Lakshya Batch 2020-21

Ch-03
Current Electricity
Formula Sheet

Sheet Credit: Kumar Satyam

CHAPTER-03
Current Electricity

No. 1

* **Electric current** → rate of flow of charge through a cross section area held ⊥ to the flow.

$\boxed{I = \frac{dq}{dt}}$; SI unit = Ampere.

$\boxed{I = \int \vec{J}\cdot\vec{dA}}$

* **Current density** = current per unit area.

$\boxed{\vec{J} = \frac{i}{\vec{A}}}$

* **drift velocity** = $\boxed{\vec{V_d} = \frac{-eE}{m_e}\tau}$ — τ = avg. relax. time

↳ is in opp. dirn to $\vec{E}$.

$|V| = E\cdot l \Rightarrow \vec{E} = V/l$

So; $\boxed{V_d = \frac{eV}{ml}\tau}$ (lower to higher potential)

$\boxed{I = V_d e n A}$

current flow through cross sec. area.

$\boxed{\vec{J} = -V_d e n}$ or $\boxed{\vec{J} = \frac{-ne^2 E\tau}{m}}$

Mobility (μ) → drift velocity per unit electric field.

$\mu = \frac{V_d}{E} = \frac{e\tau}{m}$

Ohm's Law → Current is directly prop. to the voltage applied across the ends; with condition that physical condition must not change

$\boxed{V = IR}$ or $\boxed{J = \sigma E}$ — σ → conductivity.

Resistance (R) → ratio of P.D. across the ends of conductor to (I) flowing.

$R = \frac{\Delta V}{I}$; SI unit Ω

from; $I = V_d e n A = \frac{eV}{ml}\tau \cdot enA$.

⊙ $R = \frac{V}{I} = \frac{ml}{ne^2\tau A} = \rho\frac{l}{A}$ — ρ → resistivity.

$\rho = \frac{m}{ne^2\tau}$ = (constant for given material at given conditions. like temp, pressure)

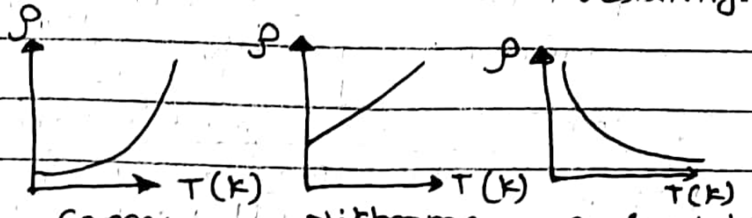
* Resistance (R) can be diff. for a given material ; ($\because R \propto l$; $R \propto \frac{1}{Area}$)

Temperature dependence of Resistance:

→ $R_T = R_0[1 + \alpha(\Delta T)]$

→ $\rho_T = \rho_0[1 + \alpha(\Delta T)]$

α = temp. coeff. of resistivity.



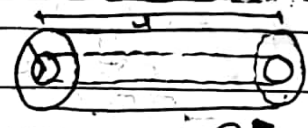
Copper — Nichrome — Semiconductor

⊙ **SHORT TRICK**

Streching/compressing length.	increasing/decreasing radius/Area.
$\frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)^2$	$\frac{R_2}{R_1} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^4$

⊙ **TRICKY PROBLEMS** (find Resistance).

1)

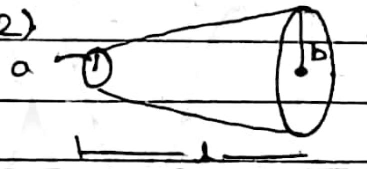


$\boxed{R = \frac{\rho}{2\pi l}\log_e\frac{b}{a}}$

Inner r = a
Outer r = b
length = l

detailed solution Lecture-4.

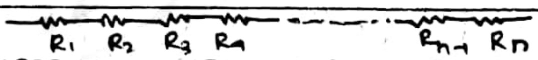
2)



$\boxed{R = \frac{\rho l}{\pi ab}}$

Combination of Resistors:

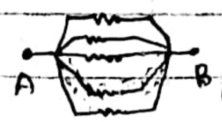
(i) **Series** → Current is same through all R.



$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n$

If all R are equal. $\boxed{R_{eq} = nR}$

(ii) **Parallel** → Potential diff. is same across all R.



