1. The integral $\int_0^{\frac{\pi}{2}} \frac{1}{3+2\sin x+\cos x}dx$ is equal to:

(1) $\tan^{-1}(2)$
(2) $\tan^{-1}(2)-\frac{\pi}{4}$
(3) $\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$
(4) $\frac{1}{2}$

2. The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x \cdot \operatorname{cosec}^{\frac{4}{3}} x dx$ is equal to

(1) $3^{\frac{7}{6}}-3^{\frac{5}{6}}$
(2) $3^{\frac{4}{3}}-3^{\frac{1}{3}}$
(3) $3^{\frac{5}{6}}-3^{\frac{2}{3}}$
(4) $3^{\frac{5}{3}}-3^{\frac{1}{3}}$

3. Let $[t]$ denote the greatest integer function. If
$\int_0^{2.4}[x^2]dx=\alpha+\beta\sqrt{2}+\gamma\sqrt{3}+\delta\sqrt{5}$, then $\alpha+\beta+\gamma+\delta$ is equal to

4. The value of $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ is equal to

(1) $\frac{\pi^2}{4}$
(2) $\frac{\pi}{4}$
(3) $\frac{\pi}{6}$
(4) $\frac{\pi^2}{2}$

5. $\int_0^{20\pi}(|\sin x|+|\cos x|)^2 dx$ is equal to:

(1) $10(\pi+4)$
(2) $10(\pi+2)$
(3) $20(\pi-2)$
(4) $20(\pi+2)$

6. For $m,\ n>0$, let $\alpha(m,n)=\int_0^2 t^m(1+3t)^n dt$. If $11\alpha(10,6)+18\alpha(11,5)=p(14)^6$, then p is equal to

7. The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x\left(\log_e\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$ is equal to

(1) $\log_e\left(\frac{\sqrt{2}\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$
(2) $\log_e\left(\frac{\left(2+\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$
(3) $\log_e\left(\frac{2\left(2+\sqrt{5}\right)}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$
(4) $\log_e\left(\frac{\sqrt{2}\left(3-\sqrt{5}\right)^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$

8. Let $f:\boldsymbol{R}\to\boldsymbol{R}$ be a differentiable function such that $f\left(\frac{\pi}{4}\right)=\sqrt{2}, f\left(\frac{\pi}{2}\right)=0$ and $f'\left(\frac{\pi}{2}\right)=1$ and let $g(x)=\int_x^{\frac{\pi}{4}}(f'(t)\sec t+\tan t\sec t f(t))dt$ for $x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right)$.

Then $\lim_{x\to\left(\frac{\pi}{2}\right)^-} g(x)$ is equal to

(1) 2
(2) 3
(3) 4
(4) -3

9. If
$\int_0^1(x^{21}+x^{14}+x^7)(2x^{14}+3x^7+6)^{1/7}dx=\frac{1}{l}(11)^{m/n}$ where $l,\ m,n\in N, m$ and n are co-prime then $l+m+n$ is equal to _____ .

10. The integral $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to

(1) $\frac{11}{6}+\log_e 4$
(2) $\frac{11}{12}+\log_e 4$
(3) $\frac{11}{12}-\log_e 4$
(4) $\frac{11}{6}-\log_e 4$

11. The integral $\int_1^2 e^x . x^x(2+\log_e x)\ dx$ equals :

(1) $e(4e+1)$
(2) $4e^2-1$
(3) $e(4e-1)$
(4) $e(2e-1)$

12. If $\int_{-0.15}^{0.15}|100x^2-1|dx=\frac{k}{3000}$, then k is equal to _____.

13. Let $[t]$ denote the greatest integer $\le t$. Then $\frac{2}{\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(8[\operatorname{cosec} x]-5[\cot x])dx$ is equal to _______

14. Let [x] denote the greatest integer $\le$ x. Consider the function $f(x)=\max\{x^2,1+[x]\}$. Then the value of the integral $\int_0^2 f(x)dx$ is :

(1) $\frac{5+4\sqrt{2}}{3}$
(2) $\frac{8+4\sqrt{2}}{3}$
(3) $\frac{1+5\sqrt{2}}{3}$
(4) $\frac{4+5\sqrt{2}}{3}$

15. Let $[t]$ denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1[-8x^2+6x-1]dx$ is equal to

(1) -1
(2) $-\frac{5}{4}$
(3) $\frac{\sqrt{17}-13}{8}$
(4) $\frac{\sqrt{17}-16}{8}$

16. The value of the integral $\int_{-2}^{2}\frac{|x^3+x|}{(e^{x|x|}+1)}dx$ is equal to

(1) $5e^2$
(2) $3e^{-2}$
(3) 4
(4) 6

17. The value of the definite integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{(1+e^{x\cos x})\left(\sin^4 x+\cos^4 x\right)}$ is equal to :

