ANSWER KEYS

1. (2)	**2.** (2)	**3.** (1)	**4.** (2)	**5.** (4)	**6.** (1)	**7.** (2)	**8.** (1)
9. (8)	**10.** (2)						

1. **(2)**

$I=\int \frac{e^{5 \log x}-e^{4 \log x}}{e^{3 \log x}-e^{2 \log x}} d x$

$I=\int \frac{e^{\log x^{5}}-e^{\log x^{4}}}{e^{\log x^{3}}-e^{\log x^{2}}} d x$

$=\int \frac{x^{5}-x^{4}}{x^{3}-x^{2}} d x \quad\left(\because a^{\log _{a} x}=x\right)$

$=\int \frac{x^{4}}{x^{2}} d x=\frac{x^{3}}{3}+C$

2. **(2)**

We have,

$\int \frac{5\left(x^{6}+1\right)}{x^{2}+1} d x$

$=\int \frac{5\left(x^{2}+1\right)\left(x^{4}-x^{2}+1\right)}{\left(x^{2}+1\right)} d x$

$\left[\because a^{3}+b^{3}=(a+b)\left(a^{2}-a b+b^{2}\right)\right]$

$=\int 5\left(x^{4}-x^{2}+1\right) d x$

$=x^{5}-\frac{5}{3} x^{3}+5 x+C.$

3. **(1)** We know that, $\sin 3 x=3 \sin x-4 \sin ^{3} x$

$\Rightarrow \sin ^{3} x=\frac{3 \sin x-\sin 3 x}{4}$

$\Rightarrow \int \sin ^{3} d x=\int \frac{3 \sin x-\sin 3 x}{4} d x$

$\Rightarrow \int \sin ^{3} d x=\frac{3}{4} \int \sin x d x-\frac{1}{4} \int \sin 3 x d x$

$\Rightarrow \int \sin ^{3} d x=\frac{3}{4}(-\cos x)-\frac{1}{4}\left(-\frac{\cos 3 x}{3}\right)+C$

$\Rightarrow \int \sin ^{3} d x=-\frac{3}{4} \cos x+\frac{\cos 3 x}{12}+C$

4. **(2)**

$\mathrm{I}=\int \frac{1+x+\sqrt{x+x^{2}}}{\sqrt{x}+\sqrt{1+x}} d x=\int \frac{\sqrt{(1+x)^{2}}+\sqrt{x} \sqrt{1+x}}{(\sqrt{x}+\sqrt{1+x})} d x$

$=\int \frac{\sqrt{1+x}(\sqrt{x}+\sqrt{x+1})}{(\sqrt{x}+\sqrt{x+1})} d x=\frac{2}{3}(1+x)^{3 / 2}+C$

5. **(4)**

$\int \sqrt{\frac{1-\mathrm{x}}{1+\mathrm{x}}} \mathrm{dx}$

Multiply and divide by $\sqrt{1-\mathrm{x}}$

$=\int \frac{1-\mathrm{x}}{\sqrt{1-\mathrm{x}^{2}}} \mathrm{dx}=\int \frac{\mathrm{dx}}{\sqrt{1-\mathrm{x}^{2}}}-\int \frac{\mathrm{x}}{\sqrt{1-\mathrm{x}^{2}}} \mathrm{dx}$

$=\sin ^{-1} \mathrm{x}+\frac{1}{2} \int \frac{(-2 \mathrm{x})}{\sqrt{1-\mathrm{x}^{2}}} \mathrm{dx}$

$=\sin ^{-1} \mathrm{x}+\sqrt{1-\mathrm{x}^{2}}+\mathrm{c}$

6. **(1)** Let $\mathrm{I}=\int \frac{\sin ^{2} x \cos ^{2} x}{\left(\sin ^{5} x+\cos ^{3} x \sin ^{2} x+\sin ^{3} x \cos ^{2} x+\cos ^{5} x\right)^{2}} d x=\int \frac{\sin ^{2} x \cos ^{2} x}{\left[\left(\sin ^{2} x+\cos ^{2} x\right)\left(\sin ^{3} x+\cos ^{3} x\right)\right]^{2}} d x=\int \frac{\sin ^{2} x \cos ^{2} x}{\left(\sin ^{3} x+\cos ^{3} x\right)^{2}} d x=\int \frac{\tan ^{2} x \cdot \sec ^{2} x}{\left(1+\tan ^{3} x\right)^{2}} d x$ Now, put $\left(1+\tan ^{3} x\right)=t$

