ANSWER KEYS

1. (1)	**2.** (2)	**3.** (1)	**4.** (3)	**5.** (3)	**6.** (2)	**7.** (1)	**8.** (1)
9. (1)	**10.** (11.5)						

1. (1)

We have to evaluate, $\lim\limits_{x \to \infty}\left[1+\frac{4}{x-1}\right]^{x+3}$

This is of the form 1^{∞}, limit $= e^{a}$ where $a = \lim\limits_{x \to \infty}(x+3)\left(1+\frac{4}{x-1}-1\right)$

$\Rightarrow a = \lim\limits_{x \to \infty}(x+3)\left(\frac{4}{x-1}\right)$

$\Rightarrow a = \lim\limits_{x \to \infty}(4)\left(\frac{1+\frac{3}{x}}{1-\frac{1}{x}}\right)$

$\Rightarrow a = 4$

Hence, limit $= e^{4}$.

2. (2)

We have,

$\lim\limits_{x \to \infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x} = e^3$

$\Rightarrow e^{\lim\limits_{x \to \infty} 2x\left(1+\frac{a}{x}-\frac{4}{x^2}-1\right)} = e^3$

$\Rightarrow e^{\lim\limits_{x \to \infty} 2x\left(\frac{a}{x}-\frac{4}{x^2}\right)} = e^3$

$\Rightarrow e^{\lim\limits_{x \to \infty}\left(2a-\frac{8}{x}\right)} = e^3$

$\Rightarrow e^{2a} = e^3$

$\Rightarrow 2a = 3 \Rightarrow a = \frac{3}{2}$

3. (1)

Given limit,

$\lim\limits_{m \to \infty}\left(\cos\frac{x}{m}\right)^{m} \quad (1^{\infty})$

$= e^{\lim\limits_{m \to \infty}\left(\cos\left(\frac{x}{m}\right)-1\right)m}$

$= e^{\lim\limits_{m \to \infty}\left(\frac{-2\sin^2\frac{x}{2m}}{\frac{x^2}{4m^2}\times\frac{4m^2}{x^2}}\times m\right)}$

$= e^{\lim\limits_{m \to \infty}\left(\frac{-2\sin^2\frac{x}{2m}}{\frac{x^2}{4m^2}}\times m\times\frac{x^2}{4m^2}\right)}$

$= e^{0}$

$= 1$

4. (3) Let $y = \lim\limits_{x \to 0}|X|^{\sin x}$ 0^0 form

$\log y = \lim\limits_{x \to 0}\sin x \log|x|0 \ \times \alpha$

$\log y = \lim\limits_{x \to 0}\frac{\log|x|\alpha}{\operatorname{cosec} x}\ \frac{\alpha}{\alpha}$

Apply L' Hospital rule

$\operatorname{Log} y = \lim\limits_{x \to 0}\frac{\frac{1}{x}}{\operatorname{cosec} x \cot x}$

$\log y = \lim\limits_{x \to 0} -\left(\frac{\sin x}{x}\right)\ \tan x$

$\log y = 0$

$y = 1$

5. **(3)**

Given,

$f(x)=\begin{cases} x+\frac{1}{2}, & x<0 \\ 2x+\frac{3}{4}, & x\geq 0 \end{cases}$

L.H.L.

$=\lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(0-h)$

$=\lim_{h\to 0}\left(0-h+\frac{1}{2}\right)=\frac{1}{2}$

R.H.L.

$=\lim_{x\to 0^+} f(x)=\lim_{h\to 0} f(0+h)$

$=\lim_{h\to 0}\left(2(0+h)+\frac{3}{4}\right)=\frac{3}{4}$

$\because$ LH.L.$\neq$R.H.L.

$\therefore \lim_{x\to 0} f(x)$ does not exists.

$\therefore \left[\lim_{x\to 0} f(x)\right]$ does not exist.

