ANSWER KEYS

1. (450)	**2.** (1)	**3.** (1)	**4.** (2)	**5.** (4)	**6.** (18)	**7.** (3)	**8.** (4)
9. (4)	**10.** (1)	**11.** (2)	**12.** (2)	**13.** (4)	**14.** (7)	**15.** (3)	**16.** (1)
17. (1)	**18.** (1)	**19.** (5)	**20.** (125)	**21.** (3)	**22.** (1)	**23.** (44)	**24.** (2)
25. (3)	**26.** (9)	**27.** (180)	**28.** (4)	**29.** (51)	**30.** (2)		

1. (450)

Given,

$P(-2,-1,1)$ and $Q\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$ be the vertices of the rhombus $PRQS$. If the direction ratios of the diagonal RS are $\alpha,-1,\beta$, where both α and β are integers of minimum absolute values, then $\alpha^2+\beta^2$ is equal to

Now $RS=(\alpha,-1,\beta)$

So, direction ratio of $PQ \equiv\left(\frac{56}{17}+2,\frac{43}{17}+1,\frac{111}{17}-1\right)$

$\equiv\left(\frac{90}{17},\frac{60}{17},\frac{94}{17}\right)$

Now we know that diagonal of rhombus are perpendicular,

So, $\frac{90}{17}\alpha+\frac{60}{17}(-1)+\frac{94}{17}\beta=0$

$\Rightarrow 90\alpha+94\beta=60$

$\Rightarrow \beta=\frac{60-90\alpha}{94}$

$\Rightarrow \beta=\frac{30(2-3\alpha)}{94}$

$\Rightarrow \beta=-30\frac{(3\alpha-2)}{94}$

$\Rightarrow \beta=\frac{-15}{47}(3\alpha-2)$

$\Rightarrow \frac{\beta}{-15}=\frac{3\alpha-2}{47}$

$\Rightarrow \beta=-15,\alpha=-15$ {as α & β are integer}

So, $\alpha^2+\beta^2=225+225=450$

2. (1) $n=\ell+m$

Now, $\ell^2+m^2=n^2=(\ell+m)^2$

$\Rightarrow\ 2\ell m=0$

$l^2+m^2+n^2=1$

If $\ell=0\Rightarrow\ 2n^2=1\Rightarrow n=\pm\frac{1}{\sqrt{2}}$

$m=n=\pm\frac{1}{\sqrt{2}}$

And, If $m=0\Rightarrow n=\ell=\pm\frac{1}{\sqrt{2}}$

$\therefore\ l^2+m^2=\frac{1}{2}\&\ l+m=\frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{2}+2\ lm=\frac{1}{2}$

$\therefore\ l=0,\ m=\frac{1}{\sqrt{2}}$ or $l=\frac{1}{\sqrt{2}},\ m=0$

So, direction cosines of two lines are

$\left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$

Thus,

$\therefore\ \cos\alpha=0+0+\frac{1}{2}=\frac{1}{2}$

$\Rightarrow \alpha=\frac{\pi}{3}$

$\therefore\ \sin^4\alpha+\cos^4\alpha=1-\frac{1}{2}\sin^2(2\alpha)=1-\frac{1}{2}\cdot\frac{3}{4}=\frac{5}{8}$

3. **(1)**

Given, $l+m-n=0$ & $3l^2+m^2+cln=0$

Now taking $n=l+m$ and putting in $3l^2+m^2+cln=0$ we get,

$3l^2+m^2+cl(l+m)=0$

$\Rightarrow 3l^2+m^2+cl^2+clm=0$

$\Rightarrow(3+c)l^2+clm+m^2=0$

$\Rightarrow(3+c)\left(\frac{l}{m}\right)^2+c\left(\frac{l}{m}\right)+1=0$ ..equation (i)

$\because$ lies are parallel.

So, roots of equation (i) must be equal

$\Rightarrow\ D=0$

$\Rightarrow c^2-4(3+c)=0$

$\Rightarrow c^2-4c-12=0$

$\Rightarrow(c-6)(c+2)=0$

$c=6$ or $c=-2$

So, positive value of $c=6$

4. **(2)**

Given equations of direction cosies

$2\ell+2m-n=0\ldots\ldots\ldots\ldots\ldots(i)$

$mn+n\ell+\ell m=0\ldots\ldots\ldots\ldots\ldots(ii)$

$\ell m+n(\ell+m)=0$

From equation (i)

