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Declaration of Authorship: I declare that all materials included in this assessment is the end result of our own work and that due acknowledgement has been given in the bibliography and references to ALL sources, be they printed, electronic or personal.

Signed:

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Does the stock market predict real activity? Time series evidence from the G-7 countries

Introduction

Project Overview

The objective of this project was to replicate the research paper *"Does the Stock Market Predict Real Activity? Time Series Evidence from the G-7 Countries."* In this study, we explored the relationship between industrial production (IP) and stock returns for G-7 countries. We implemented our analysis across three different time frames: monthly, quarterly, and annual data. The project involved the application of three types of models: a standard linear regression model, a lagged ECM (Error Correction Model), and ARIMA and GARCH models to predict industrial production growth.

This replication study provided valuable insights into the interplay between stock market performance and real economic activity, as outlined in the original research.

Paper Selection

The reason for selecting this research paper was to study the relationship between various factors that contribute to predicting industrial production. Additionally, it aimed to explore how industrial production is influenced by these factors, given its significance as a key indicator for forecasting the growth patterns of any nation.
We chose this paper to leverage data and predictions for the G-7 countries, as this group represents some of the world's largest economies. Analyzing patterns across these nations provides valuable insights that can be applied to a significant portion of the global economy. This paper also employs various time series models for prediction, offering a diverse perspective and enriching our understanding of industrial production.
The original paper analyzed data from 1950 to 1990; however, due to data constraints, we adjusted the timeframe for our predictions to 2005–2021. Beyond this, a significant reason for this change was to examine how these predictive models perform with modern data, thereby testing the applicability of older models in a contemporary context.

We used three different datasets for our predictions: Industrial Production, Stock Price Index, and Consumer Price Index for the respective G-7 countries, including Japan, Canada, the USA, the UK, Germany, France, and Italy.

Data Collection

The dataset consists of monthly observations of the Stock Index, Industrial Production Index, and Consumer Price Index for all G-7 countries. The stock index data, which varies by country, was collected from Refinitiv and TradingView. The abbreviations for the respective indices are:

- **Canada:** S&P/TSX
- **USA:** NASDAQ
- **UK:** FTSE 100
- **Japan:** Nikkei 225
- **Italy:** Italian FTSE
- **France:** CAC 40
- **Germany:** DAX

The Industrial Production Index and Consumer Price Index were sourced from the International Financial Statistics provided by the International Monetary Fund (IMF).
The time period for all data samples is one month, and the data was converted into yearly and quarterly observations as required for different calculations and analyses.

Data Description

In our analysis, we primarily focus on two main variables: the Industrial Production Index and the Nominal Stock Index. The Real Stock Index is calculated by dividing the Nominal Stock Index by the Consumer Price Index. This step is a critical aspect of our predictive model, as it adjusts the stock index for inflation, ensuring that our predictions are based on real, inflation-adjusted values rather than nominal figures.

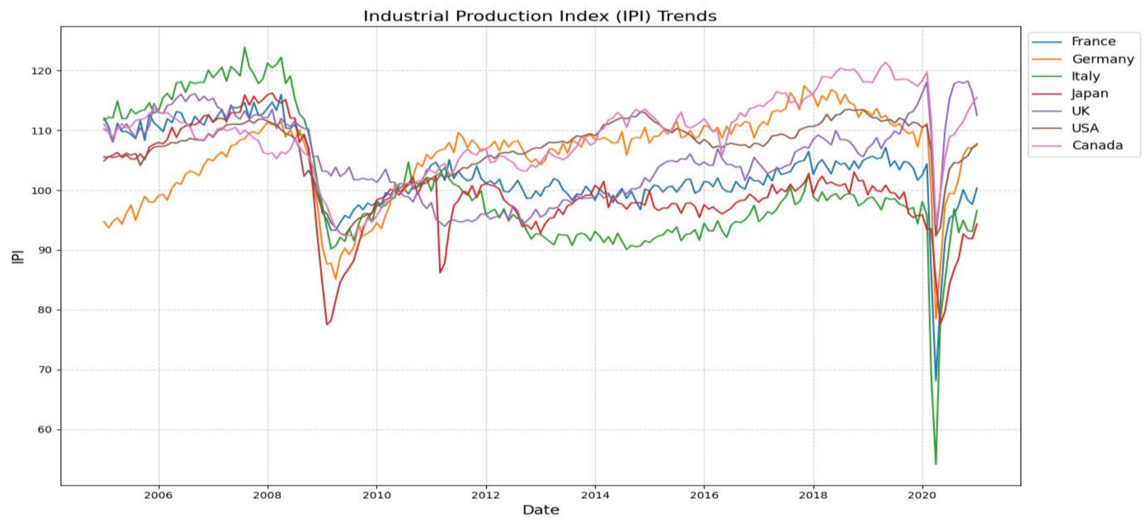

Image1: Industrial production index

In the above plot, we can observe the trend of the Industrial Production Index for all G-7 countries over the time frame from 2005 to 2020. The countries are represented using the following color scheme: France in blue, Germany in orange, Italy in green, Japan in red, the UK in purple, the USA in brown, and Canada also in purple.

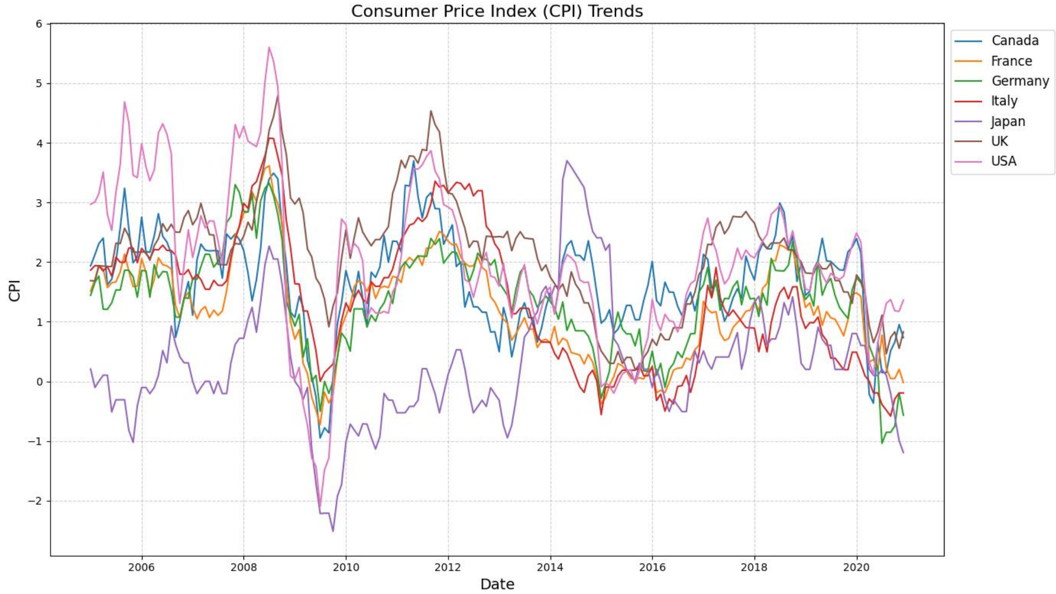

Image 2: Consumer Price Index

The above graph shows the trend lines of consumer price index over a time frame of 2005 to 2020.

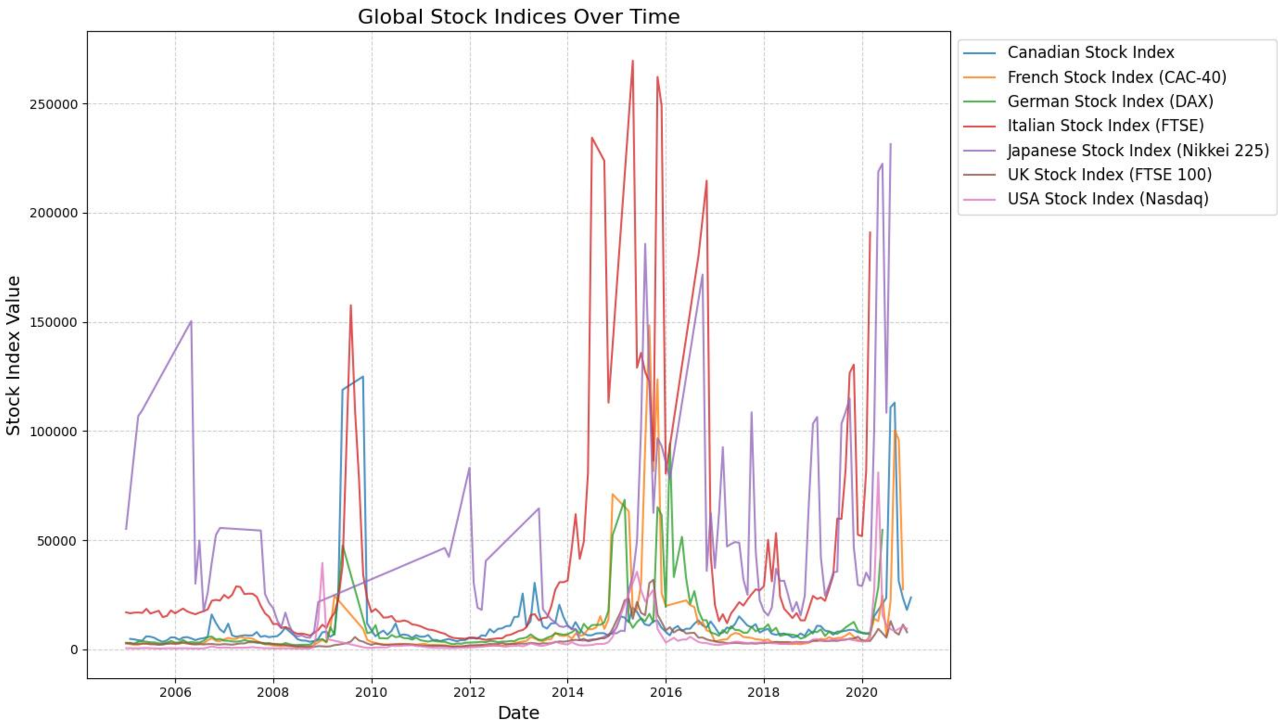

Image 3: Stock Index

In the above graph, we can observe the actual price index for all the major G-7 economies. This index is calculated by dividing the real stock index by the consumer price index. Most countries follow a relatively stable trend with minimal volatility. However, the Italian and Japanese stock indices stand out, exhibiting distinct behavior compared to others after the conversion to the actual price index. This divergence might play a critical role in the prediction of the Industrial Production Index.

In every model we use, we have applied the log growth rate for both industrial production and the real stock index. Additionally, we have handled missing data by dropping all null values in our predictive analysis. One of the key challenges faced was the conversion of the date-time format, which we successfully addressed. We also combined all the datasets into one unified dataframe. To handle different time frames, we converted the monthly data into yearly and quarterly data using the mean method.

Data Consistency

First, we performed the Augmented Dickey-Fuller (ADF) test for each of the G-7 countries. The ADF test was conducted to determine whether the data is stationary or non-stationary, which is a crucial step for ensuring the stationarity of the data. In the ADF test, the null hypothesis is that the data is non-stationary, while the alternative hypothesis is that the data is stationary. A stationary data series is one where the mean, variance, and autocorrelation remain constant over time. We conducted the ADF test on the differences in log returns of industrial production and stock indices for each of the countries.

Ho: Data points are non – Stationary

H1: Data points are Stationary

```
Performing Augmented Dickey-Fuller Test for Differenced Log Canadian Stock Index:
ADF Statistic: -9.07690205359602
p-value: 4.155610679335628e-15
Critical Values:
   1%: -3.467004502498507
   5%: -2.8776444997243558
   10%: -2.575355189707274
The series Differenced Log Canadian Stock Index is stationary (reject null hypothesis).

Performing Augmented Dickey-Fuller Test for Differenced Log Industrial Production:
ADF Statistic: -11.350799045332003
p-value: 1.0006645755130169e-20
Critical Values:
   1%: -3.4648754262570978
   5%: -2.876714157488788
   10%: -2.574858699001736
The series Differenced Log Industrial Production is stationary (reject null hypothesis).
```
.

Image 4 : ADF For Canada

In the above Augmented Dickey-Fuller (ADF) test, we conducted the test on the monthly data for the Canadian Stock Index and the Industrial Production Index. The p-value obtained from the test was very low, indicating that we can reject the null hypothesis, which suggests that the data is non-stationary. Therefore, we accept the alternative hypothesis, which states that the data is stationary. This is a crucial step, as stationarity is required for the validity of subsequent time series models.

```
Performing Augmented Dickey-Fuller Test for differenced US Stock Index:
ADF Statistic: -11.288165347123375
p-value: 1.401160214043319e-20
Critical Values:
   1%: -3.468952197801766
   5%: -2.878495056473015
   10%: -2.57580913601947
The differenced series US Stock Index is stationary (reject null hypothesis).

Performing Augmented Dickey-Fuller Test for differenced US Industrial Production:
ADF Statistic: -10.816370964837661
p-value: 1.852773619673187e-19
Critical Values:
   1%: -3.4652439354133255
   5%: -2.8768752281673717
   10%: -2.574944653739612
The differenced series US Industrial Production is stationary (reject null hypothesis).
```

Image 5: ADF For USA

In the above ADF test, we conducted the test on the US Stock Index (Nasdaq) and the Industrial Production Index. The results show that both the US Stock Index and the Industrial Production Index are stationary, as we were able to reject the null hypothesis that suggests the data is non-stationary. This indicates that the data is suitable for further analysis, and we can proceed with time series modeling for prediction purposes.

```
Performing Augmented Dickey-Fuller Test for differenced French Stock Index:
ADF Statistic: -11.492810776847932
p-value: 4.691442415482325e-21
Critical Values:
   1%: -3.4682803641749267
   5%: -2.8782017240816327
   10%: -2.5756525795918366
The differenced series French Stock Index is stationary (reject null hypothesis).

Performing Augmented Dickey-Fuller Test for differenced French Industrial Production:
ADF Statistic: -12.320492053793304
p-value: 6.768384122287175e-23
Critical Values:
   1%: -3.4682803641749267
   5%: -2.8782017240816327
   10%: -2.5756525795918366
The differenced series French Industrial Production is stationary (reject null hypothesis).
```

Image 6: ADF For France

Same goes with the French index, which is the **CAC-40**. The Augmented Dickey-Fuller (ADF) test conducted on the French Stock Index (CAC-40) and the Industrial Production Index shows that both are stationary.

```
Performing Augmented Dickey-Fuller Test for Differenced German Industrial Production (Monthly):
ADF Statistic: -11.081773269535569
p-value: 4.293485345073923e-20
Critical Values:
   1%: -3.4682803641749267
   5%: -2.8782017240816327
   10%: -2.5756525795918366
The differenced series Differenced German Industrial Production (Monthly) is stationary (reject null hypothesis).

Performing Augmented Dickey-Fuller Test for Differenced German Stock Index (Monthly):
ADF Statistic: -17.6761303608289
p-value: 3.625972323004401e-30
Critical Values:
   1%: -3.4680615871598537
   5%: -2.8781061899535128
   10%: -2.5756015922004134
The differenced series Differenced German Stock Index (Monthly) is stationary (reject null hypothesis).
```

Image 7: ADF For Germany

Same goes for the German index, which is the **DAX**. The Augmented Dickey-Fuller (ADF) test conducted on the German Stock Index (DAX) and the Industrial Production Index shows that both are stationary

```
ADF Test for Log_Diff_Japanese_Stock_Index:
ADF Statistic: -11.240467129233062
p-value: 1.812446323132547e-20
Critical Values: {'1%': -3.49181775886872, '5%': -2.8884437992971588, '10%': -2.5811201893779985}
The series is stationary.


ADF Test for Log_Diff_Japanese_Industrial_Production:
ADF Statistic: -7.086965567292661
p-value: 4.5122719269876515e-10
Critical Values: {'1%': -3.492995948509562, '5%': -2.888954648057252, '10%': -2.58139291903223}
The series is stationary.
```

Image 8: ADF For Japan

Same goes for the Japanese index, which is the **Nikkei 225**. The Augmented Dickey-Fuller (ADF) test conducted on the Nikkei 225 Stock Index and the Industrial Production Index shows that both are stationary.

```
ADF Test for Log_Diff_UK_Stock_Index:
ADF Statistic: -6.714480798598389
p-value: 3.61096887104710e-09
Critical Values: {'1%': -3.465620397124192, '5%': -2.8770397560752436, '10%': -2.5750324547306476}
The series is stationary.


ADF Test for Log_Diff_UK_Industrial_Production:
ADF Statistic: -7.910401386948031
p-value: 3.96217930210933e-12
Critical Values: {'1%': -3.4662005731940853, '5%': -2.8772932777920364, '10%': -2.575167750182615}
The series is stationary.
```

Image 9: ADF For UK

The same applies to the UK **FTSE 100** index as well. The Augmented Dickey-Fuller (ADF) test conducted on the FTSE 100 Stock Index and the Industrial Production Index shows that both are stationary.

```
ADF Test for Log_Diff_Italy_Stock_Index:
ADF Statistic: -11.969810934923128
p-value: 3.9223787817278283e-22
Critical Values: {'1%': -3.470616369591229, '5%': -2.8792214018977655, '10%': -2.57619681359045}
The series is stationary.


ADF Test for Log_Diff_Italy_Industrial_Production:
ADF Statistic: -3.2495117880321986
p-value: 0.017291762091788867
Critical Values: {'1%': -3.471118535474365, '5%': -2.8794405060097024, '10%': -2.576313761526591}
The series is stationary.
```

Image 10: ADF For Italy

The same goes for the **Italian FTSE** index as well. The Augmented Dickey-Fuller (ADF) test conducted on the Italian FTSE Stock Index and the Industrial Production Index shows that both are stationary.

After performing the Augmented Dickey-Fuller (ADF) test and confirming the stationarity of the data, we proceeded to run an **OLS regression** using the **Error Correction Model (ECM)**, which incorporates lagged values of the variables. The ECM is a model designed to capture both the short-term and long-term relationships between the variables—in this case, the log-transformed stock index and the industrial production index.

$$y_t = \sum_{j=1}^{m} a_{t-j} \cdot S_{t-j} + \sum_{i=1}^{m} b_j y_{t-j} + \phi_t$$

For the lagged values, we used:

- **Monthly data**: Lag length of 24
- **Quarterly data**: Lag length of 8
- **Yearly data**: Lag length of 2

We applied this model for each of the G-7 countries and estimated the coefficients to understand the dynamic relationships between stock prices and industrial production over the specified time frames.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.125
Model:                                  OLS   Adj. R-squared:                 -0.030
Method:                       Least Squares   F-statistic:                    0.8086
Date:                      Tue, 17 Dec 2024   Prob (F-statistic):              0.721
Time:                              23:17:12   Log-Likelihood:                 482.95
No. Observations:                       161   AIC:                            -915.9
Df Residuals:                           136   BIC:                            -838.9
Df Model:                                24
Covariance Type:                  nonrobust
==============================================================================
                          coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const                   0.0005      0.001      0.518      0.606      -0.002       0.003
Stock_Log_Return_lag_1  0.0009      0.003      0.308      0.758      -0.005       0.007
Stock_Log_Return_lag_2  0.0020      0.003      0.650      0.517      -0.004       0.008
Stock_Log_Return_lag_3  0.0006      0.003      0.197      0.844      -0.006       0.007
Stock_Log_Return_lag_4  0.0025      0.003      0.757      0.450      -0.004       0.009
Stock_Log_Return_lag_5  0.0040      0.003      1.233      0.220      -0.002       0.010
Stock_Log_Return_lag_6  0.0037      0.003      1.079      0.283      -0.003       0.010
Stock_Log_Return_lag_7  0.0052      0.003      1.534      0.127      -0.002       0.012
Stock_Log_Return_lag_8  0.0041      0.004      1.174      0.242      -0.003       0.011
Stock_Log_Return_lag_9  0.0040      0.004      1.120      0.265      -0.003       0.011
Stock_Log_Return_lag_10 0.0077      0.004      2.153      0.033       0.001       0.015
Stock_Log_Return_lag_11 0.0019      0.004      0.526      0.600      -0.005       0.009
Stock_Log_Return_lag_12 0.0046      0.004      1.289      0.200      -0.002       0.012
Stock_Log_Return_lag_13 0.0008      0.004      0.219      0.827      -0.006       0.008
Stock_Log_Return_lag_14 0.0014      0.004      0.383      0.702      -0.006       0.008
Stock_Log_Return_lag_15 0.0073      0.004      2.039      0.043       0.000       0.014
Stock_Log_Return_lag_16 -0.0021     0.004     -0.588      0.558      -0.009       0.005
Stock_Log_Return_lag_17 0.0056      0.004      1.562      0.121      -0.001       0.013
Stock_Log_Return_lag_18 0.0036      0.004      1.012      0.313      -0.003       0.011
Stock_Log_Return_lag_19 0.0039      0.004      1.104      0.272      -0.003       0.011
```

Image 11: Canadian Monthly OLS Regression

```
Stock_Log_Return_lag_20   0.0016      0.004      0.470      0.639      -0.005       0.009
Stock_Log_Return_lag_21   0.0032      0.003      0.906      0.366      -0.004       0.010
Stock_Log_Return_lag_22   0.0069      0.003      2.009      0.046       0.000       0.014
Stock_Log_Return_lag_23   0.0040      0.003      1.230      0.221      -0.002       0.011
Stock_Log_Return_lag_24   0.0046      0.003      1.395      0.165      -0.002       0.011
==============================================================================
Omnibus:                       48.609   Durbin-Watson:                   2.014
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              368.191
Skew:                          -0.813   Prob(JB):                     1.12e-80
Kurtosis:                      10.228   Cond. No.                         6.77
==============================================================================
```

Image 12: *Canadian Monthly OLS Regression*

In the first two figures, we observe the monthly OLS regression over the Canadian Stock Index and Industrial Production log. The "coefficient estimates" are mostly positive, indicating a positive correlation with the dependent variable. The "standard error" across all 24 lags is relatively low, suggesting that the independent variable has low variability. However, the "confidence intervals" for the majority of the lags include 0 in their range, indicating that the relationship between the independent and dependent variables is not as strong as it should be.

The "F-statistic" is quite high at 0.721, which leads us to accept the null hypothesis, again suggesting that the model is not statistically significant. Additionally, the "R-squared" value is 0.125, which means only 12.5% of the data is being explained by the model, a rather poor result.

When comparing with the paper, the original R-squared value was 0.24, which was also not a good sign for acceptance. However, the original model was statistically significant due to its p-value of 0.

Regarding "time instability," we applied the Augmented Dickey-Fuller test, which shows that the variables are stationary in nature. The Durbin-Watson test also returns a value near 2, indicating no autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.14, suggesting that the model does not suffer from multicollinearity.

```
                                 OLS Regression Results
===========================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                      0.267
Model:                                          OLS   Adj. R-squared:                 0.139
Method:                               Least Squares   F-statistic:                    2.091
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):            0.0562
Time:                                      23:17:12   Log-Likelihood:                198.03
No. Observations:                                55   AIC:                           -378.1
Df Residuals:                                    46   BIC:                           -360.0
Df Model:                                         8
Covariance Type:                          nonrobust
=========================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------------------
const                   -0.0009      0.001     -0.856      0.397      -0.003       0.001
Stock_Log_Return_lag_1   0.0032      0.004      0.738      0.464      -0.005       0.012
Stock_Log_Return_lag_2   0.0128      0.005      2.780      0.008       0.004       0.022
Stock_Log_Return_lag_3   0.0132      0.005      2.666      0.011       0.003       0.023
Stock_Log_Return_lag_4   0.0077      0.005      1.486      0.144      -0.003       0.018
Stock_Log_Return_lag_5   0.0083      0.005      1.616      0.113      -0.002       0.019
Stock_Log_Return_lag_6   0.0073      0.005      1.440      0.157      -0.003       0.018
Stock_Log_Return_lag_7   0.0076      0.005      1.503      0.140      -0.003       0.018
Stock_Log_Return_lag_8   0.0119      0.005      2.629      0.012       0.003       0.021
==============================================================================
Omnibus:                        4.766   Durbin-Watson:                   1.529
Prob(Omnibus):                  0.092   Jarque-Bera (JB):                3.708
Skew:                          -0.535   Prob(JB):                        0.157
Kurtosis:                       3.689   Cond. No.                         8.54
==============================================================================
```

Image 13: Canadian Quarterly OLS Regression

In the above figures, we observe the quarterly OLS regression over the log Canadian Stock Index return and log Industrial Production return. The "coefficient estimates" are mostly positive, indicating a positive correlation with the dependent variable. The "standard error" across all 8 lags is relatively low, suggesting that the independent variable has low variability. However, the "confidence intervals" for 5 of the lags include 0, showing that they don't have a strong correlation with the model. On the other hand, 3 of the lags do not include 0 within their range, indicating a strong correlation with the model.

The "F-statistic" is quite low at 0.0562, which leads us to reject the null hypothesis, suggesting that the model is statistically significant at the 1% and 5% significance levels. Additionally, the "R-squared" value is 0.267, meaning only 26.7% of the data is being explained by the model, which is acceptable.

When comparing with the paper, the original R-squared value was 0.30, which was also not very high. However, the original model was statistically significant at all levels due to its p-value of 0.

Regarding "time instability," we applied the Augmented Dickey-Fuller test, which shows that the variables are stationary in nature. The Durbin-Watson test also returns a value near 1.5, indicating positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.36, suggesting that the model does not suffer from multicollinearity.

