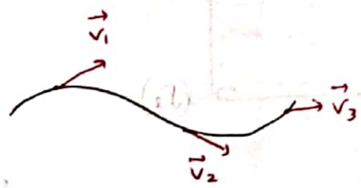
26/12 Fluid Dynamics (Flow of fluids)

(i) Line of flow:- The path taken by the particle in flowing fluid is called line of flow.

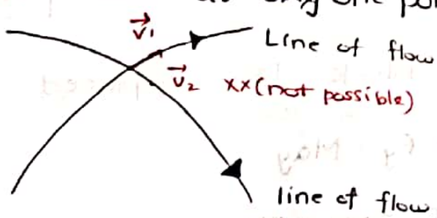
(ii) ~~String~~ Stream line flow/steady flow/Laminar flow:- If velocity of fluid particle at any point remains constant with time, then flow is said to be steady. The velocity at different point may be different. In the case of static flow all the particle's passing through a given points follow the same path, hence there is unique line of flow passing through a given point which is also called ~~Strong~~ Streamline flow.

Characteristics of ~~String~~ Stream line flow:-

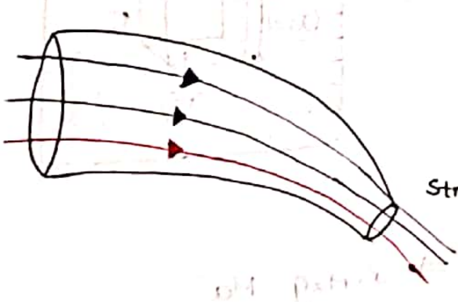
1. A tangent at any point of string line gives direction of velocity of fluid at that point



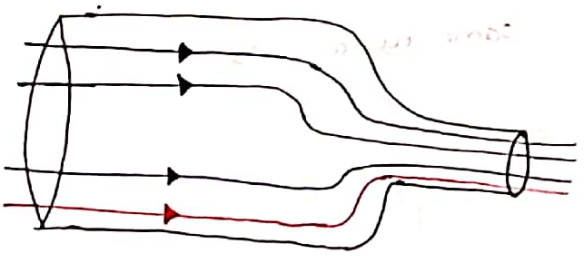
2 Any two stream line never intersect each other because at only one point it will show two direction of velocity.



Stream line flow:-



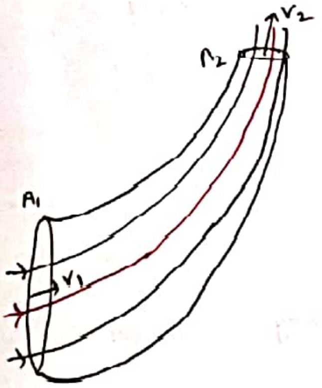
Stream line flow.

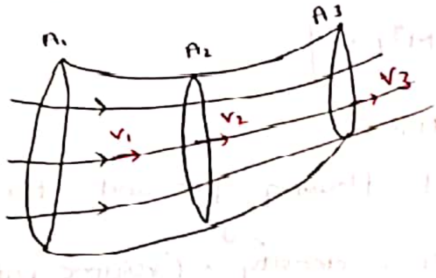


Equation of continuity

Equation of Continuity represents the law of conservation of mass in fluid dynamics

Principle of continuity:- It states that when an ideal liquid [imcompressible, non viscous] flows in a stream line motion through a tube of non uniform cross section then product of area of crosssection and the velocity of flow is same at everypoint, in the fluid





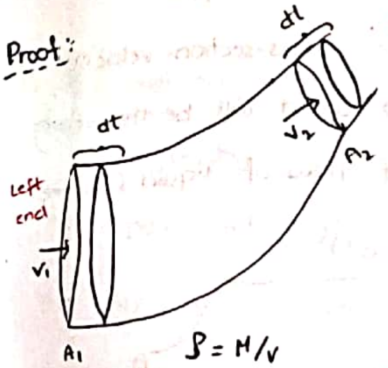
$AV = \text{constant}$

$A_1V_1 = A_2V_2 = A_3V_3$

$\boxed{A_1V_1 = A_2V_2}$

eqn of continuity.

