ANSWER KEYS

1. (1)	**2.** (2)	**3.** (2)	**4.** (3)	**5.** (2)	**6.** (2)	**7.** (3)	**8.** (2)
9. (3)	**10.** (2)						

1. (1)

Since, $(b,\ c) \in R_1$ but $(c,\ b) \notin R_1$

Hence, R_1 is not symmetric.

In $R_1\ :\ (b,\ c)\in R_1$ and $(c,\ a)\in R_1$ but $(b,\ a)\notin R_1$

So, R_1 is not transitive.

R_2 is symmetric.

In $R_2\ :\ (b,\ a)\in R_2$ and $(a,\ c)\in R_1$ but $(b,\ c)\notin R_2$

So, R_2 is not transitive.

2. (2)

Let $B,\ H,\ F$ be the sets of members in the basketball team, hockey team, football team, respectively.

Given $n(B)=21,\ n(H)=26,\ n(F)=29$

$n(H\cap B)=14,\ n(H\cap F)=15$

$n(F\cap B)=12,\ n(B\cap H\cap F)=8$

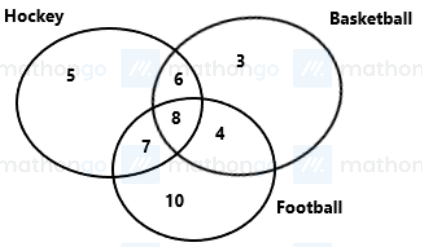
$\therefore\ n(B\cup H\cup F)=n(B)+n(H)+n(F)$

$-n(B\cap H)-n(H\cap F)-n(B\cap F)+n(B\cap H\cap F)$

$=21+26+29-14-15-12+8=43$

Alternative solution

Draw the Venn Diagram



Only plays basketball and hockey$=6$

Only plays basketball and football$=4$

Only plays football and hockey$=7$

From the Venn diagram number of members

$=n(H\cup B\cup F)=43$

3. (2)

$\sum_{i=1}^{30} n(Ai)=5\times 30=150$

Let S consist of m distinct elements. Since each elements of S belongs to exactly 10 of the Ai', so, we have $\sum_{i=1}^{30} n(A_i)=10m=150 \Rightarrow m=15$

Now each B_i has 3 elements and each elements of S belongs to exactly 9 of the B_i's. So, we have $\sum_{j=1}^{n} n(B_j)=3n=9m \Rightarrow 3n=9\times 15 \Rightarrow n=45$

4. (3)

Let $A=\{1,2,3,\ldots n\}$

Total subsets $=2^n$

Number of even integers in $1,\ 2,\ 3,\ldots\ldots n=\left[\frac{n}{2}\right]$

Number of odd integers in $=n-\left[\frac{n}{2}\right]$

$\Rightarrow$ Number of subsets having even integers $=2^{\left[\frac{n}{2}\right]}$

$\Rightarrow$ Number of subsets having atleast one odd integers

$=2^n-2^{\left[\frac{n}{2}\right]}$

5. (2) Total numbers of elements in the set $A=$ The selection of two distinct elements from given 10 elements.

$\Rightarrow\ n(A)={}^{10}C_1\times {}^{9}C_1=10\times 9=90$

6. **(2)** No. of subsets if a set contain r elements $= 2^r$

$\therefore\ 2^m - 2^n = 56 \Rightarrow 2^n\left(2^{m-n} - 1\right) = 8 \times 7 = 2^3 \times 7$

$\therefore\quad n = 3\quad and\ 2^{m-n} = 8 = 2^3$

$\Rightarrow\quad m - n = 3\quad \Rightarrow m - 3 = 3\quad \Rightarrow\quad m = 6$

$\therefore\quad m = 6,\ n = 3$

7. **(3)** For R_1 let $a = 1 + \sqrt{2},\ b = 1 - \sqrt{2},\ c = 8^{\frac{1}{4}}$

