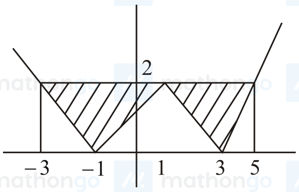
ANSWER KEYS

1. (8)	**2.** (2)	**3.** (3)	**4.** (3)	**5.** (16)	**6.** (4)	**7.** (1)	**8.** (62)
9. (3)	**10.** (4)	**11.** (2)	**12.** (2)	**13.** (2)	**14.** (17)	**15.** (25)	**16.** (72)
17. (19)	**18.** (6)	**19.** (4)	**20.** (1)				

1. **(8)**

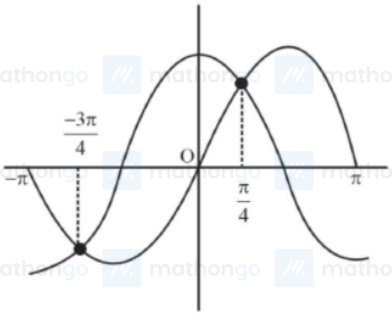
The area bounded by

$y = ||x-1|-2|$ and $y = 2$



Area $= 2\left(\frac{1}{2}.4.2\right) = 8$ sq unit.

2. **(2)**



Area =

$$\left|\int_{-\pi}^{\frac{-3\pi}{4}} \sin x dx\right| + \left|\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}} \cos x dx\right| + \int_{\frac{-\pi}{2}}^{\frac{\pi}{4}} \cos x dx + \int_{\frac{\pi}{4}}^{\pi} \sin x dx$$

$= 4$

3. **(3)**

The given curves are $x^2 \leq y,\ y \leq 8 - x^2$ and $y \leq 7$

The point of intersection of the curves $x^2 \leq y$ and $y \leq 8 - x^2$ is obtained by,

$\Rightarrow x^2 = 8 - x^2$

$\Rightarrow x = \pm 2$ and $y = 4$.

Hence the points are $(2,4),(-2,4)$.

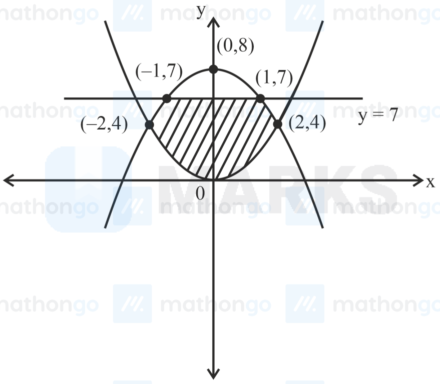
The points of intersection of the curves $y \leq 8 - x^2$ and $y \leq 7$ is obtained by,

$\Rightarrow 8 - x^2 = 7$

$\Rightarrow x = \pm 1$ and $y = 7$

Hence the points are $(1,7),(-1,7)$.

The required graph is



The required area is symmetrical about $y-$axis.

Hence required area is $A = 2\left[\int_0^4 \sqrt{y}\,\mathrm{d}\,y + \int_4^7 \sqrt{8-y}\,\mathrm{d}\,y\right]$

$A = 2\left[\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 - \left[\frac{(8-y)^{\frac{3}{2}}}{\frac{3}{2}}\right]_4^7\right]$

$= \frac{4}{3}\left[(4)^{\frac{3}{2}} - (0)^{\frac{3}{2}} - \left\{(8-7)^{\frac{3}{2}} - (8-4)^{\frac{3}{2}}\right\}\right]$

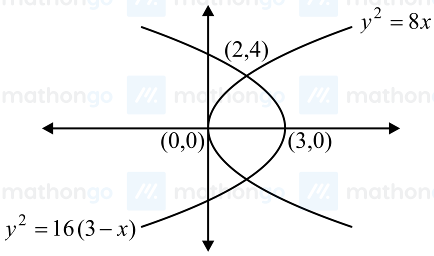
$= \frac{4}{3}(8 - 0 - 1 + 8)$

$= 20$ sq units.

Hence, the required area is 20 sq units.

