ANSWER KEYS

1. (2)	**2.** (1)	**3.** (2)	**4.** (2)	**5.** (1)	**6.** (1)	**7.** (3)	**8.** (1)
9. (3)	**10.** (2)						

1. **(2)** $\because A^{-1}=\frac{1}{|A|}(adj(A))$

$\therefore adj(adjA)=|adj\ A|(adj\ A)^{-1}=|A|^2(adj\ A)^{-1}\ldots\ldots(\text{i})$

$\because A(adj\ A)=|A|\mathrm{I}$

$\therefore adj(A)=|A|A^{-1}$

$\therefore (adj\ A)^{-1}=\frac{1}{|A|}A$

$\therefore$ from (i)

$adj(adj\ A)=|A|.\ A$

2. **(1)** $|adj\ (adj\ A)|=|A|^{(n-1)^2}$

$=(5)^{(3-1)^2}=5^4=625$

3. **(2)**

Given, $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$

$\Rightarrow\ A^2=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$

$\Rightarrow\ A^2=\begin{bmatrix}4+12 & -6-3\\ -8-4 & 12+1\end{bmatrix}$

$\Rightarrow\ A^2=\begin{bmatrix}16 & -9\\ -12 & 13\end{bmatrix}$

$\Rightarrow\ 3A^2=\begin{bmatrix}48 & -27\\ -36 & 39\end{bmatrix}$ and $12A=\begin{bmatrix}24 & -36\\ -48 & 12\end{bmatrix}$

$\therefore\ (3A^2+12A)=\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$

$\mathrm{Adj}(3A^2+12A)=\begin{bmatrix}51 & 84\\ 63 & 72\end{bmatrix}^T=\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$

4. **(2)**

$|A^{-1}adjB^{-1}adj(3A^{-1})|=|A|^{-1}|adjB^{-1}||adj(3A^{-1})|$

$=\frac{1}{|A|}|B^{-1}|^2\times|3A^{-1}|^2$

$=\frac{1}{|A|}\times\frac{1}{|B|^2}\times\frac{3^6}{|A|^2}$

$=\frac{3^6}{3^3\times2^2}=\frac{27}{4}$

5. **(1)** $\because\ |A|=1$

and $A^c=\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

and adj $A=(A^c)'=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$

$\therefore\ A^{-1}=\frac{\text{adj }A}{|A|}=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}=f(-x)$

6. **(1)**

$A^{-1}=\frac{1}{1+10}\begin{bmatrix}1 & -2\\ 5 & 1\end{bmatrix}=\frac{1}{11}\begin{bmatrix}1 & -2\\ 5 & 1\end{bmatrix}$

Also, $A^{-1}=xA+yI$

$\Rightarrow\ \frac{1}{11}\begin{bmatrix}1 & -2\\ 5 & 1\end{bmatrix}=\begin{bmatrix}x & 2x\\ -5x & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 0 & y\end{bmatrix}$

$\Rightarrow\ x+y=\frac{1}{11},\ 2x=\frac{-2}{11}$

$\Rightarrow\ x=\frac{-1}{11},\ \ y=\frac{2}{11}$

7. **(3)**
Given $A+B=2I$ ... (i)
and $A^{-1}+B^{-1}=3I$ ...(ii)
Multiplying by matrix A in eq. (ii), we get,
$AA^{-1}+AB^{-1}=3AI$
$I+AB^{-1}=3A$
Now, multiplying by matrix B, we get,
$IB+AB^{-1}B=3AB$
$\Rightarrow B+A=3AB$
$\Rightarrow 3AB=2I$ (from (i))
$\Rightarrow AB=\frac{2I}{3}$

8. **(1)**
Given,
$\left[A(A+B)^{-1}\cdot B\right]^{-1}\cdot\left(AB\right)$
$\left[A(A+B)^{-1}\cdot B\right]^{-1}\cdot\left(AB\right)=(B^{-1}(A+B)\cdot A^{-1})\left(AB\right)$
$\left[A(A+B)^{-1}\cdot B\right]^{-1}\cdot\left(AB\right)=B^{-1}\left(A+B\right)\cdot B\,(\because A^{-1}A=I)$
$\left[A(A+B)^{-1}\cdot B\right]^{-1}\cdot\left(AB\right)=B^{-1}AB+B^{-1}B\cdot B$
$\left[A(A+B)^{-1}\cdot B\right]^{-1}\cdot\left(AB\right)=B^{-1}BA+B^{-1}B\cdot B\,(\because AB=BA$ as A and B commute)
$\left[A(A+B)^{-1}\cdot B\right]^{-1}\cdot\left(AB\right)=A+B\,(\because B^{-1}B=I)$

9. **(3)** $\because \mathrm{B}=\mathrm{CAC}^{-1}$
$\Rightarrow BC=CA$
$\Rightarrow C^{-1}BC=A$
$\therefore CA^3C^{-1}=CAAAC^{-1}$
$=\mathrm{C}(\mathrm{C}^{-1}\mathrm{BC})(\mathrm{C}^{-1}\mathrm{BC})(\mathrm{C}^{-1}\mathrm{BC})\mathrm{C}^{-1}$
$=\mathrm{B(I)(B)I(B)I}=\mathrm{B}^3$

10. **(2)**
Given, $\mathrm{Z}=\mathrm{PQ}^{-1}$
$|\mathrm{P}|=-5,\ |\mathrm{Z}|=10$
$|\mathrm{Z}|=\frac{|\mathrm{P}|}{|\mathrm{Q}|}\Rightarrow|\mathrm{Q}|=\frac{-5}{10}=-\frac{1}{2}$
$Z|\mathrm{Q}|=\mathrm{P.\,adj(Q)}\Rightarrow\mathrm{P.\,adj(Q)}=-\frac{1}{2}\mathrm{Z}$
$\mathrm{Tr((adjQ).\,P)=Tr(P.\,adjQ)}$
$=\left(-\frac{1}{2}\mathrm{Tr}\left(\mathrm{Z}\right)\right)$
$=-1$