1. Let a curve $y=f(x)$ pass through the point $\left(2,(\log_e 2)^2\right)$ and have slope $\frac{2y}{x\log_e x}$ for all positive real value of x. Then the value of $f(e)$ is equal to ______.

2. A continuous function $f: R\rightarrow R$ satisfies the differential equation $f(x)=(1+x^2)\left[1+\int_0^x \frac{(f(t))^2}{1+t^2}dt\right]$, then $f(-3)$ is

(1) $\frac{13}{10}$
(2) $\frac{8}{5}$
(3) $\frac{10}{13}$
(4) $\frac{5}{8}$

3. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$. If $p(0)=100$, then $p(t)$ equals :

(1) $600-500e^{\frac{t}{2}}$
(2) $400-300e^{-\frac{t}{2}}$
(3) $400-300e^{\frac{t}{2}}$
(4) $300-200e^{-\frac{t}{2}}$

4. If $\cos^2 x\frac{dy}{dx}-y\tan 2x=\cos^4 x$, where $|x|<\frac{\pi}{4}$ and $y\left(\frac{\pi}{6}\right)=\frac{3\sqrt{3}}{8}$, then the constant term in the solution of the differential equation is

(1) 1
(2) 0
(3) -1
(4) 2

5. The solution of the differential equation $(1-x^2)\cdot\frac{dy}{dx}+xy=(x-x^3)y^{\frac{1}{2}}$, $(\forall|x|<1)$ is $\sqrt{9y}=-f(x)+c(1-x^2)^{\frac{1}{4}}$, where c is an arbitrary constant and $f\left(\frac{1}{2}\right)=\frac{3}{4}$. Then, $f(x)$ is

(1) an odd function
(2) an even function
(3) a periodic function
(4) symmetric about line $x=1$

6. The solution of the equation $(xy^4+y)dx-xdy=0$ is

(1) $4x^4y^3+3x^3=Cy^3$
(2) $3x^3y^4+4x^3=Cx^3$
(3) $3x^4y^3+4x^3=Cy^3$
(4) $3x^4y^3+4x^3=Cx^3$

7. The solution of the differential equation $y(\sin^2 x)dy+(\sin x\cos x)y^2dx=xdx$ is (where C is the constant of integration)

(1) $\sin^2 x\cdot y=x^2+C$
(2) $\sin^2 x\cdot y^2=x^2+C$
(3) $\sin x\cdot y^2=x^2+C$
(4) $\sin^2 x\cdot y^2=x+C$

8. The solution of $\frac{x^3dx+yx^2dy}{\sqrt{x^2+y^2}}=ydx-xdy$ is: { Where C is an arbitrary constant }

(1) $\sqrt{x^2+y^2}=Cx$
(2) $\sqrt{x^2+y^2}+y/x=C$
(3) $\sqrt{x^2+y^2}+y/x^2=C$
(4) $(x^2+y^2)^2+xy^2=C$

9. The curve satisfying the differential equation $\frac{dx}{dy}=\frac{x+2yx^2}{y-2x^3}$ and passing through $(1,\ 0)$ is given by

(1) $x^2+y^2=1$
(2) $x^2+y^2+\frac{y}{x}=1$
(3) $y^2-\frac{y}{x}-x^2=-1$
(4) $x^2-y^2=1$

10. If the solution of the differential equation $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$ is $x+kye^{\frac{x}{y}}=C$ (where, C is an arbitrary constant), then the value of k is equal to