ANSWER KEYS

1. (2)	**2.** (3)	**3.** (2)	**4.** (3)	**5.** (4)	**6.** (3)	**7.** (1)	**8.** (3)
9. (4)	**10.** (4)						

1. (2)

$I = \int_1^2 \{x\} dx + \int_2^3 \{x\}^2 dx + \int_2^3 \{x\}^3 dx$

$= \int_0^1 (x + x^2 + x^3) dx = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6+4+3}{12}$

$= \frac{13}{12}$

$\Rightarrow 24\left(\frac{I}{13}\right) = 2.$

2. (3)

$I = \int_0^{2\pi} [\sin 2x(1 + \cos 3x)] dx \quad \ldots(1)$

$I = \int_0^{2\pi} [\sin(2\pi - 2x)(1 + \cos(2\pi - 3x))] dx$

Applying $\left(\int_0^a f(x) = \int_0^a f(a - x) dx\right)$

$I = \int_0^{2\pi} [-\sin 2x(1 + \cos 3x)] dx \quad \ldots(2)$

Adding (1) and (2)

$2I = \int_0^{2\pi} ([\sin 2x(1 + \cos 3x)] + [-\sin 2x(1 + \cos 3x)]) dx$

$\Rightarrow 2I = \int_0^{2\pi} -1 dx \quad \left\{\because \ [x] + [-x] = \begin{cases} 0 & : x \in \mathrm{I} \\ -1 & : x \notin \mathrm{I} \end{cases}\right.$

$\Rightarrow 2I = (-x)_0^{2\pi}$

$\Rightarrow 2I = -2\pi$

$\Rightarrow I = -\pi$

3. (2)

Let $I = \int_{1/e}^{e} |\log x| dx$

$= -\int_{1/e}^{1} \log x dx + \int_1^e \log x dx$

$= (x - x\log x)_{1/e}^{1} + (x\log x - x)|_1^e$

$= \left[(1 - 0) - \left(\frac{1}{e} + \frac{1}{e}\right)\right] + [(e - e) - (0 - 1)]$

$= 2\left(1 - \frac{1}{e}\right)$

4. **(3)** Let $I=\int_{-3}^{2}(|x+1|+|x+2|+|x-1|)dx$

Again, let $f(x)=|x+1|+|x+2|+|x-1|$

$$=\begin{cases}-(x+1)-(x+2)-(x-1), & -3<x\leq -2\\ -(x+1)+x+2-(x-1), & -2<x\leq -1\\ 1+x+x+2-(x-1) & -1<x\leq 0\\ 1+x+x+2-(x-1) & 0\leq x<1\\ 1+x+x+2+x-1 & 1\leq x<2\end{cases}$$

$$=\begin{cases}-3x-2, & -3<x\leq -2\\ -x+2, & -2<x\leq -1\\ x+4, & -1\leq x<1\\ 3x+2, & 1\leq x<2\end{cases}$$

$\therefore I=\int_{-3}^{-2}(-3x-2)dx+\int_{-2}^{-1}(-x+2)dx$

$+\int_{-1}^{1}(x+4)dx+\int_{1}^{2}(3x+2)dx$

$=\left[-\frac{3x^2}{2}-2x\right]_{-3}^{-2}+\left[-\frac{x^2}{2}+2x\right]_{-2}^{-1}$

$+\left[\frac{x^2}{2}+4x\right]_{-1}^{1}+\left[\frac{3x^2}{2}+2x\right]_{1}^{2}$

$=\left[-6+4-\left(-\frac{27}{2}+6\right)\right]+\left[-\frac{1}{2}-2-\left(-2-4\right)\right]$

$+\left[\frac{1}{2}+4-\left(\frac{1}{2}-4\right)\right]\left[6+4-\left(\frac{3}{2}+2\right)\right]$

$=\frac{11}{2}+\frac{7}{2}+8+\frac{13}{2}$

$=\frac{31}{2}+8=\frac{47}{2}$

Alternate

Let $I=\int_{-3}^{2}\{|x+1|+|x+2|+|x-1|\}dx$

$=\int_{-3}^{-1}|x+1|dx+\int_{-1}^{2}|x+1|dx+\int_{-3}^{-2}|x+2|dx$

$+\int_{-2}^{2}|x+2|dx+\int_{-3}^{1}|x-1|dx$

$+\int_{1}^{2}|x-1|dx$

$=-\int_{-3}^{-1}(x+1)dx+\int_{-1}^{2}(x+1)dx-\int_{-3}^{-2}(x+2)dx$

$+\int_{-2}^{2}(x+2)dx-\int_{-3}^{1}(x-1)dx+\int_{1}^{2}(x-1)dx$

$=-\left(\frac{x^2}{2}+x\right)_{-3}^{-1}+\left(\frac{x^2}{2}+x\right)_{-1}^{2}-\left(\frac{x^2}{2}+2x\right)_{-3}^{-2}$

$+\left(\frac{x^2}{2}+2x\right)_{-2}^{2}-\left(\frac{x^2}{2}-x\right)_{-3}^{1}-\left(\frac{x^2}{2}-x\right)_{1}^{2}$

$=\frac{47}{2}$

5.
(4) Let $I=\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx$ ....(i)

$I=\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}{1+\sin\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}$

$=\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{(\pi-x) dx}{1+\sin x}$ ....(ii)

$\left[\because \int_a^b f(x) dx=\int_a^b f(a+b-x) dx\right]$

By adding Equations (i) and (ii), we get

$2I=\int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi\, dx}{1+\sin x}$

$\Rightarrow 2I=\pi \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{(1+\sin x)(1-\sin x)} dx$

