ANSWER KEYS

1. (1)	**2.** (1)	**3.** (1)	**4.** (4)	**5.** (4)	**6.** (2)	**7.** (11)	**8.** (2)
9. (80)	**10.** (2)	**11.** (1)	**12.** (3)	**13.** (2)	**14.** (4)	**15.** (4)	**16.** (4)
17. (1)	**18.** (4)	**19.** (3)	**20.** (4)	**21.** (1)	**22.** (1)	**23.** (2)	**24.** (3)
25. (3)							

1. (1)

Given:

$2a = \tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ$

$\Rightarrow 2a = \tan 15^\circ + \frac{1}{\tan(90^\circ - 15^\circ)} + \frac{1}{\tan(90^\circ + 15^\circ)} + \tan(180^\circ + 15^\circ)$

$\Rightarrow 2a = \tan 15^\circ + \frac{1}{\cot 15^\circ} - \frac{1}{\cot 15^\circ} + \tan 15^\circ$

$\Rightarrow 2a = 2\tan 15^\circ$

$\Rightarrow a = \tan 15^\circ$

$\Rightarrow a = \tan(45^\circ - 30^\circ)$

$\Rightarrow a = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$

$\Rightarrow a = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

So,

$a + \frac{1}{a} = \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\Rightarrow a + \frac{1}{a} = \frac{\left(\sqrt{3}-1\right)^2 + \left(\sqrt{3}+1\right)^2}{2}$

$\Rightarrow a + \frac{1}{a} = \frac{8}{2} = 4$

2. (1)

Given,

$\cot\alpha = 1, \sec\beta = \frac{-5}{3}$

So, $\cos\beta = \frac{-3}{5}, \tan\beta = \frac{-4}{3}$ and $\tan\alpha = 1$

Now using formula $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\Rightarrow \tan(\alpha+\beta) = \frac{1-\frac{4}{3}}{1+\frac{4}{3}\times 1} = \frac{-1}{7}$

We know that $\tan\theta$ is negative in 2^{nd} & 4^{th} quadrant but as $\pi < \alpha < \frac{3\pi}{2}$, so $\alpha+\beta$ will lie in $\mathrm{IV}^{\mathrm{th}}$ quadrant.

3. (1)

Given,

$\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}, \ \alpha, \beta \in \left(0, \frac{\pi}{2}\right)$

Now using $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$, we can write

$\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$ and $\frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$

$\tan\alpha = \frac{1}{7}$ and $\sin\beta = \frac{1}{\sqrt{10}}$ or $\tan\beta = \frac{1}{3}$

$tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = 2.\frac{\left(\frac{1}{3}\right)}{\left[1-\left(\frac{1}{9}\right)\right]} = \frac{3}{4}$

$\tan(\alpha+2\beta) = \frac{tan\alpha + tan 2\beta}{1 - tan\alpha\tan 2\beta} = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$

4. (4) The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$\Rightarrow \tan 9^\circ + \cot 9^\circ - \tan 27^\circ - \cot 27^\circ$

$\Rightarrow \dfrac{2}{\sin 18^\circ} - \dfrac{2}{\sin 54^\circ}$

$\Rightarrow \dfrac{2\times 4}{\sqrt{5}-1} - \dfrac{2\times 4}{\sqrt{5}+1}$

$\Rightarrow 4$

5. (4)

We need to find the value of

$36\left(4 \cos^2 9^\circ - 1\right)\left(4 \cos^2 27^\circ - 1\right)\left(4 \cos^2 81^\circ - 1\right)\left(4 \cos^2 243^\circ - 1\right)$

$= 36\left[\left(4 \cos^2 9^\circ - 1\right)\left(4 \sin^2 9^\circ - 1\right)\left(4 \cos^2 27^\circ - 1\right)\left(4 \sin^2 27^\circ - 1\right)\right]$

$= 36\left[\left(4(2\cos 9^\circ \sin 9^\circ)^2 - 4\cos^2 9^\circ - 4\sin^2 9^\circ + 1\right)\left(4(2\sin 27^\circ \cos 9^\circ)^2 - 4\cos^2 27^\circ - 4\sin^2 27^\circ + 1\right)\right]$

$= 36\left[\left(4 \sin^2 18^\circ - 4 + 1\right)\left(4 \sin^2 54^\circ - 4 + 1\right)\right]$

