ANSWER KEYS

1. (4)	**2.** (1)	**3.** (3)	**4.** (1)	**5.** (2.00)	**6.** (2)	**7.** (1)	**8.** (1)
9. (2)	**10.** (1)						

1. **(4)**

Given, $2\left(\frac{d^2y}{dx^2}\right)+2\left(\frac{dy}{dx}\right)=x\sin\left(\frac{d^2y}{dx^2}\right)$

As we know the degree of any differential equation can be found when it is in the form of polynomial of differential coefficients otherwise, the degree can not be defined.

So it is clear that given differential equation cannot be represented in the form of polynomial, so its degree cannot be defined.

2. **(1)** Given that

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=k^2\left(\frac{d^2y}{dx^2}\right)^2$$

Here highest derivative is $\frac{d^2y}{dx^2}$ and it's exponent is 2.

So required order and degree both are 2

3. **(3)** The given family of curves is $y=k_1 2^{k_2 x}+k_3 3^{k_4} 3^x=k_1 2^{k_2 x}+k_5 3^x$

which has '3' arbitrary constants.

Hence, the order will be 'three'

4. **(1)**

Equation of parabolas whose axes is parallel to X-axis is given as

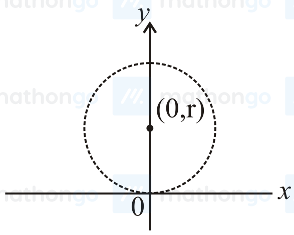
$(y-k)^2=4a(x-h)$

Here in this equation arbitrary constants are a, k , h which are 3

Therefore Order of the equation is 3.

Order of a given equation is equal to number of arbitrary constants.

5. **(2.00)**



The equation of the family of circles is $x^2+(y-r)^2=r^2$

i.e. only one arbitrary constant.

Hence, the order of its differential equation will be '1'

$\therefore 2k=2$

6. **(2)** We have $\frac{dy}{dx}=\left(e^{3x}+x^2\right)e^{-2y}$

$\Rightarrow e^{2y}dy=\left(e^{3x}+x^2\right)dx$

$\Rightarrow \int e^{2y}dy=\int\left(e^{3x}+x^2\right)dx+a$

$\Rightarrow \frac{e^{2y}}{2}=\frac{e^{3x}}{3}+\frac{x^3}{3}+a$

$\Rightarrow 3e^{2y}=2\left(e^{3x}+x^3\right)+c$

7. (1)

Given: $\frac{dy}{dx}=\frac{y-1}{x^2+x}$

$\Rightarrow \frac{dy}{y-1}=\frac{dx}{x(1+x)}$

Integrating,

$\Rightarrow \int \frac{dy}{y-1}=\int \frac{(1+x-x)dx}{x(1+x)}$

$\Rightarrow \ln\left(y-1\right)=\int \frac{1}{x}dx-\int \frac{1}{1+x}dx$

$\Rightarrow \ln(y-1)=\ln x-\ln(1+x)+\ln c$

$\Rightarrow \ln\left(y-1\right)=\ln \frac{cx}{1+x}$

$\Rightarrow y-1=\frac{cx}{1+x}\ldots\left(1\right)$

Put $(1,\ 0)$ in equation(1), we get,

$\Rightarrow 0-1=\frac{c}{1+1}$

$\Rightarrow c=-2$

$\therefore$ From equation (1), $2x+(y-1)(x+1)=0$

8. (1) $\frac{dy}{dx}+\frac{y}{x}=\frac{1}{(1+\ln xy)^2}$

Let $xy=u$ so that $\frac{du}{dx}=\frac{x}{(1+\ln u)^2}$

$\therefore \int(1+\ln u)^2du=\int x\,dx+c$

$\Rightarrow u\left(1+\ln u\right)^2-\int \frac{2(1+\ln u)}{u}\cdot udu=\frac{x^2}{2}+c$

$\Rightarrow u\left(1+2\ \ln u+2u(\ln u)^2\right)-2u\ln u=\frac{x^2}{2}+c$

$\therefore xy\left(1+\left(\ln(xy)\right)^2\right)=\frac{x^2}{2}+c$

9. (2)

Given equation

$\frac{dy}{dx}+\sin\left(\frac{x+y}{2}\right)=\sin\left(\frac{x-y}{2}\right)$

$\Rightarrow \frac{dy}{dx}=\sin\left(\frac{x-y}{2}\right)-\sin\left(\frac{x+y}{2}\right)$

$\Rightarrow \frac{dy}{dx}=-2\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)$

$\Rightarrow \text{cosec}\left(\frac{y}{2}\right)dy=-2\cos\left(\frac{x}{2}\right)dx$

On integrating both sides, we get

$\int \text{cosec}\left(\frac{y}{2}\right)dy=-\int 2\cos\left(\frac{x}{2}\right)dx$

$\Rightarrow \frac{\ln\left(\tan\frac{y}{4}\right)}{\frac{1}{2}}=-\frac{2\sin\left(\frac{x}{2}\right)}{\frac{1}{2}}+c$

$\Rightarrow \ln\left(\tan\frac{y}{4}\right)=c-2\sin\left(\frac{x}{2}\right)$

10. (1)

Given,

$\log_e\left(\frac{dy}{dx}\right)=3x+4y$

$\Rightarrow \frac{dy}{dx}=e^{3x}\cdot e^{4y}$

$\Rightarrow \int e^{-4y}dy=\int e^{3x}dx$

$\Rightarrow \frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+C$

Given,

$y(0)=0$

So,

$-\frac{1}{4}-\frac{1}{3}=C\Rightarrow C=-\frac{7}{12}$

So, the particular solution is

$\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}-\frac{7}{12}$

$\Rightarrow e^{-4y}=\frac{4e^{3x}-7}{-3}$

$\Rightarrow e^{4y}=\frac{3}{7-4e^{3x}}\Rightarrow 4y=\ln\left(\frac{3}{7-4e^{3x}}\right)$

$4y=\ell n\left(\frac{3}{6}\right)$ when $x=-\frac{2}{3}\ell n2$

$\Rightarrow y=\frac{1}{4}\ell n\left(\frac{1}{2}\right)$

$\Rightarrow y=-\frac{1}{4}\ell n2$

So, $\alpha=-\frac{1}{4}$