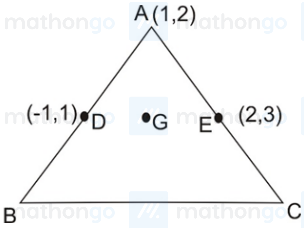
ANSWER KEYS

1. (3)	**2.** (1)	**3.** (122)	**4.** (1)	**5.** (5)	**6.** (1)	**7.** (3)	**8.** (3)
9. (2)	**10.** (2)	**11.** (2)	**12.** (2)	**13.** (4)	**14.** (2)	**15.** (2)	**16.** (2)
17. (2)	**18.** (4)	**19.** (1)	**20.** (31)	**21.** (4)	**22.** (1)	**23.** (4)	**24.** (1)
25. (2)	**26.** (2)	**27.** (3)	**28.** (1)	**29.** (2)	**30.** (48)		

1. **(3)**



Let co-ordinates of $B = (x_1, y_1)$

$\therefore\ \frac{x_1+1}{2} = -1\ \&\ \frac{y_1+2}{2} = 1$

$\Rightarrow\ x_1 = -3,\quad y_1 = 0$

$\therefore B = (-3,\ 0)$

Let co-ordinates of $C = (x_2, y_2)$

$\therefore\ \frac{x_2+1}{2} = 2\ \&\ \frac{y_2+2}{2} = 3$

$\therefore\ C = (3,\ 4)$

Let centroid $G = (h, k)$

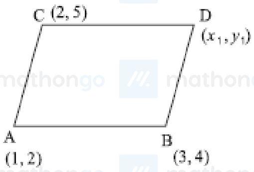
$h = \frac{1+x_1+x_2}{3} = \frac{1-3+3}{3} = \frac{1}{3}$

$k = \frac{2+y_1+y_2}{3} = \frac{2+0+4}{3} = 2$

$\therefore$ Centroid, G $= \left(\frac{1}{3}, 2\right)$

2. **(1)** Since, in parallelogram mid points of both diagonals coinsides.

$\therefore$ mid-point of AD = mid-point of BC



$$\left(\frac{x_1+1}{2}, \frac{y_1+2}{2}\right) = \left(\frac{3+2}{2}, \frac{4+5}{2}\right)$$

$\therefore\quad (x_1, y_1) = (4, 7)$

Then, equation of AD is,

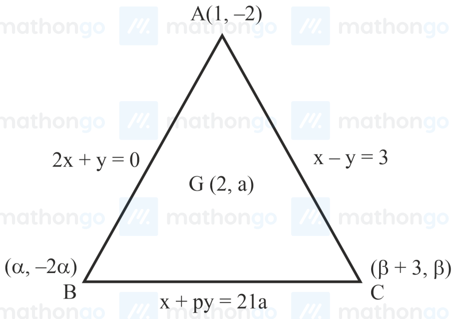
$$y - 7 = \frac{2-7}{1-4}(x-4)$$

$$y - 7 = \frac{5}{3}(x-4)$$

$$3y - 21 = 5x - 20$$

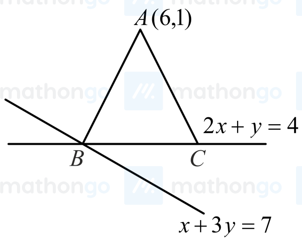
$$5x - 3y + 1 = 0$$

3. **(122)**
We have,



Since, $G(2, a)$, so
$\frac{1+\alpha+\beta+3}{3} = 2$ and $\frac{-2-2\alpha+\beta}{3} = a$
$\Rightarrow \alpha + \beta = 2$ and $-2\alpha + \beta = 3a + 2$
So, $\alpha = -a,\ \beta = 2 + a$
So, $B \equiv (-a, 2a),\ C \equiv (5 + a, 2 + a)$
Now, B and C lies on the line $x + py = 21a$, so
$-a + 2pa = 21a$
$\Rightarrow p = 11$
Also,
$5 + a + 11(2 + a) = 21a$
$\Rightarrow 27 = 9a \Rightarrow a = 3$
So,
$B \equiv (-3, 6),\ C \equiv (8, 5)$
So,
$BC^2 = 121 + 1 = 122$

