ANSWER KEYS

1. (3)	**2.** (2)	**3.** (2)	**4.** (3)	**5.** (3)	**6.** (3)	**7.** (4)	**8.** (1)
9. (3)	**10.** (2)						

1. **(3)** We have,

$\phi(x)=\log_8 \log_3 x=\log_8\left(\frac{\log x}{\log 3}\right)$

$=\log_8(\log x)-\log_8(\log 3)$

$=\frac{\log(\log x)}{\log 8}-\log_8(\log 3)$

$\phi'(x)=\frac{1}{\log 8}\cdot\frac{1}{\log x}\cdot\frac{1}{x}-0$

$\therefore \phi'(e)=\frac{1}{\log 8}\cdot\frac{1}{\log e}\cdot\frac{1}{e}=\frac{1}{e\log 8}$

2. **(2)**

$y=3\log x+3\sin^{-1}x+kx^2$ (using $\log(ab)=\log a+\log b,\ \log(a^p)=p\log a$)

$\frac{dy}{dx}=\frac{3}{x}+\frac{3}{\sqrt{1-x^2}}+k(2x)$

$\Rightarrow \frac{dy}{dx}\left(_{x=\frac{1}{2}}\right)=6+\frac{6}{\sqrt{3}}+k$

Now $\frac{dy}{dx}\left(_{x=\frac{1}{2}}\right)=2\sqrt{3}$ given

$\therefore\ 6+\left(6/\sqrt{3}\right)+k=2\sqrt{3}\ \Rightarrow\ k=-6$

3. **(2)** $\because xy=1\Rightarrow y=\frac{1}{x}$

$\frac{dy}{dx}=\frac{-1}{x^2}$

at $x=\frac{1}{\sqrt{3}},\ \frac{dy}{dx}=-3$

4. **(3)**

Given that,

$f(x)=2^x\ \ldots\ldots\ldots(1)$

and, $g(x)=3^x\ \ldots\ldots\ldots(2)$

Differentiate (1) and (2) w.r.t x

$f'(x)=2^x\log 2$ and $g'(x)=3^x\log 3$

Hence, $f'(0)=2^0\log 2\Rightarrow f'(0)=\log 2$

$g'(0)=3^0\log 3\Rightarrow \log 3$

Now, $\frac{f'(0)-g'(0)}{1+f'(0)g'(0)}=\frac{\log 2-\log 3}{1+\log 2\ \log 3}$

$\Rightarrow \frac{\log\left({}^{2}/{}_{3}\right)}{1+\log 2\ \log 3}$

5. **(3)** $\frac{dy}{dz}=\frac{\frac{d}{dx}(\tan^{-1}x)}{\frac{d}{dx}(\cot^{-1}x)}$

$=\frac{\frac{1}{1+x^2}}{\frac{-1}{1+x^2}}=-1.$

6. **(3)**

$y=f(\tan x)\qquad z=g(\sec x)$

$dy/dx\frac{dy}{dx}=f'(\tan x).\sec^2 x dz/dx\frac{dz}{dx}=g'(\sec x)\ \sec x\ \tan x$

put $x=\frac{\pi}{4}$

$\frac{dy}{dx}=f'(1).2\qquad \frac{dz}{dx}=g'\left(\sqrt{2}\right).\sqrt{2}$

$\frac{dy}{dz}=\frac{2f'(1)}{g'\left(\sqrt{2}\right).\sqrt{2}}=\frac{2\times 2}{4\sqrt{2}}=\frac{1}{\sqrt{2}}$

7. **(4)** $\ell ny=(\ell nx)^2+\ell n\left(\tan^{-1}x\right)$

$\frac{y'}{y}=\frac{2\ell nx}{x}+\frac{1}{(\tan^{-1}x)(1+x^2)}$

at $x=1,\ y=\frac{\pi}{4}\Rightarrow y'=\frac{1}{2}$

8. **(1)**

$y=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\infty$

$\Rightarrow y=e^x$

$\Rightarrow \frac{dy}{dx}=e^x$

$\Rightarrow \frac{dy}{dx}=y.$

9. (3)

Given that,

$f'(x) > \phi'(x)$

$\Rightarrow 2^{2x-1}(2\log 2) > -2^{x}(\log 2) + 2\log 2$

$\Rightarrow 2^{2x} > -2^{x} + 2$

$\Rightarrow 2^{2x} + 2^{x} - 2 > 0$

$\Rightarrow (2^{x} - 1)(2^{x} + 2) > 0$

$\Rightarrow 2^{x} - 1 > 0 \ (\because 2^{x} + 2 > 0, \ \forall x \in R)$

$\Rightarrow 2^{x} > 1$

$\Rightarrow 2^{x} > 2^{0}$

$\therefore x > 0.$

10. (2)

We have,

$(\cos x)^{y} = (\sin y)^{x}$

Taking logarithms on both sides, we get

$y \log \ \cos x = x \log \sin y$

Differentiate with respect to x we get

$y \log \ \cos x = x \log \sin y$