ANSWER KEYS

1. (3)	**2.** (4)	**3.** (4)	**4.** (1)	**5.** (235)	**6.** (3)	**7.** (2)	**8.** (0.2)
9. (2)	**10.** (3)						

1. **(3)**

When two dice are thrown, we get the sum of the points in $2 \leq S \leq 12$, where $S \in N$.

The event A : Consists of four cases.

(i) Sum is $9 = \{(3,6),(6,3),(4,5),(5,4)\}$

(ii) Sum is $10 = \{(4,6),(6,4),(5,5)\}$

(iii) Sum is $11 = \{(6,5),(5,6)\}$

(iv) Sum is $12 = \{(6,6)\}$

The event B : Two occurs on either die.

$= \{(2,1),(1,2),(2,2),(2,3),(3,2),(2,4),(4,2),(2,5),(5,2),(2,6),(6,2)\}$

The event C : Consists of two cases.

(i) Sum is $9 = \{(3,6),(6,3),(4,5),(5,4)\}$

(ii) Sum is $12 = \{(6,6)\}$.

Hence, the set A and the set B are mutually exclusive because, there is no common elements between them.

$\Rightarrow A \cap B = \emptyset$.

2. **(4)** Let coin is tossed n times

P (at least are head) $= 1 - P$ (no head)

$= 1 - \left(\frac{1}{2}\right)^n \geq 0.99 \Rightarrow \left(\frac{1}{2}\right)^n \leq \frac{1}{100}$

Clearly minimum value of n is 7 .

3. **(4)**

$P\left(A\right) = \frac{3x+1}{3},\ P\left(B\right) = \frac{1-x}{4}$

$P\left(C\right) = \frac{1-2x}{2}$

$\therefore$ For any event $E, 0 \leq P(E) \leq 1$

$\Rightarrow 0 \leq \frac{3x+1}{3} \leq 1,\ \ 0 \leq \frac{1-x}{4} \leq 1$ and $0 \leq \frac{1-2x}{2} \leq 1$

$\Rightarrow -1 \leq 3x \leq 2, -3 \leq x \leq 1$ and $-1 \leq 2x \leq 1$

$\Rightarrow -\frac{1}{3} \leq x \leq \frac{2}{3} \leq -3 \leq x \leq 1$ and $-\frac{1}{2} \leq x \leq \frac{1}{2}$

Also for mutually exclusive events A, B, C

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$\Rightarrow P\left(A \cup B \cup C\right) = \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2}$

$\therefore 0 \leq \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \leq 1$

$0 \leq 13 - 3x \leq 12 \Rightarrow 1 \leq 3x \leq 13$

$\Rightarrow \frac{1}{3} \leq x \leq \frac{13}{3}$

Considering all inequations, we get

$\max\left\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\right\} \leq x \leq \min\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\}$

$\frac{1}{3} \leq x \leq \frac{1}{2} \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2}\right]$

4. **(1)**

$P\left(\bar{A} \cap \left(B \cap \bar{C}\right)\right\} = P\left(B \cap \bar{C}\right) - P\left(A \cap B \cap \bar{C}\right)$

$= P(B) - P(B \cap C) - P\left(A \cap B \cap \bar{C}\right)$

$\Rightarrow -P\left(\bar{A} \cap B \cap \bar{C}\right) - P\left(A \cap B \cap \bar{C}\right) + P(B) = P(B \cap C)$

$\Rightarrow P(B \cap C) = \frac{3}{4} - \frac{1}{3} - \frac{1}{3} = \frac{1}{12}$

5. **(235)**

P(problem solved by atleast one) $= 1 - P$(problem is not solved by all)

$= 1 - P(\overline{A})P(\overline{B})P(\overline{C})P(\overline{D})$

$= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{7}{8}\right) = 1 - \frac{21}{256} = \frac{235}{256}$