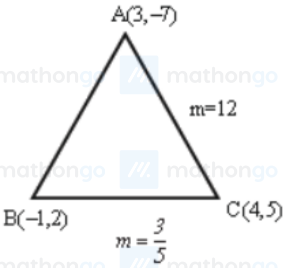
4. **(1)**



In $\triangle ABC,\ AB = AC$
Solving $2x + y = 4$ & $x + 3y = 7$, we get
$B \equiv (1, 2)$
Let $C \equiv (h, k)$ and as it lies on $2x + y = 4$
so $2h + k = 4$
Now, $AB^2 = AC^2$
$26 = \left(h - 6\right)^2 + (k - 1)^2 \Rightarrow 26 = (h - 6)^2 + (3 - 2h)^2$
$\Rightarrow 26 = 5h^2 - 24h + 45 \Rightarrow (h - 1)(5h - 19) = 0$
$\Rightarrow h = \frac{19}{5}$ (as $h = 1$ rejected)
so $k = -\frac{18}{5}$
Hence, centroid $= \left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3}\right) \equiv \left(\frac{18}{5}, -\frac{1}{5}\right)$
i.e. $15(\alpha + \beta) = 15 \times \frac{17}{5} = 51$

5. **(5)** P must be centroid of ΔABC
$\therefore P\left(\frac{17}{6}, \frac{8}{3}\right) \Rightarrow PQ = \sqrt{\left(\frac{24}{6}\right)^2 + \left(\frac{9}{3}\right)} = 5$ units

6. **(1)**



Altitude of BC: $y+7=\frac{-5}{3}(x-3)$

$3y+21=-5x+15$

$5x+3y+6=0$

Altitude of AC: $y-2=\frac{-1}{12}(x+1)$

$12y-24=-x-1$

$x+12y=23$

$\alpha=\frac{-47}{19},\quad \beta=\frac{121}{57}$

$9\alpha-6\beta+60=25$

7. **(3)**

Let $AB\equiv x-2y+1=0$ and $AC\equiv 2x-y-1=0$

So, $A(1,\ 1)$

Now, altitude from B which is perpendicular to line $2x-y-1=0$ and which passes through $\left(\frac{7}{3},\frac{7}{3}\right)$ is $BH=x+2y-7=0,$

Now intersection of BH & AB will give point $B(3,\ 2)$

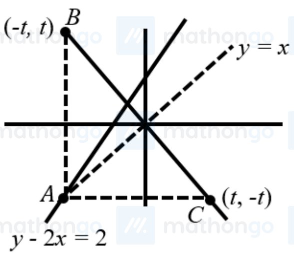
Similarly, altitude from C is $CH=2x+y-7=0\Rightarrow C(2,\ 3)$

Centroid of $\triangle\ ABC=E(2,\ 2)$ & $OE=2\sqrt{2}$

where O is origin.

8. **(3)**

On plotting the diagram of the given data we get,



Now given, x $=$ y and $y-2x=2$

On solving we get, $A(-2,-2)$

Now height h of $\Delta ABC=$ Distance of point A to origin,

$h=\sqrt{(-2)^2+\left(-2\right)^2}$

$\Rightarrow h=2\sqrt{2}$

Given ΔABC is an equilateral triangle,

So, $\sin(B)=\frac{h}{AB}$

$\Rightarrow \sin(60^\circ)=\frac{h}{AB}\quad\left(\because \angle A=\angle B=\angle C=60^\circ\right)$

Area of $\Delta=\frac{\sqrt{3}}{4}(AB)^2$

$\Rightarrow$Area of $\Delta=\frac{\sqrt{3}}{4}\frac{h^2}{\sin^2 60^\circ}=\frac{\sqrt{3}}{4}\times\frac{32}{3}$

So, Area of $\Delta ABC=\frac{8}{\sqrt{3}}$

9. **(2)**

Given,

$A(\alpha,-2),\ B(\alpha,6)\ \&\ C\left(\frac{\alpha}{4},-2\right)$

Now slope of $AC=0$ and slope of $AB=\frac{8}{0}\rightarrow\infty$

Now we can see AC is perpendicular to AB.