$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$

If all R are same/equal → $\boxed{R_{eq} = \frac{R}{n}}$

$R_{eq} = \frac{R_1R_2}{R_1+R_2}$ → for two resistors in parallel.

⊙ **Wheatstone Bridge.**

↳ is balanced;

⊙ If $\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow V_C = V_D$

↳ no current through G.

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* KIRCHHOFF'S LAW

Junction law (or) Current law

⊙ Conservation of charge.

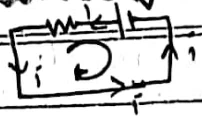
→ $i_{in} = i_{out}$

→ $I_1 + I_2 + I_4 = I_3 + I_5$

→ $\sum I_P$ (at P) $= 0$ junction.

Loop law (or) Voltage law

⊙ Conservation of energy.



~~* algebraic sum~~ of P.D. along a closed loop is zero.

$\sum V = 0$ (closed loop)

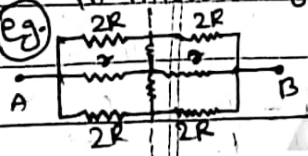
↳ $\boxed{V_i = V_f}$

⊙ Combination of Resistors (Symmetry problem)

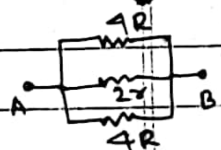
⊡ Mirror Symmetry (⊥ axis)

→ current is same in mirror img.

eg.



→ cut the wire, which touch branch only once.

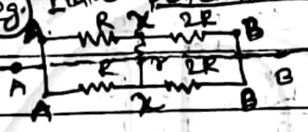


$\boxed{R_{eq} = \frac{2Rr}{(R+r)}}$

⊡ Folding Symmetry (∥ axis)

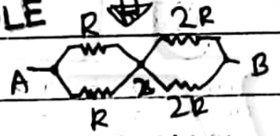
→ symm. points will have potential same

eg.



→ resistors b/w the same potential can be cut out.

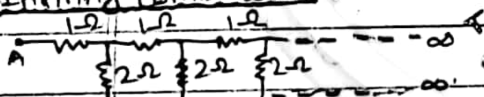
EXAMPLE



$R_{eq} = \frac{3}{2}R$

* Infinity combination

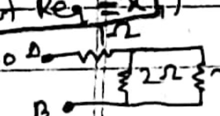
eg.



This is an example.

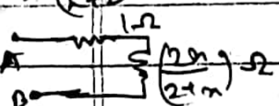
$\boxed{\text{Let } R_{eq} = x}$ → 1 unit is removed, then also $r_{eq} = x$ (for remaining)

So



→ 2, x → parallel.
↳ $R_{eq} = \left(\frac{2x}{x+2}\right)$.

now, $\left(\frac{2x}{x+2}\right)$ & 1 in series.

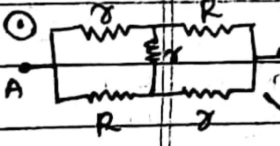


$R_{eq} = x = 1 + \frac{2x}{2+x}$

Solve quad. eq.

$R_{eq} = 2\,\Omega$ (just for example)

⊙



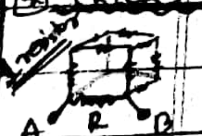
→ unbalanced wheatstone bridge.

using Kirchhoff's law

$R_{eq} = \frac{r(3R+r)}{(R+3r)}$

⊡ TRICKS USING KCL & KVL

1 resistor

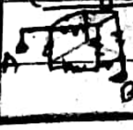


$R_{eq} = \frac{7}{12}R$

2 resistors (min) (face diagonal)

$R_{eq} = \frac{3}{4}R$

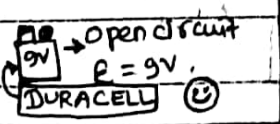
3 resistors (min.) (body diagonal)



$R_{eq} = \frac{5}{6}R$

⊡ Electric Cell → provides electrical energy to all the circuit elements.

EMF (ε) → electromotive force – P.D. b/w the electrodes of a cell, when no current is drawn. (OPEN CIRCUIT)



→ open circuit
$\varepsilon = 9V$.

Int. resistance (r)
↳ due to material of electrolyte (inside cell).

ε → EMF = work done by cell in carrying a unit charge in closed circuit.

