ANSWER KEYS

1. (4.00)	**2.** (1)	**3.** (3)	**4.** (1)	**5.** (2)	**6.** (3)	**7.** (3)	**8.** (1)
9. (3)	**10.** (3)						

1. (4.00)

Given, $\left(\frac{1+i}{1-i}\right)^n = 1$

$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n = 1$

$\Rightarrow \left(\frac{(1+i)^2}{1^2-i^2}\right)^n = 1$

$\Rightarrow \left(\frac{1^2+i^2+2i}{1-i^2}\right)^n = 1$

$\Rightarrow \left(\frac{2i}{2}\right)^n = 1$

$\Rightarrow i^n = 1$

Hence, n is an integer multiple of 4.

So, the smallest positive integer value of n is 4.

2. (1)

We have, $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow z = \frac{3+2i\cos\theta}{1-3i\cos\theta} \times \frac{1+3i\cos\theta}{1+3i\cos\theta}$

$\Rightarrow z = \frac{(3+2i\cos\theta)(1+3i\cos\theta)}{1+9\cos^2\theta}$

$\Rightarrow z = \frac{(3-6\cos^2\theta+8i\cos\theta)}{1+9\cos^2\theta}$

Now, $\text{Re}(z) = \frac{3-6\cos^2\theta}{1+9\cos^2\theta} = 0$

$\Rightarrow 3-6\cos^2\theta = 0$

$\Rightarrow \theta = \frac{\pi}{4}$

Hence, $\sin^2 3\theta + \cos^2\theta = \sin^2 3\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{4}\right) = 1.$

3. (3) Let $z = x + iy$

$\therefore \quad \left|\frac{z-25}{z-1}\right| = 5$

$\Rightarrow \quad \left|\frac{(x-25)+iy}{(x-1)+iy}\right| = 5$

$\Rightarrow \quad |(x-25)+iy| = 5|(x-1)+iy|$

$\Rightarrow \quad \sqrt{(x-25)^2+y^2} = 5\sqrt{(x-1)^2+y^2}$

On squaring both sides, we get

$(x-25)^2+y^2 = 25\left\{(x-1)^2+y^2\right\}$

$\Rightarrow \quad x^2-50x+625+y^2 = 25x^2-50x+25+25y^2$

$\Rightarrow \quad 24x^2+24y^2 = 600$

$\Rightarrow \quad x^2+y^2 = 25$

$\Rightarrow \quad \sqrt{x^2+y^2} = 5 \qquad \left[\because |z| = \sqrt{(x^2+y^2)}\right]$

$\Rightarrow \quad |z| = 5$

4. **(1)**
If a complex number if purely imaginary, then it must be equal to minus times its conjugate.
$\Rightarrow \frac{z-\alpha}{z+\alpha}=-\left(\frac{\bar{z}-\alpha}{\bar{z}+\alpha}\right)$
$\Rightarrow z\bar{z}+\alpha z-\alpha\bar{z}-\alpha^2=-\left(z\bar{z}-\alpha z+\alpha\bar{z}-\alpha^2\right)$
$\Rightarrow |z|^2=\alpha^2$
$\Rightarrow \alpha^2=4$
$\Rightarrow \alpha=\pm 2$

5. **(2)** $\sqrt{x^2+y^2}-x\leq 1$
$\Rightarrow \sqrt{x^2+y^2}\leq x+1$
$\Rightarrow \quad x^2+y^2\leq x^2+2x+1$
$\Rightarrow \quad y^2\leq 2x+1$

6. **(3)** $\frac{(1+\mathrm{i})^5\left(1+\sqrt{3}\mathrm{i}\right)^2}{-2\mathrm{i}\left(-\sqrt{3}+\mathrm{i}\right)}=\frac{\left(\sqrt{2}\right)^5\left(\frac{1}{\sqrt{2}}+\frac{\mathrm{i}}{\sqrt{2}}\right)^5.2^2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\mathrm{i}\right)^2}{(2\mathrm{i})\,2\left(\frac{\sqrt{3}}{2}-\frac{\mathrm{i}}{2}\right)}$

$\therefore$ argument $=\frac{5\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{2}+\frac{\pi}{6}=\frac{19\pi}{12}$

$\therefore$ principal argument is $-\frac{5\pi}{12}$

7. **(3)** Taking modulus and squaring on both sides
$|1+\mathrm{i}|^2\,.\,|1+2\mathrm{i}|^2\,\ldots\ldots\ldots\,|1+\mathrm{ni}|^2=|\alpha+\mathrm{i}\beta|^2$
$(1+1).\,(1+4)\ldots\ldots\ldots(1+\mathrm{n}^2)=\alpha^2+\beta^2$
$2\,.\,5\,.\,10\,.\,\ldots\ldots\ldots\ldots\,(1+\mathrm{n}^2)=\alpha^2+\beta^2$

8. **(1)**
Given that $|z_1|=|z_2|+|z_1-z_2|,\ |z_1|>|z_2|$
$\therefore\ |z_1-z_2|=|z_1|-|z_2|$
$=||z_1|-|z_2||$
Now, $arg z_1=arg z_2$
$\Rightarrow \arg\left(\frac{z_1}{z_2}\right)=0$
$\Rightarrow Im\left(\frac{z_1}{z_2}\right)=0.$

9. **(3)**
We have, $z^2=\bar{z}$
Let $z=x+iy$
$z^2=(x+iy)^2=x^2-y^2+i2xy$ (i)
$\bar{z}=x-iy\quad\ldots$(ii)
From (i) and (ii)
on equating imaginary parts
$\Rightarrow 2xy=-y$
$\Rightarrow y(2x+1)=0$
$\Rightarrow y=0$ or $x=-\frac{1}{2}$
on equating real parts
$\Rightarrow x^2-y^2=x$
Case 1: when $y=0$
$\Rightarrow x^2-x=0$
$\Rightarrow x(x-1)=0$
$\Rightarrow x=0$ or $x=1$
Case 2: when $x=-\frac{1}{2}$
$\Rightarrow \frac{1}{4}-y^2=-\frac{1}{2}$
$\Rightarrow y^2=\frac{3}{4}$
$\Rightarrow y=\pm\frac{\sqrt{3}}{2}$
hence there exist 4 solution of z

10. **(3)** $x+i(y+1)+\sqrt{2}\,|x+iy+1|=0$

$y+1=0 \Rightarrow\ y=-1 \because\ |x+i\,y+1|$ is real

so $x+\sqrt{2}\,|x\ -\ i+1|=0$

$(x+2)^2=0 \Rightarrow\ x=-2$