ANSWER KEYS

1. (2)	**2.** (1)	**3.** (2)	**4.** (1)	**5.** (2)	**6.** (2)	**7.** (4)	**8.** (3)
9. (2)	**10.** (1)						

1. **(2)** Let $f(x)=x^{25}(1-x)^{75},\ x\ \epsilon\ [0,\ 1]$

$\Rightarrow\quad f'(x)=25x^{24}(1-x)^{75}-75x^{25}(1-x)^{74}$

$=25x^{24}(1-x)^{74}\{(1-x)-3x\}$

$=25x^{24}(1-x)^{74}(1-4x)$

Which shows that $f'(x)$ is positive for $x<\frac{1}{4}$ and $f'(x)$ is negative for $x>\frac{1}{4}$

Hence, $f(x)$ attains maximum at $x=\frac{1}{4}$

2. **(1)**

$f(x)=x\log x$

$\Rightarrow f'(x)=x\times\frac{1}{x}+\log x\times 1$

$\Rightarrow f'(x)=1+\log x$

Now for f(x) to be minimum,

$f'(x)=0$

$\Rightarrow 1+\log x=0$

$\Rightarrow \log_e x=-1$

$\Rightarrow x=e^{-1}=\frac{1}{e}$

Also $f''(x)=\frac{1}{x}$

$\Rightarrow f''\left(\frac{1}{e}\right)=\frac{1}{\frac{1}{e}}=3>0$

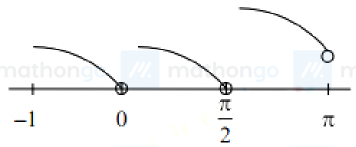
$\Rightarrow f(x)$ is minimum at $x=\frac{1}{e}$ and the minimum value is $f\left(\frac{1}{e}\right)=\frac{1}{e}\log\frac{1}{e}=\frac{-1}{e}$

3. **(2)**

$$f'(x)=\begin{cases}3x^2+2x-10 & -1\le x\le 0\\ -\sin x & 0<x<\frac{\pi}{2}\\ \cos x & \frac{\pi}{2}<x<\pi\end{cases}$$

It is clear that all functions are decreasing in their respective interval

Displaying the trend of values of the function in different intervals, we get the adjoining graph.



Here it is clear that

$\therefore f\left(\frac{\pi}{2}-h\right)<\left(2=f\left(\frac{\pi}{2}\right),f\left(\frac{\pi}{2}+h\right)<f\left(\frac{\pi}{2}\right)\right)$

$f(x)$ has a local maximum at $x=\frac{\pi}{2}$

4. **(1)**

Given function $f\left(x\right)=\begin{cases}|x-1|+a & \text{if } x\le 1\\ 2x+3 & \text{if } x>1\end{cases}$

The function has minima at $x=1$,

$\therefore\ f(1+h)>f(1)$ and $f(1-h)>f(1)$

Now, at $x=1,\ f(1)=a.$

and $2(1+h)+3\ge a$

$\Rightarrow a\le 5$

5. **(2)** Here, the given function is

$f(x)=x^3+3(a-7)x^2+3(a^2-9)x-2$

Differentiating w.r.t. x,

$f'(x)=3[x^2+2(a-7)x+(a^2-9)]$

Both roots should be positive, $D>0 \Rightarrow a<\frac{29}{7}$

$-\frac{b}{2a}>0 \Rightarrow a<7,\ f(0)>0$

$\Rightarrow a \in(-\infty,-3)\cup(3,\infty)$

$a\in\left(3,\frac{29}{7}\right)$ as $a>0$

6. **(2)**

$f:(1,3)\rightarrow R,\ f(x)=\frac{x[x]}{1+x^2}$

$$f(x)=\begin{cases}\frac{x}{1+x^2}, & x\in(1,\ 2)\\ \frac{2x}{1+x^2}, & x\in[2,\ 3)\end{cases}$$

$$f'(x)=\begin{cases}\frac{(1+x^2)(1)-x(2x)}{(1+x^2)^2}, & x\in(1,\ 2)\\ \frac{(1+x^2)(2)-2x(2x)}{(1+x^2)^2}, & x\in[2,3)\end{cases}$$

$$f'(x)=\begin{cases}\frac{1-x^2}{1+x^2}, & x\in(1,\ 2)\\ \frac{1-2x^2}{1+x^2}, & x\in[2,3)\end{cases}$$

$\therefore\ f(x)$ is decreasing function.

$\therefore\ R_f\in\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{6}{10},\frac{4}{5}\right]$

7. **(4)**

$f(x)=(3x^2+ax-2-a)e^x$

$f'(x)=(3x^2+ax-2-a)e^x+e^x(6x+a)=e^x(3x^2+(a+6)x-2)$

$\because\ x=1$ is a critical point $\therefore\ f'(1)=0$

$\therefore 3+a+6-2=0$

$a=-7$

$\therefore f'(x)=e^x(3x^2-x-2)=e^x(3x^2-3x+2x-2)=e^x(3x+2)(x-1)$

$\therefore$ maxima at $x=\frac{-2}{3}$ $\therefore$ minima at $x=1$

8. **(3)**

Given function $f(x)=\frac{x^2}{x^3+200}$ (Let)

Now, differentiating $f(x)$, we get,

$f'(x)=\frac{x(400-x^3)}{(x^3+200)^2}$

For critical points, $f'(x)=0$

$\Rightarrow \frac{x(400-x^3)}{(x^3+200)^2}=0$

$\Rightarrow\ x=(400)^{\frac{1}{3}}$

$\therefore\ 7<(400)^{\frac{1}{3}}<8$

$\therefore\ a_7=\frac{49}{543},\ a_8=\frac{8}{89}.$

$\therefore a_7>a_8$

Hence, the greatest term is a_7.

9. **(2)**

$f'(x)=2a^2x^2-5ax+3=(ax-1)(2ax-3)=0$

$x=\frac{1}{a},\ \frac{3}{2a}$

If $a>0$, then local maxima occurs at $x=\frac{1}{a}$ and minima at $x=\frac{3}{2a}$

$\because$ maxima occurs at $x=\frac{1}{a}=\frac{1}{3}\ \Rightarrow\ a=3$

Minima occurs at $x=\frac{3}{2a}=\frac{1}{2}$

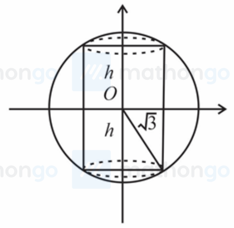
$f\left(\frac{1}{2}\right)>0\Rightarrow\ \frac{3}{8}+b>0\Rightarrow\ b>-\frac{3}{8}$

If $a<0$, then maxima shall occur at $x=\frac{3}{2a}$ and minima at $x=\frac{1}{2a}$

i.e. $\frac{3}{2a}=\frac{1}{3}\Rightarrow\ a=\frac{9}{2}>0$ (not acceptable)

Hence, $b>-\frac{3}{8}$

10. **(1)** Let r be the radius of the cylinder and $2h$ be the height.



$\Rightarrow \quad V=\pi r^2(2h)$

Also $h^2+r^2=\left(\sqrt{3}\right)^2=3$

$V=2\pi h\left(3-h^2\right)$

$\quad =2\pi\left(3h-h^3\right)$

$\frac{dV}{dh}=0 \Rightarrow 3-3h^2=0$

$\Rightarrow \quad h=1$

$V_{\max}=2\pi(3-1)=4\pi.$