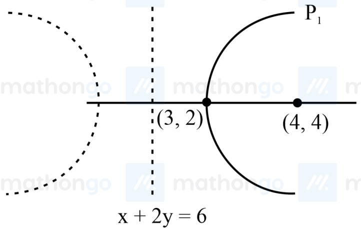
ANSWER KEYS

1. (10)	**2.** (4)	**3.** (3)	**4.** (3)	**5.** (16)	**6.** (4)	**7.** (2)	**8.** (2)
9. (2)	**10.** (1)	**11.** (9)	**12.** (2)	**13.** (1)	**14.** (2)	**15.** (1)	**16.** (4)
17. (3)	**18.** (3)	**19.** (27)	**20.** (1)	**21.** (3)	**22.** (4)	**23.** (2)	**24.** (4)
25. (2)	**26.** (42)	**27.** (3)	**28.** (306)	**29.** (1552)	**30.** (80)		

1. (10)

Plotting the diagram as per given data we get,



P_1: Directrix

$x+2y=k$

$x+2y-k=0$

$\left|\frac{3+4-K}{\sqrt{5}}\right|=\sqrt{5}$

$|7-k|=5$

$7-K=5 \qquad 7-K=-5$

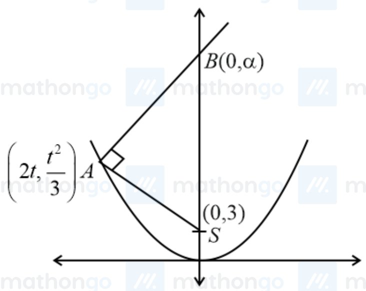
$k=2 \qquad k=12$

Accepted Rejected Passes through focus

$$\left.\begin{array}{l} D_1=x+2y=2 \\ =x+2y=6 \\ D_2=x+2y=C \end{array}\right] \Rightarrow d \Rightarrow d \Rightarrow c=10$$

2. (4)

The parametric form $x=2t, y=\frac{t^2}{3}$ represents the parabola $x^2=12y$



Given $AS \perp AB$

So, $m_{AS} \cdot m_{AB}=-1$

$\Rightarrow \frac{\left(3-\frac{t^2}{3}\right)}{(0-2t)} \cdot \frac{\left(\alpha-\frac{t^2}{3}\right)}{(0-2t)}=-1$

$\Rightarrow 3\alpha=\frac{27t^2+t^4}{t^2-9}$

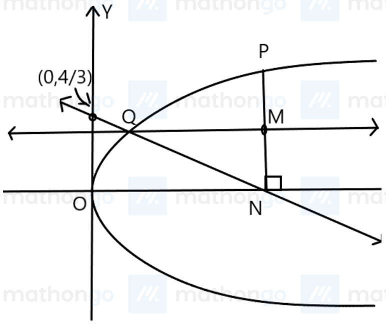
Ordinate of centroid of $\Delta SAB=k=\frac{\alpha+\frac{t^2}{3}+3}{3}=\frac{9+3\alpha+t^2}{9}$

$\lim_{t \rightarrow 1} k=\lim_{t \rightarrow 1} \frac{1}{9}\left(9+t^2+\frac{27t^2+t^4}{\left(t^2-9\right)}\right)=\frac{13}{18}$

3. (3)

Comparing the equation of the parabola, with standard equation $y^2 = 4ax$, we get $a = 3$.

Figure is drawn on the basis of the given information.



Parametric form of parabola is $x = at^2,\ y = 2at$

$\therefore\ P = (3t^2, 6t),\ N = (3t^2, 0),\ M = (3t^2, 3t)$

$\Rightarrow Q = \left(\frac{3t^2}{4}, 3t\right)$

Equation of NQ: $y - 0 = \left(\frac{3t - 0}{\frac{3t^2}{4} - 3t^2}\right)(x - 3t^2)$

i.e.,

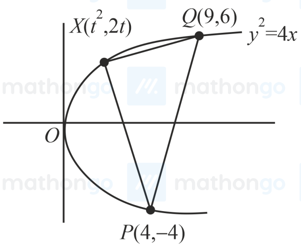
$y = \frac{-4}{3t} \cdot (x - 3t^2) \Rightarrow y = \frac{-4}{3t} \cdot x + 4t$

$\Rightarrow 4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$

$PN = 6t = 2$

$MQ = 3t^2 - \frac{3t^2}{4} = \frac{9t^2}{4} = \frac{1}{4}$

4. (3)



Two different approaches we can use here.

