ANSWER KEYS

1. (4)	**2.** (2)	**3.** (1)	**4.** (5151)	**5.** (1)	**6.** (2)	**7.** (2)	**8.** (1)
9. (6)	**10.** (4)						

1. (4)

Explanation: At first we have to arrange the given aletters in alphabetical order

E, H, M, O, R, T

5! words will begin with E

5! words will begin with H .

Then M series will begin.($Remember\ we\ should\ not\ count\ all\ the\ words\ in\ M\ series, as\ we\ would\ leave\ behind\ word\ MOTHER$)

4! words will begin with ME

4! words will begin with MH

$Next\ series\ beginning\ with\ MOE\ has\ 3!$

$Next\ series\ beginning\ with\ MOH\ has\ 3!$

$Next\ series\ beginning\ with\ MOR\ has\ 3!$

$\Rightarrow$ Total before MOT $= 120 + 120 + 4! + 4! + 3! + 3! + 3! = 306$

$Now\ let\ us\ consider\ the\ series\ beginning\ with\ MOT$

If we write all words of the series MOT , we get

MOTEHR,

MOTERH,

MOTHER.

$\Rightarrow$ The rank of MOTHER Is 309

2. (2)

Letters of the word PARKAR written in alphabetical order are AAKPRR.

Number of words starting with A$= \frac{5!}{2!} = 60$

Number of words starting with K$= \frac{5!}{2!2!} = 30$

Number of words starting with PAA$= \frac{3!}{2!} = 3$

Number of words starting with PAK$= \frac{3!}{2!} = 3$

Number of words starting with PARA$= 2! = 2$

Number of words starting with PARKAR$= 1$

$\therefore$ Rank of word PARKAR is 99.

3. (1) There are 12 letters in the word PERMUTATIONS. which have T two times. Now the vowels a, e, i,o,u are together Let it be considered in one block.

The letters of vowels can be arranged in 5! ways

Thus, there are 7 letters and 1 block of vowel with T two times

$\therefore$ Number of arrangements $= \frac{8!}{2!} \times 5!$

4. (5151)

Given red balls$(R) = 100$, blue balls$(B) = 100$,

white balls$(W) = 100$

selection of 100 balls out of 300 is sum of the red balls(R), blue balls(B), white balls(W).

$R + W + B = 100$

then total number of ways of selection of 100 balls from identical balls of blue, red , white is

${}^{n+r-1}C_{r-1}$

here $n = 100,\ r = 3$

then we get ${}^{100+3-1}C_{3-1} = {}^{102}C_2 = \frac{102!}{100!\cdot 2!} = 5151$

5. (1) There is only one way in which we can choose one, two or more objects from 10 identical objects.

Number of ways of selecting 10 objects

$= {}^{21}C_{10} + {}^{21}C_9 + {}^{21}C_8 + {}^{21}C_7 + {}^{21}C_6 + {}^{21}C_5 + {}^{21}C_4 + {}^{21}C_3 + {}^{21}C_2 + {}^{21}C_1 + {}^{21}C_0$

We know ${}^nC_r = {}^nC_{n-r}$ and

$$\Rightarrow 2^{21} = {}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + {}^{21}C_4 + \dots\dots\dots + {}^{21}C_{21} \qquad \left\{\begin{array}{l} \because {}^{21}C_{21} = {}^{21}C_0 \\ {}^{21}C_{20} = {}^{21}C_1 \\ {}^{21}C_{19} = {}^{21}C_2 \end{array}\right.$$

$= 2\left({}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \dots\dots\dots\dots\dots + {}^{21}C_{10}\right)$

$\Rightarrow {}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \dots\dots\dots + {}^{21}C_{10} = 2^{20}$

6. **(2)** $\because 720 = 2^4 \times 3^2 \times 5^1$

$\therefore$ Sum of all odd divisors $= (1)\left(1+3+3^2\right)\left(1+5^1\right)$

$= 13 \times 6 = 78$

7. **(2)**

On prime factorization we have, $N = 12600 = 2^3 3^2 5^2 7^1$.

Number of factors $=(3+1)(2+1)(2+1)(1+1)= 72$.

Removing power of 2 remaining factors which are odd $=(2+1)(2+1)(1+1)= 18$.

$\therefore$ Even factors $= 72 - 18 = 54$.

Hence, the answer is 54.

8. **(1)**

$4n + 2 = 2(2n+1)$

Means factor must contain one 2 and atleast one odd.

Required no. of divisor are = (no. of ways to select one 2) x (No. of ways to select atleast one odd

Since, $240 = 2^4 . 3.5$

$\therefore$ Total number of such divisors $= \underbrace{(1)}_{\text{to select one 2}} \times (1+1) \times (1+1) = 4$

2nd approach can be by directly finding such divisors

We see that 2, 6, 10 and 30 are of the form $4n + 2$

9. **(6)** Any natural number is either of the form 3k or 3k - 1 or 3k + 1.

Sum of two numbers will be divisible by 3 if and only if, either both are of the form 3k ($^4C_2$ ways).

OR

One is of the form 3k - 1 and other is of the form 3k + 1($^4C_1$ x $^4C_1$ ways).

Hence, total number of required ways

$= {}^4C_2 + {}^4C_1 \times {}^4C_1 = 6 + 16 = 22$

$\Rightarrow \quad k = 22$

$\therefore \; k - 16 = 6$

10. **(4)** Required number of ways

$=(10+1)(8+1)(6+1)-1$

$= 11 \times 9 \times 7 - 1$

$= 693 - 1$

$= 692$