

**ANSWER KEYS**

1. (2)      2. (4)      3. (9)      4. (1)      5. (1)      6. (3)      7. (4)      8. (4)  
 9. (1)      10. (1)

1. (2)

Mean is the ratio of sum of all the observations to total number of observations.

$$\text{Here, mean, } \bar{x} = \frac{6+8+12+15+10+9}{6} = 10$$

The arithmetic average of the deviations (all taking positive) from the mean, median or mode is known as mean deviation.

$$\begin{aligned} \therefore \text{Mean deviation from mean} &= \frac{\sum |x_i - \bar{x}|}{n} \\ &= \frac{|6-10| + |8-10| + |12-10| + |15-10| + |10-10| + |9-10|}{6} \\ &= \frac{4+2+2+5+0+1}{6} = \frac{14}{6} = 2.33 \end{aligned}$$

2. (4) We have,  $\sum_{i=1}^n (x_i + 1)^2 = 11n \dots \dots (i)$   
 and  $\sum_{i=1}^n (x_i - 1)^2 = 7n \dots \dots (ii)$

Adding (i) and (ii), we get

$$\begin{aligned} 2 \sum_{i=1}^n (x_i^2 + 1) &= 18n \\ \Rightarrow \sum_{i=1}^n (x_i^2 + 1) &= 9n \end{aligned}$$

$$\begin{aligned} \Rightarrow \sum_{i=1}^n x_i^2 + n &= 9n \\ \Rightarrow \sum_{i=1}^n x_i^2 &= 8n \end{aligned}$$

$$\Rightarrow \frac{\sum_{i=1}^n x_i^2}{n} = 8$$

Subtracting (i) and (ii), we get

$$\begin{aligned} 4 \sum_{i=1}^n x_i &= 4n \Rightarrow \sum_{i=1}^n x_i = n \\ \Rightarrow \frac{\sum_{i=1}^n x_i}{n} &= 1 \end{aligned}$$

$$\text{Now, variance} = \frac{1}{n} \left[ \sum_{i=1}^n x_i^2 \right] - \left[ \frac{\sum_{i=1}^n x_i}{n} \right]^2 = 8 - 1 = 7$$

3. (9)

We have,

$$n_1 = 20, \bar{x}_1 = 50, \sigma_1^2 = 1$$

$$n_2 = 40, \bar{x}_2 = 50, \sigma_2^2 = 4$$

Then,

$$\begin{aligned} \bar{x} &= \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} \\ \Rightarrow \bar{x} &= \frac{20 \times 50 + 40 \times 50}{20 + 40} = 50 \end{aligned}$$

Then,

$$d_1 = \bar{x} - \bar{x}_1 = 0$$

$$d_2 = \bar{x} - \bar{x}_2 = 0$$

Then,

$$\begin{aligned} \sigma^2 &= \frac{n_1 (\sigma_1^2 + d_1^2) + n_2 (\sigma_2^2 + d_2^2)}{n_1 + n_2} \\ \Rightarrow \sigma^2 &= \frac{20 \times 1 + 40 \times 2^2}{60} = 3 \end{aligned}$$

4. (1)

Given,  $\sigma = 9$

Let a student obtains  $x$  out of 75.

Then his marks out of 100 are  $\frac{4x}{3}$ .

$\Rightarrow$  Each observation is multiplied by  $\frac{4}{3}$

$$\Rightarrow \text{New } \sigma = \frac{4}{3} \times 9 = 12,$$

$$\Rightarrow \text{New Variance} = \sigma^2 = 144$$

5. (1)

Series  $A = 101, 102, 103, \dots, 200$

Series  $B = 151, 152, 153, \dots, 250$

Series  $B$  is obtained by adding a fixed quantity to each item of series  $A$ , we know that variance is independent of change of origin both series have the same variance so ratio of their variances is 1.

6. (3)

Observations are 2021, so median is  $\left(\frac{2021+1}{2}\right)^{th}$  observation, i.e.  $(1011)^{th}$ .

Now increasing the value of largest 40 observations won't affect the median.

Hence, the median of the data would remain same as 20.19

7. (4) We have  $n = 20$ ,  $\bar{x}_{old} = 10$ ,  $\text{Var}_{old} = 4$

$$\Rightarrow \bar{x} = \frac{\sum x_i}{20} \Rightarrow \sum x_i = 200$$

$$\Rightarrow (\sum x_i)_{new} = 192, \bar{X}_{new} = \frac{192}{9}$$

$$\text{Var}_{(old)} = \frac{\sum x_i^2}{20} - (\bar{x}_{old})^2$$

$$4 = \frac{\sum x_i^2}{20} - 100$$

$$\sum x_i^2 (old) = 2080$$

$$\sum x_i^2_{new} = 2080 - 64 = 2016$$

$$\text{Var}_{(new)} = \frac{2016}{19} - \left(\frac{192}{19}\right)^2 = \frac{1440}{361}$$

8. (4)

For  $a, b, c$

$$\text{mean} = \frac{a+b+c}{3} (= \bar{x})$$

$$b = a + c$$

$$\Rightarrow \bar{x} = \frac{2b}{3} \dots (1)$$

$$\text{S.D.}(a+2, b+2, c+2) = \text{S.D.}(a, b, c) = d$$

$$\Rightarrow d^2 = \frac{a^2+b^2+c^2}{3} - (\bar{x})^2$$

$$\Rightarrow d^2 = \frac{a^2+b^2+c^2}{3} - \frac{4b^2}{9}$$

$$\Rightarrow 9d^2 = 3(a^2 + b^2 + c^2) - 4b^2$$

$$\Rightarrow b^2 = 3(a^2 + c^2) - 9d^2$$

9. (1)

Let observations are denoted by  $x_i$  for  $1 \leq i \leq 2n$

$$\bar{x} = \frac{\sum x_i}{2n} = \frac{(a+a+\dots+a)-(a+a+\dots+a)}{2n}$$

$$\Rightarrow \bar{x} = 0$$

$$\text{and } \sigma_x^2 = \frac{\sum x_i^2}{2n} - (\bar{x})^2 = \frac{a^2+a^2+\dots+a^2}{2n} - 0 = a^2$$

$$\Rightarrow \sigma_x = a$$

Now, adding a constant  $b$  then  $\bar{y} = \bar{x} + b = 5$

$$\Rightarrow b = 5$$

and  $\sigma_y = \sigma_x$  (No change in S.D.)  $\Rightarrow a = 20$

$$\Rightarrow a^2 + b^2 = 425$$

10. (1)  $\bar{x} = A + \frac{\sum fd}{n} = 4 - \frac{11}{100} = 3.87$

$$\text{And } \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} = \sqrt{\frac{257}{100} - \left(-\frac{11}{100}\right)^2} = 1.6$$

Coefficient of variation

$$= \frac{\sigma}{\bar{x}} \times 100 = \frac{1.6}{3.89} \times 100 = 41.13\%$$