc., at different speeds v_1, v_2, v_3, ... etc., respectively, then

$v_{av} = \frac{\Delta s}{\Delta t} = \frac{\Sigma s_i}{\Sigma(s_i / v_i)}$

If $s_1 = s_2 = ... = s_n = s$,

Then $\frac{1}{v_{av.}} = \frac{1}{n}\left[\frac{1}{v_1} + \frac{1}{v_2} + ...\right] = \frac{1}{n}\Sigma\frac{1}{v_i}$

Special case: If a particle moves a distance at speed v_1 and comes back to initial position with speed v_2, then

$v_{av} = \frac{2v_1v_2}{v_1 + v_2}$

☆ If a particle travels at speeds v_1, v_2, ..., etc., for time intervals t_1, t_2, ..., then

$v_{av} = \frac{\Delta s}{\Delta t} = \frac{v_1t_1 + v_2t_2 + ... + v_nt_n}{t_1 + t_2 + ...t_n} = \frac{\Sigma v_it_i}{\Sigma t_i}$

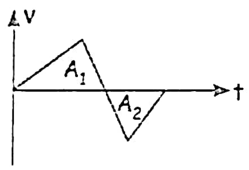
Special case: If a particle moves for two equal intervals of time at different speeds, then

$v_{av} = \frac{v_1 + v_2}{2}$.

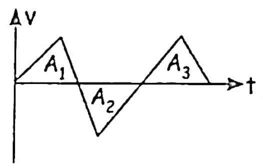
⁂ काम का डब्बा

$$x \underset{\text{integration}}{\overset{\text{differentiation}}{\rightleftarrows}} v \underset{\text{integration}}{\overset{\text{differentiation}}{\rightleftarrows}} a$$

$\int y \, . \, dx$ = Area under curve
curve और x-axis के बीच का Area

+ a-t graph का area change in velocity देता है।
+ v-t graph का area change in position देता है बोले तो
  v-t graph का area displacement देता है
+ 

  Displacement = $A_1 - A_2$
  Distance = $A_1 + A_2$

  Displacement = $A_1 - A_2 + A_3$
  Distance = $A_1 + A_2 + A_3$
+ (x - t) graph ka slope ⇒ v
+ (v - t) graph ka slope ⇒ $\bar{a}$
+ $v = \frac{dx}{dt}$, $a = \frac{dv}{dt}$
+ Displacement = $\int v dt$ = Area
+ change in velocity = $\int a dt$
+ St line → slope → const
+ (v - t) is st. line → a = constant
+ If (x-t) is straight line → v = constant and a = 0.
+ If a = 0 → v = constant → (x-t) straight line
+ If a = const → (v-t) straight line → (x-t) parabola
+ If a body moves with uniform acceleration and velocity changes from u to v in a time interval, then average velocity = $\frac{v+u}{2}$.
+ If a body moving with uniform acceleration has velocities u and v at two points in its path, then the velocity at the midpoint of given two points = $\sqrt{\frac{u^2+v^2}{2}}$

Q. At t = 0, particle is at x = 10 particle is moving such that its velocity vs time relation is given as $V = 3t^2 + 2t$. Find location of particle t = 1 and x = f(t)

Sol. $x = \int v dt = \int (3t^2 + 2t) dt = t^3 + t^2 + c$

at t = 0, x = 10 (put the value)

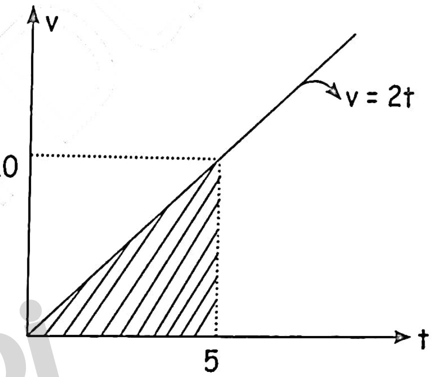
$x = t^3 + t^2 + c$

$10 = 0 + 0 + c \Rightarrow c = 10$

$x = t^3 + t^2 + 10$ (now you can find x for any t)

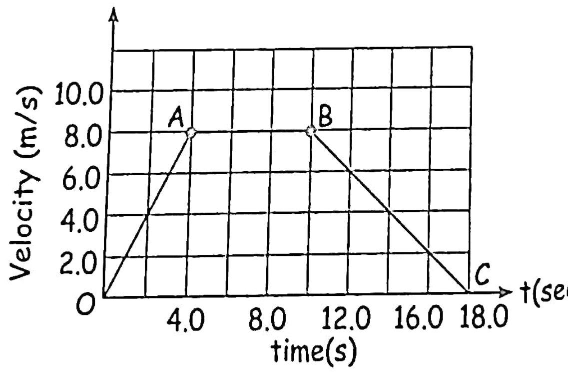
Q. A particle is moving along x-axis. Its velocity vs time relation is given as v = 2t. Find displacement from t = 0, t = 5

Sol.



Area = 1/2 × 5 × 10 = 25 = displacement

Q. What is the acceleration for each graph segment in figure? Describe the motion of the object over the total time interval. Also calculate displacement.



Sol. Segment OA; $a = \frac{8-0}{4-0} = 2 \, m/s^2$

Segment AB; graph horizontal i.e., slope zero i.e., a = 0

Segment BC; $a = \frac{0-8}{18-10} = -1$ m/s2

The graph is trapezium. Its area between t = 0 to t = 18s is displacement.

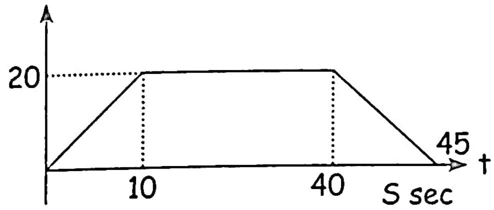
Area of v-t graph = displacement = $\frac{1}{2}(18+6) \times 8$

= 96 m

Particle accelerates uniformly for first 4 sec., then moves with uniform velocity for next 6 sec. and then retards uniformly to come to rest in next 8 sec.

Q. A particle starts from rest, accelerates at 2 m/s^2 for 10 s & then goes at constant velocity for 30s and then deaccelerate at 4 m/s^2 till it stops. What is the distance travelled by it.

Sol.



$$\text{Area} = \frac{1}{2}\times(30+45)\times 20 = 750$$

$$\left.\begin{aligned} v &= u + at \\ s &= ut + \frac{1}{2}at^2 \\ v^2 &= u^2 + 2as \end{aligned}\right\} a = \text{cons}$$

u = intial velocity
v = final velocity
a = acc
s = displacement

भाई ये eqn तभी लगाना जब a constant हो

Q. A particle start motion having initial velocity 20 m/s and it move with const acc of 10 m/s^2

1. Find velocity at t = 4 sec
2. Find displacement of particle from t = 0 to t = 4 sec
3. Find displacement in 4^{th} sec

Sol. 1. t = 4, v = u + at

= 20 + 10 × 4

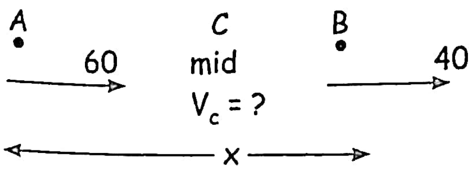
v = 60

2. $s = ut + 1/2\ at^2$

$$= 20\times 4 + \frac{1}{2}\times 10\times 4^2 = 160$$

Q. A truck travelling with uniform acceleration crosses two points A & B with velocities 60 m/s & 40 m/s respectively. The speed of the body at the midpoint of A & B is nearest to.

Sol.



$\boxed{A \to B}\ 40^2 = 60^2 + 2ax \Rightarrow 2ax = -2000$ (i)

$\boxed{A \to C}\ V_C^2 = 60^2 + 2a\, x/2$ (ii)

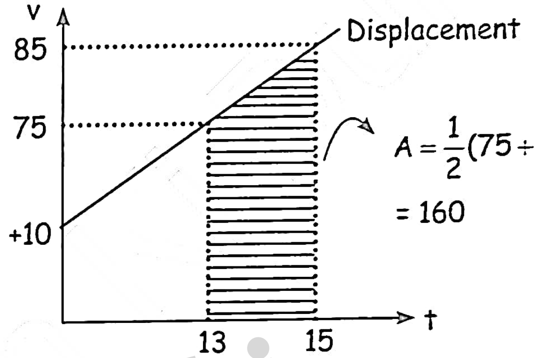
from (i) & (ii)

$V_C^2 = 3600 - 1000 = 2600$

$V_c = 10\sqrt{26}$

Q. A particle having initial velocity 10 m/s move with a constant acceleration 5 ms^{-2}, for a time 15 second along a straight line, what is the displacement of the particle in last 2 second?

Sol.



$$A = \frac{1}{2}(75+85)2$$

= 160

Q. A bullet is moving with a velocity of 200 cm/s. penetrates a wooden block & comes to rest after travelling 4cm insides it. What velocity is needed for travelling distance of 9cm in same block?

Sol. $$0^2 = 2^2 + 2a\times\frac{4}{100}$$

$$-4 = \frac{8}{100}a \Rightarrow a = -50$$

$0^2 = u^2 + 2as$

$$0 = u^2 + 2\times(-50)\frac{9}{100}$$

$u^2 = 9$

u = 3 m/s

Q. A bullet going with speed 350 m/s enters a concrete wall and penetrates a distance of 5.0 cm before coming to rest. Find the deaceleration

Sol. $U_f = 0$, a = ?

$$0^2 = (350)^2 + 2\times a\times\frac{5}{100}$$

$a = -(350)^2 \times 10$

$a = -1225000\ m/s^2$

Q. A particle covered 100 m distance in first 10 s of its joumey and in next 10 s it travel 200 m. Find distance travelled in next 10s? (acc is const)

Sol. $t = 0 \to t = 10 \qquad 100 = u \times 10 + \frac{1}{2} a \times 10^2 \quad ...(i)$

$t = 0 \to t = 20 \qquad 300 = u \times 20 + \frac{1}{2} a (20)^2 \quad ...(ii)$

Sol. e (i) and (ii) and get $a = 1$, $u = 5$

Let it travel x_3 in last 10s so in 30s a travel $300 + x_3$

$$300 + x_3 = 5 \times 30 + \frac{1}{2} \times 1 \times (30)^2$$

Sol. e and get $x_3 = 300$

Q. A particle moving with initial velocity of 10 m/s towards East has an acceleration of 5 m/s^2 towards west. Find the displacement and distance travelled by the particle in first 4 seconds?

Sol. $u = 10 m/s$, $a = -5 m/s^2$, $t = 2$

$v = u + at \Rightarrow 0 = 10 - 5t \Rightarrow t = 2s$

The direction of velocity changes after two seconds.

$S = 10 \times 4 + \frac{1}{2}(-5) \times 4^2 = 0 =$ displacement

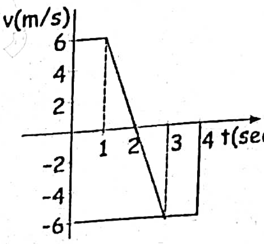
Distance travelled is not equal to displacement because during course of journey, velocity changes direction.