$n=2(\ell+m)$

$\ell m+2(\ell+m)^2=0$

$2\ell^2+2m^2+5\ell m=0$

Dividing by m^2 on both sides

$2\left(\frac{\ell}{m}\right)^2+2+5\left(\frac{\ell}{m}\right)=0$

Let $\frac{\ell}{m}=t$

$2t^2+5t+2=0$

$2t^2+4t+t+2=0$

$(t+2)(2t+1)=0$

$t=-2,-\frac{1}{2}$

Case 1

$\frac{\ell}{m}=-\frac{1}{2}$

$m=-2\ell, n=-2\ell$

$(\ell,-2\ell,-2\ell)\Rightarrow(1,-2,-2)$

Case 2

$\frac{\ell}{m}=-2$

$\ell=-2m,\ n=-2m$

$(-2m,m,-2m)\Rightarrow(-2,1,-2)$

$\cos\theta=\frac{1\times(-2)+(-2)\times1+(-2)\times(-2)}{\sqrt{1^2+(-2)^2+(-2)^2}\sqrt{(-2)^2+1^2+(-2)^2}}$

$\cos\theta=\frac{-2-2+4}{9}=0$

$\theta=\frac{\pi}{2}$

5. **(4)** Given lines

$\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ & $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$

Formula for shortest distance

$$\text{S.D.}=\frac{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}}{\left|\begin{vmatrix}i & j & k\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}\right|}$$

$$=\frac{\begin{vmatrix}6 & 1 & -8\\ 1 & -2 & 2\\ 1 & 2 & 0\end{vmatrix}}{\left|\begin{vmatrix}i & j & k\\ 1 & -2 & 2\\ 1 & 2 & 0\end{vmatrix}\right|}=\frac{54}{6}=9$$

6. **(18)** If the lines $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$

And $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$ intersect

Point on first line $(1,2,3)$ and point on second line $(4,1,0)$.

Vector joining both points is $-3\hat{i}+\hat{j}+3\hat{k}$

Now vector along first line is $2\hat{i}+3\hat{j}+\alpha\hat{k}$

Also vector along second line is $5\hat{i}+2\hat{j}+\beta\hat{k}$

Now these three vectors must be coplanar

$$\Rightarrow\begin{vmatrix}2 & 3 & \alpha\\ 5 & 2 & \beta\\ -3 & 1 & 3\end{vmatrix}=0$$

$\Rightarrow 2(6-\beta)-3(15+3\beta)+\alpha(11)=0$

$\Rightarrow \alpha-\beta=3$

Now $\alpha=3+\beta$

Given expression $8(3+\beta)\cdot\beta=8\left(\beta^2+3\beta\right)$

$=8\left(\beta^2+3\beta+\frac{9}{4}-\frac{9}{4}\right)=8\left(\beta+\frac{3}{2}\right)^2-18$

So magnitude of minimum value $=18$

7. **(3)**

Shortest distance between two lines

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given as

$$\frac{\begin{vmatrix}x_1-x_2 & y_1-y_2 & z_1-z_2\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}}{\sqrt{\left(a_1b_2-a_2b_1\right)^2+\left(b_1c_2-b_2c_1\right)^2+\left(c_1a_2-c_2a_1\right)^2}}$$

$$=\frac{\begin{vmatrix}5-(-3) & 2-(-5) & 4-1\\ 1 & 2 & -3\\ 1 & 4 & -5\end{vmatrix}}{\sqrt{(4-2)^2+(-10+12)^2+(-3+5)^2}}$$

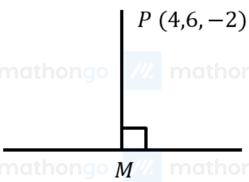
$$=\frac{\begin{vmatrix}8 & 7 & 3\\ 1 & 2 & -3\\ 1 & 4 & -5\end{vmatrix}}{\sqrt{(2)^2+(2)^2+(2)^2}}$$

$=\frac{|8(-10+12)-7(-5+3)+3(4-2)|}{\sqrt{4+4+4}}$

$=\frac{16+14+6}{\sqrt{12}}=\frac{36}{2\sqrt{3}}$

$=\frac{18}{\sqrt{3}}=6\sqrt{3}$

8. **(4)**



The equation of line passing through the point $(-3,2,3)$ and parallel to a line with direction ratios $3,3,-1$ will be,

$\frac{x+3}{3}=\frac{y-2}{3}=\frac{z-3}{-1}=\lambda$

Now let any point on the line will be, $M(3\lambda-3,3\lambda+2,3-\lambda)$

We know the direction ratios of the line joining the points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

$(x_2-x_1,y_2-y_1,z_2-z_1)$

Therefore, the direction ratios of line PM,

D.R of PM$(3\lambda-7,3\lambda-4,5-\lambda)$

Since, PM is perpendicular to the line

$\Rightarrow\ 3(3\lambda-7)+3(3\lambda-4)-1(5-\lambda)=0$

$\Rightarrow\ \lambda=2$

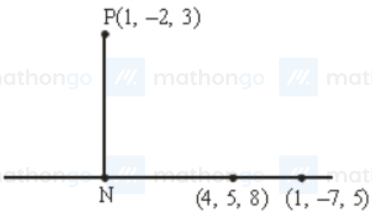
$\Rightarrow$ M$(3,8,1)$

Now distance of point P from point M,

$\Rightarrow$ PM $=\sqrt{(3-4)^2+(8-6)^2+(1+2)^2}$

$\Rightarrow$ PM $=\sqrt{14}$

9. (4)



