ANSWER KEYS

1. (36)	**2.** (171)	**3.** (2)	**4.** (98)	**5.** (4)	**6.** (1120)	**7.** (2)	**8.** (4)
9. (2)	**10.** (405)	**11.** (1)	**12.** (1)	**13.** (1)	**14.** (3)	**15.** (4)	**16.** (29)
17. (4)	**18.** (3)	**19.** (2)	**20.** (7)	**21.** (1)	**22.** (1)	**23.** (96)	**24.** (960)
25. (1080)	**26.** (4)	**27.** (99)	**28.** (2)	**29.** (3)	**30.** (45)		

1. (36)

$T_{k+1}={ }^n C_k(x)^{\frac{n-k}{2}}(-6)^k(x)^{\frac{-3}{2} k}$

$\frac{n-k}{2}-\frac{3}{2} k=0$

$n-4 k=0$

$(-5)^n-\left({ }^n C_{\frac{n}{4}}(-6)^{\frac{n}{4}}\right)=649$

By observation $(625+24=649)$, we get $n=4$

$\because n=4 \& k=1$

Required is coefficient of x^{-4} is $\left(\sqrt{4}-\frac{6}{x^{\frac{3}{2}}}\right)^4$

${ }^4 C_1(-6)^3$

By calculating we will get $\lambda=36$

2. (171) The number of integral term in the expression of $\left(3^{\frac{1}{2}}+5^{\frac{1}{4}}\right)^{680}$ is equal to

General term $={ }^{600} C_x\left(3^{\frac{1}{2}}\right)^{680-x}\left(5^{\frac{1}{4}}\right)^x$

$={ }^{680} C_r 3^{\frac{680-x}{2}} 5^{\frac{x}{4}}$

Value's of r, where $\frac{r}{4}$ goes to integer $r=0,4,8,12, .680$

All value of r are accepted for $\frac{680-r}{2}$ as well so No of integral terms $=171$.

3. (2) T_{1011} from beginning $=T_{1010+1}$

$={ }^{2082} C_{1010}\left(\frac{4 x}{5}\right)^{1012}\left(\frac{-5}{2 x}\right)^{1010}$

T_{1011} from end

$={ }^{20 e 2} C_{1010}\left(\frac{-5}{2 x}\right)^{1012}\left(\frac{4 x}{5}\right)^{1010}$

Given: ${ }^{2022} C_{1010}\left(\frac{-5}{2 x}\right)^{1012}\left(\frac{4 x}{5}\right)^{1010}$

$=2^{10} \cdot{ }^{2022} C_{1010}\left(\frac{-5}{2 x}\right)^{1010}\left(\frac{4 x}{5}\right)^{1012}$

$\left(\frac{-5}{2 x}\right)^2=2^{10}\left(\frac{4 x}{5}\right)^2$

$x^4=\frac{5^4}{2^{16}}$

$|x|=\frac{5}{16}$

4. (98)

Given,

The constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2}-\frac{4}{x^l}\right)^9$ is -84 and the coefficient of x^{-3} is $2^\alpha \beta$

Now General Term of the binomial is given by, $T_{r+1}={ }^9 C_r\left(\frac{x^{\frac{5}{2}}}{2}\right)^{9-r}\left(\frac{-4}{x^l}\right)^r=(-1)^r C_r x^{\frac{45}{2}-\frac{5 r}{2}-l r} 2^{3-9}$

Now for constant term $45-5 r-2 l r=0 \Rightarrow r=\frac{45}{5+2 l}$

So, coefficient of constant term will be,

$(-1)^9 \times{ }^9 C_r 2^{3 r-9}=-84 \Rightarrow r=3$

Now putting the value of r in $r=\frac{45}{5+2 l}$ we get, $l=5$

Now coefficient of $x^{-3 l}=x^{-15} \Rightarrow \frac{45}{2}-\frac{5 r}{2}-l r=-15 \Rightarrow r=5$ when $l=5$

Now coefficient of $x^{-15}=(-1)^{59} C_5 2^6=-63\left(2^7\right)=\beta\left(2^\alpha\right)$

Hence, on comparing we get, $\alpha=7, \beta=-63 \& l=5$

So, the value of $|\alpha l-\beta|=|7 \times 5-(-63)|=98$

5. (4)

We have,

$(1+x)^{500}+x(1+x)^{499}+x^2(1+x)^{498}+\ldots..+x^{500}$

This is a geometric progression.