$D = S(\text{at } 2s) + |S(\text{at } 4s) - S(\text{at } 2s)|$

$$= \left(10 \times 2 - \frac{1}{2} \times 5 \times 2^2\right) + \left|0 - (10 \times 2) - \frac{1}{2} \times 5 \times 2^2\right|$$

$= 10 + 10 = 20$ m

Q. A particle moves along a straight line along x-axis. At time $t = 0$, its position is at $x = 0$. The velocity v m/s of the object changes as a function of time t seconds as shown in the figure.



(i) What is x at $t = 1$ sec?

(ii) What is the acceleration at $t = 2$ sec?

(iii) What is x at $t = 4$ sec?

(iv) What is the average speed between $t = 0$ and $t = 3$ sec?

Sol. (i) x is displacement at $t = 1$ sec.

Area under the $v - t$ curve gives displacement From $t = 0$ to $t = 1$ sec.

$x = 6 \times 1 = 6$ m

(ii) Slope of the v-t curve gives acceleration from the given v-t curve

Slope at $t = 2$ sec. gives acceleration at $t = 2$ sec.

$\tan\theta = a = -\frac{6}{1} = -6 \; m/s^2$

(iii) x (at $t = 4$ sec):

Area under the curve from $t = 0$ to $t = 4$ sec

$= 6 \times 1 + \frac{1}{2} \times 6 \times 1 - \frac{1}{2} \times 6 \times 1 - 6 \times 1 = 0$

$\Rightarrow x(t = 4) = 0$ m

(iv) Average speed from $t = 0$ to $t = 3$ sec.

Displacement from $t = 0$ to $t = 2$ sec. = Area under the curve $= 6 + \frac{1}{2} \times 6 \times 1 = 9$ m

Displacement form $t = 2$ to $t = 3$ sec.

$= -\frac{1}{2} \times 6 \times 1 = -3$ m

Distance from $t = 0$ to $t = 3$ sec $= |9| + |-3| = 12$ m

Average speed $= \frac{\text{Distance}}{\text{Time}} = \frac{12}{3} = 4$ m/s

Question Practice on Variable Acceleration and Utptang Integration

Q. The acceleration a of a particle moving in one dimension is given by $a = 6 - 2t$. If the particle is initially at $x = 0$ and its velocity is 2 m/s, find its position and velocity at time t.

Sol. $\frac{dv}{dt} = 6 - 2t$

$$\int_2^v dv = \int_0^t (6 - 2t)dt$$

$\Rightarrow v - 2 = (6t - t^2)|_0^t = 6t - t^2 \Rightarrow v(t) = 2 + 6t - t^2$

To find position, we integrate velocity.

$v = \frac{dx}{dt} = 2 + 6t - t^2 \Rightarrow dx = (2 + 6t - t^2)dt$

$$\int_0^x dx = \int_0^t (2 + 6t - t^2)dt = 2t + 3t^2 - \frac{t^3}{3}$$

or $x(t) = 2t + 3t^2 - \frac{t^3}{3}$

Q. The retardation of a c[...] off depends on its ve[...] α is positive constant. travelled by the car i[...] m/s and $\alpha = 0.5/s$.

Sol. $\frac{dv}{dt} = -\alpha v$

$\frac{dv}{dx}\left(\frac{dx}{dt}\right) = -\alpha v \Rightarrow \frac{vdv}{dx}$ [...]

or $dv = -\alpha dx$

$\int_{20}^{0} dv = -\alpha \int_0^d dx \Rightarrow v|_{20}^{0} = -$ [...]

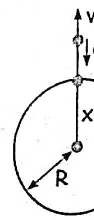
$-20 = -\alpha d$

$d = \frac{20}{\alpha} = \frac{20}{0.5} = 40$ m

Q. With what velocity in [...] should a body be proje[...] earth so that it reache[...] of earth? The acceler[...]

$a = -\frac{GM}{x^2}$ where x is t[...] of earth and M is the [...]

Sol. Note that acceleratio[...] constant near the sur[...] height become too l[...] distance can not be ig[...]



$a = \frac{dv}{dt} = -\frac{GM}{x^2}$

or $\frac{dv}{dx} \cdot \frac{dx}{dt} = -\frac{GM}{x^2} \Rightarrow vd$[...]

At the highest point, v[...] $x_i = R$ and $x_f = 2R$.

$$\int_u^0 v dv = -GM \int_R^{2R} \frac{dx}{x^2}$$

$\frac{v^2}{2}\Big|_u^0 = -GM \int_R^{2R} x^{-2} dx = \frac{GM}{x}$ [...]

or $x(t) = 2t + 3t^2 - \frac{t^3}{3}$

Q. The retardation of a car when its engine is shut off depends on its velocity as $a = -\alpha v$ where α is positive constant. Find the total distance travelled by the car if its initial velocity is 20 m/s and $\alpha = 0.5/s$.

Sol. $\frac{dv}{dt} = -\alpha v$

$$\frac{dv}{dx}\left(\frac{dx}{dt}\right) = -\alpha v \Rightarrow \frac{vdv}{dx} = -\alpha v$$

or $dv = -\alpha dx$

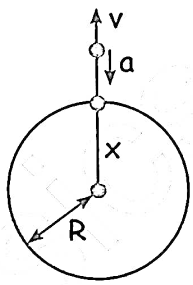
$$\int_{20}^{0} dv = -\alpha\int_{0}^{d} dx \Rightarrow v\big|_{20}^{0} = -\alpha x\big|_{0}^{d}$$

$-20 = -\alpha d$

$$d = \frac{20}{\alpha} = \frac{20}{0.5} = 40 \text{ m}$$

Q. With what velocity in vertical upward direction should a body be projected from the surface of earth so that it reaches a height equal to radius of earth? The acceleration of body is given by $a = -\frac{GM}{x^2}$ where x is the distance from centre of earth and M is the mass of earth.

Sol. Note that acceleration due to gravity is nearly constant near the surface of earth. But if the height become too large its dependence on distance can not be ignored.



$$a = \frac{dv}{dt} = -\frac{GM}{x^2}$$

or $\frac{dv}{dx}\cdot\frac{dx}{dt} = -\frac{GM}{x^2} \Rightarrow vdv - \frac{GM}{x^2}dx$

At the highest point, velocity is zero. Also note $x_i = R$ and $x_f = 2R$.

$$\int_{u}^{0} vdv = -GM\int_{R}^{2R}\frac{dx}{x^2}$$

$$\frac{v^2}{2}\bigg|_{u}^{0} = -GM\int_{R}^{2R} x^{-2}dx = \frac{GM}{x}\bigg|_{R}^{2R}$$

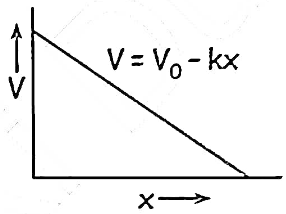
$$\Rightarrow -\frac{u^2}{2} = GM\left[\frac{1}{2R} - \frac{1}{R}\right]$$

$$\Rightarrow u^2 = \frac{GM}{R} \Rightarrow u = \sqrt{\frac{GM}{R}} = \sqrt{\frac{GM}{R^2}R}$$

$$\because g = \left(\frac{GM}{R^2}\right)$$

$$\therefore u = \sqrt{gR} = 8 \text{ km/s } [\because R = 6400 \text{ km}, g = 10 \text{ m/s}^2]$$

Q. A particle is moving along x-axis with velocity V which varies according to the law $V = V_0 - Kx$ here V_0 and K are constants. Choose the correct acceleration vs time plot for the time interval when particle moves from $x = 0$ to $x = \frac{V_0}{K}$.



Sol. $V = V_0 - Kx$

$$\frac{dx}{dt} = (V_0 - Kx) \Rightarrow \int_0^x \frac{dx}{(V_0 - Kx)} = \int_0^t dt$$

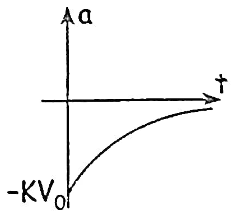
$$x = \frac{V_0}{K}(1 - e^{-Kt})$$

$$\therefore \frac{dx}{dt} = +V_0 e^{-Kt} \Rightarrow a = \frac{d^2x}{dt^2} = -KV_0 e^{-Kt}$$

At $t = 0$, $a = -KV_0$

At $t = \infty$, $a = 0$

Therefore, graph is as shown



Q. A block of mass m is fired horizontally along a level surface that is lubricated with oil. The oil provides a viscous resistance that varies as the 3/2 power of the speed. If the initial speed of the block is v_0 at $x = 0$, find the maximum distance reached by the block. Assume no resistance to motion other than that provided by the oil.

Sol. $F = -v^{3/2}$

$$a = -\frac{1}{m}v^{3/2} \qquad [\because F = ma]$$

$$v\frac{dv}{dx} = -\frac{1}{m}v^{3/2}$$

$$\int_{v_0}^{0} v^{-\frac{1}{2}} dv = -\frac{1}{m}\int_0^d dx$$

$2mv_0^{1/2} = d$

Q. Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where x is position in metre and a in m/s^2). It is at rest at $x = 0$. At what position x (in metre) will the particle again come to instantaneous rest?

Sol. $\frac{vdv}{dx} = 4 - 2x$

$$\int_0^v vdv = \int_0^x (4-2x)dx \Rightarrow \frac{v^2}{2} = 4x - x^2$$

when $v = 0$, $4x - x^2 = 0$

$x = 0, 4$

$\therefore$ At $x = 4$ m, the particle will again come to rest.

Question for Practice

Q. A particle start motion at $t = 0$ from $x = +10$, such that its 'v' vs t relation is given as $\bar{v} = 4t^3 + 3t^2 + 2t$. Find location of particle at $t = 1$ sec.