Approach 1:

Let X be $(t^2,\ 2t)$, then

Area of $\Delta PXQ = \frac{1}{2}\begin{vmatrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix}$

$\Delta = \frac{1}{2}.10|t^2 - t - 6| \quad \ldots(i)$

For maxima, differentiating both the sides with respect to t and equating it to zero, we get

$\Delta' = 0 \Rightarrow 5(2t - 1) = 0 \Rightarrow t = \frac{1}{2}$

Hence, area of $\Delta PXQ = 5\left|\left(\frac{1}{2}\right)^2 - \frac{1}{2} - 6\right| = 5\left|\frac{1}{4} - \frac{1}{2} - 6\right| = 5\left|\frac{1-2-24}{4}\right| = \frac{125}{4}$ sq.units (using equation (i))

Approach 2:

For maximum area tangent to the parabola at X must be parallel to PQ. Let $X(t^2,\ 2t)$, then

$2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(t^2,2t)} = \frac{1}{t}$

Also, slope of line $PQ = \frac{6+4}{9-4} = 2$

Since, both the slopes are equal.

Thus, $t = \frac{1}{2} \Rightarrow X\left(\frac{1}{4},\ 1\right)$

Therefore, area of $\Delta PXQ = \frac{1}{2}\begin{vmatrix} 4 & -4 & 1 \\ \frac{1}{4} & 1 & 1 \\ 9 & 6 & 1 \end{vmatrix} = \frac{1}{2}\left|4(1-6) - (-4)\left(\frac{1}{4} - 9\right) + 1\left(\frac{3}{2} - 9\right)\right| = \frac{1}{2}\left|-20 - 35 - \frac{15}{2}\right| = \frac{125}{4}$ sq.units.

5. (16)

Given,

The x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3,

Now simplifying equation of the given parabola we get,

$y^2 = 8x + 4y + 4$

$\Rightarrow (y-2)^2 = 8(x+1)$

Now on comparing with standard parabola $y^2 = 4ax$ we get,

$a = 2,\ X = x + 1,\ Y = y - 2$

Hence, the vertex is $(-1, 2)$ and focus is $(1, 2)$

So, equation of line passing through focus is given by $y - 2 = m(x - 1)$

Now, putting $(3, 0)$ {as given $x-$intercept of focal chord is 3} in the above line we get, $m = -1$

So, equation of chord will be $y = 3 - x$,

Now putting the value of $y = 3 - x$ in parabola $(y-2)^2 = 8(x+1)$ we get,

$\Rightarrow (1-x)^2 = 8(x+1)$

$\Rightarrow x^2 - 10x - 7 = 0$

$\Rightarrow |x_1 - x_2| = \sqrt{10^2 + 4 \times 7} = \sqrt{128}$

$\Rightarrow |y_1 - y_2| = |3 - x_1 - (3 - x_2)| = |x_2 - x_1| = \sqrt{128}$

Length of focal chord $\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} = \sqrt{256} = 16.$

6. (4)

$9\left(t + \frac{1}{t}\right)^2 = 100$

$t = 3$

$\Rightarrow P(81, 54) \& Q(1, -6)$

$M(21, 9)$

$\Rightarrow L$ is $(y - 9) = \frac{-4}{3}(x - 21)$

$3y - 27 = -4x + 84$

$4x + 3y = 111$

7. (2)

Given,

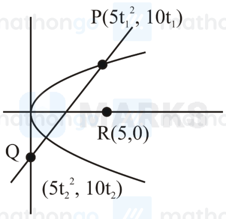
R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q,

And the points $G(10,\ 10)$ be the centroid of the triangle PQR,

Now focus of the parabola $y^2 = 20x$ will be, $R(5, 0)$

And parametric points of PQ be $(5t^2, 10t)$,

Now plotting the diagram we get,



Now finding the centroid of the triangle we get,

For $x-$ coordinate we get,

$\Rightarrow \frac{5{t_1}^2 + 5{t_2}^2 + 5}{3} = 10$

$\Rightarrow {t_1}^2 + {t_2}^2 = 5 \quad \ldots(i)$

Now for $y-$coordinate we get,

$\frac{10(t_1 + t_2)}{3} = 10$

$\Rightarrow t_1 + t_2 = 3 \quad \ldots(ii)$

Now solving both equations we get, $t_1 = 1,\ t_2 = 2$

So, points will be, $P \equiv (5,\ 10)$ & $Q \equiv (20, 20)$

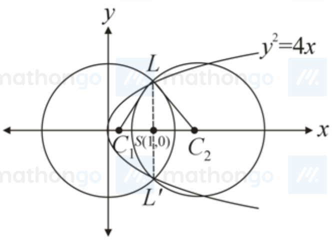
Hence, equation of $PQ = y - 10 = \frac{10}{15}(x - 5)$

$\Rightarrow 3y - 30 = 2x - 10$

$\Rightarrow y = \frac{2}{3}x + \frac{20}{3}$, so on comparing with $y = mx + c$, we get $c - m = 6$

Hence, $PQ^2 = 225 + 100 = 325$

8. **(2)**



Co-ordinates of focus of parabola will be $S\equiv(a,\ 0)\equiv(1,\ 0)$.