So, ΔABC is right angled at A.

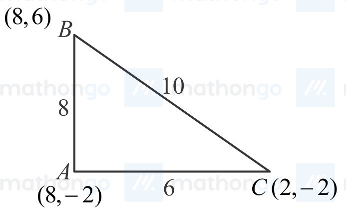
So, Circumcentre $=$mid point of $BC=\left(\frac{5\alpha}{8},2\right)$

Now given circumcentre $\equiv\left(5,\frac{\alpha}{4}\right)$

So on comparing we get, $\frac{5\alpha}{8}=5\ \&\ \frac{\alpha}{4}=2$

$\Rightarrow\alpha=8$

Now plotting the diagram and finding the sides with distance formula we get,



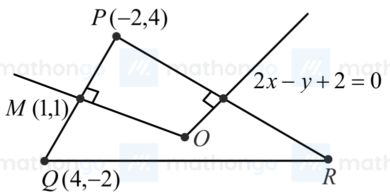
Area $=\frac{1}{2}(6)(8)=24$

Perimeter $=24$

Circumradius $=\frac{10}{2}=5$

Inradius $=\frac{\Delta}{s}=\frac{24}{12}=2$

10. **(2)**



Perpendicular bisector of PR is $2x-y+2=0$

Midpoint of PQ is $M(1,1)$

Equation of perpendicular bisector of PQ is $y=x$.

Point of intersection of equations PR and PQ is circumcentre.

So, circumcentre is $(-2,-2)$

11. **(2)**

Given, point A lies on $L_2\ :\ -4x+3y=12$

Take $x=\alpha$, so $y=4+\frac{4}{3}\alpha$, $A\left(\alpha,4+\frac{4}{3}\alpha\right)$

Points B lies on $L_1\ :\ 2x+5y=10$

Take $x=\beta$, so $y=2-\frac{2}{5}\beta$, $B\left(\beta,2-\frac{2}{5}\beta\right)$

Now point P divides AB internally in the ratio $1:3$

$\Rightarrow P(2,3)=\mathrm{P}\left(\frac{3\alpha+\beta}{4},\frac{3\left(4+\frac{4}{3}\alpha\right)+1\left(2-\frac{2}{5}\beta\right)}{4}\right)$

$\Rightarrow\alpha=\frac{3}{13},\beta=\frac{95}{13}$

We get, point $A\left(\frac{3}{13},\frac{56}{13}\right),B\left(\frac{95}{13},-\frac{12}{13}\right)$

Vertex C of triangle is the point of intersection of L_1 and L_2

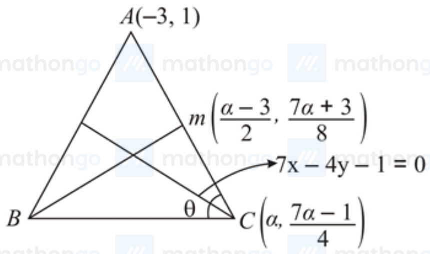
$\Rightarrow C\left(-\frac{15}{13},\frac{32}{13}\right)$

area $\Delta ABC=\frac{1}{2}\left\|\begin{vmatrix}\frac{3}{13}&\frac{56}{13}&1\\\frac{95}{13}&-\frac{12}{13}&1\\-\frac{15}{13}&\frac{32}{13}&1\end{vmatrix}\right\|$

$=\frac{1}{2\times13^3}\left\|\begin{matrix}3&56&13\\95&-12&13\\-15&32&13\end{matrix}\right\|$

area $\Delta ABC=\frac{132}{13}$sq. units

12. **(2)**



Let, $x-$coordinate of the point C is α, So coordinate of the point $C\left(\alpha, \frac{7\alpha-1}{4}\right)$ $\because$ (C lies on $7x-4y-1=0$)

Hence, mid point of AC is $m\left(\frac{\alpha-3}{2}, \frac{7\alpha+3}{8}\right)$

As, point m lies on the line $2x+y-3=0$, Substituting, we get $\alpha=3$.

