ANSWER KEYS

1. (3)	**2.** (4)	**3.** (1)	**4.** (1)	**5.** (2)	**6.** (2)	**7.** (1)	**8.** (3)
9. (2)	**10.** (2)						

1. **(3)**

The numbers between 100 and 500 that are divisible by 7 are, $105,\ 112,\ 119,\ 126,\ 133,\ 140,\ 147, \ldots, 483,\ 490,\ 497.$

Let such numbers be n.

Then, $497 = 105 + (n-1) \times 7$

$\Rightarrow n = 57$

The number between 100 and 500 that are divisible by 21 are, $105,\ 126,\ 147, \ldots, 483.$

Let such numbers be m.

Then, $483 = 105 + (m-1) \times 21$

$\Rightarrow m = 19$

$\therefore$ Required number $= n - m = 57 - 19 = 38$

2. **(4)**

We know that n^{th} term $T_n = S_n - S_{n-1}$, and difference of any two consecutive terms is common difference d.

$S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$

$= (S_{n+3} - S_{n+2}) - 2(S_{n+2} - S_{n+1}) + (S_{n+1} - S_n)$

$= T_{n+3} - 2T_{n+2} + T_{n+1}$

$= (T_{n+3} - T_{n+2}) - (T_{n+2} - T_{n+1})$

$= d - d = 0.$

3. **(1)**

If $a,\ b,\ c$ are in AP, then b is A.M of a & c.

$\therefore\ 2b = a + c$

$\Rightarrow 28 = 3^{2\sin 2\alpha - 1} + 3^{4 - 2\sin 2\alpha}$

Putting, $3^{2\sin 2\alpha} = x$ we get,

$28 = \frac{x}{3} + \frac{81}{x} \Rightarrow x^2 - 84x + 243 = 0$

$\Rightarrow (x-3)(x-81) = 0$

$\therefore\ 3^{2\sin 2\alpha} = 3$ or 3^4

$\sin 2\alpha = \frac{1}{2}, \because \sin 2\alpha \neq 2$

Terms are $1, 14, 27, \ldots\ldots$ then

$T_6 = 1 + 5(13) = 66$

4. **(1)**

Let, the first 3 terms of the $A.P.$ are $a-d,\ a,\ a+d$

$\Rightarrow$ sum $= 3a = 33 \Rightarrow a = 11$

And product $(a-d)\ a\ (a+d) = a(a^2 - d^2) = 1155$

$\Rightarrow 11(11^2 - d^2) = 1155$

$\Rightarrow 121 - d^2 = \frac{1155}{11} = 105$

$\Rightarrow d^2 = 121 - 105 = 16$

$\Rightarrow d = \pm 4$

For $a = 11,\ d = 4$, the $A.P.$ is $7,\ 11,\ 15, \ldots$

The n^{th} term of an $A.P.$, with first term A and common difference d is $T_n = A + (n-1)d$

So, $T_{11} = 7 + 10 \times 4 = 47$

For $a = 11,\ d = -4$, the $A.P.$ is $15,\ 11,\ 7, \ldots$

So, $T_{11} = 15 + 10(-4) = -25.$

5. **(2)**

$a_1 + a_2 = 4 \Rightarrow a_1 + a_1 r = 4 \ldots(1)$

$a_3 + a_4 = 16 \Rightarrow a_1 r^2 + a_1 r^3 = 16 \ldots(2)$

$\frac{1}{r^2} = \frac{1}{4} \Rightarrow r^2 = 4 \Rightarrow r = -2\ (a_1 < 0)$

$\sum_{i=1}^{a} a_i = \frac{a_1(r^9 - 1)}{r-1} = \frac{(-4)\left[(-2)^9 - 1\right]}{(-2-1)} = \frac{4}{3}(-513) = 4\lambda$

$\lambda = -171.$

6. (2)
Let a be the first term and r the common ratio of the $G.P.$ Then
sum$=57 \Rightarrow \frac{a}{1-r}=57$ ....(i)
Sum of the cubes $=9747$
$\Rightarrow a^3+a^3r^3+a^3r^6+\ldots=9747$
$\Rightarrow \frac{a^3}{1-r^3}=9747$ ....(ii)
Dividing the cube of (i) by (ii), we
get $\frac{a^3}{(1-r)^3}\cdot\frac{(1-r^3)}{a^3}=\frac{(57)^3}{9747}$
$\Rightarrow \frac{1-r^3}{(1-r)^3}=19$
$\Rightarrow \frac{1+r+r^2}{(1-r)^2}=19$
$\Rightarrow 18r^2-39r+18=0$
$\Rightarrow \left(3r-2\right)\left(6r-9\right)=0$
$\Rightarrow r=\frac{2}{3}\ or\ r=\frac{3}{2}$
$\Rightarrow r=\frac{2}{3}$
$\left(\because\ r\neq\frac{3}{2}\right.$ because $-1<r<1$ for an infinite $\left.G.P\right)$

7. (1) $\frac{a+ar+ar^2+ar^3+ar^4}{\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3}+\frac{1}{ar^4}}=49$
$\Rightarrow \frac{a\left[1+r+r^2+r^3+r^4\right]}{\frac{1}{ar^4}\left[r^4+r^3+r^2+r+1\right]}=49$
$\Rightarrow a^2r^4=49$
$\Rightarrow ar^2=7$
$\Rightarrow T_3=7$
$T_1+T_3=35$
$T_1+7=35$
$T_1=28$

8. (3) We know that some of infinite geometric progression $a+ar+ar^2+\ldots\ldots\ldots=\frac{a}{1-r}$, $|r|<1$

Hence, $(32)(32)^{1/6}(32)^{1/36}\ldots=32^{1+\frac{1}{6}+\frac{1}{36}+\cdots}=32^{\frac{1}{1-1/6}}$
$=\left(2^5\right)^{6/5}=64$

9. (2) Let the three numbers of the GP be $a,\ ar$ and ar^2 where r is the common ratio.
According to the given condition, $a,\ 2ar$ and ar^2 are in AP.
$\therefore\ 2ar=\frac{a+ar^2}{2}$
$\therefore\ a+ar^2=4ar$
$\therefore\ r^2+1=4r$
$\therefore\ r^2-4r+1=0$
$\therefore\ r=\frac{4\pm\sqrt{16-4}}{2}$
$\quad=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$
As it is an 'increasing' GP, $r>1$.
$\therefore\ r=2+\sqrt{3}$.

10. (2)

a, b, c are in A.P., so $2b = a + c \ \ldots(1)$

Given, $a + b + c = 60$

$\Rightarrow 3b = 60 \Rightarrow b = 20, \ a + c = 40$

So, $c = 40 - a \ \ldots(2)$

We know, if p, q, r are in G.P., then $q^2 = pr$.

Now, $a - 2, b, c + 3$ are in G.P.

$\therefore \ a - 2, \ 20, \ 43 - a$ are in G.P.

$\Rightarrow 20^2 = (a - 2)(43 - a)$

$\Rightarrow 400 = (a - 2)(43 - a)$

$\Rightarrow a^2 - 45a + 486 = 0$

$\Rightarrow (a - 18)(a - 27) = 0$

If $a = 27$, then $b = 20$, $c = 13$

$\Rightarrow a^2 + b^2 + c^2 = (27)^2 + (20)^2 + (13)^2 = 1298$

If $a = 18$, then $b = 20$, $c = 22$

$\Rightarrow a^2 + b^2 + c^2 = (18)^2 + (20)^2 + (22)^2 = 1208$

Hence, only possible values are

1208 & 1298.