## ANSWER KEYS

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **1.** (2) | **2.** (24) | **3.** (12) | **4.** (1) | **5.** (1) | **6.** (3) | **7.** (4) | **8.** (3) |
| **9.** (7) | **10.** (165) | **11.** (07.00) | **12.** (1) | **13.** (2) | **14.** (1) | **15.** (10) | **16.** (7) |
| **17.** (3) | **18.** (1) | **19.** (3) | **20.** (121.00) | **21.** (4) | **22.** (2) | **23.** (61) | **24.** (25) |
| **25.** (1) | **26.** (816) | **27.** (1) | **28.** (3) | **29.** (3) | **30.** (3) | | |

**1. (2)**

Given,

$C_1 \ : \ x_2 + \ y_2 - 4x - \ 2y + (5 - \ \alpha) = \ 0$

So, its centre will be, $O_1 = (2, \ 1)$ and radius $= \sqrt{\alpha}$

And $C_2 \ : \ 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$

$\Rightarrow C_2 \ : \ x^2 + y^2 - 2fx - 2gy \ + \frac{36}{5} = 0$

So, Centre $O_2 \ = \ (f, \ g)$ and radius $\ r = \sqrt{f^2 + g^2 - \frac{36}{5}}$

Also given $O_2$ is reflection of $O_1$ in $2x - y + 1 = 0$, so image formula we get,

$\Rightarrow \frac{f-2}{2} = \frac{g-1}{-1} = -2 \cdot \left(\frac{2 \times 2 - 1 + 1}{2^2 + 1^2}\right)$

$\Rightarrow f = \frac{-6}{5}$ and $g = \frac{13}{5}$

So, radius $r = \sqrt{\left(\frac{-6}{5}\right)^2 + \left(\frac{13}{5}\right)^2 - \frac{36}{5}} = \frac{\sqrt{25}}{5} = 1$

$\Rightarrow r = 1$ and $\alpha \ = 1$ as they both are same radius circle,

Hence, $r + \alpha = 2$.

**2. (24)**

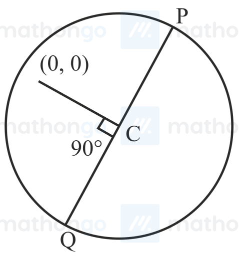
Given,

$P(a_1, \ b_1)$ and $Q(a_2, \ b_2)$ be two distinct points on a circle with center $C\left(\sqrt{2}, \sqrt{3}\right)$,

And $O$ be the origin and $OC$ be perpendicular to both $CP$ and $CQ$,

So, $PCQ$ will be a straight line

Now on plotting the diagram we get,




So, $OC = \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2} = \sqrt{5}$

Now let $CP = CQ = I$

Now by using area of triangle $OCP = \frac{1}{2} \times OC \times I$ we get,

$\frac{\sqrt{35}}{2} = \frac{1}{2} \times \sqrt{5} \times I \Rightarrow I = \sqrt{7}$

And $OP = OQ = \sqrt{OC^2 + I^2} = \sqrt{12}$

So, $a_1^2 + b_1^2 + a_2^2 + b_2^2 = OP^2 + OQ^2 = 12 + 12 = 24$

**3.** **(12)**

Let the mirror image of a circle $c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$ in line $y = x + 1$ be $c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$. If $r$ is the radius of circle $c_2$, then $\alpha + 6r^2$ is equal to ______

Now Image of centre $c_1 \equiv (1, 3)$ in $x - y + 1 = 0$ is given by

$\frac{x_1-1}{1} = \frac{y_1-3}{-1} = \frac{-2(1-3+1)}{1^2+1^2}$

$\Rightarrow x_1 = 2, y_1 = 2$

So, centre of circle $c_2 \equiv (2, 2)$

Now radius of circle $c_2$ will be $\sqrt{4 + 4 - \frac{38}{5}} = \sqrt{\frac{2}{5}} = r$

Now $\left(\text{radius of } c_1{}^2\right) = \left(\text{radius of } c_2\right)^2$ as they are image to one another,

$\Rightarrow \sqrt{1^2 + 3^2 - \alpha} = \sqrt{\frac{2}{5}}$

$\Rightarrow 10 - \alpha = \frac{2}{5} \Rightarrow \alpha = \frac{48}{5}$

Now putting the value of $\alpha$ & $r$ in $\alpha + 6r^2$ we get,

$\alpha + 6r^2 = \frac{48}{5} + \frac{12}{5} = 12$

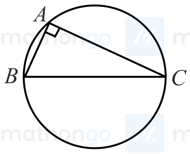
**4.** **(1)**

Given,

circle $x^2 - \sqrt{2}(x + y) + y^2 = 0$

With centre $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and radius 1.