Proof:-



$\rho = M/V$

at left end of tube:-

Distance covered by fluid through area A_1 in 'dt' time interval $= V_1 dt$

Volume of liquid in 'dt' time interval = Area x distance

$= V_1 dt \times A_1$

$= V_1 dt A_1$

=) So, mass entered in tube is small time (dt) = density x volume

$$\boxed{dM_1 = \rho \times A_1V_1dt}$$

Similarly at right end:-

Distance covered by the fluid through area A_2 in 'dt' time interval $= V_2 \cdot dt$

Volume of liquid in 'dt' time interval = Area x distance

$= A_2 \times V_2 \cdot dt$

Mass come out from tube in small time interval = density of liquid x Vol

$$\boxed{dM_2 = \rho \times A_2V_2\, dt}$$

according to law of conservation of mass,

Mass entered in 'dt' time = Mass come out in 'dt' time

$dM_1 = dM_2$

$$\boxed{\rho A_1V_1dt = \rho A_2V_2dt}$$

$$\boxed{A_1V_1 = A_2V_2}$$

$\hookrightarrow A \times V = \text{constant}$

The quantity (area × velocity) represents volume of liquid flowing per unit time (112)
that is known as volume-rate flow. and also called rate of flow of liquid

$$\boxed{\underset{\text{velocity}}{AV} = A \times \frac{dx}{dt} = \frac{d(vol)}{dt}}$$ → volume of liquid flowing per unit time
↳ Volume rate flow
↳ Rate of flow

$$\boxed{\text{SI unit} = M^3/sec}$$

→ Mass rate flow (M·R·F)

Mass of liquid flowing per unit time.

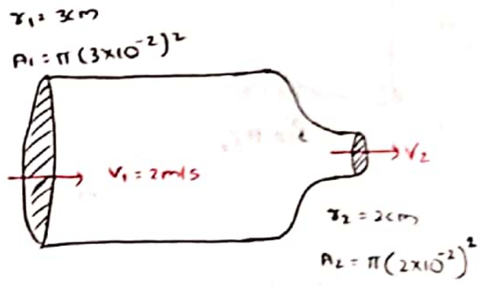
$$MRF = \overset{\rho}{\text{density}} \times (\underbrace{\text{volume rate flow}}_{\text{Area} \times \text{velocity}})$$

$$\boxed{MRF = \rho \times A \times \text{Velocity}}$$ → S.I unit = Kg/sec

If the liquid is compressible ($\rho \neq$ constant)

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$ → eqn of continuity for compressible liquid.

Q Water is flowing through horizontal tube of non uniform cross-section. velocity of water is 2m/sec at a point where radius of tube = 3cm. what will be the velocity of water at a point where radius of tube = 2cm. find rate of flow of liquid (volume rate flow)



$$A_1 V_1 = A_2 V_2$$

$$(\pi \times 9\times10^{-4}) \times 2 = (\pi \times 4 \times 10^{-4}) \times 3$$

$$\frac{18}{4} = V_2$$

$$\boxed{V_2 = 4.5m/sec}$$

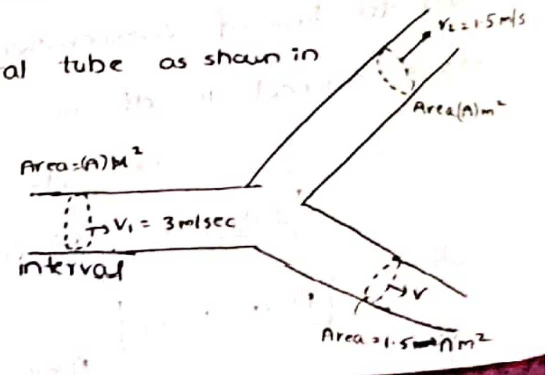
Rate of flow / Volume rate flow $= AV = \frac{d(vol)}{dt}$

$= A_1V_1$ (or) A_2V_2

$= \pi(9\times10^{-4}) \times 2$

$$\boxed{AV = 18\pi \times 10^{-4} \ M^3/sec}$$

Q An incompressible liquid flows through horizontal tube as shown in figure. Find velocity 'v' of liquid



