1. The domain of the function, $f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$ is:

(1) $\left[0, \frac{1}{2}\right]$ (2) $\left[0, \frac{1}{4}\right]$
(3) $\left[\frac{1}{4}, \frac{1}{2}\right]\cup\{0\}$ (4) $[-2, 0]\cup\left[\frac{1}{4}, \frac{1}{2}\right]$

2. If the domain of the function $f(x)=\log_e\left(4x^2+11x+6\right)+\sin^{-1}(4x+3)+\cos^{-1}\left(\frac{10x+6}{3}\right)$ is $(\alpha, \beta]$, then $36|\alpha+\beta|$ is equal to

(1) 54 (2) 72
(3) 63 (4) 45

3. Let $[x]$ denote the greatest integer $\leq x$, where $x \in R$. If the domain of the real valued function $f(x)=\sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a)\cup[b, c)\cup[4, \infty)$, $a<b<c$, then the value of $a+b+c$ is:

(1) 8 (2) 1
(3) -2 (4) -3

4. Let $f:\mathbb{R}\rightarrow\mathbb{R}$ be a function defined by $f(x)=\log_{\sqrt{m}}\left\{\sqrt{2}(\sin x-\cos x)+m-2\right\}$, for some m, such that the range of f is $[0,2]$. Then the value of m is _____.

(1) 5 (2) 3
(3) 2 (4) 4

5. If the domain of the function $f(x)=\frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2,6)$, then its range is

(1) $\left(\frac{5}{26}, \frac{2}{5}\right]-\left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$ (2) $\left(\frac{5}{26}, \frac{2}{5}\right]$
(3) $\left(\frac{5}{37}, \frac{2}{5}\right]-\left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$ (4) $\left(\frac{5}{37}, \frac{2}{5}\right]$

6. The sum of the maximum and minimum values of the function $f(x)=|5x-7|+[x^2+2x]$ in the interval $\left[\frac{5}{4}, 2\right]$, where $[t]$ is the greatest integer $\leq t$, is _____.

7. Let $f:\boldsymbol{R}\rightarrow\boldsymbol{R}$ be a function defined by $f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right)^{\frac{1}{50}}$. If the function $g(x)=f(f(f(x)))+f(f(x))$, then the greatest integer less than or equal to $g(1)$ is _____.

8. Let $f^1(x)=\frac{3x+2}{2x+3}, x\in R-\left\{-\frac{3}{2}\right\}$. For $n\geq 2$, define $f^n(x)=f^1 o f^{n-1}(x)$. If $f^5(x)=\frac{ax+b}{bx+a}$, $\gcd(a,b)=1$, then $a+b$ is equal to _____

9. Let $S=\{1,2,3,4,5,6,7,8,9,10\}$. Define $f:S\rightarrow S$ as $f(n)=\begin{cases} 2n, & \text{if } n=1,2,3,4,5 \\ 2n-11 & \text{if } n=6,7,8,9,10\end{cases}$

Let $g:S\rightarrow S$ be a function such that $\operatorname{fog}(n)=\begin{cases} n+1 & \text{, if } n \text{ is odd} \\ n-1 & \text{, if } n \text{ is even}\end{cases}$, then $g(10)(g(1)+g(2)+g(3)+g(4)+g(5))$ is equal to

10. Let $f:\mathbb{R}\rightarrow\mathbb{R}$ be defined as $f(x)=x-1$ and $g:R-\{1,-1\}\rightarrow\mathbb{R}$ be defined as $g(x)=\frac{x^2}{x^2-1}$. Then the function fog is:

(1) One-one but not onto (2) onto but not one-one
(3) Both one-one and onto (4) Neither one-one nor onto

11. Let a function $f:(0,\infty)\rightarrow(0,\infty)$ be defined by $f(x)=\left|1-\frac{1}{x}\right|$. Then f is :

(1) not injective but it is surjective (2) injective only
(3) neither injective nor surjective (4) not a function

12. Let $f:R\rightarrow R$ be a function such that $f(x)=\frac{x^2+2x+1}{x^2+1}$. Then

(1) $f(x)$ is many-one in $(-\infty,-1)$ (2) $f(x)$ is many-one in $(1,\infty)$
(3) $f(x)$ is one-one in $[1,\infty)$ but not in $(-\infty,\infty)$ (4) $f(x)$ is one-one in $(-\infty,\infty)$

13. Let a function $f:\mathbb{N}\rightarrow\mathbb{N}$ be defined by $f(n)=\begin{bmatrix} 2n, & n=2,4,6,8,\ldots.. \\ n-1, & n=3,7,11,15,\ldots.. \\ \frac{n+1}{2}, & n=1,5,9,13,\ldots..\end{bmatrix}$

then, f is

(1) One-one and onto (2) One-one but not onto
(3) Onto but not one-one (4) Neither one-one nor onto

14. Let $A=\{1,2,3,\ldots,10\}$ and $f:A\rightarrow A$ be defined as $f(k)=\begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even}\end{cases}$ Then the number of possible functions $g:A\rightarrow A$ such that $gof=f$ is:

(1) ${}^{10}C_5$ (2) 5^5
(3) $5!$ (4) 10^5

15. Let $A=\{1,2,3,4,5\}$ and $B=\{1,2,3,4,5,6\}$. Then the number of functions $f:A\rightarrow B$ satisfying $f(1)+f(2)=f(4)-1$ is equal to........

