ANSWER KEYS

1. (3)	**2.** (4)	**3.** (4)	**4.** (2)	**5.** (2)	**6.** (6)	**7.** (1)	**8.** (4)
9. (672)	**10.** (4)						

1. **(3)**

$$2A+4B=\begin{bmatrix}2 & 4 & 0\\ 12 & -6 & 6\\ -10 & 6 & 2\end{bmatrix}\ \ldots(1)$$

$$2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}\ \ldots(2)$$

Subtracting equation (2) from equation (1), we get

$$5B=\begin{bmatrix}0 & 5 & -5\\ 10 & -5 & 0\\ -10 & 5 & 0\end{bmatrix}$$

Hence, $B=\begin{bmatrix}0 & 1 & -1\\ 2 & -1 & 0\\ -2 & 1 & 0\end{bmatrix}$ & $A=\begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 3\\ -1 & 1 & 1\end{bmatrix}$

So $tr(A)-tr(B)=1-(-1)=2$

2. **(4)** $T_r\left(A^2\right)=T_r(A)^2$ cannot hold in general.

3. **(4)** $AB=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}a+2c & b+2d\\ 3a+4c & 2c+4d\end{bmatrix}$

$BA=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}a+3b & 2a+4b\\ c+3d & 2c+4d\end{bmatrix}$

if $AB=BA$, then $a+2c=a+3b$

$\Rightarrow 2c=3b \Rightarrow\ b\neq 0$

$b+2d=2a+4b$

$\Rightarrow 2a-2d=-3b$

$\frac{A-d}{3b-c}=\frac{-\frac{3}{2}b}{3b-\frac{3}{2}b}=\ -1$

4. **(2)** We have $A=iB$

$\Rightarrow A^2=\ (iB)^2=\ i^2B^2=\ -B^2$

$=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}\ =-2B$

$\Rightarrow A^4=\ (-2B)^2=\ 4B^2=\ 4(2B)=\ 8B$

$\Rightarrow A^8=\ \left(A^4\right)^2=\ (8B)^2=\ 64B^2=\ 128B$

5. **(2)** $f(A)=\ I+A+A^2+\ldots\ldots..+A^{16}$

$A=\begin{bmatrix}0 & 5\\ 0 & 0\end{bmatrix}\Rightarrow A^2=\begin{bmatrix}0 & 5\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 5\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$A^3=A^2.A=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

Similarly $A^4=A^5=\ldots\ldots\ldots\ldots\ =A^{16}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$f(A)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 5\\ 0 & 0\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}+\ldots+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$=\begin{bmatrix}1 & 5\\ 0 & 1\end{bmatrix}$

6. **(6)**

$P=\begin{bmatrix}2 & -1\\ 5 & -3\end{bmatrix}$

$5I-8P=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}-\begin{bmatrix}16 & -8\\ 40 & -24\end{bmatrix}=\begin{bmatrix}-11 & 8\\ -40 & 29\end{bmatrix}$

$P^2=\begin{bmatrix}-1 & 1\\ -5 & 4\end{bmatrix}$

$P^3=\begin{bmatrix}3 & -2\\ 10 & -7\end{bmatrix}\Rightarrow P^6=\begin{bmatrix}-11 & 8\\ -40 & 29\end{bmatrix}=P^n$

$\Rightarrow n=6$

7. **(1)**

Let A be a symmetric matrix, then A' = A

Now consider

$X=B'AB$

$X=B'\ AB$

$\Rightarrow X'=\ (B'\ A\ B)'$

$\Rightarrow X'=\ B'\ A'\ (B')'$

$\Rightarrow X'=\ B'\ AB=X$

Hence it is symmetric.

8. **(4)**

$A=\operatorname{diag}(d_1,d_2,\ldots,d_n)$

Given, $A^3=A$

$\Rightarrow \operatorname{diag}(d_1^3,d_2^3,\ldots,d_n^3)=\operatorname{diag}(d_1,d_2,\ldots,d_n)$

$\Rightarrow d_1^3=d_1, d_2^3=d_2,\ldots,d_n^3=d_n$

Hence, all $d_1,d_2,d_3,\ldots,d_n$ have three possible values ±1 and 0. Each diagonal element can be selected in three ways. Hence, the number of different matrices is 3^n.

9. **(672)** Let, $A=[a_{ii}]_{3\times3}$

$tr(AA^T)=3$

$a_{11}^2+a_{12}^2+a_{13}^2+a_{21}^2+\ldots\ldots+a_{33}^2=3$

Possible cases

$$\left.\begin{array}{rl}0,0,0,0,0,0,1,1,1 & \to 1\\ 0,0,0,0,0,0,-1,-1,-1 & \to 1\\ 0,0,0,0,0,0,1,1,-1 & \to 3\\ 0,0,0,0,0,0,-1,1,-1 & \to 3\end{array}\right\}{}^9C_6\times8=84\times8=672$$

10. **(4)**

Matrices are symmetric. So, we can only arrange entries of either upper right portion of matrices or lower left portion of matrices.

Assume we are arranging lower left portion and diagonal. So, two cases arise.

Case1: When non-diagonal elements have 2 zeros and 4 ones and diagonal elements have 2 zeros and 1 one

$\therefore$ Number of ways $=\left(\frac{3!}{2!}\times1\right)\times\frac{3!}{2!}=9$ (arrangement of non-diagonal followed by diagonal)

Case 2: When non-diagonal elements has 4 zeros and 2 ones and diagonal elements have 3 ones

$\therefore$ Number of ways $=\left(\frac{3!}{2!}\times1\right)\times\frac{3!}{3!}=3$ (arrangement of non-diagonal followed by diagonal)

Hence, number of matrices in $A=9+3=0$