Sol. According to law of mass conservation

Mass entered = Mass comeout in equal time interval

$$\boxed{\text{Mass} = \text{Area} \times \text{velocity}}$$

$A_1V_1 = A_1 \times V_2 + A_3 \times V_3$

$A \times 3 = A \times 1.5 + 1.5 \times V \times A$

$3A - 1.5A = 1.5VA$

$1.5A = 1.5VA$

$\boxed{V = 1m/s}$

TYPES OF ENERGY OF LIQUID :- 27/12

KE of liquid — PE of liquid — Pressure energy of liquid

① KE of liquid :-

$KE = \frac{1}{2}mv^2$

→ KE of liquid per unit mass $= \frac{KE}{mass} = \frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2$

, KE of liquid per unit volume $= \frac{KE}{Volume} = \frac{\frac{1}{2}mv^2}{Vol} = \frac{1}{2}\rho v^2$

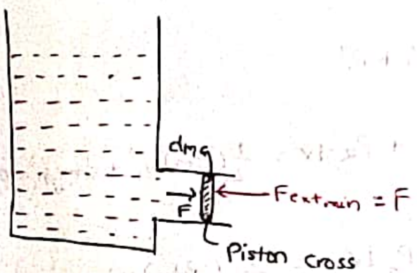
② Gravitational PE of liquid :-

$PE = mgh$

PE per unit mass $= \frac{PE}{M} = \frac{mgh}{m} = gh$

PE per unit volume $: \frac{PE}{V} = \frac{mgh}{V} = \rho gh$

③ Pressure energy of liquid :-



Work done by external agent on body = energy stored in body

Work done by external agent against Force of pressure $= F_{ext}|_{min} \times dx$ = Pressure energy of liquid

$= F \times dx = P\underbrace{Adx}_{dv}$

$P = F/A$

$\boxed{F = P \times A}$

$\boxed{\text{* Pressure energy of liquid} = Pdv}$

Pressure energy per unit mass $= \frac{Pdv}{dm} = \frac{d}{dm/dv} = P/\rho$

Pressure energy per unit volume $= \frac{Pdv}{dv} = P$

BERNOULLI'S EQUATION :-

→ It is a basic consequence (result) of energy conservation principle for a flowing liquid. It states that the sum of pressure energy, KE & PE per unit mass/volume is always constant for an ideal fluid having stream line flow

1. (Pressure energy + PE + KE) per unit mass = constant

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}.$$

2. (Pressure energy + PE + KE) per unit volume = constant

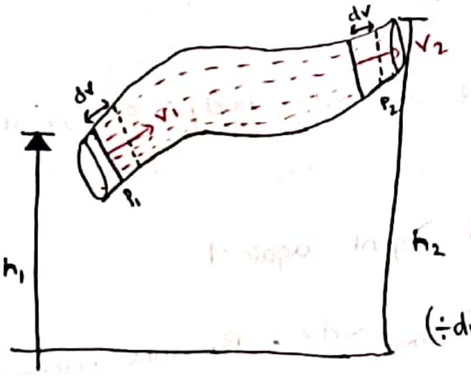
$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

$$\div(\rho g) \rightarrow \underbrace{\frac{P}{\rho g}}_{\text{Pressure head}} + \underbrace{h}_{\text{Gravitational head}} + \underbrace{\frac{v^2}{2g}}_{\text{velocity head}} = \text{constant}$$

☆ In other words, Bernoulli theorem states that the sum of pressure head, gravitational head and velocity head is constant for an ideal fluid.

Proof of Bernoulli theorem :-



$\rho = \frac{dm}{dv}$

$(dm = \rho \cdot dv)$

$$K_2 + P_2 + P_2' = K_1 + P_1 + P_1'$$

$$\Rightarrow \frac{1}{2}(dm)v_2^2 + (dm)gh_2 + P_2 dv = \frac{1}{2}(dm)v_1^2 + dmgh_1 + P_1 dv$$

$$\Rightarrow \frac{1}{2}(\rho dv)v_2^2 + (\rho dv)gh_2 + P_2 dv = \frac{1}{2}(\rho dv)v_1^2 + (\rho dv)gh_1 + P_1 dv$$

$(\div dv)$

$$\frac{1}{2}\rho v_2^2 + \rho gh_2 + P_2 = \frac{1}{2}\rho v_1^2 + \rho gh_1 + P_1$$

$$\boxed{P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2 = P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2}$$

