

- The difference between degree and order of a differential equation that represents the family of curves given by  $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$ ,  $a > 0$  is \_\_\_\_\_.
- Let  $y = y_1(x)$  and  $y = y_2(x)$  be the solution curves the differential equation  $\frac{dy}{dx} = y + 7$  with initial conditions  $y_1(0) = 0$  and  $y_2(0) = 1$  respectively. Then the curves  $y = y_1(x)$  and  $y = y_2(x)$  intersect at
  - no point
  - two points
  - one point
  - infinite number of points
- If  $\frac{dy}{dx} + \frac{2^x - y(2^y - 1)}{2^x - 1} = 0$ ,  $x, y > 0$ ,  $y(1) = 1$ , then  $y(2)$  is equal to
  - $2 + \log_2 3$
  - $2 + \log_2 2$
  - $2 - \log_2 3$
  - $2 - \log_2 2$
- If the solution curve of the differential equation  $\frac{dy}{dx} = \frac{x+y-2}{x-y}$  passes through the point  $(2, 1)$  and  $(k+1, 2)$ ,  $k > 0$ , then
  - $2 \tan^{-1}\left(\frac{1}{k}\right) = \log_e(k^2 + 1)$
  - $\tan^{-1}\left(\frac{1}{k}\right) = \log_e(k^2 + 1)$
  - $2 \tan^{-1}\left(\frac{1}{k+1}\right) = \log_e(k^2 + 2k + 2)$
  - $2 \tan^{-1}\left(\frac{1}{k}\right) = \log_e\left(\frac{k^2+1}{k^2}\right)$
- If  $y = y(x)$  is the solution of the differential equation  $(1 + e^{2x})\frac{dy}{dx} + 2(1 + y^2)e^x = 0$  and  $y(0) = 0$ , then  $6\left(y'(0) + \left(y(\log_e \sqrt{3})\right)^2\right)$  is equal to:
  - 2
  - 2
  - 4
  - 1
- Let a curve  $y = y(x)$  be given by the solution of the differential equation  $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \left(\sqrt{e^{2x}-1}\right)dy$ . If it intersects  $y$ -axis at  $y = -1$ , and the intersection point of the curve with  $x$ -axis is  $(\alpha, 0)$ , then  $e^\alpha$  is equal to
- Let  $y = y(x)$  be the solution of the differential equation  $x \tan\left(\frac{y}{x}\right)dy = \left(y \tan\left(\frac{y}{x}\right) - x\right)dx$ ,  $-1 \leq x \leq 1$ ,  $y\left(\frac{1}{2}\right) = \frac{\pi}{6}$ . Then the area of the region bounded by the curves  $x = 0$ ,  $x = \frac{1}{\sqrt{2}}$  and  $y = y(x)$  in the upper half plane is:
  - $\frac{1}{8}(\pi - 1)$
  - $\frac{1}{12}(\pi - 3)$
  - $\frac{1}{4}(\pi - 2)$
  - $\frac{1}{6}(\pi - 1)$
- The solution of the differential equation  $\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$  is  
(where  $C$  is a constant of integration)
  - $x - \frac{1}{2}(\log_e(y+3x))^2 = C$
  - $x - \log_e(y+3x) = C$
  - $y + 3x - \frac{1}{2}(\log_e x)^2 = C$
  - $x - 2 \log_e(y+3x) = C$
- The slope of tangent at any point  $(x, y)$  on a curve  $y = y(x)$  is  $\frac{x^2+y^2}{2xy}$ ,  $x > 0$ . If  $y(2) = 0$ , then a value of  $y(8)$  is
  - $-4\sqrt{2}$
  - $2\sqrt{3}$
  - $-2\sqrt{3}$
  - $4\sqrt{3}$
