ANSWER KEYS

1. (4)	**2.** (1)	**3.** (1)	**4.** (1)	**5.** (4)	**6.** (1)	**7.** (4)	**8.** (1)
9. (3)	**10.** (0)						

1. (4)

$gof(x) = 2e^x - 5 = y$

As gof is an invertible function, $x = gof^{-1}(y) \ldots (1)$

$2e^x - 5 = y \Rightarrow x = \ln\left(\frac{y+5}{2}\right) \ldots (2)$

From (1) & (2), $gof^{-1}(y) = \ln\left(\frac{y+5}{2}\right)$

$\Rightarrow \quad (gof)^{-1}(\mathrm{x}) = \ln\left(\frac{x+5}{2}\right)$

2. (1)

$y = f(x) = \sqrt{3}\sin x + \cos x + 4$

$= 2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) + 4$

$y = 2\sin\left(x + \frac{\pi}{6}\right) + 4 \quad \ldots (1)$

$-1 \leq \sin\left(x + \frac{\pi}{6}\right) \leq 1$

$-2 \leq 2\sin\left(x + \frac{\pi}{6}\right) \leq 2$

$-2 + 4 \leq 2\sin\left(x + \frac{\pi}{6}\right) + 4 \leq 2 + 4$

$2 \leq y \leq 6$

$y \in [2, 6] = B$

Using equation (1):

$y - 4 = 2\sin\left(x + \frac{\pi}{6}\right)$

$\sin^{-1}\left(\frac{y-4}{2}\right) - \frac{\pi}{6} = x$

$\Rightarrow -\frac{\pi}{2} \leq \sin^{-1}\left(\frac{y-4}{2}\right) \leq \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} - \frac{\pi}{6} \leq \sin^{-1}\left(\frac{y-4}{2}\right) - \frac{\pi}{6} \leq \frac{\pi}{2} - \frac{\pi}{6}$

$\Rightarrow -\frac{\pi}{2} - \frac{\pi}{6} \leq x \leq \frac{\pi}{2} - \frac{\pi}{6}$

$\Rightarrow x \in \left[\frac{-2\pi}{3}, \frac{\pi}{3}\right] = A$

3. (1) Let $y = f(t) \therefore t = f^{-1}(y)$

Now, $y = f(t) = \frac{1-t}{1+t} \Rightarrow y + ty = 1 - t$

$\Rightarrow t + ty = 1 - y \Rightarrow t = \frac{1-y}{1+y}$

i.e., $f^{-1}(y) = \frac{1-y}{1+y}$ or $f^{-1}(t) = \frac{1-t}{1+t}$

Thus, this function is inverse of itself.

4. (1) Given a set containing 10 distinct elements and $f : A \rightarrow A$ Now, every element of a set A can make an image in10 ways.

$\therefore$ The total number of ways in which each element make images $= 10^{10}$.

5. (4)

If a function is defined $f : A \rightarrow B$ such that

$n(A) = m,\ n(B) = n$

If $m \leq n$ then the total number of one-one functions $= {}^nP_m = \frac{n!}{(n-m)!}$

Total number of one-one onto function $= {}^3P_3 = 3! = 6$

6. (1)

Given, $f : (-\infty,\ 1] \rightarrow (-\infty,\ 1]$ and

$f(x) = x(2 - x)$

Let, $y = x(2 - x)$

$\Rightarrow x^2 - 2x + y = 0$

$\Rightarrow x = \frac{2 \pm 2\sqrt{1-y}}{2}$

$\Rightarrow x = 1 \pm 1\sqrt{1 - y}$

$\Rightarrow x = 1 - \sqrt{1 - y}\ \left[\because\ 1 + \sqrt{1 - y} > 1\ ,\ \mathrm{y} < 1\right]$

Now replace 'x' by 'y' and 'y' by 'x'

$\Rightarrow y = 1 - \sqrt{1 - x}$

$\Rightarrow\ f^{-1}(x) = 1 - \sqrt{1 - x}$

7. (4)

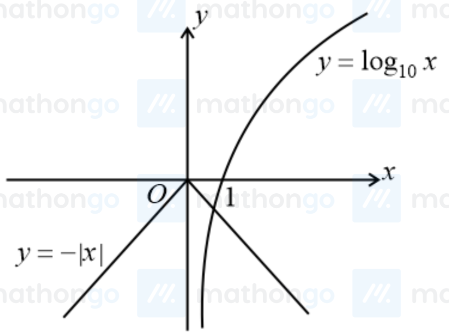
Given:

$\log_{10} x + |x| = 0$

$\Rightarrow \log_{10} x = -|x|$

We know that the graph of $\log_a x$ is monotonically increasing if $a > 1$ and graph of $-|x|$ can be drawn by taking image of $|x|$ in the x-axis as plane mirror.

