1. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7 is _____ .

2. The sum of the common terms of the following three arithmetic progressions. $3, 7, 11, 15, \ldots\ldots\ldots\ldots, 399$

$2, 5, 8, 11, \ldots\ldots\ldots 359$ and

$2, 7, 12, 17, \ldots\ldots, 197$, is equal to _____ .

3. Suppose $a_1, a_2, \ldots, a_n, \ldots$ be an arithmetic progression of natural numbers. If the ratio of the sum of the first five terms to the sum of first nine terms of the progression is $5:17$ and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to

(1) 290 (2) 380
(3) 460 (4) 510

4. If $a_1, a_2, a_3 \ldots\ldots\ldots, a_n$ are in A. P. and $a_1 + a_4 + a_7 \ldots\ldots\ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

(1) 64 (2) 98
(3) 38 (4) 76

5. Let $a_1, a_2, a_3, \ldots., a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then

$\lim\limits_{n\to\infty} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ is

(1) $\frac{1}{\sqrt{d}}$ (2) $\sqrt{d}$
(3) 1 (4) 2

6. Let $a_1, a_2, \ldots, a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \ldots + a_n$. If $a_1 = 1, a_n = 300$ and $15 \le n \le 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to:

(1) $(2490, 249)$ (2) $(2480, 249)$
(3) $(2480, 248)$ (4) $(2490, 248)$

7. If $a_1(>0), a_2, a_3, a_4, a_5$ are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to _____.

8. Let $\{a_k\}$ and $\{b_k\}$, $k \in \mathbb{N}$, be two G.P.s with common ratio r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to

9. Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x+y+z)^2$ is equal to

10. The 8^{th} common term of the series

$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots S_2 = 1 + 6 + 11 + 16 + 21 + \ldots.$ is

11. Let $3, 6, 9, 12, \ldots$ upto 78 terms and $5, 9, 13, 17, \ldots$ upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to ______.

12. If for $x, y \in R$, $x > 0$, $y = \log_{10} x + \log_{10} x^{1/3} + \log_{10} x^{1/9} + \ldots$upto ∞ terms and $\frac{2+4+6+\ldots+2y}{3+6+9+\ldots+3y} = \frac{4}{\log_{10} x}$, then the ordered pair (x, y) is equal to

(1) $(10^6, 6)$ (2) $(10^6, 9)$
(3) $(10^2, 3)$ (4) $(10^4, 6)$

13. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1:7$ and $a + n = 33$, then the value of n is

(1) 21 (2) 22
(3) 23 (4) 24

14. Consider two G.Ps. $2, 2^2, 2^3, \ldots$ and $4, 4^2, 4^3, \ldots$ of 60 and n terms respectively. If the geometric mean of all the $60 + n$ terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to:

(1) 560 (2) 1540
(3) 1330 (4) 2600

15. Let $x_1, x_2, \ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200 . If $y_i = i(x_i - i), 1 \le i \le 100$, then the mean of $y_1, y_2, \ldots, y_{100}$ is

(1) 10100 (2) 10101.50
(3) 10049.50 (4) 10051.50

16. Let A_1 and A_2 be two arithmetic means and G_1, G_2 and G_3 be three geometric means of two distinct positive numbers. Then $G_1^4 + G_2^4 + G_3^4 + G_1^2G_3^2$ is equal to

(1) $(A_1 + A_2)^2 G_1 G_3$ (2) $2(A_1 + A_2) G_1 G_3$
(3) $(A_1 + A_2) G_1^2 G_3^2$ (4) $2(A_1 + A_2) G_1^2 G_3^2$

17. Let $a_1, a_2, a_3, \ldots\ldots$ be an A.P. If $a_7 = 3$, the product $(a_1 a_4)$ is minimum and the sum of its first n terms is zero then $n! - 4a_{n(n+2)}$ is equal to

(1) $\frac{381}{4}$ (2) 9
(3) $\frac{33}{4}$ (4) 24

18. For $p, q \in R$, consider the real valued function $f(x) = (x-p)^2 - q, x \in R$ and $q > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $|f(a_i)| = 500$ for all $i = 1, 2, 3, 4$, then the absolute difference between the roots of $f(x) = 0$ is

19. Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as $A, A+1, A+2$, respectively. Let a, b, c be the $7^{\text{th}}, 9^{\text{th}}, 17^{\text{th}}$ terms of A_1, A_2, A_3, respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$. If $a = 29$, then the sum of first 20 terms of an AP whose first term is $c-a-b$ and common difference is $\frac{d}{12}$, is equal to _____ .

20. Consider the sequence $a_1, a_2, a_3, \ldots\ldots$ such that $a_1 = 1, a_2 = 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_n$ for $n = 1, 2, 3, \ldots$
If $\left(\frac{a_1+\frac{1}{a_2}}{a_3}\right)\cdot\left(\frac{a_2+\frac{1}{a_3}}{a_4}\right)\cdot\left(\frac{a_3+\frac{1}{a_4}}{a_5}\right)\ldots\left(\frac{a_{30}+\frac{1}{a_{31}}}{a_{32}}\right) = 2^{\alpha}\left({}^{61}C_{31}\right)$ then α is equal to

(1) -30
(2) -31
(3) -60
(4) -61

21. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1, \forall n \geq 0$.Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to

(1) 483
(2) 528
(3) 575
(4) 624

22. Let $a_1 = b_1 = 1, a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to _____ .

23. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1, a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47\sum_{n=1}^{\infty}\left(\frac{a_n}{2^{3n}}\right)$ is equal to ________.

24. If the minimum value of $f(x) = \frac{5x^2}{2} + \frac{\alpha}{x^5}, x > 0$, is 14, then the value of α is equal to

(1) 32
(2) 64
(3) 128
(4) 256

25. Let $x, y > 0$. If $x^3y^2 = 2^{15}$, then the least value of $3x + 2y$ is

(1) 30
(2) 32
(3) 36
(4) 40

26. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$, $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to

(1) -1
(2) $-\sqrt{2}$
(3) $\sqrt{2}$
(4) 0

27*. The sum $\sum_{n=1}^{21}\frac{3}{(4n-1)(4n+3)}$ is equal to

(1) $\frac{7}{87}$
(2) $\frac{7}{29}$
(3) $\frac{14}{87}$
(4) $\frac{21}{29}$

28*. The sum to 10 terms of the series $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots$ is :-

(1) $\frac{59}{111}$
(2) $\frac{55}{111}$
(3) $\frac{56}{111}$
(4) $\frac{58}{111}$

29*. The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \ldots$ upto 11^{th} term is:

(1) 945
(2) 916
(3) 946
(4) 915

30*. Let $[\alpha]$ denote the greatest integer $\leq \alpha$. Then $\left[\sqrt{1}\right] + \left[\sqrt{2}\right] + \left[\sqrt{3}\right] + \ldots\ldots\ldots\ldots + \left[\sqrt{120}\right]$ is equal to

*Note: Question with * denotes it is optional but good to solve.*