ANSWER KEYS

1. (16)	**2.** (2997)	**3.** (30)	**4.** (1492)	**5.** (100)	**6.** (2)	**7.** (1)	**8.** (2)
9. (21)	**10.** (4)	**11.** (432)	**12.** (3000)	**13.** (120)	**14.** (31650)	**15.** (3)	**16.** (2)
17. (54)	**18.** (2)	**19.** (1)	**20.** (4)	**21.** (32)	**22.** (576)	**23.** (18915)	**24.** (924)
25. (1)	**26.** (45)	**27.** (4)	**28.** (56)	**29.** (40)	**30.** (44)		

1. **(16)** For number to be divisible by ' 6 ' unit digit should be even and sum of digit is divisible by 3 .

$(2,1,3),(2,3,4),(2,5,5),(2,2,5),(2,2,2)$
$(4,1,1),(4,4,1),(4,4,4),(4,3,5)$
$2,1,3 \Rightarrow 312,132$
$2,3,4 \Rightarrow 342,432,234,324$
$2,5,5 \Rightarrow 552$
$2,2,5 \Rightarrow 252,522$
$2,2,2 \Rightarrow 222$
$4,1,1 \Rightarrow 114$
$4,4,1 \Rightarrow 414,144$
$4,4,4 \Rightarrow 444$
$4,3,5 \Rightarrow 354,534$

Total 16 numbers.

2. **(2997)**

Number of five-digit numbers formed using the digits $0,\ 2,\ 3,\ 4,\ 7,\ 9$ with 2 at the first place is

$=6 \times 6 \times 6 \times 6=1296$

Since, $2 \underset{6}{+} \underset{6}{+} \underset{6}{+} \underset{6}{+} = 1296$

Number of five-digit numbers formed using the given digits with 3 at the first place is

$=1296$

Since, $3 \underset{6}{+} \underset{6}{+} \underset{6}{+} \underset{6}{+} = 1296$

Number of five-digit numbers formed using the given digits with 40 at the first place is

$=216$

Since, $40 \underset{6}{+} \underset{6}{+} \underset{6}{+} = 216$

And,

$420 \underset{6}{+} \underset{6}{+} = 36$

$\underline{4}\ \underline{2}\ \underline{2} \underset{6}{+} \underset{6}{+} = 36$

$423 \underset{6}{+} \underset{6}{+} = 36$

$424 \underset{6}{+} \underset{6}{+} = 36$

$427 \underset{6}{+} \underset{6}{+} = 36$

$429\ \underline{0} \underset{6}{+} = 6$

$429\ 2\ 0 = 1$

$429\ 2\ 2 = 1$

$429\ 2\ 3 = 1$

Required serial number is

$=1296 \times 2+216+5 \times 36+6+1+1+1+1=2997$

3. **(30)**

To find total numbers between 1000 and 3000 divisible by 4 using the digits $1,2,3,4,5,6$,
We will solve in two cases.

Case I : When first digit is 1.

Then last two digits can be $24,32,36,52,56$ or 64

So, total ways of choosing last two digit is 6 and second digit will be chosen in 3 ways

So, number of such numbers $=6 \times 3=18$

Case II: When first digit is 2

Then last two digits can be $16,36,56$ or 64

So, total ways of choosing last two digit is 4 and second digit will be chosen in 3

So, number of such numbers $=4 \times 3=12$

Total numbers of numbers $=18+12=30$

4. (1492)

M	A	N	K	I	N	D

Arranging the letters alphabetically, we get
ADIKMNN
When the word starts with any of the letters A/D/I/K, the number of possibilities= $\frac{6!}{2!} \times 4 = 1440$
Now when the word starts with MA, then the number of possibilities= $\frac{4!}{2!} \times 3 = 36$
Now when the word starts with MAN, then the number of possibilities= $3! \times 2 = 12$
Now when the word starts with MANK, then the number of possibilities= 4
Hence, rank of the work MANKIND is 1492

5. (100)

