ANSWER KEYS

1. (1)	**2.** (4)	**3.** (2)	**4.** (3)	**5.** (1)	**6.** (3)	**7.** (2)	**8.** (2)
9. (2)	**10.** (3)						

1. (1)

$\lim_{x \to 3} \frac{x^n - 3^n}{x-3} = 108$

Using standard limit $\lim_{x \to a} \left(\frac{x^n - a^n}{x-a}\right) = na^{n-1}$, we get

$n \times 3^{n-1} = 108$

$\Rightarrow \quad n \cdot 3^{n-1} = 4 \cdot 3^{4-1}$

Hence, $n = 4$.

2. (4)

$\lim_{x \to 5} \frac{xf(5) - 5f(x)}{x-5}$

Given limit is of the form $\frac{0}{0}$, so apply L' Hospital Rule.

$= \lim_{x \to 5} \frac{f(5) - 5f'(x)}{1-0}$

$= f(5) - 5f'(5)$

$= 7 - 5 \cdot 7 = 7 - 35 = -28.$

3. (2) $\lim_{x \to -\infty} \frac{2x-1}{\sqrt{x^2+2x+1}} = \lim_{y \to \infty} \frac{-2-\frac{1}{y}}{\sqrt{1-\frac{2}{y}+\frac{1}{y^2}}}$

[put $x = -y \;\therefore\; \quad x \to -\infty \; ie, \; y \to \infty$]

$= -\frac{2}{1} = -2$

4. (3)

We have to evaluate $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$

$= \lim_{x \to 0} \left[\frac{(e^{\sin x} - 1)}{\sin x} \times \frac{\sin x}{x}\right]$

$= \lim_{x \to 0} \frac{e^{\sin x} - 1}{\sin x} \times \lim_{x \to 0} \frac{\sin x}{x}$

$= 1 \times 1 = 1.$

5. (1)

Let, $I = \lim_{x \to 0} \frac{\log(1+3x^2)}{x(e^{5x}-1)}$

$\Rightarrow I = \lim_{x \to 0} \frac{\log(1+3x^2)}{3x^2} \times \frac{5x}{(e^{5x}-1)} \times \frac{3}{5}$

$\Rightarrow I = \frac{3}{5} \lim_{x \to 0} \frac{\log(1+3x^2)}{3x^2} \times \lim_{x \to 0} \frac{5x}{(e^{5x}-1)}$

$\because \lim_{x \to 0} \frac{\log(1+x)}{x} = 1$ and $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

$\therefore \; I = \frac{3}{5} \times 1 \times 1$

$\Rightarrow I = \frac{3}{5}$.

6. (3)

Let $l = \lim_{x \to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$

$\Rightarrow l = \lim_{x \to 0} \left(\frac{(1-cos 2x)(2x)^2}{(2x)^2}\right)\left(\frac{3+1}{x\left(\frac{\tan 4x}{4x}\right) \cdot 4x}\right)$

$\because \lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

and $\lim_{x \to 0} \frac{\tan x}{x} = 1$

$\Rightarrow l = 2$

7. **(2)**

$k=\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}$

$k=\lim_{x\to 0}\frac{\log\left(3\left(1+\frac{x}{3}\right)\right)-\log\left(3\left(1-\frac{x}{3}\right)\right)}{x}$

Using $\log(mn)=\log m+\log n$, we get

$k=\lim_{x\to 0}\frac{\log\left(1+\frac{x}{3}\right)+\log 3-\left[\log\left(1-\frac{x}{3}\right)+\log 3\right]}{x}$

$k=\lim_{x\to 0}\frac{\log\left(1+\frac{x}{3}\right)-\log\left(1-\frac{x}{3}\right)}{x}$

$k=\lim_{x\to 0}\left[\frac{\log\left(1+\frac{x}{3}\right)}{x}-\frac{\log\left(1-\frac{x}{3}\right)}{x}\right]$

$k=\lim_{x\to 0}\left[\frac{\log\left(1+\frac{x}{3}\right)}{3\left(\frac{x}{3}\right)}+\frac{\log\left(1-\frac{x}{3}\right)}{3\left(-\frac{x}{3}\right)}\right]$

Using, $\lim_{x\to 0}\frac{\log(1+x)}{x}=1$, we get

$k=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

8. **(2)**

Let,

$L=\lim_{x\to 2^{+}}\{x\}\frac{\sin(x-2)}{(x-2)^{2}}$

Put $x=2+h$

$=\lim_{h\to 0}\frac{\{2+h\}\cdot\sin(2+h-2)}{(2+h-2)^{2}},\ [\because\ \{2+h\}=h]$

$=\lim_{h\to 0}\frac{h\sin h}{h^{2}}$

We know that Standard limit:

$\lim_{x\to 0}\frac{\sin x}{x}=1$

$\therefore\ L=\lim_{h\to 0}\frac{\sin h}{h}=1$

9. **(2)** $\lim_{x\to-\infty}\frac{2x-1}{\sqrt{x^{2}+2x+1}}=\lim_{y\to\infty}\frac{-2-\frac{1}{y}}{\sqrt{1-\frac{2}{y}+\frac{1}{y^{2}}}}$

[put $x=-y\ \therefore\ \ x\to-\infty\ ie,\ y\to\infty$]

$=-\frac{2}{1}=-2$

10. **(3)**

$l=\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$

Put, $1-x=y$ as $x\to 1,\ y\to 0$

$\therefore\ \ l=\lim_{y\to 0}y\tan\frac{\pi(1-y)}{2}$

$\Rightarrow\ \ l=\lim_{y\to 0}y\tan\left(\frac{\pi}{2}-\frac{\pi y}{2}\right)$

$\Rightarrow\ \ l=\lim_{y\to 0}y\cot\left(\frac{\pi y}{2}\right)$

$\Rightarrow\ l=\lim_{y\to 0}\frac{2}{\pi}\frac{\left(\frac{\pi y}{2}\right)}{\tan\left(\frac{\pi y}{2}\right)}$

Using $\lim_{x\to 0}\frac{x}{\tan x}=1$, we get

$l=\frac{2}{\pi}$