ANSWER KEYS

1. (2)	**2.** (1)	**3.** (4)	**4.** (1)	**5.** (3)	**6.** (2)	**7.** (1)	**8.** (1)
9. (4)	**10.** (1)	**11.** (3)	**12.** (4)	**13.** (2)	**14.** (1)	**15.** (1)	**16.** (1)
17. (1)	**18.** (3)	**19.** (2)	**20.** (1)	**21.** (1)	**22.** (1)	**23.** (2)	**24.** (1)
25. (3)	**26.** (1)	**27.** (2)	**28.** (1)	**29.** (4)	**30.** (4)		

1. (2)

$y^2 = a\left(x + \frac{\sqrt{a}}{2}\right) = ax + \frac{a^{3/2}}{2} \ldots(1)$

$\Rightarrow 2yy' = a$

Put in equation (1) we get,

$y^2 = (2yy')x + \frac{(2yy')^{3/2}}{2}$

$\left(y^2 - 2xyy'\right) = \frac{(2yy')^{3/2}}{2}$

Squaring both the sides we get,

$\left(y^2 - 2xyy'\right)^2 = 2y^3(y')^3$

$\therefore$ Order $= 1$

Degree $= 3$

Degree $-$ order $= 3 - 1 = 2$

2. (1)

$\frac{dy}{dx} = y + 7 \Rightarrow \frac{dy}{dx} - y = 7$

I.F. $= e^{-x}$

$ye^{-x} = \int 7e^{-x}dx$

$\Rightarrow ye^{-x} = -7e^{-x} + c$

$\Rightarrow y = -7 + ce^x$

$-7 + 7e^x = -7 + 8e^x$

$\Rightarrow e^x = 0$

3. (4)

Given, $\frac{dy}{dx} + \frac{2^{x-y}(2^y-1)}{2^x-1} = 0 \ x,\ y > 0,\ y(1) = 1$

Now rearranging and integrating both side of $\frac{dy}{dx} = -\frac{2^x(2^y-1)}{2^y(2^x-1)}$, we get

$\Rightarrow \int \frac{2^y}{2^y-1}dy = -\int \frac{2^x}{2^x-1}dx$

$\Rightarrow \frac{1}{\ln 2}\int \frac{2^y \ln 2}{2^y-1}dy = -\frac{1}{\ln^2}\int \frac{2^x \ln 2}{2^x-1}dx$

$\Rightarrow \frac{1}{\ln 2}\ln|2^y - 1| = \frac{-1}{\ln 2}\ln|2^x - 1| + C$

At $x = 1, y = 1$

Putting this values in above equation we get $C = 0$

So, $\ln|2^y - 1| + \ln|2^x - 1| = 0$

$\Rightarrow (2^x - 1)(2^y - 1) = 1$

$\Rightarrow 2^y - 1 = \frac{1}{2^x+1}$

At $x = 2$

$\Rightarrow 2^y = \frac{1}{3} + 1 = \frac{4}{3}$

$y = \log_2 \frac{4}{3} = \log_2 4 - \log_2 3 = 2 - \log_2 3$

4. (1)

Given,

$\frac{dy}{dx}=\frac{x+y-2}{x-y}=\frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

Now let $x-1=X, y-1=Y$

So, $\frac{dy}{dx}=\frac{X+Y}{X-Y}$ ....(1)

Now let $Y=VX\ \frac{dY}{dX}=V+X\frac{dV}{dX}$

Putting the value in equation (1) we get,

$V+X\frac{dV}{dX}=\frac{1+V}{1-V}$

$\Rightarrow X\frac{dV}{dX}=\frac{V^2+1}{1-V}$

$\Rightarrow \int\frac{1-V}{1+V^2}dV=\int\frac{dX}{X}$

$\Rightarrow \int\frac{dV}{1+V^2}-\frac{1}{2}\int\frac{2VdV}{1+V^2}=\int\frac{dX}{X}$

$\Rightarrow \tan^{-1}V-\frac{1}{2}\ln(1+V^2)=\ln X+c$

$\Rightarrow \tan^{-1}\left(\frac{Y}{X}\right)-\frac{1}{2}\ln\left(1+\frac{Y^2}{X^2}\right)=\ln(X)+c$

$\Rightarrow \tan^{-1}\left(\frac{y-1}{x-1}\right)-\frac{1}{2}\ln\left(1+\frac{(y-1)^2}{(x-1)^2}\right)=\ln(x-1)+c$

Now given curve passes through $(2,1)$

So, $0-\frac{1}{2}\ln 1=\ln 1+c\Rightarrow c=0$

Now given curve also passes through $(k+1,2)$

So, $\tan^{-1}\left(\frac{1}{k}\right)-\frac{1}{2}\ \text{In}\left(1+\frac{1}{k^2}\right)=\ln k$

$\Rightarrow 2\tan^{-1}\left(\frac{1}{k}\right)=\text{In}\left(\frac{1+k^2}{k^2}\right)+2\ln k$

$\Rightarrow 2\tan^{-1}\left(\frac{1}{k}\right)=\text{In}(1+k^2)$

5. (3)

Given,

$(1+e^{2x})\frac{dy}{dx}+2(1+y^2)e^x=0$

$\Rightarrow \frac{dy}{1+y^2}+\frac{2e^x}{1+e^{2x}}dx=0$ ....(i)

