ANSWER KEYS

1. (3)	**2.** (4)	**3.** (3)	**4.** (3)	**5.** (4)	**6.** (3)	**7.** (1)	**8.** (2)
9. (1)	**10.** (14)	**11.** (45)	**12.** (3)	**13.** (2)	**14.** (3)	**15.** (4)	**16.** (4)
17. (5)	**18.** (2)	**19.** (5)	**20.** (1)	**21.** (1)	**22.** (3)	**23.** (5)	**24.** (4)
25. (1)	**26.** (4)	**27.** (2)	**28.** (3)	**29.** (4)	**30.** (2)		

1. (3)

Let the volume of the cone be V c. c.

Given $\frac{dv}{dt}=1\,\text{cc}/\text{sec},\ h=35\,\text{cm},\ r=7\,\text{cm}$

i.e. $\frac{h}{r}=5$

We know for a cone $l^2=r^2+h^2$

Lateral surface area, $S=\pi r\sqrt{r^2+h^2}$

$S=\pi\frac{h}{5}\sqrt{\frac{h^2}{25}+h^2}=\pi\frac{\sqrt{26}}{25}h^2$

Also $V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{h}{5}\right)^2h=\frac{\pi}{75}h^3$

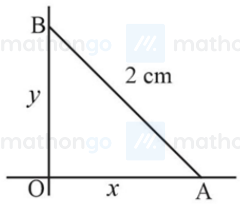
$\Rightarrow\frac{dV}{dt}=\frac{\pi}{25}h^2\frac{dh}{dt}\Rightarrow\frac{\pi}{25}h^2\frac{dh}{dt}=1$

$\Rightarrow\frac{dh}{dt}=\frac{25}{\pi h^2}\quad\ldots(i)$

Now $\frac{dS}{dt}=\frac{\pi\sqrt{26}}{25}\times2h\frac{dh}{dt}=\frac{2\sqrt{26}}{h}\quad(\text{from }(i))$

$\left(\frac{dS}{dt}\right)_{h=10}=\frac{\sqrt{26}}{5}$

2. (4)



$\frac{dy}{dt}=-25\,cm/\text{sec}$, $\frac{dx}{dt}=?$

Now, $x^2+y^2=2^2=4$

differentiating w.r.t. t both side

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$ $x^2+y^2=4$ when $y=1\Rightarrow x^2=3\Rightarrow x=\sqrt{3}$

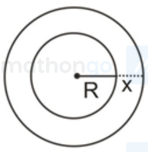
$\Rightarrow\frac{dx}{dt}=-\frac{y}{x}\frac{dy}{dt}$

$\Rightarrow\frac{dx}{dt}_{\left(\sqrt{3},1\right)}=-\frac{1}{\sqrt{3}}\times(-25)=\frac{25}{\sqrt{3}}\ cm/\text{sec}$

3. (3)

We know that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Given the radius of the spherical iron ball is $R=10\ cm$ and the thickness of the ice is $x\ cm$.



Volume of ice: $V=\frac{4}{3}\pi(R+x)^3-\frac{4}{3}\pi R^3$

$\Rightarrow V=\frac{4}{3}\pi(10+x)^3-\frac{4}{3}\pi(10)^3$

Differentiating with respect to t, we get

$\frac{dV}{dt}=4\pi(10+x)^2\frac{dx}{dt}-0$

Given $\frac{dV}{dt}=50\ cm^3/min$

$\Rightarrow50=4\pi(10+x)^2\frac{dx}{dt}$

$\Rightarrow\frac{dx}{dt}=\frac{50}{4\pi(10+x)^2}$

$\Rightarrow\frac{dx}{dt}\Big|_{x=5}=\frac{50}{4\pi(15)^2}=\frac{1}{18\pi}\ cm/min.$

4. (3)

Given: position of the moving car at time t is $= f(t) = at^2 + bt + c$

So,

$v_{avg} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$

$\Rightarrow v_{avg} = \frac{a(t_2^2 - t_1^2) + b(t_2 - t_1)}{t_2 - t_1}$

$\Rightarrow v_{avg} = a(t_1 + t_2) + b$

The instantaneous speed is given by:

$f'(t) = 2at + b$

So, to get the point where average speed is equal to car's actual speed,

$f'(t) = v_{avg}$

$\Rightarrow a(t_1 + t_2) + b = at + b$

$\Rightarrow t = \frac{t_1 + t_2}{2}$

5. (4)

