ANSWER KEYS

1. (514)	**2.** (321)	**3.** (2)	**4.** (4)	**5.** (3)	**6.** (4)	**7.** (40)	**8.** (9)
9. (150)	**10.** (151)	**11.** (2223)	**12.** (2)	**13.** (3)	**14.** (3)	**15.** (3)	**16.** (1)
17. (4)	**18.** (50)	**19.** (495)	**20.** (3)	**21.** (2)	**22.** (27560)	**23.** (7)	**24.** (3)
25. (4)	**26.** (2)	**27.** (2)	**28.** (2)	**29.** (3)	**30.** (825)		

1. (514)

We know that total number of three-digit number will be 900,

Now let $n(A)$ be number of three-digit number which are divisible by 2, i.e,

$A = \{100, 102, \ldots, 998\}$

$\Rightarrow n(A) = 450$

$n(B)$ be number of three-digit number which are divisible by 3, i.e, $B = \{102, 105, \ldots, 999\}$

$n(B) = 300$

Numbers divisible by both 2 and 3 is

$\{102, 108, \ldots 996\}$ i.e, 150

Numbers divisible by both 2 and 7 are $\{112, 126, \ldots, 994\}$ i.e., 64 numbers.

Numbers divisible by both 3 and 7 is $\{105, 126, \ldots 987\}$ i.e, 43 numbers.

Numbers divisible by $2, 3$ & 7 is $\{126, 168, \ldots, 966\}$ i.e, 21 numbers.

Required number

$= 450 + 150 - 64 - 43 + 21 = 514$

Hence, 514 three-digit number are there which are divisible by 2 or 3 but not divisible by 7.

2. (321)

We have,

$S_1 : \ 2, 5, 8, 11, \ldots, 359$

Common difference is $d_1 = 3$

$S_2 : \ 3, 7, 11, 15 \ldots, 399$

Common difference is $d_2 = 4$

$S_3 : 2, 7, 12, 17 \ldots, 197$

Common difference is $d_3 = 5$

Now taking LCM of common difference we get,

$\operatorname{LCM}(d_1,\ d_2,\ d_3) = 60$

So, common terms are $47, 107, 167$,

Hence, the sum will be $47 + 107 + 167 = 321$.

3. (2)

Given the ratio of the sum of the first five terms to the sum of the first nine terms is 5 : 17,

So, $\frac{S_5}{S_9} = \frac{5}{17} \Rightarrow \frac{\frac{5}{2}(2a+4d)}{\frac{9}{2}(2a+8d)} = \frac{5}{17}$

$\Rightarrow 17(2a + 4d) = 9(2a + 8d)$

$\Rightarrow 34a + 68d = 18a + 72d$

$\Rightarrow d = 4a$

Now $a_{15} = a + 14d = 57a$

Also given $110 < a_{15} < 120$

$\Rightarrow 110 < 57a < 120$

$\Rightarrow a = 2 \ \therefore \ d = 8$

So, $S_{10} = \frac{10}{2}(2 \times 2 + 9 \times 8) = 380$

4. (4)

$\therefore$ We know that sum of equidistant term from end & beginning is same in any A. P.$\Rightarrow$ $a_1 + a_{16} = a_2 + a_{15} = a_3 + a_{14} = a_4 + a_{13} = a_5 + a_{12} = \ldots = a_7 + a_{10}$

$\because \ a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$\Rightarrow 3(a_1 + a_{16}) = 114 \Rightarrow a_1 + a_{16} = 38$

Then,

$a_1 + a_6 + a_{11} + a_{16}$

$= 2(a_1 + a_{16}) = 76$

5. **(3)** $\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$

On rationalising each term

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_n}-\sqrt{a_1}}{d}\right)$$

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{(n-1)d}{\left(\sqrt{a_n}+\sqrt{a_1}\right)d}\right)=1$$

6. **(4)**

$a_n=a_1+(n-1)d$

$300=1+(n-1)d$

$\Rightarrow d=\frac{299}{(n-1)}=\frac{13\times 23}{(n-1)}=$ integer

so $n-1=\pm 13,\pm 23,\pm 299,\pm 1$

$\Rightarrow \quad n=14,-12,24,-22,300,-298,2,0$

But $n\in[15,\ 50]\ \Rightarrow n=24\ \Rightarrow d=13$

Hence,

$S_{n-4}=S_{20}=\frac{20}{2}[2(1)+(20-1)(13)]$

$\Rightarrow S_{n-4}=2490$

And,

$a_{n-4}=a_{20}=a_1+19d$

$=1+19\times 13$

$=248$

7. **(40)**

Given, $a_1>0, a_2, a_3, a_4, a_5\rightarrow$G.P.