16. Let $A=\{1,2,3,5,8,9\}$. Then the number of possible functions $f:A\rightarrow A$ such that $f(m\cdot n)=f(m)\cdot f(n)$ for every $m,n\in A$ with $m\cdot n\in A$ is equal to

17. The number of bijective function $f(1,3,5,7,\cdots,99)\rightarrow(2,4,6,8,\cdots,100)$ if $f(3)>f(5)>f(7)\cdots>f(99)$ is

(1) ${}^{50}C_1$ (2) ${}^{50}C_2$
(3) $\frac{50!}{2}$ (4) ${}^{50}C_3\times 3!$

18. The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set $\{1, 2, 3, 4, 5\}$ satisfied $f(a)+2f(b)-f(c)=f(d)$ is

(1) $\frac{1}{24}$ (2) $\frac{1}{40}$

(3) $\frac{1}{30}$ (4) $\frac{1}{20}$

19. Let $f: R-\{0,1\} \rightarrow R$ be a function such that $f(x)+f\left(\frac{1}{1-x}\right)=1+x$. Then $f(2)$ is equal to :

(1) $\frac{9}{2}$ (2) $\frac{9}{4}$

(3) $\frac{7}{4}$ (4) $\frac{7}{3}$

20. A function $f(x)$ is given by $f(x)=\frac{5^x}{5^x+5}$, then the sum of the series $f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+\ldots+f\left(\frac{39}{20}\right)$ is equal to:

(1) $\frac{19}{2}$ (2) $\frac{49}{2}$

(3) $\frac{39}{2}$ (4) $\frac{29}{2}$

21. If $f(x)=x^3-x^2 f'(1)+x f''(2)-f'''(3)$, $x \in \mathrm{R}$, then

(1) $3f(1)+f(2)=f(3)$ (2) $f(3)-f(2)=f(1)$

(3) $2f(0)-f(1)+f(3)=f(2)$ (4) $f(1)+f(2)+f(3)=f(0)$

22. Let $f(x)$ be a polynomial of degree 3 such that $f(k)=-\frac{2}{k}$ for $k=2,\ 3,\ 4,\ 5$. Then the value of $52-10\,f(10)$ is equal to _____ .

23. Let $f(x)$ be a function such that $f(x+y)=f(x)\cdot f(y)$ for all $x, y \in \mathrm{N}$, If $f(1)=3$ and $\sum_{k=1}^{n} f(k)=3279$, then the value of n is

(1) 6 (2) 8

(3) 7 (4) 9

24. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function that satisfies the relation $f(x+y)=f(x)+f(y)-1, \forall\, x, y \in \mathbb{R}$. If $f'(0)=2$, then $|f(-2)|$ is equal to

25. Let $f: N \rightarrow N$ be a function such that $f(m+n)=f(m)+f(n)$ for every $m,\ n \in N$. If $f(6)=18$ then $f(2)\cdot f(3)$ is equal to :

(1) 54 (2) 6

(3) 36 (4) 18

26. Suppose a differentiable function $f(x)$ satisfies the identity $f(x+y)=f(x)+f(y)+xy^2+x^2y$, for all real x and y. If $\lim_{x \rightarrow 0} \frac{f(x)}{x}=1$, then $f'(3)$ is equal to :

27. The number of functions $f:\{1,2,3,4\} \rightarrow \{\mathrm{a} \in \mathbb{Z}: |\mathrm{a}| \leq 8\}$ satisfying $f(\mathrm{n})+\frac{1}{\mathrm{n}} f(\mathrm{n}+1)=1, \forall\, \mathrm{n} \in \{1,2,3\}$ is

(1) 3 (2) 4

(3) 1 (4) 2

28. For $x \in R$, Let $[x]$ denotes the greatest integer $\leq x$, then the sum of the series $\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right]$ is

(1) -131 (2) -153

(3) -135 (4) -133

29. For some $a, b, c \in \mathbb{N}$, let $f(x)=ax-3$ and $g(x)=x^b+c, x \in \mathbb{R}$. If $(fog)^{-1}(x)=\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$, then $(f \circ g)(ac)+(g \circ f)(b)$ is equal to _____ .

30. The equation $x^2-4x+[x]+3=x[x]$, where $[x]$ denotes the greatest integer function, has:

(1) exactly two solutions in $(-\infty,\ \infty)$ (2) no solution

(3) a unique solution in $(-\infty,\ 1)$ (4) a unique solution in $(-\infty,\ \infty)$