$=(1+x)^{500}\cdot\left\{\frac{1-\left(\frac{x}{1+x}\right)^{501}}{1-\frac{x}{1+x}}\right\}$

$=(1+x)^{500}\frac{\left((1+x)^{501}-x^{501}\right)}{(1+x)^{501}}\cdot(1+x)$

$=(1+x)^{501}-x^{501}$

Coefficient of x^{301} in $(1+x)^{501}-x^{501}$ is given by

${}^{501}C_{301}={}^{501}\mathrm{C}_{200}$

6. (1120)

Given,

The coefficients of three consecutive terms in the binomial expansion of $(1+2x)^{\mathrm{n}}$ be in the ratio $2:\ 5:\ 8$.

Now r^{th} term in expansion of $(1+2x)^{\mathrm{n}}$ is given by, $\mathrm{t}_{\mathrm{r}+1}={}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}(2\mathrm{x})^{\mathrm{r}}$

Now let $T_r,\ T_{r+1},\ T_{r+2}$ are in the ratio 2 : 5 : 8

$\Rightarrow\frac{T_r}{T_{r+1}}=\frac{{}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}-1}(2)^{\mathrm{r}-1}}{{}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}(2)^{\mathrm{r}}}=\frac{2}{5}$

$\Rightarrow\frac{\frac{\mathrm{n}!}{(\mathrm{r}-1)!(\mathrm{n}-\mathrm{r}+1)!}}{\frac{\mathrm{n}!(2)}{\mathrm{r}!(\mathrm{n}-\mathrm{r})!}}=\frac{2}{5}$

$\Rightarrow\frac{r}{n-r+1}=\frac{4}{5}\Rightarrow 5r=4n-4r+4$

$\Rightarrow 9r=4(n+1)\ldots\ldots(1)$

Now taking other ratio $\frac{T_{r+1}}{T_{r+2}}=\frac{{}^{n}C_{\mathrm{r}}(2)^{\mathrm{r}}}{{}^{n}C_{\mathrm{r}+1}(2)^{\mathrm{r}+1}}=\frac{5}{8}$

$\Rightarrow\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}}=\frac{5}{4}\Rightarrow\frac{r+1}{n-r}=\frac{5}{4}$

$\Rightarrow 4r+4=5n-5r$

$\Rightarrow 5n-4=9r\ldots\ldots\ldots(2)$

From equation (1) & (2) we get,

$4\mathrm{n}+4=5\mathrm{n}-4\Rightarrow\mathrm{n}=8$ and $r=4$

So, coefficient of middle term will be

${}^{8}\mathrm{C}_{4}2^{4}=16\times\frac{8\times7\times6\times5}{4\times3\times2\times1}=16\times70=1120$

7. (2)

General term of $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ is

$T_{r+1}={}^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(-\frac{1}{5x^2}\right)^r={}^{11}C_r(-1)^r\cdot\frac{5^{11-2r}}{2^{11-r}}\cdot x^{33-5r}$

The term independent of x in the expansion of $\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ will be the coefficient of x^0 in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}-$ coefficient of x^{-2} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}+3\times$ coefficient of x^{-3} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

$=-{}^{11}C_7(-1)^7\cdot\frac{5^{-3}}{2^4}=\frac{330}{5^3\cdot2^4}=\frac{33}{200}$

8. (4)

$$\frac{{}^{n}C_4 2^{\frac{n-4}{4}}\cdot\left(3^{\frac{-1}{4}}\right)^4}{{}^{n}C_4 3^{-\left(\frac{n-4}{4}\right)}\cdot\left(2^{\frac{1}{4}}\right)^4}=\frac{\sqrt{6}}{1}$$

$\Rightarrow n=10$

So $T_3={}^{10}C_2 2^{\frac{1}{4}\cdot8}\cdot3^{\frac{1}{4}\cdot2}=\frac{45\cdot4}{\sqrt{3}}=60\sqrt{3}$

9. (2)

$(r+1)^{th}$ term in the expansion of $\left(ax^3+\frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is

$T_{r+1}={ }^{15}C_r\left(ax^3\right)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r$

$\Rightarrow T_{r+1}={ }^{15}C_r(a)^{15-r}(x)^{45-\frac{10r}{3}}\left(\frac{1}{b}\right)^r$

For the coefficient of x^{15} in $\left(ax^3+\frac{1}{bx^{1/3}}\right)^{15}$:

