ANSWER KEYS

1. (4)	**2.** (2)	**3.** (3)	**4.** (3)	**5.** (3)	**6.** (1)	**7.** (2)	**8.** (4)
9. (3)	**10.** (1)						

1. **(4)** Change the given equation in standard form, we get, $\frac{x+\frac{1}{3}}{-1}=\frac{y+\frac{2}{3}}{2}=\frac{z}{-1}$

So direction cosine are, $\left(\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$.

2. **(2)** We know, $\cos\theta=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$=\frac{|(2)(3)+(3)(-4)+(-6)(5)|}{\sqrt{2^2+3^2+(-6)^2}\sqrt{3^2+(-4)^2+(5)^2}}$

$=\frac{|6-12-30|}{\sqrt{4+9+36}\sqrt{9+16+25}}$

$=\frac{36}{(7)\cdot\left(5\sqrt{2}\right)}=\frac{18\sqrt{2}}{35}$

$\Rightarrow\ \theta=\cos^{-1}\left(\frac{18\sqrt{2}}{35}\right)$

3. **(3)**

Since point $N\left(\vec{n}\right)$ divides LM in ratio 2:1 externally is

$\vec{n}=\frac{a\left(\vec{m}\right)-b\left(\vec{l}\right)}{a-b}$ (Use section formula)

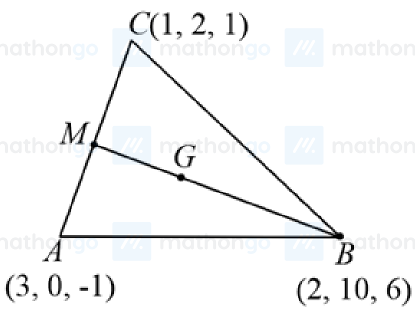
$\Rightarrow \vec{n}=\frac{2\left(\vec{m}\right)-\vec{l}}{2-1}$

$\Rightarrow \vec{n}=2\left(\vec{a}+2\vec{b}\right)-\left(2\vec{a}-\vec{b}\right)$

$\Rightarrow \vec{n}=0\vec{a}+5\vec{b}$

$\vec{n}=5\vec{b}$

4. **(3)**



$\because$ M is mid point of AC & G divides BM in the ratio 2 : 1 internally

$\therefore$ G is centroid of $\triangle ABC$

$\therefore G=\left(\frac{3+1+2}{3}, \frac{0+2+10}{3}, \frac{-1+1+6}{3}\right)=(2,4,2)$

$\therefore$ $\overrightarrow{OA}=3\vec{i}+0\vec{j}-\vec{k}$ & $\overrightarrow{OG}=2\vec{i}+4\vec{j}+2\vec{k}$

$\therefore$ $\cos\angle GOA=\frac{\overrightarrow{OA}.\overrightarrow{OG}}{\left|\overrightarrow{OA}\right|\left|\overrightarrow{OG}\right|}=\frac{6-2}{\sqrt{10}\sqrt{24}}=\frac{4}{\sqrt{10}\,2\sqrt{6}}$

$=\frac{1}{\sqrt{15}}$

5. **(3)** Let $A=(3,4,5)$, $B=(4,6,3)$, $C=(-1,2,4)$, $D\equiv(1,0,5)$

For AB, $x_2-x_1=4-3=1$, $y_2-y_1=6-4=2$

$z_2-z_1=3-5=-2$

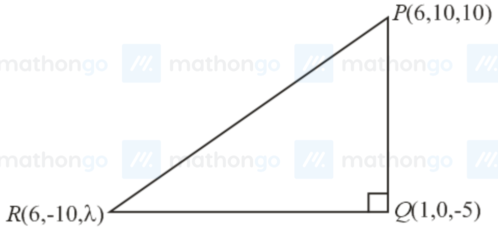
Let l, m, n for CD are $\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}$.

$\therefore$ Projection of AB on $CD=\sum l\left(x_2-x_1\right)$

$=\frac{2(1)}{3}+\left(-\frac{2}{3}\right)2+\left(\frac{1}{3}\right)\left(-2\right)$

$=-\frac{4}{3}$

6. (1)



In ΔPQR is right angled, at θ

$\therefore PR^2 = PQ^2 + RQ^2$

$\Rightarrow (6-6)^2 + (-10-10)^2 + (\lambda - 10)^2$

$= \left[(1-6)^2 + (0-10)^2 + (-5-10)^2\right] + \left[(1-6)^2 + (0+10)^2 + (-5-\lambda)^2\right]$

$\Rightarrow 400 + \lambda^2 + 100 - 20\lambda = (25 + 100 + 225) + \left(25 + 100 + 25 + \lambda^2 + 10\lambda\right)$

