ANSWER KEYS

1. (4)	**2.** (1)	**3.** (2)	**4.** (3)	**5.** (1)	**6.** (2)	**7.** (4)	**8.** (2)
9. (4)	**10.** (4)	**11.** (4)	**12.** (11)	**13.** (1)	**14.** (4)	**15.** (2)	**16.** (2)
17. (479)	**18.** (2)	**19.** (1)	**20.** (1)	**21.** (3)	**22.** (2)	**23.** (10)	**24.** (1)
25. (1)	**26.** (1)	**27.** (11)	**28.** (1)	**29.** (3)	**30.** (19)		

1. **(4)** $=$ Sum of the numbers when two dice are rolled such that $2^N < N$!

$\Rightarrow 4 \leq N \leq 12$

Probability that $2^N \geq N$!

Now $P(N=2)+P(N=3)=\frac{1}{36}+\frac{2}{36}=\frac{3}{36}=\frac{1}{12}$

Required probability $=1-\frac{1}{12}=\frac{11}{12}=\frac{m}{n}\ 4m-3n=8$

2. **(1)**

Total No of 3-digit Number $=900$

Case -1 When there are 2 odd digit and 1 even digit

(i) Even odd odd $=4\times5\times5=100$

{As 0 cannot come at 1st place}

(ii) Odd even odd $=5\times5\times5=125$

(iii) Odd odd even $=5\times5\times5=125$

Case-2

All 3 odd digit $=5\times5\times5=125$

So total number of selecting 3-digit number having atleast 2 odd digit will be addition of both cases which is$=100+125+125+125=475$

So probability $=\frac{475}{900}=\frac{19}{36}$

3. **(2)**

We know $x_1<7<x_3<11<x_5$

$x_1 \in\{1,2,3,4,5,6\}$

$x_3 \in\{8,9,10\}$

$x_5 \in\{12,13,14,15,16,17,18\}$

Required probability $P=\frac{{}^6C_1\times{}^3C_1\times{}^7C_1}{{}^{18}C_5}=\frac{1}{68}$

4. **(3)**

$2b=a+c \Rightarrow$sum of a & c is even and b depends on a & c

Thus, we can not choose all three independently, but we can choose a & c

Moreover, a and c should be both either even or odd as their sum is even.

Out of 11 consecutive natural numbers, either five are even and six are odd or six are even and five are odd.

Hence, $P=\frac{1}{2}\frac{{}^5C_2+{}^6C_2}{{}^{11}C_3}+\frac{1}{2}\frac{{}^6C_2+{}^5C_2}{{}^{11}C_3}=\frac{5}{33}$

5. **(1)**

Total sample space of two dice will be equal to $6\times6=36$

Now for $x^2+\alpha x+\beta>0$ its discriminant D must be less than zero,

So $D=b^2-4ac<0$ or $\alpha^2-4\beta<0 \Rightarrow \alpha^2<4\beta$

Now, if $\alpha=1, \beta$ can take values $1,2,3,4,5,6$

If $\alpha=2, \beta$ can take values $2,3,4,5,6$

If $\alpha=3, \beta$ can take values $3,4,5,6$

If $\alpha=4, \beta$ can take values $5,6$

If $\alpha=5$ and 6, no value of β possible

So number of favourable ways $=17$

Required probability$=\frac{\text{favourable ways}}{\text{total sample space}}=\frac{17}{36}$

6. **(2)**

We have,

EXAMINATION

AAEIIMNNOTX

Given word has A's $\rightarrow 2$, I's $\rightarrow 2$, N's $\rightarrow 2$ and rest other are distinct alphabets.

