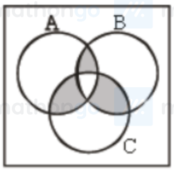
ANSWER KEYS

1. (2)	**2.** (1)	**3.** (1)	**4.** (1633)	**5.** (2)	**6.** (4)	**7.** (2)	**8.** (4)
9. (1)	**10.** (112)	**11.** (1)	**12.** (1)	**13.** (4)	**14.** (3)	**15.** (13)	**16.** (4)
17. (7)	**18.** (15)	**19.** (702)	**20.** (8)				

1. (2)

$|A|=48$
$|B|=25$
$|C|=18$
$|A\cup B\cup C|=60 \quad [\text{Total}]$
$|A\cap B\cap C|=5$



$|A\cup B\cup C|=\sum|A|-\sum|A\cap B|+|A\cap B\cap C|$
$\Rightarrow \sum|A\cap B|=48+25+18+5-60$
$=36$

No. of men who received exactly 2 medals
$=\sum|A\cap B|-3|A\cap B\cap C|$
$=36-15$
$=21$

2. (1)

Let H and L is set of people suffered from heart ailment and lungs infections respectively.

$n(H)=89\%$

$n(L)=98\%$

$n(H\cap L)=K\%$

$\max\{n(H),\ n(L)\}\leq n(H\cup L)\leq n(U)$

Where, U is the universal set.

$\Rightarrow 98\leq n(H\cup L)\leq 100$

$\Rightarrow 98\leq n(H)+n(L)-n(H\cap L)\leq 100$

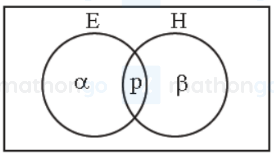
$\Rightarrow 98\leq 89+98-k\leq 100$

$\Rightarrow 0\leq 89-k\leq 2$

$\Rightarrow -89\leq -k\leq -87$

$\Rightarrow 87\leq K\leq 89$

3. (1)



$\alpha+p=75$
$\beta+p=40$
$\alpha+\beta+p=100$

From (1), (2) and (3) $P=15, \alpha=60$ and $\beta=25$

Now equation of ellipse: $25\left(\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}\right)=1$

$$\frac{x^2}{144}+\frac{y^2}{25}=1$$
$$\Rightarrow e=\frac{\sqrt{119}}{12}$$

4. (1633)

Given $S = \{1, 2, 3 \ldots 100\}$

Now finding the sum of $S = \frac{100 \times 101}{2}$

Prime factors of $24 = 2^3 \times 3$

Let $n(A) =$ Multiples of 2

$n(B) =$ Multiples of 3

$n(A \cap B) =$ Multiples of 2 & 3

So $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

To have H.C.F to be 1 we need to subtract the sum of multiples of 2 & 3 from sum of set S to get required answer,

So required answer

$= \frac{100 \times 101}{2} -$ Sum of $n(A \cup B)$

$= \frac{100 \times 101}{2} - \left\{2 \times \frac{50 \times 51}{2} + \frac{33}{2}(102) - \frac{16}{2} \times 102\right\} = 1633$

5. (2)

We have, $A = \{1, 2, 3, 4, 5, 6, 7\}$

Reflexive: A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, R is reflexive $\Leftrightarrow (a, a) \in R$ for all $a \in A$

$\because (1, 1), (2, 2), (3, 3), \ldots\ldots (7, 7)$ does not satisfy $x + y = 7$

Hence R is not reflexive.

Symmetric: A relation R is symmetric on a set A iff

$(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

$\Rightarrow x + y = 7$

Now on interchanging y and x we get, $y + x = 7$ which is always true for given set,

Hence R is symmetric.

Transitive: A relation R on A is said to be transitive relation iff

$(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R$ for all $a, b, c \in A$

Now taking $(a, b) \equiv (3, 4)$ and $(b, c) \equiv (4, 3)$ so $(a, c) \equiv (3, 3)$ does not satisfy $x + y = 7$,

Hence, R is not transitive and not equivalence.

Therefore, R is only Symmetric.