Now, length of latus-rectum $=4$.

So, length of semi latus-rectum $=2$.

Also, $\angle LSC_1=90^{\circ}$

Now, radius of circle $LC_1=2\sqrt{5}$

Now, applying pythagoras theorem in $\triangle\ LSC_1$,

$(LC_1)^2=(C_1S)^2+(LS)^2$

$\Rightarrow (C_1S)^2=(LC_1)^2-(LS)^2=\left(2\sqrt{5}\right)^2-(2)^2=16$

$\Rightarrow C_1S=\sqrt{16}=4$

$\therefore C_1C_2=2C_1S=2\times4=8.$

9. **(2)**

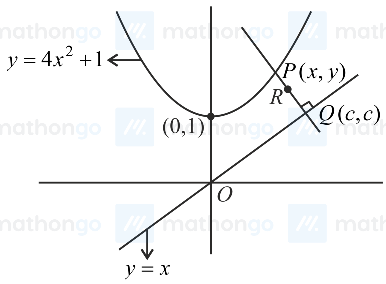
We have,

$y=4x^2+1$

$L: y=x$

Let the foot of perpendicular from P to line $y=x$ is Q.

Let $P\equiv(x,y)$, $Q\equiv(c,c)$ and $R\equiv(h,k)$ where, R is the mid-point of PQ.



Clearly,

$PQ\perp L$

$\Rightarrow \frac{k-c}{h-c}=-1$

$\Rightarrow c=\left(\frac{h+k}{2}\right)$

And,

$R\equiv\left(\frac{x+c}{2},\frac{y+c}{2}\right)$

$\Rightarrow R\equiv\left(\frac{x}{2}+\frac{h}{4}+\frac{k}{4},\frac{y}{2}+\frac{h}{4}+\frac{k}{4}\right)$

Hence,

$h=\frac{x}{2}+\frac{h}{4}+\frac{k}{4}\Rightarrow x=\frac{3h}{2}-\frac{k}{2}$

$k=\frac{y}{2}+\frac{h}{4}+\frac{k}{4}\Rightarrow y=\frac{3k}{2}-\frac{h}{2}$

Now,

$y=4x^2+1$

$\Rightarrow\left(\frac{3k-h}{2}\right)=4\left(\frac{3h-k}{2}\right)^2+1$

$\Rightarrow(3k-h)=2(3h-k)^2+2$

Required locus is

$2(3x-y)^2+(x-3y)+2=0$

10. (1)

Given:

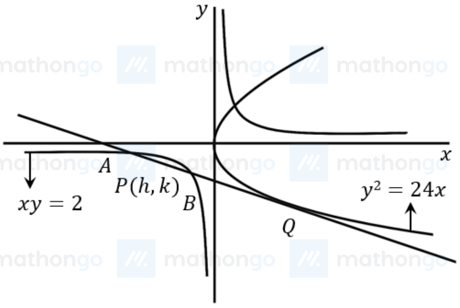
$y^2 = 24x$

Comparing with $y^2 = 4ax$, we get

$4a = 24 \Rightarrow a = 6$

Also given $xy = 2$

Let any point $Q \equiv (at^2, 2at) \equiv (6t^2, 12t)$ on the parabola.



Equation of tangent at Q is

$12yt = 12(x + 6t^2)$

$\Rightarrow yt = x + 6t^2$

$\Rightarrow x - yt + 6t^2 = 0 \quad \ldots.(i)$

Let mid-point of AB be $P(h, k)$.

Equation of chord of hyperbola is

$\frac{xk+yh}{2} = hk$

$\Rightarrow xk + hy - 2hk = 0 \quad \ldots(ii)$

Since, (i) & (ii) are same, so on comparing, we get

$\frac{k}{1} = \frac{h}{-t} = \frac{-2hk}{6t^2}$

$\Rightarrow t = -\frac{h}{k}$ & $\frac{h}{-t} = \frac{-2hk}{6t^2} \Rightarrow t = \frac{k}{3}$

So,

$\frac{k}{3} = -\frac{h}{k}$

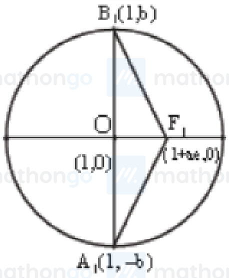
$\Rightarrow k^2 = -3h$

Hence, locus is $y^2 = -3x$.

Therefore, directrix is $4x = 3$.

