ANSWER KEYS

1. (2)	**2.** (4)	**3.** (2)	**4.** (2)	**5.** (2)	**6.** (2)	**7.** (3)	**8.** (0.00)
9. (1)	**10.** (1)						

1. **(2)** Here, $2x \in[-1,1]$ and $\sin^{-1}(2x)+\frac{\pi}{3} \geq 0$

Hence, $x \in\left[-\frac{1}{2}, \frac{1}{2}\right]$ and $x \geq-\frac{\sqrt{3}}{4}$

So the domain is $\left[-\frac{\sqrt{3}}{4}, \frac{1}{2}\right]$

2. **(4)** $\sin^{-1} \sin 17=\sin^{-1} \sin(17-5\pi+5\pi)$

$=5\pi-17$

$\cos^{-1}(\cos 10)=\cos^{-1} \cos(10-3\pi+3\pi)$

$=\cos^{-1} \cos\{3\pi+(10-3\pi)\}$

$=\cos^{-1}\{-\cos(10-3\pi)\}$

$=\pi-\cos^{-1} \cos(10-3\pi)$

$=\pi-(10-3\pi)=4\pi-10$

Hence, $\sin^{-1} \sin 17+\cos^{-1}(\cos 10)=9\pi-27$

3. **(2)**

Given,

$\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3} \quad \ldots(i)$

and $\cos^{-1}x-\cos^{-1}y=\frac{\pi}{3} \quad \ldots(ii)$

We know that $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

Using the above concept, we can write

$\Rightarrow\left(\frac{\pi}{2}-\sin^{-1}x\right)-\left(\frac{\pi}{2}-\sin^{-1}y\right)=\frac{\pi}{3}$

$\Rightarrow \quad -\sin^{-1}x+\sin^{-1}y=\frac{\pi}{3} \quad \ldots(iii)$

On adding equation (i) and (iii), we get

$\sin^{-1}y=\frac{\pi}{2} \Rightarrow y=1$

On subtracting equation (i) from equation (iii), we get

$\sin^{-1}x=\frac{\pi}{6} \Rightarrow x=\frac{1}{2}$

$\therefore \quad (x, y)=\left(\frac{1}{2}, 1\right)$

4. **(2)**

As we know that $\sin^{-1} x+\cos^{-1} x=\frac{\pi}{2}, \quad -1 \leq x \leq 1$.

Therefore, $\sin\left\{\sin^{-1} \frac{1}{2}+\cos^{-1} \frac{1}{2}\right\}=\sin\left(\frac{\pi}{2}\right)=1$.

5. **(2)** Given, $4 \sin^{-1} x+\cos^{-1} x=\pi$

$\Rightarrow 4 \sin^{-1} x+\frac{\pi}{2}-\sin^{-1} x=\pi$

$\Rightarrow \quad 3 \sin^{-1} x=\frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1} x=\frac{\pi}{6}$

$\Rightarrow \quad x=\frac{1}{2}$

6. **(2)**

$\cot\left(\operatorname{cosec}^{-1} \frac{5}{3}+\tan^{-1} \frac{2}{3}\right)$

$=\cot\left(\tan^{-1} \frac{3}{4}+\tan^{-1} \frac{2}{3}\right)$

$=\cot\left(\tan^{-1} \frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4} \frac{2}{3}}\right)$

$=\cot\left(\tan^{-1} \frac{\frac{17}{12}}{\frac{1}{2}}\right)$

$=\cot\left(\tan^{-1} \frac{17}{6}\right)$

$=\frac{6}{17}$.

7. **(3)**

Let $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

$\therefore \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)-\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$=3\tan^{-1}x-2\tan^{-1}x, |x|<\frac{1}{\sqrt{3}}$

$=\tan^{-1}x$

8. **(0.00)** $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$

$\Rightarrow \sin^{-1}x=\sin^{-1}y=\sin^{-1}z=\frac{\pi}{2}$

$\Rightarrow \cos^{-1}x=\cos^{-1}y=\cos^{-1}z=0$

9. **(1)** $\because \tan^{-1}\left(\frac{1}{1+r+r^2}\right)=\tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)$

$=\tan^{-1}(r+1)-\tan^{-1}(r)$

$\therefore \sum_{r=0}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]$

$=\tan^{-1}(n+1)-\tan^{-1}(0)$

$=\tan^{-1}(n+1)$

$\Rightarrow \sum_{r=0}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)=\tan^{-1}(\infty)=\frac{\pi}{2}$

10. **(1)**

Let $T_r=\sum_{r=1}^{\infty}\cot^{-1}\left(3r^2-r-\frac{1}{3}\right)=\tan^{-1}\left(\frac{3}{9r^2-3r-1}\right)$

$=\tan^{-1}\left(\frac{(3r+1)-(3r-2)}{1+(3r+1)(3r-2)}\right)$

$\sum_{r=1}^{\infty}T_r=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}=\tan^{-1}1=\cot^{-1}1$