6. (4)

Given relation is

$R = \{(a, b) : gcd(a, b) = 1,\ 2a \neq b,\ a,\ b \in \mathbb{Z}\}$

Now,

Reflexive:

$\gcd(a, a) \neq 1\ \forall a \in Z$, hence R is not reflexive.

Symmetric:

$a = 2,\ b = 1 \Rightarrow \gcd(2, 1) = 1$

Also, $2a = 4 \neq b$

Now when $a = 1,\ b = 2 \Rightarrow gcd(1,\ 2) = 1$

But, $2a = 2 = b$

Hence, $a = 2b$

$\Rightarrow R$ is not Symmetric

Transitive:

Let $a = 14,\ b = 19,\ c = 21$

$gcd(a,\ b) = 1$

$gcd(b,\ c) = 1$

$gcd(a,\ c) = 7$

Hence, not transitive

So, R is neither symmetric nor transitive.

7. (2)

To check if R_1 is transitive -

Let $a=16,\ b=4,\ c=1$

Here $|a-b|=|16-4|\leq 13$ holds true

now $|b-c|=|4-1|\leq 13 \Rightarrow |3|\leqslant 13$ also holds true

but $|a-c|=|16-1|\nleq 13$

Hence R_1 is not transitive

Similarly, to check transitivity of R_2

Let $a=16,\ b=4,\ c=3$

Here $|a-b|=|16-4|\neq 13$ and $|b-c|=|4-3|\neq 13$

but $|a-c|=|16-3|=13$

Hence, R_2 is also not transitive.

Hence neither R_1 nor R_2 is an equivalence relation

8. (4)

Given,

$a,\ b\ \in\ I$ and relation T is defined as $a^2-b^2\in I$

Now if $a^2-b^2\in I$ so b^2-a^2 will also be integer, for example if $5^2-4^2=9$ is integer then $4^2-5^2=-9$ is also a integer,

Now checking transitive,

If $4^2-3^2=7$ is an integer, $3^2-2^2=5$ is an integer then $4^2-2^2=12$ is also an integer, hence we can say that T is symmetric and transitive,

Now checking relation S which is defined as $2+\frac{a}{b}>0$,

So, if we replace $\frac{a}{b}$ by $\frac{-1}{9}$ then $2+\frac{a}{b}>0$ is true but we take $\frac{b}{a}=-9$ for symmetric we get $2+\frac{b}{a}=2-9=-7\ngtr 0$ hence, the relation is not symmetric,

Now checking transitive, now if $2+\frac{a}{b}>0\Rightarrow\frac{a}{b}>-2$ and $\frac{b}{c}>-2$ then we cannot say that $\frac{a}{c}>-2$,

For example if we take $\frac{4}{1}>-2,\ \frac{1}{-1}>-2$ then $\frac{4}{-1}\ngtr -2$, hence it is not transitive,

Hence, we can say that T is symmetric and S is not.

9. (1)

Given:

$R=\left\{(a,b):\ 3a-3b+\sqrt{7}\ \text{is irrational}\right\}$

Reflexive:

$3a-3a+\sqrt{7}=\sqrt{7}$ is irrational.

So, $(a,a)\in R$, hence R is reflexive.

Given:

$R=\left\{(a,b):\ 3a-3b+\sqrt{7}\ \text{is irrational}\right\}$

Symmetric:

Let $(a,b)\in R$, then

$3a-3b+\sqrt{7}$ is irrational.

So, $3b-3a+\sqrt{7}$ is not necessarily irrational, since if $3a=\sqrt{7}$ and $3b\in Z$, then $3a-3b+\sqrt{7}$ is irrational., but $3b-3a+\sqrt{7}$ is not irrational.

So, $(a,b)\in R \not\Rightarrow (b,a)\in R$, hence R is not symmetric

Transitive:

Take (a,b) as $\left(\frac{\sqrt{7}}{3},1\right)$ and (b,c) as $\left(1,\frac{2\sqrt{7}}{3}\right)$

So, $(a,b)\in R$ and $(b,c)\in R$ but $(a,c)\notin R$ which means relation is not transitive.