Sol. $x = \int vdt$

$x = t^4 + t^3 + t^2 + c$

$t = 0, x = 10$ (put $t = 0, x = 10$) $\boxed{c = 10}$

$10 = 0 + 0 + 0 + c$

$x = t^4 + t^3 + t^2 + 10$

$t = 1, \ x = 13$

Q. A particle start motion from $x = 5$, at $t = 0$ such that $v = t^2 + t$. Find location of particle at $t = 6$ sec.

Sol. $dx = v\,dt$

$$\int_{x=5}^{x_f} dx = \int_{t=0}^{t=6} (t^2 + t)dt$$

$$x\Big|_5^{x_f} = (t^3/3 + t^2/2)\Big|_0^6$$

$$x_f - 5 = \left(\frac{6^3}{3} + \frac{6^2}{2}\right) - \left(\frac{0^3}{3} + \frac{0^2}{2}\right)$$

$x_f - 5 = 72 + 18 - 0$

$x_f = 90 + 5$

$x_f = 95$ Ans

Q. A particle start motion from rest from origin at $t = 0$ such that $a = 6t$ find v, x (location) at, $t = 2$ sec

Sol. $a = 6t$

$v = \int adt = \int 6tdt$

$v = 6t^2/2 + c$

$v = 3t^2 + c$

$t = 0, v = 0$

$0 = 0 + c$

$c = 0$

$v = 3t^2$ $\boxed{t = 2, v = 12}$

$x = \int 3t^2\,dt = 3\,t^3/3 + c'$

$x = t^3 + c'$

$t = 0, x = 0, \quad 0 = 0 + c \quad \rightarrow c' = 0$

$x = t^3$

$t = 2, x = 8, v = 12$

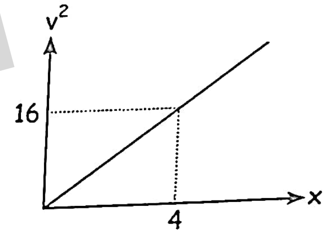
Q. If $v = x^2 + 3x$ Find acc at $x = 2$

Sol. $a = v\frac{dv}{dx} \quad \frac{dv}{dx} = 2x + 3$

$a = (x^2 + 3x)(2x + 3)$

put $x = 2$, $a = 70$

Q. Find acc at $x = 4$

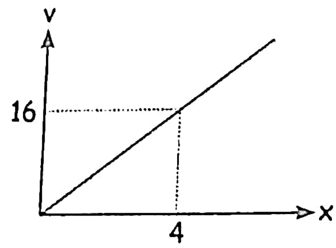


Sol. $\text{slope} = \frac{dy}{dx} = \frac{d(v^2)}{dx}$

$16/4 = 2v\frac{dv}{dx}$

$4 = 2a \rightarrow \boxed{a = 2}$

Q. Find acc at $x = 4$



Sol. $\text{slope} = \frac{dy}{dx} = \frac{dv}{dx} = 16/4$

$a = v\frac{dv}{dx}$

$a = 16 \times \frac{16}{4} = \boxed{64}$

Q. $x = \sqrt{v+1}$ find acc at $x = 5$

Sol. $x^2 = v + 1$

$v = x^2 - 1$

$a = v\frac{dv}{dx} = (x^2 - 1)(2x)$

$x = 5 \Rightarrow a = 240$

Q. Acceleration of a particle moving on x-axis having initial speed v_0 with distance from origin is given by $a = \sqrt{x}$ Distance covered by particle where its speed become thrice that of initial speed.

Sol. $a = \sqrt{x}$ $\qquad$ $a > 0$, speed 1, $v > 0$

$v\frac{dv}{dx} = \sqrt{x}$

$\int v dv = \sqrt{x}$

$$\int_{V_0}^{3V_0} v dv = \int_0^{x_f} x^{1/2} dx$$

$$\left.\frac{v^2}{2}\right|_{V_0}^{3V_0} = \left.\frac{x^{1/2+1}}{1/2+1}\right|_0^{x_f}$$

$$\frac{9V_0^2}{2} - \frac{V_0^2}{2} = \left.\frac{2}{3}x^{3/2}\right|_0^{xf}$$

$$\frac{8V_0^2}{2} = 2/3 x_f^{3/2}$$

Q. $x = a \sin \omega t$

$y = a(1 - \cos \omega t) = a - a\cos \omega t$

Find $\vec{v}$, speed of particle

Sol. $v_x = a\omega \cos \omega t$

$v_y = 0 - a\omega[-\sin \omega t]$

$v_y = a\omega \sin \omega t$

$\vec{v} = (a\omega \cos \omega t)\hat{i} + (a\omega \sin \omega t)\hat{j}$

speed = ? magnitude

$|\vec{v}| = \sqrt{(a\omega\cos\omega t)^2 + (a\omega\sin\omega t)^2}$

$= a\omega\sqrt{\cos^2 \omega t + \sin^2 \omega t}$

speed = $a\omega$

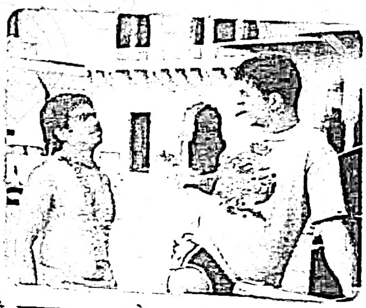
MOTION UNDER GRAVITY

It's very important article जिसमे हम particle का motion under the effect of gravity पढ़ेंगे.....

Assumption

1. Air resistance force is neglected until mention.
2. Variation of gravity g is neglected until mention.

भईया kinematics के ques मे समझ नही आता कैसे करना है sign convention मे भी दिक्कत होती है sol. तो समझ आ जाते है पर सवाल खुद से नही बनते इसका कोई इलाज.....



हाँ भाई सबसे पहले तो जोर से घुटना लगाओ..... फिर जो given है वो लिख लो जो पूछा है वो भी लिखलो और देखो कौन-सी eqn of motion connect हो रही है।

+/- जहाँ मन करे ऊपर या नीचे मान सकते हो ans. same आएगा बस इस बात का ध्यान रखना u, s, a को with sign लिखना है।

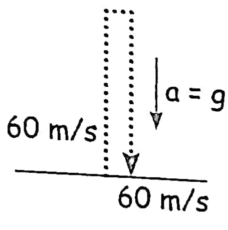
Q. A particle is projected from ground vertically upward with velocity 60 m/s. (1) Find velocity and location of particle at t = 10 (2). When will particle come to rest (3). Find h_{max}, time of flight.

Sol. Sign convention

Let upward direction (+ve)

u = 60 upar = +60

a = 10 neeche = -10



✦ $t = 10, v = u + at = 60 + (-10)(10)$

$v = \underbrace{-}_{\text{नीचे}}40$

✦ location $= y = 60 \times 10 + 1/2(-10)(10)^2 = \underbrace{+}_{\text{उपर}}100$

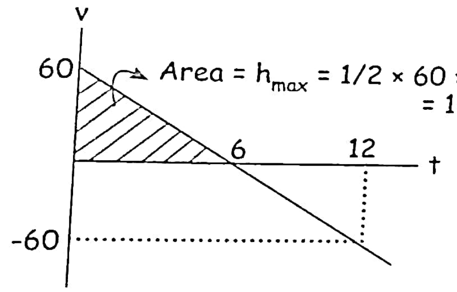
✦ When particle will come to rest

$v = u + at \Rightarrow 0 = 60 - 10t \Rightarrow t = 6$

✦ $0^2 = 60^2 + 2 \times (-10) h_{max}$

$h_{max} = 180$

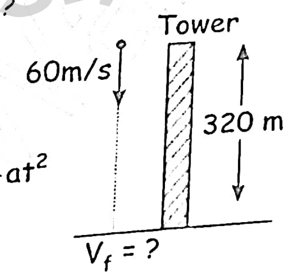
✦ Time of fight = 6 + 6 = 12 sec



भईया जब particle ऊपर जाता है तब उसका acceleration नीचे होता है और जब नीचे आता है तब भी उसका acceleration नीचे होता है।

हाँ भाई अभी के लिए बस तु ये याद रख अगर particle हवा मे है तो उसका acceleration g होगा चाहे ऊपर जाए या नीचे..... particle उधर जाता है जिस तरफ उसकी velocity होती है और उसका acceleration उधर होता है जिस तरफ उस पर net force होता है।

Q. A particle is thrown in vertically downward with velocity 60 m/s from top of a tower of height 320 m. Find when particle will hit the ground & with what velocity & V_f = ?



Sol. Let (Downward +ve)

$u = +60 \qquad a = +10$

✦ displacement $s = ut + \frac{1}{2}at^2$

$+320 = 60t + \frac{1}{2} \times 10 \times t^2$

$t^2 + 12t - 64 = 0$

$(t + 16)(t - 4) = 0 \Rightarrow t = 4s$

✦ V_f = ?

$V^2 = u^2 + 2as$

$V^2 = 60^2 + 2 \times 10 \times 320$

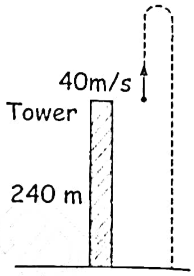
$V^2 = 3600 + 6400$

$V^2 = 10000$

V_f = velocity $V = 100$ (नीचे)

Q. A particle is thrown in vertically upward with velocity 40 m/s from top of a tower of height 240 m. Find when particle will hit the ground & with what velocity & V_f = ?

Sol. (1) let (up = +ve)



$u = +40,\ a = -10,\ s = -240$

$s = ut + \frac{1}{2}at^2$

$-240 = 40t + 1/2\,(-10)\,t^2$

$5t^2 - 40t - 240 = 0$

$(t - 12)(t + 4) = 0$

$t = 12$

(2) $V^2 = u^2 + 2as = (40)^2 + 2 \times (-10)(-240)$

$V = 80$

Q. A balloon is rising upward with constant velocity 40 m/s. When it reaches at a height of 240 m from ground a particle is drop from it. Find when will particle hit the ground with what velocity?

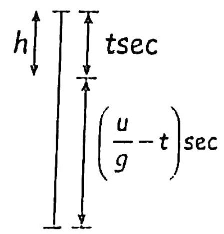
Ans. t = 12, v = 80 m/s

SKC

अगर हम किसी चलती हुई गाडी लिफ्ट और उड़ते हुए गुब्बारे से किसी particle को drop करे तो drop के just बाद particle की velocity वही होती जो उस गाडी, लिफ्ट और गुब्बारे की हो। So, अब ये सवाल तो वही टावर वाला सवाल है।

Q. If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

Sol. If ball is thrown with velocity u, then time of ascent $= \frac{u}{g}$



Velocity after $\left(\frac{u}{g} - t\right)$sec, $\quad v = u - g\left(\frac{u}{g} - t\right) = gt.$

So, distance in last 't' sec, $\quad 0^2 = (gt)^2 - 2(g)h.$

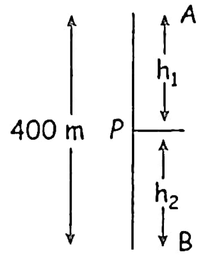
$h = \frac{1}{2}gt^2.$

Q. A man drops a ball downside from the roof of a tower of height 400 m. At the same time another ball is thrown upside with a velocity 50 m/s from the foot of tower. What is the height from the foot of the tower where the two balls would meet?

Sol. Let both balls meet at point P after time t.

The distance travelled by ball A

$h_1 = \frac{1}{2}gt^2$ ...(i)



The distance travelled by ball B

$h_2 = ut - \frac{1}{2}gt^2$ ...(ii)

By adding (i) and (ii) $h_1 + h_2 = ut = 400$

(Given $h = h_1 + h_2 = 400$)

$\therefore$ t = 400/50 = 8 s and $h_1 = 320$ m, $h_2 = 80$ m

Q. Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant?

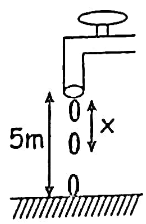
Sol. Let the interval between each drop be t then from question

For first drop $\frac{1}{2}g(2t)^2 = 5$ ...(i)

For second drop $x = \frac{1}{2}gt^2$ ...(ii)

By solving (i) and (ii) $x = \frac{5}{4}$ and

Hence required height $h = 5 - \frac{5}{4} = 3.75$m.