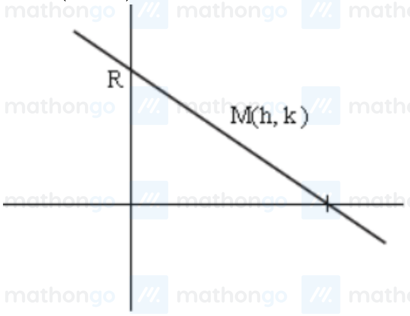
So. $C(3,5)$

Hence, slope of AC is $\frac{2}{3}$ and slope of $7x-4y-1=0$ is $\frac{7}{4}$

$\therefore \tan\frac{\theta}{2}=\frac{\frac{7}{4}-\frac{2}{3}}{1+\frac{7}{4}\times\frac{2}{3}}=\frac{1}{2}$

$\Rightarrow \tan\theta=\frac{2\cdot\frac{1}{2}}{1-\frac{1}{4}}=\frac{4}{3}\Rightarrow\tan\theta=\frac{4}{3}$

13. **(4)** pt $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$



$$x\cos\theta+y\sin\theta=7$$

$$x-\text{ intercept }=\frac{7}{\cos\theta}$$

$$y-\text{ intercept }=\frac{7}{\sin\theta}$$

$A:\left(\frac{7}{\cos\theta},0\right)$ $B:\left(0,\frac{7}{\sin\theta}\right)$

Locus of mid pt $M:(h,k)$

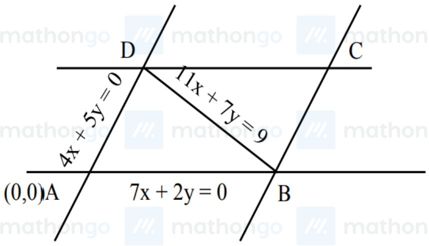
$\mathrm{h}=\frac{7}{2\cos\theta}, \mathrm{k}=\frac{7}{2\sin\theta}$

$\frac{7}{2\sin\theta}=\frac{7\sqrt{3}}{3}\Rightarrow\sin\theta=\frac{\sqrt{3}}{2}\Rightarrow\theta=\frac{\pi}{3}$

$\alpha=\frac{7}{2\cos\theta}=7$

14. (2)

Both the lines $4x+5y=0, 7x+2y=0$ pass through origin.



D is the point of intersection of $4x+5y=0$ & $11x+7y=9$

So upon solving we get the coordinates of point $D=\left(\frac{5}{3},-\frac{4}{3}\right)$

Also, point B is the point of intersection of $7x+2y=0$ & $11x+7y=9$

So, coordinates of point $B=\left(-\frac{2}{3},\frac{7}{3}\right)$

The diagonals of parallelogram bisect each other, so the mid point of BD is

$\left(\frac{\frac{5}{3}-\frac{2}{3}}{2},\frac{\frac{-4}{3}+\frac{7}{3}}{2}\right)=\left(\frac{1}{2},\frac{1}{2}\right)$

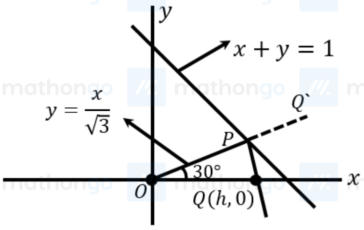
The equation of diagonal AC is

$(y-0)=\frac{\frac{1}{2}-0}{\frac{1}{2}-0}(x-0)$ i.e., $y=x$

Therefore, the diagonal AC passes through $(2,2)$.

15. (2)

On plotting the diagram of given data we get,



Let $Q(h,0)$

$\because OP$ reflected by $x+y=1$

So, the image of Q lies on $y=\frac{x}{\sqrt{3}}$

$\therefore \frac{x-h}{1}=\frac{y}{1}=\frac{-2(h-1)}{2}$

$\Rightarrow x=1,\ y=1-h$

Since it lies on $y=\frac{x}{\sqrt{3}}$

$\therefore\ 1-h=\frac{1}{\sqrt{3}}$

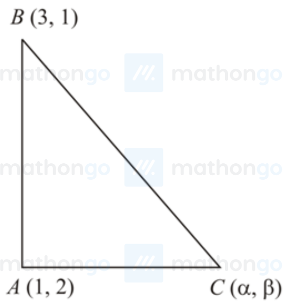
$\Rightarrow h=1-\frac{1}{\sqrt{3}}=\frac{\sqrt{3}-1}{\sqrt{3}}$