4. **(3)** Given curves are $y^2 = 8x$ and $y^2 = 16(3-x)$

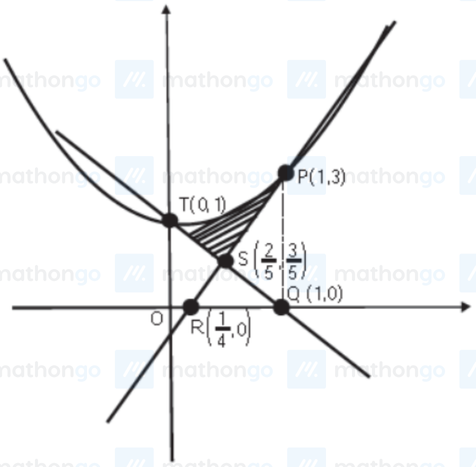
$\Rightarrow 8x = 48 - 16x \Rightarrow x = 2$, so $y = \pm 4$



Required area $= 2\int_0^4 \left(\left(3 - \frac{y^2}{16}\right) - \frac{y^2}{8}\right) dy$

$= 2\left[3y - \frac{y^3}{48} - \frac{y^3}{24}\right]_0^4 = 2\left[12 - \frac{64}{48} - \frac{64}{24}\right] = 16$

5. **(16)**



$y=2x^2+1$

Tangent at $(1,3)$

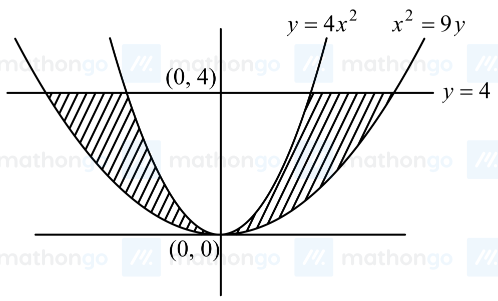
$y=4x-1$

$A=\int_0^1\left(2x^2+1\right)dx$ - area of $(\triangle QOT)-$ area of $(\triangle PQR)+$ area of $(\triangle QRS)$

$A=\left(\frac{2}{3}+1\right)-\frac{1}{2}-\frac{9}{8}+\frac{9}{40}=\frac{16}{60}$

6. **(4)**

Plotting the required region



Enclosed area $=2\int_0^4\left(3\sqrt{y}-\frac{\sqrt{y}}{2}\right)dy$

$=2\int_0^4\frac{5\sqrt{y}}{2}dy$

$=5\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^4$

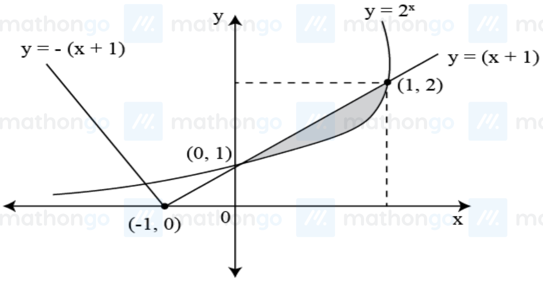
$=\frac{10}{3}(4)^{\frac{3}{2}}=\frac{80}{3}$

7. **(1)**

We know that $|x|=\begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$

Hence, $|x+1|=\begin{cases} x+1, & x \geq -1 \\ -(x+1), & x<-1 \end{cases}$

The graph of the given functions is



The shaded area is the required area.

Area $=\int_0^1\left(y_{line}-y_{curve}\right) dx$

Area $=\int_0^1\left(x+1-2^x\right) dx$

Using $\int x^n dx=\frac{x^{n+1}}{n+1}$ & $\int a^x dx=\frac{a^x}{\log_e a}$, we get

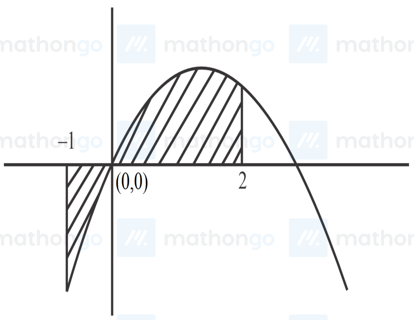
Area $=\left[\frac{x^2}{2}+x-\frac{2^x}{\log_e 2}\right]_0^1$

Area $=\left(\frac{1}{2}+1-\frac{2}{\log_e 2}\right)-\left(0+0-\frac{1}{\log_e 2}\right)$

Area $=\frac{3}{2}-\frac{1}{\log_e 2}$ sq units.