Since $\angle BAC = \frac{\pi}{2}$, so $BC$ will act as diameter,

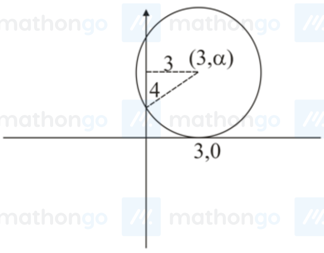



$BC$ = diameter $= 2$ (given)

So, $AC = \sqrt{BC^2 - AB^2} = \sqrt{2}$ and $AB = \sqrt{2}$

Now, $\triangle ABC = \frac{1}{2} AB \cdot AC = 1$

**5.** **(1)**




Since circle touches the x-axis at $(3,\ 0)$, hence centre can be taken as $(3, \alpha)$ and radius $= |\alpha|$

Since y-intercept is 8, by applying Pythagorean theorem

$3^2 + 4^2 = \alpha^2 \Rightarrow \alpha = 5, -5$

Hence two circles are possible

$(x - 3)^2 + (y - 5)^2 = 25$ and $(x - 3)^2 + (y + 5)^2 = 25$.

Clearly $(3,\ 10)$ satisfies the given equation of circle.

**6.** **(3)** $3 - x \leq y \leq \sqrt{9 - x^2}$

Points $(p, p + 1)$ lies on $y = x + 1$

So point of intersection between

$y = x + 1 \& y = 3 - x$ is $x = 1, y = 2$

and point of intersection between

$x + 1 = \sqrt{9 - x^2}$ is $x = \frac{-1+\sqrt{17}}{2}$

Hence p $\in \left(1, \frac{-1+\sqrt{17}}{2}\right)$

Hence $b^2 + b - a^2 = 3$

**7. (4)**

Given equation of circle $C: 4x^2+4y^2-12x+8y+k=0$

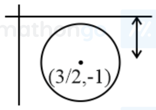
$\Rightarrow x^2+y^2-3x+2y+\left(\frac{k}{4}\right)=0$

Centre $\left(\frac{3}{2},-1\right); r=\sqrt{\frac{13-k}{2}} \Rightarrow k \leq 13 \quad \ldots(1)$

case (i) Point $\left(1, \frac{-1}{3}\right)$ lies on or inside circle $C$

$\Rightarrow S_1 \leq 0 \Rightarrow k \leq \frac{92}{9} \quad \cdots(2)$

case (ii) $C$ lies in $4^{\text{th}}$ quadrant




$r<1$

$\Rightarrow \frac{\sqrt{13-k}}{2}<1$

$\Rightarrow k<9 \quad \cdots(3)$

Hence from equation $(1)\cap(2)\cap(3) \Rightarrow k \in \left(9, \frac{92}{9}\right]$

**8. (3)**

Given circles $x^2+y^2-2x-2y+1=0 \Rightarrow (x-1)^2+(y-1)^2=1^2$ and

$x^2+y^2-18x-2y+78=0 \Rightarrow (x-9)^2+(y-1)^2=2^2$

Thus, centers of two circles are $(1,\ 1)$ & $(9,\ 1)$ and radii are 1 & 2 respectively.

Since, center of circles are on the opposite side of line $3x+4y-\lambda=0$.

$\Rightarrow (3+4-\lambda)(27+4-\lambda)<0$

$\Rightarrow (7-\lambda)(31-\lambda)<0$

$\Rightarrow \lambda \in (7,\ 31) \ \ldots\ldots(i)$

Since, circles should not intersect given line.

