ANSWER KEYS

1. (1)	**2.** (2)	**3.** (0.60)	**4.** (2)	**5.** (3)	**6.** (1)	**7.** (4)	**8.** (1)
9. (2)	**10.** (1)						

1. **(1)** $y=\log_{10} x+\log_x 10+\log_x x+\log_{10} 10$

$=\log_{10} x+\frac{\log_e 10}{\log_e x}+1+1$

$\Rightarrow \frac{dy}{dx}=\frac{1}{x}\log_{10} e-\frac{\log_e 10}{x(\log_e x)^2}$

2. **(2)**

Given $f(x)=e^x$ and $g(x)=\sin^{-1} x$ and $h(x)=f[g(x)]$

$\Rightarrow h(x)=f(\sin^{-1} x)=e^{\sin^{-1} x}$

$\therefore\ h(x)=e^{\sin^{-1} x}$

$\Rightarrow h'(x)=e^{\sin^{-1} x}\times\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{h'(x)}{h(x)}=\frac{1}{\sqrt{1-x^2}}$

3. **(0.60)**

Given that

$y=\tan^{-1}\left(\frac{2^x}{1+2^{2x+1}}\right)$

$\Rightarrow y=\tan^{-1}\left(\frac{2.2^x-2^x}{1+2^x.(2.2^x)}\right)$

We know that

$\tan^{-1} x-\tan^{-1} y=\tan^{-1}\left(\frac{x-y}{1+xy}\right);\ x>0\ \&\ y>0.$

$\Rightarrow y=\tan^{-1}(2^{x+1})-\tan^{-1}(2^x)$

$\Rightarrow \frac{dy}{dx}=\frac{1}{1+\left(2^{x+1}\right)^2}2.2^x\ln(2)-\frac{1}{1+(2^x)^2}2^x\ln(2)$

Put $x=0$

$\frac{dy}{dx}=\frac{2\ln(2)}{5}-\frac{\ln(2)}{1}=\ln(2)\left(\frac{-3}{5}\right)$

$=\frac{3}{5}(-\ln(2))=\frac{3}{5}\ln\left(\frac{1}{2}\right)$

So required value of $k=\frac{3}{5}=0.6$

4. **(2)**

We have,

$f(x)=e^x+x$

Differentiating both sides w.r.t. x, we get

$f'(x)=e^x+1\quad\ldots$(i)

Let $g(x)$ be the inverse of $f(x)$. Then,

$f(g(x))=x$

$\Rightarrow f'(g(x))g'(x)=1$

$\Rightarrow\left(e^{g(x)}+1\right)g'(x)=1$

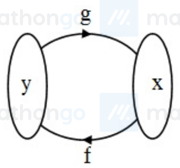
$\Rightarrow\left(e^{g(f(\log 2))}+1\right)g'\left(f(\log 2)\right)=1$

$\Rightarrow\left(e^{\log 2}+1\right)g'\left(f(\log 2)\right)=1$

$\Rightarrow 3\times g'(f(\log 2))=1$

$\Rightarrow g'\left(f\left(\log 2\right)\right)=\frac{1}{3}$

5.



(3)

So, that we can write $f-1(y)=g(y)$

Now $f(x)=x+\tan x$

Put $x=f-1(y)$

$f(f-1\,y) \;=\; f-1(y) \;+\; \tan(f-1\,y)$

$y=g(y)+\tan g(y)$

$y \Rightarrow 1$

$x=g(x)+\tan g(x)\ \ldots(1)$differentiate the function

$1=g'(x)+\sec^2 g(x)\cdot g'(x)$

$g'(x)=\frac{1}{1+\sec^2 g(x)}=\frac{1}{2+(x-g(x))^2}$

From equation (1), $\tan g(x)=[x-g(x)]$

$\sec^2 g(x)=1+\tan^2 g(x)$

$=1+[x-g(x))2]$

6. **(1)**

On taking log on both sides of the given equation, we get,

$\log y=\log(1-x)+\log(2-x)+\ldots+\log(n-x)$

Differentiating both sides w.r.t. to y, we get,

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{1}{(1-x)}(-1)+\frac{1}{(2-x)}(-1)+\ldots+\frac{1}{(n-x)}(-1)$

$\Rightarrow \frac{dy}{dx}=y\left[\frac{(2-x)(3-x)\ldots(n-x)+(1-x)(3-x)\ldots(n-x)+\ldots}{y}\right](-1)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1}=1.2\ldots..(n-1)(-1)$