Equation of line

$$\frac{x-4}{4-1}=\frac{y-5}{5-(-7)}=\frac{z-8}{8-5}$$

$$\frac{x-4}{3}=\frac{y-5}{12}=\frac{z-8}{3}$$

Let point $N(3\lambda+4, 12\lambda+5, 3\lambda+8)$

$\overrightarrow{\mathrm{PN}}=(3\lambda+4-1)\hat{i}+(12\lambda+5-(-2))\hat{j}+(3\lambda+8-3)\hat{k}$

$\overrightarrow{\mathrm{PN}}=(3\lambda+3)\hat{i}+(12\lambda+7)\hat{j}+(3\lambda+5)\hat{k}$

And parallel vector to line (say $\vec{a}=3\hat{i}+12\hat{j}+3\hat{k}$)

Now, $\overrightarrow{\mathrm{PN}}\cdot\vec{a}=0$

$(3\lambda+3)3+(12\lambda+7)12+(3\lambda+5)3=0$

$162\lambda+108=0\Rightarrow\lambda=\frac{-108}{162}=\frac{-2}{3}$

So point N is $(2,-3,6)$

Now distance is $=\left|\frac{2(2)-2(-3)+6+5}{\sqrt{4+4+1}}\right|=7$

10. (1)

Given,

Equation of the lines

$x+1=2y=-12z$ and $x=y+2=6z-6$

$\Rightarrow\frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{\frac{-1}{12}}$ and $\frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}$

We know that, shortest distance between the lines is given by $\mathrm{S.D}=\frac{\left(\vec{b}-\vec{a}\right)\cdot\left(\vec{p}\times\vec{q}\right)}{\left|\vec{p}\times\vec{q}\right|}$

Here $\vec{p}=\hat{i}+\frac{\hat{j}}{2}+\frac{-\hat{k}}{12}$ & $\vec{q}=\hat{i}+\hat{j}+\frac{\hat{k}}{6}$ and $\vec{a}=\hat{i}$ & $\vec{b}=2\hat{j}-\hat{k}$

$\Rightarrow\mathrm{S.D}=\left(-\hat{i}+2\hat{j}-\hat{k}\right)\cdot\frac{\left(\vec{p}\times\vec{q}\right)}{\left|\vec{p}\times\vec{q}\right|}$

Now solving $\vec{p}\times\vec{q}\equiv\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&\frac{1}{2}&\frac{-1}{12}\\1&1&\frac{1}{6}\end{vmatrix}$

$\Rightarrow\vec{p}\times\vec{q}\equiv\frac{1}{6}\hat{i}-\frac{1}{4}\hat{j}+\frac{1}{2}\hat{k}$

$\Rightarrow\vec{p}\times\vec{q}\equiv2\hat{i}-3\hat{j}+6\hat{k}$

Hence, $\mathrm{S.D}=\frac{\left(-\hat{i}+2\hat{j}-\hat{k}\right)\cdot\left(2\hat{i}-3\hat{j}+6\hat{k}\right)}{\sqrt{2^2+3^2+6^2}}=\left|\frac{-14}{7}\right|=2$

11. (2)

D.R. 's of $\vec{a}_1=(-1,0,3)$

D.R. 's of $\vec{a}_2=(0,-1,2)$

D.R. 's of $\vec{b}_1=(1,-a,0)$

D.R. 's of $\vec{b}_2=(1,-1,1)$

Now, $\vec{a}_2-\vec{a}_1=\hat{i}-\hat{j}-\hat{k}$

and $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -a & 0\\ 1 & -1 & 1\end{vmatrix}$

$\vec{b}_1\times\vec{b}_2=\hat{i}(-a)-\hat{j}+\hat{k}(a-1)$

i.e. $\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{a^2+1+(a-1)^2}$

So, $\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)=2-2a$

Shortest distance between the lines, $\frac{2(1-a)}{\sqrt{a^2+1+(a-1)^2}}=\sqrt{\frac{2}{3}}$

Squaring on both the sides, we get,

$3(1-a)^2=a^2-a+1$

i.e. $a=2,\frac{1}{2}$

12. (2)

