Formula Sheet -04

Moving Charges and Magnetism

Sheet Credit: Aakriti Maurya
PHYSICSWALLAH Lakshya Batch

MOVING CHARGES AND MAGNETISM

- **Oersted's Experiment** :- A magnetic compass (or magnetic needle) shows deflection when placed near a current carrying wire. SNOW → west, south, North, wire is over

★ If direction of current is reversed, the deflection is also reversed. NSOE → East

SNOW SNBE (If wire is held below needle)

★ (1 change ⇒ deflection change
2 change ⇒ deflection same)

conclusion :- moving charges produce magnetic field.

- **Right hand thumb rule** :- thumb ⇒ direction of current, fingers ⇒ direction of $\vec{B}$.
- **Biot-Savart Law** :- [Calculation of Magnetic Field ($\vec{B}$) and its direction due to a current element.

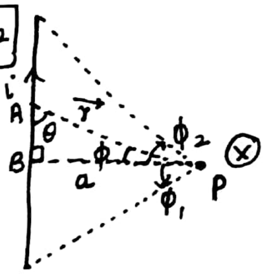
vector form : $\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{(i)(\vec{dl} \times \vec{r})}{r^2}}$ **magnitude** : $\boxed{|\vec{dB}| = \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2}}$

(For direction of $\vec{dB} \Rightarrow \vec{dl} \times \vec{r}$)

★ Magnetic field is zero on the axis of current element.

★ Units of Magnetic field Intensity ⇒ N/A-m or Tesla $\boxed{1\,T = Weber/m^2}$

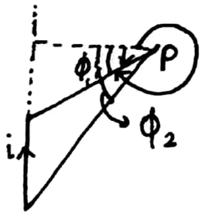
- **Magnetic field due to Straight Current carrying wire** :-



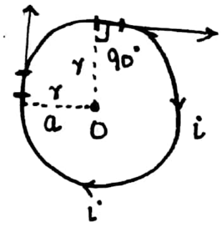
★ $\boxed{B = \frac{\mu_0}{4\pi} \frac{i}{a} [\sin\phi_2 + \sin\phi_1]}$ r = variable, ϕ = variable (ϕ_1 and ϕ_2 are taken at the ends of wire).

- **If Point P lies outside the line of wire** :-

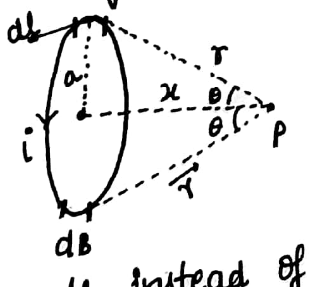
$\boxed{B = \frac{\mu_0}{4\pi} \frac{i}{a} [\sin\phi_2 - \sin\phi_1]}$ (ϕ_1 and ϕ_2 are measured in opposite directions from perpendicular ⇒



- **Magnetic field due to straight wire of infinite length** - $\boxed{B = \frac{\mu_0}{4\pi} \frac{2i}{a}}$
- **Magnetic field due to straight wire of semi-infinite length** :- $\boxed{B = \frac{\mu_0}{4\pi} \frac{i}{a}}$ ($\because \phi_1 = 0°$ and $\phi_2 = 90°$)
- **Magnetic field at the centre of circular loop / circular arc** :- $\boxed{B_{net} = \frac{\mu_0}{2} \frac{i}{a}}$



- **For N loops** :- $\boxed{\left(\frac{\mu_0}{2} \frac{i}{a}\right) N = B_{N\,loops}}$
- **B at the centre of a semi-circular loop** ⇒ $\boxed{B = \frac{\mu_0}{4} \frac{i}{a}}$
- **B at centre of circular arc** ⇒ $\boxed{B_{net} = \frac{\mu_0}{4\pi} \frac{i\theta}{a}}$ radians
- **Magnetic Field on the Axis of a circular current loop**



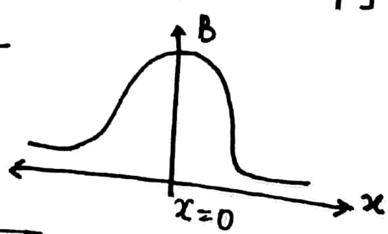
$\vec{dB} \perp \vec{dl}$

$\vec{dB} \perp \vec{r}$

$\boxed{B = \frac{\mu_0 i a^2}{2(a^2+x^2)^{3/2}}}$ (axis) [direction of $\vec{B}$ is same on either side of the loop].

