1. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2. Then $\alpha + r$ is equal to _____

2. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C\left(\sqrt{2}, \sqrt{3}\right)$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to _____

3. Let the mirror image of a circle $c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$ in line $y = x + 1$ be $c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$. If r is the radius of circle c_2, then $\alpha + 6r^2$ is equal to _____

4. Let a triangle ABC be inscribed in the circle $x^2 - \sqrt{2}(x + y) + y^2 = 0$ such that $\angle BAC = \frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the $\triangle ABC$ is equal to:
(1) 1
(2) $\frac{\left(\sqrt{6}+\sqrt{3}\right)}{2}$
(3) $\frac{\left(\sqrt{3}+\sqrt{3}\right)}{2}$
(4) $\frac{\left(\sqrt{6}+2\sqrt{3}\right)}{4}$

5. A circle touching the $x-$ axis at $(3, 0)$ and making an intercept of length 8 on the $y-$ axis passes through the point:
(1) $(3, 10)$
(2) $(2, 3)$
(3) $(3, 5)$
(4) $(1, 5)$

6. Let the point $(p, p + 1)$ lie inside the region $E = \left\{(x, y): 3 - x \le y \le \sqrt{9 - x^2}, 0 \le x \le 3\right\}$. If the set of all values of p is the interval (a, b), then $b^2 + b - a^2$ is equal to _____.

7. The set of values of k for which the circle $C : 4x^2 + 4y^2 - 12x + 8y + k = 0$ lies inside the fourth quadrant and the point $\left(1, -\frac{1}{3}\right)$ lies on or inside the circle C is
(1) An empty set
(2) $\left(6, \frac{95}{9}\right]$
(3) $\left[\frac{80}{9}, 10\right)$
(4) $\left(9, \frac{92}{9}\right]$

8. If a variable line $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval :
(1) $[13, 23]$
(2) $(23, 31)$
(3) $[12, 21]$
(4) $(2, 17)$

9. Let a circle $C : (x - h)^2 + (y - k)^2 = r^2, k > 0$, touch the x-axis at $(1, 0)$. If the line $x + y = 0$ intersects the circle C at P and Q such that the length of the chord PQ is 2, then the value of $h + k + r$ is equal to _____.

10. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x - 1)^2 + (y - 1)^2 = 1$ and $(x - 9)^2 + (y - 1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is

11. Two circles in the first quadrant of radii r_1 and r_2 touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line $x + y = 2$. Then ${r_1}^2 + {r_2}^2 - r_1 r_2$ is equal to ____.

12. A circle C_1 passes through the origin O and has diameter 4 on the positive x-axis. The line $y = 2x$ gives a chord OA of a circle C_1. Let C_2 be the circle with OA as a diameter. If the tangent to C_2 at the point A meets the x-axis at P and y-axis at Q, then $QA : AP$ is equal to
(1) 1 : 4
(2) 1 : 5
(3) 2 : 5
(4) 1 : 3

13. Choose the incorrect statement about the two circles whose equations are given below:$x^2 + y^2 - 10x - 10y + 41 = 0$ and $x^2 + y^2 - 16x - 10y + 80 = 0$
(1) Distance between two centres is the average of radii of both the circles.
(2) Both circles' centres lie inside region of one another.
(3) Both circles pass through the centre of each other.
(4) Circles have two intersection points.

14. The number of common tangents, to the circles $x^2 + y^2 - 18x - 15y + 131 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$, is
(1) 3
(2) 1
(3) 4
(4) 2

15. If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $\left(x - 2\sqrt{2}\right)^2 + \left(y - 2\sqrt{2}\right)^2 = r^2$, then the value of r^2 is equal to

16. Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2\left(x^2 + y^2\right) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to _____.

17. A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius
(1) $\frac{3}{5}\lambda$
(2) $\frac{2}{3}\lambda$
(3) $\frac{\sqrt{19}}{5}\lambda$
(4) $\frac{\sqrt{19}}{7}\lambda$

18. Let A be the point $(1, 2)$ and B be any point on the curve $x^2 + y^2 = 16$. If the centre of the locus of the point P, which divides the line segment $A\,B$ in the ratio $3 : 2$ is the point $C(\alpha, \beta)$, then the length of the line segment AC is

(1) $\frac{3\sqrt{5}}{5}$ (2) $\frac{4\sqrt{5}}{5}$

(3) $\frac{2\sqrt{5}}{5}$ (4) $\frac{6\sqrt{5}}{5}$

19. Let the locus of the centre (α, β), $\beta > 0$, of the circle which touches the circle $x^2 + (y-1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is

(1) $\frac{32\sqrt{2}}{3}$ (2) $\frac{40\sqrt{2}}{3}$

(3) $\frac{64}{3}$ (4) $\frac{32}{3}$

20. A circle passing through the point $P(\alpha, \beta)$ in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then the value of $\alpha\beta$ is _____

21. The locus of the middle points of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_i, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ and ${r_1}^2 = {r_2}^2 + {r_3}^2$, then θ_2 is equal to

(1) $\frac{\pi}{4}$ (2) $\frac{3\pi}{4}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

22. Let C be a circle passing through the points $A(2, -1)$ and $B(3, 4)$. The line segment AB is not a diameter of C. If r is the radius of C and its centre lies on the circle $(x-5)^2 + (y-1)^2 = \frac{13}{2}$, then r^2 is equal to

(1) 32 (2) $\frac{65}{2}$

(3) $\frac{61}{2}$ (4) 30

23. Let the equation $x^2 + y^2 + px + (1-p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2$ and q is an integer$\}$ is _____

24. If the circles $x^2 + y^2 + 6x + 8y + 16 = 0$ and $x^2 + y^2 + 2\left(3 - \sqrt{3}\right)x + 2\left(4 - \sqrt{6}\right)y = k + 6\sqrt{3} + 8\sqrt{6}$, $k > 0$, touch internally at the point $P(\alpha, \beta)$, then $\left(\alpha + \sqrt{3}\right)^2 + \left(\beta + \sqrt{6}\right)^2$ is equal to _____.

25*. Let $y = x + 2$, $4y = 3x + 6$ and $3y = 4x + 1$ be three tangent lines to the circle $(x-h)^2 + (y-k)^2 = r^2$. Then $h + k$ is equal to :

(1) 5 (2) $5\left(1 + \sqrt{2}\right)$

(3) 6 (4) $5\sqrt{2}$

26*. Let the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle $C : (x-h)^2 + (y-k)^2 = r^2$. If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle C, then the value of $(5h - 8k)^2 + 5r^2$ is equal to _____.

27*. The line $2x - y + 1 = 0$ is a tangent to the circle at the point $(2,\ 5)$ and the centre of the circle lies on $x - 2y = 4$. Then, the radius of the circle is:

(1) $3\sqrt{5}$ (2) $5\sqrt{3}$

(3) $5\sqrt{4}$ (4) $4\sqrt{5}$

28*. Let O be the origin and OP and OQ be the tangents to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$ at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha, \frac{1}{2}\right)$, then a value of α is

(1) $\frac{3}{2}$ (2) $-\frac{1}{2}$

(3) $\frac{5}{2}$ (4) 1

29*. Let the tangent to the circle $C_1 : x^2 + y^2 = 2$ at the point $M(-1, 1)$ intersect the circle $C_2 : (x-3)^2 + (y-2)^2 = 5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) $\frac{1}{6}$ (4) $\frac{5}{3}$

30*. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point:

(1) $(4, -2)$ (2) $(-4, 6)$

(3) $(6, -2)$ (4) $(-6, 4)$

*Note: Question with * denotes it is optional but good to solve.*