Given $l_1:\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z-2}{0}$

$l_2:\frac{x-1}{1}=\frac{y+\frac{3}{2}}{\frac{\alpha}{2}}=\frac{z+5}{2}$

$l_3:\frac{x-1}{-3}=\frac{y-\frac{1}{2}}{-2}=\frac{z-0}{4}$

Since l_1 is perpendicular to l_2, so $\frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}}=0\Rightarrow\alpha=3$

Now for angle between l_2 & l_3,

$\cos\theta=\frac{\left|1\times(-3)+(-2)\left(\frac{\alpha}{2}\right)+2\times4\right|}{\sqrt{1+\frac{\alpha^2}{4}+4}\sqrt{9+4+16}}$

$\cos\theta=\frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$

Putting $\alpha=3$

$\cos\theta=\frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}}=\frac{4}{29}$

$\theta=\cos^{-1}\left(\frac{4}{29}\right)\Rightarrow\theta=\sec^{-1}\left(\frac{29}{4}\right)$

13. (4)

Given, $\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ and $\frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3}$ lies on the plane $x+2y-z=8$.

Let $\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3}=\emptyset$

and $\frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3}=q$

Any point on the first line can be considered as $(\phi+\alpha,\ 2\phi+1,\ 3\phi+1)$

and a point on the second line can be $(q\beta+4,3q+6,3q+7)$.

For intersection of the lines $\phi+\alpha=q\beta+4\quad\ldots(1)$

$2\phi+1=3q+6\quad\ldots(2)$

$3\phi+1=3q+7\quad\ldots(3)$

For (2) & (3) $\phi=1$, $q=-1$

So, from (1) $\alpha+\beta=3$

Now, point of intersection is $(\alpha+1,3,4)$

It lies on the given plane, therefore, $\alpha+1+2\times3-4=8\Rightarrow\alpha=5$

$\Rightarrow\beta=-2$ from $\alpha+\beta=3$

Hence, $\alpha-\beta=5-(-2)=7$

14. **(7)**

We have, point $P(3,4,4)$

Equation of line joining the points $Q(3,-4,-5)$ and $R(2,-3,1)$ is $\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}=r$

Any point on above line is $(-r+3, r-4, 6r-5)$

Now, satisfying it in the given plane $2x+y+z=7$, we get

$2(-r+3)+(r-4)+(6r-5)=7$

$\Rightarrow r=2$

So, required point of intersection is $T(1,-2,7)$.

Hence, $PT=\sqrt{(3-1)^2+(4+2)^2+(4-7)^2}=7$.

15. **(3)** $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}=r$

$\Rightarrow P(x,y,z)=(2r+1,3r-1,-2r+1)$

Since, $\overrightarrow{QP}\perp\left(2\hat{i}+3\hat{j}-2\hat{k}\right)$

$\Rightarrow 4r+2+9r-6+4r+2=0$

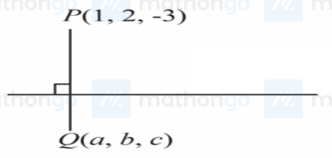
$\Rightarrow r=\frac{2}{17}$

$\Rightarrow P\left(\frac{21}{17},\frac{-11}{17},\frac{13}{17}\right)$

$\Rightarrow \overrightarrow{PQ}=\frac{21\hat{i}-28\hat{j}-21\hat{k}}{17}$

So, $\overrightarrow{QP}:\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$

16. **(1)**



R$(-1+2\text{r}, 3-2\text{r}, -\text{r})$

$dr's\ of\ PR\ are\ (2-2r,-1+2r,-3+r)$

Then, $2(2-2r)+2(1-2r)+1(3-r)=0$

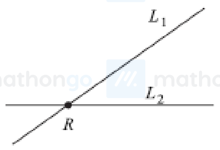
$9-9r=0\Rightarrow r=1$

$R(1,1,-1)$

then $a+1=2, b+2=2, c-3=-2$ i.e., $\text{a}=1, \text{b}=0, \text{c}=1$

$\therefore\ a+b+c=2$

17. **(1)** Let the coordinate of P with respect to line



$$\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}=\lambda$$
$$\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}=\mu$$

$L_1=(\lambda+3,3\lambda-1,-\lambda+6)$

and coordinate of P w.r.t.

line $L_2=(7\mu-5,-6\mu+2,4\mu+3)$

$\therefore\quad \lambda-7\mu=-8, 3\lambda+6\mu=3, \lambda+4\mu=3$

From above equation : $\lambda=-1,\mu=1$

$\therefore$ Coordinate of point of intersection $R=(2,-4,7)$.

Image of R w.r.t. xy plane $=(2,-4,-7)$.

18.

