ANSWER KEYS

1. (2)	**2.** (2)	**3.** (269)	**4.** (151)	**5.** (3)	**6.** (4)	**7.** (6)	**8.** (211)
9. (4)	**10.** (25)	**11.** (2)	**12.** (37)	**13.** (238)	**14.** (3)	**15.** (4)	**16.** (4)
17. (5)	**18.** (164)	**19.** (2)	**20.** (1)				

1. **(2)**

Since, number of observations are even $n=10$

Hence, the median is $=\frac{\left(\frac{n}{2}\right)^{th} term+\left(\frac{n}{2}+1\right)^{th} term}{2}=\frac{5^{th} term+6^{th} term}{2}$

Thus, median $=\frac{34+x}{2}$

$\Rightarrow \frac{34+x}{2}=35$

$\Rightarrow x=36 \quad \ldots(i)$

And, mean$=\frac{sum\ of\ terms}{number\ of\ terms}$

$\Rightarrow \frac{10+22+26+29+34+x+42+67+70+y}{10}=42$

$\Rightarrow x+y+300=420$

$\Rightarrow x+y=120 \quad \ldots(ii)$

Put the value of x from (i) in (ii), to get

$36+y=120$

$\Rightarrow y=84$

$\Rightarrow \frac{y}{x}=\frac{84}{36}=\frac{7}{3}$.

2. **(2)**

$\mu=20, \sigma=8$

$\mu_{\text{Corrected}}=\frac{200-50+40}{10}=19$

$\sigma^2=\frac{1}{10}\sum x_i^2-20^2$

$(64+400)10=\sum x_i^2$

$\sigma_{\text{carrected}}^2=\frac{1}{10}[(64+400)10-2500+1600]-19^2$

$=374-361$

$=13$

3. **(269)** $\bar{x}=50$

$\sum x_i=500$

$\sum x_{i_{\text{correct}}}=500+20+25-45-50=450$

$\sigma^2=144$

$\frac{\sum x_i^2}{10}-(50)^2=144$

$\sum x_{\text{icorrect}}^2=\left(144+(50)^2\right)\times 10-(45)^2-(50)^2+(20)^2+(25)^2$

$=22940$

Correct variance $=\frac{\sum\left(x_{\text{icorrect}}\right)^2}{10}-\left(\frac{\sum x_{\text{icorred}}}{10}\right)^2$

$=2294-(45)^2$

$=2294-2025=269$

4. **(151)** Given mean is $=28$

$\frac{2\times 5+3\times 15+x\times 25+5\times 35+4\times 45}{14+x}=28$

x $=6$

Variance $=\left(\frac{\sum x_i^2 f_i}{\sum f_i}\right)-(\text{ mean })^2$

Variance $==\frac{2\times 5^2+3\times 15^2+6\times 25^2+5\times 35^2+4\times 45^2}{20}-(28)^2$

$=151$

5. (3)

Given,

a_1 be any natural number

So, $a_1, a_1+1, a_1+2, \ldots, a_1+99$ are values of a_i or observation.

Now mean of the observation is given by,

$\bar{x}=\frac{a_1+(a_1+1)+(a_1+2)+\ldots+a_1+99}{100}$

$\Rightarrow \bar{x}=\frac{100a_1+(1+2+\ldots+99)}{100}$

$\Rightarrow \bar{x}=a_1+\frac{99\times100}{2\times100}$

$\Rightarrow \bar{x}=a_1+\frac{99}{2}$

Mean deviation about mean$=\frac{\sum_{i=1}^{100}|x_i-\bar{x}|}{100}$

$=\frac{2\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\ldots+\frac{1}{2}\right)}{100}$

$=\frac{1+3+\ldots+99}{100}$

$=\frac{\frac{50}{2}[1+99]}{100}$

$=25$

So, it is true for every natural no. ' a_1 ' as there is no term of a_1 in mean deviation.

