

## **ANSWER KEYS 2.** (2) **5.** (4) **6.** (1) 7. (2) 1. (2) **3.** (1) **4.** (2) 9. (8) nathongo 10. (2) athongo 11. mathongo 1. (2) $I = \int rac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}} dx \ I = \int rac{e^{\log x^{5}} - e^{\log x^{4}}}{e^{\log x^{3}} - e^{\log x^{2}}} dx$ $=\int \frac{e^{\log x \cdot \cdot \cdot} - e^{\log x \cdot \cdot}}{x^3 - x^2} dx \qquad (\because a^{\log_a x} = x) \qquad \text{mathongo} \qquad$ $=\int \frac{x^4}{3} dx = \frac{x^3}{3} + C$ 2. (2)athongo ///. mathongo ///. We have, $\int \frac{5 \left(x^6+1\right)}{x^2+1} dx$ mathong $\frac{5 \left(x^6+1\right)}{x^2+1} dx$ mathong $\frac{7}{x^2} dx$ $[:: a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$ $=\int 5(x^4-x^2+1)dx$ athongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// $=x^5-\frac{5}{3}x^3+5x+C.$ 3. (1) We know that, $\sin 3x = 3 \sin x - 4 \sin^3 x$ $\Rightarrow \sin^3 x = \frac{3\sin x - \sin 3x}{x}$ $\Rightarrow \int \sin^3 dx = \int \frac{\frac{4}{3} \sin x - \sin 3x}{4} dx$ $\Rightarrow \int \sin^3 dx = \frac{3}{4} \int \sin x \, dx - \frac{1}{4} \int \sin 3x \, dx$ $\Rightarrow \int \sin^3 dx = \frac{3}{4}(-\cos x) - \frac{1}{4}\left(-\frac{\cos 3x}{3}\right) + C$ $\Rightarrow \int \sin^3 dx = -\frac{3}{4} \cos x + \frac{\cos 3x}{12} + C$ $$\begin{split} & I = \int \frac{1 + x + \sqrt{x + x^2}}{\sqrt{x} + \sqrt{1 + x}} dx = \int \frac{\sqrt{(1 + x)^2} + \sqrt{x} \sqrt{1 + x}}{(\sqrt{x} + \sqrt{1 + x})} dx \\ & = \int \frac{\sqrt{1 + x} \left(\sqrt{x} + \sqrt{x + 1}\right)}{\left(\sqrt{x} + \sqrt{x + 1}\right)} dx = \frac{2}{3} \left(1 + x\right)^{3/2} + C \end{split}$$ 5. (4) $\int \sqrt{\frac{1-x}{1+x}} dx$ Multiply and divide by $\sqrt{1-x}$ $= \int \frac{1-x}{\sqrt{1-x^2}} \, dx = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x}{\sqrt{1-x^2}} \, dx$ $= \sin^{-1} x + \frac{1}{2} \int \frac{(-2x)}{\sqrt{1-x^2}} \, dx$ mathongo /// $=\sin^{-1}x+\sqrt{1-x^2}+c$ $= \sin^{-5}x + \sqrt{1 - x^{2} + c}$ 6. (1) Let $I = \int \frac{\sin^{2}x \cos^{2}x}{\left(\sin^{5}x + \cos^{3}x \sin^{2}x + \sin^{3}x \cos^{2}x + \cos^{5}x\right)^{2}} dx = \int \frac{\sin^{2}x \cos^{2}x}{\left(\sin^{3}x + \cos^{3}x\right)^{2}} dx = \int \frac{\sin^{2}x \cos^{2}x}{\left(\sin^{3}x + \cos^{3}x\right)^{2}} dx = \int \frac{\tan^{2}x \sec^{2}x}{\left(1 + \tan^{3}x\right)^{2}} dx$ Now, put $(1 + \tan^{3}x) = t$ $\Rightarrow 3 an^2x\sec^2x dx = dt$ .: $I = rac{1}{3}\intrac{dt}{t^2} = -rac{1}{3t} + C = rac{-1}{3(1+ an^3x)} + C$ 7. (2) $\int \sqrt[3]{\frac{\sin^n x}{\cos^{n+6} x}} dx = \int \sqrt[3]{\frac{\sin^n x}{\cos^n x \cdot \cos^6 x}} dx$ $=\int \sqrt[3]{\tan^n x} \sec^2 x dx$ $=\int ( an x)^{n/3} d( an x) \quad \left\{ \begin{array}{c} \therefore \ d( an x) = \sec^2 x dx \end{array} \right\}$ $\left\{ : \int_{\mathbb{R}^n} x^n dx = \frac{x^{n+1}}{n+1} + c \right\}_{\text{hongo}} \text{ $\frac{1}{n}$ mathongo $\frac{1}{$ $= \frac{(\tan x)^{\frac{n}{3}+1}}{\frac{n}{3}+1} + c$ $=\frac{3}{3+n}(\tan x)^{\frac{n}{3}+1}+c$ Hence (B) is the correct answer.



