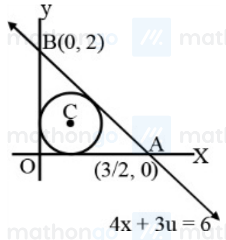
ANSWER KEYS

1. (1)	**2.** (3)	**3.** (1)	**4.** (1)	**5.** (3)	**6.** (4)	**7.** (4)	**8.** (1)
9. (1)	**10.** (1)						

1. (1)

Given, $4x+3y=6$, is a line that touches the circle.

Let the radius of the circle is r.



From the above figure, $C \equiv (r, r)$.

Since line $4x+3y-6=0$ touches the circle at point P.

$CP = r \Rightarrow \frac{|4r+3r-6|}{5}$

$\Rightarrow 7r - 6 = \pm 5r$

$\Rightarrow 12r = 6, \quad 2r = 6$

$\Rightarrow r = 1/2, \quad r = 3$

$\because r = 3$ not possible, as distance can never be a negative number.

$\Rightarrow r = 1/2$

Hence, the equation $(x-1/2)^2 + (y-1/2)^2 = 1/4$

2. (3)

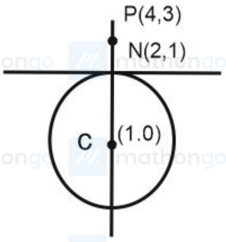
Equation of the diameter of the circle is given as

$2x - y - 2 = 0$ ...(i)

If $\mathrm{P}(4,3)$ and $\mathrm{N}(2,1)$ are the given points, then

slope of PN $= \frac{3-1}{4-2} = 1$

Equation of normal through PN is



$y - 1 = (x-2)$

$x - y - 1 = 0$ ...(ii)

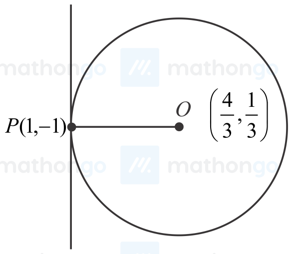
solving (i) and (ii), we get, the centre as $(1, 0)$

Hence, the equation of the circle is

$(x-1)^2 + y^2 = (2-1)^2 + 1$

$x^2 + y^2 - 2x - 1 = 0$

3. (1)



Point of intersection of lines

$x-y=1$ and $2x+y=3$

O is $\left(\frac{4}{3},\frac{1}{3}\right)$

Slope of $OP=\frac{\frac{1}{3}+1}{\frac{4}{3}-1}=\frac{\frac{4}{3}}{\frac{1}{3}}=4$

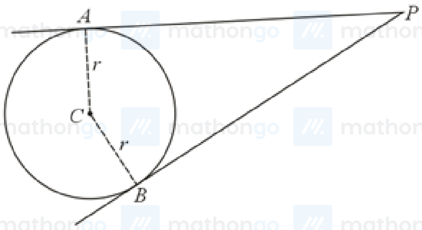
Slope of tangent $=-\frac{1}{4}$

$\Rightarrow$ Equation of tangent :$y+1=\ -\frac{1}{4}(x-1)$

$\Rightarrow 4y+4=-x+1$

$\Rightarrow x+4y+3=0$

4. (1)



The given circle is $x^2+y^2+4x+6y+8=0$

$\Rightarrow (x+2)^2+(y+3)^2=5$, with centre $C(-2,-3)$ and radius $r=\sqrt{5}$.

Now, any line through the point $P(-5,-4)$ is

$y+4=m(x+5)$

$\Rightarrow mx-y+(5m-4)=0 \quad \ldots(1)$

If the equation (1) is a tangent to the circle, then the perpendicular distance of the centre $C(-2,-3)$ is equal to radius $r=\sqrt{5}$.

$\Rightarrow \frac{|m(-2)-(-3)+(5m-4)|}{\sqrt{m^2+1}}=\sqrt{5}$

$\Rightarrow |-2m+3+5m-4|=\sqrt{5}\sqrt{m^2+1}$

$\Rightarrow (3m-1)^2=5\left(m^2+1\right)$

$\Rightarrow 9m^2+1-6m=5m^2+5$

$\Rightarrow 4m^2-6m-4=0$

$\Rightarrow 2m^2-3m-2=0$

$\Rightarrow m=-\frac{1}{2},\ 2.$

Hence, the required equations of tangents are

$2x-y+6=0$ and $x+2y+13=0.$

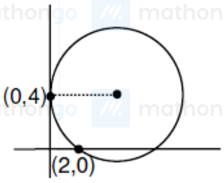
5. (3)

The line $5x-2y+6=0$ meets the y-axis at the point $(0,\ 3)$ and therefore the tangent has to pass through the point $(0,3)$ and required length

$=\sqrt{x_1^2+y_1^2+6x_1+6y_1-2}$

$=\sqrt{0^2+3^2+6(0)+6(3)-2}$

$=\sqrt{25}=5$

6. 

