

CO11

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Tutorial - 5

Q1) Difference b/w DFS & BFS. Write applications of both algorithms.

BFS

12) Breadth First Search
Uses queue

Suitable when destination is close to start node.

Not suitable for decision making trees used in games & puzzles

13) Siblings visited before children
Requires more memory
No concept of backtracking.

Applications -

BFS → Bipartite graph and shortest path, pccs to peer networking, crawler in search engine of GPS navigation system.

DFS acyclic graph, topological order, scheduling problems, sudoku puzzle

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6 7 10 11 12

Q. Which data structure are used to implement BFS & DFS and how do we use them?

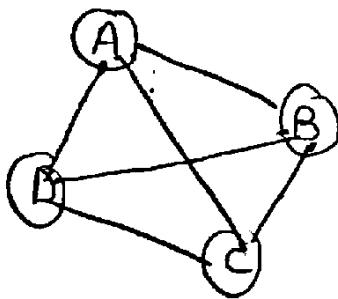
For BFS we use queue

For DFS we use a stack

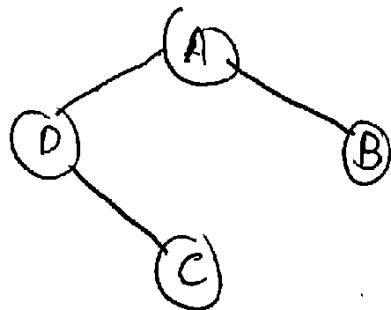
Q. Now what do you mean by sparse and dense graphs. Which representation of graph is better for sparse & dense graph?

Dense graph is where no. of edges is close to maximal no. of edges.

Sparse graph has less no. of edges.



Dense



Sparse

For sparse graph we use adjacency list.

For dense graph we use adjacency matrix

BFS and DFS

How do we detect a cycle in a graph using BFS & DFS

For detecting a cycle in graph using BFS, we use Kahn's algorithm for topological sorting.

Steps involved are:

1) Compute in-degree (no. of incoming edges) for each of vertex present in graph & initialise no. of visited nodes as 0.

2) Pick all vertices with in-degree as 0 & add them in queue.

3) Remove a vertex from queue and then,

1) ++ visited nodes

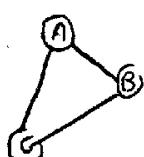
2) Decrease in-degree by 1 for all its neighbouring node

3) If in-degree of neighbouring nodes is reduced to zero then add to queue.

$\log 4)$ Repeat 3) until queue is empty.

$T_{CN} =$

5) If count of visited nodes is equal to no. of nodes in graph, then cycle, otherwise not.



A \rightarrow 2
B \rightarrow 2
C \rightarrow 2

(a) To use DFS for its sake,

DFS for a connected graph produces a tree. There is a cycle in a graph if there is a back edge present in graph. A back edge is an edge that is from a node to itself (self-loop) or one of its ancestors in the tree produced by DFS. For a disconnected graph, get DFS forest as output. To detect cycle, check for a cycle in individual trees by checking back edges.

(b) To detect a back edge, keep track of vertices currently in recursion stack for DFS traversal. If a vertex reached has already in recursion stack, then there is a cycle.

$\therefore \log_2$

$\frac{f(n)}{2} > n$ - Q. What do you mean by disjoint set data structure? Explain 3 operations along with examples which can be performed on disjoint sets?

A disjoint set is a data structure that keeps track of set of elements partitioned into several disjoint subsets. In other words, a disjoint set is a group of sets where no item can be in more than one set.

3 operations

Find Implemented recursively traversing parent array until we hit a node who is parent to itself.

```
int find(int i)
{
    if (parent[i] == i)
        return i;
    else
        return find(parent[i]);
```

union takes 2 elements as input and find representative of their set, by finding the find operation of them, puts either one of the tree under root of other tree, effectively merging the trees & sets.

