ANSWER KEYS

1. (1)	**2.** (9.9)	**3.** (0)	**4.** (210)	**5.** (1)	**6.** (2)	**7.** (3)	**8.** (3)
9. (2)	**10.** (4)						

1. **(1)** In $(1+x)^n$, if n is even, then the middle term is $\left(\frac{n}{2}+1\right)^{th}$ term.

So, Coefficient of middle term in $(1+x)^{20} = {}^{20}C_{10}$

In $(1+x)^n$, if n is odd, then the middle term is $\left(\frac{n+1}{2}\right)^{th}$ & $\left(\frac{n+3}{2}\right)^{th}$ term.

So, Sum of Coefficient of two middle terms in

$(1+x)^{19} = {}^{19}C_9 + {}^{19}C_{10}$

So required ratio $= \frac{{}^{20}C_{10}}{{}^{19}C_9+{}^{19}C_{10}} = \frac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$

2. **(9.9)** $(3-5x)^{11} = 3^{11}\left(1-\frac{5}{3}x\right)^{11}$

Now, $m = \frac{\left|-\frac{5x}{3}\right|(11+1)}{\left|-\frac{5x}{3}\right|+1} = \frac{\frac{1}{3}\times 12}{\frac{4}{3}} = 3$

$\Rightarrow$ The greatest terms in the expansion are T_3 and T_4

Greatest term $T_3 = T_{2+1}$

$= 3^{11}\left|{}^{11}C_2\left(-\frac{1}{3}\right)^2\right|$

$= 3^9 \times {}^{11}C_2 = \left(\frac{11\times 10}{2}\times 3^3\right)\times 3^6$

$= 55 \times 27 \times (729)$

$\Rightarrow \lambda = 55 \times 27$

$\Rightarrow \frac{\lambda}{150} = 9.9$

3. **(0)** Here, $\mathrm{n} = 10,$ which is even.

Middle term $= \left(\frac{10}{2}+1\right)^{\mathrm{th}}$ term $= 6^{\mathrm{th}}$ term

$\mathrm{T}_6 = {}^{10}\mathrm{C}_5\left(\frac{1}{\mathrm{x}}\right)^5(\mathrm{x}\sin\mathrm{x})^5$

$\Rightarrow \frac{63}{8} = 252(\sin x)^5$

$\Rightarrow (\sin x)^5 = \frac{1}{32}$

$\Rightarrow (\sin x)^5 = \left(\frac{1}{2}\right)^5$

$\Rightarrow \sin x = \frac{1}{2}$

$\Rightarrow 2\sin x - 1 = 0$

$\Rightarrow 6\sin^2 x + \sin x - 2 = (2\sin x - 1)(3\sin x + 2) = 0$

4. **(210)**

$\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$

$= \left((x^{1/3}+1) - \left(\frac{x^{1/2}+1}{x^{1/2}}\right)\right)^{10}$

$= \left(x^{1/3} - \frac{1}{x^{1/2}}\right)^{10}$

Now the $(r+1)^{th}$ Term

$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r}\cdot\left(-\frac{1}{x^{1/2}}\right)^r$

For independent term

$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 4$

$\Rightarrow T_5 = {}^{10}C_4 = 210$

5. **(1)**

To get sum of coefficients put $x = 0$. Given that sum of coefficients is 64

$2^n = 64 \Rightarrow n = 6$

The greatest binomial coefficients is ${}^6C_3$.

Now given that $T_4 - T_3 = 6 - 1 = 5$

$\Rightarrow {}^6C_3\left(3^{\frac{-x}{4}}\right)^3\left(3^{\frac{5x}{4}}\right)^3 - {}^6C_2\left(3^{\frac{-x}{4}}\right)^2\left(3^{\frac{5x}{4}}\right)^4 = 5$

which is satisfied by $x = 0$.

6. **(2)**

Given

$P+\beta=\left(\sqrt{2}+1\right)^{n} \quad \ldots(1)$

where p in integer and β is fraction.

