ANSWER KEYS

1. (1)	**2.** (2)	**3.** (2)	**4.** (3)	**5.** (1)	**6.** (3)	**7.** (4)	**8.** (1)
9. (1)	**10.** (4)						

1. **(1)** $A(1,5,35), B(7,5,5), C(1,\lambda,7), D(2\lambda,1,2)$

$\overrightarrow{AB}=6\hat{i}-30\hat{k},\ \overrightarrow{BC}=-6\hat{i}(\lambda-5)\hat{j}+2\hat{k}$

$\overrightarrow{CD}=(2\lambda-1)\hat{i}+(1-\lambda)\hat{j}-5\hat{k}$

Points are coplanar

$\Rightarrow 0=\begin{vmatrix}6 & 0 & -30\\ -6 & \lambda-5 & 2\\ 2\lambda-1 & 1-\lambda & -5\end{vmatrix}$

$=6(-5\lambda+25-2+2\lambda)$

$-30\left(-6+6\lambda-\left(2\lambda^2-\lambda-10\lambda+5\right)\right)$

$=6(-3\lambda+23)-30\left(-2\lambda^2+11\lambda-5-6+6\lambda\right)$

$=6(-3\lambda+23)-30\left(-2\lambda^2+17\lambda-11\right)$

$=6\left(-3\lambda+23+10\lambda^2-85\lambda+55\right)$

$=6\left(10\lambda^2-88\lambda+78\right)=12\left(5\lambda^2-44\lambda+39\right)$

$\Rightarrow 0=12\left(5\lambda^2-44\lambda+39\right)$

$\lambda_1+\lambda_2=\frac{44}{5}$

2. **(2)**

Given the three vectors are coplanar.

Hence, $\begin{vmatrix}\mu & 1 & 1\\ 1 & \mu & 1\\ 1 & 1 & \mu\end{vmatrix}=0$

$\Rightarrow \mu\left(\mu^2-1\right)-1(\mu-1)+1(1-\mu)=0$

$\Rightarrow \mu(\mu-1)(\mu+1)-2(\mu-1)=0\ \left[\because a^2-b^2=(a-b)(a+b)\right]$

$\Rightarrow(\mu-1)(\mu\,(\mu+1)-2)=0$

$\Rightarrow(\mu-1)\left(\mu^2+\mu+2\right)=0$

$\Rightarrow(\mu-1)(\mu-1)(\mu+2)=0$

$\Rightarrow \mu=1,\ 1,\ -2$

Therefore, the sum of the distinct real values of $\mu=-2+1=-1$.

3. **(2)** $\pm1=\begin{vmatrix}1 & 1 & \lambda\\ 1 & 1 & 3\\ 2 & 1 & 1\end{vmatrix}\Rightarrow=-\lambda+3=\pm1\Rightarrow\lambda=2$ or $\lambda=4$

For $\lambda=4$

$\cos\theta=\frac{2+1+4}{\sqrt{6}\sqrt{18}}=\frac{7}{6\sqrt{3}}$

4. **(3)** $\left(2\vec{a}-\vec{b}\right)\cdot\left\{\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right\}$

$=\left(2\vec{a}-\vec{b}\right)\cdot\left\{\left(\vec{a}\times\vec{b}\right)\times\vec{a}+2\left(\vec{a}\times\vec{b}\right)\times\vec{b}\right\}$

$=\left(2\vec{a}-\vec{b}\right)\cdot\left\{\left(\vec{a}\cdot\vec{a}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{a}+2\left(\vec{a}\cdot\vec{b}\right)\vec{b}-2\left(\vec{b}\cdot\vec{b}\right)\vec{a}\right\}\left[\vec{a}\cdot\vec{b}=0,\ \left|\vec{a}\right|=\left|\vec{b}\right|=1\right]$

$=\left(2\vec{a}-\vec{b}\right)\cdot\left(\vec{b}-2\vec{a}\right)=-4\vec{a}\cdot\vec{a}-\vec{b}\cdot\vec{b}=-5$

5. **(1)**

We have,

$\frac{\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)}{\left(\vec{c}\times\vec{a}\right)\cdot\vec{b}}+\frac{\vec{b}\cdot\left(\vec{a}\times\vec{c}\right)}{\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)}$

$=\frac{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}{\left[\vec{c}\ \vec{a}\ \vec{b}\right]}+\frac{\left[\vec{b}\ \vec{a}\ \vec{c}\right]}{\left[\vec{c}\ \vec{a}\ \vec{b}\right]}$

