ANSWER KEYS

1. (1)	**2.** (4)	**3.** (2)	**4.** (1)	**5.** (.25)	**6.** (1)	**7.** (4)	**8.** (4)
9. (2)	**10.** (2)						

1. **(1)** $\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{\mathrm{y}^2}{\mathrm{xy}-\mathrm{x}^2}$

Put, $\mathrm{y}=\mathrm{vx}$

$\Rightarrow \quad \frac{\mathrm{dy}}{\mathrm{dx}}=\mathrm{v}+\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}$

$\Rightarrow \quad \mathrm{v}+\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{\mathrm{v}^2}{\mathrm{v}-1}$

$\Rightarrow \quad \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{\mathrm{v}^2}{\mathrm{v}-1}-\mathrm{v}=\frac{\mathrm{v}}{\mathrm{v}-1}$

$\Rightarrow \quad \int\left(\frac{\mathrm{v}-1}{\mathrm{v}}\right)\mathrm{dv}=\int\frac{\mathrm{dx}}{\mathrm{x}}$

$\Rightarrow \quad \int\left(1-\frac{1}{\mathrm{v}}\right)\mathrm{dv}=\int\frac{\mathrm{dx}}{\mathrm{x}}$

$\Rightarrow \quad \mathrm{v}-\ell\mathrm{n}\,\mathrm{v}=\ell\mathrm{n}\,\mathrm{x}+\mathrm{c}$

$\Rightarrow \quad \frac{\mathrm{y}}{\mathrm{x}}=\ell\mathrm{n}\,\mathrm{y}+\mathrm{c}$

Alternate Solution :

We have, $\left(xy-x^2\right)\frac{dy}{dx}=y^2$

$\Rightarrow \quad y^2\frac{dx}{dy}=xy-x^2$

$\Rightarrow \quad \frac{1}{x^2}\frac{dx}{dy}-\frac{1}{x}\cdot\frac{1}{y}=-\frac{1}{y^2}$

Put $\frac{1}{x}=\mathrm{v} \Rightarrow -\frac{1}{x^2}\frac{dx}{dy}=\frac{d\mathrm{v}}{dy}$

$\therefore \quad \frac{d\mathrm{v}}{dy}+\frac{\mathrm{v}}{y}=\frac{1}{y^2}$, which is linear

$\therefore \quad \mathrm{IF}=e^{\int\frac{1}{y}dy}=e^{\log \mathrm{y}}=y$

$\therefore$ The solution is $\mathrm{vy}=\int\frac{1}{y^2}\cdot y\;\; dy+c$

$\Rightarrow \quad \frac{y}{x}=\log|\mathrm{y}|+c$

$\Rightarrow \quad y=x(\log|\mathrm{y}|+c)$

This posses through the point (–1, 1).

$\therefore \quad 1=-1(\log 1+c)$

ie, $\quad c=-1$

Thus, the equation of the curve is

y = x (log |y| – 1)

2. **(4)** Given differential equation is $x\frac{dy}{dx}=y+xe^{\frac{y}{x}}$

$\frac{dy}{dx}=\frac{y}{x}+e^{\frac{y}{x}}$

It is homogeneous differential equation.

$\therefore$ Put $y=vx \quad \Rightarrow \quad \frac{dy}{dx}=v+\frac{dv}{dx}$

$\therefore v+x\frac{dv}{dx}=\frac{vx}{x}+e^{\frac{vx}{x}}$

$\Rightarrow v+x\frac{dv}{dx}=v+e^{v}$

$\Rightarrow x\frac{dv}{dx}=e^{v}$

$\Rightarrow e^{-v}dv=\frac{1}{x}dx$

On integrating both sides, we get

$-e^{-v}=\log x+c$

$-e^{-\frac{y}{x}}=\log x+c$

Given, $y(1)=0$

$\therefore e^{-\frac{0}{1}}=\log 1+c$

$-1=0+c \Rightarrow \quad c=\; -1$

$\therefore -e^{-\frac{y}{x}}=\log x-1$

$\Rightarrow 1=\log x+e^{-\frac{y}{x}}$

3. **(2)**

Given differential equation is

$2x^4 y\frac{dy}{dx} + y^4 = 4x^6$

On substituting $y = u^m$ in the given differential equation, we get

$2x^4 \cdot u^m \cdot mu^{m-1} \cdot \frac{du}{dx} + u^{4m} - 4x^6 = 0$

$\left(\because y = u^m \Rightarrow \frac{dy}{dx} = mu^{m-1}\frac{du}{dx}\right)$

