ANSWER KEYS

1. (2)	**2.** (4)	**3.** (4)	**4.** (3)	**5.** (3)	**6.** (4)	**7.** (3)	**8.** (1)
9. (1)	**10.** (1)						

1. **(2)**

We have, $f(x)=\begin{cases} 1-x & , \quad x<1 \\ (1-x)(2-x) & , \quad 1 \leq x \leq 2 \\ 3-x & , \quad x>2 \end{cases}$ Now,

check the continuity at $x=1$

$\text{LHL}=\lim\limits_{x \rightarrow 1^{-}}(1-x)=0$

$\text{RHL}=\lim\limits_{x \rightarrow 1^{+}}((1-x)(2-x))=0$

$\because$ LHL $=$ RHL, $f(x)$ is continuous at $x=1$.

Now, check the continuity at $x=2$

$\text{LHL}=\lim\limits_{x \rightarrow 2^{-}}((1-x)(2-x))=0$

$\text{RHL}=\lim\limits_{x \rightarrow 2^{+}}(3-x)=1$

$\because$ L. H. L $\neq$ R. H. L, $f(x)$ is discontinuous at $x=2$.

2. **(4)**

We have function,

$$f(x)=\begin{cases} \frac{\left(e^{kx}-1\right) \tan kx}{4x^2}, x \neq 0 \\ 16, \quad x=0 \end{cases}$$

is continuous at $x=0$

$\therefore \lim\limits_{x \rightarrow 0} f(x)=f(0)$

$\Rightarrow \lim\limits_{x \rightarrow 0}\left(\frac{\left(e^{kx}-1\right) \tan kx}{4x^2}\right)=16$; $\left(\frac{0}{0} form\right)$

$\Rightarrow \lim\limits_{x \rightarrow 0}\left(\frac{e^{kx}-1}{kx}\right)\left(\frac{\tan kx}{kx}\right) \times \frac{k^2}{4}=16$

$\Rightarrow \frac{k^2}{4}=16 \Rightarrow k^2=64 \Rightarrow k=\pm 8$

3. **(4)**

Given, $\mathrm{f}(\mathrm{x})=\begin{cases} \frac{\sqrt{4+\mathrm{ax}}-\sqrt{4-\mathrm{ax}}}{\mathrm{x}}, & -1 \leq \mathrm{x}<0 \\ \frac{3\mathrm{x}+2}{\mathrm{x}-8}, & 0 \leq \mathrm{x} \leq 1 \end{cases}$

Checking for continuity at $x=0$

RHL at $x=0^{+}$

$\Rightarrow \mathrm{f}\left(0^{+}\right)=\lim\limits_{x \rightarrow 0^{+}} \frac{3\mathrm{x}+2}{\mathrm{x}-8}=\frac{-1}{4}$

LHL at $x=0^{-}$

$\mathrm{f}\left(0^{-}\right)=\frac{\sqrt{4+\mathrm{ax}}-\sqrt{4-\mathrm{ax}}}{\mathrm{x}}$

Divide and multiply by $\sqrt{4+\mathrm{ax}}+\sqrt{4-\mathrm{ax}}$

$\Rightarrow \mathrm{f}\left(0^{-}\right)=\frac{\sqrt{4+\mathrm{ax}}-\sqrt{4-\mathrm{ax}}}{\mathrm{x}} \times \frac{\sqrt{4+\mathrm{ax}}+\sqrt{4-\mathrm{ax}}}{\sqrt{4+\mathrm{ax}}+\sqrt{4-\mathrm{ax}}}$

$\Rightarrow \mathrm{f}\left(0^{-}\right)=\frac{(\sqrt{4+\mathrm{ax}})^2-(\sqrt{4-\mathrm{ax}})^2}{\mathrm{x}\{\sqrt{4+\mathrm{ax}}+\sqrt{4-\mathrm{ax}}\}}$

$\Rightarrow \mathrm{f}\left(0^{-}\right)=\frac{(4+\mathrm{ax})-(4-\mathrm{ax})}{\mathrm{x}\{\sqrt{4+\mathrm{ax}}+\sqrt{4-\mathrm{ax}}\}}$

$\Rightarrow \mathrm{f}\left(0^{-}\right)=\frac{4+\mathrm{ax}-4+\mathrm{ax}}{\mathrm{x}\{\sqrt{4+\mathrm{ax}}+\sqrt{4-\mathrm{ax}}\}}$

Apply the limit ,

$\Rightarrow \mathrm{f}\left(0^{-}\right)=\lim\limits_{\mathrm{x} \rightarrow 0^{-}} \frac{2\mathrm{ax}}{\mathrm{x}\{\sqrt{4+\mathrm{ax}}+\sqrt{4-\mathrm{ax}}\}}$

$\Rightarrow \lim\limits_{\mathrm{x} \rightarrow 0^{-}} \frac{2\mathrm{a}}{4}=\frac{\mathrm{a}}{2}$

Since, function is continuous at $x=0$

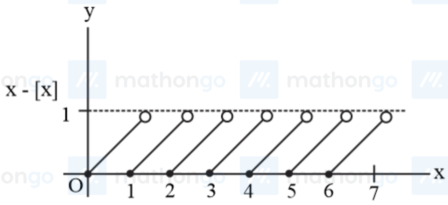
Hence,LHL=RHL

$\Rightarrow \frac{\mathrm{a}}{2}=\frac{-1}{4}$

$\Rightarrow \mathrm{a}=\frac{-1}{2}$

4. **(3)**

Function represented by $f(x)=x-[x]$ is a fractional part function shown as $\{x\}$.
The graph of the function $f(x)=x-[x]$ for the interval $(0,7)$ is shown below:



It is obvious from the above graph that the function $x-[x]$ is discontinuous at the points $x=1,\ 2,\ 3,\ 4,\ 5,\ 6$. Therefore, number of points of discontinuity of the given function in the given interval are 6.