Now integrating both side we get,

$\Rightarrow \int\frac{dy}{1+y^2}+\int\frac{2e^x}{1+e^{2x}}dx=\int 0$

$\Rightarrow \tan^{-1}y+2\tan^{-1}e^x=c$

$\because y(0)=0$

so, $C=\frac{\pi}{2}\Rightarrow \tan^{-1}\ y+2\ \tan^{-1}e^x=\frac{\pi}{2}$ ....(ii)

Now from equation (i), we get $\left(\frac{dy}{dx}\right)_{x=0}=-1$

From equation (ii) we get, $y\ (\text{In}\ \sqrt{3})=-\frac{1}{\sqrt{3}}$

So, $6[y'(0)+(y\ \text{In}\ \sqrt{3})^2]=6\left[-1+\frac{1}{3}\right]=-4.$

6. **(2)**

We have,

$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx=\left(\sqrt{e^{2x}-1}\right)dy\ \ldots(i)$

Put $\cos^{-1}(e^{-x})=\theta,\ \theta\in[0,\pi]$

$\Rightarrow \cos\theta=e^{-x}$

$\Rightarrow 2\cos^2\frac{\theta}{2}-1=e^{-x}$

$\Rightarrow \cos\left(\frac{\theta}{2}\right)=\sqrt{\frac{e^{-x}+1}{2}}=\sqrt{\frac{e^x+1}{2e^x}}$

Hence, by (i), we have

$\cos\left(\frac{\theta}{2}\right)dx=\left(\sqrt{e^{2x}-1}\right)dy$

$\Rightarrow \sqrt{\frac{e^x+1}{2e^x}}dx=\sqrt{e^{2x}-1}dy$

$\Rightarrow \left(\sqrt{\frac{e^x+1}{2e^x}}\right)dx=\left(\sqrt{e^x-1}\right)\left(\sqrt{e^x+1}\right)dy$

$\Rightarrow \frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{e^x}\sqrt{e^x-1}}=\int dy$

Put $e^x=t\Rightarrow dt=e^x dx$

$\frac{1}{\sqrt{2}}\int\frac{dt}{e^x\sqrt{e^x}\sqrt{e^x-1}}=\int dy$

$\int\frac{dt}{t\sqrt{t^2-t}}=\sqrt{2}\int dy$

Put $t=\frac{1}{z}\Rightarrow\frac{dt}{dz}=-\frac{1}{z^2}$

$\int\frac{-\frac{dz}{z^2}}{\frac{1}{z}\sqrt{\frac{1}{z^2}-\frac{1}{z}}}=\sqrt{2}\int dy$

$\Rightarrow -\int\frac{dz}{\sqrt{1-z}}=\sqrt{2}\int dy$

$\Rightarrow \frac{-2(1-z)^{1/2}}{-1}=\sqrt{2}y+c$

$\Rightarrow 2\left(1-\frac{1}{t}\right)^{1/2}=\sqrt{2}y+c$

$\Rightarrow 2(1-e^{-x})^{1/2}=\sqrt{2}y+c$

Now, it meets $y-$ axis at $(0,-1)$, hence

$0=-\sqrt{2}+c\Rightarrow c=\sqrt{2}$

Hence,

$2(1-e^{-x})^{1/2}=\sqrt{2}(y+1)$

It passes through $(\alpha,\ 0)$

$2(1-e^{-\alpha})^{1/2}=\sqrt{2}$

$\Rightarrow \sqrt{1-e^{-\alpha}}=\frac{1}{\sqrt{2}}$

$\Rightarrow 1-e^{-\alpha}=\frac{1}{2}$

$\Rightarrow e^{-\alpha}=\frac{1}{2}\Rightarrow e^{\alpha}=2$

7. **(1)** We have,

$\frac{dy}{dx}=\frac{x\left(\frac{y}{x}\cdot\tan\left(\frac{y}{x}\right)-1\right)}{x\tan\left(\frac{y}{x}\right)}$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}-\cot\left(\frac{y}{x}\right)$

Put $y=vx$

$\Rightarrow \frac{dy}{dx}=v+x\left(\frac{dv}{dx}\right)$

Now, we get

$v+x\left(\frac{dv}{dx}\right)=v-\cot v$

$\Rightarrow \int \tan v dv=-\int\frac{dx}{x}$

$\Rightarrow \ln|\sec v|=-\ln|x|+c$

$\Rightarrow \ln\left|\sec\left(\frac{y}{x}\right)\right|=-\ln|x|+c$

$\Rightarrow \ln\left|\sec\left(\frac{y}{x}\right)\right|+\ln|x|=c$

Now, $y\left(\frac{1}{2}\right)=\frac{\pi}{6}$, then

$\ln\left|\sec\left(\frac{\pi}{3}\right)\right|+\ln\left(\frac{1}{2}\right)=c$

$\Rightarrow \ln 2+\ln\left(\frac{1}{2}\right)=c$

$\Rightarrow \ln 2-\ln 2=c$

$\Rightarrow c=0$

Hence,

$\therefore\ \sec\left(\frac{y}{x}\right)=\frac{1}{x}$

$\Rightarrow \cos\left(\frac{y}{x}\right)=x$

$\Rightarrow y=x\cos^{-1}(x)$

So, required bounded area

$=\int_0^{\frac{1}{\sqrt{2}}}\left(\underbrace{\cos^{-1}x}_{\text{I}}\right)\underbrace{xdx}_{\text{II}}$

$=\left[\cos^{-1}x\cdot\left(\frac{x^2}{2}\right)\right]_0^{\frac{1}{\sqrt{2}}}+\frac{1}{2}\int_0^{\frac{1}{\sqrt{2}}}\left(\frac{x^2}{\sqrt{1-x^2}}\right)dx$