$3a_2+a_3=2a_4$

Using the formula $a_n=ar^{n-1}$ and assuming $a_1=a$ we get, $3ar+ar^2=2ar^3$

$\Rightarrow 3+r=2r^2$

$\Rightarrow 2r^2-r-3=0$

$\Rightarrow r=-1\ \&\ r=\frac{3}{2}$

Also, $a_2+a_4=2a_3+1$

$\Rightarrow ar+ar^3=2ar^2+1$

$\Rightarrow a\left(r+r^3-2r^2\right)=1$

$\Rightarrow a\left(\frac{3}{2}+\frac{27}{8}-\frac{18}{4}\right)=1$

$\Rightarrow a=\frac{8}{3}$

When $r=-1, a=-\frac{1}{4}$ (rejected, $a_1>0$)

So, at $r=\frac{2}{3},\ a=\frac{8}{3}$

Now $a_2+a_4+2a_5$

$=\frac{8}{3}\times\frac{3}{2}+\frac{8}{3}\times\frac{27}{8}+2\times\frac{8}{3}\times\frac{81}{16}$

$=4+9+27=40$

8. (9)

Given:

$c_k = a_k + b_k$ and $a_1 = b_1 = 4$

Also,

$a_2 = 4r_1$ and $a_3 = 4r_1^2$

$b_2 = 4r_2$ and $b_3 = 4r_2^2$

Now,

$c_2 = a_2 + b_2 = 5$

$\Rightarrow 4r_1 + 4r_2 = 5$

$\Rightarrow r_1 + r_2 = \frac{5}{4}$

And,

$c_3 = a_3 + b_3 = \frac{13}{4}$

$\Rightarrow r_1^2 + r_2^2 = \frac{13}{16}$

$\Rightarrow (r_1 + r_2)^2 - 2r_1r_2 = \frac{13}{16}$

$\Rightarrow \frac{25}{16} - 2r_1r_2 = \frac{13}{16}$

$\Rightarrow 2r_1r_2 = \frac{12}{16}$

$\Rightarrow r_1r_2 = \frac{3}{8}$

$\Rightarrow 8r_1\left(\frac{5}{4} - r_1\right)3$

$\Rightarrow 10r_1 - 8r_1^2 = 3$

$\Rightarrow 8r_1^2 - 10r_1 + 3 = 0$

$\Rightarrow r_1 = \frac{10 \pm \sqrt{100-96}}{16}$

$\Rightarrow r_1 = \frac{3}{4}, \frac{1}{2}$

$\Rightarrow r_2 = \frac{1}{2}, \frac{3}{4}$

Now,

$\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$

$= (c_1 + c_2 + c_3 + \ldots) + \left[12 \times \left\{4 \times \left(\frac{1}{2^5}\right)\right\} + 8\left\{4 \times \left(\frac{3}{4}\right)^3\right\}\right]$

$= (a_1 + a_2 + a_3 + \ldots) + (b_1 + b_2 + b_3 + \ldots) + \left[\left(\frac{3}{2}\right) + \left(\frac{27}{2}\right)\right]$

$= \frac{4}{1-r_1} + \frac{4}{1-r_2} - 15$

$= 24 - 15 = 9$

9. (150)

Given that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in AP and $x, \sqrt{2}y, z$ are in GP.

As given, $\frac{2}{y} = \frac{1}{x} + \frac{1}{y}$ ....(i)

Also, $2y^2 = xz$ ....(ii)

Also given that $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{\sqrt{2}}$ .....(iii)

From (i) and (iii) we get $\frac{3}{y} = \frac{3}{\sqrt{2}}$

$y = \sqrt{2}$ .....(iv)

Now from (ii) $xz = 4$ ....(v)

Now using (ii), (iv) and (v)

$\Rightarrow x + z = 4\sqrt{2}$

Hence $3\left(x + y + z\right)^2 = 3\left(\sqrt{2} + 4\sqrt{2}\right)^2$

$= 150$

Therefore, this is the required answer.

