## ANSWER KEYS

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|---|---|---|---|---|---|---|---|
| **1.** (2) | **2.** (4) | **3.** (3) | **4.** (4) | **5.** (3) | **6.** (3) | **7.** (12) | **8.** (2) |
| **9.** (2) | **10.** (1) | **11.** (2) | **12.** (3) | **13.** (1) | **14.** (2) | **15.** (2) | **16.** (130) |
| **17.** (3) | **18.** (1) | **19.** (1) | **20.** (1) | | | | |

**1. (2)**

Given,

$\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right)$

$=\tan^{-1}\left(\frac{\cos\left(4\pi-\frac{\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right)$

$=\tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}-1}{\frac{1}{\sqrt{2}}}\right)$

$=\tan^{-1}\left(1-\sqrt{2}\right)\ =-\tan^{-1}\left(\sqrt{2}-1\right)$

$=-\frac{\pi}{8}$

**2. (4)**

We have,

$\tan^{-1}\sqrt{x^2+x}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{4}$

For equation to be defined,

for $\tan^{-1}\sqrt{x(x+1)}$,

$x^2+x\geq 0$

$\Rightarrow x^2+x+1\geq 1\ \ldots(1)$

And, from domain of $\sin^{-1}\sqrt{x^2+x+1}$, $x^2+x+1\leq 1\quad\ldots(2)$

$\therefore$ from $(1)$ & $(2)$ only possibility that the equation is defined is

$x^2+x=0$

$\Rightarrow x=0,\ -1$

at $x=0$, $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=0+\frac{\pi}{2}\neq\frac{\pi}{4}$

and at $x=-1$, $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=0+\frac{\pi}{2}\neq\frac{\pi}{4}$

None of these values satisfy

$\therefore$ Number of roots$=0$

**3. (3)**

$\cos^{-1}(\cos(-5))+\sin^{-1}(\sin(6)-\tan^{-1}(\tan(12))$

$=2\pi-5+(-2\pi+6)-(12-4\pi)$

$=4\pi-11$

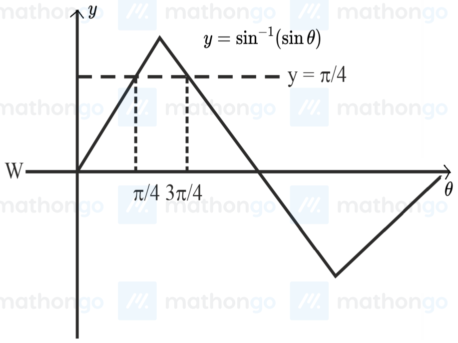
**4.** **(4)**

Given, $\sin^{-1}(\sin\theta)-\cos^{-1}(\sin\theta)>0, \theta\in(0,2\pi)$

$\because \sin^{-1}(\theta)+\cos^{-1}(\theta)=\frac{\pi}{2}$

$\therefore \sin^{-1}(\sin\theta)-\left(\frac{\pi}{2}-\sin^{-1}(\sin\theta)\right)>0$

$\sin^{-1}(\sin\theta)>\frac{\pi}{4}\Rightarrow\theta\in\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$




$(a,b)=\left(\frac{\pi}{4},\frac{3\pi}{4}\right)\Rightarrow b-a=\frac{\pi}{2}$

Given $b-a=\alpha-\beta$

$\therefore\ \alpha-\beta=\frac{\pi}{2}\ \ldots..(1)$

Now $\alpha x^2+\beta x+\sin^{-1}(x^2-6x+10)+\cos^{-1}(x^2-6x+10)=0$

Now defining $x^2-6x+10=1+(x-3)^2\geq 1$

Hence $x=3$ is the only possible solution

$9\alpha+3\beta+\frac{\pi}{2}=0\quad\ldots(2)$

On solving equations (1) and (2) we get,

$\alpha=\frac{\pi}{12}$

**5.** **(3)**

$\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}$

$=\tan^{-1}\left(\frac{1+\sqrt{3}}{\sqrt{3}\left(1+\sqrt{3}\right)}\right)+\sec^{-1}\sqrt{\frac{4\left(2+\sqrt{3}\right)}{3\left(2+\sqrt{3}\right)}}$