$=\frac{\pi}{16}-\frac{1}{2}\int_0^{\frac{1}{\sqrt{2}}}\left(\frac{1-x^2-1}{\sqrt{1-x^2}}\right)dx$

$=\frac{\pi}{16}-\frac{1}{2}\int_0^{\frac{1}{\sqrt{2}}}\left(\sqrt{1-x^2}-\frac{1}{\sqrt{1-x^2}}\right)dx$

$=\frac{\pi}{16}-\frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x-\sin^{-1}x\right]_0^{\frac{1}{\sqrt{2}}}$

$=\frac{\pi}{16}-\frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}x\right]_0^{\frac{1}{\sqrt{2}}}$

$=\frac{\pi}{16}-\frac{1}{2}\left[\frac{1}{4}-\frac{\pi}{8}\right]$

$=\left(\frac{\pi-1}{8}\right)$ sq. units

8. **(1)** $\frac{dy}{dx}-\frac{y+3x}{\ln(y+3x)}+3=0$

$\frac{dy}{dx}+3=\frac{y+3x}{\ln(y+3x)}$

$\frac{d}{dx}\left(y+3x\right)=\frac{y+3x}{\ln(y+3x)}$

$\int\frac{\ln(y+3x)}{(y+3x)}d\left(y+3x\right)=\int dx$

Let $\ln(y+3x)=t$

$\frac{1}{(y+3x)}d\left(y+3x\right)=dt$

$\int \text{tdt}=\int \text{dx}$

$\frac{t^2}{2}=x+c$

$\frac{(\ln(y+3x))^2}{2}=x+c$

9. **(4)** $\frac{dy}{dx}=\frac{1+\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)}$

Let $y=tx$

$\Rightarrow t+x\frac{dt}{dx}=\frac{1+t^2}{2t}$

$\Rightarrow x\frac{dt}{dx}=\frac{1-t^2}{2t}$

$\Rightarrow \int\frac{2t}{1-t^2}dt=\int\frac{dx}{x}$

$\Rightarrow \ln\left|1-t^2\right|=\ln x+\ln c$

$\Rightarrow \left(1-t^2\right)(cx)=1$

$\Rightarrow \left(1-\frac{y^2}{x^2}\right)cx=1$

$y(2)=0\Rightarrow c=\frac{1}{2}$

$\left(1-\frac{y^2}{x^2}\right)\cdot\frac{1}{2}x=1$

at $x=8$

$\left(1-\frac{y^2}{64}\right)\times\frac{8}{2}=1$

$y=\pm4\sqrt{3}$

10. **(1)**

$\frac{dy}{dx}=\frac{y}{x}+\frac{y^2}{2x^2}$

$\Rightarrow y^{-2}\frac{dy}{dx}-\frac{1}{y}\cdot\frac{1}{x}=\frac{1}{2x^2}$

Put $-\frac{1}{y}=t \quad \Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx}+\left(\frac{1}{x}\right)t=\frac{1}{2x^2}$

This is a linear differential equation,

with Integrating Factor : $e^{\int\frac{1}{x}dx}=e^{\ln(x)}=x$

So, solution of the linear differential equation is $tx=\int\frac{1}{2x^2}\cdot x dx+C$

$\Rightarrow -\frac{x}{y}=\frac{1}{2}\ln(x)+C$

The curve passes through $(1,2)$

$\Rightarrow -\frac{1}{2}=\frac{1}{2}\ln(1)+\text{C}\Rightarrow \text{C}=-\frac{1}{2}$

Hence, the particular solution to the differential equation is $-\frac{x}{y}=\frac{1}{2}\ln(x)-\frac{1}{2}$

$\Rightarrow \frac{x}{y}=\frac{1-\ln(x)}{2}\Rightarrow y=\frac{2x}{1-\ln(x)}$

$\Rightarrow f\left(\frac{1}{2}\right)=\frac{2\times\frac{1}{2}}{1-\ln\left(\frac{1}{2}\right)}=\frac{1}{1+\ln(2)}=\frac{1}{1+\log_e(2)}$

11. (3)

Given equation is:

$\left(x^2 - 3y^2\right)dx + 3xydy = 0$

We can re-write equation as

$\frac{dy}{dx} = -\frac{\left(x^2-3y^2\right)}{3xy}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{3}\frac{x}{y}$ ....(1)

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Equation (1) can be written as

$v + x\frac{dv}{dx} = v - \frac{1}{3}\frac{1}{v}$

$\Rightarrow vdv = -\frac{1}{3x}$

Integrating both sides, we get

$\frac{v^2}{2} = -\frac{1}{3}\ln(x)+c$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln(x)+c$ .....(2)

$\because$ $y(1)= 1$ (given)

$\therefore$ $c = \frac{1}{2}$ (from equation 2)

Equation (2) can be written as

$\frac{y^2}{2x^2} = \frac{-1}{3}\ln(x)+\frac{1}{2}$

$\Rightarrow y^2 = \frac{-2}{3}x^2\ln(x)+x^2$

Now $y^2\left(e\right) = \frac{-2}{3}e^2\ln(e)+e^2 = \frac{-2}{3}e^2 + e^2 = \frac{e^2}{3}$

$\Rightarrow 6y^2\left(e\right) = 2e^2$

12. (4)

Given: $\frac{y}{x}\frac{dy}{dx} = \left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$ ....(1)

Let $\frac{y}{x} = t$

$\Rightarrow y = xt$

$\Rightarrow \frac{dy}{dx} = t + x\cdot\frac{dt}{dx}$

$\therefore t\left(t + x\frac{dt}{dx}\right) = \left(t^2 + \frac{\phi\left(t^2\right)}{\phi'\left(t^2\right)}\right)$