Surface area of cube $(s) = 6a^2$

$\Rightarrow \frac{ds}{dt} = 12a\frac{da}{dt} = 3.6$

$\Rightarrow 12(10)\frac{da}{dt} = 3.6 \Rightarrow \frac{da}{dt} = 0.03 cm/sec$

Now, volume of cube $(V) = a^3$

$\Rightarrow \frac{dv}{dt} = 3a^2 \cdot \frac{da}{dt} = 3\left(10\right)^2(0.03) = 9cm^3/sec$

6. (3)

Let the volume of the cone be V c. c.

Given $\frac{dv}{dt} = 1\,\text{cc}/\text{sec},\ h = 35\,\text{cm},\ r = 7\,\text{cm}$

i.e. $\frac{h}{r} = 5$

We know for a cone $l^2 = r^2 + h^2$

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Also $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{5}\right)^2 h = \frac{\pi}{75}h^3$

$\Rightarrow \frac{dV}{dt} = \frac{\pi}{25}h^2\frac{dh}{dt} \Rightarrow \frac{\pi}{25}h^2\frac{dh}{dt} = 1$

$\Rightarrow \frac{dh}{dt} = \frac{25}{\pi h^2} \quad \ldots(i)$

Now $\frac{dS}{dt} = \frac{\pi\sqrt{26}}{25} \times 2h\frac{dh}{dt} = \frac{2\sqrt{26}}{h} \quad (\text{from } (i))$

$\left(\frac{dS}{dt}\right)_{h=10} = \frac{\sqrt{26}}{5}$

7. (1)

Let $f(x) = xe^{x(1-x)}$ then $f(x)$ is:

Now $f(x) = xe^{x(1-x)}$

Differentiating both side w.r.t x we get,

$f'(x) = xe^{x(1-x)}(1 - 2x) + e^{x(1-x)}$

$= -e^{x(1-x)}\left(2x^2 - x - 1\right)$

$= -e^{x(1-x)}(2x + 1)(x - 1)$

Now finding critical point by equating $f'(x) = 0$

We get, $x = \frac{-1}{2}$ & $x = 1$

$\therefore\ f(x)$ is decreasing in $\left(-\infty, -\frac{1}{2}\right) \cup (1, \infty)$

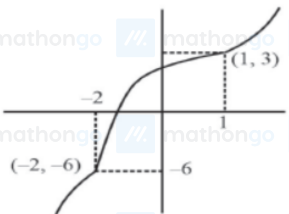
And increasing in $\left(-\frac{1}{2}, 1\right)$

8. (2)

$f(x) = x|x - 1| + |x + 2|$

$\mathrm{x}|\mathrm{x} - 1| + |\mathrm{x} + 2| + \mathbf{a} = 0$

$\mathrm{x}|\mathrm{x} - 1| + |\mathrm{x} + 2| = -a$



9. (1)

$g(x) = f(x) + f(1-x) \& f''(x) > 0, x \in (0,1)$

$g'(x) = f'(x) - f'(1-x) = 0$

$\Rightarrow f'(x) = f'(1-x)$

$x = 1 - x$

$x = \frac{1}{2}$

$g'(x) = 0$

at $x = \frac{1}{2}$

$g''(x) = f''(x) + f''(1-x) > 0$

g is concave up

hence $\alpha = \dfrac{1}{2}$

$\tan^{-1} 2\alpha + \tan^{-1} \dfrac{1}{\alpha} + \tan^{-1} \dfrac{\alpha+1}{\alpha}$

$\Rightarrow \quad \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

10. (14)

Given:

