## ANSWER KEYS

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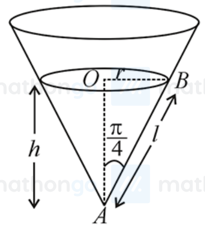
|---|---|---|---|---|---|---|---|
| **1.** (1) | **2.** (4) | **3.** (1) | **4.** (3) | **5.** (3) | **6.** (2) | **7.** (2) | **8.** (3) |
| **9.** (1) | **10.** (3) | | | | | | |

**1. (1)**




Let at any time $t, h, r$, and $l$ be the height, radius and slant height of cone respectively. Let $V$ be the volume of cone then

$\frac{dV}{dt}=-2$

Now, in $\Delta AOB$,

$\frac{r}{l}=\sin 45^{\circ}$

$\Rightarrow r=\frac{l}{\sqrt{2}}$

And,

$\frac{h}{l}=\cos 45^{\circ}$

$\Rightarrow h=\frac{l}{\sqrt{2}}$

Now, volume of cone is

$V=\frac{1}{3}\pi r^2 h$

$\Rightarrow V=\frac{1}{3}\pi\left(\frac{l^2}{2}\right)\left(\frac{l}{\sqrt{2}}\right)$

$\Rightarrow V=\frac{\pi}{6\sqrt{2}}l^3$

Differentiating both sides w.r.t. $t$, we get

$\Rightarrow \frac{dV}{dt}=\frac{\pi}{6\sqrt{2}}\left(3l^2\right)\frac{dl}{dt}$

$\Rightarrow -2=\frac{\pi}{2\sqrt{2}}\left(l^2\right)\frac{dl}{dt}$

$\Rightarrow \frac{dl}{dt}=\left(\frac{-4\sqrt{2}}{\pi}\right)\frac{1}{l^2}$

$\Rightarrow \left(\frac{dl}{dt}\right)_{l=4}=\left(\frac{-4\sqrt{2}}{\pi}\right)\frac{1}{\left(4^2\right)}$

$\Rightarrow \left(\frac{dl}{dt}\right)_{l=4}=-\frac{\sqrt{2}}{4\pi}\frac{\mathrm{cm}}{\mathrm{s}}$

**2. (4)** Given, $\mathrm{V}=\pi \mathrm{r}^2 \mathrm{h}$

Differentiating both sides, we get

$\frac{\mathrm{dV}}{\mathrm{dt}}=\pi\left(\mathrm{r}^2\frac{\mathrm{dh}}{\mathrm{dt}}+2\mathrm{r}\frac{\mathrm{dr}}{\mathrm{dt}}\mathrm{h}\right)=\pi \mathrm{r}\left(\mathrm{r}\frac{\mathrm{dh}}{\mathrm{dt}}+2\mathrm{h}\frac{\mathrm{dr}}{\mathrm{dt}}\right)$

$\frac{\mathrm{dr}}{\mathrm{dt}}=\frac{1}{10}$ and $\frac{\mathrm{dh}}{\mathrm{dt}}=-\frac{2}{10}$

$\frac{\mathrm{dV}}{\mathrm{dt}}=\pi \mathrm{r}\left(\mathrm{r}\left(-\frac{2}{10}\right)+2\mathrm{h}\left(\frac{1}{10}\right)\right)=\frac{\pi \mathrm{r}}{5}(-\mathrm{r}+\mathrm{h})$

Thus, when r = 2 and h = 3.