$45-\frac{10r}{3}=15$

$\Rightarrow 30=\frac{10r}{3}$

$\Rightarrow r=9$

Coefficient of $x^{15}={ }^{15}C_9a^6b^{-9}$

$(r+1)^{th}$ term in the expansion of $\left(ax^{1/3}-\frac{1}{bx^3}\right)^{15}$ is:

$T_{r+1}={ }^{15}C_r\left(ax^{1/3}\right)^{15-r}\left(-\frac{1}{bx^3}\right)^r$

For the coefficient of x^{-15} in $\left(ax^{1/3}-\frac{1}{bx^3}\right)^{15}$ is:

$5-\frac{r}{3}-3r=-15$

$\Rightarrow \frac{10r}{3}=20$

$\Rightarrow r=6$

Coefficient of $x^{-15}={ }^{15}\mathrm{C}_6\mathrm{a}^9\times\mathrm{b}^{-6}$

Hence,

${ }^{15}C_9a^6b^{-9}={ }^{15}\mathrm{C}_6\mathrm{a}^9\times b^{-6}$

$\Rightarrow \frac{a^9}{b^6}=\frac{a^6}{b^9}$

$\Rightarrow a^3b^3=1\Rightarrow ab=1$

10. (405)

We know that general term in the expansion of $\left(x-\frac{3}{x^2}\right)^n$ is given by,

$T_{r+1}={ }^nC_rx^{n-r}\left(\frac{-3}{x^2}\right)^r$

$=(-1)^r\times{ }^nC_r3^r\,x^{n-r-2r}$

$T_{r+1}=(-1)^r\times{ }^nC_r3^r\,x^{n-3r}$..............(1)

So, $T_1=T_{0+1}={ }^nC_03^0\,x^n=x^n$

$T_2=(-1)\times{ }^nC_13^1\,x^{n-3}$

$T_3={ }^nC_23^2\,x^{n-6}$

Now given sum is

$1-{ }^nC_1\cdot 3+{ }^nC_2\cdot 3^2=376$

$\Rightarrow 1-3n+\frac{n(n-1)}{2}\cdot 9=376$

$\Rightarrow 1-3n+\frac{n^2-n}{2}\cdot 9=376$

$\Rightarrow 2-6n+9n^2-9n=752$

$\Rightarrow 9n^2-15n-750=0$

$\Rightarrow 3n^2-5n-250=0$

$\Rightarrow n=\frac{5\pm\sqrt{25+3000}}{6}$

$\Rightarrow n=\frac{5\pm 55}{6}$

$\Rightarrow n=10$ {ignoring negative sign}

Now, $T_{r+1}=(-1)^r\,{ }^{10}C_r\,3^r\,x^{10-3r}$

So, for coefficient of x^4 we take

$10-3r=4$

$\Rightarrow 3r=6$

$\Rightarrow r=2$

So, coefficient of x^4 is given by,

$T_{2+1}=(-1)^2\cdot{ }^{10}C_2\,3^2=45\times 9=405.$

11. **(1)**

Given,

$(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+\ldots\ldots+x^{500}$

Using sum of the G.P in the given expression we get,

$\Rightarrow \frac{(5+x)^{500}\left[\left(\frac{x}{5+x}\right)^{501}-1\right]}{\left(\frac{x}{5+x}\right)-1}=\frac{x^{501}-(5+x)^{501}}{-5}$

$=\frac{1}{5}(5+x)^{501}-\frac{x^{501}}{5}$

Now coefficient of x^{101} in $\frac{1}{5}(5+x)^{501}$ will be $\frac{1}{5}\left({}^{501}C_{101}\right)(5)^{501-101}=5^{399}\left({}^{501}C_{101}\right)$

12. **(1)** In $(1+x)^n$, if n is even, then the middle term is $\left(\frac{n}{2}+1\right)^{th}$ term.

So, Coefficient of middle term in $(1+x)^{20}={}^{20}C_{10}$

In $(1+x)^n$, if n is odd, then the middle term is $\left(\frac{n+1}{2}\right)^{th}$ & $\left(\frac{n+3}{2}\right)^{th}$ term.

