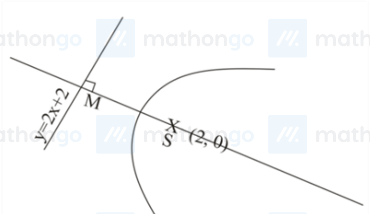
ANSWER KEYS

1. (1) **2.** (32) **3.** (16) **4.** (2) **5.** (8.00) **6.** (2) **7.** (3) **8.** (1)
9. (2) **10.** (4)

1.



(1)

Length of LR $= 2(\mathrm{SM})$

$= 2\frac{|2(2)+2-0|}{\sqrt{2^2+1}} = \frac{12}{\sqrt{5}}$

$\Rightarrow \frac{1}{\mathrm{K}} = \frac{12}{\sqrt{5}} \Rightarrow \mathrm{K} = \frac{\sqrt{5}}{12}$

2. **(32)**

$y^2 - kx + 8 = 0$

$\Rightarrow\ y^2 = kx - 8$

$\Rightarrow \boxed{y^2 = k\left(x - \frac{8}{k}\right)}$

$Y^2 = 4aX$

Directrix : $X = -a$

$\Rightarrow x - \frac{8}{k} = \frac{-k}{4}$

$\Rightarrow x = \frac{8}{k} - \frac{k}{4} \ldots(1)$

Given directrix $x = 1 \ldots(2)$

$\therefore\ \frac{8}{k} - \frac{k}{4} = 1$

$\Rightarrow k = -8,\ 4$

3. **(16)**

$x^2 - ky + 32 = 0$

$\Rightarrow x^2 = k\left(y - \frac{32}{k}\right)$

Put, $x = X,\ y - \frac{32}{k} = Y$

The equation of directrix is $Y + \frac{k}{4} = 0$

i.e., $y - \frac{32}{k} + \frac{k}{4} = 0$

But, $y - 2 = 0$ is the directrix.

$\Rightarrow \frac{32}{k} - \frac{k}{4} = 2$

$\Rightarrow k^2 + 8k - 128 = 0$

$\Rightarrow k = -16$ or $k = 8$

For $k = 8$, the parabola is $x^2 = 8(y-4)$ which does not intersect the circle.

For $k = -16$, the parabola is $x^2 = -16(y+2)$ which intersects the circle at two real distinct points.

$\Rightarrow$ Absolute value of $k = |-16| = 16$

4. **(2)**

Given Equation of parabola is $(x-1)^2 = 4\left(y - \frac{1}{2}\right)$

It is upward parabola with vertex $= \left(1, \frac{1}{2}\right)$

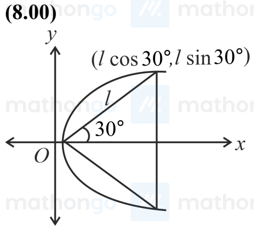
and for the origin, parabola $x^2 - 2x - 4y + 3 > 0$ as it lies outside the parabola.

So for $(2a, a)$, Parabola $x^2 - 2x - 4y + 3 < 0$ should be negative, as it lies inside the parabolic region.

i.e., $4a^2 - 8a + 3 < 0$

Thus, a belongs to the $\left(\frac{1}{2}, \frac{3}{2}\right)$.

5. **(8.00)**



Point at a 'l ' unit distance from origin will be

$\left(\frac{l\sqrt{3}}{2}, \frac{l}{2}\right)$

It lies on parabola, so

$\frac{l^2}{4} = 4a\frac{l\sqrt{3}}{2} \Rightarrow l = 8a\sqrt{3}$

6. **(2)**

Given, curve is

$(7x+5)^2 + (7y+3)^2 = \lambda^2(4x+3y-24)^2$,

$\Rightarrow \left\{7\left(x+\frac{5}{7}\right)\right\}^2 + \left\{7\left(y+\frac{3}{7}\right)\right\}^2 = \lambda^2\left\{\left(\sqrt{4^2+3^2}\right)^2 \times \left(\frac{4x+3y-24}{\sqrt{4^2+3^2}}\right)\right\}^2$,

$\Rightarrow 49\left[\left(x+\frac{5}{7}\right)^2 + \left(y+\frac{3}{7}\right)^2\right] = 25\lambda^2\left(\frac{4x+3y-24}{\sqrt{4^2+3^2}}\right)^2$,

$\Rightarrow \left(x+\frac{5}{7}\right)^2 + \left(y+\frac{3}{7}\right)^2 = \frac{25\lambda^2}{49}\left(\frac{4x+3y-24}{\sqrt{4^2+3^2}}\right)^2$,

$\Rightarrow \sqrt{\left(x+\frac{5}{7}\right)^2 + \left(y+\frac{3}{7}\right)^2} = \frac{5|\lambda|}{7} \times \left|\frac{4x+3y-24}{\sqrt{4^2+3^2}}\right|$.

The L. H. S. of the above equation represent distance of a point (x, y) from a point $\left(-\frac{5}{7}, -\frac{3}{7}\right)$ and R. H. S. represent its perpendicular distance from a line $4x+3y-24=0$.

Hence, the above equation satisfy the basic definition of a conic section, i.e., $(PS) = e(PM)$, where PS is the distance of a variable point from a fixed point (focus) and PM is its perpendicular distance from a fixed line (directrix) and e is the eccentricity.

And, we know that, for parabola $e = 1$.

$\therefore |\lambda| = \frac{7}{5}, \Rightarrow \lambda = \pm\frac{7}{5}$.

