ANSWER KEYS

1. (4)	**2.** (1)	**3.** (2)	**4.** (8)	**5.** (3)	**6.** (7.50)	**7.** (4)	**8.** (1)
9. (2)	**10.** (2)						

1. **(4)** Given, $S(K,\ 0)$ and $S'(-K,\ 0)$,

Now, $SS' = |2K|$ and $PS + PS' = 4$ (given)

Now for the ellipse we know that,

$PS + PS' > SS'$

$\Rightarrow 4 > |2K|$

$\Rightarrow K \in (-2,\ 2).$

If P, S & S' are collinear, then

$PS + PS' = SS'$

$\Rightarrow K = \pm 2.$

If $PS + PS' < SS'$, then no locus exists.

$\Rightarrow K \in (-\infty,\ -2) \cup (2,\ \infty).$

2. **(1)** We have $\left|\sqrt{x^2 + (y-1)^2} - \sqrt{x^2 + (y+1)^2}\right| = K$

Which is equivalent to $|S_1P - S_2P|$=Const.

Where $S_1 \equiv (0,\ 1),\ S_2 \equiv (0,\ -1)$ .and $P \equiv (x,\ y)$

Using properties of a hyperbola, the above equation represents a hyperbola, then we have.

$2a = K$ [where 2a is the transverse axis and e is the eccentricity]

and $2ae = S_1\ S_2 = 2$

Dividing, we have $e\ = \frac{2}{K}$

Since, $e\ >\ 1$ for a hyperbola, therefore $K\ <\ 2$

Also, K must be a positive quantity. Hence, we have, $K \in\ (0,\ 2)$.

3. **(2)**

Let equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore$ It passes through $\left(4,\ -2\sqrt{3}\right)\ \Rightarrow \frac{16}{a^2} - \frac{12}{b^2} = 1$

$\Rightarrow 16 - 12 \times \frac{a^2}{b^2} = a^2 \ldots (1)$

Equation of directrix is $x = \frac{a}{e} = \frac{4}{\sqrt{5}}$, given in question.

$\Rightarrow a^2 = \frac{16}{5}e^2 \ldots (2)$

And we know that $b^2 = a^2\left(e^2 - 1\right)$

$\Rightarrow \frac{b^2}{a^2} = e^2 - 1 \ldots (3)$

$\therefore$ From (1),(2) & (3)

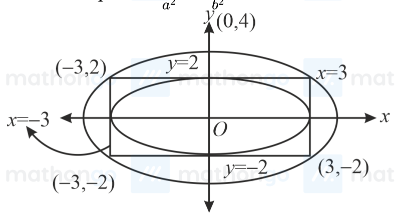
$16 - \frac{12}{e^2-1} = \frac{16}{5}e^2$

$\Rightarrow 16e^2 - 16 - 12 = \frac{16e^2}{5}\left(e^2 - 1\right)$

$\Rightarrow\ 4e^4 - 24e^2 + 35 = 0$

4. **(8)**

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



As it is passing through $(0, 4)$ and $(3, 2)$. So, $b^2 = 16$ and

$\frac{9}{a^2} + \frac{4}{16} = 1$ or $a^2 = 12$

So, the length of major axis $= 2b = 8$

5. (3)

$3x^2+4y^2=12 \Rightarrow \frac{x^2}{4}+\frac{y^2}{3}=1$

Electricity of ellipse $=\sqrt{\frac{1-3}{4}}=\frac{1}{2}$

Hence, the focus of ellipse is $\left(2\times\frac{1}{2},0\right)=(1,0)$

As the hyperbola is confocal with the ellipse, the focus of the hyperbola is $(1,0)$.

Now for hyperbola $ae=1$ and $2a=1 \Rightarrow a=\frac{1}{2}$

$ae=\frac{1}{2}e=1 \Rightarrow e=2$

$e=\sqrt{1+\frac{b^2}{a^2}} \Rightarrow 2=\sqrt{1+\frac{b^2}{\frac{1}{4}}} \Rightarrow 4=1+4b^2$

$b^2=\frac{3}{4} \Rightarrow b=\frac{\sqrt{3}}{2}$

Length of conjugate axis $=2\times\frac{\sqrt{3}}{2}=\sqrt{3}$

$\therefore\ (2b)^2=\left(\sqrt{3}\right)^2=3$

6. (7.50)