(1) Shor test distance $= \dfrac{\begin{vmatrix} 0 & 4 & 1 \\ 3 & -4 & 0 \\ 2\lambda & 3 & -12 \end{vmatrix}}{\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 4 & 1 \\ 3 & -4 & 0 \end{vmatrix}}$

$13 = \dfrac{|153+8\lambda|}{\left|4\bar{i}+3\bar{j}-12\hat{k}^{-}\right|}$

$= \dfrac{|153+8\lambda|}{13}$

$|153+8\lambda| = 169$

$153+8\lambda = 169, -169$

$\lambda = \dfrac{16}{8} \cdot \dfrac{-322}{8}$

$8\left|\sum_{\lambda \in S} \lambda\right| = 306$

19. **(5)** Let $\ell = (0\hat{\mathrm{i}} + 0\hat{\mathrm{j}} + 0\hat{\mathrm{k}}) + y(\mathbf{a}\hat{\mathrm{i}} + \mathrm{b}\hat{\mathrm{j}} + \mathrm{ck})$

$= \gamma(a\hat{i} + \hat{b}\hat{j} + \hat{k})$

$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$

$= \hat{i}(2-6) - \hat{j}(1-6) + \hat{k}(2-4)$

$= -4\hat{i} - 5\hat{j} - 2\hat{k}$

$\ell = \gamma(-4\hat{i} + 5\hat{j} - 2\hat{k})$

P is intersection of ℓ and ℓ_1

$-4\gamma = 1+\lambda, 5\gamma = -11+2\lambda, -2\gamma = -7+3\lambda$

By solving there equation $y = -1, P(4, -5, 2)$

Let $\mathrm{Q}(-1+2\mu, 2\mu, 1+\mu)$

$\overrightarrow{\mathrm{PQ}} \cdot (2\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \hat{\mathrm{k}}) = 0$

$-2+4\mu+4\mu+1+\mu = 0$

$9\mu = 1$

$\mu = \dfrac{1}{9}$

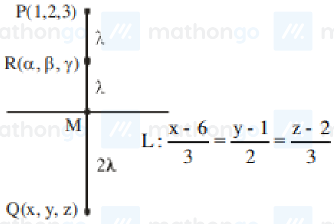
$\mathrm{Q}\left(\dfrac{-7}{9}, \dfrac{2}{9}, \dfrac{10}{9}\right)$

$9(\alpha+\beta+\gamma) = 9\left(\dfrac{-7}{9} + \dfrac{2}{9} + \dfrac{10}{9}\right)$

$= 5$

20. (125)

Plotting the rough graph of the given data we have,



Let M be the mid-point of PQ

$\therefore\ M = (3\lambda+6, 2\lambda+1, 3\lambda+2)$

Now, $\overrightarrow{PM} = (3\lambda+5)\hat{i} + (2\lambda-1)\hat{j} + (3\lambda-1)\hat{k}$

$\because\ \overrightarrow{PM} \perp \left(3\hat{i}+2\hat{j}+3\hat{k}\right)$

$\therefore\ 3(3\lambda+5)+2(2\lambda-1)+3(3\lambda-1)=0$

$\lambda = \frac{-5}{11}$

$\therefore\ M\left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)$

Since R is mid-point of PM

So by using midpoint formula in P & M we get,

$R = \left(\frac{62}{22}, \frac{23}{22}, \frac{40}{22}\right)$, now comparing with $R(\alpha, \beta, \gamma)$

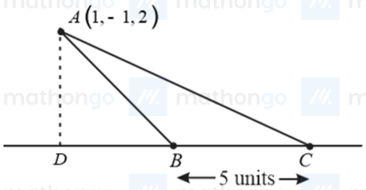
We get, $\alpha = \frac{62}{22},\ \beta = \frac{23}{22}$ & $\gamma = \frac{40}{22}$

So, $22(\alpha+\beta+\gamma) = 125$

21. (3)

The points B and C lie on the line $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$.

Draw perpendicular AD on the line BC.



Clearly area of $\Delta ABC = \frac{1}{2} \cdot AD \cdot BC$

To find a point on the line, let $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4} = r$

$\Rightarrow x+2 = 3r,\ y-1 = 0,\ z = 4r$

$\Rightarrow x = 3r-2,\ y = 1,\ z = 4r$

Thus, the point $D \equiv (3r-2,\ 1,\ 4r)$

The direction ratios of a line joining two points $(x_1,\ y_1,\ z_1)$ and $(x_2,\ y_2,\ z_2)$ are $< x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1 >$

Thus, the direction ratios of AD are $< 3r-2-1,\ 1+1,\ 4r-2 > = < 3r-3,\ 2,\ 4r-2 >$

Since AD is perpendicular to given line, hence the dot product of their direction ratios is zero.

