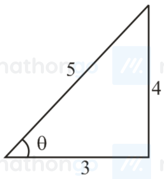
ANSWER KEYS

1. (2.30) **2.** (4) **3.** (2) **4.** (2) **5.** (1) **6.** (4) **7.** (4.00) **8.** (1)
9. (3) **10.** (1)

1. **(2.30)**

$3\tan\theta + 4 = 0 \Rightarrow \tan\theta = \frac{-4}{3}$



$\cot\theta = -3/4$

$\cos\theta = -3/5$

$\sin\theta = 4/5$

$= 2\cot\theta - 5\cos\theta + \sin\theta$

$= 2\left(-3/4\right) - 5\left(-3/5\right) + 4/5 = \frac{23}{10}$

2. **(4)**

$\cos(\alpha+\beta) = \frac{3}{5} \Rightarrow \tan(\alpha+\beta) = \frac{4}{3}$

$\sin\left(\alpha-\beta\right) = \frac{5}{13} \Rightarrow \tan\left(\alpha-\beta\right) = \frac{5}{12}$

Now,

$\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta))$

$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$

$= \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}} = \frac{63}{16}$

3. **(2)**

$\lambda = \sin^2(5^\circ)+\sin^2(10^\circ)+ \ldots\ldots\ldots + \sin^2(85^\circ) = \left(\sin^2(5^\circ)+\sin^2 85^\circ\right)+\left(\sin^2(10^\circ)+\sin^2(80^\circ)\right)+ \ldots\ldots\ldots + \left(\sin^2(40^\circ)+\sin^2(50^\circ)\right)+\sin^2(45^\circ)$

$= \left(\sin^2(5^\circ)+\sin^2(90^\circ-5^\circ)\right)+\left(\sin^2(10^\circ)+\sin^2(90^\circ-10^\circ)\right)+ \ldots\ldots\ldots + \left(\sin^2(40^\circ)+\sin^2(90^\circ-40^\circ)\right)+\sin^2(45^\circ)$

$= \left(\sin^2(5^\circ)+\cos^2 5^\circ\right)+\left(\sin^2(10^\circ)+\cos^2(10^\circ)\right)+ \ldots\ldots\ldots + \left(\sin^2(40^\circ)+\cos^2(40^\circ)\right)+\sin^2(45^\circ)$

$= 1+1+1+1+1+1+1+1+\frac{1}{2}$

$\lambda = 8+\frac{1}{2} = \frac{17}{2}$

$2\lambda - 8 = 9$

Number of the positive divisor of 9 are $1,\ 3,\ 9$

total 3 divisions

4. **(2)** $3(\cos\theta-\sin\theta)^4 + 6(\cos\theta+\sin\theta)^2 + 4\sin^6\theta$

$= 3\left((\cos\theta-\sin\theta)^2\right)^2 + 6(\cos\theta+\sin\theta)^2 + 4\sin^6\theta$

$= 3\left(\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta\right)^2 + 6\left(\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta\right)+4\sin^6\theta$

$= 3(1-2\sin\theta\cos\theta)^2 + 6(1+2\sin\theta\cos\theta)+4\sin^6\theta$

$= 3\left(1-4\sin\theta\cos\theta+4\sin^2\theta\cos^2\theta\right)+6(1+2\sin\theta\cos\theta)+4\sin^6\theta$

$= 9+12\sin^2\theta\cos^2\theta+4\sin^6\theta$

$= 9+12\left(1-\cos^2\theta\right)\cos^2\theta+4\left(1-\cos^2\theta\right)^3$

$= 9+12\cos^2\theta-12\cos^4\theta+4\left(1-3\cos^2\theta+3\cos^4\theta-\cos^6\theta\right)$

$= 13-4\cos^6\theta$

5. **(1)**

Let the given expression is

$E=\frac{\sin 22^{\circ} \cos 8^{\circ}+\cos 158^{\circ} \cos 98^{\circ}}{\sin 23^{\circ} \cos 7^{\circ}+\cos 157^{\circ} \cos 97^{\circ}}$

As we know $\cos(180^{\circ}-\theta)=-\cos\theta$ and

$\cos(90^{\circ}+\theta)=-\sin\theta$, So

$E=\frac{\sin 22^{\circ} \cos 8^{\circ}+\cos\left(180^{\circ}-22^{\circ}\right) \cos(90^{\circ}+8^{\circ})}{\sin 23^{\circ} \cos 7^{\circ}+\cos\left(180^{\circ}-23^{\circ}\right) \cos(90^{\circ}+7^{\circ})}$

$=\frac{\sin 22^{\circ} \cos 8^{\circ}+\cos 22^{\circ} \sin 8^{\circ}}{\sin 23^{\circ} \cos 7^{\circ}+\cos 23^{\circ} \sin 7^{\circ}}$

Now, using

$\sin(A)(+B)=\sin A\cos B+\cos A\sin B$, above expression becomes

$E=\frac{\sin\left(22^{\circ}+8^{\circ}\right)}{\sin\left(23^{\circ}+7^{\circ}\right)}$

$E=\frac{\sin 30^{\circ}}{\sin 30^{\circ}}=1$

6. **(4)** $F(1)=\left(1+\sin\frac{\pi}{2}\right)(1+\sin 0)\left(1+\sin\frac{3\pi}{2}\right)(1+\sin\pi)$

$=2\times 1\times 0\times 1=0$

$F(2)=\left(1+\sin\frac{\pi}{4}\right)\left(1+\sin\frac{\pi}{4}\right)\left(1+\sin\frac{5\pi}{4}\right)\left(1+\sin\frac{5\pi}{4}\right)$