10. (112)

Given,

$A=\{1,2,3,4,5,6,7\}$ and $B=\{3,6,7,9\}$

We know that if any set has n elements then total number of subset is given by 2^n

So, total subset of $A=2^7=128$

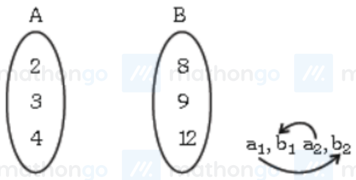
$C\cap B=\phi$ when set C contains the element $1,2,4,5$

$\therefore\quad S=\{C\subseteq A; C\cap B\neq\phi\}$

$=\text{Total}-(C\cap B=\phi)$

$=128-2^4=128-16=112$

11. **(1)**



a_1 divides b_2

Each element has 2 choices

$\Rightarrow 3 \times 2 = 6$

a_2 divides b_1

Each element has 2 choices

$\Rightarrow 3 \times 2 = 6$

Total $= 6 \times 6 = 36$

12. **(1)**

Given, $(a, b) R(c, d) \Rightarrow ad(b-c) = bc(a-d)$

$\Rightarrow (a, b) R(c, d) \Leftrightarrow \frac{b-c}{bc} = \frac{a-d}{ad}$

$\Rightarrow \frac{1}{c} - \frac{1}{b} = \frac{1}{d} - \frac{1}{a}$

$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{d} - \frac{1}{c}$

Reflexive:

$(a, b) R(a, b) \Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a}$

It is false. So, given relation is not reflexive.

Symmetric:

$(a, b) R(c, d) \Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{d} - \frac{1}{c}$

$\Rightarrow \frac{1}{c} - \frac{1}{d} = \frac{1}{b} - \frac{1}{a}$

$\therefore (c, d) R(a, b)$

It is symmetric.

Transitive:

$(a, b) R(c, d) \Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{d} - \frac{1}{c}$

$(c, d) R(e, f) = \frac{1}{c} - \frac{1}{d} = \frac{1}{e} - \frac{1}{f}$

$\frac{1}{a} - \frac{1}{b} = \frac{1}{e} - \frac{1}{f}$

$\Rightarrow (a, b) \not R (e, f)$

So, not transitive.

13. (4)

Given, a relation R on N given by $R=\{(x, y): 3x+\alpha y$ is a multiple of $7\}$,

Now for R to be reflexive $\Rightarrow xRx$

$\Rightarrow 3x+\alpha x=7x$

$\Rightarrow (3+\alpha) x=7K$

$\Rightarrow 3+\alpha=7\lambda$

$\Rightarrow \alpha=7\lambda-3=7N+4,\ \{$where $K, \lambda, N \in I\}$

So, when α divided by 7, remainder is 4.

Now R to be symmetric $xRy \Rightarrow yRx$

$3x+\alpha y=7N_1, 3y+\alpha x=7N_2$

$\Rightarrow (3+\alpha)(x+y)=7(N_1+N_2)=7N_3$

Which holds when $3+\alpha$ is multiple of 7

So, $\alpha=7N+4$ (as did earlier)

Now, for R to be transitive

xRy & $yRz \Rightarrow xRz$.

$\Rightarrow 3x+\alpha y=7N_1 \ldots.(1)$

$\Rightarrow 3y+\alpha z=7N_2 \ldots\ldots(2)$

And $3x+\alpha z=7N_3 \ldots..(3)$

Now subtracting equation $(3)-(2)$ we get,

$3x+7N_2-3y=7N_3$

Now putting the value of $3x$ from equation (1) we get, $7N_1-\alpha y+7N_2-3y=7N_3$

$\Rightarrow 7(N_1+N_2)-(3+\alpha) y=7N_3$

$\Rightarrow (3+\alpha) y=7N$

Which is true again when $3+\alpha$ divisible by 7, i.e. when α divided by 7, remainder is 4 .

14. (3)

Given A and B are matrices of $n \times n$ order and ARB iff there exists a non-singular matrix P ($\operatorname{det}(P) \neq 0$) such that $PAP^{-1}=B$.

