1. The value of $\tan^{-1}\left[\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right]$ is equal to

(1) $-\frac{\pi}{4}$
(2) $-\frac{\pi}{8}$
(3) $-\frac{5\pi}{12}$
(4) $-\frac{4\pi}{9}$

2. The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{4}$ is:

(1) 1
(2) 2
(3) 4
(4) 0

3. $\cos^{-1}(\cos(-5))+\sin^{-1}(\sin(6))-\tan^{-1}(\tan(12))$ is equal to :
(The inverse trigonometric functions take the principal values)

(1) $3\pi+1$
(2) $3\pi-11$
(3) $4\pi-11$
(4) $4\pi-9$

4. Let $(a,b)\subset(0,2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta)-\cos^{-1}(\sin\theta)>0, \theta\in(0,2\pi)$, holds . If $\alpha x^2+\beta x+\sin^{-1}(x^2-6x+10)+\cos^{-1}(x^2-6x+10)=0$ and $\alpha-\beta=b-a$, then α is equal to;

(1) $\frac{\pi}{8}$
(2) $\frac{\pi}{48}$
(3) $\frac{\pi}{16}$
(4) $\frac{\pi}{12}$

5. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}=$

(1) $\frac{\pi}{4}$
(2) $\frac{\pi}{2}$
(3) $\frac{\pi}{3}$
(4) $\frac{\pi}{6}$

6. Let $S=\left\{x\in R: 0<x<1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$. If $n(S)$ denotes the number of elements in S then :

(1) n(S) $=2$ and only one element in S is less then $\frac{1}{2}$
(2) n(S) $=1$ and the element in S is more than $\frac{1}{2}$
(3) n(S) $=1$ and the element in S is less then $\frac{1}{2}$
(4) n(S) $=0$

7. For $k\in\mathbb{R}$, let the solutions of the equation $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right)\right)\right)=k, 0<|x|<\frac{1}{\sqrt{2}}$ be α and β, where the inverse trigonometric functions take only principal values. If the solutions of the equation $x^2-bx-5=0$ are $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ and $\frac{\alpha}{\beta}$, then $\frac{b}{k^2}$ is equal to ______.

8. If $\left(\sin^{-1}x\right)^2-\left(\cos^{-1}x\right)^2=a;\ 0<x<1,\ a\neq 0$, then the value of $2x^2-1$ is

(1) $\cos\left(\frac{2a}{\pi}\right)$
(2) $\sin\left(\frac{2a}{\pi}\right)$
(3) $\cos\left(\frac{4a}{\pi}\right)$
(4) $\sin\left(\frac{4a}{\pi}\right)$

9. If $0<x<\frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1}x}{\alpha}=\frac{\cos^{-1}x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is

(1) $4\sqrt{(1-x^2)}(1-2x^2)$
(2) $4x\sqrt{(1-x^2)}(1-2x^2)$
(3) $2x\sqrt{(1-x^2)}(1-4x^2)$
(4) $4\sqrt{(1-x^2)}(1-4x^2)$

10. If $\sin^{-1}\frac{\alpha}{17}+\cos^{-1}\frac{4}{5}-\tan^{-1}\frac{77}{36}=0, 0<\alpha<13$, then $\sin^{-1}(\sin\alpha)+\cos^{-1}(\cos\alpha)$ is equal to

(1) π
(2) 16
(3) 0
(4) $16-5\pi$

11. $\tan\left(2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{\sqrt{5}}{2}+2\tan^{-1}\frac{1}{8}\right)$ is equal to:

(1) 1
(2) 2
(3) $\frac{1}{4}$
(4) $\frac{5}{4}$

12. If $\alpha=\cos^{-1}\left(\frac{3}{5}\right), \beta=\tan^{-1}\left(\frac{1}{3}\right)$, where $0<\alpha,\beta<\frac{\pi}{2}$, then $\alpha-\beta$ is equal to

(1) $\tan^{-1}\left(\frac{9}{14}\right)$
(2) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$
(3) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$
(4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

13. Let S be the set of all solutions of the equation $\cos^{-1}(2x)-2\cos^{-1}\left(\sqrt{1-x^2}\right)=\pi, x\in\left[-\frac{1}{2},\frac{1}{2}\right]$. Then $\sum_{x\in S}2\sin^{-1}(x^2-1)$ is equal to

(1) 0
(2) $\frac{-2\pi}{3}$
(3) $\pi-\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$
(4) $\pi-2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

14. The number of solutions of the equation $\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$ for $x\in[-1,1]$, and $[x]$ denotes the greatest integer less than or equal to x, is :

(1) 2
(2) 0
(3) 4
(4) Infinite

15. For $x\in(-1,1]$, the number of solutions of the equation $\sin^{-1}x=2\tan^{-1}x$ is equal to

16. Let $x=\sin\left(2\tan^{-1}\alpha\right)$ and $y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$. If $S=\{\alpha\in\mathbb{R}: y^2=1-x\}$, then $\sum_{\alpha\in S}16\alpha^3$ is equal to _______.

17. The value of $\lim_{n\to\infty} 6\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$ is equal to

(1) 1
(2) 2
(3) 3
(4) 6

18. Let $a_1 = 1,\ a_2,\ a_3,\ a_4, \ldots..$ be consecutive natural numbers. Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)+\ldots..+\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

(1) $\frac{\pi}{4}-\cot^{-1}(2022)$
(2) $\cot^{-1}(2022)-\frac{\pi}{4}$
(3) $\tan^{-1}(2022)-\frac{\pi}{4}$
(4) $\frac{\pi}{4}-\tan^{-1}(2022)$

19. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is:

(1) $\frac{21}{19}$
(2) $\frac{19}{21}$
(3) $\frac{23}{22}$
(4) $\frac{22}{23}$

20. If $\cot^{-1}(\alpha) = \cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\cot^{-1}32+\ldots.$ upto 100 terms, then α is:

(1) 1.01
(2) 1.00
(3) 1.02
(4) 1.03