$\boxed{B_{max.} = \frac{\mu_0}{2} \frac{i}{a}}$

Graph for B_{axis} v/s x :-

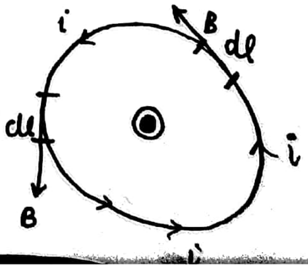


★ If instead of one loop, there is a coil of N turns ⇒ $\boxed{B = \frac{\mu_0 i a^2 N}{2(a^2+x^2)^{3/2}}}$

- **Magnetic field on axis of a current loop at very far distance from centre** ($x >>> a$) ⇒ $\boxed{B_{axis} = \frac{\mu_0 i a^2}{2x^2}}$

$\boxed{B_{axis\ (x>>>a)} = \frac{\mu_0}{4\pi} \frac{2iA}{x^3}}$ OR $\boxed{B_{axis} = \frac{\mu_0}{4\pi} \frac{2M}{x^3}}$ where M = Magnetic Moment.

- **AMPERE'S CIRCUITAL LAW** (when current distribution is symmetric) $\boxed{\oint_{closed\ path} \vec{B} \cdot \vec{dl} = \mu_0 i_{inside}}$ ⟹



★ Ampere's Circuital law states that the line integral of magnetic field along a closed path is equal to μ_0 times the total current enclosed by the closed path.

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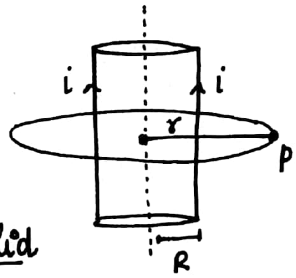
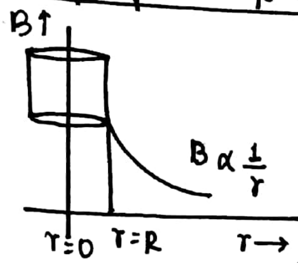
★ Note :- ($\oint \vec{B}\cdot\vec{dl} = \mu_0 i_{inside}$)
↳ at P, is due to all currents (inside/outside).

◉ Magnetic field due to hollow Cylindrical wire (long cylinders) ⇒



⇒ outside the wire ($r > R$) ⇒ $\boxed{B = \frac{\mu_0 i}{2\pi r}}$ and inside ⇒ $\boxed{B = 0}$

◉ Graph for B v/s r -

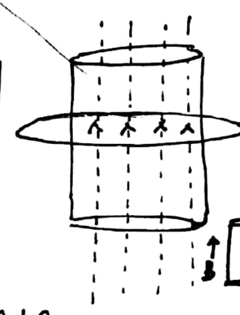


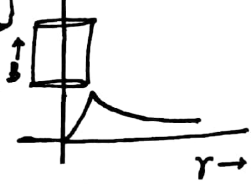
$\boxed{B_{surface} = \frac{\mu_0}{4\pi}\frac{2i}{R}}$

◉ Magnetic field due to solid cylindrical wire :-

⇒ outside = $\boxed{B = \frac{\mu_0}{4\pi}\frac{2i}{r}}$

⇒ inside ⇒ $\boxed{i' = \frac{ir^2}{R^2}}$



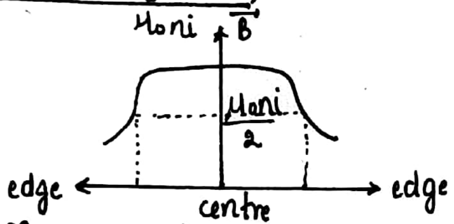


◉ Magnetic field due to a Solenoid (ideal solenoid ⇒ $L \ggg R$)

★ Magnetic field at any point 'P' on the axis of solenoid - (Biot Savart Law) $dB = \frac{-\mu_0 n i \sin\theta \, d\theta}{2}$

⇒ $\boxed{B = \frac{\mu_0 n i}{2}[\cos\theta_2 - \cos\theta_1]}$ n = no. of turns per unit length.