$\Rightarrow xt\frac{dt}{dx} = \frac{\phi\left(t^2\right)}{\phi'\left(t^2\right)}$

$\Rightarrow \frac{t\cdot\phi'\left(t^2\right)}{\phi\left(t^2\right)}dt = \frac{1}{x}dx$

Integrating both sides

$\int\frac{t\cdot\phi'\left(t^2\right)}{\phi\left(t^2\right)}dt = \int\frac{1}{x}dx$

Let $\phi\left(t^2\right) = p$

$\Rightarrow \phi'\left(t^2\right).2t = dp$

$\Rightarrow \frac{1}{2}\int\frac{1}{p}dp = \int\frac{1}{x}dx$

$\Rightarrow \frac{1}{2}\ln p = \ln x + C$

$\Rightarrow \frac{1}{2}\ln\phi\left(t^2\right) = \ln x + C$

$\Rightarrow \frac{1}{2}\ln\left(\phi\left(\frac{y^2}{x^2}\right)\right) = \ln x + C \ldots(2)$

If $x = 1,\ y = -1$ then $C = \frac{1}{2}\ln(\phi(1))$

Substituting value of C in (2)

$\frac{1}{2}\ln\left(\phi\left(\frac{y^2}{x^2}\right)\right) = \ln x + \frac{1}{2}\ln(\phi(1))$

$\Rightarrow\ln\left(\phi\left(\frac{y^2}{x^2}\right)\right) = \ln x^2 + \ln(\phi(1))$

If $x = 2$ then

$\ln\left(\phi\left(\frac{y^2}{4}\right)\right) = \ln 4 + \ln(\phi(1))$

SO, $\phi\left(\frac{y^2}{4}\right) = 4\phi(1)$

13. (2)

Given, $\frac{dy}{dx} \propto \frac{-y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{-Ky}{x}$ (where K is proportionality constant)

Now integrating both side,

$\Rightarrow \int \frac{dy}{y} = -K \int \frac{dx}{x}$

$\Rightarrow \ln|y| = -K \ln|x| + C$

If the above equation satisfy $(1, 2)$

$\Rightarrow \ln 2 = -K \times 0 + C \Rightarrow C = \ln 2$

So, $\ln|y| = -K \ln|x| + \ln 2$

Now it also passes through $(8, 1)$

$\Rightarrow \ln 1 = -K \ln 8 + \ln 2 \Rightarrow K = \frac{1}{3}$

So, equation becomes $\ln|y| = -\frac{1}{3}\ln|x| + \ln 2$

So, at $x = \frac{1}{8}$

$\Rightarrow \ln|y| = -\frac{1}{3}\ln\left(\frac{1}{8}\right) + \ln 2 = 2\ln 2$

$\Rightarrow |y| = 4$

14. (1)

Given:

$\frac{dy}{dx} + y \tan x = x \sec x$

This is a linear differential equation.

$\text{I. F.} = e^{\int \tan x dx} = \sec x$

Then solution of differential equation is

$y(\sec x) = \int x \sec^2 x dx$

$\Rightarrow y(\sec x) = x \tan x - \int \tan x dx$

$\Rightarrow y(\sec x) = x \tan x - \ln(\sec x) + C$

Given:

$y(0) = 1 \Rightarrow c = 1$

$\therefore y(\sec x) = x \tan x - \ln(\sec x) + 1$

At $x = \frac{\pi}{6}$, we get

$y\left(\sec\left(\frac{\pi}{6}\right)\right) = \left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right) - \ln\left(\sec\left(\frac{\pi}{6}\right)\right) + 1$

$\Rightarrow y\left(\frac{2}{\sqrt{3}}\right) = \left(\frac{\pi}{6}\right)\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{2}{\sqrt{3}}\right) + 1$

$\Rightarrow y = \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ln\left(\frac{2}{\sqrt{3}}\right) + \frac{\sqrt{3}}{2}$

$\Rightarrow y = \frac{\pi}{12} - \frac{\sqrt{3}}{2}\left[\ln\left(\frac{2}{\sqrt{3}}\right) - \ln e\right]$

$\Rightarrow y = \frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

15. (1)

Given,

$y(x+1)dx - x^2 dy = 0$

$\Rightarrow \frac{x+1}{x^2}dx = \frac{dy}{y}$

$\Rightarrow \left(\frac{1}{x} + \frac{1}{x^2}\right)dx = \frac{dy}{y}$

Now integrating both side we get,

$\log_e x - \frac{1}{x} = \log_e y + c$

Now on using $y(1) = e$ we get, $c = -2$

So, the equation of curve becomes $\log_e x - \frac{1}{x} = \log_e y - 2$

$\Rightarrow y = e^{\ln x} - \frac{1}{x} + 2$

Hence, $\lim_{x \to 0^+} e^{\ln x - 1} - \frac{1}{x} + 2 = e^{-\infty} = 0.$

16. (1)

$\frac{dy}{dx}=\frac{x^2-2y}{x}$

$\Rightarrow \frac{dy}{dx}+\frac{2}{x}y=x$. It is in linear form.

I.F.$=e^{\int \frac{2}{x}dx}=e^{2\ln x}=x^2$

$\therefore y.x^2=\int x.x^2\,dx=\frac{x^4}{4}+c$

$\because$At $x=1,\ y=-2,$

$\therefore -2\times 1^2=\frac{(1)^4}{4}+c$

$\Rightarrow c=-2-\frac{1}{4}=-\frac{9}{4}$

$\therefore$curve is $y.x^2=\frac{x^4}{4}-\frac{9}{4}$

$\therefore$It passes through $\left(\sqrt{3},\ 0\right)$.