$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sec^{-1}\sqrt{\frac{4}{3}}$

$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\cos^{-1}\frac{\sqrt{3}}{2}=\frac{\pi}{6}+\frac{\pi}{6}=\frac{\pi}{3}$

**6.** **(3)**

Given, $0<x<1$

$2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put $x=\tan(\theta)\quad;\ 0<\theta<\frac{\pi}{4}$

$2\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)=\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow 2\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)=\cos^{-1}\left(\cos 2\theta\right)$

$2\left(\frac{\pi}{4}-\theta\right)=2\theta\Rightarrow\theta=\frac{\pi}{8}$

Put $\theta=\tan^{-1}x$ we get,

$\mathrm{x}=\tan\frac{\pi}{8}\therefore \mathrm{x}=\sqrt{2}-1\simeq 0.414<\frac{1}{2}$

**7. (12)**

Given,

$\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right)\right)\right)=k$

Now simplifying $\cos\left(\sin^{-1}x\right)=\cos\left(\cos^{-1}\sqrt{1-x^2}\right)=\sqrt{1-x^2}$

So, $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right)\right)\right)=k$

becomes $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\sqrt{1-x^2}\right)\right)\right)=k$

And now solving $\cot\left(\tan^{-1}\sqrt{1-x^2}\right)=\cot\cot^{-1}\left(\sqrt{\frac{1}{\sqrt{1-x^2}}}\right)=\frac{1}{\sqrt{1-x^2}}$

So, $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\sqrt{1-x^2}\right)\right)\right)=k$ becomes

$\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right)=k$

Now solving $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right)=\frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}$

So, $\frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}=k$

$\Rightarrow 1-2x^2=k^2\left(1-x^2\right)$

$\Rightarrow\left(k^2-2\right)x^2=k^2-1$

$\Rightarrow x^2=\frac{k^2-1}{k^2-2}$

So, roots are $\alpha=\sqrt{\frac{k^2-1}{k^2-2}}\Rightarrow\alpha^2=\frac{k^2-1}{k^2-2}$

And $\beta=\sqrt{\frac{k^2-1}{k^2-2}}\Rightarrow\beta^2=\frac{k^2-1}{k^2-2}$

Now finding $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=2\left(\frac{k^2-2}{k^2-1}\right)$ and $\frac{\alpha}{\beta}=-1$

So, sum of roots of $x^2-bx-5=0$ will be $=\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{\alpha}{\beta}=b$

$\Rightarrow\frac{2\left(k^2-2\right)}{k^2-1}-1=b\quad\cdots(1)$

Product of roots of $x^2-bx-5=0$ will be $=\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)\frac{\alpha}{\beta}=-5$

$\Rightarrow\frac{2\left(k^2-2\right)}{k^2-1}(-1)=-5$

$\Rightarrow 2k^2-4=5k^2-5$

$\Rightarrow 3k^2=1\Rightarrow k^2=\frac{1}{3}\quad\cdots\text{Put in }(1)$

$\Rightarrow b=\frac{2\left(k^2-2\right)}{k^2-1}-1=5-1=4$

$\frac{b}{k^2}=\frac{4}{\frac{1}{3}}=12$

**8. (2)**

$\left(\sin^{-1}x\right)^2-\left(\cos^{-1}x\right)^2=a$

Let $\sin^{-1}(x)=t$

$x=\sin\ t$

$\left(\sin^{-1}\sin\ t\right)^2-\left(\cos^{-1}\sin\ t\right)^2=a$

$t^2-\left(\frac{\pi}{2}-t\right)^2=a$

$t^2-\left(\frac{\pi^2}{4}+t^2-\pi t\right)=a$

$\pi t-\frac{\pi^2}{4}=a$

$t=\frac{a}{\pi}+\frac{\pi}{4}$

$x=\sin\left(\frac{a}{\pi}+\frac{\pi}{4}\right)$

$2x^2-1=2\sin^2\left(\frac{a}{\pi}+\frac{\pi}{4}\right)-1$

$=2\left(\sin\frac{a}{\pi}\ \cos\frac{\pi}{4}+\cos\frac{a}{\pi}\ \sin\frac{\pi}{4}\right)^2-1$

