1. $\int_{-\frac{\pi}{2}}^{0} \sqrt{\cos x - \cos^3 x}\, dx$

(1) $\frac{2}{3}$
(2) $-\frac{2}{3}$
(3) $\frac{3}{2}$
(4) $-\frac{3}{2}$

2. The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ is

(1) -1
(2) 1
(3) $\frac{\pi}{2} - 1$
(4) $\frac{\pi}{2} + 1$

3. Evaluate the following $\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

(1) $\frac{\sqrt{3}\pi}{12}$
(2) $\frac{\sqrt{3}\pi}{6}$
(3) $\frac{6\pi-\sqrt{3}}{12}$
(4) $\frac{6-\pi\sqrt{3}}{12}$

4. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t}\, dt$, Then $F(e)$ equals

(1) 1
(2) 2
(3) 1/2
(4) 0

5. The value of $x > 1$ satisfying the equation $\int_1^x t \log t dt = \frac{1}{4}$, is

(1) $\sqrt{e}$
(2) $e^{\frac{3}{2}}$
(3) e^2
(4) $2e - 1$

6. If for $n \geqslant 1$, $P_n = \int_1^e (\log\ x)^n dx$, then $P_{10} - 90P_8$ is equal to

(1) $10e$
(2) -9
(3) 10
(4) $-9e$

7. If $\int_0^\infty e^{-ax} dx = \frac{1}{a}$, then $\int_0^\infty x^n e^{-ax} dx$ is

(1) $\frac{(-1)^n n!}{a^{n+1}}$
(2) $\frac{(-1)^n (n-1)!}{a^n}$
(3) $\frac{n!}{a^{n+1}}$
(4) none of these

8. If $m, n \in N$, then $I_{m,n} = \int_0^1 x^m (1-x^n) dx$ is equal to

(1) $\frac{m!n!}{(m+n+2)!}$
(2) $\frac{2m!n!}{(m+n+1)!}$
(3) $\frac{m!n!}{(m+n+1)!}$
(4) None of these

9. If $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m\left(\pi + n\right)$, then mn is equal to

(1) 1
(2) $\frac{1}{2}$
(3) -1
(4) $-\frac{1}{2}$

10. The equation $\int_0^x (t^2 - 8t + 13) dt = x \sin\left(\frac{a}{x}\right)$ has a solution if $\sin\left(\frac{a}{6}\right)$ is

(1) zero
(2) -1
(3) 1
(4) none of these