So, Sum of Coefficient of two middle terms in

$(1+x)^{19}={}^{19}C_9+{}^{19}C_{10}$

So required ratio $=\frac{{}^{20}C_{10}}{{}^{19}C_9+{}^{19}C_{10}}=\frac{{}^{20}C_{10}}{{}^{20}C_{10}}=1$

13. **(1)** $(1+x)(1-x)^{10}\left(1+x+x^2\right)^9$

$=(1+x)(1-x)(1-x)^9\left(1+x+x^2\right)^9$

$=\left(1-x^2\right)\left[(1-x)\left(1+x+x^2\right)\right]^9$

$=\left(1-x^2\right)\left(1-x^3\right)^9$

So, Coefficient of x^{18} in $(1+x)(1-x)^{10}\left(1+x+x^2\right)^9$

$=$ Coefficient of x^{18} in $\left(1-x^2\right)\left(1-x^3\right)^9$

$=$ Coefficient of x^{18} in $\left(1-x^2\right)\left[{}^9C_0+{}^9C_1\left(-x^3\right)+{}^9C_2\left(-x^3\right)^2+\ldots\right]$

$={}^9C_6(-1)^6={}^9C_3=\frac{9.8.7}{1.2.3}=84$

14. **(3)**

Using the binomial expansion in the given expression we get,

$7^{2022}+3^{2022}=(50-1)^{1011}+(10-1)^{1011}$

$={}^{1011}C_0(50)^{1011}(-1)^0+{}^{1011}C_1(50)^{1010}(-1)^1+\ldots.\ {}^{1011}C_{1011}(50)^0(-1)^{1011}+{}^{1011}C_0(10)^{1011}(-1)^0+{}^{1011}C_1(10)^{1010}(-1)^1+\ldots.\ {}^{1011}C_{1011}(10)^0(-1)^{1011}$

$=5k-2$

Using binomial expansion we get the remainder as $-2+5=3$

15. **(4)**

The remainder when 2021 divided by 7 is -2. Hence, the problem reduces to finding the remainder when $(-2)^{2023}$ is divided by 7.

$=\frac{(-2)^{2022}\times(-2)}{7}$

$=-\frac{2\times 2^{2022}}{7}$

$=\frac{-2\times\left(2^3\right)^{674}}{7}$

$=\frac{-2\times(8)^{674}}{7}$

$=\frac{-2\times(1+7)^{674}}{7}$

Using binomial theorem, we get

$=\frac{-2\times(1+7k)}{7}$ where k is an integer.

$=\frac{-2-14k}{7}$

Clearly, the remainder when $-14k$ is divisible by 7 is 0.

$=\frac{-2}{7}$

$=\frac{-2-5+5}{7}=\frac{5}{7}$

Hence, the remainder is 5.

16. (29)

Solving $19^{200}+23^{200}=(21-2)^{200}+(21+2)^{200}$

$=2\left\{{}^{200}C_0(21)^{200}\times 2^0+{}^{200}C_2(21)^{198}\times 2^2\ldots.{}^{200}C_{200}(21)^0\times 2^{200}\right\}$

Now we know that $21^2=441$ is divisible by 49,

So rest of the terms will be divisible by 49

So lets consider for $2\left\{{}^{200}C_{200}(21)^0\times 2^{200}\right\}=2^{201}$

Now rewriting the term $2^{201}=\left(2^3\right)^{67}=(7+1)^{67}$

Now $(1+7)^{67}={}^{67}C_0\times 1+{}^{67}C_1 7+{}^{67}C_2 7^2\ldots\ldots\ldots$

$=1+67\times 7+49k$

Now when dividing $(1+7)^{67}$ by 49 we consider $1+67\times 7$ as $49k$ is divisible by 49

Now $1+67\times 7=470$ which when divided by 49 leaves remainder as 29.

17. (4)

$(2023-2)^{3762}=2023k_1+2^{3762}$

$=17k_2+2^{3762}$ (as$2023=17\times 17\times 9$)

$=17k_2+4\times 16^{940}$

$=17k_2+4\times(17-1)^{940}$

$=17k_2+4(17k_3+1)$

$=17k+4\Rightarrow$remainder$=4$

18. (3) $2^{403}=2^3.\left(2^4\right)^{100}$

$=8.(15+1)^{100}$

$=8\left[{}^{100}C_0 15^{100}+{}^{100}C_1 15^{99}+\ldots.+{}^{100}C_{99}15+{}^{100}C_{100}\right]$

$=8.[15\lambda+1]$

$=8.15\lambda+8$

$\Rightarrow\frac{2^{403}}{15}=8\lambda+\frac{8}{15}$

Hence, $k=8$

19. (2) $S_1=(1999+24)^{2022}-(1999)^{2022}$

$\Rightarrow{}^{2022}\mathrm{C}_1(1999)^{2021}(24)+{}^{2022}\mathrm{C}_2(1999)^{2020}(24)^2+\ldots.50$ on