Length of latus rectum is $= 4a = 3$

11. (9)



L.R. $= \dfrac{2b^2}{a} = \dfrac{1}{2}$

$4b^2 = a$

Ellipse $\dfrac{(x-1)^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$m_{e_{-2}-1} = \dfrac{1}{\sqrt{3}}$

$\dfrac{b}{ae} = \dfrac{1}{\sqrt{3}}$

$3b^2 = a^2e^2 = a^2 - b^2$

$4b^2 = a^2$

$From(i)$ and (ii)

$a = a^2$

$\therefore a = 1$

$b^2 = \dfrac{1}{4}$

$((2a) + (2b))^2 = 9$

12. (2)

Given,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ a > b

Now using eccentricity formula, $e^2 = 1 - \frac{b^2}{a^2}$

We get, $\frac{1}{16} = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow b^2 = \frac{15}{16}a^2$

Now again $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is passing through $\left(-4\sqrt{\frac{2}{5}}, 3\right)$ on satisfying the point we get,

$\Rightarrow \frac{16 \times \frac{2}{5}}{a^2} + \frac{9}{b^2} = 1$

$\Rightarrow \frac{32}{5a^2} + \frac{9}{b^2} = 1$

Now putting the value $b^2 = \frac{15}{16}a^2$ in above equation we get,

$\frac{32}{5a^2} + \frac{9}{\frac{15}{16}a^2} = 1$

$\frac{80}{5a^2} = 1$

$16 = a^2$

So, $b^2 = 15$ and $a^2 + b^2 = 15 + 16 = 31$

13. (1)

Given,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the line $\frac{x}{7} + \frac{y}{2\sqrt{6}} = 1$ on the x-axis, so at $x-$axis $y = 0$ putting in both equation and comparing we get, $a = 7$

And $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the line $\frac{x}{7} - \frac{y}{2\sqrt{6}} = 1$ on the y-axis where $x = 0$ so putting in both equation and comparing we get, $b = 2\sqrt{6}$

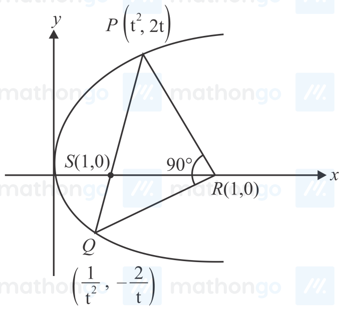
Therefore, $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{24}{49}$

$\Rightarrow e = \sqrt{\frac{25}{49}} = \frac{5}{7}$

14. (2)

Since PQ is focal chord of the parabola $y^2 = 4ax$

so let $P(t^2, 2t)$, $Q\left(\frac{1}{t^2}, -\frac{2}{t}\right)$ and $R(3,0)$



Given $m_{PR} \cdot m_{PQ} = -1$

$\frac{2t}{t^2-3} \times \frac{-\frac{2}{t}}{\frac{1}{t^2}-3} = -1$

$\left(t^2-1\right)^2 = 0$

$\Rightarrow t = 1$

i.e. PQ must be the latus rectum

We get $P(1,2)$ & $Q(1,-2)$

For ellipse

$\frac{2b^2}{a} = 4$ & $ae = 1$

Also $b^2 = a^2\left(1-e^2\right)$

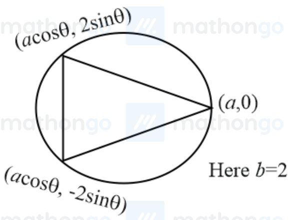
$\therefore\ a = 1+\sqrt{2}$

and $e^2 = 3-2\sqrt{2}$

Hence $\frac{1}{e^2} = \frac{1}{3-2\sqrt{2}} = 3+2\sqrt{2}$

15. (1)

Plotting the diagram as per given information we get,



Now finding area of the triangle we get,

$A = \frac{1}{2}a(1-\cos\theta)(4\sin\theta)$

$\mathrm{A} = 2\mathrm{a}(1-\cos\theta)\sin\theta$

Differentiating to get maxima and minima we get,

$\frac{dA}{d\theta} = 2a\left(\sin^2\theta + \cos\theta - \cos^2\theta\right)$

$\frac{dA}{d\theta} = 0 \Rightarrow 1+\cos\theta - 2\cos^2\theta = 0$

$\cos\theta = 1(\text{Reject})$

OR

$\cos\theta = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

$\frac{d^2A}{d\theta^2} = 2a\left(2\sin^2\theta - \sin\theta\right)$

$\frac{d^2A}{d\theta^2} < 0$ for $\theta = \frac{2\pi}{3}$

Now, $A_{\max} = \frac{3\sqrt{3}}{2}a = 6\sqrt{3}$

$\Rightarrow a = 4$

Now, $e = \sqrt{\frac{a^2-b^2}{a^2}} = \frac{\sqrt{3}}{2}$

16. **(4)** Let $R(2\cos\theta, 3\sin\theta)$

as $\bigcirc P \perp \bigcirc R$

so $\frac{3\sin\theta}{2\cos\theta} \times \frac{\frac{6}{\sqrt{7}}}{\frac{2\sqrt{3}}{\sqrt{7}}} = -1$