- If a curve  $y = f(x)$ , passing through the point  $(1, 2)$ , is the solution of the differential equation  $2x^2 dy = (2xy + y^2)dx$ , then  $f\left(\frac{1}{2}\right)$  is equal to
  - $\frac{1}{1+\log_e 2}$
  - $\frac{1}{1-\log_e 2}$
  - $1 + \log_e 2$
  - $\frac{-1}{1+\log_e 2}$
- Let  $y = y(x)$  be the solution of the differential equation  $(x^2 - 3y^2)dx + 3xy dy = 0$ ,  $y(1) = 1$ . Then  $6y^2(e)$  is equal to
  - $3e^2$
  - $e^2$
  - $2e^2$
  - $\frac{3e^2}{2}$
- If  $y \frac{dy}{dx} = x \left[ \frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)} \right]$ ,  $x > 0$ ,  $\phi > 0$ , and  $y(1) = -1$ , then  $\phi\left(\frac{y^2}{4}\right)$  is equal to:
  - $2\phi(1)$
  - $\phi(1)$
  - $4\phi(2)$
  - $4\phi(1)$
- Let a smooth curve  $y = f(x)$  be such that the slope of the tangent at any point  $(x, y)$  on it is directly proportional to  $\left(\frac{-y}{x}\right)$ . If the curve passes through the points  $(1, 2)$  and  $(8, 1)$ , then  $\left|y\left(\frac{1}{8}\right)\right|$  is equal to
  - $2 \log_e 2$
  - 4
  - 1
  - $4 \log_e 2$
- If  $y = y(x)$  is the solution curve of the differential equation  $\frac{dy}{dx} + y \tan x = x \sec x$ ,  $0 \leq x \leq \frac{\pi}{3}$ ,  $y(0) = 1$ , then  $y\left(\frac{\pi}{6}\right)$  is equal to
  - $\frac{\pi}{12} - \frac{\sqrt{3}}{2} \log_e\left(\frac{2}{e\sqrt{3}}\right)$
  - $\frac{\pi}{12} + \frac{\sqrt{3}}{2} \log_e\left(\frac{2\sqrt{3}}{e}\right)$
  - $\frac{\pi}{12} - \frac{\sqrt{3}}{2} \log_e\left(\frac{2\sqrt{3}}{e}\right)$
  - $\frac{\pi}{12} + \frac{\sqrt{3}}{2} \log_e\left(\frac{2}{e\sqrt{3}}\right)$
- Let  $y = f(x)$  be the solution of the differential equation  $y(x+1)dx - x^2 dy = 0$ ,  $y(1) = e$ . Then  $\lim_{x \rightarrow 0^+} f\left(x\right)$  is equal to
  - 0
  - $\frac{1}{e}$
  - $e^2$
  - $\frac{1}{e^2}$

16. If a curve passes through the point  $(1, -2)$  and has slope of the tangent at any point  $(x, y)$  on it as  $\frac{x^2-2y}{x}$ , then the curve also passes through the point
- (1)  $(\sqrt{3}, 0)$  (2)  $(-1, 2)$   
 (3)  $(-\sqrt{2}, 1)$  (4)  $(3, 0)$
17. Let  $y = y(x)$ ,  $y > 0$ , be a solution curve of the differential equation  $(1+x^2)dy = y(x-y)dx$ . If  $y(0)=1$  and  $y(2\sqrt{2}) = \beta$ , then
- (1)  $e^{3\beta-1} = e(3+2\sqrt{2})$  (2)  $e^{3\beta-1} = e(5+\sqrt{2})$   
 (3)  $e^{\beta-1} = e^{-2}(3+2\sqrt{2})$  (4)  $e^{\beta-1} = e^{-2}(5+\sqrt{2})$
18. Suppose  $y = y(x)$  be the solution curve to the differential equation  $\frac{dy}{dx} - y = 2 - e^{-x}$  such that  $\lim_{x \rightarrow \infty} y(x)$  is finite. If  $a$  and  $b$  are respectively the  $x$  and  $y$ -intercept of the tangent to the curve at  $x = 0$ , then the value of  $a - 4b$  is equal to \_\_\_\_\_.