S_1 is divisible by 8

$S_2: 13\left(13^n\right)-11n-13$

$13^n=(1+12)^n=1+12n+{}^nC_2 12^2+{}^nC_3 12^3\ldots\ldots$

$13\left(13^n\right)-11n-13=145n+{}^nC_2 12^2+{}^nC_3 12^3\ldots\ldots$

If $(n=144m, m\in N)$, then it is divisible by 144 For infinite value of n.

20. (7)

To find the remainder when 2023^{2023} is divided by 35 we will use binomial expansion,

Now rewriting above expression we get,

$2023^{2023}=(2030-7)^{2023}$

$\Rightarrow(2030-7)^{2023}={}^{2023}C_0 2030^{2023}-{}^{2023}C_1(2030)^{2022}\times 7+\ldots\ldots-7^{2023}$

$\Rightarrow(2030-7)^{2023}=35k-7^{2023};\ k\in Z.$

[as 2030 is multiple of 35]

Now remainder will be -7^{2023}. Rewriting -7^{2023}, we get

$-7^{2023}=-\left(7^3\right)^{674}\times 7$

$=-\left(7^3\right)^{674}\times 7=-(343)^{674}\times 7$

$=-(343)^{674}\times 7=-(350-7)^{674}\times 7$

$=\left(35k_1+7^{674}\right)\times(-7)$

So, again using binomial we get remainder as -7×7^{674}

Again rewriting the expression as

$-7\times 7^{674}=-7^{675}$

$\Rightarrow -7^{675}=-\left(7^3\right)^{225}=(350-7)^{75}$

Again using binomial we get remainder as

$(350-7)^{75}=(-7)^{75}=-(350-7)^{25}$

Again using binomial we get remainder as 7^{25}.

Now again rewriting $\left(7^3\right)^8\times 7=(350-7)^8\times 7$

Using binomial remainder will be 7^9 which can be written as $\left(7^3\right)^3=(350-7)^3$,

Using binomial remainder will be -7^3 which can be written as -343, so remainder will be $350-343=7$,

Hence, the remainder when 2023^{2023} when divided by 35 is 7.

21. (1)

$x=4k+3$

$\therefore(2020+x)^{2022}=(2020+4k+3)^{2022}$

$=(4(505+k)+3)^{2022}$

$=(4\lambda+3)^{2022}=\left(16\lambda^2+24\lambda+9\right)^{1011}$

$=\left(8\left(2\lambda^2+3\lambda+1\right)+1\right)^{1011}$

$=(8p+1)^{1011}$

$\therefore$ Remainder when divided by $8=1$.

22. (1)

We know the binomial expansion of $(a+b)^n$ is,

$(a+b)^n={}^nC_0a^nb^0+{}^nC_1a^{n-1}b^1+{}^nC_2a^{n-2}b^2+\ldots\ldots+{}^nC_na^0b^n$

Using the above expansion,

$x=\left(8\sqrt{3}+13\right)^{13}={}^{13}C_0\left(8\sqrt{3}\right)^{13}+{}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)^1+\ldots$

Let $x'=\left(8\sqrt{3}-13\right)^{13}={}^{13}C_0\left(8\sqrt{3}\right)^{13}-{}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)^1+\ldots$

So, $x-x'=2\left[{}^{13}C_1\cdot\left(8\sqrt{3}\right)^{12}(13)^1+{}^{13}C_3\left(8\sqrt{3}\right)^{10}\cdot(13)^3\ldots\right]$

Therefore, $x-x'$ is an even integer, hence $[x]$ is even

Now, $y=\left(7\sqrt{2}+9\right)^9={}^9C_0\left(7\sqrt{2}\right)^9+{}^9C_1\left(7\sqrt{2}\right)^8(9)^1+{}^9C_2\left(7\sqrt{2}\right)^7(9)^2+\ldots$ Now let,

$y'=\left(7\sqrt{2}-9\right)^9={}^9C_0\left(7\sqrt{2}\right)^9-{}^9C_1\left(7\sqrt{2}\right)^8\left(9\right)^1+{}^9C_2\left(7\sqrt{2}\right)^7(9)^2$

