1. Let α and β be real numbers. Consider a 3×3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

(1) $\alpha = 1$
(2) $\alpha = 4$
(3) $\beta = 8$
(4) $\beta = -8$

2. Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, \ldots, 100\} : A^n = A\}$ is

3. Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$. If $P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $2a + b - 3c - 4d$ is equal to

(1) 2004
(2) 2005
(3) 2007
(4) 2006

4. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$ and $B = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}$, $\alpha, \beta \in R$. Let α_1 be the value of α which satisfies $(A+B)^2 = A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and α_2 be the value of α which satisfies $(A+B)^2 = B^2$. Then $|\alpha_1 - \alpha_2|$ is equal to

5. Let $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ and $B = A - I$. If $\omega = \frac{\sqrt{3}i - 1}{2}$, then the number of elements in the set $\{n \in \{1, 2, \ldots, 100\}: A^n + (\omega B)^n = A + B\}$ is equal to _____ .

6. Let $S = \left\{ \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}; a, b \in \{1, 2, 3, \ldots 100\} \right\}$ and let $T_n = \{A \in S : A^{n(n+1)} = I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is _____.

7. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to ______.

8. Let A, B, C be 3×3 matrices such that A is symmetric and B and C are skew-symmetric.Consider the statements

$(S1)$ $A^{13} B^{26} - B^{26} A^{13}$ is symmetric

(S2) $A^{26} C^{13} - C^{13} A^{26}$ is symmetric

Then,

(1) Only $S2$ is true
(2) Only S1 is true
(3) Both $S1$ and $S2$ are false
(4) Both $S1$ and $S2$ are true

9. Let $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is

(1) a non-identity symmetric matrix
(2) a skew-symmetric matrix
(3) neither symmetric nor skew-symmetric matrix
(4) an identity matrix

10. The number of symmetric matrices of order 3, with all the entries from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is

(1) 6^{10}
(2) 10^6
(3) 9^{10}
(4) 10^9

11. The total number of 3×3 matrices A having enteries from the set $(0, 1, 2, 3)$ such that the sum of all the diagonal entries of AA^T is 9, is equal to

12. Let $A = \left[a_{\hat{i}\hat{j}}\right] \cdot a_{ij} \in Z \cap [0, 4]$, $1 \leq i, j \leq 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2, 13)$ is _____.

13. The number of matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $a, b, c, d \in \{-1, 0, 1, 2, 3, \ldots\ldots, 10\}$, such that $A = A^{-1}$, is ______.

14. Let A be a 3×3 matrix having entries from the set $\{-1, 0, 1\}$. The number of all such matrices A having sum of all the entries equal to 5, is _____

15. Let for $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$. If $|2\,\mathrm{adj}(2\,\mathrm{adj}(2A))| = 32^n$, then $3n + \alpha$ is equal to

(1) 9
(2) 11
(3) 12
(4) 10

16. Let the determinant of a square matrix A of order m be $m - n$, where m and n satisfy $4m + n = 22$ and $17m + 4n = 93$. If $det(n\, adj(adj(mA))) = 3^a 5^b 6^c$, then $a + b + c$ is equal to

(1) 84
(2) 96
(3) 101
(4) 109

17. Let A be a matrix of order 3×3 and $\det(A) = 2$. Then $\det(\det(A)\, \mathrm{adj}(5\, \mathrm{adj}(A^3)))$ is equal to _____.

(1) 256×10^6
(2) 1024×10^6
(3) 512×10^6
(4) 256×10^{11}

18. Consider a matrix A $= \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$, where α, β, γ are three distinct natural numbers.

If $\frac{\det(\mathrm{adj}(\mathrm{adj}(\mathrm{adj}(\mathrm{adj}\, A))))}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}} = 2^{32} \times 3^{16}$, then the number of such 3- tuples (α, β, γ) is _______.

19. Let $A=\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$. If $A^{-1}=\alpha I+\beta A,\ \alpha,\beta\in R,\ I$ is a 2×2 identity matrix, then $4(\alpha-\beta)$ is equal to :

(1) 5
(2) $\frac{8}{3}$
(3) 2
(4) 4

20. Let A and B be two 3×3 real matrices such that $\left(A^2-B^2\right)$ is invertible matrix. If $A^5=B^5$ and $A^3\,B^2=A^2\,B^3$, then the value of the determinant of the matrix A^3+B^3 is equal to :

(1) 2
(2) 4
(3) 1
(4) 0

21. If $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}\cdots\begin{bmatrix}1 & n-1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\ 0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$ is:

(1) $\begin{bmatrix}1 & -12\\ 0 & 1\end{bmatrix}$
(2) $\begin{bmatrix}1 & 0\\ 12 & 1\end{bmatrix}$
(3) $\begin{bmatrix}1 & 0\\ 13 & 1\end{bmatrix}$
(4) $\begin{bmatrix}1 & -13\\ 0 & 1\end{bmatrix}$

22. Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha\in R$. Suppose $Q=\left[q_{ij}\right]$ is a matrix satisfying $PQ=kI_3$ for some non-zero $k\in R$. If $q_{23}=-\frac{k}{8}$ and $|Q|=\frac{k^2}{2}$, then α^2+k^2 is equal to_________.