Hence, $\left|\frac{7-\lambda}{\sqrt{3^2+4^2}}\right| \geq 1$ & $\left|\frac{31-\lambda}{\sqrt{3^2+4^2}}\right| \geq 2$

$\Rightarrow |7-\lambda| \geq 5$ & $|31-\lambda| \geq 10$

$\Rightarrow \lambda \leq 2$ or $\lambda \geq 12 \ \ldots\ldots(ii)$

and $\lambda \leq 21$ or $\lambda \geq 41 \ \ldots\ldots(iii)$

Thus, $\lambda \in (i)\cap(ii)\cap(iii)$

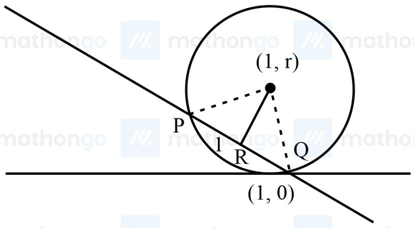
$\Rightarrow \lambda \in [12,\ 21]$

**9. (7)**

Since the circle is touching $x$-axis so, $k=r$ and $h=1$

Now by diagram, $OP=r, PR=1$

$OR=\left|\frac{r+1}{\sqrt{2}}\right|$




$r^2=1+\frac{(r+1)^2}{2}$

$2r^2=2+r^2+1+2r$

$r^2-2r-3=0$

$(r-3)(r+1)=0$

$r=3,-1$

$h+k+r=1+3+3$

$=7$

**10.** **(165)**

Given line $3x + 4y = \alpha$

Given circles $(x-1)^2 + (y-1)^2 = 1$ & $(x-9)^2 + (y-1)^2 = 4$

According to the given information centres of both the circles must lie on opposite side of line.

$L_{11} \cdot L_{22} < 0$

$\Rightarrow (3 + 4 - \alpha)(27 + 4 - \alpha) < 0$

$\alpha \in (7, 31)$.

Line is neither touching nor intersecting that means perpendicular distance from centre to line must more than or equal to radius.

$\Rightarrow \frac{|3+4-\alpha|}{5} \geq 1$ & $\frac{|27+4-\alpha|}{5} \geq 2$

$\Rightarrow \alpha \in (-\infty, 2] \cup [12, \infty)$ & $\alpha \in (-\infty, 21] \cup [41, \infty)$

$\Rightarrow \alpha \in [12, 21]$

Sum of integer values of $\alpha = 12 + 13 + --21 = 165$.

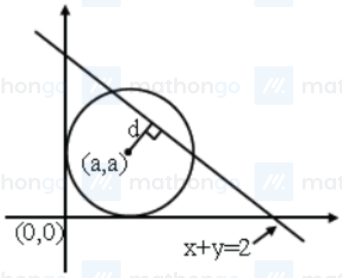
**11.** **(07.00)**

Sol Circle$(x-a)^2 + (y-a)^2 = a^2$

$x^2 + y^2 - 2ax - 2ay + a^2 = 0$

intercept $= 2$

$\Rightarrow 2\sqrt{a^2 - d^2} = 2$




Where $d =$ perpendicular distance of centre from line $x + y = 2$

$$\Rightarrow 2\sqrt{a^2 - \left(\frac{a+a-2}{\sqrt{2}}\right)^2} = 2$$

$$\Rightarrow a^2 - \frac{(2a-2)^2}{2} = 1 \Rightarrow 2a^2 - 4a^2 + 8a - 4 = 2$$

$\Rightarrow 2a^2 - 8a + 6 = 0 \Rightarrow a^2 - 4a + 3 = 0$

$\therefore r_1 + r_2 = 4$ and $r_1 r_2 = 3$

$\therefore r_1^2 + r_2 - r_1 r_2 = (r_1 + r_2)^2 - 3r_1 r_2$

$= 16 - 9 = 7$

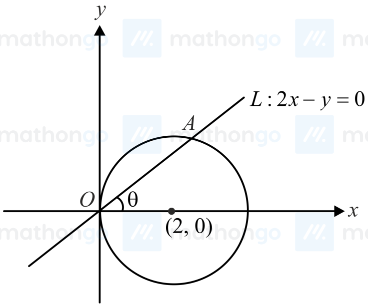
**12. (1)**

The centre of the circle $C_1$ will be $(2,0)$ and radius is 2

So, the equation of the circle $C_1$ is $x^2+y^2-4x=0$

Let $\theta$ be the angle made by the line $y=2x$ with positive $x$-axis.

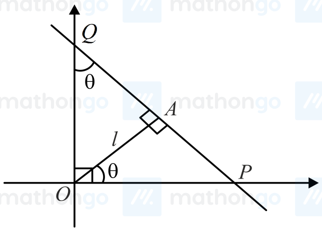
i.e. $\tan\theta=2$




Now $C_2$ is a circle with $OA$ as diameter.