```
                                 OLS Regression Results
============================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.267
Model:                                          OLS   Adj. R-squared:                  0.133
Method:                               Least Squares   F-statistic:                     1.999
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):              0.182
Time:                                      23:17:12   Log-Likelihood:                 61.748
No. Observations:                                14   AIC:                            -117.5
Df Residuals:                                    11   BIC:                            -115.6
Df Model:                                         2
Covariance Type:                          nonrobust
==========================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
const                 2.674e-05      0.001      0.030      0.977      -0.002       0.002
Stock_Log_Return_lag_1   0.0243      0.030      0.803      0.439      -0.042       0.091
Stock_Log_Return_lag_2   0.0601      0.030      1.988      0.072      -0.006       0.127
==============================================================================
Omnibus:                        0.629   Durbin-Watson:                   1.110
Prob(Omnibus):                  0.730   Jarque-Bera (JB):                0.340
Skew:                          -0.354   Prob(JB):                        0.844
Kurtosis:                       2.716   Cond. No.                         38.9
==============================================================================
```

Image 14: Canadian Yearly OLS Regression

In the above figures, we observe the yearly OLS regression over the log Canadian Stock Index return and log Industrial Production return. The "coefficient estimates" are all positive, indicating a positive correlation with the dependent variable. The "standard error" across all 2 lags is relatively low, suggesting that the independent variable has low variability. However, the "confidence intervals" for all the lags include 0, which shows that the variables do not have a strong correlation within their range, indicating a weak relationship.
The "F-statistic" is quite low at 0.18, leading us to accept the null hypothesis, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.267, meaning only 26.7% of the data is being explained by the model, which is acceptable but not ideal. When comparing with the paper, the original R-squared value was 0.86, which is much higher. However, the original model was statistically significant at all levels, with a p-value of 0.022. Regarding "time instability," we applied the Augmented Dickey-Fuller test, which shows that the variables are stationary in nature. The Durbin-Watson test also returns a value near 1.10, indicating positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.36, suggesting that the model does not suffer from multicollinearity.

```
                                        OLS Regression Results
==========================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.264
Model:                                          OLS   Adj. R-squared:                  0.122
Method:                               Least Squares   F-statistic:                     1.853
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):             0.0157
Time:                                      23:18:03   Log-Likelihood:                 399.79
No. Observations:                               149   AIC:                            -749.6
Df Residuals:                                   124   BIC:                            -674.5
Df Model:                                        24
Covariance Type:                          nonrobust
==========================================================================================
                             coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
const                      0.0008      0.002      0.520      0.604      -0.002       0.004
Stock_Log_Return_lag_1     0.0218      0.004      5.466      0.000       0.014       0.030
Stock_Log_Return_lag_2    -0.0057      0.004     -1.371      0.173      -0.014       0.003
Stock_Log_Return_lag_3    -0.0003      0.004     -0.063      0.950      -0.009       0.008
Stock_Log_Return_lag_4     0.0031      0.004      0.706      0.481      -0.006       0.012
Stock_Log_Return_lag_5    -0.0062      0.004     -1.417      0.159      -0.015       0.002
Stock_Log_Return_lag_6 -1.907e-06      0.004     -0.000      1.000      -0.009       0.009
Stock_Log_Return_lag_7    -0.0031      0.005     -0.662      0.509      -0.012       0.006
Stock_Log_Return_lag_8     0.0011      0.005      0.229      0.820      -0.009       0.011
Stock_Log_Return_lag_9    -0.0047      0.005     -0.941      0.349      -0.015       0.005
Stock_Log_Return_lag_10   -0.0025      0.005     -0.505      0.615      -0.012       0.007
Stock_Log_Return_lag_11    0.0001      0.005      0.024      0.981      -0.010       0.010
Stock_Log_Return_lag_12   -0.0025      0.005     -0.500      0.618      -0.012       0.007
Stock_Log_Return_lag_13    0.0005      0.005      0.101      0.920      -0.009       0.010
Stock_Log_Return_lag_14   -0.0018      0.005     -0.353      0.724      -0.012       0.008
Stock_Log_Return_lag_15    0.0030      0.005      0.596      0.552      -0.007       0.013
Stock_Log_Return_lag_16   -0.0039      0.005     -0.772      0.442      -0.014       0.006
Stock_Log_Return_lag_17    0.0008      0.005      0.155      0.877      -0.009       0.011
Stock_Log_Return_lag_18    0.0044      0.005      0.890      0.375      -0.005       0.014
Stock_Log_Return_lag_19   -0.0075      0.005     -1.503      0.135      -0.017       0.002
```

Image 15: USA Monthly OLS Regression

```
Stock_Log_Return_lag_20    0.0070      0.005      1.402      0.163      -0.003       0.017
Stock_Log_Return_lag_21    0.0027      0.005      0.552      0.582      -0.007       0.013
Stock_Log_Return_lag_22   -0.0043      0.005     -0.865      0.389      -0.014       0.005
Stock_Log_Return_lag_23   -0.0010      0.005     -0.213      0.832      -0.010       0.008
Stock_Log_Return_lag_24   -0.0023      0.005     -0.506      0.614      -0.011       0.007
==============================================================================
Omnibus:                      142.913   Durbin-Watson:                   1.840
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4364.284
Skew:                          -3.155   Prob(JB):                         0.00
Kurtosis:                      28.752   Cond. No.                         4.91
==============================================================================
```

Image 16: USA Monthly OLS Regression

In the above figures, we observe the monthly OLS regression over the log American Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of positive and negative values, indicating varying relationships with the dependent variable — some having a positive correlation and others having a negative correlation. The "standard error" across all 24 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all lags include 0, which indicates that the variables do not exhibit a strong correlation within their range, reflecting a weak relationship.

The p-value of the "F-statistic" is quite low at 0.015, leading us to reject the null hypothesis, which suggests that the model is statistically significant at any level. Additionally, the "R-squared" value is 0.264, meaning that only 26.4% of the variability in the data is explained by the model. While this is acceptable, it is not ideal.
When comparing with the paper, the original R-squared value was 0.36, which is slightly higher. However, the original model was statistically significant at all levels, with a p-value of 0.000.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.80, indicating positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.35, suggesting that the model does not suffer from multicollinearity.

```
                                OLS Regression Results
==========================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                      0.200
Model:                                          OLS   Adj. R-squared:                 0.055
Method:                               Least Squares   F-statistic:                    1.379
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):             0.232
Time:                                      23:18:03   Log-Likelihood:                188.96
No. Observations:                                53   AIC:                           -359.9
Df Residuals:                                    44   BIC:                           -342.2
Df Model:                                         8
Covariance Type:                          nonrobust
========================================================================================
                             coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------
const                   8.005e-05      0.001      0.075      0.941      -0.002       0.002
Stock_Log_Return_lag_1     0.0083      0.003      2.876      0.006       0.002       0.014
Stock_Log_Return_lag_2     0.0050      0.003      1.553      0.128      -0.001       0.012
Stock_Log_Return_lag_3     0.0028      0.003      0.833      0.410      -0.004       0.010
Stock_Log_Return_lag_4     0.0015      0.004      0.426      0.672      -0.006       0.009
Stock_Log_Return_lag_5     0.0043      0.004      1.198      0.237      -0.003       0.012
Stock_Log_Return_lag_6     0.0005      0.004      0.151      0.881      -0.007       0.008
Stock_Log_Return_lag_7    -0.0010      0.003     -0.292      0.771      -0.008       0.006
Stock_Log_Return_lag_8     0.0024      0.003      0.807      0.424      -0.004       0.008
==============================================================================
Omnibus:                        4.166   Durbin-Watson:                   1.513
Prob(Omnibus):                  0.125   Jarque-Bera (JB):                3.151
Skew:                          -0.448   Prob(JB):                        0.207
Kurtosis:                       3.791   Cond. No.                         5.83
==============================================================================
```

Image 17: USA Quarterly OLS Regression

In the above figures, we observe the quarterly OLS regression over the log American Stock Index return and log Industrial Production return. The "coefficient estimates" predominantly include positive and one negative value, indicating positive relationships with the dependent variable. The "standard error" across all 8 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for 7 lags include 0, which indicates that the variables do not show a strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is quite high at 0.232, leading us to accept the null hypothesis, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.200, meaning that only 20% of the variability in the data is explained by the model. While this is acceptable, it is far from ideal.

When comparing with the paper, the original R-squared value was 0.53, which is notably higher. Moreover, the original model was statistically significant at all levels, with a p-value of 0.000. Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.80, indicating positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.25, suggesting that the model does not suffer from multicollinearity.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.455
Model:                                  OLS   Adj. R-squared:                  0.356
Method:                       Least Squares   F-statistic:                     4.590
Date:                      Tue, 17 Dec 2024   Prob (F-statistic):             0.0355
Time:                              23:18:03   Log-Likelihood:                 63.825
No. Observations:                        14   AIC:                            -121.7
Df Residuals:                            11   BIC:                            -119.7
Df Model:                                 2
Covariance Type:                  nonrobust
==============================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const                   -0.0005      0.001     -0.569      0.581      -0.002       0.001
Stock_Log_Return_lag_1   0.0091      0.003      3.030      0.011       0.002       0.016
Stock_Log_Return_lag_2   0.0038      0.003      1.265      0.232      -0.003       0.010
==============================================================================
Omnibus:                        1.015   Durbin-Watson:                   1.458
Prob(Omnibus):                  0.602   Jarque-Bera (JB):                0.558
Skew:                           0.471   Prob(JB):                        0.756
Kurtosis:                       2.738   Cond. No.                         4.73
==============================================================================
```

Image 18: American Yearly OLS Regression

In the above figures, we observe the yearly OLS regression over the log American Stock Index return and log Industrial Production return. The "coefficient estimates" are all positive, indicating a positive relationship with the dependent variable. The "standard error" across the 2 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for 1 lag include 0, indicating that the variables do not show a strong correlation within their range, reflecting a weak relationship.

The p-value of the "F-statistic" is 0.0355, leading us to reject the null hypothesis at the 5% and 10% significance levels, suggesting that the model is statistically significant at some level. Additionally, the "R-squared" value is 0.455, meaning that 45.5% of the variability in the data is explained by the model. While this is acceptable, it is not ideal.

When compared with the paper, the original R-squared value was 0.84, which is notably higher. Furthermore, the original model was statistically significant at the 1% level, with a p-value of 0.052. Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.458, indicating positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.83, suggesting that the model does not suffer from multicollinearity.

OLS Regression Results

Dep. Variable:	Industrial_Production_Log_Change	R-squared:	0.262
Model:	OLS	Adj. R-squared:	-0.079
Method:	Least Squares	F-statistic:	0.7689
Date:	Wed, 18 Dec 2024	Prob (F-statistic):	0.756
Time:	22:39:44	Log-Likelihood:	197.18
No. Observations:	77	AIC:	-344.4
Df Residuals:	52	BIC:	-285.8
Df Model:	24		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-0.0024	0.003	-0.922	0.361	-0.008	0.003
Stock_Log_Return_lag_1	0.0003	0.006	0.047	0.963	-0.011	0.011
Stock_Log_Return_lag_2	0.0087	0.006	1.580	0.120	-0.002	0.020
Stock_Log_Return_lag_3	-0.0024	0.005	-0.451	0.654	-0.013	0.008
Stock_Log_Return_lag_4	-0.0026	0.006	-0.477	0.636	-0.014	0.009
Stock_Log_Return_lag_5	0.0015	0.006	0.261	0.795	-0.010	0.013
Stock_Log_Return_lag_6	-0.0017	0.006	-0.296	0.768	-0.013	0.010
Stock_Log_Return_lag_7	0.0015	0.006	0.265	0.792	-0.010	0.013
Stock_Log_Return_lag_8	-0.0031	0.006	-0.567	0.573	-0.014	0.008
Stock_Log_Return_lag_9	-0.0036	0.006	-0.618	0.539	-0.015	0.008
Stock_Log_Return_lag_10	-0.0018	0.006	-0.320	0.750	-0.013	0.010
Stock_Log_Return_lag_11	-0.0013	0.006	-0.224	0.824	-0.013	0.010
Stock_Log_Return_lag_12	0.0106	0.006	1.864	0.068	-0.001	0.022
Stock_Log_Return_lag_13	0.0048	0.006	0.846	0.401	-0.007	0.016
Stock_Log_Return_lag_14	0.0047	0.006	0.825	0.413	-0.007	0.016
Stock_Log_Return_lag_15	-0.0101	0.006	-1.769	0.083	-0.022	0.001
Stock_Log_Return_lag_16	-0.0040	0.006	-0.706	0.484	-0.015	0.007
Stock_Log_Return_lag_17	-0.0050	0.006	-0.897	0.374	-0.016	0.006
Stock_Log_Return_lag_18	-0.0028	0.006	-0.483	0.631	-0.014	0.009
Stock_Log_Return_lag_19	0.0077	0.006	1.354	0.182	-0.004	0.019

Image 18: Japan Monthly OLS Regression

Stock_Log_Return_lag_20	0.0014	0.006	0.253	0.801	-0.010	0.013
Stock_Log_Return_lag_21	-0.0026	0.006	-0.464	0.645	-0.014	0.009
Stock_Log_Return_lag_22	0.0046	0.006	0.798	0.428	-0.007	0.016
Stock_Log_Return_lag_23	0.0038	0.005	0.702	0.486	-0.007	0.015
Stock_Log_Return_lag_24	-0.0007	0.005	-0.132	0.895	-0.012	0.010

Omnibus:	25.809	Durbin-Watson:	1.986
Prob(Omnibus):	0.000	Jarque-Bera (JB):	53.258
Skew:	-1.161	Prob(JB):	2.72e-12
Kurtosis:	6.348	Cond. No.	2.90

Image 19: Japan Monthly OLS Regression

In the above figures, we observe the montly OLS regression over the log Japanese Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of positive and negative values, indicating a mixed relationship with the dependent variable. The "standard error" across the 24 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all 24 lags include 0, indicating that the variables do not show a strong correlation within their range, reflecting a weak relationship.

The p-value of the "F-statistic" is 0.756, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.262, meaning that 26.2% of the variability in the data is explained by the model. While this is somewhat acceptable, it is far from ideal.
When compared with the paper, the original R-squared value was 0.44, which is notably higher. Furthermore, the original model was statistically significant at all levels, with a p-value of 0.027. Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.986, indicating no significant autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.35, suggesting that the model does not suffer from multicollinearity.

```
Best Lag for Quarterly Data: 8 with R-squared: 0.3308307226923949
                            OLS Regression Results
==============================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.331
Model:                                OLS   Adj. R-squared:                  0.076
Method:                     Least Squares   F-statistic:                     1.298
Date:                    Wed, 18 Dec 2024   Prob (F-statistic):              0.297
Time:                            22:39:44   Log-Likelihood:                 86.733
No. Observations:                      30   AIC:                            -155.5
Df Residuals:                          21   BIC:                            -142.9
Df Model:                               8
Covariance Type:                nonrobust
==============================================================================
                         coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const                 -0.0025      0.003     -0.831      0.415      -0.009       0.004
Stock_Log_Return_lag_1  0.0071     0.010      0.733      0.472      -0.013       0.027
Stock_Log_Return_lag_2  0.0020     0.010      0.204      0.840      -0.019       0.023
Stock_Log_Return_lag_3  0.0008     0.009      0.087      0.931      -0.019       0.020
Stock_Log_Return_lag_4 -0.0004     0.010     -0.037      0.971      -0.021       0.020
Stock_Log_Return_lag_5  0.0066     0.010      0.662      0.515      -0.014       0.027
Stock_Log_Return_lag_6 -0.0023     0.010     -0.235      0.817      -0.023       0.018
Stock_Log_Return_lag_7  0.0199     0.007      2.736      0.012       0.005       0.035
Stock_Log_Return_lag_8 -0.0070     0.008     -0.922      0.367      -0.023       0.009
==============================================================================
Omnibus:                       12.030   Durbin-Watson:                   2.185
Prob(Omnibus):                  0.002   Jarque-Bera (JB):               12.837
Skew:                          -1.060   Prob(JB):                      0.00163
Kurtosis:                       5.403   Cond. No.                         4.09
==============================================================================
```

Image 20: Japan Quarterly OLS Regression

In the above figures, we observe the quarterly OLS regression over the log Japanese Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of positive and negative values, indicating a mixed relationship with the dependent variable. The "standard error" across the 8 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for 7 lags include 0, indicating that the variables do not show a strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is 0.297, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.331, meaning that 33.1% of the variability in the data is explained by the model. While this is somewhat acceptable, it is not ideal.

When compared with the paper, the original R-squared value was 0.56, which is significantly higher. Furthermore, the original model was statistically significant at all levels, with a p-value of 0.003. Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 2.185, indicating slightly negative autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.49, suggesting that the model does not suffer from multicollinearity.

```
Best Lag for Yearly Data: 2 with R-squared: 0.1636691784601233
                            OLS Regression Results
==============================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.164
Model:                                  OLS   Adj. R-squared:                 -0.022
Method:                       Least Squares   F-statistic:                    0.8806
Date:                      Wed, 18 Dec 2024   Prob (F-statistic):              0.447
Time:                              22:39:44   Log-Likelihood:                 45.840
No. Observations:                        12   AIC:                            -85.68
Df Residuals:                             9   BIC:                            -84.22
Df Model:                                 2
Covariance Type:                  nonrobust
==========================================================================================
                             coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
const                     -0.0002      0.002     -0.072      0.944      -0.005       0.005
Stock_Log_Return_lag_1     0.0102      0.008      1.248      0.244      -0.008       0.029
Stock_Log_Return_lag_2     0.0064      0.008      0.792      0.449      -0.012       0.025
==============================================================================
Omnibus:                        4.429   Durbin-Watson:                   2.395
Prob(Omnibus):                  0.109   Jarque-Bera (JB):                1.980
Skew:                          -0.977   Prob(JB):                        0.372
Kurtosis:                       3.376   Cond. No.                         5.33
==============================================================================
```

Image 21: *Japanese Yearly OLS Regression*

In the above figures, we observe the yearly OLS regression over the log Japanese Stock Index return and log Industrial Production return. The "coefficient estimates" are all positive, indicating a consistent positive relationship with the dependent variable. The "standard error" across the 2 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for both lags include 0, indicating that the variables do not show a statistically strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is 0.447, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.164, meaning that only 16.4% of the variability in the data is explained by the model. While this is somewhat acceptable, it falls short of providing robust explanatory power.
When compared with the paper, the original R-squared value was 0.69, which is significantly higher. Moreover, the original model was statistically significant at all levels, with a p-value of 0.002, showcasing better model performance and significance.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 2.395, indicating slightly negative autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.19, suggesting that the model does not suffer from multicollinearity.

```
Best Lag for Monthly Data: 24 with R-squared: 0.10098676067273449
                            OLS Regression Results
==============================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.101
Model:                                          OLS   Adj. R-squared:                 -0.074
Method:                               Least Squares   F-statistic:                    0.5757
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):              0.941
Time:                                      23:18:50   Log-Likelihood:                 328.49
No. Observations:                               148   AIC:                            -607.0
Df Residuals:                                   123   BIC:                            -532.0
Df Model:                                        24
Covariance Type:                          nonrobust
==============================================================================================
                              coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------------
const                      -0.0013      0.002     -0.553      0.582      -0.006       0.003
Stock_Log_Return_lag_1      0.0152      0.007      2.065      0.041       0.001       0.030
Stock_Log_Return_lag_2      0.0058      0.008      0.752      0.454      -0.009       0.021
Stock_Log_Return_lag_3     -0.0053      0.008     -0.641      0.522      -0.022       0.011
Stock_Log_Return_lag_4      0.0005      0.008      0.060      0.952      -0.016       0.017
Stock_Log_Return_lag_5     -0.0072      0.008     -0.874      0.384      -0.024       0.009
Stock_Log_Return_lag_6      0.0086      0.008      1.043      0.299      -0.008       0.025
Stock_Log_Return_lag_7      0.0045      0.008      0.544      0.587      -0.012       0.021
Stock_Log_Return_lag_8      0.0022      0.008      0.258      0.797      -0.014       0.019
Stock_Log_Return_lag_9      0.0030      0.008      0.352      0.726      -0.014       0.020
Stock_Log_Return_lag_10    -0.0035      0.008     -0.416      0.678      -0.020       0.013
Stock_Log_Return_lag_11     0.0024      0.009      0.284      0.777      -0.014       0.019
Stock_Log_Return_lag_12    -0.0023      0.009     -0.268      0.790      -0.019       0.015
Stock_Log_Return_lag_13    -0.0084      0.009     -0.987      0.326      -0.025       0.008
Stock_Log_Return_lag_14     0.0005      0.009      0.061      0.951      -0.016       0.017
Stock_Log_Return_lag_15    -0.0017      0.008     -0.201      0.841      -0.018       0.015
Stock_Log_Return_lag_16     0.0034      0.008      0.400      0.690      -0.013       0.020
Stock_Log_Return_lag_17    -0.0040      0.008     -0.472      0.638      -0.021       0.013
Stock_Log_Return_lag_18    -0.0072      0.008     -0.861      0.391      -0.024       0.009
```

Image 22: Germany Monthly OLS Regression

```
Stock_Log_Return_lag_19    -0.0107      0.008     -1.280      0.203      -0.027       0.006
Stock_Log_Return_lag_20    -0.0031      0.008     -0.377      0.707      -0.020       0.013
Stock_Log_Return_lag_21     0.0058      0.008      0.700      0.485      -0.011       0.022
Stock_Log_Return_lag_22    -0.0040      0.008     -0.487      0.627      -0.020       0.012
Stock_Log_Return_lag_23    -0.0048      0.008     -0.590      0.557      -0.021       0.011
Stock_Log_Return_lag_24    -0.0107      0.008     -1.389      0.167      -0.026       0.005
==============================================================================
Omnibus:                      152.598   Durbin-Watson:                   1.959
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             5514.579
Skew:                          -3.466   Prob(JB):                         0.00
Kurtosis:                      32.090   Cond. No.                         5.72
==============================================================================
```

Image 22: Germany Monthly OLS Regression

In the above figures, we observe the monthly OLS regression over the log German Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of positive and negative values, indicating a varied relationship with the dependent variable. The "standard error" across the 24 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for 23 out of the 24 lags include 0, with one exception, indicating that the

variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is 0.941, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.101, meaning that only 10.1% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.
When compared with the paper, the original R-squared value was 0.38, which is notably higher. Furthermore, the original model was statistically significant at all levels, with a p-value of 0.00, highlighting stronger model performance and significance in the paper.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.959, indicating slightly positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.12, suggesting that the model does not suffer from multicollinearity.

```
Best Lag for Quarterly Data: 8 with R-squared: 0.07296539518044987
                            OLS Regression Results
==============================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.073
Model:                                          OLS   Adj. R-squared:                 -0.100
Method:                               Least Squares   F-statistic:                    0.4231
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):              0.901
Time:                                      23:18:50   Log-Likelihood:                 173.37
No. Observations:                                52   AIC:                            -328.7
Df Residuals:                                    43   BIC:                            -311.2
Df Model:                                         8
Covariance Type:                          nonrobust
============================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------
const                   -0.0010      0.001     -0.692      0.492      -0.004       0.002
Stock_Log_Return_lag_1   0.0066      0.005      1.241      0.221      -0.004       0.017
Stock_Log_Return_lag_2   0.0034      0.006      0.604      0.549      -0.008       0.015
Stock_Log_Return_lag_3   0.0024      0.006      0.409      0.685      -0.009       0.014
Stock_Log_Return_lag_4   0.0013      0.006      0.220      0.827      -0.011       0.013
Stock_Log_Return_lag_5  -0.0001      0.006     -0.022      0.983      -0.012       0.012
Stock_Log_Return_lag_6  -0.0046      0.006     -0.777      0.441      -0.016       0.007
Stock_Log_Return_lag_7  -0.0057      0.006     -0.990      0.328      -0.017       0.006
Stock_Log_Return_lag_8   0.0007      0.005      0.131      0.897      -0.010       0.012
==============================================================================
Omnibus:                       40.719   Durbin-Watson:                   0.893
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              122.573
Skew:                          -2.218   Prob(JB):                     2.42e-27
Kurtosis:                       9.075   Cond. No.                         7.34
==============================================================================
```

Image 23: Germany Quarterly OLS Regression

In the above figures, we observe the quarterly OLS regression over the log German Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of positive and negative values, indicating a varied relationship with the dependent variable. The "standard error" across the 8 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all 8 lags include 0, indicating that the variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.

The p-value of the "F-statistic" is 0.901, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.073, meaning that only 7.3% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.
When compared with the paper, the original R-squared value was 0.35, which is notably higher. Furthermore, the original model was statistically significant at all levels, with a p-value of 0.001, highlighting stronger model performance and significance in the paper.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 0.893, indicating a positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.07, suggesting that the model does not suffer from multicollinearity.

```
Best Lag for Yearly Data: 2 with R-squared: 0.03506502247217802
                            OLS Regression Results
==============================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.035
Model:                                    OLS   Adj. R-squared:                 -0.140
Method:                         Least Squares   F-statistic:                    0.1999
Date:                        Tue, 17 Dec 2024   Prob (F-statistic):              0.822
Time:                                23:18:50   Log-Likelihood:                 55.058
No. Observations:                          14   AIC:                            -104.1
Df Residuals:                              11   BIC:                            -102.2
Df Model:                                   2
Covariance Type:                    nonrobust
==============================================================================
                          coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const                6.593e-06      0.001      0.005      0.996      -0.003       0.003
Stock_Log_Return_lag_1  0.0117      0.019      0.630      0.542      -0.029       0.053
Stock_Log_Return_lag_2  0.0017      0.019      0.090      0.930      -0.039       0.043
==============================================================================
Omnibus:                        2.055   Durbin-Watson:                   2.258
Prob(Omnibus):                  0.358   Jarque-Bera (JB):                0.361
Skew:                          -0.133   Prob(JB):                        0.835
Kurtosis:                       3.741   Cond. No.                         13.4
==============================================================================
```

Image 24: *German Yearly OLS Regression*

In the above figures, we observe the yearly OLS regression over the log German Stock Index return and log Industrial Production return. The "coefficient estimates" are all positive values, indicating a positive relationship with the dependent variable. The "standard error" across the 2 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all 2 lags include 0, indicating that the variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is 0.822, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.035, meaning that only 3.5% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.

When compared with the paper, the original R-squared value was 0.71, which is notably higher. Furthermore, the original model was statistically significant at all levels, with a p-value of 0.005, highlighting stronger model performance and significance in the paper.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 2.258, indicating a positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.03, suggesting that the model does not suffer from multicollinearity.