Q. A balloon is at a height of 81 m and is ascending vertically upward with a velocity of 12 m/sec. A body of 2 kg weight is dropped from it. If g = 10 m/s^2 the body will reach the surface of the earth in

Sol. As the balloon is going up so initial velocity of balloon

= + 12 m/s,

$\Delta y = -81$ m; $a = -g = -10$ m/s^2

By applying $h = ut + \frac{1}{2}gt^2$; $-81 = 12t - \frac{1}{2}(10)t^2$

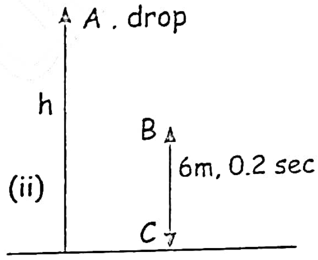
$\Rightarrow 5t^2 - 12t - 81 = 0$

$\Rightarrow t = \frac{12 \pm \sqrt{144 + 1620}}{10} = \frac{12 \pm \sqrt{1764}}{10} = 5.4$ s

Q. A ball dropped from a height h from ground, if it take 0.2 sec to cross the last 6m before hitting the ground. Find height from which it was dropped.

Sol. $t_{A \to C} = t = \sqrt{\frac{2h}{g}}$ ...(i)

$t_{A \to B} = t - .2 = \frac{\sqrt{2(h-6)}}{g}$ (ii)



PROJECTILE MOTION

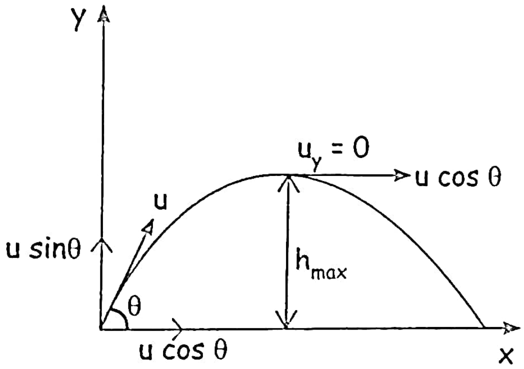
भाई ये बहुत मजेदार chapter है बस यहाँ x और y दोनो मे motion हो रहा है, बस ये याद रखना x और y दो independent axis है तो दोनो के साथ independently खेलना है जब x के साथ हो तो y को भुल जाओ और जब y के साथ हो तो x को भूल जाओ it's like your gf... अब समझ गया ना।

In this chapter

- Air Resistance neglected until mention
- variation of g neglected g = 10m/s^2 (downward)

  g = 9.8 m/s^2
- Since, $a_x = 0 \Rightarrow v_x$ = const = u cos θ
- Particle चाहे ऊपर जा रहा हो या नीचे आ रहा हो उसका acc नीचे की तरफ g होगा

A particle is projected with speed u at angle θ with the horizontal as shown in fig.



1. Time of flight (T)

$u_y = u\sin\theta,$

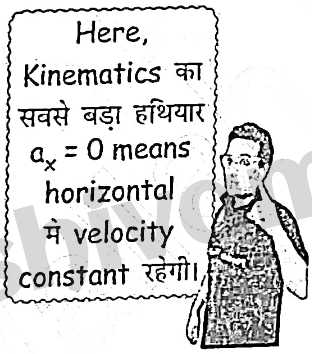
$a_y = -g$

$v = u + at$

$0 = u\sin\theta - gt$

$t = u\sin\theta/g$ (time to reach highest point)

$$\boxed{T = 2t = \frac{2u\sin\theta}{g}}$$



2. Maximum height (h_{max})

$v^2 = u^2 + 2as$ (y)

$0 = (u\sin\theta)^2 - 2g\,h_{max}$

$$\boxed{h_{max} = \frac{u^2\sin^2\theta}{2g}}$$

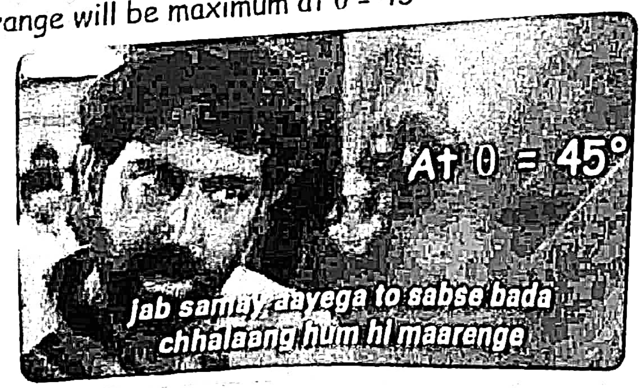
3. Range = $u\cos\theta \times T$

$$R = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{u^2 2\sin\theta\cos\theta}{2}$$

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

For R to be maximum by keeping u fix sin 2θ should be maximum ⇒ 2θ = 90° ⇒ θ = 45°

If u is fixed then from ground to ground projectile range will be maximum at θ = 45°



काम का डब्बा ($a_x = 0$, ground to ground)

$$T = \frac{2u\sin\theta}{g} = \frac{2U_y}{a_y}$$

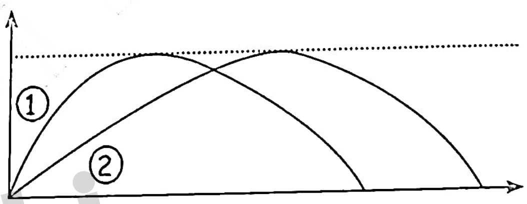
$$H_{max} = \frac{(u\sin\theta)^2}{2g} = \frac{U_y^2}{2a_y}$$

$$R = \frac{u^2\sin 2\theta}{g} = U_x T = \frac{2U_xU_y}{a_y}$$

☆ If vertical velocity same ⇒ T, h_{max}, same.

☆ If U → fix, at θ and 90-θ ⇒ Range Same

☆

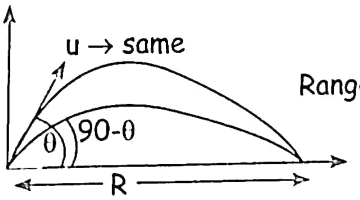


H_{max} → same

U_y → same

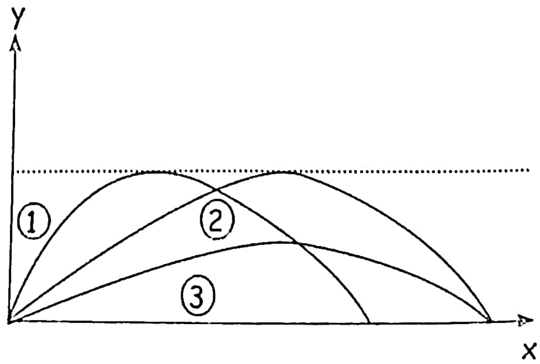
$u_1\sin\theta = u_2\sin\theta_2$

☆



Range → same

☆



☆ $(H_{max})_1 = (H_{max})_2 > (H_{max})_3$

☆ $T_1 = T_2 > T_3$

$(U_y)_1 = (U_y)_2 > (U_y)_3$

$R_1 < R_2 = R_3$

Q. A particle is projected from ground with speed at an angle θ with horizontal such that its maximum height is half of range. Find θ

Sol. $h_{max} = 1/2\ R$

$$\frac{u^2\sin^2\theta}{2g} = \frac{1}{2}\frac{u^2\sin 2\theta}{g}$$

$\sin^2\theta = \sin 2\theta$

$\sin^2\theta = 2\sin\theta\cos\theta$

$\tan\theta = 2$

$\theta = \tan^{-1}(2)$

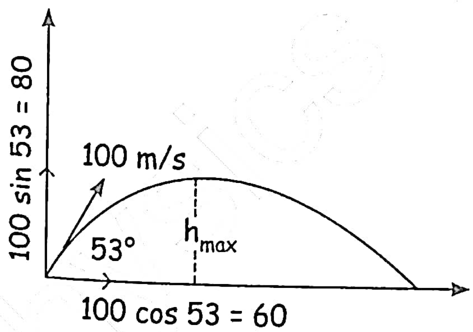
Q. Two particle are projected with same speed at angle 30 & 60 Find Ratio of their max height, range and time of flight respectively.

Sol. (1) $$\frac{(h_{max})_1}{(h_{max})_2} = \frac{\left(\dfrac{u^2\sin^2 30}{2g}\right)}{\left(\dfrac{u^2\sin^2 60}{2g}\right)} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3}$$

(2) $$\frac{R_1}{R_2} = \frac{\left(\dfrac{u^2\sin(2\times 30)}{g}\right)}{\left(\dfrac{u^2\sin(2\times 60)}{g}\right)} = \frac{\sin 60}{\sin 120} = \frac{\sqrt{3}/2}{\sqrt{3}/2} = 1$$

(3) $$T = \frac{2u\sin\theta}{g} \therefore \frac{T_1}{T_2} = \frac{\sin 30}{\sin 60} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

Q. A particle is projected with velocity 100m/s at angle of 53° with the horizontal, answer following parts.



1. Find initial velocity, acc

Sol. $\vec{u} = 60\hat{i} + 80\hat{j}$

2. Acceleration.

$\vec{a} = g$ नीचे

$\vec{a} = 0\hat{i} - 10\hat{j}$

3. Find time of flight

Sol. $T = 8 + 8 = 16$ sec.

4. Range = ?

Sol. $60 \times 16 = 960$

5. Find velocity at

Sol.

$t = 2$	$v_y = 60\hat{j}$	$\vec{v} = 60\hat{i} + 60\hat{j}$
$t = 8$	$v_y = 0\hat{j}$	$\vec{v} = 60\hat{i} + 0\hat{j}$
$t = 10$	$v_y = -20\hat{j}$	$\vec{v} = 60\hat{i} - 20\hat{j}$
$t = 16$	$v_y = -80\hat{j}$	$\vec{v} = 60\hat{i} - 80\hat{j}$

6. Find change in momentum for entire path (m = 1 kg) (momentum = mass × velocity)

Sol. $\Delta\vec{P} = \vec{P}_f - \vec{P}_i$

$\vec{P}_i = m\vec{v}_i = 1(60\hat{i} + 80\hat{j})$ $\quad \vec{P}_f = 60\hat{i} - 80\hat{j}$

$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = 0\hat{i} - 160\hat{j}$

$\Delta\vec{P} = -160\hat{j}$

7. Find h_{max}

Sol. $v^2 = u^2 + 2as$

$0^2 = (80)^2 + 2(-10)\, h_{max}$

$h_{max} = 320$

8. Find $\vec{V}$ at any time

Sol. $V = u + at$

$V_y = 80 - 10t$

$\vec{V} = 60\hat{i} + (80 - 10t)\hat{j}$

9. Find time when $\vec{v}$ become perpendicular to $\vec{a}$

Sol. $\vec{V}_i = 60\hat{i} + (80 - 10t)\hat{j}$

$\vec{a} = \qquad -10\hat{j}$

$\vec{v}_i \cdot \vec{a} = -10(80 - 10t) = 0$

$t = 8$

10. Find the time when velocity of particles become perpendicular to initial velocity.

Sol. $\vec{V} = 60\hat{i} + 80\hat{j}$

$\vec{V}_f = 60\hat{i} + (80 - 10t)\hat{j}$

$\vec{V}_i \cdot \vec{V}_f = 0$

$60 \times 60 + 80(80 - 10t) = 0$

$3600 + 6400 - 800\,t = 0$

$$t = \frac{10000}{800} = \frac{100}{8} = 12.5$$

11. Find location of particle at t = 8

Sol. $t = 8, \quad x = 60 \times 8 = 480$

$$y = 80 \times 8 - \frac{1}{2} \times 10 \times 8^2$$

$$y = 640 - 320 = 320$$

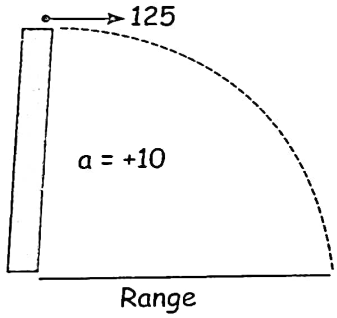
$$(x, y) = (480, 320)$$

Q. A particle is projected horizontally with velocity 40m/s from top of a tower of height 500 m above ground. Calculate range.