19. If  $y = y(x)$  is the solution of the differential equation  $2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$  such that  $y(e) = \frac{e}{3}$ , then  $y(1)$  is equal to
- (1)  $\frac{1}{3}$  (2)  $\frac{2}{3}$   
 (3)  $\frac{3}{2}$  (4)  $3$
20. Let  $y = y(x)$  be the solution of the differential equation  $x(1-x^2) \frac{dy}{dx} + (3x^2y - y - 4x^3) = 0$ ,  $x > 1$  with  $y(2) = -2$ . Then  $y(3)$  is equal to
- (1)  $-18$  (2)  $-12$   
 (3)  $-6$  (4)  $-3$
21. Let  $y = y(x)$  be the solution of the differential equation  $x \log_e \frac{dy}{dx} + y = x^2 \log_e x$ ,  $(x > 1)$ . If  $y(2) = 2$ , then  $y(e)$  is equal to
- (1)  $\frac{4+e^2}{4}$  (2)  $\frac{1+e^2}{4}$   
 (3)  $\frac{2+e^2}{2}$  (4)  $\frac{1+e^2}{2}$
22. Let  $y = y(x)$  be the solution curve of the differential equation  $\frac{dy}{dx} = \frac{y}{x}(1+x^2(1+\log_e x))$ ,  $x > 0$ ,  $y(1) = 3$ . Then  $\frac{y^2(x)}{9}$  is equal to :
- (1)  $\frac{x^2}{5-2x^3(2+\log_e x^3)}$  (2)  $\frac{x^2}{2x^3(2+\log_e x^3)-3}$   
 (3)  $\frac{x^2}{3x^3(1+\log_e x^2)-2}$  (4)  $\frac{x^2}{7-3x^3(2+\log_e x^2)}$
23. Let  $f$  be a differentiable function such that  $x^2 f(x) - x = 4 \int_0^x t f(t) dt$ ,  $f(1) = \frac{2}{3}$ . Then  $18 f(3)$  is equal to
- (1)  $210$  (2)  $160$   
 (3)  $150$  (4)  $180$
24. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function such that  $f'(x) + f(x) = \int_0^2 f(t) dt$ . If  $f(0) = e^{-2}$ , then  $2f(0) - f(2)$  is equal to \_\_\_\_\_.
25. If  $\frac{dy}{dx} + e^x(x^2 - 2)y = (x^2 - 2x)(x^2 - 2)e^{2x}$  and  $y(0) = 0$ , then the value of  $y(2)$  is
- (1)  $-1$  (2)  $1$   
 (3)  $0$  (4)  $e$
26. Let  $y = y(x)$  be the solution of the differential equation  $xy dy = (y + x^3 \cos x) dx$  with  $y(\pi) = 0$ , then  $y(\frac{\pi}{2})$  is equal to:
- (1)  $\frac{\pi^2}{4} + \frac{\pi}{2}$  (2)  $\frac{\pi^2}{2} + \frac{\pi}{4}$   
 (3)  $\frac{\pi^2}{2} - \frac{\pi}{4}$  (4)  $\frac{\pi^2}{4} - \frac{\pi}{2}$
27. Let  $y = y(x)$  be the solution of the differential equation  $(x - x^3) dy = (y + yx^2 - 3x^4) dx$ ,  $x > 2$  If  $y(3) = 3$ , then  $y(4)$  is equal to:
- (1)  $4$  (2)  $12$   
 (3)  $8$  (4)  $16$
28. Let  $y = y(x)$  be the solution curve of the differential equation  $\sin(2x^2) \log_e(\tan x^2) dy + (4xy - 4\sqrt{2}x \sin(x^2 - \frac{\pi}{4})) dx = 0$ ,  $0 < x < \sqrt{\frac{\pi}{2}}$ , which passes through the point  $(\sqrt{\frac{\pi}{6}}, 1)$ . Then  $|y(\sqrt{\frac{\pi}{3}})|$  is equal to \_\_\_\_\_.
29. Let  $x = x(y)$  be the solution of the differential equation  $2ye^{\frac{x}{y^2}} dx + (y^2 - 4xe^{\frac{x}{y^2}}) dy = 0$  such that  $x(1) = 0$ . Then,  $x(e)$  is equal to
- (1)  $e \log_e(2)$  (2)  $-e \log_e(2)$   
 (3)  $e^2 \log_e(2)$  (4)  $-e^2 \log_e(2)$
30. Let  $y = y(x)$  be the solution of the differential equation  $((x+2)e^{\frac{y+1}{x+2}} + (y+1)) dx = (x+2) dy$ ,  $y(1) = 1$ . If the domain of  $y = y(x)$  is an open interval  $(\alpha, \beta)$ , then  $|\alpha + \beta|$  is equal to \_\_\_\_\_.