ANSWER KEYS

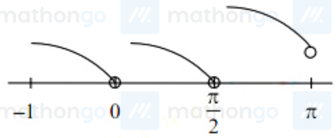
1. (2)	**2.** (1)	**3.** (3)	**4.** (1)	**5.** (3)	**6.** (1)	**7.** (3)	**8.** (2)
9. (2)	**10.** (2)						

1. (2)

$$f'(x)=\begin{cases} 3x^2+2x-10 & -1\le x\le 0 \\ -\sin x & 0<x<\frac{\pi}{2} \\ \cos x & \frac{\pi}{2}<x<\pi \end{cases}$$

It is clear that all functions are decreasing in their respective interval

Displaying the trend of values of the function in different intervals, we get the adjoining graph.

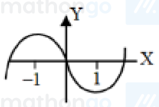


Here it is clear that

$\therefore f\left(\frac{\pi}{2}-h\right)<\left(2=f\left(\frac{\pi}{2}\right), f\left(\frac{\pi}{2}+h\right)<f\left(\frac{\pi}{2}\right)\right)$

$f(x)$ has a local maximum at $x=\frac{\pi}{2}$

2. (1)



Let $f\left(x\right)=\frac{x^3}{3}-x-b$

$\Rightarrow f'(x)=x^2-1=0$

$\Rightarrow x=\pm 1$

$f(-1)f(1)<0$ for three distinct zeroes

$\Rightarrow (3b+2)(3b-2)<0$

$\Rightarrow b\in\left(-\frac{2}{3},\frac{2}{3}\right)$

3. (3)

Given statement is

$f(x)=\int_{-1}^{x} t\left(e^t-1\right)(t-1)(t-2)^3(t-3)^5\,\mathrm{d}t$

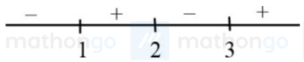
Now for local maximum put $f'(x)=0$ and to get $f'(x)$ we need to apply Leibnitz theorem. So differentiating both sides

$f'(x)=x(e^x-1)(x-1)(x-2)^3(x-3)^5,\ f'(x)=0$

$\Rightarrow x=0,1,2,3$

Ignoring $x=0$ as it has occurred two times so no sign change in derivative of function.

Applying wavy-curve method



So at x $=2$,the function increases to its local maximum and then starts decreasing.

4. (1)

Let $f(x)=x^3+2x^2+5x+2\cos x$

$\Rightarrow f'(x)=3x^2+4x+5-2\sin x$

$\Rightarrow f'(x)=3\left(x^2+\frac{4}{3}x+\frac{5}{3}\right)-2\sin x$

$\Rightarrow f'x=3\left\{\left(x+\frac{2}{3}\right)^2+\frac{11}{9}\right\}-2\sin x$

$\Rightarrow f'x=3\left(x+\frac{2}{3}\right)^2+\frac{11}{3}-2\sin x$

Now $\frac{11}{3}-2\sin x>0\ x\ (as-1\le\sin x\le 1)$

$\Rightarrow f'(x)>0\ x\ \Rightarrow f\left(x\right)$ is an increasing function.

Now $f(0)=2$

$\Rightarrow f(x)=0$ has no solution in $[0,2\pi]$

Hence (A) is the correct answer.

5. (3)

Given,

$f(x)=|x-1|+|x-4|+|x-9|+\ldots+|x-2500|\ \forall\ x \in R$

$\because |x-a|$ is non-differentiable at $x=a$,

$\Rightarrow f(x)$ is non-differentiable at $x=1^2,\ 2^2, 3^2\ldots 25^2, 26^2\ldots 50^2$.

Let, $g(x)=|x-a|+|x-b|+|x-c|+|x-d|$ (where $a<b<c<d$),

We know that, the function $g(x)$ will have minimum value $\forall\ x \in [b,\ c]$.

Since, 25 and 26 lies in the middle of numbers $1,\ 2,\ 3,\ 4\ldots\ldots 48,\ 49,\ 50$.

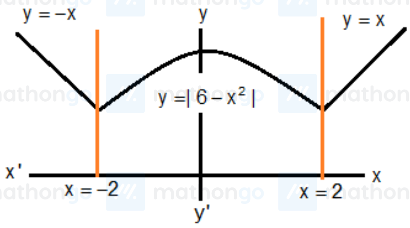
$\Rightarrow f(x)$ is minimum at $\forall x \in [25^2,\ 26^2]$.

So, $f(x)$ is minimum for the range $[625,\ 676]$.

6. (1)

Given function $f(x)=max.\{|6-x^2|,\ |x|\}$

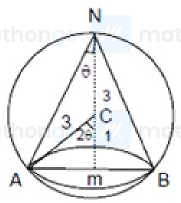
Graph of $f(x)$ is given by:



Now, at $x=2,\ y=2$ and at $x=-2,\ y=2$.

Hence, function has minima at $x=\pm 2$ and minimum value of the function $f(2)=2$.

