ANSWER KEYS

1. (3)	**2.** (1)	**3.** (2)	**4.** (2)	**5.** (1)	**6.** (3)	**7.** (2)	**8.** (2)
9. (1)	**10.** (4)						

1. **(3)**

Point $\left(\frac{3}{2}a,\ 1\right)$ lies between two different lines $x+y=a$ and $x+y=2a$

$\therefore\left(\frac{3}{2}a+1-a\right)\left(\frac{3}{2}a+1-2a\right)<0$

⇒a∈-∞,-2∪2,∞

$\Rightarrow|a|\in(2,\infty)$

Hence, the least integral value of $|a|$ is 3

2. **(1)**

We know that if $A(x_1,y_1)$ & $B(x_2,y_2)$ lies on the same side to line $L=0$, then $L_{A(x_1,y_1)}\ L_{B(x_2,y_2)}>0$.

Given, the points $A(1,2)$ & $B(\sin\theta,\cos\theta)$ lies on the same side of the line $x+y-1=0$.

$\therefore(1+2-1)(\sin\theta+\cos\theta-1)>0$

$\Rightarrow\sin\theta+\cos\theta>1$

$\Rightarrow\sin\theta\times\frac{1}{\sqrt{2}}+\cos\theta\times\frac{1}{\sqrt{2}}>1\times\frac{1}{\sqrt{2}}$

$\Rightarrow\sin\theta.\cos\frac{\pi}{4}+\cos\theta.\sin\frac{\pi}{4}>\frac{1}{\sqrt{2}}$

$\Rightarrow\sin\left(\theta+\frac{\pi}{4}\right)>\frac{1}{\sqrt{2}}$

$\Rightarrow\frac{\pi}{4}<\left(\theta+\frac{\pi}{4}\right)<\frac{3\pi}{4}$

$\Rightarrow 0<\theta<\frac{\pi}{2}$

$\Rightarrow\theta\in\left(0,\frac{\pi}{2}\right)$

3. **(2)**

Let $A\equiv(-2,0),\ B=\left(-1,\frac{1}{\sqrt{3}}\right),\ C\equiv(\cos\theta,\sin\theta)$

Equation to AB,

$y-0=\left(\frac{\frac{1}{\sqrt{3}}-0}{-1+2}\right)(x+2)$

$\Rightarrow\sqrt{3}y=x+2$, Point C lies on this line.

$\Rightarrow\sqrt{3}\sin\theta-\cos\theta=2$

$\Rightarrow\frac{\sqrt{3}}{2}\sin\theta-\frac{1}{2}\cos\theta=1$

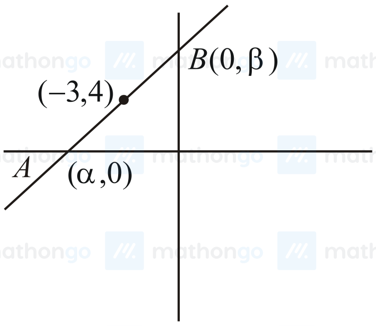
$\Rightarrow\sin\left(\theta-\frac{\pi}{6}\right)=1$, where $\theta\in[0,2\pi]$

$\theta-\frac{\pi}{6}=\frac{\pi}{2}$

$\Rightarrow\theta=\frac{\pi}{2}+\frac{\pi}{6}=\frac{2\pi}{3}$

So, only one value of θ is possible.

4. **(2)**



Let the line passing through $(-3,\ 4)$ intersect the coordinate axes at A & B.

By mid-point formula, co-ordinates of $P\left(\frac{\alpha+0}{2},\ \frac{0+\beta}{2}\right)$.

Comparing with given co-ordinate values, we get

$\frac{\alpha+0}{2}=-3\Rightarrow\alpha=-6$

and $\frac{\beta+0}{2}=4\Rightarrow\beta=8$

Hence, equation of line is $\frac{x}{-6}+\frac{y}{8}=1$

$\Rightarrow 4x-3y+24=0$

5. **(1)** Let slope of a line be m.

Now, the equation of a line passing through (1, 0) is

$y-0=m(x-1)$

$\Rightarrow mx-y-m=0$

Distance from origin $=\frac{\sqrt{3}}{2}$

$\Rightarrow\frac{|-m|}{\sqrt{1+m^2}}=\frac{\sqrt{3}}{2}$

$\Rightarrow 4m^2=3\left(1+m^2\right)$

$\Rightarrow m^2=3$

$\Rightarrow m=\pm\sqrt{3}$

$\therefore$ Equations of lines are

$\sqrt{3}x-y-\ \sqrt{3}=0$

and $-\sqrt{3}x-y+\ \sqrt{3}=0$

$\Rightarrow\sqrt{3}x-y-\sqrt{3}=0$

and $\sqrt{3}x+y-\ \sqrt{3}=0$

6. **(3)** Let the equation of the line parallel to $x-2y=1$ is $x-2y+\lambda=0$

Since, it passes through (3, 5)

$\therefore 3-10+\lambda=0\ \Rightarrow\lambda=7$

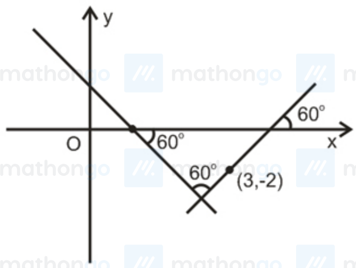
$\therefore$ The lines is $x-2y+7=0$.

The point of intersection of $x-2y+7=0$ and $2x+3y-14=0$ is (1, 4).

