ANSWER KEYS

1. (1)	**2.** (8)	**3.** (8)	**4.** (4)	**5.** (4)	**6.** (1)	**7.** (2)	**8.** (2)
9. (2)	**10.** (3)	**11.** (4)	**12.** (256)				

1. (1) $\log_4(x-1)=\log_2(x-3)$

$\Rightarrow \frac{1}{2}\log_2(x-1)=\log_2(x-3)$

$\Rightarrow \log_2(x-1)^{1/2}=\log_2(x-3)$

$\Rightarrow (x-1)^{1/2}=x-3$

$\Rightarrow x-1=x^2+9-6x$

$\Rightarrow x^2-7x+10=0$

$\Rightarrow (x-2)(x-5)=0$

$\Rightarrow x=2,5$

But $x\neq 2$ because it is not satisfying the domain of given equation i.e $\log_2(x-3)\rightarrow$ its domain $x>3$ finally x is 5

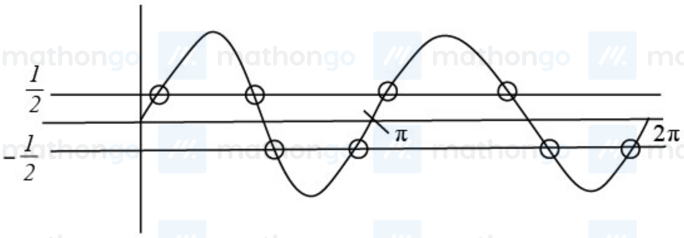
$\therefore$ No. of solutions $=1$.

2. (8) $\log_{1/2}|\sin x|=2-\log_{1/2}|\cos x|$

$\log_{1/2}|\sin x\cos x|=2$

$|\sin x\cos x|=\frac{1}{4}$

$\sin 2x=\pm\frac{1}{2}$



We can see in the diagram above that graph of $\sin 2x$ intersects the lines $y=\left(\frac{1}{2}\right)$ and $y=\left(-\frac{1}{2}\right)$ 8 times in the range $[0,2\pi]$.

Therefore,
Number of solution $=8$.

3. **(8)** $(2a)^{\ln a} = (bc)^{\ln b} 2a > 0, bc > 0 \quad b^{\ln 2} = a^{\ln c}$

$\ln a(\ln 2 + \ln a) = \ln b(\ln b + \ln c) \quad \ln 2 . \ln b = \ln c . \text{In } a$

$\ln 2 = \alpha_1 \ln a = x_1 \ln b = y_1 \ln c = z \quad \alpha y = yz$

$x(a + x) = y(y + 2)$

$\alpha = \frac{xz}{y} \quad (2a)^{\ln a} = (2a)^0$

$x\left(\frac{xz}{y} + x\right) = y(y + z)$

$x^2(z + y) = y^2(y + z)$

$y + z = 0 \text{ or } x^2 = y^2 \Rightarrow x = -y$

$bc = 1 \text{ or } ab = 1$

(1) if bc $= 1 \Rightarrow (2a)^{\ln a} = 1 \longrightarrow_{a=1/2}^{a=1}$

$(a, b, c) = \left(\frac{1}{2}, \lambda, \frac{1}{\lambda}\right), \lambda \neq 1, 2, \frac{1}{2}$

then $6a + 5bc = 3 + 5 = 8$

(II) $(a, b, c) = \left(\lambda, \frac{1}{\lambda}, \frac{1}{2}\right), \lambda \neq 1, 2, \frac{1}{2}$

In this situation infinite answer are possible

So, Bonus.

It's given in question statement that we have to Ignore the case where we are getting infinite solutions for a, b, c

NOTE: This question was BONUS in JEE Mains, We have modified the question statement

4. **(4)**

Given:

$\log_{\cos x}(\cot x) + 4 \log_{\sin x}(\tan x) = 1, \ x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \frac{\text{l}n\cos x - \text{l}n\sin x}{\text{l}n\cos x} + 4\left(\frac{\text{l}n\sin x - \text{l}n\cos x}{\text{l}n\sin x}\right) = 1$

$\Rightarrow 1 - \left(\frac{\text{l}n\sin x}{\text{l}n\cos x}\right) + 4\left(1 - \frac{\text{l}n\cos x}{\text{l}n\sin x}\right) = 1$

$\Rightarrow \left(\frac{\text{l}n\sin x}{\text{l}n\cos x}\right) + 4\left(\frac{\text{l}n\cos x}{\text{l}n\sin x}\right) - 4 = 0$

$\Rightarrow (\text{l}n\sin x)^2 - 4(\text{l}n\sin x)(\text{l}n\cos x) + 4(\text{l}n\cos x)^2 = 0$