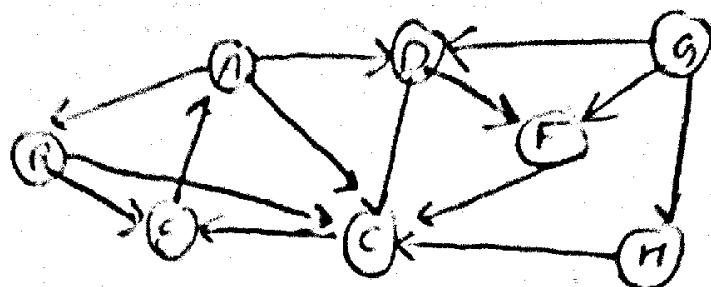
```
void union(int i, int j)
{
    int irep = this->find(i);
    int jrep = this->find(j);
    this->parent[irep] = jrep;
}
```

union by rank → we need a new array rank[], size of array same as parent array. If i is representative of set, rank[i] is height of tree. We need to minimize height of tree. If we are uniting 2 trees, we call them left & right. Then it all depends on the rank of left & right. If rank of left is less than right then it's best to move left under right of vice versa.

If ranks are equal, rank of result will always be one greater than rank of tree.

```
void union(int i, int j)
{
    int irep = this->find(i);
    int jrep = this->find(j);
    if (irep == jrep) return;
    irank = RANK[irep];
    jrank = RANK[jrep];
    if (irank < jrank)
        this->parent[irep] = jrep;
    else if (jrank < irank)
        this->parent[jrep] = irep;
    else {
        this->parent[irep] = jrep;
        RANK[jrep] += 1;
    }
}
```

Q1 Run BFS & DFS on below graph



BFS

child of H P F C E A B

parent of G G G H C E A

Path: G → H → C → E → A → B

DFS

G
D
H
F
G
E
A
B

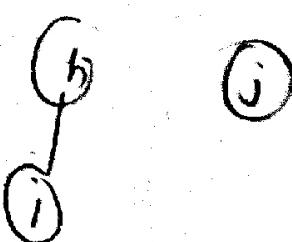
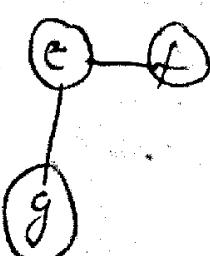
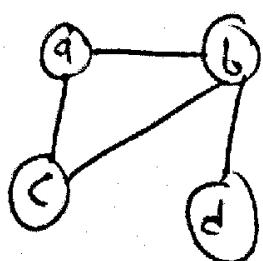
Node visited

G
C
E
A
B

STACK

Path: G → F → C → E → A → D

Find no. of connected components and vertices in each component using disjoint set data structure



$V = \{a, b, c, d, e, f, g, h, i, j\}$

$E = \{a,b\}, \{a,c\}, \{b,c\}, \{b,d\}, \{c,f\}, \{e,g\}, \{b,i\}, \{b,j\}$

(a,b) $\{a,b\} \{c\} \{d\} \{e\} \{f\} \{g\} \{i\} \{j\}$

(a,c) $\{a,b\} \{c\} \{d\} \{e\} \{f\} \{g\} \{h\} \{i\} \{j\}$

(b,c) $\{a,b\} \{c\} \{d\} \{e\} \{f\} \{g\} \{h\} \{i\} \{j\}$

(b,d) $\{a,b\} \{c\} \{d\} \{e\} \{f\} \{g\} \{h\} \{i\} \{j\}$

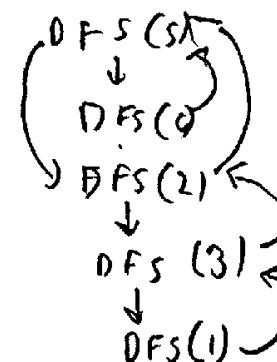
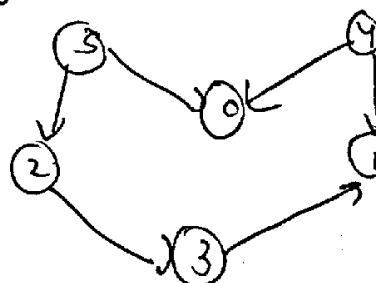
(c,f) $\{a,b\} \{c\} \{d\} \{e\} \{f\} \{g\} \{h\} \{i\} \{j\}$