$\beta^{1}=\left(\sqrt{2}-1\right)^{n}$

β^{1} will be fraction $\quad \ldots(2)$

added (1) & (2)

$p+\beta+\beta^{1}=\left(\sqrt{2}+1\right)^{n}+\left(\sqrt{2}-1\right)^{n}$

$\because\ \beta\ \&\ \beta^{1}\ \in(0,1)$

$[\beta+\beta^{1}]=1$ (integral Part of $\beta+\beta^{1}$)

$P+1=\left(\sqrt{2}+1\right)^{6}+\left(\sqrt{2}-1\right)^{6}$

$P+1=2\left[\left(\sqrt{2}\right)^{6}={}^{6}C_{2}\left(\sqrt{2}\right)^{4}+{}^{6}C_{4}\left(\sqrt{2}\right)^{2}+{}^{6}C_{6}\left(\sqrt{2}\right)^{0}\right]$

$P+1=2\left[8+15\times4+15\times2+1\right]$

$P+1=2\times[8+60+30+1]$

$=\ 2\times99$

$P+1=198$

$\Rightarrow\ P=197$

7. **(3)** General term in the expansion of

$\left(1+3x+2x^{2}\right)^{6}=\Sigma\frac{6!}{r_1!r_2!r_3!}(1)^{r_1}(3x)^{r_2}\left(2x^{2}\right)^{r_3}$

Where $r_1+r_2+r_3=6\ldots(i)$

For coefficient of x^{11}, we have

$r_2+2\,r_3=11\ldots(ii)$

Now, from Eqs, (i),(ii), we get

$r_1=r_3-5$

For $\quad r_3=5,\ \ r_1=0$

And $r_2=1$

$\therefore$ Coefficient of $x^{11}=\frac{6!}{0!1!5!}(1)^{0}(3)^{1}(2)^{5}$

$=6\times3\times2^{5}=18\times32=576$

8. **(3)**

Given expansion: $\left(x+y^{2}\right)^{13}+\left(x^{2}+y\right)^{14}$

Total no. of terms $=$ no. of terms in $\left(x+y^{2}\right)^{13}+$ no. of terms in $\left(x^{2}+y\right)^{14}-$ no. of common terms

No of terms in $\left(x+y^{2}\right)^{13}=14$

No. of terms in $\left(x^{2}+y\right)^{14}=15$

For common terms, powers of x and y must be same i.e., power of x and y in the terms of $\left(x+y^{2}\right)^{13}=$ power of x and y in the terms of $\left(x^{2}+y\right)^{14}$

$\Rightarrow x^{r_1}\left(y^{2}\right)^{13-r_1}=\left(x^{2}\right)^{r_2}\left(y\right)^{14-r_2}$

Comparing powers of x and y on both sides, we get

$r_1=2r_2\quad\ldots(1)$

and

$26-2r_1=14-r_2\quad\ldots(2)$

Solving (1) and (2) $r_1=8$ and $r_2=4$

Hence, we have only one ordered pair $(r_1,\ r_2)$ for which terms are common.

$\therefore$ No. of common terms $=1$

Hence, Total terms $=14+15-1=28$

9. **(2)** $\left(1-\frac{2}{x}+\frac{4}{x^{2}}\right)^{n}$ as the question is having three variables the total number of terms would be $\frac{(n+1)(n+2)}{1.2}$ which is equal to 28

$\therefore\quad(n+1)(n+2)=56$

Which gives n = 6 , and sum of coefficients would be $(1-2+4)^{6}=3^{6}=729$.

10. (4)

$\left({}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \ldots + {}^{21}C_{10}\right)$

$-\left({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \ldots + {}^{10}C_{10}\right)$

$= \frac{1}{2}\left[{}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \ldots + {}^{21}C_{10} + {}^{21}C_{11} + \ldots + {}^{21}C_{20}\right] - \left(2^{10} - 1\right)$

$= \frac{1}{2}\left[2^{21} - 2\right] - \left(2^{10} - 1\right)$

$= 2^{20} - 1 - 2^{10} + 1$

$= 2^{20} - 2^{10}.$