ANSWER KEYS

1. (3)	**2.** (4)	**3.** (3)	**4.** (1)	**5.** (4)	**6.** (15)	**7.** (2)	**8.** (3125)
9. (190)	**10.** (4)	**11.** (4)	**12.** (3)	**13.** (1)	**14.** (4)	**15.** (360)	**16.** (432)
17. (1)	**18.** (4)	**19.** (2)	**20.** (3)	**21.** (3)	**22.** (26)	**23.** (3)	**24.** (3)
25. (1)	**26.** (10)	**27.** (4)	**28.** (4)	**29.** (2039)	**30.** (4)		

1. (3)

The domain of the function

$f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$

For, $\cos^{-1}\left(\frac{x-1}{x+1}\right)$

$-1 \le \frac{x-1}{x+1} \le 1$

$\Rightarrow -1 \le 1-\frac{2}{x+1} \le 1$

$\Rightarrow -2 \le \frac{-2}{x+1} \le 0$

$\Rightarrow 0 \le \frac{1}{x+1} \le 1$

$\Rightarrow x+1 \in [1,\infty)$

$\Rightarrow x \in [0,\infty) \ldots .(i)$

For, $\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)$

$-1 \le \frac{3x^2+x-1}{(x-1)^2} \le 1$

$\Rightarrow -(x-1)^2 \le 3x^2+x-1 \le (x-1)^2,\ x \ne 1$

Now, $-(x-1)^2 \le 3x^2+x-1,\ x \ne 1$

$\Rightarrow 4x^2-x \ge 0\ ,\ x \ne 1$

$\Rightarrow x(4x-1) \ge 0,\ x \ne 1$

$\Rightarrow x \in \left(-\infty,\ 0\right] \cup \left[\frac{1}{4},\ \infty\right)-\{1\} \ \ldots .(ii)$

And $3x^2+x-1 \le (x-1)^2,\ x \ne 1$

$\Rightarrow 2x^2+3x-2 \le 0,\ x \ne 1$

$\Rightarrow (x+2)(2x-1) \le 0,\ x \ne 1$

$\Rightarrow x \in \left[-2,\ \frac{1}{2}\right] \ \ldots .(iii)$

Domain of the function $\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)$ from the equations (ii) & (iii) is

$\Rightarrow x \in [-2,\ 0] \cup \left[\frac{1}{4},\ \frac{1}{2}\right] \ldots .(iv)$

Now the domain of the given function will be the intersection of the equation (i) & (iv)

Hence, domain is $x \in \left[\frac{1}{4},\ \frac{1}{2}\right] \cup \left\{0\right\}$

2. (4)

$f(x)=\ln\left(4x^2+11x+6\right)+\sin^{-1}(4x+3)$

$+\cos^{-1}\left(\frac{10x+6}{3}\right)$

(i) $4x^2+11x+6>0$

$4x^2+8x+3x+6>0$

$(4x+3)(x+2)>0$

$x \in(-\infty,-2) \cup\left(-\frac{3}{4}, \infty\right)$

(ii) $4x+3 \in[-1,1]$

$x \in[-1,-1/2]$

(iii) $\frac{10x+6}{3} \in[-1,1]$

$x \in\left[-\frac{9}{10},-\frac{3}{10}\right]$

$x \in\left(-\frac{3}{4},-\frac{1}{2}\right] \quad \alpha=-\frac{3}{4}, \beta=-\frac{1}{2}$

