ANSWER KEYS

1. (3)	**2.** (3)	**3.** (1)	**4.** (4)	**5.** (2)	**6.** (1)	**7.** (2)	**8.** (3)
9. (30)	**10.** (3)						

1. (3)

Given, $\vec{A} = 3\hat{i} + 4\hat{j},\ \vec{B} = 6\hat{i} + 8\hat{j}$ & $\vec{B} = 2\left(\vec{A}\right)$

$\therefore\ \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 6 & 8 & 0 \end{vmatrix} = 0$

$\therefore\ \left|\vec{A}\right| = A = \sqrt{3^2 + 4^2} = 5,\ \left|\vec{B}\right| = B = \sqrt{6^2 + 8^2} = 10$

$\Rightarrow \frac{A}{B} = \frac{5}{10} = \frac{1}{2}$

$\therefore\ \vec{A} . \vec{B} = 3 \cdot 6 + 4 \cdot 8 = 50$

2. (3)

A unit perpendicular to the plane $\vec{a}$ and $\vec{b} = \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|}$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$

$= \hat{i}(6+9) - \hat{j}(-2+12) + \hat{k}(6+24)$

$= 15\hat{i} - 10\hat{j} + 30\hat{k}$

and $\left|\vec{a} \times \vec{b}\right| = \sqrt{15^2 + (-10)^2 + (30)^2}$

$= \sqrt{1225} = 35$

$\therefore$ Required vector

$= \frac{15\hat{i} - 10\hat{j} + 30\hat{k}}{35} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$

3. (1)

Since $\vec{b} = 2\vec{a}$, so $3 - \lambda_2 = 2\lambda_1$

$\lambda_2 = 3 - 2\lambda_1 \quad \ldots(1)$

We know that if two vectors $a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ are perpendicular, then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Since, $\vec{a}$ is perpendicular to $\vec{c}$ so

$6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$

$\Rightarrow 6 + 6\lambda_1 + 3\lambda_3 - 3 = 0$

$\Rightarrow \lambda_3 = -1 - 2\lambda_1 \quad \ldots(2)$

From equations (1) and (2), we get

$(\lambda_1,\ \lambda_2,\ \lambda_3) = (\lambda_1,\ 3 - 2\lambda_1,\ -1 - 2\lambda_1)$ where $\lambda_1 \in R$

$\Rightarrow \left(-\frac{1}{2},\ 4,\ 0\right)$ satisfies the above triplet.

4. **(4)**
We have,
$\left|\vec{a}\right|=\left|\vec{b}\right|=\left|\vec{c}\right|=1$
And,
$\vec{a}+2\vec{b}+2\vec{c}=\vec{0}$
$\Rightarrow \vec{a}+2\vec{c}=-2\vec{b}$
$\Rightarrow\left(\vec{a}+2\vec{c}\right)\cdot\left(\vec{a}+2\vec{c}\right)=-\left(2\vec{b}\right)\cdot\left(-2\vec{b}\right)$
$\Rightarrow\left|\vec{a}\right|^2+4\left|\vec{c}\right|^2+4\vec{a}\cdot\vec{c}=4\left|\vec{b}\right|^2$
$\Rightarrow 1+4+4\vec{a}\cdot\vec{c}=4$
$\Rightarrow \vec{a}\cdot\vec{c}=\frac{-1}{4}$
$\Rightarrow\left|\vec{a}\right|\left|\vec{c}\right|\cos\theta=-\frac{1}{4}$
$\Rightarrow\cos\theta=-\frac{1}{4}$
Then,
$\left|\vec{a}\times\vec{c}\right|=\left|\vec{a}\right|\left|\vec{c}\right|\sin\theta$
$\Rightarrow\left|\vec{a}\times\vec{c}\right|=\sin\theta$
$\Rightarrow\left|\vec{a}\times\vec{c}\right|=\sqrt{1-\cos^2\theta}$
$\Rightarrow\left|\vec{a}\times\vec{c}\right|=\sqrt{1-\frac{1}{16}}=\frac{\sqrt{15}}{4}$

5. **(2)**
Given, $\left|\vec{a}\times\vec{b}\right|^2+\left|\vec{a}\cdot\vec{b}\right|^2=144$
$\Rightarrow\ \left(\left|\vec{a}\right|\left|\vec{b}\right|\sin\theta\right)^2+\left(\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta\right)^2=144$
$\Rightarrow\ \left|\vec{a}\right|^2\left|\vec{b}\right|^2\left(\sin^2\theta+\cos^2\theta\right)=144$
$\Rightarrow\ \left|\vec{a}\right|^2\left|\vec{b}\right|^2=144$
$\Rightarrow\ 16\left|\vec{b}\right|^2=144\Rightarrow\left|\vec{b}\right|^2=9$
$\Rightarrow\ \left|\vec{b}\right|=3$

6. **(1)** Adjacent sides of parallelogram are $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$. We know that vector area of parallelogram.