→ Bernoulli Theorem

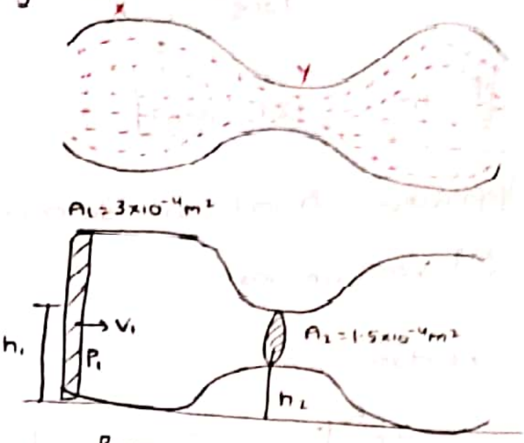
→ Total energy per unit volume = constant

For solving problems based on dynamic fluids

Use → (i) equation of continuity → $A_1V_1 = A_2V_2$

(ii) $P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 = P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2$

Q Water flows in horizontal tube as shown in figure, the pressure of water changes by 600 N/m² between x & y where the area of cross section are 3cm² & 1.5cm² respectively. Find the rate of flow of water through the tube?



$P_1 > P_2$

$h_1 = h_2$

Sol: ① $A_1V_1 = A_2V_2$

$3\times10^{-4} \times V_1 = 1.5\times10^{-4} V_2$

$(2V_1 = V_2)$

② $P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 = P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2$

$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = \underbrace{P_1 - P_2}_{600}$

$\rho \frac{1}{2}(v_2^2 - v_1^2) = 600$

$\rightarrow \frac{1}{2}\times1000((2v_1)^2 - v_1^2) = 600 \rightarrow 4v_1^2 - v_1^2 = \frac{1200}{1000} \Rightarrow 3v_1^2 = \frac{1200}{1000} = \frac{\cancel{1200}\,400}{\cancel{3}\times1000}$

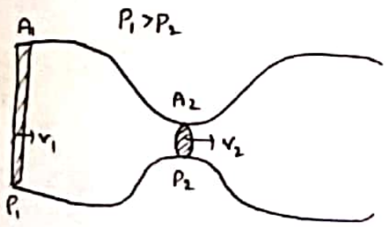
$\rightarrow \boxed{V_1 = \sqrt{0.4} = 0.63}$

$V_2 = 2V_1 = 2\sqrt{0.4}$

$= 2(0.63)$

$\boxed{V_2 = 1.26}$

NOTE:



$V_1 < V_2$

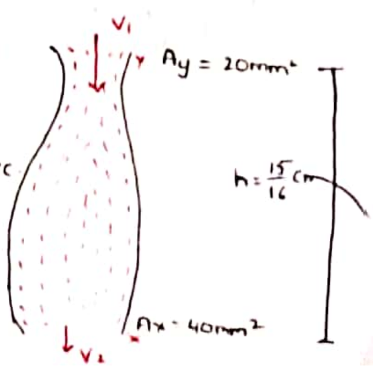
$P_1 > P_2$

$AV = \text{constant} \rightarrow A \propto \frac{1}{V}$ [A ↑ V ↓ P ↑]

$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$

Q Find the pressure difference b/w x & y.

water enter through 'y' at the rate of 10cm³/sec



$P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 = P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2$

$P_2 + \rho g \times 0 + \frac{1}{2}\rho \times v_2^2 = P_1 + \rho g \times \frac{15}{16}\times10^{-2} + \frac{1}{2}\rho \times (2v_2)^2$

$P_2 - P_1 = \frac{1}{2}\rho \times 4v_2^2 - \frac{1}{2}\rho v_2^2 + \rho g \times \frac{15}{16}\times10^{-2}$

$$P_2 - P_1 = \frac{3\rho v_2^2}{2} + \rho g \times \frac{15}{16} \times 10^{-2}$$

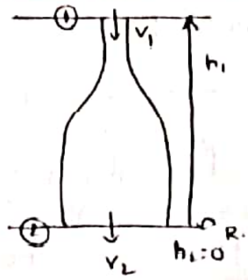
$$P_2 - P_1 = \rho\left(\frac{3}{2}v_2^2 + g \times \frac{15}{16} \times 10^{-2}\right)$$

$$= 1000\left[\frac{3}{2} \times \frac{1}{16} + \frac{150}{16} \times 10^{-2}\right]$$

