ANSWER KEYS

1. (3)	**2.** (3)	**3.** (1)	**4.** (3)	**5.** (2)	**6.** (2)	**7.** (1)	**8.** (1)
9. (2)	**10.** (1)						

1. **(3)** Let the boxes be marked as A, B, C. We have to ensure that no box remains empty and all five balls have to put in. There will be two possibilities.

(i) Any two box containing one ball each and 3rd box containing 3 balls. Number of ways

$= A(1)B(1)C(3)$

$= {}^5C_1.{}^4C_1.{}^3C_3 = 5.4.1 = 20$

Since, the box containing 3 balls could be any of the three a boxes A, B, C. Hence, the required number of ways $20 \times 3 = 60$

(ii) Any two box containing 2 balls each and $3rd$ containing 1 ball, the number of ways

$= A(2)B(2)C(1) = {}^5C_2.\ {}^3C_2.\ {}^1C_1$

$= 10 \times 3 \times 1 = 30$

Since, the box containing 1 ball could be any of the three boxes A, B, C. Hence, The required number of ways

$= 30 \times 3 = 90$

Hence, total number of ways $= 60 + 90 = 150$

2. **(3)**

As we can select p alike objects any number of times in $p+1$ ways. And we can select an object in two ways, i.e., either rejection or acceptance.

Therefore, total number of ways of selecting any number of fruits $= 11 \times 6 \times 3 \times 2 \times 2 \times 2 = 1584$

Number of ways in which no fruit is selected$= 1$

Number of ways in which only one fruit is selected $= 6$

Number of ways in which two fruit are selected (there are two cases, i.e., both are identical and both are different)$= {}^6C_2 + {}^3C_2 = 18$

$\therefore$Number of ways in which at least three fruits are selected $= 1584 - (1+6+18) = 1559$

3. **(1)**

Positive integral solution of the equation

$x_1 + x_2 + x_3 + \ldots\ldots + x_r = n$ is ${}^{n-1}C_{r-1}$.

Given, $6 \leq a+b+c \leq 10$

$\therefore\ a+b+c = 6,\ 7,\ 8,\ 9,\ 10.$

Since $a,\ b,\ c$ are natural numbers, we need to find the positive integral solutions for $a,\ b,\ c$.

Number of positive integral solutions of the equation

$a+b+c=6$ is ${}^{6-1}C_{3-1} = {}^5C_2$

For $a+b+c=7$ is ${}^{7-1}C_{3-1} = {}^6C_2$

This will be continued upto positive integral solution of $a+b+c=10$ is ${}^9C_2$

$\therefore$ Required number of ways

$= {}^5C_2 + {}^6C_2 + {}^7C_2 + {}^8C_2 + {}^9C_2$

$= 110$

4. **(3)**

Let $X_1, X_2,\ X_3$ and X_4 be the number of students watching four movies, respectively.

$X_1 + X_2 + X_3 + X_4 = 10$

$X_1 \geq 1, X_2 \geq 1, X_3 \geq 1$ and $X_4 \geq 1$.

$X_1 - 1 \geq 0, X_2 - 1 \geq 0, X_3 - 1 \geq 0$ and $X_4 - 1 \geq 0$.

Let $X_1 = 1 + t_1,\ X_2 = 1 + t_2,\ X_3 = 1 + t_3,\ X_4 = 1 + t_4$

$t_1 \geq 0, t_2 \geq 0, t_3 \geq 0$ and $t_4 \geq 0$.

$\therefore t_1 + t_2 + t_3 + t_4 = 6$

$\therefore {}^{6+4-1}C_{4-1} = {}^9C_3$

${}^{6+4-1}C_{4-1} = 84$

5. **(2)**

We have been asked to find the highest exponent of 2 in 33!

$\left[\frac{33}{2}\right]+\left[\frac{33}{4}\right]+\left[\frac{33}{8}\right]+\left[\frac{33}{16}\right]+\left[\frac{33}{32}\right]$

$= 16 + 8 + 4 + 2 + 1 = 31$

6. **(2)**

${}^{n+1}C_3 - {}^nC_3 = 21 \Rightarrow n = 7.$

7. **(1)** A polygon of n sides has number of diagonals

$= \frac{n(n-3)}{2} = 275$ [given]

$\Rightarrow \ n^2 - 3n - 550 = 0$

$\Rightarrow \ (n-25)(n+22) = 0$

$\Rightarrow \ n = 25 \ [\because \ n \neq -22]$

8. **(1)**

In the given diagram, all Paralelograms are either Rectangles or Squares

$\therefore$ *Number of Rectangles* $=$ *Number of Parallelograms* $-$ *Number of Squares*

$\because$ *there are 5 Horizantal & 7 Vertical lines*

Number of Parallelograms $= {}^7C_2 \times {}^5C_2 = 210$

Squares :

Let the Horizantal lines be $H_1, H_2, H_3 \ldots\ldots\ldots H_5$

Let the Vertical lines be $V_1, V_2, V_3, \ldots\ldots\ldots\ldots V_7$

No of 1 unit squares : We require one pair from each of the following brackets

$$\begin{pmatrix} V_1 & V_2 \\ V_2 & V_3 \\ & \\ & \\ & \\ V_6 & V_7 \end{pmatrix} \quad \begin{pmatrix} H_1 & H_2 \\ H_2 & H_3 \\ & \\ & \\ H_5 & H_6 \end{pmatrix} \quad \Rightarrow \textit{No of one unit squares} = 6 \times 5$$

For two unit squares : we require one pair from each of the following

$$\begin{pmatrix} V_1 & V_3 \\ V_2 & V_4 \\ & \\ & \\ V_5 & V_7 \end{pmatrix} \quad \begin{pmatrix} H_1 & H_3 \\ H_2 & H_4 \\ H_3 & H_5 \end{pmatrix} \quad \Rightarrow \textit{No of two unit squares} = 5 \times 3$$

similar counting can be done for other squares and the largest size square possible is 5×5

$\therefore$ *total no of squares* $= 6 \times 4 + 5 \times 3 + 4 \times 2 + 3 \times 1 = 50$

$\therefore$ *total no of Rectangles which are not Squares is* $= 210 - 50 = 160$

9. **(2)**

If we fix 1 at one's place then number of words formed is 3!.

Similarly, if we fix 2 at one's place then the number of words formed is 3! and so on.

Required sum $= 3!(1+2+3+4) = 6(10) = 60$.

10. **(1)**

Given, numbers are $6, \ 7, \ 8$ & 9, repetition of the numbers are allowed.

Here, all places can be occupied by all given numbers since the repetition is allowed.

Total numbers formed $4 \times 4 \times 4 \times 4 = 256$.

Number of times each digit will appear at each place will be $= \frac{256}{4} = 64$.

Sum of the digits $= 64(6+7+8+9) = 1920$.

Hence, sum of all the four-digit numbers $1920(0+10+100+1000) = 2133120$.