$f(x) = x^2 + g'(1)x + g''(2)$

$\Rightarrow f'(x) = 2x + g'(1)$

$\Rightarrow f''(x) = 2$

Now,

$g(x) = f(1)x^2 + xf'(x) + f''(x)$

$\Rightarrow g(x) = f(1)x^2 + x[2x + g'(1)] + 2$

$\Rightarrow g(x) = [f(1)+2]x^2 + g'(1)x + 2$

$\Rightarrow g'(x) = 2f(1)x + 4x + g'(1) \quad \ldots.(1)$

$\Rightarrow g''(x) = 2f(1) + 4 \quad \ldots(2)$

Put $x = 1$ in (1), then we get

$2f(1) + 4 = 0$

$\Rightarrow f(1) = -2$

$\therefore g''(2) = 2f(1) + 4 = 0$

Now,

$f(x) = x^2 + g'(1)x + g''(2)$

Put $x = 1$, then we get

$f(1) = 1 + g'(1) + g''(2)$

$\Rightarrow g'(1) = -3$

So,

$f'(x) = 2x - 3$

$f(x) = x^2 - 3x + c$

Since, $f(1) = -2 \Rightarrow c = 0$

So,

$f(x) = x^2 - 3x$

And,

$g(x) = f(1)x^2 + xf'(x) + f''(x)$

$\Rightarrow g(x) = -2x^2 + x(2x-3) + 2$

$\Rightarrow g(x) = -3x + 2$

Hence, $f(4) - g(4) = 4 + 10 = 14$

11. (45)

Given, $f(x) = 2x^2 - \log_e x$

$\Rightarrow f'(x) = 4x - \frac{1}{x}$

$\Rightarrow f'(x) = \frac{4x^2-1}{x}$

$\Rightarrow f'(x) = 0 \Rightarrow 4x^2 - 1 = 0 \Rightarrow x = \pm\frac{1}{2}$

But given $x > 0$ so $x = \frac{1}{2}$

So function is decreasing in $\left(0, \frac{1}{2}\right)$ and increasing in the interval $\left(\frac{1}{2}, \infty\right)$

So, $a = \frac{1}{2}$

Now equation of parabola will be $y^2 = 2x$

Now tangent to $y^2 = 2x$ will be given by,

$y = mx + \frac{1}{2m}$, given this tangent passes through $(8a, 8a-1) \equiv (4, 3)$,

So $3 = 4m + \frac{1}{2m}$

$\Rightarrow m = \frac{1}{2}$ or $\frac{1}{4}$

So equation of tangent are $y = \frac{x}{2} + 1$ or $y = \frac{x}{4} + 2$

But $y = \frac{x}{2} + 1$ passes through $(-2, 0)$ so rejected as given in the question.

Now equation of normal at P will be,

$y = -4x - 2\left(\frac{1}{2}\right)(-4) - \frac{1}{2}(-4)^3 \left\{\text{as slope of normal} = \frac{-1}{\frac{1}{4}} = -4\right\}$

$\Rightarrow y = -4x + 4 + 32$

$\Rightarrow y + 4x = 36$

$\Rightarrow \frac{x}{9} + \frac{y}{36} = 1$

So, $\alpha = 9, \beta = 36$

So, $\alpha + \beta = 45$

12. (3)

Given $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5, x > 1$

$f'(x) = \frac{4}{x-1} - 4(x-1)$

For $1 < x < 2 \Rightarrow f'(x) > 0$

For $x > 2 \Rightarrow f'(x) < 0$

Hence, $f(x)$ has a maxima at $x = 2$

$f(x) = -1$ has two solution as the curve of $f(x)$ will cut it at exactly two points.

Now, $f(e) > 0$ and $f(e+1) < 0$

i.e. $f(e) \cdot f(e+1) < 0$

$f'(e) - f''(2) = \frac{4}{e-1} - 4(e-1) + 8 > 0$

13. (2)

$f(x) = (4a-3)(x + \log_e 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right), x \neq 2n\pi, n \in N$

$f(x) = (4a-3)(x + \log_e 5) + (a-7)\sin x$

$f'(x) = (4a-3)(1) + (a-7)\cos x = 0$

$\Rightarrow \cos x = \frac{3-4a}{a-7}$

$-1 \leq \frac{3-4a}{a-7} \leq 1$

$\frac{3-4a}{a-7} + 1 \geq 0 \quad \frac{3-4a}{a-7} \leq 1$

$\frac{3-4a+a-7}{a-7} \geq 0 \quad \frac{3-4a}{a-7} - 1 \leq 0$

$\frac{-3a-4}{a-7} \geq 0 \quad \frac{3-4a-a+7}{a-7} \leq 0$

$\frac{3a+4}{a-7} \leq 0 \quad \frac{-5a+10}{a-7} \leq 0$

$\frac{3a+4}{a-7} \leq 0 \quad \frac{5a-10}{a-7} \geq 0$

$\frac{3a+4}{a-7} \leq 0 \quad \frac{5(a-2)}{a-7} \geq 0$

$\alpha \in \left[-\frac{4}{3}, 7\right) \quad \alpha \in \left(-\infty, 2\right] \cup \left(7, \infty\right)$