$$= 1000\left[\frac{3 + 300 \times 10^{-2}}{32}\right]$$

$$P_2 - P_1 = 1000 \times \frac{6}{32}$$

$$\boxed{P_2 - P_1 = 187.5\ N/M^2}$$



at end ⊗

Rate of flow = $A_1 v_1 = 10 cm^3/sec$

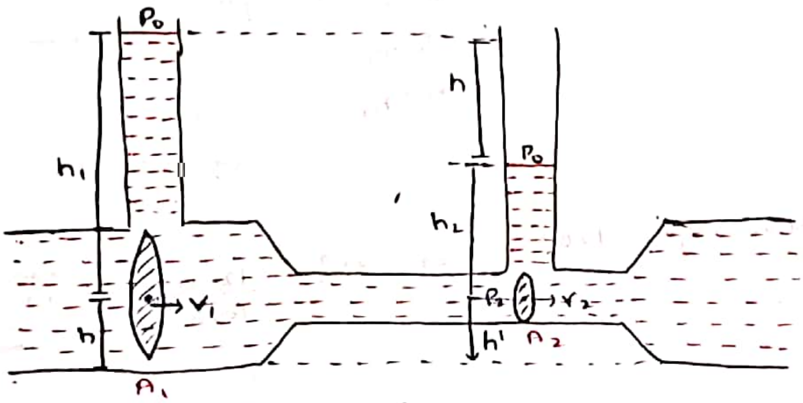
$20 \times 10^{-6} \times v_1 = 10 \times 10^{-6}$

$(v_1 = 1/2)$ $(v_2 = 1/4)$

Application based on Bernoulli's equation:

(a) Venturimeter speed of Efflux

(a) Venturimeter :-



$(h_1 - h_2 = h)$

$$\begin{cases} P_1 = P_0 + \rho g h_1 \\ P_2 = P_0 + \rho g h_2 \end{cases}$$

(I) $A_1 v_1 = A_2 v_2$

$$\boxed{v_2 = \frac{A_1 v_1}{A_2}}$$

(II) $P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g} h' = P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g} h'$

$$(P_0 + \rho g h_2) + \frac{1}{2}\rho \times \left(\frac{A_1 v_1}{A_2}\right)^2 = \cancel{P_0} + \cancel{\rho} g h_1 + \frac{1}{2}\cancel{\rho} v_1^2$$

$$\frac{1}{2}\frac{A_1^2 v_1^2}{A_2^2} - \frac{1}{2}v_1^2 = gh_1 - gh_2 \Rightarrow \frac{v_1^2}{2}\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right) = g\,\underbrace{(h_1 - h_2)}_{h}$$

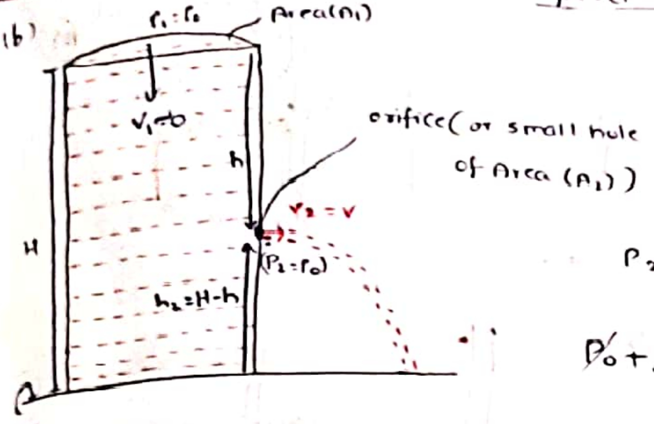
$$\frac{v_1^2}{2}\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right) = gh \Rightarrow v_1^2 = \frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}$$

$$\boxed{v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}}$$

Rate of flow $= A_1 v_1$

$$= A_1\sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

(b)



as area $A_1 >> A_2 \to$ liquid level decrease slowly so speed of liquid at top surface $(V_1 \simeq 0)$

$$P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 = P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2$$

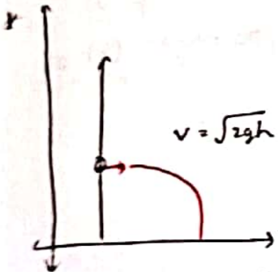
$$\cancel{P_0} + \rho g (H-h) + \frac{1}{2}\rho v^2 = \cancel{P_0} + \rho g H + \frac{1}{2}\rho \times 0^2$$

$$\rho g H - \rho g h + \frac{1}{2}\rho v^2 = \rho g H$$

$$v^2 = 2gh$$

$$\boxed{v = \sqrt{2gh}}$$

Speed of Efflux = $v = \sqrt{2gh}$ where h is measured from Top level of liquid.



