1. Water is being filled at the rate of 1 $cm^3 \ sec^{-1}$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in $cm^2 \ sec^{-1}$) at which the wet conical surface area of the vessel increases is

(1) 5
(2) $\frac{\sqrt{21}}{5}$
(3) $\frac{\sqrt{26}}{5}$
(4) $\frac{\sqrt{26}}{10}$

2. A 2m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25cm/sec , then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is:

(1) 25
(2) $25\sqrt{3}$
(3) $\frac{25}{3}$
(4) $\frac{25}{\sqrt{3}}$

3. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

(1) $\frac{1}{9\pi}$
(2) $\frac{1}{36\pi}$
(3) $\frac{1}{18\pi}$
(4) $\frac{5}{6\pi}$

4. The position of a moving car at time t is given by $f(t) = at^2 + bt + c, \ t > 0$, where $a, \ b$ and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point:

(1) $\frac{(t_2 - t_1)}{2}$
(2) $a(t_2 - t_1) + b$
(3) $\frac{(t_1 + t_2)}{2}$
(4) $2a(t_1 + t_2) + b$

5. If the surface area of a cube is increasing at a rate of $3.6 cm^2/sec$, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is $10 cm$, is:

(1) 20
(2) 10
(3) 18
(4) 9

6. Water is being filled at the rate of 1 $cm^3 \ sec^{-1}$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in $cm^2 \ sec^{-1}$) at which the wet conical surface area of the vessel increases is

(1) 5
(2) $\frac{\sqrt{21}}{5}$
(3) $\frac{\sqrt{26}}{5}$
(4) $\frac{\sqrt{26}}{10}$

7. The function $f(x) = xe^{x(1-x)}, x \in R$, is

(1) increasing in $\left(-\frac{1}{2}, 1\right)$
(2) decreasing in $\left(\frac{1}{2}, 2\right)$
(3) increasing in $\left(-1, -\frac{1}{2}\right)$
(4) decreasing in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

8. The set of all $a \in \mathbb{R}$ for which the equation $x|x-1| + |x+2| + a = 0$ has exactly one real root, is

(1) $(-7, \infty)$
(2) $(-\infty, \infty)$
(3) $(-6, -3)$
(4) $(-\infty, -3)$

9. Let $g(x) = f(x) + f(1-x)$ and $f''(x) > 0, x \in (0, 1)$. If g is decreasing in the interval $(0, \alpha)$ and increasing in the interval $(\alpha, 1)$, then $\tan^{-1} 2\alpha + \tan^{-1}\left(\frac{1}{\alpha}\right) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to

(1) π
(2) $\frac{5\pi}{4}$
(3) $\frac{3\pi}{4}$
(4) $\frac{3\pi}{2}$

10. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to _____ .

11. Let the function $f(x) = 2x^2 - \log_e x, x > 0$, be decreasing in $(0, a)$ and increasing in $(a, 4)$. A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point $(8a, \ 8a-1)$ but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha + \beta$ is equal to

12. For the function $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5, x > 1$, which one of the following is NOT correct?

(1) $f(x)$ is increasing in $(1, 2)$ and decreasing in $(2, \infty)$
(2) $f(x) = -1$ has exactly two solutions
(3) $f'(e) - f''(2) < 0$
(4) $f(x) = 0$ has a root in the interval $(e, e+1)$

13. The range of $a \in R$ for which the function $f(x) = (4a-3)(x + \log_e 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right), x \neq 2n\pi, n \in N$, has critical points, is

(1) $(-3, 1)$
(2) $\left[-\frac{4}{3}, 2\right]$
(3) $[1, \infty)$
(4) $(-\infty, -1]$

14. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is

(1) 0
(2) 1
(3) 2
(4) 4

15. The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is

(1) 5
(2) 7
(3) 1
(4) 3

16. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is _________.

17. The number of points, where the curve $y=x^5-20x^3+50x+2$ crosses the x-axis, is _____.

18. Let $f: \boldsymbol{R} \rightarrow \boldsymbol{R}$ and $g: \boldsymbol{R} \rightarrow \boldsymbol{R}$ be two functions defined by $f(x)=\log_e\left(x^2+1\right)-e^{-x}+1$ and $g(x)=\frac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right)>f\left(g\left(\alpha-\frac{5}{3}\right)\right)$ holds?