$\Rightarrow 3 \tan ^{2} x \sec ^{2} x d x=d t \therefore \quad I=\frac{1}{3} \int \frac{d t}{t^{2}}=-\frac{1}{3 t}+C=\frac{-1}{3\left(1+\tan ^{3} x\right)}+C$

7. **(2)**

$\int \sqrt[3]{\frac{\sin ^{\mathrm{n}} \mathrm{x}}{\cos ^{\mathrm{n}+6} \mathrm{x}}} \mathrm{dx}=\int \sqrt[3]{\frac{\sin ^{\mathrm{n}} \mathrm{x}}{\cos ^{\mathrm{n}} \mathrm{x} \cdot \cos ^{6} \mathrm{x}}} \mathrm{dx}$

$=\int \sqrt[3]{\tan ^{\mathrm{n}}} \mathrm{x} \sec ^{2} \mathrm{x} \mathrm{dx}$

$=\int(\tan \mathrm{x})^{\mathrm{n} / 3} \mathrm{d}(\tan \mathrm{x}) \quad\left\{\because \mathrm{d}(\tan \mathrm{x})=\sec ^{2} \mathrm{xdx}\right\}$

$\left\{\because \int x^{n} d x=\frac{x^{n+1}}{n+1}+c\right\}$

$=\frac{(\tan \mathrm{x})^{\frac{\mathrm{n}}{3}+1}}{\frac{\mathrm{n}}{3}+1}+\mathrm{c}$

$=\frac{3}{3+\mathrm{n}}(\tan \mathrm{x})^{\frac{\mathrm{n}}{3}+1}+\mathrm{c}$.

Hence (B) is the correct answer.

8. **(1)**

$I=\int \frac{\cos x-\sin x}{7-9 \sin 2 x} d x$

$\Rightarrow \quad I=\int \frac{\cos x-\sin x}{7-9[(1+\sin 2 x)-1]} d x$

$\Rightarrow \quad I=\int \frac{\cos x-\sin x}{7-9\left[\left(\sin ^2 x+\cos ^2 x+2 \sin x \cos x\right)-1\right]}$

$\Rightarrow \quad I=\int \frac{\cos x-\sin x}{7-9\left[(\sin x+\cos x)^2-1\right]} d x$

Let $\sin x+\cos x=t \Rightarrow(\cos x-\sin x) d x=d t$

$\Rightarrow \quad I=\int \frac{d t}{7-9\left(t^2-1\right)}$

$\Rightarrow \quad I=\int \frac{d t}{4^2-(3 t)^2}$

$\Rightarrow \quad I=\frac{1}{2 \cdot 4} \cdot \frac{1}{3} \log \left|\frac{4+3 t}{4-3 t}\right|+c$

$\Rightarrow \quad I=\frac{1}{24} \log \left|\frac{4+3(\sin x+\cos x)}{4-3(\sin x+\cos x)}\right|+c$

9. **(8)**

$I=\int \frac{d x}{\left(\frac{x-1}{x+2}\right)^{\frac{3}{4}}(x+2)^2}$ put $\frac{x-1}{x+2}=t$

On solving, we get,

$I=\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+c$

$k=\frac{4}{3} \Rightarrow 30 \mathrm{k}=40=2^3 \times 5^1$

Hence, total divisors are $4 \times 2=8$

10. **(2)**

$\int \frac{\sin x}{\sin (x-\alpha)} d x=A x+B \log \sin (x-\alpha)+C$

$\Rightarrow$ Differentiating w.r.t. x both sides

$\Rightarrow \frac{\sin x}{\sin (x-\alpha)}=A+\frac{B \cos (x-\alpha)}{\sin (x-\alpha)}$

$\Rightarrow \sin x=A \sin (x-\alpha)+B \cos (x-\alpha)$

$\sin x=A(\sin x \cos \alpha-\cos x \sin \alpha)+B(\cos x \cos \alpha+\sin x \sin \alpha)$

$\sin x=\sin x(A \cos \alpha+B \sin \alpha)+\cos x(B \cos \alpha-A \sin \alpha)$

Comparing coefficients of $\sin x$ and $\cos x$ both side

$A \cos \alpha+B \sin \alpha=1$ and $B \cos \alpha-A s \text{ in } \alpha=0$

$(A, B)=(\cos \alpha, \sin \alpha)$