17. (1)

$\left(1+x^2\right)dy=y(x-y)dx$

$y(0)=1\cdot y(2\sqrt{2})=\beta$

$$\frac{dy}{dx}=\frac{yx-y^2}{1+x^2}$$

$$\frac{dy}{dx}+y\left(\frac{-x}{1+x^2}\right)=\left(\frac{-1}{1+x^2}\right)y^2$$

$$\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{y}\left(\frac{-x}{1+x^2}\right)=\frac{-1}{1+x^2}$$

put $\frac{1}{y}=t$ then $\frac{-1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

$$\frac{dt}{dx}+t\frac{x}{1+x^2}=\frac{1}{1+x^2}$$

$$I.F=e^{\int\frac{x}{1+x^2}dx}=e^{\frac{1}{2}\ln(1+x^2)}=\sqrt{1+x^2}$$

$$t\sqrt{1+x^2}=\int\frac{1}{\sqrt{1+x^2}}dx$$

$$\frac{\sqrt{1+x^2}}{y}=\ln\left(x+\sqrt{x^2+1}\right)+c$$

$y(0)=1\quad\Rightarrow c=1$

$$\Rightarrow\sqrt{1+x^2}=y\ln\left(e\left(x+\sqrt{x^2+1}\right)\right)$$

$$\beta=\frac{3}{\ln(e(3+2\sqrt{2}))}\Rightarrow\frac{3}{\beta}=\ln(e(3+2\sqrt{2}))$$

$$e^{\frac{3}{\beta}}=e(3+2\sqrt{2})$$

18. (3)

Given,

$\frac{dy}{dx}-y=2-e^{-x}$

Linear differential equation,

So, $I.F.\ =\mathrm{e}^{-\int dx}=\mathrm{e}^{-x}$

Now solution of differential equation is given by

$y\times IF=\int(2-e^{-x})\times IFdx$

$\Rightarrow ye^{-x}=\int(2-e^{-x})e^{-x}dx$

$\Rightarrow ye^{-x}=\int\left(2e^{-x}-e^{-2x}\right)dx$

$\Rightarrow ye^{-x}=-2e^{-x}+\frac{e^{-2x}}{2}+C$

$\Rightarrow y=-2+\frac{e^{-x}}{2}+Ce^{x}$

Given $\lim_{x\to\infty}y$ is finite

So, $\lim_{x\to\infty}\left(-2+\frac{e^{-x}}{2}+C\cdot e^{x}\right)\to$finite

This is possible only when $C=0$

Hence $y=y(x)=-2+\frac{e^{-x}}{2}$

Now finding the slope $\frac{dy}{dx}=-\frac{1}{2}e^{-x}$

$\left.\frac{dy}{dx}\right|_{x=0}=-\frac{1}{2}=m, y(0)=-2+\frac{1}{2}=\frac{-3}{2}$

Now equation of tangent will be,

$y+\frac{3}{2}=-\frac{1}{2}(x-0)$

$\Rightarrow x+2y=-3$

x-intercept $a=-3$, y-intercept $b=\frac{-3}{2}$

So, $a-4b=-3+6=3$

19. (2)

Given $2x^2\frac{dy}{dx}-2xy+3y^2=0$

Dividing by $2x^2y^2$, we get,

$-\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{xy}=\frac{3}{2x^2}$

Let $\frac{1}{y}=p, \frac{-1}{y^2}\frac{dy}{dx}=\frac{dp}{dx}$

$\Rightarrow \frac{dp}{dx}+\frac{1}{x}(p)=\frac{3}{2x^2}$

General solution will be $p\cdot e^{\int\frac{1}{x}dx}=\int\frac{3}{2x^2}\cdot e^{\int\frac{1}{x}dx}dx+c$

$\Rightarrow \frac{x}{y}=\int\frac{3}{2x}dx+c \Rightarrow \frac{x}{y}=\frac{3}{2}\ln x+c$

$\because \quad y(e)=\frac{e}{3}\Rightarrow c=\frac{3}{2} \quad \Rightarrow \frac{x}{y}=\frac{3}{2}\ln x+\frac{3}{2}$ is the particular solution.

when $x=1;\ y(1)=\frac{2}{3}$

20. (1)