So, tangent at $A$ on $C_2$ is perpendicular to $OA$

Let $OA=l$




$\therefore \frac{QA}{AP}=\frac{l\cot\theta}{l\tan\theta}$

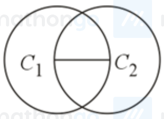
$\Rightarrow \frac{QA}{AP}=\frac{1}{\tan^2\theta}=\frac{1}{4}$

**13. (2)**

$r_1=3, c_1(5,5)$

$r_2=3, c_2(8,5)$

$C_1C_2=3, r_1=3, r_2=3$




**14. (1)**

$C_1\left(9,\frac{15}{2}\right) \quad r_1=\sqrt{81+\frac{225}{4}-131}=\frac{5}{2}$

$C_2(3,3) r_2=5$

$C_1C_2=\sqrt{6^2+\frac{81}{4}}=\frac{15}{2}$

$r_1+r_2=\frac{15}{2}$

$C_1C_2=r_1+r_2$

Number of common tangents $=3$

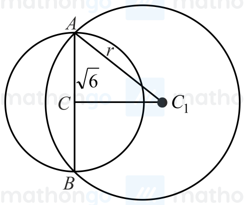
**15. (10)**

Given that, one of the diameter of the circle $\left(x-\sqrt{2}\right)^2+\left(y-3\sqrt{2}\right)^2=6$ is a chord of another circle $\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$.

Let, $C$ and $C_1$ be the centres of the circles $\left(x-\sqrt{2}\right)^2+\left(y-3\sqrt{2}\right)^2=6$ and $\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$ respectively.

So, $C\left(\sqrt{2},\ 3\sqrt{2}\right)$ & $C_1\left(2\sqrt{2},\ 2\sqrt{2}\right)$

The position of the two circles as shown in figure.




Now, $CC_1=\sqrt{\left(2\sqrt{2}-\sqrt{2}\right)^2+\left(2\sqrt{2}-3\sqrt{2}\right)^2}=\sqrt{2+2}=2$

It is clear from the above diagram, $\Delta ACC_1$ is a right angled triangle.$\Rightarrow r^2=(AC)^2+(CC_1)^2=\left(\sqrt{6}\right)^2+2^2\Rightarrow r^2=10$

**16. (7)**

Let the roots of $2x^2-rx+p=0$ are $x_1,\ x_2$ and roots of $y^2-sy-q=0$ are $y_1,\ y_2$
So, $x_1+x_2=\frac{r}{2},\ x_1x_2=\frac{p}{2},\ y_1+y_2=s,\ y_1y_2=-q$
Equation of the circle with $PQ$ as diameter will be
$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$
i.e. $2\left(x^2+y^2\right)-rx-2sy+p-2q=0$
On comparing with the given equation $r=11, s=7$
$p-2q=-22$
$\therefore\ 2r+s-2q+p=22+7-22=7$

**17. (3)** $\left(\frac{\lambda}{\sqrt{2}}\sin\theta,\frac{-\lambda}{\sqrt{2}}\cos\theta\right)$ A $\frac{\text{A}}{\text{P(h,k)}}{}^2B\left(\frac{\lambda}{\sqrt{2}}\cos\theta,\frac{\lambda}{\sqrt{2}}\sin\theta\right)$

$h=\dfrac{\frac{2\lambda}{\sqrt{2}\sin\theta}+3\times\frac{\lambda}{\sqrt{2}}\cos\theta}{5}$

$k=\dfrac{\frac{-2\lambda}{\sqrt{2}}2\cos\theta+\frac{3\lambda}{\sqrt{2}}\sin\theta}{5}$

$h^2+k^2=\dfrac{19\lambda^2}{5}$

$r=\dfrac{\sqrt{19}\lambda}{5}$

**18. (1)**


$\frac{12\cos\theta+2}{5}=\mathrm{h}\quad\Rightarrow 12\cos\theta=5\mathrm{~h}-2$
$\frac{12\sin\theta+4}{5}=k\quad\Rightarrow 12\sin\theta=5k4$
Sq \& add :
$144=(5h-2)2+(5k-4)2$
$\left(x-\frac{2}{5}\right)^2+\left(y-\frac{4}{5}\right)^2=\frac{144}{25}$