$=\left(1+\frac{1}{\sqrt{2}}\right)^{2}\left(1-\frac{1}{\sqrt{2}}\right)^{2}=\left(1-\frac{1}{2}\right)^{2}=\frac{1}{4}$

$F(3)=\left(1+\sin\frac{\pi}{6}\right)\left(1+\sin\frac{\pi}{3}\right)\left(1+\sin\frac{7\pi}{6}\right)\left(1+\sin\frac{4\pi}{3}\right)$

$=\left(1+\frac{1}{2}\right)\left(1+\frac{\sqrt{3}}{2}\right)\left(1-\frac{1}{2}\right)\left(1-\frac{\sqrt{3}}{2}\right)$

$=\left(1-\frac{1}{4}\right)\left(1-\frac{3}{4}\right)=\frac{3}{4}\times\frac{1}{4}=\frac{3}{16}$

$\Rightarrow F(1)+F(2)+F(3)=0+\frac{1}{4}+\frac{3}{16}=\frac{7}{16}$

7. **(4.00)**

$2\cos^{2}\theta+\cos\theta+1=2\left(\cos^{2}\theta+\frac{\cos\theta}{2}+\frac{1}{2}\right)$

$2\left\{\left(\cos\theta+\frac{1}{4}\right)^{2}+\frac{7}{16}\right\}$

Given expression is maximum when $\cos\theta=1$ and minimum when $\cos\theta=-\frac{1}{4}$

$\Rightarrow M=2\left(\left(\frac{5}{4}\right)^{2}+\frac{7}{16}\right)=2\left(\frac{32}{16}\right)=4$

and $m=2\left(\frac{7}{16}\right)=\frac{7}{8}$

Hence, $\left[\frac{M}{m}\right]=\left[\frac{32}{7}\right]=4$

8. **(1)**

Given $f(\theta)=3\cos\theta+5\sin\left(\theta-\frac{\pi}{6}\right)$

$\Rightarrow f(\theta)=3\cos\theta+5\left(\sin\theta\cdot\frac{\sqrt{3}}{2}-\cos\theta\cdot\frac{1}{2}\right)$

$\Rightarrow f(\theta)=\frac{5\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta$

Now using the concept $a\cos x+b\sin x+c\in\left[c-\sqrt{a^{2}+b^{2}},\ c+\sqrt{a^{2}+b^{2}}\right]$, we can write

Maximum value of $f(\theta)$ is $\sqrt{\left(\frac{5\sqrt{3}}{2}\right)^{2}+\left(\frac{1}{2}\right)^{2}}=\sqrt{\frac{75}{4}+\frac{1}{4}}=\sqrt{19}$

9. **(3)**

Given expression is $3\sin^{2}x+3\sin x\ \cos x+7\cos^{2}x$

Now using formula $\cos 2x=\cos^{2}x-\sin^{2}x\Rightarrow 2\sin^{2}x=1-\cos 2x$ & $2\cos^{2}x=1+\cos 2x$, we get$\Rightarrow\frac{3(1-\cos 2x)}{2}+\frac{3\sin 2x}{2}+\frac{7(1+\cos 2x)}{2}$

$\Rightarrow\frac{3-3\cos 2x+3\sin 2x+7+7\cos 2x}{2}$

$\Rightarrow\frac{3\sin 2x+4\cos 2x}{2}+\frac{10}{2}$

Now using $-\sqrt{a^{2}+b^{2}}\leq a\sin x+b\cos x\leq\sqrt{a^{2}+b^{2}}$, we get

$-\sqrt{3^{2}+4^{2}}\leq 3\sin 2x+4\cos 2x\leq\sqrt{3^{2}+4^{2}}$

$\Rightarrow -5\leq 3\sin 2x+4\cos 2x\leq 5$

$\Rightarrow -\frac{5}{2}+5\leq\frac{3\sin 2x+4\cos 2x}{2}+5\leq\frac{5}{2}+5$

$\therefore\frac{3\sin 2x+4\cos 2x}{2}+5\in\left[\frac{5}{2},\frac{15}{2}\right]$

Hence, the number of integers are 5 that is $\{3,\ 4,\ 5,\ 6,\ 7\}$.

10. (1)

$\alpha = \max\left\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\right\}$

$= \max\{2^{6\sin 3x} \cdot 2^{8\cos 3x}\}$

$= \max\{2^{6\sin 3x+8\cos 3x}\}$

$= \max\{2^{6\sin 3x+8\cos 3x}\}$

and $\beta = \min\left\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\right\} = \min\{2^{6\sin 3x+8\cos 3x}\}$

Now range of $6\sin 3x + 8\cos 3x$

$= \left[-\sqrt{6^2+8^2}, +\sqrt{6^2+8^2}\right] = \left[-10, 10\right]$

$\alpha = 2^{10} \& \beta = 2^{-10}$

So, $\alpha^{1/5} = 2^2 = 4$

$\Rightarrow \beta^{1/5} = 2^{-2} = \frac{1}{4}$

The quadratic $8x^2 + bx + c = 0$,

sum of roots $= \frac{-b}{8}$ and product of roots $= \frac{c}{8}$

$\therefore c - b = 8 \times$[(product of roots)+(sum of roots)]

$= 8 \times \left[4 \times \frac{1}{4} + 4 + \frac{1}{4}\right] = 8 \times \left[\frac{21}{4}\right] = 42$