Total number of ways to form words using the alphabets of EXAMINATION is

$=\frac{11!}{2!\cdot 2!\cdot 2!}$

Now, putting M at 4^{th} position

$\_ \ \_ \ \_ \ M \ \_ \ \_ \ \_ \ \_$

Total words with M at fourth Place $=\frac{10!}{2!\,2!\,2!}$

Required probability $=\frac{10!}{11!}=\frac{1}{11}$

7. **(4)** $\mathbf{M}\begin{bmatrix}\mathbf{a} & \mathbf{b}\\ \mathbf{c} & \mathrm{d}\end{bmatrix}$, where $a, b, c, d, \in\{0,1,2\}$

$n(s)=3^4=81$

we first bound $\mathbf{p}(\overline{\mathbf{A}})$

$|\mathrm{m}|=0 \Rightarrow \mathrm{ad}=\mathrm{bc}$

$\mathrm{ad}=\mathrm{bc}=0 \Rightarrow$ no. of $(a, b, c, d)=\left(3^2-2^2\right)^2=25$

$\mathrm{ad}=\mathrm{bc}=1 \Rightarrow$ no. of $(a, b, c, d)=1^2=1$

$\mathrm{ad}=\mathrm{bc}=2 \Rightarrow$ no. of $(a, b, c, d)=2^2=4$

$\mathrm{ad}=\mathrm{bc}=4 \Rightarrow$ no. of $(a, b, c, d)=1^2=1$

$: \mathrm{P}(\overline{\mathrm{A}})=\frac{31}{81} \Rightarrow \mathrm{p}(\mathrm{A})=\frac{50}{81}$

8. **(2)**

Given,

Sum of two integer is 66, so one number will be x and other will be $66-x$,

And given M is maximum value of their product,

So let $y=x(66-x)$

$\Rightarrow y=66x-x^2$

Now differentiating to find maxima and minima we get,

$\frac{dy}{dx}=66-2x$

Now equating with zero to find point of maxima as $y=66x-x^2$ represents a downward parabola so it will give maxima,

So $\frac{dy}{dx}=0 \Rightarrow 66-2x=0 \Rightarrow x=33$,

Hence, the value of $M=33\times 33=1089$

Now solving $x(66-x) \geq \frac{5M}{9}$

$\Rightarrow x(66-x) \geq \frac{5\times 1089}{9}$

$\Rightarrow x(66-x) \geq 605$

$\Rightarrow x^2-66x+605 \leq 0$

$\Rightarrow(x-11)(x-55) \leq 0$

So $x \in[11,55] \rightarrow$ total 45 numbers,

Now for probability of A, favourable outcomes will be $x=3k \Rightarrow x=\{12,15,18,\ldots\ldots 54\} \rightarrow$ total 15 numbers,

So probability will be $\frac{15}{45}=\frac{1}{3}$

9. (4)

Given,

$S = \{1, 2, 3, \ldots, 2022\}$

So, total number of elements$=2022$

Now factors of $2022 = 2 \times 3 \times 337$

Now HCF$(n, 2022) = 1$ is feasible only when the value of 'n' and 2022 has no common factor.

Now let $A =$Number which are divisible by 2 from $\{1, 2, 3, \ldots 2022\}$

So, $n(A) = 1011$

Now let $B =$Number which are divisible by 3by 3 from $(1, 2, 3, \ldots 2022)$

So, $n(B) = 674$

Now $A \cap B =$Number which are divisible by 6

Now from $\{1, 2, 3, \ldots 2022\}$ number multiple of 6 are $6, 12, 18, \ldots .2022$

So, $n(A \cap B) = 337$

Now applying the formula we get, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 1011 + 674 - 337$

$= 1348$

Now let $C =$Number which divisible by 337 from $\{1, \ldots .. 1022\}$

C = {337,674,1011,1348,1658,20222}

Already counted in Set(A∪B) Already counted in Set(A∪B) Already counted in Set(A∪B)

Total elements which are divisible by 2 or 3 or $337 = 1348 + 2 = 1350$

Favourable cases$=$elements which are neither divisible by $2, 3$ or 337

$= 2022 - 1350 = 672$

Required probability$= \frac{672}{2022} = \frac{112}{337}$

10. (4)

Given:$S = \{1, 2, 3, 4, 5, 6\}$

Total number of onto functions from S tto $S = 6!$

Now, for $g(3) = 2g(1)$:

$g(3)$	$g(1)$
2	1
4	2
6	3

$\therefore\ g(3) = \ 2g(1)$ can be defined in 3 ways.

$g(2), g(4), g(5)$ & $g(6)$ can be anything and can be defined in 4! ways.

