1. The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i=\sqrt{-1}$, is a positive integer, is ______.

2. For two non-zero complex number z_1 and z_2, if $\operatorname{Re}(z_1 z_2)=0$ and $\operatorname{Re}(z_1+z_2)=0$, then which of the following are possible?

(A) $\operatorname{Im}(z_1)>0$ and $\operatorname{Im}(z_2)>0$
(B) $\operatorname{Im}(z_1)<0$ and $\operatorname{Im}(z_2)>0$
(C) $\operatorname{Im}(z_1)>0$ and $\operatorname{Im}(z_2)<0$
(D) $\operatorname{Im}(z_1)<0$ and $\operatorname{Im}(z_2)<0$

Choose the correct answer from the options given below:

(1) B and D
(2) B and C
(3) A and B
(4) A and C

3. If the set $\left\{Re\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right): z\in\mathbb{C},\ Re\,z=3\right\}$ is equal to the interval $(\alpha,\ \beta]$, then $24(\beta-\alpha)$ is equal to

(1) 36
(2) 27
(3) 30
(4) 42

4. Let $u=\frac{2z+i}{z-ki}, z=x+iy$ and $k>0$. If the curve represented by $\operatorname{Re}(u)+\operatorname{Im}(u)=1$ intersects the y-axis at points P and Q where PQ $=5$ then the value of k is

(1) $\frac{3}{2}$
(2) $\frac{1}{2}$
(3) 4
(4) 2

5. Let for some real numbers α and $\beta, a=\alpha-i\beta$. If the system of equations $4ix+(1+i)y=0$ and $8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)x+\bar{a}y=0$ has more than one solution then $\frac{\alpha}{\beta}$ is equal to

(1) $2-\sqrt{3}$
(2) $2+\sqrt{3}$
(3) $-2+\sqrt{3}$
(4) $-2-\sqrt{3}$

6. Let a complex number $z,\ |z|\neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right)\leq 2$. Then, the largest value of $|z|$ is equal to ________.

(1) 8
(2) 7
(3) 6
(4) 5

7. Let $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z, then

(1) $I(z)=0$
(2) $R(z)<0$ and $I(z)>0$
(3) $R(z)>0$ and $I(z)>0$
(4) $R(z)=-3$

8. Let $S=\{z\in\mathbb{C}: \bar{z}=i(z^2+\operatorname{Re}(\bar{z}))\}$. Then $\sum_{z\in S}|z|^2$ is equal to

(1) $\frac{5}{2}$
(2) 4
(3) $\frac{7}{2}$
(4) 3

9. If for $z=\alpha+i\beta,\ |z+2|=z+4(1+i)$, then $\alpha+\beta$ and $\alpha\beta$ are the roots of the equation

(1) $x^2+3x-4=0$
(2) $x^2+7x+12=0$
(3) $x^2+x-12=0$
(4) $x^2+2x-3=0$

10*. Let the minimum value v_0 of $v=|z|^2+|z-3|^2+|z-6i|^2, z\in\mathbb{C}$ is attained at $z=z_0$. Then $|2z_0^2-\bar{z}_0^3+3|^2+v_0^2$ is equal to

(1) 1000
(2) 1024
(3) 1105
(4) 1196

11. For $z\in\mathbb{C}$ if the minimum value of $\left(|z-3\sqrt{2}|+|z-p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of p is _______.

(1) 3
(2) $\frac{7}{2}$
(3) 4
(4) $\frac{9}{2}$

12. Let n denote the number of solutions of the equation $z^2+3\bar{z}=0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty}\frac{1}{n^k}$ is equal to

(1) 1
(2) $\frac{4}{3}$
(3) $\frac{3}{2}$
(4) 2

13. Let $z=a+ib$, $b\neq 0$ be complex numbers satisfying $z^2=\bar{z}\cdot 2^{1-|z|}$. Then the least value of $n\in N$, such that $z^n=(z+1)^n$, is equal to _____.

14. Let $S=\left\{z\in\mathbb{C}-\{i,2i\}: \frac{z^2+8iz-15}{z^2-3iz-2}\in\mathbb{R}\right\}$. $\alpha-\frac{13}{11}i\in S, \alpha\in\mathbb{R}-\{0\}$, then $242\alpha^2$ is equal to

15. Let $p,\ q\in\mathbb{R}$ and $\left(1-\sqrt{3}i\right)^{200}=2^{199}(p+iq), i=\sqrt{-1}$. Then, $p+q+q^2$ and $p-q+q^2$ are roots of the equation.