Hence, $\alpha \in \left[-\frac{4}{3}, 2\right]$

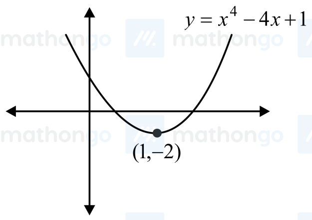
14. (3)

Let $f(x)=x^4-4x+1$

$f'(x)=4x^3-4$ and $f'(x)=0 \Rightarrow x=1$

So the extrema of the function is at $x=1$

Plotting the graph we get



$f''(x)=12x^2 \quad \therefore\ f''(1)>0$

And at $x=1,\ f(1)=-2<0$

So, graph of $f(x)$ will cut x-axis at two points,

$\therefore$ Number of real roots of $f(x)=0$ is equal to 2

15. (4)

Given,

$x^7-7x-2=0$

$x^7-7x=2$

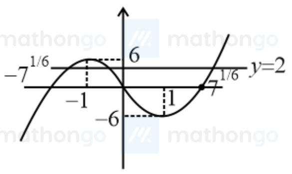
Now taking $f(x)=x^7-7x$ (odd) & $y=2$

Now differentiating $f(x)$ w.r.t x we get,

$f'(x)=7\left(x^6-1\right)=7\left(x^2-1\right)\left(x^4+x^2+1\right)$

$f'(x)=0 \Rightarrow x=\pm 1$

So points of extrema are -1 & 1 , Now plotting the graph to check number of solution we get,



$f(x)=2$ has 3 real distinct solution.

16. (4)

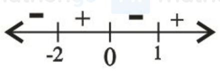
$3x^4+4x^3-12x^2+4=0$

So,

let $f(x)=3x^4+4x^3-12x^2+4$

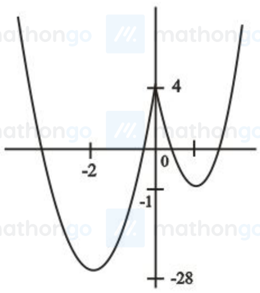
$\therefore\ f'(x)=12x\left(x^2+x-2\right)$

$=12x(x+2)(x-1)$



$f(0)=4$

$f(1)=-1$



Ans. 4

17. (5)

$y = x^5 - 20x^3 + 50x + 2$

$\frac{dy}{dx} = 5x^4 - 60x^2 + 50 = 5\left(x^4 - 12x^2 + 10\right)$

$\frac{dy}{dx} = 0 \Rightarrow x^4 - 12x^2 + 10 = 0$

$\Rightarrow x^2 = \frac{12 \pm \sqrt{144 - 40}}{2}$

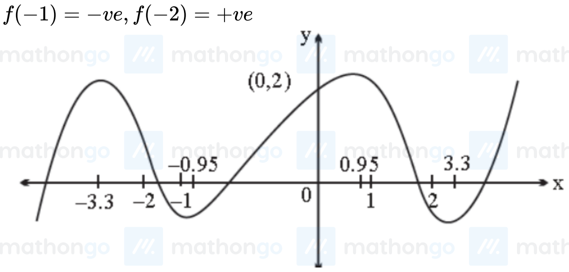
$\Rightarrow x^2 = 6 \pm \sqrt{26} \Rightarrow x^2 \approx 6 \pm 5.1$

$\Rightarrow x^2 \approx 11.1, 0.9$

$\Rightarrow x \approx \pm 3.3, \pm 0.95$

$f(0) = 2, f(1) = +ve, f(2) = -ve$

$f(-1) = -ve, f(-2) = +ve$



18. (2)

$f(x) = \log_e\left(x^2+1\right) - e^{-x} + 1 \Rightarrow f'(x) = \frac{2x}{x^2+1} + e^{-x} > 0, \forall x \in R$

So $f(x)$ increasing

$g(x) = e^{-x} - 2e^x \quad \Rightarrow \quad g'(x) = -(e^{-x} + 2e^x) < 0 \ \forall x \in R$

So $g(x)$ decreasing

$\Rightarrow \quad f(g(x))$ is decreasing

$\Rightarrow \quad f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$

$\Rightarrow \quad \frac{(\alpha-1)^2}{3} < \alpha - \frac{5}{3}$

$\Rightarrow \quad (\alpha - 2)(\alpha - 3) < 0 \quad \Rightarrow \alpha \in (2, 3)$