$\therefore$Number of onto functions for which $[g(3) = 2\ g(1)] = 3.\ 4!$

Now required probability$= \frac{3.4!}{6!} = \frac{3\times4!}{30\times4!} = \frac{1}{10}$

11. (4)

$P(\text{target is hit}) = 1 - P(\text{no one hit the target})$

$= 1 - \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{4}\right) \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{8}\right)$

$= 1 - \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{7}{8}$

$= 1 - \frac{7}{32} = \frac{25}{32}$.

Thus, the required probability is $\frac{25}{32}$.

12. (11)

let probability of hitting the target $= \mathrm{p} \Rightarrow \mathrm{p} = \frac{1}{2}$

Let n be the minimum number of bombs

According to given condition

$1 - \left({}^{n}C_0 P^0 (1-P)^n + {}^{n}C_1 P^1 (1-P)^{n-1}\right) \geq \frac{99}{100}$

$\Rightarrow\ \ 2^n \geq (n+1)100$

$n = 10\ \ \Rightarrow\ \ 2^{10} \geq 1100$ Reject

$n = 11\ \ \Rightarrow\ \ 2^{11} \geq 1200$ Select

13. (1)

Given, A: No. on 1^{st} die < No. on 2^{nd} die

$A=\{(1,2),(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6),(4,5),(4,6),(5,6)\}$

$\therefore\ n(A)=15$

B: No. on 1^{st} die $=$even & No. of 2^{nd} die $=$ odd

$B=\{(2,1),(2,3),(2,5),(4,1),(4,3),(4,5),\ (6,1),(6,3),(6,5)\}$

$\therefore\ n(B)=9$

C : No. on 1^{st} die $=$odd & No. on 2^{nd} die $=$even

$C=\{(1,2),(1,4),(1,6),(1,2),(1,4),(1,6),(5,2),(5,4),(5,6)\}$

$\therefore\ n(C)=9$

Now,

$n(A\cap B)=3,\ n(A\cap C)=6,\ n(B\cap C)=0$

$n(A\cap B\cap C)=0$

Since $(4,5)\in A$ and $(4,5)\in B$

$\therefore A$ and B are not exclusive events

Now,

$\mathrm{n}((\mathrm{A}\cup\mathrm{B})\cap\mathrm{C})=n(\mathrm{A}\cap\mathrm{C})+n(\mathrm{B}\cap\mathrm{C})-n(A\cap B\cap C)$

$=6$

Since, $P\left(B\right)=\frac{9}{36},\ P\left(C\right)=\frac{9}{36},\ P\left(B\cap C\right)=0$

$\Rightarrow P(B).P(C)\neq P(B\cap C)$

$\therefore B$ and C are not independent

14. (4)

Given the probability that the digit 0 appear at the even place is $\frac{1}{2}$, hence the probability that the digit 1 appear at the even place is $1-\frac{1}{2}=\frac{1}{2}$.

And the probability that the digit 0 appear at the odd place is $\frac{1}{3}$, hence the probability that the digit 1 appear at the odd place is $1-\frac{1}{3}=\frac{2}{3}$.

For the digits '10' and '01' we have the following possibilities

$\underset{odd\ place}{1}\quad\underset{even\ place}{0}\quad\underset{odd\ place}{0}\quad\underset{even\ place}{1}$

Or

$\underset{even\ place}{1}\quad\underset{odd\ place}{0}\quad\underset{even\ place}{0}\quad\underset{odd\ place}{1}$

Thus, the required probability is $\left(\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}\right)+\left(\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\right)$

$=\frac{1}{18}+\frac{1}{18}=\frac{1}{9}$.