$\Rightarrow 2I=\pi \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-\sin x}{\cos^2 x} dx$

$\Rightarrow 2I=\pi \int\limits_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left[\sec^2 x-\sec x \tan x\right] dx$

$\Rightarrow 2I=\pi[\tan x-\sec x]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$\Rightarrow 2I=\pi\left[-1-(-\sqrt{2})-(1-\sqrt{2})\right]$

$\Rightarrow 2I=\pi\left[-1+\sqrt{2}-1+\sqrt{2}\right]$

$\Rightarrow 2I=\pi\left[-2+2\sqrt{2}\right]$

$\therefore I=\pi\left[\sqrt{2}-1\right]$

$=\frac{\pi(\sqrt{2}-1)}{(\sqrt{2}+1)}(\sqrt{2}+1)$

$=\frac{\pi}{\sqrt{2}+1}$

6. **(3)**

$\mathrm{I}_1=\int_0^\pi \frac{(\pi-\mathrm{x}) \sin (\pi-\mathrm{x})}{1+(\cos (\pi-\mathrm{x}))^2} \mathrm{dx}$

$=\pi \int_0^\pi \frac{\sin \mathrm{x}}{1+\cos^2 \mathrm{x}} \mathrm{dx}-\int_0^\pi \frac{\mathrm{x} \sin \mathrm{x}}{1+\cos^2 \mathrm{x}} \mathrm{dx}$

$2 \mathrm{I}_1=\pi \int_0^\pi \frac{\sin \mathrm{x}}{1+\cos^2 \mathrm{x}} \mathrm{dx}=2\pi \int_0^{\pi/2} \frac{\sin \mathrm{x}}{1+\cos^2 \mathrm{x}} \mathrm{dx}$

$\Rightarrow \mathrm{I}_1=\pi \int_0^{\pi/2} \frac{\sin \mathrm{x}}{1+\cos^2 \mathrm{x}} \mathrm{dx}$

$\mathrm{t}=\cos\ \mathrm{x} \Rightarrow \mathrm{dt}=\text{-sinxdx}$

$x=0, t=1\ \&\ x=\frac{\pi}{2}, t=0$

$=\pi \int_0^1 \frac{\mathrm{dt}}{1+\mathrm{t}^2}$

$=\pi\ \left[\tan^{-1} \mathrm{t}\right]_0^1=\frac{\pi^2}{4}.$

$\mathrm{I}_2=\int_0^\pi(\pi-\mathrm{x}) \sin^4 \mathrm{x}\ \mathrm{dx}$

$=\pi \int_0^\pi \sin^4 \mathrm{x}\ \mathrm{dx}-\mathrm{I}_2$

$\Rightarrow 2 \mathrm{I}_2=2\pi \int_0^{\pi/2} \sin^4 \mathrm{x}\ \mathrm{dx}=2\pi \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

$\Rightarrow \mathrm{I}_2=\frac{3}{16} \pi^2$

Therefore, $\mathrm{I}_1 : \mathrm{I}_2=\frac{1}{4} : \frac{3}{16}=4:3$.

7. **(1)**

We have $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x]+[\sin x]+4}$

$=\int_{-\frac{\pi}{2}}^0 \frac{dx}{[x]+3}+\int_0^{\frac{\pi}{2}} \frac{dx}{[x]+4}$

$=\int_{-\frac{\pi}{2}}^{-1} \frac{dx}{1}+\int_{-1}^0 \frac{dx}{2}+\int_0^1 \frac{dx}{4}+\int_1^{\frac{\pi}{2}} \frac{dx}{5}$

$=[x]_{-\frac{\pi}{2}}^{-1}+\left[\frac{x}{2}\right]_{-1}^0+\left[\frac{x}{4}\right]_0^1+\left[\frac{x}{5}\right]_1^{\frac{\pi}{2}}=\left(-1+\frac{\pi}{2}\right)+\left(0+\frac{1}{2}\right)+\frac{1}{4}+\frac{\pi}{10}-\frac{1}{5}$

$=\frac{-20+10\pi+10+5+2\pi-4}{20}=\frac{12\pi-9}{20}=\frac{3}{20}(4\pi-3)$

8. **(3)**

Given,

$\int_{-1}^{1} \frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{\sqrt{1+x+x^2}+\sqrt{1-x+x^2}} dx$

Let $f(x) = \frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{\sqrt{1+x+x^2}+\sqrt{1-x+x^2}}$

$f(-x) = -\left(\frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{\sqrt{1+x+x^2}+\sqrt{1-x+x^2}}\right) = -f(x)$

$\Rightarrow f(x)$ is an odd function.

$\Rightarrow \int_{-1}^{1} \frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{\sqrt{1+x+x^2}+\sqrt{1-x+x^2}} dx = 0$

9. **(4)**

We have,

$\int_0^{200} [\tan^{-1} x] dx$

$= \int_0^{\tan 1} [\tan^{-1} x] dx + \int_{\tan 1}^{200} [\tan^{-1} x] dx$

$= 0 + \int_{\tan 1}^{200} 1 dx$

$= (x)_{\tan 1}^{200} = 200 - \tan 1.$

10. **(4)**

Since, $\sin^2 x - \sin x + \frac{1}{2} = \left(\sin x - \frac{1}{2}\right)^2 + \frac{1}{4} > 0 \; \forall x \in \left(0, \frac{\pi}{2}\right)$

$\therefore \; sgn\left(\sin^2 x - \sin x + \frac{1}{2}\right) = 1$

Thus, $\mathrm{I} = \int_0^{\frac{\pi}{2}} 1 dx$

$= (x)_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2}$