(4)

Equation of family of circle

$(x-0)^2+(y-4)^2+\lambda x=0 \Rightarrow passes(2,0)$

$4+16+2\lambda=0 \Rightarrow \lambda=-10$

$x^2+y^2-10x-8y+16=0$

Centre (5,4), $R=\sqrt{25+16-16}=5$

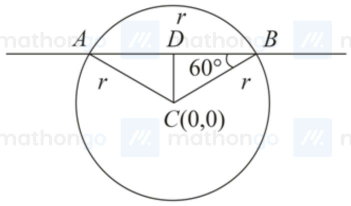
Check the options.

Option (4)

$\left|\frac{4\times5+3\times4-8}{5}\right|=\frac{24}{5}\neq 5$

7. **(4)**

$AB=r, AD=\frac{r}{2}$



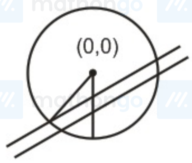
$\mathrm{CD}=\mathrm{r}\sin 60^\circ=\frac{\sqrt{3}\mathrm{r}}{2}$

$\Rightarrow \frac{|0+0-3|}{\sqrt{1^2+2^2}}=\frac{\sqrt{3}\mathrm{r}}{2} \Rightarrow \mathrm{r}=2\sqrt{\frac{3}{5}} \Rightarrow \mathrm{r}^2=\frac{12}{5}$

8. **(1)**

$\because$ We know that for $ax^2+bx+c=0$, the roots are real then $b^2-4ac>0$.

For the intersection of the line $x+y=n$ and circle $x^2+y^2=4$. We must have



$x^2+(n-x)^2=4x^2+(\mathrm{n}-\mathrm{x})^2=4$

$\Rightarrow 2x^2+2nx+n^2-4=0$

For $D>0$

$\Rightarrow\ 4n^2-8\left(n^2-4\right)>0$

$\Rightarrow n^2-8<0$

$-2\sqrt{2}<\mathrm{n}<2\sqrt{2}$

$\Rightarrow n=1$ or 2

Hence, lines are

$x+y=1$ or $x+y=2$

$\therefore$Length of chord

$l_1=2\left(\sqrt{4-\frac{1}{2}}\right)=\sqrt{14}$

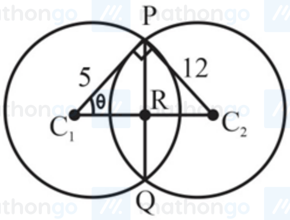
and $l_2=2\sqrt{4-2}=\sqrt{8}$

According to the given condition

$11k=l_1^2+l_2^2=22$

$\Rightarrow\quad k=2$

9.



(1)

$\because C_1P = 5$

$C_2P = 12 \,\&\, \angle C_1PC_2 = 90^o$

$\therefore C_1C_2 = 13$

$\therefore\ \Delta CC_2P$

$\tan\theta = \frac{12}{5} \Rightarrow \sin\theta = \frac{12}{13}$

C_1C_2 biscets the PQ at R & $C_1C_2 \perp PQ$

in ΔPC_1R, $\ \sin\theta = \frac{PR}{5}$

$\Rightarrow \frac{12}{13} = \frac{PR}{5} \Rightarrow PR = \frac{60}{13}$

& $PQ = 2PR = \frac{120}{13}$

10. (1)

Given,

$x^2 + y^2 = 32$

The centre is $(0,\ 0)$ and the radius is $\sqrt{32}$.

As per the given condition, the equation of the chord of the circle making intercept of length l should be $\pm\frac{x}{l} \pm \frac{y}{l} = 1$.

Now, for a line to be a chord of a circle the perpendicular distance from the centre of the circle to the line should be less than the radius of the circle.

$\left|\frac{l}{\sqrt{2}}\right| < \sqrt{32}$

$\Rightarrow |l| < 8$

But, it is given that l is the length of the intercept and so it will only have positive values.

Hence, we get $l < 8$.