15. (2)

Given,

$P(A)=\frac{1}{3},P(B)=\frac{1}{5}$ and $P(A\cup B)=\frac{1}{2}$

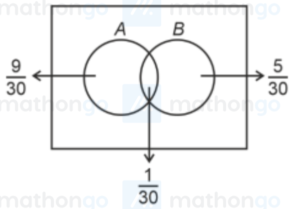
We know that,

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cap B)=\frac{1}{3}+\frac{1}{5}-\frac{1}{2}=\frac{1}{30}$

Now $P(A\cap B')=$ Only A $=\frac{1}{3}-\frac{1}{30}=\frac{9}{30}$

And $P(B\cap A')=$ Only B $=\frac{1}{5}-\frac{1}{30}=\frac{5}{30}$



Now, $P(A\mid B')+P(B\mid A')=\frac{P(A\cap B')}{P(B')}+\frac{P(B\cap A')}{P(A')}$

$=\frac{\frac{9}{30}}{\frac{4}{5}}+\frac{\frac{5}{30}}{\frac{2}{3}}=\frac{5}{8}$

16. (2)

The unbiased die is thrown five times such that the product of the outcomes is positive, then either all outcomes are positive or any two are negative.

Either all outcomes are positive or any two are negative.

Now,

$p=p\,(\text{positive})=\frac{3}{6}=\frac{1}{2}$

$q=p\,(\text{negative})=\frac{2}{6}=\frac{1}{3}$

Required probability

$={}^{5}\mathrm{C}_5\left(\frac{1}{2}\right)^5+{}^{5}\mathrm{C}_2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3+{}^{5}\mathrm{C}_4\left(\frac{1}{3}\right)^4\left(\frac{1}{2}\right)^1$

$=\frac{521}{2592}$

17. (479)

Given,

$A=\{1,2,3,4\}$: $P(A)=\frac{3}{4}$ $\rightarrow$Correct answer

$B=\{5,6,7,8,9,10\}$; $P(B)=\frac{1}{4}$ Correct answer

Now total ways in which 8 Correct answer is given will be,

$\Rightarrow$ Case (i) $(4,4)$: ${}^{4}C_4\left(\frac{3}{4}\right)^4\cdot{}^{6}C_4\cdot\left(\frac{1}{4}\right)^4\cdot\left(\frac{3}{4}\right)^2$

$\Rightarrow$ Case (ii) $(3,5)$: ${}^{4}C_3\left(\frac{3}{4}\right)^3\cdot\left(\frac{1}{4}\right)^1\cdot{}^{6}C_5\left(\frac{1}{4}\right)^5\cdot\left(\frac{3}{4}\right)$

$\Rightarrow$ Case (iii) $(2,6)$: ${}^{4}C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2\cdot{}^{6}C_6\left(\frac{1}{4}\right)^6$

Total probability $=\frac{1}{4^{10}}\left[3^4\times15\times3^2+4\times3^3\times6\times3+6\times3^2\right]$

$=\frac{27}{4^{10}}[2.7\times15+72+2]$

Now comparing with $\frac{27k}{4^{10}}$

We get, $K=479$

18. (2)

$P(B_1)=\frac{1}{2}=P(B_2)$

Box B_1 have 20 non-prime numbers $\{1,\ 4,\ 6,\ 8,\ 9,\ 10,\ 12,\ 14,\ 15,\ 16,\ 18,\ 20,\ 21,\ 22,\ 24,\ 25,\ 26,\ 27,\ 28,\ 30\}$

$\Rightarrow P(NP/B_1)=\frac{20}{30}$

Box B_1 have 15 non-prime numbers $\{32,\ 33,\ 34,\ 35,\ 36,\ 38,\ 39,\ 40,\ 42,\ 44,\ 45,\ 46,\ 48,\ 49,\ 50\}$

$\Rightarrow P(NP/B_2)=\frac{15}{20}$

Probability of non-prime$=P(NP)$.

$P(NP)=P(B_1).\,P(NP/B_1)+P(B_2).\,P(NP/B_2)$

$=\frac{1}{2}\cdot\frac{20}{30}+\frac{1}{2}\cdot\frac{15}{20}$

$P(B_1/NP)=\frac{\frac{1}{2}\cdot\frac{20}{30}}{\frac{1}{2}\cdot\frac{20}{30}+\frac{1}{2}\cdot\frac{15}{20}}=\frac{8}{17}$

19. (1)

Let $X\equiv$Event that product is defective, then

$P\left(\frac{X}{A}\right)=\frac{3}{100}$

$P\left(\frac{X}{B}\right)=\frac{4}{100}$

$P\left(\frac{X}{C}\right)=\frac{2}{100}$

Now,

$P\left(\frac{C}{X}\right)=\frac{P(C)P\left(\frac{X}{C}\right)}{P(A)P\left(\frac{X}{A}\right)+P(B)P\left(\frac{X}{B}\right)+P(C)P\left(\frac{X}{C}\right)}$