On rationalising the numerator we get,

$\Rightarrow h=\frac{\sqrt{3}-1}{\sqrt{3}}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow h=\frac{2}{3+\sqrt{3}}$

16. (2)

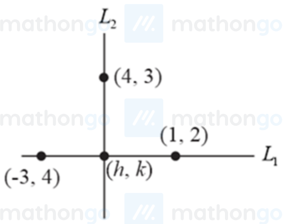


$m_{AC} = \frac{\beta-2}{\alpha-1}$
$m_{AB} = \frac{2-1}{1-3} = -\frac{1}{2}$
$AB \perp AC$
$\therefore \frac{\beta-2}{\alpha-1}\left(-\frac{1}{2}\right) = -1$
$\beta = 2\alpha - 2 + 2$
$\beta = 2\alpha$
Now area of $\Delta ABC = 5\sqrt{5} = \frac{1}{2}AB.\,AC$
$\Rightarrow \frac{1}{2}\sqrt{(3-1)^2+(1-2)^2}.\sqrt{(\alpha-1)^2+(\beta-2)^2} = 5\sqrt{5}$
$\Rightarrow \sqrt{(\alpha-1)^2+(2\alpha-2)^2} = 10$
$\Rightarrow \sqrt{(\alpha-1)^2}\sqrt{5} = 10$
$\Rightarrow |\alpha-1| = 2\sqrt{5}$
$\Rightarrow \alpha = 1 \pm 2\sqrt{5}.$

17. (2)

The equation of a line parallel to the line $ax+by+c=0$ is $ax+by+k=0$.
Thus, the equation of a line parallel to $4x-3y+2=0$ is $4x-3y+c=0$.
The distance of a line $ax+by+c=0$ from origin is $\frac{|c|}{\sqrt{a^2+b^2+c^2}}$.
Given the line $4x-3y+c=0$ is at a distance of $\frac{3}{5}$ units from origin
$\Rightarrow \frac{|c|}{\sqrt{16+9}} = \frac{3}{5}$
$\Rightarrow \frac{|c|}{5} = \frac{3}{5}$
$\Rightarrow c = \pm 3.$
Hence, the equation of the lines are $4x-3y+3=0$ and $4x-3y-3=0$.
Clearly point $\left(-\frac{1}{4}, \frac{2}{3}\right)$ satisfy the given equation $4x-3y+3=0$.

18. (4)



Slope of line L_1 is $\frac{4-2}{-3-1} = \frac{2}{-4} = \frac{-1}{2}$
Hence slope of L_2 is 2.
From line L_1, $\frac{k-2}{h-1} = \frac{-1}{2}$
$\Rightarrow h+2k=5$ .....(1)
From line L_2, $\frac{k-3}{h-4} = 2$
$\Rightarrow k = 2h-5$ .....(2)
From (1) and (2) $h=3$ and $k=1$
$\Rightarrow \frac{k}{h} = \frac{1}{3}$

19. (1)

We know that if $A(x_1, y_1)$ & $B(x_2, y_2)$ lies on the same side to line $L = 0$, then $L_{A(x_1, y_1)}\ L_{B(x_2, y_2)} > 0$.

Given, the points $A(1, 2)$ & $B(\sin\theta, \cos\theta)$ lies on the same side of the line $x + y - 1 = 0$.

