ANSWER KEYS

1. (3)	**2.** (2)	**3.** (1)	**4.** (1)	**5.** (3)	**6.** (1)	**7.** (4)	**8.** (3)
9. (2)	**10.** (4)						

1. **(3)** Given equation is

$\lambda x^2 + (2\lambda - 3)y^2 - 4x - 1 = 0$

Here, $a = \lambda,\ b = (2\lambda - 3)$

It represents a circle, if $a = b$

$\Rightarrow\ \lambda = 2\lambda - 3$

$\Rightarrow\ \lambda = 3$

Also, $h = 0$

Then, equation becomes

$3x^2 + 3y^2 - 4x - 1 = 0$

$\Rightarrow x^2 + y^2 - \frac{4}{3}x - \frac{1}{3} = 0$

Here, $g = -\frac{2}{3},\ c = -\frac{1}{3},\ f = 0$

$\therefore$ Radius$= \sqrt{\left(-\frac{2}{3}\right)^2 + 0 - \left(-\frac{1}{3}\right)} = \sqrt{\frac{4}{9} + \frac{1}{3}}$

$= \frac{\sqrt{7}}{3}$

2. **(2)** The radius of the circle will be minimum when line segment joining given two points is the diameter of the circle.

Hence, the equation of circle is,

$(x-2)\ (x-0) + (y-0)(y-4) = 0$

$\Rightarrow x^2 - 2x + y^2 - 4y = 0$

3. **(1)** Given x_1, x_2 are the roots of the equation.

$$\begin{aligned} & \mathrm{x}^2 + 2\mathrm{x} - 3 = 0 \\ \Rightarrow\ & \mathrm{x}^2 + 3\mathrm{x} - \mathrm{x} - 3 = 0 \\ \Rightarrow\ & \mathrm{x}(\mathrm{x}+3) - 1(\mathrm{x}+3) = 0 \\ \Rightarrow\ & (\mathrm{x}-1)(\mathrm{x}+3) = 0 \\ \Rightarrow\ & \mathrm{x}_1 = -3, \mathrm{x}_2 = 1 \end{aligned}$$

And y_1, y_2 are the roots of the equation.

$$\begin{aligned} & \mathrm{y}^2 + 4\mathrm{y} - 12 = 0 \\ \Rightarrow\ & \mathrm{y}^2 + 6\mathrm{y} - 2\mathrm{y} - 12 = 0 \\ \Rightarrow\ & \mathrm{y}(\mathrm{y}+6) - 2(\mathrm{y}+6) = 0 \\ \Rightarrow\ & (\mathrm{y}-2)(\mathrm{y}+6) = 0 \\ \Rightarrow\ & \mathrm{y}_1 = -6, \mathrm{y}_2 = 2 \end{aligned}$$

Points are $\mathrm{P}(-3,-6)$ & $\mathrm{Q}(1,2)$

Since P and Q are the endpoints of a diameter.

$\therefore$ Centre $=$ Mid point of PQ

$= \left(\frac{-3+1}{2}, \frac{-6+2}{2}\right)$

$= (-1, -2)$

4. **(1)** From figure, we have

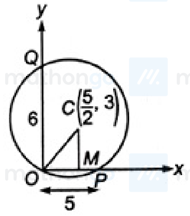
$OP = 5,\ OQ = 6$

and $OM = \frac{5}{2},\ CM = 3$

$\therefore$ In $OMC,\ OC^2 = OM^2 + MC^2$

$\Rightarrow OC^2 = \left(\frac{5}{2}\right)^2 + 3^2$

$\Rightarrow OC = \frac{\sqrt{61}}{2}$



Thus, the required circle has its centre $\left(\frac{5}{2},\ 3\right)$ and radius $\frac{\sqrt{61}}{2}$.

So, its equation is $\left(x - \frac{5}{2}\right)^2 + (y-3)^2 = \left(\frac{61}{4}\right)$.

Hence, $\lambda = \frac{61}{4}$

5. **(3)**

The centre of a circle which touches the $X-$axis at a point $(a,\ 0)$ is $(a,\ r)$ and radius is r and the equation of the circle with centre $(h,\ k)$ and radius r is $(x-h)^2+(y-k)^2=r^2$

Hence, the centre of the required circle is $(1,\ k)$ and radius k

And, the equation of the required circles is $(x-1)^2+(y-k)^2=k^2$

Given, it passes through $(2,\ 3)$

$\Rightarrow\ (2-1)^2+(3-k)^2=k^2$

$\Rightarrow\ 1+9-6k+k^2=k^2$

$\Rightarrow\ k=\frac{5}{3}$

Thus, diameter $=\frac{10}{3}$.