Sol. $u_y = 0, a_y = +10, s_y = +500$

$$s = ut + 1/2\, at^2$$

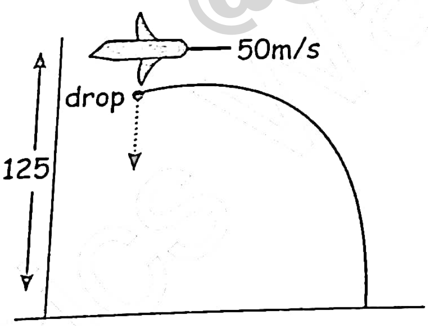
$$500 = 0 + 1/2 \times 10 \times t^2$$



t = 10 sec.

Range: $40 \times 10 = 400$ m

Q. A body is dropped from plane. Calculate range.

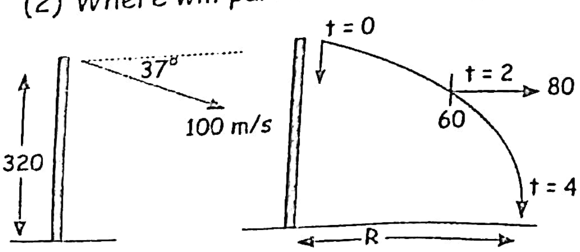


Sol. $t = \sqrt{\frac{2h}{a}} = \sqrt{2 \times \frac{125}{10}} = 5$

$R = 50 \times 5 = 250$m

Q. (1) When will particle strike the ground?

(2) Where will partice strike the ground



Sol. (1) y = ↓+ve $\quad s = ut + 1/2\, at^2$

$$320 = 60\,t + (1/2)10\,at^2$$

$$5t^2 + 60t - 320 = 0$$

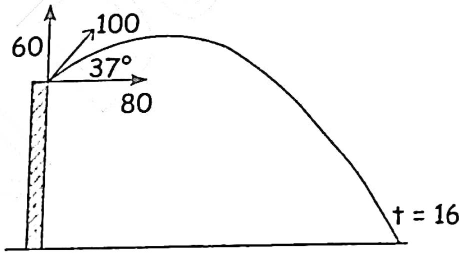
$$\Rightarrow t = 4 \text{ sec}$$

(2) Range: → $80 \times 4 = 320$

Q. A particle is thrown with velocity 100 m/s at an angle of 37° with horizontal from a tower of height 320 m.

(A) Find when particle will hit the ground and where. (Take upward direction is positive)

Sol. (a) $u_y = +60, a_y = -10, s_y = -320$



$$-320 = 60t - 1/2 \times 10 \times t^2$$

$$\boxed{t = 16 \text{ sec}}$$

$$R = 80 \times 16$$

(B) Find speed of particle at t = 16 sec

$$\vec{v} = 80\hat{i} - 100\hat{j}$$

$$|\vec{v}| = \sqrt{80^2 + 100^2}$$

(C) Find angle made by horizontal at t = 16 sec

$t = 16, \; v = 80\hat{i} - 100\hat{j}$

$$\tan\alpha = 100/80$$

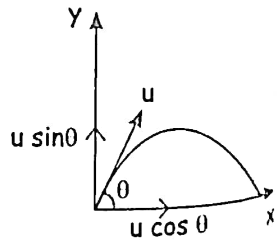
$$\alpha = \tan^{-1} 5/4$$

Equation of Trajectory

$$x = u\cos\theta \cdot t \;\Rightarrow\; t = \frac{x}{u\cos\theta}$$

$$y = u\sin\theta t - 1/2\, gt^2$$

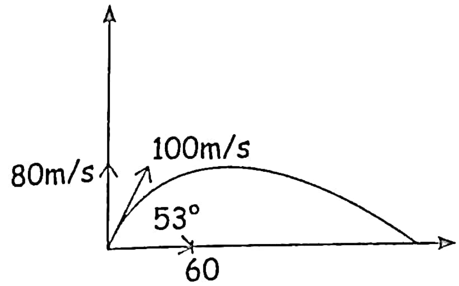
$$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$$



$$y = x\tan\theta - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$$

$$y = x\tan\theta(1 - x/R)$$

Q. Find equation of trajectory.



$x = 60t \Rightarrow t = x/60$

$y = 80t - 1/2 \times 10 \times t^2$

$$y = 80\frac{x}{60} - \frac{1}{2} \times 10 \times \left(\frac{x}{60}\right)^2$$

$$\boxed{y = \frac{4x}{3} - \frac{x^2}{720}}$$

Q. If equation of trajectory is given by $y = x\sqrt{3} - \frac{x^2}{20}$. Find u, θ.

Sol. $y = x\tan\theta - 1/2\, g \frac{x^2}{u^2\cos^2\theta}$ (Formula)

$\Rightarrow \sqrt{3} = \tan\theta$

$\theta = 60°$

$$\frac{1}{20} = \frac{g}{2u^2\cos^2\theta}$$

$$\frac{1}{20} = \frac{10}{2u^2\cos^2 60}$$

Solve and get u = 20

To find the range directly

$$y = x\sqrt{3} - \frac{x^2}{20} = x\sqrt{3}\left(1 - \frac{x}{20\sqrt{3}}\right)$$

Compare this with $y = x\tan\theta(1 - x/R)$

$\tan\theta = \sqrt{3},\ R = 20\sqrt{3}$

Q. If eqn of trajectory of a projectile from ground to ground is given as $y = 4x - \frac{x^2}{4}$.

Find Range = ?

Sol. $y = 4x(1-x/16)$

$y = x\tan\theta\,(1-x/R)$

$R = 16$

Q. $y = ax - bx^2$ find θ and Range

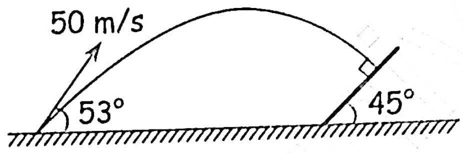
Sol. $y = ax\,(1 - bx^2/ax)$ $\quad R = a/b$

$y = ax\,(1 - bx/a)$ $\quad \tan\theta = a$

$y = 2\tan\theta\,(1 - x/R)$ $\quad \theta = \tan^{-1} a$

Q. In following figure particle strike the inclined plane perpendicularly. Find

Time of collision and coordinate of point where particle collide if projection point is origin.

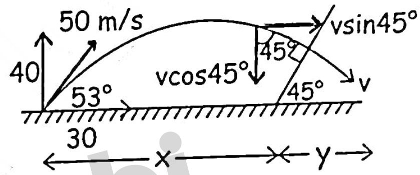


Sol. At the time of collision,

$v\sin45° = 50\cos53° \Rightarrow v = 30\sqrt{2}$

$v_y = 50\sin53° - 10T \Rightarrow -30\sqrt{2}\cos45° = 40 - 10T$

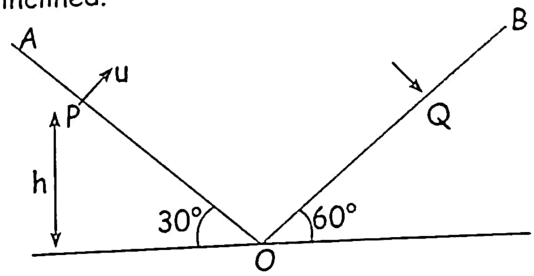
$\Rightarrow T = 7$ sec



$x = u\cos\theta \times T = 30 \times 7 = 2010$

$$y = 40 \times 7 - \frac{1}{2} \times 10 \times 7^2 = 35$$

Q. A particle thrown perpendicularly to an inclined plane and it's strike perpendicularly to another inclined plane with speed 100 m/s as shown in fig. Find with what velocity it will strike the inclined.



Sol. 100 sin 30° = v sin 60° (Horizontal velocity)

SKC

Projectile motion का सबसे बड़ा हथियार horizontal में velocity constant रहता।

$$100 \times 1/2 = v\frac{\sqrt{3}}{2} \Rightarrow v = \frac{100}{\sqrt{3}}$$

Q. A particle is moving such that
$x = 4t^2$
$y = 2t^3$
$z = 4t^4$
Find velocity at t = 2 sec →

Sol. $u_x = \frac{dx}{dt} = 8t$

$u_y = \frac{dy}{dx} = 6t^2$

$u_z = \frac{dz}{dt} 16t^3$

Velocity $= 8t\hat{i} + 6t^2\hat{j} + 16t^3\hat{k}$
acc $= 8\hat{i} + 12t\hat{j} + 48t^2\hat{k}$

$\vec{V} = 8t\hat{i} + 6t^2\hat{j} + 16t^3\hat{k}$

Q. A particle is moving such that $\vec{r} = 3t^2 + 2t^3\hat{j} + 10t\hat{k}$
Find angle between $\vec{v}$ & $\vec{a}$ at t = 1 sec.

Sol. $\vec{v} = 6t\hat{i} + 6t^2\hat{j} + 10\hat{k}$

$\vec{a} = 6\hat{i} + 12t\hat{j} + 0\hat{k}$

At t = 1, $\vec{v} = 6\hat{i} + 6\hat{j} + 10\hat{k}$

$\vec{a} = 6\hat{i} + 12\hat{j}$

$\vec{a} . \vec{v} = av \cos\theta$

$$\cos\theta = \frac{\vec{a}.\vec{v}}{av} = \frac{36 + 72 + 0}{\sqrt{6^2 + 12^2}\sqrt{6^2 + 6^2 + 10^2}}$$

RELATIVE MOTION

Velocity of A wrt ground = 10 m/s i
Velocity of B wrt ground = 6 m/s i

$\boxed{\text{Velocity of A wrt B} = \vec{v}_{A/B} = \vec{v}_A - \vec{v}_B}$

Velocity of A wrt B = $\vec{V}_{A/B} \equiv$ B की खोपडी पर बैठकर A को देख

$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

Similarly $\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B$

☆ (A) → 6m/s   (B) → 8m/s

$v_{B/A} = 8\hat{i} - 6\hat{i} = 2\hat{i}$

$v_{A/B} = 6\hat{i} - 8\hat{i} = -2\hat{i}$

☆ ← 10m/s (B)   → 6m/s (A)

$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = (-10\hat{i} - (6\hat{i})) = -16\hat{i}$

☆ (A) → 10m/s   (B) → 10m/s

$\vec{V}_{B/A} = 10 - 10 = 0$

☆ (A) → 10m/s   (B) ↑ 20m/s

$\vec{V}_{B/A} = 20\hat{j} - 10\hat{i}$

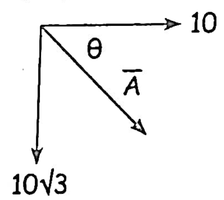
☆ $\boxed{\begin{array}{l} \vec{V}_{A/B} = \vec{V}_A - \vec{V}_B \\ \vec{V}_{B/A} = \vec{V}_B - \vec{V}_A \\ \vec{V}_{A/B} = -\vec{V}_{B/A} \end{array}}$

अब आगे हमे relative motion, rain man, river boat problems पढ़ने है उसके लिए बहुत जरूरी है की हम vector को draw करना सीखें इसके लिए i am attaching few fundamental things grab it.