5	a	b	b	a	5

Palindrome Numbers only in this form will be divisible by 55.
Numbers can be between $0 - 9$
Number of ways of selecting value of $a = {}^{10}_{1}P$
Similarly number of ways of choosing value of $b = {}^{10}_{1}P$ (As a and b can also be same)
Therefore, Total number of ways of getting palindrome numbers divisible by 55 is $= 1 \times 10 \times 10 \times 1 = 100$

6. (2)

5 x x x 0
7 x x x 0
7 x x x 5
9 x x x 0
9 x x x 5

So Required numbers $= 5 \times {}^4P_3 = 120$

7. (1)

There are 5 vowels in the given word which are $4E's$ & $1I$.
Since they have to always occur together we take them as a single object $E\,E\,E\,E\,I$ for the time being.
This single object together with 7 remaining object will account for 8 objects.
There 8 objects in which there are $3N's$ & $2D's$ can be arrangement in $\frac{8!}{3!2!}$ ways.
Corresponding to each of their arrangements the 5 vowels E, E, E, E & I which can be arranged in $\frac{5!}{4!}$
Hence, required number of arrangements.
$= \frac{8!}{3!2!} \times \frac{5!}{4!} = 16800$
Hence this is the correct option.

8. (2)

The word 'MATHEMATICS' consists of 11 letters including two $M's$, two $T's$ and two $A's$,
Now, first we will arrange letters $\_M\_A\_T\_H\_E\_M\_A\_T\_I\_$.
Here, we have 10 gaps to fill C & S.
The number of words with or without meaning can be formed from the word MATHEMATICS when $C,\ S$ does not come together is
$= \frac{9!}{2!\cdot 2!\cdot 2!} \times {}^{10}C_2 \times 2!$
$= \frac{9\times 8\times 7\times 6!}{2!\cdot 2!} \times \frac{10\times 9}{2}$
$= 5670$
Hence this is the correct option.

9. **(21)**

Given,

We have to form four-digit number using the digits $1, 2, 3$ & 5

Now for number to be divisible by 15 we need to fix the last digit as 5 as $\_\ \_\ \_\ 5$, so that number can be divisible by 5 and for divisibility by 3 the addition of all number should be divisible by 3,

So making cases where sum is divisible by 3 we get,

Case $1 -\ \{1,1,2\}$ {as 1 ,1,2 & 5 sum is divisible by 3}

So, total ways for case 1 is 3 ways,

Case $2 - \{5,1,1\} \rightarrow 3$ ways, $\{3,3,1\} \rightarrow 3$ ways & $\{3,2,2\} \rightarrow 3$ ways

So, total ways for case 2 is 9 ways.

Case $3 -\ \{5,3,2\} \rightarrow 6$ ways

Case $4 -\ \{5,5,3\} \rightarrow 3$ ways

So, adding all cases we get, $3+9+6+3=21$ ways.

10. **(4)**

As we have to make a six-digit number by using all the digits $1, 3, 5, 7, 9$, hence exactly one of the digits is repeating.

For the digit to repeat we have ${}^5C_1 = 5$ choices.

And, six digits in which one is repeating can be arranged in $\frac{6!}{2!}$ ways.

Hence, total such numbers $= 5 \times \frac{6!}{2!} = \frac{5}{2}(6!)$.

11. **(432)** UNIVERSE

Vowels: E, I, U

Consonants: N, V, R, S

$\rightarrow {}^3C_2 \times {}^4C_2 \times 4! = 3 \times 6 \times 24 = 432$

12. **(3000)**

Four-digit numbers are $\{1000, 1001, \ldots, 9999\}$

Total number of four digit numbers are 9000.

Now,

$54 = 2 \times 3^3$

So, for getting gcd with 2, we must have numbers multiple of 2 but not of 3.

Multiple of 2 are $\{1000, 1002, \ldots\ldots, 9998\}$

Let total numbers be m, then

$9998 = 1000 + (m-1)2$

$\Rightarrow m = 4500$

Total numbers are 4500.

Multiple of 6 are $\{1002, 1008 \ldots, 9996\}$

Let total numbers be n, then

$9996 = 1002 + (n-1)6$

$\Rightarrow n = 1500$

So, required number is $= 4500 - 1500 = 3000$

13. (120)

Given:

$1 \le x, y \le 25$

And, x, y are distinct integers.

