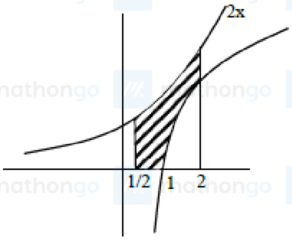
ANSWER KEYS

1. (2)	**2.** (1)	**3.** (2)	**4.** (1)	**5.** (1)	**6.** (3)	**7.** (2)	**8.** (2)
9. (1)	**10.** (4.00)						

1. **(2)** $R=\left\{(x,y)\cdot \max(0,\log_e x)\le y\le 2^x, \frac{1}{2}\le x\le 2\right\}$



$\int_{\frac{1}{2}}^{2} 2^x dx-\int_1^2 \ln x dx$

$\Rightarrow \left[\frac{2^x}{\ln 2}\right]_{1/2}^{2}-[x\ln x-x]_1^2$

$\Rightarrow \frac{\left(2^2\right)-2^{1/2}}{\log_0 2}-(2\ln 2-1)$

$\Rightarrow \frac{\left(2^2-\sqrt{2}\right)}{\log_2 2}-2\ln 2+1$

$\therefore \alpha=2^2-\sqrt{2}, \beta=-2, \gamma=1$

$\Rightarrow (\alpha+\beta+2\gamma)^2$

$\Rightarrow \left(2^2-\sqrt{2}-2-2\right)^2$

$\Rightarrow (\sqrt{2})^2=2$

2. **(1)**

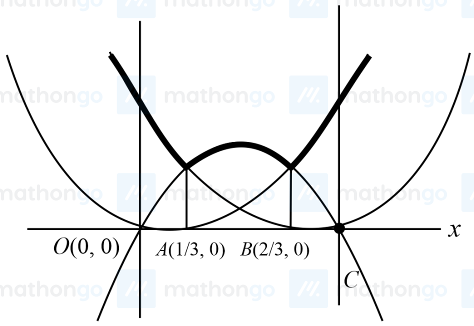
$f(x)=\text{Max}\left\{x^2,(1-x)^2,2x(1-x)\right\}$

coordinate of A, $(1-x)^2=2x(1-x)$

$(1-x)(1-x-2x)=0$

$x=\frac{1}{3}$, $A\left(\frac{1}{3},0\right)$ coordinate of B

$2x(1-x)=x^2, 2(1-x)=x, 2-3x=0,\ x=\frac{2}{3}$

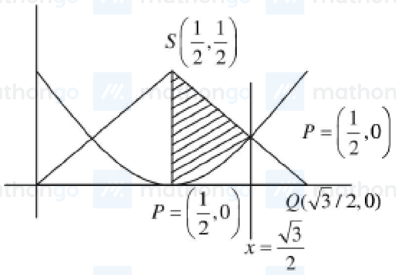


$B\left(\frac{2}{3},0\right)$ Req. Area $=\int_0^{1/3}(1-x)^2dx+\int_{1/3}^{2/3}2x(1-x)dx+\int_{\frac{2}{3}}^{1}x^2dx$

$=\frac{17}{27}$

3. **(2)** Coordinates of $P\left(\frac{1}{2}, 0\right), Q\left(\frac{\sqrt{3}}{2}, 0\right), R\left(\frac{\sqrt{3}}{2}, 1-\frac{\sqrt{3}}{2}\right)$

and $S\left(\frac{1}{2}, \frac{1}{2}\right)$



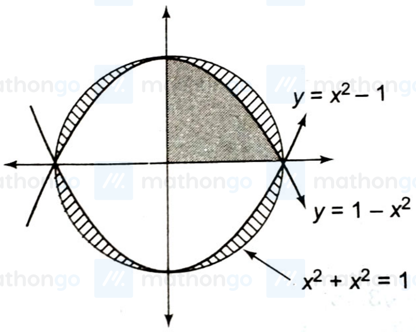
Required area $=$ Area of trapezium $PQRS$

$-\int_{1/2}^{\sqrt{3}/2}\left(x-\frac{1}{2}\right)^2 dx$

$=\frac{1}{2}\left(\frac{\sqrt{3}-1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right)-\frac{1}{3}\left(\left(x-\frac{1}{2}\right)^3\right)_{1/2}^{\sqrt{3}/2}$

$=\frac{\sqrt{3}}{4}-\frac{1}{3}$

4.



