ANSWER KEYS

1. (2) **2.** (4) **3.** (2) **4.** (1) **5.** (1) **6.** (1) **7.** (4) **8.** (3)

9. (4) **10.** (4)

1. (2)

We have,

$|x-1| \geq |x-3|$

Squaring both sides, we get

$(x-1)^2 \geq (x-3)^2$

$\Rightarrow x^2 - 2x + 1 \geq x^2 - 6x + 9$

$\Rightarrow -2x + 1 \geq -6x + 9$

$\Rightarrow 4x \geq 8$

$\Rightarrow x \geq 2.$

2. (4)

We have,

$\left| \frac{x^2+6}{5x} \right| \geq 1$

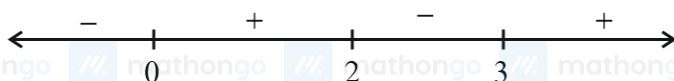Now, we know that if $|x| \geq a \Rightarrow x \leq -a$ or $x \geq a$.

Therefore, first we will solve for

$\frac{x^2+6}{5x} \geq 1$

$\Rightarrow \frac{x^2+6-5x}{5x} \geq 0$

$\Rightarrow \frac{(x-2)(x-3)}{5x} \geq 0$

From the wavy curve, we have
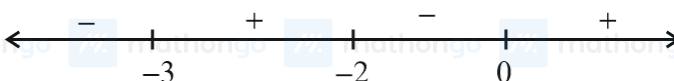

$\therefore x \in (0, 2] \cup [3, \infty) \ \dots(i)$

Now,

$\frac{x^2+6}{5x} \leq -1$

$\Rightarrow \frac{x^2+6+5x}{5x} \leq 0$

$\Rightarrow \frac{(x+2)(x+3)}{5x} \leq 0$

From the wavy curve, we have

$\therefore x \in (-\infty, -3] \cup [-2, 0) \ \dots(ii)$

Combining (i) and (ii), we get

$(-\infty, \ -3] \cup [-2, \ 0) \cup (0, \ 2] \cup [3, \infty).$

3. **(2)**

$2 - \log_2\left(x^2 + 3x\right) \geq 0$

$\Rightarrow 2 \geq \log_2\left(x^2 + 3x\right)$
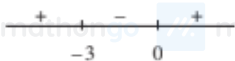
$\Rightarrow x^2 + 3x - 4 \leq 0$

$\Rightarrow (x + 4)(x - 1) \leq 0$

$x \in (-4, 1]$ ...(i)

Also $x^2 + 3x > 0$

$\Rightarrow \quad x(x + 3) > 0$

$\Rightarrow \quad x \in (-\infty, -3) \cup (0, \infty)$ ...(ii)

Combining equations (1) and (2)

$x \in (-4, -3) \cup (0, 1]$

Hence, (B) is correct.

4. **(1)** Here, it is given that

$\dfrac{\log_2\left(x^2 - 5x + 4\right)}{\log_2\left(x^2 + 1\right)} > 1$

$x^2 - 5x + 4 > 0 \Rightarrow (x - 4)(x - 1) > 0$

$\Rightarrow x \in (-\infty, 1) \cup (4, \infty) \qquad ...(1)$

$x^2 + 1 > 0$ which is true $\forall x \in R \qquad ...(2)$

$\log_2\left(x^2 - 5x + 4\right) > \log_2\left(x^2 + 1\right)$

$x^2 - 5x + 4 > x^2 + 1$

$-5x + 3 > 0$

$x < \dfrac{3}{5} \qquad ...(3)$

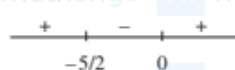From equations, $(1),(2)$ and (3) we get

$x \in \left(-\infty, \dfrac{3}{5}\right) - \{0\}$

5. **(1)** $\log_{0.2}\dfrac{x+2}{x} \leq 1$

$\Rightarrow \dfrac{x+2}{x} \geq 0.2$

$\dfrac{5(x+2)-x}{x} \geq 0$

$\Rightarrow \dfrac{4x+10}{x} \geq 0$

$x \in \left(-\infty, -\dfrac{5}{2}\right) \cup (0, \infty)$

Hence, (A) is correct.

6. **(1)** $x - \sqrt{1 - |x|} < 0$

Firstly $|x| \leq 1 \Rightarrow x \in [-1, 1]$
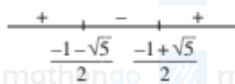
If $x \in [-1, 0]$, inequality will always hold true.

If $x \leq [0, 1]$

$x < \sqrt{1 - |x|}$

Squaring, we get

$x^2 \leq 1 - x$

$\Rightarrow x^2 + x - 1 < 0$

$$\underbrace{\quad +\quad}_{} \overset{\displaystyle -}{\underset{\frac{-1-\sqrt{5}}{2}}{\quad}} \quad \overset{\displaystyle +}{\underset{\frac{-1+\sqrt{5}}{2}}{\quad}}$$

$\Rightarrow \quad x \in \left(0, -1 + \frac{\sqrt{5}}{2} \right)$

So, $x \in \left(-1, -1 + \frac{\sqrt{5}}{2} \right)$

Hence, (A) is correct.

7. **(4)** Given $f(x) = [x]$ and $g(x) = |x|$

Now, $f\left(g\left(\frac{8}{5} \right) \right) = f\left(\frac{8}{5} \right) = \left[\frac{8}{5} \right] = 1$

And $g\left(f\left(-\frac{8}{5} \right) \right) = g\left(\left[-\frac{8}{5} \right] \right) = g(-2) = 2$

$\therefore \quad f\left(g\left(\frac{8}{5} \right) \right) - g\left(f\left(-\frac{8}{5} \right) \right) = 1 - 2 = -1$

8. **(3)**

We have,

$y = 3[x] + 1 = 4[x - 1] - 10$

$y = 3[x] + 1 = 4[x] - 14; \ (\because [X + a] = [X] + a, \ a \in Z)$

$\Rightarrow [x] = 15$ and $y = 46$

Then,

$[x + 2y] = [x] + 2y$

$= 15 + 92$

$= 107$

9. **(4)**

Given, $[x]^2 - 5[x] + 6 = 0$

$\Rightarrow [x]^2 - 3[x] - 2[x] + 6 = 0$

(Using middle term factorisation).

$\Rightarrow [x]([x] - 3) - 2([x] - 3) = 0$

$\Rightarrow ([x] - 3)([x] - 2) = 0$

$\Rightarrow [x] = 3$ or $[x] = 2$

$\Rightarrow x \in [3, 4)$ or $x \in [2, 3)$

Combining we get, $x \in [2, 4)$.

10. (4)

$y = \{x\} + \{-x\}$

We know that,

$\{x\} + \{-x\} = 0; x \in I$

& $\{x\} + \{-x\} = 1;$ otherwise

$\therefore y = 0 \,\forall\, x \in I$ and $y = 1$ otherwise

For $x \in [-1, \, 2]$

$y = 0$ at $x = -1, \, 0, \, 1, \, 2$

$y = 1 \,\forall\, x \in [-1, \, 2] - \{-1, \, 0, \, 1, \, 2\}$