6. (4)

Median is the central value (or middle observation) of statistical data if it is arranged in ascending or descending order.

Thus, if there are n observations (variates) $x_1, x_2, x_3, \ldots, x_n$ arranged in ascending or descending order, then

median $=\left(\frac{n+1}{2}\right)^{\text{th}}$ observation, if n is odd

and median $=\frac{\frac{n}{2}\text{ th observation}+\left(\frac{n}{2}+1\right)\text{ th observation}}{2}$, if n is even.

Here we can see there are even number of terms,

So, median $=\frac{2k+12}{2}=k+6$

Mean deviation $=\sum\frac{|x_i-M|}{n}=6$

$\Rightarrow \frac{(k+3)+(k+1)+(k-1)+(6-k)+(6-k)+(10-k)+(15-k)+(18-k)}{8}=6$

$\Rightarrow \frac{58-2k}{8}=6$

$\Rightarrow k=5$

So, median $=\frac{2\times5+12}{2}=11$

7. (6)

x_i (observation)	0	2	2^2................	2^n
f_i (frequency)	${}^nC_0$	${}^nC_1$	${}^nC_2$............	${}^nC_n$

$\bar{x}=\frac{\Sigma f_i x_i}{\Sigma f_i}$

$\frac{0\times{}^nC_0+2\times{}^nC_1+2^2\times{}^nC_2\ldots..2^n\times{}^nC_n}{{}^nC_0+{}^nC_1+{}^nC_2\ldots..+{}^nC_n}=\frac{3^n-1}{2^n}=\frac{728}{2^n}$

$\Rightarrow\ 3^n=3^6$

$\Rightarrow n=6$

8. (211) Let $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Given $\frac{a}{r^2}+\frac{a}{r}+a+ar+ar^2=5\times\frac{31}{10}$

And $\frac{r^2}{a}+\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}=5\times\frac{31}{40}$

$a^2=4 \Rightarrow a=2 \quad \therefore r+\frac{1}{r}=5/2$

$\Rightarrow \mathrm{r}=2$

$\therefore$ Now $\frac{1}{2}, 1, 2.4, 8$

$\therefore \sigma^2=\frac{\sum_{x^2}}{N}-\left(\frac{\sum \mathrm{x}}{\mathrm{N}}\right)^2$

$=\frac{186}{25}=\frac{M}{N} \Rightarrow 211=\mathrm{m}+\mathrm{n}$

9. **(4)**

Given,

The mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively,

So, mean will be,

$\bar{x}=\frac{x_1+x_2+\ldots\ldots+9+10+\ldots\ldots+x_{12}}{12}=\frac{9}{2}$

$\Rightarrow x_1+x_2\ldots\ldots+x_{12}+19=54$

Now removing the observation 9 & 10 and adding the observation 7 & 14,

So, the new total sum of the observation will be,

$x_1+x_2+\ldots\ldots+x_{12}+7+14=54-19+7+14$

Now new mean will be,

$\frac{x_1+x_2+\ldots+x_{12}+7+14}{12}=\frac{56}{12}$

$\Rightarrow \bar{x}_{\text{new}}=\frac{14}{3}$

Now using the formula of the variance we get,

$\frac{x_1^2+x_2^2+\ldots\ldots+x_{12}^2+9^2+10^2}{12}-\left(\frac{9}{2}\right)^2=4$

$\Rightarrow x_1^2+x_2^2+\ldots\ldots+x_{12}^2+9^2+10^2-81\times 3=4\times 12$

$\Rightarrow x_1^2+x_2^2+\ldots\ldots+x_{12}^2+9^2+10^2=291$

Now removing 9^2 & 10^2 and adding 7^2 & 14^2 we get,

$\Rightarrow x_1^2+x_2^2+\ldots\ldots+x_{12}^2+7^2+14^2=355$

New variance $=\frac{\sum x_i^2}{N}-\left(\bar{x}\right)^2$

$=\frac{355}{12}-\left(\frac{14}{3}\right)^2$

$=\frac{281}{36}$

Now on comparing with $\frac{m}{n}=\frac{281}{36}$ we get,

$\Rightarrow m=281$ and $n=36$

$\therefore\ m+n=317$

10. **(25)**

x_i	f_i	f_ix_i	$f_ix_i^2$
2	4	8	16
4	4	16	64
6	α	6α	36α
8	15	120	960
10	8	80	800
12	β	12β	144β
14	4	56	784
16	5	80	1280