$$t = \sqrt{\frac{2(H-h)}{g}}$$

$$R = v \times t = \sqrt{2gh} = \sqrt{\frac{2(H-h)}{g}}$$

$$\boxed{R = 2\sqrt{h(H-1)}}$$

* The speed of efflux of a liquid passing through orifice is same as it would attain if allowed to fall freely by the same vertical height.

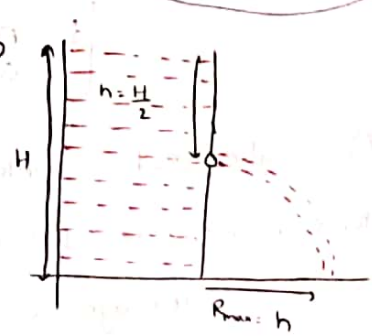
Q Find the value of 'h', so that range should be max^m.

Sol:- $\frac{dR}{dh} = 0$

$$\frac{d}{dh}\left(2\sqrt{h(H-h)}\right) = 0 \Rightarrow 2 \cdot \frac{d}{dh}(h(H-h))^{1/2} = 0 \Rightarrow \frac{1}{2}(h(H-h))^{\frac{1}{2}-1} \times \frac{d}{dh}(h-(H-h)) = 0$$

$$\frac{d}{dh}(h(H-h)) = 0 \Rightarrow \frac{d}{dh}(Hh - h^2) = 0 \Rightarrow \frac{d}{dh}(hH) - \frac{d}{dh}(h^2) = 0$$

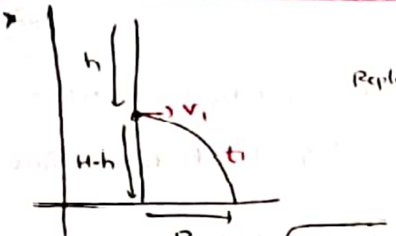
$$H \cdot \frac{d}{dh}(h) - 2h = 0 \Rightarrow H \times 1 - 2h = 0 \Rightarrow \left(h = H/2\right)$$



$$R_{max} = 2\sqrt{h(H-h)}$$

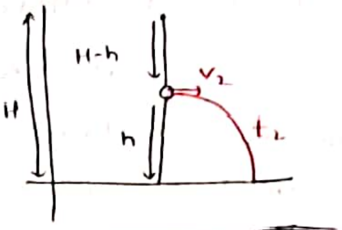
$$R_{max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} = H$$

{Range will be max^m when $h = \frac{H}{2}$
$R_{max} = H$}



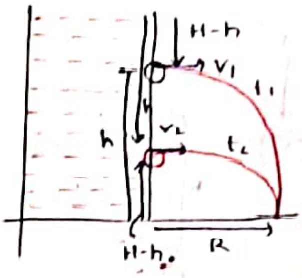
$R = 2\sqrt{h(H-h)}$

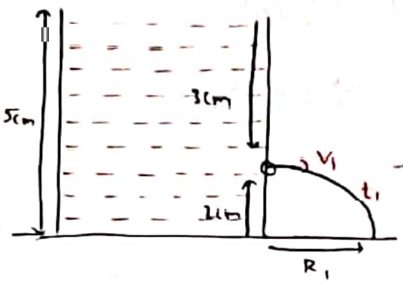
Replace $h \xrightarrow{by} H-h$ $\Longrightarrow$