(1) $(-2,-1)$
(2) $(2,3)$
(3) $(1,2)$
(4) $(-1,1)$

19. If a_α is the greatest term in the sequence $a_n=\frac{n^3}{n^4+147}$, $n=1,\ 2,\ 3, \ldots$, then α is equal to ______

20. Let $x=2$ be a local minima of the function $f(x)=2x^4-18x^2+8x+12, x \in(-4,4)$. If M is local maximum value of the function f in $(-4,4)$, then $M=$

(1) $12\sqrt{6}-\frac{33}{2}$
(2) $12\sqrt{6}-\frac{31}{2}$
(3) $18\sqrt{6}-\frac{33}{2}$
(4) $18\sqrt{6}-\frac{31}{2}$

21. The sum of the absolute minimum and the absolute maximum values of the function $f(x)=\left|3x-x^2+2\right|-x$ in the interval $[-1,2]$ is

(1) $\frac{\sqrt{17}+3}{2}$
(2) $\frac{\sqrt{17}+5}{2}$
(3) 5
(4) $\frac{9-\sqrt{17}}{2}$

22. The sum of all the local minimum values of the twice differentiable function $f: R \rightarrow R$ defined by $f(x)=x^3-3x^2-\frac{3f''(2)}{2}x+f''(1)$ is:

(1) -22
(2) 5
(3) -27
(4) 0

23. Let $f:[-1,1] \rightarrow R$ be defined as $f(x)=ax^2+bx+c$ for all $x \in[-1,\ 1]$, where $a,\ b,\ c \in R$ such that $f(-1)=2,\ f'(-1)=1$ and for $x \in(-1,\ 1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \leq \alpha,\ x \in[-1,\ 1]$, then the least value of α is equal to

24. Let $f:[0,\ 2] \rightarrow R$ be a twice differentiable function such that $f''(x)>0$, for all $x \in[0,\ 2]$. If $\phi(x)=\ f(x)+f(2-x)$, then ϕ is

(1) decreasing on (0,2)
(2) increasing on (0,2)
(3) increasing on (0,1) and decreasing on (1,2)
(4) decreasing on (0,1) and increasing on (1,2)

25. The sum of the abosolute maximum and minimum values of the function $f(x)=\left|x^2-5x+6\right|-3x+2$ in the interval $[-1,3]$ is equal to :

(1) 10
(2) 12
(3) 13
(4) 24

26. If $x=1$ is a critical point of the function $f(x)=\left(3x^2+ax-2-a\right)e^x$, then

(1) $x=1$ and $x=-\frac{2}{3}$ are local minima of f
(2) $x=1$ and$x=-\frac{2}{3}$ is a local maxima of f
(3) $x=1$ is a local maxima and$x=-\frac{2}{3}$ is a local minima of f
(4) $x=1$ is a local minima and$x=-\frac{2}{3}$ are local maxima of f

27. The number of real solutions of $x^7+5x^3+3x+1=0$ is equal to _____.

(1) 0
(2) 1
(3) 3
(4) 5

28. Let $f:[2,4] \rightarrow \mathbb{R}$ be a differentiable function such that $(x\log_e x)f'(x)+(\log_e x)f(x)+f(x) \geq 1, x \in[2,4]$ with $f(2)=\frac{1}{2}$ and $f(4)=\frac{1}{2}$.

Consider the following two statements:

(A) $f(x) \leq 1$, for all $x \in[2,4]$
(B) $f(x) \geq 1/8$, for all $x \in[2,4]$

Then,

(1) Neither statement (A) nor statement (B) is true
(2) Only statement (B) is true
(3) Both the statements $(A)$and (B) are true
(4) Only statement (A) is true

29. $\max\limits_{0 \leq x \leq \pi}\left\{x-2\sin x\cos x+\frac{1}{3}\sin 3x\right\}=$

(1) $\frac{\pi+2-3\sqrt{3}}{6}$
(2) π
(3) 0
(4) $\frac{5\pi+2+3\sqrt{3}}{6}$

30. The sum of absolute maximum and absolute minimum values of the function $f(x)=\left|2x^2+3x-2\right|+\sin x\cos x$ in the interval $[0,1]$ is

(1) $3+\frac{\sin(1)\cos^2\left(\frac{1}{2}\right)}{2}$
(2) $3+\frac{1}{2}(1+2\cos(1))\sin(1)$
(3) $5+\frac{1}{2}(\sin(1)+\sin(2))$
(4) $2+\sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$