

ANSWER KEYS

1. (1) 2. (4) 3. (1) 4. (4) 5. (2) 6. (3) 7. (4) 8. (1)
9. (1) 10. (1)

1. (1) $x^2 + 2ax + 10 - 3a > 0$

Above inequality will satisfy if

$$D < 0$$

$$\Rightarrow (2a)^2 - 4(10 - 3a) < 0$$

$$\Rightarrow 4a^2 - 4(10 - 3a) < 0$$

$$\Rightarrow a^2 - 10 + 3a < 0$$

$$\Rightarrow a^2 + 3a - 10 < 0$$

$$\Rightarrow a^2 + 5a - 2a - 10 < 0$$

$$\Rightarrow a(a + 5) - 2(a + 5) < 0$$

$$\Rightarrow (a - 2)(a + 5) < 0$$

$$\therefore a \in (-5, 2)$$

2. (4)

As $f(x) > 0$

$$\therefore a^2 - 1 > 0 \quad \text{and} \quad D < 0$$

$$a \in (-\infty, -1) \cup (1, \infty) \quad \dots (1)$$

$$\text{For } D < 0 \Rightarrow \{2(a - 1)\}^2 - 8(a^2 - 1) < 0$$

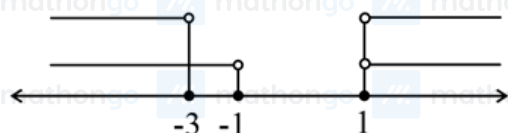
$$\Rightarrow a^2 + 1 - 2a - 2a^2 + 2 < 0$$

$$-a^2 - 2a + 3 < 0$$

$$a^2 + 2a - 3 > 0$$

$$(a - 1)(a + 3) > 0$$

$$a \in (-\infty, -3) \cup (1, \infty) \quad \& \quad a \in (-\infty, -1) \cup (1, \infty)$$



$$\therefore a \leq -3 \text{ or } a > 1$$

3. (1)

\therefore Quadratic expression is positive, hence $1 + 2m > 0$ and $D < 0$

$$\Rightarrow m > -\frac{1}{2} \text{ and }$$

$$4(1 + 3m)^2 - 16(1 + m)(1 + 2m) < 0$$

$$\Rightarrow 9m^2 + 1 + 6m - 4(2m^2 + 3m + 1) < 0$$

$$\Rightarrow m^2 - 6m - 3 < 0$$

$$\Rightarrow (m - 3 + 2\sqrt{3})(m - 3 - 2\sqrt{3}) < 0$$

$$\Rightarrow m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$$

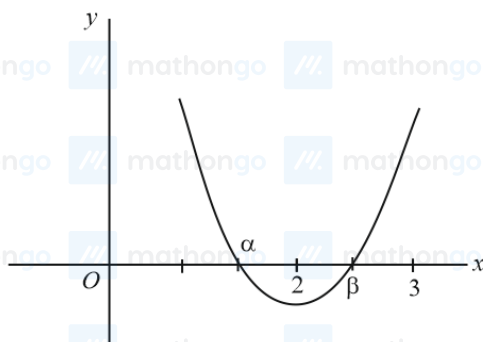
$$\therefore m = 0, 1, 2, 3, 4, 5, 6$$

\therefore Number of integral values = 7

4. (4)

Since, α and β are the roots of $4x^2 - 16x + \lambda = 0$, $\lambda \in R$ such that $1 < \alpha < 2$ and $2 < \beta < 3$.

Therefore, we have



Clearly,

$$f(1) > 0, f(2) < 0, f(3) > 0$$

$$f(1) = -12 + \lambda > 0 \Rightarrow \lambda > 12$$

$$f(2) = -16 + \lambda < 0 \Rightarrow \lambda < 16$$

$$f(3) = -12 + \lambda > 0$$

$$\text{And for } x \in R \Rightarrow D > 0$$

$$\Rightarrow (16)^2 - 16\lambda > 0$$

$$\Rightarrow 16(16 - \lambda) > 0$$

$$\therefore 12 < \lambda < 16$$

$$\therefore \lambda \in \{13, 14, 15\}.$$

Hence, three integral solutions.