Given $x(1-x^2)\frac{dy}{dx}+(3x^2y-y-4x^3)=0$

$(x-x^3)\frac{dy}{dx}+(3x^2-1)y=4x^3$

$\frac{dy}{dx}+\frac{(3x^2-1)}{(x-x^3)}y=\frac{4x^3}{(x-x^3)}$

This is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$

Here $IF=e^{\int Pdx}=e^{\int\frac{3x^2-1}{x-x^3}dx}$

Let $x-x^3=t\Rightarrow IF=e^{\int-\frac{dt}{t}}$

$=e^{-\ln t}=\frac{1}{t}$

$\therefore\ IF=\frac{1}{x-x^3}$

So the general solution of the differential equation will be

$y\times IF=\int Q\times IFdx$

$y\left(\frac{1}{x-x^3}\right)=\int\frac{4x^3}{x-x^3}\times\frac{1}{(x-x^3)}dx$

$=\int\frac{4x^3}{(x-x^3)^2}dx$

$=\int\frac{4x}{(1-x^2)^2}dx$

Now let $I=\int\frac{4x}{(1-x^2)^2}dx$

Substituting $1-x^2=z$

$I=2\int\frac{-dz}{z^2}$

$=-2\left(-\frac{1}{z}\right)+c=\frac{2}{1-x^2}+c$

Hence the solution of the differential equation becomes

$\frac{y}{x-x^3}=\frac{2}{1-x^2}+c$

At $x=2,\ y=-2$

$\frac{-2}{2-8}=\frac{2}{1-4}+c$

$\frac{1}{3}=\frac{-2}{3}+c$

$\therefore c=1$

Hence the particular solution will be $\frac{y}{x-x^3}=\frac{2}{1-x^2}+1$

Put $x=3$

$\frac{y}{3-27}=\frac{2}{1-9}+1$

$\frac{y}{-24}=-\frac{1}{4}+1$

$-\frac{y}{24}=\frac{3}{4}$

$y=\frac{3}{4}(-24)=-18$

21. (1)

Given,

$x \ln x \frac{dy}{dx} + y = x^2 \ln x$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x \ln x} = x$

This is a linear differential equation.

Now, finding integrating factor we get

$\mathrm{I.F} = \mathrm{e}^{\int \frac{1}{\mathrm{xlnx}} \mathrm{dx}} = \mathrm{e}^{\ln(\ln \mathrm{x})} = \ln \mathrm{x}$

Now, solution of the differential equation is given by

$y \times \mathrm{I.F} = \int (\mathrm{x} \times \mathrm{I.F}) dx$

$\Rightarrow y \times \ln x = \int \mathrm{xlnx}\ dx$

$\Rightarrow y \times \ln x = \ln x \cdot \frac{x^2}{2} - \frac{x^2}{4} + c$

Now given $y(2) = 2$,

$2 \times \ln 2 = \ln 2 \cdot \frac{2^2}{2} - \frac{2^2}{4} + c$

$\Rightarrow c = 1$

So, equation will be $y \times \ln x = \ln x \cdot \frac{x^2}{2} - \frac{x^2}{4} + 1$

Now finding $y(e)$ we get,

$y \times \ln e = \ln e \cdot \frac{e^2}{2} - \frac{e^2}{4} + 1$

$\Rightarrow y \times 1 = 1 \cdot \frac{e^2}{2} - \frac{e^2}{4} + 1$

$\Rightarrow y = \frac{4+e^2}{4}$

22. (1)

Given differential equation can be rewritten as,

$\frac{dy}{dx} - \frac{y}{x} = y^3(1 + \log_e x)$

$\frac{1}{\mathrm{y}^3} \frac{\mathrm{dy}}{\mathrm{dx}} - \frac{1}{\mathrm{xy}^2} = 1 + \log_e \mathrm{x}$

Let $\left(-\frac{1}{\mathrm{y}^2}\right) = \mathrm{t} \Rightarrow \frac{2}{\mathrm{y}^3} \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{dt}}{\mathrm{dx}}$

$\therefore \frac{\mathrm{dt}}{2\,\mathrm{dx}} + \frac{\mathrm{t}}{\mathrm{x}} = (1 + \log_e \mathrm{x})$

$\Rightarrow \frac{\mathrm{dt}}{\mathrm{dx}} + \frac{2\mathrm{t}}{\mathrm{x}} = 2(1 + \log_e \mathrm{x}) \ldots\ldots\ldots \left(1\right)$

We know the solution of the differential equation $\frac{dy}{dx} + Py = Q$ is given by,

$\mathrm{ye}^{\int \mathrm{Pdx}} = \int \mathrm{Qe}^{\int \mathrm{Pdx}} + \mathrm{C}\ \left(\text{Where I.F.} = \mathrm{e}^{\int \mathrm{Pdx}}\right)$

Therefore, on solving equation(1) we get,

$\mathrm{I.F.} = \mathrm{e}^{\int \frac{2}{x} dx} = x^2$

$\frac{-x^2}{y^2} = \frac{2}{3}\left((1 + \log_e x)x^3 - \frac{x^3}{3}\right) + C \ldots\ldots \left(2\right)$

Given, $y(1) = 3$

$\Rightarrow \frac{1}{9} = -2\left(\frac{1}{3} - \frac{1}{9}\right) + C$

$\therefore C = \frac{5}{9}$

Now putting value of C in equation(2) we get,

$\frac{y^2}{9} = \frac{x^2}{5 - 2x^3\left(2 + \log_e x^3\right)}$

23. **(2)** Differentiate the given equation

$\Rightarrow 2xf(x)+x^2f'(x)-1=4xf(x)$

$\Rightarrow x^2\frac{dy}{dx}-2xy=1$

$\Rightarrow \frac{dy}{dx}+\left(-\frac{2}{x}\right)y=\frac{1}{x^2}$

$I.F.=e^{\int -\frac{2}{2}L_{nx}}=\frac{1}{x^2}$

$\therefore y\left(\frac{1}{x^2}\right)=\int\frac{1}{x^4}dx$

$\Rightarrow \frac{y}{x^2}=\frac{-1}{3x^3}+c$

$\Rightarrow y=-\frac{1}{3x^3}+c$

$\Rightarrow y=-\frac{1}{3x}+cx^2$

$\because f(1)=\frac{2}{3}=-\frac{1}{3}+c\Rightarrow c=1$

$f(x)=-\frac{1}{3x}+x^2$

$18f(3)=160$

24. **(1)**

Let

$\int_0^2 f(t)dt=k$

$y=f(x)$

Now, we have

$f(x)+f'(x)=\int_0^2 f(t)dt$

$\Rightarrow \frac{dy}{dx}+y=k$

This is a linear differential equation.

