ANSWER KEYS

1. (1)	**2.** (3)	**3.** (3)	**4.** (2)	**5.** (2)	**6.** (3)	**7.** (2)	**8.** (2)
9. (2)	**10.** (1)						

1. **(1)** Let a curve $y=f(x)$ pass through the point $\left(2,(\log_c 2)^2\right)$ and have slope $\frac{2y}{x\log_e x}$ for all positive real value of x.

Therefore, $y' = \frac{2y}{x'\ln x}$

$\Rightarrow \frac{dy}{y} = \frac{2dx}{x\ell nx}$

$\Rightarrow \ell n|y| = 2\ell n|\ell nx| + C$

Put $x=2$, $y=(\ell n2)^2$

$\Rightarrow c=0$

$\Rightarrow y=(\ell nx)^2$

$\Rightarrow f(\mathrm{e})=1.$

2. **(3)**

$\frac{f(x)}{1+x^2} = 1+\int_0^x \frac{(f(t))^2}{1+t^2}dt$

Differentiating w.r.t. x

$\frac{(1+x^2)f'(x)-2x\,f(x)}{(1+x^2)^2} = \frac{f^2(x)}{(1+x^2)} \Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \left(\frac{2x}{1+x^2}\right)\frac{1}{y} = 1$

Put $-\frac{1}{y}=t$

$\therefore \frac{dt}{dx} + \left(\frac{2x}{1+x^2}\right)t = 1,$

$\mathrm{IF} = e^{\int \frac{2x}{1+x^2}dx} = 1+x^2$

$\therefore$ Solution is $-\frac{(1+x^2)}{y} = x+\frac{x^3}{3}+c$

Now $f(0)=1$

$\therefore\ c=-1$

$\Rightarrow y = \frac{-3(1+x^2)}{x^3+3x-3} = f(x)$

$f(-3) = \frac{-3\left(1+3^2\right)}{3^3+3\times3-3} = \frac{10}{13}$

3. **(3)**

It is given that,

$\frac{dp(t)}{dt} = \frac{1}{2}\{p(t)-400\},\ p(0)=100$

$\frac{2\,dp(t)}{p(t)-400} = dt$

Integrating,

$\int_{100}^{p(t)} \frac{dp(t)}{p(t)-400} = \int_0^t \frac{1}{2}\,dt$

$\Rightarrow\ (\ln|p(t)-400|)_{100}^{p(t)} = \frac{t}{2}$

$\Rightarrow\ \ln\left|\frac{p(t)-400}{300}\right| = \frac{t}{2}$

$\Rightarrow |p(t)-400| = 300e^{\frac{t}{2}}$

$\Rightarrow\ 400-p(t) = 300e^{\frac{t}{2}}\quad (\because p(t)<400)$

$\Rightarrow\ p(t) = 400-300e^{\frac{t}{2}}$

4. (2)

We have

$\cos^2 x \frac{dy}{dx} - y \tan 2x = \cos^4 x$

$\Rightarrow \frac{dy}{dx} - \left(\frac{\tan 2x}{\cos^2 x}\right) y = \cos^2 x$

This is the linear differential equation of the form $\frac{dy}{dx} + Py = Q$.

Therefore,

I.F. $= e^{\int P dx}$

$\Rightarrow$ I.F. $= e^{-\int\left(\frac{\tan 2x}{\cos^2 x}\right) dx} = e^{I_1}$

where,

$I_1 = -\int \frac{\tan 2x}{\cos^2 x} dx$

$\Rightarrow I_1 = -2\int \frac{\tan 2x}{2\cos^2 x} dx$

$\Rightarrow I_1 = -2\int \frac{\sin 2x}{\cos 2x\,(\cos 2x+1)} dx$

Let $\cos 2x = t \Rightarrow -2\sin 2x dx = dt$

$\Rightarrow I_1 = \int \frac{dt}{t\,(t+1)}$

$\Rightarrow I_1 = \int\left(\frac{1}{t} - \frac{1}{t+1}\right) dt$

$\Rightarrow I_1 = \ln\left|\frac{t}{t+1}\right|$

$\Rightarrow I_1 = \ln\left|\frac{\cos 2x}{\cos 2x+1}\right|$.

Therefore,

$\Rightarrow$ I.F. $= e^{\ln\left|\frac{\cos 2x}{\cos 2x+1}\right|} = \left(\frac{\cos 2x}{\cos 2x+1}\right) = \frac{\cos 2x}{2\cos^2 x}$

Hence, the solution is

$y \times$[I.F.]$= \int [Q \times$[I.F.]$] dx + C$

$\Rightarrow y \times \frac{\cos 2x}{2\cos^2 x} = \int\left(\cos^2 x \times \frac{\cos 2x}{2\cos^2 x}\right) dx + C$

$\Rightarrow \frac{y\cos 2x}{2\cos^2 x} = \frac{1}{2}\int \cos 2x dx + C$

$\Rightarrow \frac{y\cos 2x}{2\cos^2 x} = \frac{1}{4}\sin 2x + C$

At $x = \frac{\pi}{6}$, $y = \frac{3\sqrt{3}}{8}$.