Let

$x + y = 5k$, where $k \in N$

So, we have

x	y		Number of ways
5λ i.e, 5, 10, 15, 20, 25	5λ i.e, 5, 10, 15, 20, 25	Since, x and y are distinct integers, so we cannot pair $(5,5)$, $(10,10), \ldots (25,25)$	20
$5\lambda + 1$ i.e., 1, 6, 11, 16, 21	$5\lambda + 4$ i.e., 4, 9, 14, 19, 14		25
$5\lambda + 2$ i.e, 2, 7, 12, 17, 22	$5\lambda + 3$ i.e., 3, 8, 13, 18, 23		25
$5\lambda + 3$ i.e., 3, 8, 13, 18, 23	$5\lambda + 2$ i.e, 2, 7, 12, 17, 22		25
$5\lambda + 4$ i.e., 4, 9, 14, 19, 14	$5\lambda + 1$ i.e, 1, 6, 11, 16, 21		25

Total number of ways $= 120$

14. (31650)

If group C has one student then number of groups

${}^{10}C_1\left[2^9 - 2\right] = 5100$

If group C has two students then number of groups

${}^{10}C_2\left[2^8 - 2\right] = 11430$

If group C has three students then number of groups

$= {}^{10}C_3 \times \left[2^7 - 2\right] = 15120$

So total groups$= 31650$

15. (3)

Each box contains 10 balls numbered from 1 to 10.

n_1, n_2, n_3 are numbers on the balls drawn from the box B_1, B_2 and B_3 respectively such that $n_1 < n_2 < n_3$.

i.e., all 3 numbers n_1, n_2, n_3 must be different and can be arranged only in one way (increasing).

Now n_1, n_2, n_3 can be selected in ${}^{10}C_3$ ways.

Hence, total number of ways $= ^{10}C_3 . 1 = ^{10}C_3 = \frac{(10!)}{(3!)\,(7!)} = \frac{10 \times 9 \times 8}{3 \times 2} = 120.$

16. (2)

We have,

$xyz = 24$

$\Rightarrow xyz = 2^3 \times 3^1$

Let

$x = 2^{\alpha_1} \times 3^{\beta_1}$

$y = 2^{\alpha_2} \times 3^{\beta_2}$

$z = 2^{\alpha_3} \times 3^{\beta_2}$

where,

$\alpha_1 + \alpha_2 + \alpha_3 = 3 \quad \ldots$(i)

$\beta_1 + \beta_2 + \beta_3 = 1 \quad \ldots.$(ii)

Number of non-negative integral solution for (i)

$= {}^5C_2 = 10$

Number of non-negative integral solution for (ii)

$= {}^3C_2 = 3$

Total number of positive integral solutions (x, y, z) such that $xyz = 24$ is

$= 10 \times 3 = 30$

17. (54)

Let xyz be the three-digit number such that

$x+y+z=10,\ x\geq 1,\ y\geq 0,\ z\geq 0$

Let $x-1=t \Rightarrow x=1+t$

So, $x\geq 1 \Rightarrow t+1\geq 1 \Rightarrow t\geq 0$

Also, $x+y+z=10 \Rightarrow t+y+z=10-1=9$

So, $t+y+z=9,\ 0\leq t,\ y,\ z\leq 9$

Thus, total number of non-negative integral solution $= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = \frac{11.10}{2} = 55$

But for $t=9,\ x=10$, so required number of integers $=55-1=54$.

18. (2)

Given,

7 boys and 5 girls are to be seated around a circular such that no two girls to be seated together,

Now we know that n objects can be arranged in a circle in $(n-1)!$ ways.

Let us first arrange 7 boys in circular arrangement in $(7-1)!$ ways.

Now there will be 7 gaps.

So let us select any 5 gaps out of 7 gaps and arrange 5 girls in the chosen gaps. This can be done in ${}^7C_5\times 5!$ ways.

Hence, required arrangements are $6!\times {}^7C_5\times 5!$

$=6\times 5!\times \frac{7\times 6}{2}\times 5!$

$=126(5!)^2$.