(1)

The dotted area is

$\mathrm{A}=\int_0^1\left(1-\mathrm{x}^2\right) \mathrm{dx}=\left(\mathrm{x}-\frac{\mathrm{x}^3}{3}\right)_0^1=1-\frac{1}{3}=\frac{2}{3}$

Hence, area bounded by circle $\mathrm{x}^2+\mathrm{y}^2=1$ and $|\mathrm{y}|=1-\mathrm{x}^2$

$=$ Lined area

$=$ Area of curcle $-$ Area bounded by $|\mathrm{y}|=1-\mathrm{x}^2$

$=\pi-4 .\left(\frac{2}{3}\right)=\frac{3 \pi-8}{3}$ sq. units

5. **(1)**

We have, $y=2,\ y=f(x)$ and $x=1,\ x=a$

Now, the area under the curve$=\int_1^a(f(x)-2) dx$

$\Rightarrow \int_1^a(f(x)-2) d x=\frac{2}{3}\left((2 a)^{\frac{3}{2}}-3 a+3-2 \sqrt{2}\right)$ (Given)

Differentiate the above equation w.r.t. a we get,

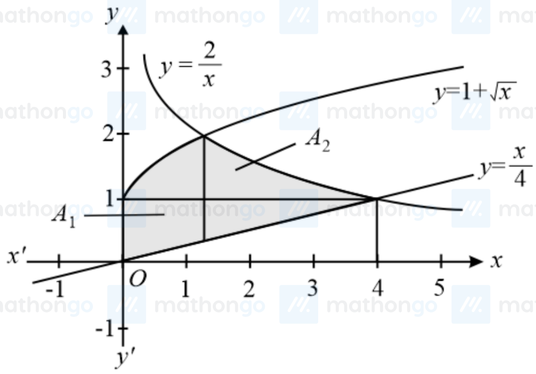
$f(a)-2=\frac{2}{3}\left(\frac{3}{2} \sqrt{2 a} \cdot 2-3\right)$ (Using Leibtniz rule)

$\Rightarrow f(a)=2 \sqrt{2 a},\ a \geq 1$

$\therefore\ f(x)=2 \sqrt{2 x},\ x \geq 1$

6. **(3)**

Point of intersection of $y=\frac{2}{\sqrt{x}}$ and $y=1+\sqrt{x}$ is $(1,2)$ and that of $y=\frac{2}{\sqrt{x}}$ and $y=\frac{x}{4}$ is $(4,\ 1)$



$$A_1=\int_0^1\left(1+\sqrt{x}-\frac{x}{4}\right)dx$$

$$=\left[x+\frac{2x^{\frac{3}{2}}}{3}-\frac{x^2}{8}\right]_0^1$$

$$=1+\frac{2}{3}-\frac{1}{8}=\frac{37}{24}$$

$$=A_2=\int_1^4\left(\frac{2}{\sqrt{x}}-\frac{x}{4}\right)dx$$

$$=\left[4\sqrt{x}-\frac{x^2}{8}\right]_1^4$$

$$=\left[8-2-4+\frac{1}{8}\right]=\frac{17}{8}$$

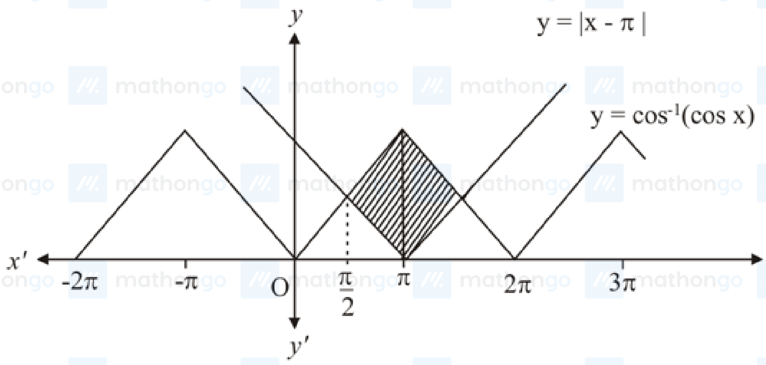
$$\Rightarrow A=A_1+A_2$$

$$=\frac{88}{24}=\frac{11}{3}\text{ sq. unit}$$

7. **(2)**

Given, $f(x)=\cos^{-1}(\cos x)=\begin{cases} x, & x\in[0,\ \pi] \\ 2\pi-x, & x\in[\pi,\ 2\pi]\end{cases}$

