ANSWER KEYS

1. (1)	**2.** (3)	**3.** (1)	**4.** (2)	**5.** (4)	**6.** (3)	**7.** (3)	**8.** (4)
9. (1)	**10.** (2)						

1. **(1)** Given lines are

$\vec{r} = 2\hat{i} - 3\hat{j} + \hat{k} + \lambda\left(\hat{i} + 4\hat{j} + 3\hat{k}\right)$

and $\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$

Here DR's of given lines are $(1,\ 4,\ 3)$ and $(1,\ 2,\ -3)$.

$\therefore$ Angle between these lines is

$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$= \frac{1\times1+4\times2+3\times(-3)}{\sqrt{1^2+4^2+3^2}\sqrt{1^2+2^2+(-3)^2}}$

$= \frac{1+8-9}{\sqrt{1+16+9}\sqrt{1+4+9}} = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

2. **(3)**

Given lines are

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda_1$(Let) ….. (i)

and $\frac{x-3}{1} = \frac{y-\lambda}{2} = \frac{z}{1} = \lambda_2$ (Let)…… (ii)

Then, any point on line (i) is

$(2\lambda_1+1, 3\lambda_1-1, 4\lambda_1+1)$ and any point on line (ii) is $(\lambda_2+3, 2\lambda_2+\lambda, \lambda_2)$

Clearly, then lines (i) and (ii) will intersect if

$(2\lambda_1+1, 3\lambda_1-1, 4\lambda_1+1) = (\lambda_2+3, 2\lambda_2+\lambda, \lambda_2)$

For some particular value of λ_1 and λ_2

$\Rightarrow 2\lambda_1+1 = \lambda_2+3, 3\lambda_1-1 = 2\lambda_2+\lambda$

and $4\lambda_1+1 = \lambda_2$

$\Rightarrow 2\lambda_1-\lambda_2 = 2, 3\lambda_1-2\lambda_2 = \lambda+1$ and $4\lambda_1-\lambda_2 = -1$

after solving equations $2\lambda_1-\lambda_2 = 2$ and $4\lambda_1-\lambda_2 = -1$.

We get $\lambda_1 = -\frac{3}{2}$ and $\lambda_1 = -5$

Now, putting the values of λ_1 and λ_2 in

$3\lambda_1 - 2\lambda_2 = \lambda+1$

$\Rightarrow 3\left(\frac{-3}{2}\right) - 2(-5) = \lambda+1 \Rightarrow \frac{-9}{2} + 10 = \lambda+1$

$\Rightarrow \lambda+1 = \frac{11}{2} \Rightarrow \lambda = \frac{9}{2}$

3. **(1)** Variable points on the 1^{st} and the 2^{nd} line can be taken as $(t+3,\ 3t-1,\ -t+6)$ and $(7s-5,\ -6s+2,\ 4s+3)$ respectively. If both the lines intersect then for some value of t and s

$7s-5 = t+3,\ -6s+2 = 3t-1$ and $4s+3 = -t+6$

$\Rightarrow\ t = -1$ and $s = 1$.

Hence point of intersection is $(2,\ -4,\ 7)$ and its image in xy-plane is $(2,\ -4,\ -7)$

4. **(2)** Let point is $\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z+5}{-2} = \lambda$

$P \equiv (\lambda+2,\ -2\lambda-3, -2\lambda-5)$

Let, $Q \equiv (2,\ -3, -5)$

Given, $PQ = 6$

$\Rightarrow PQ^2 = 36$

$\Rightarrow (\lambda+2-2)^2 + (-2\lambda-3+3)^2 + (2\lambda-5+5)^2 = 36$

$\Rightarrow 9\lambda^2 = 36$

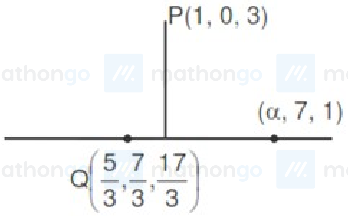
$\Rightarrow \lambda^2 = 4$

$\Rightarrow \lambda = \pm 2$

$\therefore$ From $\lambda = 2,\ P \equiv (4, -7, -9)$.

And from $\lambda = -2,\ P \equiv (0,\ 1,\ -1)$.