8. **(62)**

Plotting the $y=x|x-3|$ and $x-$axis between $x=-1$ and $x=2$ we get,



Now area of the above region is given by $A=\left|\int_{-1}^0\left(3x-x^2\right) dx\right|+\int_0^2\left(3x-x^2\right) dx$

$\Rightarrow A=\left|\left[3 \times \frac{x^2}{2}-\frac{x^3}{3}\right]_{-1}^0\right|+\left[3 \times \frac{x^2}{2}-\frac{x^3}{3}\right]_0^2$

$\Rightarrow A=\frac{11}{6}+\frac{10}{3}=\frac{11+20}{6}=\frac{31}{6}$

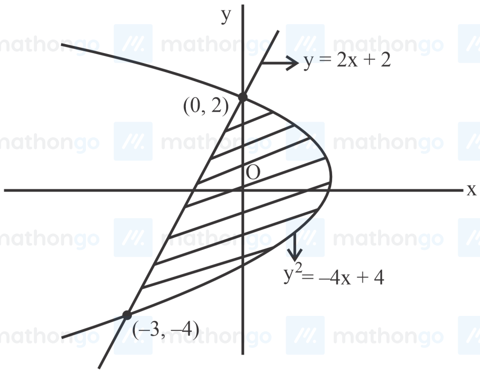
Hence, the value of $12A=62$

9. **(3)**

Given:

$y^2 = -4x + 4$

$y = 2x + 2$



Required area

$= \int_{-4}^{2}\left(\frac{4-y^2}{4} - \frac{y-2}{2}\right)dy$

$= \int_{-4}^{2}\left(2 - \frac{y^2}{4} - \frac{y}{2}\right)dy$

$= \left[2y - \frac{y^3}{12} - \frac{y^2}{4}\right]_{-4}^{2}$

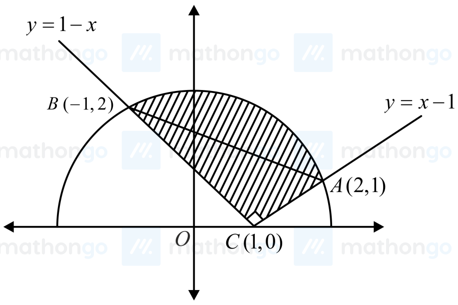
$= \left[\left(4 - \frac{8}{12} - 1\right) - \left(-8 + \frac{64}{12} - \frac{16}{4}\right)\right]$

$= 15 - 6$

$= 9$ sq. units

10. **(4)**

Plotting region for $|x-1| \leq y \leq \sqrt{5-x^2}$, we get,



The intersection points of the curves $x^2 + y^2 = 5$ and $y = |x-1|$ will be $(-1, 2)$ & $(2, 1)$

Here chord AB subtends a right angle at the centre of the circle.

So, required area $=$ area of $\Delta ABC +$ area of segment of circle on chord AB

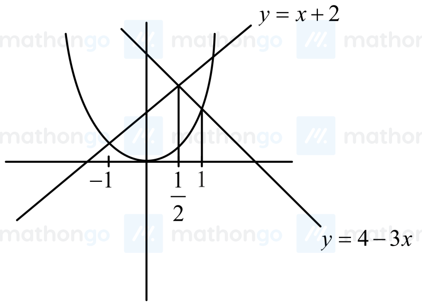
$= \frac{1}{2} AC \cdot BC +$ [area of quarter circle $-$ area of ΔAOB]

$= \frac{1}{2} \times \sqrt{2} \times 2\sqrt{2} + \left(\frac{90^\circ}{360^\circ} \times \pi\left(\sqrt{5}\right)^2 - \frac{1}{2} \times \left(\sqrt{5}\right)^2\right)$

