ANSWER KEYS

1. (4)	**2.** (1)	**3.** (14)	**4.** (2)	**5.** (39)	**6.** (2)	**7.** (2)	**8.** (4)
9. (2)	**10.** (1)	**11.** (4)	**12.** (62)	**13.** (7)	**14.** (4)	**15.** (1)	**16.** (25)
17. (79)	**18.** (2)	**19.** (4)	**20.** (3)	**21.** (2)	**22.** (1)	**23.** (2)	**24.** (1)
25. (2)	**26.** (4)	**27.** (2)	**28.** (3)	**29.** (2)	**30.** (4)		

1. (4)

Given,

$f(x)$ is continuous at $x=\frac{\pi}{2}$

Now, solving L. H. L. at $x=\frac{\pi}{2}$ we get,

$$\lim_{x\to\frac{\pi}{2}^{+}} e^{\frac{\cot 6x}{\cot 4x}}=\lim_{x\to\frac{\pi}{2}^{+}} e^{\frac{\sin 4x\cdot\cos 6x}{\sin 6x\cdot\cos 4x}}=e^{2/3}$$

Similarly, on simplification for R. H. L. we get

$$\lim_{x\to\frac{\pi}{2}^{-}}(1+|\cos x|)^{\frac{\lambda}{|\cos x|}}=e^{\lim\limits_{x\to\frac{\pi}{2}^{-}}\lambda}=e^{\lambda}$$

$\because f\left(\frac{\pi}{2}\right)=\mu$

For continuous function,

$f\left(\frac{\pi}{2}^{-}\right)=f\left(\frac{\pi}{2}\right)=f\left(\frac{\pi}{2}^{+}\right)$

$\Rightarrow e^{\frac{2}{3}}=e^{\lambda}=\mu$

$\Rightarrow \lambda=\frac{2}{3},\quad \mu=e^{\frac{2}{3}}$

Now,

$9\lambda+6\log_e\mu+\mu^6-e^{6\lambda}$

$=9\left(\frac{2}{3}\right)+6\left(\frac{2}{3}\right)+\left(e^{\frac{2}{3}}\right)^6-e^{6\left(\frac{2}{3}\right)}$

$=6+4+e^4-e^4$

$=10$

2. (1)

Given,

$$f(x)=\begin{cases}\frac{\log_e\left(1-x+x^2\right)+\log_e\left(1+x+x^2\right)}{\sec x-\cos x}, & x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)-\{0\}\\ k & , x=0\end{cases}$$

$$\Rightarrow \lim_{x\to 0}\frac{\left(\ln\left(1+x^2+x^4\right)\right)\cos x}{1-\cos^2 x}=k$$

Taking L.H.S$=\lim\limits_{x\to 0}\frac{\left(\ln\left(1+x^2+x^4\right)\right)\cos x}{1-\cos^2 x}$

$$=\lim_{x\to 0}\frac{\left(\frac{\ln\left(1+x^2+x^4\right)}{x^2+x^4}\right)x^2\left(1+x^2\right)\cos x}{\left(\frac{\sin^2 x}{x^2}\right)x^2}$$

$$=\lim_{x\to 0}\frac{\left(\frac{\ln\left(1+x^2+x^4\right)}{x^2+x^4}\right)\left(1+x^2\right)\cos x}{\left(\frac{\sin^2 x}{x^2}\right)}$$

$$=\frac{(1)(1+0)1}{(1)}=1$$

Now equating with R.H.S we get, $k=1$

3. **(14)**

We have,

$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), & \text{for } x\le 0 \\ \frac{\tan 2x-\sin 2x}{bx^3}, & \text{for } x>0 \end{cases}$