$=\frac{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}{\left[\vec{c}\ \vec{a}\ \vec{b}\right]}-\frac{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}{\left[\vec{c}\ \vec{a}\ \vec{b}\right]}=0$

6. **(3)**

Given, $\left|\vec{a}\right|=1,\left|\vec{b}\right|=2,\left|\vec{c}\right|=3$ and

$\vec{a}\cdot\vec{b}=0=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}$ (as the three vectors are mutually perpendicular)

So,

$\left[\left(\vec{a}+\vec{b}+\vec{c}\right)\times\left(\vec{b}-\vec{a}\right)\right]\cdot\vec{c}$

$=\left[\vec{a}\times\vec{b}-0+0-\vec{b}\times\vec{a}+\vec{c}\times\vec{b}-\vec{c}\right]\cdot\vec{c}$

$=\left[2\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}+\left(\vec{c}\times\vec{b}\right)\cdot\vec{c}-\left(\vec{c}\times\vec{a}\right)\cdot\vec{c}\right]$

$=2\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}+0-0$

$=2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$=2\cdot1\cdot2\cdot3$

$=12$

7. **(4)**

$\left|\vec{a}\right|=\left|\vec{b}\right|=\left|\vec{c}\right|=2$

Given: $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=2$

$\Rightarrow\left|\vec{a}\right|^2=\left|\vec{b}\right|^2=\left|\vec{c}\right|^2=4$

$\Rightarrow\vec{a}\cdot\vec{a}=\vec{b}\cdot\vec{b}=\vec{c}\cdot\vec{c}=4$

Volume of tetrahedron $=\frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

Now, $\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2=\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}=\begin{vmatrix}4 & 2 & 2\\ 2 & 4 & 2\\ 2 & 2 & 4\end{vmatrix}$

$=4(12)+2(-4)+2(-4)=32$

$\therefore\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\sqrt{32}=4\sqrt{2}$

Volume $=\frac{1}{6}\times4\sqrt{2}=\frac{2\sqrt{2}}{3}$

8. **(1)** $\left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{b}\cdot\vec{c}\right)\vec{a}=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow0-\left(\vec{b}\cdot\vec{c}\right)\vec{a}=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow-\left|\vec{b}\right|\left|\vec{c}\right|\cos\theta=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|$

$\Rightarrow\ \cos\theta=-\frac{1}{3}$

$\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{1}{9}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$

9. **(1)** Given, $\vec{r}\times\vec{a}=\vec{b}$

Taking right cross product with $\vec{c}$, we get,

$\left(\vec{r}\times\vec{a}\right)\times\vec{c}=\vec{b}\times\vec{c}\Rightarrow\left(\vec{r}\cdot\vec{c}\right)\vec{a}-\left(\vec{a}\cdot\vec{c}\right)\vec{r}=\vec{b}\times\vec{c}$

$\Rightarrow3\vec{a}-\vec{r}=\vec{b}\times\vec{c}\Rightarrow\vec{r}=3\left(2\hat{i}+3\hat{j}+4\hat{k}\right)-\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$\Rightarrow\vec{r}=5\hat{i}+7\hat{j}+9\hat{k}\Rightarrow\left|\vec{r}\right|=\sqrt{155}$

10. **(4)**

The given expression

$$=\left\{\left\{\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{b}\times\vec{c}\right\}\times\left(\vec{b}\times\vec{c}\right)\right\}\cdot\left(\vec{b}+\vec{c}\right)$$

$$=\left\{\left(\vec{a}\times\vec{c}\right)\times\left(\vec{b}\times\vec{c}\right)+\left(\vec{b}\times\vec{a}\right)\times\left(\vec{b}\times\vec{c}\right)\right\}\cdot\left(\vec{b}+\vec{c}\right)$$

$$=\left[\left(\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)\right)\vec{c}-\left(\vec{c}\cdot\left(\vec{b}\times\vec{c}\right)\right)\vec{a}+\left(\vec{b}\cdot\left(\vec{b}\times\vec{c}\right)\right)\vec{a}-\left(\vec{a}\left(\left(\vec{b}\times\vec{c}\right)\vec{b}\right)\right)\right]\cdot\left(\vec{b}+\vec{c}\right)$$

$$=\left[\left(\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)\right)\left(\vec{c}-\vec{b}\right)\cdot\left(\vec{b}+\vec{c}\right)\right]=\left(\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)\right)\left[\left|\vec{c}\right|^2-\left|\vec{b}\right|^2\right]=0$$

$$\left[\left|\vec{b}\right|=\left|\vec{c}\right|=1\right].$$