$= 36\left[\left(4 \sin^2 18^\circ - 3\right)\left(4 \sin^2 54^\circ - 3\right)\right]$

$= 36\left[\left\{\frac{\left(\sqrt{5}-1\right)^2}{4} - 3\right\}\left\{\frac{\left(\sqrt{5}+1\right)^2}{4} - 3\right\}\right]$

$= 36\left[\left\{\frac{\left(6-2\sqrt{5}-12\right)}{4}\frac{\left(6+2\sqrt{5}-12\right)}{4}\right\}\right]$

$= -36\left[\left\{\frac{\left(2\sqrt{5}\right)^2 - 36}{16}\right\}\right] = 36$

Hence this is the correct option.

6. (2)

$\Rightarrow 2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$\Rightarrow 2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\pi - \frac{3\pi}{8}\right)\sin\left(\pi - \frac{2\pi}{8}\right)\sin\left(\pi - \frac{\pi}{8}\right)$

$\Rightarrow 2\sin^2\frac{\pi}{8}\sin^2\left(\frac{2\pi}{8}\right)\sin^2\left(\frac{3\pi}{8}\right)$

$\Rightarrow 2\sin^2\frac{\pi}{8}\sin^2\left(\frac{2\pi}{8}\right)\sin^2\left(\frac{3\pi}{8}\right)$

$\Rightarrow 2\sin^2\frac{\pi}{8}\sin^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{3\pi}{8}\right)$

$\Rightarrow \sin^2\frac{\pi}{8}\sin^2\left(\frac{3\pi}{8}\right)$

$\Rightarrow \sin^2\frac{\pi}{8}\sin^2\left(\frac{\pi}{2} - \frac{\pi}{8}\right)$

$\Rightarrow \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$

$\Rightarrow \frac{1}{4}\left(2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)^2$

$\frac{1}{4}\sin^2\frac{\pi}{4} = \frac{1}{8}$

7. (11) $3\sin x + 4\cos x = k + 1$

$\Rightarrow k + 1 \in \left[-\sqrt{3^2+4^2}, \sqrt{3^2+4^2}\right]$

$\Rightarrow k + 1 \in [-5, 5]$

$\Rightarrow k \in [-6, 4]$

No. of integral values of $k = 11$

8. (2)

We know that $\sin\theta\sin(60-\theta)\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

Now given $16\sin 20^\circ \sin 40^\circ \sin 80^\circ$

Comparing with above formula $\theta = 20^\circ$

we get $16\sin 20^\circ \sin 40^\circ \sin 80^\circ = 16 \times \frac{1}{4} \times \sin(3 \times 20^\circ)$

$= 16 \times \frac{1}{4} \times \sin 60^\circ = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

9. (80)

We have, $\sin^2 10^\circ \times \sin 20^\circ \times \sin 40^\circ \times \sin 50^\circ \times \sin 70^\circ$

$=(\sin 10^\circ \times \sin 50^\circ \times \sin 70^\circ)(\sin 10^\circ \times \sin 20^\circ \times \sin 40^\circ)$

Using, $\sin\theta = \cos(90^\circ - \theta)$

$= [\cos 80^\circ \times \cos 40^\circ \times \cos 20^\circ][\sin 10^\circ \times \sin(30^\circ - 10^\circ) \times \sin(30^\circ + 10^\circ)]$

We know that $\cos\theta \cos 2\theta \cos 2^2\theta \ldots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta}$

and $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B \Rightarrow \left(\frac{\sin 2^3(20^\circ)}{2^3 \sin 20^\circ}\right)\left[\sin 10^\circ \times \left(\frac{1}{4} - \sin^2 10^\circ\right)\right]$

$\Rightarrow \left(\frac{\sin 160^\circ}{2^3 \sin 20^\circ}\right)\left(\frac{\sin 10^\circ - 4\sin^3 10^\circ}{4}\right)$

$\Rightarrow \left(\frac{\sin(180^\circ - 20^\circ)}{8\sin 20^\circ}\right)\left(\frac{3\sin 10^\circ - 4\sin^3 10^\circ - 2\sin 10^\circ}{4}\right)$

$\Rightarrow \left(\frac{\sin 20^\circ}{8\sin 20^\circ}\right)\left(\frac{\sin 30^\circ - 2\sin 10^\circ}{4}\right)$

$\Rightarrow \frac{1}{8}\left(\frac{\frac{1}{2} - 2\sin 10^\circ}{4}\right)$

$\Rightarrow \frac{1}{8}\left(\frac{1 - 4\sin 10^\circ}{8}\right) = \alpha - \frac{\sin 10^\circ}{16}$ (given)

$\Rightarrow \frac{1 - 4\sin 10^\circ}{64} = \frac{16\alpha - \sin 10^\circ}{16}$

$\Rightarrow 1 - 4\sin 10^\circ = 64\alpha - 4\sin 10^\circ$

$\Rightarrow \alpha = \frac{1}{64}$.

