ANSWER KEYS

1. (4)	**2.** (3)	**3.** (4)	**4.** (2)	**5.** (3)	**6.** (1)	**7.** (1)	**8.** (3)
9. (4)	**10.** (4)						

1. (4)

$$\frac{1+i}{\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)}=\frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)}{e^{\frac{-i\pi}{4}}}$$
$$=\sqrt{2}e^{\frac{i\pi}{4}+\frac{i\pi}{4}}=\sqrt{2}e^{\frac{2i\pi}{4}}=\sqrt{2}e^{\frac{i\pi}{2}}$$
$$=\sqrt{2}\left[\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right]$$

2. (3)

We know that
$e^{i\theta}=\cos\theta+i\sin\theta$
Then,
$z_r=\cos\frac{2r\pi}{5}+i\sin\frac{2r\pi}{5}$
$=e^{\frac{i2r\pi}{5}}$
So, $z_1z_2z_3z_4z_5=e^{i\frac{2\pi}{5}}\cdot e^{i\frac{4\pi}{5}}\cdot e^{i\frac{6\pi}{5}}\cdot e^{i\frac{8\pi}{5}}\cdot e^{i\frac{10\pi}{5}}$
$=e^{2i\left(\frac{\pi}{5}+\frac{2\pi}{5}+\frac{3\pi}{5}+\frac{4\pi}{5}\right)}\cdot e^{i2\pi}$
$=e^{2i(2\pi)}\cdot e^{i2\pi}$
$=e^{i6\pi}=\cos 6\pi+\text{i}\sin 6\pi=1$
Thus $z_1z_2z_3z_4z_5=1$ where $z_r=\cos\frac{2r\pi}{5}+i\sin\frac{2r\pi}{5}$

3. (4)

$$\left(\frac{-1+\sqrt{3}i}{1-i}\right)^{30}=\left(\frac{2\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)}{\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)}\right)^{30}$$
$$=\frac{2^{30}\left(\cos 20\pi+i\sin 20\pi\right)}{2^{15}\left(\cos\frac{15\pi}{2}-i\sin\frac{15\pi}{2}\right)}$$
$$=\frac{2^{15}\left(1+0i\right)}{(0+i)}=-2^{15}i$$

4. (2)

$(\mathrm{x_1x_2x_3}\ldots\ldots\infty)^2(\mathrm{z_1z_2z_3}\ldots\ldots\infty)^4$

$$=\left[\left(\cos\frac{\pi}{2}+\mathrm{i}\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2^2}+\mathrm{i}\sin\frac{\pi}{2^2}\right)\left(\cos\frac{\pi}{2^3}+\mathrm{i}\sin\frac{\pi}{2^3}\right)\ldots\infty\right]^2\cdot\left[\left(\cos\frac{\pi}{3}+\mathrm{isin}\frac{\pi}{3}\right)\left(\cos\frac{\pi}{3^2}+\mathrm{isin}\frac{\pi}{3^2}\right)+\ldots\infty\right]^4$$

$$=\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{2^2}+\frac{\pi}{2^3}+\ldots\right)+\mathrm{i}\sin\left(\frac{\pi}{2}+\frac{\pi}{2^2}+\frac{\pi}{2^3}+\ldots\right)\right]^2\cdot\left[\cos\left(\frac{\pi}{3}+\frac{\pi}{3^2}+\frac{\pi}{3^3}+\ldots\right)+\mathrm{isin}\left(\frac{\pi}{3}+\frac{\pi}{3^2}+\frac{\pi}{3^3}+\ldots\right)\right]^4$$

$$=\left[\cos\left(\frac{\pi/2}{1-\frac{1}{2}}\right)+\mathrm{i}\sin\left(\frac{\pi/2}{1-\frac{1}{2}}\right)\right]^2\cdot\left[\cos\left(\frac{\pi/3}{1-\frac{1}{3}}\right)+\mathrm{isin}\left(\frac{\pi/3}{1-\frac{1}{3}}\right)\right]^4$$

$$=(\cos\pi+\mathrm{i}\sin\pi)^2\left(\frac{\cos\frac{\pi}{2}+\mathrm{i}\sin\pi}{2}\right)^4=(-1)^2(\mathrm{i})^4=1.$$

