Fast and Robust Ellipse Detection

This document summarizes a sub-task from a project I worked on at <u>Oneirix Labs</u>. The objective was to extract the geometrical parameters of blood cells from microscopic images by fitting ellipses to noisy, edge-detected binary images. While the code and results are subject to an NDA and cannot be disclosed, I will outline the problem formulation, core idea, and the modifications I introduced to improve computational performance.

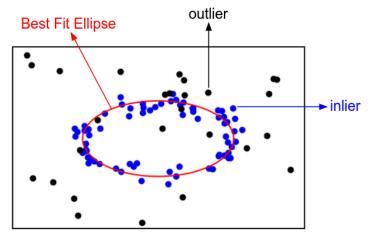


Figure: Simulated example illustrating RANSAC-based ellipse fitting

I) Problem:

- 1. **Input**: Noisy, edge-detected binary microscopic images containing multiple blood cells.
- 2. **Task**: Rapidly, accurately identify ellipse-shaped cells and fit ellipses to their boundaries.
- 3. **Output**: Geometric parameters of each fitted ellipse (e.g., major/minor axes, orientation, centroid).

II) Mathematical Formulation:

The problem involved detecting ellipses in a set of 2D points (x, y) which may contain significant noise and outliers. An ellipse is represented by the general conic equation:

$$Ax^{2} + Bxy + Cy^{2} + Dx + Ey + F = 0; b^{2} - 4ac < 0$$

Given a set of points, the goal was to find the parameters (A, B, C, D, E, F) that best fit these points, while also being robust to noise. For instance, given N points, we need to minimize:

$$\Sigma (Ax_i^2 + Bx_iy_i + Cy_i^2 + Dx_i + Ey_i + F)^2$$

This is formulated as a least squares problem, and the non-trivial solution is obtained using Singular Value Decomposition.

III) Challenge:

- 1. **Large number of points**: Increasing *N* directly impacts computation time.
- 2. Noise and outliers: Standard least squares is sensitive to outliers

IV) Existing Solutions:

Using tools from standard libraries like OpenCV:

- 1. Hough Transform
- 2. Active Contours

Both methods operated directly on binary images without requiring a mathematical formulation, but were highly sensitive to outliers and performed poorly in terms of speed and convergence.

V) Proposed Solution:

- 1. **RANSAC** (Random Sample Consensus): Iteratively select minimal subsets of 5 data points (sufficient to uniquely define a general ellipse), fit an ellipse model to these points, and evaluate the number of inliers to identify the best-fitting model.
- 2. **Vectorization**: Eliminate ALL explicit loops by operating on entire NumPy arrays

VI) Programming Stack:

- 1. **NumPy**: Numerical computing and efficient array operations.
- 2. **OpenCV**: Image processing and binary image handling.

VII) Results: The detection time for a single ellipse decreased by ~2-5×. While this improvement may appear modest for a single detection, it translated to a considerable performance gain when processing multiple ellipses across numerous microscopic slides.

VIII) Reference: The core idea was adapted from two literature sources, while I developed a custom implementation from the ground up with a focus on vectorization and loop reduction to improve computational efficiency.