Therefore, required arrangements are $126(5!)^2$

19. (1)

Digits are $1,\ 2,\ 2,\ 3$

total distinct numbers $\frac{4!}{2!}=12$.

total numbers when

1 at unit place is 3.

2 at unit place is 6

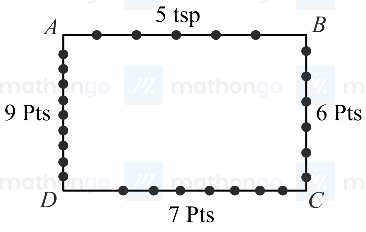
3 at unit place is 3.

So, sum $=(3+12+9)(10^3+10^2+10+1)$

$=(1111)\times 24$

$=26664$

20. (4)



$\alpha=$ Number of triangles

$\alpha=5\cdot 6\cdot 7+5\cdot 7\cdot 9+5\cdot 6\cdot 9+6\cdot 7\cdot 9$

$=210+315+270+378$

$=1173$

$\beta=$ Number of Quadrilateral

$\beta=5\cdot 6\cdot 7\cdot 9=1890$

$\beta-\alpha=1890-1173=717$

21. (32)

Taking case (1) fixing 1 & 2 at first two places,

1	2				

So, other number can be selected in ${}^7C_4 = 35$ ways {as number are from 1-9}

Case (2) fixing 1 & 3 at first two place,

1	3				

So, other number can be selected in ${}^6C_4 = 15$ ways

Similarly,

1	4				

So, other number can be selected in ${}^5C_4 = 5$ ways

Now fixing 1 & 5 we get,

1	5				

Other number can be selected in ${}^4C_4 = 1$

Now fixing 2 & 3 in first two place,

2	3				

So, other number can be selected in ${}^6C_4 = 15$

So, the 72^{nd} number will be 245678

Hence, sum will be $2+4+5+6+7+8=32$

22. (576)

Digits given are $1, 2, 3, 4, 5, 7, 9$

Now multiple of 11 $\rightarrow$ Difference of sum at even & odd place should be divisible by 11.

Let number of the form $abcdefg$

$\therefore\ (a+c+e+g)-(b+d+f)=11x$

$a+b+c+d+e+f=31$

$\therefore$ either $a+c+e+g=21$ or 10

$\therefore b+d+f=10$ or 21

Case-1

$a+c+e+g=21$

$b+d+f=10$

$(b, d, f) \in \{(1, 2, 7)(2, 3, 5)(1, 4, 5)\}$

$(\text{a, c, e, g}) \in \{(1, 4, 7, 9), (3, 4, 5, 9), (2, 3, 7, 9)\}$

$\therefore$ Total number in case-1 $=(3! \times 3)(4!)=432$

Case-2

$a+c+e+g=10$

$b+d+f=21$

$(a, b, e, g) \in \{1, 2, 3, 4\}$

(b, d, f) & $\{(5, 7, 9)\}$

$\therefore$ Total number in case $2=3! \times 4!=144$

$\therefore$ Total numbers $=144+432=576$

23. (18915)

$b_i \in \{1, 2, 3 \ldots\ldots\ldots 100\}$

Let $A=$ set when $b_1\ b_2\ b_3$ are consecutive

$n(A)=97+97+\ldots\ldots+97$ (added 98 times) $=97 \times 98=9506$

Similarly when $b_2\ b_3\ b_4$ are consecutive

$n(B)=97 \times 98=9506$

Now when $b_1\ b_2\ b_3\ b_4$ are consecutive then $n(A \cap B)=97$

$n(AUB)=n(A)+n(B)-n(A \cap B)$

Number of required permutation $=9506+9506-97=18915$

24. (924)

Let $N = 2^{10} \times 5^{10} \times 11^{11} \times 13^{13}$

Now, power of 2 must be zero,

Power of 5 can be anything,

Power of 13 can be anything.

But, power of 11 should be even.

So, required number of divisors is

$1 \times 11 \times 14 \times 6 = 924.$

25. (1)

$y + z = 5$

$\frac{1}{y} + \frac{1}{z} = \frac{5}{6} \ \ y > z$

$\Rightarrow y = 3, z = 2$

$\Rightarrow n = 2^x . 3^3 . 5^2 = (2.2.2\ldots)(3.3.3)(5.5)$

For calculating the odd divisor of $n = 2^x 3^y 5^z$, x must be 0.