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&3\\-3&-2&1\end{vmatrix}=\hat{i}\,(2+6)-\hat{j}(1+9)+\hat{k}\,(-2+6)$$

$=8\hat{i}-10\hat{j}+4\hat{k}$.

Therefore area of parallelogram $=\left|\vec{a}\times\vec{b}\right|=\sqrt{(8)^2+(-10)^2+(4)^2}=\sqrt{64+100+16}$

$=\sqrt{180}$ sq. unit.

7. **(2)** Let $\vec{p}=2\vec{a}-\vec{b}$ and $\vec{q}=4\vec{a}-5\vec{b}$.

Then $\vec{p}\times\vec{q}=\left(2\vec{a}-\vec{b}\right)\times\left(4\vec{a}-5\vec{b}\right)=-6\left(\vec{a}\times\vec{b}\right)$

$=-6\left|\vec{a}\right|\left|\vec{b}\right|\sin\frac{\pi}{4}\,\hat{n}=-6\times\frac{1}{\sqrt{2}}\,\hat{n}=-3\sqrt{2}\,\hat{n}$.

Hence the area of the given parallelogram

$=\frac{1}{2}\left|\vec{p}\times\vec{q}\right|=\frac{3}{\sqrt{2}}$.

8. (3)

It is given that the angle between $\vec{a}$ and $\vec{c}$ is $\cos^{-1}\frac{1}{4}$

So,

$\vec{a}\cdot\vec{c} = |\vec{a}||\vec{c}|\cos\left(\cos^{-1}\frac{1}{4}\right)$

$\Rightarrow \vec{a}\cdot\vec{c} = \frac{1}{4}$ .....(i)

Taking dot product with $\vec{a}, \vec{b}, \vec{c}$ we have

$\vec{a}\cdot\vec{b} - 2\left(\vec{a}\cdot\vec{c}\right) = \lambda\left(\vec{a}\cdot\vec{a}\right)$

$\Rightarrow \vec{a}\cdot\vec{b} - \frac{1}{2} = \lambda$

$\Rightarrow \vec{a}\cdot\vec{b} = \lambda + \frac{1}{2}$ .....(ii)

Similarly, $\vec{b}\cdot\vec{c} = 8 - \frac{\lambda^2}{2} - \frac{\lambda}{4}$ .....(iii)

and $\vec{b}\cdot\vec{c} - 2 = \lambda\left(\vec{a}\cdot\vec{c}\right)$ .....(iv)

From equations (ii), (iii) and (iv), we get

$8 - \frac{\lambda^2}{2} - \frac{\lambda}{4} - 2 = \lambda\left(\frac{1}{4}\right)$

$\Rightarrow \lambda = 3, \ -4$

9. (30)

$\vec{b}\,.\,\vec{c} = 10 \Rightarrow |\vec{b}||\vec{c}|\cos\left(\frac{\pi}{3}\right) = 10 \Rightarrow 5.|\vec{c}|.\frac{1}{2} = 10$

$\Rightarrow |\vec{c}| = 4$

Also, $\vec{a}.\left(\vec{b}\times\vec{c}\right) = 0$

$\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right| = |\vec{a}||\vec{b}\times\vec{c}|\sin\left(\frac{\pi}{2}\right)$

$\Rightarrow \sqrt{3}\times|\vec{b}||\vec{c}|\sin\frac{\pi}{3}\times 1$

$\Rightarrow \sqrt{3}\times 5\times 4\times\frac{\sqrt{3}}{2} = 30$

10. (3)

$\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \vec{a}\times\left(\vec{b}\times\vec{a}\right)$

$\Rightarrow -\left(\vec{a}\,.\,\vec{b}\right)\vec{c} = \left(\vec{a}\,.\,\vec{a}\right)\vec{b} - \left(\vec{a}\,.\,\vec{b}\right)\vec{a}$

Using given information $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, we can write $\vec{a}\,.\,\vec{b} = 4$, $\vec{a}\,.\,\vec{a} = 6$

$\Rightarrow -4\vec{c} = 6\left(\hat{i} - \hat{j} + \hat{k}\right) - 4\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

$\Rightarrow -4\vec{c} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

$\Rightarrow \vec{c} = -\frac{1}{2}\left(\hat{i} + \hat{j} + \hat{k}\right)$

$\Rightarrow \vec{b}\,.\,\vec{c} = -\frac{1}{2}.$