$= \frac{5\pi}{4} - \frac{1}{2}$

11. (2)

The graph of $y=x^2$ & $y=\min\{x+2,4-3x\}$ will be

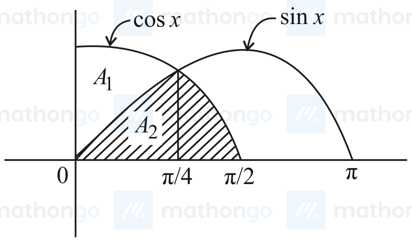


From the graph,

required area $A=\int_{-1}^{\frac{1}{2}}(x+2-x^2)dx+\int_{\frac{1}{2}}^{1}(4-3x-x^2)dx$

$=\left(\frac{x^2}{2}+2x-\frac{x^3}{3}\right)_{-1}^{\frac{1}{2}}+\left(4x-3\frac{x^2}{2}-\frac{x^3}{3}\right)_{\frac{1}{2}}^{1}=\frac{17}{6}$

12. (2)



$A_1=\int_0^{\pi/4}(\cos x-\sin x)dx$

$A_1=(\sin x+\cos x)_0^{\pi/4}=\sqrt{2}-1$

$A_2=\int_0^{\pi/4}\sin x dx+\int_{\pi/4}^{\pi/2}\cos x dx$

$=(-\cos x)_0^{\pi/4}+(\sin x)_{\pi/4}^{\pi/2}$

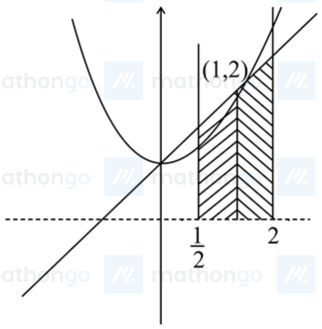
$A_2=\left(1-\frac{1}{\sqrt{2}}\right)+\left(1-\frac{1}{\sqrt{2}}\right)=2-\sqrt{2}$

$A_1:A_2=1:\sqrt{2},\ A_1+A_2=1$

13. (2)

$0\le y\le x^2+1,\ 0\le y\le x+1,\ \frac{1}{2}\le x\le 2$

Drawing the graph according to the given information,



Area is, $A=\int_{\frac{1}{2}}^{1}(x^2+1)\mathrm{d}x+\int_1^2(x+1)\mathrm{d}x$

$A=\left[\frac{x^3}{3}+x\right]_{\frac{1}{2}}^{1}+\left[\frac{x^2}{2}+x\right]_1^2$

$A=\left[\frac{1}{3}+1\right]-\left[\left(\frac{1}{2}\right)^3\cdot\frac{1}{3}+\frac{1}{2}\right]+\left[\frac{4}{2}+2\right]-\left[\frac{1}{2}+1\right]$

$A=\frac{4}{3}-\left(\frac{1}{24}+\frac{1}{2}\right)+(2+2)-\frac{3}{2}$

$A=\frac{4}{3}-\frac{13}{24}+4-\frac{3}{2}$

$A=\frac{32-13+96-36}{24}$

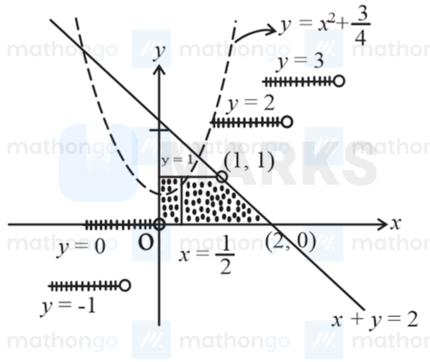
$A=\frac{79}{24}$ sq. units.