$N=\sum f_i=40+\alpha+\beta$

$\sum f_ix_i=360+6\alpha+12\beta$

$\sum f_ix_i^2=3904+36\alpha+144\beta$

$\text{Mean}(\bar{x})=\frac{\sum f_ix_i}{\sum f_i}=9$

$\Rightarrow 360+6\alpha+12\beta=9(40+\alpha+\beta)$

$3\alpha=3\beta\Rightarrow\alpha=\beta$

$\sigma^2=\frac{\sum f_ix_i^2}{\sum f_i}-\left(\frac{\sum f_ix_i}{\sum f_i}\right)^2$

$\Rightarrow\frac{3904+36\alpha+144\beta}{40+\alpha+\beta}-(\bar{x})^2=15.08$

$\Rightarrow\frac{3904+180\alpha}{40+2\alpha}-(9)^2=15.08$

$\Rightarrow\alpha=5$

Now, $\alpha^2+\beta^2-\alpha\beta=\alpha^2=25$

11. (2)

Given,

$9 = x_1 < x_2 < \ldots < x_7$ be in an A.P. with common difference d

And the standard deviation of $x_1, x_2 \ldots, x_7$ is 4 and the mean is $\bar{x}$,

Now solving, $9 = x_1 < x_2 < \ldots\ldots < x_7$ which is an A.P, we get,

$9,\ 9+d,\ 9+2d, \ldots\ldots\ldots 9+6d$

Now subtracting 9 from the series we get,

$0,\ d,\ 2d, \ldots\ldots . 6d$

So, mean will be $\bar{x}_{\text{new}} = \frac{21d}{7} = 3d$

Now using the formula of variance we get,

$\sigma^2 = \sum_{i=1}^{n} (x_i)^2 - (\bar{x})^2$

$\Rightarrow 16 = \frac{1}{7}\left(0^2 + 1^2 + \ldots\ldots\ldots + 6^2\right)d^2 - 9d^2$

$\Rightarrow 16 = \frac{1}{7}\left(\frac{6\times7\times13}{6}\right)d^2 - 9d^2$

$\Rightarrow 16 = 4d^2$

$\Rightarrow d^2 = 4$

$\Rightarrow d = 2$

So, mean $\bar{x} = \frac{9+9+d+9+2d\ldots\ldots\ldots 9+6d}{7} = 15$

Hence, $\bar{x} + x_6 = 15 + 9 + 5d = 15 + 9 + 10 = 34$

12. (37)

Given,

Mean and variance of 7 observation is given as 8 & 16 respectively,

So using the formula of mean we get, $\bar{x} = \frac{S}{7} \Rightarrow S = 8 \times 7 = 56$

Now if we remove 14 sum will be $S = 56 - 14 = 42$ and new mean will be $a = \frac{42}{6} = 7$

Now using the formula of variance we get,

$\frac{\sum\limits_{i=1}^{6} x_i^2 + 14^2}{7} - (\bar{x})^2 = 16$

$\Rightarrow \frac{\sum\limits_{i=1}^{6} x_i^2 + 14^2}{7} - 8^2 = 16$

$\Rightarrow \frac{\sum\limits_{i=1}^{6} x_i^2 + 14^2}{7} = 80$

$\Rightarrow \sum_{i=1}^{6} x_i^2 + 14^2 = 560$

$\Rightarrow \sum_{i=1}^{6} x_i = 364$

Now, new variance $b = \frac{\sum_{i=1}^{6} x_i^2}{6} - a^2 = \frac{364}{6} - 7^2 = \frac{35}{3}$