If function is continuous at $x=0$, then

$\mathrm{LHL}=\mathrm{RHL}=f(0)$

Now,

$\mathrm{LHL}=\lim_{x\to 0^-}\left[a\sin\frac{\pi}{2}(x-1)\right]$

$\Rightarrow \mathrm{LHL}=\lim_{h\to 0}\left[a\sin\frac{\pi}{2}(0-h-1)\right]$

$\Rightarrow \mathrm{LHL}=\lim_{h\to 0}\left[a\sin\frac{\pi}{2}(-1)\right]$

$\Rightarrow \mathrm{LHL}=-a$

Now,

$\mathrm{RHL}=\lim_{x\to 0^+}\left(\frac{\tan 2x-\sin 2x}{bx^3}\right)$

$\because \sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}+\ldots\ldots$

$\because \tan x=x+\frac{x^3}{3}+\frac{2x^5}{15}+\ldots.$

$\mathrm{RHL}=\lim_{x\to 0^+}\left[\frac{\frac{(2x)^3}{3}+\frac{(2x)^3}{6}}{bx^3}\right]=\frac{\frac{8}{3}+\frac{8}{6}}{b}=\frac{4}{b}$

And,

$f(0)=a\sin\left[\frac{\pi}{2}(0-1)\right]=a\sin\left(-\frac{\pi}{2}\right)=-a$

Then,

$\mathrm{RHL}=f(0)$

$\Rightarrow \frac{4}{b}=-a\Rightarrow -ab=4\Rightarrow 10-ab=14$

4. **(2)**

Given $f(x)=\lim_{n\to\infty}\frac{\cos 2\pi x-x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$

$=\lim_{n\to\infty}\frac{\cos 2\pi x-(x^2)^n\sin(x-1)}{1+x(x^2)^n-(x^2)^n}$

Now for $-1<x<1$, as $0<x^2<1\Rightarrow \lim_{n\to\infty}x^{2n}\to 0$

i.e. $f(x)=\cos 2\pi x$

Now again rewriting $f(x)=\lim_{n\to\infty}\frac{(x^2)^{-n}\cos 2\pi x-\sin(x-1)}{(x^2)^{-n}+x-1}$

For $x>1$ or $x<-1$, $\lim_{n\to\infty}x^{-2n}\to 0$

i.e. $f(x)=-\frac{\sin(x-1)}{x-1}$

For $x=\pm 1$, $f(x)=\begin{cases} 1 & \text{if } x=1 \\ \frac{(1+\sin 2)}{-1} & \text{if } x=-1 \end{cases}$

i.e. $\lim_{x\to 1^+}f(x)=-1$, $\lim_{x\to 1^-}f(x)=1$

So f is discontinuous at $x=1$

$\lim_{x\to -1^+}f(x)=1$, $\lim_{x\to -1^-}f(x)=-\frac{\sin 2}{2}$

So $f(x)$ is discontinuous at $x=-1$

5. **(39)**

We have, $f(x)=\begin{cases}\frac{P(x)}{\sin(x-2)}, & x \neq 2 \\ 7, & x=2\end{cases}$

$P''(x)=$ Constant

$\Rightarrow P(x)$ is a 2 degree polynomial

Let $P(x)=(x-2)(ax+b)$

$f(x)$ is continuous at $x=2$

$f(2^+)=f(2^-)$

Now, $\lim_{x\to 2^+} \frac{P(x)}{\sin(x-2)}=7$

$\Rightarrow \lim_{x\to 2^+} \frac{(x-2)(ax+b)}{\sin(x-2)}=7$

$\Rightarrow 2a+b=7$

$\Rightarrow P(3)=(3-2)(3a+b)=9$

$\Rightarrow 3a+b=9$

$\Rightarrow a=2, b=3$

Hence, $P(5)=(5-2)(2.5+3)$

$=3.13$

$=39.$

6. **(2)** Need to check at doubtful points discont at $x \in I$ only

at $x=-1 \Rightarrow f\left(-1^+\right)=1+0=1$

$\Rightarrow f\left(-1^-\right)=2+1=3$

at $x=0 \Rightarrow f\left(0^+\right)=0+0=0$

$\Rightarrow f\left(0^-\right)=1+1=2$

at $x=1 \Rightarrow f\left(1^+\right)=1+0=1$

$\Rightarrow f\left(1^-\right)=0+1=1$

discont. at two points

7. **(2)** Here $f(x)=[x(x-1)]+\{x\}$

$f\left(0^+\right)=-1+0=-1$ $f(0)=0$	$f\left(1^+\right)=0+0=0$ $f(1)=0$ $f\left(1^-\right)=-1+1=0$

