ANSWER KEYS

1. (4) **2.** (3) **3.** (4) **4.** (1) **5.** (4) **6.** (1) **7.** (3) **8.** (2)
9. (2) **10.** (1)

1. **(4)** P (exactly one) $= \frac{2}{5}$

$= P(A)+P(B)-2P(A \cap B)= \frac{2}{5} \ \ldots(1)$

$P(A \cup B)= \frac{1}{2}$

$= P(A)+P(B)-P(A \cap B)= \frac{1}{2} \ \ldots(2)$

$\therefore P(A \cap B)= \frac{1}{2}-\frac{2}{5}=\frac{5-4}{10}=\frac{1}{10}$.

2. **(3)**

Total number of ways to form the numbers of five digits with $1,\ 2,\ 3,\ 4,\ 5$ are $= 5! = n(S)$

We know number divisible by 4 when the number formed by the last two digits is divisible by 4.

Hence, the possible last two digits of the number are $12,\ 24,\ 32,\ 52$.

And, for each case the remaining three places can be filled in 3! ways.

Thus, the total number of numbers which are divisible by 4 are $n(E)= 3! \times 4 = 4!$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{4!}{5!} = \frac{1}{5}$

3. **(4)** P (selecting an apple from both baskets)

$= P(\text{apple from first basket}) \cdot P(\text{apple from second basket})$

$= \frac{{}^5C_1}{{}^{12}C_1} \cdot \frac{{}^4C_1}{{}^{12}C_1}$

P (selecting a orange from both baskets)

$= P(\text{orange from first basket}) \cdot P(\text{orange from second basket})$

$= \frac{{}^7C_1}{{}^{12}C_1} \cdot \frac{{}^8C_1}{{}^{12}C_1}$

Required probability $= \frac{{}^5C_1\,{}^4C_1}{{}^{12}C_1\,{}^{12}C_1} + \frac{{}^7C_1\,{}^8C_1}{{}^{12}C_1\,{}^{12}C_1}$

$= \frac{20+56}{144} = \frac{76}{144}$

4. **(1)**

The number of ways in which 8 peoples (excluding A snd B) sit in a round table with peoples between A and $B = {}^{18}C_6(2!)(13-1)!$

Thus, the required probability is $= \frac{{}^{18}C_6\,(2!)\,(13-1)!\,(6!)}{(20-1)!} = \frac{(18!)\,(2!)\,(12!)\,(6!)}{(19!)\,(12!)\,(6!)} = \frac{2}{19}$.

Here, ${}^{18}C_6$ refers to selection of 6 people out of $18,\ 2!$ is arranging A and B, $6!$ is arranging the 6 people in between A and B.

5. **(4)**

Given 15 coupons.

Now we have to select 7 coupons.

Number of ways of selecting a coupon $= 15$

Total Number of ways of selecting 7 coupons $= 15 \cdot 15 \cdot 15 \cdot 15 \cdot 15 \cdot 15 \cdot 15 = 15^7$

Here, we have to ensure that maximum number selected on the coupon is 9.

Now, number of ways of selecting 7 coupons from coupons numbering from 1 to $9 = 9^7$

Here, we have to subtract those cases where coupon numbering 9 is not selected (select all seven coupon from numbers ranging from 1 to 8) $= 8^7$

$\Rightarrow$ Favourable Ways $= 9^7 - 8^7$

So, desired probability

$= \frac{\text{Favourable Cases}}{\text{Total Cases}}$

$= \frac{9^7-8^7}{15^7}$

6. (1)

$S = \{1, 2, ..., 20\}$

Total outcome $= {}^{20}C_3 = \frac{20!}{3!17!}$

$= \frac{20\times19\times18}{3\times2} = 1140$

$\because\ a, b, c$ are in A.P.

$2b = a + c =$even number

$\Rightarrow\ a, c$ both are odd or both are even

As 10 odd numbers are there in S

So, selecting 2 number $= {}^{10}C_2$

and 10 even number are there in S

so, selecting 2 numbers $= {}^{10}C_2$

$\Rightarrow$Favourable cases $= {}^{10}C_2 + {}^{10}C_2$

$= 2 \times {}^{10}C_2$

$= 2 \times \frac{10!}{2!\times8!} = 90$

Probability $= \frac{\text{favorable outcome}}{\text{total number of outcome}}$

$= \frac{30}{1140} = \frac{3}{38}$

7. (3)

$P_1 + P_2 + P_3 = \frac{27}{20}$

$P_1P_2 + P_2P_3 + P_3P_1 = \frac{14}{20}$

$P_1P_2P_3 = \frac{2}{20}$

Probability that the student passes in exactly one of A, B, C is

$= P_1 + P_2 + P_3 - 2(P_1P_2 + P_2P_3 + P_3P_1) + 3P_1P_2P_3$

$= \frac{27}{20} - 2\left(\frac{14}{20}\right) + \frac{6}{20} = \frac{5}{20} = \frac{1}{4}$

8. (2)

$P(A) = \frac{1}{2}\ P(B) = \frac{1}{2}$

$P(AA + ABA + BBA + ABBA + BBAA + BABA) = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

9. (2)

Let,

R_1 represents a Red ball is drawn from urn A in an attempt and that ball is placed in urn B

B_1 represents a Black ball is drawn from urn A in an attempt and that ball is placed in urn B

Similarly, R_2 represents a Red ball is drawn from urn B in an attempt and that ball is placed in urn A, and

B_2 represents a Black ball is drawn from urn B in an attempt and that ball is placed in urn A

Now consider that R represents that a Red ball is drawn in the second attempt from urn A

Then the required probability is,

$P(R_1R_2R) + P(R_1B_2R) + P(B_1R_2R) + P(B_1B_2R)$

$= P(R_1)\times P(R_2)\times P(R) + P(R_1)\times P(B_2)\times P(R) + P(B_1)\times P(R_2)\times P(R) + P(B_1)\times P(B_2)\times P(R)$

$= \frac{6}{10}\times\frac{5}{11}\times\frac{6}{10} + \frac{6}{10}\times\frac{6}{11}\times\frac{5}{10} + \frac{4}{10}\times\frac{4}{11}\times\frac{7}{10} + \frac{4}{10}\times\frac{7}{11}\times\frac{6}{10}$

$= \frac{180+180+112+168}{1100} = \frac{640}{1100} = \frac{32}{55}$

10. (1) $\because f(x) = x^3 + ax^2 + bx + c$

$\therefore f'(x) = 3x^2 + 2ax + b$

$y = f(x)$ is increasing.

$\Rightarrow f'(x) \geq 0, \forall x$

And for $f'(x) = 0$ should not form an interval.

$\Rightarrow 4a^2 - 4 \times 3 \times b \leq 0$

$\Rightarrow a^2 - 3b \leq 0$

This is true for exactly 16 ordered pairs $(a, b),\ 1 \leq a, b \leq 6$ namely

$(1, 1), (1, 2), (1, 3), (1, 4); (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6); (3, 3), (3, 4), (3, 5), (3, 6)$ & $(4, 6)$.

Thus, required probability $= \frac{16}{36} = \frac{4}{9}$