So, $y-y'=2\left[{}^9C_1\left(7\sqrt{2}\right)^8(9)^1+{}^9C_3\left(7\sqrt{2}\right)^6(9)^3+\ldots\right]$

$y-y'=$ Even integer hence $[y]$ is even

23. **(96)**

We have to check $11^n > 10^n + 9^n$

$\Rightarrow 11^n - 9^n > 10^n$

$\Rightarrow (10+1)^n - (10-1)^n > 10^n$

Using $(a+b)^n - (a-b)^n = 2\left[{}^nC_1 \cdot a^{n-1} \cdot b + {}^nC_3 \cdot a^{n-3} \cdot b^3 + {}^nC_5 \cdot a^{n-5} \cdot b^5 + \ldots\right]$

$\Rightarrow 2\left[{}^nC_1 \cdot 10^{n-1} + {}^nC_3 \cdot 10^{n-3} + {}^nC_5 \cdot 10^{n-5} + \cdots\cdots\right] > 10^n$

$\Rightarrow 2n \cdot 10^{n-1} + 2\left[{}^nC_3 \cdot 10^{n-3} + {}^nC_5 \cdot 10^{n-5} + \cdots\cdots\right] > 10^n \ldots(1)$

For $n = 5$, the $L.H.S.$ is $2 \cdot 5 \cdot 10^4 + 2\left[{}^5C_3 \cdot 10^2 + {}^5C_5\right]$

$\Rightarrow 10^5 + 2\left[{}^5C_3 \cdot 10^2 + {}^5C_5\right] > 10^5$ which is true.

Now, for $n = 6,\ 7,\ 8,\ \ldots\ldots,\ 100$

$2n \cdot 10^{n-1} > 10^n$

$\Rightarrow 2n \cdot 10^{n-1} + 2\left[{}^nC_3 \cdot 10^{n-3} + {}^nC_5 \cdot 10^{n-5} + \cdots\cdots\right] > 10^n$

$\Rightarrow 11^n - 9^n > 10^n$ for all $n = 5,\ 6,\ 7,\ \ldots\ldots,\ 100$

For $n = 4$, the inequality (1) is not satisfied.

Hence, the inequality does not hold good for $n = 1,\ 2,\ 3,\ 4$

So, required number of elements$= 96$.

24. **(960)** General term $= \frac{10!}{r_1! \cdot r_2! \cdot r_3!}(-1)^{r_2} \cdot (2)^{r_3} x^{r_2+3r_3}$

where $r_1 + r_2 + r_3 = 10$ and $r_2 + 3r_3 = 7$

r_1	r_2	r_3
3	7	0
5	4	1
7	1	2

Required coefficient

$= \dfrac{10!}{3! \cdot 7!}(-1)^7 + \dfrac{10!}{5! \cdot 4!}(-1)^4(2) + \dfrac{10!}{7! \cdot 2!}(-1)^1(2)^2$

$= -120 + 2520 - 1440 = 960$

25. **(1080)**

Any term in the expansion of $\left(2x + \frac{1}{x^7} + 3x^2\right)^5$ is given by

$\frac{5!}{a! \cdot b! \cdot c!}(2x)^a\left(x^{-7}\right)^b\left(3x^2\right)^c$

where $a + b + c = 5 \quad \ldots$(i)

$= 2^a(3)^c\left(\frac{5!}{a! \cdot b! \cdot c!}\right)x^a x^{-7b} x^{2c}$

$= 2^a(3)^c\left(\frac{5!}{a! \cdot b! \cdot c!}\right)x^{a-7b+2c}$

Now for term to be independent of x, we know

$a - 7b + 2c = 0 \quad \ldots$(ii)

Solving (i) & (ii), we get

$b = \frac{c+5}{8}$

Also, $a,\ b,\ c \in \{1, 2, 3, 4, 5\}$

So, $a = 1,\ b = 1,\ c = 3$

Therefore, the term independent of x will be

$2^1(3)^3 \cdot \frac{5!}{1! \cdot 1! \cdot 3!} = 1080$

26. **(4)**

For $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ the general term is given by

$T_{r+1} = \frac{10!}{r_1! r_2! r_3!}(3)^{r_1}(-2)^{r_2}(5)^{r_3}(x)^{3r_1+2r_2-5r_3}$