$\alpha+\beta=-\frac{5}{4}$

$36|\alpha+\beta|=45$

3. **(3)** We have,

$f(x)=\sqrt{\frac{|[x]|-2}{|[x]|-3}}$

For domain,

$\left(\frac{|[x]|-2}{|[x]|-3}\right)\geq 0$

Case I : When $|[x]|-2\geq 0$ and $|[x]|-3>0$, then

$\therefore$ x $\in(-\infty,-3)\cup[4,\infty)$ $\quad\ldots(1)$

Case II : When $|[x]|-2\leq 0$ and $|[x]|-3<0$, then

$\therefore\ x\in[-2,3)\quad\ldots(2)$

So, from (1) and (2), we get

Domain of function is

$(-\infty,-3)\cup[-2,3)\cup[4,\infty)$

$\therefore\ (a+b+c)=-3+(-2)+3=-2$

$(\because\ a<b<c)$

4. **(1)**

Given,

$f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x)=\log_{\sqrt{m}}\left\{\sqrt{2}(\sin x-\cos x)+m-2\right\}$, for some m,

Also given the range of f is $[0,2]$,

Now we know that,

$-\sqrt{2}\leq\sin x-\cos x\leq\sqrt{2}$

$\Rightarrow -2\leq\sqrt{2}(\sin x-\cos x)\leq 2$

(Assuming $\sqrt{2}(\sin x-\cos x)=k$)

$\Rightarrow -2\leq k\leq 2\qquad\ldots(1)$

Now taking function $f(x)=\log_{\sqrt{m}}(k+m-2)$

Given, $0\leq f(x)\leq 2$

$\Rightarrow 0\leq\log_{\sqrt{m}}(k+m-2)\leq 2$

$\Rightarrow 1\leq k+m-2\leq m$

$\Rightarrow -m+3\leq k\leq 2\qquad\ldots(2)$

Now from equations (1) & (2), we get

$-m+3=-2$

$\Rightarrow$ m $=5$

5. **(4)**

Given:

$f(x)=\frac{[x]}{1+x^2}$

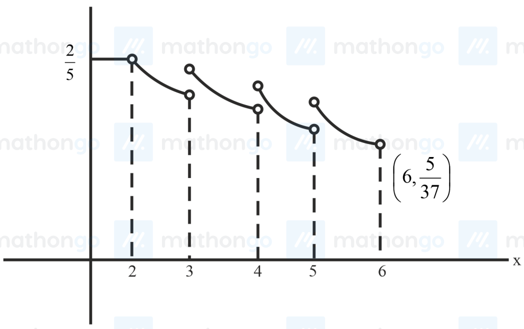
So,

$f(x)=\frac{2}{1+x^2};\qquad x\in[2,3)$

$f(x)=\frac{3}{1+x^2};\qquad x\in[3,4)$

$f(x)=\frac{4}{1+x^2};\qquad x\in[4,5)$

$f(x)=\frac{5}{1+x^2};\qquad x\in[5,6)$



Hence,

$f(x)\in\left(\frac{5}{37},\frac{2}{5}\right]$

6. (15)

Given,

$f(x)=|5x-7|+\left[x^2+2x\right]$

$=|5x-7|+\left[(x+1)^2-1\right]$

$=|5x-7|+\left[(x+1)^2\right]-1$

(as $[x-1]=[x]$-1 where [.] is greatest integer function)

Now critical points of

$f(x)=\frac{7}{5}, \sqrt{5}-1, \sqrt{6}-1, \sqrt{7}-1, \sqrt{8}-1, 2$

$\therefore$ Maximum or minimum value of $f(x)$ occur at critical points or boundary points.

So, $f\left(\frac{5}{4}\right)=\frac{3}{4}+4=\frac{19}{4}$ and $f\left(\frac{7}{5}\right)=0+4=4$

As both $|5x-7|$ and x^2+2x are increasing in nature after $x=\frac{7}{5}$

So, $f(2)=|10-7|+[4+4]=3+8=11$

$\therefore f\left(\frac{7}{5}\right)_{\min}=4$ and $f(2)_{\max}=11$

So sum of minimum and maximum value is $4+11=15$

7. (2)

Given $f(x)=\left[2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}$

$f(x)=\left[\left(2-x^{25}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}$