I. F. $=e^{\int dx}=e^x$

Solution of the given differential equation is

$ye^x=k\int e^x dx$

$\Rightarrow ye^x=ke^x+C$

Now, $f(0)=e^{-2}$, so

$C=e^{-2}-k$

Hence,

$ye^x=ke^x+\left(e^{-2}-k\right)$

$\Rightarrow y=f(x)=k+\left(e^{-2}-k\right)e^{-x}$ ....(i)

Now,

$k=\int_0^2 f(t)dt$

$\Rightarrow k=\int_0^2\left[k+\left(e^{-2}-k\right)e^{-t}\right]dt$

$\Rightarrow k=\left[kt-\left(e^{-2}-k\right)e^{-t}\right]_0^2$

$\Rightarrow k=2k-\left(e^{-2}-k\right)\left(e^{-2}-1\right)$

$\Rightarrow\left(e^{-2}-k\right)\left(e^{-2}-1\right)=k$

$\Rightarrow k=\left(e^{-2}-1\right)$

So,

$f(x)=\left(e^{-2}-1\right)+e^{-x}$

$\therefore f(2)=2e^{-2}-1$

$f(0)=e^{-2}$

So,

$2f(0)-f(2)=1$

25. (3)

Given

$\frac{dy}{dx}+e^x(x^2-2)y=(x^2-2x)(x^2-2)e^{2x}$

It is linear differential equation so,

$IF=e^{\int(x^2-2)e^x dx}$

$=e^{x^2e^x-2\int xe^x dx-2e^x}=e^{x^2e^x-2[xe^x-e^x]-2e^x}$

$IF=e^{(x^2-2x)e^x}$

Now solution is given by

$y\times e^{(x^2-2x)e^x}=\int e^{(x^2-2x)e^x}\times(x^2-2x)(x^2-2)e^{2x}$

$=\int e^{(x^2-2x)e^x}\times(x^2-2x)e^x(x^2-2)e^x$

Now let $(x^2-2x)e^x=t$

We get $e^x(x^2-2x)+e^x(2x-2)dx=dt$

$e^x(x^2-2x+2x-2)dx=dt$

$e^x(x^2-2)dx=dt$

$y\times e^{(x^2-2x)e^x}=\int e^t\times t\times dt$

$y\times e^{(x^2-2x)e^x}=e^t(t-1)+c$

$y\times e^{(x^2-2x)e^x}=e^{(x^2-2x)e^x}((x^2-2x)e^x-1)+c$

Now at $x=0$ $y=0$

$0\times e^0=e^0(0xe^x-1)+c$

$0=-1+c$ $c=1$

Putting the value of $c=1$ we get the equation as

$\Rightarrow y\times e^{(x^2-2x)e^x}=e^{(x^2-2x)e^x}((x^2-2x)e^x-1)+1$

Now putting $x=2$ in equation we get

$y\times e^{(4-4)e^2}=e^{(4-4)e^2}((4-4)e^2-1)+1$

$\Rightarrow y\times e^0=e^0((0)e^2-1)+1$

$\Rightarrow y=-1+1=0$

26. (1)

We have, $xdy=(y+x^3\cos x)dx$

$\Rightarrow xdy=ydx+x^3\cos xdx$

$\Rightarrow \frac{xdy-ydx}{x^2}=\frac{x^3\cos xdx}{x^2}$

$\Rightarrow \frac{d}{dx}\left(\frac{y}{x}\right)=\int x\cos xdx$

$\Rightarrow \frac{y}{x}=x\sin x-\int 1.\sin xdx$

Therefore, $\frac{y}{x}=x\sin x+\cos x+C$

At $x=\pi,\ y=0,$

$0=-1+C$

$\Rightarrow C=1, x=\pi, y=0$

So, $\frac{y}{x}=x\sin x+\cos x+1$

$\Rightarrow y=x^2\sin x+x\cos x+x$

Hence, $y\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}+\frac{\pi}{2}$.

27. (2)

$(x-x^3)dy=(y+yx^2-3x^4)dx$

$\Rightarrow xdy-ydx=(yx^2-3x^4)dx+x^3dy$

$\Rightarrow \frac{xdy-ydx}{x^2}=\left(ydx+xdy\right)-3x^2dx$

$\Rightarrow d\left(\frac{y}{x}\right)=d(xy)-d(x^3)$

Integrate

$\Rightarrow \frac{y}{x}=xy-x^3+c$

given $f(3)=3$

$\Rightarrow \frac{3}{3}=3\times3-3^3+c$

$\Rightarrow c=19$

$\therefore \frac{y}{x}=xy-x^3+19$

at $x=4, \frac{y}{4}=4y-64+19$

$15y=4\times45$

$\Rightarrow y=12$

28. (1)

Given differential equation, $\sin(2x^2)\ln(\tan x^2)dy+\left(4xy-4\sqrt{2}x\sin\left(x^2-\frac{\pi}{4}\right)\right)dx=0$

$\Rightarrow \ln(\tan x^2)dy+\frac{4xydx}{\sin(2x^2)}-\frac{4\sqrt{2}x\sin\left(x^2-\frac{\pi}{4}\right)}{\sin(2x^2)}dx=0$

$\Rightarrow d(y\cdot\ln(\tan x^2))-4\sqrt{2}x\frac{(\sin x^2-\cos x^2)}{\sqrt{2}(2\sin x^2\cos x^2)}dx=0$