Hence, the value of $a + 3b - 5 = 7 + 35 - 5 = 37$

13. (238)

Given,

Wrong mean $= \mu_1 = 30$

Wrong standard deviation $= \sigma_1 = 5$

Now mean ig given by $\frac{\sum x_i}{40} = 30$

$\Rightarrow \sum x_i = 1200$

Variance is given by $\sigma_1^2 = 25$

$\Rightarrow \frac{\sum x_i^2}{40} - 30^2 = 25$

$\Rightarrow \sum x_i^2 = 925 \times 40 = 37000$

New sum of mean $= \sum x_i' = 1200 - 10 - 12 = 1178$

So, new mean $= \mu_1' = \frac{1178}{38} = 31$

Now new variance $\sum x_i^2 = 37000 - (10)^2 - (12)^2 = 36756$

So, new S.D. $\sigma_1' = \sqrt{\frac{36756}{38} - (31)^2} = \sigma$

$36756 - (31)^2 \times 38 = 38\sigma^2$

$\Rightarrow 38\sigma^2 = 238$

14. (3)

We have, number of terms in the first group $n_1=100$ with their mean $\overline{X}_1=15$ and number of terms in whole group is $n_1+n_2=250$ and their mean $\overline{X}=15.6$.

Variance of first group $V_1(x)=9$ and whole group is $\operatorname{Var}(x)=13.44$.

Now, $\sigma^2=\frac{n_1\sigma_1^2+n_2\sigma_2^2}{n_1+n_2}+\frac{n_1n_2}{(n_1+n_2)^2}(\bar{x}_1-\bar{x}_2)^2$

and $n_2=150, \bar{x}_2=16, V_2(x)=\sigma_2$

Therefore, $13.44=\frac{100\times 9+150\times\sigma_2^2}{250}+\frac{100\times 150}{(250)^2}\times 1$

$\Rightarrow \sigma_2^2=16$

$\Rightarrow \sigma_2=4$

15. (4)

SD, $\sigma=\sqrt{\frac{\sum x_i^2}{n}-\left(\frac{\sum x_i}{n}\right)^2}$

SD is independent of change of origin, so, for given data

$\sigma=\sqrt{\frac{\Sigma(x_i-a)^2}{n}-\left(\frac{\Sigma(x_i-a)}{n}\right)^2}$

$=\sqrt{\frac{na}{n}-\left(\frac{n}{n}\right)^2}=\sqrt{a-1}$

16. (4)

$\sum_{i=1}^{18}(x_i-\alpha)=36, \sum_{i=1}^{18}(x_i-\beta)^2=90$

$\Rightarrow \sum_{i=1}^{18}x_i=18(\alpha+2), \sum_{i=1}^{18}x_i^2-2\beta\sum_{i=1}^{18}x_i+18\beta^2=90$

Hence $\sum x_i^2=90-18\beta^2+36\beta(\alpha+2)$

Given $\frac{\sum x_i^2}{18}-\left(\frac{\sum x_i}{18}\right)^2=1$

$\Rightarrow 90-18\beta^2+36\beta(\alpha+2)-18(\alpha+2)^2=18$

$\Rightarrow 5-\beta^2+2\alpha\beta+4\beta-\alpha^2-4\alpha-4=1$

$\Rightarrow(\alpha-\beta)^2+4(\alpha-\beta)=0 \Rightarrow|\alpha-\beta|=0$ or 4

As α and β are distinct $|\alpha-\beta|=4$

17. (5)

$\sigma^2=\frac{n_1\sigma_1^2+n_2\sigma_2^2}{n_1+n_2}+\frac{n_1n_2}{(n_1+n_2)}(\bar{x}_1-\bar{x}_2)^2$