$\Rightarrow P\left(\frac{C}{X}\right)=\frac{\frac{50}{100}\times\frac{2}{100}}{\frac{20}{100}\times\frac{3}{100}+\frac{30}{100}\times\frac{4}{100}+\frac{50}{100}\times\frac{2}{100}}$

$\Rightarrow P\left(\frac{C}{X}\right)=\frac{100}{60+120+100}$

$\Rightarrow P\left(\frac{C}{X}\right)=\frac{5}{14}$

Hence this is the correct option.

20. (1)

Let the bag contains black balls(B) and non-black balls $(\overline{B})$, then we can have the following possibilities:

$\{(6B, 0\overline{B}),\ (5B, 1\overline{B}),\ (4B, 2\overline{B}),\ (3B, 3\overline{B}),$
$(2B, 4\overline{B})\}$

All the events are equally likely, i.e., $\frac{1}{5}$.

Required probability

$$= \frac{\frac{1}{5}\times\frac{{}^6C_2}{{}^6C_2}+\frac{1}{5}\times\frac{{}^5C_2}{{}^6C_2}}{\frac{1}{5}\times\frac{{}^6C_2}{{}^6C_2}+\frac{1}{5}\times\frac{{}^5C_2}{{}^6C_2}+\frac{1}{5}\times\frac{{}^4C_2}{{}^6C_2}+\frac{1}{5}\times\frac{{}^3C_2}{{}^6C_2}+\frac{1}{5}\times\frac{{}^2C_2}{{}^6C_2}}$$

$$= \frac{25}{35} = \frac{5}{7}$$

21. (3)

Let E_1 event denotes the selection for 1^{st} bag

and E_2 event denotes the selection for 2^{nd} bag

Given $P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}$

Let the event $A =$ selected balls are 1 red & 1 black

$$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_1\times{}^1C_1}{{}^6C_2} = \frac{1}{5}$$

$$P\left(\frac{A}{E_2}\right) = \frac{{}^3C_1\times{}^2C_1}{{}^{(n+5)}C_2} = \frac{12}{(n+5)(n+4)}$$

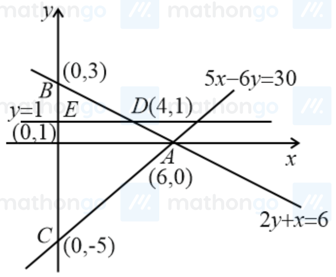
$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\times P\left(\frac{A}{E_1}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right)+P(E_2)\times P\left(\frac{A}{E_2}\right)}$$

$$\Rightarrow \frac{\frac{1}{2}\times\frac{1}{5}}{\frac{1}{2}\times\frac{1}{5}+\frac{6}{(n+5)(n+4)}} = \frac{6}{11}$$

$$\Rightarrow n = 4$$

22. (2)

Plotting the diagram of the given straight line we get,



Now by diagram, required probability $= \frac{\text{ar}(ADEC)}{\text{ar}(ABC)}$

$$= 1 - \frac{\text{ar}(BDE)}{\text{ar}(ABC)}$$

$$= 1 - \frac{\frac{1}{2}\times2\times4}{\frac{1}{2}\times8\times6} = 1 - \frac{1}{6} = \frac{5}{6}$$

23. **(10)**

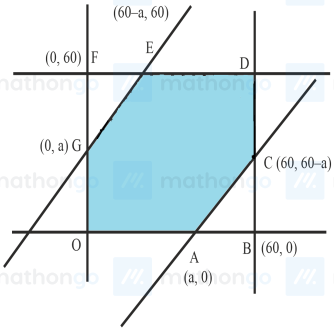
Given,

A be the event that the absolute difference between two randomly chosen real numbers in the sample space $[0,60]$ is less than or equal to a,

