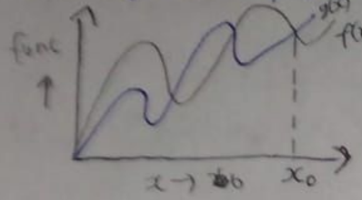
Tutorial 1 (1)

Name → Aditya Thapyal
Sec - F
Roll - 40
Uni - Roll → 2016593

Q1) Asymtotic notation and define it's diff typs.

(i) Big O(n) $f(n) \Rightarrow O(g(n))$ if $f(n) \le g(x) \times c \ \forall\ x \ge 0$, $c > 0$ $g(x)$ is upper bound of $f(x)$

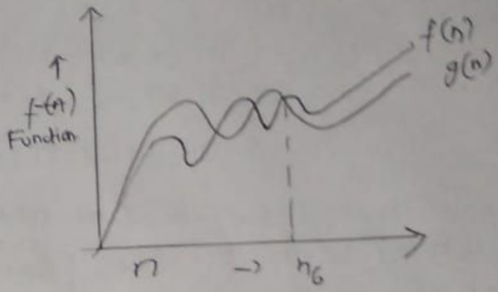


eg. $f(n) \Rightarrow n^2+n$ $g(x) \Rightarrow n^3$

$n^2+n \le c * n^3$,

$n^2+n = O(n^3)$

2) Big omega Ω



when $f(n) = \Omega(g(n))$

$\Rightarrow g(n)$ is "tight" lower bound of $f(n)$ i.e. $f(n)$ can go beyond $g(n)$

i.e $f(n) = \Omega g(n)$ if and only if $f(x) \ge c \cdot g(n)$

$\forall\ n \ge n_0$ & $c > 0$

Ex $f(n) \Rightarrow n^3+4n^2$ $g(n) = n^2$ ie. $f(n) \ge c * g(n)$ $n^3+4n^2 = \Omega(n^2)$

3) Theta given us a range of $f(x)$ $c_1 \cdot g(n) \le f(n) \le c_2\, g(n)$

$f(n) = \Theta(g(n))$

4) Small oh (o): Upper bound of $f(n)$ $f(n) = o(g(n))$ $f(n) < c \cdot g(n)$

5) small omega (ω) lower bound of $f(n)$ $f(n) > c \cdot g(n)$

$f(n) = \omega(g(n))$,

2) What should be the time complexity of

for (i=1 to n) { i = i*2; }

1, 2, 4, 8, 16 - - - - . n

$\sum_{i=1}^{n} = 1+2+4+8 \ldots n$ k^{th} Term $= T_K = ar^{K-1}$

$a = 1,\ r = 2$

$T_K = 1 \times 2^{K-1} = 2^{K-1}$ Put $n = 2^{K-1}$ $2n = 2^K$

$\log_2 2n = K \log_2 2 \Rightarrow \log_2 2 + \log_2 n = K \Rightarrow \log_2 n + 1 = k \Rightarrow \log n$

$\Rightarrow O(\log n)$

Q3) $T(n) = \{ 3T(n-1)$ if $n > 0$, otherwise $1 \}$

$T(0) = 1$ $T(n) = 3T(n-1)$ — (1) Put $n = n-1$

$T(n-1) = 3T(n-2)$ — (2)

n-4n

Put value of T(n-1) in 1

$T(n) = 3 \times 3T(n-2) = 9T(n-2)$ — 3   Put $n = n-2$ in 1   2

$T(n-2) = 3T(n-3)$ — 4   Put value of $T(n-2)$ in 3

$T(n) = 27T(n-3) \Rightarrow T(n) = 3^k T(n-k)$ Put $k = n$ $T(n) = 3^n T(0)$

$T(n) = 3^n$

(Q4) $T(n) = \{ 2T(n-1) - 1$ for $n > 0$, else $1 \}$

$T(n) = 2T(n-1) - 1$ — 1

$T(0) = 1$

Put $n = n-1$ in 1

$T(n-1) = 2T(n-2) - 1$ — 2

Put value of $T(n-1)$ in 1

$T(n) = 2 \times 2T(n-2) - 2 - 1$

$T(n) = 4T(n-2) - 3$ — 3

Put $n = n-2$ in 1

$T(n-2) = 2T(n-3) - 1$ — 4

Put $T(n-2)$ in 3

$T(n) = 4(2T(n-3) - 1) - 2 - 1$

$= 8T(n-3) - 4 - 2 - 1$

$T(n) = 2^k T(n-k) - (2^{k-1} + 2^{k-2} + 2^{k-3} \dots 1)$

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - 2^{k-3} \dots 1$

$= 2^k T(n-k) - (1 + 2 + 4 \dots 2^{k-1}) \Rightarrow 2^k - \left[1 \frac{(2^k - 1)}{2 - 1} \right]$