Therefore,

$C = \frac{\left(\frac{3\sqrt{3}}{8}\right)\cos\left(\frac{2\pi}{6}\right)}{2\left[\cos\left(\frac{\pi}{6}\right)\right]^2} - \frac{1}{4}\sin\left(\frac{2\pi}{6}\right)$

$\Rightarrow C = \frac{\left(\frac{3\sqrt{3}}{8}\right)\cos\left(\frac{\pi}{3}\right)}{2\left[\frac{\sqrt{3}}{2}\right]^2} - \frac{1}{4}\sin\left(\frac{\pi}{3}\right)$

$\Rightarrow C = \frac{\left(\frac{3\sqrt{3}}{8}\right)\times\frac{1}{2}}{2\times\frac{3}{4}} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow C = \frac{\sqrt{3}}{8} - \left(\frac{\sqrt{3}}{8}\right) = 0$.

5. (2)

Given equation is $\frac{1}{\sqrt{y}}\frac{dy}{dx} + \frac{x}{1-x^2}\sqrt{y} = x$

Let $2\sqrt{y} = \nu$

$\Rightarrow \frac{1}{\sqrt{y}}\frac{dy}{dx} = \frac{d\nu}{dx}$

Thus, we have $\frac{d\nu}{dx} + \frac{x}{2\,(1-x^2)} \cdot \nu = x$

$\therefore$ I. F. $= e^{\int \frac{x}{2\,(1-x^2)} dx}$

$= e^{-\frac{1}{4}\ln(1-x^2)}$

$= (1-x^2)^{-\frac{1}{4}}$

Thus, the solution is $\nu(1-x^2)^{-\frac{1}{4}} = \int x(1-x^2)^{-\frac{1}{4}} dx$

or $\nu \cdot (1-x^2)^{-\frac{1}{4}} = -\frac{2}{3}(1-x^2)^{\frac{3}{4}} + c''$

or $2\sqrt{y} = -\frac{2}{3}(1-x^2) + c''(1-x^2)^{\frac{1}{4}}$

$\Rightarrow \sqrt{y} = -\frac{(1-x^2)}{3} + c'(1-x^2)^{\frac{1}{4}}$

$\Rightarrow \sqrt{9y} = -(1-x^2) + c(1-x^2)^{\frac{1}{4}}$

$\Rightarrow f(x) = 1 - x^2$

$\therefore f(x)$ is an even function

6. **(3)**

We have,

$xy^4dx + ydx - xdy = 0.$

$\Rightarrow xdx + \frac{ydx - xdy}{y^4} = 0$

$\Rightarrow x^3dx + \left(\frac{x}{y}\right)^2 \cdot \frac{ydx - xdy}{y^2} = 0$

$\Rightarrow x^3dx + \left(\frac{x}{y}\right)^2 \cdot d\left(\frac{x}{y}\right) = 0$

On integrating, we get

$\frac{x^4}{4} + \frac{1}{3}\left(\frac{x}{y}\right)^3 = c$

$\Rightarrow 3x^4y^3 + 4x^3 = Cy^3$

which is the required solution.

7. **(2)** The given equation is $\left(\sin^2 x\right)(2ydy) + (2\sin x\cos x dx)y^2 = 2xdx$

or $d\left(\sin^2 x \cdot y^2\right) = 2xdx$

On integrating, we get

$\sin^2 x \cdot y^2 = x^2 + C$

8. **(2)**

$\frac{x^2(x\,dx + y\,dy)}{\sqrt{x^2+y^2}} = ydx - x\,dy$

or $\int \frac{1}{2}\frac{d\left(x^2+y^2\right)}{\sqrt{x^2+y^2}} = \int \frac{ydx - xdy}{x^2}$

$\Rightarrow \frac{1}{2}2.\sqrt{x^2+y^2} = -\frac{y}{x} + C$

9. **(2)**

Given equation can be rewritten as

$2xdx + 2ydy + d\left(\frac{y}{x}\right) = 0$

Integrating, we get,

$x^2 + y^2 + \frac{y}{x} = c$

As $(1,\ 0)$ lies on it, hence, $c = 1$

The required equation of the curve is $x^2 + y^2 + \frac{y}{x} = 1$

10. **(1)**

The given equation is

$e^{\frac{x}{y}}\left(dx - \frac{x}{y}dy\right) + e^{\frac{x}{y}}dy + dx = 0$

or $e^{\frac{x}{y}}yd\left(\frac{x}{y}\right) + e^{\frac{x}{y}}dy + dx = 0$

$\Rightarrow\ d\left(e^{\frac{x}{y}}y\right) + dx = 0$

On integrating, we get,

$e^{\frac{x}{y}}y + x = C$

$\Rightarrow k = 1$