$=\left(\sin\frac{a}{\pi}+\cos\frac{a}{\pi}\right)^2-1$

$=\sin^2\frac{a}{\pi}+\cos^2\frac{a}{\pi}+2\cos\frac{a}{\pi}\sin\frac{a}{\pi}-1$

$=2\cos\frac{a}{\pi}\sin\frac{a}{\pi}$

$=\sin\frac{2a}{\pi}$

**9. (2)**

Let $\frac{\sin^{-1} x}{\alpha}=\frac{\cos^{-1} x}{\beta}=k$

i.e. $\sin^{-1} x=k\alpha$ and $\cos^{-1} x=k\beta$

$\Rightarrow \sin^{-1} x+\cos^{-1} x=k(\alpha+\beta)$

i.e. $k(\alpha+\beta)=\frac{\pi}{2}$

$\Rightarrow k=\frac{\pi}{2(\alpha+\beta)}$

Now $\frac{2\pi\alpha}{\alpha+\beta}=4k\alpha=4\sin^{-1} x$

So $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)=\sin\left(4\sin^{-1} x\right)$

$=2\sin\left(2\sin^{-1} x\right)\cos\left(2\sin^{-1} x\right)$

$=2\sin\left(\sin^{-1} 2x\sqrt{1-x^2}\right)\cos\left(\sin^{-1} 2x\sqrt{1-x^2}\right)$

$=2\sin\left(\sin^{-1} 2x\sqrt{1-x^2}\right)\cos\left(\cos^{-1}\left(1-2x^2\right)\right)$

$=4x\sqrt{1-x^2}\left(1-2x^2\right)$

**10. (1)**

Given,

$\sin^{-1}\left(\frac{\alpha}{17}\right)+\cos^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{77}{36}\right)=0$

$\Rightarrow \sin^{-1}\left(\frac{\alpha}{17}\right)=\tan^{-1}\left(\frac{77}{36}\right)-\cos^{-1}\left(\frac{4}{5}\right)$

$\Rightarrow \sin^{-1}\left(\frac{\alpha}{17}\right)=\tan^{-1}\left(\frac{77}{36}\right)-\tan^{-1}\left(\frac{3}{4}\right)$

$\Rightarrow \sin^{-1}\left(\frac{\alpha}{17}\right)=\tan^{-1}\left(\frac{\frac{77}{36}-\frac{3}{4}}{1+\frac{77}{36}\times\frac{3}{4}}\right)$

$\Rightarrow \sin^{-1}\left(\frac{\alpha}{17}\right)=\tan^{-1}\left(\frac{8}{15}\right)$

$\Rightarrow \sin^{-1}\left(\frac{\alpha}{17}\right)=\sin^{-1}\left(\frac{8}{17}\right)$

$\Rightarrow \alpha=8$

Now solving

$\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$

$=\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$

$\Rightarrow \sin^{-1}(\sin 8)+\cos^{-1}(\cos 8)=3\pi-8+8-2\pi=\pi$

**11. (2)**

Given,

$\tan\left(2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{\sqrt{5}}{2}+2\tan^{-1}\frac{1}{8}\right)$

$=\tan\left(2\left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right)$

$=\tan\left(2\left(\tan^{-1}\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{40}}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right)$

$=\tan\left[2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]$

$=\tan\left[\tan^{-1}\left(\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]$

$=\tan\left[\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]$

$=\tan\left[\tan^{-1}\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{8}}\right]$

