

# **ANSWER KEYS**

1. (4)	<b>2.</b> (1)	<b>3.</b> (1)	<b>4.</b> (1)	<b>5.</b> (4)	<b>6.</b> (1)	7. (4)	<b>8.</b> (1)	
77.								

$$gof(x) = 2e^x - 5 = y$$

As 
$$gof$$
 is an invertible function,  $x = gof^{-1}(y)$ ...(1)

$$gof(x) = 2e^x - 5 = y$$
As  $gof$  is an invertible function,  $x = gof^{-1}(y) \dots (1)$ 

$$2e^x-5=y\Rightarrow x=\ln\Bigl(rac{y+5}{2}\Bigr).\,..(2)$$

$$2e^x-5=y\Rightarrow x=\ln\left(rac{y+5}{2}
ight)\ldots(2)$$
 From  $(1)\ \&\ (2),\ gof^{-1}(y)=\ln\left(rac{y+5}{2}
ight)$  mathongo /// mathongo ///

$$\Rightarrow (gof)^{-1}(\mathbf{x}) = \ln\left(\frac{x+5}{2}\right)$$

$$\Rightarrow (gof)^{-1}(x) = \ln\left(\frac{x+5}{2}\right) \text{ mathongo } \text{ mathon$$

$$y=f(x)=\sqrt{3}\sin x+\cos x+4$$
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$$y=2\sin\left(x+rac{\pi}{6}
ight)+4\dots(1)$$
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$$+2 \le 2\sin\left(x+\frac{\pi}{6}\right) \le 2$$
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$$-2+4 \le 2\sin\left(x+\frac{\pi}{6}\right)+4 \le 2+4$$

$$y\in [2,6]=B$$

$$y - 4 = 2\sin\left(x + \frac{\pi}{6}\right)$$

$$\sin^{-1}\left(\frac{y-4}{2}\right) - \frac{\pi}{6} \stackrel{\text{\tiny def}}{=} x$$
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$$\Rightarrow -\frac{\pi}{2} \le \sin^{-1}\left(\frac{y-4}{2}\right) \le \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{2} - \frac{\pi}{2} < \sin^{-1}\left(\frac{y-4}{2}\right) - \frac{\pi}{2} < \frac{\pi}{2} - \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{2} \leq \sin^{-1}\left(\frac{y-2}{2}\right) \leq \frac{\pi}{2}$$

$$= \frac{\pi}{2} - \frac{\pi}{6} \leq \sin^{-1}\left(\frac{y-4}{2}\right) - \frac{\pi}{6} \leq \frac{\pi}{2} - \frac{\pi}{6}$$
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$$\Rightarrow -\frac{\pi}{2} - \frac{\pi}{6} \le x \le \frac{\pi}{2} - \frac{\pi}{6}$$

$$\Rightarrow x \in \left[\frac{-2\pi}{3}, \frac{\pi}{3}\right] = A$$
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3. (1) Let 
$$y = f(t)$$
 :  $t = f^{-1}(y)$  mathongo /// mathongo /// mathongo /// mathongo ///

Now, 
$$y = f(t) = \frac{1-t}{1+t} \Rightarrow y + ty = 1-t$$
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$$\Rightarrow t+ty=1-y \Rightarrow t=rac{1-y}{1+y}$$
 mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo ///

i.e., 
$$f^{-1}(y) = \frac{1-y}{1+y}$$
 or  $f^{-1}(t) = \frac{1-t}{1+t}$  mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo ///

Thus, this function is inverse of itself.

4. (1) Given a set containing 10 distinct elements and 
$$f: A \to A$$
 Now, every element of a set  $A$  can make an image in 10 ways.