$\Rightarrow 2^k - 2^k + 1 \rightarrow T(n) = 1$

Q5 what should be the time complexity of int i=1, s=1;

while (s<=n) { i++; ~~s++~~ s=s+i; print ("#"); }

we can define the term s according to $s_i = s_{i-1} + i$.

value in s at the end of ith iteration is the sum of the first 'i' positive integer

If k is the total no. of iterations taken by the program, then while loop terminates i

$1 + 2 + 3 \dots k = \frac{k(k+1)}{2} > N$

So $k = O(\sqrt{n}) \Rightarrow O(\sqrt{n})$

Q6 Time complexity of

```
void function (int n)
{ int i, count = 0;
  for (i=1 ; i*i<=n; ++i) { count ++; } }
```

$i^2 <= n \Rightarrow i <= \sqrt{n}$

$i = 1, 2, 3, 4 \dots \sqrt{n}$

$\sum_{i=1}^{n} = 1 + 2 + 3 + 4 \dots \sqrt{n}$ $T(n) = \frac{\sqrt{n} \times (\sqrt{n} + 1)}{2} \Rightarrow T(n) = \frac{n \times \sqrt{n}}{2}$

$T(n) = O(n)$

7. Time complexity of

```
void function (int n)
{ int i, j, k, count=0;    for (i=n/2; i<=n; ++i)
                           for (j=1; j<=n; j*=2)
                           for (k=1 ; k<=n; k*=2)
                           ++count;
}
```

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sol^n => I^st loop n/2 times

II^nd loop log n times

III^rd loop log n times

$T(n) = \frac{n}{2} \times \log n \times \log n$

$T(n) = O(n \log^2 n)$

8) Time complexity of

function (int n)

{ if (n==1) return ; for (i=1 to n) { for (j=1 to n) Print("*"); }

function (n-3); }

$T(n) = T(n-3) + O(n^2)$

$T(n-3) + n^2$ — ①

$T(n-3) = T(n-6) + (n-3)^2$

from eq ①:

$T(n) = T(n-6) + (n-3)^2 + n^2$

$T(n-6) = T(n-9) + (n-6)^2$

$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n^2$

$T(n) = T(n-3k) + (n-(3k-3))^2 + [n-(3k-6)]^2 + n^2$

$n - 3k = 0$

$k = n/3$

$T(n) = T(n-n) + 1^2 + 2^2 + 3^2 + \dots n^2$

$= T(0) + \frac{n(n+1)(n+2)}{6}$

$T(n) = \frac{n \times n \times n}{6}$

$T(n) = O(n^3)$

a) - Time complexity of - void function (int n)

{ for (i=1 to n)

{ for (j=1 ; j<=n; j=j+i)

Printf ("*");

} }

for $i = 1 \Rightarrow j = 1, 2, 3, 4 \dots n = n$

$i = 2 \Rightarrow j = 1 \dots \quad n = n/2$

$i = 3 \Rightarrow j = 1, 4, 7 \dots n = n/3$

for $i = n = j = 1 \quad = 1$

$\Rightarrow \sum_{j=n}^{1} = n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \dots 1$

$\Rightarrow \sum_{j=n}^{1} = n\left(1 + \frac{1}{2} + \frac{1}{3} \dots \frac{1}{n}\right) \Rightarrow T(n) = n \log n$

10) For the function n^k & C^n, what is the asymptotic relationship between these functions.
Assume $K >= 1$ & $C>1$ are constants. Find C & n for which relation hold,

n^K & c^n

$n^K = O(C^n)$

as $n^K < a c^n$

$\forall$ $x > x_0$ & some constant $a>0$

for $n_0 = 1$

$c = 2$

$1^n \leq a_2^n$

$n_0 = 1$ & $c = 2$.