6. **(2)**

We know that,

$\lim_{x\to 0}\frac{\sin x}{x}\to 1^-$

then,

$\lim_{x\to 0}\left(\left[\frac{100x}{\sin x}\right]\right)\Rightarrow \lim_{x\to 0}\left(\left[\frac{100}{\frac{\sin x}{x}}\right]\right)=100$

$\lim_{x\to 0}\left(\left[\frac{99\sin x}{x}\right]\right)\Rightarrow \lim_{x\to 0}\left(\left[99\frac{\sin x}{x}\right]\right)=98$

Hence $\Rightarrow \lim_{x\to 0}\left[\frac{100x}{\sin x}\right]+\left[\frac{99\sin x}{x}\right]=100+98=198$

$99\lambda=198$

Hence, $\lambda=2$

7. **(1)**

We know $[x-1]=[x]-1$

Since, $[x+n]=[x]+n$ for $n\in Z$

$[1-x]=1+[-x]$

$[x-1]+[1-x]=[x]+[-x]$

$[x]+[-x]=\begin{cases} 0 & x\in Z \\ -1 & x\notin Z \end{cases}$

Since here $x\to 1$, $\therefore x\notin Z$

$\therefore [x]+[-x]=-1$

Using the above property the given equation reduces to

$\lim_{x\to 1}(1-x+[x-1]+[1-x])$

$=\lim_{x\to 1}(1-x-1)$

$=\lim_{x\to 1}(-x)$

$=-1.$

8. **(1)** As f is a positive increasing function, we have

$f(x)<f(2x)<f(3x)$

Dividing by $f(x)$ leads to $1<\frac{f(2x)}{f(x)}<\frac{f(3x)}{f(x)}$

As $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$, we have by squeeze theorem

or sandwich theorem, $\lim_{x\to\infty}\frac{f(2x)}{f(x)}=1$

9. **(1)** $\operatorname{Lim}_{x\to\infty}\frac{(\log x)^3+x.3(\log x)^2\times\frac{1}{x}}{1+2x}$

(By D.L. Hospital rule)

$\Rightarrow \operatorname{Lim}_{x\to\infty}\frac{3(\log x)^2\times\frac{1}{x}+6(\log x)\times\frac{1}{x}}{2}$

(By D.L. Hospital rule)

$\Rightarrow \operatorname{Lim}_{x\to\infty}\frac{3(\log x)^2+6\log x}{2x}$

(By D.L. Hospital rule)

$\Rightarrow \operatorname{Lim}_{x\to\infty}\frac{6\log x\times\frac{1}{x}+\frac{6}{x}}{2}$

(By D.L. Hospital rule)

$\Rightarrow \operatorname{Lim}_{x\to\infty}\frac{6\log x+6}{2x}$

(By D.L. Hospital rule)

$\Rightarrow \operatorname{Lim}_{x\to\infty}\frac{6\left(\frac{1}{x}\right)+0}{2}$

(By D.L. Hospital rule)

$=\frac{\frac{6}{\infty}}{2}=0$

10. **(11.5)**

$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=L$, say.

$L=\lim_{x\to 0}\frac{x\left\{1+a\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\right)-b\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots\right)\right\}}{x^3}$

$=\lim_{x\to 0}\frac{(1+a-b)+x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+\ldots}{x^2}\quad\ldots$(i)

$L=1\ \Rightarrow$ To get finite limit, $1+a-b=0\quad\ldots$(ii)

$\lim_{x\to 0}\left[\frac{x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+\ldots}{x^2}\right]=1$

...[From (i) and (ii)]

$\Rightarrow \frac{b}{6}-\frac{a}{2}=1$

$\Rightarrow b-3a=6\quad\ldots$(iii)

Solving equations (ii) and (iii), we get

$a=-\frac{5}{2},\ b=-\frac{3}{2}$

$\Rightarrow 4a+b=-11.5$

$\Rightarrow |4a+b|=11.5$