6. **(1)**

As $y-$ axis is tangent to the circle therefore the centre of the circle would be $(x,2)$ and distance of centre from $(0,2)$ and $(-2,4)$ should be equal.

$\therefore\ \sqrt{h^2+0^2}=\sqrt{(h+2)^2+2^2}$

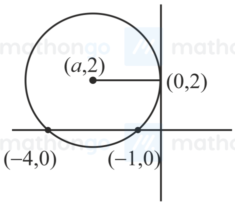
$\Rightarrow h^2=h^2+4h+4+4$

$\Rightarrow h=-2$

Hence, centre is $(-2,2)$.

$\therefore\ 2x-3y+10=0$ is the diameter, as centre lies on it.

7. **(4)**



Let $(a,\ 2)$ be the centre of the circle

Equation of the circle is

$(x-a)^2+(y-2)^2=a^2$

Put $(-1,0)$ in the equation of circle $(-1-a)^2+(-2)^2=a^2$

$1+2a+a^2+4=a^2$

$2a=-5$

$a=-\frac{5}{2}$

Equation of circle is

$\left(x+\frac{5}{2}\right)^2+(y-2)^2=\frac{25}{4}$

x intercept $=2\sqrt{g^2-c}$

$=2\sqrt{\frac{25}{4}-4}$

$=2\sqrt{\frac{9}{4}}$

$=3$

8. **(3)**

$x^2+y^2+ax+2ay+c=0$

$2\sqrt{g^2-c}=2\sqrt{\frac{a^2}{4}-c}=2\sqrt{2}$

$\Rightarrow\ \frac{a^2}{4}-c=2\ldots(1)$

$2\sqrt{f^2-c}=2\sqrt{a^2-c}=2\sqrt{5}$

$\Rightarrow a^2-c=5\ldots(2)$

(1) & (2)

$\frac{3a^2}{4}=3\Rightarrow a=-2\ (a<0)$

$\therefore\ c=-1$

Circle $\Rightarrow x^2+y^2-2x-4y-1=0$

$\Rightarrow(x-1)^2+(y-2)^2=6$

Given $x+2y=0\Rightarrow m=-\frac{1}{2}$

$m_{\text{tangent}}=2$

Equation of tangent $\Rightarrow(y-2)=2(x-1)\pm\sqrt{6}\sqrt{1+4}$

$\Rightarrow 2x-y\pm\sqrt{30}=0$

Perpendicular distance from $(0,0)=\left|\frac{\pm\sqrt{30}}{\sqrt{4+1}}\right|=\sqrt{6}$

9. (2)

Consider equation of circle as

$x^2+y^2+2gx+2fy+c=0 \ldots(i)$

The equation of the circle passing through the points $A(2,0)$, $B(0,1)$ and $C(4,5)$

$\Rightarrow 4+4g+c=0 \ldots(ii)$

$\Rightarrow 1+2f+c=0 \ldots(iii)$

$\Rightarrow 41+8g+10f+c=0 \ldots(iv)$

Upon solving, we get the equation of circle, $3\left(x^2+y^2\right)-13x-17y+14=0 \ldots(v)$

Also, circle is passes through $D(0,k)$

$\Rightarrow 3k^2-17k+14=0$

$\Rightarrow k=1, \frac{14}{3}$

Since, $k=1$ is already there, for point $C(0,1)$

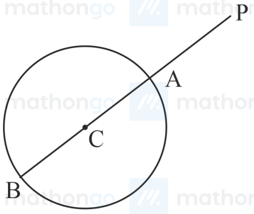
Therefore, we take $k=\frac{14}{3}$.

10. (4)

The equation of the given circle is $x^2+y^2-4x-2y-20=0$.

$\therefore\ S_1=10^2+7^2-4\times 10-2\times 7-20>0$

$\Rightarrow P(10,7)$ lies outside the circle.



Now, centre of the circle $C(2,1)$ with radius $BC=r=5$, where points $P,\ C$ & B are collinear and B lies diametrically opposite to that of point P.

Again, $PC=\sqrt{(10-2)^2+(7-1)^2}=10$

$\therefore$ Greatest distance $=10+5=15$ units.

and least distance $=10-5=5$ units.