5.



(2)

$$a - 1 < -1 \text{ and } a^2 + 2 > 3$$

$$a < 0 \text{ and } a^2 > 1$$

$$a < 0 \text{ and } a > 1 \text{ or } a < -1$$

$$\Rightarrow a < -1$$

6. (3) The given equation can be written as

$$x^2 - (m-1)x - (m+1)x + m^2 - 1 = 0$$

$$\Rightarrow x(x-m-1) - (m-1)(x-m-1) = 0$$

$$\Rightarrow (x-m-1)(x-m+1) = 0$$

$$\Rightarrow \text{either } x = m-1 \text{ or } x = m+1.$$

By given condition $-2 < m-1 < m+1 < 4$

$$\Rightarrow -1 < m < 3$$

7. (4) Let, $f(x) = 4x^2 - 20kx + (25k^2 + 15k - 66) = 0 \dots\dots\dots (i)$

Let the roots of $f(x) = 0$ be α, β

Since α, β are real.

$$\therefore \Delta \geq 0$$

$$\Rightarrow 400k^2 - 4.4(25k^2 + 15k - 66) \geq 0$$

$$\Rightarrow -15k + 66 \geq 0 \Rightarrow k \leq \frac{22}{5} \dots\dots\dots (ii)$$

We have $\alpha, \beta < 2$

$$\therefore \alpha + \beta < 4$$

$$\Rightarrow -\frac{(-20k)}{4} < 4 \Rightarrow k < \frac{4}{5} \dots\dots\dots (iii)$$

$$f(x) = 4(x-\alpha)(x-\beta)$$

$$\therefore f(2) = 4(2-\alpha)(2-\beta) = 4(+)(+) = +ve$$

$$\therefore f(2) = 16 - 40k + (25k^2 + 15k - 66) > 0$$

$$\Rightarrow 25k^2 - 25k - 50 > 0 \Rightarrow k^2 - k - 2 > 0$$

$$\Rightarrow (k+1)(k-2) > 0 \Rightarrow k < -1 \text{ or } k > 2 \dots\dots\dots (iv)$$

Combining (ii), (iii) & (iv), we get $k \in (-\infty, -1)$

8. (1) Both roots are less than 3

$$\Rightarrow D \geq 0, -\frac{B}{2A} < 3, f(3) > 0$$

$$D = 4a^2 - 4(a^2 + a - 3) = -4(a - 3) \geq 0$$

$$\Rightarrow a \leq 3 \dots (i)$$

$$-\frac{B}{2A} = \frac{2a}{2} = a < 3 \dots (ii)$$

$$f(3) = 9 - 6a + a^2 + a - 3 > 0$$

$$a^2 - 5a + 6 > 0$$

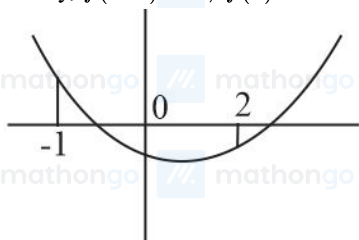
$$(a - 2)(a - 3) > 0 \Rightarrow a < 2 \text{ or } a > 3 \dots (iii)$$

Taking the intersection of (i), (ii), (iii), we get,

$$a < 2$$

9. (1)

Clearly, $f(-1) > 0, f(2) < 0$



$$\text{since, } f(0) = -4 < 0$$

$$\Rightarrow 1 - a - 4 > 0 \Rightarrow -a - 3 > 0 \Rightarrow -a > 3$$

$$\text{or } a < -3$$

$$\text{and } 4 + 2a - 4 < 0$$

$$\Rightarrow a < 0$$

$$\text{Hence, } a \in (-\infty, -3).$$

10. (1)

$ax^2 + 2bx - 3c = 0$ has no real roots

$$f(2) = 4a + 4b - 3c$$

$$= 4\left(a + b - \frac{3c}{4}\right) > 0 \text{ (given)}$$

$$\text{So, } f(x) > 0 \forall x$$

$$\Rightarrow f(0) > 0$$

$$\Rightarrow -3c > 0 \Rightarrow c < 0$$