$=\left(4-x^{50}\right)^{\frac{1}{50}}$

i.e. $f(f(x))=\left(4-\left(\left(4-x^{50}\right)^{\frac{1}{50}}\right)^{50}\right)^{\frac{1}{50}}=x$

Now $g(x)=f(f(f(x)))+f(f(x))$

$=f(x)+x$

So $g(1)=f(1)+1=3^{\frac{1}{50}}+1$

Hence $[g(1)]=\left[3^{\frac{1}{50}}+1\right]=2$

8. (3125)

Given:

$f^1(x)=\frac{3x+2}{2x+3}$

Now,

$f^2(x)=f^1 o f^1(x)$

$\Rightarrow f^2(x)=\frac{3\left(\frac{3x+2}{2x+3}\right)+2}{2\left(\frac{3x+2}{2x+3}\right)+3}$

$\Rightarrow f^2(x)=\frac{9x+6+4x+6}{6x+4+6x+9}$

$\Rightarrow f^2(x)=\frac{13x+12}{12x+13}$

$\Rightarrow f^3(x)=\frac{3\left(\frac{13x+12}{12x+13}\right)+2}{2\left(\frac{13x+12}{12x+13}\right)+3}$

$\Rightarrow f^3(x)=\frac{63x+62}{62x+63}$

$\Rightarrow f^4(x)=\frac{3\left(\frac{63x+62}{62x+63}\right)+2}{2\left(\frac{63x+62}{62x+63}\right)+3}$

$\Rightarrow f^4(x)=\frac{313x+312}{312x+313}$

$\therefore f^5(x)=\frac{1563x+1562}{1562x+1563}=\frac{ax+b}{bx+a}$

$\therefore a+b=1563+1562=3125$

9. (190)

Given, $f(n)=\begin{cases} 2n, & \text{if } n=1,2,3,4,5 \\ 2n-11 & \text{if } n=6,7,8,9,10 \end{cases}$

So, $f(1)=2$, $f(2)=4$, ...$f(5)=10$

And $f(6)=1$, $f(7)=3$, $f(8)=5$ ......$f(10)=9$

$f(g(n))=\begin{cases} n+1 & ; \ n\in \text{ odd} \\ n-1 & ; \ n\in \text{ even} \end{cases}$

So, $f(g(10))=9 \Rightarrow g(10)=10$

$f(g(1))=2 \Rightarrow g(1)=1$

$f(g(2))=1 \Rightarrow g(2)=6$

$f(g(3))=4 \Rightarrow g(3)=2$

$f(g(4))=3 \Rightarrow g(4)=7$

$f(g(5))=6 \Rightarrow g(5)=3$

So, $g(10)\cdot[g(1)+g(2)+g(3)+g(4)+g(5)]$

$=10\cdot[1+6+2+7+3]=190$

10. (4)

$fog(x)=f(g(x)) \ = f\left(\frac{x^2}{x^2+1}\right)$

$=\frac{x^2}{x^2+1}-1=\frac{x^2-x^2-1}{x^2+1}=\frac{-1}{x^2+1}$

We know that, $0\le x^2<\infty$, $\forall x\in R$

$\Rightarrow 1\le x^2+1<\infty$, $\forall x\in R \Rightarrow 1\ge \frac{1}{x^2+1}>0$, $\forall x\in R \ \Rightarrow -1\le \frac{-1}{x^2+1}<0$, $\forall x\in R$

So, range of $fog(x)$ is $[-1,\ 0)\subset R$.

Hence, the function $fog(x)$ is into function and $fog(-x)=f(g(-x))=\frac{-1}{(-x)^2+1}=\frac{-1}{x^2+1}=f(g(x))$

$\therefore$ $fog(x)$ is an even function. So, it is a many one function.

Hence, $fog(x)$ is neither one-one nor onto function.

11. (4) $f:(0,\infty)\to(0,\infty)$

$f(x)=\left|1-\frac{1}{x}\right|$ is not a function

$\because$ $f(1)=0$ and $1\in$ domain but $0\notin$ co-domain

Hence, $f(x)$ is not a function.