10. (2)

$\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$

$= \sin 10^\circ \times \frac{1}{2}\left(\frac{1}{2} \times 2\sin 50^\circ \sin 70^\circ\right)$

$= \frac{1}{4}\sin 10^\circ(2\sin 50^\circ \sin 70^\circ)$

Using $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$= \frac{1}{4}\sin 10^\circ(\cos 20^\circ - \cos 120^\circ)$

$= \frac{1}{4}\sin 10^\circ(\cos 20^\circ - \cos(90^\circ + 30^\circ))$

$= \frac{1}{4}\sin 10^\circ(\cos 20^\circ + \sin 30^\circ)$

$= \frac{1}{4}\sin 10^\circ\left(\cos 20^\circ + \frac{1}{2}\right)$

$= \frac{1}{8}(2\sin 10^\circ \cos 20^\circ + \sin 10^\circ)$

Again, using $2\sin A \cos B = \sin(A+B) - \sin(A-B)$

$= \frac{1}{8}(\sin 30^\circ - \sin 10^\circ + \sin 10^\circ)$

$= \frac{1}{8}\left(\frac{1}{2}\right) = \frac{1}{16}$.

11. (1)

Sol. $P = 96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$

$2P \times \sin\frac{\pi}{33} = 96 \times 2\sin\frac{\pi}{33}\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$

$2P \times \sin\frac{\pi}{33} = 6 \times \sin\frac{32\pi}{33} = 6\sin\frac{\pi}{33}$

$P = 3$

12. (3)

Let $\theta + (m-1)\frac{\pi}{6} = x$ and $\theta + m\frac{\pi}{6} = y$

So, $y - x = \frac{\pi}{6}$

Now, $\sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right)\sec\left(\theta + \frac{m\pi}{6}\right)$

$= \sum_{m=1}^{9} \sec x \sec y = \sum_{m=1}^{9} \frac{1}{\cos x \cos y}$

$= 2\sum_{m=1}^{9} \frac{\sin(y-x)}{\cos x \cos y} = 2\sum_{m=1}^{9}(\tan y - \tan x)$

$= 2\sum_{m=1}^{9}\left(\tan\left(\theta + m\frac{\pi}{6}\right) - \tan\left(\theta + \left(m-1\right)\frac{\pi}{6}\right)\right)$

$= 2\left(\tan\left(\theta + \frac{\pi}{6}\right) - \tan\left(\theta + \frac{0\cdot\pi}{6}\right)\right) + 2\left(\tan\left(\theta + 2\frac{\pi}{6}\right) - \tan\left(\theta + \frac{\pi}{6}\right)\right)$

$+2\left(\tan\left(\theta + 3\frac{\pi}{6}\right) - \tan\left(\theta + 2\frac{\pi}{6}\right)\right) + \ldots + 2\left(\tan\left(\theta + 9\frac{\pi}{6}\right) - \tan\left(\theta + 8\frac{\pi}{6}\right)\right)$

$= 2\left(\tan\left(\theta + \frac{9\pi}{6}\right) - \tan\theta\right) = 2(-\cot\theta - \tan\theta)$

i.e. $2(-\cot\theta - \tan\theta) = -\frac{8}{\sqrt{3}}$ (Given)

$\therefore \tan\theta + \cot\theta = \frac{4}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$ or $\sqrt{3}$ (as $\theta \in \left(0, \frac{\pi}{2}\right)$)

So, $S = \left\{\frac{\pi}{6}, \frac{\pi}{3}\right\}$

Hence, $\sum_{\theta \in S}\theta = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$

13. (2)

We know that $\cot\theta=\frac{1+\cos 2\theta}{\sin 2\theta}$

Since, $\theta=\frac{\pi}{24}$

Therefore, $\cot\theta=\frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$

$\Rightarrow \cot\left(\frac{\pi}{24}\right)=\frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$

