1. Let a be an integer such that $\lim_{x\to7}\frac{18-[1-x]}{[x-3a]}$ exists, where $[t]$ is greatest integer $\le t$. Then a is equal to

(1) -2
(2) 6
(3) -6
(4) -7

2. Let $[t]$ denote the greatest integer $\le t$ and $\{t\}$ denote the fractional part of t. Then integral value of α for which the left hand limit of the function

$f(x)=[1+x]+\frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$ at $x=0$ is equal to $\alpha-\frac{4}{3}$ is _____

3. If $\lim_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$ then $8(\alpha+\beta)$ is equal to

(1) 4
(2) -8
(3) -4
(4) 8

4. Let $f(x)$ be a polynomial function such that $f(x)+f'(x)+f''(x)=x^5+64$. Then, the value of $\lim_{x\to1}\frac{f(x)}{x-1}$ is equal to

(1) -15
(2) 15
(3) -60
(4) 60

5. The value of $\lim_{n\to\infty}\frac{1+2-3+4+5-6+\ldots+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is

(1) $\frac{\sqrt{2}+1}{2}$
(2) $3\left(\sqrt{2}+1\right)$
(3) $\frac{3}{2}\left(\sqrt{2}+1\right)$
(4) $\frac{3}{2\sqrt{2}}$

6. $\lim_{x\to0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to

(1) $\frac{1}{3}$
(2) $\frac{1}{6}$
(3) $\frac{1}{4}$
(4) $\frac{1}{12}$

7. $\lim_{x\to\frac{\pi}{4}}\frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ is equal to

(1) 14
(2) 7
(3) $14\sqrt{2}$
(4) $7\sqrt{2}$

8. $\lim_{x\to0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is

(1) 3
(2) 1
(3) 2
(4) 6

9. The value of $\lim_{h\to0}\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cos h-\sin h\right)}\right\}$ is :

(1) $\frac{4}{3}$
(2) $\frac{2}{\sqrt{3}}$
(3) $\frac{2}{3}$
(4) $\frac{3}{4}$

10. Let f, g and h be the real valued functions defined on $\mathbb{R}$ as $f(x)=\begin{cases}\frac{x}{|x|}, & x\ne0\\ 1, & x=0\end{cases}$, $g(x)=\begin{cases}\frac{\sin(x+1)}{(x+1)}, & x\ne-1\\ 1, & x=-1\end{cases}$ and $h(x)=2[x]-f(x)$, where $[x]$ is the greatest integer $\le x$. Then the value of $\lim_{x\to1}g(h(x-1))$ is

(1) 1
(2) $\sin(1)$
(3) -1
(4) 0

11. If $\lim_{x\to0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$ then the value of k is

12. For each $t\in R$, let $[t]$ be the greatest integer less than or equal to t. Then, $\lim_{x\to1^+}\frac{(1-|x|+\sin|1-x|)\sin\left([1-x]\frac{\pi}{2}\right)}{|1-x|\,[1-x]}$

(1) equals 0
(2) equals -1
(3) does not exist
(4) equal 1

13. If $\lim_{x\to0}\frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of $a-2b$ is ___ .

14. If the value of $\lim_{x\to0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to_____.

15. Let $f: R\to R$ be a differentiable function satisfying $f'(3)+f'(2)=0$. Then $\lim_{x\to0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to

(1) 1
(2) e
(3) e^2
(4) e^{-1}

16. $\lim_{x\to0}\left(\frac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)}{(x+2)^3+2(x+2)^2+3\sin(x+2)}\right)^{\frac{100}{x}}$ is equal to

17. If $\lim\limits_{x \to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \frac{2}{3}$, where $\alpha, \beta, \gamma \in R$, then which of the following is NOT correct?

(1) $\alpha^2 + \beta^2 + \gamma^2 = 6$
(2) $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$
(3) $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$
(4) $\alpha^2 - \beta^2 + \gamma^2 = 4$

18. If $\lim\limits_{x \to 0} \left[\frac{\alpha x e^x - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x}\right] = 10,\ \alpha,\ \beta,\ \gamma \in R$, then the value of $\alpha + \beta + \gamma$ is __________.

19. $\lim\limits_{n \to \infty} \left\{ \left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right) \ldots \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right) \right\}$ is equal to

(1) 1
(2) 0
(3) $\sqrt{2}$
(4) $\frac{1}{\sqrt{2}}$

20. The value of $\lim\limits_{n \to \infty} \frac{[r] + [2r] + \ldots + [nr]}{n^2}$, where r is non-zero real number and $[r]$ denotes the greatest integer less than or equal to r, is equal to :

(1) $\frac{r}{2}$
(2) r
(3) $2r$
(4) 0