ANSWER KEYS

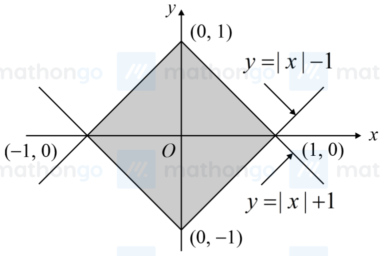
1. (2) **2.** (3) **3.** (1) **4.** (2) **5.** (3) **6.** (2) **7.** (3) **8.** (4)
9. (1) **10.** (41)

1. **(2)** Length of side of square $= \sqrt{2}$

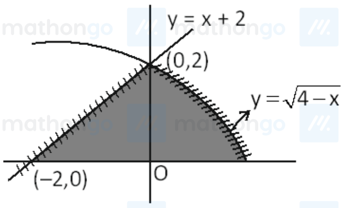
Required area $=$ shaded Area

$= \left(\sqrt{2}\right)^2$

$= 2$ sq. units



2. **(3)**



Required area $= \int_0^2 \left[\left(4-y^2\right)-(y-2)\right] dy$

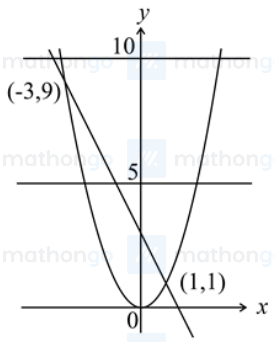
$= \int_0^2 \left(6-y^2-y\right) dy$

$= \left[6y - \frac{y^3}{3} - \frac{y^2}{2}\right]_0^2 = 12 - \frac{8}{3} - 2$

$= \frac{22}{3}$ sq. units

3. **(1)**

Point of intersection of $y = x^2$ & $y = -2x + 3$, is obtained by $x^2 + 2x - 3 = 0 \Longrightarrow x = -3, 1$.

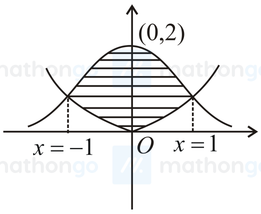


So, area of the required region is

$\int_{-3}^{1} \left((3-2x)-x^2\right)\, \mathrm{d}x$

$= \left[3x - x^2 - \frac{x^3}{3}\right]_{-3}^{1} = \frac{32}{3}$.

4. **(2)**



For point of intersection $x^2 = \frac{2}{1+x^2}$

$\Rightarrow x^4 + x^2 - 2 = 0$

$\Rightarrow (x^2 - 1)(x^2 + 2) = 0$

$\Rightarrow x = 1, -1, \quad \Rightarrow y = 1, -1$

$\therefore$ area $= 2\int_0^1 \left(\frac{2}{1+x^2} - x^2\right) dx$

$= 4\left[\tan^{-1} x\right]_0^1 - \frac{2}{3}\left[x^3\right]_0^1$

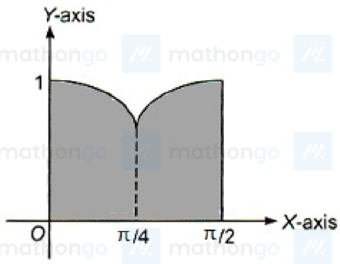
$= 4\left(\frac{\pi}{4}\right) - \frac{2}{3}$

$= \pi - \frac{2}{3}$ sq. units

5. **(3)** $f(x) = \begin{cases} \cos x \text{ for } 0 \le x \le \pi/4 \\ \sin x \text{ for } \pi/4 < x \le \pi/2 \end{cases}$

$\therefore$ Required Area $= 2\int_0^{\pi/4} \cos x dx = 2[\sin x]_0^{\pi/4}$

$= \sqrt{2}$ sq units



$\Rightarrow \quad k = \sqrt{2}$

$\Rightarrow [k + 3] = \left[\sqrt{2} + 3\right] = 4$

6. **(2)** Here, area between 0 to b is R_1 and b to 1 is R_2.