5. **(4)** Since PQ, is perpendicular to L, therefore

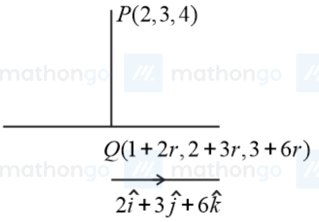


$\left(1-\frac{5}{3}\right)\left(\alpha-\frac{5}{3}\right)+\left(\frac{-7}{3}\right)\left(7-\frac{7}{3}\right)+\left(3-\frac{17}{3}\right)\left(1-\frac{17}{3}\right)=0$

$\frac{-2\alpha}{3}+\frac{10}{9}-\frac{98}{9}+\frac{112}{9}=0$

$\frac{2\alpha}{3}=\frac{24}{9} \Rightarrow \alpha=4.$

6. **(3)**



Let the foot of perpendicular be $Q(1+2r,2+3r,3+6r)$.

Direction ratios of PQ are $(2r-1,3r-1,6r-1)$.

Since, PQ is perpendicular to the line,

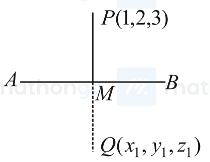
$\therefore\ 2(2r-1)+3(3r-1)+6(6r-1)=0$

i.e., $49r-11=0$ i.e, $r=\frac{11}{49}$

$\therefore$ Coordinate of the foot are $\left(\frac{71}{49},\frac{131}{49},\frac{213}{49}\right)$.

7. **(3)**

Equation of given line is $\frac{x}{2}=\frac{y-1}{3}=\frac{z-1}{3}=k$ (say)



Any point on the line is $M(2k,\ 3k+1,\ 3k+1)$

Direction ratio of PM are $(2k-1,3k-1,3k-2)$ Since, the line PM is perpendicular to AB

$\therefore 2(2k-1)+3(3k-1)+3(3k-2)=0$

$\Rightarrow 22k-11=0$

$\Rightarrow k=\frac{1}{2}$

$\therefore$ Point M is $\left(1,\frac{5}{2},\frac{5}{2}\right)$

Let the image of P about the line AB is Q, where M is the mid point of PQ

$\therefore\ \frac{x_1+1}{2}=1,\ \frac{y_1+2}{2}=\frac{5}{2},\ \frac{z_1+3}{2}=\frac{5}{2}$

$\Rightarrow x_1=1,\ y_1=3,\ z_1=2$

8. **(4)** S.D. $=\dfrac{\begin{vmatrix}6 & 15 & -3\\ 3 & -1 & 1\\ -3 & 2 & 4\end{vmatrix}}{\sqrt{(-4-2)^2+(12+3)^2+(6-3)^2}}$

$=\frac{270}{\sqrt{270}}=\sqrt{270}=3\sqrt{30}$.

9. (1)

The equation of the given plane is $x+2y+3z-4=0\ \ldots(i)$

The equation of plane passing through the point $P(1,\ 1,\ 1)$ having normal $1,\ 1,\ 1$ is given by

$1(x-1)+1(y-1)+1(z-1)=0$

Or $x+y+z-3=0\ \ldots(ii)$

Now, locus of Q is the line of intersection of the planes (i) and (ii).

Hence, the direction ratios of required line is given by

$$\overrightarrow{n_1}\times\overrightarrow{n_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 1 & 1 & 1\end{vmatrix}=-\hat{i}+2\hat{j}-\hat{k}$$

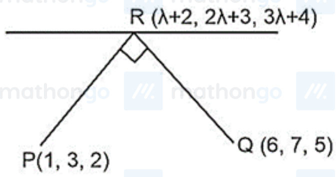
Substituting $x=0$ in (i) and (ii), we get,

$y=5,\ z=-2$

So, the equation of line can be written as $\frac{x}{1}=\frac{y-5}{-2}=\frac{z+2}{1}$.

10. (2)

Any general point on the line is $(\lambda+2, 2\lambda+3, \lambda+4)$



$\overrightarrow{PR}=<\lambda+1, 2\lambda, 3\lambda+2>$

$\overrightarrow{RQ}=<4-\lambda, 4-2\lambda, 1-3\lambda>$

From the diagram, $\overrightarrow{PR}\cdot\overrightarrow{RQ}=0$

$\Rightarrow(\lambda+1)(4-\lambda)+2\lambda(4-2\lambda)+(3\lambda+2)(1-3\lambda)=0$

$\Rightarrow \lambda^2-3\lambda-4+4\lambda^2-8\lambda+9\lambda^2+3\lambda-2=0$

$\Rightarrow 14\lambda^2-8\lambda-6=0$

$\Rightarrow \lambda=1,\ -\frac{3}{7}$

Coordinates of R are $(3,5,7)$ or $\left(\frac{11}{7},\frac{15}{7},\frac{19}{7}\right)$

Least value of $7(a+b+c)=11+15+19=45$