10. **(151)**

Given,

Two A.P's as

$3, 7, 11, 15, \ldots$

First term$(a_1) = 3$

Common difference$(d_1) = 4$

And,

$1, 6, 11, 16, \ldots,$

First term$(a_2) = 1$

Common difference $(d_2) = 5$

Now, common difference of the series of the common terms of the given A.P's is $d = \text{LCM}(d_1, d_2) = \text{LCM}(4, 5) = 20$

Now, series of common term is

$11, 31, 51, 71, \ldots$

So, 8^{th} common term appearing in the both the series is

$a_8 = 11 + (8-1)d$

$\Rightarrow a_8 = 11 + 7 \times 20 = 151$

11. **(2223)**

Given,

$3, 6, 9, \cdots$ upto 78 terms

$\Rightarrow t_{78} = 3 + 77 \times 3 = 234$

$5, 9, 13, \cdots$ upto 59 terms

$\Rightarrow t_{59} = 5 + 58 \times 4 = 237$

Common difference of common terms $= \text{LCM}\{3, 4\} = 12$

First common term is 9 and last common term is 225

So series will be $9, 21, 33, \cdots, 225 \ \Rightarrow n = 19$

$S = \frac{n}{2}[a + l] = \frac{19}{2}[9 + 225] = 2223$

12. **(2)**

$\frac{2+4+6+\ldots+2y}{3+6+9+\ldots+3y} = \frac{4}{\log_{10} x}$

$\frac{2(1+2+3+----y)}{3(1+2+3+----y)} = \frac{4}{\log_{10} x}$

$\log_{10} x = 6$

$x = 10^6$

Now

$y = \log_{10} x + \log_{10} x^{1/3} + \log_{10} x^{1/9} - - - \infty$

$= \left(1 + \frac{1}{3} + \frac{1}{9} - - - \infty\right)\log_{10} x$

$= \left[\frac{1}{1-\frac{1}{3}}\right]\log_{10} x = 9$

So, $(x, y) = (10^6, 9)$

13. **(3)**

Let the A.P. be $a, \ A_1, \ A_2 \ldots A_n, \ 100$

Here, common difference, $d = \frac{100-a}{n+1}$

Given $\frac{A_1}{A_n} = \frac{1}{7} \Rightarrow \frac{a+d}{100-d} = \frac{1}{7} \quad \ldots(i)$

Also $a + n = 33$

From options, when $n = 23$, $a = 10$ and $d = \frac{90}{24} = \frac{15}{4}$

from (i) $\frac{10+\frac{15}{4}}{100-\frac{15}{4}} = \frac{55}{385} = \frac{1}{7}$

14. **(3)**

Given two G.Ps. $2, 2^2, 2^3, \ldots$ and $4, 4^2, 4^3, \ldots$ of 60 and n terms respectively,

Also given the geometric mean of all the $60+n$ terms is $(2)^{\frac{225}{8}}$,

So, $\left(\left(2^1 2^2 \ldots . 2^{60}\right)\left(4^1 \cdot 4^2 \ldots \ldots . 4^n\right)\right)^{\frac{1}{60+n}}=2^{\frac{225}{8}}$

$\Rightarrow\left(2^{30 \times 61} 4^{\frac{n(n+1)}{2}}\right)^{\frac{1}{60+n}}=2^{\frac{225}{8}}$

$\Rightarrow 2^{1830+n^2+n}=2^{\frac{(225)(60+n)}{8}}$

On comparing both side we get,

$\Rightarrow 1830+n^2+n=\frac{225(60+n)}{8}$

$\Rightarrow 8 n^2-217 n+1140=0$

$\Rightarrow n=20, \frac{57}{8}$

Now $\sum_{k=1}^n k(n-k)=\sum_{k=1}^{20} n k-k^2=\frac{n^2(n+1)}{2}-\frac{n(n+1)(2 n+1)}{6}$