By the graph of $y = -|x|$ and $y = \log_{10} x$, we have



Both graphs intersect at one point only. So, required number of solution is 1.

8. **(1)**

Given $\cot x=\frac{\pi}{2}+x,\ \ x\in\left[-\pi,\ \frac{3\pi}{2}\right]$

(i) $y=\cot x$

Domain $=R-\{n\pi,\ n\in Z\}$

Range$=(-\infty,\ \infty)$

Fundamental period is π.

$\left(\text{ii}\right)\ y=x+\frac{\pi}{2}$

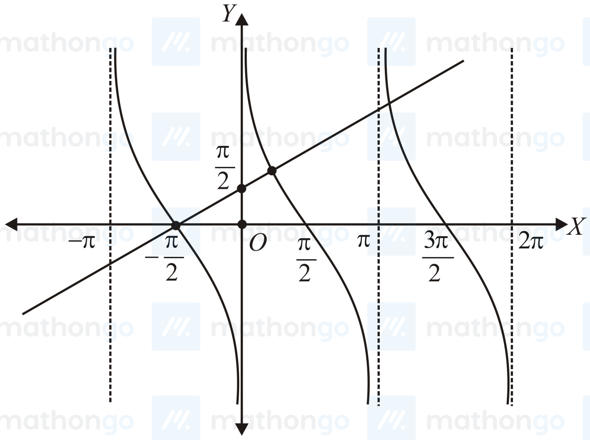
It is an equation of a line.

Slope$=1$,

at $x=0,\ \ y=\frac{\pi}{2}$

at $y=0,\ \ x=-\frac{\pi}{2}$

So combined graph



So, there are 3 intersection points for $x\in\left[-\pi,\ \frac{3\pi}{2}\right]$, which are at $x=-\frac{\pi}{2}$, $2^{\text{nd}}\in\left(0,\ \frac{\pi}{2}\right)$, $\ 3^{\text{rd}}\in\left(\pi,\ \frac{3\pi}{2}\right)$

Hence, the number of solutions$=3$.

9. **(3)** A function whose graph is symmetrical about the origin must be odd.

$\left(2^x+2^{-x}\right)$ is an even function.

Since, $\log\left(x+\sqrt{1+x^2}\right)$ is an odd function,

$\therefore\ \left[\log\left(x+\sqrt{1+x^2}\right)\right]^2$ is an even function.

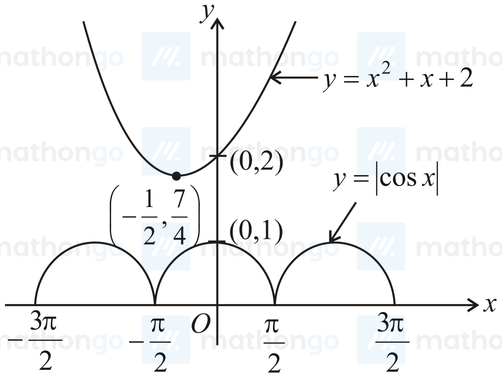
If $f(x+y)=f(x)+f(y)\forall x,y\in R$, then

Put $x=y=0\Rightarrow f(0)=0$

Now, put $y=-x\Rightarrow f(x)+f(-x)=0$

$\therefore\ f(x)$ is an odd function

10. (0)



we know, $0 \leq |\cos x| \leq 1$ and minimum value of $x^2 + x + 2$ is $\frac{-\left(1^2 - 4.1.2\right)}{4.1} = \frac{7}{4}$

$\because$ Minimum value of $x^2 + x + 2$ is greater than the maximum value of $|\cos x|$.

$\therefore$ No common value of x exist for which $|\cos x| = x^2 + x + 2$