5. **(3)** $u=\emptyset(x)=\frac{1}{x-1}$ is discontinuous at $x=1,\ y=f(u)=\frac{1}{u^2+u-2}=\frac{1}{(u+2)(u-1)}$ is discontinuous at $u=-2,\ u=1$. If $u=-2$ then $-2=\frac{1}{x-1}\ \Rightarrow x=\frac{1}{2}$. If $u=1$ then $1=\frac{1}{x-1}\ \Rightarrow x=2$. Hence the composite function is discontinuous only at $x=1,\ \frac{1}{2},\ 2$.

6. **(4)**

$$\because\quad f(x)=\begin{cases}\frac{\sin(p+1)x+\sin x}{x} & : x<0\\ q & : x=0\\ \frac{\sqrt{x^2+x}-\sqrt{x}}{x^{3/2}} & : x>0\end{cases}$$

$\because f$ is continuous at $x=0$

$L.H.L=$

$\lim\limits_{h\to 0}\frac{\sin(p+1)(-h)+\sin(-h)}{-h}=\lim\limits_{h\to 0}\frac{\sin(p+1)h+\sin h}{h}$

$=\lim\limits_{h\to 0}\frac{\sin(p+1)h}{h(p+1)}(p+1)+\lim\limits_{h\to 0}\frac{\sin h}{h}=p+1+1$

$=p+2$

$R.H.L=\lim\limits_{h\to 0}\frac{\sqrt{h^2+h}-\sqrt{h}}{h^{\frac{3}{2}}}\times\frac{\sqrt{h^2+h}+\sqrt{h}}{\sqrt{h^2+h}+\sqrt{h}}=\lim\limits_{h\to 0}\frac{h^2+h-h}{h^{\frac{3}{2}}h^{\frac{1}{2}}\left(\sqrt{h+1}+1\right)}$

$\lim\limits_{h\to 0}\frac{1}{\sqrt{1+h}+1}=\frac{1}{2}$

$\Rightarrow L.H.L.=R.H.L.=f(0)\Rightarrow p+2=\frac{1}{2}=q$

$=p=-\frac{3}{2}\quad \&\ q=\frac{1}{2}$

7. **(3)**

Define the limit near by the unity

Case-1$(x=1)$

$f\left(x\right)=\lim\limits_{n\to\infty}\frac{\log(2+1)-(1)^n\sin 1}{1+(1)^n}=\frac{\log(3)-\sin 1}{2}$

Case-2 $(0<x<1)$

$f\left(x\right)=\lim\limits_{n\to\infty}\frac{\log(2+x)-(x)^{2n}\sin x}{1+(x)^{2n}}$

Here $(x)^{2n}\to 0$ so

$f(x)=\log(2+x)$

Case-3 $(x>1)$

$f\left(x\right)=\lim\limits_{n\to\infty}\frac{\frac{\log(2+x)}{(x)^{2n}}-\sin x}{\frac{1}{(x)^{2n}}+1}$

Here $(x)^{2n}\to\infty$

so $f(x)=-\sin x$

$\lim\limits_{x\to 1^+}f(x)=-\sin 1$

$f(1)\neq f(1^+)\neq f(1^-)$

so function is not continuous at $x=1$

8. **(1)** Given, $f(x)= x - |x - x^2|$

At $x = 1$, $f(1)= 1 -|1 - 1|= 1$

$\lim_{x\to1^{-1}} f(x)= \lim_{h\to0}\left[(1 - h)-\left|\left(1 - h\right)-(1 - h)^2\right|\right]$

$= \lim_{h\to0} [(1 - h)-|h - h^2|]= 1$

$\lim_{x\to1^{+}} f(x)= \lim_{h\to0}\left[(1 + h)-\left|(1 + h)-(1 + h)^2\right|\right]$

$= \lim_{h\to0} [1 + h -|-h^2 - h|]= 1$

$\because \lim_{x\to1^{-1}} f(x)= \lim_{x\to1^{+}} = f\left(1\right)$

9. **(1)** (i) $f(x)=|x^3|$ is continuous and differentiable

(ii) $f(x)= \sqrt{|x|}$ is continuous

$f'(x)= \frac{1}{2\sqrt{|x|}} \cdot \frac{x}{|x|}$, does not exist at $x = 0$

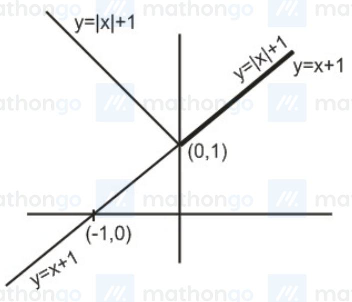
(iii) $f(x)=|\sin^{-1} x|$ is continuous

$f'(x)= \frac{\sin^{-1} x}{|\sin^{-1} x|} \cdot \frac{1}{\sqrt{1-x^2}}$ does not exist at $x = 0$

(iv) $f(x)= \cos^{-1}|x|$ is continuous

$f'(x)= \frac{-1}{\sqrt{1-x^2}} \cdot \frac{x}{|x|}$ does not exist at $x = 0$

10. **(1)** $f(x)$ = min $\{x + 1, |x|+1\}$



$\Rightarrow f(x)= x + 1, x \in R$

Hence $f(x)$ is differentiable for all ,$x \in R$