19. (5)

Let,

$y = \frac{x^3}{x^4+147} = f(x)$

We know that, for increasing function $\frac{dy}{dx} > 0$

So, differentiating the given function $f(x) = \frac{x^3}{x^4+147}$ we get,

$\frac{dy}{dx} = \frac{-x^2\left(x^4-441\right)}{\left(x^4+147\right)^2}$

$\Rightarrow \frac{-x^2\left(x^4-441\right)}{\left(x^4+147\right)^2} > 0$

$\Rightarrow \frac{x^2\left(x^4-441\right)}{\left(x^4+147\right)^2} < 0$

$\Rightarrow x^4 < 441$

Now for maxima/minima $\frac{dy}{dx} = 0$

$\Rightarrow x^4 = 441$

$\Rightarrow x = \alpha, \ 4 < \alpha < 5$

$\Rightarrow$Maximum value of $f(x)$ is at $x = 4$ or $x = 5$

$f(4) = \frac{64}{403}, \ f(5) = \frac{125}{772}$

$\because f(5) > f(4)$

$\Rightarrow \alpha = 5$

20. **(1)**

Given,

$x=2$ be a local minima of the function $f(x)=2x^4-18x^2+8x+12, x\in(-4,4)$

Now differentiating the above function we get,

$f'(x)=8x^3-36x+8$

$\Rightarrow f'(x)=4\left(2x^3-9x+2\right)$

Now equating it to zero we get, $f'(x)=0$

$\Rightarrow 2x^3-9x+2=0$

$\Rightarrow(x-2)\left(2x^2+4x-1\right)=0$

$\Rightarrow x=\frac{-4\pm\sqrt{24}}{4}$

$\Rightarrow x=\frac{-2\pm\sqrt{6}}{2}$

$\therefore\ x=\frac{\sqrt{6}-2}{2}$ {by checking sign change of $f'(x)$ for maxima}

Now rewriting $f(x)=2x^4-18x^2+8x+12$ we get,

$f(x)=\left(x^2-2x-\frac{9}{2}\right)\left(2x^2+4x-1\right)+24x+7.5$

Hence, $f\left(\frac{\sqrt{6}-2}{2}\right)=M=12\sqrt{6}-\frac{33}{2}$

21. **(1)**

Given $f(x)=\begin{cases} x^2-4x-2, & \forall x\in\left(-1,\frac{3-\sqrt{17}}{2}\right) \\ -x^2+2x+2, & \forall x\in\left[\frac{3-\sqrt{17}}{2},2\right) \end{cases}$

When $x\in\left(-1,\frac{3-\sqrt{17}}{2}\right)$

$f'(x)=2x-4=0\Rightarrow x=2$

Now $f(2)=2,\ f(-1)=3$

$f\left(\frac{3-\sqrt{17}}{2}\right)=\frac{\sqrt{17}-3}{2}$

When $x\in\left[\frac{3-\sqrt{17}}{2},2\right)$

$f'(x)=-2x+2$

$f'(x)=-2(x-1)$

$f'(x)=0$ when $x=1$

$f(1)=3$

Hece, absolute minimum value $=\frac{\sqrt{17}-3}{2}$

and absolute maximum value $=3$

Sum $=\frac{\sqrt{17}-3}{2}+3=\frac{\sqrt{17}+3}{2}$

22. (3)

$f(x)= x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1) \quad \ldots$(i)

$\Rightarrow f'(x)= 3x^2 - 6x - \frac{3}{2}f''\left(2\right) \quad \ldots$(ii)

$\Rightarrow f''(x)= 6x - 6 \quad \ldots$(iii)

Then,

$f''(2) = 12 - 6 = 6$

$f''(1) = 0$

Now,

$f'(x)= 3x^2 - 6x - \frac{3}{2}f''\left(2\right)$

$\Rightarrow f'(x)= 3x^2 - 6x - \frac{3}{2} \times 6$

$\Rightarrow f'\left(x\right) = 3x^2 - 6x - 9$

For maxima/minima, we have

$f'(x)= 0$

$\Rightarrow 3x^2 - 6x - 9 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0$

