ANSWER KEYS

1. (2) **2.** (3) **3.** (3) **4.** (-1) **5.** (2) **6.** (1) **7.** (4) **8.** (2)
9. (1) **10.** (2)

1. (2)

Since, $5x-1<x^2+2x+1$

$\Rightarrow \quad x^2-3x+2>0$

$\Rightarrow \quad (x-1)(x-2)>0$

$\therefore \quad x<1$ or $x>2 \quad \ldots(1)$

Again, $x^2+2x+1<7x-3$

$\Rightarrow \quad x^2-5x+4<0$

$\Rightarrow \quad (x-1)(x-4)<0$

$\Rightarrow 1<x<4 \qquad \ldots(2)$

From Eqs. (1) and (2), we get

$2<x<4$

Since, x is integral, then the required value is $x=3$.

2. (3)

We have,

$(x-2)^4(x-3)^3(x-4)^2(1-x)\leq 0$

$\Rightarrow (x-2)^4(x-3)^3(x-4)^2(x-1)\geq 0$

Using wavy-curve method, we get

$x\in(-\infty,1]\cup[3,\infty)$

3. (3)

$\frac{x+1}{2x-1}\geq 1$ and $\frac{x+1}{2x-1}-2<0$

$\Rightarrow x\in\left(\frac{1}{2},2\right]$ and $x\in\left(-\infty,\frac{1}{2}\right)\cup\left(1,\infty\right)\Rightarrow x\in\left(1,2\right]$

4. (-1) $\frac{1}{x-2}-\frac{1}{x}\leq\frac{2}{x+2}$

$\frac{2}{x(x-2)}\leq\frac{2}{x+2}$

$\frac{(x+2)-x(x-2)}{x(x-2)(x+2)}\leq 0$

$\frac{-x^2+3x+2}{x(x-2)(x+2)}\leq 0\Rightarrow\frac{x^2-3x-2}{x(x-2)(x+2)}\geq 0$

$\left(-2,\frac{3-\sqrt{17}}{2}\right]\cup\left(0,2\right)\cup\left[\frac{3+\sqrt{17}}{2},\infty\right)$

5. (2)

$\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(2x-7)^6} \leq 0$

$\Rightarrow x=0, \frac{4}{3}, 2, 3x-4<0, x-5>0$

or $3x-4>0, x-5<0$

$\left[\because x^2,(x-2)^4,(2x-7)^6>0\right]$

$\Rightarrow x=0, \frac{4}{3}, 2,\ x<\frac{4}{3},\ x>5$ or $x\frac{4}{3},\ x<5$

$\Rightarrow x=0,2$ and integral value between

$\frac{4}{3}<x<5$ i.e., $x=2,3,4$.

Hence, positive integral solutions are $2,3,4$.

So, three positive integral solutions are possible.

6. (1)

The given expression is $7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$

$\Rightarrow \log\left(\frac{16}{15}\right)^7+\log\left(\frac{25}{24}\right)^5+\log\left(\frac{81}{80}\right)^3$

Since bases of every logarithmic terms in addition are equal. So, we can follow the following property of logarithm-

$\log_m(x)+\log_m(y)=\log_m(xy)$ So, by property of logarithm, we have $\Rightarrow \log\left[\left(\frac{16}{15}\right)^7\times\left(\frac{25}{24}\right)^5\times\left(\frac{81}{80}\right)^3\right]=\log 2$ Hence, required value is $\log 2$.

7. (4)

Here, $5^{2\log_{10}x}=5+4\times 5^{\log_5 x^{\log_{10}5}}$ $\left\{\because a=b^{\log_b a}\right\}$

$=5+4\times 5^{\log_{10}5^{\log_5 x}}=5+4\times 5^{\log_{10}x}$

Note : Our purpose was to write $x^{\log_{10}(5)}$ in terms of an exponential with the common base 5.

So, the equation becomes

$\left(5^{\log_{10}x}\right)^2-4\left(5^{\log_{10}x}\right)-5=0$

$\Rightarrow\left(5^{\log_{10}x}-5\right)\left(5^{\log_{10}x}+1\right)=0$

But $5^{\log_{10}x}+1\neq 0$ $\left(\because 5^{\log_{10}x}=+ve\right)$

$\therefore 5^{\log_{10}x}-5=0$

$\therefore 5^{\log_{10}x}=5$; or $\log_{10}x=1$

Hence, $x=10$

8. (2) $abc+1=\frac{\log 12}{\log 24}\times\frac{\log 34}{\log 36}\times\frac{\log 36}{\log 48}+1$

$=\frac{\log(48\times 12)}{\log 48}$

$=\frac{2\log 24}{\log 48}=2bc$

Hence, (B) is correct.

9. $\log_x 2\log_{2x}2=\log_{4x}2$

$\Rightarrow (\log_x)(\log_2 2x)=\log_2 4x$

Let $\log_\tau x=t$

(1) $\Rightarrow t(1+t)=2+t$

$\Rightarrow t^2=2$

$\Rightarrow t=\pm\sqrt{2}$

$\Rightarrow x=2^{\pm}\sqrt{2}$

Hence, (A) is correct.

10. (2) $\log_{10}(7x-9)^2+\log_{10}(3x-4)^2=2$

$\Rightarrow \quad (7x-9)^2(3x-4)^2=(10)^2$

$\Rightarrow \quad (7x-9)(3x-4)=\pm 10$

$\Rightarrow \quad \left(21x^2-55x+36\right)=\pm 10$

Either $21x^2-55x+46=0$ or $21x^2-55x+26=0$

$21x^2-55x+46=0$ has no real root.

and $21x^2-55x+26=0$ has 2 real roots

Hence, (B) is correct.