Centre $\equiv\left(\frac{2}{5},\frac{4}{5}\right)\equiv(\alpha,\beta)$

$\mathrm{AC}=\sqrt{\left(1-\frac{2}{5}\right)^2+\left(2-\frac{4}{5}\right)^2}$
$=\sqrt{\frac{9}{25}+\frac{36}{25}}=\frac{\sqrt{45}}{5}=\frac{3\sqrt{5}}{5}$

**19. (3)**

Since the circle with centre $(\alpha, \beta)$ touches the circle $x^2+(y-1)^2=1$ externally and also touches $x-$ axis, therefore the distance between the centres will be equal to the sum of the radius of both circles

i.e. $\sqrt{(\alpha-0)^2+(\beta-1)^2}=\beta+1$

$\Rightarrow \alpha^2=4\beta$

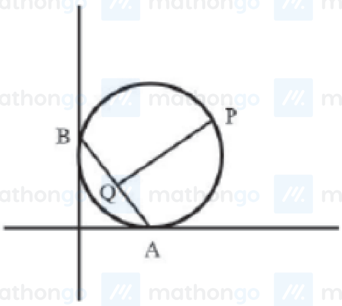
Hence the locus of $L$ will be $x^2=4y$

Now, the area bounded by $x^2=4y$ & $y=4$ will be

$A=2\int_0^4\left(4-\frac{x^2}{4}\right)dx$

$=2\left(4x-\frac{x^3}{12}\right)_0^4=\frac{64}{3}$

**20. (121.00)**




Let equation of circle is $(x-a)^2+(y-a)^2=a^2$ which is passing through $P(\alpha, \beta)$ then $(\alpha-a)^2+(\beta-a)^2=a^2$

$\alpha^2+\beta^2-2\alpha a-2\beta a+a^2=0$

Here equation of $AB$ is $x+y=a$

Let $Q(\alpha', \beta')$ be foot of perpendicular of $P$ on $AB$

$$\frac{\alpha'-\alpha}{1}=\frac{\beta'-\beta}{1}=\frac{-(\alpha+\beta-a)}{2}$$

$$PQ^2=(\alpha'-\alpha)^2+(\beta'-\beta)=\frac{1}{4}(\alpha+\beta-a)^2+\frac{1}{4}(\alpha+\beta-a)^2$$

$$121=\frac{1}{2}(\alpha+\beta-a)^2$$

$242=\alpha^2+\beta^2-2\alpha a-2\beta a+a^2+2\alpha\beta$

$242=2\alpha\beta$

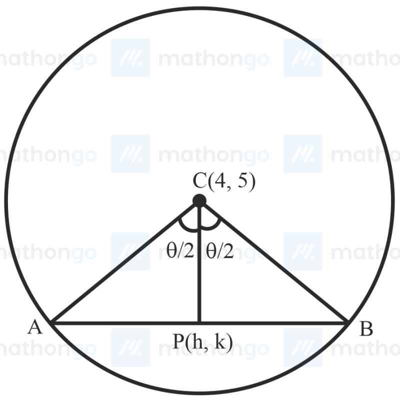
$\Rightarrow \alpha\beta=121$

**21. (4)**

Given,

$C_1: (x-4)^2+(y-5)^2=4$ which subtend an angle $\theta_i$ at the centre of the circle $C_i$, is a circle of radius $r_i$,

Now plotting the diagram we get,




Now given If $\theta_1=\frac{\pi}{3}, \theta_3=\frac{2\pi}{3}$ and ${r_1}^2={r_2}^2+{r_3}^2$,

Now taking triangle $CPB$ we get,

$\cos\frac{\theta}{2}=\frac{PC}{2}$, so $PC=2\cos\frac{\theta}{2}$

Now using distance formula we get,

$(h-4)^2+(k-5)^2=4\cos^2\frac{\theta}{2}$

So, locus will be,

$(x-4)^2+(y-5)^2=4\cos^2\frac{\theta}{2}$

Now radius $r_1=2\cos\frac{\pi}{6}=\sqrt{3}, r_2=2\cos\frac{\theta_2}{2}$ and $r_3=2\cos\frac{\pi}{3}=1$