$\therefore \quad f(x)$ is continuous at $x=1$, discontinuous at $x=0$

8. **(4)**

Doubtful points are $x=n,\ n \in I$

$L.H.L=\lim_{x\to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi=(n-2)\cos\left(\frac{2n-1}{2}\right)\pi=0$

$R.H.L=\lim_{x\to n^+}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi=(n-1)\cos\left(\frac{2n-1}{2}\right)\pi=0$

$f(n)=0$

Hence continuous.

9. (2)

As $[.]$ is discontinuous at integers so we will check at $x=-1,\ 0,\ 1$ only

at $x=-1$

$[-1^{-}]=-2$ and $[-1^{+}]=-1$

we have

LHL $=(-2)\cdot(0)+\sin(\pi)+1=1$

RHL $=\left(-1\right)\cdot\left(0\right)+\sin\left(\frac{\pi}{2}\right)-0=1$

$f(-1)=1 \Rightarrow$ Continuous at $x=-1$

Again at $x=0$

$[0^{-}]=-1$ & $[0^{+}]=0$

LHL $=\left(-1\right)\cdot\left(1\right)+\sin\left(\frac{\pi}{2}\right)-0=0$

RHL $=0+\sin\left(\frac{\pi}{3}\right)-1\neq$ LHL

Hence, discontinuous at $x\ =\ 0$

Again at $x=1$,

$[1^{-}]=0$ & $[1^{+}]=1$

We have

LHL $=0+\sin\left(\frac{\pi}{3}\right)-1$

RHL $=0+\sin\left(\frac{\pi}{4}\right)-2\neq$ LHL

Hence, discontinuous at $x=1$

Hence, discontinuous at exactly 2 points

10. (1)

$\log_{x\to0}\left(\frac{4}{x}-\left\{\frac{4}{x}\right\}\right)=A \Rightarrow \log_{x\to O}4-x\left\{\frac{4}{x}\right\}=A$

$\Rightarrow 4-0$

$\Rightarrow A=4$

Hence, for $\sqrt{5}$, $f(x)$ is discontinuous as it is an non-integral value.

11. (4)

Given $f(x)=\begin{cases}x+a; & x\leq0\\ |x-4|; & x>0\end{cases}$ and $g(x)=\begin{cases}x+1; & x<0\\ (x-4)^2+b; & x\geq0\end{cases}$

Given $f(x)$ & $g(x)$ are continuous on R

So, for continuity

$f(0^{-})=f(0)=f(0^{+})$

$\Rightarrow a=|0-4|$

$\Rightarrow a=4$

And $g(0^{-})=g(0)=g(0^{+})$

$\Rightarrow 1=(0-4)^2+b$

$\Rightarrow b=1-16=-15$

Then, $g(f(2))+f(g(-2))$

$=g(2)+f(-1)$

$=(2-4)^2-15+(-1+4)$

$=4-15+3=-8$

12. (62)

Given,

$f(x)=[2x^2+1]$ and $g(x)=\begin{cases}2x-3, & x<0\\ 2x+3, & x\geq0\end{cases}$

Now, $f(g(x))=[2g^2(x)]+1$

$=\begin{cases}\left[2(2x-3)^2\right]+1; x<0\\ \left[2(2x+3)^2\right]+1; x\geq0\end{cases}$

If $x<0$ then $-1<x<0 \Rightarrow -2<x<0$

$\Rightarrow -5<2x-3<-3 \Rightarrow\Rightarrow 9<(2x-3)^2<25$

$\Rightarrow 18<2(2x-3)^2<50$, So there will be 31 values where $2(2x-3)^2$ is integer, Similarly for $2(2x+3)^2$ there will be 31, so total 62 points of discontinuity

13. (7)

Here both the modulus function is continuous in its domain.