$=(-1)(n-1)!$

7. **(4)**

$y=a\ \sin\ x+b\ \cos\ x \quad -(i)$

$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}(a\ \sin\ x+b\ \cos\ x)$

$=a\ \cos\ x-b\ \sin\ x \quad$ (ii)

Now , $y^2+\left(\frac{dy}{dx}\right)^2=(a\ \sin\ x+b\ \cos\ x)^2+(a\ \cos\ x-b\ \sin\ x)^2$

putting values from (i) & (ii), we get

$y^2+\left(\frac{dy}{dx}\right)^2=a^2\ \sin^2\ x+b^2\ \cos^2\ x+2ab\ \sin\ x\ \cos\ x+a^2\ \cos^2\ x+b^2\ \sin^2\ x-2ab\ \cos\ x\ \sin\ x$

$=a^2(\sin^2 x+\cos^2 x)+b^2(\cos^2 x+\sin^2 x)$

$=a^2+b^2$

which is a constant.

8. (1)

Given, $g(x) = g(y)g(x-y)\ \forall\ x, y \in R$

Differentiating both sides w.r.t. x, keeping y as constant, we get, $g'(x) = g(y)g'(x-y)$

Put $y = x$

$g'(x) = g(x)g'(0) = ag(x)\quad [\because g'(0) = a]$

$\Rightarrow \frac{g'(x)}{g(x)} = a$

Integrating in both sides w.r.t x, we get,

$\int \frac{g'(x)}{g(x)} dx = a \int 1 dx$

$\Rightarrow \log_e g(x) = ax + c$, where, c is the constant of integration.

$\Rightarrow g(x) = e^{ax+c} = e^c e^{ax}$

$\Rightarrow g(x) = ke^{ax}$

Differentiating both sides w.r.t. x, we get,

$g'(x) = kae^{ax}$

$\because g'(0) = a \Rightarrow ka = a \Rightarrow k = 1.$

$\Rightarrow g'(x) = ae^{ax}$

$\because g'(3) = ae^{3a} = b \Rightarrow e^{3a} = \frac{b}{a}$

Thus, $g'(-3) = ae^{-3a} = \frac{a^2}{b}$.

9. (2)

$\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$

$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$

$\therefore \frac{dy}{dx} = \frac{\sin 2\theta - \sin\theta}{\cos\theta - \cos 2\theta}$

$= \frac{2\sin\frac{\theta}{2}.\cos\frac{3\theta}{2}}{2\sin\frac{\theta}{2}.\sin\frac{3\theta}{2}} = \cot\frac{3\theta}{2}$

$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx} = -\frac{3}{2}\operatorname{cosec}^2\frac{3\theta}{2}.\frac{d\theta}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-\frac{3}{2}\operatorname{cosec}^2\frac{3\theta}{2}}{2(\cos\theta - \cos 2\theta)}$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\theta=\pi} = \frac{3}{4(-1-1)} = \frac{3}{8}$

10. (1)

Given that, $x = e^t \sin t, y = e^t \cos t \quad \ldots(1)$

At point $(1, 1), 1 = e^t \sin t, 1 = e^t \cos t$

$\tan t = 1 \Rightarrow t = \frac{\pi}{4}$

On differentiating Equation (1) w.r.t. x, we get

$\frac{dy}{dt} = e^t(\cos t - \sin t)$

and $\frac{dx}{dt} = e^t(\sin t + \cos t)$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t - \sin t}{\cos t + \sin t}$

Again differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = \frac{dx}{dt}\left(\frac{\cos t - \sin t}{\cos t - \sin t}\right)\frac{dt}{dx}$

$= \left[\frac{[(\cos t + \sin t)(-\sin t - \cos t) - (\cos t - \sin t)(-\sin t + \cos t)]}{(\cos t + \sin t)^2}\right]\frac{dt}{dx}$

$= \frac{-2}{(\cos t + \sin t)^2}.\frac{1}{e^t(\sin t + \cos t)}$

$= \frac{-2}{(e^t\cos t + e^t\sin t)}.\frac{1}{(\cos t + \sin t)^2}$

$= \frac{-2}{x+y}.\frac{1}{(\cos t + \sin t)^2}\quad$ [from Equation(1)]

At $t = \frac{\pi}{4}, x = 1,\ y = 1$

$\therefore \frac{d^2y}{dx^2} = \frac{-2}{1+1}.\frac{1}{\left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)^2}$

$= \frac{-1}{\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]} = -\frac{1}{2}$