$=\frac{\left(2\sqrt{2}+\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)}\times\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$

$=\frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}$

$=\sqrt{6}+\sqrt{2}+\sqrt{3}+2.$

14. (4) $3\cos^4\theta-5\cos^2\theta-2\sin^6\theta+2=0$

$\Rightarrow\quad 3\cos^4\theta-3\cos^2\theta-2\cos^2\theta-2\sin^6\theta+2=0$

$\Rightarrow\quad 3\cos^4\theta-3\cos^2\theta+2\sin^2\theta-2\sin^6\theta=0$

$\Rightarrow\quad 3\cos^2\theta\left(\cos^2\theta-1\right)+2\sin^2\theta\left(\sin^4\theta-1\right)=0$

$\Rightarrow\quad -3\cos^2\theta\sin^2\theta+2\sin^2\theta\left(1+\sin^2\theta\right)\cos^2\theta-1$

$\Rightarrow\quad \sin^2\theta\cos^2\theta\left(2+2\sin^2\theta-3\right)=0$

$\Rightarrow\quad \sin^2\theta\cos^2\theta\left(2\sin^2\theta-1\right)=0$

(C1) $\sin^2\theta=0\rightarrow 3$ solution; $\theta=\{0,\pi,2\pi\}$

(C2) $\cos^2\theta=0\rightarrow 2$ solution ; $\theta=\left\{\frac{\pi}{2},\frac{3\pi}{2}\right\}$

(C3) $\sin^2\theta=\frac{1}{2}\rightarrow 4$ solution: $\theta=\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$

No. of solution $=9$

15. (4) Let $g^{\operatorname{tn}\Gamma^2 x}=P$

$\frac{g}{P}+P=10$

$P^2-10P+9=0$

$(P-9)(P-1)=0$

$P=1,9$

$9^{\tan^2 x}=1, 9^{\operatorname{trn}^2 x}=9$

$\tan^2 x=0, \tan^2 x=1$

$x=0,\pm\frac{\pi}{4}\quad\therefore x\in\left(-\frac{\pi}{2},\frac{P}{2}\right)$

$\beta=\tan^2(0)+\tan^2\left(+\frac{\pi}{12}\right)+\tan^2\left(-\frac{\pi}{12}\right)$

$=0+2(\tan 15^{\circ})^2$

$2(2-\sqrt{3})^2$

$2(7-4\sqrt{3})$

Than $\frac{1}{6}(14-8\sqrt{3}-14)^2=32$

16. (4)

Given,

$\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{4}\cos^2(2x)$

$\Rightarrow 2\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{2}\cos^2(2x)$

$\Rightarrow \cos(2x)+\cos\left(\frac{2\pi}{3}\right)=\frac{1}{2}\cos^2 2x$

$\Rightarrow \cos 2x+\left(-\frac{1}{2}\right)=\frac{1}{2}\cos^2 2x$

$\Rightarrow \cos^2 2x-2\cos 2x+1=0$

$\Rightarrow (\cos 2x-1)^2=0$ or $\cos 2x=1$

$\Rightarrow 2x=-6\pi,-4\pi,-2\pi,0,2\pi,4\pi,6\pi$

So, $x\in\{-3\pi,-2\pi,-\pi,0,\pi,2\pi,3\pi\}$

So total 7 solutions.

17. (1)

Given,

$2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}$

Now we know that maximum value of cos function is 1, so $2\cos\left(\frac{x^2+x}{6}\right)\leq 2$ and using $A.M \geq G.M$ we get $4^x+4^{-x}\geq 2$

So, $\mathrm{L.H.S.} \leq 2.$ & $\mathrm{R.H.s} \geq 2$

Hence $\mathrm{L.H.S.}=2$ & $\mathrm{R.H.S.}=2$

Now equating both to 2 we get,

$2\cos\left(\frac{x^2+x}{6}\right)=2$ and $4^x+4^{-x}=2$

Now solving $2\cos\left(\frac{x^2+x}{6}\right)=2$

$\Rightarrow \cos\left(\frac{x^2+x}{6}\right)=1$

$\Rightarrow \frac{x^2+x}{6}=0$

$\Rightarrow x^2+x=0$

$\Rightarrow x=0$ or -1 ....(1)

Now solving $4^x+4^{-x}=2$ we get $x=0$ .....(2)

Now from equation (1) & (2) we can see common solution is $x=0$

Hence, possible solution is only one.