$\Rightarrow (\text{l}n\sin x - 2\,\text{l}n\cos x)^2 = 0$

$\Rightarrow \text{l}n\sin x = 2\,\text{l}n\cos x$

$\Rightarrow \text{l}n\sin x = \text{l}n\cos^2 x$

$\Rightarrow \cos^2 x = \sin x$

$\Rightarrow 1 - \sin^2 x = \sin x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{-1+\sqrt{5}}{2}$

So, $\alpha = -1, \ \beta = 5$

$\therefore \alpha + \beta = 4$

5. **(4)**

Given, $x+1-2\log_2(3+2^x)+2\log_4\left(10-2^{-x}\right)=0$

$\Rightarrow x+1-2\log_2(3+2^x)+2\log_{2^2}\left(\frac{10.2^x-1}{2^x}\right)=0$

$\Rightarrow x+1-2\log_2(3+2^x)+\log_2\left(\frac{10.2^x-1}{2^x}\right)=0$

$\Rightarrow x+1-2\log_2(3+2^x)+\log_2(10.\,2^x-1)-\log_2 2^x=0$

$\Rightarrow x+1-\log_2(3+2^x)^2+\log_2(10.\,2^x-1)-x\log_2 2=0$

$\Rightarrow x+1-\log_2(3+2^x)^2+\log_2(10.\,2^x-1)-x=0$

$\Rightarrow x+1+\log_2\left[\frac{10.2^x-1}{(3+2^x)^2}\right]-x=0$

$\Rightarrow 1+\log_2\left[\frac{10.2^x-1}{(3+2^x)^2}\right]=0$

$\Rightarrow \log_2\left[\frac{10.2^x-1}{(3+2^x)^2}\right]=-1$

$\Rightarrow \log_2\left[\frac{10.2^x-1}{9+(2^x)^2+6.2^x}\right]=\log_2\left(\frac{1}{2}\right)$

$\Rightarrow \frac{10.2^x-1}{9+(2^x)^2+6.2^x}=\frac{1}{2}$

$\Rightarrow 2(10.\,2^x-1)=9+(2^x)^2+6.\,2^x$

$\Rightarrow (2^x)^2-14.\,2^x+11=0$

Let $2^x=y$

$\Rightarrow y^2-14y+11=0$

Let roots are $y_1=2^{x_1}$ & $y_2=2^{x_2}$

Product of roots, $y_1y_2=2^{x_1}\times 2^{x_2}=2^{x_1+x_2}=11$

$\Rightarrow \log 2^{x_1+x_2}=\log 11$

$\Rightarrow (x_1+x_2)\log 2=\log 11$

$\Rightarrow x_1+x_2=\frac{\log 11}{\log 2}$

Hence, $x_1+x_2=\log_2 11$

6. (1)

We have,

$\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}\left(x+1\right)^2-4=0,\ x>0$

$\Rightarrow \log_{(x+1)}(2x+5)(x+1)+2\log_{(2x+1)}(x+1)=4$

$\Rightarrow \log_{(x+1)}(2x+5)+\log_{(x+1)}(x+1)+2\log_{(2x+1)}(x+1)=4$

$\Rightarrow \log_{(x+1)}(2x+5)+1+2\log_{(2x+1)}(x+1)=4$

$\Rightarrow \log_{(x+1)}(2x+5)+2\log_{(2x+1)}(x+1)=3$

$\Rightarrow \log_{(x+1)}(2x+5)+\frac{2}{\log_{(x+1)}(2x+5)}=3$

Put $\log_{(x+1)}\left(2x+5\right)=t$, then

$t+\frac{2}{t}=3$

$\Rightarrow t^2-3t+2=0$

$\Rightarrow t=1,\ 2$

Then,

$\log_{(x+1)}\left(2x+5\right)=1$ and $\log_{(x+1)}\left(2x+5\right)=2$

$\Rightarrow 2x+5=x+1$ and $2x+5=(x+1)^2$

$x=-4$ (rejected) as $x>0$

And,

$2x+5=(x+1)^2$

$\Rightarrow x^2=4$

$\Rightarrow x=2,-2$ (rejected)

So, $x=2$

Number of solution is 1.

7. (2)

Given $x\in\left(0,\frac{\pi}{2}\right)$

$\log_{10}\sin x+\log_{10}\cos x=-1$

$\Rightarrow \log_{10}\sin x\cdot\cos x=-1$

$\Rightarrow \sin x\cdot\cos x=\frac{1}{10}\ldots(1)$

$\log_{10}(\sin x+\cos x)=\frac{1}{2}(\log_{10}n-1)$

$\Rightarrow \sin x+\cos x=10^{\left(\log_{10}\sqrt{n}-\frac{1}{2}\right)}=10^{\left(\log_{10}\sqrt{n}-\log_{10}\sqrt{10}\right)}=\sqrt{\frac{n}{10}}$

By squaring we get,

$\sin^2 x+\cos^2 x+2\sin x\cos x=\frac{n}{10}$

$1+2\sin x\cdot\cos x=\frac{n}{10}$

From (1), we know $\sin x\cdot\cos x=\left(\frac{1}{10}\right)$. So-

$\Rightarrow 1+\frac{1}{5}=\frac{n}{10}\Rightarrow n=12$

8. **(2)**

Given,

$A = \{x \in R : |x+1| < 2\}$

$\Rightarrow A = (-3, 1)$

Now, $B = \{x \in R : |x-1| \geq 2\}$

$\Rightarrow B = (-\infty, -1] \cup [3, \infty)$

Now, $B - A = (-\infty, -3] \cup [3, \infty)$

$\Rightarrow B - A = R - (-3, 3)$

So, option B is not true.

