## ANSWER KEYS

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|---|---|---|---|---|---|---|---|
| **1.** (4) | **2.** (1) | **3.** (4) | **4.** (997.50) | **5.** (2) | **6.** (1) | **7.** (2) | **8.** (4) |
| **9.** (3) | **10.** (2) | | | | | | |

**1.** **(4)** $f(x+y)=f(x)+f(y)$

$f(1)=7 \Rightarrow f(1+1)=f(1)+f(1)$

$f(2)=14$

Similarly, $f(3)=21$

$f(n)=7n$

$\sum_{\gamma=1}^{n} f(\gamma)=f(1)+f(2)+$ ............... $f(n)$

$=7+14+$ ............. $7n$

$=7(1+2+............n)$

$=7\frac{n(n+1)}{2}$

**2.** **(1)**

We have, $f(x)=\frac{a^x+a^{-x}}{2}$, $(a>2)$, where $x \in R$ is the domain of the function.

$\Rightarrow f(x)=\frac{a^{2x}+1}{2a^x}$

Now, $f(x+y)=\frac{a^{x+y}+a^{-x-y}}{2}$ and

$f(x-y)=\frac{a^{x-y}+a^{-x+y}}{2}$

$\therefore f(x+y)+f(x-y)=\frac{a^x a^y+\frac{1}{a^x a^y}}{2}+\frac{\frac{a^x}{a^y}+\frac{a^y}{a^x}}{2}$

$=\frac{(a^x a^y)^2+1+(a^x)^2+(a^y)^2}{2a^x a^y}$

$=\frac{(a^{2x}+1)(a^{2y}+1)}{2a^x a^y}$

$=2\cdot\frac{a^{2x}+1}{2a^x}\cdot\frac{a^{2y}+1}{2a^y}$

$=2f(x)f(y)$

**3.** **(4)**

Given that, $f$ is a real valued function and $f(x+y)=f(x)+f(y)$ and $f(1)=5$.

Now put here, $x=1$ & $y=99$

$f(100)=f(1)+f(99)$

$f(100)=f(1)+f(1)+f(98)$

$f(100)=2f(1)+f(1)+f(97)$

$f(100)=3f(1)+f(1)+f(96)$

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$f(100)=100f(1)$

Hence, $f(100)=500$ ; $f(1)=5$

**4.** **(997.50)**

We have, $f(x)=\frac{9^x}{9^x+3}$

$\therefore\ f(x)+f(1-x)=1$

$\therefore\ f\left(\frac{1}{1996}\right)+f\left(1-\frac{1}{1996}\right)=1\ \ldots(i)$

$f\left(\frac{2}{1996}\right)+f\left(1-\frac{2}{1996}\right)=1\ \ldots(ii)$

and so on $f\left(\frac{1995}{1996}\right)+f\left(1-\frac{1995}{1996}\right)=1\ \ldots(iii)$

Adding all the equation we get,

$2\left(f\left(\frac{1}{1996}\right)+f\left(\frac{2}{1996}\right)+\ldots+f\left(\frac{1995}{1996}\right)\right)=1995$

$\Rightarrow\ f\left(\frac{1}{1996}\right)+f\left(\frac{2}{1996}\right)+\ldots+f\left(\frac{1995}{1996}\right)=997.5$

**5.** **(2)** $5f(x)+3f\left(\frac{1}{x}\right)=x+2 \quad \ldots(1)$

Replacing $x$ by $\frac{1}{x}$

$\therefore\ 5f\left(\frac{1}{x}\right)+3f(x)=\frac{1}{x}+2 \quad \ldots(2)$

From (1)

$25f(x)+15f\left(\frac{1}{x}\right)=5x+10 \quad \ldots(3)$

and from (2)

$9f(x)+15f\left(\frac{1}{x}\right)=\frac{3}{x}+6 \quad \ldots(4)$

Subtracting(4) from (3)

$\therefore 16f(x)=5x-\frac{3}{x}+4$

$\therefore xf(x)=\frac{5x^2-3+4x}{16}=y$

$\therefore \frac{dy}{dx}=\frac{10x+4}{16}$

$\left.\frac{dy}{dx}\right|_{x=1}=\frac{10+4}{16}=\frac{7}{8}$.

**6.** **(1)**

$f(x)=2x+\sin x,\ \forall x\in R$

$\Rightarrow f'(x)=2+\cos x>0$, since $\cos x\in[-1,1]$

$\Rightarrow f$ is increasing

As, the function is increasing in its domain.

So, it is a one-one function.

As $x\in R$, therefore $2x+\cos x\in R$

$\Rightarrow$ Range=Co-domain.

So, it is an onto function.

Hence, the function is One-one and onto.

**7.** **(2)**

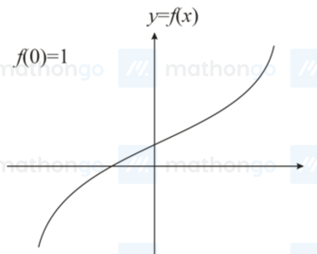
The function is $R\rightarrow R$

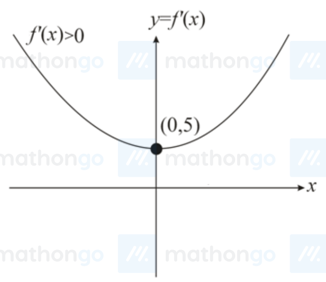
$f(x)=x^3+5x+1$

$f'(x)=3x^2+5>0\ \forall x\in R$

$\therefore$ f(x) is strictly increasing on R

$\therefore$ f is one-one and onto R




**8.** **(4)** We have, $f(x)=\frac{x^2-8}{x^2+2}$

Clearly, $f(-x)=f(x)$ . Therefore, f is not one - one

Again,

$f(x)=\frac{x^2-8}{x^2+2}=1-\frac{10}{x^2+2}$

$\Rightarrow \quad f(x)<1$ for all $x\in R$

$\Rightarrow \quad$ Range of $f\neq$ codomain of f, i.e.R

So, f is not onto

Hence, f is neither one - one nor onto

**9.** **(3)** Rewriting the given function, we get, $f(x) = (x - 6a)^2 + 15 - 2a$

$\because f(x)$ is surjective on $R$

$\Rightarrow 15 - 2a = 2 \Rightarrow 2a = 13 \Rightarrow a = \frac{13}{2}$

**10.** **(2)** Given, $f(x) = (x-1)(x-2)(x-3)$

Clearly $f(1) = f(2) = f(3) = 0$

$\Rightarrow$ $f(x)$ is not one-one.

We know that an odd degree polynomial function ranges from $-\infty$ to $\infty$.

Therefore, $f$ is onto.