7. (3)



In $\triangle AMC,\ AM=3\sin 2\theta \,\&\, MC=3\cos 2\theta$

$V=\frac{1}{3}\pi r^2 h$ where, r is radius and h is height of cone

$\Rightarrow V=\frac{1}{3}\pi(3\sin 2\theta)^2(3+3\cos 2\theta)$

(since, radius of cone $=AM$ and height of cone $=MC$)

$\Rightarrow V=\pi\left(36\sin^2\theta\cos^2\theta\right)\left(2\cos^2\theta\right)\ \left[\because \sin 2\theta=2\sin\theta\cos\theta \,\&\, \cos 2\theta=2\cos^2\theta-1\right]$

$=72\pi\sin^2\theta\cos^4\theta$

Differentiating both sides with respect to θ, we get

$\frac{dv}{d\theta}=72\pi\left[2\sin\theta\cos^5\theta-4\sin^3\theta\cos^3\theta\right]$

For maximum value, $\frac{dV}{d\theta}=0$

$\Rightarrow 72\pi\left[2\sin\theta\cos^5\theta-4\sin^3\theta\cos^3\theta\right]=0 \Rightarrow \tan^2\theta=\frac{1}{2}$

Thus, volume is maximum when $\tan\theta=\frac{1}{\sqrt{2}}$

Hence, curved surface area $S=\pi rl$

$=\pi r\sqrt{(3+3\cos 2\theta)^2+(3\sin 2\theta)^2}\ \left[\because l=\sqrt{r^2+h^2}\right]$

$=\pi(3\sin 2\theta)\sqrt{36\cos^2\theta}=18\pi\left(2\sin\theta\cos^2\theta\right)$

$=36\pi\frac{1}{\sqrt{3}}\cdot\frac{2}{3}=\frac{24\pi}{\sqrt{3}}=8\sqrt{3}\pi$

8. **(2)**

We have to calculate the maximum value of modulus function.

Consider $f(\mathrm{x})=\mathrm{x}^2-3\mathrm{x}+2$ on $\left[1,\ \frac{5}{2}\right]$.

Differentiating both sides with respect to x

$f'(\mathrm{x})=2\mathrm{x}-3$

for smallest value $f'(x)=0$

$2x-3=0\Rightarrow x=\frac{3}{2}$

The only critical point is $\frac{3}{2}$

$f(1)=1^2-3\times1+2=0$

$f\left(\frac{3}{2}\right)=\left(\frac{3}{2}\right)^2-3\times\frac{3}{2}+2=-\frac{1}{4}$

$f\left(\frac{5}{2}\right)=\frac{25}{4}-3\times\frac{5}{2}+2\Rightarrow f\left(\frac{5}{2}\right)=\frac{3}{4}$

Hence the maximum value of $f(x)=\frac{3}{4}$.

Thus $M=3/4$.

9. **(2)**

We have,

$f''(x)>0\ \forall\ x\in R$

$\Rightarrow f'(x)$ is an increasing function.

Now, $g(x)=2f\left(2x^3-3x^2\right)+f\left(6x^2-4x^3-3\right)$

$g'(x)=\left(6x^2-6x\right)\cdot 2f'\left(2x^3-3x^2\right)+f'\left(6x^2-4x^3-3\right)\cdot\left(12x-12x^2\right)$

$g'(x)=12\,x(x-1)\left[f'\left(2x^3-3x^2\right)-f'\left(6x^2-4x^3-3\right)\right]$

For $g(x)$ to be increasing, $g'(x)\geq 0$

$\Rightarrow 12(x)(x-1)\left[f'\left(2x^3-3x^2\right)-f'\left(6x^2-4x^3-3\right)\right]\geq 0$

Case-I:

When $x(x-1)\geq 0\Rightarrow x\in(-\infty,0)\cup(1,\infty)\ldots(1)$

We must have

$\left[f'\left(2x^3-3x^2\right)-f'\left(6x^2-4x^3-3\right)\right]\geq 0$

$\Rightarrow f'\left(2x^3-3x^2\right)\geq f'\left(6x^2-4x^3-3\right)$

$\Rightarrow 2x^3-3x^2>6x^2-4x^3-3$

[$f'(x)$ is an increasing function]

$6x^3-9x^2+3>0$

$2x^3-3x^2+1>0$

$(x-1)^2(2x+1)>0$

$2x+1>0$

[$(x-1)^2$ is always non-negative except at $x=1$]

$x>-\frac{1}{2}$

$x\in\left(-\frac{1}{2},\infty\right)-\{1\}\ldots(2)$

From (1) and (2) we have:

$x\in\left(-\frac{1}{2},0\right)\cup(1,\infty)$

Case -II:

When $x(x-1)\leq 0\Rightarrow x\in[0,1]\ldots(3)$

We must have

$f'\left(2x^3-3x^2\right)-f'\left(6x^2-4x^3-3\right)\leq 0$

$\Rightarrow (x-1)^2(2x+1)<0$

[as $(x-1)^2$ is always non negative except $x=1$]

$2x+1<0$

$x<-\frac{1}{2}$

$x\in\left(-\infty,-\frac{1}{2}\right)\ldots\left(4\right)$

From (3) and (4), we get:

$x\in\phi$

$\therefore g(x)$ is increasing for all x belonging to $\left(-\frac{1}{2},0\right)\cup(1,\infty)$.

10. **(2)**

Given equation is:

$(x-41)^{49}+(x-49)^{41}+(x-2009)^{2009}=0$

Let, $P(x)=(x-41)^{49}+(x-49)^{41}+(x-2009)^{2009}$

$P'(x)=49(x-41)^{48}+41(x-49)^{40}+2009(x-2009)^{2008}$

Since, all powers are even.

Hence, $P'(x)>0\ \forall x\in R$

$\therefore\ P(x)$ is strictly increasing.

$\therefore\ P(x)$ cuts x-axis only once.

Therefore, only one real root.

Here, $p(0)=(-41)^{49}+\ldots+(-2009)^{2009}$

So, $p(0)$ is negative.

$p(2009)=(2009-41)^{49}+(2009-49)^{41}+\ldots$

So, $p(2009)$ is positive.

As $p(0)$ is negative and $p(x)$ is increasing function, $p(x)$ has only one positive real root.