12. (3)

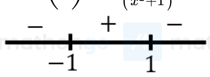
Given,

$f(x)=\frac{x^2+2x+1}{x^2+1}$

$f'\left(x\right)=\frac{(x^2+1)(2x+2)-(x^2+2x+1)2x}{(x^2+1)^2}$

$\Rightarrow f'\left(x\right)=\frac{\cancel{2x^3}+2x^2+\cancel{2x}+2-\cancel{2x^3}-4x^2-\cancel{2x}}{(x^2+1)^2}$

$\Rightarrow f'\left(x\right)=\frac{2(1-x^2)}{(x^2+1)^2}=-\frac{2(x+1)(x-1)}{(x^2+1)^2}$



Clearly $f(x)$ is one-one in $(-\infty,-1)$ and also in $(1,\infty)$ but $f(x)$ is not one-one in $(-\infty,\infty)$

13. (1)

For one value of n we will get only one corresponding value of $f(n)$, so $f(n)$ is one-one

Now, for $n=2,4,6\cdots$; $f(n)=4,8,12\ldots$

for $n=3,7,11\cdots$; $f\ (n)=2,6,10\ldots$

for $n=1,5,9\cdots$; $f(n)=1,3,5,7\ldots$

So range of $f(n)$ is N

Hence $f(n)$ is onto

14. (4)

$f(x)=\begin{cases} x+1 & \text{if } x \text{ is odd} \\ x & \text{if } x \text{ is even} \end{cases}$

$\because$ $g:A\to$A such that $g(f(x))=f(x)$

$\Rightarrow$ If x is even then $g(x)=x \quad \ldots(1)$

If x is odd then $g(x+1)=x+1 \quad \ldots(2)$

from (1) and (2) we can say that $g(x)=x$ if x is even

$\Rightarrow$ If x is odd then $g(x)$ can take any value in set A

so number of $g(x)=10^5\times 1$

15. (360) $f(1)+f(2)+1=f(4)\leq 6$

$f(1)+f(2)\leq 5$

Case (i) $f(1)=1\Rightarrow f(2)=1,2,3,4\Rightarrow 4$ mappings

Case (ii) $f(1)=2\Rightarrow f(2)=1,2,3\Rightarrow 3$ mappings

Case (iii) $f(1)=3\Rightarrow f(2)=1,2\Rightarrow 2$ mappings

Case (iv) $f(1)=4\Rightarrow f(2)=1\Rightarrow 1$ mapping

$f(5)$ & $f(6)$ both have 6 mappings each

Number of functions $=(4+3+2+1)\times 6\times 6=360$

16. (432)

Given,

$f(m\cdot n)=f(m)\cdot f(n)$

Taking $m=1$

$\Rightarrow f(1\cdot n)=f(1)\cdot f(n)$

$\Rightarrow f(1)=1$

Now taking $m=3$ & $n=3$ we get,

$f(9)=f(3)\times f(3)$,

So, $(f(3),\ f(9))$ have two possibility $(1,1)$ & $(3,9)$

Now taking $m=2$ & $n=1$ we get,

$f(2\cdot 1)=f(2)\cdot f(1)$

Now here $f(1)=1$, so $f(2)$ can take all 6 numbers,

Similarly, for $f(5)$ & $f(8)$ there will be 6 ways each,

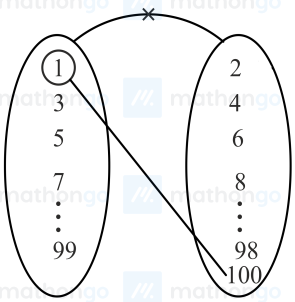
So, total possible function will be, $=1\times 6\times 2\times 6\times 6\times 1=432$

17. (1)

Given, $f(1,3,5,7,\cdots,99)\rightarrow(2,4,6,8,\cdots,100)$

The number of elements in domain and codomain is 99.