$\Rightarrow \tan\theta = \frac{-2}{3\sqrt{3}}$

$\Rightarrow R\left(\frac{-6\sqrt{3}}{\sqrt{31}}, \frac{6}{\sqrt{31}}\right)$ or $R\left(\frac{6\sqrt{3}}{\sqrt{31}}, \frac{-6}{\sqrt{31}}\right)$

Now $= \frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{1}{4}\left(\frac{1}{(OP)^2} + \frac{1}{(OR)^2}\right)$

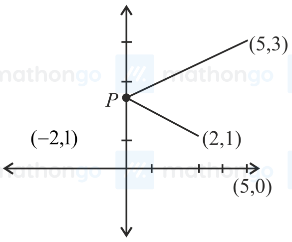
$= \frac{1}{4}\left(\frac{1}{\frac{48}{7}} + \frac{1}{\frac{144}{31}}\right) = \frac{1}{4}\left(\frac{7}{48} + \frac{31}{144}\right)$

$= \frac{13}{144}$

$\Rightarrow p + q = 157$

17. **(3)**

The reflected point of $(2,1)$ is $(-2,1)$.



Equation of reflected ray

$y - 1 = \frac{3-1}{5+2}(x+2)$

$\Rightarrow y - 1 = \frac{2}{7}(x+2)$

$\Rightarrow 7y - 7 = 2x + 4$

$\Rightarrow 2x - 7y + 11 = 0$

Let the equation of other directrix is

$2x - 7y + \lambda = 0$

Distance of directrix from focus

$\frac{a}{e} - ae = \frac{8}{\sqrt{53}}$, where e is the eccentricity

$\Rightarrow 3a - \frac{a}{3} = \frac{8}{\sqrt{53}}$ or $a = \frac{3}{\sqrt{53}}$

Distance from other focus $\frac{a}{e} + ae$

$= 3a + \frac{a}{3} = \frac{10a}{3} = \frac{10}{3} \times \frac{3}{\sqrt{53}} = \frac{10}{\sqrt{53}}$

Distance between two directrix $= \frac{2a}{e}$

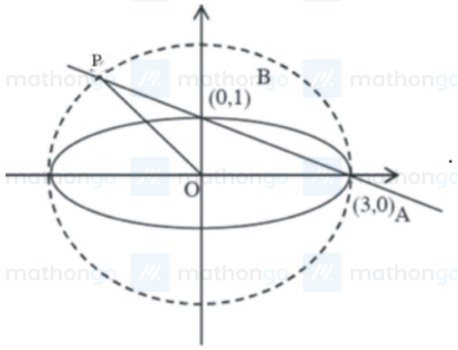
$= 2 \times 3 \times \frac{3}{\sqrt{53}} = \frac{18}{\sqrt{53}}$

So, $\left|\frac{\lambda-11}{\sqrt{53}}\right| = \frac{18}{\sqrt{53}}$

$\Rightarrow \lambda - 11 = 18$ or -18

$\Rightarrow 2x - 7y - 7 = 0$ or $2x - 7y + 29 = 0$

18. (3)



For line $AB x + 3y = 3$ and circle is $x^2 + y^2 = 9$

$(3 - 3y)^2 + y^2 = 9$

$\Rightarrow 10y^2 - 18y = 0$

$\Rightarrow y = 0, \frac{9}{5}$

$\therefore$ Area $= \frac{1}{2} \times 3 \times \frac{9}{5} = \frac{27}{10}$

$m - n = 17$

19. (27)

Given,

$S: \ 9\left(x-3\right)^2 + 16(y-4)^2 \leq 144$ where $(x, y) \in N \times N$

On rearranging we get,

$S: \frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} \leq 1$ where $x, y \in \{1, 2, 3, \ldots\ldots\}$

Now $T: (x-7)^2 + (y-4)^2 \leq 36$ where $x, y \in R$

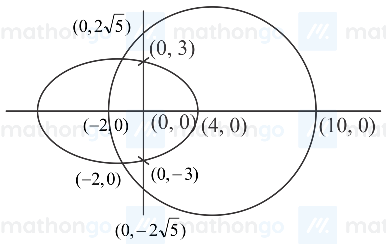
Now let $x - 3 = X : y - 4 = Y$

So, new equation and domain will become,

$S: \frac{X^2}{16} + \frac{Y^2}{9} \leq 1;\ X \in \{-2, -1, 0, 1, \ldots\ldots\}$