$=\frac{20^2 \times 21}{2}-\frac{20 \times 21 \times 41}{6}=1330$

15. **(3)** Mean $=200$

$$\Rightarrow \frac{\frac{100}{2}(2 \times 2+99 d)}{100}=200$$

$\Rightarrow 4+99 d=400$

$\Rightarrow d=4$

$y_i=i(x i-i)$

$=i(2+(i-1) 4-i)=3 i^2-2 i$

$$\text{Mean }=\frac{\sum y_i}{100}$$

$$=\frac{1}{100} \sum_{i=1}^{100} 3 i^2-2 i$$

$$=\frac{1}{100}\left\{\frac{3 \times 100 \times 101 \times 201}{6}-\frac{2 \times 100 \times 101}{2}\right\}$$

$$=101\left\{\frac{201}{2}-1\right\}=101 \times 99.5$$

$=10049 \cdot 50$

16. **(1)** $a_1 A_1, A_2, b$ are in A.P.

$$d=\frac{b-a}{3} ; A_1=a+\frac{b-a}{3}=\frac{2 a+b}{3}$$

$$A_2=\frac{a+2 b}{3}$$

$A_1+A_2=a+b$

$a_1 G_1 \cdot G_2 \cdot G_3 b$ are in $G \cdot P$.

$$r=\left(\frac{b}{a}\right)^{\frac{1}{4}}$$

$G_1=\left(a^3 b\right)^{\frac{1}{4}}$

$G_2=\left(a^2 b^2\right)^{\frac{1}{4}}$

$G_3=\left(a b^3\right)^{\frac{1}{4}}$

$G_1^4+G_2^4+G_3^4+G_1^2 G_3^2=$

$a^3 b+a^2 b^2+a b^3+\left(a^3 b\right)^{\frac{1}{2}} \cdot\left(a b^3\right)^{\frac{1}{2}}$

$=a^3 b+a^2 b^2+a b^3+a^2 \cdot b^2$

$=a b\left(a^2+2 a b+b^2\right)$

$=a b(a+b)^2$

$=G_1 \cdot G_3 \cdot\left(A_1+A_2\right)^2$

17. (4)

We know the n^{th} term of an A.P. is given by,

$a_n = a + (n-1)d$

Given, $a_7 = 3$

$\Rightarrow a + 6d = 3$

$\Rightarrow\ a = 3 - 6d$

And, $a_1 a_4 = a(a+3d)$

$=(3-6d)(3-3d)$

$= 18d^2 - 27d + 9$

Given product $(a_1 a_4)$ is minimum then,

Let $f(d) = 18d^2 - 27d + 9$

$f'(d) = 36d - 27$

Product to be minimum, $f'(d) = 0$

$\Rightarrow 36d - 27 = 0$

$\Rightarrow d = \frac{27}{36} = \frac{3}{4}$

So, $a = 3 - \frac{9}{2} = \frac{-3}{2}$

Given, $S_n = 0$

$S_n = \frac{n}{2}[2a + (n-1)d] = 0$

$-3 + (n-1)\frac{3}{4} = 0$

$\Rightarrow\ n = 5$

Now $n! - 4a_{n(n+2)} = 5! - 4a_{35}$

$= 120 - 4(a + 34d)$

$= 120 - 4\left(\frac{-3}{2} + 34 \times \frac{3}{4}\right)$

$= 120 + 6 - 102 = 24$

18. (50)

Given,

$f(x) = 0 \Rightarrow (x-p)^2 - q = 0$

So, roots are $p + \sqrt{q}, p - \sqrt{q}$

Now absolute difference between roots will be $2\sqrt{q}$.

Now given a_1, a_2, a_3, a_4 are in $A.P$ and its mean is p

Now let a_1, a_2, a_3, a_4 be $a_1 = p - 3d,\ a_2 = p - d,\ a_3 = p + d\ \&\ a_4 = p + 3d$

Now given $|f(a_i)| = 500$

So, $|f(a_4)| = 500$

$\Rightarrow \left|(a_4 - p)^2 - q\right| = 500$

$\Rightarrow (a_4 - p)^2 - q = 500$

$\Rightarrow 9d^2 - q = 500 \quad \ldots(1)$

And using $|f(a_i)| = 500\ \forall\ i = 1, 2, 3, 4$

We get $|f(a_4)|^2 = |f(a_3)|^2$

$\Rightarrow \left((a_4 - p)^2 - q\right)^2 = \left((a_3 - p)^2 - q\right)^2$

$\Rightarrow 9d^2 - q + d^2 - q = 0$

So, $2q = 10d^2 \Rightarrow q = 5d^2$

$\Rightarrow d^2 = \frac{q}{5}$

From equation (1) we get,

$9\left(\frac{q}{5}\right) - q = 500$

$\Rightarrow \frac{4q}{5} = 500$

$\Rightarrow q = \frac{500 \times 5}{4}$

Now absolute difference is $2\sqrt{q} = 2 \times \sqrt{\frac{500 \times 5}{4}} = 2 \times \frac{50}{2} = 50$