(1) $x^2+4x-1=0$
(2) $x^2-4x+1=0$
(3) $x^2+4x+1=0$
(4) $x^2-4x-1=0$

16. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(1) $\frac{-1}{2}\left(1-i\sqrt{3}\right)$
(2) $\frac{1}{2}\left(1-i\sqrt{3}\right)$
(3) $\frac{-1}{2}\left(\sqrt{3}-i\right)$
(4) $\frac{1}{2}\left(\sqrt{3}+i\right)$

17. If z and ω are two complex numbers such that $|z\omega|=1$ and $\arg(z)-\arg(\omega)=\frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is:
(Here $\arg(z)$ denotes the principal argument of complex number z)

(1) $\frac{\pi}{4}$
(2) $-\frac{3\pi}{4}$
(3) $-\frac{\pi}{4}$
(4) $\frac{3\pi}{4}$

18. Let $z=\frac{1-i\sqrt{3}}{2}, i=\sqrt{-1}$. Then the value of $21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+\ldots+\left(z^{21}+\frac{1}{z^{21}}\right)^3$ is______.

19. If $z^2+z+1=0, z\in C$, then $\left|\sum_{n=1}^{15}\left(z^n+\left(-1\right)^a\frac{1}{z^n}\right)^2\right|$ is equal to _____.

20. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x)=f(x^3)+xg(x^3)$ is divisible by x^2+x+1, then $P(1)$ is equal to ___ .

21. The number of elements in the set $\{z=a+ib\in\mathbb{C}: a,b\in\mathbb{Z}$ and $1<|z-3+2i|<4\}$ is _____.

22. Let $w=z\bar{z}+k_1z+k_2iz+\lambda(1+i), k_1,k_2\in\mathbb{R}$. . Let $Re(w)=0$ be the circle C of radius 1 in the first quadrant touching the line $y=1$ and the $y-$axis. If the curve $Im(w)=0$ intersects C at A and B, then $30(AB)^2$ is equal to _______.

23. If the center and radius of the circle $\left|\frac{z-2}{z-3}\right|=2$ are respectively (α,β) and γ, then $3(\alpha+\beta+\gamma)$ is equal to

(1) 11
(2) 9
(3) 10
(4) 12

24. For $n\in N$, let $S_n=\left\{z\in C:|z-3+2i|=\frac{n}{4}\right\}$ and $T_n=\left\{z\in C:|z-2+3i|=\frac{1}{n}\right\}$. Then the number of elements in the set $\{n\in N: S_n\cap T_n=\phi\}$ is

(1) 0
(2) 2
(3) 3
(4) 4

25. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z}=iz^2$ is

(1) $\frac{3\sqrt{3}}{2}$
(2) $\frac{3\sqrt{3}}{4}$
(3) $\frac{\sqrt{3}}{4}$
(4) $\frac{\sqrt{3}}{2}$

26. Let $A=\left\{z\in C:\left|\frac{z+1}{z-1}\right|<1\right\}$ and $B=\left\{z\in C:\arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}\right\}$. Then $A\cap B$ is

(1) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only
(2) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only
(3) an empty set
(4) a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

27. Let z_1, z_2 be the roots of the equation $z^2+az+12=0$ and $z_1,\ z_2$ form an equilateral triangle with origin. Then, the value of $|a|$ is

28. Let z be those complex numbers which satisfy $|z+5|\leq4$ and $z(1+i)+\bar{z}(1-i)\geqslant-10, i=\sqrt{-1}$. If the maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha+\beta)$ is

29. Let $S=\{z\in C:|z-2|\leq1, z(1+i)+\bar{z}(1-i)\leq2\}$. Let $|z-4i|$ attains minimum and maximum values, respectively, at $z_1\in S$ and $z_2\in S$. If $5\left(|z_1|^2+|z_2|^2\right)=\alpha+\beta\sqrt{5}$, where α and β are integers, then the value of $\alpha+\beta$ is equal to ______.

30. Let $S_1,\ S_2$ and S_3 be three sets defined as
$S_1=\left\{z\in\mathbb{C}:|z-1|\leq\sqrt{2}\right\}$,
$S_2=\{z\in\mathbb{C}:Re((1-i)z)\geq1\}$ and
$S_3=\{z\in\mathbb{C}:Im(z)\leq1\}$.
Then, the set $S_1\cap S_2\cap S_3$

(1) is a singleton
(2) has exactly two elements
(3) has infinitely many elements
(4) has exactly three elements

*Note: Question with * denotes it is optional but good to solve.*