$\Rightarrow 3 \cdot (3r-3) + 0 \cdot 2 + 4 \cdot (4r-2) = 0$

$\Rightarrow 9r - 9 + 16r - 8 = 0$

$\Rightarrow 25r - 17 = 0$

$\Rightarrow r = \frac{17}{25}$

$\Rightarrow D \equiv \left(3 \times \frac{17}{25} - 2,\ 1,\ 4 \times \frac{17}{25}\right) = \left(\frac{1}{25},\ 1,\ \frac{68}{25}\right)$

The distance between the points $(x_1,\ y_1,\ z_1)$ & $(x_2,\ y_2,\ z_2)$ is $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$

$\Rightarrow AD = \sqrt{\left(\frac{1}{25}-1\right)^2 + (1+1)^2 + \left(\frac{68}{25}-2\right)^2}$

$\Rightarrow AD = \sqrt{\frac{576}{625} + 4 + \frac{324}{625}}$

$\Rightarrow AD = \sqrt{\frac{136}{25}}$

$\Rightarrow AD = \frac{2}{5}\sqrt{34}$ units

Hence, the area of $\Delta ABC = \frac{1}{2} \cdot \left(\frac{2}{5}\sqrt{34}\right) \cdot 5 = \sqrt{34}$ sq units.

22. (1)

Centroid $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$D(2,2)$

Point of intersection $P\left(-\frac{1}{5}, \frac{2}{5}\right)$

Equation of line DP $8x-11y+6=0$.

Hence, $(-9,-6)$ satisfy above equation.

23. (44)

$l_1 : \overrightarrow{r} = (3+t)\hat{\mathrm{i}} + (-1+2t)\hat{\mathrm{j}} + (4+2t)\widehat{\mathrm{k}}$

$l_2 : \overrightarrow{r} = (3+2s)\hat{\mathrm{i}} + (3+2s)\hat{\mathrm{j}} + (2+s)\widehat{\mathrm{k}}$

DR of $l_1 \equiv (1,2,2)$

DR of $l_2 \equiv (2,2,1)$

$l_1 \times l_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

$= i(2-4) - j(1-4) + k(2-4)$

$= -2i + 3j - 2k$

Hence, DR of l (line $\perp$ to l_1 & l_2)

$\equiv (-2,3,-2)$

$\therefore\ l : \overrightarrow{r} = -2\mu\hat{\mathrm{i}} + 3\mu\hat{\mathrm{j}} - 2\mu\widehat{\mathrm{k}}$

for intersection of l & l_1

$3+t=-2\mu$

$-1+2t=3\mu$

$4+2\mathrm{t}=-2\mu$

$\Rightarrow \mathrm{t}=-1$ & $\mu=-1$

$\therefore$ Point of intersection $P \equiv (2, \ -3, \ 2)$

Let point on l_2 be $Q(3+2s, \ 3+2s, \ 2+s)$

Given $PQ=\sqrt{17} \Rightarrow (PQ)^2=17$

$\Rightarrow (2s+1)^2+(6+2s)^2+(s)^2=17$

$\Rightarrow 9s^2+28s+20=0$

$\Rightarrow \mathrm{s}=-2, -\frac{10}{9}$

$s \neq -2$ as point lies on 1^{st} octant.

$\therefore a = 3+2\left(-\frac{10}{9}\right) = \frac{7}{9}$

$b = 3+2\left(-\frac{10}{9}\right) = \frac{7}{9}$

$c = 2+\left(-\frac{10}{9}\right) = \frac{8}{9}$

$\therefore 18(a+b+c) = 18\left(\frac{22}{9}\right) = 44$

24. (2)

$(3, 5, 7)$ satisfy the line L_1

$\frac{3-a}{\ell} = \frac{5-2}{3} = \frac{7-b}{4}$

$\frac{3-a}{\ell} = 1$ & $\frac{7-b}{4} = 1$

$a + \ell = 3 \ldots(1)$ & $b = 3 \ldots(2)$

$\vec{v}_1 = <4, 3, 8> - <3, 5, 7>$

$\vec{v}_1 = <1, -2, 1>$

$\vec{v}_2 = <\ell, 3, 4>$

$\vec{v}_1 \cdot \vec{v}_2 = 0 \Rightarrow \ell - 6 + 4 = 0 \Rightarrow \ell = 2$

$a + \ell = 3 \Rightarrow a = 1$

$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

$A = <1, 2, 3>$

$B = <2, 4, 5>$

$\overrightarrow{AB} = <1, 2, 2>$

$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

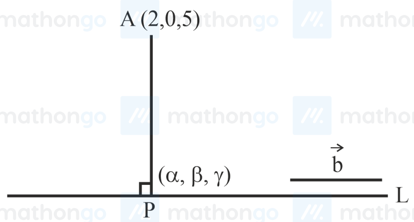
$\vec{p} \times \vec{q} = -\hat{i} + 2\hat{j} - \hat{k}$

Shortest distance $= \left|\frac{\overrightarrow{AB} \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right| = \frac{1}{\sqrt{6}}$