Now using ${r_1}^2={r_2}^2+{r_3}^2$ we get,

$\left(\sqrt{3}\right)^2={r_2}^2+1^2$

$\Rightarrow {r_2}^2=2$

$\Rightarrow 4\cos^2\frac{\theta}{2}=2$

$\Rightarrow \left|\cos\frac{\theta}{2}\right|=\frac{1}{\sqrt{2}}$

$\Rightarrow \theta=\frac{\pi}{2}$

**22.** **(2)**

Equation of line joining $AB$ is

$y+1=5(x-2)$

i.e. $5x-y-11=0$

Also $AB=\sqrt{(3-2)^2+(4+1)^2}=\sqrt{26}$

$r$ is the radius of the circle $C$

For the circle $(x-5)^2+(y-1)^2=\frac{13}{2}$

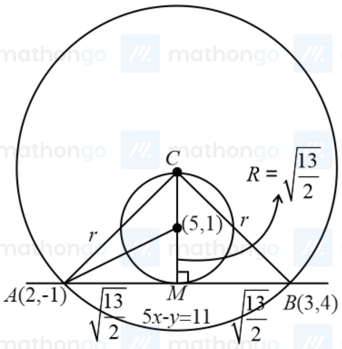
centre is $(5,1)$ and radius $R=\sqrt{\frac{13}{2}}$

Equation of perpendicular bisector of $AB$ will be $y-1=-\frac{1}{5}(x-5)\Rightarrow x+5y-10=0$

Solving this equation of line with $(x-5)^2+(y-1)^2=\frac{13}{2}$, we get

$M$ as $\left(\frac{5}{2},\frac{3}{2}\right)$

So $M$ lies on the line joining $AB$.




Now $r^2=CM^2+AM^2$

$=\left(2\times\sqrt{\frac{13}{2}}\right)^2+\left(\sqrt{\frac{13}{2}}\right)^2$

$r^2=\frac{65}{2}$

**23.** **(61)**

$r=\sqrt{\frac{p^2}{4}+\frac{(1-p)^2}{4}-5}$

$=\frac{\sqrt{2p^2-2p-19}}{2}$

Since $r\in(0,5]$

So, $0<\frac{\sqrt{2p^2-2p-19}}{2}\leq 5$

$0<2p^2-2p-19\leq 100$

$\Rightarrow 2p^2-2p-19>0\quad\ldots(i)$

For, $2p^2-2p-19>0$

$p=\frac{2\pm\sqrt{4-4\times2\times(-19)}}{4}$

$=\frac{2\pm\sqrt{156}}{4}$

$\Rightarrow\left(p-\frac{1+\sqrt{39}}{2}\right)\left(p-\frac{1-\sqrt{39}}{2}\right)>0\ldots(ii)$

$p^2=10\pm\frac{\sqrt{39}}{2}$

Also, $2p^2-2p-119\leq 0\quad\ldots(iii)$

For, $2p^2-2p-119=0$

$p=\frac{2\pm\sqrt{4-4\times2\times(-119)}}{4}$

$=\frac{1\pm\sqrt{239}}{2}$

$\Rightarrow\left(p-\frac{1+\sqrt{239}}{2}\right)\left(p-\frac{1-\sqrt{239}}{2}\right)\leq 0\ldots(iv)$

$p^2=60\pm\frac{\sqrt{239}}{2}$

So, from $(ii)$ & $(iv)$

$\Rightarrow p\in\left[\left(\frac{1-\sqrt{239}}{2},\frac{1-\sqrt{39}}{2}\right)\cup\left(\frac{1+\sqrt{39}}{2},\frac{1+\sqrt{239}}{2}\right)\right]$

$\Rightarrow p^2\in\left[\left(10-\frac{\sqrt{39}}{2},60-\frac{\sqrt{239}}{2}\right)\cup\left(10+\frac{\sqrt{39}}{2},60+\frac{\sqrt{239}}{2}\right)\right]$

So, number of integral values of $p^2$ is 61.

**24. (25)**

The circle $x^2+y^2+6x+8y+16=0$ has centre $(-3,-4)$ and radius $\sqrt{9+16-16}=3$ units.