19. (495)

From the definition of A.P., we have

$a=A+6d \quad \ldots.(i)$

$b=A+1+8d \quad \ldots.(ii)$

$c=A+2+16d \quad \ldots.(iii)$

Now, we are given

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix}+70=0$$

$$\Rightarrow \begin{vmatrix} A+6d & 7 & 1 \\ 2A+2+16d & 17 & 1 \\ A+2+16d & 17 & 1 \end{vmatrix}+70=0$$

$$\Rightarrow \begin{vmatrix} A+6d & 7 & 1 \\ 2+4d & 3 & -1 \\ -A & 0 & 0 \end{vmatrix}+70=0$$

$\Rightarrow 10A+70=0$

$\Rightarrow A=-7$

So,

$a=A+6d$

$\Rightarrow 29=-7+6d$

$\Rightarrow d=6$

$\therefore c-a-b=1+2d-A=13+7=20$

And, $\frac{d}{12}=\frac{6}{12}=\frac{1}{2}$

So,

$S_{20}=\frac{20}{2}\left(40+19\times\frac{1}{2}\right)=5\times 99=495$

20. (3)

Given $a_{n+2}=\frac{2}{a_{n+1}}+a_n$

$\Rightarrow a_{n+2}a_{n+1}-a_n a_{n+1}=2$

Now, $T_r=a_{r+1}a_r$ is an A.P. with common difference 2

Now, $T_1=a_1a_2=2,\ T_2=a_2a_3=4,\ \ldots.,\ T_r=2r$

So $\left(\frac{a_1+\frac{1}{a_2}}{a_3}\right)\cdot\left(\frac{a_2+\frac{1}{a_3}}{a_4}\right)\cdots\left(\frac{a_{30}+\frac{1}{a_{31}}}{a_{32}}\right)=\prod_{i=1}^{30}\left(\frac{a_i+\frac{1}{a_{i+1}}}{a_{i+2}}\right)$

$=\prod_{i=1}^{30}\left(\frac{a_ia_{i+1}+1}{a_{i+1}a_{i+2}}\right)=\prod_{i=1}^{30}\frac{T_r+1}{T_{r+1}}$

$=\prod_{i=1}^{30}\left(\frac{2r+1}{2r+2}\right)=\frac{3\cdot5\cdot7\cdots61}{4\cdot6\cdot8\cdots62}=\frac{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdots61\cdot62}{2(4\cdot6\cdot8\cdots62)^2}$

$=\frac{62!}{2(4\cdot6\cdot8\cdots62)^2}=\frac{62!}{2^{61}\cdot(31!)^2}=\frac{62(61!)}{2^{61}\cdot31(31!)(30!)}$

$=2^{-60}\cdot{}^{61}C_{30}$

$=2^{-60}\cdot{}^{61}C_{31}$

Now on comparing with $2^{\alpha}\left({}^{61}C_{31}\right)$

We get $\alpha=-60$

21. (2)

Given,

$a_0 = 0, a_1 = 0$

And $a_{n+2} = 3a_{a+1} - 2a_{n+1} : n \geq 0$

$\Rightarrow a_{n+2} - a_{n+1} = 2(a_{n+1} - \text{a}_n) + 1$

Now for $n = 0$ $a_2 - a_1 = 2(a_1 - a_0) + 1 \ldots.(1)$

For $n = 1$ $a_3 - a_2 = 2(a_2 - a_1) + 1 \ldots\ldots..(2)$

For $n = 2$ $a_4 - a_3 = 2(a_3 - a_2) + 1 \ldots\ldots..(3)$

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For $n = n$ $a_{n+2} - a_{n+1} = 2(a_{n+1} - a_n) + 1 \ldots\ldots(n)$

