1. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6\left(\alpha^3 + \beta^3\right)^2$ is equal to

(1) 46 (2) 36

(3) 24 (4) 18

2. The sum of all integral values of $k(k \neq 0)$ for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is_____.

3. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to:

(1) -7 (2) 7

(3) -3 (4) 3

4. The value of $3 + \cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots\infty}}}}$ is equal to

(1) $1.5 + \sqrt{3}$ (2) $2 + \sqrt{3}$

(3) $3 + 2\sqrt{3}$ (4) $4 + \sqrt{3}$

5. Let $p,\ q \in Q$. If $2 - \sqrt{3}$ is a root of the quadratic equation $x^2 + px + q = 0$, then

(1) $p^2 - 4q + 12 = 0$ (2) $q^2 + 4p + 14 = 0$

(3) $p^2 - 4q - 12 = 0$ (4) $q^2 - 4p - 16 = 0$

6. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to:

(1) 1 (2) α

(3) $1 + \alpha$ (4) $1 + 2\alpha$

7. Let $\alpha,\ \beta$ be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$ is equal to

(1) 81 (2) 9

(3) 72 (4) 729

8. If α and β are the roots of the equation $375\ x^2 - 25x - 2 = 0$, then $\lim\limits_{n\to\infty} \sum_{r=1}^{n} \alpha^r + \lim\limits_{n\to\infty} \sum_{r=1}^{n} \beta^r$ is equal to:

(1) $\frac{1}{12}$ (2) $\frac{21}{346}$

(3) $\frac{7}{116}$ (4) $\frac{29}{358}$

9. Let $\alpha, \beta(\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n, n \in \mathbb{N}$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to _____.

10. If α, β are roots of the equation $x^2 + 5\left(\sqrt{2}\right)x + 10 = 0, \alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2}\right)$ is equal to

11. The sum of all the roots of the equation $\left|x^2 - 8x + 15\right| - 2x + 7 = 0$ is

(1) $9 - \sqrt{3}$ (2) $9 + \sqrt{3}$

(3) $11 - \sqrt{3}$ (4) $11 + \sqrt{3}$

12. The number of the real roots of the equation $(x + 1)^2 + \left|x - 5\right| = \frac{27}{4}$ is _____.

13. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\frac{1}{\alpha^2} + 1, \frac{1}{\beta^2} + 1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are :

(1) non-real complex numbers (2) real and both negative

(3) real and both positive (4) real and exactly one of them is positive

14. Let α, β be the roots of the equation $x^2 - 4\lambda x + 5 = 0$ and α, γ be the roots of the equation $x^2 - \left(3\sqrt{2} + 2\sqrt{3}\right)x + 7 + 3\lambda\sqrt{3} = 0$. If $\beta + \gamma = 3\sqrt{2}$, then $(\alpha + 2\beta + \gamma)^2$ is equal to

15. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x+2| - 4 = 0$ respectively, where $[x]$ denotes the greatest integer $\leq x$. Then $m^2 + mn + n^2$ is equal to

16. If $x^2 + 9y^2 - 4x + 3 = 0,\ x, y \in R$, then x and y respectively lie in the intervals

(1) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$ (2) $[1,\ 3]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

(3) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $[1,\ 3]$ (4) $[1,\ 3]$ and $[1,\ 3]$

17. The minimum value of the sum of the squares of the roots of $x^2 + (3-a)x = 2a - 1$ is

(1) 6 (2) 4

(3) 5 (4) 8

18. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in R$, is:

(1) $\frac{7}{36}$ (2) $\frac{2}{9}$

(3) $\frac{1}{6}$ (4) $\frac{1}{4}$

19. If for some $p, q, r \in R$, not all have same sign, one of the roots of the equation $(p^2+q^2)x^2 - 2q(p+r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 - 2x - 8 = 0$, then $\frac{q^2+r^2}{p^2}$ is equal to-

20. Let $a, b \in R$ be such that the equation $ax^2 - 2bx + 15 = 0$ has repeated root α and if α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to:

(1) 37 (2) 58

(3) 68 (4) 92

21. Let $f(x)$ be a quadratic polynomial with leading coefficient 1 such that $f(0) = p, p \neq 0$, and $f(1) = \frac{1}{3}$. If the equations $f(x) = 0$ and $fofofof(x) = 0$ have a common real root, then $f(-3)$ is equal to _____.

22. The set of all real values of λ for which the quadratic equation $(\lambda^2+1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval $(0,\ 1)$ is :

(1) $(-3,\ -1)$ (2) $(0,\ 2)$

(3) $(1,\ 3]$ (4) $(2,\ 4]$

23. The sum of 162^{th} power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is _____.

24. Let $S = \left\{x : x \in \mathbb{R} \text{ and } \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10\right\}$. Then $n(S)$ is equal to

(1) 2 (2) 4

(3) 6 (4) 0

25. Let $S = \left\{\alpha : \log_2\left(9^{2\alpha-4} + 13\right) - \log_2\left(\frac{5}{2} \cdot 3^{2\alpha-4} + 1\right) = 2\right\}$. Then the maximum value of β for which the equation $x^2 - 2\left(\sum_{\alpha \in s} \alpha\right)^2 x + \sum_{\alpha \in s} (\alpha+1)^2 \beta = 0$ has real roots, is _____.

26. If $a + b + c = 1, ab + bc + ca = 2$ and $abc = 3$, then the value of $a^4 + b^4 + c^4$ is equal to:

27. The number of points, where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in \mathbb{R}$ cuts x-axis, is equal to.....

28. The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0, x \in R$ has :

(1) four solutions two of which are negative (2) two solutions and both are negative

(3) no solution (4) two solutions and only one of them is negative

29. Let $\alpha_1, \alpha_2, \ldots, \alpha_7 \alpha_1, \alpha_2, \ldots, \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq \ldots \geq |\alpha_7|$. Then, $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to _____

30. The number of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is :

(1) 6
(2) 8
(3) 4
(4) 2