ANSWER KEYS

1. (1)	**2.** (3)	**3.** (2)	**4.** (4)	**5.** (1)	**6.** (1)	**7.** (2)	**8.** (2)
9. (4)	**10.** (2)						

1. (1)

$^7C_3 x^4 x^{(3\log_2^x)} = 4480$

$\Rightarrow x^{(4+3\log_2^x)} = 2^7$

$\Rightarrow (4+3t)t = 7; t = \log_2^x$

$\Rightarrow t = 1, \frac{-7}{3} \Rightarrow x = 2$

2. (3)

The general term of the binomial expansion, $\left(\sqrt{2}+3^{\frac{1}{5}}\right)^{10}$ is given as,

$T_{r+1} = {}^{10}C_r \left(2\right)^{\frac{10-r}{2}} \left(3\right)^{\frac{r}{5}}$ where, $0 \leq r \leq 10$

For integral term both $(n-r)$ and r should be a positive multiple of 2 and 5 respectively,

$LCM\ of\ 2\ and\ 5\ is\ 10$

Therefore the values of r for which both $\frac{(n-r)}{2}$ and $\frac{r}{5}$ gives an integer value are,

$r = 0\ \ and\ r = 10$

Therefore, the rational terms correspond for $r\ =\ 0,\ r\ =\ 10$

Hence the sum of rational terms $=\ {}^{10}C_0 2^5 + {}^{10}C_{10} 3^2 = 32 + 9 = 41.$

3. (2)

We have the expansion $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n \therefore \frac{T_{4+1}}{T_{n-4+1}} = \frac{{}^nC_4}{{}^nC_{n-4}} \frac{(2)^{\frac{n-4}{4}} \cdot \left(\frac{1}{3}\right)^{\frac{4}{4}}}{(2)^{\frac{n-(n-4)}{4}} \cdot \left(\frac{1}{3}\right)^{\frac{n-4}{4}}} = \left(\sqrt{6}\right)^5 \Rightarrow (2)^{\frac{n-8}{4}} \cdot (3)^{\frac{n-8}{4}} = \left(\sqrt{6}\right)^5$

$\Rightarrow (6)^{\frac{n-8}{4}} = (6)^{\frac{5}{2}}$

$\Rightarrow \frac{n-8}{4} = \frac{5}{2}$

$\Rightarrow n = 18.$

4. (4)

From the given condition

${}^nC_r :{}^n C_{r+1} :{}^n C_{r+2} = 2:15:70$

$\Rightarrow \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{2}{15}$ and $\frac{{}^nC_{r+1}}{{}^nC_{r+2}} = \frac{15}{70}$

$\Rightarrow \frac{\left(\frac{n!}{(n-r)!\cdot r!}\right)}{\left(\frac{n!}{(n-r-1)!\cdot(r+1)!}\right)} = \frac{2}{15}$ and $\frac{\left(\frac{n!}{(n-r-1)!\cdot(r+1)!}\right)}{\left(\frac{n!}{(n-r-2)!\cdot(r+2)!}\right)} = \frac{15}{70}$

$\Rightarrow \frac{(n-r-1)!\cdot(r+1)!}{(n-r)!\cdot r!} = \frac{2}{15}$ and $\frac{(n-r-2)!\cdot(r+2)!}{(n-r-1)!\cdot(r+1)!} = \frac{3}{14}$

$\Rightarrow \frac{(n-r-1)!\cdot(r+1)\cdot r!}{(n-r)\cdot(n-r-1)!\cdot r!} = \frac{2}{15}$ and $\frac{(n-r-2)!\cdot(r+2)\cdot(r+1)!}{(n-r-1)\cdot(n-r-2)!\cdot(r+1)!} = \frac{3}{14}$

$\Rightarrow \frac{r+1}{n-r} = \frac{2}{15}$ and $\frac{r+2}{n-r-1} = \frac{3}{14}$

$\Rightarrow\ 17r = 2n - 15$ and $17r = 3n - 31$

$\Rightarrow 3n - 31 = 2n - 15,\ \ \Rightarrow n = 16$ and $r = 1$

Hence, average $= \frac{{}^nC_r + {}^nC_{r+1} + {}^nC_{r+2}}{3}$

$= \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3}$

$= 232.$

5. (1)

Here, coefficient of t^{24} in $\left(1+t^2\right)^{12}\left(1+t^{12}\right)\left(1+t^{24}\right)$