$256P = 235$

6. **(3)**

Let, A and B are two independent events

$P(A\cap B)=P(A)\cdot P(B)=\frac{8}{49}\ \ldots(1)$

$P(A\cap B^c)+P(B\cap A^c)=\frac{26}{49}$

$\Rightarrow P(A)+P(B)-2P(A)\cdot P(B)=\frac{26}{49}$

$\Rightarrow P(A)+P(B)=\frac{42}{49}=\frac{6}{7}\ \ldots(2)$

From (1) and (2), we get,

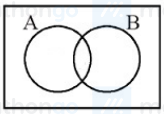
$P(A)+\frac{8}{49P(A)}=\frac{6}{7}\Rightarrow 49\{P(A)\}^2-42P(A)+8=0$

$\Rightarrow(7P(A)-2)(7P(A)-2)=0$

$P(A)=\frac{4}{7},\ P(B)=\frac{2}{7}$ or $P(A)=\frac{2}{7},\ P(B)=\frac{4}{7}$

Hence, the probability of the more probable event is $\lambda=\frac{4}{7}\Rightarrow 14\lambda=8$

7.



(2)

Given $P(A)=\frac{2}{5}=\frac{8}{20}\ \&\ P(A\cap B)=\frac{3}{20}$

$\therefore\ P(A'\cup B')=P(A\cap B)'$

$=1-P(A\cap B)$

$=1-\frac{3}{20}=\frac{17}{20}$

$P(A\cap(A'\cup B'))=P(A\cap B')$

$=P(A)-P(A\cap B)$

$=\frac{8}{20}-\frac{3}{20}=\frac{5}{20}$

$\therefore\ P(A|(A'\cup B'))=\frac{P(A\cap(A'\cup B'))}{P(A'\cup B')}$

$=\frac{\left(\frac{5}{20}\right)}{\left(\frac{17}{20}\right)}=\frac{5}{17}$

8. **(0.2)**

A_i : Event that number i appears on the die.

E : Event that only white balls are drawn.

$P(A_1)=P(A_2)=\ldots=P(A_6)=\frac{1}{6}$

$P(E|A_i)=\frac{^6C_i}{^{10}C_i},\ i=1,\ 2,\ldots,\ 6$

$\Rightarrow$Required Probability$=P(E)$

$=P\left(\bigcup_{i=1}^{6}(E\cap A_i)\right)$

$=\sum_{i=1}^{6}P(E\cap A_i)$

$=\sum_{i=1}^{6}P(A_i)P(E|A_i)$

$=\frac{1}{6}\left[\frac{6}{10}+\frac{15}{45}+\frac{20}{120}+\frac{15}{210}+\frac{6}{252}+\frac{1}{210}\right]$

$=\frac{1}{6}\left[\frac{3}{5}+\frac{1}{3}+\frac{1}{6}+\frac{1}{14}+\frac{1}{42}+\frac{1}{210}\right]$

$=\frac{1}{6}\left[\frac{33}{30}+\frac{4}{42}+\frac{1}{210}\right]$

$=\frac{1}{6}\left[\frac{11}{10}+\frac{2}{21}+\frac{1}{210}\right]$

$=\frac{1}{6}\left[\frac{(11)(21)+20+1}{210}\right]$

$=\frac{1}{6}\left[\frac{(21)(12)}{210}\right]=0.2$

9. **(2)**

Req. Probability

$=\frac{\frac{^{48}C_2}{^{52}C_2}\times\frac{4}{50}}{\frac{^{48}C_2}{^{52}C_2}\times\frac{4}{50}+\frac{^{48}C_1\,^4C_1}{^{52}C_2}\times\frac{3}{50}+\frac{^4C_2}{^{52}C_2}\times\frac{2}{50}}$

$=\frac{4.^{48}C_2}{4.^{48}C_2+12^{48}C_1+12}$

10. (3)

We have,

$P(\text{spade}) = p = \frac{13}{52} = \frac{1}{4}$

$P(\text{non-spade}) = q = 1 - \frac{1}{4} = \frac{3}{4}$

Now,

$P(A \text{ win}) = \frac{1}{4} + \left(\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}\right) + \left(\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}\right) + \ldots \infty$

$\Rightarrow P(A \text{ win}) = \frac{\frac{1}{4}}{1 - \frac{9}{16}} = \frac{4}{7}$

Hence,

$P(B \text{ win}) = 1 - \frac{4}{7} = \frac{3}{7}$