14. (17)

Given,

The lines $x+y=2,\ y=0,\ x=0$ and the curve $f\left(x\right)=\min\left\{x^2+\frac{3}{4},1+\left[x\right]\right\}$ where $[x]$ denotes the greatest integer $\leq x$,

Now plotting the diagram of given function we get,



Now from above diagram, the area enclosed is given by,

$A=\left[\int_0^{\frac{1}{2}}\left(x^2+\frac{3}{4}\right)dx\right]+\frac{1}{2}\left(\frac{1}{2}+\frac{3}{2}\right)\times 1=\frac{5}{12}+1$

$\Rightarrow\ 12A=17$

15. (25)

Given,

A be the area of the region

$\left\{\left(x,y\right):y\geq x^2,y\geq\left(1-x\right)^2,y\leq 2x\left(1-x\right)\right\}$

Now solving $y=x^2$ & $y=2x(1-x)$ we get, $x=0,\ \frac{2}{3}$

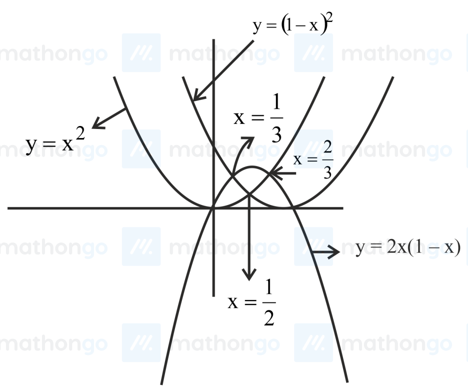
And solving $y=\left(1-x\right)^2$ & $y=2x(1-x)$ we get,

$\Rightarrow 1+x^2-2x=2x-2x^2$

$\Rightarrow 3x^2-4x+1=0$

$\Rightarrow x=1,\ \frac{1}{3}$

Now on plotting the diagram of the above region we get,



Now from the above diagram area of the shaded region will be,

$A=\int_{\frac{1}{3}}^{\frac{2}{3}}\left(2x-2x^2\right)dx-\left\{\int_{\frac{1}{3}}^{\frac{1}{2}}\left(1-x\right)^2dx+\int_{\frac{1}{2}}^{\frac{2}{3}}x^2dx\right\}$

$\Rightarrow A=\left[x^2-\frac{2x^3}{3}\right]_{\frac{1}{3}}^{\frac{2}{3}}-\left\{\left[\frac{(x-1)^3}{3}\right]_{\frac{1}{3}}^{\frac{1}{2}}+\left[\frac{x^3}{3}\right]_{\frac{1}{2}}^{\frac{2}{3}}\right\}$

$\Rightarrow A=\frac{5}{108}$

$\therefore A=\frac{5}{108}\Rightarrow 540\ A=\frac{5}{108}\times 540=25$

16. (72)

We have,

$f(x)=\frac{x+|x|}{2}=\begin{cases} x; & x\ge 0 \\ 0; & x<0 \end{cases}$

$g(x)=\begin{cases} x^2; & x\ge 0 \\ x; & x<0 \end{cases}$

So,

$fog(x)=f\{g(x)\}=\begin{bmatrix} g(x); & g(x)\ge 0 \\ 0; & g(x)<0 \end{bmatrix}$

$\Rightarrow fog(x)=\begin{cases} x^2; & x\ge 0 \\ 0; & x<0 \end{cases}$

And,

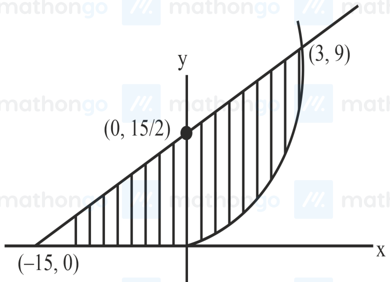
$2y-x=15$

Solving $2y-x=15$ and $y=x^2$, we get

$2x^2-x-15=0$

$\Rightarrow x=\frac{1\pm 11}{4}$

$\Rightarrow x=3,\ -\frac{5}{2}$



Required area

$A=\int_0^3\left(\frac{x+15}{2}-x^2\right)\mathrm{d}x+\frac{1}{2}\times\frac{15}{2}\times 15$

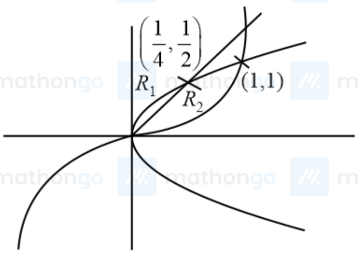
$\Rightarrow A=\left[\frac{x^2}{4}+\left(\frac{15}{2}\right)x-\frac{x^3}{3}\right]_0^3+\frac{225}{4}$