$\therefore (1 + 2 - 1)(\sin\theta + \cos\theta - 1) > 0$

$\Rightarrow \sin\theta + \cos\theta > 1$

$\Rightarrow \sin\theta \times \frac{1}{\sqrt{2}} + \cos\theta \times \frac{1}{\sqrt{2}} > 1 \times \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\theta.\cos\frac{\pi}{4} + \cos\theta.\sin\frac{\pi}{4} > \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\pi}{4} < \left(\theta + \frac{\pi}{4}\right) < \frac{3\pi}{4}$

$\Rightarrow 0 < \theta < \frac{\pi}{2}$

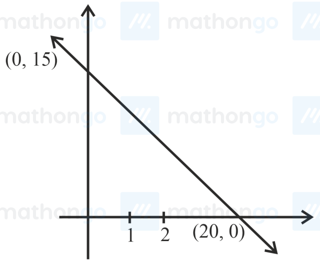
$\Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

20. (31)

Given:

$3x + 4y = 60$

$\Rightarrow \frac{x}{20} + \frac{y}{15} = 1$



If $x = 1, y = \frac{57}{4} = 14.25$

So, points are

$(1, 1)(1, 2)-(1, 14)\quad \Rightarrow$ 14 points.

If $x = 2, y = \frac{27}{2} = 13.5$

So, points are

$(2, 2)(2, 4)\ldots(2, 12)\quad \Rightarrow$ 6 points.

If $x = 3, y = \frac{51}{4} = 12.75$

So, points are

$(3, 3)(3, 6)-(3, 12)\quad \Rightarrow$ 4 points.

If $x = 4,\ y = 12$

So, points are

$(4, 4)(4, 8)\quad \Rightarrow$ 2 points.

If $x = 5, y = \frac{45}{4} = 11.25$

So, points are

$(5, 5), (5, 10)\quad \Rightarrow$ 2 points.

If $x = 6, y = \frac{21}{2} = 10.5$

So, point is

$(6, 6) \Rightarrow$ 1 point

If $x = 7, y = \frac{39}{4} = 9.75$

So, point is

$(7, 7) \Rightarrow$ 1 point

If $x = 8,\ y = 9$

So, point is

$(8, 8) \Rightarrow$ 1 point

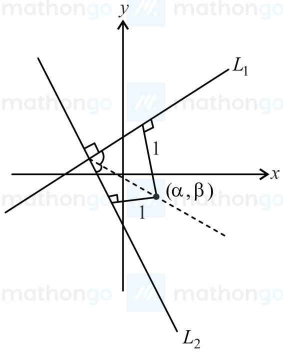
If x $= 9 \Rightarrow$ y $= \frac{33}{4} = 8.25 \Rightarrow$ no point

Total points inside the triangle

$= 31$ points

21. (4)

Given, the point $P(\alpha)(,\beta)$ be at a unit distance from each of the two lines $L_1 : 3x - 4y + 12 = 0$, and $L_2 : 8x + 6y + 11 = 0$, so on plotting the diagram we get,



Now, $L_1 : 3x - 4y + 12 = 0$ and $L_2 : 8x + 6y + 11 = 0$

Since L_1 & L_2 are perpendicular so it will form square of unit length, so equation of angle bisector of L_1 and L_2 of angle containing origin and will pass through (α, β), so

Equation of angle bisector will be, $\frac{(3x-4y+12)}{\sqrt{3^2+4^2}} = \frac{8x+6y+11}{\sqrt{8^2+6^2}}$

$\Rightarrow 2(3x - 4y + 12) = 8x + 6y + 11$

$\Rightarrow 2x + 14y - 13 = 0$

$\Rightarrow 2\alpha + 14\beta - 13 = 0 \quad \cdots$(i)

Also given perpendicular distance is one unit so, $\frac{3\alpha-4\beta+12}{5} = 1$

$\Rightarrow 3\alpha - 4\beta + 7 = 0 \quad \cdots$(ii)

Solving equation (i) & (ii)

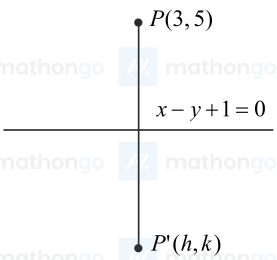
$2\alpha + 14\beta - 13 = 0$

$3\alpha - 4\beta + 7 = 0$

$\alpha = \frac{-23}{25}, \beta = \frac{53}{50}$

So, $100(\alpha + \beta) = 14$

22. (1)



The slope of $PP' = -1$ (As, PP' is perpendicular to the given line)

$\Rightarrow \frac{k-5}{h-3} = -1 \Rightarrow k + h = 8 \; \ldots(1)$

Mid-point of PP' lies on the given line,

$\Rightarrow \left(\frac{h+3}{2}\right) - \left(\frac{k+5}{2}\right) + 1 = 0 \Rightarrow h + 3 - k - 5 + 2 = 0 \Rightarrow h - k = 0 \; \ldots(2)$Solving equation (1) and (2) we get, $h = 4,\ k = 4$

Which satisfy the equation $(x-2)^2 + (y-4)^2 = 4$.