Hence, Number of odd divisors $= (3+1) \times (2+1) = 4 \times 3 = 12.$

26. (45)

$\because 7^n = (10-3)^n = 10k + (-3)^n$

$7^n + 3^n = 10k + (-3)^n + 3^n$

$10k + (-3)^n + (3)^n = \begin{cases} 10k \text{ if n=odd} \\ 10k + 2.3^n \text{ if n=even} \end{cases}$

Let $n = 2t;\ t \in N$

$\therefore 3^n = 3^{2t} = (10-1)^t$

$= 10p + (-1)^t$

$.= 10p \pm 1$

$\therefore$ if $n =$ even then $7^n + 3^n$ will not be multiply of 10

So if n is odd then only $7^n + 3^n$ will be multiply of 10

$\therefore n = 11, 13, 15, \ldots\ldots\ldots, 99$

$\therefore$ Number of odd two digit numbers $= 45$

27. (4)

Given, $C_1 + C_2 + C_3 + C_4 = 30$

Now it has given that $4 \leq C_2 \leq 7$ & $2 \leq C_3 \leq 6$

and C_1 & C_4 can take any value.

Finding coefficient of x^{30} in

$(x^0 + x^1 + \ldots\ldots x^{30})(x^4 + x^5 + \ldots\ldots x^7)(x^2 + \ldots x^6)(x^0 + \ldots . x^{30})$

$= \left(\frac{1-x^{31}}{1-x}\right)^2 x^4 \left(\frac{1-x^4}{1-x}\right) x^2 \left(\frac{1-x^5}{1-x}\right)$

{Ignoring higher power more than 30}

$= \left(\frac{1}{1-x}\right)^2 x^4 \left(\frac{1-x^4}{1-x}\right) x^2 \left(\frac{1-x^5}{1-x}\right)$

$= x^6(1-x^4)(1-x^5)(1-x)^{-4} = x^6(1 - x^5 - x^4 + x^9)(1-x)^{-4}$

$= (x^6 - x^{11} - x^{10} + x^{15})(1-x)^{-4}$

Required coefficient $= {}^{4+24-1}C_{24} - {}^{4+19-1}C_{19} - {}^{20+4-1}C_{20} + {}^{15+4-1}C_{15}$

$= {}^{27}C_{24} - {}^{22}C_{19} - {}^{23}C_{20} + {}^{18}C_{15} = 430$

28. (56)

Given, 4 blue & 5 red spherical ball

Now according to question there should be minimum 2 blue ball in between any two red ball.

So assume a, b, c, d, e, f are number of blue balls,

$\underline{a} \bigcirc \underline{b} \bigcirc \underline{c} \bigcirc \underline{d} \bigcirc \underline{e} \bigcirc \underline{f}$

So, $a+b+c+d+e+f=11$ ......(i)

Where $b, c, d, e \geqslant 2$

Now let $b=b'+2$

$c=c'+2$

$d=d'+2$

$e=e'+2$

So equation (i) becomes

$a+b'+2+c'+2+d'+2+e'+2+f=11$

$\Rightarrow \; a+b'+c'+d'+e'+f=3$

Now total ways will be

$={ }^{3+6-1} C_{6-1}={ }^8 C_5={ }^8 C_3=56$

29. (40)

Let the marks obtained in each of the five MCQ be $x_1,\ x_2,\ x_3,\ x_4,\ x_5$

Now, $x_1+x_2+x_3+x_4+x_5=5$

Only one possibility satisfies the above equation i.e. $3,3,3,-2,-2$

Hence, total number of ways $=\frac{5!}{3!2!} \times 2 \times 2=40$

30. (44) Derangement of 5 students

$$\mathrm{D}_5=5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

$$=120\left(\frac{1}{2}-\frac{1}{6}+\frac{1}{24}-\frac{1}{120}\right)$$

$=60-20+5-1$

$=40+4$

$=44$