$\therefore \int_0^b (1-x)^2 dx - \int_b^1 (1-x)^2 dx = \frac{1}{4}$

$\Rightarrow \left(\frac{(1-x)^3}{-3}\right)_0^b - \left(\frac{(1-x)^3}{-3}\right)_b^1 = \frac{1}{4}$

$\Rightarrow -\frac{1}{3}\left\{(1-b)^3 - 1\right\} + \frac{1}{3}\left\{0 - (1-b)^3\right\}$

$= \frac{1}{4}$

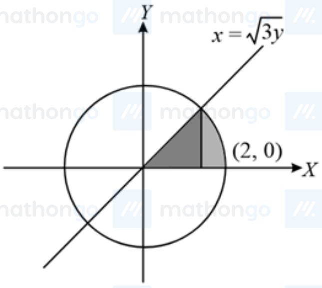
$\Rightarrow -\frac{2}{3}(1-b)^3 = -\frac{1}{3} + \frac{1}{4} = -\frac{1}{12}$

$\Rightarrow (1-b)^3 = \frac{1}{8}$

$\Rightarrow (1-b) = \frac{1}{2} \Rightarrow b = \frac{1}{2}$

7. (3)

Required area $\Delta=\int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} d x+\int_{\sqrt{3}}^2 \sqrt{4-x^2} d x$



$=\frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}}+\left[\frac{x}{2} \sqrt{4-x^2}+\frac{4}{2} \sin ^{-1} \frac{x}{2}\right]_{\sqrt{3}}^2$

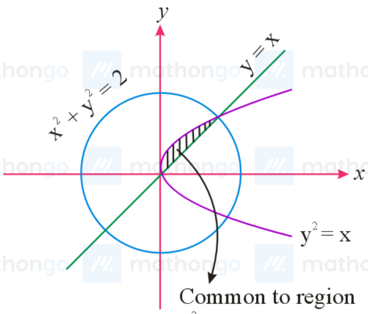
$=\frac{\sqrt{3}}{2}+\left[\pi-\frac{\sqrt{3}}{2}-\frac{2 \pi}{3}\right]$

$=\frac{\pi}{3}$ sq. units.

Trick: Area of sector made by an arc $=\frac{\theta R^2}{2}$ (where, θ is in radian and R is the radius of the circle).

$=\frac{\pi}{6} \cdot \frac{4}{2}=\frac{\pi}{3}$.

8.



(4)

Required area

$=2 \pi-\int_0^1(\sqrt{x}-x) d x$

$=2 \pi-\left(\frac{2 x^{\frac{3}{2}}}{3}-\frac{x^2}{2}\right)_0^1$

$=2 \pi-\left(\frac{2}{3}-\frac{1}{2}\right)$

$=2 \pi-\left(\frac{1}{6}\right)$

$=\frac{12 \pi-1}{6}$ sq. units

9. (1)

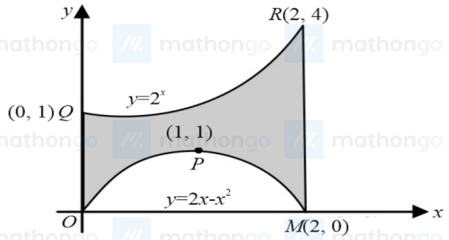
Given curves are

$y = 2^x \ \ldots(i)$

$(x-1)^2 = -(y-1) \ \ldots(ii)$

The required area

$$\Delta = \int_0^2 (y_{upper} - y_{lower})dx$$



(where, $y_{upper} = 2^x$ & $y_{lower} = 2x - x^2$).

$$\therefore \Delta = \int_0^2 (2^x - 2x + x^2)dx$$

$$= \left[\frac{2^x}{\log 2} - x^2 + \frac{1}{3}x^3\right]_0^2$$

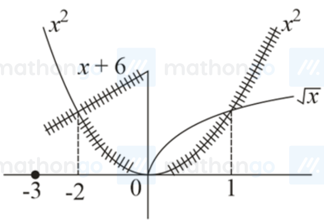
$$= \left(\frac{4}{\log 2} - 4 + \frac{8}{3}\right) - \frac{1}{\log 2}$$

$$= \left(\frac{3}{\log 2} - \frac{4}{3}\right) \text{ sq. units.}$$

10. (41)

$f : [-3, 1] \to R$

$$f(x) = \begin{cases} \min\{(x+6), x^2\} & , -3 \le x \le 0 \\ \max\{\sqrt{x}, x^2\} & , 0 \le x \le 1 \end{cases}$$



area bounded by $y = f(x)$ and $x - axis$

$$= \int_{-3}^{-2} (x+6)dx + \int_{-2}^{0} x^2\, dx + \int_0^1 \sqrt{x}\, dx$$

$A = \frac{41}{6}$

$6A = 41$