UK

```
Monthly Data - OLS Model Summary with 24 Lags:
                                 OLS Regression Results
==========================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                     0.110
Model:                                          OLS   Adj. R-squared:               -0.042
Method:                               Least Squares   F-statistic:                  0.7240
Date:                              Wed, 18 Dec 2024   Prob (F-statistic):            0.820
Time:                                      22:49:17   Log-Likelihood:               417.10
No. Observations:                               166   AIC:                          -784.2
Df Residuals:                                   141   BIC:                          -706.4
Df Model:                                        24
Covariance Type:                          nonrobust
==========================================================================================
                              coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
const                      -0.0001      0.002     -0.080      0.937      -0.003      0.003
Stock_Log_Return_lag_1      0.0142      0.009      1.537      0.127      -0.004      0.032
Stock_Log_Return_lag_2     -0.0166      0.009     -1.776      0.078      -0.035      0.002
Stock_Log_Return_lag_3     -0.0206      0.009     -2.192      0.030      -0.039     -0.002
Stock_Log_Return_lag_4     -0.0178      0.010     -1.828      0.070      -0.037      0.001
Stock_Log_Return_lag_5      0.0050      0.010      0.476      0.634      -0.016      0.026
Stock_Log_Return_lag_6      0.0152      0.011      1.430      0.155      -0.006      0.036
Stock_Log_Return_lag_7     -0.0094      0.011     -0.868      0.387      -0.031      0.012
Stock_Log_Return_lag_8     -0.0045      0.011     -0.417      0.677      -0.026      0.017
Stock_Log_Return_lag_9      0.0044      0.011      0.381      0.703      -0.018      0.027
Stock_Log_Return_lag_10     0.0031      0.011      0.270      0.787      -0.020      0.026
Stock_Log_Return_lag_11    -0.0061      0.012     -0.532      0.595      -0.029      0.017
Stock_Log_Return_lag_12     0.0017      0.011      0.150      0.881      -0.021      0.024
Stock_Log_Return_lag_13     0.0043      0.011      0.376      0.707      -0.018      0.027
Stock_Log_Return_lag_14    -0.0056      0.011     -0.498      0.620      -0.028      0.017
Stock_Log_Return_lag_15    -0.0056      0.011     -0.493      0.623      -0.028      0.017
Stock_Log_Return_lag_16    -0.0069      0.011     -0.607      0.545      -0.029      0.015
Stock_Log_Return_lag_17     0.0075      0.011      0.656      0.513      -0.015      0.030
Stock_Log_Return_lag_18    -0.0017      0.011     -0.145      0.885      -0.024      0.021
Stock_Log_Return_lag_19     0.0028      0.011      0.243      0.808      -0.020      0.025
```

Image 24: UK Monthly OLS Regression

Stock_Log_Return_lag_20	0.0048	0.011	0.424	0.672	-0.018	0.027
Stock_Log_Return_lag_21	0.0038	0.011	0.330	0.742	-0.019	0.026
Stock_Log_Return_lag_22	-0.0027	0.011	-0.240	0.811	-0.025	0.020
Stock_Log_Return_lag_23	0.0016	0.011	0.145	0.885	-0.021	0.024
Stock_Log_Return_lag_24	-0.0006	0.011	-0.057	0.955	-0.023	0.022

Omnibus:	132.421	Durbin-Watson:	1.831
Prob(Omnibus):	0.000	Jarque-Bera (JB):	3910.025
Skew:	-2.450	Prob(JB):	0.00
Kurtosis:	26.266	Cond. No.	9.98

Image 25: UK Monthly OLS Regression

In the above figures, we observe the monthly OLS regression over the log UK Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of positive and negative values, indicating a mixed relationship with the dependent variable. The "standard error" across the 24 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all 24 lags include 0, indicating that the variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is 0.820, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.110, meaning that only 11% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.
When compared with the paper, the original R-squared value was 0.18, which is slightly higher. Furthermore, the original model was statistically significant at all levels, with a p-value of 0.016, highlighting stronger model performance and significance in the paper.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.83, indicating a positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.12, suggesting that the model does not suffer from multicollinearity.

```
Quarterly Data - OLS Model Summary with 8 Lags:
                                  OLS Regression Results
==============================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                         0.110
Model:                                          OLS   Adj. R-squared:                   -0.041
Method:                               Least Squares   F-statistic:                      0.7280
Date:                              Wed, 18 Dec 2024   Prob (F-statistic):                0.666
Time:                                      22:49:17   Log-Likelihood:                   206.15
No. Observations:                                56   AIC:                              -394.3
Df Residuals:                                    47   BIC:                              -376.1
Df Model:                                         8
Covariance Type:                          nonrobust
============================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------
const                -2.019e-05      0.001     -0.023      0.982      -0.002       0.002
Stock_Log_Return_lag_1  -0.0059      0.009     -0.699      0.488      -0.023       0.011
Stock_Log_Return_lag_2  -0.0123      0.009     -1.439      0.157      -0.029       0.005
Stock_Log_Return_lag_3   0.0025      0.009      0.285      0.777      -0.015       0.020
Stock_Log_Return_lag_4  -0.0012      0.009     -0.137      0.892      -0.019       0.016
Stock_Log_Return_lag_5  -0.0083      0.009     -0.943      0.350      -0.026       0.009
Stock_Log_Return_lag_6   0.0103      0.009      1.175      0.246      -0.007       0.028
Stock_Log_Return_lag_7   0.0025      0.009      0.294      0.770      -0.015       0.020
Stock_Log_Return_lag_8  -0.0011      0.009     -0.128      0.899      -0.019       0.016
==============================================================================
Omnibus:                        8.051   Durbin-Watson:                   1.927
Prob(Omnibus):                  0.018   Jarque-Bera (JB):               11.179
Skew:                           0.429   Prob(JB):                      0.00374
Kurtosis:                       5.014   Cond. No.                         12.2
==============================================================================
```

Image 26: UK Quarterly OLS Regression

In the above figures, we observe the quarterly OLS regression over the log UK Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of positive and negative values, indicating a mixed relationship with the dependent variable. The "standard error" across the 8 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all 8 lags include 0, indicating that the variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is 0.666, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.110, meaning that only 11% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.
When compared with the paper, the original R-squared value was 0.28, which is slightly higher. Furthermore, the original model was statistically significant at all levels, with a p-value of 0.002, highlighting stronger model performance and significance in the paper.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.927, indicating a positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.12, suggesting that the model does not suffer from multicollinearity.

```
Yearly Data - OLS Model Summary with 2 Lags:
                                 OLS Regression Results
==============================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                         0.104
Model:                                          OLS   Adj. R-squared:                   -0.059
Method:                               Least Squares   F-statistic:                      0.6350
Date:                              Wed, 18 Dec 2024   Prob (F-statistic):                0.548
Time:                                      22:49:17   Log-Likelihood:                   62.583
No. Observations:                                14   AIC:                              -119.2
Df Residuals:                                    11   BIC:                              -117.2
Df Model:                                         2
Covariance Type:                          nonrobust
============================================================================================
                            coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------
const                  9.888e-07      0.001      0.001      0.999      -0.002       0.002
Stock_Log_Return_lag_1   -0.0164      0.020     -0.807      0.437      -0.061       0.028
Stock_Log_Return_lag_2   -0.0145      0.020     -0.715      0.490      -0.059       0.030
==============================================================================
Omnibus:                        1.919   Durbin-Watson:                   1.158
Prob(Omnibus):                  0.383   Jarque-Bera (JB):                0.482
Skew:                           0.404   Prob(JB):                        0.786
Kurtosis:                       3.418   Cond. No.                         25.3
==============================================================================
```

Image 27: UK Yearly OLS Regression

In the above figures, we observe the yearly OLS regression over the log UK Stock Index return and log Industrial Production return. The "coefficient estimates" are all positive values, indicating a positive relationship with the dependent variable. The "standard error" across the 2 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for both 2 lags include 0, indicating that the variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is 0.548, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.104, meaning that only 10.4% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.
When compared with the paper, the original R-squared value was 0.83, which is considerably higher. Furthermore, the original model was statistically significant at all levels, with a p-value of 0.000, highlighting stronger model performance and significance in the paper.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.158, indicating a positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.11, suggesting that the model does not suffer from multicollinearity.

```
Best Lag for Monthly Data: 24 with R-squared: 0.07280083058963749
                            OLS Regression Results
==============================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.073
Model:                                          OLS   Adj. R-squared:                 -0.107
Method:                               Least Squares   F-statistic:                    0.4057
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):              0.994
Time:                                      23:19:44   Log-Likelihood:                 299.12
No. Observations:                               149   AIC:                            -548.2
Df Residuals:                                   124   BIC:                            -473.1
Df Model:                                        24
Covariance Type:                          nonrobust
=============================================================================================
                             coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------------------
const                     -0.0017      0.003     -0.569      0.570      -0.008       0.004
Stock_Log_Return_lag_1     0.0121      0.008      1.559      0.121      -0.003       0.027
Stock_Log_Return_lag_2    -0.0065      0.008     -0.841      0.402      -0.022       0.009
Stock_Log_Return_lag_3    -0.0089      0.008     -1.090      0.278      -0.025       0.007
Stock_Log_Return_lag_4    -0.0056      0.009     -0.650      0.517      -0.022       0.011
Stock_Log_Return_lag_5    -0.0112      0.009     -1.267      0.207      -0.029       0.006
Stock_Log_Return_lag_6    -0.0035      0.009     -0.393      0.695      -0.021       0.014
Stock_Log_Return_lag_7     0.0050      0.009      0.565      0.573      -0.013       0.023
Stock_Log_Return_lag_8     0.0040      0.009      0.441      0.660      -0.014       0.022
Stock_Log_Return_lag_9    -0.0028      0.009     -0.307      0.759      -0.021       0.015
Stock_Log_Return_lag_10   -0.0003      0.009     -0.035      0.972      -0.019       0.018
Stock_Log_Return_lag_11   -0.0004      0.009     -0.043      0.966      -0.019       0.018
Stock_Log_Return_lag_12   -0.0011      0.009     -0.116      0.908      -0.019       0.017
Stock_Log_Return_lag_13    0.0016      0.009      0.178      0.859      -0.017       0.020
Stock_Log_Return_lag_14   -0.0003      0.009     -0.038      0.970      -0.019       0.018
Stock_Log_Return_lag_15   -0.0082      0.009     -0.886      0.377      -0.027       0.010
Stock_Log_Return_lag_16    0.0010      0.009      0.105      0.917      -0.017       0.019
Stock_Log_Return_lag_17   -0.0044      0.009     -0.480      0.632      -0.023       0.014
Stock_Log_Return_lag_18    0.0007      0.009      0.077      0.938      -0.017       0.019
Stock_Log_Return_lag_19   -0.0087      0.009     -0.954      0.342      -0.027       0.009
```

Image 28: *France Monthly OLS Regression*

```
Stock_Log_Return_lag_20    0.0015      0.009      0.159      0.874      -0.017       0.020
Stock_Log_Return_lag_21   -0.0098      0.009     -1.064      0.289      -0.028       0.008
Stock_Log_Return_lag_22    0.0050      0.009      0.545      0.587      -0.013       0.023
Stock_Log_Return_lag_23   -0.0022      0.009     -0.243      0.808      -0.020       0.016
Stock_Log_Return_lag_24    0.0027      0.009      0.293      0.770      -0.016       0.021
==============================================================================
Omnibus:                      110.595   Durbin-Watson:                   1.773
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3370.860
Skew:                          -2.074   Prob(JB):                         0.00
Kurtosis:                      25.929   Cond. No.                         4.38
==============================================================================
```

Image 29: *France Monthly OLS Regression*

In the above figures, we observe the monthly OLS regression over the log French Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of positive and negative values, indicating a mixed relationship with the dependent variable. The "standard error" across the 24 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all 24 lags include 0, indicating that the variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.

The p-value of the "F-statistic" is 0.994, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.073, meaning that only 7.3% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.
When compared with the paper, the original R-squared value was 0.26, which is notably higher. Furthermore, the original model was also not statistically significant at all levels, with a p-value of 0.365, highlighting weak model performance and significance in the paper as well.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.773, indicating a positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.078, suggesting that the model does not suffer from multicollinearity.

```
Best Lag for Quarterly Data: 8 with R-squared: 0.04872139907066153
                            OLS Regression Results
==============================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                     0.049
Model:                                          OLS   Adj. R-squared:               -0.128
Method:                               Least Squares   F-statistic:                  0.2753
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):            0.971
Time:                                      23:19:44   Log-Likelihood:               163.33
No. Observations:                                52   AIC:                          -308.7
Df Residuals:                                    43   BIC:                          -291.1
Df Model:                                         8
Covariance Type:                          nonrobust
============================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------
const                   -0.0016      0.002     -0.980      0.332      -0.005       0.002
Stock_Log_Return_lag_1  -0.0020      0.006     -0.319      0.752      -0.015       0.011
Stock_Log_Return_lag_2  -0.0046      0.007     -0.645      0.522      -0.019       0.010
Stock_Log_Return_lag_3   0.0007      0.007      0.093      0.926      -0.014       0.015
Stock_Log_Return_lag_4   0.0011      0.007      0.146      0.884      -0.014       0.016
Stock_Log_Return_lag_5  -0.0044      0.007     -0.605      0.548      -0.019       0.010
Stock_Log_Return_lag_6  -0.0081      0.007     -1.125      0.267      -0.023       0.006
Stock_Log_Return_lag_7  -0.0053      0.007     -0.733      0.468      -0.020       0.009
Stock_Log_Return_lag_8 8.849e-05     0.007      0.012      0.990      -0.015       0.015
==============================================================================
Omnibus:                       57.722   Durbin-Watson:                   1.847
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              425.393
Skew:                          -2.809   Prob(JB):                     4.24e-93
Kurtosis:                      15.836   Cond. No.                         5.65
==============================================================================
```

Image 30: France Quarterly OLS Regression

In the above figures, we observe the quarterly OLS regression over the log French Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of positive and negative values, indicating a varied relationship with the dependent variable. The "standard error" across the 8 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all 8 lags include 0, indicating that the variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is 0.971, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-

squared" value is 0.049, meaning that only 4.9% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.
When compared with the paper, the original R-squared value was 0.26, which is notably higher. Furthermore, the original model was statistically significant at all levels, with a p-value of 0.015, highlighting a strong model performance and significance in the paper.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.847, indicating positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.051, suggesting that the model does not suffer from multicollinearity.

```
Best Lag for Yearly Data: 2 with R-squared: 0.001913266591000884
                              OLS Regression Results
==============================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                         0.002
Model:                                          OLS   Adj. R-squared:                   -0.180
Method:                               Least Squares   F-statistic:                     0.01054
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):                0.990
Time:                                      23:19:44   Log-Likelihood:                   60.183
No. Observations:                                14   AIC:                              -114.4
Df Residuals:                                    11   BIC:                              -112.4
Df Model:                                         2
Covariance Type:                          nonrobust
============================================================================================
                             coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------
const                     -0.0009      0.001     -0.868      0.404      -0.003       0.001
Stock_Log_Return_lag_1    -0.0011      0.009     -0.122      0.905      -0.020       0.018
Stock_Log_Return_lag_2    -0.0006      0.009     -0.070      0.946      -0.020       0.019
==============================================================================
Omnibus:                        5.824   Durbin-Watson:                   1.756
Prob(Omnibus):                  0.054   Jarque-Bera (JB):                2.637
Skew:                          -0.771   Prob(JB):                        0.268
Kurtosis:                       4.464   Cond. No.                         9.11
==============================================================================
```

Image 31: *French Yearly OLS Regression*

```
Best Lag for Monthly Data: 24 with R-squared: 0.17507684767450393
                                OLS Regression Results
===========================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                       0.175
Model:                                          OLS   Adj. R-squared:                 -0.003
Method:                               Least Squares   F-statistic:                    0.9816
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):              0.496
Time:                                      23:20:08   Log-Likelihood:                 369.58
No. Observations:                               136   AIC:                            -689.2
Df Residuals:                                   111   BIC:                            -616.3
Df Model:                                        24
Covariance Type:                          nonrobust
==========================================================================================
                            coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
const                    -0.0017      0.002     -1.043      0.299      -0.005       0.001
Stock_Log_Return_lag_1    0.0071      0.006      1.292      0.199      -0.004       0.018
Stock_Log_Return_lag_2   -0.0052      0.006     -0.866      0.388      -0.017       0.007
Stock_Log_Return_lag_3    0.0049      0.006      0.822      0.413      -0.007       0.017
Stock_Log_Return_lag_4   -0.0048      0.006     -0.794      0.429      -0.017       0.007
Stock_Log_Return_lag_5    0.0065      0.006      1.098      0.275      -0.005       0.018
Stock_Log_Return_lag_6   -0.0035      0.006     -0.593      0.554      -0.015       0.008
Stock_Log_Return_lag_7   -0.0003      0.006     -0.047      0.962      -0.013       0.012
Stock_Log_Return_lag_8    0.0012      0.006      0.192      0.848      -0.012       0.014
Stock_Log_Return_lag_9    0.0025      0.007      0.390      0.697      -0.010       0.015
Stock_Log_Return_lag_10  -0.0091      0.007     -1.403      0.163      -0.022       0.004
Stock_Log_Return_lag_11   0.0020      0.006      0.303      0.762      -0.011       0.015
Stock_Log_Return_lag_12  -0.0006      0.006     -0.089      0.929      -0.013       0.012
Stock_Log_Return_lag_13  -0.0038      0.006     -0.596      0.553      -0.016       0.009
Stock_Log_Return_lag_14   0.0028      0.006      0.430      0.668      -0.010       0.016
Stock_Log_Return_lag_15   0.0011      0.007      0.165      0.869      -0.012       0.014
Stock_Log_Return_lag_16  -0.0185      0.007     -2.774      0.007      -0.032      -0.005
Stock_Log_Return_lag_17   0.0184      0.007      2.754      0.007       0.005       0.032
Stock_Log_Return_lag_18  -0.0054      0.006     -0.849      0.398      -0.018       0.007
Stock_Log_Return_lag_19  -0.0104      0.006     -1.707      0.091      -0.022       0.002
```

Image 32: Italian Monthly OLS Regression

```
Stock_Log_Return_lag_20   0.0074      0.006      1.182      0.240      -0.005       0.020
Stock_Log_Return_lag_21   0.0082      0.006      1.268      0.207      -0.005       0.021
Stock_Log_Return_lag_22  -0.0185      0.006     -2.856      0.005      -0.031      -0.006
Stock_Log_Return_lag_23   0.0123      0.007      1.843      0.068      -0.001       0.026
Stock_Log_Return_lag_24  -0.0049      0.006     -0.798      0.427      -0.017       0.007
==============================================================================
Omnibus:                        1.863   Durbin-Watson:                   2.202
Prob(Omnibus):                  0.394   Jarque-Bera (JB):                1.699
Skew:                          -0.274   Prob(JB):                        0.428
Kurtosis:                       2.979   Cond. No.                         7.95
==============================================================================
```

Image 33: Italian Monthly OLS Regression

In the above figures, we observe the monthly OLS regression over the log Italian Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of negative and positive values, indicating a mixed relationship with the dependent variable. The "standard error" across the 24 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all 24 lags include 0, indicating that the variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.

The p-value of the "F-statistic" is 0.496, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.175, meaning that only 17.5% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.
When compared with the paper, the original R-squared value was 0.29, which is notably higher. Furthermore, the original model was not statistically significant at all levels, with a p-value of 0.411, highlighting a weak model performance and significance in the paper.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 2.202, indicating negative autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.21, suggesting that the model does not suffer from multicollinearity.

```
Best Lag for Quarterly Data: 8 with R-squared: 0.10444284134034387
                            OLS Regression Results
==============================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                        0.104
Model:                                          OLS   Adj. R-squared:                  -0.079
Method:                               Least Squares   F-statistic:                     0.5685
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):               0.797
Time:                                      23:20:08   Log-Likelihood:                  128.32
No. Observations:                                48   AIC:                             -238.6
Df Residuals:                                    39   BIC:                             -221.8
Df Model:                                         8
Covariance Type:                          nonrobust
============================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------
const                   -0.0044      0.003     -1.536      0.133      -0.010       0.001
Stock_Log_Return_lag_1  -0.0047      0.010     -0.469      0.642      -0.025       0.016
Stock_Log_Return_lag_2   0.0066      0.010      0.650      0.520      -0.014       0.027
Stock_Log_Return_lag_3   0.0102      0.010      0.989      0.329      -0.011       0.031
Stock_Log_Return_lag_4   0.0056      0.010      0.546      0.588      -0.015       0.026
Stock_Log_Return_lag_5  -0.0025      0.010     -0.243      0.810      -0.023       0.018
Stock_Log_Return_lag_6  -0.0013      0.011     -0.119      0.906      -0.023       0.020
Stock_Log_Return_lag_7  -0.0101      0.011     -0.954      0.346      -0.032       0.011
Stock_Log_Return_lag_8   0.0133      0.011      1.245      0.221      -0.008       0.035
==============================================================================
Omnibus:                       61.845   Durbin-Watson:                   1.027
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              477.460
Skew:                          -3.283   Prob(JB):                    2.09e-104
Kurtosis:                      16.986   Cond. No.                         4.95
==============================================================================
```

Image 34: Italian Quarterly OLS Regression

In the above figures, we observe the quarterly OLS regression over the log Italian Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of negative and positive values, indicating a mixed relationship with the dependent variable. The "standard error" across the 8 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all 8 lags include 0, indicating that the variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is 0.797, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-

squared" value is 0.104, meaning that only 10.4% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.
When compared with the paper, the original R-squared value was 0.24, which is notably higher. Furthermore, the original model was not statistically significant at all levels, with a p-value of 0.164, highlighting a weak model performance and significance in the paper.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.027, indicating positive autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.11, suggesting that the model does not suffer from multicollinearity.

```
Best Lag for Yearly Data: 2 with R-squared: 0.14736585350086984
                                     OLS Regression Results
==============================================================================================
Dep. Variable:     Industrial_Production_Log_Change   R-squared:                         0.147
Model:                                          OLS   Adj. R-squared:                   -0.008
Method:                               Least Squares   F-statistic:                      0.9506
Date:                              Tue, 17 Dec 2024   Prob (F-statistic):                0.416
Time:                                      23:20:08   Log-Likelihood:                   55.804
No. Observations:                                14   AIC:                              -105.6
Df Residuals:                                    11   BIC:                              -103.7
Df Model:                                         2
Covariance Type:                          nonrobust
============================================================================================
                            coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------
const                    -0.0017      0.001     -1.181      0.263      -0.005       0.001
Stock_Log_Return_lag_1    0.0053      0.004      1.237      0.242      -0.004       0.015
Stock_Log_Return_lag_2   -0.0028      0.004     -0.647      0.531      -0.012       0.007
==============================================================================
Omnibus:                        7.436   Durbin-Watson:                   1.836
Prob(Omnibus):                  0.024   Jarque-Bera (JB):                3.803
Skew:                          -1.029   Prob(JB):                        0.149
Kurtosis:                       4.512   Cond. No.                         3.25
==============================================================================
```

Image 35: Italian Yearly OLS Regression

In the above figures, we observe the yearly OLS regression over the log Italian Stock Index return and log Industrial Production return. The "coefficient estimates" are a mix of negative and positive values, indicating a mixed relationship with the dependent variable. The "standard error" across the 2 lags is relatively low, suggesting that the independent variable exhibits low variability. However, the "confidence intervals" for all 2 lags include 0, indicating that the variables generally do not show a statistically strong correlation within their range, reflecting a weak relationship.
The p-value of the "F-statistic" is 0.416, leading us to accept the null hypothesis at all significance levels, suggesting that the model is not statistically significant at any level. Additionally, the "R-squared" value is 0.147, meaning that only 14.7% of the variability in the data is explained by the model. This indicates that the model's explanatory power is quite limited.
When compared with the paper, the original R-squared value was 0.60, which is notably higher. Furthermore, the original model was not statistically significant at all levels, with a p-value of 0.276, highlighting a weak model performance and significance in the paper.
Regarding "time instability," the Augmented Dickey-Fuller test confirms that the variables are stationary in nature. The Durbin-Watson test returns a value near 1.836, indicating positive

autocorrelation. Lastly, for multicollinearity, the Variance Inflation Factor (VIF) is 1.17, suggesting that the model does not suffer from multicollinearity.