(e,g) $\{a,b\} \{c\} \{d\} \{e\} \{f\} \{g\} \{h\} \{i\} \{j\}$

(b,i) $\{a,b\} \{c\} \{d\} \{e\} \{f\} \{g\} \{h\} \{i\} \{j\}$

(3)

Apply topological sort & DFS on graph having vertices from 0-5



DFS(4)
↓
Not Visited

4
3
2
1
0

4 → 2 → 3 → 1 → 0

Heaps can be used to implement priority queue.
 Now few graph algorithms where you

Yes heaps are used to implement priority queue.
 It will take $O(\log N)$ time to insert & delete each element in priority queue.

Based on heap structure there are

- 1) Max Heap
- 2) Min Heap,

Graph algorithms like Dijkstra's, Prim's used priority queue.

- 1) Dijkstra → Priority queue is used to extract minimum
- 2) Prim's → used to store keys of nodes & extract min key node at every step

Ques

Min Heap

Key present at root node must be less than or equal to keys present at all of its children

Min key element is present at root.

Uses ascending priority

Smallest element is popped first

Max Heap:

Key present at root node must be greater than or equal to keys present at all of its children

Max key element is present at root.

Uses descending priority

Largest element is popped first

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TUTORIAL - 6

over what is Minimum Spanning Tree? What are applications of MST

MST is the subset of edges of a connected edge-weighted undirected graph that connects all vertices together, without any cycles & with min possible edge weight.

Applications

- 1) Designing LAN
- 2) To construct highways connecting multiple cities.
- 3) Laying pipelines in a city.

2) ANALYZE THE time complexity of Prim, Kruskal, Dijkstra and Bellman Ford

Time Complexity of Prim's Space Complexity

$O(|E| \log |V|)$

$O(|V|)$

Prim

$O(|E| \log |E|)$

$O(|V|)$

Kruskal

$O(V^2)$

$O(|V|^2)$

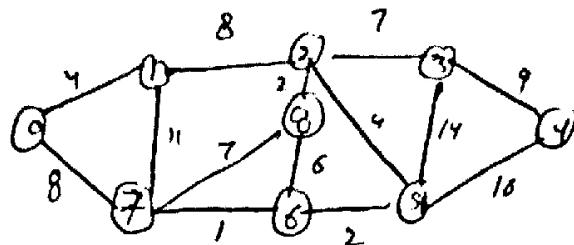
Dijkstra

$O(\sqrt{E})$

$O(E)$

Bellman Ford

Q. Apply Kruskal & Prim's Algorithm on given graph and find MST

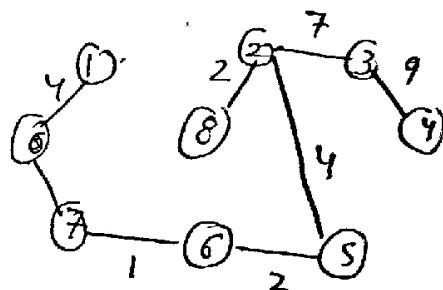


Kruskal

i	v_i	w_i	status
0	0	4	✓
1	0	8	✓
2	1	11	✓
3	1	7	✓
4	2	2	✓
5	2	8	✓
6	3	7	✓
7	3	6	✓
8	4	9	✓
9	4	14	✓
10	5	10	X
11	5	1	X
12	6	2	X
13	7	8	X
14	7	1	X
15	8	3	✓
16	8	2	✓
17	9	4	✓
18	9	10	X

