ANSWER KEYS

1. (1)	**2.** (4)	**3.** (2)	**4.** (1)	**5.** (2)	**6.** (12)	**7.** (2)	**8.** (3)
9. (1)	**10.** (1)						

1. (1)

$$\begin{vmatrix} 3a & 3b & c \\ x & 2y & z \\ p & 5 & 5 \end{vmatrix} = \begin{vmatrix} 3a & x & p \\ 3b & 2y & 5 \\ c & z & 5 \end{vmatrix} \text{[changing rows into columns]}$$

$$= \frac{1}{3}\begin{vmatrix} 3a & x & p \\ 3b & 2y & 5 \\ 3c & 3z & 15 \end{vmatrix} = \frac{3}{3} \times \frac{1}{5}\begin{vmatrix} a & 5x & p \\ b & 10y & 5 \\ c & 15z & 15 \end{vmatrix} = \frac{1}{5}(125)$$

$= 25.$

2. (4)

Doing $R_3 \longrightarrow \mathrm{R}_3 - \mathrm{x}\mathrm{R}_2$ and $R_2 \longrightarrow R_2 - xR_1$

we get $f\left(x\right) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & 0 & a+x \end{vmatrix}$

$= a(a+x)^2$

So that

$f(2x) - f(x)$

$= a\left[(a+2x)^2 - (a+x)^2\right]$

$= a(a+2x-a-x)(a+2x+a+x)$

$= ax(2a+3x)$

3. (2)

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+xyz)\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$$

$\Rightarrow (1+xyz)\left[x\left(y^2-z^2\right) - y\left(x^2-z^2\right) + z\left(x^2-y^2\right)\right] = 0$

$\Rightarrow (1+xyz)[(x-y)(y-z)(z-x)] = 0$

$\Rightarrow 1+xyz = 0$ or $x = y = z$

As $x,\ y,\ z$ is distinct, $x \neq y \neq z$

$\Rightarrow xyz = -1$

4. (1)

Given that

$$\begin{vmatrix} x^2+x+1 & x+1 & 2x-3 \\ 3x^2 & x+2 & x-1 \\ x^2+5x+1 & 2x+3 & x+4 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$$

It is a polynomial of degree 4

To get constant term $f(0)$=constant.

$$\begin{vmatrix} 1 & 1 & -3 \\ 0 & 2 & -1 \\ 1 & 3 & 4 \end{vmatrix} = e$$

$e = 1(11) - 1(1) - 3(-2) = 16.$

5. (2)

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix} \xrightarrow{C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1} \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & b^2-4 & c^2-4 \end{vmatrix}$$

$=(b-2)(c-2)(c-b)$

Let common difference of $AP = d$

$\therefore\ b = 2 + d, \quad c = 2 + 2d$

$|A| = d \times 2d \times d = 2d^3 \in [2,\ 16]$

$\Rightarrow d^3 \in [1,\ 8] \Rightarrow d \in [1,\ 2] \Rightarrow 2d \in [2,\ 4]$

$\Rightarrow 2 + 2d \in [4,\ 6]$

$\therefore\ c \in [4,\ 6]$

6. (12)

$|A| = 2, |B| = 3, |C| = 5$

$\Rightarrow |A^2| = |A|^2, |ABC| = |A||B||C|$

and $|\mathrm{A}^{-1}| = \frac{1}{|\mathrm{A}|}$

$\Rightarrow |A^2BC^{-1}| = |A|^2 \cdot |B| \cdot \frac{1}{|C|}$

$= (2)^2 (3) \times \frac{1}{5}$

$|A^2BC^{-1}| = \frac{12}{5}$

$|\mathrm{A}^2\mathrm{BC}^{-1}| = \frac{\lambda}{5} \ \ldots \left[\text{Given}\right]$

$\Rightarrow \lambda = 12$

7. (2)

Here we will use the properties of matrices as given below.

$|ABC| = |A|\,|B||C|$ and $|A^{-1}| = \frac{1}{|A|}$

$\Rightarrow |A^2\,BC^{-1}| = |A|^2|B|\,\frac{1}{|C|} = 2^2 \times 3 \times \frac{1}{5} = \frac{12}{5}$

8. (3) We have,

$$ax^4 + bx^3 + cx^2 + 50x + d = \begin{vmatrix} x^3 - 14x^2 & -x & 3x + \lambda \\ 4x + 1 & 3x & x - 4 \\ -3 & 4 & 0 \end{vmatrix}$$

On differentiating with respect to x, we get

$$4ax^3 + 3bx^2 + 2cx + 50 = \begin{vmatrix} 3x^2 - 28x & -1 & 3 \\ 4x + 1 & 3x & x - 4 \\ -3 & 4 & 0 \end{vmatrix} + \begin{vmatrix} x^3 - 14x^2 & -x & 3x + \lambda \\ 4 & 3 & 1 \\ -3 & 4 & 0 \end{vmatrix}$$

Now, put $x = 0$, we get

$$50 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & \lambda \\ 4 & 3 & 1 \\ -3 & 4 & 0 \end{vmatrix}$$

$\Rightarrow 50 = 25\lambda$

$\Rightarrow \lambda = 2$

9. (1)

Given that,

$|A - B| \neq 0$

$A^4 = B^4, B^3A = A^3B$

and $C^3A = C^3B$

Adding all relations, we get

$(A^3 + B^3 + C^3)A = (B^3 + A^3 + C^3)B$

$\Rightarrow (A^3 + B^3 + C^3) \cdot (A - B) = 0$

$|A^3 + B^3 + C^3| \cdot |A - B| = 0$

But, $|A - B| \neq 0$ so $|A^3 + B^3 + C^3| = 0$

10. (1)

$$\Delta_r=\begin{vmatrix} r & (2r-1) & (3r-2) \\ \frac{n}{2} & (n-1) & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \end{vmatrix}$$

Since, the second and the third rows are independent of r, hence the sum is applied to the first row only.

$$\Rightarrow \sum_{r=1}^{n-1} \Delta_r=\begin{vmatrix} \sum_{r=1}^{n-1} r & 2\sum_{r=1}^{n-1} r-\sum_{r=1}^{n-1} 1 & 3\sum_{r=1}^{n-1} r-2\sum_{r=1}^{n-1} 1 \\ \frac{n}{2} & (n-1) & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \end{vmatrix}$$

Using $\sum_{r=1}^{n} r=\frac{n(n+1)}{2}$, we get

$$\sum_{\mathrm{r}=1}^{\mathrm{n}-1} \Delta_{\mathrm{r}}=\begin{vmatrix} \frac{(n-1)n}{2} & \frac{2(n-1)n}{2}-(n-1) & \frac{3(n-1)n}{2}-2(n-1) \\ \frac{n}{2} & (n-1) & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \end{vmatrix}$$

$$\Rightarrow \sum_{\mathrm{r}=1}^{\mathrm{n}-1} \Delta_{\mathrm{r}}=\begin{vmatrix} \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \\ \frac{n}{2} & (n-1) & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n+4) \end{vmatrix}$$

$\Rightarrow \sum_{r=1}^{n-1} \Delta_r=0$, ($\because$ R_1 and R_3 are identical)

Hence, the sum is independent of both n and a.