★ Graph of $\vec{B}$ v/s x on axis of a long straight solenoid ($l \ggg R$)



★ Magnetic field inside a long straight solenoid (near its centre) ⇒

($\oint \vec{B}\cdot\vec{dl} = \mu_0 i_{inside}$) = Ampere's Circuital law (no. of turns = nx)

$\boxed{B = \mu_0 n i}$ ⇒ inside a long solenoid (near its centre)

◉ Magnetic field due to Toroid and large thick sheet (Toroid ⇒ solenoid bent in the form of closed ring)

⇒ For points in the open space interior to Toroid ⇒ $\boxed{\vec{B} = 0}$

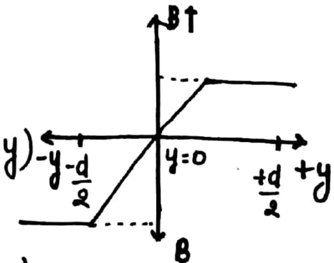
⇒ For points in the open space exterior to Toroid ⇒ $\boxed{\vec{B} = 0}$

⇒ For points inside the toroid $\boxed{B = \frac{\mu_0 N i}{2\pi r}}$

$\left(B_{avg.(inside)} = \frac{\mu_0 N i}{2\pi r_{avg.}}\right)\left(B_{inside\ avg.} = \frac{\mu_0 N i}{L}\right)\left(B_{inside\ avg.} = \mu_0 n i\right)$

◉ Magnetic field due to a large and thick current carrying sheet -

(i) Point outside (P) ⇒ $\boxed{B_{out.} = \frac{\mu_0 J d}{2}}$ (ii) Point inside (Q) $\boxed{B = \mu_0 J y}$ ($B \propto y$)



◉ Force experienced by a moving charge in a Magnetic field

$\boxed{|F| = qvB\sin\theta}$ $\boxed{\vec{F} = q(\vec{v}\times\vec{B})}$ ($\vec{F}\perp\vec{v}$) and ($\vec{F}\perp\vec{B}$) when, ($v = 0$ ⇒ $|F| = 0$)
($\theta = 0°$ & $\theta = 180°$ ⇒ $|F| = 0$)

◉ Fleming's left hand rule ⇒ F = thumb ③
B = forefinger ①
I = middle finger ②

◉ Right hand palm rule]
① fingers - $\vec{B}$ ② Thumb-current,
palm = force.

Work done by Magnetic force on moving charge ⇒ $\boxed{W = 0}$ [F → changes direction of velocity, magnitude ⇒ same]

◉ Lorentz force ⇒ $\boxed{F_{lorentz} = q\vec{E} + q(\vec{v}\times\vec{B})}$ F_l = electromagnetic force.

◉ Motion of a charged particle in Magnetic field $\boxed{R = \frac{mv}{qB}}$ $\boxed{T = \frac{2\pi m}{qB}}$ (T ⇒ independent of velocity)

($\omega = \frac{qB}{m}$) where (ω = angular velocity)

$\boxed{R = \sqrt{\frac{2mV}{qB^2}}}$ V → voltage. $\boxed{R = \frac{\sqrt{2mk}}{qB}}$ $\boxed{p = \sqrt{2mk}}$ p ↳ momentum. $\boxed{R = \frac{p}{qB}}$

◉ Helical path of a charged particle in a Magnetic field ⇒ $\boxed{R = \frac{mv\sin\theta}{qB}}$ $\boxed{R = \frac{mv_\perp}{qB}}$

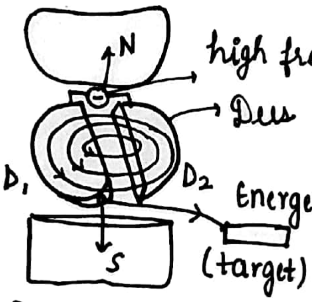
Pitch :- horizontal distance covered in one Time Period ⇒ $\left(\text{pitch} = V\cos\theta \times \frac{2\pi m}{qB}\right)$ 3

★ **Path of Charged Particle in both (Electric and Magnetic field)**

⇒ Case I: $\vec{E}$ & $\vec{B}$ are parallel ⇒ moves in a straight line path with constant acceleration.

⇒ Case II : $\vec{E}$ & $\vec{B}$ are perpendicular ⇒ (i) $F_m > F_e$ ⇒ $\boxed{V > \frac{E}{B}}$ (ii) $F_m < F_e$ ⇒ $\boxed{V < \frac{E}{B}}$ (iii) $F_e = F_m$ $\boxed{V = \frac{E}{B}}$ velocity selector

● **CYCLOTRON** :- used to accelerate charged particles to a very high velocities ⇒ also known as Particle accelerator.



Frequency of oscillator

Time. Period of A.C. = T.P. of circular motion

$f_{A.C.} = \frac{1}{T} = \frac{qB}{2\pi m}$

($f_{oscillator} = f_{circular\ motion}$)

$\boxed{f = \frac{qB}{2\pi m}}$ cyclotron frequency or Magnetic Resonance frequency.

