ANSWER KEYS

1. (2)	**2.** (2)	**3.** (4)	**4.** (3)	**5.** (2)	**6.** (1)	**7.** (2)	**8.** (2)
9. (3)	**10.** (4)						

1. **(2)**

$$f(x)=x|x|=\begin{cases}-x^2, & x<0\\ x^2, & x\geq 0\end{cases}$$

$$f'(x)=\begin{cases}-2x, & x<0\\ 2x, & x\geq 0\end{cases}$$

$f'(0^-)=0=f'(0^+)$

$\Rightarrow f'(0)=0$

2. **(2)**

$f(x)=[x^3+1]$

By property of GIF, we know that $[x]$ is discontinuous at every integral point.

So we have to just check that for how many values of $x\in(1,2)$, (x^3+1) is taking integral values.

$1<x<2\Rightarrow\ 1<x^3<8\Rightarrow\ 2<x^3+1<9$

As $(x^3+1)\in(2,9)$, then integer lying in this range are

$[x^3+1]=3,4,5,6,7,8\longleftarrow$ 6 points are there at which $f(x)$ is becoming discontinuous.

3.

(4) $f(x)=\begin{cases}\frac{e^{[x]+|x|}-1}{[x]+|x|} & x\neq 0\\ -1 & x=0\end{cases}$

$\lim\limits_{x\to 0^-}f(x)=\lim\limits_{x\to 0^-}\frac{e^{[x]+|x|}-1}{[x]+|x|}=\frac{e^{-1}-1}{-1}=\frac{e-1}{e}$

$\lim\limits_{x\to 0^+}f(x)=\lim\limits_{x\to 0^+}\frac{e^{[x]+|x|}-1}{[x]+|x|}=\lim\limits_{x\to 0^+}\frac{e^x-1}{x}=1$

$\because$ LHL $\neq$ RHL at $x=0\Rightarrow f(x)$ is discontinuous at $x=0$

4. **(3)**

$$f(x)=\begin{bmatrix}1 & , & x<1\\ 1 & , & x=1\\ x^2 & , & x>1\end{bmatrix}$$

$\because f(1^+)=f(1^-)=f(1)=1$ conti. at $x=1$

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$$f'(x)=\begin{bmatrix}0 & , & x<1 & \\ 0 & , & x=1 & f'(1^-)=0\\ 2x & , & x>1 & f'(1^+)=2\times 1=2\end{bmatrix}$$

not diff. at $x=1$

5. **(2)** For f to be continuous everywhere, we must have $x_0^2 = f(x_0) = \lim_{x \to x_0^+} f(x) = ax_0 + b$

Also, f has a derivative at x_0 if $f'(x_0^+) = f'(x_0^-)$

Now, $f'(x_0^+) \lim_{h \to 0^+} \frac{f(x_0+h)-f(x_0)}{h}$

$= \lim_{h \to 0^+} \frac{a(x_0+h)+b-x_0^2}{h}$

$= \lim_{h \to 0^+} \left(\frac{ax_0+b-x_0^2}{h} + a\right) = \lim_{h \to 0^+} a = a$

$[\because x_0^2 = ax_0 + b]$

and $f'(x_{0-}) = \lim_{h \to 0-} \frac{f(x_0+h)-f(x_0)}{h}$

$= \lim_{h \to 0-} \frac{(x_0+h)^2-x_0^2}{h}$

$= \lim_{h \to 0-} \frac{x_0^2+h^2+2x_0h-x_0^2}{h}$

$= \lim_{h \to 0-} (h + 2x_0) = 2x_0$

That is, $a = 2x_0$, and hence $b = x_0^2 - ax_0 = x_0^2 - 2x_0^2 = -x_0^2$

6. **(1)** Since $f(1-0) = \lim_{x \to 1} 3^x = 3$

$f(1+0) = \lim_{x \to 1}(4-x) = 3$

and $f(1) = 3^1 = 3$

$\therefore\ f(1-0) = f(1+0) = f(1)$

$\Rightarrow$ f(x) is continuous at x = 1

Again $f'(1+0) = \lim_{x \to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x \to 1} \frac{3^x-3}{x-1} = \lim_{h \to 0} \frac{3^{1+h}-3}{h} = 3\lim_{h \to 0} \frac{3^h-1}{h} = 3\log 3$

and $f'(1+0) = \lim_{x \to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x \to 1} \frac{4-x-3}{x-1} = -1$