(1) $-\frac{\pi}{2}$
(2) $\frac{\pi}{2\sqrt{2}}$
(3) $-\frac{\pi}{4}$
(4) $\frac{\pi}{\sqrt{2}}$

18. Let $f(x)$ and $g(x)$ be two functions satisfying $f(x^2)+g(4-x)=4x^3$ and $g(4-x)+g(x)=0$, then the value of $\int_{-4}^{4} f(x^2)dx$ is

19. If the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$, where $\alpha,\ \beta,\ \gamma$ are integers and $[x]$ denotes the greatest integer less than or equal to x, then the value of $\alpha+\beta+\gamma$ is equal to:

(1) 0
(2) 20
(3) 25
(4) 10

20. Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\le x$ respectively of a real number x. if $\int_0^n \{x\}dx,\ \int_0^n [x]dx$ and $10(n^2-n),\ (n \in N, n>1)$ are three consecutive terms of a G.P. then n is equal to__

21. Let $[t]$ denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101}\left([\sin(\pi x)]+e^{[\cos(2\pi x)]}\right)dx$ is equal to

(1) $\frac{52(1-e)}{e}$
(2) $\frac{52}{e}$
(3) $\frac{52(2+e)}{e}$
(4) $\frac{104}{e}$

22. Let f be a twice differentiable function on R. If $f'(0)=4$ and $f(x)+\int_0^x (x-t)f'(t)dt = \left(e^{2x}+e^{-2x}\right)\cos 2x + \frac{2}{a}x$, then $(2a+1)^5 a^2$ is equal to ______.

23. The function $f(x)$, that satisfies the condition $f\left(x\right) = x + \int_0^{\pi/2} \sin x \cos y f\left(y\right) \mathrm{d}y$, is :

(1) $x + \frac{\pi}{2} \sin x$
(2) $x + (\pi+2)\sin x$
(3) $x + \frac{2}{3}\left(\pi-2\right)\sin x$
(4) $x + (\pi-2)\sin x$

24. The integral $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}}dx$, where $[\cdot]$ denotes the greatest integer function, is equal to

(1) $1-6\ln\left(\frac{6}{7}\right)$
(2) $1+6\ln\left(\frac{6}{7}\right)$
(3) $1-7\ln\left(\frac{6}{7}\right)$
(4) $1+7\ln\left(\frac{6}{7}\right)$

25. Let $f(x)=\min\{[x-1],[x-2],\ldots,[x-10]\}$ where $[t]$ denotes the greatest integer $\le t$. Then $\int_0^{10} f(x)dx + \int_0^{10} (f(x))^2 dx + \int_0^{10} |f(x)|dx$ is equal _______. to

26. Consider the integral $I = \int_0^{10} \frac{[x]e^{[x]}}{e^{x-1}}dx$ where $[x]$ denotes the greatest integer less than or equal to x. Then the value of I is equal to :

(1) $9(e-1)$
(2) $45(e+1)$
(3) $45(e-1)$
(4) $9(e+1)$

27. For any real number x, let $[x]$ denote the largest integer less than or equal to x. Let f be a real-valued function defined on the interval $[-10, 10]$ by

$$f(x)=\begin{cases} x-[x], & \text{if } [x] \text{ is odd} \\ 1+[x]-x, & \text{if } [x] \text{ is even} \end{cases}$$

Then, the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x dx$ is

(1) 4
(2) 2
(3) 1
(4) 0

28. Let $\alpha \in (0,1)$ and $\beta = \log_e(1-\alpha)$. Let $P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots + \frac{x^n}{n}, x \in (0,1)$. Then the integral $\int_0^\alpha \frac{t^{50}}{1-t}dt$ is equal to

(1) $\beta - P_{50}(\alpha)$
(2) $-(\beta + P_{50}(\alpha))$
(3) $P_{50}(\alpha) - \beta$
(4) $\beta + P_{50}(\alpha)$

29. Let $a_n = \int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{3}+\ldots+\frac{x^{n-1}}{n}\right)dx$ for every $n \in N$. Then the sum of all the elements of the set $\{n \in \mathrm{N} : a_n \in (2,30)\}$ is ________.

30. Let $\underset{0\leqslant x\leqslant 2}{\mathrm{Max}}\left\{\frac{9-x^2}{5-x}\right\} = \alpha$ and $\underset{0\leqslant x\leqslant 2}{\mathrm{Min}}\left\{\frac{9-x^2}{5-x}\right\} = \beta$. If $\int_{\beta-\frac{8}{3}}^{2\alpha-1} \mathrm{Max}\left\{\frac{9-x^2}{5-x}, x\right\}dx = \alpha_1 + \alpha_2 \log_e\left(\frac{8}{15}\right)$, then $\alpha_1+\alpha_2$ is equal to ______