$T: (X-4)^2 + Y^2 \leq 36;\ \mathrm{Y} \in \{-3, -2, -1, 0, \ldots\ldots\}$

Now on plotting the diagram we get,



Now number of point in $n(S \cap T)$ will be given by,

Take $Y = -3, -2, -1, 0,\ 1,\ 2,\ 3$ and check for X

So, possible for $S \cap T$ will be,

When $Y = 0 (-2, 0), (-1, 0), \ldots\ldots (4, 0) \rightarrow$ 7 possible case

When $Y = 1$

$(-1, 1), (0, 1), \ldots\ldots. (3, 1) \rightarrow$ 5 possible case

When $Y = -1$

$(-1, -1), (0, -1), \ldots\ldots. (3, -1) \rightarrow$ 5 possible case

When $Y = 2$

$(-1, 2), (0, 2), (1, 2), (2, 2) \rightarrow$ 4 possible case

When $Y = -2$

$(-1, -2), (0, -2), (1, -2), (2, -2) \rightarrow$ 4 possible case

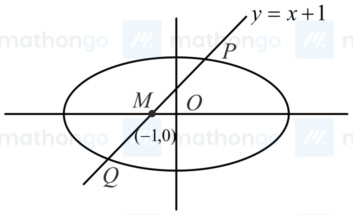
When $Y = 3\ \&\ -3$

$(0, 3)(0, -3) \rightarrow$ Total 2 possible case

So, total cases will be $7 + 5 + 5 + 4 + 4 + 2 = 27$

20. (1)

Plotting the diagram of given data we have,



Let P be the point $\left(-1+\frac{p}{\sqrt{2}}, \frac{p}{\sqrt{2}}\right)$

Since it lies on the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$, we get

$\frac{\left(-1+\frac{p}{\sqrt{2}}\right)^2}{4}+\frac{\left(\frac{p}{\sqrt{2}}\right)^2}{2}=1 \Rightarrow \frac{3p^2}{8}-\frac{p}{2\sqrt{2}}-\frac{3}{4}=0$

Here $\left(p_1-p_2\right)^2=\left(\frac{2\sqrt{2}}{3}\right)^2-4(-2)=\frac{8}{9}+8=\frac{80}{9}$

We know, $\left(p_1-p_2\right)^2=PQ^2$

$\Rightarrow PQ^2=\frac{80}{9}$

Now $r=\frac{PQ}{2} \Rightarrow 9r^2=9\frac{(PQ)^2}{4}=\frac{9}{4}\times\frac{80}{9}=20$

21. (3)

Given,

$\frac{x^2}{4}+\frac{y^2}{2}=1$

$P(4,3)$ ——•—— $Q\left(2\cos\theta, \sqrt{2}\sin\theta\right)$
D

Let coordinate of D $\left(\frac{2\cos\theta+4}{2}, \frac{\sqrt{2}\sin\theta+3}{2}\right)\equiv(h, k)$

So, $\frac{2h-4}{2}=\cos\theta \quad \ldots(1)$

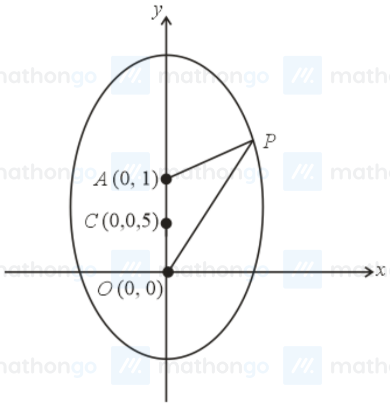
And $\frac{2k-3}{\sqrt{2}}=\sin\theta \quad \ldots(2)$

Equation $(1)^2+$ equation $(2)^2$, then we get

$\left(\frac{2h-4}{2}\right)^2+\left(\frac{2k-3}{\sqrt{2}}\right)^2=1 \Rightarrow \frac{(x-2)^2}{1}+\frac{\left(y-\frac{3}{2}\right)^2}{\left(\frac{1}{2}\right)}=1$

$\therefore$ Required eccentricity is $e=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$

22. (4)



Given, $OA+PO+PA=4$

$\Rightarrow PO+PA=3$

$\Rightarrow$Locus of P is ellipse with foci at O & A and major axis $(2b)=3$

Distance between foci $=2be=1 \Rightarrow e=\frac{1}{3}$

$\Rightarrow$Minor axis $2a=2b\sqrt{1-e^2}=3.\frac{2\sqrt{2}}{3}=2\sqrt{2}$

$\Rightarrow$ Equation of Locus of P is :

