1. The integral $\int\left(\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x\right)\log_2 x\,dx$ is equal to

(1) $\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x+C$
(2) $\left(\frac{x}{2}\right)^x-\left(\frac{2}{x}\right)^x+C$
(3) $\left(\frac{x}{2}\right)^x\log_2\left(\frac{x}{2}\right)+C$
(4) $\left(\frac{x}{2}\right)^x\log_2\left(\frac{2}{x}\right)+C$

2. If $\int\frac{dx}{x^3\left(1+x^6\right)^{\frac{2}{3}}}=xf(x)\left(1+x^6\right)^{\frac{1}{3}}+C$, where C is a constant of integration, then the function $f(x)$ is equal to

(1) $\frac{3}{x^2}$
(2) $-\frac{1}{2x^3}$
(3) $-\frac{1}{6x^3}$
(4) $-\frac{1}{2x^2}$

3. If $\int x^5e^{-x^2}dx=g(x)e^{-x^2}+c$, where c is a constant of integration, then $g(-1)$ is equal to

(1) $-\frac{5}{2}$
(2) -1
(3) 1
(4) $-\frac{1}{2}$

4. Let $f(x)=\int\frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx$. If $f(3)=\frac{1}{2}\left(\log_e 5-\log_e 6\right)$, then $f(4)$ is equal to

(1) $\frac{1}{2}\left(\log_e 17-\log_e 19\right)$
(2) $\log_e 17-\log_e 18$
(3) $\frac{1}{2}\left(\log_e 19-\log_e 17\right)$
(4) $\log_e 19-\log_e 20$

5. Let $I(x)=\int\sqrt{\frac{x+7}{x}}\,dx$ and $I(9)=12+7\log_e 7$. If $I(1)=\alpha+7\log_e\left(1+2\sqrt{2}\right)$, then α^4 is equal to _____.

6. Let $I(x)=\int\frac{x+1}{x\left(1+xe^x\right)^2}dx,\ x>0$. If $\lim\limits_{x\to\infty}I(x)=0$ then $I(1)$ is equal to

(1) $\frac{e+2}{e+1}-\log_e(e+1)$
(2) $\frac{e+1}{e+2}+\log_e(e+1)$
(3) $\frac{e+1}{e+2}-\log_e(e+1)$
(4) $\frac{e+2}{e+1}+\log_e(e+1)$

7. If $\int\frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx=g(x)+c, g(1)=0$, then $g\left(\frac{1}{2}\right)$ is equal to

(1) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+\frac{\pi}{3}$
(2) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)+\frac{\pi}{3}$
(3) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)-\frac{\pi}{3}$
(4) $\frac{1}{3}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)-\frac{\pi}{6}$

8. $\int\frac{2e^x+3e^{-x}}{4e^x+7e^{-x}}dx=\frac{1}{14}\left(ux+v\log_e\left(4e^x+7e^{-x}\right)\right)+C$, where C is a constant of integration, then $u+v$ is equal to

9. If $f(x)=\int\frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx,\ (x\geq 0),\ f(0)=0$ and $f(1)=\frac{1}{K}$, then the value of K is

10. The value of the integral $\int\frac{\sin\theta\cdot\sin 2\theta\left(\sin^6\theta+\sin^4\theta+\sin^2\theta\right)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta$ is (where c is a constant of integration)

(1) $\frac{1}{18}\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\frac{3}{2}}+c$
(2) $\frac{1}{18}\left[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta\right]^{\frac{3}{2}}+c$
(3) $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}}+c$
(4) $\frac{1}{18}\left[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{-\frac{3}{2}}+c$

11. If $\int\frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx=a\sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+c$, where c is a constant of integration, then the ordered pair (a,b) is equal to:

(1) $(1,-3)$
(2) $(3,1)$
(3) $(-1,3)$
(4) $(1,3)$

12. If $\int\frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta=A\log_e\left|B\left(\theta\right)\right|+C$, where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be:

(1) $\frac{2\sin\theta+1}{\sin\theta+3}$
(2) $\frac{2\sin\theta+1}{5(\sin\theta+3)}$
(3) $\frac{5(\sin\theta+3)}{2\sin\theta+1}$
(4) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$

13. If $\int\sin^{-1}\left(\frac{\sqrt{x}}{1+x}\right)dx=A(x)\tan^{-1}\left(\sqrt{x}\right)+B(x)+C$, where C is a constant of integration, then the ordered pair (A(x), B(x)) can be :

(1) $\left(x-1,\ \sqrt{x}\right)$
(2) $\left(x-1,\ -\sqrt{x}\right)$
(3) $\left(x+1,\ \sqrt{x}\right)$
(4) $\left(x+1,\ -\sqrt{x}\right)$

14. The integral $\int\frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to: (where C is a constant of integration)

(1) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+C$
(2) $\left(\frac{x-3}{x+4}\right)^{\frac{-1}{7}}+C$
(3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}}+C$
(4) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{\frac{-13}{7}}+C$

15. If $\int\frac{d\theta}{\cos^2\theta\left(\tan 2\theta+\sec 2\theta\right)}=\lambda\tan\theta+2\log_e|f(\theta)|+C$ where C is a constant of integration, then the ordered pair $(\lambda,f(\theta))$ is equal to:

(1) $(1,1-\tan\theta)$
(2) $(-1,1-\tan\theta)$
(3) $(-1,1+\tan\theta)$
(4) $(1,1+\tan\theta)$

16. If $\int\frac{\cos x\,dx}{\sin^3 x\left(1+\sin^6 x\right)^{\frac{2}{3}}}=f(x)\left(1+\sin^6 x\right)^{\frac{1}{\lambda}}+c$, where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to

