ANSWER KEYS

1. (3) **2.** (3) **3.** (4) **4.** (4) **5.** (3) **6.** (3) **7.** (1) **8.** (4)
9. (3) **10.** (2)

1. (3)

Here, $\overrightarrow{OA} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{OB} = 3\,\hat{i} + 4\hat{j} + 2\hat{k}$

And $\overrightarrow{OC} = 4\hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore\ \overrightarrow{AB} = \hat{i} + \hat{j} - 2\,\hat{k}$

$\overrightarrow{BC} = \hat{i} - 2\,\hat{j} + \hat{k}$

$\overrightarrow{CA} = 2\,\hat{i} - \hat{j} - \hat{k}$

Clearly, $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right| = \left|\overrightarrow{CA}\right| = \sqrt{6}$

So, these points are vertices of an equilateral triangle.

2. (3)

$\vec{\alpha} = \vec{a} + \vec{b} + \vec{c} = 6\hat{i} + 12\hat{j} \ldots (i)$

Let $\vec{\alpha} = x\,\vec{a} + y\,\vec{b}$

$\vec{\alpha} = x\left(6\hat{i} - 3\hat{j}\right) + y\left(2\hat{i} - 6\hat{j}\right)$

$\Rightarrow \vec{\alpha} = (6x + 2y)\hat{i} - (3x + 6y)\hat{j} \ldots (ii)$

From (i) & (ii)

$\Rightarrow 6x + 2y = 6$ and $-3x - 6y = 12$

$\therefore\ x = 2,\ y = -3$

$\therefore\ \alpha = 2\vec{a} - 3\,\vec{b}$

3. (4)

We know, if $\vec{P}$ is collinear with $\vec{Q}$, then $\vec{P} = \beta\vec{Q}$, where β is a non-zero scalar.

Given, $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero vectors which are pairwise non-collinear.

Also, given $\vec{a} + 3\,\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 2\,\vec{c}$ is collinear with $\vec{a}$

$\therefore\ \vec{a} + 3\,\vec{b} = \lambda\vec{c}\ \ \ldots (i)$

And $\vec{b} + 2\,\vec{c} = \mu\,\vec{a}\ \ \ldots\ldots (ii)$

where $\lambda,\ \mu$ are non-zero scalars.

From equation (i) and (ii)

$\Rightarrow \vec{b} = \frac{\lambda}{3}\vec{c} - \frac{1}{3}\vec{a} = -2\,\vec{c} + \mu\,\vec{a}$

$\Rightarrow \lambda = -6,\ \mu = \frac{-1}{3}$

Put, $\lambda = -6$ in the equation (i)

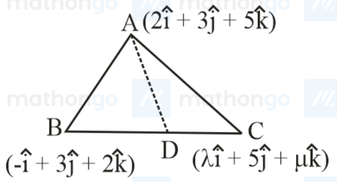
$\Rightarrow \vec{a} + 3\,\vec{b} = -6\,\vec{c}$

$\Rightarrow \vec{a} + 3\,\vec{b} + 6\,\vec{c} = 0$

4. **(4)**

Given that $A\left(2\hat{i}+3\hat{j}+5\hat{k}\right)$, $B\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$ and $C\left(\lambda\hat{i}+5\hat{j}+\mu\hat{k}\right)$ are vertices of a triangle

Since, D is the mid-point of B and C



So, position vector of D

$=\left(\frac{\lambda-1}{2}\right)\hat{i}+4\hat{j}+\left(\frac{\mu+2}{2}\right)\hat{k}$

Direction ratio of AD is $\left(\frac{\lambda-5}{2},\ 1,\ \frac{\mu-8}{2}\right)$ or $\left(\frac{\lambda-5}{2\sqrt{3}},\ \frac{1}{\sqrt{3}},\ \frac{\mu-8}{2\sqrt{3}}\right)$.

