

- If the function $f(x) = \begin{cases} (1 + |\cos x|) \frac{\lambda}{|\cos x|}, & 0 < x < \frac{\pi}{2} \\ \mu, & x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}}, & \frac{\pi}{2} < x < \pi \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6 \log_e \mu + \mu^6 - e^{6\lambda}$ is equal to

(1) 11 (2) 8
(3) $2e^4 + 8$ (4) 10
- If the function $f(x) = \begin{cases} \frac{\log_e(1-x+x^2) + \log_e(1+x+x^2)}{\sec x - \cos x}, & x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\} \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then k is equal to:

(1) 1 (2) -1
(3) e (4) 0
- Let $a, b \in \mathbb{R}, b \neq 0$. Defined a function, $f(x) = \begin{cases} a \sin \frac{\pi}{2}(x-1), & \text{for } x \leq 0 \\ \frac{\tan 2x - \sin 2x}{bx^3}, & \text{for } x > 0 \end{cases}$ If f is continuous at $x = 0$, then $10 - ab$ is equal to
- The function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \lim_{n \rightarrow \infty} \frac{\cos(2\pi x) - x^{2n} \sin(x-1)}{1 + x^{2n+1} - x^{2n}}$ is continuous for all x in

(1) $\mathbb{R} - \{-1\}$ (2) $\mathbb{R} - \{-1, 1\}$
(3) $\mathbb{R} - \{1\}$ (4) $\mathbb{R} - \{0\}$
- Consider the function $f(x) = \frac{P(x)}{\sin(x-2)}$, $x \neq 2$, and $f(x) = 7$, $x = 2$ where $P(x)$ is a polynomial such that $P''(x)$ is always a constant and $P(3) = 9$. If $f(x)$ is continuous at $x = 2$, then $P(5)$ is equal to _____.
- Let $[x]$ be the greatest integer $\leq x$. Then the number of points in the interval $(-2, 1)$ where the function $f(x) = [x] + \sqrt{x - [x]}$ is discontinuous, is _____.
- Let $f(x) = [x^2 - x] + [-x + [x]]$, where $x \in \mathbb{R}$ and $[t]$ denotes the greatest integer less than or equal to t . Then, f is

(1) continuous at $x = 0$, but not continuous at $x = 1$ (2) continuous at $x = 1$, but not continuous at $x = 0$
(3) continuous at $x = 0$ and $x = 1$ (4) not continuous at $x = 0$ and $x = 1$
- If $f: \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by $f(x) = [x-1] \cos\left(\frac{2x-1}{2}\pi\right)$, where $[\cdot]$ denotes the greatest integer function, then f is:

(1) discontinuous only at $x = 1$ (2) discontinuous at all integral values of x except at $x = 1$
(3) continuous only at $x = 1$ (4) continuous for every real x
- Let $[t]$ denote the greatest integer $\leq t$. The number of points where the function $f(x) = [x]^2 - 1 + \sin\left(\frac{\pi}{[x]+3}\right) - [x+1]$, $x \in (-2, 2)$ is not continuous is _____.
- Let $[t]$ denote the greatest integer $\leq t$ and $\lim_{x \rightarrow 0} x \left[\frac{4}{x} \right] = A$. Then the function, $f(x) = [x^2] \sin(\pi x)$ is discontinuous, when x is equal to:

(1) $\sqrt{A+1}$ (2) $\sqrt{A+5}$
(3) $\sqrt{A+21}$ (4) \sqrt{A}
- If $f(x) = \begin{cases} x+a, & x \leq 0 \\ |x-4|, & x > 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & x < 0 \\ (x-4)^2 + b, & x \geq 0 \end{cases}$ are continuous on \mathbb{R} , then $(g \circ f)(2) + (f \circ g)(-2)$ is equal to:

(1) -10 (2) 10
(3) 8 (4) -8
- Let $f(x) = [2x^2 + 1]$ and $g(x) = \begin{cases} 2x-3, & x < 0 \\ 2x+3, & x \geq 0 \end{cases}$, where $[t]$ is the greatest integer $\leq t$. Then, in the open interval $(-1, 1)$, the number of points where $f \circ g$ is discontinuous is equal to _____.
- The number of points where the function $f(x) = \begin{cases} |2x^2 - 3x - 7| & \text{if } x \leq -1 \\ [4x^2 - 1] & \text{if } -1 < x < 1, \text{ where } [t] \text{ denotes the greatest integer } \leq t, \\ |x+1| + |x-2| & \text{if } x \geq 1 \end{cases}$ is discontinuous is _____
- Let $[x]$ denote the greatest integer function and $f(x) = \max\{1+x+[x], 2+x, x+2[x]\}$, $0 \leq x \leq 2$, where f is not continuous and n be the number of points in $(0, 2)$, where f is not differentiable. Then $(m+n)^2 + 2$ is equal to

(1) 2 (2) 11
(3) 6 (4) 3
- Let $f(x) = \begin{cases} \max(|x|, x^2), & |x| \leq 2 \\ 8-2|x|, & 2 < |x| \leq 4 \end{cases}$. Let S be the set of points in the interval $(-4, 4)$ at which f is not differentiable. Then S

(1) equals $\{-2, -1, 0, 1, 2\}$ (2) equals $\{-2, 2\}$
(3) is an empty set (4) equal $\{-2, -1, 1, 2\}$
- Let $a \in \mathbb{Z}$ and $[t]$ be the greatest integer $\leq t$, then the number of points, where the function $f(x) = [a + 13 \sin x]$, $x \in (0, \pi)$ is not differentiable, is _____.
- If $[t]$ denotes the greatest integer $\leq t$, then number of points, at which the function $f(x) = 4[2x+3] + 9\left[x + \frac{1}{2}\right] - 12[x+20]$ is not differentiable in the open interval $(-20, 20)$, is _____.