25. (3)

Given line is

$L : \frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} = \lambda$ (let)



Let foot of perpendicular is $P(2\lambda - 1, 5\lambda + 1, -\lambda - 1)$

$\overrightarrow{PA} = (3 - 2\lambda)\hat{i} - (5\lambda + 1)\hat{j} + (6 + \lambda)\hat{k}$

Direction ratio of line is

$\vec{b} = 2\hat{i} + 5\hat{j} - \hat{k}$

Now, $\overrightarrow{PA} \perp L$, so

$\overrightarrow{PA} \cdot \vec{b} = 0$

$\Rightarrow 2(3 - 2\lambda) - 5(5\lambda + 1) - (6 + \lambda) = 0$

$\Rightarrow 6 - 4\lambda - 25\lambda - 5 - 6 - \lambda = 0$

$\Rightarrow -30\lambda - 5 = 0$

$\Rightarrow \lambda = -\frac{1}{6}$

So,

$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1) \equiv P(\alpha, \beta, \gamma)$

Now,

$\alpha = 2\left(-\frac{1}{6}\right) - 1 = -\frac{4}{3} \Rightarrow \boxed{\alpha = -\frac{4}{3}}$

$\beta = 5\left(-\frac{1}{6}\right) + 1 = \frac{1}{6} \Rightarrow \boxed{\beta = \frac{1}{6}}$

$\gamma = -\lambda - 1 = \frac{1}{6} - 1 \Rightarrow \boxed{\gamma = -\frac{5}{6}}$

So,

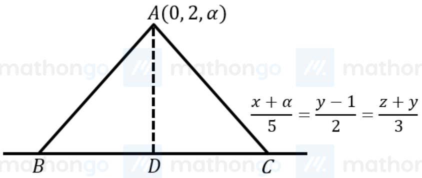
$\frac{\beta}{\gamma} = -\frac{1}{5} \neq -5$

26. (9)

Given,

The co-ordinates of one vertex of ΔABC be $A(0,2,\alpha)$ and the other two vertices lie on the line $\frac{x+\alpha}{5}=\frac{y-1}{2}=\frac{z+4}{3}$

Now plotting the diagram we get,



Now any point on the plane $\frac{x+\alpha}{5}=\frac{y-1}{2}=\frac{z+4}{3}=k$ is given by $(5k-\alpha,\ 2k+1,\ 3k-4)$

Now direction ratio of line AD is given by, $(5k-\alpha-0,\ 2k+1-2,\ 3k-4-\alpha)$

$\equiv(5k-\alpha,\ 2k-1,\ 3k-4-\alpha)$

Now AD is perpendicular to BC,

So, $5(5k-\alpha)+2(2k-1)+3(3k-4-\alpha)=0$

$\Rightarrow 19k-4\alpha-7=0\ldots..(1)$

Also given area, $21=\frac{1}{2}\times 2\sqrt{21}\times AD$

$\Rightarrow AD=\sqrt{21}$

$\Rightarrow(5k-\alpha)+(2k-1)+(3k-4-\alpha)=21$

$\Rightarrow 19k^2-8k\alpha+\alpha^2-14k+4\alpha=2\ldots\ldots(2)$

Now on solving equation (1) & (2) we get,

$\alpha=3\Rightarrow\alpha^2=9.$

27. (180)

We have,

$L : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1} = \lambda$

Any point on L is $P((2\lambda+1),(-\lambda-1),(\lambda+3))$

It lies on plane $2x+y+3z=16$, so

$\Rightarrow 2(2\lambda+1)+(-\lambda-1)+3(\lambda+3)=16$

$\Rightarrow 6\lambda+10=16 \Rightarrow \lambda=1$

$\therefore P \equiv (3,-2,4)$

Let $Q((2\mu+1),(-\mu-1),(\mu+3))$

Direction ratios of QR is

$<2\mu, -\mu, \mu+6>$

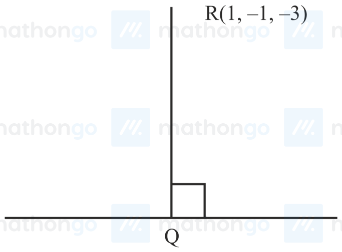
Direction ratios of L is $<2,-1,1>$.