At $x=-1$,

$\lim_{x\to-1^-}|2x^2-3x-7|=2$ & $\lim_{x\to-1^+}[4x^2-1]=2$

Hence $f(x)$ is continuous at $x=-1$

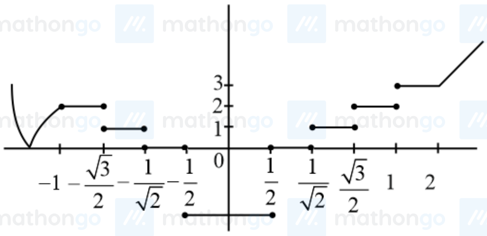
At $x=1$,

$\lim_{x\to1^-}[4x^2-1]=2$ & $\lim_{x\to1^+}|x+1|+|x-2|=3$

Hence, $f(x)$ is discontinuous at $x=1$

For $x\in(-1,1)$ the function $f(x)$ is discontinuous when $[4x^2-1]=0/\,1/\,2$

i.e when $x\in\left\{\pm\frac{1}{\sqrt{2}},\pm\frac{1}{2},\pm\frac{\sqrt{3}}{2}\right\}$



14. (4)

Let $g(x)=1+x+[x]=\begin{cases}1+x; & x\in[0,1)\\ 2+x; & x\in[1,2)\\ 5; & x=2\end{cases}$

$\lambda(x)=x+2[x]=\begin{cases}x; & x\in[0,1)\\ x+2; & x\in[1,2)\\ 6; & x=2\end{cases}$

$r(x)=2+x$

$f(x)=\begin{cases}2+x; & x\in[0,2)\\ 6; & x=2\end{cases}$

$f(x)$ is discontinuous only at $x=2\Rightarrow m=1$

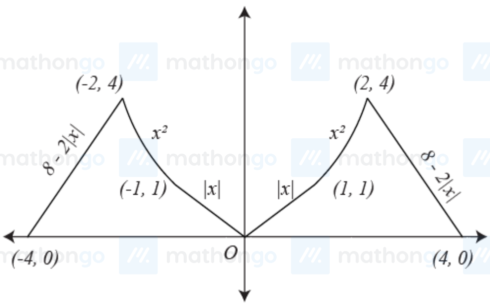
$f(x)$ is differentiable in $(0,2)\Rightarrow n=0$

$(m+n)^2+2=3$

15. (1)

Given function is $f(x)=\begin{cases}max(|x|,\ x^2), & |x|\le 2\\ 8-2|x|, & 2<|x|\le 4\end{cases}$

The graph of the function, is



We know that, a function is not differentiable at a point where its graph has a sharp corner.

And, from the graph we can easily conclude that $f(x)$ is non-derivable at $x=-2,\ -1,\ 0,1,\ 2$

Hence, the set $S=\{-2,\ -1,\ 0,1,\ 2\}$.

16. (25) $f(x)=[a+13\sin x], x\in(0,\pi)$

For $[n\sin x]$; Total number of non differentiable points are $=2n-1$ for $x\in(0,\pi)$