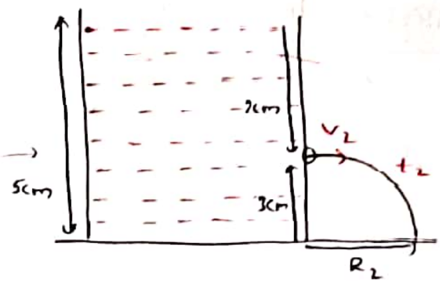
$R = 2\sqrt{(H-h)\,(H-(H-h))}$

$R = 2\sqrt{(H-h)\,(h)}$

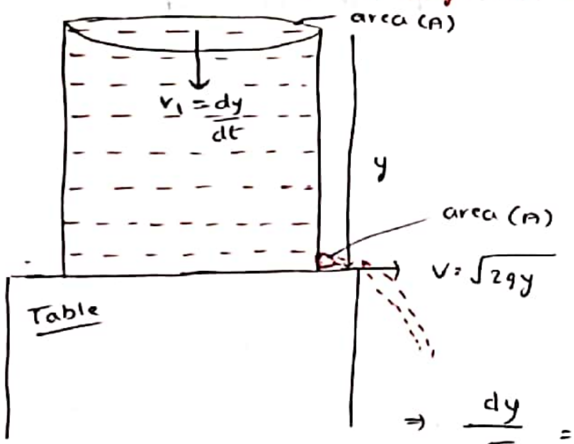




$R_1 = R_2$



Time taken to empty the tank:-



Initially container is filled upto height (h) at time (t) the liquid level (y)

Rate of flow:-

$$A_1V_1 = A_2V_2$$

$$A\left(\frac{-dy}{dt}\right) = a \times v \Rightarrow -A \times \left(\frac{dy}{dt}\right) = a \times \sqrt{2gy}$$

$$\Rightarrow \frac{dy}{\sqrt{y}} = \frac{-a}{A} \times \sqrt{2g}\, dt \Rightarrow \int_H^0 y^{-1/2}\, dy = \int_0^t \frac{-a}{A}\sqrt{2g}\, dt$$

$$\Rightarrow \left(\frac{y^{-1/2+1}}{-\frac{1}{2}+1}\right)_0^H = \frac{-a}{A} \times \sqrt{2g}\,(t)_0^t \Rightarrow \left[\frac{\sqrt{y}}{1/2}\right]_0^H = \frac{-a}{A} \times \sqrt{2g}\,(t-0)$$

$$\Rightarrow (2\sqrt{y})_0^H = -\frac{a}{A}\sqrt{2g}\,(t) \qquad \Rightarrow (2\sqrt{H} - 2\sqrt{0}) = \frac{-a}{A}\sqrt{2g}\, t$$

$$t = \left|\frac{-A \times 2\sqrt{H}}{a\sqrt{2g}}\right|$$

$$\boxed{t = \frac{A}{a}\sqrt{\frac{2H}{g}}}$$

Q A cylindrical tank of height 0.4m is opened at the top and have diameter 0.16m, water is filled in it upto height 0.16m, How long it will take to empty the tank through a hole of radius, $r = 5 \times 10^{-3}$m when at it's bottom.

Sol.- $t = \frac{A}{a}\sqrt{\frac{2H}{g}}$

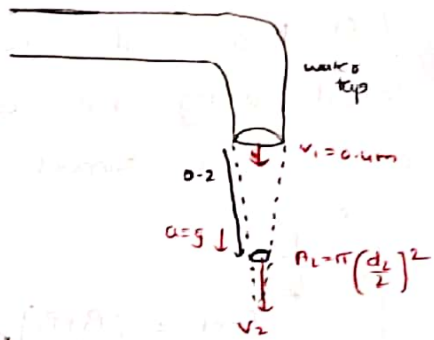
$A = \pi (0.08)^2$

$a = \pi (5\times10^{-3})^2$, $H = 0.16$

$$t = \frac{\pi (0.08)^2}{\pi (5\times10^{-3})^2} \times \sqrt{\frac{2\times0.16}{10}}$$

$\boxed{t = 46.26 \text{ sec}}$

Q Water is flowing continously from a tap having an internal diameter 8×10^{-3}m. The water velocity as it leaves the tap is 0.4 m/s. The diameter of tap water stream at a distance 2×10^{-1}m below the tap is close to:



$v_y^2 = u_y^2 + 2a_y s_y$

$v_2^2 = (0.4)^2 + 2\times g\times0.2 \Rightarrow 0.16 + 4 \rightarrow 4.16$

$v_2 = \sqrt{4.16} \simeq \sqrt{4}$

$\boxed{v_2 \simeq 2}$

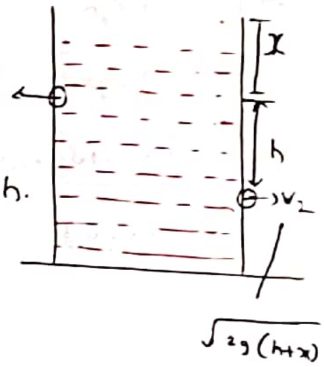
$d_1 = 8\times10^{-3}$m ; $v_1 = 0.4$m/s $d_2 = ?$