9. **(2)** We know that, $A = \{x \in R : |x| < 2\}$ and $B = \{x \in R : |x-2| \geq 3\}$. Therefore-

$A = \{x : x \in (-2, 2)\}$

$B = \{x : x \in (-\infty, -1] \cup [5, \infty)\}$

$A \cap B = \{x : x \in (-2, -1]\}$

$A \cup B = \{x : x \in (-\infty, 2) \cup [5, \infty)\}$

$A - B = \{x : x \in (-1, 2)\}$

$B - A = \{x : x \in (-\infty, -2] \cup [5, \infty)\}$

So, (2) is the correct option

10. **(3)** $\log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$

Feasible region : $x+\frac{7}{2}>0 \Rightarrow x>-\frac{7}{2}$

And $x+\frac{7}{2} \neq 1 \Rightarrow x \neq -\frac{5}{2}$

And $\frac{x-7}{2x-3} \neq 0 \Rightarrow x \neq 7$

and $2x-3 \neq 0 \Rightarrow x \neq \frac{3}{2}$

Taking intersection : $x \in\left(\frac{-7}{2}, \infty\right)-\left\{-\frac{5}{2}, \frac{3}{2}, 7\right\}$

Now $\log_a b \geq 0$ if $a>1$ and $b \geq 1$ (Condition i)

Or $a \in(0,1)$ and $b \in(0,1)$ (Condition ii)

As per condition i,

Firstly, $x+\left(\frac{7}{2}\right)>1 \Rightarrow x>\left(-\frac{5}{2}\right)$

and $\left(\frac{x-7}{2x-3}\right)^2 \geq 1$

$(2x-3)^2-(x-7)^2 \leq 0$

$(2x-3+n-7)(2x-3-x+7) \leq 0$

$(3x-10)(x+4) \leq 0$

$x \in\left[-4, \frac{10}{3}\right]$

Intersection : $x \in\left(\frac{-5}{2}, \frac{10}{3}\right]$

As per condition ii, $x+\frac{7}{2} \in(0,1)$ and $\left(\frac{x-7}{2x-3}\right)^2 \in(0,1)$

$0<x+\frac{7}{2}<1$ and $\left(\frac{x-7}{2x-3}\right)^2<1$

$-\frac{7}{2}<x<\frac{-5}{2}$ and $(x-7)^2<(2x-3)^2$

$x \in(-\infty,-4) \cup\left(\frac{10}{3}, \infty\right)$

No common values of x.

Hence intersection with feasible region

We get $x \in\left(\frac{-5}{2}, \frac{10}{3}\right]-\left\{\frac{3}{2}\right\}$

Integral value of x are $\{-2,-1,0,1,2,3\}$

No. of integral values $=6$

11. **(4)**

Given, $T=|x|^2-7|x|+9 \leq 0$

$\Rightarrow |x| \leq \frac{7 \pm \sqrt{13}}{2}$

$\Rightarrow |x| \in\left[\frac{7-\sqrt{13}}{2}, \frac{7+\sqrt{13}}{2}\right]$

Also given $x \in$ Integers, so x can be $\pm 2, \pm 3, \pm 4, \pm 5$

Now out of these values of x only $3,-4,-5$ will satisfy $S=\frac{|x+3|-1}{|x|-2} \geq 0$

So, $S \cap T \in\{3,-4,-5\}$

So, $n(S \cap T)=3$

12. (256)

For First set $A=\{x \in R:|x-2|>1\}$
$\Rightarrow$ Value which A contains lies from $x \in(-\infty, 1) \cup(3, \infty)$
For second set $B=\left\{x \in R: \sqrt{x^2-3}>1\right\}$
$\Rightarrow$ Value which B contains lies from $x \in(-\infty,-2) \cup(2, \infty)$
For third set $C=\{x \in R:|x-4| \geqslant 2\}$
$\Rightarrow$ Value which C contains lies from $x \in(-\infty, 2] \cup[6, \infty)$
Now, $(-\infty,-2) \cup[6, \infty) \in(A \cap B \cap C)$
Therefore, $(A \cap B \cap C)^c \cap Z=\{-2,-1,0,1,2,3,4,5\}$
Number of Subsets of above set $=2^8=256$