ANSWER KEYS

1. (1)	**2.** (2)	**3.** (4)	**4.** (1)	**5.** (3)	**6.** (3)	**7.** (4)	**8.** (4)
9. (2)	**10.** (2)						

1. (1)

$-1 \le \log_3 \frac{x}{3} \le 1$

$\frac{1}{3} \le \frac{x}{3} \le 3$

$1 \le x \le 9$

$x \in [1,9]$

2. (2)

For f (x) to be defined

$|\tan x| - \tan x > 0$

$\Rightarrow |\tan x| > \tan x$

If $\tan x > 0$ then $\tan x > \tan x$ which is not possible.

If $\tan x < 0$, then

$-\tan x > \tan x \Rightarrow 2\tan x < 0$

$\Rightarrow \tan x < 0$

$\Rightarrow \frac{\pi}{2} < x < \pi$

For general solution, $n\pi + \frac{\pi}{2} < x < n\pi + \pi,\ n \in I$

3. (4) Here $x + 3 > 0$ & $x^2 + 3x + 2 \neq 0$

$\therefore x > -3$ & $(x+1)(x+2) \neq 0$, i.e., $x \neq -1, -2$

$\therefore$ The domain$= (-3, \infty) - \{-1, -2\}$.

4. (1)

Domain of $f(x)$ is $[1, 3]$

$1 \le \log_2 (x^2 + 3x - 2) \le 3$

$\Rightarrow 2 \le x^2 + 3x - 2 \le 8$

It is the same as solving two inequalities.

Case 1 : $x^2 + 3x - 2 \ge 2$

or, $x^2 + 3x - 4 \ge 0$

or, $(x+4)(x-1) \ge 0$

$\therefore\ x \in (-\infty, -4] \cup [1, \infty)$

Case 2 : $x^2 + 3x - 2 \le 8$

or, $x^2 + 3x - 10 \le 0$

or, $(x+5)(x-2) \le 0$

$\therefore\ x \in [-5, 2]$

Considering both Case 1 and Case 2 we have, $x \in [-5, -4] \cup [1, 2]$

5. (3)

The range of $\cos x$ is $[-1, 1]$.

$-1 \le \cos 3x \le 1 \Rightarrow -1 \le -\cos 3x \le 1$

$1 \le 2 - \cos 3x \le 3$

$\Rightarrow 1 \ge \frac{1}{2 - \cos 3x} \ge \frac{1}{3}$

$\therefore \frac{1}{2 - \cos 3x} \in \left[\frac{1}{3}, 1\right]$

6. (3) $\frac{x^2+14x+9}{x^2+2x+3} = y$

$\Rightarrow x^2 + 14x + 9 = x^2 y + 2xy + 3y$

$\Rightarrow x^2(y-1) + 2x(y-7) + (3y-9) = 0$

Since x is real, $b^2 - 4ac \ge 0$

$\therefore 4(y-7)^2 - 4(3y-9)(y-1) \ge 0$

$\Rightarrow 4(y^2 + 49 - 14y) - 4(3y^2 + 9 - 12y) \ge 0$

$\Rightarrow (y+5)(y-4) \le 0;$

$\therefore y$ lies between -5 and 4.

7. **(4)**

Given, $f(x)=\log_e\left(3x^2+4\right)$

Let, $y=\log_e\left(3x^2+4\right)$

$\Rightarrow 3x^2+4=e^y$

$\Rightarrow |x|=\sqrt{\frac{e^y-4}{3}}$

Here, $\frac{e^y-4}{3}\geq 0$ (Because, a square root function is defined for non-negative real numbers).

$\Rightarrow e^y\geq 4$

$\Rightarrow \log_e e^y\geq \log_e 4$ (applying log on both sides).

$\Rightarrow y\geq 2\log_e 2$

$\Rightarrow y\in[2\log_e 2,\infty)$.

8. **(4)** Period of $\sin\frac{\theta}{3}=6\pi$

And period of $\cos\frac{\theta}{2}=4\pi$

$\therefore$ Period of $f(x)=\text{LCM}(6\pi,\ 4\pi)=12\pi$

9. **(2)** $f\left(\frac{\pi}{2}+x\right)=\left|\sin\left(\frac{\pi}{2}+x\right)\right|+\left|\cos\left(\frac{\pi}{2}+x\right)\right|$

$=|\cos\ x|+|\sin\ x|$ for all x.

Hence, $f(x)$ is periodic with period $\frac{\pi}{2}$.

10. **(2)** Given, $f(x)=\log\left(x+\sqrt{x^2+1}\right)$

$\therefore\ f(x)+f(-x)=\log\left(x+\sqrt{x^2+1}+\log\left(-x+\sqrt{x^2+1}\right)\right.$

$=\log(1)=0$

Hence, $f(x)$ is an odd function.