Now adding all above equation upto n we get,

$(a_{n+2} - a_1) - 2(a_{n+1} - a_0) - (n+1) = 0$

$\Rightarrow a_{n+2} = 2a_{n+1} + (n+1)$

Now replacing $n \to n-2$ we get,

$\Rightarrow a_n - 2a_{n-1} = n - 1$

Now putting $n = 25$ we get,

$\Rightarrow a_{25} - 2a_{24} = 25 - 1 = 24$

Now putting $n = 23$ we get,

$\Rightarrow a_{23} - 2a_{22} = 23 - 1 = 22$

So, $(a_{25} - 2a_{24})(a_{23} - 2a_{22}) = (24)(22) = 528$

22. (27560)

Given $a_n - a_{n-1} = 2$

So the sequence is an A.P. with first term $a_1 = 1$ and common difference $d = 2$

i.e., $a_n = a_1 + (n-1)d = 2n - 1$

Also, $a_n = b_n - b_{n-1}$

We know $2n - 1 = n^2 - (n-1)^2$

So, $b_n = n^2$

Now, $\sum_{n=1}^{15} a_n \cdot b_n = \sum_{n=1}^{15} (2n^3 - n^2) = 2\sum\limits_{n=1}^{15} n^3 - \sum_{n=1}^{15} n^2$

$= 2\left(\frac{15 \cdot 16}{2}\right)^2 - \frac{15 \cdot 16 \cdot 31}{6}$

$= 27560$

23. (7)

We have,

$47 \sum_{n=1}^{\infty} \left(\frac{a_n}{2^{3n}}\right)$

$\Rightarrow \sum_{n=1}^{\infty} \left(\frac{a_n}{8^n}\right) = P(\text{let})$

And,

$a_{n+2} = 2a_{n+1} + a_n$

$\Rightarrow \frac{a_{n+2}}{8^n} = \frac{2a_{n+1}}{8^n} + \frac{a_n}{8^n}$

$\Rightarrow 64\left(\frac{a_{n+2}}{8^{n+2}}\right) = \frac{16a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n}$

$\Rightarrow 64 \sum_{n=1}^{\infty} \left(\frac{a_{n+2}}{8^{n+2}}\right) = 16 \sum_{n=1}^{\infty} \left(\frac{a_{n+1}}{8^{n+1}}\right) + \sum_{n=1}^{\infty} \left(\frac{a_n}{8^n}\right)$

$\Rightarrow 64\left(P - \frac{a_1}{8} - \frac{a_2}{8^2}\right) = 16\left(P - \frac{a_1}{8}\right) + P$

$\Rightarrow 64\left(P - \frac{1}{8} - \frac{1}{64}\right) = 16\left(P - \frac{1}{8}\right) + P$

$\Rightarrow 64P - 8 - 1 = 16P - 2 + P$

$\Rightarrow 47P = 7$

$\Rightarrow 47 \sum_{n=1}^{\infty} \left(\frac{a_n}{8^n}\right) = 7$

24. (3)

From weighted A. M. $-$G. M. inequality, we know

$\frac{5\left(\frac{x^2}{2}\right)+2\left(\frac{\alpha}{2x^5}\right)}{5+2} \geq \left(\left(\frac{x^2}{2}\right)^5 \cdot \left(\frac{\alpha}{2x^5}\right)^2\right)^{\frac{1}{7}}$

$\Rightarrow \frac{5x^2}{2} + \frac{\alpha}{x^5} \geq \frac{7}{2}(\alpha)^{\frac{2}{7}}$

Given that the least value of $\frac{5x^2}{2} + \frac{\alpha}{x^5}$ is 14

i.e. $\frac{7}{2}(\alpha)^{\frac{2}{7}} = 14 \Rightarrow (\alpha)^{\frac{1}{7}} = 2 \Rightarrow \alpha = 128$

25. (4)

Using $A.M \geq G.M$, we get

$\frac{x+x+x+y+y}{5} \geq \sqrt[5]{x^3 \cdot (y)^2}$

$\Rightarrow \frac{3x+2y}{5} \geq \sqrt[5]{x^3y^2}$

$\Rightarrow \frac{3x+2y}{5} \geq \sqrt[5]{(2^{15})} \quad (\because \ x^3y^2 = 2^{15})$