7. **(3)**

$y^2 - 8x = 2y - 17$

$y^2 - 2y = 8x - 17$

$\Rightarrow (y-1)^2 = 8x - 16$

$(y-1)^2 = 8(x-2)$

Length of Lactus rectum of parabola $= 4a = 8$

$a = 2$

From the property of parabola $2a$ is harmonic mean between SP and SQ

$\therefore \frac{1}{SP} + \frac{1}{SQ} = \frac{2}{2a}$

$\Rightarrow \frac{1}{6} + \frac{1}{SQ} = \frac{2}{4}$

$\therefore SQ = 3$

8. **(1)** Equation of parabola, $y^2 = 6x \Rightarrow y^2 = 4 \times \frac{3}{2}x$ $\therefore$ Focus $= \left(\frac{3}{2}, 0\right)$ Let equation of chord passing through focus be $ax + by + c = 0$ Since chord is passing through $\left(\frac{3}{2}, 0\right)$ $\therefore$ Put $x = \frac{3}{2}, y = 0$ in cqn (1), we get $\frac{3}{2}a + c = 0 \Rightarrow c = -\frac{3}{2}a \ldots (2)$ distance of chord from origin is $\frac{\sqrt{5}}{2}\ \frac{\sqrt{5}}{2} = \left|\frac{a(0)+b(0)+c}{\sqrt{a^2+b^2}}\right| = \frac{c}{\sqrt{a^2+b^2}}$ Squaring both sides $\frac{5}{4} = \frac{c^2}{a^2+b^2} \Rightarrow a^2 + b^2 = \frac{4}{5}c^2$ Putting value of c from (2), we get $a^2 + b^2 = \frac{4}{5} \times \frac{9}{4}a^2$ $b^2 = \frac{9}{5}a^2 - a^2 = \frac{4}{5}a^2$ $\frac{a^2}{b^2} = \frac{5}{4}, \frac{a}{b} = \pm\frac{\sqrt{5}}{2}$ Slope of chord, $\frac{dy}{dx} = -\frac{a}{b} = -\left(\frac{\pm\sqrt{5}}{2}\right) = \mp\frac{\sqrt{5}}{2}$

9. **(2)** Let, $P = (a, 4b) = (4t_1^2, 8t_1)$ & $Q = \left(c, -\frac{16}{b}\right) = (4t_2^2, 8t_2)$

$\Rightarrow t_1 = \frac{b}{2}$ and $t_2 = -\frac{2}{b}$

$\Rightarrow t_1 t_2 = -1$

$\Rightarrow$ Chord PQ is a focal chord that passes through the focus

$\Rightarrow (\alpha, \beta) = (4, 0)$

10. **(4)** Let AB be any chord joining the points $A(t_1^2, 2t_1)$ and $B(t_2^2, 2t_2)$ of the given parabola $y^2 = 4x$ and having slope 2.

$\Rightarrow \quad \frac{2}{t_1+t_2} = 2$

$\Rightarrow \quad t_1 + t_2 = 1$

Let us assume that the coordinates of the point P are $P(h, k)$

Since, P divides AB 1 : 2 in the ratio 1 : 2 internally

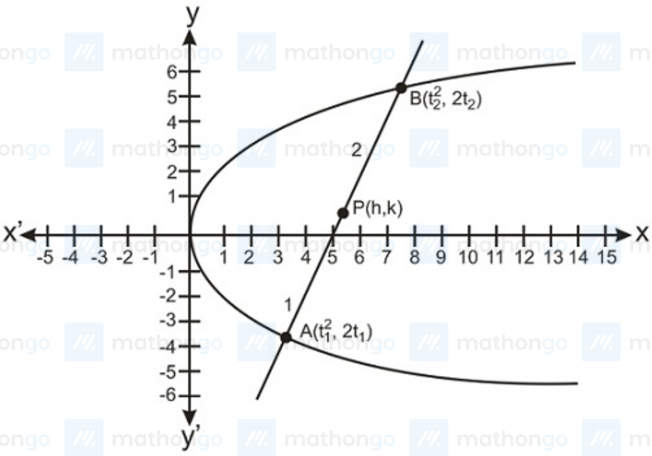
Hence, the coordinates of P are given by

$h = \frac{2t_1^2+t_2^2}{2+1}, k = \frac{4t_1+2t_2}{2+1}$

$\Rightarrow \quad 3h = 2t_1^2 + t_2^2, 3k = 4t_1 + 2t_2$

Using $t_1 + t_2 = 1$ and $3k = 4t_1 + 2t_2$, we get

$t_1 = \frac{3k-2}{2}$ and $t_2 = \frac{4-3k}{2}$



On substituting the values of t_1 and t_2 in $3h = 2t_1^2 + t_2^2$, we get

$3h = 2\left(\frac{3k-2}{2}\right)^2 + \left(\frac{4-3k}{2}\right)^2$

$\Rightarrow \quad 3h = 2\left(\frac{9k^2}{4} + 1 - 3k\right) + \left(4 + \frac{9k^2}{4} - 6k\right)$

$\Rightarrow \quad 3h = \frac{27k^2}{4} - 12k + 6$

$\Rightarrow \quad 12h = 27k^2 - 48k + 24$

$\Rightarrow \quad k^2 - \frac{16}{9}k - \frac{4}{9}h + \frac{8}{9} = 0$

Hence, the required locus is

$y^2 - \frac{16}{9}y - \frac{4}{9}x + \frac{8}{9} = 0$

$\Rightarrow \quad \left(y - \frac{8}{9}\right)^2 = \frac{4}{9}\left(x - \frac{2}{9}\right)$

The focus of the above locus is

$\left(x - \frac{2}{9} = \frac{1}{9}, y - \frac{8}{9} = 0\right) \equiv \left(x = \frac{1}{3}, y = \frac{8}{9}\right)$