$aR_1b \Rightarrow a^2 + b^2 = \left(1 + \sqrt{2}\right)^2 + \left(1 - \sqrt{2}\right)^2 = 6 \in Q$

$bR_1c \Rightarrow b^2 + c^2 = \left(1 - \sqrt{2}\right)^2 + \left(8^{\frac{1}{4}}\right)^2 = 3 \in Q$

$aR_1c \Rightarrow a^2 + c^2 = \left(1 + \sqrt{2}\right)^2 + \left(8^{\frac{1}{4}}\right)^2 = 3 + 4\sqrt{2} \notin Q$

$\therefore R_1$ is not transitive.

For R_2 let $a = 1 + \sqrt{2},\ b = \sqrt{2},\ c = 1 - \sqrt{2}$

$aR_2b \Rightarrow a^2 + b^2 = \left(1 + \sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 = 5 + 2\sqrt{2} \notin Q$

$bR_2c \Rightarrow b^2 + c^2 = \left(\sqrt{2}\right)^2 + \left(1 - \sqrt{2}\right)^2 = 5 - 2\sqrt{2} \notin Q$

$aR_2c \Rightarrow a^2 + c^2 = \left(1 + \sqrt{2}\right)^2 + \left(1 - \sqrt{2}\right)^2 = 6 \in Q$

$\therefore R_2$ is not transitive.

8. **(2)** Here $\alpha R \beta \Leftrightarrow \alpha \perp \beta$

$\therefore \alpha \perp \beta \Leftrightarrow \beta \perp \alpha$

Hence, R is symmetric.

9. **(3)** We have $(x,\ x) \in R$ for $w\ =\ 1$ implying that R is reflexive.

For a $\neq 0$, (a, 0)$\notin$ R for any w but $(0,\ a) \in R$. Thus R is not symmetric.

Hence R is not an equivalence relation.

As $\left(\frac{m}{n}, \frac{m}{n}\right) \in$ S since $mn\ =\ mn,\ S$ is reflexive.

$\left(\frac{m}{n}, \frac{p}{q}\right) \in \mathrm{S} \Rightarrow \mathrm{qm} = \mathrm{pn}$

But this can be written as $np\ =\ mq,$

giving $\left(\frac{p}{q}, \frac{m}{n}\right) \in$ S.Thus S is symmetric.

Again, $\left(\frac{m}{n}, \frac{p}{q}\right) \in$ S and $\left(\frac{p}{q}, \frac{a}{b}\right) \in$ S

means $qm\ =\ pn$ and $bp\ =\ aq.$

i.e. $\frac{m}{n} = \frac{p}{q}$ and $\frac{p}{q} = \frac{a}{b}$.i.e.$\frac{m}{n} = \frac{a}{b}$

Thus $\left(\frac{m}{n}, \frac{a}{b}\right) \in$ S

This means S is transitive.

10. **(2)** Given, $r = \{(a,\ b)|a,\ b \in R$ and $a - b + \sqrt{3}$ is an irrational number$\}$

(i) Reflexive

$ara = a - a + \sqrt{3} = \sqrt{3}$ which is irrational number.

(ii) Symmetric

Now, $2r\sqrt{3} = 2 - \sqrt{3} + \sqrt{3} = 2$

Which is not an irrational number.

Also, $\sqrt{3}r2 = \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3} - 2$ which is an irrational number.

$2r\sqrt{3}$ does not implies $\sqrt{3}r2$

Therefore r is not symmetric.

(iii) Transitive

Now, $\sqrt{3}r2$ and $2r4\sqrt{5},\ i.e.,$

$\sqrt{3} - 2 + \sqrt{3} + 2 - 4\sqrt{5} + \sqrt{3}$

$= 2\sqrt{3} - 4\sqrt{5} + \sqrt{3}$ does not implies $\sqrt{3}r4\sqrt{5}$

$\therefore$ It is not transitive.