Tutorial 4

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F-70

Q1) $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$ $\quad d=2$

$\log_2 3 = 1.5$ $\quad 2 > 1.5$ hence $O(n^2)$

$c = \log_b a$ $\quad n^c = n^{1.5} < n^2$

$f(n) > n^c$

$T(n) = O(n^2)$

2) $T(n) = 4T(n/2) + n^2$

$\log_2 4 = 2$ $\quad d=2$

$2 == 2$

$O(n^d \log n) = O(n^2 \log n)$

3) $T(n) = T(n/2) + 2^n$ $\quad 2^n = n^{\log_n 2^n}$

$\log_2 1 = 0$ $\quad 0 < n$

$c = \log_b a = \log_2 c = 0$

$n^c = n^0 = 1$

$f(n) > n^c$ $\quad T(n) = O(2^n)$

$\log_n 2^n > 0$

4) $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$

$b = 2$ $\quad d = n^n$

$c = n$

$n = n$ hence $n^n \log_2 n$

Q1) $T(n) = 16\,T(n/4) + n$

$a = 16, b = 4$

$f(n) = \log_4 16 = 2$

$c = 2$

$d = 1$

$c > d$

$O(n^2)$ $O(n^{\log_b a}) = O(n^2)$ $O(n^2 \log_2 n)$

Q2) $2T(n/2) + n\log n$ $T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2$

$f(n) = n\log n$

$c = \log 2 = 1$

$n^c = n^1 = n$

$n\log n > n$

$f(n) > n^c$

$T(n) = O(n\log n)$

$n^{\log n}$ $n\log n$

$\log_n n\log n$

$\log_n n + \log_n \log n$

$1 + \log_n \log n$

Q7) $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2$ $f(n) = n/\log n$

$c = 1$

$n^c = n^1 = n$

$\frac{n}{\log n} < n$

$f(n) < n^c$

$T(n) = O(n)$

Q8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4, f(n) = n^{0.31}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$f(n) < n^c$

$T(n) = O(n^{0.51})$

Q10) $T(n) = 16\,T(n/4) + n!$ $a = 16, b = 4$ $f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$ As $n! > n^2$

$T(n) = \Theta(n!)$

(11) $4T(n/2) + \log n$

$a=4,\ b=2,\ f(n)=\log n$

$c = \log_b a = \log_2 4 = 2$

$n^2 = n^2$

$f(n) = \log n$

$\therefore \log n < n^2 \quad f(n) < n^c$

$T(n) = \Theta(n^c) = \Theta(n^2)$

(12) $T(n) = \sqrt{n}\,T(n/2) + \log n$

$a=\sqrt{n},\ b=2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{\log_2 n}{2} < \log(n)$

$f(n) > n^c$

$T(n) = \Theta(f(n)) = \Theta(\log(n))$

(13) $T(n) = 3T(n/2) + n$

$a=3,\ b=2,\ f(n)=n$

$n^c = n^{1.54}$

$n < n^{1.58}$

$f(n) < n^c$

$T(n) = \Theta(n^{1.5})$

(14) $aT(n) = 3T(n/3) + \text{something}(n)$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\sqrt{n} < n$

$f(n) < n^c$

$T(n) = \Theta(n)$

(18) $T(n) = 3T(n/4) + n\log n$

$a=2,\ b=4,\ f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79} \quad n^{0.79} < n\log n$

$T(n) = \Theta(n\log n)$

(17) $T(n) = 3T(n/3 + n/2)$

$a=3;\ b=3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$n^c = n^1 = n \quad$ As $n/2 < n$

$f(n) < n^c$

$T(n) = \Theta(n)$

(19) $T(n) = 4T(n/2) + n(\log n)$

$a=4\ \ b=2\ \ f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \Theta(n^2)$

(20) $T(n) = 7T(n/3) + n^2$

$a=7,\ b=3,\ f(n)=n^2$

$c = \log_b a = \log_3 7 = 1.77 12$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \Theta(n^2)$

(21) $T(n) = T(n/2) + n(2-\cos n)$

$a=1,\ b=2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1 \quad n(2-\cos n) > 1$

$T(n) = \Theta(n(2-\cos n))$