$\frac{x^2}{8}+\frac{\left(y-\frac{1}{2}\right)^2}{9}=\frac{1}{4}$

By simplifying the above equation, we get

$9x^2+8y^2-8y=16$

23. (2)

$e_H=\sqrt{2}$

$e_E=\dfrac{1}{\sqrt{2}}$

Since the curves intersect each other orthogonally The ellipse and the hyperbola are confocal

$H: \dfrac{x^2}{1/2}-\dfrac{y^2}{1/2}=1$

$\Rightarrow$ foci $=(1,0)$

For ellipse a. $e_E=1$

$\Rightarrow a=\sqrt{2}$

$\left(e_{\mathrm{E}}\right)^2=\dfrac{1}{2} \Rightarrow 1-\dfrac{b^2}{a^2}=\dfrac{1}{2} \Rightarrow \dfrac{b^2}{a^2}=\dfrac{1}{2}$

$\Rightarrow b^2=1$

Length of $L \cdot R=\frac{2 b^2}{a}=\frac{2}{\sqrt{2}}=\sqrt{2}$

24. (4)

A line $y=m x+c$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if $c^2=a^2 m^2-b^2$.

Given that, tangent to hyperbola $\frac{x^2}{3}-\frac{y^2}{3}=1$ is $2 x+y=k$ or $y=-2 x+k$

Thus, we have, slope $m=-2,\ c=\ k\ \&\ a^2=b^2=3$

$k^2=3(-2)^2-3$

$\Rightarrow\ k^2=9$

Given $k>0,\ \Rightarrow\ k=3$.

Thus, the equation of the tangent to the hyperbola is $y=-2 x+3$.

Given this line is also a tangent to the parabola, $y^2=\alpha x$ and a line $y=m x+c$ is tangent to the parabola $y^2=4 A x$, if $c=\frac{A}{m}$

Thus, we have $3=\dfrac{\left(\frac{\alpha}{4}\right)}{-2}$

$\Rightarrow\ \frac{\alpha}{-8}=3$

$\Rightarrow\ \alpha=-24$.

25. (2)

Given,

$H: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

So coordinates of foci will be : $S(a e, 0), S^{\prime}(-a e, 0)$

Now foot of directrix of parabola will be $(-a e, 0)$

Also focus of parabola is which is same as focus of H will be $(a e,\ 0)$

Now, semi latus rectum of parabola $=\left|S S^{\prime}\right|=2 a e$

Given, $4 a e=e\left(\frac{2 b^2}{a}\right)$

$\Rightarrow b^2=2 a^2 \quad \ldots(1)$

Also given, $(2 \sqrt{2},-2 \sqrt{2})$ lies on $H:\ \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow \dfrac{(2 \sqrt{2})^2}{a^2}-\dfrac{(2 \sqrt{2})^2}{b^2}=1$

$\Rightarrow \frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{8} \quad \ldots(2)$

Now from equation (1) & (2) we get,

$a^2=4,\ b^2=8$

$\because b^2=a^2\left(e^2-1\right)$

$\therefore\ e=\sqrt{3}$

So, the equation of parabola is $y^2=4 \times(a e) x \Rightarrow y^2=8 \sqrt{3} x$

So, only $(3 \sqrt{3},-6 \sqrt{2})$ will satisfy the parabola $y^2=8 \sqrt{3} x$

26. (42)

Given

$E \equiv 3x^2 + 4y^2 = 12 \;\Rightarrow\; \frac{x^2}{4} + \frac{y^2}{3} = 1$

Now, $e_E = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

Length of L.R. $= \frac{2b^2}{a} = \frac{2\times 3}{2} = 3$

Now $H \equiv \frac{x^2}{a^2} - \frac{y^2}{1} = 1$

Length of L.R.$= \frac{2b^2}{a} = \frac{2\times 1}{a}$

Given length of L.R. of E and H are equal

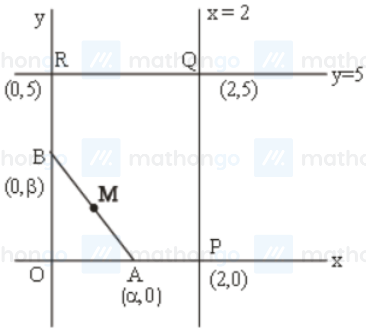
So $\frac{2}{a} = 3 \;\Rightarrow\; a = \frac{2}{3}$

Now $e_H = \sqrt{\frac{b^2}{a^2} + 1} = \sqrt{\frac{1}{\left(\frac{2}{3}\right)^2} + 1} \;\Rightarrow\; e_H = \sqrt{\frac{9}{4} + 1} = \sqrt{\frac{13}{4}}$

So $12\left(e_H^2 + e_E^2\right) = 12\left(\frac{13}{4} + \frac{1}{4}\right) = 12\left(\frac{14}{4}\right) = 14 \times 3 = 42$

27. (3) $\frac{\text{ar(OPQR)}}{\text{or(OAB)}} = \frac{4}{1}$

Let M be the mid-point of AB.