(1) $-\frac{9}{8}$
(2) 2
(3) $\frac{9}{8}$
(4) -2

17. The integral $\int \frac{2x^3-1}{x^4+x} dx$, is equal to

(1) $\frac{1}{2}\log_e \frac{(x^3+1)^2}{|x^3|}+C$ (2) $\log_e \frac{|x^3+1|}{x^2}+C$

(3) $\log_e\left|\frac{(x^3+1)}{x}\right|+C$ (4) $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2}+C$

18. $\int \sec^2 x \cdot \cot^{\frac{4}{3}} x dx$ is equal to

(1) $3\tan^{-\frac{1}{3}} x+C$ (2) $-\frac{3}{4}\tan^{-\frac{4}{3}} x+C$

(3) $-3\tan^{-\frac{1}{3}} x+C$ (4) $-3\cot^{-\frac{1}{3}} x+C$

19. $\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx$, is equal to

(1) $x+2\sin x+\sin 2x+c$ (2) $2x+\sin x+\sin 2x+c$

(3) $x+2\sin x+2\sin 2x+c$ (4) $2x+\sin x+2\sin 2x+c$

20. If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(\mathrm{x})\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(\mathrm{A(x)})^{\mathrm{m}}$ equals :

(1) $\frac{-1}{27x^9}$ (2) $\frac{-1}{3x^3}$

(3) $\frac{1}{27x^6}$ (4) $\frac{1}{9x^4}$

21. If $I(x) = \int e^{\sin^2 x}(\cos x \ \sin 2x - \sin x)dx$ and $I(0) = 1$, then $I\left(\frac{\pi}{3}\right)$ is equal to

(1) $-\frac{1}{2}e^{\frac{3}{4}}$ (2) $\frac{1}{2}e^{\frac{3}{4}}$

(3) $-e^{\frac{3}{4}}$ (4) $e^{\frac{3}{4}}$

22. For $I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x} dx$, if $I\left(\frac{\pi}{4}\right) = 2^{1011}$, then

(1) $3^{1010} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$ (2) $3^{1010} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

(3) $3^{1011} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$ (4) $3^{1011} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

23. $\int \frac{(x^2+1)e^x}{(x+1)^2} dx = f(x)e^x + C$, where C is a constant, then $\frac{d^3 f}{dx^3}$ at $x=1$ is equal to

(1) $\frac{3}{4}$ (2) $\frac{3}{8}$

(3) $-\frac{3}{2}$ (4) $\frac{7}{8}$

24. If $I_1 = \int_0^1 \left(1-x^{50}\right)^{100} dx$ and $I_2 = \int_0^1 \left(1-x^{50}\right)^{101} dx$ such that $I_2 = \alpha I_1$ then α equals to :

(1) $\frac{5049}{5050}$ (2) $\frac{5050}{5049}$

(3) $\frac{5050}{5051}$ (4) $\frac{5051}{5050}$

25. If $\int e^{secx}\left(secx \tan x f(x) + \left(secx \tan x + sec^2 x\right)\right) dx = e^{secx} f(x) + C$, then a possible choice of $f(x)$ is:

(1) $\mathrm{sec}x - \mathrm{tan}x - \frac{1}{2}$ (2) $\mathrm{sec}x + \mathrm{tan}x + \frac{1}{2}$

(3) $x\mathrm{sec}x + \mathrm{tan}x + \frac{1}{2}$ (4) $\mathrm{sec}x + x\mathrm{tan}x - \frac{1}{2}$

26. Let $f(x) = \int \frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$, $|x| < \frac{2}{\sqrt{3}}$. If $f(0) = 0$ and $f(1) = \frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\alpha}{\beta}\right)$, $\alpha, \ \beta > 0$, then $\alpha^2 + \beta^2$ is equal to ______.

27. If $\int \frac{\sin x}{\sin^3 x + \cos^3 x} \mathrm{d}x = \alpha \log_e \left|1+\tan x\right| + \beta \log_e \left|1-\tan x+\tan^2 x\right| + \gamma \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$, when C is constant of integration, then the value of $18\left(\alpha+\beta+\gamma^2\right)$ is

28. For real numbers α, β, γ and δ, if $\int \frac{(x^2-1)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)} dx = \alpha \log_e\left(\tan^{-1}\left(\frac{\mathrm{x}^2+1}{\mathrm{x}}\right)\right) + \beta \tan^{-1}\left(\frac{\gamma(\mathrm{x}^2-1)}{\mathrm{x}}\right) + \delta \tan^{-1}\left(\frac{\mathrm{x}^2+1}{\mathrm{x}}\right) + \mathrm{C}$ where C is an arbitrary constant, then the value of $10(\alpha+\beta\gamma+\delta)$ is equal to ______ .

29. Let, $n \geq 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$. Then $\int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta} d\theta$, is equal to

(1) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+c$ (2) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+c$

(3) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+c$ (4) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+c$

30. Let $g : (0,\infty) \to \boldsymbol{R}$ be a differentiable function such that $\int\left(\frac{x(\cos x - \sin x)}{\mathrm{e}^x+1} + \frac{g(x)(\mathrm{e}^x+1-x\mathrm{e}^x)}{(\mathrm{e}^x+1)^2}\right)\mathrm{d}x = \frac{xg(x)}{\mathrm{e}^x+1} + C$, for all $x > 0$, where C is an arbitrary constant. Then

(1) g is decreasing in $\left(0, \frac{\pi}{4}\right)$ (2) $g-g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$

(3) g' is increasing in $\left(0, \frac{\pi}{4}\right)$ (4) $g+g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$