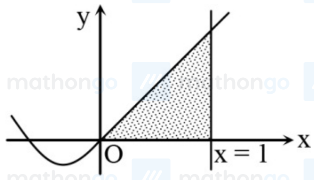
Graph of $y=f(x)$ has been shown in the figure.



$$\therefore\ A=2\int_{\frac{\pi}{2}}^{\pi}[x-(\pi-x)]dx=2\left[x^2-\pi x\right]_{\frac{\pi}{2}}^{\pi}=\frac{\pi^2}{2}\text{ sq. units}$$

Hence, $n=2$.

8. **(2)**



Since at point (x, y) of the curve, slope of the tangent to the curve is $\frac{dy}{dx}$, so as given

$\frac{dy}{dx} = 2x + 1$

Integrating, we get

$y = x^2 + x + c$

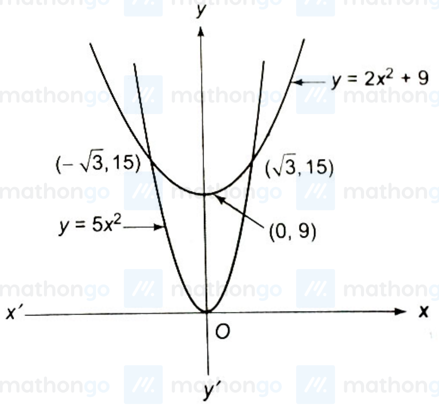
But the curve passes through the point $(1, 2)$, so we have

$2 = 1 + 1 + c \Rightarrow c = 0$

$\therefore$ equation of the curve is $y = x^2 + x$ which is a parabola. Also the curve cuts x -axis at $x = -1$, and at $x = 0$. So the required area

$= \int_0^1 (x^2 + x)\, dx = \left(\frac{x^3}{3} + \frac{x^2}{2}\right)_0^1 = \frac{1}{3} + \frac{1}{2} =$

9. **(1)** Given $5x^2 - y = 0$ and



$2x^2 - y + 9 = 0$

Eliminating y, we get

$5x^2 - (2x^2 + 9) = 0$

$\Rightarrow 3x^2 = 9 \Rightarrow x = -\sqrt{3}, \sqrt{3}$

$\therefore$ Required area

$= 2\int_0^{\sqrt{3}} \left((2x^2 + 9) - 5x^2\right) dx$

$= 2\int_0^{\sqrt{3}} (9 - 3x^2)\, dx$

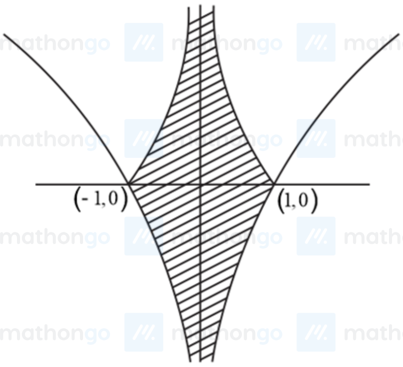
$= 2\left[9x - x^3\right]_0^{\sqrt{3}}$

$= 2\left[9\sqrt{3} - 3\sqrt{3}\right]$

$= 12\sqrt{3}$ sq. units

10. (4.00)

The required shaded region is as shown in figure



As the graph is symmetric in all quadrants, we calculate area in one quadrant and multiply by 4.

Hence, required area $= 4\left|\int_0^1 (\ln x).\,dx\right|$

$A = 4\left|[x\ln\ x - x]_0^1\right|$

$A = 4\left|\left[(1\ln 1 - 1) - \lim_{x\to 0^+}(x\ln x - x)\right]\right|$

$A = 4\left|\left[(0-1) - \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}} - 0\right]\right| = 4\left|\left[-1 - \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}}\right]\right|$

$A = 4\left|\left[-1 + \lim_{x\to 0^+} x\right]\right|$

$A = 4|-1|$

$A = 4$ sq. units