ANSWER KEYS

1. (4)	**2.** (4)	**3.** (3)	**4.** (3)	**5.** (7)	**6.** (3)	**7.** (2)	**8.** (1)
9. (2)	**10.** (2)						

1. **(4)** Given, $\cos x+\cos y=\frac{3}{2}$

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=\frac{3}{2}$

and $\sin x+\sin y=\frac{3}{4}$

$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=\frac{3}{4}$

$\therefore \frac{2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}=\frac{\frac{3}{4}}{\frac{3}{2}}$

$\Rightarrow \tan\left(\frac{x+y}{2}\right)=\frac{1}{2}$

$\therefore \sin(x+y)=\frac{2\tan\left(\frac{x+y}{2}\right)}{1+\tan^2\left(\frac{x+y}{2}\right)}$

$=\frac{2\times\frac{1}{2}}{1+\left(\frac{1}{2}\right)^2}=\frac{4}{4+1}=\frac{4}{5}$

2. **(4)**

$L=\sin\left(\frac{\pi}{16}+\frac{\pi}{8}\right)\sin\left(\frac{\pi}{16}-\frac{\pi}{8}\right)$

$=\sin\frac{3\pi}{16}\cdot\sin\left(-\frac{\pi}{16}\right)$

$=\frac{1}{2}\left(\cos\left(\frac{3\pi}{16}+\frac{\pi}{16}\right)-\cos\left(\frac{3\pi}{16}-\frac{\pi}{16}\right)\right)=\frac{1}{2}\left(\frac{1}{\sqrt{2}}-\cos\frac{\pi}{8}\right)$

$M=\cos\left(\frac{\pi}{16}+\frac{\pi}{8}\right)\cos\left(\frac{\pi}{16}-\frac{\pi}{8}\right)$

$=\cos\frac{3\pi}{16}\cdot\cos\left(-\frac{\pi}{16}\right)$

$=\frac{1}{2}\left(\cos\left(\frac{3\pi}{16}+\frac{\pi}{16}\right)+\cos\left(\frac{3\pi}{16}-\frac{\pi}{16}\right)\right)=\frac{1}{2}\left(\frac{1}{\sqrt{2}}+\cos\frac{\pi}{8}\right)$

3. **(3)**

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$\therefore A+B=\pi,\ A=\pi-B$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\left(\pi-\frac{3\pi}{8}\right)\right)\times\left(1+\cos\left(\pi-\frac{\pi}{8}\right)\right)$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\times\left(1-\cos\frac{\pi}{8}\right)$

$\left(1+\cos\frac{\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)$

$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$=\sin^2\frac{\pi}{8}.\sin^2\frac{3\pi}{8}=\frac{1}{4}\left[2\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right]^2$

$\therefore 2sinAsinB=cos(A-B)-cos(A+B)$

$=\frac{1}{4}\left[\cos\frac{\pi}{4}-\cos\frac{\pi}{2}\right]^2=\frac{1}{4}\left[\frac{1}{\sqrt{2}}-0\right]^2=\frac{1}{8}$

4. **(3)**

$\sin\theta+\cos\theta=\frac{1}{2}$

$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\frac{1}{4}\Rightarrow\sin 2\theta=-\frac{3}{4}$

Now:

$\cos 4\theta=1-2\sin^2 2\theta$

$=1-2\left(-\frac{3}{4}\right)^2$

$=1-2\times\frac{9}{16}=-\frac{1}{8}$

And $\sin 6\theta=3\sin 2\theta-4\sin^3 2\theta$

$=\left(3-4\sin^2 2\theta\right)\cdot\sin 2\theta$

$=\left[3-4\left(\frac{9}{16}\right)\right]\cdot\left(-\frac{3}{4}\right)$

$\Rightarrow\left[\frac{3}{4}\right]\times\left(-\frac{3}{4}\right)=-\frac{9}{16}$

So, $16[\sin 2\theta+\cos 4\theta+\sin 6\theta]$

$16\left(-\frac{3}{4}-\frac{1}{8}-\frac{9}{16}\right)=-23$

5. **(7)** Let, $\tan 20^{\circ}=k$

Then, $\tan 60^{\circ}=\frac{3\tan 20^{\circ}-\tan^{3}20^{\circ}}{1-3\tan^{2}20^{\circ}}$