The circle $x^2+y^2+2\left(3-\sqrt{3}\right)x+2\left(4-\sqrt{6}\right)y=k+6\sqrt{3}+8\sqrt{6}, k>0$ has centre $\left(\sqrt{3}-3, \sqrt{6}-4\right)$ and radius

$\sqrt{\left(\sqrt{3}-3\right)^2+\left(\sqrt{6}-4\right)^2+k+6\sqrt{3}+8\sqrt{6}}=\sqrt{k+34}$

Given that these two circles touch internally, so

distance between their centres=|difference of radii|

$\sqrt{3+6}=\left|\sqrt{k+34}-3\right|$

$\Rightarrow \sqrt{k+34}-3=\pm 3$

Here, $k=2$ is only possible value $(\because \ k>0)$

Now the equation of common tangent to both the circles is given by $2\sqrt{3}x+2\sqrt{6}y+16+k+6\sqrt{3}+8\sqrt{6}=0$

$\because \ k=2$ then equation becomes

$x+\sqrt{2}y+3\sqrt{3}+3+4\sqrt{2}=0 \quad \cdots$(i)

$\because \ (\alpha,\beta)$ are foot of perpendicular from $(-3,-4)$ to this common tangent, then

$\frac{\alpha+3}{1}=\frac{\beta+4}{\sqrt{2}}=\frac{-\left(-3-4\sqrt{2}+3+4\sqrt{2}+3\sqrt{3}\right)}{1+2}$

$\therefore \ \alpha+3=-\sqrt{3}$ & $\frac{\beta+4}{\sqrt{2}}=-\sqrt{3}$

$\Rightarrow \left(\alpha+\sqrt{3}\right)^2=9$ and $\left(\beta+\sqrt{6}\right)^2=16$

Hence, $\left(\alpha+\sqrt{3}\right)^2+\left(\beta+\sqrt{6}\right)^2=25$

**25. (1)**

Lines $y=x+2, 4y=3x+6$ and $3y=4x+1$ are tangents to the circle $(x-h)^2+(y-k)^2=r^2$.

Centre of the circle is $(h,k)$.

Equation of bisector of lines $4y=3x+6,\ 3y=4x+1$ is:

$\frac{4x-3y+1}{5}=\pm\left(\frac{3x-4y+6}{5}\right)$

$\Rightarrow 4x-3y+1=\pm(3x-4y+6)$

Taking positive sign, we get

$4x-3y+1=3x-4y+6$

$\Rightarrow x+y=5$

Since, centre $(h,k)$ lies on the bisector, therefore

$h+k=5$

**26. (816)**

Equations of normal are

$y+2x=\sqrt{11}+7\sqrt{7} \quad \ldots(i)$

$2y+x=2\sqrt{11}+6\sqrt{7} \quad \ldots(ii)$

Now the center of the circle is point of intersection of the normals i.e. solving $(i)$ & $(ii)$, we get the point of intersection as

$\left(\frac{8\sqrt{7}}{3}, \sqrt{11}+\frac{5\sqrt{7}}{3}\right)\equiv(h,k)$

The equation of tangent is $\sqrt{11}y-3x=\frac{5\sqrt{77}}{3}+11$

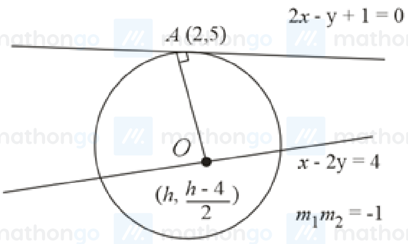
The radius will be perpendicular distance of tangent from center

i.e. $r=\frac{\left|\sqrt{11}\frac{8\sqrt{7}}{3}-3\left(\sqrt{11}+\frac{5\sqrt{7}}{3}\right)-\frac{5\sqrt{77}}{3}-11\right|}{\sqrt{11+9}}=4\sqrt{\frac{7}{5}}$

Hence $(5h-8k)^2+5r^2=816$

**27. (1)**

Given




Slope of $OA = \frac{5-\frac{(h-4)}{2}}{2-h} = m_1$Slope of the given tangent equation$= m_2 = 2$

$\Rightarrow \left(\frac{5-\frac{(h-4)}{2}}{2-h}\right)(2) = -1$

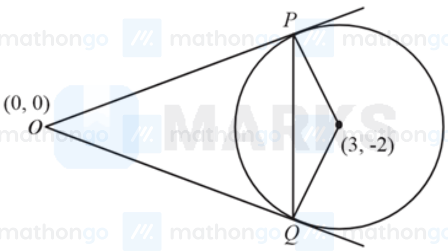
$\Rightarrow h = 8$

Center$=(8,\ 2)$

Radius$= \sqrt{(8-2)^2 + (2-5)^2} = 3\sqrt{5}$

**28. (3)**

The given information can be represented in the form of the diagram below.