Residual Diagonstic and correction

Below are the results of the statistical tests applied to each of the G7 countries. We conducted tests for normality, serial correlation, and heteroskedasticity, as well as the Ramsey RESET test.
For normality, the Jarque-Bera test was used. For serial correlation, we applied the Ljung-Box test, and for heteroskedasticity, the Breusch-Pagan test was implemented.
These tests were conducted across three categories: monthly, quarterly, and yearly data for each country. At the end, we compared all the results to provide a comprehensive analysis.

```
-------- Monthly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 3.3741, p-value: 0.0000

Serial Correlation Test (Ljung-Box):
       lb_stat      lb_pvalue
10  734.188299  2.862070e-151

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 20.7828, p-value: 0.6515

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 1.1330, p-value: 0.5675
```

Image 36: Monthly Report for Canada

```
-------- Quarterly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 3.1249, p-value: 0.0057

Serial Correlation Test (Ljung-Box):
      lb_stat     lb_pvalue
10  77.545456  1.517222e-12

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 5.8207, p-value: 0.6673

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 0.1530, p-value: 0.9264
```

Image 37: Quarterly Report for Canada

```
-------- Yearly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 1.7593, p-value: 0.2210

Serial Correlation Test (Ljung-Box):
      lb_stat  lb_pvalue
10  32.908389   0.000282

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 0.7114, p-value: 0.7007

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 0.4629, p-value: 0.7934
```

Image 38: Yearly Report for Canada

```
-------- Monthly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 3.4032, p-value: 0.0000

Serial Correlation Test (Ljung-Box):
       lb_stat     lb_pvalue
10  123.440056  1.012564e-21

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 36.3153, p-value: 0.0511

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 124.6595, p-value: 0.0000
```

Image 39: Monthly Report for USA

```
-------- Quarterly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 0.8256, p-value: 0.6082

Serial Correlation Test (Ljung-Box):
      lb_stat  lb_pvalue
10  16.967782    0.07508

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 5.9542, p-value: 0.6524

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 43.4761, p-value: 0.0000
```

Image 40: Quarterly Report for USA

```
-------- Yearly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 0.4527, p-value: 0.7686

Serial Correlation Test (Ljung-Box):
      lb_stat  lb_pvalue
10  11.276863   0.336359

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 0.6385, p-value: 0.7267

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 2.8646, p-value: 0.2388
```

Image 41: Yearly Report for USA

```
-------- Monthly Data Residual Diagnostics for Japan --------

Normality Test (Shapiro-Wilk):
Statistic: 0.9764, p-value: 0.6554

Normality Test (Jarque-Bera):
Statistic: 0.8680, p-value: 0.6479

Serial Correlation Test (Ljung-Box):
Final Ljung-Box p-value: 0.0003

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 22.4375, p-value: 0.3173
```

Image 42: Monthly Report for Japan

```
-------- Quarterly Data Residual Diagnostics for Japan --------

Normality Test (Shapiro-Wilk):
Statistic: 0.9501, p-value: 0.6383

Normality Test (Jarque-Bera):
Statistic: 0.7444, p-value: 0.6892

Serial Correlation Test (Ljung-Box):
Final Ljung-Box p-value: 0.2977

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 5.4211, p-value: 0.7118
```

Image 43: *Quarterly Report for Japan*

```
-------- Yearly Data Residual Diagnostics for Japan --------

Normality Test (Shapiro-Wilk):
Statistic: 0.9720, p-value: 0.9130

Normality Test (Jarque-Bera):
Statistic: 0.1144, p-value: 0.9444

Serial Correlation Test (Ljung-Box):
Final Ljung-Box p-value: 0.9701

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 5.2166, p-value: 0.1566
```

Image 44: Yearly Report for Japan

```
-------- Monthly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 3.2600, p-value: 0.0001

Serial Correlation Test (Ljung-Box):
      lb_stat     lb_pvalue
10  70.696996  3.251765e-11

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 19.6141, p-value: 0.7185

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 222.4726, p-value: 0.0000
```

Image 45: Monthly Report for Germany

```
-------- Quarterly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 1.1415, p-value: 0.3621

Serial Correlation Test (Ljung-Box):
      lb_stat  lb_pvalue
10  20.105489   0.028271

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 1.4811, p-value: 0.9930

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 110.9962, p-value: 0.0000
```

Image 46: Quarterly Report for Germany

```
-------- Yearly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 1.2639, p-value: 0.3522

Serial Correlation Test (Ljung-Box):
     lb_stat  lb_pvalue
10  4.061094   0.944549

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 1.5235, p-value: 0.4669

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 7.8656, p-value: 0.0196
```

Image 47: Yearly Report for Germany

```
-------- Monthly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 1.3905, p-value: 0.1154

Serial Correlation Test (Ljung-Box):
       lb_stat      lb_pvalue
10  912.917182  1.056237e-189

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 19.5055, p-value: 0.7245

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 11.3036, p-value: 0.0035
```

Image 48: Monthly Report for UK

```
-------- Quarterly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 1.1602, p-value: 0.3419

Serial Correlation Test (Ljung-Box):
       lb_stat     lb_pvalue
10  157.330523  1.151843e-28

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 7.7779, p-value: 0.4555

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 4.4907, p-value: 0.1059
```

Image 49: *Quarterly Report for UK*

```
-------- Yearly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 1.5299, p-value: 0.2733

Serial Correlation Test (Ljung-Box):
      lb_stat   lb_pvalue
10  25.345912   0.004727

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 2.8826, p-value: 0.2366

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 0.9405, p-value: 0.6249
```

Image 50: Yearly Report for UK

```
-------- Monthly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 0.5755, p-value: 0.9411

Serial Correlation Test (Ljung-Box):
       lb_stat     lb_pvalue
10  152.166327  1.335287e-27

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 28.6271, p-value: 0.2345

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 85.8071, p-value: 0.0000
```

Image 51: *Monthly Report for France*

```
-------- Quarterly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 0.3255, p-value: 0.9668

Serial Correlation Test (Ljung-Box):
     lb_stat  lb_pvalue
10  20.53502    0.02458

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 7.2162, p-value: 0.5135

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 36.5945, p-value: 0.0000
```

Image 52: *Quartely Report for France*

```
-------- Yearly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 0.0915, p-value: 0.9828

Serial Correlation Test (Ljung-Box):
     lb_stat  lb_pvalue
10  2.471183   0.991289

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 1.3879, p-value: 0.4996

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 0.9595, p-value: 0.6189
```

Image 53: Yearly Report for France

```
-------- Monthly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 3.7859, p-value: 0.0000

Serial Correlation Test (Ljung-Box):
      lb_stat     lb_pvalue
10  75.479739  3.837059e-12

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 10.4567, p-value: 0.9924

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 3.5923, p-value: 0.1659
```

Image 54: Monthly Report for Italy

```
-------- Quarterly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 1.6115, p-value: 0.1546

Serial Correlation Test (Ljung-Box):
      lb_stat     lb_pvalue
10  69.154238  6.455843e-11

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 1.8642, p-value: 0.9849

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 12.6806, p-value: 0.0018
```

Image 55: Quartely Report for Canada

```
-------- Yearly Data Residual Diagnostics --------

Manually Implemented Ramsey RESET Test:
F-statistic: 0.2519, p-value: 0.9014

Serial Correlation Test (Ljung-Box):
     lb_stat  lb_pvalue
10  3.788772   0.956373

Heteroskedasticity Test (Breusch-Pagan):
LM Statistic: 1.3868, p-value: 0.4999

Normality Test (Jarque-Bera):
Jarque-Bera Statistic: 4.4087, p-value: 0.1103
```

Image 56: Yearly Report for Italy

Based on the results above, we have created a table summarizing the entire analysis. Different tests have been marked as follows:

- ***1** stands for the Ramsey RESET test*
- ***2** stands for the Ljung-Box test*
- ***3** stands for the Breusch-Pagan test*
- ***4** stands for the Jarque-Bera test*

The table highlights where the alternative hypothesis is accepted for each test. This provides a clear and structured overview of the outcomes across the tests.

Country	***Monthly***	***Quarterly***	***Yearly***
Canada	*1,2*	*1,2*	*2*
USA	*1,2,3,4*	*2,4*	
UK	*2,4*	*2*	*2*
Germany	*1,2,4*	*2,4*	*4*
Japan	*3*		
France	*2,4*	*2,4*	
Italy	*1,2*	*2,4*	

Table1:Outcomes of Summarize analysis

Based on the table results, we can see that the Canadian model faces issues with heteroskedasticity and normality in the monthly and quarterly time frames. In the yearly time frame, it experiences serial correlation, heteroskedasticity, and normality problems as well. The US results show that the OLS model is perfectly fine and does not violate any assumptions in the monthly time frame. However, it suffers from heteroskedasticity at the quarterly intervals, and in the yearly time frame, the model performs poorly, failing all tests and accepting the null hypothesis. The UK model suffers from heteroskedasticity at all time frames and also faces normality issues at the quarterly and yearly intervals. The German model experiences heteroskedasticity at all time frames, and in the yearly time

frame, it also struggles with serial correlation. The Japanese model is affected by serial correlation and normality issues across all time frames, with heteroskedasticity being an exception in the monthly time frame. The French model suffers from heteroskedasticity, with the yearly model accepting all null hypotheses in the tests. The Italian model shows lagging in the monthly time frame and suffers from both heteroskedasticity and normality. The quarterly model experiences heteroskedasticity, while the yearly model accepts the null hypothesis for all tests.
so the possible correction we can implement is using the ARIMA model as it's a time series forecasting so using ARIMA models over a multiple regression model gives us more accurate results better fit over the residuals , we have implemented the ARIMA and GARCH model in the next part.

Residuals Analysis By Using ARIMA and GARCH Models :

In the ARIMA and GARCH models, we applied both models to the monthly data for all G-7 countries. Now, we will walk through the different outcomes that each model presents.

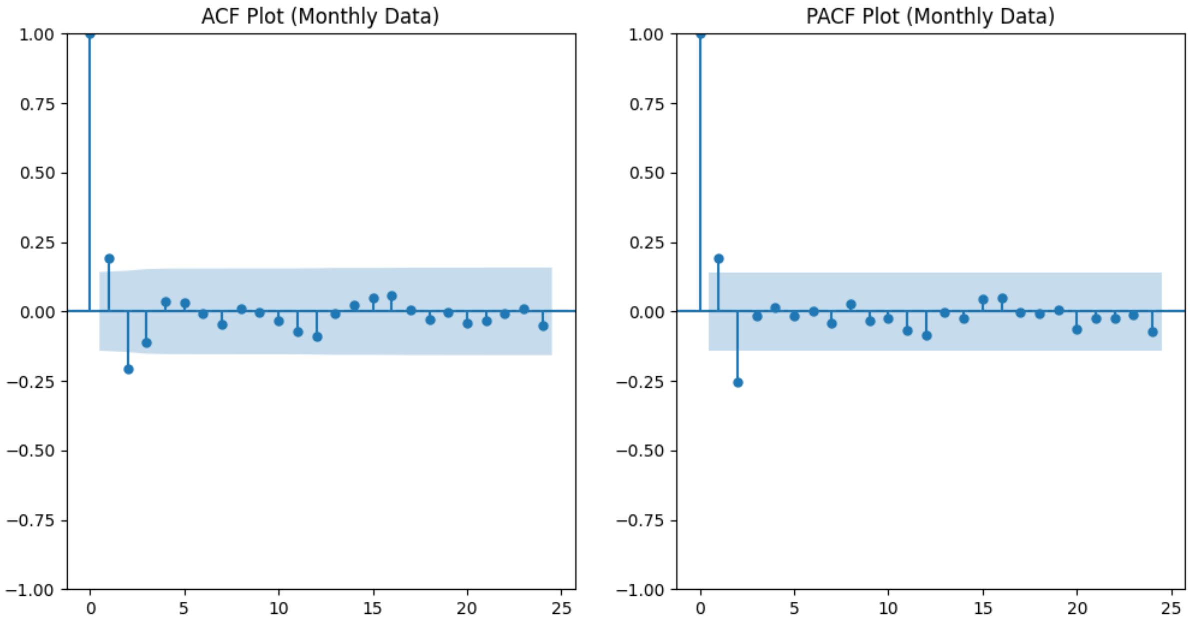

Image 57: ACF And PACF Graph

```
Best ARIMA model order for Monthly data: (2, 0, 0)
Best AIC for Monthly data: -1019.2442424566116
                               SARIMAX Results
==============================================================================
Dep. Variable:             Log_Return   No. Observations:                  192
Model:                 ARIMA(2, 0, 0)   Log Likelihood                 513.622
Date:                Thu, 19 Dec 2024   AIC                          -1019.244
Time:                        12:59:47   BIC                          -1006.214
Sample:                    02-28-2005   HQIC                         -1013.967
                         - 01-31-2021
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0002      0.002      0.113      0.910      -0.003       0.004
ar.L1          0.2387      0.042      5.690      0.000       0.156       0.321
ar.L2         -0.2524      0.108     -2.339      0.019      -0.464      -0.041
sigma2         0.0003   1.51e-05     18.355      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):              9223.76
Prob(Q):                              0.97   Prob(JB):                         0.00
Heteroskedasticity (H):               5.21   Skew:                            -3.52
Prob(H) (two-sided):                  0.00   Kurtosis:                        36.22
===================================================================================
```

Image 58: ARIMA Model Output

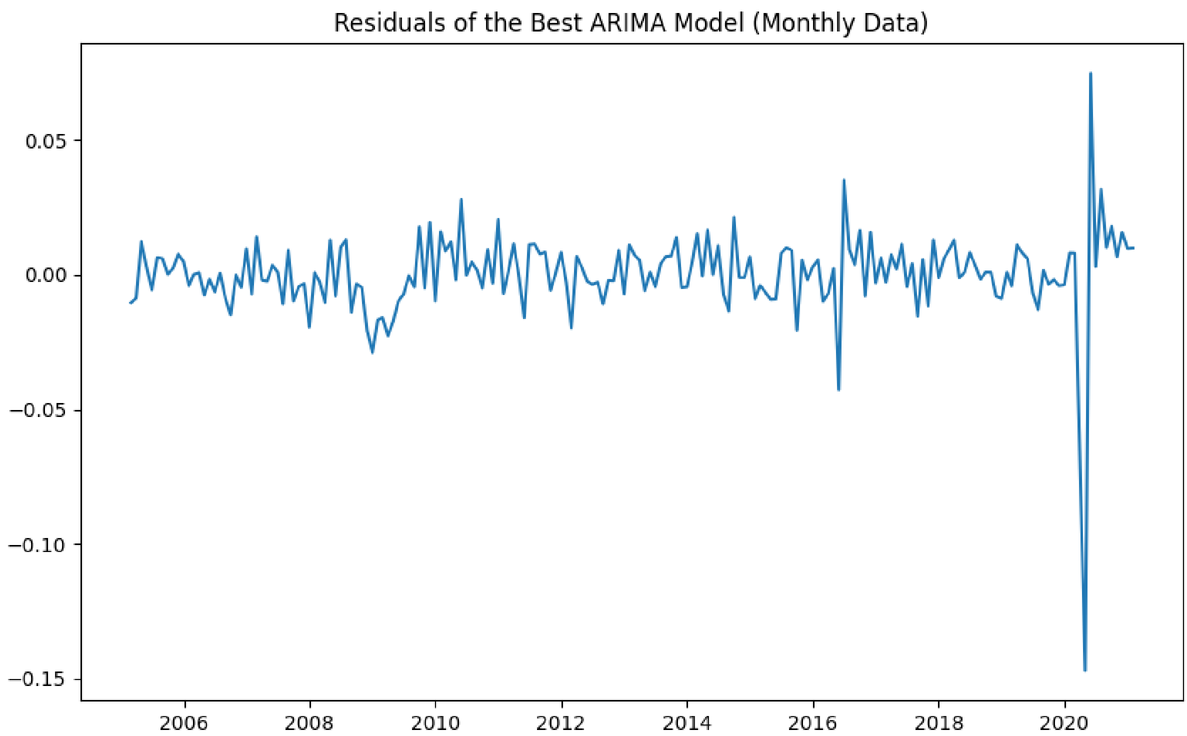

Image 59: Residuals of ARIMA Model

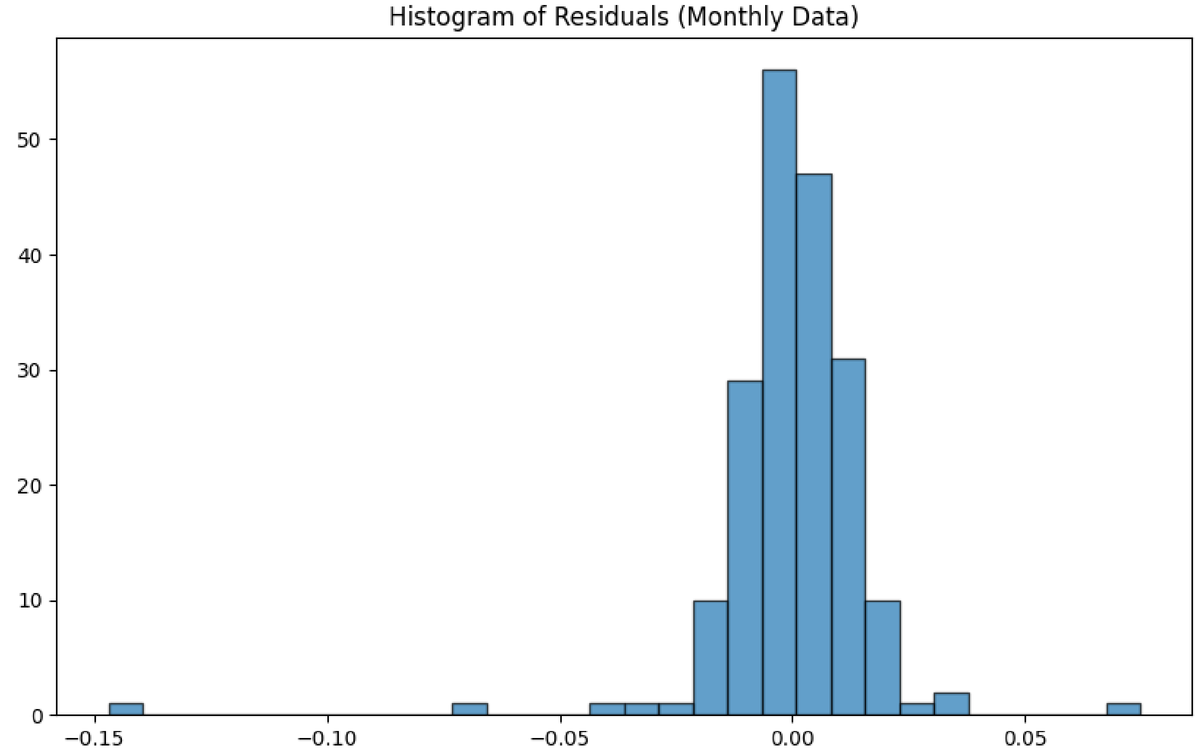

Image 60: Histogram

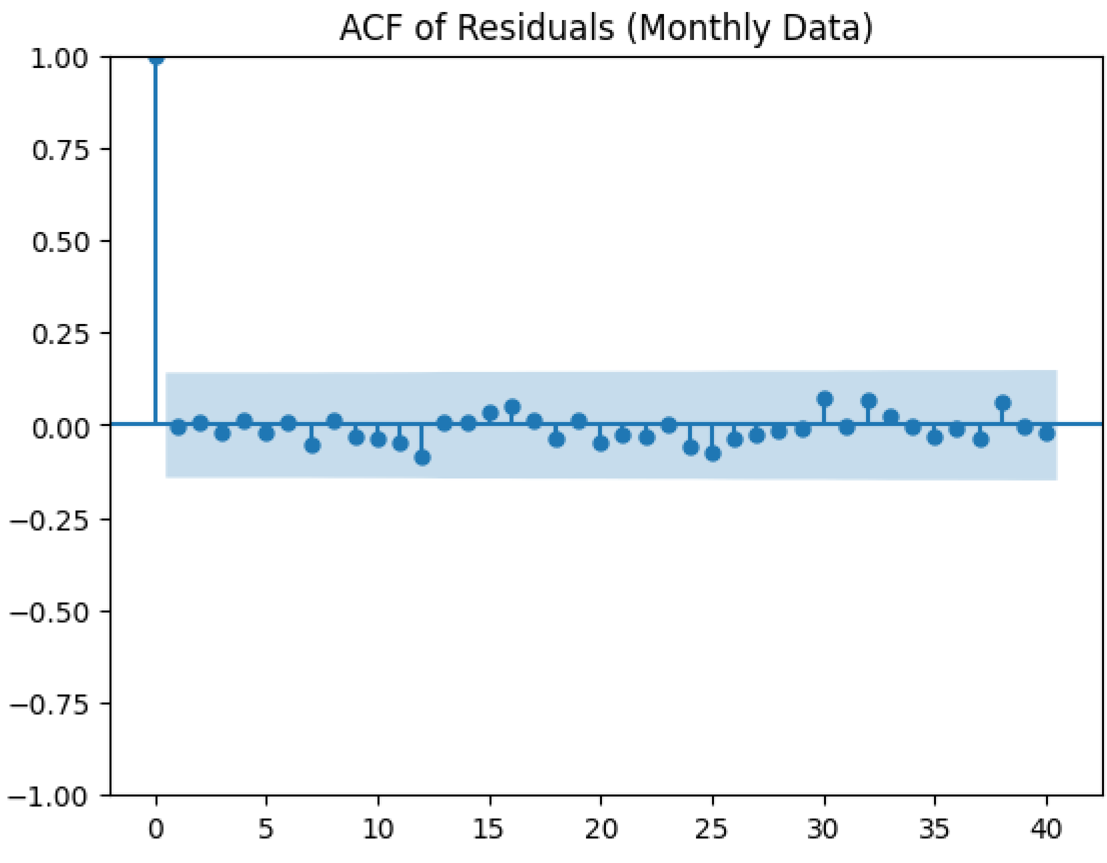

Image 61: ACF Residuals

As we can see in the ACF and PACF graph in Figure No. "57", the spikes at the first two lags are above the confidence bands, indicating significant autocorrelation at these lags. Overall, in the long prediction, the ACF and PACF decay quickly, but some autocorrelation remains at certain lags, both in the positive and negative directions. We ran our ARIMA model with different parameters, and out of these, we selected the best one, ARIMA(2,0,0), which had the best log-likelihood and other comparative parameters such as AIC and BIC. The model also shows heteroskedasticity, as depicted by the Breusch-Pagan test. After fitting the model, you can see in Figure- "60" that the histogram appears normally distributed, and the ACF of the residuals is very low compared to the original data.

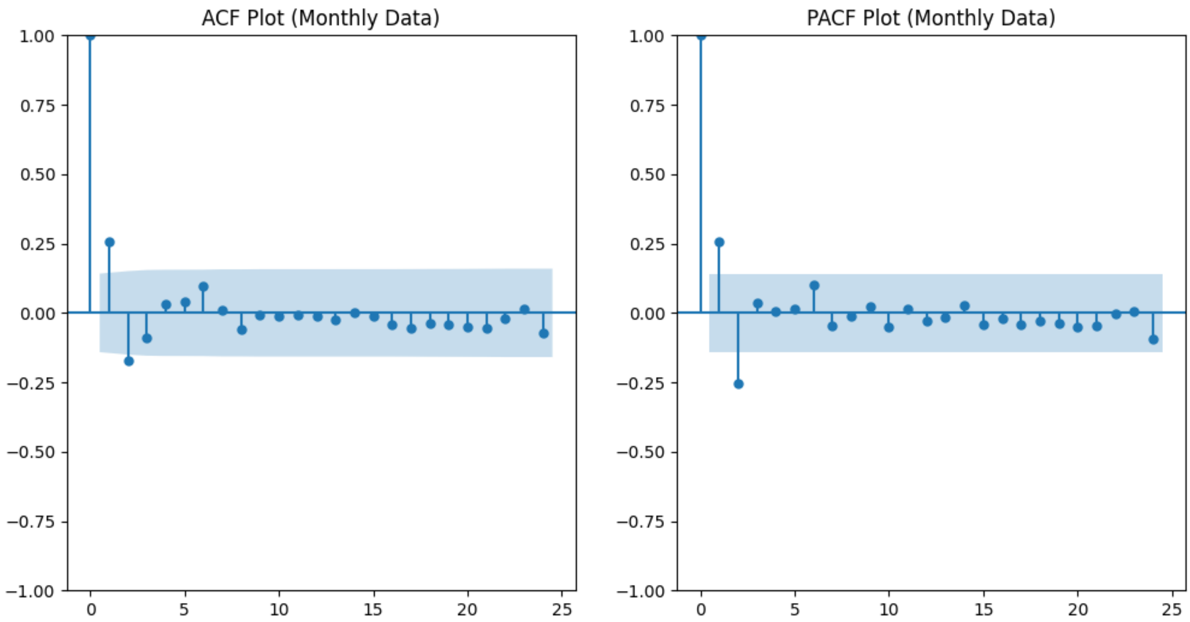

Image 62: ACF And PACF Graph

```
Best ARIMA model order for Monthly data: (2, 0, 0)
Best AIC for Monthly data: -1116.1052939945384
                               SARIMAX Results
==============================================================================
Dep. Variable:             Log_Return   No. Observations:                  192
Model:                 ARIMA(2, 0, 0)   Log Likelihood                 562.053
Date:                Thu, 19 Dec 2024   AIC                          -1116.105
Time:                        13:00:15   BIC                          -1103.075
Sample:                    02-28-2005   HQIC                         -1110.828
                         - 01-31-2021
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0001      0.002      0.075      0.940      -0.003       0.004
ar.L1          0.3212      0.025     12.810      0.000       0.272       0.370
ar.L2         -0.2558      0.048     -5.359      0.000      -0.349      -0.162
sigma2         0.0002   5.39e-06     31.082      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   0.02   Jarque-Bera (JB):             20778.86
Prob(Q):                              0.90   Prob(JB):                         0.00
Heteroskedasticity (H):               3.48   Skew:                            -5.11
Prob(H) (two-sided):                  0.00   Kurtosis:                        52.93
===================================================================================
```