Now, let us assume $f(1)=100$



Now as per diagram we have only 1 way for arranging $f(3)>f(5)>f(7)\cdots>f(99)$,

Similarly if we choose $f(1)=98$ then again we have only 1 way for arranging $f(3)>f(5)>f(7)\cdots>f(99)$,

So we can see the arrangement $f(3)>f(5)>f(7)\cdots>f(99)$ depends upon $f(1)$,

So number of ways for choosing $f(1)$ is ${}^{50}C_1$

18. (4)

Total cases will be ${}^5C_4\times 4!$

Now favourable cases for $2f(b)=f(c)+f(d)-f(a)$ will be,

Case (I) if $f(b)=1$ then $f(c),\ f(d),\ f(a)$ can take the value $3,4,5$ & $4,3,5$ respectively

Case (II) if $f(b)=2$ then $f(c),\ f(d),\ f(a)$ can take value $3,5,4$ & $5,3,4$ respectively

Case (III) if $f(b)=3$ then $f(c),\ f(d),\ f(a)$ can take value $2,5,1$ & $5,2,1$ respectively

So total favourable case will be 6

So probability$=\frac{\text{favourable cases}}{\text{total outcomes}}=\frac{6}{5\times 4!}=\frac{1}{20}$

19. **(2)**

Given,

$f(x)+f\left(\frac{1}{1-x}\right)=1+x$

$x=2 \Rightarrow f(2)+f(-1)=3 \quad \ldots(1)$

$x=-1 \Rightarrow f(-1)+f\left(\frac{1}{2}\right)=0 \quad \ldots(2)$

$x=\frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right)+f(2)=\frac{3}{2} \quad \ldots(3)$

On subtracting equation (3) and equation (2) we get,

$f\left(2\right)-f\left(-1\right)=\frac{3}{4} \ldots\ldots\ldots\left(4\right)$

On adding equation (1) and equation (4) we get,

$2f(2)=\frac{9}{2}$

$\Rightarrow f(2)=\frac{9}{4}$

20. **(3)**

$f(x)=\frac{5^x}{5^x+5}$ and $f(2-x)=\frac{5}{5^x+5}$

$f(x)+f(2-x)=1$

$\Rightarrow f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+\ldots+f\left(\frac{39}{20}\right)$

$=\left(f\left(\frac{1}{20}\right)+f\left(\frac{39}{20}\right)\right)+\ldots+\left(f\left(\frac{19}{20}\right)+f\left(\frac{21}{20}\right)\right)+f\left(\frac{20}{20}\right)$

$=19+\frac{1}{2}=\frac{39}{2}$

21. **(3)**

Let

$f(x)=x^3-ax^2+bx-c$

$f'(x)=3x^2-2ax+b$

$f''(x)=6x-2a$

$f'''(x)=6$

Also,

$f'(1)=a$

$\Rightarrow 3-2a+b=a$

$\Rightarrow 3a=b+3$

Also,

$f''(2)=12-2a=b$

$\Rightarrow 12-2a=3a-3$

$\Rightarrow 5a=15 \Rightarrow a=3$

$\Rightarrow b=6$

And,

$f'''(3)=c \Rightarrow c=6$

Hence,

$f(x)=x^3-3x^2+6x-6$

Now,

$f(3)+2f(0)=27-27+18-6-12=0$

$f(2)+f(1)=8-12+12-6+1-3+6-6=0$

So,

$f(3)+2f(0)=f(2)+f(1)$

$\Rightarrow 2f(0)-f(1)+f(3)=f(2)$

22. (26)

Given that: $kf(k)+2=0$ for $k=2,3,4,5$ which means $(k-2),(k-3),(k-4),(k-5)$ are the factors of this expression.