So number of non differentiable points for $[13\sin x]\Rightarrow 25$ Points

17. (79)
Given,
$f(x)= 4|2x+3|+9\left[x+\frac{1}{2}\right]-12[x+20]$
$x\in(-20,20)$
Now we know that $[x]$ is non-differentiable when $x\in I$
So, $12[x+20]$ is not differentiable at $x=I\in\{-19,-18,\ldots 0,\ldots 19\}=39$
Now $9\left[x+\frac{1}{2}\right]$ is not differentiable when $x=I+\frac{1}{2}$
So, here also total non-differentiable points are 39
Now checking modulus function
We know that modulus function are non-differentiable at point where modulus function is zero,
So at $x=\frac{-3}{2}$ modulus function is zero
Also $f(x)$ at x $=\frac{-3}{2}$ is not continuous and non-differentiable,
So, number of point of non-differentiability $=39+39+1=79$

18. (2)
Given that $f(x)=|x^2-2x-3|e^{(9x^2-12x+4)}$
Differentiating on both sides
$f'(x)=|x^2-2x-3|.e^{(9x^2-12x+4)}.(18x-12)+\frac{|x^2-2x-3|}{x^2-2x-3}.(2x-2).e^{(9x^2-12x+4)}$
$\Rightarrow f'(x)=|x^2-2x-3|.e^{(9x^2-12x+4)}\left\{(18x-12)+\frac{(2x-2)}{x^2-2x-3}\right\}$
$\Rightarrow f'(x)=|x^2-2x-3|.e^{(9x^2-12x+4)}.\frac{\{18x^3-36x^2-54x-12x^2+24x+36+2x-2\}}{x^2-2x-3}$
$\Rightarrow f'(x)=|x^2-2x-3|.e^{(9x^2-12x+4)}.\frac{\{18x^3-48x^2-28x+34\}}{(x-3)(x+1)}$
Here $f'(x)$ is not defined at $x=-1$ & $x=3$.
$f(x)$ is not differentiable at two points.

19. (4)
$f(x)=\begin{cases}\frac{1}{|x|}, & |x|\geq 1\\ ax^2+b, & |x|<1\end{cases}$
at $x=1$ function must be continuous
So, $1=a+b\ \ldots(1)$
differentiability at $x=1$
$\left(-\frac{1}{x^2}\right)_{x=1}=\left(2\cdot ax\right)_{x=1}$
$\Rightarrow -1=2a\Rightarrow a=-\frac{1}{2}$
Put in $(1)\Rightarrow b=1+\frac{1}{2}=\frac{3}{2}$

20. (3)
Check non-differentiability of function, we get,
$\because f(x)$ is non differentiable at $x=1,3,5$
$\sum f(f(x))=f(f(1))+f(f(3))+f(f(5))$
$=1+1+1$
$=3$

21. (2) $f(x)=|2x+1|-3|x+2|+|x^2+x-2|$
$=|2x+1|-3|x+2|+|x+2||x-1|$
$=|2x+1|+|x+2|(|x-1|-3)$
Critical points are $x=\frac{-1}{2},-2,-1$
but $x=-2$ is making a zero twice in product so, points of non differentiability are $x=\frac{-1}{2}$ and $x=-1$
$\therefore$ Number of points of non-differentiability $=2$

22. **(1)** $f(x)=\sin|x|-|x|+2(x-\pi)\cos|x|$

There are two cases,

Case (1), $x>0$

$f(x)=\sin x-x+2(x-\pi)\cos x$

$f'(x)=\cos x-1+2(1-0)\cos x-2\sin(x-\pi)$

$f'(x)=3\cos x-2(x-\pi)\sin x-1$

Then, function $f(x)$ is differentiable for all $x>0$

Case (2) $x<0$

$f(x)=-\sin x+x+2(x-\pi)\cos x$

$f'(x)=-\cos x+1-2(x-\pi)\sin x+2\cos x$

$f'(x)=\cos x+1-2(x-\pi)\sin x$

Then, function $f(x)$ is differentiable for all $x<0$

Now check for $x=0$

$f'\left(0^{+}\right)$ R.H.D. $=3-1=2$

$f'\left(0^{-}\right)$L.H.D. $=1+1=2$

L.H.D. = R.H.D.