Image 63: ARIMA Model Output

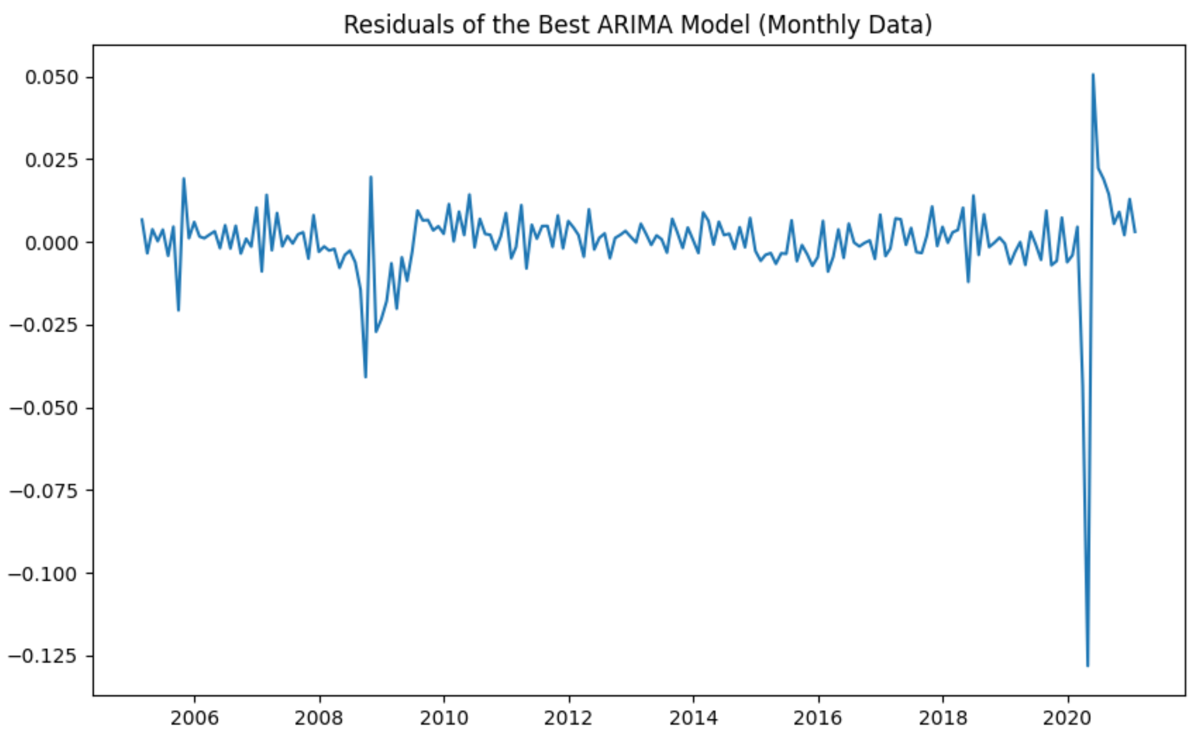

Image 64: Residuals of ARIMA Model

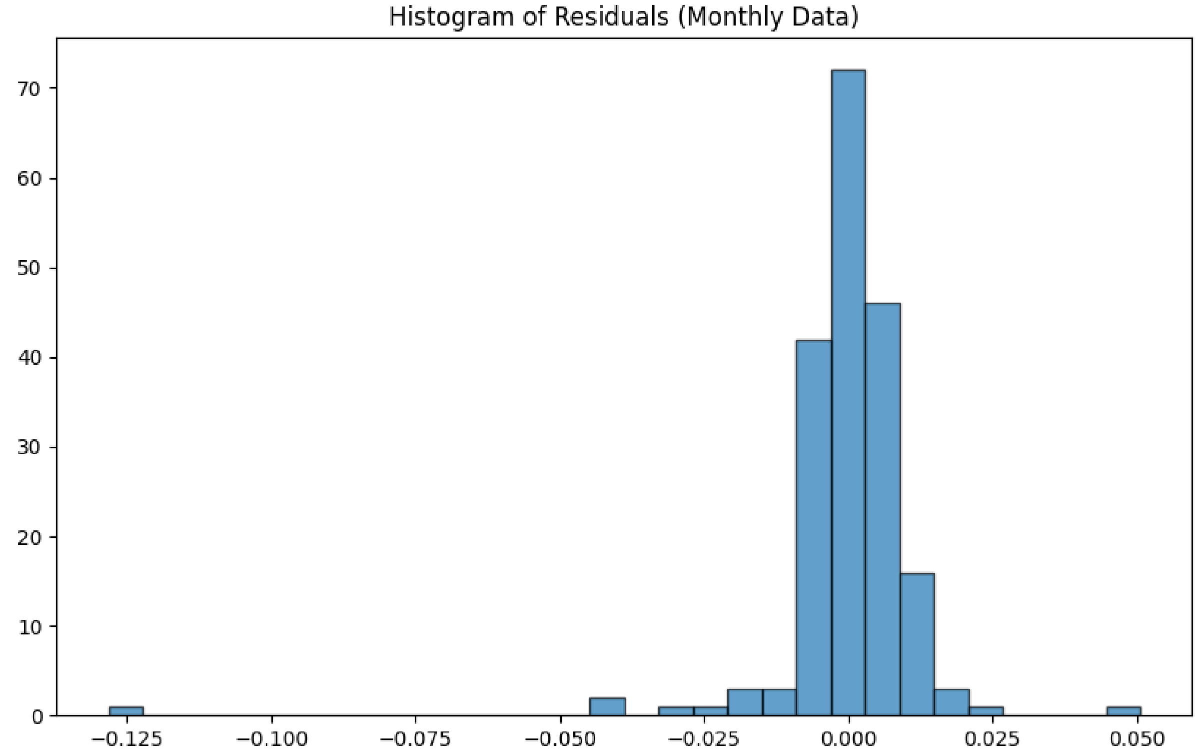

Image 65: Histogram

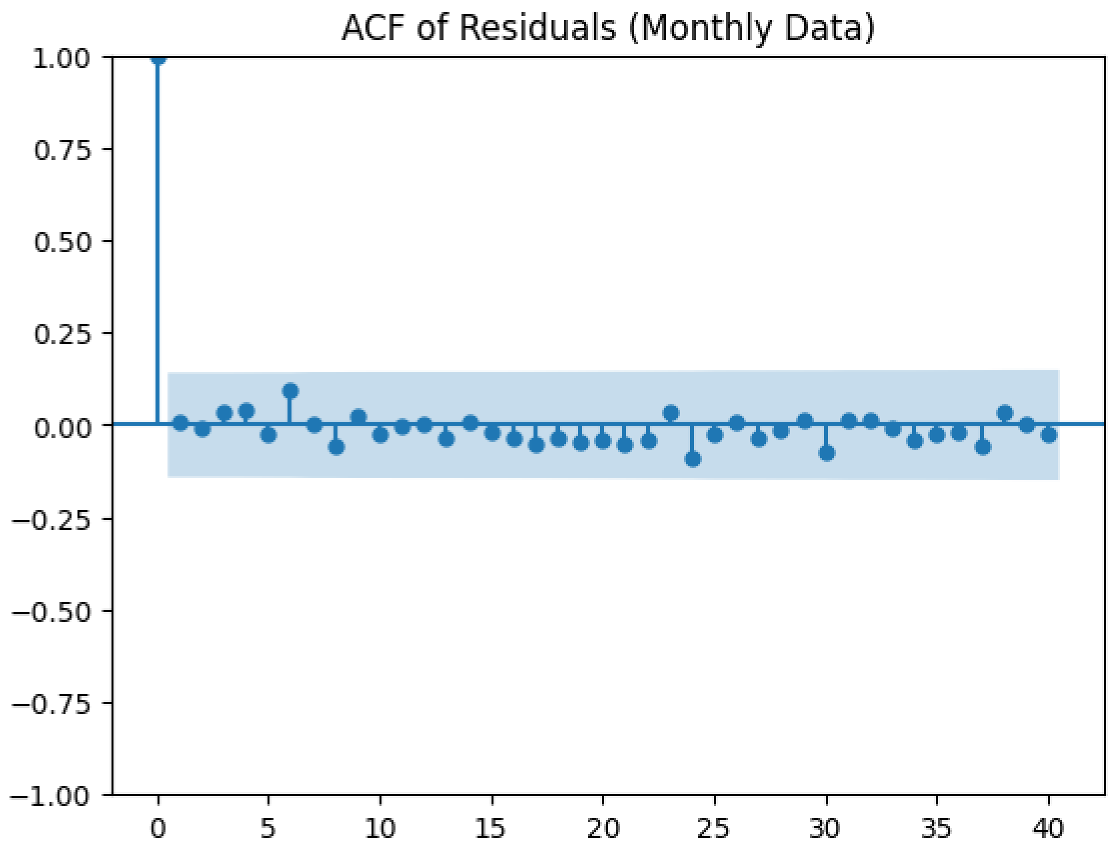

Image 66: ACF Residuals

The results for the US ARIMA model. As we can see in the ACF and PACF graph in Figure No. "62", the spikes at the first three lags are quite significant, and some are above the confidence bands, indicating significant autocorrelation at these lags. Overall, in the long prediction, the ACF and PACF decay quickly, but some autocorrelation remains at certain lags, both in the positive and negative directions. We ran our ARIMA model with different parameters, and out of these, we selected the best one, ARIMA(2,0,0), which had the best log-likelihood and other comparative parameters such as AIC and BIC. The model also shows heteroskedasticity, as depicted by the Breusch-Pagan test. After fitting the model, you can see in Figure- "65" that the histogram appears normally distributed and is negatively skewed, and the ACF of the residuals is very low compared to the original data, with some exceptions at the initial lags, where spikes are above the confidence intervals.

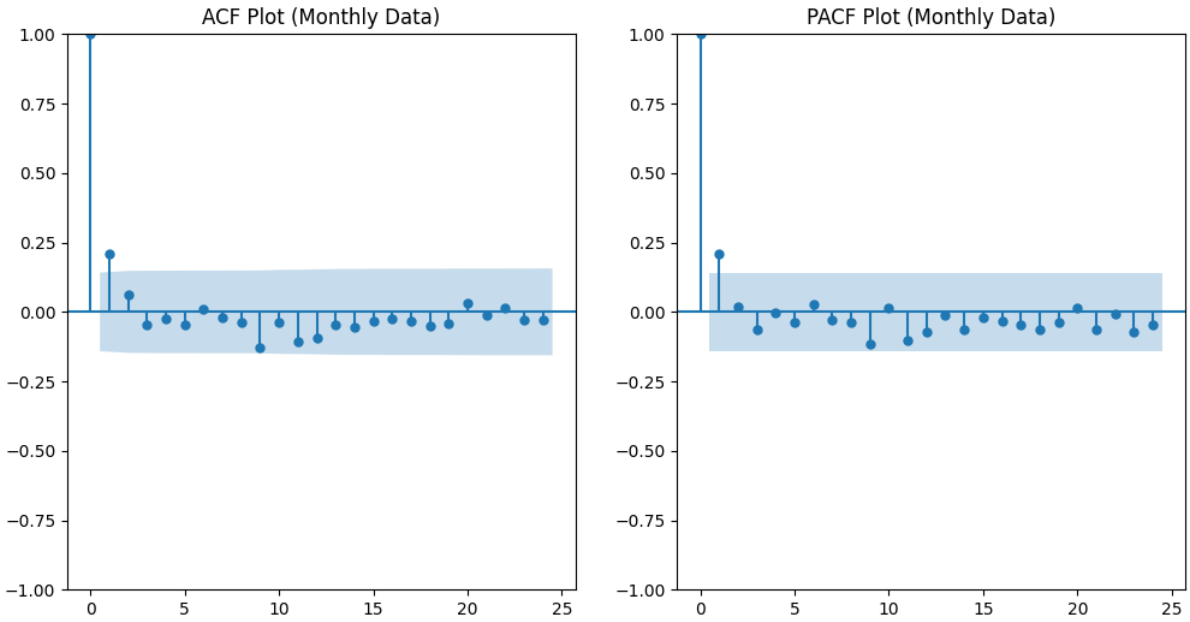

Image 67: ACF And PACF Graph

```
Best ARIMA model order for Monthly data: (3, 0, 2)
Best AIC for Monthly data: -873.8613907906481
                               SARIMAX Results
==============================================================================
Dep. Variable:             Log_Return   No. Observations:                  192
Model:                 ARIMA(3, 0, 2)   Log Likelihood                 443.931
Date:                Thu, 19 Dec 2024   AIC                           -873.861
Time:                        13:00:38   BIC                           -851.059
Sample:                    02-28-2005   HQIC                          -864.626
                         - 01-31-2021
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0006      0.001     -1.237      0.216      -0.002       0.000
ar.L1          0.2711      0.182      1.487      0.137      -0.086       0.628
ar.L2          0.8377      0.187      4.486      0.000       0.472       1.204
ar.L3         -0.2621      0.074     -3.565      0.000      -0.406      -0.118
ma.L1         -0.1254      0.174     -0.722      0.471      -0.466       0.215
ma.L2         -0.8601      0.168     -5.112      0.000      -1.190      -0.530
sigma2         0.0006   3.35e-05     17.353      0.000       0.001       0.001
===================================================================================
Ljung-Box (L1) (Q):                   0.04   Jarque-Bera (JB):              2401.79
Prob(Q):                              0.84   Prob(JB):                         0.00
Heteroskedasticity (H):               1.08   Skew:                            -2.91
Prob(H) (two-sided):                  0.76   Kurtosis:                        19.32
===================================================================================
```

Image 68: ARIMA Model Output

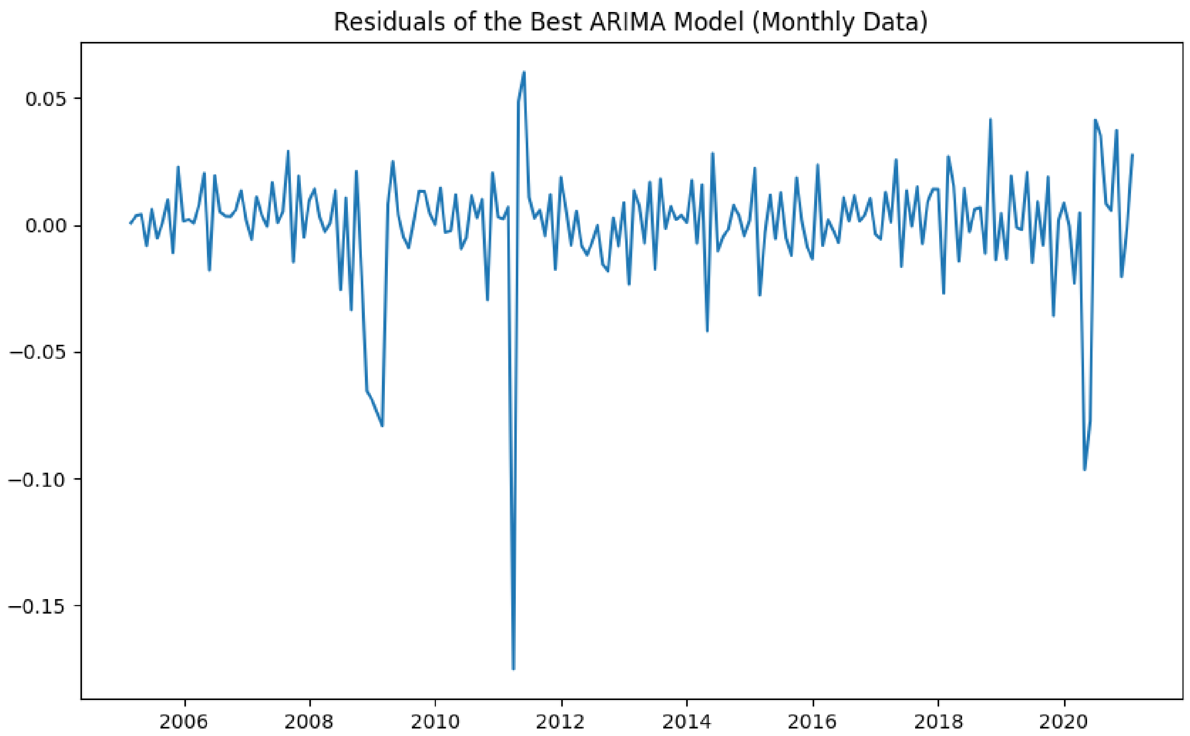

Image 69: Residuals of ARIMA Model

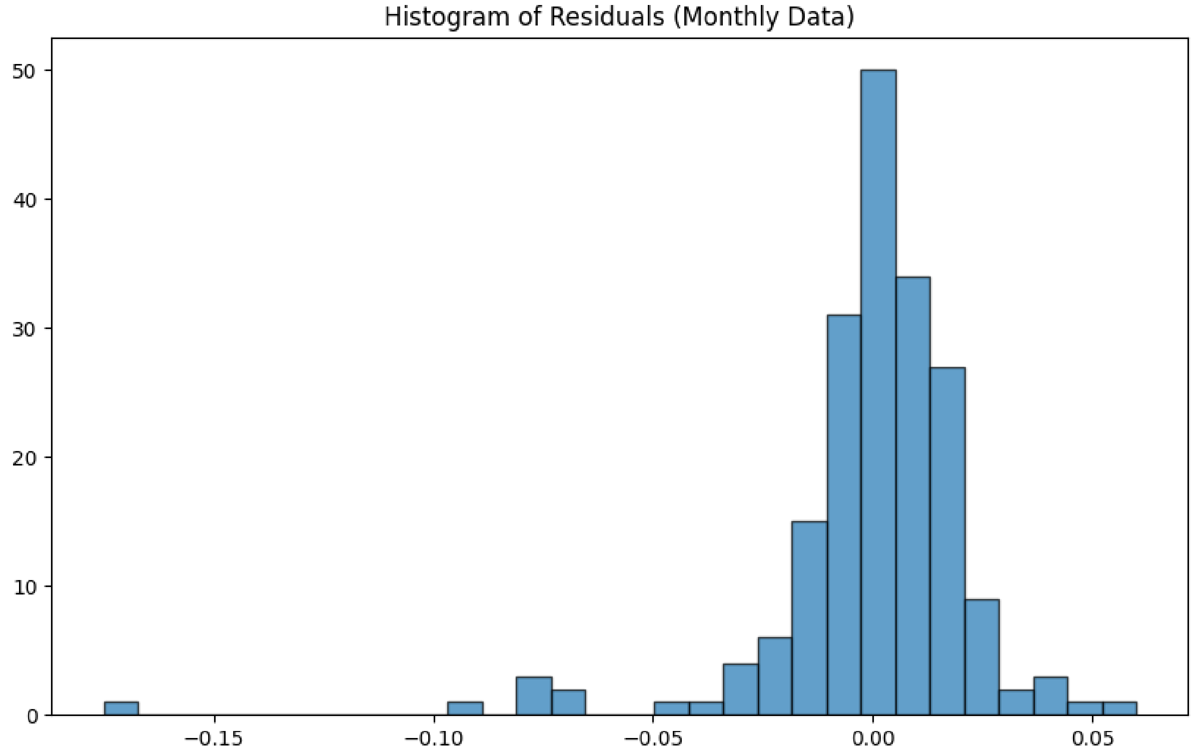

Image 70: Histogram

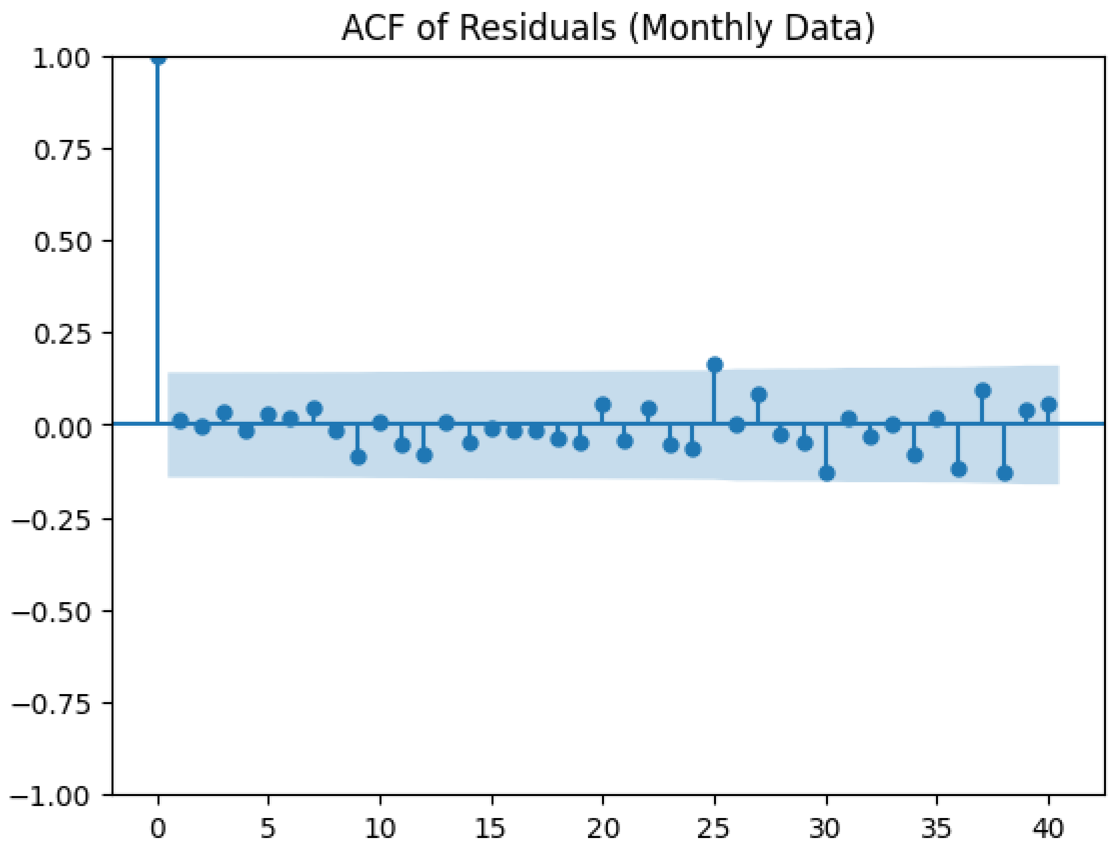

Image 71: ACF Residuals

The results for the Japanese ARIMA model are as follows. As we can see in the ACF and PACF graph in Figure No. "67", the spikes at the first two lags are quite significant and are above the confidence interval, indicating significant autocorrelation at these lags. Overall, in the long prediction, the ACF and PACF decay quickly, but some autocorrelation remains at certain lags, both in the positive and negative directions. We ran our ARIMA model with different parameters, and out of these, we selected the best one, ARIMA(3,0,2), which had the best log-likelihood and other comparative parameters such as AIC and BIC. The model also shows homoskedasticity, as depicted by the Breusch-Pagan test. After fitting the model, you can see in Figure- "70" that the histogram appears normally distributed and is negatively skewed, and the ACF of the residuals is very low compared to the original data, with some exceptions at the initial lags, where spikes are above the confidence intervals. Additionally, there are some spikes in the middle that are near the confidence interval, but overall, the autocorrelation appears low.

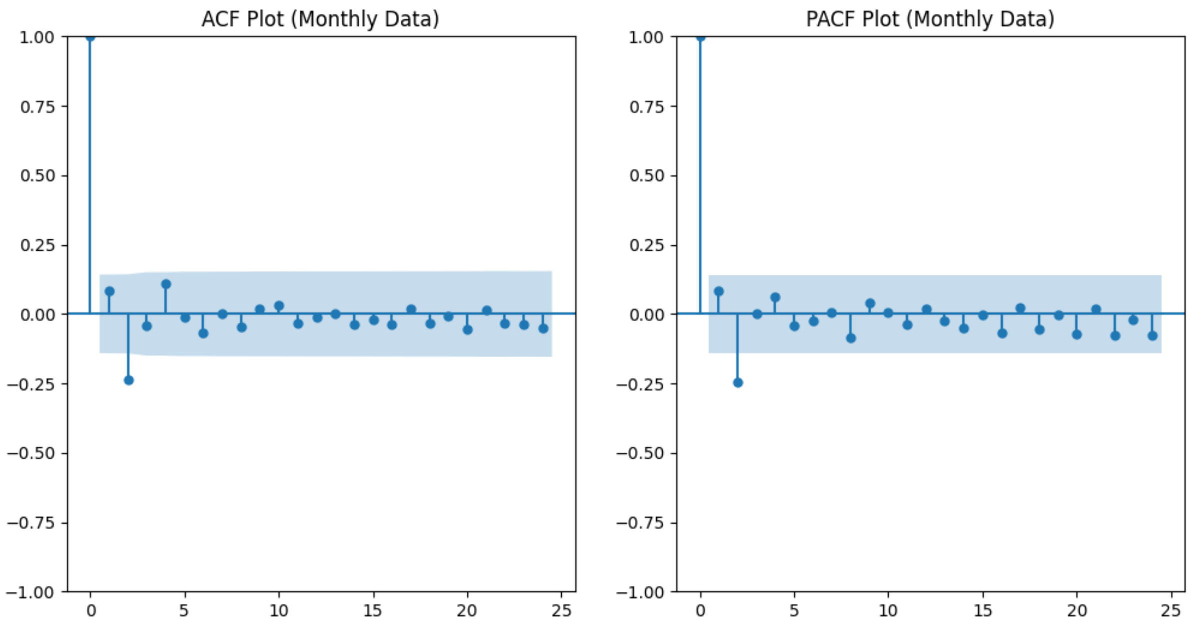

Image 72: ACF And PACF Graph

```
Best ARIMA model order for Monthly data: (2, 0, 0)
Best AIC for Monthly data: -848.1997100268466
                               SARIMAX Results
==============================================================================
Dep. Variable:             Log_Return   No. Observations:                  192
Model:                 ARIMA(2, 0, 0)   Log Likelihood                 428.100
Date:                Thu, 19 Dec 2024   AIC                           -848.200
Time:                        13:01:02   BIC                           -835.170
Sample:                    02-28-2005   HQIC                          -842.922
                         - 01-31-2021
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0007      0.002      0.267      0.789      -0.004       0.005
ar.L1          0.1070      0.030      3.581      0.000       0.048       0.166
ar.L2         -0.2453      0.055     -4.431      0.000      -0.354      -0.137
sigma2         0.0007   2.68e-05     25.249      0.000       0.001       0.001
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):              8386.87
Prob(Q):                              1.00   Prob(JB):                         0.00
Heteroskedasticity (H):               3.32   Skew:                            -3.60
Prob(H) (two-sided):                  0.00   Kurtosis:                        34.57
===================================================================================
```