☆ $\vec{A} = 10\hat{i} - 10\sqrt{3}\hat{j}$

$\tan\theta = \frac{10\sqrt{3}}{10}$

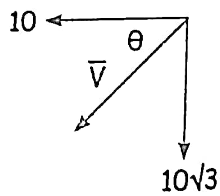
$\theta = 60°$



☆ $\vec{v} = -10\hat{i} - 10\sqrt{3}\hat{j}$

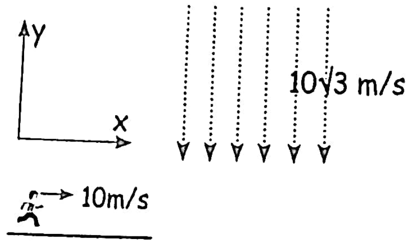
$\tan\theta = \frac{10\sqrt{3}}{10}$

$\theta = 60°$



Q. A man is moving along +x-axis (east) with speed 10 m/s & rain is falling vertically downward with speed 10√3 m/s. In which direction man should hold umbrella to protect himself.

Sol.

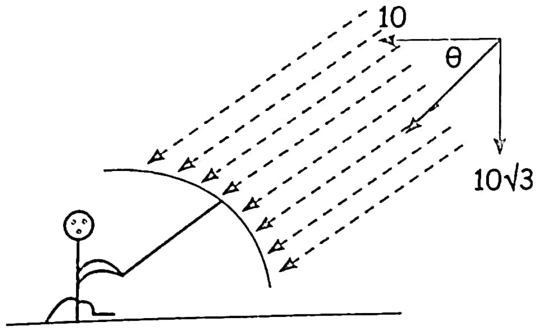


$\vec{v}_{man} = 10\hat{i}$

$\vec{v}_{rain} = -10\sqrt{3}\hat{j}$

$\vec{v}_{rain/man} = \overline{v_r} - \overline{v_m}$

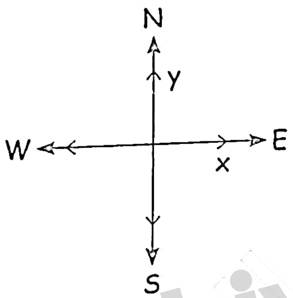
$\vec{v}_{r/m} = -10\sqrt{3}\hat{j} - 10\hat{i} = -10\hat{i} - 10\sqrt{3}\hat{j}$



Q. A man is moving in east direction with speed 10 m/s in a car A bird is flying with speed 10√3 in south direction

(1) Find velocity of bird observed by man

Sol. $\vec{v}_{man} = 10\hat{i}$



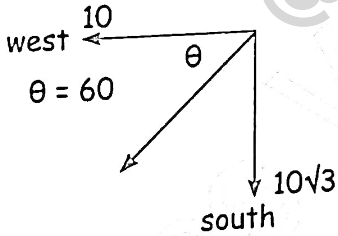
$\vec{v}_{bird} = -10\sqrt{3}\,\hat{j}$

$\bar{v}_{b/m} = \vec{v}_b - \vec{v}_m$

$= -10\sqrt{3}\hat{j} - 10\hat{i}$

$|\bar{v}_{b/m}| = \sqrt{10^2 + (10\sqrt{3})^2}$

= 20 (60° south of west)

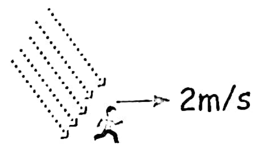


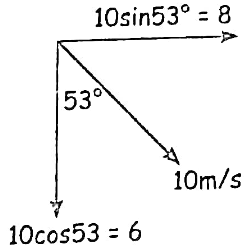
SKC

Rainman prob. जैसे ques के लिए ये 4 step follow करो

1. सबसे पहले velocity of man and rain निकालो
2. अब vectorly velocity of rain wrt man $\vec{V}_{r/m}$ निकालो
3. अब $\vec{V}_{r/m}$ को draw करके नया fig. बनाओ
4. अब हमें पता चल गया कि man को rain कहाँ आती दिख रही है so अब छाता लगा दो

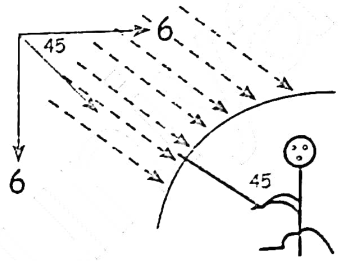
Q. Rain is falling with speed 10 m/s at an angle 53° with vertical. A man is moving with speed 2 m/s along east as shown in diagram.





(1) In which direction man should hold umbrella to protect himself

Sol. $\vec{V}_r = 8\hat{i} - 6\hat{j}$



$\vec{V}_m = 2\hat{i}$

$\vec{v}_{r/m} = (8\hat{i} - 6\hat{j}) - (2\hat{i})$

$= 6\hat{i} - 6\hat{j}$

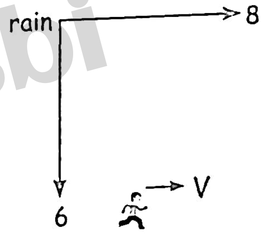
$\tan\theta = \frac{6}{6}$

$\tan\theta = 1$

$\tan 45 = 1$

$\theta = 45°$

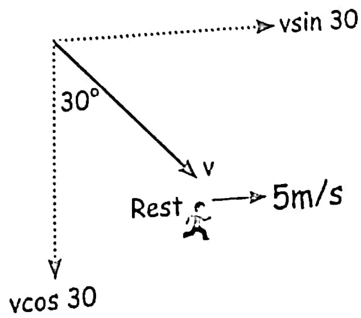
(2) What should be velocity of man so that rain appear falling vertically downward. to him?



V= 8 Ans.

Q. To a stationary man, rain appear to be falling at an angle 30° with the vertical As he start moving with speed of 5 m/s he feels that rain is falling vertically. Find speed of rain.

Sol.



v sin 30 = 5

v = 10 speed of rain

Q. A man holding a flag is running with speed 10 m/s along east. If wind speed is 10√3 m/s along south then. In which direction flag will flutter?

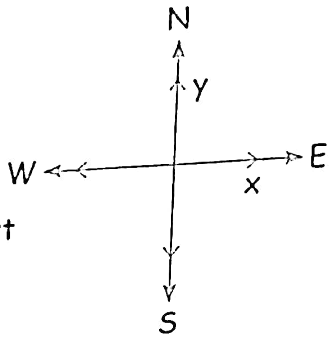
Sol. $\vec{V}_{man} = 10\hat{i}$

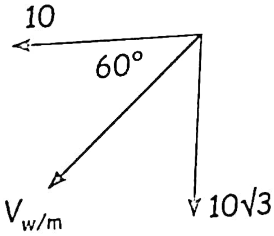
$\vec{V}_{wind} = -10\sqrt{3}\hat{j}$

$\vec{V}_{w/m} = -10\sqrt{3}\hat{j} - 10\hat{i}$

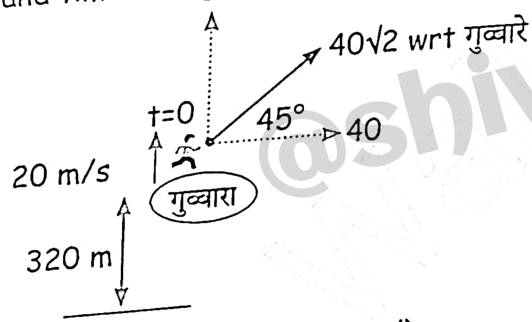
Ans = 60° south of west

30° west of south





Q. A balloon is rising upward with constant velocity 20 m/s. When it is at a height 320 m from ground a particle is projected from balloon with velocity $40\sqrt{2}$ m/s at an angle of 45° with horizontal at t = 0.
Find velocity of particle just after projection and time of flight of particle.



t = 0+ $\quad \vec{V}_p = ?$ (wrt ground)

Sol. $\vec{V}_{P/ballon} = 40\hat{i} + 40\hat{j}$

$\vec{V}_p - \vec{V}_{ballon} = 40\hat{i} + 40\hat{j}$

$\vec{V}_p - 20\hat{j} = 40\hat{i} + 40\hat{j}$

$\vec{V}_p = 40\hat{i} + 60\hat{j}$

$S = ut + \frac{1}{2}at^2$ (upward +ve)

$-320 = 60t - \frac{1}{2} \times 10 \times t^2$

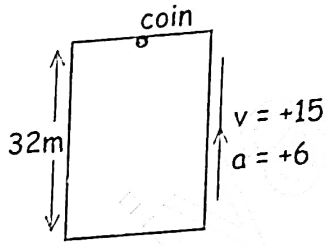
Sol.e and get t = 16 sec

अबे ये तो अब 320 m के tower से पत्थर फेकने वाला ques. बन गया।

SKC

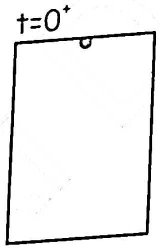
Question में अगर किसी भी lift का, गुब्बारे का acc given होता है, तो वो g को consider करने के बाद given होता है

Q. A lift of height 32 m is going up with cons... acc $a = 6$ m/s^2. When velocity of lift is 15... upward, a coin is drop from cealing of lif... t = 0. Find time when coin will hit the floor.



Sol. चुपचाप lift ke अंदर जाके बैठ जाओ.

wrt lift



(In lift frame or wrt lift)

At t = 0 $\vec{V}_{coin/lift} = 0$

$\vec{a}_{coin/lift} = -10 - (+6) = -16$

$\vec{s}_{coin/lift} = -32$

$S = ut + 1/2 \times 16 \times t^2$

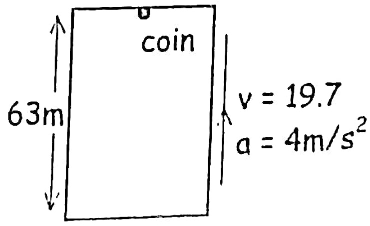
$-32 = 0 - 1/2 \times 16 \times t^2$

t = 2 sec

SKC

ऐसे सवालो मे सबसे पहले चुप चाप lift के अंदर जाके बैठ जाओ... और coin की velocity, coin का acc lift के respect मे... लिखकर eqn of motion लगाओ. बस ये याद रखना अगर coin... हवा मे है तो ground के respect मे उसका acc नीचे g होगा...