$=\tan\tan^{-1}\frac{\frac{5}{4}}{\frac{5}{8}}$

$=\tan\tan^{-1} 2$

$=2$

**12. (3)**

$\alpha=\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{4}{3}$

$\therefore a-\beta=\tan^{-1}\frac{4}{3}-\tan^{-1}\frac{1}{3}$

$=\tan^{-1}\frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{3}\times\frac{1}{3}}$

$=\tan^{-1}\frac{1}{\frac{13}{9}}$

$=\tan^{-1}\frac{9}{13}$

$=\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

**13. (1)**

Given equation is:

$\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi, x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

It is possible if

$\cos^{-1}(2x) = \pi$ ....(1)

and $2\cos^{-1}\left(\sqrt{1-x^2}\right) = 0$ ....(2)

From equation (1), $x = -\frac{1}{2}$ which is not satisfying equation (2).

So, no such $x$ exists.

**14. (2)**

Given equation is

$\sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$

Now, $\sin^{-1}\left[x^2 + \frac{1}{3}\right]$ is defined if $-1 \le \left[x^2 + \frac{1}{3}\right] \le 1$

$\Rightarrow -1 \le x^2 + \frac{1}{3} < 2$

$\Rightarrow \frac{-4}{3} \le x^2 < \frac{5}{3}$

$\Rightarrow 0 \le x^2 < \frac{5}{3}$ ...(1)

Also, and $\cos^{-1}\left[x^2 - \frac{2}{3}\right]$ is defined if $-1 \le \left[x^2 - \frac{2}{3}\right] \le 1$

$\Rightarrow -1 \le x^2 - \frac{2}{3} < 2$

$\Rightarrow \frac{-1}{3} \le x^2 < \frac{8}{3}$

$\Rightarrow 0 \le x^2 < \frac{8}{3}$ ...(2)

So, from (1) and (2) we can conclude $0 \le x^2 < \frac{5}{3}$

Case -$I$ : If $0 \le x^2 < \frac{2}{3}$

$\Rightarrow \sin^{-1}(0) + \cos^{-1}(-1) = x^2$

$\Rightarrow 0 + \pi = x^2$

$\Rightarrow x^2 = \pi$ but $\pi \notin \left[0, \frac{2}{3}\right)$

$\Rightarrow$ No value of '$x$'

Case - $II$ : If $\frac{2}{3} \le x^2 < \frac{5}{3}$

$\Rightarrow \sin^{-1}(1) + \cos^{-1}(0) = x^2$

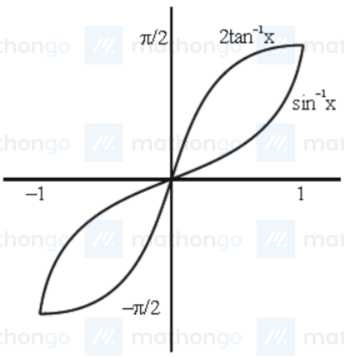
$\Rightarrow \frac{\pi}{2} + \frac{\pi}{2} = x^2$

$\Rightarrow x^2 = \pi$ but $\pi \notin \left[\frac{2}{3}, \frac{5}{3}\right)$

$\Rightarrow$ No value of '$x$'

So, number of solutions of the equation is zero.

**15.**




**(2)**

**16. (130)**

Given $x = \sin\left(2\tan^{-1}\alpha\right) = \sin\left(\tan^{-1}\frac{2\alpha}{1-\alpha^2}\right) = \sin\left(\sin^{-1}\frac{2\alpha}{1+\alpha^2}\right) = \frac{2\alpha}{1+\alpha^2}$ ...(i)

and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right) = \sin\left(\tan^{-1}\frac{1}{2}\right) = \sin\left(\sin^{-1}\frac{1}{\sqrt{5}}\right) = \frac{1}{\sqrt{5}}$ ...(ii)

Now, $y^2 = 1 - x$

$\Rightarrow \frac{1}{5} = 1 - \frac{2\alpha}{1+\alpha^2}$ (from (i) & (ii))