Then, function $f(x)$ is differentiable for $x=0$. So it is differentiable everywhere

Hence, $k=\phi$

23. **(2)**

$$f(g(x))=\begin{cases} g(x)+2, & g(x)<0\\ (g(x))^{2}, & g(x)\geq 0\end{cases}$$

$$=\begin{cases} x^{3}+2, & x<0\\ x^{6}, & x\in[0,1)\\ (3x-2)^{2}, & x\in[1,\infty)\end{cases}$$

$$(fog(x))'=\begin{cases} 3x^{2}, & x<0\\ 6x^{5}, & x\in[0,1)\\ 2(3x-2)\times 3, & x\in[1,\infty)\end{cases}$$

At $x=0$

L. H. L. $\neq$ R. H. L. (Discontinuous)

At $x=1$

L. H. L. = R. H. L. (continuous)

L. H. D. = 6 = R. H. D.

$\Rightarrow fog(x)$ is differentiable for all $x\in R-\{0\}$

24. **(1)**

Given $f(1)=2$ and $f(x+y)=f(x)\cdot f(y)$

Put $x=y=1$

$\Rightarrow f(2)=f(1)\cdot f(1)=2^{2}$

Now, put $x=1,\ y=2$

$\Rightarrow f(3)=f(1)\cdot f(2)=2^{3}$

Now, put $x=1,\ y=3$

$\Rightarrow f(4)=f(1)\cdot f(3)=2^{4}$

Similarly, $f(10)=2^{10}$

Now $\sum_{k=1}^{10} f(a+k)=16\left(2^{10}-1\right)$

$\Rightarrow \sum_{k=1}^{10} f(a)f(k)=16\left(2^{10}-1\right)$

$\Rightarrow f(a)\sum_{k=1}^{10} f(k)=16\left(2^{10}-1\right)$

$\Rightarrow f(a)(f(1)+f(2)\ldots+f(10))=16\left(2^{10}-1\right)$

$\Rightarrow f(a)\left(2^{1}+2^{2}+2^{3}+\ldots+2^{10}\right)=16\left(2^{10}-1\right)$

Using the sum of n terms of a geometric progression, i.e. $a+ar+ar^{2}+ar^{3}+\ldots+ar^{n-1}=\frac{a\left(r^{n}-1\right)}{r-1}$, we get

$f(a)\frac{2\left(2^{10}-1\right)}{2-1}=16\left(2^{10}-1\right)$

$\Rightarrow f(a)=8=2^{3}$

Also, we know that $f(3)=8$

$\Rightarrow f(a)=f(3)$

$\Rightarrow a=3.$

25. **(2)** Given functional equality is

$|f(x)-f(y)| \leq 2\,|x-y|^{3/2}$

Put $x=y+h$, we get

$|f(y+h)|-f(y)| \leq 2.\ |h|^{3/2}$

$\Rightarrow \left|\frac{f(y+h)-f(y)}{h}\right| \leq 2\,|h|^{1/2}$

$\Rightarrow \lim\limits_{h \rightarrow 0}\left|\frac{f(y+h)-f(y)}{h}\right| \leq \lim\limits_{h \rightarrow 0} 2\,|h|^{1/2}$

$\Rightarrow |f'(y)| \leq 0$

$\Rightarrow |f'(y)|=0$

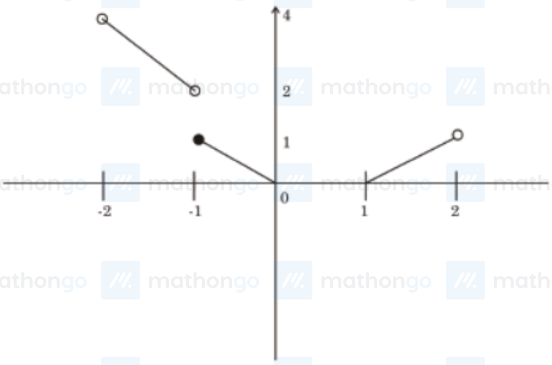
$\Rightarrow f'(y)=0$

$\Rightarrow f(y)=c$

$\Rightarrow f(y)=1$ (since $f(0)=1$)