$\Rightarrow x = -1$ and 3

Now,

$f''(x) = 6x - 6$

$f''(-1) = -12 < 0$ (maxima)

$f''(3) = 12 > 0$ (minima)

Again

$f(x)= x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$

$\Rightarrow f(x)= x^3 - 3x^2 - \frac{3}{2} \times 6x + 0$

$\Rightarrow f\left(x\right) = x^3 - 3x^2 - 9x$

$\Rightarrow f(3) = 27 - 27 - 9 \times 3 = -27$

23. (5)

Given $f : [-1,\ 1] \rightarrow R,\ f\left(x\right) = ax^2 + bx + c,\ f'\left(x\right) = 2ax + b$ and $f''\left(x\right) = 2a$

$\Rightarrow f(-1) = a - b + c = 2 \quad \ldots(1),$

$\Rightarrow f'(-1) = -2a + b = 1 \quad \ldots(2)$ and

$f''\left(x\right) = 2a$

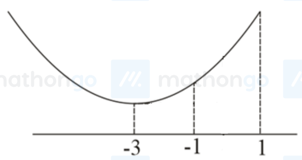
Given the maximum value of $f''\left(x\right) = 2a = \frac{1}{2} \ \Rightarrow a = \frac{1}{4},$

From the equations (1) and (2) we get $b = \frac{3}{2}$ and $c = \frac{13}{4}$.

$\therefore f\left(x\right) = \frac{x^2}{4} + \frac{3x}{2} + \frac{13}{4}$

We know that, the vertex of the quadratic $Ax^2 + Bx + C$ is at $\left(-\frac{B}{2A},\ -\frac{B^2-4AC}{4A}\right)$

Thus, the vertex of $\frac{x^2}{4} + \frac{3x}{2} + \frac{13}{4}$ is at $(-3,\ 1)$, hence both the numbers ± 1 are on the same side of the vertex and the graph of $f(x)$ is given below.



For, $x \in [-1,\ 1]$, we have $f(-1)= 2$ & $f(1)= 5$

$\Rightarrow 2 \leq f(x) \leq 5$

$\therefore$ Least value of α is 5.

24. (4)

$f(x):[0,2] \to R$ and $f''(x) > 0$ for $x \in [0,2]$

$\Rightarrow f'(x)$ is increasing for $x \in [0,2]$

Now, $\phi(x) = f(x) + f(2-x)$

$\Rightarrow \phi'(x) = f'(x) - f'(2-x)$

For $x \in [0,1)$, $x < 2-x$

$\Rightarrow f'(x) < f'(2-x) \Rightarrow \phi'(x) < 0$

For $x \in (1, 2]$, $x > 2-x$

$\Rightarrow f'(x) > f'(2-x) \Rightarrow \phi'(x) > 0$

Hence, ϕ is decreasing on $(0, 1)$ and increasing on $(1, 2)$.

25. (1)

Given,

$f(x) = |x^2 - 5x + 6| - 3x + 2$

$\Rightarrow f(x) = |(x-3)(x-2)| - 3x + 2$

$$\Rightarrow f(x) = \begin{cases} x^2 - 8x + 8 & ; x \in [-1, 2] \\ -x^2 + 2x - 4 & ; x \in (2, 3] \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 2x - 8 & ; x \in (-1, 2) \\ -x + 2 & ; x \in (2, 3) \end{cases}$$

Now point of extrema will be $2x - 8 = 0 \Rightarrow x = 4$which is not in domain and $-x + 2 = 0 \Rightarrow x = 2$

Now for finding the value of global minima and maxima we will check the value of function at extrema points and boundary points,

So, $f(x) = x^2 - 8x + 8$ will give, $f(-1) = 17$ & $f(2) = -4$

And $f(x) = -x^2 + 2x - 4$ will give, $f(2) = 4$, $f(3) = -7$

Hence, absolute minima is -7 and maxima is 17

So, their sum is $-7 + 17 = 10$

26. (4)

$f(x) = (3x^2 + ax - 2 - a)e^x$

$f'(x) = (3x^2 + ax - 2 - a)e^x + e^x(6x + a) = e^x(3x^2 + (a+6)x - 2)$

$\because$ $x = 1$ is a critical point $\therefore$ $f'(1) = 0$

$\therefore 3 + a + 6 - 2 = 0$

$a = -7$

$\therefore f'(x) = e^x(3x^2 - x - 2) = e^x(3x^2 - 3x + 2x - 2) = e^x(3x + 2)(x - 1)$

$\therefore$ maxima at x $= \frac{-2}{3}$ $\therefore$ minima at x $= 1$

27. (2)

$f(x) = x^7 + 5x^3 + 3x + 1$

For number of real solutions, we need to find the number of times the curve cuts $x-$axis.