$\Rightarrow 1 + \alpha^2 = 5 + 5\alpha^2 - 10\alpha$

$\Rightarrow 2\alpha^2 - 5\alpha + 2 = 0$

$\therefore \alpha = 2, \frac{1}{2}$

$\therefore \sum_{\alpha \in S} 16\alpha^3 = 16 \times 2^3 + 16 \times \frac{1}{2^3} = 16\left(\frac{65}{8}\right)$

$= 130$

**17. (3)**

Let $S=\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+3r+3}\right)$

So, $T_r=\tan^{-1}\frac{1}{r^2+3r+2+1}$

$=\tan^{-1}\left(\frac{(r+2)-(r+1)}{1+(r+1)(r+2)}\right)$

$=\tan^{-1}(r+2)-\tan^{-1}(r+1)$

$T_1=\tan^{-1}3-\tan^{-1}2$

$T_2=\tan^{-1}4-\tan^{-1}3$

.

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$T_n=\tan^{-1}(n+2)-\tan^{-1}(n+1)$

$\Rightarrow\sum_{r=1}^{n}\tan^{-1}\frac{1}{r^2+3r+3}=\tan^{-1}(n+2)-\tan^{-1}2$

i.e. $6\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\frac{1}{r^2+3r+3}\right)=6\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(n+2)-\tan^{-1}2\right]\right)$

$=6\tan\left(\frac{\pi}{2}-\tan^{-1}2\right)$

$=6\tan\left(\tan^{-1}\frac{1}{2}\right)=3$

**18. (1)**

Given,

$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)\ldots\ldots\ldots\ldots\ldots\ldots\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$

$=\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)\ldots\ldots\ldots\ldots\ldots\ldots\tan^{-1}\left(\frac{a_{2022}-a_{2021}}{1+a_{2021}a_{2022}}\right)$

$=\tan^{-1}a_2-\tan^{-1}a_1+\tan^{-1}a_3-\tan^{-1}a_2\ldots\ldots\ldots+\tan^{-1}a_{2022}-\tan^{-1}a_{2021}$

$\left\{\text{using the formula }\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right\}$

$=\tan^{-1}a_{2022}-\tan^{-1}a_1$

$=\tan^{-1}a_{2022}-\tan^{-1}1$ {as given $a_1=1$}

$=\tan^{-1}a_{2022}-\frac{\pi}{4}$

$=\frac{\pi}{2}-\cot^{-1}a_{2022}-\frac{\pi}{4}$

$=\frac{\pi}{4}-\cot^{-1}a_{2022}$

$=\frac{\pi}{4}-\cot^{-1}2022$

As $a_1=1$, $a_2=2$ .... and so on $a_{2022}=2022$

**19. (1)** We have, $\cot\sum_{n=1}^{19}\left(\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$

$=\cot\sum_{n=1}^{19}\tan^{-1}\left(\frac{1}{1+n(n+1)}\right)$

$=\cot\sum_{n=1}^{19}\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)$

$=\cot\sum_{n=1}^{19}\left(\tan^{-1}(n+1)-\tan^{-1}(n)\right)$

$=\cot\left(\tan^{-1}20-\tan^{-1}1\right)$

$=\frac{1}{\tan\left(\tan^{-1}20-\tan^{-1}1\right)}$

$=\frac{1}{\frac{20-1}{1+(20).1}}=\frac{21}{19}$

**20. (1)**

Given $\cot^{-1}(\alpha)=\cot^{-1}(2)+\cot^{-1}(8)+\cot^{-1}(18)+\ldots.$

$=\sum_{n=1}^{100}\tan^{-1}\left(\frac{2}{4n^2}\right)$

$=\sum_{n=1}^{100}\tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$=\sum_{n=1}^{100}\tan^{-1}(2n+1)-\tan^{-1}(2n-1)$

$=\tan^{-1}201-\tan^{-1}1$

$=\tan^{-1}\left(\frac{200}{202}\right)$

$\therefore\cot^{-1}(\alpha)=\cot^{-1}\left(\frac{202}{200}\right)$

$\alpha=1.01$