ANSWER KEYS

1. (3)	**2.** (1)	**3.** (1)	**4.** (4)	**5.** (1)	**6.** (1)	**7.** (3)	**8.** (1)
9. (3)	**10.** (2)						

1. **(3)**

We know that, if

$I_n=\int \sin^n x\, dx$, then

$I_n=\int \sin^{(n-1)} x \sin x dx$

$I_n=-\cos x \sin^{(n-1)} x+(n-1) \int \sin^{(n-2)} x\left(1-\sin^2 x\right) dx$

$I_n=-\cos x \sin^{(n-1)} x+(n-1) I_{n-2}-(n-1) I_n$

$n I_n=-\cos x \sin^{(n-1)} x+(n-1) I_{n-2}$

$I_n=\frac{-(\cos x) \sin^{(n-1)} x}{n}+\frac{(n-1)}{n} I_{n-2}$

$\Rightarrow n I_n-(n-1) I_{n-2}=-(\cos x)\left(\sin^{(n-1)}(x)\right)$

2. **(1)** $I=\int \frac{b x \cos 4 x-a \sin 4 x}{x^2} d x$

By integration by parts

$=(b x \cos 4 x-a \sin 4 x) \times\left(-\frac{1}{x}\right)+\int \frac{b \cos 4 x-4 b x \sin 4 x-4 a \cos 4 x}{x} d x+k$

$=-\frac{b x \cos 4 x-a \sin 4 x}{x}+\int\left[\left(\frac{b-4 a}{x}\right) \cos 4 x-4 b \sin 4 x\right] d x+k$

$=-b \cos 4 x+\frac{a \sin 4 x}{x}+b \cos 4 x+\int\left(\frac{b-4 a}{x}\right) \cos 4 x d x+c$

Given,

$I=\frac{a \sin 4 x}{x}+c$

By comparing we can say, $b-4 a=0 \Rightarrow b=4 a$

Hence, we can write $a=1$ & $b=4$

3.

$$I=\int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right) dx=\int \underset{\text{I}}{\tan^{-1}\sqrt{x}} \cdot \underset{\text{II}}{1}\, dx$$

(1)

$=x \tan^{-1} \sqrt{x}-\int \frac{1}{1+x} \cdot \frac{1}{2 \sqrt{x}} x d x+C=x \tan^{-1} \sqrt{x}-\frac{1}{2} \int \frac{t \cdot 2 t d t}{1+t^2}+C$ (Put $x=t^2 \Rightarrow d x=2 t d t$) $=x \tan^{-1} \sqrt{x}-\int \frac{t^2}{1+t^2} d t+C$

$=x \tan^{-1} \sqrt{x}-t+\tan^{-1} t+C=x \tan^{-1} \sqrt{x}-\sqrt{x}+\tan^{-1} \sqrt{x}+C=(x+1) \tan^{-1} \sqrt{x}-\sqrt{x}+C \Rightarrow A(x)=x+1 \Rightarrow B(x)=-\sqrt{x}$

4. **(4)**

Let

$I=\int\left(e^{\tan^{-1} x} \frac{\left(1+x^2\right)}{\left(1+x^2\right)} d x+e^{\tan^{-1} x} \cdot \frac{x}{\left(1+x^2\right)}\right) d x$

$I=\int\left(e^{\tan^{-1} x}+e^{\tan^{-1} x} \frac{x}{1+x^2}\right) d x$

As we know,

$d\left(e^{\tan^{-1} x}\right)=e^{\tan^{-1} x}\left(d\left(\tan^{-1} x\right)\right)$

$\Rightarrow d\left(e^{\tan^{-1} x}\right)=e^{\tan^{-1} x}\left(\frac{1}{1+x^2}\right)$

So,

$I=\int\left(e^{\tan^{-1} x} d x+x\, d\left(e^{\tan^{-1} x}\right)\right)$

$\Rightarrow I=\int \frac{d}{d x}\left(x\, e^{\tan^{-1} x}\right) d x+c$

$I=x e^{\tan^{-1} x}+c$

5. (1)

Let, $I=\int e^{x}\left[\frac{2+\sin 2x}{1+\cos 2x}\right] dx$

We know that $\sin 2x=2\sin x\cos x$ & $\cos 2x=2\cos^{2}x-1$

Using the above formulas we can write

$I=\int e^{x}\left(\frac{2+2\sin x\cos x}{2\cos^{2}x}\right)dx$

$I=\int e^{x}\left(\frac{2(1+\sin x\cos x)}{2\cos^{2}x}\right)dx$

$I=\int e^{x}\left(\frac{1}{\cos^{2}x}+\frac{\sin x\cos x}{\cos^{2}x}\right)dx$

$I=\int e^{x}\left(\tan x+\sec^{2}x\right)dx$

We know that $\int e^{x}(f(x)+f'(x))dx=e^{x}\cdot f+C$

Using the above formula we can write

$I=e^{x}\tan x+C$

6. (1) Let $I=\int e^{x}\left(\frac{1+\sin x}{1+\cos x}\right)dx$

$=\int e^{x}\frac{\left(1+2\sin\frac{x}{2}\cos\frac{x}{2}\right)}{2\cos^{2}\frac{x}{2}}dx$