$\boxed{\varepsilon = \frac{Work}{q}}$

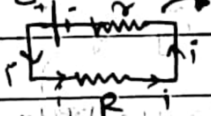
⊙ Terminal Voltage ⇒ $V = \varepsilon - ir$

↳ discharging of cell

$\boxed{V = \varepsilon + ir}$ – charging of cell.

$\boxed{V = \varepsilon\,;\; i = 0}$ open circuit.

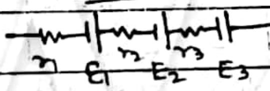
$\boxed{V = 0\,;\; i = \frac{\varepsilon}{r}}$ – short circuit.



$\boxed{i = \frac{\varepsilon}{R+r}}$ – for circuit with ext. resistance.

⊡ Combination of Cells

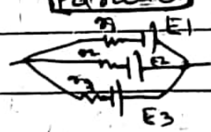
Series



$r_{eq} = r_1 + r_2 + r_3 + \dots r_n$

$E_{eq} = E_1 + E_2 + E_3 + \dots E_n$

Parallel



$E_{eq} = \dfrac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$

$r_{eq} = \dfrac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$

⊙ Infinity in Series.
(n – Identical cells)

$\boxed{E_{eq} = nE}$; $\boxed{r_{eq} = nr}$

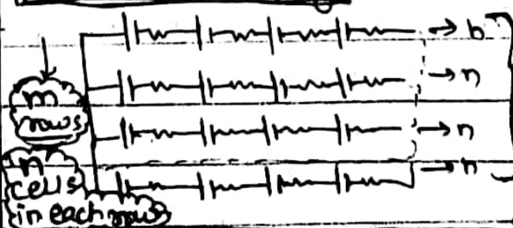
$\boxed{I = \frac{nE}{nr + R}}$ → ext load

⊙ Infinity (identical)

$\boxed{E_{eq} = E}$; $\boxed{r_{eq} = \frac{r}{n}}$

$\boxed{I = \frac{nE}{r + nR}}$ → ext. Load.

⊡ Mixed Grouping



Total cells = mn.

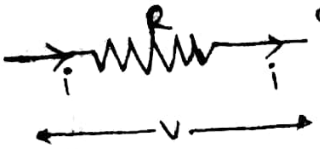
$\boxed{E_{eq} = nE}$

$\boxed{r_{eq} = \frac{nr}{m}}$

$I = \frac{mnE}{mR + nr}$ → ext load.

; for $mR = nr$
↳ I will be max.

$\boxed{I_{max} = \frac{nE}{2r}}$



energy absorbed → $dE = V dq$;

power consumed

So, for Resistor of Resistance (R) $P = \frac{dE}{dt} = Vi$

$\boxed{\text{Power} \to P}$

1. $P = i^2 R$
2. $P = \frac{V^2}{R}$
3. $P = Vi$

Energy → $\boxed{E = \int P \cdot dt}$

$\boxed{E = P \times t}$ $\boxed{E = Vit}$

$E = i^2 R t$

$E = \frac{V^2}{R} t$

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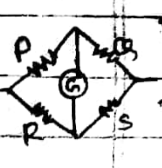
*Meter Bridge / Slide Wire Bridge

→ principle of Wheatstone Bridge.

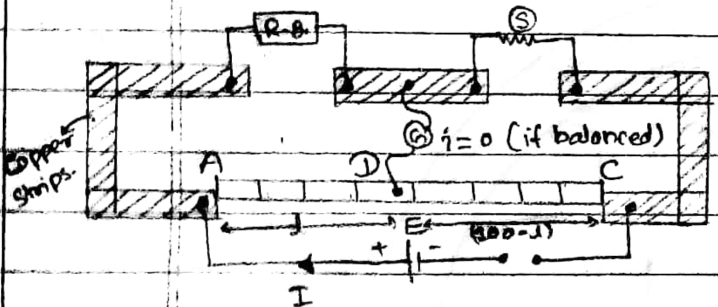


$\boxed{\text{if } \frac{P}{R} = \frac{Q}{S}}$ then $i = 0$ through (G)

USED TO →

→ measure unknown resistance (99%)

→ Compare two unknown resistances.



* R.B = Resistance box = ~~unknown~~ known.
* S = unknown Resistance ⇒ To find.
* Slide the jockey smoothly on wires till galvanometer shows No Deflection, → NULL POINT.