Image 73: ARIMA Model Output

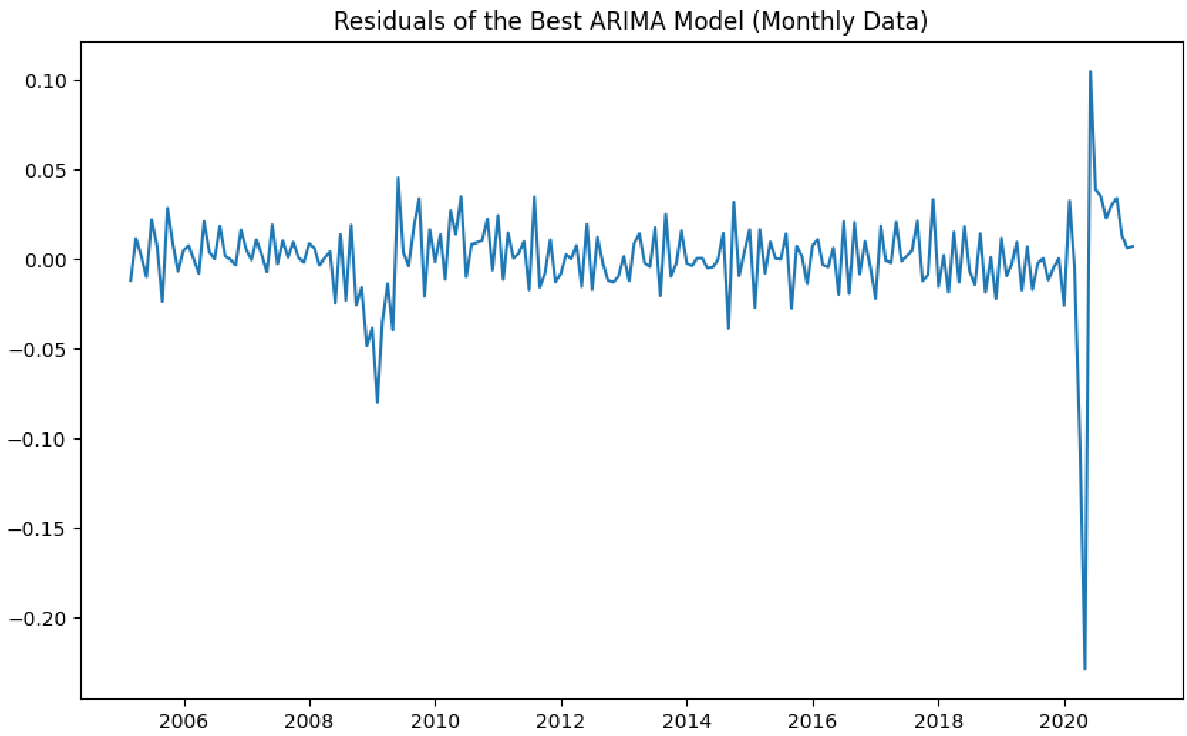

Image 74: Residuals of ARIMA Model

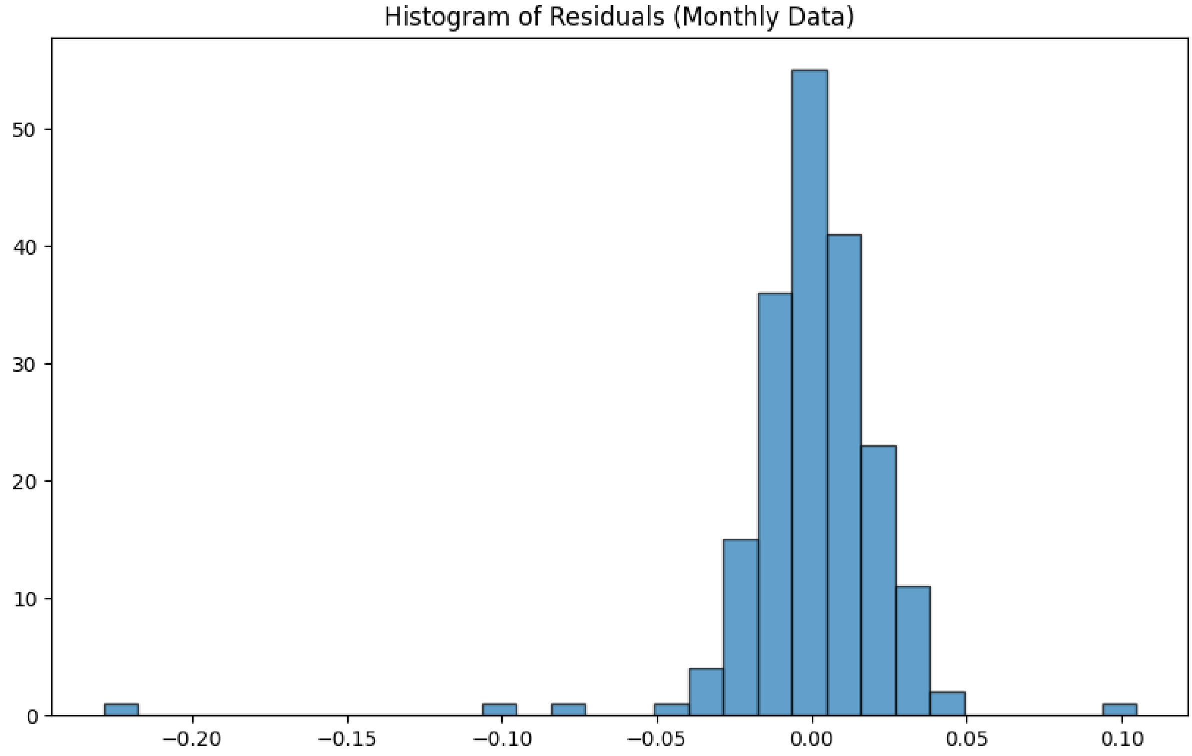

Image 75: Histogram

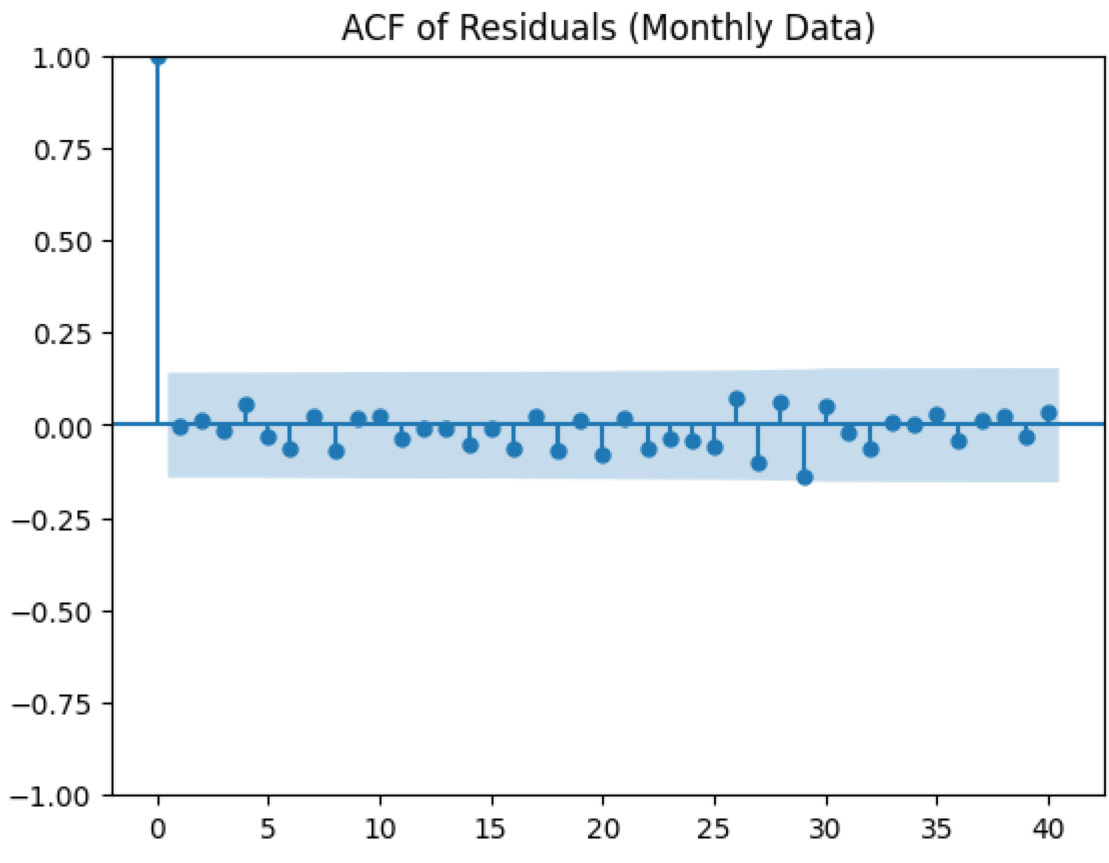

Image 76: ACF Residuals

The results for the German ARIMA model are as follows. As we can see in the ACF and PACF graph in Figure No. "72", the spikes at the first three lags are quite significant, with two of them above the confidence interval, indicating significant autocorrelation at these lags. Overall, in the long prediction, the ACF and PACF decay quickly, but some autocorrelation remains at certain lags, both in the positive and negative directions. We ran our ARIMA model with different parameters, and out of these, we selected the best one, ARIMA(2,0,0), which had the best log-likelihood and other comparative parameters such as AIC and BIC. The model also shows heteroskedasticity, as depicted by the Breusch-Pagan test. After fitting the model, you can see in Figure- "75" that the histogram appears normally distributed and is negatively skewed, and the ACF of the residuals is very low compared to the original data, with some exceptions at the initial lags, where spikes are above the confidence intervals. Overall, the autocorrelation appears to be low.

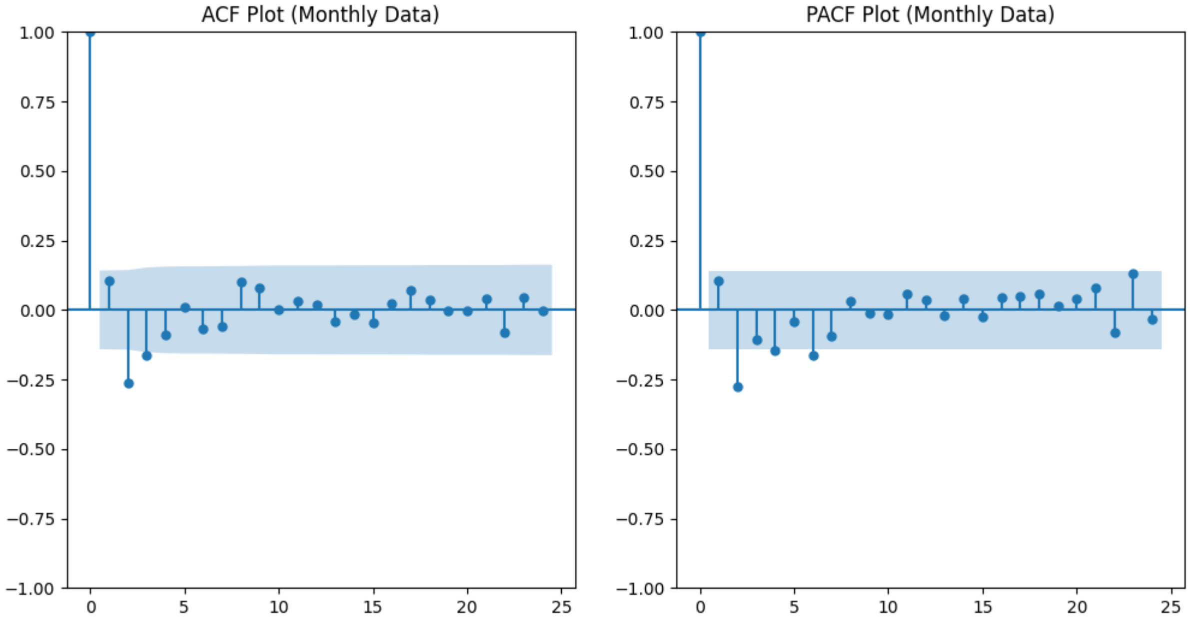

Image 77: ACF And PACF Graph

```
Best ARIMA model order for Monthly data: (2, 0, 1)
Best AIC for Monthly data: -973.5942383354711
                               SARIMAX Results
==============================================================================
Dep. Variable:             Log_Return   No. Observations:                  192
Model:                 ARIMA(2, 0, 1)   Log Likelihood                 491.797
Date:                Thu, 19 Dec 2024   AIC                           -973.594
Time:                        13:01:27   BIC                           -957.307
Sample:                    02-28-2005   HQIC                          -966.998
                         - 01-31-2021
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0002      0.001      0.199      0.842      -0.002       0.002
ar.L1          0.7448      0.095      7.859      0.000       0.559       0.931
ar.L2         -0.3254      0.042     -7.717      0.000      -0.408      -0.243
ma.L1         -0.6920      0.112     -6.176      0.000      -0.912      -0.472
sigma2         0.0003   1.75e-05     19.944      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   0.02   Jarque-Bera (JB):              4173.39
Prob(Q):                              0.89   Prob(JB):                         0.00
Heteroskedasticity (H):               3.25   Skew:                            -2.82
Prob(H) (two-sided):                  0.00   Kurtosis:                        25.13
===================================================================================
```

Image 78: ARIMA Model Output

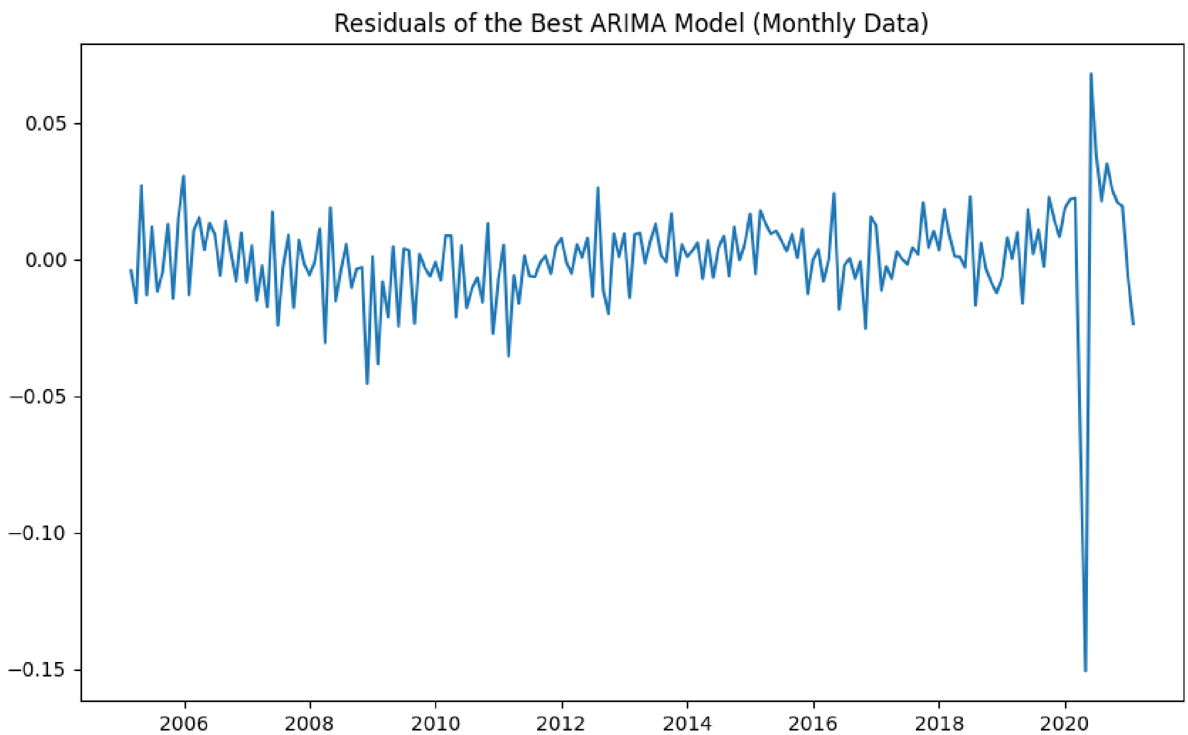

Image 79: Residuals of ARIMA Model

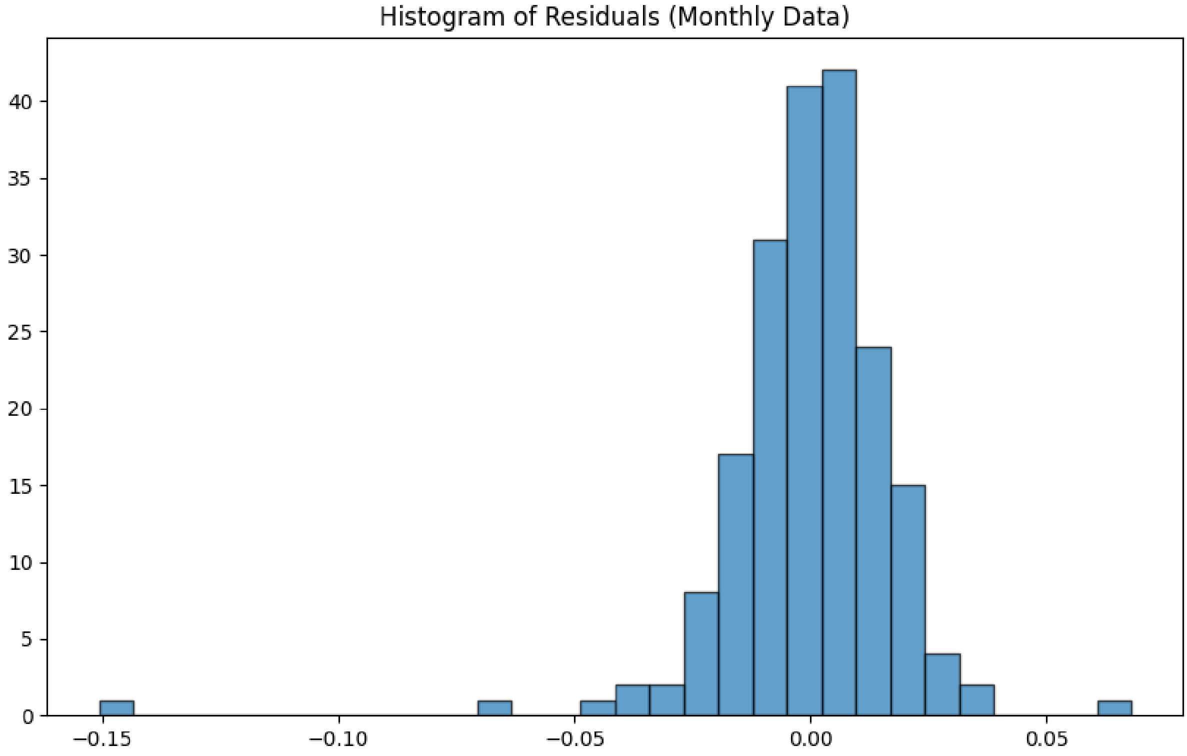

Image 80: Histogram

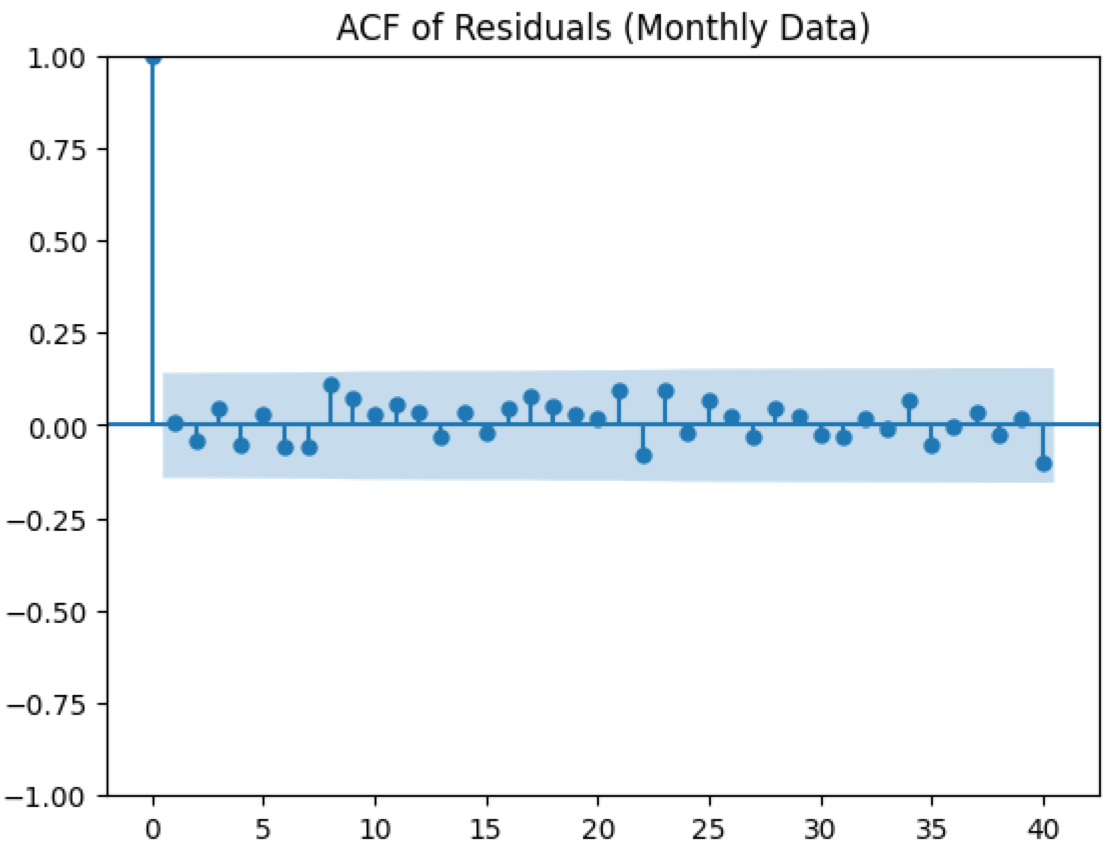

Image 81: ACF Residuals

The results for the German ARIMA model are as follows. As we can see in the ACF and PACF graph in Figure No. "77", the spikes at the first four lags are quite significant, with two of them above the confidence interval, and some in the middle also surpassing the significance level, indicating not a good significant autocorrelation at these lags. Overall, in the long prediction, the ACF and PACF decay quickly, but they do not remain very low, as some autocorrelation remains at certain lags, both in the positive and negative directions. We ran our ARIMA model with different parameters, and out of these, we selected the best one, ARIMA(2,0,1), which had the best log-likelihood and other comparative parameters such as AIC and BIC. The model also shows heteroskedasticity, as depicted by the Breusch-Pagan test. After fitting the model, you can see in Figure- "80" that the histogram appears normally distributed and is negatively skewed, and the ACF of the residuals is very low compared to the original data, with some exceptions at the initial lags, where spikes are above the confidence intervals. Overall, the autocorrelation appears to be low and better fitted than the initial result.

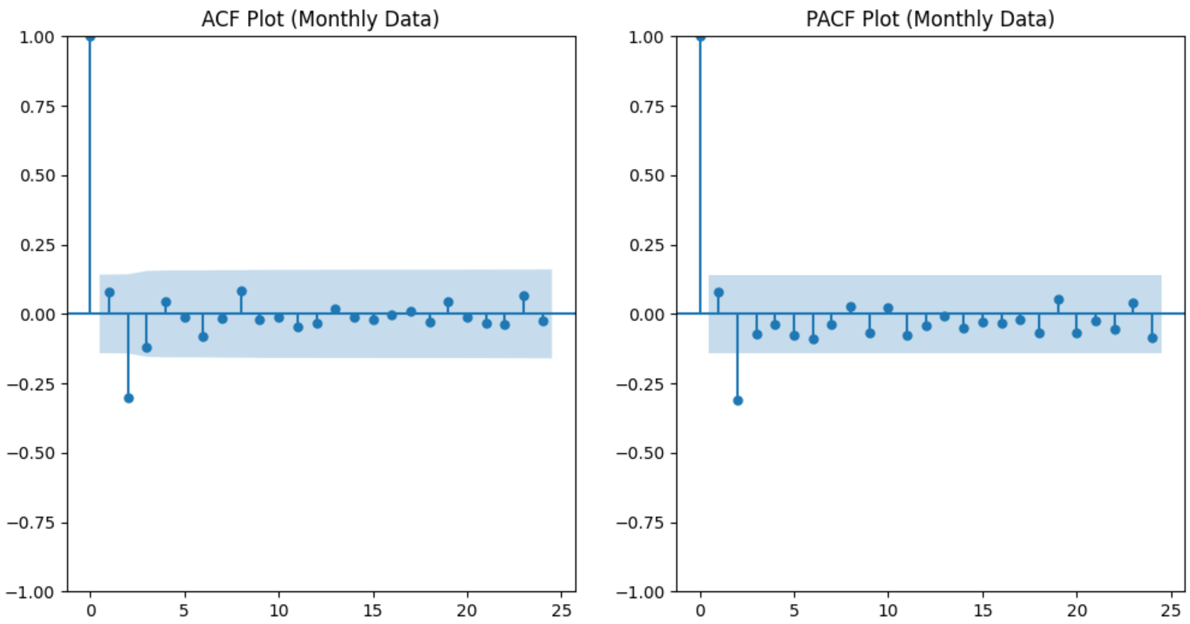

Image 82: ACF And PACF Graph

```
Best ARIMA model order for Monthly data: (3, 0, 1)
Best AIC for Monthly data: -807.5287874840184
                               SARIMAX Results
==============================================================================
Dep. Variable:             Log_Return   No. Observations:                  192
Model:                 ARIMA(3, 0, 1)   Log Likelihood                 409.764
Date:                Thu, 19 Dec 2024   AIC                           -807.529
Time:                        13:01:54   BIC                           -787.984
Sample:                    02-28-2005   HQIC                          -799.613
                         - 01-31-2021
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0007      0.002     -0.450      0.653      -0.004       0.002
ar.L1          0.9763      0.187      5.227      0.000       0.610       1.342
ar.L2         -0.3737      0.072     -5.216      0.000      -0.514      -0.233
ar.L3          0.2002      0.089      2.240      0.025       0.025       0.375
ma.L1         -0.9182      0.184     -4.988      0.000      -1.279      -0.557
sigma2         0.0008   4.78e-05     17.289      0.000       0.001       0.001
===================================================================================
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):              6244.89
Prob(Q):                              0.92   Prob(JB):                         0.00
Heteroskedasticity (H):               4.30   Skew:                            -3.34
Prob(H) (two-sided):                  0.00   Kurtosis:                        30.13
===================================================================================
```