$\Rightarrow \quad \frac{du}{dx} = \frac{4x^6 - u^{4m}}{2mx^4u^{2m-1}} = \frac{4x^6 - u^{4m}}{2mx^4u^{2m-1}}$

If the above differential equation is homogeneous, then

$\Rightarrow \quad 4m = 6$

$\Rightarrow \quad m = \frac{3}{2}$

4. **(1)** $\frac{\mathrm{dy}}{\mathrm{dx}} - \left(\frac{\mathrm{x}}{1-\mathrm{x}^2}\right)\mathrm{y} = \frac{1}{1-\mathrm{x}^2}$

I. F $= \sqrt{\mathrm{x}^2 - 1}$

$\therefore \; \mathrm{y}\sqrt{\mathrm{x}^2 - 1} = -\int \frac{\mathrm{dx}}{\sqrt{\mathrm{x}^2-1}} + \mathrm{C} = -\log\left(\mathrm{x} + \sqrt{\mathrm{x}^2 - 1}\right) + \mathrm{C}$

$\therefore \; y\sqrt{x^2 - 1} = -\log\left(\mathrm{x} + \sqrt{\mathrm{x}^2 - 1}\right) + \mathrm{C}$

5. **(.25)** Given, $\frac{\mathrm{dy}}{\mathrm{dt}} - \left(\frac{\mathrm{t}}{1+\mathrm{t}}\right)\mathrm{y} = \frac{1}{(1+\mathrm{t})}$ and $\mathrm{y}(0) = -1$

Which represents linear differential equation of first order.

$\therefore \quad \mathrm{IF} = \mathrm{e}^{\int -\left(\frac{\mathrm{t}}{1+\mathrm{t}}\right)\mathrm{dt}} = \mathrm{e}^{-\mathrm{t}+\log(1+\mathrm{t})} = \mathrm{e}^{-\mathrm{t}} \cdot (1+\mathrm{t})$

Required solution is,

$\mathrm{ye}^{-\mathrm{t}}(1+\mathrm{t}) = \int \frac{1}{(1+\mathrm{t})} \cdot \mathrm{e}^{-\mathrm{t}}(1+\mathrm{t})\mathrm{dt} + \mathrm{c} = \int \mathrm{e}^{-\mathrm{t}}\mathrm{dt} + \mathrm{c}$

$\Rightarrow \mathrm{ye}^{-\mathrm{t}}(1+\mathrm{t}) = -\mathrm{e}^{-\mathrm{t}} + \mathrm{c}$