23. Let $x=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$ and $A=\begin{bmatrix}-1 & 2 & 3\\ 0 & 1 & 6\\ 0 & 0 & -1\end{bmatrix}$. For $k\in\mathbb{N}$, if $X'A^kX=33$, then k is equal to

24. Let $X=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$, $Y=\alpha I+\beta X+\gamma X^2$ and $Z=\alpha^2 I-\alpha\beta X+\left(\beta^2-\alpha\gamma\right)X^2,\alpha,\beta,\gamma\in\mathbb{R}$. If $\mathrm{Y}^{-1}=\begin{bmatrix}\frac{1}{5} & \frac{-2}{5} & \frac{1}{5}\\ 0 & \frac{1}{5} & \frac{-2}{5}\\ 0 & 0 & \frac{1}{5}\end{bmatrix}$, then $(\alpha-\beta+\gamma)^2$ is equal to ______.

25. Let $A=\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}$. Then the number of elements in the set $(\{n,m): n,m\in\{1,2,\ldots\ldots 10\}$ and $nA^n+mB^m=I\}$ is _____.

26. Let A be a $n\times n$ matrix such that $|A|=2$. If the determinant of the matrix $\operatorname{Adj}\left(2.\operatorname{Adj}\left(2\,\mathrm{A}^{-1}\right)\right)$ is 2^{84}, then n is equal to _____ .

27. If $A=\frac{1}{5!6!7!}\begin{bmatrix}5! & 6! & 7!\\ 6! & 7! & 8!\\ 7! & 8! & 9!\end{bmatrix}$, then $|\operatorname{adj}(\operatorname{adj}(2A))|$ is equal to

(1) 2^{20}
(2) 2^{8}
(3) 2^{12}
(4) 2^{16}

28. Let $A=\begin{pmatrix}1+i & 1\\ -i & 0\end{pmatrix}$ where $i=\sqrt{-1}$. Then, the number of elements in the set $\{n\in\{1,2,\ldots,100\}:A^n=A\}$ is

29. Let $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 1 & 0 & 0\end{bmatrix}$. Then $A^{2025}-A^{2020}$ is equal to

(1) A^6-A
(2) A^6
(3) A^5
(4) A^5-A

30. If $P=\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}$, then P^{50} is:

(1) $\begin{bmatrix}1 & 0\\ 25 & 1\end{bmatrix}$
(2) $\begin{bmatrix}1 & 50\\ 0 & 1\end{bmatrix}$
(3) $\begin{bmatrix}1 & 25\\ 0 & 1\end{bmatrix}$
(4) $\begin{bmatrix}1 & 0\\ 50 & 1\end{bmatrix}$

31. Let $A=\begin{pmatrix}0 & 2q & r\\ p & q & -r\\ p & -q & r\end{pmatrix}$. If $\mathrm{AA}^{\mathrm{T}}=\mathrm{I}_3$, then $|\mathrm{p}|$ is :

(1) $\frac{1}{\sqrt{5}}$
(2) $\frac{1}{\sqrt{3}}$
(3) $\frac{1}{\sqrt{2}}$
(4) $\frac{1}{\sqrt{6}}$

32. Let A be a 3×3 matrix having entries from the set $\{-1,0,1\}$. The number of all such matrices A having sum of all the entries equal to 5, is _____

33. Let $M=\left\{A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}: a,b,c,d\in(\pm 3,\pm 2,\pm 1,0)\right\}$. Define $f:M\to Z$, as $f(A)=\det(A)$, for all $A\in M$ where Z is set of all integers. Then the number of $A\in M$ such that $f(A)=15$ is equal to ________.

34. If the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 3 & 0 & -1\end{bmatrix}$ satisfies the equation $A^{20}+\alpha A^{19}+\beta A=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 1\end{bmatrix}$ for some real numbers α and β, then $\beta-\alpha$ is equal to ______.

35. Let the matrix $A=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$ and the matrix $B_0=A^{49}+2A^{98}$. If $B_n=\operatorname{Adj}(B_{a-1})$ for all $n\geq 1$, then $\det(B_4)$ is equal to

(1) 3^{28}

(2) 3^{30}

(3) 3^{32}

(4) 3^{36}