18. (4)

Given,

$\sin x+\sin 2x+\sin 3x+\sin 4x=0$

$\Rightarrow (\sin x+\sin 4x)+(\sin 2x+\sin 3x)=0$

$\Rightarrow 2\sin\frac{5x}{2}\cos\frac{3x}{2}+2\sin\frac{5x}{2}\cos\frac{x}{2}=0$

$\Rightarrow 2\sin\frac{5x}{2}\left\{\cos\frac{3x}{2}+\cos\frac{x}{2}\right\}=0$

$\Rightarrow 2\sin\frac{5x}{2}\left\{2\cos x\cos\frac{x}{2}\right\}=0$

$2\sin\frac{5x}{2}=0 \Rightarrow \frac{5x}{2}=0,\pi,2\pi,3\pi,4\pi,5\pi$

$\Rightarrow x=0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5},2\pi$

and $\cos\frac{x}{2}=0 \Rightarrow \frac{x}{2}=\frac{\pi}{2} \Rightarrow x=\pi$

and $\cos x=0 \Rightarrow x=\frac{\pi}{2},\frac{3\pi}{2}$

So sum $=6\pi+\pi+2\pi=9\pi$

19. (3)

We know that $0\leq \sin^2 x\leq 1$ and $0\leq \cos^2 x\leq 1$

Also, we know for a number which lie between 0 to 1, higher the power of the number the less is its value.

$\Rightarrow \sin^7 x\leq \sin^2 x\leq 1 \quad \ldots(1)$

and $\cos^7 x\leq \cos^2 x\leq 1 \quad \ldots(2)$

Also, we know that $\sin^2 x+\cos^2 x=1$

This means the equality must hold for (1) and (2)

$\Rightarrow \sin^7 x=\sin^2 x$ and $\cos^7 x=\cos^2 x$

$\Rightarrow \sin^2 x\left(\sin^5 x-1\right)=0$ and $\cos^2 x\left(\cos^5 x-1\right)=0$

$\Rightarrow \sin x=0$ or $\sin x=1$ and $\cos x=0$ or $\cos x=1 \Rightarrow x=0,\ 2\pi,\ 4\pi,\ \frac{\pi}{2},\ \frac{5\pi}{2}.$

Thus, there are total 5 solutions.

20. (4)

$\cos x+\cos y-\cos(x+y)=\frac{3}{2}$

$2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)-\left[2\cos^2\left(\frac{x+y}{2}\right)-1\right]=\frac{3}{2}$

$2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)-2\cos^2\left(\frac{x+y}{2}\right)+1=\frac{3}{2}$

$2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)-2\cos^2\left(\frac{x+y}{2}\right)=\frac{1}{2}$

$4\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)-4\cos^2\left(\frac{x+y}{2}\right)=1$

$\Rightarrow 4\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)-4\cos^2\left(\frac{x+y}{2}\right)=1=\sin^2\left(\frac{x-y}{2}\right)+\cos^2\left(\frac{x-y}{2}\right)$

$\Rightarrow 4\cos^2\left(\frac{x+y}{2}\right)+\cos^2\left(\frac{x-y}{2}\right)-4\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)+\sin^2\left(\frac{x-y}{2}\right)=0$

$\Rightarrow \left(2\cos\left(\frac{x+y}{2}\right)-\cos\left(\frac{x-y}{2}\right)\right)^2+\sin^2\left(\frac{x-y}{2}\right)=0$

$\Rightarrow \sin\left(\frac{x-y}{2}\right)=0$ and $\cos\left(\frac{x+y}{2}\right)=\frac{1}{2}\cos\left(\frac{x-y}{2}\right)$

$\Rightarrow x=y$ and $\cos x=\frac{1}{2}=\cos y$

$\therefore\ \sin x=\frac{\sqrt{3}}{2}$

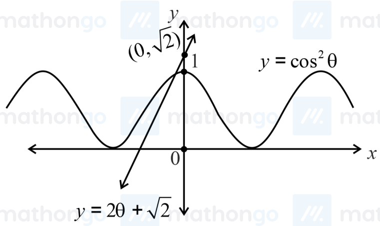
$\Rightarrow \sin x+\cos y=\frac{1+\sqrt{3}}{2}$

option (4)

21. (1)

$2\theta-\cos^2\theta+\sqrt{2}=0$

$\Rightarrow\ 2\theta+\sqrt{2}=\cos^2\theta$



As per the graph drawn there is only one point of intersection, so only one solution of the given equation.

