1. If $y^2+\log_e(\cos^2 x)=y,\ \ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ then :

(1) $y''(0)=0$
(2) $|y'(0)|+|y''(0)|=1$
(3) $|y''(0)|=2$
(4) $|y'(0)|+|y''(0)|=3$

2. If $y=y(x)$ is an implicit function of x such that $\log_e(x+y)=4xy$, then $\frac{d^2y}{dx^2}$ at $x=0$ is equal to

3. Let f be a differentiable function such that $f(1)=2$ and $f'(x)=f(x)$ for all $x\in R$. If $h(x)=f(f(x))$, then $h'(1)$ is equal to :

(1) $4e^2$
(2) $2e$
(3) $4e$
(4) $2e^2$

4. If $e^y+xy=e$, the ordered pair $\left(\frac{dy}{dx},\frac{d^2y}{dx^2}\right)$ at $x=0$ is equal to

(1) $\left(-\frac{1}{e},-\frac{1}{e^2}\right)$
(2) $\left(-\frac{1}{e},\frac{1}{e^2}\right)$
(3) $\left(\frac{1}{e},-\frac{1}{e^2}\right)$
(4) $\left(\frac{1}{e},\frac{1}{e^2}\right)$

5. Let $x(t)=2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t)=2\sqrt{2}\sin t\sqrt{\sin 2t}, t\in\left(0,\frac{\pi}{2}\right)$. Then $\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t=\frac{\pi}{4}$ is equal to

(1) $\frac{-2\sqrt{2}}{3}$
(2) $\frac{2}{3}$
(3) $\frac{1}{3}$
(4) $\frac{-2}{3}$

6. If $x=2\sin\theta-\sin 2\theta$ and $y=2\cos\theta-\cos 2\theta$, $\theta\in[0,2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta=\pi$ is:

(1) $\frac{3}{4}$
(2) $\frac{3}{8}$
(3) $\frac{3}{2}$
(4) $-\frac{3}{4}$

7. If $x\log_e(\log_e x)-x^2+y^2=4(y>0)$, then $\frac{dy}{dx}$ at $x=\mathrm{e}$ is equal to:

(1) $\frac{(1+2e)}{2\sqrt{4+e^2}}$
(2) $\frac{(2e-1)}{2\sqrt{4+e^2}}$
(3) $\frac{(1+2e)}{\sqrt{4+e^2}}$
(4) $\frac{e}{\sqrt{4+e^2}}$

8. If $2x^y+3y^x=20$, then $\frac{dy}{dx}$ at $(2,2)$ is equal to:

(1) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$
(2) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$
(3) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$
(4) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

9. The value of $\log_e 2\frac{\mathrm{d}}{\mathrm{dx}}(\log_{\cos x}\operatorname{cosec} x)$ at $x=\frac{\pi}{4}$ is

(1) $-2\sqrt{2}$
(2) $2\sqrt{2}$
(3) -4
(4) 4

10. Let $y=f(x)=\sin^3\left(\frac{\pi}{3}\left(\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{\frac{3}{2}}\right)\right)\right)$. Then, at $x=1$,

(1) $2y'+\sqrt{3}\pi^2 y=0$
(2) $2y'+3\pi^2 y=0$
(3) $\sqrt{2}y'-3\pi^2 y=0$
(4) $y'+3\pi^2 y=0$

11. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=\frac{1}{2}$ is :

(1) $\frac{2\sqrt{3}}{5}$
(2) $\frac{\sqrt{3}}{12}$
(3) $\frac{2\sqrt{3}}{3}$
(4) $\frac{\sqrt{3}}{10}$

12. Let $y=y(x)$ be a function of x satisfying $y\sqrt{1-x^2}=k-x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right)=-\frac{1}{4}$.Then $\frac{dy}{dx}$ at $x=\frac{1}{2}$, is equal to

(1) $-\frac{\sqrt{5}}{4}$
(2) $-\frac{\sqrt{5}}{2}$
(3) $\frac{2}{\sqrt{5}}$
(4) $\frac{\sqrt{5}}{2}$

13. If $y(x)=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$, $x\in\left(\frac{\pi}{2},\pi\right)$, then $\frac{dy}{dx}$ at $x=\frac{5\pi}{6}$ is:

(1) 0
(2) -1
(3) $\frac{-1}{2}$
(4) $\frac{1}{2}$

14. Let $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$, $0<x<1$. Then:

(1) $(1-x)^2 f'(x)+2(f(x))^2=0$
(2) $(1+x)^2 f'(x)+2(f(x))^2=0$
(3) $(1-x)^2 f'(x)-2(f(x))^2=0$
(4) $(1+x)^2 f'(x)-2(f(x))^2=0$

15. Let $f:\boldsymbol{R}\rightarrow\boldsymbol{R}$ be defined as $f(x)=x^3+x-5$. If $g(x)$ is a function such that $f(g(x))=x,\forall x\in\boldsymbol{R}$, then $g'(63)$ is equal to ______

(1) 49
(2) $\frac{1}{49}$
(3) $\frac{43}{49}$
(4) $\frac{3}{49}$

16. Let f and g be differentiable functions on R such that fog is the identity function. If for some $a, b \in R, g^{'}(a)=5$ and $g(a)=b$, then $f^{'}(b)$ is equal to:

(1) $\frac{1}{5}$

(2) 1

(3) 5

(4) $\frac{2}{5}$

17. If $y=\tan^{-1}(\sec x^3-\tan x^3), \frac{\pi}{2}<x^3<\frac{3\pi}{2}$, then

(1) $xy''+2y'=0$

(2) $x^2y''-6y+\frac{3\pi}{2}=0$

(3) $x^2y''-6y+3\pi=0$

(4) $xy''-4y'=0$

18. For the curve $C:(x^2+y^2-3)+(x^2-y^2-1)^5=0$, the value of $3y'-y^3y''$, at the point $(\alpha, \alpha), \alpha>0$, on C, is equal to ________.

19. Let $f(x)=(\sin(\tan^{-1}x)+\sin(\cot^{-1}x))^2-1, |x|>1$. If $\frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}(\sin^{-1}(f(x)))$ and $y(\sqrt{3})=\frac{\pi}{6}$, then $y(-\sqrt{3})$ is equal to:

(1) $\frac{2\pi}{3}$

(2) $-\frac{\pi}{6}$

(3) $\frac{5\pi}{6}$

(4) $\frac{\pi}{3}$

20. Let $f:\mathbb{R}\to\mathbb{R}$ satisfy $f(x+y)=2^x f(y)+4^y f(x), \forall x, y \in \mathbb{R}$. If $f(2)=3$, then $14\cdot\frac{f'(4)}{f'(2)}$ is equal to _____.