$A_1 v_1 = A_2 v_2$

$= \pi\left(\frac{d_1}{2}\right)^2 \times 0.4 = \pi\left(\frac{d_2}{2}\right)^2 \times 2 \Rightarrow \pi\left(\frac{8\times10^{-3}}{2}\right)^2 \times 0.4 = \pi\left(\frac{d_2^2}{4}\right)\times 2$

$\boxed{d_2 = 3.6\times10^{-3}\text{m}}$

Q There are two identical small holes containing the liquid as shown in figure, when the liquid comes from 2 holes. The tank experience net horizontal force. Prove that force is proportional to h.



Rate of mass flowing $= \frac{dm}{dt} = \rho A v$

Let the area of hole $= (a) = \frac{dm}{dt} = \rho a v$

F = Rate of change of momentum

$F = \frac{dp}{dt} = \frac{d}{dt}(M\times v)$; $F_1 = v_1 \times \frac{dm}{dt} = v_1 \times \rho a v_1$

$(F_1 = \rho a v_1^2)$

Similarly

$F_2 = v_2 \times \frac{dm}{dt} \rightarrow \rho a v_2$

$(F_2 = \rho a v_2^2)$

Net horizontal force on tank :- $|F_2 - F_1|$

$$= |\rho a v_2^2 - \rho a v_1^2|$$

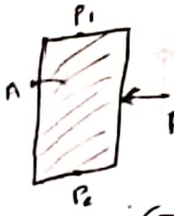
$$= \rho a (v_2^2 - v_1^2)$$

$$= \rho a \left(\left(\sqrt{2g(h+x)}\right)^2 - \left(\sqrt{2gx}\right)^2\right)$$

$$= \rho a \left(2gh + \cancel{2gx} - \cancel{2gx}\right)$$

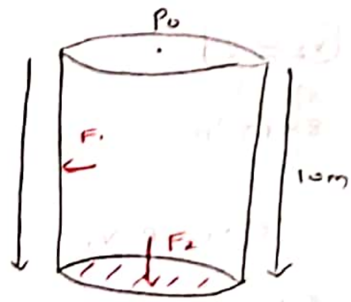
$$\boxed{F_{net} = 2g\rho a h} \qquad \boxed{F_{net} \propto h}$$

Q A large cylindrical vessel contain water of height 10m. It is found that the thrust acting on the curved surface is equal to that of the bottom. If atmospheric pressure can support a water column of 10m, the radius of vessel is.



$$F_{net} = \left(\frac{P_1 + P_2}{2}\right) A$$

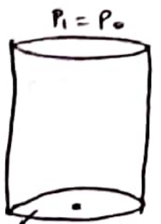
$$(F_{net} = P_{avg} \times A)$$



$$\Delta P = \rho g h = P_0$$

$$\boxed{\rho \times g \times 10 = P_0}$$

$$(F_1 = F_2)$$



$$P_2 = P_0 + \rho g h$$

$$P_2 = P_0 + \rho g \times 10$$

$$\boxed{P_2 = \rho g \times 10 + \rho g \times 10}$$

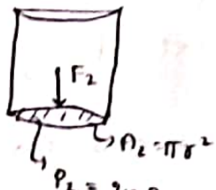
$$(P_2 = 20\rho g)$$

$$F_1 = \left(\frac{P_1 + P_2}{2}\right)\left(\begin{array}{l}\text{area of curved} \\ \text{surface of} \\ \text{cylinder}\end{array}\right)$$

$$F_1 = \left(\frac{10\rho g + 20\rho g}{2}\right)(2\pi r \times h)$$

$$F_1 = 15\rho g \times 2\pi r \times 10$$

$$\boxed{F_1 = 300\rho g \times \pi r}$$



$$F_2 = P_2 \times A_2$$

$$F_2 = 20\rho g \times \pi r^2$$

$$F_1 = F_2$$

$$\overset{15}{\cancel{300}} \rho g \pi r = \cancel{20} \cancel{\rho g} \cancel{\pi} r^{\cancel{2}}$$

$$\boxed{r = 15m}$$