$\Rightarrow \ 3x + 2y \geq 5 \cdot (2)^{15/5}$

$\Rightarrow \ 3x + 2y \geqslant 5 \times 2^3$

$\Rightarrow \ 3x + 2y \geqslant 40$

$\therefore$ The minimum value of $3x + 2y$ is 40

26. (2)

We have,

$\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\ \alpha\cos\beta$

Applying AM $\geq$ GM, we get

$\left(\frac{\sin^4\alpha+4\cos^4\beta+1+1}{4}\right) \geq (4\sin^4\alpha \cdot \cos^4\beta \cdot 1 \cdot 1)^{\frac{1}{4}}$

$\Rightarrow \sin^4\alpha + 4\cos^4\beta + 2 \geq 8\sin\alpha\cos\beta$

And, AM $=$ GM $\Rightarrow$numbers are equal and positive

$\Rightarrow \sin^4\alpha = 4\cos^4\beta = 1$

$\therefore \sin^4\alpha = 1 \Rightarrow \alpha = \frac{\pi}{2}$ and

$\cos\beta = \frac{1}{\sqrt{2}} \Rightarrow \beta = \frac{\pi}{4}$

Hence,

$\cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= -2\sin\alpha\sin\beta$

$= -2 \times 1 \times \frac{1}{\sqrt{2}} = -\sqrt{2}$

27. (2)

$\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)} = \frac{3}{4}\sum_{n=1}^{21}\left(\frac{1}{4n-1} - \frac{1}{4n+3}\right)$

$= \frac{3}{4}\left[\left(\frac{1}{3} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{11}\right) + \ldots + \left(\frac{1}{83} - \frac{1}{87}\right)\right]$

$= \frac{3}{4}\left[\frac{1}{3} - \frac{1}{87}\right] = \frac{3}{4}\frac{84}{261} = \frac{7}{29}$

28. (2)

We have,

$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\ldots$

Here,

$t_n=\frac{n}{1+n^2+n^4}$

$=\frac{n}{(n^2+n+1)(n^2-n+1)}$

So,

$t_1=\frac{1}{2}\left(1-\frac{1}{3}\right)$

$t_2=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{7}\right)$

$\vdots \quad \vdots \quad \vdots$

$\vdots \quad \vdots \quad \vdots$

$\vdots \quad \vdots \quad \vdots$

$t_{10}=\frac{1}{2}\left(\frac{1}{91}-\frac{1}{111}\right)$

Now,

$S_{10}=t_1+t_2+t_3+\ldots..+t_{10}$

$\Rightarrow S_{10}=\frac{1}{2}\left(1-\frac{1}{111}\right)$

$\Rightarrow S_{10}=\frac{55}{111}$

29. (3)

Given series is $1+2\times 3+3\times 5+4\times 7+5\times 9+\ldots$

Thus, the general term is $T_r=r(2r-1)$

Hence, the sum of 11 terms of the series is $S_{11}=\sum_{r=1}^{11} r(2r-1)$

$\Rightarrow S_{11}=2\sum r^2-\sum r$

Using the formulae for $\sum\limits_{r=1}^{n} r^2=\frac{n(n+1)(2n+1)}{6}$ and $\sum\limits_{r=1}^{n} r=\frac{n(n+1)}{2}$, we get

$S_{11}=2\cdot\frac{11(11+1)(22+1)}{6}-\frac{11(11+1)}{2}$

$\Rightarrow S_{11}=44\times 23-66$

$\Rightarrow S_{11}=1012-66=946.$

30. (825)

Sol. $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+\ldots\ldots.\sqrt{120}]$

$\Rightarrow 1+1+1+2+2+2+2+2+3+3+\ldots\ldots+$
$3=7$ times
$+4+4+\ldots\ldots+4=9$ times $+\ldots\ldots 10+10+$
$\ldots\ldots+10=21$ times

$\Rightarrow \sum\limits_{r=1}^{10}(2r+1).r$

$\Rightarrow 2\sum\limits_{r=1}^{10} r^2+\sum\limits_{r=1}^{10} r$

$\Rightarrow 2\times\frac{10\times 11\times 21}{6}+\frac{10\times 11}{2}$

$\Rightarrow 770+55$

$\Rightarrow 825$