Let $kf(k)+2=a(k-2)(k-3)(k-4)(k-5)\ldots.$(i)

Put $k=0$

$2=a(-2)(-3)(-4)(-5)$

$a=\frac{1}{60}$

Put $a=\frac{1}{60}$ in (i), we get

$kf\left(k\right)+2=\frac{1}{60}\left(k-2\right)\left(k-3\right)\left(k-4\right)\left(k-5\right)$

Now, put $k=10$

$10f\left(10\right)+2=\frac{1}{60}\times 8\times 7\times 6\times 5$

$10\ f(10)=26$

So, $52-10\ f(10)=26$

23. (3)

Given,

$f(x+y)=f(x)\cdot f(y)$ and $f(1)=3$

Now taking $x=1$ & $y=1$ we get,

$f(1+1)=f(1)\cdot f(1)\Rightarrow f(2)=3^2$

Similarly $f(2+1)=f(2)\cdot f(1)\Rightarrow f(3)=3^2\times 3=3^3$

And so on $f(n)=3^n$

So, $\sum_{r=1}^{n} f(r)=3279$

$\Rightarrow f(1)+f(2)+f(3).\ldots\ldots f(n)=3279$

$\Rightarrow 3+3^2+3^3.\ldots\ldots+3^n=3279$

$\Rightarrow 3\times\frac{3^n-1}{3-1}=3279$

$\Rightarrow\frac{3^n-1}{2}=1093$

$\Rightarrow 3^n-1=2186$

$\Rightarrow 3^n=2187$

$\Rightarrow 3^n=3^7$

$\Rightarrow n=7$

24. (3)

Given,

Functional equation $f(x+y)=f(x)+f(y)-1$

Now taking $x=0$ & $y=0$ in above equation we get,

$f(0+0)=f(0)+f(0)-1\Rightarrow f(0)=1$

Now we know that,

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x)=\lim_{h\to 0}\frac{f(x)+f(h)-1-f(x)}{h}$

$\Rightarrow f'(x)=\lim_{h\to 0}\frac{f(h)-1}{h}$

Now let $\lim_{h\to 0}\frac{f(h)-1}{h}=k$

So, the equation becomes $f'(x)=k$

Now putting $x=0$ in above equation we get,

$f'(0)=k\Rightarrow k=2$ {as given $f'(0)=2$}

So, $f'(x)=2$

Now integrate both side we get,

$f(x)=2x+c$

Now again taking $x=0$ we get,

$f(0)=2\times 0+c\Rightarrow c=1$ {as $f(0)=1$}

So, $f(x)=2x+1$

And $f(-2)=2\times(-2)+1\Rightarrow f(-2)=-3$

Hence, $|f(-2)|=3$

25. (1)

Given functional equation is

$f(m+n)= f(m)+f(n); m, n \in N \ldots .(1)$

Put $m=n=3$

$f(3+3)=f(3)+f(3)$

$\Rightarrow f(3)=9[\because f(6)=18]$

Put $m=2, n=1$ in equation (1)

$\therefore f(3)=f(2+1)=f(2)+f(1)$

$=f(1+1)+f(1)$

$=f(1)+f(1)+f(1)$

$9=3 f(1)$

$\Rightarrow f(1)=3$

$\therefore f(2)=f(1+1)=f(1)+f(1)=6$

Now,

$f(2) \cdot f(3)=(6)(9)=54$

26. (10)

$\because \lim\limits_{x \rightarrow 0} \frac{f(x)}{x}=1 \Rightarrow f'(0)=1$

$f(x+y)= f(x)+f(y)+x y^2+x^2 y$

Differentiate w.r.t. x keeping y constant

$f'(x+y)= f'(x)+0+y^2+2 x y$

put $y=-x$

$f'(0)= f'(x)+x^2-2 x^2$

$1=f'(x)-x^2$

$f'(x)=1+x^2$

$f'(3)=10.$

27. (4)