$n_1=10, n_2=n, \sigma_1^2=2, \sigma_2^2=1$

$\bar{x}_1=2, \bar{x}_2=3, \sigma^2=\frac{17}{9}$

$\frac{17}{9}=\frac{10\times 2+n}{n+10}+\frac{10n}{(n+10)^2}(3-2)^2$

$\Rightarrow \frac{17}{9}=\frac{(n+20)(n+10)+10n}{(n+10)^2}$

$\Rightarrow 17n^2+1700+340n=90n+9\left(n^2+30n+200\right)$

$\Rightarrow 8n^2-20n-100=0$

$2n^2-5n-25=0$

$\Rightarrow(2n+5)(n-5)=0 \Rightarrow n=\underset{(\text{Rejected})}{\underset{\downarrow}{\frac{-5}{2}}}, 5$

Hence $n=5$

18. (164)

$\because$ Sum of frequencies $=584$

$\alpha+110+54+30+\beta=584$

$\Rightarrow \alpha+\beta=390 \quad \ldots(1)$

Now, Median is at $\frac{584}{2}=292^{\text{th}}$ term

$\because$ Median $=45$ (lies in class $40-50$)

$\Rightarrow \alpha+110+54+15=292$

$\Rightarrow \alpha=113 \quad \ldots(2)$

On solving, equations (1) and(2), we get

$\Rightarrow \alpha=113$ & $\beta=390-113=277$

$\Rightarrow|\alpha-\beta|=164$

19. (2)

Sol. $\sum f_i = 62$

$\Rightarrow \quad 3k^2 + 16k - 12k - 64 = 0$

$\Rightarrow \quad k = \text{ or } -\frac{16}{3} \text{ (rejected}\Big)$

$\mu = \frac{\sum f_i x_i}{\sum f_i}$

$\mu = \frac{8 + 2(15) + 3(15) + 4(17) + 5}{62} = \frac{156}{62}$

$\sigma^2 = \sum \mathrm{f_i x_i^2} - \left(\sum \mathrm{f_i x_i}\right)^2$

$= \frac{8 \times 1^2 + 15 \times 13 + 17 \times 16 + 25}{62} - \left(\frac{156}{62}\right)^2$

$\sigma^2 = \frac{500}{62} - \left(\frac{156}{62}\right)^2$

$\sigma^2 + \mu^2 = \frac{500}{62}$

$[\sigma^2 + \mu^2] = 8$

20. (1)

Let the mean is denoted by $\overline{x_i}$, variance by ${\sigma_i}^2$, so standard deviation will be σ_i, now as per given data we get,

A	B	$A+B$
$\bar{x}_1 = 40$	$\bar{x}_2 = 55$	$\bar{x} = 50$
$\sigma_1 = \alpha$	$\sigma_2 = 30 - \alpha$	$\sigma_2 = 350$
$n_1 = 100$	$n_2 = n$	$100 + n$

So, using the formula of mean in $A + B$ we get,

$\bar{x} = \frac{100 \times 40 + 55n}{100 + n} = 50$

$\Rightarrow \quad n = 200$

Now using the formula of variance we get,

$\sigma_1^2 = \frac{\sum x_i^2}{100} - 40^2 = \alpha^2$

And $\sigma_2^2 = \frac{\sum y_i^2}{200} - 55^2 = (30 - \alpha)^2$

So, variance of $A + B$ will be,

$\sigma^2 = \frac{\sum x_i^2 + \sum y_i^2}{300} - 50^2 = 350^2$

$\Rightarrow 350^2 = \frac{(1600 + \alpha^2) \times 100 + \left[3025 + (30 - \alpha)^2\right] \times 200}{300} - 50^2$

$\Rightarrow \alpha^2 - 40\alpha + 300 = 0$

$\Rightarrow \alpha = 10 \text{ or } 30$

And $\alpha = 30$ is not possible

So, ${\sigma_1}^2 + {\sigma_2}^2 = 10^2 + 20^2 = 500$