$\mathbf{M}(\mathrm{h}, \mathrm{k}) \equiv \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

$\Rightarrow \quad \frac{10 - \frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta} = 4$

$\Rightarrow \quad \frac{5}{2}\alpha\beta = 10 \Rightarrow \alpha\beta = 4$

$\Rightarrow \quad (2\,\mathrm{h})(2\,\mathrm{K}) = 4$

$\therefore$ Locus of M is $\mathrm{xy} = 1$

Which is a hyperbola.

Which is a hyperbola.

28. (306)

$\mathrm{Hn} \Rightarrow \frac{\mathrm{x}^2}{1+\mathrm{n}} - \frac{\mathrm{y}^2}{3+\mathrm{n}} = 1$

$\mathrm{e} = \sqrt{1 + \frac{\mathrm{b}^2}{\mathrm{a}^2}} = \sqrt{1 + \frac{3+\mathrm{n}}{1+\mathrm{n}}} = \sqrt{\frac{2\mathrm{n}+4}{\mathrm{n}+1}}$

$\mathrm{e} = \sqrt{\frac{2\mathrm{n}+4}{\mathrm{n}+1}}$

$\mathrm{n} = 48$ (smallest even value for which $\mathrm{e} \in \mathrm{Q}$)

$\mathrm{e} = \frac{10}{7}$

$\mathrm{a}^2 = \mathrm{n}+1 \quad \mathrm{b}^2 = \mathrm{n}+3$

$= 49 \quad = 51$

$1 = 1$ length of $LR = \frac{2\,\mathrm{b}^2}{\mathrm{a}}$

$\mathrm{L} = 2 \cdot \frac{51}{7}$

$1 = \frac{102}{7}$

$21\ell = 306$

29. (1552)

Given,

Hyperbola: $\frac{y^2}{64} - \frac{x^2}{49} = 1$

And ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H : \frac{x^2}{49} - \frac{y^2}{64} = -1$, so vertices will be $V \equiv (0, \pm 8)$

So $b^2 = 64$

Now eccentricity of hyperbola will be $e_H = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{49}{64}}$

And eccentricity of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ will be

$e_E = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{a^2}{64}}$

And using $b = 8$

We get, $e_H \times e_E = \frac{1}{2}$ (given)

$\Rightarrow \sqrt{\frac{1-a^2}{64}} \times \frac{\sqrt{113}}{8} = \frac{1}{2}$

$\Rightarrow \sqrt{64 - a^2} \times \sqrt{113} = 32$

$\Rightarrow (64 - a^2) = \frac{32^2}{113}$

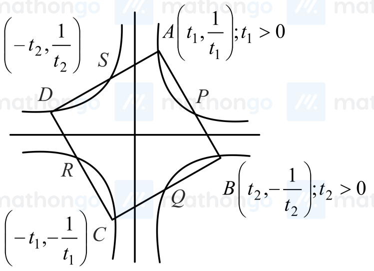
$\Rightarrow a^2 = 64 - \frac{32^2}{113}$

Now length of latus rectum will be $l = \frac{2a^2}{b} = \frac{2}{8}\left(64 - \frac{32^2}{113}\right) = \frac{1552}{113}$

$\Rightarrow 113l = 1552$

30. (80)

$xy = 1, \ -1$



$\frac{t_1+t_2}{2} \cdot \frac{\frac{1}{t_1} - \frac{1}{t_2}}{2} = 1$

$\Rightarrow t_2^2 - t_1{}^2 = 4t_1t_2$

Product of slope $= -1$

$\frac{1}{t_1^2} \times \left(-\frac{1}{t_2^2}\right) = -1 \Rightarrow t_1t_2 = 1$

$\Rightarrow (t_1t_2)^2 = 1 \Rightarrow t_1t_2 = 1$

$\Rightarrow t_1^2 + t_2^2 = \sqrt{4^2 + 4} = 2\sqrt{5}$

$\Rightarrow t_1^2 = 2 + \sqrt{5} \Rightarrow \frac{1}{t_1^2} = \sqrt{5} - 2$

$AB^2 = (t_1 - t_2)^2 + \left(\frac{1}{t_1} + \frac{1}{t_2}\right)^2$

$= 2\left(t_1^2 + \frac{1}{t_1^2}\right) = 4\sqrt{5} \Rightarrow$ Area $^2 = 80$ sq. unit