Now $\int_0^1 f^2(x) dx = \int_0^1 1\, dx = 1.$

26. **(4)** $f(x)=\begin{cases} x[x] & ,-2<x<0 \\ (x-1)[x] & ,0 \leq x<2 \end{cases}$



$|f(x)|=$ Remain same

$m=1, n=3$

$m+n=4$

27. (2)

Given functions are:

$f''(x)=g''(x)+6x \quad \ldots(1)$

$f'(1)=4g'(1)-3=9 \quad \ldots(2)$

$f(2)=3g(2)=12 \quad \ldots(3)$

By integrating (1), we get

$f'(x)=g'(x)+3x^2+C$

At $x=1$

$f'(1)=g'(1)+3+C$

$\Rightarrow 9=3+3+C \Rightarrow C=3$

$\therefore \ f'(x)=g'(x)+3x^2+3$

Again by integrating, we get

$f(x)=g(x)+x^3+3x+D$

At $x=2$, we get

$f(2)=g(2)+8+3(2)+D$

$\Rightarrow 12=4+8+6+D \Rightarrow D=-6$

So,

$f(x)=g(x)+x^3+3x-6$

Option (A):

At $x=-2$,

$\Rightarrow g(-2)-f(-2)=20$

So, this option is true.

Option (B)

If $-1<x<2$

Let $h(x)=f(x)-g(x)=x^3+3x-6$

$\Rightarrow h'(x)=3x^2+3$

$\Rightarrow h'(x)>0$ for all values of x.

So, $h(-1)<h(x)<h(2)$

$\Rightarrow -10<h(x)<8$

$\Rightarrow |h(x)|<10$

So, this option is NOT true.

Option (C)

$h'(x)=f'(x)-g'(x)=3x^2+3$

If $|h'(x)|<6 \Rightarrow |3x^2+3|<6$

$\Rightarrow 3x^2+3<6$ and $-6<3x^2+3$

$\Rightarrow x^2<1$ and $x^2>-3(\text{always true})$

$\Rightarrow -1<x<1$

So, If $x \in(-1,1)$ then $|f'(x)-g'(x)|<6$

So, this option is true.

Option (D)

$f(x)-g(x)=0$

$\Rightarrow x^3+3x-6=0$

$h(x)=x^3+3x-6$

Here, $h(1)=-$ ve and $h\left(\frac{3}{2}\right)=+$ ve

So, there exists $x_0 \in\left(1, \frac{3}{2}\right)$ such that $f(x_0)=g(x_0)$

So, this option is true.

28. (3)

$$f(x)=\begin{cases} x^3-3x, x\le -1 \\ 2, -1<x\le 2 \\ x^2+2x-6, 2<x<3 \\ 9, 3\le x<4 \\ 10, 4\le x<5 \\ 11, x=5 \\ 2x+1, x>5 \end{cases}$$

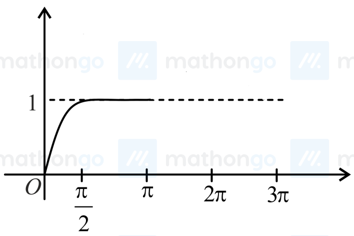
Clearly $f(x)$ is not differentiable at $x=2,3,4,5 \Rightarrow m=4$

$I=\int_{-2}^{2} f(x)dx=\int_{-2}^{-1}(x^3-3x)dx+\int_{-1}^{2} 2\cdot dx=\frac{27}{4}$

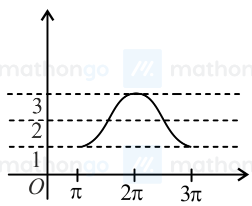
Hence the ordered pair is $\left(4, \frac{27}{4}\right)$