Since, $\mathrm{y}(0) = -1$

$\mathrm{c} = 0$

$\therefore \mathrm{y} = -\frac{1}{(1+\mathrm{t})}$

$\Rightarrow \mathrm{y}(-5) = \frac{1}{4} = .25$

6. **(1)**

Given equation can be rewritten as

$\frac{dx}{dy} + \frac{1}{(1+y^2)}x = \frac{e^{\tan^{-1}y}}{(1+y^2)}$

$\therefore$ If $= e^{\int \frac{1}{1+y^2}dy} = e^{\tan^1 y}$

$\therefore$ Required solution is

$xe^{\tan^1 y} = \int \frac{e^{\tan^{-1}y}e^{\tan^{-1}y}}{1+y^2}dy$

Put $e^{\tan^{-1}y} = t \Rightarrow e^{\tan^{-1}y}\frac{1}{1+y^2}dy = dt$

$\therefore \; xe^{\tan^{-1}y} = \int t\,dt = \frac{t^2}{2} + c$

$\Rightarrow \; 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$

7. **(4)**

$\frac{dy}{dx} + 2y\tan x = \sin x$

$\mathrm{I.F.} = e^{\int 2\tan x dx} = e^{2\ell n \sec x}$

$\mathrm{I.F.} = \sec^2 x$

$y.(\sec^2 x) = \int \sin x.\sec^2 x dx$

$y.(\sec^2 x) = \int \sec x \tan x dx$

$y.(\sec^2 x) = \sec x + C$

$x = \frac{\pi}{3}; y = 0$

$\Rightarrow C = -2$

$\Rightarrow y = \frac{\sec x - 2}{\sec^2 x} = \cos x - 2\cos^2 x$

$y = t - 2t^2 \Rightarrow \frac{dy}{dt} = 1 - 4t = 0 \Rightarrow t = \frac{1}{4}$

$\therefore \; \max = \frac{1}{4} - \frac{1}{8} = \frac{2-1}{8} = \frac{1}{8}$

8. **(4)** Given that $\frac{dy}{dx}=\frac{y}{2y\log y+y-x}$

$\Rightarrow \frac{dx}{dy}=\frac{2y\log y+y-x}{y}$

$\Rightarrow \frac{dx}{dy}=2\log y+1-\frac{x}{y}$

$\Rightarrow \frac{dx}{dy}+\frac{x}{y}=2logy+1$

It is linear equation in x and y, we have

$IF=e^{\int\frac{1}{y}dy}=y$

$\Rightarrow \frac{d}{dy}(xy)=2ylogy+y$

Now, $xy=2\int ylogydy+\frac{y^2}{2}+c$

$=2\left[\frac{y^2}{2}logy-\frac{y^2}{4}\right]+\frac{y^2}{2}+c$

$\Rightarrow xy=y^2logy+c$

9. **(2)**

Let, $2x+y=t\Rightarrow \frac{dy}{dx}+2=\frac{dt}{dx}$

$\frac{dt}{dx}+xt=x^3t^3\Rightarrow \frac{1}{t^3}\frac{dt}{dx}+\frac{1}{t^2}x=x^3$

Let, $\frac{1}{t^2}=u\Rightarrow \frac{-2}{t^3}\frac{dt}{dx}=\frac{du}{dx}$

$\frac{du}{dx}+(-2x)u=-2x^3$

I.F. $=e^{-\int 2xdx}=e^{-x^2}\Rightarrow u.e^{-x^2}=\int e^{-x^2}\left(-2x^3\right)dx$

$\frac{e^{-x^2}}{(2x+y)^2}=-2\int e^{-x^2}.x^3dx$

$\frac{e^{-x^2}}{(2x+y)^2}=\int e^{-x^2}.x^2(-2x)dx$

Let, $-x^2=v$

$-2xdx=dv$

$\Rightarrow \frac{e^{-x^2}}{(2x+y)^2}=-\int e^v vdv$

$\Rightarrow \frac{e^{-x^2}}{(2x+y)^2}+v\cdot e^v-e^v=C$

$\Rightarrow \frac{e^{-x^2}}{(2x+y)^2}-x^2e^{-x^2}-e^{-x^2}=C$

$\Rightarrow \frac{1}{(2x+y)^2}=\left(x^2+1\right)+Ce^{x^2}$

10. **(2)**

Given differential equation is

$2y\frac{dy}{dx}+y^2\left(\frac{1}{x\ln x}\right)=\frac{\cos x}{\ln x}$

$\Rightarrow \frac{dz}{dx}+\frac{z}{x\ln x}=\frac{\cos x}{\ln x}$

On substituting $y^2=z$ and $2y\frac{dy}{dx}=\frac{dz}{dx}$, we get

$\Rightarrow \frac{dz}{dx}+\frac{z}{x\ln x}=\frac{\cos x}{\ln x}$

Which is a LDE,

$\therefore y^2\ln x=z\ln x=\sin x+C$

Now, $y\left(\frac{3\pi}{2}\right)=0\Rightarrow C=1$

$\therefore y=\pm\sqrt{\frac{1+\sin x}{\ln x}}$

$y=\sqrt{\frac{1+\sin x}{\ln x}}\quad(\because\ y>0)$

Required solution $y=a\sqrt{\frac{1+\sin x}{\ln x}}$ where a is any constant.