$\frac{\mathrm{dV}}{\mathrm{dt}}=\frac{\pi(2)}{5}(-2+3)=\frac{2\pi}{5}$

**3. (1)** Given, $s=t^3+2t^2+t$

$\Rightarrow \quad v=\frac{ds}{dt}=3t^2+4t+1$

Speed of the particle after 1 sec

$v_{(t=1)}=\left(\frac{ds}{dt}\right)_{(t=1)}$

$=3\times 1^2+4\times 1+1=3+5=8$ cm/s

**4. (3)**

Let $r$ be the base radius and $h$ be the height of the cone. Then,

$\frac{r}{h} = \frac{1}{2}$

$\Rightarrow 2r = h$

Let $V$ be the volume of the cone, then

$V = \frac{1}{3}\pi r^2 h = \frac{2\pi r^3}{3}$

Differentiating both sides w.r.t. $r$, we get

$\Rightarrow \ \frac{dV}{dr} = 2\pi r^2$

Now,

$\Delta V = \frac{dV}{dr}\Delta r$

$\Rightarrow \Delta V = 2\pi r^2 \Delta r$

$\Rightarrow \frac{\Delta V}{V} = \frac{2\pi r^2 \Delta r}{\frac{2\pi r^3}{3}}$

$\Rightarrow \ \frac{\Delta V}{V} \times 100 = \frac{2\pi r^2 \Delta r}{\frac{2}{3}\pi r^3} \times 100$

$\Rightarrow \frac{\Delta V}{V} \times 100 = 3\left(\frac{\Delta r}{r} \times 100\right) = 3\lambda\%$

**5. (3)**

Given:

$\Rightarrow f(x) = \sin x - \cos x - kx + b \Rightarrow f'(x) = \cos x + \sin x - k$

For decreasing function $f'(x) \ < \ 0$

So, $\cos\ x \ + \ \sin\ x \ - \ k \ < \ 0$

The maximum value of $\cos\ x \ + \ \sin\ x$ is $\sqrt{2}$.

$\Rightarrow \sqrt{2} - k < 0$

$\Rightarrow k > \sqrt{2}$

**6. (2)** $f(x) = x^4 - 4x^3 + 4x^2 + 40$

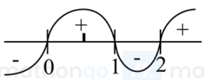
$f'(x) = 4x^3 - 12x^2 + 8x$

For monotonic decreasing $f'(x) < 0$

$\Rightarrow x\left(4x^2 - 12x + 8\right) < 0$

$\Rightarrow x\left(x^2 - 3x + 2\right) < 0$

$\Rightarrow x(x-1)(x-2) < 0$




$\Rightarrow x \in (-\infty,\ 0) \cup (1,\ 2)$

**7. (2)** $(-\infty, \infty)$

**8. (3)** $h(x) = fog(x) = e^{g(x)} - g(x)$

$= e^{x^2 - x} - \left(x^2 - x\right)$

$h'(x) = e^{x^2-x} \times (2x-1) - 2x + 1$

$= (2x-1)\left(e^{x^2-x} - 1\right)$

$\because h(x)$ is increasing $\therefore h'(x) \geq 0 \Rightarrow (2x-1)\left(e^{x^2-x} - 1\right) \geq 0$

Case 1: $(2x-1) \geq 0$ & $x^2 - x \geq 0$

$\Rightarrow x \geq \frac{1}{2}$ & $x \in (-\infty,\ 0] \cup [1, \infty) \Rightarrow x \in [1,\ \infty)$

Case 2: $(2x-1) \leq 0$ and $x^2 - x \leq 0$

$x < \frac{1}{2}$ and $x \in (0,\ 1) \Rightarrow x \in \left(0, \frac{1}{2}\right]$

$\therefore \ \ x \in \left[0, \frac{1}{2}\right] \cup \left[1, \infty\right)$

**9.** **(1)**

We have, $f(x) = xe^{x(1-x)}$

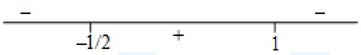
On differentiating, we get

$f'(x) = e^{x(1-x)} + xe^{x(1-x)}(1-2x)$

$\Rightarrow f'(x) = e^{x(1-x)}\left(1 + x - 2x^2\right)$

$\Rightarrow f'(x) = -e^{x(1-x)}(2x+1)(x-1)$

Wavy curve Method sign of $f'(x)$




$f(x)$ is increasing in $\left[-\frac{1}{2}, 1\right]$

$f(x)$ is decreasing in $\left(-\infty, -\frac{1}{2}\right] \cup \left[1, \infty\right)$.

**10.** **(3)**

Given:

$\Rightarrow f(x) = \sin x - \cos x - kx + b \Rightarrow f'(x) = \cos x + \sin x - k$

For decreasing function $f'(x) < 0$

So, $\cos x + \sin x - k < 0$

The maximum value of $\cos x + \sin x$ is $\sqrt{2}$.

$\Rightarrow \sqrt{2} - k < 0$

$\Rightarrow k > \sqrt{2}$