Reflexivity Check :

$ARA \Rightarrow PAP^{-1}=A$ which is true for $P=I$.

So, R is reflexive relation.

Symmetric Check :

$ARB \Rightarrow PAP^{-1}=B \Rightarrow P^{-1}PAP^{-1}P=P^{-1}BP \Rightarrow IAI=P^{-1}BP \Rightarrow P^{-1}BP=A \Rightarrow BRA$ for matrix P^{-1}.

So, R is symmetric relation.

Transitivity Check :

$ARB \Rightarrow PAP^{-1}=B$ and $BRC \Rightarrow PBP^{-1}=C$.

So, $PPAP^{-1}P^{-1}=C$.

$\Rightarrow P^2A\left(P^2\right)^{-1}=C \Rightarrow ARC$

So, R is a transitive relation.

Since, R is reflexive, symmetric and transitive all.

Hence, R is an equivalence relation.

15. (13)

Let

$A=\{a,\ b,\ c, d\},\ R\ :\ A \rightarrow A$

$R=\{(a, b),(b, c),(b, d)\}$

Reflexive:

For reflexivity, we must add $(a, a),\ (b, b),\ (c, c),\ (d, d)$

Symmetric and Transitive:

For symmetric and transitivity, we must add

$(b, a),\ (c, b),\ (d, b)$

$(a, c),\ (a, d),\ (c, d),\ (d, c),\ (c, a),\ (d, a)$

So, we can say that minimum 13 elements are required to make an equivalence relation.

16. **(4)** $A=\{1,2,3\}$

For Reflexive $(1,1)(2,2),(3,3)\in \mathrm{R}$

For transitive : $(1,2)$ and $(2,3)\in R\Rightarrow(1,3)\in R$

Not symmetric : $(2,1)$ and $(3,2)\notin \mathrm{R}$

$\mathrm{R}_1=\{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)\}$

$\mathrm{R}_2=\{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)(2,1)\}$

$\mathrm{R}_3=\{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)(3,2)\}$

17. **(7)** $R=[(-4,4),(-3,3),(3,-2),(0,1),(0,0),(1,1),(4,4),(3,3)\}$

For reflexive, add $\Rightarrow(-2,-2),(-4,-4),(-3,-3)$

For symmetric, add $\Rightarrow(4,-4),(3,-3),(-2,3),(1,0)$

18. **(15)** $n\in[10,100]$

3^n-3 is multiple of 7

$3^n=7\lambda+3$

$n=1,7,13,20,\ldots.97$

Number of possible values of $n=15$

19. **(702)**

$A+A=(1+1,1+a_1,\ldots.1+a_{18},1+77,\ a_1+77,\ a_2+77,\ \ldots.77+77)$ has exactly 39 terms,

$1,a_1,a_2,a_3,\ldots,a_{18},77$ are in AP as the sum of terms equidistant from beginning and the end is always same for an AP

i.e. $1,5,9,13,\ldots,77.$

Hence $a_1+a_2+a_3+\ldots+a_{18}=5+9+13+\ldots$till 18 terms $=\frac{18}{2}(2\times5+17\times4)=702$

20. **(8)**

Here, $p,p^n\in\{1,2,\ldots50\}$

Now p can take values

2, 3, 5, 7, 11, 13, 17, 23, 29, 31, 37, 41, 43 and 47.

$\therefore R_1=\{(2,2^0),(2,2^1)\ldots(2,2^5),(3,3^0),\ldots(3,3^3),(5,5^0),\ldots(5,5^2),(7,7^0),\ldots(7,7^2),(11,11^0),(11,11^1),\ldots(47,47^0),(47,47^1)\}$

$R_2=\{(2,2^0),(2,2^1),(3,3^0),(3,3^1)\ldots.(47,47^0),(47,47^1)\}$

Now $R_1-R_2\equiv R_1\cap R_2{}'$

So $R_1-R_2=\{(2,2^2),(2,2^3),(2,2^4),(2,2^5),(3,3^2),(3,3^3),(5,5^2),(7,7^2)\}$

$\therefore\ n(R_1-R_2)=8$