ANSWER KEYS

1. (4)	**2.** (3)	**3.** (1)	**4.** (39)	**5.** (3)	**6.** (2)	**7.** (2)	**8.** (3)
9. (4)	**10.** (3)						

1. **(4)** $S_1 \equiv 3,\ 7,\ 11,\ 15, \ldots\ldots \Rightarrow\ c.d = 4 \Rightarrow d_1 = 4$

$S_2 \equiv 1,\ 6,\ 11,\ 16, \ldots\ldots \Rightarrow\ c.d = 5 \Rightarrow d_2 = 5$

$\Rightarrow$Every 5^{th} term of S_1 and 4^{th} term of S_2 will be same

$\Rightarrow$ Terms common to both AP will have $a = 11$ and $d = 20$

Hence, $S_{20} = \frac{20}{2}[(2 \times 11) + (20 - 1)\,20]$

$= 10 \times 402$

$= 4020$

2. **(3)** Given, AP is $3,\ a_1, a_2, a_3, a_4, a_5, a_6, 31$

$\therefore\ 31 = 3 + 7d$

$\Rightarrow d = 4$

$\therefore\ a_1 = 3 + 4 = 7$

$a_5 = a + 5d = 3 + 20 = 23$

and $a_6 = a + 6d = 3 + 24 = 27$

$\therefore\ a_6 - a_5 = 27 - 23 = 4$

and $a_1 + a_6 = 7 + 27 = 34$

3. **(1)** We know that geometric mean of numbers $a_1, a_2 \ldots\ldots a_n\ = (a_1 . a_2 \ldots\ldots a_n)^{\frac{1}{n}}$

Hence, Geometric mean $= \left(7.7^2.7^3 \ldots 7^n\right)^{1/n}$

$= \left(7^{1+2+3+\ldots+n}\right)^{1/n}$

$= \left(7^{\frac{n(n+1)}{2}}\right)^{1/n} = 7^{\left(\frac{n+1}{2}\right)}$

4. **(39)**

Let $3,\ A_1,\ A_2,\ A_3, \ldots\ldots A_m,\ 243$ are in arithmetic progression with m arithmetic means.

Common difference $d = \frac{243-3}{m+1} = \frac{240}{m+1}$

Let $3,\ G_1,\ G_2,\ G_3,\ 243$ are in geometric progression with 3 geometric means.

Common ratio $r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{\frac{1}{4}} = 3$

Given $G_2 = A_4$

$\Rightarrow 3(3)^2 = 3 + 4\left(\frac{240}{m+1}\right)$

$\Rightarrow 27 = 3 + \frac{960}{m+1}$

$\Rightarrow m + 1 = 40$

$\Rightarrow m = 39$

5. **(3)**

$\log_2 6 = \log_2(3 \times 2) = \log_2 3 + \log_2 2 = 1 + \log_2 3$

and $\log_2 12 = \log_2\left(2^2 \times 3\right)$

$= \log_2 3 + 2\log_2 2$

$= 2 + \log_2 3$

Since, $\log_2 3, 1 + \log_2 3,\ 2 + \log_2 3$, $(\because\ 2(1 + \log_2 3) = \log_2 3 + 2 + \log_2 3)$ are in AP.

$\Rightarrow \log_2 3,\ \log_2 6,\ \log_2 12$ are in AP.

Taking reciprocals,

$\Rightarrow \log_3 2, \log_6 2, \log_{12} 2$ are in HP.

6. **(2)** Let the first term of an AP be a and common difference be d.

Since, $a+3d=\frac{5}{3}.....(i)$

and $a+7d=3......(ii)$

On solving an equation (i) and (ii), we get

$a=\frac{2}{3}\ ,\quad d=\frac{1}{3}$

$\therefore T_6=a+5d=\frac{2}{3}+\frac{5}{3}=\frac{7}{3}$

$\Rightarrow$ 6th term of HP is $\frac{3}{7}$.

7. **(2)** AM $\geq$ GM for positive numbers. So,

$$\frac{4^x+\frac{4}{4^x}}{2}\geq\sqrt{4^x\cdot\frac{4}{4^x}}=2.$$

$4^x+\frac{4}{4^x}\geq 4$

8. **(3)**

AM $\geq$ GM

$\frac{a+b+b+c+c+c}{6}\geq\sqrt[6]{ab^2c^3}$

$\Rightarrow\frac{a+2b+3c}{6}\geq\sqrt[6]{ab^2c^3}$

$\Rightarrow\ ab^2c^3\leq 2^6$.

9. **(4)** We have,

$5\ +\ 55\ +\ 555\ +\ ....$ to n terms

$=5[1\ +\ 11\ +\ 111\ +\ ....$ to n terms$]$

$=\frac{5}{9}[9+99+999+\ldots n\text{ terms}]$

$=\frac{5}{9}\left[(10-1)+(10^2-1)+(10^3-1)+\ldots+(10^n-1)\right]$

$=\frac{5}{9}\left[\left(10+10^2+10^3+\ldots+10^n\right)-(1+1+1+\ldots n\text{ times})\right]$

$=\frac{5}{9}\left[10\times\frac{(10^n-1)}{10-1}-n\right]$

$=\frac{5}{9}\left[\frac{10}{9}(10^n-1)-n\right]$

$=\frac{5}{81}\left[10^{n+1}-10-9n\right]$

$S_{100}=\frac{5}{81}\left[10^{101}-910\right]$

10. **(3)**

$t_r=\frac{1}{r}(1\ +\ 2\ +\ 3\ .....\ \ +\ r)=\ \frac{1}{r}\ .\ \frac{1}{2}r\ (r\ +\ 1)$

$S\ =\Sigma t_r=\ \frac{1}{2}\sum_{r=1}^{n}(r+1)$

$S=\frac{1}{2}\sum_{r=1}^{20}r+\frac{1}{2}\sum_{r=1}^{20}1$

$S=\frac{1}{4}.20(20+1)+\frac{1}{2}.20$

$S=115$