$=\int\frac{1}{2}e^{x}\sec^{2}\frac{x}{2}dx+\int e^{x}\tan\frac{x}{2}dx$

$=\frac{1}{2}\left[2e^{x}\tan\frac{x}{2}-\int 2e^{x}\tan\frac{x}{2}dx\right]+\int e^{x}\tan\frac{x}{2}dx$

$=e^{x}\tan\frac{x}{2}-\int e^{x}\tan\frac{x}{2}dx+\int e^{x}\tan\frac{x}{2}dx+c$

$=e^{x}\tan\frac{x}{2}+c$

7. (3)

Method1: by cross checking the options

Consider $f(x)=\frac{x}{(\log x)^{2}+1}$

$\therefore\ f'(x)=\frac{1+(\log x)^{2}-\frac{2x\log x}{x}}{\left(1+(\log x)^{2}\right)^{2}}$

$\therefore\ f'(x)=\frac{1+(\log x)^{2}-2\log x}{\left(1+\log^{2}x\right)^{2}}=\left(\frac{(\log x-1)}{(1+\log x)^{2}}\right)^{2}$

$\therefore\ \int\left(\frac{(\log x-1)^{2}}{1+(\log x)^{2}}\right)dx=\int f'(x)dx=f(x)+C$

$\therefore\ \int\left(\frac{\log x-1}{1+(\log x)^{2}}\right)^{2}dx=\frac{x}{1+(\log x)^{2}}+C$

Hence option 3 is the correct answer and we can check the other choices by the similar argument.

Alternate solution

$\int\left\{\frac{\log(x)-1}{1+(\log x)^{2}}\right\}^{2}dx$

Put $\log(x)=t\Rightarrow x=e^{t}\Rightarrow dx=e^{t}dt$

$=\int e^{t}\left\{\frac{(t-1)^{2}}{(t^{2}+1)^{2}}\right\}dt$

$\int e^{t}\left\{\frac{t^{2}+1-2t}{(t^{2}+1)^{2}}\right\}dt$

$=\int e^{t}\left\{\frac{1}{t^{2}+1}+\left(\frac{-2t}{(t^{2}+1)^{2}}\right)\right\}dt$

$=\frac{e^{t}}{t^{2}+1}+C\quad\left[\because\int e^{x}(f(x)+f'(x))dx=e^{x}f(x)+C\right]$

$=\frac{x}{(\log x)^{2}+1}$

8. (1)

Let

$I=\int\frac{x}{(x-2)(x-1)}dx$

Put

$\frac{x}{(x-2)(x-1)}=\frac{A}{x-2}+\frac{B}{x-1}$

$\Rightarrow x=A(x-1)+B(x-2)$

$A+B=1,\ -A-2B=0$

On solving, we get $A=2$ and $B=-1$

$\therefore I=\int\left[\frac{2}{x-2}-\frac{1}{x-1}\right]dx$

$\Rightarrow I=2\log|x-2|-\log|x-1|+P$

$\Rightarrow I=\log\frac{(x-2)^{2}}{|x-1|}+P$

9. **(3)**

$\int \frac{x^2+1}{\left(x^2+2\right)\left(x^2+3\right)} dx$

let $x^2=\alpha$

$\frac{(\alpha+1)}{(\alpha+2)(\alpha+3)}=\frac{A}{\alpha+2}+\frac{B}{\alpha+3}$

$(\alpha+1)=A(\alpha+3)+B(\alpha+2)$

Put $\alpha=-3, \ -2=-B \ B=2$

$\alpha=-2, \ -1=A \quad A=-1$

Hence $I=\int \frac{\left(x^2+1\right) dx}{\left(x^2+2\right)\left(x^2+3\right)}=\int \frac{-1}{x^2+2}+\frac{2}{x^2+3} dx$

$=-\frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}}+\frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}}$

10. **(2)** $\frac{x^2+1}{(2x-1)\left(x^2-1\right)}=\frac{A}{(2x-1)}+\frac{B}{x+1}+\frac{C}{x-1}$

$\Rightarrow x^2+1=A\left(x^2-1\right)+B(2x-1)(x-1)+C(x+1)(2x-1)$

For $x=-1, \ 2=2C \Rightarrow C=1$

For $x=\frac{1}{2}, \frac{5}{4}=-\frac{3}{4} A \Rightarrow A=-\frac{5}{3}$

$\therefore$ Given expression $=-\frac{5}{3} \frac{1}{(2x-1)}+\frac{1}{3} \frac{1}{x+1}+\frac{1}{x-1}$