Image 83: ARIMA Model Output

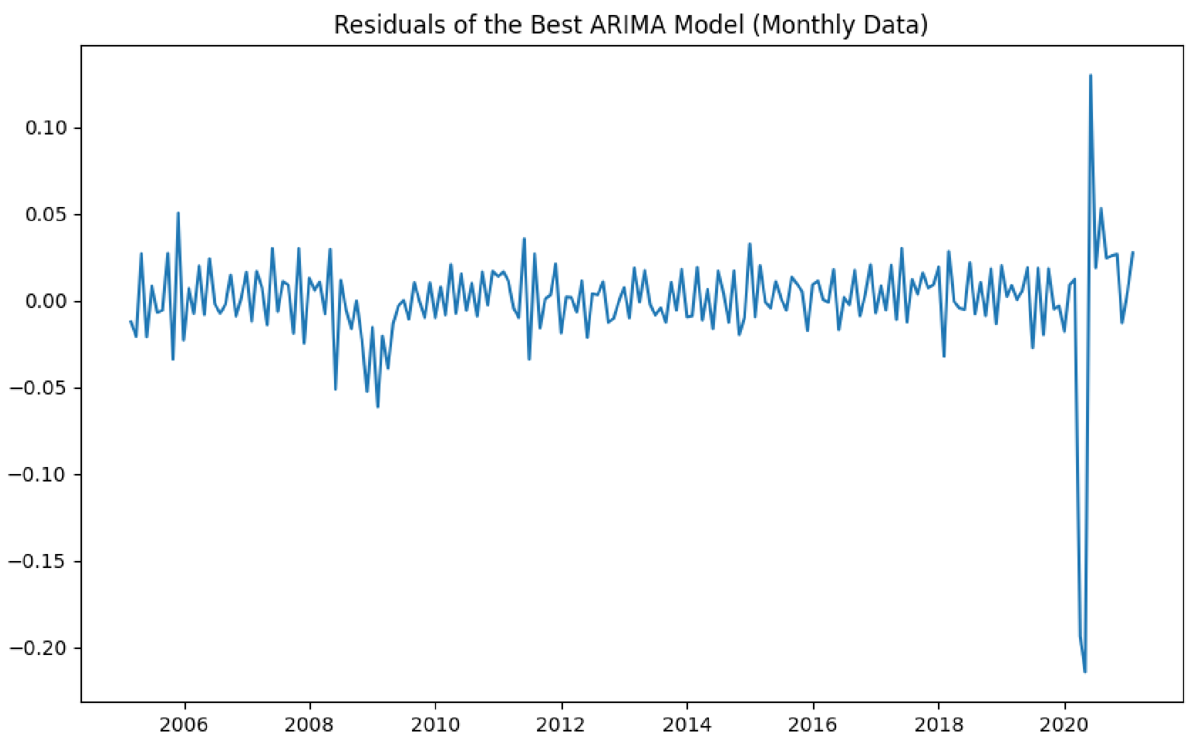

Image 84: Residuals of ARIMA Model

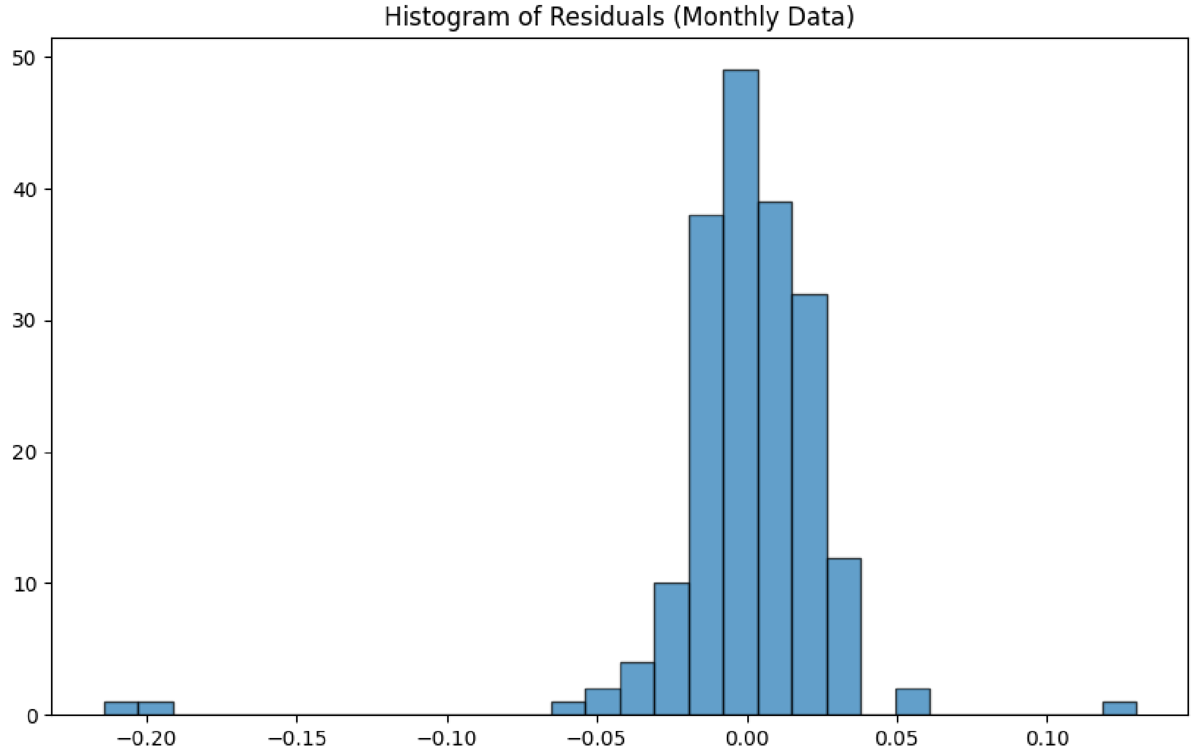

Image 85: Histogram

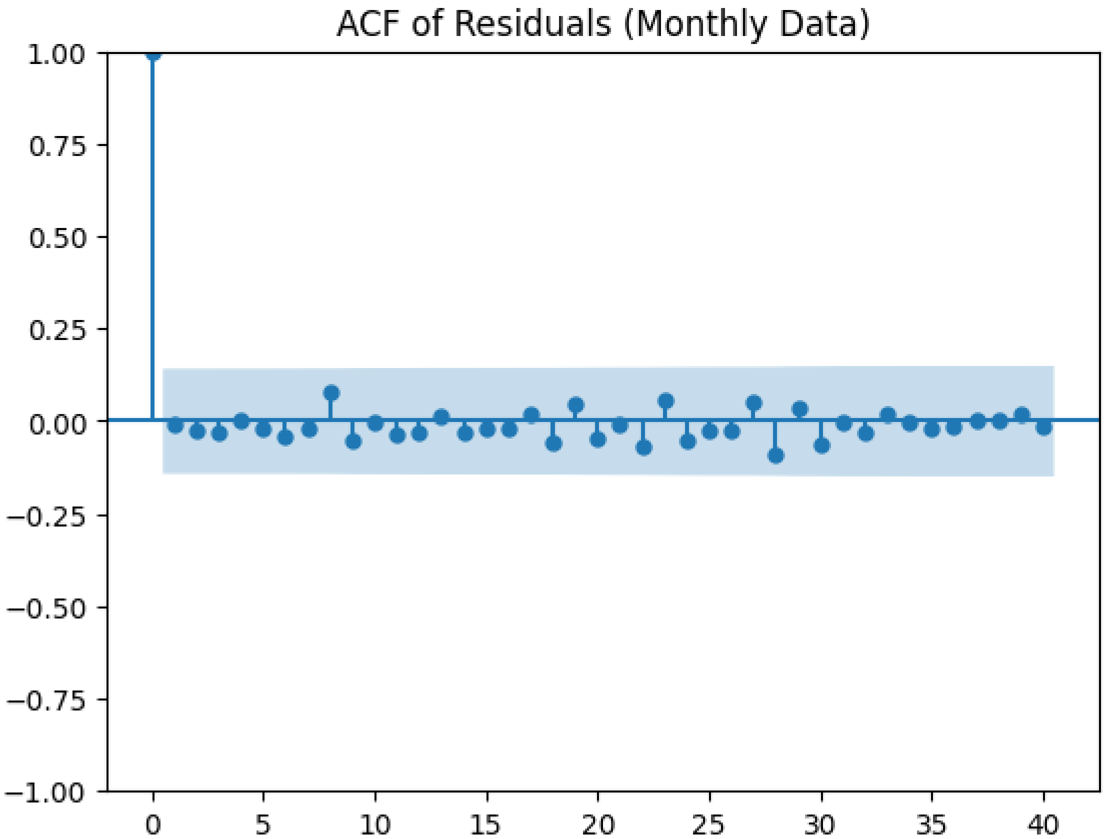

Image 86: ACF Residuals

The results for the French ARIMA model are as follows. As we can see in the ACF and PACF graph in Figure No. "82", the spikes at the first three lags are quite significant, with two of them above the confidence interval. Overall, in the long prediction, the ACF and PACF decay quickly, both in the positive and negative directions. We ran our ARIMA model with different parameters, and out of these, we selected the best one, ARIMA(3,0,1), which had the best log-likelihood and other comparative parameters such as AIC and BIC. The model also shows heteroskedasticity, as depicted by the Breusch-Pagan test. After fitting the model, you can see in Figure- "85" that the histogram appears normally distributed and is negatively skewed, and the ACF of the residuals is very low compared to the original data, with some exceptions at the initial lags, where spikes are above the confidence intervals. Overall, the autocorrelation appears to be low and better fitted than the initial result.

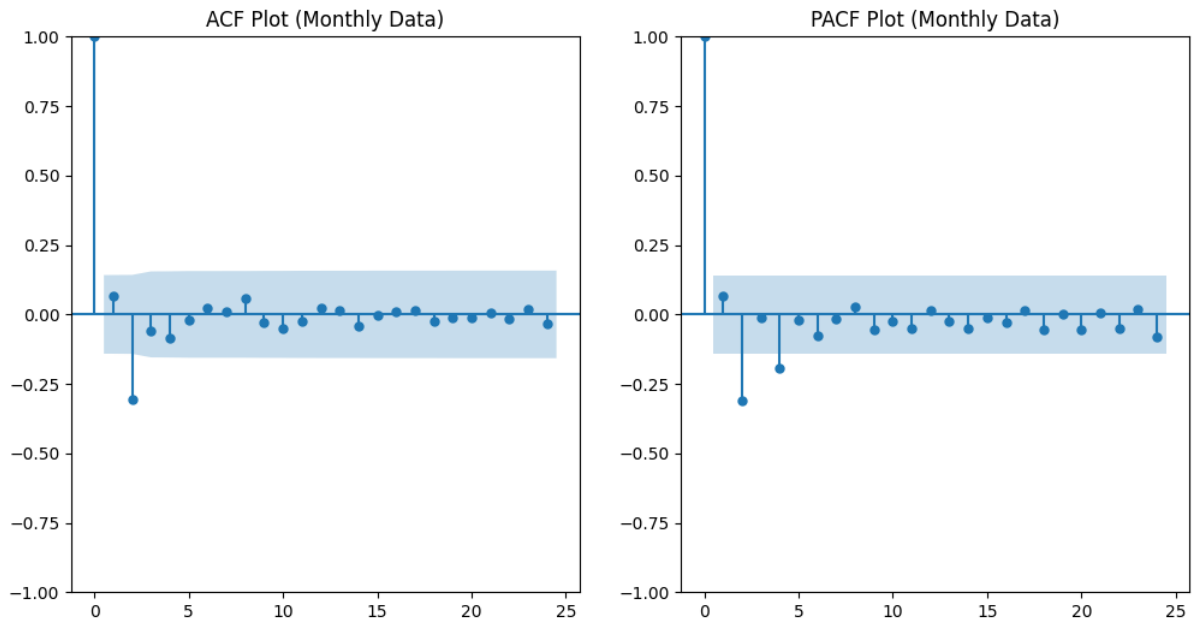

Image 87: ACF And PACF Graph

```
Best ARIMA model order for Monthly data: (1, 0, 3)
Best AIC for Monthly data: -672.7364413549667
                               SARIMAX Results
==============================================================================
Dep. Variable:             Log_Return   No. Observations:                  192
Model:                 ARIMA(1, 0, 3)   Log Likelihood                 342.368
Date:                Thu, 19 Dec 2024   AIC                           -672.736
Time:                        13:02:23   BIC                           -653.191
Sample:                    02-28-2005   HQIC                          -664.821
                         - 01-31-2021
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0012      0.001     -0.779      0.436      -0.004       0.002
ar.L1          0.8916      0.097      9.177      0.000       0.701       1.082
ma.L1         -0.8337      0.114     -7.295      0.000      -1.058      -0.610
ma.L2         -0.4339      0.069     -6.299      0.000      -0.569      -0.299
ma.L3          0.2914      0.049      5.892      0.000       0.194       0.388
sigma2         0.0016      0.000     14.311      0.000       0.001       0.002
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):              7670.88
Prob(Q):                              0.96   Prob(JB):                         0.00
Heteroskedasticity (H):               8.14   Skew:                            -1.53
Prob(H) (two-sided):                  0.00   Kurtosis:                        33.81
===================================================================================
```

Image 88: ARIMA Model Output

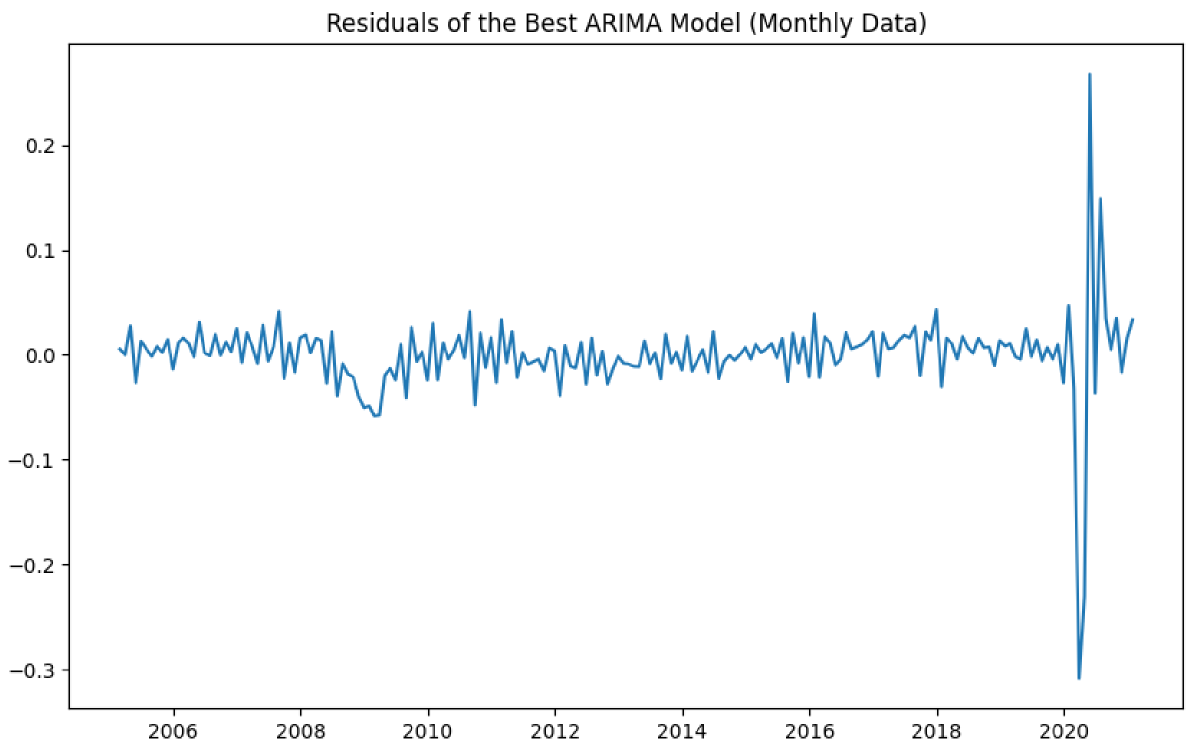

Image 89: Residuals of ARIMA Model

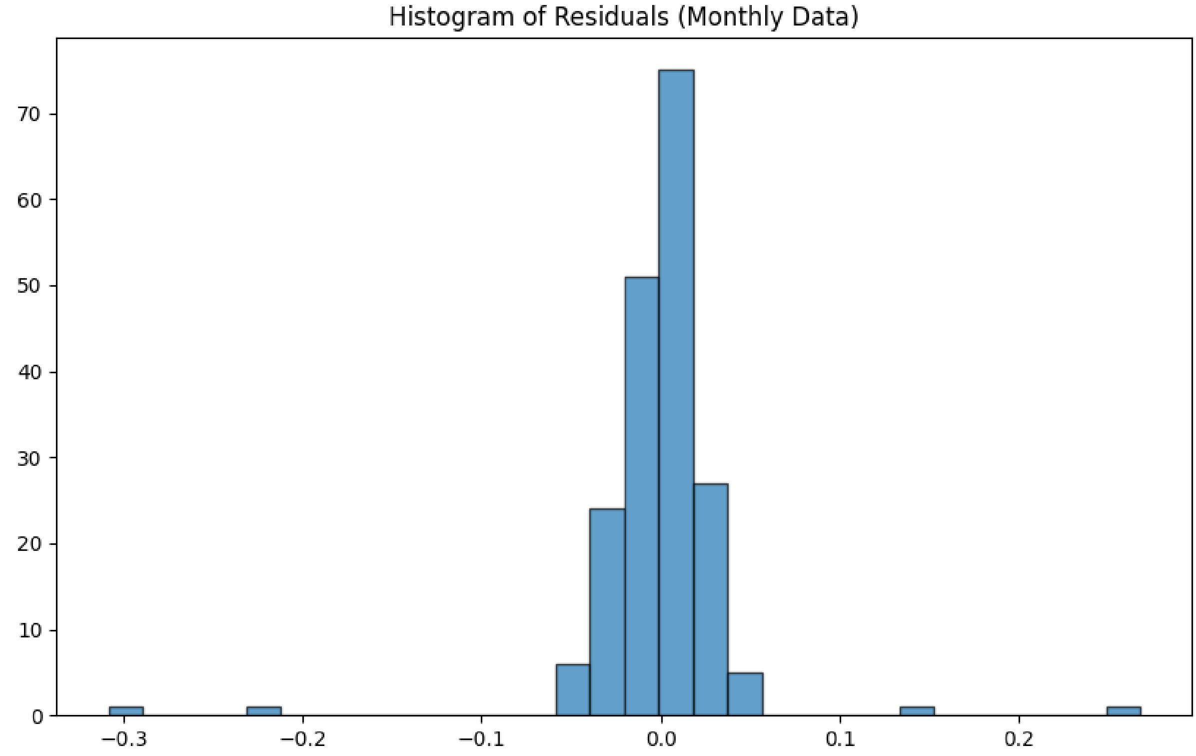

Image 90: Histogram

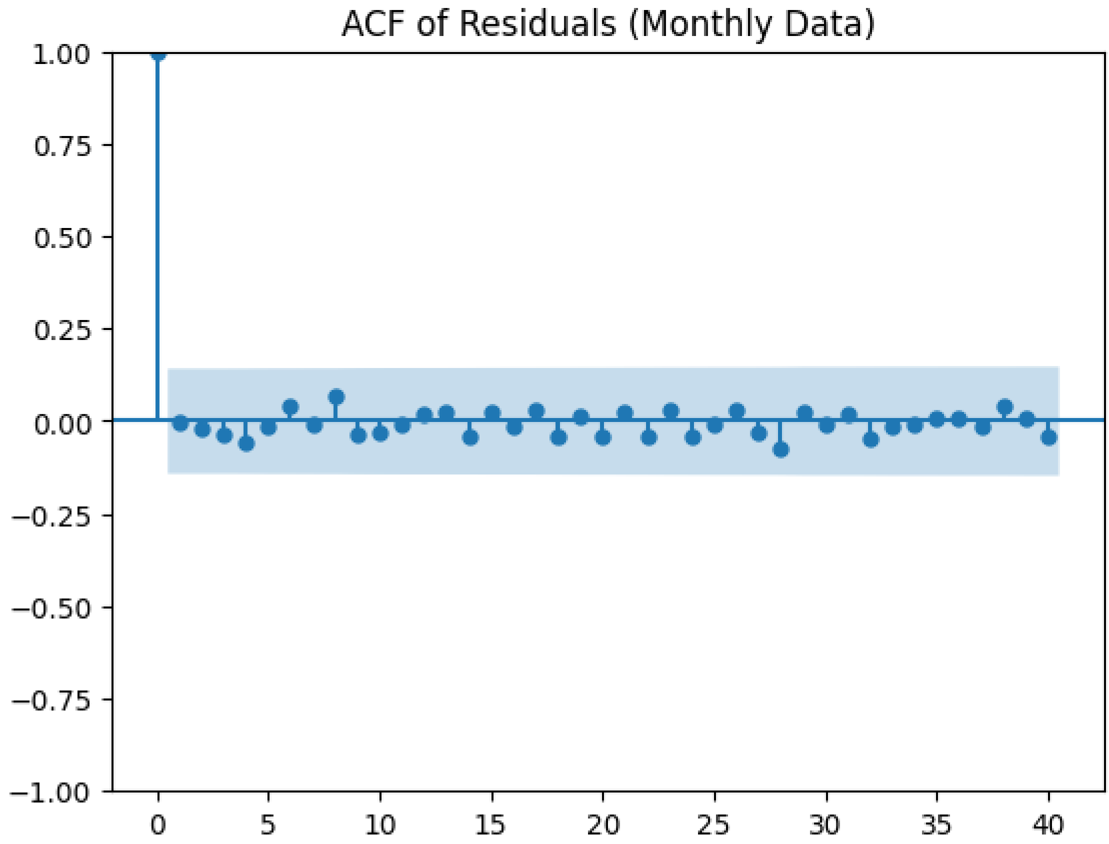

Image 91: ACF Residuals

The results for the Italian ARIMA model are as follows. As we can see in the ACF and PACF graph in Figure No. "87", the spikes at the first four lags are quite significant, with two of them above the confidence interval. Overall, in the long prediction, the ACF and PACF decay quickly, both in the positive and negative directions. We ran our ARIMA model with different parameters, and out of these, we selected the best one, ARIMA(1,0,3), which had the best log-likelihood and other comparative parameters such as AIC and BIC. The model also shows heteroskedasticity, as depicted by the Breusch-Pagan test. After fitting the model, you can see in Figure- "90" that the histogram appears normally distributed and is negatively skewed, and the ACF of the residuals is very low compared to the original data, with some exceptions at the initial lags, where spikes are above the confidence intervals. Overall, the autocorrelation appears to be very low and better fitted than the initial result.

GARCH Models

```
Best GARCH model order for Monthly data: (4, 2)
Best AIC for Monthly data: -1142.9449467090126
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:     Canada_Log_Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                579.472
Distribution:                  Normal   AIC:                          -1142.94
Method:            Maximum Likelihood   BIC:                          -1116.88
                                        No. Observations:                  192
Date:                Thu, Dec 19 2024   Df Residuals:                      191
Time:                        13:02:52   Df Model:                            1
                                  Mean Model
==============================================================================
                  coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
mu         -2.5987e-04  1.159e-03     -0.224      0.823 [-2.531e-03,2.011e-03]
                               Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega      7.8320e-05  4.191e-05      1.869  6.165e-02 [-3.820e-06,1.605e-04]
alpha[1]       0.5370  5.028e-02     10.680  1.266e-26      [  0.438,  0.635]
alpha[2]       0.1189      0.924      0.129      0.898      [ -1.692,  1.930]
alpha[3]   1.1254e-17      0.272  4.137e-17      1.000      [ -0.533,  0.533]
alpha[4]   1.5790e-17      0.222  7.126e-17      1.000      [ -0.434,  0.434]
beta[1]    1.3526e-18      1.274  1.061e-18      1.000      [ -2.497,  2.497]
beta[2]    7.9076e-18      0.691  1.145e-17      1.000      [ -1.354,  1.354]
=============================================================================
```

Image 92: *ARCH Model Output for Canada*

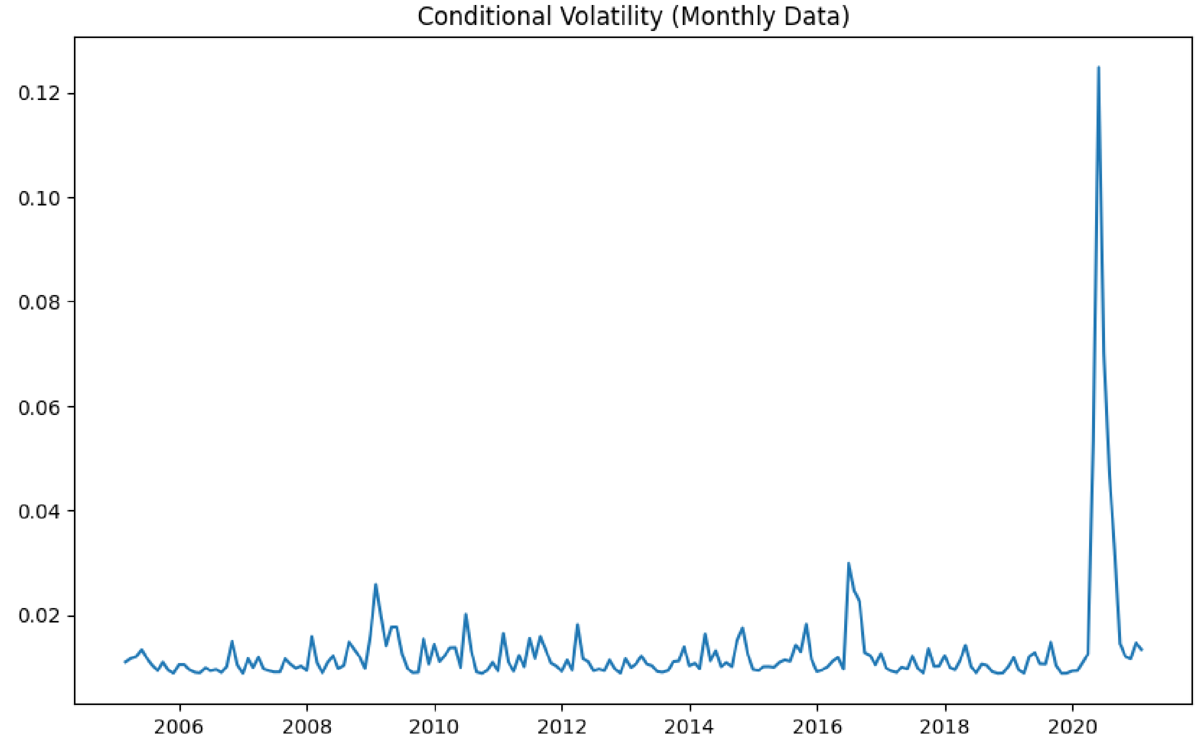

Image 93: *Conditional Volatility Curve for Canada*

*The GARCH model below is based on the Canadian industrial index, specifically a GARCH(4,2) model. This means the model includes 4 lags of the squared error term and 2 lags of the conditional variance. This model was selected after comparing various GARCH models and determining that GARCH(4,2) had the best log-likelihood and the lowest AIC and BIC values, making it the best fit. We ran our estimation for a total of 24*24 values, comparing each of them, and concluded that this was the best-fitted GARCH model. The conditional volatility, while not highly volatile overall, shows some notable spikes, particularly between 2008 to 2010 and 2016 to 2018, with a major spike observed after 2020, likely due to the aftermath of COVID-19.*

```
Best GARCH model order for Monthly data: (1, 2)
Best AIC for Monthly data: -1312.2913657314637
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:     Canada_Log_Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                661.146
Distribution:                  Normal   AIC:                          -1312.29
Method:            Maximum Likelihood   BIC:                          -1296.00
                                        No. Observations:                  192
Date:                Thu, Dec 19 2024   Df Residuals:                      191
Time:                        13:05:12   Df Model:                            1
                                 Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         4.8762e-04  8.767e-04      0.556      0.578 [-1.231e-03,2.206e-03]
                              Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega      3.3376e-05  5.559e-06      6.003  1.932e-09  [2.248e-05,4.427e-05]
alpha[1]       0.9947      0.411      2.422  1.542e-02      [  0.190,  1.799]
beta[1]    5.3333e-03  9.085e-03      0.587      0.557 [-1.247e-02,2.314e-02]
beta[2]    4.9716e-07  2.838e-02  1.752e-05      1.000 [-5.563e-02,5.563e-02]
=============================================================================
```