Since angle in a semicircle is a right angle.

Hence, the other circle will be passing through the centre of the first circle.

The two ends of diameter of the circle is $O(0,0)$ and $C(3,-2)$.

Hence, the required equation of circle is $(x-0)(x-3)+(y-0)(y-(-2)) = 0$

$\Rightarrow x^2 + y^2 - 3x + 2y = 0$

Put $\left(\alpha, \frac{1}{2}\right)$ in the above equation$\Rightarrow \alpha^2 + \frac{1}{4} - 3\alpha + 1 = 0$

$\Rightarrow \alpha^2 - 3\alpha + \frac{5}{4} = 0$

$\Rightarrow 4\alpha^2 - 12\alpha + 5 = 0$

$\Rightarrow 4\alpha^2 - 10\alpha - 2\alpha + 5 = 0$

$\Rightarrow 2\alpha(2\alpha - 5) - 1(2\alpha - 5) = 0 \Rightarrow \alpha = \frac{1}{2}, \frac{5}{2}$

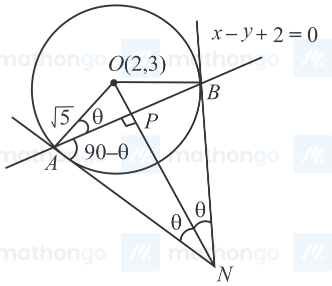
Therefore this is the correct option.

**29. (3)**

Let $O(3,2)$ be the centre of the circle $C_2$.

The tangent to the circle $C_1: x^2+y^2-2=0$ at $M(-1,1)$ is $x-y+2=0$

Now the perpendicular distance from $O$ to the tangent is $OP=\left|\frac{3-2+2}{\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$




Also $AP=\sqrt{OA^2-OP^2}=\sqrt{5-\frac{9}{2}}$

$=\frac{1}{\sqrt{2}}$

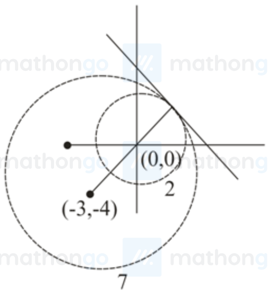
Now $\tan\theta=\frac{OP}{AP}=3$

$\therefore\ \sin\theta=\frac{3}{\sqrt{10}}=\frac{AP}{AN}$ & $\cos\theta=\frac{1}{\sqrt{10}}$

$\Rightarrow AN=\frac{\sqrt{10}}{3}AP=\frac{\sqrt{10}}{3}\times\frac{1}{\sqrt{2}}=\frac{\sqrt{5}}{3}=BN$

Area of $\Delta ANB=\frac{1}{2}\cdot\left(AN^2\right)\sin 2\theta=\frac{1}{2}\times\frac{10}{9}\times\frac{3}{5}=\frac{1}{6}$

**30. (3)**




The centre and radius of the circle $x^2+y^2+2gx+2fy+c=0$ are respectively $(-g,\ -f)$ and $\sqrt{g^2+f^2-c}$.

Let, for the circle $S_1: x^2+y^2=4$ centre $C_1:(0,\ 0),\ r_1=2$ units.

And, for the circle $S_2: x^2+y^2+6x+8y-24=0$ centre

$C_2(-3,\ -4),\ r_2=\sqrt{3^2+4^2+24}=7$ units.

The distance between the points $(x_1,\ y_1)$ and $(x_2,\ y_2)$ is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Hence, the distance between the centres of the circles is $C_1C_2=\sqrt{(-3-0)^2+(-4-0)^2}=5$ units.

Also, $|r_1-r_2|=5$ units.

Since, $C_1C_2=|r_1-r_2|$, hence both the circles touches each other internally.

Therefore, the common tangent is same as the radical axis of the two circles i.e. $S_1-S_2=0$

Thus, the common tangent is $x^2+y^2-4-\left(x^2+y^2+6x+8y-24\right)=0$

$\Rightarrow\quad -6x-8y+20=0$

$\Rightarrow\quad 3x+4y-10=0.$

Clearly, the point $(6,-2)$ lies on the common tangent.