Q. At t = 0 coin drop. Find time when coin will hi... the floor



Sol. $\vec{u}_{coin/lift} = 0$

$\vec{a}_{coin/lift} = -10 - (+4) = -14$

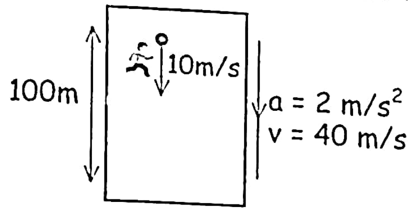
$\vec{s}_{coin/lift} = -63m$

$s = ut + 1/2\ at^2$

$-63 = 0 + 1/2 \times (-14)\ t^2$

$t = 3$ sec

Q. Ball thrown downward at 10m/s w.r.t lift.



When will coin strike the floor?

Sol. Lift ke अंदर आकर wrt lift.

$v_{coin/lift} = -10$

$a_{coin/lift} = -10 - (-2) = -8$

$S_{coin/lift} = -100$

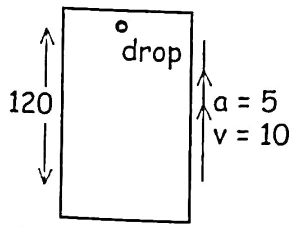
$-100 = -10t + 1/2\ (-8) \times t^2$

$100 = 10t + 4t^2$

$2t^2 + 5t - 50 = 0$

$t = 4$

Q. When will coin strike the floor of left.



Sol. $\vec{V}_{coin/lift} = 0$

$\vec{a}_{coin/lift} = -10 - (+5) = -15$

$\vec{S}_{coin/lift} = -120$

$-120 = 0 + 1/2\ (-15)t^2$

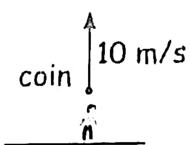
$t = 4$ sec

lift $\quad t = 4, \quad S = ?, \quad u = 10, \quad a = 5$

$s = 10 \times 4 + 1/2 \times 5 \times 4^2 = 80$

Find displacement of lift w.r.t ground before coin strike the floor?

Displacement of coin in t = 4 sec in ground frame



$S = ut + 1/2\ at^2$

$s = 10 \times 4 + 1/2(-10) \times 4^2$

$= 40 - 80 = -40$ Ans

Q. Car A and car B start moving simultaneously in the same direction along the line joining them. Car A moves with a constant acceleration $a = 4\ m/s^2$, while car B moves with a constant velocity $v = 1$ m/s. At time $t = 0$, car A is 10 m behind car B. Find the time when car A overtakes car B. How much time A take to overtake.

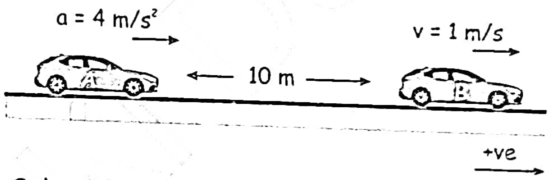
Sol. Given: $u_A = 0$, $u_B = 1$ m/s, $a_A = 4\ m/s^2$ and $a_B = 0$

Assuming car B to be at rest, we have

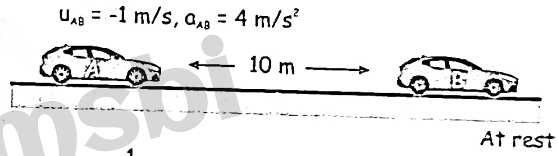
$u_{AB} = u_A - u_B = 0 - 1 = -1$ m/s

$a_{AB} = a_A - a_B = 4 - 0 = 4\ m/s^2$

Now, the problem can be assumed in simplified form as follow:



Substituting the proper values in equation



$S = ut + \frac{1}{2}at^2$

we get $10 = -t + \frac{1}{2}(4)(t^2)$ or $2t^2 - t - 10 = 0$

Ignoring the negative value, the desired time is 2.5s.

Note: The above problem can also be solved without using the concept of relative motion as under. At the time when A overtakes B,

$S_A = S_B + 10$

$\therefore \frac{1}{2} \times 4 \times t^2 = 1 \times t + 10$

or $2t^2 - t - 10 = 0$

Which on solving gives $t = 2.5$ s and -2 s, the same as we found above.

SKC

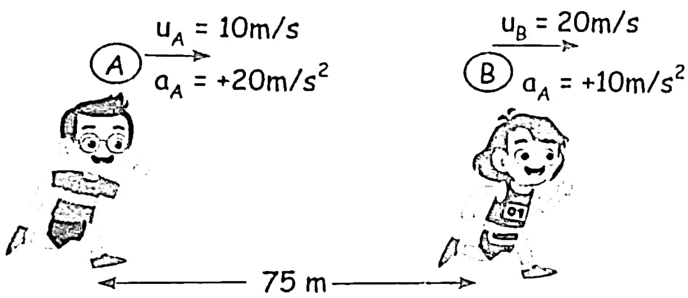
Catching वाले सवालो मे

✦ सबसे पहले आगे वाले पर बैठ जाओ

✦ बाद आगे वाले के respect मे Initial v, a, s लिखो

✦ equ^n of motion लगा दो

Q. At t = 0 gap between saleemian boy A and girl B is 75 m. Find when boy will catch the girl.



Sol. M-1 (relative वाला method, आगे वाले के ऊपर जाके वैठ जाओ)

$\vec{u}_{A/B} = 10 - 20 = -10$

$\vec{a}_{A/B} = 20 - 10 = 10$

$s = ut + 1/2\ at^2$

$75 = -10t + (1/2)10\ t^2$

$5t^2 - 10t - 75 = 0$

Sol.e and get t = 5

M-2

$75 + x_b = x_a$

$75 + 20t + 1/2 \times 10 \times t^2 = 10t + 1/2 \times 20 \times t^2$

$75 + 20t + 5t^2 = 10t + 10t^2$

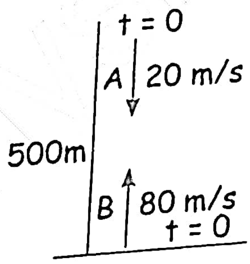
$5t^2 - 10t - 75 = 0$

$t^2 - 2t - 15 = 0$

$(t - 5)(t + 3) = 0$

$t = 5$

Q. A & B are thrown as shown simantaneously at t = 0 when will they meet?



Sol. $v_{A/B} = -20 - 80 = -100$

$\vec{a}_{A/B} = -10 - (-10) = 0$

$s_{A/B} = -500$

$-500 = -100t + 1/2 \times 0 \times t^2$

t = 5 sec

Direct^n

$\vec{a}_{rel} = 0$

$\vec{v}_{rel} = 100$

$500 = 100t$

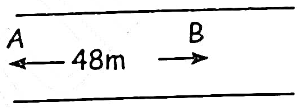
t = 5 sec

RIVER-BOAT-MAN PROBLEM

भाई ये JEE-Mains और NEET का favourite article है वेशरमो की तरह वार-वार आता रहता है इसे अच्छे से कर लेना

Q. River is flowing with speed 6 m/s, and $\vec{v}_{man/river}$ is 10 m/s.

Man start swimming from A to B and return back to A. Find time taken



Sol. $(A \to B)\ t_{A\to B} = \frac{48}{10+6} = 3\,sec$

$(B \to A)\ t_{B\to A} = \frac{48}{10-6} = 12\,sec$

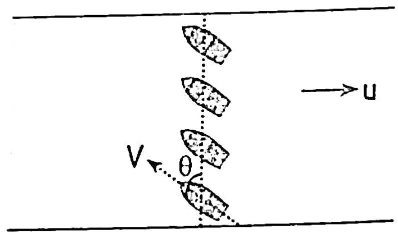
total time = 3 + 12 = 15 sec

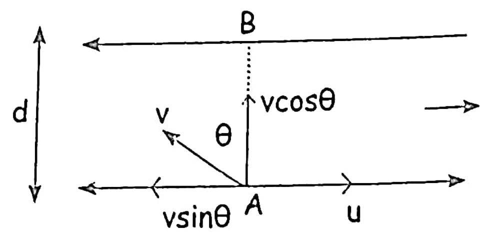
Average speed $= \frac{48+48}{15}$

Avrg velocity = 0

Case-I

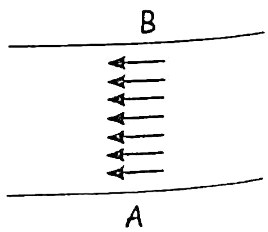
Crossing of river in min distance.





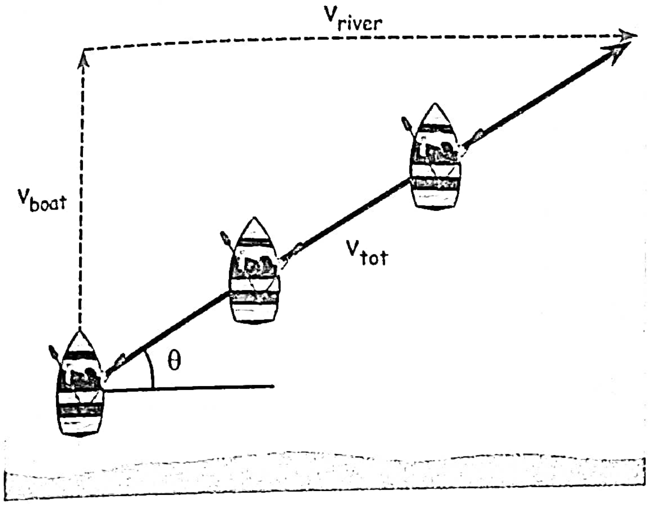
Drift = 0

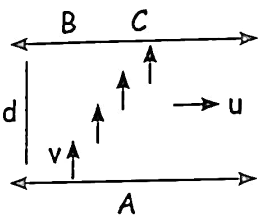
$$\boxed{\begin{gathered} V\sin\theta = u \\ t = \frac{d}{u\cos\theta} \end{gathered}}$$



Case-II

Crossing of river in min time

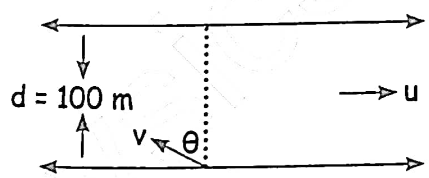




$$\boxed{\begin{array}{l}\text{Min time} = d/v \\ \text{Drift} = CB = ut\end{array}}$$

Q. A river is flowing with velocity 5 m/s wrt ground. A man can swim with velocity 10 m/s wrt river. The width of the river is 100 m.

(1) In which direction man should swim so that he crosses that river in min distance travel also find time taken to cross the river.