For term independent of x the exponent $3r_1 + 2r_2 - 5r_3 = 0 \quad \ldots(1)$

Also we know $r_1 + r_2 + r_3 = 10 \quad \cdots(2)$

From (1) and (2), we get

$r_1 + 2(10 - r_3) - 5r_3 = 0$

i.e. $r_1 + 20 = 7r_3$

So $r_1, r_2, r_3 = 1, 6, 3$

Hence the constant term $= \frac{10!}{1!6!3!}(3)^1(-2)^6(5)^3$

$= 2^9 \cdot 3^2 \cdot 5^4 \cdot 7^1$

$\Rightarrow k = 9$

27. (99)

Let $I=1+\left(1+{}^{49}C_0+{}^{49}C_1+\ldots+{}^{49}C_{49}\right)\left({}^{50}C_2+{}^{50}C_4+\ldots+{}^{50}C_{50}\right)$

As ${}^{49}C_0+{}^{49}C_1+\ldots+{}^{49}C_{49}=2^{49}$

and ${}^{50}C_0+{}^{50}C_2+\ldots+{}^{50}C_{50}=2^{49}$

$\Rightarrow {}^{50}C_2+{}^{50}C_4+\ldots.+{}^{50}C_{50}=2^{49}-1$

$\therefore\ I=1+\left(2^{49}+1\right)\left(2^{49}-1\right)=2^{98}$

Now comparing $I=2^n m$ we get, $m=1$ and $n=98$

So, $m+n=99$

28. (2)

Given $\left(1+x+2x^2\right)^{20}=a_0+a_1x+\ldots+a_{40}x^{40}$

Put $x=1$ we get,

$a_0+a_1+a_2+\ldots+a_{40}=4^{20}$

Put $x=-1$ we get,

$a_0-a_1+a_2+\ldots.+a_{40}=2^{20}$

$\Rightarrow a_1+a_3+\ldots.+a_{39}=\frac{4^{20}-2^{20}}{2}$

$\Rightarrow a_1+a_3+\ldots+a_{37}=2^{39}-2^{19}-a_{39}$

Here $a_{39}=\frac{20!(2)^{19}\times 1}{19!}=20\times 2^{19}$

$\Rightarrow a_1+a_3+\ldots+a_{37}=2^{19}\left(2^{20}-1-20\right)$

$=2^{19}\left(2^{20}-21\right)$

29. (3)

Given,

$\sum_{r=0}^{6}{}^{51-k}C_3$

$={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{48}C_3+{}^{47}C_3+{}^{46}C_3+{}^{45}C_3$

Now we know that ${}^nC_r+{}^nC_{r+1}={}^{n+1}C_{r+1}$ or ${}^nC_r={}^{n+1}C_{r+1}-{}^nC_{r+1}$

Now using the above formula in given expression we get,

$\sum_{r=0}^{6}{}^{51-k}C_3={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{48}C_3+{}^{47}C_3+\underbrace{{}^{46}C_3+{}^{46}C_4}-{}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6}{}^{51-k}C_3={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{48}C_3+\underbrace{{}^{47}C_3+{}^{47}C_4}-{}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6}{}^{51-k}C_3={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{48}C_3+{}^{48}C_4-{}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6}{}^{51-k}C_3={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{49}C_4-{}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6}{}^{51-k}C_3={}^{51}C_3+{}^{50}C_3+{}^{50}C_4-{}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6}{}^{51-k}C_3={}^{51}C_3+{}^{51}C_4-{}^{45}C_4$

$\Rightarrow \sum_{r=0}^{6}{}^{51-k}C_3={}^{52}C_4-{}^{45}C_4.$

30. (45) $30\left({}^{30}C_0\right)+29\left({}^{30}C_1\right)+\ldots+2\left({}^{30}C_{28}\right)+1\left({}^{30}C_{29}\right)$

$=30\left({}^{30}C_{30}\right)+29\left({}^{30}C_{29}\right)+\ldots\ldots+2\left({}^{30}C_2\right)+1\left({}^{30}C_1\right)$

$=\sum_{r=1}^{30} r\left({}^{30}C_r\right)$

$=\sum_{r=1}^{30} r\left(\frac{30}{r}\right)\left({}^{29}C_{r-1}\right)$

$=30\sum_{r=1}^{30}{}^{29}C_{r-1}$

$=30\left({}^{29}C_0+{}^{29}C_1+{}^{29}C_2+\ldots+{}^{29}C_{29}\right)$

$=30\left(2^{29}\right)=15(2)^{30}=n(2)^m$

$\therefore\ n=15, m=30$

$n+m=45$