$\sqrt{3}=\frac{3k-k^{3}}{1-3k^{2}}$

$\left(\sqrt{3}\left(1-3k^{2}\right)\right)^{2}=\left(3k-k^{3}\right)^{2}$

$3\left(1+9k^{4}-6k^{2}\right)=9k^{2}+k^{6}-6k^{4}$

$3+27k^{4}-18k^{2}=9k^{2}+k^{6}-6k^{4}$

$k^{6}-33k^{4}+27k^{2}-3=0$

$k^{6}-33k^{4}+27k^{2}+4=7$

Hence, $\tan^{6}20^{\circ}-33\tan^{4}20^{\circ}+27\tan^{2}20^{\circ}+4=7$

6. **(3)**

Let $E=\frac{2\cos 8\theta+1}{2\cos\theta+1}$

$=\frac{2\left(2\cos^{2}4\theta-1\right)+1}{(2\cos\theta+1)}$

$=\frac{(2\cos 4\theta-1)(2\cos 4\theta+1)}{(2\cos\theta+1)}$

$=\frac{(2\cos 4\theta-1)(2\cos 2\theta-1)(2\cos 2\theta+1)}{(2\cos\theta+1)}$

$=\frac{[(2\cos 4\theta-1)(2\cos 2\theta-1)(2\cos\theta-1)(2\cos\theta+1)]}{(2\cos\theta+1)}$

$=(2\cos 4\theta-1)(2\cos 2\theta-1)(2\cos\theta-1)$

7. **(2)**

Given,

$\cos\frac{\pi}{7}+\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{5\pi}{7}+\cos\frac{6\pi}{7}+\cos\frac{7\pi}{7}$

$=\left(\cos\frac{\pi}{7}+\cos\frac{6\pi}{7}\right)+\left(\cos\frac{2\pi}{7}+\cos\frac{5\pi}{7}\right)+\left(\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}\right)+\cos\pi$

$=\left(\cos\frac{\pi}{7}+\cos\left(\pi-\frac{\pi}{7}\right)\right)+\left(\cos\frac{2\pi}{7}+\cos\left(\pi-\frac{2\pi}{7}\right)\right)+\left(\cos\frac{3\pi}{7}+\cos\left(\pi-\frac{3\pi}{7}\right)\right)+\cos\pi$

$=\left(\cos\frac{\pi}{7}-\cos\frac{\pi}{7}\right)+\left(\cos\frac{2\pi}{7}-\cos\frac{2\pi}{7}\right)+\left(\cos\frac{3\pi}{7}-\cos\frac{3\pi}{7}\right)+\cos\pi$

$=\cos\pi=-1$

8. **(1)**

$16\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right)=16\cos A\cos 2A\cos 2^{2}A\cos 2^{3}A$ with $A=\frac{2\pi}{5}$

$=16\frac{\sin\left(2^{4}A\right)}{2^{4}\sin A}=\frac{\sin\left(\frac{32\pi}{15}\right)}{\sin\left(\frac{2\pi}{15}\right)}=\frac{\sin\left(2\pi+\frac{2\pi}{15}\right)}{\sin\left(\frac{2\pi}{15}\right)}$

$=\frac{\sin\left(\frac{2\pi}{15}\right)}{\sin\left(\frac{2\pi}{15}\right)}=1\Rightarrow n=1$

9. **(2)** $\cos\frac{4\pi}{5}\cos\frac{6\pi}{5}\cos\frac{8\pi}{5}=\cos\left(\frac{4\pi}{5}\right)\left[-\cos\frac{\pi}{5}\right]\cos\left(\frac{8\pi}{5}\right)$

$=\frac{-\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}}{\cos\frac{2\pi}{5}}=-\frac{\sin\left(2^{4}\frac{\pi}{5}\right)}{2^{4}\sin\left(\frac{\pi}{5}\right)\cos\frac{2\pi}{5}}$

$=-\frac{\sin\left(3\pi+\frac{\pi}{5}\right)}{16\sin\left(\frac{\pi}{5}\right)\cos 72^{\circ}}=\frac{\sin\left(\frac{\pi}{5}\right)}{16\sin\left(\frac{\pi}{5}\right)\sin 18^{\circ}}=\frac{1}{16\left(\frac{\sqrt{5}-1}{4}\right)}=\frac{1}{4\left(\sqrt{5}-1\right)}$

10. **(2)** Total number of given numbers $=90$

Sum of numbers,

$S=2\sin 2^{\circ}+4\sin 4^{\circ}+\ldots\ldots\ldots+178\sin 178^{\circ}+180\sin 180^{\circ}$

$S=178\sin 2^{\circ}+176\sin 4^{\circ}+\ldots\ldots\ldots+2\sin 178^{\circ}$

Adding both the equation, we get,

$2S=180(\sin 2^{\circ}+\sin 4^{\circ}+\ldots\ldots\ldots+\sin 178^{\circ})+0$

$S=90(\sin 2^{\circ}+\sin 4^{\circ}+\ldots\ldots\ldots+\sin 178^{\circ})$

$\Rightarrow$ Mean $=\frac{S}{90}=\sin 2^{\circ}+\sin 4^{\circ}+\ldots\ldots+\sin 178^{\circ}$

$=\frac{\sin 89^{\circ}}{\sin 1^{\circ}}\sin\left(\frac{2^{\circ}+178^{\circ}}{2}\right)$

$=\frac{\cos 1^{\circ}}{\sin 1^{\circ}}\times\sin 90^{\circ}=\cot 1^{\circ}$