Limitation

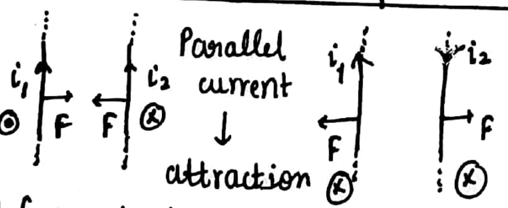
[It cannot be used to accelerate e^-]

$\boxed{K.E._{max.} = \frac{q^2B^2R^2}{2m}}$

● **Force on a current carrying wire in a Magnetic field** –

$\left(F = ilB\sin\theta = i(\vec{l} \times \vec{B})\right)$ when l & B = same direction ⇒ $\theta = 0°$ ⇒ $(|\vec{F}| = 0)$.

● **Force between two parallel current carrying wires** :-



$\boxed{|\vec{F_{12}}| = |\vec{F_{21}}|}$ and $\boxed{\vec{F_{12}} = -\vec{F_{21}}}$

Newton's third law.

● **Current loop as Magnetic dipole** $\vec{M} = m(2l)$ $\boxed{N \quad S}$ (m, m; $\vec{m}$) direction ⇒ South to North.

● **Magnetic moment of current loop** (unit = Am^2)

$\boxed{\vec{M} = i\vec{A}}$ If n loops, $\boxed{\vec{M} = ni\vec{A}}$ Magnetic dipole moment of revolving e^- ⇒ $\boxed{\vec{M} = \frac{evr}{2}}$

● **Relationship with Angular Momentum** $(\vec{L} = \vec{r} \times \vec{P})$

vector form ⇒ $\boxed{\vec{M} = \frac{-e}{2m}\vec{L}}$ Scalar / magnitude ⇒ $\boxed{|\vec{M}| = \frac{e}{2m}|\vec{L}|}$

Bohr's postulate

$\boxed{M = \frac{eh}{4\pi m_e}}$ → B_m.

● **Torque on a current carrying loop (magnetic dipole) in a Uniform magnetic field** –

i) When $\vec{M}$ is along $\vec{B}$ $(\theta = 0°)$ ⇒ $\boxed{F_{net} = 0}$ and $\boxed{\tau_{net} = 0}$ (same line of action)

ii) If loop is rotated by angle θ ⇒ $\boxed{F_{net} = 0}$ and $\boxed{\tau_{net} \neq 0}$ $\boxed{\tau_{net} = \vec{M} \times \vec{B}}$ $\boxed{|\tau| = MB\sin\theta}$

Max. torque ⇒ $\theta = 90°$ ⇒ $\boxed{|\tau_{max}| = MB}$ Min. Torque ⇒ $\boxed{|\tau| = 0}$ 0° = stable equi., 180° = unstable eq.

● **Potential Energy of a Magnetic dipole in Uniform Magnetic field** :- $\boxed{U_{(\theta)} = -\vec{M} \cdot \vec{B}}$

U_{min} ⇒ $-MB$ (stable equilibrium), $U_{(max)}$ ⇒ MB (unstable equilibrium). when $(U = 0)$ at 90°.

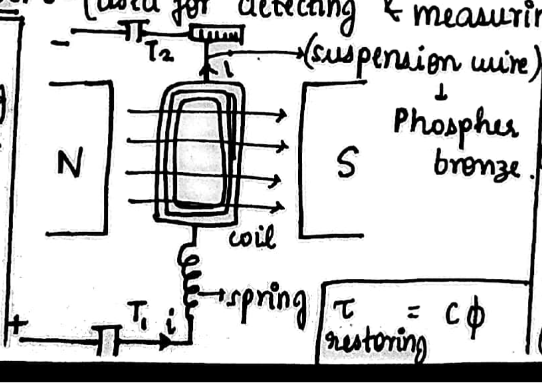
● **Moving Coil Galvanometer** :- (used for detecting & measuring small current in circuit).

↳ works on principle

'Torque on a current carrying coil placed in a magnetic field'. ⇒ $\boxed{\tau = niAB\sin\theta}$

θ ⇒ angle b/w $\vec{B}$ and $\vec{M}$.

N ⇒ no. of turns in coil.



$\tau_{restoring} = C\phi$

To eliminate $\sin\theta$, radial field is generated ⇒ (i) concave pole mag. (ii) soft iron core.

Increase current sensitivity

N ↑ A ↑ B ↑ C ↓

Increase voltage sensitivity

$(N\uparrow)^{\times}$ A ↑ B ↑ C ↓ R ↓