The total number of ways in which each element make images 
$$=10^{10}$$
. In mathon 2007, and the mathon 2007 mathon

# Answer Kevs and Solutions

- 5.' (4) athongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo
  - If a function is defined  $f: A \rightarrow B$  such that
  - $n(A) = m, \ n(B) = n_{\text{1}}$  at hongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// If  $m \leq n$  then the total number of one-one functions  $= {}^{n}P_{m} = \frac{n!}{(n-m)!}$
  - Total number of one-one onto function  $= {}^3P_3 = 3! = 6$  mathongo mathon
- 6. (1)
- Given,  $f:(-\infty,\ 1]\to (-\infty,\ 1]$  and mathongo mathongo mathongo mathongo mathongo mathongo mathongo
- Let, y = x(2-x) mathongo /// mathongo // mathongo /// mathongo /// mathongo /// mathongo /// mathongo // mathong
- $\Rightarrow x^2 2x + y = 0$
- mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo  $\Rightarrow x = rac{2\pm 2\sqrt{1-y}}{2}$
- $\Rightarrow x = 1 \pm 1 \sqrt{1-y}$  mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo
- $\Rightarrow x = 1 \sqrt{1 y}$  [:: 1 +  $\sqrt{1 y}$  > 1 , y < 1] o /// mathongo /// mathongo /// mathongo ///
- Now replace 'x' by 'y' and 'y' by 'x' mathongo m
- mathongo ///. mathongo ///.
- 7. (4) athongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo ///
- Given:  $\log_{10} x + |x| = 0$ 
  - $\Rightarrow \log_{10} x = -|x|$  mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. We know that the graph of  $\log_a x$  is monotonically increasing if a > 1 and graph of -|x| can be drawn by taking image of |x| in

the x-axis as plane mirror. 100g0 /// mothongo /// mothongo /// mothongo /// mothongo /// mothongo /// mothongo

- By the graph of y=-|x| and  $y=\log_{10}x$ , we have
- mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///.
- $y = \log_{10} x$ ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. matho
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- Both graphs intersect at one point only. So, required number of solution is 1. // mathongo // mathongo // mathongo //

## **Answer Kevs and Solutions**

8	(1) athongo								
	Given cot m —	$\pi$	$m$ $m \in \begin{bmatrix} -\pi & \frac{3}{2} \end{bmatrix}$	$\pi$					

Given 
$$\cot x = \frac{\kappa}{2} + x, \;\; x \in \left[-\pi, \; \frac{5\kappa}{2}\right]$$
 (i)  $y = \cot x$  /// mathongo // mathon

(i) 
$$y=\cot x$$
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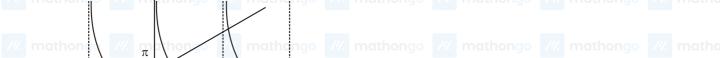
Range=
$$(-\infty, \infty)$$
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It is an equation of a line.

Slope 
$$=$$
 1, we should be mathong  $=$  mathon  $=$  matho

at 
$$y=0,\;x=-\frac{\pi}{2}$$
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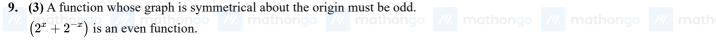




So, there are 3 intersection points for 
$$x \in \left[-\pi, \frac{3\pi}{2}\right]$$
, which are at  $x = -\frac{\pi}{2}$ ,  $2^{\mathrm{nd}} \in \left(0, \frac{\pi}{2}\right)$ ,  $3^{\mathrm{rd}} \in \left(\pi, \frac{3\pi}{2}\right)$ 

Hence, the number of solutions  $= 3$ 





Since, 
$$\log\left(x+\sqrt{1+x^2}\right)$$
 is an odd function, mathon matho

If 
$$f(x+y)=f(x)+f(y) \forall x,y \in R$$
, then

Put 
$$x=y=0 \Rightarrow f(0)=0$$
  
Now, put  $y=-x \Rightarrow f(x)+f(-x)=0$ 

$$f(x)$$
 is an odd function mathongo mat

# 10. (0) athongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathon with matho (0,2) ngo (0,2) mathongo (0,2) ma //. mathongo //. mathongo //. mathongo //. mathongo //. matho $\frac{1}{3\pi} \xrightarrow{3\pi} \text{mathongo} \text{ ///. mathongo} \text$ we know, $0 \le |\cos x| \le 1$ and minimum value of $x^2 + x + 2$ is $\frac{-\left(1^2 - 4.1.2\right)}{4.1} = \frac{7}{4}$ mathongo /// mathongo /// : Minimum value of $x^2 + x + 2$ is greater than the maximum value of $|\cos x|$ . $\therefore$ No common value of x exist for which $|\cos x| = x^2 + x + 2$ nothing y mathons y mathons y mathons y mathons y