But direction cosine of AD should be

$\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}}$

Therefore,

$\left(\frac{\lambda-5}{2}\right)=1=\left(\frac{\mu-8}{2}\right)$

$\Rightarrow \lambda=7,\ \mu=10$

$\Rightarrow 2\lambda-\mu=4$

5. **(3)**

Since $\vec{r}$ is collinear with $\vec{c}$, we can write,

$\vec{r}=k\vec{c}$

$\left|\vec{r}\right|=|k|\left|\vec{c}\right|$

Given,

$\left|\vec{r}\right|=\frac{\left|\vec{a}+\vec{b}\right|}{2}$

$\therefore \frac{\left|\vec{a}+\vec{b}\right|}{2}=|k|\left|\vec{c}\right|$

$\left|6\hat{i}-2\hat{j}-4\hat{k}\right|=2|k|\left|3\hat{i}-\hat{j}+2\hat{k}\right|$

$\sqrt{56}=2|k|\sqrt{14}$

$\therefore k=\pm 1$

$\therefore \vec{r}=\pm\vec{c}$

6. **(3)** $\vec{a}+\vec{b}=-\sqrt{3}\,\vec{c}$

7. **(1)** Given $\left|\vec{A}+\vec{B}\right|=n\left|\vec{A}-\vec{B}\right|$

$\sqrt{A^2+B^2+2AB\cos\theta}=n\left(\sqrt{A^2+B^2-2AB\cos\theta}\right)$

Also $|A|=|B|$

$\sqrt{2A^2+2A^2\cos\theta}=n\sqrt{2A^2-2A^2\cos\theta}$

Squaring both sides:

$2A^2(1+\cos\theta)=n^2 2A^2(1-\cos\theta)$

$\cos\theta=\frac{n^2-1}{n^2+1}$

i.e., $\theta=\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

8. **(4)**

Since, $\vec{a} \cdot \left(\vec{b} + \vec{c}\right) = 0$

$\Longrightarrow \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$

Similarly, $\vec{b} \cdot \vec{c} + \vec{b} \cdot \vec{a} = 0 \& \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} = 0$

$\therefore \ \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} = 0 \quad \ldots(1)$

Given, $\left|\vec{a} + \vec{b}\right| = 6$

$\Longrightarrow \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a} \cdot \vec{b} = 36 \quad \ldots(2)$

Similarly, $\left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\vec{b} \cdot \vec{c} = 64 \quad \ldots(2)$

$\& \left|\vec{c}\right|^2 + \left|\vec{a}\right|^2 + 2\vec{c} \cdot \vec{a} = 100 \quad \ldots(4)$

On adding Eqs. (2),(3) and (4), we get

$2|\vec{a}||^2 + 2|\vec{b}|^2 + 2|\vec{c}|^2 + 2\left(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}\right) = 200$

$\Longrightarrow \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 = 100 \quad \ldots(5)$ [from Eqs. (1)]

Now, $\left|\vec{a} + \vec{b} + \vec{c}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\left(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}\right)$

$\Longrightarrow \left|\vec{a} + \vec{b} + \vec{c}\right|^2 = 100$ [from Eqs. (1) and (5)]

$\Longrightarrow \left|\vec{a} + \vec{b} + \vec{c}\right| = 10$

9. **(3)**

Given that, $\left|\vec{\mathbf{a}}\right| = 2\sqrt{2}, \left|\vec{\mathbf{b}}\right| = 3$

The longer vectors is $5\vec{\mathbf{a}} + 2\vec{\mathbf{b}} + \vec{\mathbf{a}} - 3\vec{\mathbf{b}} = 6\vec{\mathbf{a}} - \vec{\mathbf{b}}$

Length of one diagonal

$= \left|6\vec{\mathbf{a}} - \vec{\mathbf{b}}\right|$

$= \sqrt{36\vec{\mathbf{a}}^2 + \vec{\mathbf{b}}^2 - 2 \times 6\left|\vec{\mathbf{a}}\right|\left|\vec{\mathbf{b}}\right|\cos 45^\circ}$

$= \sqrt{36 \times 8 + 9 - 12 \times 2\sqrt{2} \times 3 \times \frac{1}{\sqrt{2}}}$

$= \sqrt{288 + 9 - 12 \times 6} = \sqrt{225} = 15$

Other diagonal is $4\vec{\mathbf{a}} + 5\vec{\mathbf{b}}$.

Its length $= \sqrt{16 \times 8 + 25 \times 9 + 40 \times 6} = \sqrt{593}$

10.

(2) Required value $= \dfrac{\frac{\vec{b} \,.\, \vec{a}}{\left|\vec{b}\right|}}{\frac{\vec{a} \,.\, \vec{b}}{\left|\vec{a}\right|}} = \dfrac{\left|\vec{a}\right|}{\left|\vec{b}\right|} = \frac{7}{3}$.