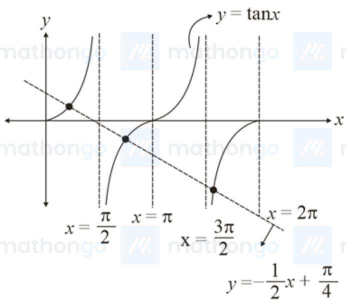
22. (1)

We have $x+2\tan x=\frac{\pi}{2}$

$\Rightarrow 2\tan x=\frac{\pi}{2}-x$

$\Rightarrow \tan x=-\frac{1}{2}x+\frac{\pi}{4}$

The graph of the two functions is given as



It is clear from the graph that the number of solutions of the given equation is 3.

23. (2)

Given:

$S=\{\theta\in[0,2\pi):\ \tan(\pi\cos\theta)+\tan(\pi\sin\theta)=0\}$

So,

$\tan(\pi\cos\theta)+\tan(\pi\sin\theta)=0$

$\Rightarrow \tan(\pi\cos\theta)=-\tan(\pi\sin\theta)$

$\Rightarrow \tan(\pi\cos\theta)=\tan(-\pi\sin\theta)$

$\Rightarrow \pi\cos\theta=n\pi-\pi\sin\theta;\ n\in Z$

$\Rightarrow \sin\theta+\cos\theta=n$

Now,

$-\sqrt{2}\le\sin\theta+\cos\theta\le\sqrt{2}$

$\Rightarrow -\sqrt{2}\le n\le\sqrt{2}$

But $n\in Z$, so $n=-1,\ 0,\ 1$

So,

$\theta\in\left\{0,\frac{\pi}{2},\frac{3\pi}{4},\frac{7\pi}{4},\frac{3\pi}{2},\pi\right\}$

So,

$\sum_{\theta\in S}\sin^2\left(\theta+\frac{\pi}{4}\right)=\frac{1}{2}+\frac{1}{2}+0+0+\frac{1}{2}+\frac{1}{2}=2$

24. (3) Given inequality is

$2^{\sqrt{\sin^2 x-2\sin x+5}}\le 4^{\sin^2 y}$

$\Rightarrow \sqrt{\sin^2 x-2\sin x+1+4}\le 2\sin^2 y$

$\Rightarrow \sqrt{(\sin x-1)^2+4}\le 2\sin^2 y$

$\because 2\sin^2 y\in[0,\ 2]\ \ \&\ \ \sqrt{(\sin x-1)^2+4}\in\left[2,\ 2\sqrt{2}\right]$

$\because$ Only equality holds

$\Rightarrow \sqrt{(\sin x-1)^2+4}=2\sin^2 y=2$

$\Rightarrow (\sin x-1)^2+4=4\sin^4 y=4$

$\Rightarrow \sin^2 y=1\ \ \Rightarrow\ \ |\sin y|=1\ \ \&\ \ (\sin x-1)^2=0\Rightarrow\sin x=1$

$\therefore |\sin y|=\sin x$

25. (3) $\sin 2\theta+\tan 2\theta>0$

$\Rightarrow \sin 2\theta+\frac{\sin 2\theta}{\cos 2\theta}>0$

$\Rightarrow \sin 2\theta\frac{(\cos 2\theta+1)}{\cos 2\theta}>0\Rightarrow\tan 2\theta\left(2\cos^2\theta\right)>0$

Note: $\cos 2\theta\ne 0$

$\Rightarrow 1-2\sin^2\theta\ne 0\Rightarrow\sin\theta\ne\pm\frac{1}{\sqrt{2}}$

Now, $\tan 2\theta(1+\cos 2\theta)>0$

$\Rightarrow \tan 2\theta>0$ (as $\cos 2\theta+1>0$)

$\Rightarrow 2\theta\in\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)\cup\left(2\pi,\frac{5\pi}{2}\right)\cup\left(3\pi,\frac{7\pi}{2}\right)$

$\Rightarrow \theta\in\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

As $\sin\theta\ne\pm\frac{1}{\sqrt{2}}$; which has been already considered