29. (2)

Graph of $\max\{\sin t: 0\le t\le x\}$ in $x\in[0,\pi]$

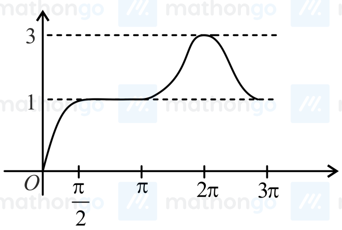


& the graph of $y=2+\cos x$ for $x\in[\pi,\infty)$



So graph of

$$f(x)=\begin{cases} \sin x, & 0\le x\le \frac{\pi}{2} \\ 1, & \frac{\pi}{2}\le x\le \pi \\ 2+\cos x, & x>\pi \end{cases}$$



$f(x)$ is differentiable everywhere in $(0,\infty)$

30. (4)

The given function is $f(x)=\begin{cases} 3\left(1-\frac{|x|}{2}\right) & if \quad |x| \leq 2 \\ 0 & if \quad |x|>2 \end{cases}$

By using the definition of modulus function, we have $|x|=\begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$ and also, we know that if $|x| \leq a, \Rightarrow -a \leq x \leq a$ and if $|x|>a, \Rightarrow x<-a$ or $x>a$,

$$\Rightarrow f(x)=\begin{cases} 3\left(1+\frac{x}{2}\right), & -2 \leq x \leq 0 \\ 3\left(1-\frac{x}{2}\right), & 0<x \leq 2 \\ 0, & -2>x \text{ or } x>2 \end{cases}$$

Now, $f(x-2)=\begin{cases} 3\left(1+\frac{x-2}{2}\right), & -2 \leq x-2 \leq 0 \\ 3\left(1-\frac{x-2}{2}\right), & 0<x-2 \leq 2 \\ 0, & -2>x-2 \text{ or } x-2>2 \end{cases}$

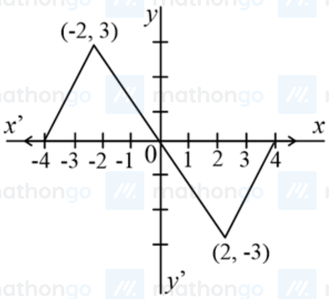
$$\Rightarrow f(x-2)=\begin{cases} \frac{3x}{2}, & 0 \leq x \leq 2 \\ 6-\frac{3x}{2}, & 2<x \leq 4 \\ 0, & 0>x \text{ or } x>4 \end{cases}$$

And, $f(x+2)=\begin{cases} 3\left(1+\frac{x+2}{2}\right), & -2 \leq x+2 \leq 0 \\ 3\left(1-\frac{x+2}{2}\right), & 0<x+2 \leq 2 \\ 0, & -2>x+2 \text{ or } x+2>2 \end{cases}$

$$\Rightarrow f(x+2)=\begin{cases} 6+\frac{3x}{2}, & -4 \leq x \leq -2 \\ -\frac{3x}{2}, & -2<x \leq 0 \\ 0, & -4>x \text{ or } x>0 \end{cases}$$

Hence, $g(x)=f(x+2)-f(x-2)=\begin{cases} \frac{3x}{2}+6, & -4 \leq x \leq -2 \\ -\frac{3x}{2}, & -2<x<2 \\ \frac{3x}{2}-6, & 2 \leq x \leq 4 \\ 0, & x \in(-\infty,-4) \cup(4, \infty) \end{cases}$

The graph of $g(x)$ is given below



From the graph we can observe that the function $g(x)$ is continuous everywhere, hence the number of points where $g(x)$ is not continuos is $n=0$ and the graph has sharp corners at the points $(-4,\ 0),\ (4,\ 0),\ (-2,\ 3)$ & $(2,\ -3)$ thus the number of points where $g(x)$ is not differentiable are $m=4$

$\Rightarrow n+m=4$.