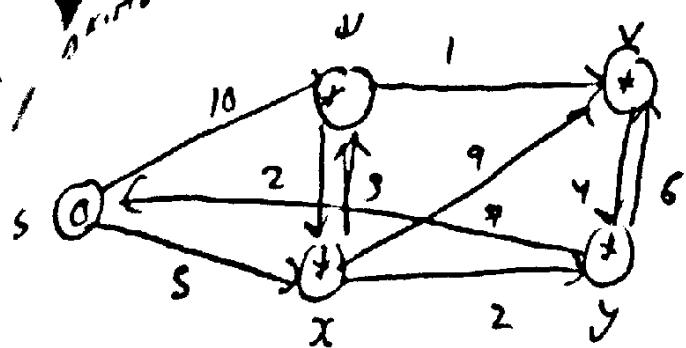
Prim's

$$\begin{aligned} \text{Weight} &= 4 + 8 + 2 + 4 + 2 + 7 + 9 + 3 \\ &= 37 \end{aligned}$$



$$\begin{aligned} \text{Weight} &= 1 + 2 + 2 + 4 + 14 + 7 + 8 + 3 \\ &= 37 \end{aligned}$$

1. Dijkstra & Bellman Ford on graph given



Dijkstra

No. 4

4

X

V

Y

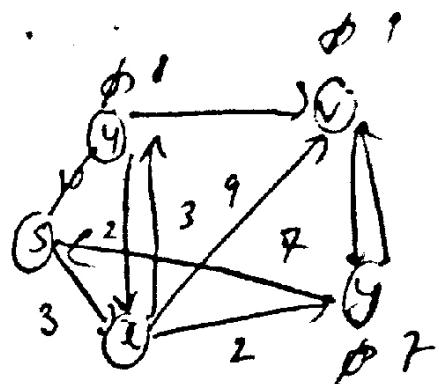
Shortest Dist from source

8

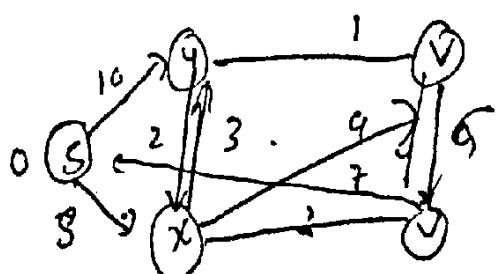
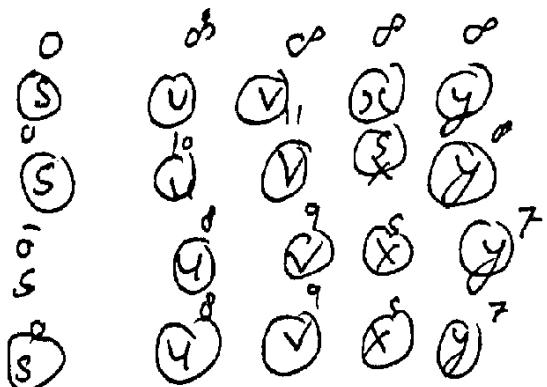
S

9

Z

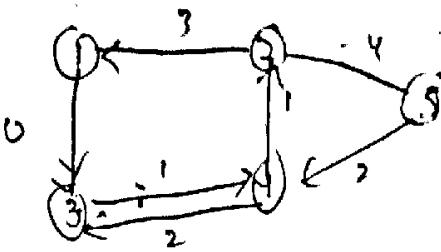


BELLMAN FORD



Final graph

Q: Apply all pair shortest path edge Floyd Warshall. Also analyze space & time complexity of



$$\begin{bmatrix} \infty & 2 & 3 & 4 & 5 \\ 0 & \infty & 6 & 3 & 0 \\ 1 & 0 & \infty & 3 & 0 \\ 2 & 6 & 0 & \infty & 2 \\ 3 & 3 & 2 & 0 & \infty \\ 4 & 0 & 1 & 0 & 0 \\ 5 & 0 & 2 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \infty & 6 & 3 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & \infty \\ 0 & 1 & 1 & 0 & 0 \\ \infty & 1 & 0 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & \infty & 6 & 3 & 0 \\ 2 & 0 & \infty & 5 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 0 & 4 & 12 & 20 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \infty & 6 & 3 & 0 \\ 2 & 0 & \infty & 5 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 6 & 4 & 12 & 20 & 0 \end{bmatrix}$$

Time Comp $\rightarrow O(|V|^3)$
 Space $\rightarrow O(|V|^2)$