18. The function $f(x) = |x^2 - 2x - 3| \cdot e^{9x^2 - 12x + 4}$ is not differentiable at exactly :
 (1) Four points (2) Two points
 (3) three points (4) one point
19. If $f(x) = \begin{cases} \frac{1}{|x|} & ; |x| \geq 1 \\ ax^2 + b & ; |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively:
 (1) $\frac{1}{2}, \frac{1}{2}$ (2) $\frac{1}{2}, -\frac{3}{2}$
 (3) $\frac{5}{2}, -\frac{3}{2}$ (4) $-\frac{1}{2}, \frac{3}{2}$
20. Let S be the set of points where the function, $f(x) = |2 - |x - 3||$, $x \in \mathbb{R}$, is not differentiable. Then $\sum_{x \in S} f(f(x))$ equal to
21. The number of points, at which the function $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$, $x \in \mathbb{R}$ is not differentiable, is
22. Let K be the set of all real values of x where the function $f(x) = \sin |x| - |x| + 2(x - \pi) \cos |x|$ is not differentiable. Then the set K is equal to :
 (1) ϕ (an empty set) (2) $\{\pi\}$
 (3) $\{0\}$ (4) $\{0, \pi\}$
23. Let the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined as :
 $f(x) = \begin{cases} x + 2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x^3, & x < 1 \\ 3x - 2, & x \geq 1 \end{cases}$
 Then, the number of points in \mathbb{R} where $(f \circ g)(x)$ is NOT differentiable is equal to :
 (1) 3 (2) 1
 (3) 0 (4) 2
24. Let $\sum_{k=1}^{10} f(a + k) = 16(2^{10} - 1)$, where the function f satisfies $f(x + y) = f(x)f(y)$ for all natural numbers x, y and $f(1) = 2$. Then the natural number 'a' is:
 (1) 3 (2) 16
 (3) 4 (4) 2
25. Let f be a differentiable function from \mathbb{R} to \mathbb{R} such that $|f(x) - f(y)| \leq 2|x - y|^{3/2}$, for all $x, y \in \mathbb{R}$. If $f(0) = 1$ then $\int_0^1 f^2(x) dx$ is equal to
 (1) 0 (2) 1
 (3) 2 (4) $\frac{1}{2}$
26. Let $f : (-2, 2) \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} x[x] & , -2 < x < 0 \\ (x - 1)[x] & , 0 \leq x < 2 \end{cases}$ where $[x]$ denotes the greatest integer function. If m and n respectively are the number of points in $(-2, 2)$ at which $y = |f(x)|$ is not continuous and not differentiable, then $m + n$ is equal to _____.
27. Let f and g be twice differentiable functions on \mathbb{R} such that $f''(x) = g''(x) + 6x$, $f'(1) = 4g'(1) - 3 = 9$
 $f(2) = 3$, $g(2) = 12$
 Then which of the following is NOT true ?
 (1) $g(-2) - f(-2) = 20$ (2) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$
 (3) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$ (4) There exists $x_0 \in (1, \frac{3}{2})$ such that $f(x_0) = g(x_0)$
28. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by : $f(x) = \begin{cases} \max_{t \leq x} \{t^3 - 3t\}; & x \leq 2 \\ x^2 + 2x - 6; & 2 < x < 3 \\ [x - 3] + 9; & 3 \leq x \leq 5 \\ 2x + 1; & x > 5 \end{cases}$
 Where $[t]$ is the greatest integer less than or equal to t . Let m be the number of points where f is not differentiable and $I = \int_{-2}^2 f(x) dx$. Then the ordered pair (m, I) is equal to
 (1) $(3, \frac{27}{4})$ (2) $(3, \frac{23}{4})$
 (3) $(4, \frac{27}{4})$ (4) $(4, \frac{23}{4})$
29. Let $f : [0, \infty) \rightarrow [0, 3]$ be a function defined by $f(x) = \begin{cases} \max\{\sin t : 0 \leq t \leq x\}, & x \in [0, \pi] \\ 2 + \cos x, & x > \pi \end{cases}$. Then which of the following is true ?
 (1) f is continuous everywhere but not differentiable exactly at one point in $(0, \infty)$
 (2) f is differentiable everywhere in $(0, \infty)$
 (3) f is not continuous exactly at two points in $(0, \infty)$
 (4) f is continuous everywhere but not differentiable exactly at two points in $(0, \infty)$
30. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as $f(x) = \begin{cases} 3(1 - \frac{|x|}{2}) & \text{if } |x| \leq 2 \\ 0 & \text{if } |x| > 2 \end{cases}$. Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be given by $g(x) = f(x + 2) - f(x - 2)$. If n and m denote the number of points in \mathbb{R} where g is not continuous and not differentiable, respectively, then $n + m$ is equal to _____.