$\Rightarrow A=\frac{9}{4}+\frac{45}{2}-9+\frac{225}{4}$

$\Rightarrow A=\frac{99-36+225}{4}$

$\Rightarrow A=72$

17. (19)



Intersection point of $y^2=x$ & $y=2|x|$ in the first quadrant will be $\left(\frac{1}{4},\frac{1}{2}\right)$

Also, intersection point of $y=x^3$ & $y^2=x$ in first quadrant will be $(1,1)$

Area of the region S $=\int_0^1\left(\sqrt{x}-x^3\right)dx=\left[\frac{2x^{\frac{3}{2}}}{3}-\frac{x^4}{4}\right]_1^0=\frac{5}{12}$

Area of region $R_1=\int_0^{\frac{1}{4}}(\sqrt{x}-2x)dx=\left[\frac{2x^{\frac{3}{2}}}{3}-x^2\right]_0^{\frac{1}{4}}=\frac{1}{48}$

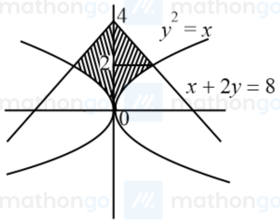
$\therefore\ R_2=S-R_1=\frac{19}{48}$

So, $\frac{R_2}{R_1}=19$

18. (6)

$A_1 = \{(x, y): |x| \leq y^2, |x| + 2y \leq 8\}$ and

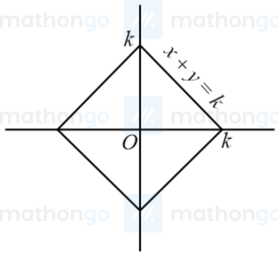
$A_2 = \{(x, y): |x| + |y| \leq k\}$



Area $(A_1) = 2\left[\int_0^2 y^2 dy + \int_2^4 (8 - 2y) dy\right]$

$= 2\left[\left(\frac{y^3}{3}\right)_0^2 + \left(8y - y^2\right)_2^4\right]$

Area $(A_1) = 2 \times \frac{20}{3} = \frac{40}{3}$



Area $(A_2) = 4 \times \frac{1}{2} k^2$

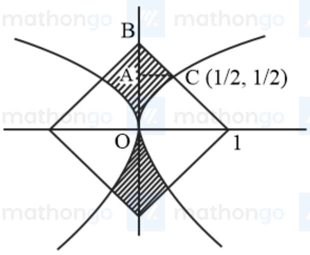
Area $(A_2) = 2k^2$

Given $27(\text{Area } A_1) = 5(\text{Area } A_2)$

$\Rightarrow 27 \times \frac{40}{3} = 5 \times 2k^2$

$\Rightarrow k = 6$

19. (4)



Required area is, $A = \operatorname{ar}(OAC) + \operatorname{ar}(\Delta ABC)$

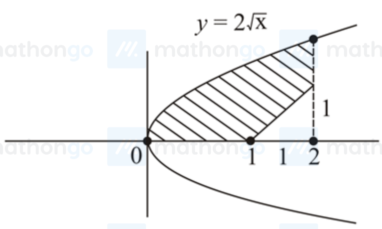
$\Rightarrow A = 4\left[\int_0^{1/2} 2y^2 dy + \frac{1}{2} \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)\right]$

$\Rightarrow A = 4\left[\left(\frac{2}{3} y^3\right)_0^{1/2} + \left(\frac{1}{8}\right)\right] = \frac{5}{6}$

20. (1)

$y = [x](x - 1)$

$= \begin{cases} 0 & 0 \leq x < 1 \\ x - 1 & 1 \leq x < 2 \end{cases}$



Area$= \int_0^2 2\sqrt{x} \cdot dx - \frac{1}{2}\left(1\right)\left(1\right) = \left(\frac{4x^{3/2}}{3}\right)_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2}$