So, $|x-y|<a \Rightarrow -a<x-y<a$

$\Rightarrow \quad x-y<a$ and $x-y>-a$

Now on plotting the diagram we get,



Now from diagram probability is given by,

$\mathrm{P(A)}=\frac{\text{area(OACDEG)}}{\text{area(OBDF)}}$

$\Rightarrow \mathrm{P(A)}=\frac{\text{area(OBDF)}-\text{area(ABC)}-\text{area(EFG)}}{\text{area(OBDF)}}$

$\Rightarrow \quad \frac{11}{36}=\frac{(60)^2-\frac{1}{2}(60-a)^2-\frac{1}{2}(60-a)^2}{3600}$

$\Rightarrow \quad 1100=3600-(60-a)^2$

$\Rightarrow (60-a)^2=2500 \Rightarrow 60-a=50$

$\Rightarrow \quad a=10$

24. **(1)**

Given,

The probability that the random variable X takes values x is given by $P(X=x)=k(x+1)3^{-x}$, $x=0,1,2,3,\ldots\ldots$, where k is a constant,

Now we know that,

$P(X=0)+P(x=1)+P(x=2)+\ldots\ldots\ldots=1$

$\Rightarrow \frac{k}{3^0}+\frac{2k}{3^1}+\frac{3k}{3^2}+\ldots\ldots=1$

$\Rightarrow k\left(1+\frac{2}{3}+\frac{3}{3^2}+\ldots\right)=1$

Now finding,

$S=1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+\ldots \quad \ldots(1)$

$\Rightarrow \frac{S}{3}=\frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^2}+\ldots\ldots \quad \ldots\ldots(2)$

Now on subtracting above two equation we get,

$\Rightarrow \frac{2S}{3}=1+\frac{1}{3}+\frac{1}{3^2}+\ldots\ldots$

$\Rightarrow S=\frac{9}{4}$

Hence, $k\left(1+\frac{2}{3}+\frac{3}{3^2}+\ldots\right)=1$

$\Rightarrow k=\frac{4}{9}$

Now finding,

$P(X\geq 2)=P(2)+P(3)+\ldots\ldots$

$\Rightarrow P(X\geq 2)=1-P(0)-P(1)$

$\Rightarrow P\left(X\geq 2\right)=1-\left(\frac{k}{1}+\frac{2k}{3}\right)=1-\frac{20}{27}=\frac{7}{27}$

25. (1)

There rotten apples are mixed accidently with seven good apples,

And four apples are drawn one by one without replacement.

Now the random variable X denote the number of rotten apples.

And μ and σ^2 represent mean and variance of X, respectively,

So, now plotting the table of given data we get,

x	$P(x)$	$xP(X)$	$x^2P(X)$
0	$\frac{1}{6}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{12}{10}$
3	$\frac{1}{30}$	$\frac{1}{10}$	$\frac{9}{30}$

Now mean is given by, $\sum xP\left(x\right) = \frac{6}{2} = \mu$

And variance is given by $\sigma^2 = \sum x^2P(x) - \mu^2$

So, by putting the value from above table we get, $\sigma^2 + \mu^2 = 0 + \frac{1}{2} + \frac{12}{10} + \frac{9}{30} = 2$

Hence, $10\left(\sigma^2 + \mu^2\right) = 20$

26. (1)

To find $P\left(\frac{1<x<4}{x\leq 2}\right)$ or $P\left(\frac{A}{B}\right)$

We know that

$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$

Given

x	0	1	2	3	4
$P(x)$	k	$2k$	$4k$	$6k$	$8k$

$P(A)=\{2,3\}$

$P(B)=\{0,1,2\}$

$P(A\cap B) = P(x=2)$

$P(B) = P(x=0)+P(x=1)+P(x=2)$

So $P\left(\frac{A}{B}\right) = \frac{P(x=2)}{P(x=0)+P(x=1)+P(x=2)}$

$= \frac{4k}{k+2k+4k} = \frac{4k}{7k} = \frac{4}{7}$

Hence option A is correct.

27. (11)

There are only two outcomes, success which stands for getting either 3 or 5, and failure, stands for getting any other outcome.