$e=\frac{\sqrt{13}}{3}$

$\frac{2b^2}{a}=\frac{10}{3} \Rightarrow \frac{b^2}{a}=\frac{5}{3} \Rightarrow b^2=\frac{5a}{3}$

also $e^2=1+\frac{b^2}{a^2}$

$\Rightarrow \frac{13}{9}=1+\frac{5a/3}{a^2} \Rightarrow \frac{13}{9}=1+\frac{5}{3d}$

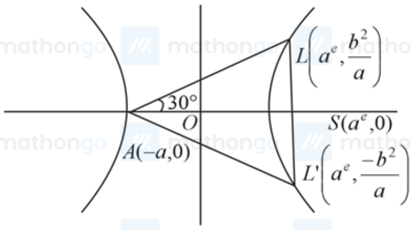
$\Rightarrow \frac{4}{9}=\frac{5}{3a} \Rightarrow a=\frac{15}{4}$

7. (4)

We have,

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Coordinates of end points of latus rectum i.e., L and L' are $L\left(ae,\frac{b^2}{a}\right)$ and $L\left(ae,\frac{-b^2}{a}\right)$



It is given that $\Delta ALL'$ is equilateral, therefore $\angle LAL'=60^\circ \Rightarrow \angle LAS=30^\circ$

Now, in right-angled ΔLAS,

$\tan 30^\circ=\frac{LS}{AS}$

$\Rightarrow \tan 30^\circ=\frac{\frac{b^2}{a}}{a+ae}$

$\Rightarrow \frac{1}{\sqrt{3}}=\frac{b^2}{a^2(1+e)}$

$\Rightarrow \frac{1+e}{\sqrt{3}}=e^2-1$

$\Rightarrow \frac{1+e}{\sqrt{3}}=(e+1)(e-1)$

$\Rightarrow e-1=\frac{1}{\sqrt{3}}$

$\Rightarrow\ e=\frac{\sqrt{3}+1}{\sqrt{3}}$

8. (1) Given that, $\frac{x^2}{25}+\frac{y^2}{16}=1$ here, $a=5,\ b=4$

The eccentricity of an ellipse,

$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

Distance between foci, $SS'=2ae=6$

$SP=8$ (Given)

$PP'=2a=10$

$PP'=SP+S'P \Rightarrow S'P=2$

Then, $SS'=4+S'P$

9. **(2)**
The chord is passing through the point $P(2,\ 2)$. So, the equation of chord in parametric form will be:
$\frac{x-2}{\cos\theta}=\frac{y-2}{\sin\theta}=r$.
Now, on solving the equation of the chord with equation of ellipse will give r of the points A and B.
$\therefore\ \frac{(r\cos\theta+2)^2}{25}+\frac{(r\sin\theta+2)^2}{16}=1$
$\Rightarrow 16(r\cos\theta+2)^2+25(r\sin\theta+2)^2=400$
$\Rightarrow 16r^2\cos^2\theta+64r\cos\theta+64+25r^2\sin^2\theta+100+100r\sin\theta=400$
$\Rightarrow r^2\left(16\cos^2\theta+25\sin^2\theta\right)+r(64\cos\theta+100\sin\theta)-236=0$, which is quadratic equation in r.
$\Rightarrow|r_1r_2|=PA\cdot PB=\left|\frac{-236}{16\cos^2\theta+25\sin^2\theta}\right|$
$=\left|\frac{236}{16+9\sin^2\theta}\right|$.
Since, range of $\sin\theta\in[-1,\ 1]$.
Maximum value occur when denominator is minimum.
Therefore, the maximum value of $PA\cdot PB=\frac{236}{16}=\frac{59}{4}$.

10. **(2)** Equation of chord connecting the points $(a\sec\theta,\ b\tan\theta)\ and\ (a\ sec,\ b\tan\phi)\ is$
$\Rightarrow \frac{x}{a}\cos\left(\frac{\theta+\phi}{2}\right)-\frac{y}{b}sin\left(\frac{\theta+\phi}{2}\right)=\cos\left(\frac{\theta-\phi}{2}\right)$
If it passes through $(ae,0)$; we have:$\Rightarrow e\cos\left(\frac{\theta-\phi}{2}\right)=\cos\left(\frac{\theta+\phi}{2}\right)$
$\Rightarrow \frac{\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}=\frac{1-\tan\frac{\theta}{2}.\tan\frac{\phi}{2}}{1+\tan\frac{\theta}{2}\tan\frac{\phi}{2}}$
$\Rightarrow \frac{\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}=\tan\frac{\theta}{2}\ .\tan\frac{\phi}{2}=\frac{1-e}{1+e}$