$\therefore$ The distance between (3, 5) and (1, 4)

$=\ \sqrt{(3-1)^2+(5-4)^2}=\ \sqrt{4+1}=\ \sqrt{5}$

7. (2)



$\sqrt{3}x + y = 1 \Rightarrow y = -\sqrt{3}x + 1$

Slope of line$=-\sqrt{3}$

Let slope of second line=m

For angle between two lines ,

$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$

$\tan 60^\circ = \pm\left(\frac{-\sqrt{3} - m}{1 - \sqrt{3}m}\right)$

$\sqrt{3} = \pm\left(\frac{-\sqrt{3} - m}{1 - \sqrt{3}m}\right)$

$\Rightarrow \sqrt{3}\left(1 - \sqrt{3}m\right) = -\sqrt{3} - m \text{ or } \sqrt{3}\left(1 - \sqrt{3}m\right) = \sqrt{3} + m$

$\Rightarrow \sqrt{3} - 3m = -\sqrt{3} - m \text{ or } \cancel{\sqrt{3}} - 3m = \cancel{\sqrt{3}} + m$

$\Rightarrow \cancel{2}\sqrt{3} = \cancel{2}m \text{ or } 4m = 0$

$\Rightarrow m = \sqrt{3} \text{ or}, m = 0$

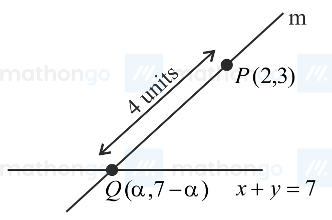
But line intersects X-axis, therefore $m \neq 0$

Slope of req. line $= \sqrt{3}$

Eq. is $(y + 2) = \sqrt{3}(x - 3)$

i.e. $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

8. **(2)**



Given, the point $P(2,\ 3)$ on the line $x+y=7$.
Let, the point of intersection of the two lines be Q
Then, Q can be taken as $Q\equiv(\alpha,\ 7-\alpha)$
Given, $PQ=4$ units and the distance between the points $(x_1,\ y_1)$ & $(x_2,\ y_2)$ is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$\Rightarrow\sqrt{(\alpha-2)^2+(7-\alpha-3)^2}=4$

$\Rightarrow\sqrt{(\alpha-2)^2+(4-\alpha)^2}=4$

$\Rightarrow(\alpha-2)^2+(4-\alpha)^2=4^2$

$\Rightarrow\alpha^2-4\alpha+4+16-8\alpha+\alpha^2=16$

$\Rightarrow 2\alpha^2-12\alpha+4=0$

$\Rightarrow\alpha^2-6\alpha+2=0$

$\Rightarrow\alpha=\frac{-(-6)\pm\sqrt{(-6)^2-4\times1\times2}}{2\times2}$

$\Rightarrow\alpha=\frac{6\pm\sqrt{36-8}}{4}$

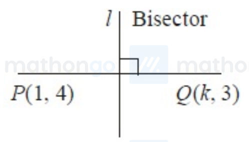
$\Rightarrow\alpha=\frac{6\pm2\sqrt{7}}{4}$

$\Rightarrow\alpha=3\pm\sqrt{7}$

The slope of a line joining the points $(x_1,\ y_1)$ & $(x_2,\ y_2)$ is $\frac{y_2-y_1}{x_2-x_1}$.
If we take $\alpha=3-\sqrt{7}$, then $Q\equiv\left(3-\sqrt{7},\ 4+\sqrt{7}\right)$
And, the slope of $PQ=m=\frac{4+\sqrt{7}-3}{3-\sqrt{7}-2}=\frac{1+\sqrt{7}}{1-\sqrt{7}}$
If we take $\alpha=3+\sqrt{7}$, then $Q\equiv\left(3+\sqrt{7},\ 4-\sqrt{7}\right)$
And, the slope of $PQ=m=\frac{4-\sqrt{7}-3}{3+\sqrt{7}-2}=\frac{1-\sqrt{7}}{1+\sqrt{7}}$.

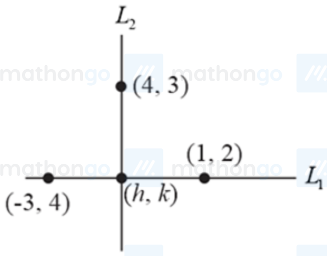
9. **(1)**

Since *l is perpendicular bisector of PQ*
Product of slopes$=-1$
The slope of line I
$=-\frac{1}{\text{Slope}(PQ)}$
$=-\frac{1}{\frac{3-4}{k-1}}=(k-1)$



The midpoint of $PQ=\left(\frac{k+1}{2},\frac{3+4}{2}\right)=\left(\frac{k+1}{2},\frac{7}{2}\right)$
The equation to the bisector l is $y-\frac{7}{2}=(k-1)\left(x-\frac{k+1}{2}\right)$
As $x=0, y=-4$ satisfies it, we have
$\left(-4-\frac{7}{2}\right)=(k-1)\left(0-\frac{k+1}{2}\right)$
$\left(\frac{-8-7}{2}\right)=(k-1)\left(-\frac{k+1}{2}\right)$
$\Rightarrow\left(-\frac{15}{2}\right)=-\left(\frac{k^2-1}{2}\right)$
$\Rightarrow k^2-1=15$
$\Rightarrow k^2=16$
$\Rightarrow k=\pm4$

10. (4)



Slope of line L_1 is $\frac{4-2}{-3-1} = \frac{2}{-4} = \frac{-1}{2}$

Hence slope of L_2 is 2.

From line L_1, $\frac{k-2}{h-1} = \frac{-1}{2}$

$\Rightarrow h + 2k = 5$ .....(1)

From line L_2, $\frac{k-3}{h-4} = 2$

$\Rightarrow k = 2h - 5$ .....(2)

From (1) and (2) $h = 3$ and $k = 1$

$\Rightarrow \frac{k}{h} = \frac{1}{3}$