$\Rightarrow \lambda^2 - 20\lambda + 500 = 350 + 150 + 10\lambda + \lambda^2$

$\Rightarrow -20\lambda = 10\lambda \Rightarrow 30\lambda = 0$

$\Rightarrow \lambda = 0$

7. (2)

Any point in YZ-plane is $(0, \beta, \gamma)$

$\therefore \quad \beta + \gamma = 3 \quad \ldots(1)$

According to information in the problem, we can write

(Its distance from XZ-plane) = 2 (Its distance from XY-plane)

$\Rightarrow \quad |\beta| = 2|\gamma| \ \Rightarrow \ \beta = \pm 2\gamma$

case(a):- when $\beta = 2\gamma$, then from equation (1), we get

$\Rightarrow \ 2\gamma + \gamma = 3$

$\Rightarrow 3\gamma = 3$

$\Rightarrow \gamma = 1$

Hence by putting the value of $\gamma = 1$ in equation (1), we get

$\Rightarrow 1 + \beta = 3$

$\beta = 2$

case(b):- when $\beta = -2\gamma$, then from equation (1), we get

$\Rightarrow \ -2\gamma + \gamma = 3$

$\Rightarrow \gamma = -3$

Hence by putting the value of $\gamma = -3$ in equation (1), we get

$\Rightarrow -3 + \beta = 3$

$\Rightarrow \beta = 6$

Hence, required coordinates are $(0, 2, 1)$ or $(0, 6, -3)$.

8. **(4)** Given, $3lm - 4ln + mn = 0$ .....(i)

and $l + 2m + 3n = 0$ .....(ii)

From Eq. (ii), $l = -(2m + 3n)$ putting in Eq. (i)

$-3(2m+3n)m + 4(2m+3n)n + mn = 0$

$\Rightarrow \quad -6m^2 + 12n^2 = 0$

$\Rightarrow \quad m = \pm\sqrt{2}n$

Now, $m = \sqrt{2}n$

$\Rightarrow \quad l = -\left(2\sqrt{2}n + 3n\right) = -\left(2\sqrt{2} + 3\right)n$

$\therefore \ l : m : n = -\left(3 + 2\sqrt{2}\right)n : \sqrt{2}n : n$

$= -\left(3 + 2\sqrt{2}\right) : \sqrt{2} : 1$

Also, $m = -\sqrt{2}n \Rightarrow l = -\left(-2\sqrt{2} + 3\right)n$

$\therefore \ l : m : n = -\left(3 - 2\sqrt{2}\right)n : -\sqrt{2} : n$

$= -\left(3 - 2\sqrt{2}\right) : -\sqrt{2} : 1$

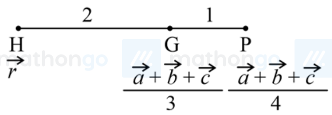
$= \cos\theta$

$= \dfrac{\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right) + \left(\sqrt{2}\right)\left(-\sqrt{2}\right) + 1\cdot 1}{\sqrt{\left(3+2\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 + 1^2}\sqrt{\left(3-2\sqrt{2}\right)^2 + \left(-\sqrt{2}\right)^2 + 1^2}}$

$= 0$

$\Rightarrow \quad \theta = \frac{\pi}{2}$

9. **(3)** Centroid divides orthocentre and circum centre in ratio 2 : 1



$\therefore \ \overrightarrow{OG} = \frac{2\times\overrightarrow{OP} + 1\times\overrightarrow{OH}}{2+1}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = \frac{\vec{a}+\vec{b}+\vec{c}}{2} + \vec{r}$

$\vec{r} = \frac{\vec{a}+\vec{b}+\vec{c}}{2}$

10. **(1)**

The given line is $\frac{1}{2}(x-1) = -y = z + 2$

$\therefore \ \frac{x-1}{2} = \frac{y-0}{-1} = \frac{z+2}{1} = \lambda$

$\therefore$ Coordinates of any point on the given line are $(2\lambda + 1, -\lambda, \lambda - 2)$

On substituting the point $(2\lambda + 1, -\lambda, \lambda - 2)$, on the given plane $2x + y - 3z = 4$, we get

$2(2\lambda + 1) + (-\lambda) - 3(\lambda - 2) = 4$

$\Rightarrow 8 = 4$ (not possible)

Hence, the point $(2\lambda + 1, -\lambda, \lambda - 2)$ cannot lie on the given plane.

The given line is parallel to the given plane. There is no point of intersection of line and the plane.

Hence, the direction cosines of the projected line are same as the direction cosine of the given line.

$\therefore$ Required direction cosines $= \left(\frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$