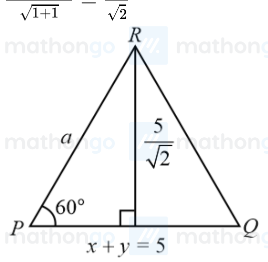
23. **(4)**

Given PQR is an equilateral triangle.

Let the side of the triangle be a

Perpendicular distance from $R(3,7)$ to the line $x+y=5$ is

$\frac{|3+7-5|}{\sqrt{1+1}}=\frac{5}{\sqrt{2}}$



So $\sin 60^{\circ}=\frac{\frac{5}{\sqrt{2}}}{a}$

$a=\frac{5\sqrt{2}}{\sqrt{3}}$

Area of $\Delta PQR=\frac{\sqrt{3}}{4}a^2=\frac{25}{2\sqrt{3}}$

24. **(1)**

Equation of line is

$\frac{x}{3}+\frac{y}{1}=1 \Rightarrow x+3y-3=0$

if image is(x_1,y_1)

Then,

$\frac{x_1+1}{1}=\frac{y_1+4}{3}=-2\left(\frac{-1-12-3}{10}\right)$

$x_1+1=\frac{y_1+4}{3}=\frac{16}{5}$

$\Rightarrow x_1=\frac{11}{5}, y_1=\frac{28}{5}$

25. **(2)**

$3x+4y=9$

$y=mx+1$

$\Rightarrow 3x+4mx+4=9$

$\Rightarrow (3+4m)x=5$

$\Rightarrow x$ will be an integer when

$3+4m=5,\ -5,\ 1,-1$

$\Rightarrow m=\frac{1}{2},-2,-\frac{1}{2},-1$

so, number of integral values of m is 2

26. **(2)** Given condition is $3p+2q+4r=0$

$\Rightarrow \frac{3}{4}p+\frac{1}{2}q+r=0\ \ \ldots\ldots(1)$

And given family of line is $px+qy+r=0\ldots(ii)$

From (i) we can say that equation (ii) always passes through $\left(\frac{3}{4},\frac{1}{2}\right)$.

Hence all lines are concurrent at $\left(\frac{3}{4},\frac{1}{2}\right)$.

27. **(3)**

Given point B lie on the line $x-y-2=0$, so the point will be $B(x_1,x_1-2)$

Now given distance between AB is $\frac{\sqrt{29}}{3}$, so by distance formula,

$\sqrt{(x_1-4)^2+(x_1-2-3)^2}=\frac{\sqrt{29}}{3}$

Now on squaring on both side we get,

$18x_1^2-162x_1+340=0$

On solving we get, $x_1=\frac{51}{9}$ or $x_1=\frac{10}{3}$

So, $y_1=\frac{33}{9}$ or $y_1=\frac{4}{3}$

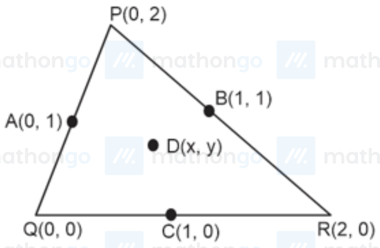
Now we can see only $x+2y=6$ will satisfy the point $\left(\frac{10}{3},\frac{4}{3}\right)$

28. (1)

Given,

$A(0,1),\ B(1,1)$ and $C(1,0)$ be the mid-points of the sides of a triangle with incentre at the point D,

Now finding the vertices of the triangle by using midpoint formula and plotting the diagram we get,