$\frac{R}{R_{AD}} = \frac{S}{R_{DC}} \to \frac{R}{l\cancel{r}} = \frac{S}{(100-l)\cancel{r}}$

$\frac{R}{l} = \frac{S}{100-l}$

R → given / chosen by us.
l → observe (balancing length)
S → find.

$\boxed{\text{Error}}$

→ max. error in Length = least count of Scale,

So, $\boxed{\frac{\Delta S}{S} = \frac{\Delta l}{l} + \frac{\Delta(100-l)}{100-l}}$ $\Delta S \to$ error in S.

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* at $l = 50$ cm ⇒ best measurement / ~~least error in~~ (S)

→ (R. Box) resistance = (Unknown) resistance

* END CORRECTIONS

→ adding neglected resistance of copper plates.

(i) In terms of eq. length.

eg. (L) → Copper Plate = 4 Ω, (R) → copper plate = 6 Ω

~~wire resistance = 2 Ω/cm~~

So, Left = 2 cm; Right = 3 cm.

Now, $\boxed{\frac{R}{l+2} = \frac{S}{100-l+3}}$ end corrected.

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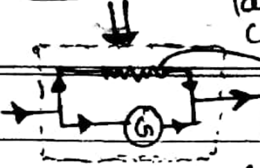
*Galvanometer → measure small current & direction of current in circuit.

(i) $\boxed{\text{Ammeter}}$ → measures large current.



(S) shunt → very very small resistance

& $\boxed{i \propto i_g}$ →

(G) (i_g)	Actual (i)
10 mA	→ 1000 mA
5 mA	→ 500 mA

for example →

$\boxed{i_g \times G = (i - i_g) S}$ ↔ $V_G = V_S$

resistance

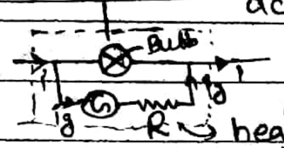
(i) Connected in series to measure current.

(i) has zero resistance (ideal Ammeter)

(i) But $i_{measured} < i_{actual}$ (practically)

(∵ R can not be zero).

(i) $\boxed{\text{Voltmeter}}$ → measures potential difference across any two points in circuit.



So, that max. current flows through bulb.

R → heavy resistance (i) connected in parallel

$\boxed{V_G = i_g (G + R)}$

(ii) has ∞ resistance (ideal)

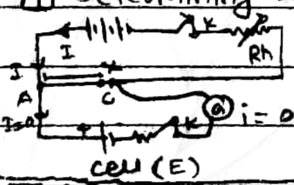
(iii) $V_{measured} < V_{actual}$ (as R is never ∞) (practically)

POTENTIOMETER → measures potential difference more accurately than voltmeter.

(because it draws not current from circuit)

(i) principle → for a wire of UNIFORM cross section Area; P.D. across any segment is proportional to its length.

[I] Determining EMF of a cell.



} At (C), ⇒ No deflection

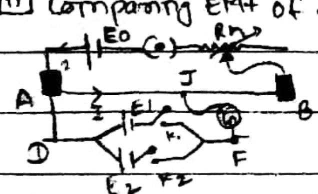
$\boxed{AC = x}$; $V_{AC} = Kx$

$V_{DF} = E$

⇒ $\boxed{E = Kx}$

$\frac{V}{L}$

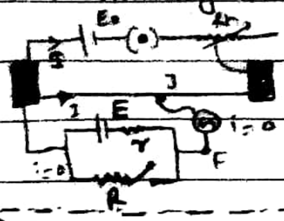
[II] Comparing EMF of two cells.



$\boxed{E_1 = K l_1}$, $\boxed{E_2 = K l_2}$

$\boxed{\frac{E_1}{E_2} = \frac{l_1}{l_2}}$

[III] Determining internal resistance of cell.



$\boxed{r = R\left(\frac{E}{V} - 1\right)}$

$\boxed{r = R\left(\frac{l_1}{l_2} - 1\right)}$

(i) how to increase sensitivity (Potentiometer)

(i) increasing length of wire → (l↑) → (K↓)

(ii) decreasing (i) in circuit using Rheostat → (i↓) → (V↓) → (K↓)

(K↓) (smaller) → more sensitive.