ANSWER KEYS

1. (1)	**2.** (2)	**3.** (22.00)	**4.** (3)	**5.** (3)	**6.** (3)	**7.** (2)	**8.** (2)
9. (37.5)	**10.** (1)						

1. (1) Let E denote the event that a six occurs and A the event that the man reports that it is a '6'. We have

$P(E)=\frac{1}{6}, P'\left(E\right)=\frac{5}{6}$

$P\left(\frac{A}{E}\right)=\frac{3}{4}$ and $P\left(\frac{A}{E'}\right)=\frac{1}{4}$

From Baye's theorem,

$$P\left(\frac{E}{A}\right)=\frac{P(E).P\left(\frac{A}{E}\right)}{P(E).P\left(\frac{A}{E}\right)+P(E').P\left(\frac{A}{E'}\right)}$$

$$=\frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}}=\frac{3}{8}$$

2. (2)

Let, A be the event that the noted number is either 7 or 8.

And, E_1 be the event that the toss of coin results in head and E_2 be the event that the toss of coin results in tail.

Then, we have $P(E_1)=P(E_2)=\frac{1}{2}$

And, $P(A|E_1)=P$(getting the sum of the numbers on the pair of dice as 7 or 8)

The total number of cases, when a pair of dice is thrown are $=6\times6=36$ and the cases of getting sum 7 or 8 are $\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(2,6),(3,5),(4,4),(5,3),(6,2)\}=$ 11 cases.

$\Rightarrow P(A|E_1)=\frac{11}{36}$

Similarly, $P(A|E_2)=$(getting the number 7 or 8 from the numbers $1,2,3,..,9$)

$\Rightarrow P(A|E_2)=\frac{2}{9}$

Now, using the total probability theorem, we get

$P(A)=P(E_1)P(A|E_1)+P(E_1)P(A|E_2)$

$\Rightarrow P(A)=\frac{1}{2}\times\frac{11}{36}+\frac{1}{2}\times\frac{2}{9}$

$\Rightarrow P(A)=\frac{11}{72}+\frac{1}{9}=\frac{19}{72}$.

3. (22.00)

E_1 : lost card is a card of hearts

E_2 : lost card is a card of clubs

E_3 : lost card is a card of spades

E_4 : lost card is a card of diamonds

A : Drawing two cards of hearts

$P(E_1)=\frac{1}{4}=P(E_2)=P(E_3)=P(E_4)$

$P(A|E_1)=\frac{^{12}C_2}{^{51}C_2}$

$P(A|E_2)=\frac{^{13}C_2}{^{51}C_2}=P(A|E_3)=P(A|E_4)$

Required probability$=P(E_1|A)$ where

$$P(E_1|A)=\frac{P(E_1)P(A|E_1)}{\sum_{i=1}^{4}P(E_i)P(A|E_i)}$$

$$=\frac{\frac{1}{4}\cdot\frac{^{12}C_2}{^{51}C_2}}{\frac{1}{4}\left[\frac{^{12}C_2}{^{51}C_2}+\frac{3\left(^{13}C_2\right)}{^{51}C_2}\right]}$$

$$=\frac{66}{66+3(78)}$$

$$=\frac{11}{50}=k$$

$\Rightarrow 100k=22$

4. (3)

Probability that box A is selected $P(A)=\frac{1}{2}$

Probability that box B is selected $P(B)=\frac{1}{2}$

Let E be the event that one ball is white while the other is red.

$\therefore P(E)=P(A)\cdot P(E/A)+P(B)\cdot P(E/B)$

$$=\frac{1}{2}\left[\frac{2\cdot3}{^7C_2}+\frac{4\cdot2}{^9C_2}\right]=\frac{1}{2}\left[\frac{6}{21}+\frac{8}{36}\right]=\frac{1}{2}\left[\frac{2}{7}+\frac{2}{9}\right]=\frac{16}{63}$$

Thus, $P(B/E)=\frac{P(B)\ P(E/B)}{P(E)}=\frac{\frac{1}{9}}{\frac{16}{63}}=\frac{7}{16}$

5. **(3)**

Let E_1, E_2 and E_3 are the events that the examinee guesses, copies and knows the answer and E is the event that he answers correctly.