Sol. $\vec{v}_{m/r} = v = 10$ m/s

$u = 5 \quad d = 100$

$v \sin\theta = u$

$10 \sin\theta = 5$

$\sin\theta = 1/2$

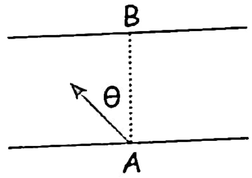
$\sin\theta = 30°$

$$\text{time} = \frac{d}{v\cos\theta} = \frac{100}{10\cos 30°}$$

(2) If he want to cross the river in min possible time find min time to cross the river & drift

Sol. $t = \frac{100}{10} = 10\,sec$

$BC = \text{drift} = 5 \times 10 = 50$ m



$$\sin\theta = \frac{\text{छोटी v}}{\text{बड़ी v}}$$

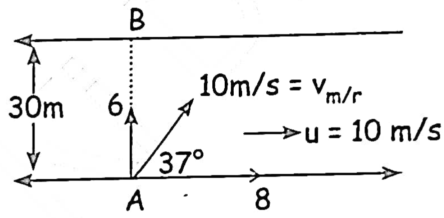
Q. $u = 10\sqrt{3}$, $\vec{V}_{m/r} = 20$

$\theta = ?$ for min distance

Sol. $\sin\theta = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$

$\theta = 60$

Q. In following figure if man is swimming with velocity 10 m/s wrt river at an angle of 37° to the river velocity. Find time taken to cross the river and drift if velocity of river is 10 m/s and width of the river is 30 m/s

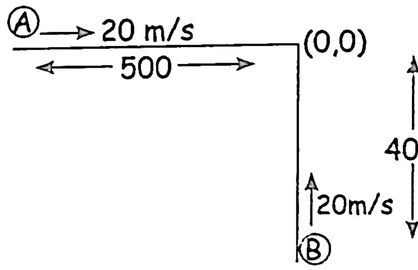


Sol. $t = \frac{30}{6} = 5\,sec$

Find drift

$BC = 18 \times 5 = 90$ [$V_{net} = 10 + 8 = 18$]

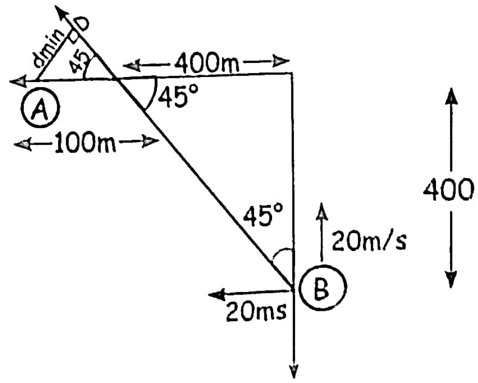
Q. Min distance between the moving particle.



Sol. $\vec{V}_A = -20\hat{i}$

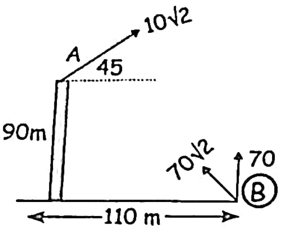
$\vec{V}_B = 20\hat{j}$

$\vec{V}_{B/A} = -20\hat{i} + 20\hat{j}$



$AD = d_{min} = 100 \sin 45$

Q. Min Distance between them

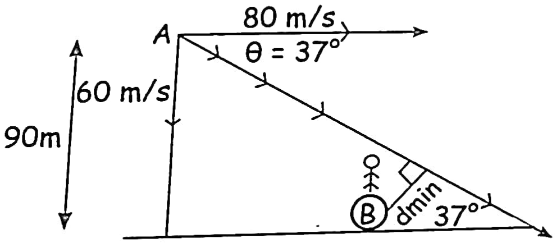


Sol. $\vec{V}_A = 10\hat{i} + 10\hat{j}$

$\vec{V}_B = -70\hat{i} + 70\hat{j}$

$\vec{V}_{A/B} = 80\hat{i} - 60\hat{j}$

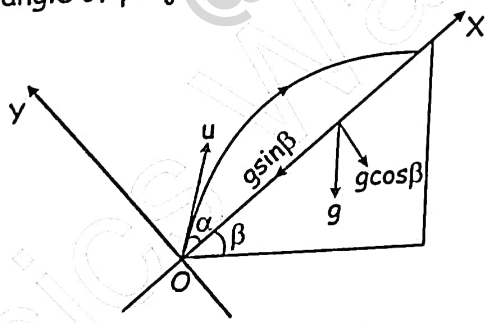
$\vec{a}_{A/B} = 0$



PROJECTION ON AN INCLINED PLANE

***Case-I:* Particle is projected up the incline**

Here α is angle of projection w.r.t. the inclined plane.



$a_x = -g\sin\beta$, $\quad u_x = u\cos\alpha$

$a_y = -g\cos\beta$, $\quad u_y = u\sin\alpha$

Time of flight (T): When the particle strikes the inclined plane, y becomes zero

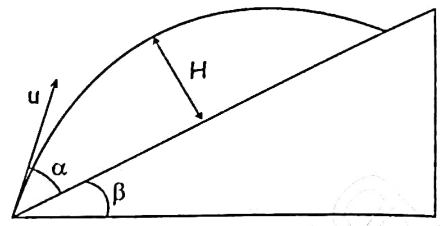
$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_y}{g_y}$$

where $u_\perp$ and $g_\perp$ are component of u and g perpendicular to the incline.

Maximum Distance from Inclined Plane (H):



When half of the time is elapsed y-coordinate is equal to maximum distance from the inclined plane of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_y^2}{2g_y}$$

Range Along the Inclined Plane (R):

When the particle strikes the inclined plane, x coordinate is equal to range of the particle

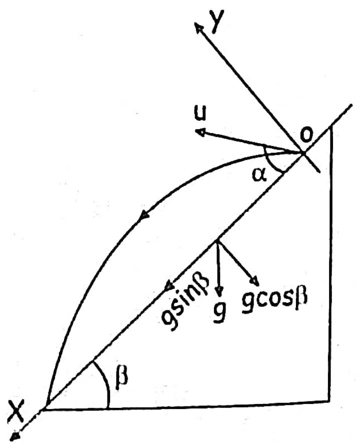
$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \boxed{R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}}$$ (इसको रटना मत बस ये देखो eqn मे value कैसे put कर रहे है)

In this case after solving we got range will be maximum when $\alpha = \frac{\pi}{4} - \frac{\beta}{2}$ and Max. Range $= \frac{u^2}{g[1+\sin\beta]}$

Case-II: Particle is projected down the incline



$a_x = g\sin\beta$ $\quad u_x = u\cos\alpha$

$a_y = -g\cos\beta$ $\quad u_y = u\sin\alpha$

Time of Flight (T):

When the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow 0 = u \sin\alpha T - \frac{1}{2} g \cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_y}{g_y}$$

Maximum Distance (H):

When half of the time is elapsed y coordinate is equal to maximum distance of the projectile from the plane.

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2} g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_y^2}{2g_y}$$

Range Along the Inclined Plane (R):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

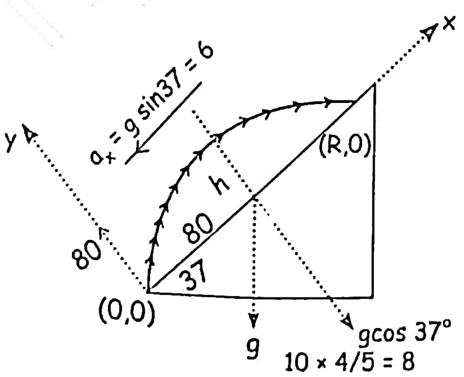
$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) + \frac{1}{2} g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\boxed{R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}}$$

In this case after solving we got range will be maximum when $\alpha = \frac{\pi}{4} + \frac{\beta}{2}$ and Max. Range $= \frac{u^2}{g[1-\sin\beta]}$

Q. A particle is projected with velocity $80\sqrt{2}$ at an angle 45° with inclined plane. If inclined plane makes angle 37° with horizontal. Time of flight and range of particle.



Time of flight = T = 10 + 10 = 20 sec

Sol. $T = \frac{2u_y}{a_y} = \frac{2 \times 80}{8} = 20\,\text{sec}$

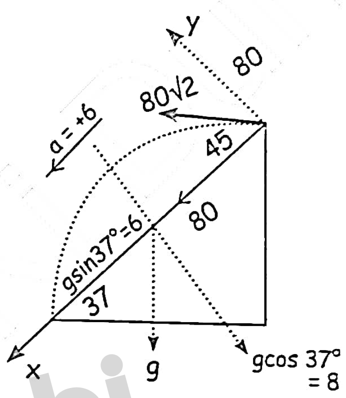
$R = ut + \frac{1}{2}at^2$ (x mai)

$R = 80 \times 20 - \frac{1}{\cancel{2}}\cancel{6}^3 \times (20)^2$

$= 1600 - (3 \times 400)$

$1600 - 1200 = 400$ m

Q. Repeat the above ques. for following fig.



Sol. $T = \frac{2 \times 80}{8} = 20$

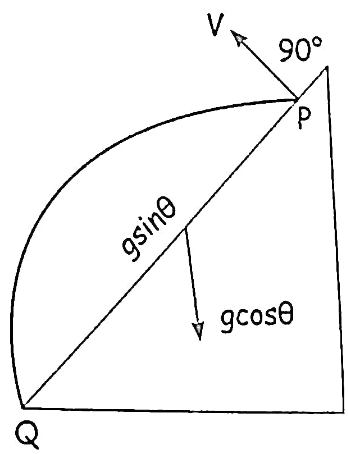
Range $= 80 \times 20 + 1/2 \times 6 \times 20^2$

$= 1600 + 3 \times 400$

$= 1600 + 1200$

$= 2800$

Q. Time of flight is T find PQ

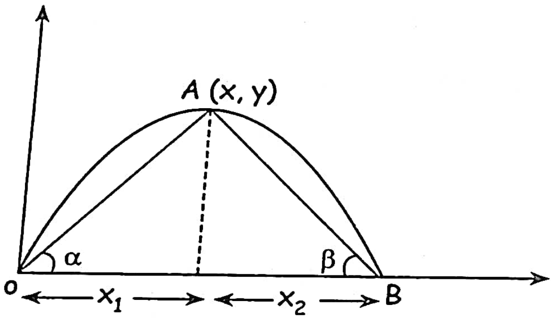


Sol. $PQ = 1/2\, g\sin\theta\, T^2$

$= \frac{1}{\cancel{2}} \cancel{g} \sin\theta T \times \frac{\cancel{2}v}{\cancel{g}\cos\theta}$ $\quad \left[T = \frac{2V}{g\cos\theta}\right]$

$TV\tan\theta$

Q. A Projectile Grazing Over a Triangle: A particle is projected over a triangular structure from one of its corners of its horizontal base. Grazing over its top vertex, it falls on other corner of base. If α and β are base angles of triangular structure and θ is angle of projection show that $\tan\theta = \tan\alpha + \tan\beta$.



Sol. $$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

$$y = x_1\tan\theta\left(1 - \frac{x_1}{x_1 + x_2}\right)$$

$$y = \frac{x_1 x_2 \tan\theta}{x_1 + x_2}$$

$$\Rightarrow \tan\theta = y\left(\frac{x_1 + x_2}{x_1 x_2}\right) = \frac{y}{x_2} + \frac{y}{x_1}$$

$$= \tan\alpha + \tan\beta$$