$\Rightarrow d(y\ln(\tan x^2))-\frac{4x(\sin x^2-\cos x^2)}{(\sin x^2+\cos^2 x)^2-1}dx=0$

Now integrating both side,

$\Rightarrow \int d(y\ln(\tan x^2))-\int\frac{4x(\sin x^2-\cos x^2)}{(\sin x^2+\cos^2 x)^2-1}dx=\int 0$

Now let $\sin x^2+\cos^2 x=t\Rightarrow -2x(\sin x^2-\cos^2 x)=dt$

$\Rightarrow \int d(y\ln(\tan x^2))+2\int\frac{dt}{t^2-1}=\int 0$

$\Rightarrow y\ln(\tan x^2)+2\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|=c$

$y\ln(\tan x^2)+\ln\left(\frac{\sin x^2+\cos x^2-1}{\sin x^2+\cos x^2+1}\right)=c$

Put $y=1$ and $x=\sqrt{\frac{\pi}{6}}$

$1\ln\left(\frac{1}{\sqrt{3}}\right)+\ln\frac{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}-1\right)}{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}+1\right)}=c$

Now at $x=\sqrt{\frac{\pi}{3}}$

$\Rightarrow y(\ln\sqrt{3})+\ln\frac{\left(\frac{\sqrt{3}}{2}+\frac{1}{2}-1\right)}{\left(\frac{\sqrt{3}}{2}+\frac{1}{2}+1\right)}=\ln\left(\frac{1}{\sqrt{3}}\right)+\ln\left(\frac{\sqrt{3}-1}{\sqrt{3}+3}\right)$

$\Rightarrow y(\ln\sqrt{3})=\ln\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad y=-1$

So, $|y|=1$

29. (4)

Given,

$2ye^{\frac{x}{y^2}}dx+\left(y^2-4xe^{\frac{x}{y^2}}\right)dy=0$

$2e^{\frac{x}{y^2}}[ydx-2xdy]+y^2dy=0$

$2e^{\frac{x}{y^2}}\left[\frac{y^2dx-x\cdot(2y)dy}{y}\right]+y^2dy=0$

Divide by y^3

$2e^{\frac{x}{y^2}}\left[\frac{y^2dx-x\cdot(2y)dy}{y^4}\right]+\frac{1}{y}dy=0$

$2e^{\frac{x}{y^2}}d\left(\frac{x}{y^2}\right)+\frac{1}{y}dy=0$

Now integrating both side we get,

$\int 2e^{\frac{x}{y^2}}d\left(\frac{x}{y^2}\right)+\int\frac{1}{y}dy=0$

$2e^{\frac{x}{y^2}}+\ln y+c=0$

Given, $(0,1)$ lies on it,

So, $2e^0+\ln 1+c=0\Rightarrow c=-2$

Hence required curve: $2e^{\frac{x}{y^2}}+lny-2=0$

For $x(e)$

$2e^{\frac{x}{e^2}}+\ln e-2=0\quad\Rightarrow\quad x=-e^2\log_e 2$

30. **(4)**

The given differential equation is $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)}+(y+1)\right)dx=(x+2)dy,\ y(1)=1$

Let, $y+1=Y \Rightarrow dy=dY$ and $x+2=X \Rightarrow dx=dX$ and at $y=1,\ Y=2$ and at $x=1,\ X=3$.

Thus, the differential equation becomes $\left(Xe^{\frac{Y}{X}}+Y\right)dX=XdY$

$\Rightarrow XdY-YdX=Xe^{\frac{Y}{X}}dX$

$\Rightarrow \frac{(XdY-YdX)}{X^2}=\left(\frac{Xe^{\frac{Y}{X}}}{X^2}\right)dX$

$\Rightarrow \frac{d}{dX}\left(\frac{Y}{X}\right)=\left(\frac{Xe^{\frac{Y}{X}}}{X^2}\right)dX$

$\Rightarrow d\left(\frac{Y}{X}\right)e^{-\frac{Y}{X}}=\frac{dX}{X}$

Integrating both sides w.r.to X, we get $\int d\left(\frac{Y}{X}\right)e^{-\frac{Y}{X}}=\int\frac{dX}{X}$

$\Rightarrow -e^{-\frac{Y}{X}}=\log_e|X|+c$

Given, at $X=3,\ Y=2$

$\Rightarrow -e^{-\frac{2}{3}}=\log_e|3|+c$

$\Rightarrow c=-e^{-\frac{2}{3}}-\log_e|3|$

$\Rightarrow -e^{-\frac{Y}{X}}=\log_e|X|-e^{-\frac{2}{3}}-\log_e 3$

$e^{-\frac{Y}{X}}=e^{\frac{2}{3}}+\log_e 3-\log_e|X|>0$

$\log_e|X|<e^{\frac{2}{3}}+\log_e 3$

Let, $\lambda=\left(e^{\frac{2}{3}}+\log_e 3\right)$ then, we have $|x+2|<e^{\lambda}$

$\Rightarrow -e^{\lambda}<x+2<e^{\lambda}$

$\Rightarrow -e^{\lambda}-2<x<e^{\lambda}-2$

Thus, the domain of the function is $\left(-e^{\lambda}-2,\ e^{\lambda}-2\right)$.

Given, the domain of the function is $(\alpha,\ \beta)$, hence $\alpha=-e^{\lambda}-2$ & $\beta=e^{\lambda}-2$

$\Rightarrow \alpha+\beta=-e^{\lambda}-2+e^{\lambda}-2=-4$

$\Rightarrow |\alpha+\beta|=4.$