$\Rightarrow$ coefficient of t^{24}in$\left(1+t^2\right)^{12}\left(1+t^{12}+t^{24}+t^{36}\right)$

$\Rightarrow$ coefficient of t^{24}in$\left(1+t^2\right)^{12} + t^{12}\left(1+t^2\right)^{12} + t^{24}\left(1+t^2\right)^{12}$

[neglecting$t^{36}\ \left(1\ +\ t^2\ \right)^{12}$]

$\Rightarrow$ coefficient of t^{24} is $t^{24}\ =\ \left({}^{12}C_{12}\ +\ {}^{12}C_6\ +\ {}^{12}C_0\right)$

$\Rightarrow\ t^{24}\ =\ 2\ +\ {}^{12}C_6$

6. (1)

Let the given series be S. Then, we get,

$S=(1+x)^{500}+x(1+x)^{499}+x^2(1+x)^{498}+\ldots+x^{500}$

The given series is clearly a geometric progression with a common ratio $\frac{x(1+x)^{499}}{(1+x)^{500}}$ which is equal to $\frac{x}{1+x}$ where the number of terms is 501.

So, the given series is

$$S=(1+x)^{500}\left[\frac{1-\left(\frac{x}{1+x}\right)^{501}}{1-\frac{x}{1+x}}\right]$$

$S=(1+x)^{501}-x^{501}$

Hence, the coefficient of x^{301} in S is ${}^{501}C_{301}$.

7. (2)

We have

$3^{37}=3^{36}\cdot 3=3\cdot(81)^9=3(80+1)^9$

$=3\left({}^9C_0 80^9+{}^9C_1 80^8+\ldots{}^9C_8\cdot 80+{}^9C_9\right)$

$=3\cdot\left({}^9C_0 80^8+{}^9C_1 80^7+\ldots{}^9C_8\cdot 80\right)+3\cdot{}^9C_9$

$=3\cdot\left({}^9C_0 80^8+{}^9C_1 80^7+\ldots{}^9C_8\cdot 80\right)+3$

$=3\cdot 80\cdot\left({}^9C_0 80^7+{}^9C_1 80^6+\ldots{}^9C_8\right)+3$

$=80k+3$

where $k=3\left({}^9C_0 80^7+{}^9C_1 80^6+\ldots{}^9C_8\right)$

Thus, required remainder is equal to 3.

8. (2)

We know, $25^{15}=(26-1)^{15}$

$={}^{15}C_0 26^{15}-{}^{15}C_1 26^{14}+\ldots\ldots-{}^{15}C_{15}$

$=26I-1=26I-13+12=13I+12$

9. (4)

$3^{2n}=9^n=(1+8)^n$

$\Rightarrow 3^{2n}={}^nC_0+{}^nC_1\cdot 8+{}^nC_2\cdot 8^2+\ldots\ldots+{}^nC_n\cdot 8^n$

$\Rightarrow \frac{3^{2n}}{8}=\frac{{}^nC_0+{}^nC_1\cdot 8+{}^nC_2\cdot 8^2+\ldots\ldots+{}^nC_n\cdot 8^n}{8}$

$\Rightarrow \frac{3^{2n}}{8}=\frac{1}{8}+\left({}^nC_1+{}^nC_2\cdot 8+\ldots\ldots+{}^nC_n\cdot 8^{n-1}\right)$

$\Rightarrow \frac{3^{2n}}{8}=\frac{1}{8}+$ integer quantity

$\{x\}\in[0,1)$ & $\{I\}=0$

$\Rightarrow \left\{\frac{3^{2n}}{8}\right\}=\frac{1}{8}+0$

10. (2)

$3^{400}=\left(3^2\right)^{200}$

$=(9)^{200}$

$=\left(10^{-1}\right)^{200}$

$={}^{200}C_0(10)^{200}+{}^{200}C_1(10)^{199}(-1)^1+{}^{200}C_2(10)^{198}(-1)^2$

$+\ldots..+{}^{200}C_{198}(10)^2(-1)^{198}+{}^{200}C_{199}(10)^1(-1)^{199}+{}^{200}C_{200}(10)^0(-1)^{200}$

$=10^2\{\lambda\}+{}^{200}C_{199}(10)(-1)+{}^{200}C_{200}$

where λ is constant term which remains after taking 10^2 common.

$=(\lambda)\left(10^2\right)+(-2000)+1$

$(\lambda-20)(100)+1$

So last two digit will be 01

sum $=0+1=1$