Given,

$f(\mathrm{n})+\frac{1}{\mathrm{n}} f(\mathrm{n}+1)=1, \forall \mathrm{n} \in\{1,2,3\}$

$\Rightarrow n f(\mathrm{n})+f(\mathrm{n}+1)=n$

At $n=1$,

$f(1)+f(2)=1 \quad \ldots .(1)$

At $n=2$,

$2 f(2)+f(3)=2 \quad \ldots .(2)$

At $n=3$,

$3 f(3)+f(4)=3 \quad \ldots .(3)$

Put the value of $f(2)$ from equation (1) in equation (2),

$2(1-f(1))+f(3)=2$

$\Rightarrow f(3)=2 f(1) \quad \ldots .(4)$

Put the value of $f(3)$ from equation (4) in equation (3),

$3(2 f(1))+f(4)=3$

$\Rightarrow f(4)=3-6 f(1)$

$\because f:\{1,2,3,4\} \rightarrow\{\mathrm{a} \in \mathbb{Z}:|\mathrm{a}| \leq 8\}$

$\Rightarrow-8 \leq f(4) \leq 8$

$\Rightarrow-8 \leq 3-6 f(1) \leq 8$

$\Rightarrow-11 \leq-6 f(1) \leq 5$

$\Rightarrow \frac{-5}{6} \leq f(1) \leq \frac{11}{6}$

$\Rightarrow f(1)=0,\ 1$

Case I: $f(1)=0$

$\Rightarrow f(2)=1,\ f(3)=0,\ f(4)=3$

Case II: $f(1)=1$

$\Rightarrow f(2)=0,\ f(3)=2,\ f(4)=-3$

Therefore, two such functions are possible.

28. **(4)** Greatest integer function

$$f(x)=[x]=\begin{cases}0 & ;\ 0\le x<1\\ -1 & ;\ -1\le x<0\\ -2 & ;\ -2\le x<-1\end{cases}$$

Given series

$$S=\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\left[-\frac{1}{3}-\frac{3}{100}\right]+\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right]$$

General term $T_r=\left[-\frac{1}{3}-\frac{r}{100}\right]=\begin{cases}-1 & 0\le r\le 66\\ -2 & r>66\end{cases}$

$\Rightarrow S=\sum_{r=0}^{66}(-1)+\sum_{r=67}^{99}(-2)=(-67)+(-2)\times 33$

$=-133$

29. **(2039)**

Given:

$f(x)=ax-3$

$g(x)=x^b+c, x\in\mathbb{R}\ (fog)^{-1}(x)=\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

Now, let

$h(x)=(f\circ g)(x)$

$\Rightarrow h^{-1}(x)=\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

Let

$y=\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

$\Rightarrow y^3=\left(\frac{x-7}{2}\right)$

$\Rightarrow x=2y^3+7$

So, inverse of $h^{-1}(x)$ is $2x^3+7$ i.e.,

$h(x)=fog(x)=2x^3+7$

Also,

$fog(x)=a\left(x^b+c\right)-3$

$\Rightarrow ax^b+ac-3=2x^3+7$

On comparing, we get

$a=2,\ b=3, c=5$

So,

$f(x)=2x-3$

$g(x)=x^3+5$

Now,

$fog\ (ac)=fog(10)=f(1005)=2007$

$(gof)(b)=gof(3)=g(3)=32$

Therefore,

$fog\ (ac)+(gof)(b)=2007+32=2039$

30. **(4)**

Given:

$x^2-4x+[x]+3=x[x]$

$\Rightarrow x^2-4x+3=x[x]-[x]$

$\Rightarrow(x-1)(x-3)=[x](x-1)$

$\Rightarrow x=1$ or $x-3=[x]$

If $x-3=[x]$

$\Rightarrow x-[x]=3$

$\Rightarrow\{x\}=3$

which is not Possible, since $\{x\}\in[0,1)$.

Hence, $x=1$ is the only one solution in $(-\infty,\infty)$.