## **Answer Keys and Solutions**

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8.	(1) Ithongo // mathongo $I = \int \frac{\cos x - \sin x}{7 - 9 \sin 2x} dx$					
	$\Rightarrow I = \int \frac{\cos x - \sin x}{7 - 9 \left[ (1 + \sin 2x) - 1 \right]} dx$ $\Rightarrow I = \int \frac{\cos x - \sin x}{7 - 9 \left[ (\sin^2 x + \cos^2 x + 2\sin x) \right]}$	$\frac{\sqrt{mathongo}}{x\cos x - 1}$				
	$\Rightarrow I = \int \frac{\cos x - \sin x}{7 - 9[(\sin x + \cos x)^2 - 1]} dx$ Let $\sin x + \cos x = t \Rightarrow (\cos x - \cos x)$	$\frac{dy}{dx} = \frac{dy}{dx}$				
	$\Rightarrow I = \int \frac{dt}{7-9(t^2-1)}$ $\Rightarrow I = \int \frac{dt}{4^2 - (3t)^2}$ The state of the state o					
14.	$\Rightarrow I = \frac{1}{2 \cdot 4} \cdot \frac{1}{3} \log \left  \frac{4 + 3t}{4 - 3t} \right  + c$ $\Rightarrow I = \frac{1}{24} \log \left  \frac{4 + 3\left(\sin x + \cos x\right)}{4 - 3\left(\sin x + \cos x\right)} \right  + c$	-c ///. mathongo				
9.	$I = \int rac{dx}{\left(rac{x-1}{x+2} ight)^{rac{3}{4}}(x+2)^2}  ext{ put } rac{x-1}{x+2}$	$\frac{1}{2} = t$ mathongo				
	On solving, we get, $I=\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+c$ $k=\frac{4}{3}\Rightarrow 30 \\ k=40=2^3\times 5^1$					
10.	Hence, total divisors are $4 \times 2 = (2)^{athongo}$ mathong					
	$\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin x$ $\Rightarrow \text{ Differentiating w.r.t. } x \text{ both s}$ $\Rightarrow \frac{\sin x}{\sin(x-\alpha)} = A + \frac{B \cos(x-\alpha)}{\sin(x-\alpha)}$					
	$\sin(x-\alpha) + \sin(x-\alpha)$ $\Rightarrow \sin x = A \sin(x-\alpha) + B \cos(x-\alpha)$ $\sin x = A(\sin x \cos \alpha - \cos x)$		$(\alpha + \sin x \sin \alpha)$			
	$\sin x = \sin x (A\cos\alpha + B\sin\alpha$	$+\cos x(B\cos\alpha-A)$	$\sin \alpha$			
	Comparing coefficients of $\sin x$ $A\cos\alpha + B\sin\alpha = 1$ and $B\cos\alpha$	and $\cos x$ both side				
	Comparing coefficients of $\sin x$ $A\cos\alpha+B\sin\alpha=1$ and $B\cos\alpha$ $(A,B)=(\cos\alpha,\sin\alpha)$	and $\cos x$ both side $\sin \alpha - As \sin \alpha = 0$				
	Comparing coefficients of $\sin x$ $A\cos\alpha+B\sin\alpha=1$ and $B\cos\alpha$ $(A,B)=(\cos\alpha,\sin\alpha)$ mathongo	and $\cos x$ both side $\sin \alpha - A\sin \alpha = 0$ mathongo				
	Comparing coefficients of $\sin x$ $A\cos\alpha+B\sin\alpha=1$ and $B\cos\alpha$ $(A,B)=(\cos\alpha,\sin\alpha)$	and $\cos x$ both side $\sin \alpha - A\sin \alpha = 0$ mathongo				
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