5. (3)

Let $z=\left[\frac{2\cos^2\left(\frac{\theta}{4}\right)-i2\sin\left(\frac{\theta}{4}\right)\cos\left(\frac{\theta}{4}\right)}{2\cos^2\left(\frac{\theta}{4}\right)+2i\sin\left(\frac{\theta}{4}\right)\cos\left(\frac{\theta}{4}\right)}\right]^{4n}$
$=\left[\frac{\cos\left(\frac{\theta}{4}\right)-i\sin\left(\frac{\theta}{4}\right)}{\cos\left(\frac{\theta}{4}\right)+i\sin\left(\frac{\theta}{4}\right)}\right]^{4n}$
$=\frac{\cos n\theta-i\sin n\theta}{\cos n\theta+i\sin n\theta}=\frac{(\cos\theta+i\sin\theta)^{-n}}{(\cos\theta+i\sin\theta)^{n}}$
$=(\cos\theta+i\sin\theta)^{-2n}=\cos 2n\theta-i\sin 2n\theta$

6. **(1)** Explanation of the correct option:

Step1. Define the cube root of unity.

Given,

$1, \omega$ and ω^2 are cube root of unity.

$1+\omega+\omega^2=0 \quad \ldots(i)$

$\left[\because \text{ From definition, } \omega=\frac{-1+\mathrm{i}\sqrt{3}}{2}, \omega^2=\frac{-1-\mathrm{i}\sqrt{3}}{2}\right]$

$1 \times \omega \times \omega^2=1 \quad \ldots(ii)$

Step2. Find the value of $\left(3+\omega^2+\omega^4\right)^6$:

$\left(3+\omega^2+\omega^4\right)^6$

$=\left(3+\omega^2+\left(\omega^3\right)(\omega)\right)^6$

$=\left(3+\omega^2+\omega\right)^6 \quad \left[\because \text{From}(2), \omega^3=1\right]$

$=\left(2+1+\omega^2+\omega\right)^6$

$=(2+0)^6 \quad \left[\because \text{From}(1), 1+\omega+\omega^2=0\right]$

$=2^6$

$=64$

Hence, Option(A) is the correct answer.

7. **(1)**

$\omega= \ -\frac{1}{2}+\frac{\sqrt{3}}{2}\, i$ (cube root of unity)

So.

$1+\omega+\omega^2=0$ and $\omega^3=1$

Now,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}=3\left(\omega^2-\omega\right)$$

So,

$k=\omega^2-\omega=\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)-\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$

$=-\sqrt{3}i= \ -z$

8. **(3)** As, $4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}-3\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$

$\Rightarrow 4+5(\omega)^{334}-3\left(\omega^2\right)^{365}$

$\Rightarrow 4+5\omega+3\omega^2$

$1+2\omega+3\left(1+\omega+\omega^2\right)$

$1+2\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$

$\sqrt{3}\, i$

9. **(4)**

$z=\frac{\sqrt{3}}{2}+\frac{i}{2}=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$

$\Rightarrow z^5=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}=\frac{-\sqrt{3}+i}{2}$

and $z^8=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\left(\frac{1+i\sqrt{3}}{2}\right)$

$\Rightarrow\left(1+iz+z^5+iz^8\right)^9=\left(1+\frac{i\sqrt{3}}{2}-\frac{1}{2}-\frac{\sqrt{3}}{2}+\frac{1}{2}-\frac{i}{2}+\frac{\sqrt{3}}{2}\right)^9$

$=\left(\frac{1+i\sqrt{3}}{2}\right)^9=\cos 3\pi+\sin 3\pi=-1$

10. **(4)**

Let $|w|=r$ and $\arg(w)=\theta$,

then $z=r(\cos(\pi-\theta)+i\sin(\pi-\theta))$

$=r(-\cos\theta+i\sin\theta)=-\overline{w}$.