Now $f'(x) = 7x^6 + 15x^2 + 3 > 0$

$\therefore$ $f(x)$ is strictly increasing function

i.e. when $x \to -\infty, y \to -\infty$ and when $x \to \infty, y \to \infty$

So, the curve will cut $x-$axis only once.

$\therefore$ number of real solution will be 1

28. **(3)** $x\ell \text{nxf} f'(x)+\ln x f(x)+f(x)\geq I, x\in[2,4]$

And f(2) $=\frac{1}{2}$, f(4) $=\frac{1}{4}$

Now $x\ln x\frac{dy}{dx}+(\ln +1)y\geq 1$

$$\frac{d}{dx}(y\cdot x\ln x)\geq 1$$

$$\frac{d}{dx}(f(x)\cdot x\ln x)\geq 1$$

$$\Rightarrow \frac{d}{dx}(x\ln x f(x)-x)\geq 0, x\in[2,4]$$

$\Rightarrow$ The function $g(x)=x\ln x f(x)-x$ is increasing in [2.4]

And $g(2)=2\ln 2f(2)-2=\ln 2-2$

$g(4)=4\ln 4f(4)-4=\ln 4-4$

$=2(\ln 2-2)$

Now $\quad g(2)\leq g(x)\leq g(4)$

$\ln 2-2\leq x\ln x f(x)-x\leq 2(\ln 2-2)$

$$\frac{\ln 2-2}{x\ln x}+\frac{1}{\ln x}\leq f(x)\leq\frac{2(\ln 2-2)}{x\ln x}+\frac{1}{\ln x}$$

Now for x $\in[2,4]$

$$\frac{2(\ln 2-2)}{x\ln x}+\frac{1}{\ln x}<\frac{2(\ln 2-2)}{2\ln 2}+\frac{1}{\ln 2}=1-\frac{1}{\ln 2}<1$$

$\Rightarrow f(x)\leq 1$ for $x\in[2,4]$

Also for $x\in[2,4]$:

$$\frac{\ln 2-2}{x\ln x}+\frac{1}{\ln x}\geq\frac{\ln 2-2}{4\ln 4}+\frac{1}{\ln 4}=\frac{1}{8}+\frac{1}{2\ln 2}>\frac{1}{8}$$

$\Rightarrow f(x)\geq\frac{1}{8}$ for $x\in[2,4]$

Hence both A and B are true.

LMVT on $(yx(\ln x))$ not satisfied.

Hence no such function exists.

Therefore it should be bonus.

29. **(4)**

$$f(x)=x-\sin 2x+\frac{1}{3}\sin 3x$$

$$f'(x)=1-2\cos 2x+\cos 3x=0$$

$$x=\frac{5\pi}{6},\frac{\pi}{6}$$

$$\therefore f''(x)=4\sin 2x-3\sin 3x$$

$$f''\left(\frac{5\pi}{6}\right)<0$$

$\Rightarrow\left(\frac{5\pi}{6}\right)$ is point of maxima

$$f\left(\frac{5\pi}{6}\right)=\frac{5\pi}{6}+\frac{\sqrt{3}}{2}+\frac{1}{3}$$

30. **(2)**

Given $f(x)=|2x^2+3x-2|+\sin x\cos x$

$\Rightarrow f(x)=|(2x-1)(x+2)|+\sin x\cos x$

Now, $f'(x)=\begin{cases}4x+3+\frac{\cos 2x}{4}, & \frac{1}{2}<x<1\\ -(4x+3)+\frac{\cos 2x}{4}, & 0\leq x<\frac{1}{2}\end{cases}$

For $0\leq x<\frac{1}{2}\Rightarrow f'(x)<0$

For $\frac{1}{2}<x\leq 1\Rightarrow f'(x)>0$

So $f(x)$ has minima at $x=\frac{1}{2}$ and maxima at $x=1$

Hence, $f\left(\frac{1}{2}\right)+f(1)=3+\frac{1}{2}(1+2\cos 1)\sin 1$