$\therefore\ f'(1+0) \neq f'(1-0)$

$\Rightarrow$ f(x) is not differentiable at x = 1

7. (2)

Given,

$f(x)=\begin{cases} e^x; x \leq 0 \\ 1-x, 0<x \leq 1 \\ x-1; x>1 \end{cases}$

Continuity at $x=0$

$LHL=\lim_{x \to 0^-} f(x)$

$=\lim_{h \to 0} f(0-h)$

$=\lim_{h \to 0} e^{0-h}=1.$

$RHL=\lim_{x \to 0^+} f(x)$

$=\lim_{h \to 0} f(0+h)$

$=\lim_{h \to 0} 1-(0+h)=1$

$f(0)=1$

Since, $(LHL)_{x=0}=(RHL)_{x=0}=f(0)$,therefore, $f(x)$ is continuous at $x=0$.

Now,

Continuity at $x=1$

$LHL=\lim_{x \to 0^-} f(x)$

$=\lim_{h \to 0} f(0-h)$

$=\lim_{h \to 0} 1-(0-h)=1.$

$RHL=\lim_{x \to 0^+} f(x)$

$=\lim_{h \to 0} f(0+h)$

$=\lim_{h \to 0}(0+h)-1=-1.$

Since, $(LHL)_{x=1} \neq (RHL)_{x=1}$, therefore, $f(x)$ is not continuous at $x=1$.

Differentiability at $x=0$

$Rf'(0)=\lim_{h \to 0} \frac{f(0+h)-f(0)}{h}$

$=\lim_{h \to 0} \frac{1-h-1}{h}=-1.$

$Lf'(0)=\lim_{h \to 0} \frac{f(0-h)-f(0)}{-h}$

$=\lim_{h \to 0} \frac{e^{-h}-1}{-h}=1$

$\because Rf'(0) \neq Lf'(0)$

So, $f(x)$ is not differentiable at $x=0$.

Differentiability at $x=1$

$Rf'(1)=\lim_{h \to 0} \frac{f(1+h)-f(1)}{h}$

$=\lim_{h \to 0} \frac{1-h-0}{h}=-1$

$L'(1)=\lim_{h \to 0} \frac{f(1-h)-f(1)}{-h}$

$=\lim_{h \to 0} \frac{(1-h)-0}{-h}=1$

$Rf'(1) \neq Lf'(1)$

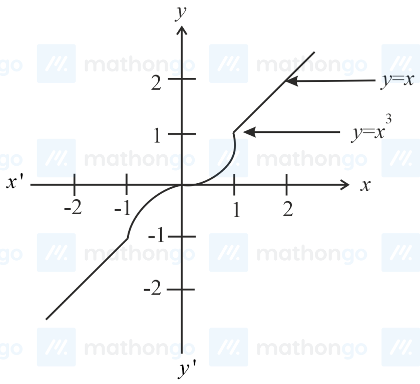
So, it is not differentiable at $x=1$.

8. **(2)**

Given, $f(x)=\begin{cases} x^3, & x^2<1 \\ x, & x^2 \geq 1 \end{cases}$

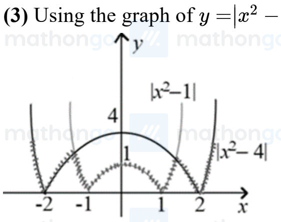
So, $f'(x)=\begin{cases} 3x^2, & x^2<1 \\ 1, & x^2>1 \end{cases}$

Drawing graph of $f(x)$



From graph, we can say that $f(x)$ is non-differentiable at $x=1, \ -1$ as there are corner points at those points.
So, $f(x)$ is differentiable at $x \in(-\infty, \infty)$-$\{1,-1\}$.

9. **(3)** Using the graph of $y=|x^2-4|, y=|x^2-1|$



Clearly, from the graph we can see $f(x)$ is non-differentiable at 6 points.

10. **(4)** Given, $f(x)=|x-1|+|x-2|+\cos x$

Since, $|x-1|, \ |x-2|$ and $\cos x$ are continuous in [0, 4]

$\therefore \ \ f(x)$ being sum of continuous functions is also continuous