Now finding the incentre of the above triangle using the formula,

$I \equiv \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

We get, Incentre $D = \left(\frac{4}{4+2\sqrt{2}}, \frac{4}{4+2\sqrt{2}}\right)$

Now given parabola $y^2 = 4ax$ passes through the incentre D we get,

$\left(\frac{4}{4+2\sqrt{2}}\right)^2 = 4a\left(\frac{4}{4+2\sqrt{2}}\right)$

$\Rightarrow a = \frac{1}{4+2\sqrt{2}}$

Now we know that focus of parabola is given by, focus $(a,\ 0)$

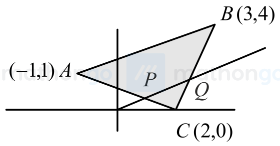
Hence, focus will be, $\left(\frac{1}{4+2\sqrt{2}}, 0\right) = \left(\frac{4-2\sqrt{2}}{8}, 0\right)$

Now comparing with $\left(\alpha + \beta\sqrt{2}, 0\right)$ we get,

$\alpha = \frac{4}{8} = \frac{1}{2},\ \beta = -\frac{1}{4}$

Hence, $\frac{\alpha}{\beta^2} = \frac{\frac{1}{2}}{\left(\frac{-1}{4}\right)^2} = \frac{16}{2} = 8$

29. (2)



$P \equiv (x_1, mx_1)$

$Q \equiv (x_2, mx_2)$

$A_1 = \frac{1}{2}\begin{vmatrix} 3 & 4 & 1 \\ 2 & 0 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{13}{2}$

$A_2 = \frac{1}{2}\begin{vmatrix} x_1 & mx_1 & 1 \\ x_2 & mx_2 & 1 \\ 2 & 0 & 1 \end{vmatrix}$

$A_2 = \frac{1}{2}|2(mx_1 - mx_2)| = m|x_1 - x_2|$

$A_1 = 3\,A_2 \Rightarrow \frac{13}{2} = 3\,m|x_1 - x_2|$

$\Rightarrow\ |x_1 - x_2| = \frac{16}{6\,m}$

$AC:\ x + 3\,y = 2$

$BC:\ y = 4\,x - 8$

$P: x + 3y = 2\ \&\ y = mx \Rightarrow x_1 = \frac{2}{1+3m}$

$Q:\ y = 4x - 8\ \&\ y = mx \Rightarrow x_2 = \frac{8}{4-m}$

$|x_1 - x_2| = \left|\frac{2}{1+3m} - \frac{8}{4-m}\right|$

$= \left|\frac{-26\,m}{(1+3m)\,(4-m)}\right| = \frac{26\,m}{(3m+1)\,|m-4|}$

$= \frac{26\,m}{(3m+1)\,(4-m)}$

$|x_1 - x_2| = \frac{13}{6\,m}$

$\frac{26\,m}{(3m+1)\,(4-m)} = \frac{13}{6\,m}$

$\Rightarrow\ 12m^2 = -(3m+1)(m-4)$

$\Rightarrow\ 12m^2 = -\left(3m^2 - 11m - 4\right)$

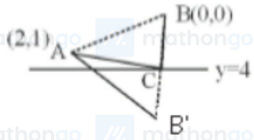
$\Rightarrow\ 15m^2 - 11m - 4 = 0$

$\Rightarrow\ 15m^2 - 15m + 4m - 4 = 0$

$\Rightarrow\ (15m + 4)(m - 1) = 0$

$\Rightarrow\ m = 1$

30. (48) $A(2,1), B(0,0), C(t,4) : t \in [0,4]$



$B_1(0,8) \equiv$ image of B w.r.t. $y = 4$

for $AC + BC + AB$ to be minimum

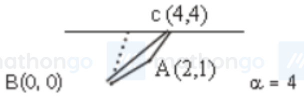
$m_{AB'} = \frac{-7}{2}$

line $AB_1 \equiv 7x + 2y = 16$

$C\left(\frac{8}{7}, 4\right)$

$\beta = \frac{8}{7}$

For max. perimeter



$AB = \sqrt{5} : BC = 4\sqrt{2}, AC = \sqrt{13}$

$6\alpha + 21\beta = 24 + 24 = 48$