Then, $P(E_1)=\frac{1}{3}, P(E_2)=\frac{1}{6}$

and $P(E_3)=1-\left(\frac{1}{3}+\frac{1}{6}\right)=\frac{1}{2}$

$\therefore$ Required probability $=P\left(\frac{E_3}{E}\right)$

$=\frac{P\left(\frac{E}{E_3}\right)\cdot P(E_3)}{P\left(\frac{E}{E_1}\right)\cdot P(E_1)+P\left(\frac{E}{E_2}\right)\cdot P(E_2)+P\left(\frac{E}{E_3}\right)\cdot P(E_3)}$

$=\frac{1\times\frac{1}{2}}{\left(\frac{1}{4}\times\frac{1}{3}\right)+\left(\frac{1}{8}\times\frac{1}{6}\right)+\left(1\times\frac{1}{2}\right)}=\frac{24}{29}$

6. **(3)**

Given, distribution is

x	0	1	2	3
$P(x)$	1/3	1/2	0	1/6

$\therefore$ Mean, $m=\sum_{i=1}^{4} p_i x_i$

$=0\times\frac{1}{3}+1\times\frac{1}{2}+2\times 0+3\times\frac{1}{6}$

$=0+\frac{1}{2}+0+\frac{1}{2}=1$

Variance, $\sigma^2=\sum_{i=1}^{4} p_i (x_1-m)^2$

$=\frac{1}{3}(0-1)^2+\frac{1}{2}(1-1)^2+0(2-1)^1+\frac{1}{6}(3-1)^2$

$=\frac{1}{3}+0+0+\frac{2}{3}=1$

$\therefore\ m=\sigma^2=1$

7. **(2)**

Given distribution is

X	1	2	3	4	5
$P(X=x)$	K	2k	3k	2k	k

$\therefore$ Variance $=\ \sum x_i^2 p-\left(\sum x_i p\right)^2$

$=(1k+8k+27k+32k+25k)-(k+4k+9k+8k+5k)^2$

$=(93k)-(27k)^2=\left(93\times\frac{1}{9}\right)-\left(27\times\frac{1}{9}\right)^2$

$\left(\because\ \ \Sigma p=1,\ \ \therefore\ \ k=\frac{1}{9}\right)$

$=\frac{93}{9}-9=\frac{93-81}{9}=\frac{12}{9}=\frac{4}{3}$

8. **(2)**

If k represents the number of consecutive heads and $P(k)$ is the corresponding probability while an unbiased coin is tossed 5 times, then

k	0	1	2	3	4	5
$P(k)$	$\frac{1}{2^5}$	$\frac{12}{2^5}$	$\frac{11}{2^5}$	$\frac{5}{2^5}$	$\frac{2}{2^5}$	$\frac{1}{2^5}$

Now, the expected value of variable X is

$(-1)\frac{1}{2^5}+(-1)\frac{12}{2^5}+(-1)\frac{11}{2^5}+(3)\frac{5}{2^5}+(4)\frac{2}{2^5}+(5)\frac{1}{2^5}$

$=\frac{1}{2^5}[-1-12-11+15+8+5]$

$=\frac{28-24}{2^5}=\frac{4}{2^5}=\frac{1}{2^3}=\frac{1}{8}$

9. **(37.5)**

2 coins can be both 10 rupee coins, both 5 rupee coins or one 5 rupee coin and one 10 rupee coin

So, expected value

$=\frac{{}^{10}C_2}{{}^{15}C_2}\times 20+\frac{{}^{5}C_2}{{}^{15}C_2}\times 10+\frac{{}^{10}C_1{}^{5}C_1}{{}^{15}C_2}\times 15=\frac{3}{7}\times 20+\frac{2}{21}\times 10+\frac{10}{21}\times 15$

$=\frac{350}{21}=\frac{50}{3}$

i.e. $\lambda=\frac{50}{3}$

Hence, $\frac{9\lambda}{4}=37.5$

10. **(1)** win Rs. 15 $\rightarrow$ number of cases $=6$

win Rs. 12 $\rightarrow$ number of cases $=4$

loss Rs. 6 $\rightarrow$ number of cases $=26$

p (expected gain/loss) $=15\times\frac{6}{36}+12\times\frac{4}{36}-6\times\frac{26}{36}=-\frac{1}{2}$