Probability for success$= p = \frac{2}{6}$

Probability for failure$= q = \frac{4}{6}$

Number of trials$= n = 27$

Number of dice$= 4$

Using Binomial Probability,

P(at least 2 shows 3 or 5) $= {}^4C_2 \cdot \left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right) + {}^4C_4\left(\frac{2}{6}\right)^4$

$= \frac{384+128+16}{6^4} = \frac{11}{27}$

Expectation is numerically same as mean.

$\therefore$ expectation$= np$

$= 27 \cdot \frac{11}{27} = 11$

28. (1) win Rs. 15 $\rightarrow$ number of cases $= 6$

win Rs. 12 $\rightarrow$ number of cases $= 4$

loss Rs. 6 $\rightarrow$ number of cases $= 26$

p (expected gain/loss) $= 15 \times \frac{6}{36} + 12 \times \frac{4}{36} - 6 \times \frac{26}{36} = -\frac{1}{2}$

29. (3)

α-β	Case	P
5	(6, 1)	1/36
4	(6, 2) (5, 1)	2/36
3	(6, 3) (5, 2) (4, 1)	3/36
2	(6, 4) (5, 3) (4, 3) (3, 1)	4/36
1	(6, 5) (5, 4) (4, 3) (3, 2) (2, 1)	5/36
0	(6, 6) (5, 5) …… (1, 1)	6/36
-1	-----	5/36
-2	-----	4/36
-3	-----	3/36
-4	(2, 6) (1, 5)	2/36
-5	(1, 6)	1/36

$$\sum\left(x^2\right)=\sum x^2 P(x)=2\left[\frac{25}{36}+\frac{32}{36}+\frac{27}{36}+\frac{16}{36}+\frac{5}{36}\right]$$

$=\dfrac{105}{18}=\dfrac{35}{6}$

$\mu=\sum(x)=0$ as data is symmetric

$\sigma^2=\sum\left(x^2\right)=\sum x^2 P(x)=\frac{35}{6} \quad P=35=5\times 7$

Sum of divisors $=\left(5^0+5^1\right)\left(7^0+7^1\right)=6\times 8=48$

30. (19)

Given,

$P(A)\geq\frac{4}{5}$

A is subset of S hence

A can have elements;

type 1:{}

type 2: $\{E_1\},\{E_2\},\ldots\ldots\{E_8\}$

type 3: $\{E_1,E_2\},\{E_1,E_3\}\ldots\ldots\{E_1,E_8\}$

$\vdots$

$\vdots$

type 6: $\{E_1,E_2,\ldots\ldots\ldots E_5\},\ldots\ldots\{E_4,E_5,E_6,E_7,E_8\}$

type 7: $\{E_1,E_2,\ldots\ldots\ldots E_6\},\ldots\ldots\{E_3,E_4,\ldots\ldots\ldots\ldots E_8\}$

type 8: $\{E_1,E_2,\ldots\ldots\ldots E_7\}\{E_2,E_3,\ldots\ldots\ldots E_8\}$

type 9: $\{E_1,E_2,\ldots\ldots E_8\}$

As $P(A)\geq\frac{4}{5}$;

Note: Type 1 to Type 4 elements can not be in set A as maximum probability of type 4 elements.

$\{E_5,E_6,E_7,E_8\}$ is $\frac{5}{36}+\frac{6}{36}+\frac{7}{36}+\frac{8}{36}=\frac{13}{18}<\frac{4}{5}$

Now for Type 5 acceptable elements let's call probability as P_5

$P_5=\frac{n_1+n_2+n_3+n_4+n_5}{36}\geq\frac{4}{5}$

$\Rightarrow n_1+n_2+n_3+n_4+n_3\geq 28.8$

Hence, 2 possible ways $\{E_5,E_6,E_7,E_8,E_3 \text{ or } E_4\}$

$P_6=n_1+n_1+n_3+n_4+n_3+n_6\geq 28.8$

$\Rightarrow$ 9 possible ways

$P_7\Rightarrow n_1+n_1+\ldots\ldots\ldots+n_7\geq 28.8$

$\Rightarrow$ 7 possible ways

$P_8\Rightarrow n_1+n_1+\ldots\ldots+n_n\geq 28.8$

$\Rightarrow$ 1 possible way

Total $=19$