ANSWER KEYS

1. (1)	**2.** (2)	**3.** (4)	**4.** (3)	**5.** (2)	**6.** (3)	**7.** (1)	**8.** (3)
9. (4)	**10.** (4)						

1. **(1)** $\alpha = \ -\omega,\ \beta = -\omega^2$

$\alpha^{2015} = \ -\omega^2,\ \beta^{2015} = \ -\omega$

2. **(2)**
As we know if $lx^2 + mx + n = 0$ is identity, then $l = 0,\ m = 0,\ n = 0$
So, $a^2 - 3a + 2 = 0$

$(a-2)\ (a-1) = 0$

$a = 2\ ,\ a = 1$

$a^2 - 5a + 6 = 0$

$(a-3)(a-2) = 0$

$a = 3,\ 2$

$a^2 - 4 = 0$

$(a-2)(a+2) = 0$

$a = \pm 2$

$\therefore\quad a = 2$

3. **(4)** Let $\alpha\ ,\ \beta$ and γ,δ are the roots of the equations

$x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ respectivaly.
$\therefore \alpha + \beta = -a, \alpha\beta = b$ and $\gamma + \delta = -b, \gamma\delta = a.$
Given $\alpha - \beta = \gamma - \delta \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$
$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$
$a^2 - 4b = b^2 - 4a \Rightarrow (a^2 - b^2) + 4(a - b) = 0$
$\Rightarrow a + b + 4 = 0.\ (\because a \neq b)$

4. **(3)**
Let α and β be the roots of the given equation. $x^2+(4-\lambda)x+3=\lambda$
$\Rightarrow x^2+(4-\lambda)x+3-\lambda=0$
$\Rightarrow \alpha+\beta=-(4-\lambda)=\lambda-4$ and $\alpha\beta=3-\lambda$
$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
$=(\lambda-4)^2-2(3-\lambda)$
$=\lambda^2-6\lambda+10$
$=(\lambda-3)^2+1$
For least value $\lambda=3$

5. **(2)** Since, 4 is a root of $x^2+ax+12=0$

$\therefore \quad 16+4a+12=0 \Rightarrow a=-7$

Let the roots of the equation $x^2+ax+b=0$ be α and α

$\therefore \quad 2\alpha=-a$

$\Rightarrow \quad \alpha=\frac{7}{2}$

And $\quad \alpha.\alpha=b$

$\Rightarrow \quad \left(\frac{7}{2}\right)^2=b$

$\Rightarrow \quad b=\frac{49}{4}$

6. **(3)** Since the given equation has no real solution.
$\Rightarrow D<0$
$\Rightarrow 4(1+3m)^2-4\times(1+m^2)(1+8m)<0$
$\Rightarrow(1+9m^2+6m)-(1+8m+m^2+8m^3)<0$
$\Rightarrow -2m(2m-1)^2<0$
Clearly, for infinite values of m given condition is true.

7. **(1)**

Given, $6x^2-11x+\alpha=0$

Roots of the quadratic equation$=\frac{-(-11)\pm\sqrt{11^2-4(6)(\alpha)}}{2\times6}=\frac{11\pm\sqrt{121-24\alpha}}{12}$

For the roots to be rational, discriminant must be non-negative and a perfect square.

Let $121-24\alpha=k^2$

$121-24\alpha\geq0\Rightarrow\alpha\leq\frac{121}{24}$

Since, we have to find positive integral values for α, range of α is $0<\alpha<\frac{121}{24}$

The positive integral values in the above range are $1,2,3,4,5$.

$\alpha=1\Rightarrow121-24\alpha=121-24(1)=97$, not a perfect square.

$\alpha=2\Rightarrow121-24\alpha=121-24(2)=73$, not a perfect square.

$\alpha=3\Rightarrow121-24\alpha=121-24(3)=49$, is a perfect square.

$\alpha=4\Rightarrow121-24\alpha=121-24(4)=25$, is a perfect square.

$\alpha=5\Rightarrow121-24\alpha=121-24(5)=1$, is a perfect square.

So, possible value of $\alpha=3,\ 4,\ 5$

Hence, there are 3 positive integral values for α for which the roots are rational.

8. **(3)** Given Equation : $x^2+|2x-3|-4=0$

$x<\frac{3}{2}\Rightarrow x^2-(2x-3)-4=0$

$\Rightarrow x^2-2x-1=0$

Roots $\alpha,\beta=\frac{2\pm\sqrt{4+4}}{2}=1\pm\sqrt{2}$

root $1+\sqrt{2}>\frac{3}{2}$, hence rejected

$\Rightarrow$ root accepted $1-\sqrt{2}$

$x\geq\frac{3}{2}\Rightarrow x^2+(2x-3)-4=0$

$\Rightarrow x^2+2x-7=0$

Roots $\gamma,\delta=\frac{-2\pm\sqrt{4+4\times7}}{2}-1\pm2\sqrt{2}$

root $-1-2\sqrt{2}<\frac{3}{2}$, hence rejected

$\Rightarrow$ root accepted $-1+2\sqrt{2}$

Sum of roots $=\left(1-\sqrt{2}\right)+\left(-1+2\sqrt{2}\right)=\sqrt{2}$

9. **(4)** Let α, β be roots $x^2-5x+3=0$.

$\alpha+\beta=5, \ \alpha\beta=3$

Now $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{25-6}{3}=\frac{19}{3}$

and $\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}=1$.

So equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$x^2-\frac{19}{3}x+1=0 \Rightarrow 3x^2-19x+3=0$.

10. **(4)** $\frac{a_{10}-2a_8}{2a_9}=\frac{(\alpha^{10}-\beta^{10})-2(\alpha^8-\beta^8)}{2(\alpha^9-\beta^9)}$

$=\frac{\alpha^8(\alpha^2-2)-\beta^8(\beta^2-2)}{2(\alpha^9-\beta^9)}$

$(\because \alpha$ is root of $x^2-6x-2=0 \Rightarrow \alpha^2-2=6\alpha)$

$(\because$ Also, β is root of $x^2-6x-2=0 \Rightarrow \beta^2-2=6\beta)$

$=\frac{\alpha^8(6\alpha)-\beta^8(6\beta)}{2(\alpha^9-\beta^9)}=\frac{6(\alpha^9-\beta^9)}{2(\alpha^9-\beta^9)}=3$