Image 94: *ARCH Model Output for USA*

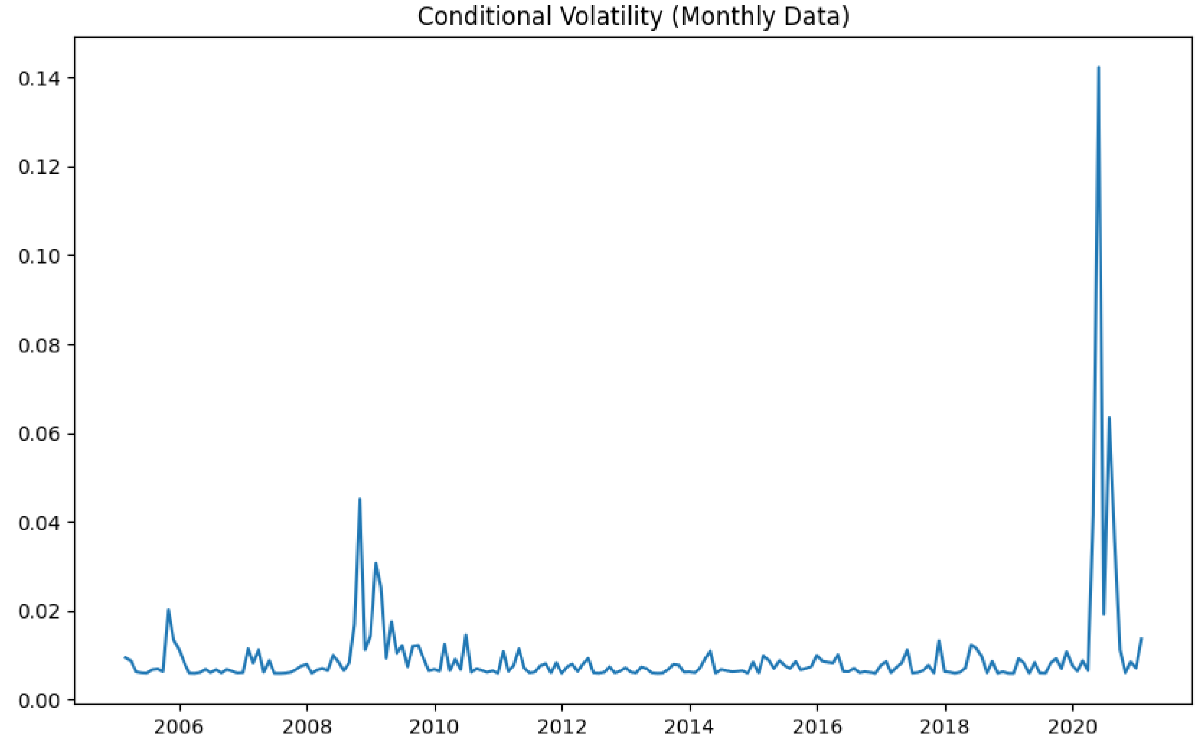

Image 95: *Conditional Volatility Curve for USA*

*The GARCH model below is based on the American industrial index, specifically a GARCH(1,2) model. This means the model includes 1 lag of the squared error term and 2 lags of the conditional variance. This model was selected after comparing various GARCH models and determining that GARCH(1,2) had the best log-likelihood and the lowest AIC and BIC values, making it the best fit. We ran our estimation for a total of 24*24 values of pp and qq, comparing each of them, and concluded that this was the best-fitted GARCH model. The conditional volatility, while not highly volatile overall, shows some notable spikes, particularly between 2008 and 2010, with a major spike observed after 2020, likely due to the aftermath of COVID-19, which may exhibit similar patterns across all economies.*

```
Best GARCH model order for Monthly data: (1, 3)
Best AIC for Monthly data: -1102.4833320418954
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:     Canada_Log_Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                557.242
Distribution:                  Normal   AIC:                          -1102.48
Method:            Maximum Likelihood   BIC:                          -1082.94
                                        No. Observations:                  192
Date:                Thu, Dec 19 2024   Df Residuals:                      191
Time:                        13:07:25   Df Model:                            1
                                  Mean Model
==============================================================================
                  coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
mu         -2.3360e-04  8.535e-04     -0.274      0.784 [-1.906e-03,1.439e-03]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      9.3086e-05  2.297e-05      4.052  5.071e-05 [4.806e-05,1.381e-04]
alpha[1]       0.6459      0.277      2.333  1.965e-02     [  0.103,  1.189]
beta[1]        0.0524  8.743e-02      0.599      0.549     [ -0.119,  0.224]
beta[2]        0.0000      0.183      0.000      1.000     [ -0.359,  0.359]
beta[3]        0.0000  5.961e-02      0.000      1.000     [ -0.117,  0.117]
============================================================================
```

Image 96: *ARCH Model Output for UK*

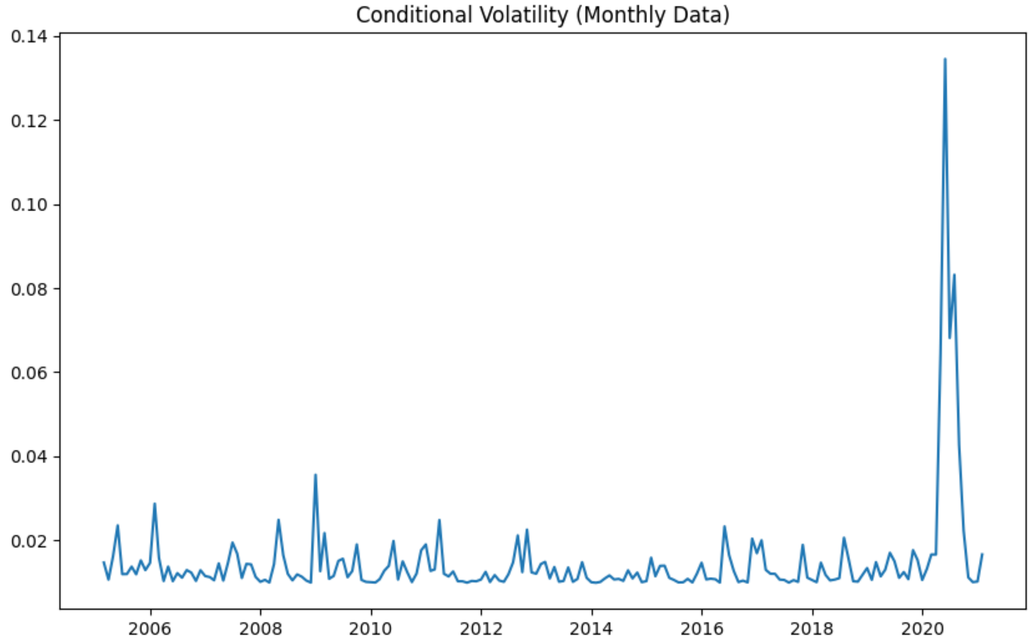

Image 97 : *Conditional Volatility Curve for UK*

*The GARCH model below is based on the UK industrial index, specifically a GARCH(1,3) model. This means the model includes 1 lag of the squared error term and 3 lags of the conditional variance. This model was selected after comparing various GARCH models and determining that GARCH(1,3) had the best log-likelihood and the lowest AIC and BIC values, making it the best fit. We ran our estimation for a total of 24*24 values of p and q, comparing each of them, and concluded that this was the best-fitted GARCH model. The conditional volatility, while not highly volatile overall, shows some notable spikes, particularly between 2008 and 2010, with a major spike observed after 2020, likely due to the aftermath of COVID-19.*

```
Best GARCH model order for Monthly data: (5, 1)
Best AIC for Monthly data: -929.6063959148719
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:     Canada_Log_Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                472.803
Distribution:                  Normal   AIC:                          -929.606
Method:            Maximum Likelihood   BIC:                          -903.546
                                        No. Observations:                  192
Date:                Thu, Dec 19 2024   Df Residuals:                      191
Time:                        13:10:14   Df Model:                            1
                                  Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         2.3991e-03  1.482e-03      1.619      0.106 [-5.061e-04,5.304e-03]
                              Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega      1.5244e-04  4.406e-05      3.459  5.414e-04  [6.607e-05,2.388e-04]
alpha[1]       0.1927  8.082e-02      2.384  1.712e-02    [3.429e-02,  0.351]
alpha[2]       0.2033      0.204      0.996      0.319      [ -0.197,  0.603]
alpha[3]   1.6921e-12  1.660e-02  1.019e-10      1.000 [-3.253e-02,3.253e-02]
alpha[4]   3.9730e-12      0.145  2.731e-11      1.000      [ -0.285,  0.285]
alpha[5]       0.6040      0.463      1.305      0.192      [ -0.303,  1.511]
beta[1]    1.1511e-12  3.901e-02  2.951e-11      1.000 [-7.645e-02,7.645e-02]
=============================================================================
```

Image 98: ARCH Model Output for japan

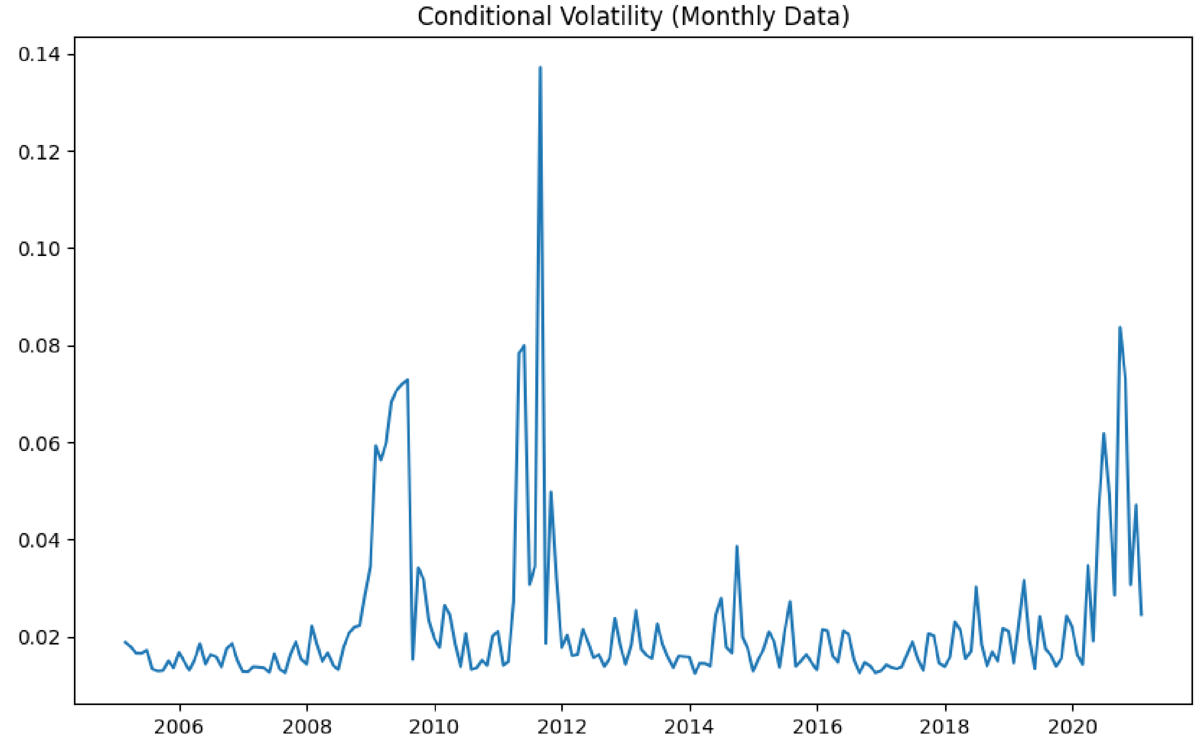

Image 99: Conditional Volatility Curve for Japan

The GARCH model below is based on the UK industrial index, specifically a GARCH(5,1) model. This means the model includes 5 lags of the squared error term and 1 lag of the conditional variance. This model was selected after comparing various GARCH models and determining that GARCH(5,1) had the best log-likelihood and the lowest AIC and BIC values, making it the best fit. We ran our estimation for a total of 24×24 values of p and q, comparing each of them, and concluded that this was the best-fitted GARCH model. The conditional volatility is relatively very high compared to other G-7 nations, showing notable spikes, particularly between 2008 and 2010, as well as between 2011 and 2012, with a less major spike observed in 2020.

```
Best GARCH model order for Monthly data: (2, 1)
Best AIC for Monthly data: -1014.0625059515214
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:      Canada_Log_Returns   R-squared:                       0.000
Mean Model:              Constant Mean   Adj. R-squared:                  0.000
Vol Model:                       GARCH   Log-Likelihood:                 512.031
Distribution:                   Normal   AIC:                           -1014.06
Method:             Maximum Likelihood   BIC:                           -997.775
                                         No. Observations:                   192
Date:                 Thu, Dec 19 2024   Df Residuals:                       191
Time:                         13:13:41   Df Model:                             1
                                  Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         1.0948e-03  8.253e-04      1.327      0.185 [-5.227e-04,2.712e-03]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      8.3832e-05  2.782e-05      3.013  2.584e-03 [2.931e-05,1.384e-04]
alpha[1]       0.5296      0.191      2.774  5.535e-03     [  0.155,  0.904]
alpha[2]       0.3149      0.406      0.775      0.438     [ -0.482,  1.111]
beta[1]        0.1265      0.238      0.531      0.595     [ -0.340,  0.593]
============================================================================
```

Image 100: ARCH Model Output for Germany

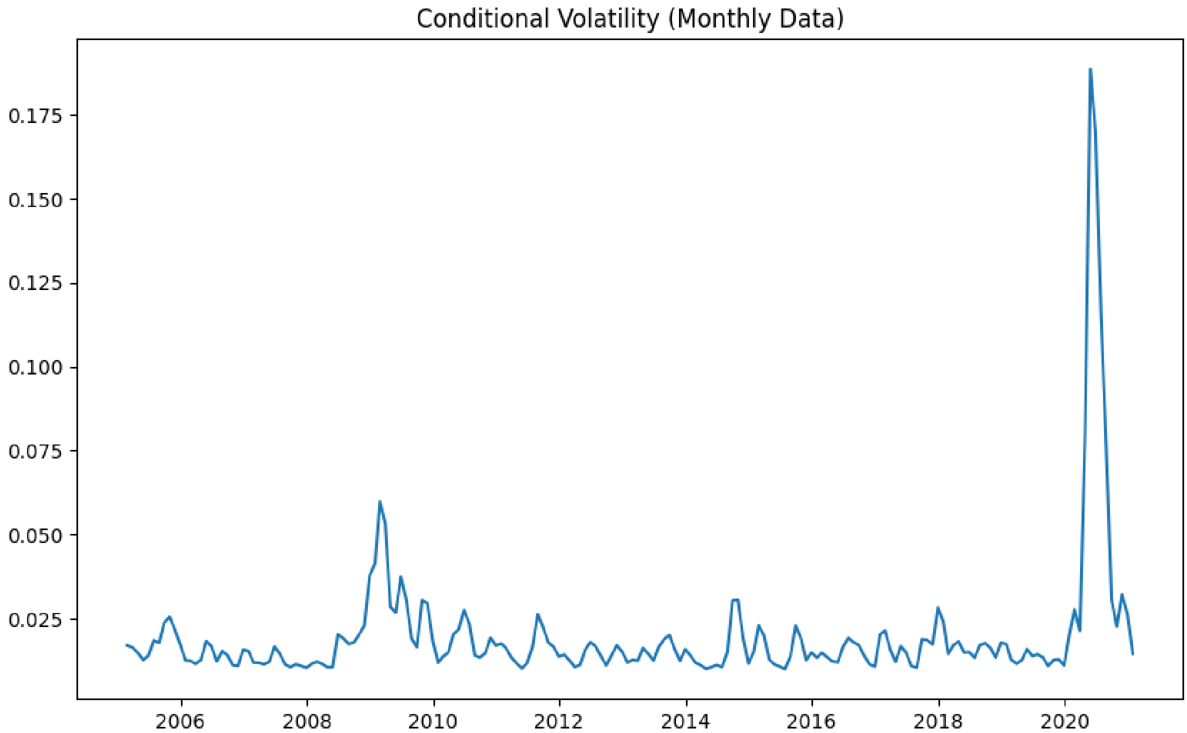

Image 101: *Conditional Volatility Curve for Germany*

The GARCH model below is based on the German industrial index, specifically a GARCH(2,1) model. This means the model includes 1 lag of the squared error term and 2 lags of the conditional variance. This model was selected after comparing various GARCH models and determining that GARCH(1,3) had the best log-likelihood and the lowest AIC and BIC values, making it the best fit. The conditional volatility, while not highly volatile overall, shows some notable spikes, particularly between 2008 and 2010, with a major spike observed after 2020, likely due to the aftermath of COVID-19.

```
Best GARCH model order for Monthly data: (10, 2)
Best AIC for Monthly data: -959.294259946728
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:     Canada_Log_Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                493.647
Distribution:                  Normal   AIC:                          -959.294
Method:            Maximum Likelihood   BIC:                          -913.689
                                        No. Observations:                  192
Date:                Thu, Dec 19 2024   Df Residuals:                      191
Time:                        13:16:06   Df Model:                            1
                                 Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         1.0774e-03  1.052e-03      1.024      0.306 [-9.841e-04,3.139e-03]
                              Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega      8.9124e-05  5.790e-05      1.539      0.124 [-2.435e-05,2.026e-04]
alpha[1]       0.3157      0.196      1.614      0.106   [-6.760e-02, 0.699]
alpha[2]       0.0000      0.275      0.000      1.000      [ -0.539, 0.539]
alpha[3]       0.0000      0.467      0.000      1.000      [ -0.915, 0.915]
alpha[4]       0.0000      0.256      0.000      1.000      [ -0.501, 0.501]
alpha[5]       0.0000  4.473e-02      0.000      1.000 [-8.766e-02,8.766e-02]
alpha[6]       0.0519      0.215      0.241      0.809      [ -0.370, 0.473]
alpha[7]       0.0000      0.247      0.000      1.000      [ -0.484, 0.484]
alpha[8]       0.0000      0.286      0.000      1.000      [ -0.561, 0.561]
alpha[9]       0.6324      0.489      1.294      0.196      [ -0.325, 1.590]
alpha[10]  1.8425e-09      1.798  1.025e-09      1.000      [ -3.524, 3.524]
beta[1]        0.0000      1.867      0.000      1.000      [ -3.659, 3.659]
beta[2]        0.0000      0.190      0.000      1.000      [ -0.372, 0.372]
```

Image 102: ARCH Model Output for France

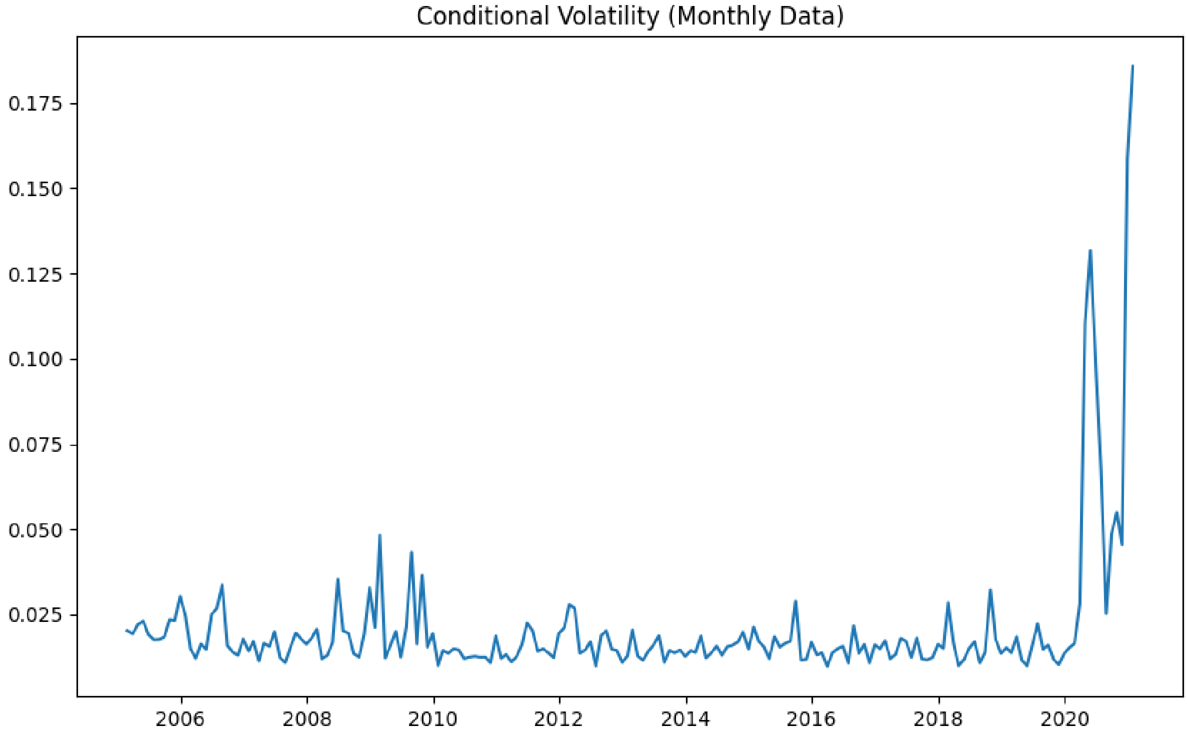

Image 103: Conditional Volatility Curve for France

The GARCH model below is based on the French industrial index, specifically a GARCH(10,2) model. This means the model includes 10 lags of the squared error term and 2 lags of the conditional variance. This model was selected after comparing various GARCH models and determining that GARCH(10,2) had the best log-likelihood and the lowest AIC and BIC values, making it the best fit. The conditional volatility, while not highly volatile overall, shows two notable spikes, particularly between 2008 and 2010, with a major spike observed after 2020, and it appears to be increasing afterward.

```
Best GARCH model order for Monthly data: (5, 1)
Best AIC for Monthly data: -891.4038206574605
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:     Canada_Log_Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                453.702
Distribution:                  Normal   AIC:                          -891.404
Method:            Maximum Likelihood   BIC:                          -865.344
                                        No. Observations:                  192
Date:                Thu, Dec 19 2024   Df Residuals:                      191
Time:                        13:17:51   Df Model:                            1
                                  Mean Model
==============================================================================
                  coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
mu         -2.0621e-03  2.206e-03     -0.935      0.350 [-6.386e-03,2.262e-03]
                               Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega      1.9167e-04  1.244e-04      1.540      0.123 [-5.223e-05,4.356e-04]
alpha[1]       0.1685      0.106      1.597      0.110    [-3.830e-02, 0.375]
alpha[2]       0.8315      0.610      1.362      0.173       [ -0.365, 2.028]
alpha[3]   2.3596e-13      0.637  3.705e-13      1.000       [ -1.248, 1.248]
alpha[4]   2.3512e-13      0.694  3.387e-13      1.000       [ -1.360, 1.360]
alpha[5]   2.3468e-13      0.173  1.354e-12      1.000       [ -0.340, 0.340]
beta[1]    3.7178e-12      0.806  4.612e-12      1.000       [ -1.580, 1.580]
=============================================================================
```

Image 103: ARCH Model Output for Italy

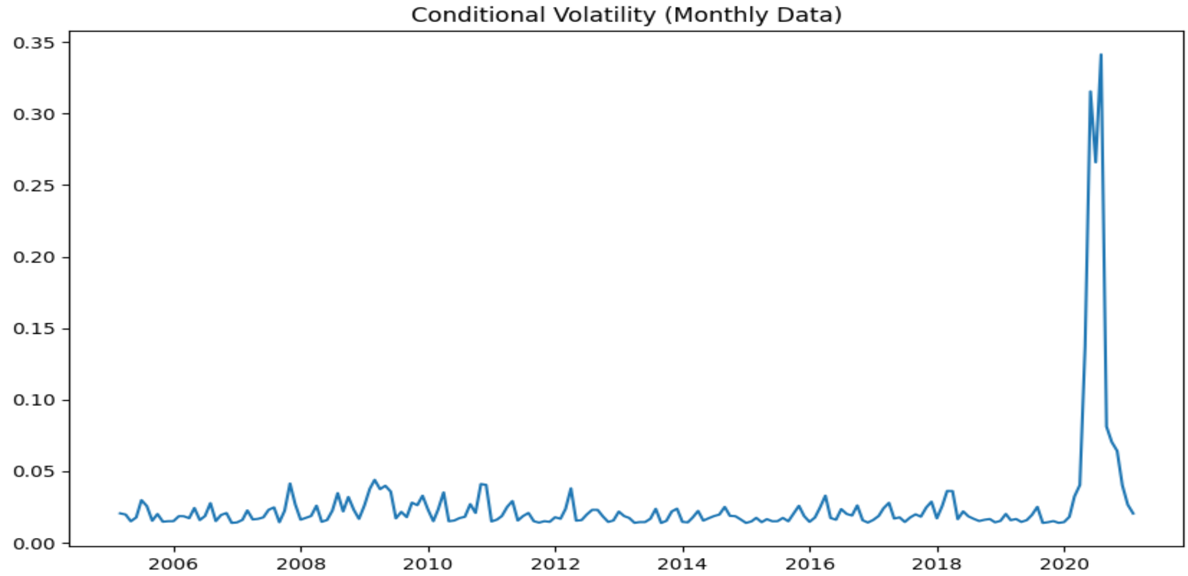

Image 104: Conditional Volatility Curve for Italy

The GARCH model below is based on the Italin industrial index, specifically a GARCH(5,1) model. This means the model includes 5 lags of the squared error term and 1 lag of the conditional variance. This model was selected after comparing various GARCH models and determining that GARCH(5,1) had the best log-likelihood and the lowest AIC and BIC values, making it the best fit. The conditional volatility, while not highly volatile overall, shows no notable spikes during the observed period, except for a major spike observed after 2020.

Conclusion

The relationship between the logarithm of industrial production in G-7 countries and the ECM (Error Correction Model) isn't always straightforward. Over time, the complexity of independent variables has evolved, making predictions and dependencies in today's economic environment quite different from those in the past. The original ECM model was developed and applied to data series from 1950 to 2000, a period when the independent variables primarily consisted of lagged values of stock price indices.

However, with the expansion of global economies and the rise of more intricate economic challenges in the modern era, the limitations of the ECM model have become more apparent. In contrast, models like ARIMA and GARCH have shown better performance in capturing the nuances of contemporary data. Their ability to handle greater complexity and volatility gives them an edge over the simpler framework of ECM. This shift highlights the need for adaptive modeling approaches as we navigate the evolving dynamics of today's world economy.

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