Since, $QR \perp L$, so

$4\mu+\mu+\mu+6=0$

$\Rightarrow \mu=-1$

$Q \equiv (-1,0,2)$



Now,

$\overrightarrow{QR} = 2\hat{i}-\hat{j}-5\hat{k}$

$\overrightarrow{QP} = 4\hat{i}-2\hat{j}+2\hat{k}$

Hence,

$\overrightarrow{QR} \times \overrightarrow{QP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i}-24\hat{j}$

Therefore,

$\alpha = \frac{1}{2} \times \sqrt{144+576}$

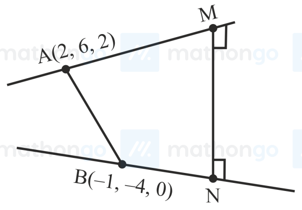
$\Rightarrow \alpha^2 = \frac{720}{4} = 180$

28. (4)

The line l_1 passes through the point $(2,\ 6,\ 2)$ and is perpendicular to the line $2x+y-2z=10$ then,

$l_1:\ \frac{x-2}{2}=\frac{y-6}{1}=\frac{z-2}{-2}$

Let the shortest distance between l_1 and the line $\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$ is S.D.



S.D. $=\left|\frac{\overrightarrow{AB}\cdot\overrightarrow{MN}}{\left|\overrightarrow{MN}\right|}\right|$

Here, $\overrightarrow{AB}=3\hat{i}+10\hat{j}+2\hat{k}$

$\overrightarrow{MN}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 2 & -3 & 2\end{vmatrix}=-4\hat{i}-8\hat{j}-8\hat{k}$

$\Rightarrow\left|\overrightarrow{MN}\right|=\sqrt{16+64+64}=12$

$\overrightarrow{AB}\cdot\overrightarrow{MN}=-12-80-16=-108$

So, shortest distance (S.D.)

$=\left|\frac{-108}{12}\right|=|-9|=9$ units

29. (51)

Given,

Line $L_1:\ \frac{x-\frac{1}{8}}{\frac{1}{8}}=\frac{y}{-\frac{1}{4\sqrt{2}}}=\frac{z}{0}$

or $\frac{x-\frac{1}{8}}{1}=\frac{y}{-\sqrt{2}}=\frac{z}{0}\quad\ldots(1)$

Equation of $L_2:\ \frac{x+\frac{1}{8}}{-6\sqrt{3}}=\frac{y}{0}=\frac{z}{8}$

Shortest distance between two lines is given by,

$d=\left|\frac{\left(\vec{c}-\vec{a}\right)\cdot\left(\vec{b}\times\vec{d}\right)}{\left|\vec{b}\times\vec{d}\right|}\right|=\frac{\left(\frac{1}{4}\hat{i}\right)\cdot\left(4\sqrt{2}\hat{i}+4\hat{j}+3\sqrt{6}\hat{k}\right)}{\sqrt{\left(4\sqrt{2}\right)^2+4^2+\left(3\sqrt{6}\right)^2}}$

$=\frac{\sqrt{2}}{\sqrt{32+16+54}}=\frac{1}{\sqrt{51}}$

$\therefore\ d^{-2}=51$

30. **(2)**

$P(3,-1,2)$

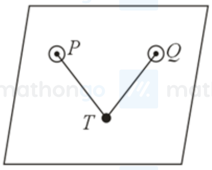
$Q(1,2,-4)$

$\overrightarrow{PR} \| 4\hat{i}-\hat{j}+2\hat{k}$

$\overrightarrow{QS} \| -2\hat{i}+\hat{j}-2\hat{k}$

Dr's of normal to the plane containing P, T and Q will be proportional to :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix} = 4\hat{j}+2\hat{k}$$



$\therefore \frac{\ell}{0}=\frac{m}{4}=\frac{n}{2}$

For point, T : $\overrightarrow{PT}=\frac{x-3}{4}=\frac{y+1}{-1}=\frac{z-2}{2}=\lambda$

$\overrightarrow{QT}=\frac{x-1}{-2}=\frac{y-2}{1}=\frac{z+4}{-2}=\mu$

$T \equiv(4\lambda+3,-\lambda-1,2\lambda+2)$

$Q \equiv(2\mu+1,\mu+2,-2\mu-4)$

$4\lambda+3=-2\mu+1 \Rightarrow 2\lambda+\mu=-1$

$\lambda+\mu=-3 \Rightarrow \lambda=2$

and $\mu=-5,\ \lambda+\mu=-3 \Rightarrow \lambda=2$

So point T :$(11,-3,6)$

$\overrightarrow{OA}=\left(11\hat{i}-3\hat{j}+6\hat{k}\right)\pm\left(\frac{2\hat{j}+\hat{k}}{\sqrt{5}}\right)\sqrt{5}$

$\overrightarrow{OA}=\left(11\hat{i}-3\hat{j}+6\hat{k}\right)\pm\left(2\hat{j}+\hat{k}\right)$

$\overrightarrow{OA}=11\hat{i}-\hat{j}+7\hat{k}$ or $9\hat{i}-5\hat{j}+5\hat{k}$

$\left|\overrightarrow{OA}\right|=\sqrt{121+1+49}=\sqrt{171}$ or $\sqrt{81+25+25}=\sqrt{131}$