## ANSWER KEYS

**1.** (1) **2.** (1) **3.** (1) **4.** (1) **5.** (2) **6.** (1) **7.** (2) **8.** (1)
**9.** (2) **10.** (2)

**1. (1)**

Let $I=\int \frac{1}{\left[(x+2)^2+1\right]^2} dx$

Put $x+2=\tan \theta ; dx=\sec ^2 \theta$

$I=\int \frac{d \theta}{\sec ^2 \theta}=\int\left(\cos ^2 \theta\right) d \theta=\int \frac{1+\cos 2 \theta}{2} d \theta=\frac{\theta}{2}+\frac{\sin 2 \theta}{4}+c$

$=\frac{\theta}{2}+\frac{2 \tan \theta}{4\left(1+\tan ^2 \theta\right)}+c$

$=\frac{\tan ^{-1}(x+2)}{2}+\frac{(x+2)}{2\left(x^2+4 x+5\right)}+c$

**2. (1)**

$\mathrm{I}=\int \frac{\mathrm{dx}}{\sec \mathrm{x}+\operatorname{cosec} \mathrm{x}}$

$=\int \frac{1}{2} \times \frac{2 \sin \mathrm{x} \cos \mathrm{x}}{\sin \mathrm{x}+\cos \mathrm{x}} \mathrm{dx}$

$=\frac{1}{2} \int \frac{(\sin \mathrm{x}+\cos \mathrm{x})^2-1}{\sin \mathrm{x}+\cos \mathrm{x}} \mathrm{dx}$

$=\frac{1}{2} \int(\sin \mathrm{x}+\cos \mathrm{x}) \mathrm{dx}-\frac{1}{2} \int \frac{1}{\sin \mathrm{x}+\cos \mathrm{x}} \mathrm{dx}$

$=\frac{1}{2}(-\cos \mathrm{x}+\sin \mathrm{x})-\frac{1}{2} \int \frac{1}{\sqrt{2}\left[\frac{1}{\sqrt{2}} \sin \mathrm{x}+\frac{1}{\sqrt{2}} \cos \mathrm{x}\right]} \mathrm{dx}$

$=\frac{1}{2}(-\cos \mathrm{x}+\sin \mathrm{x})-\frac{1}{2 \sqrt{2}} \int \frac{1}{\sin \left(\mathrm{x}+\frac{\pi}{4}\right)} \mathrm{dx}$

$=\frac{1}{2}(-\cos \mathrm{x}+\sin \mathrm{x})-\frac{1}{2 \sqrt{2}} \int \operatorname{cosec}\left(\mathrm{x}+\frac{\pi}{4}\right) \mathrm{dx}$

$=\frac{1}{2}(-\cos \mathrm{x}+\sin \mathrm{x})-\frac{1}{2 \sqrt{2}} \ln \left|\left(\operatorname{cosec}\left(\mathrm{x}+\frac{\pi}{4}\right)\text{-}\cot \left(\mathrm{x}+\frac{\pi}{4}\right)\right)\right|$

We know that, $cosec\theta-cot\theta=\frac{1}{\sin \theta}-\frac{\cos \theta}{\sin \theta}$

$=\frac{1-\cos \theta}{\sin \theta}$

$=\frac{2 \sin ^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$

$=\tan \frac{\theta}{2}$

$\therefore \mathrm{I}=\frac{1}{2}(-\cos x+\sin x)-\frac{1}{2 \sqrt{2}}\left(\log \left|\tan \left(\frac{x}{2}+\frac{\pi}{8}\right)\right|\right)+c$

**3. (1)**

$I=\int \frac{d x}{\left[x+\sqrt{x(1+x)}\right]^2}$

$=\int \frac{1}{x^2\left(1+\sqrt{1+\frac{1}{x}}\right)^2} d x$

put $1+\frac{1}{x}=t^2$

$-\frac{1}{x^2} d x=2 t d t$

or $I=\int \frac{-2 t\ d t}{(a+t)^2}=-\int \frac{2 t}{t^2+2 t+1} d t$

$=-\left[\int \frac{2 t\ +2}{t^2+2 t+1} d t-\int \frac{2}{t^2+2 t+1} d t\right]$

$=-\left[\ln (t+1)^2-2 \int \frac{1}{(t+1)^2} d t\right]$

$=-\left[2 \ln (t+1)+\frac{2}{t+1}-c\right]$

or $I=-2 \ln \left(1+\sqrt{1+\frac{1}{x}}\right)-\frac{2}{1+\sqrt{1+\frac{1}{x}}}+c$

**4.** **(1)** $A(x)\left(\sqrt{1-x^2}\right)^m + C = \int \frac{\sqrt{1-x^2}}{x^4}dx$

$= \int \frac{\sqrt{\frac{1}{x^2}-1}}{x^3}dx$

Let $\frac{1}{x^2} - 1 = u^2$

$\Rightarrow -\frac{2}{x^3} = \frac{2udu}{dx}$

$\frac{dx}{x^3} = -udu$

$A(x)\left(\sqrt{1-x^2}\right)^m + C = \int\left(-u^2\right)du = -\frac{u^3}{3} + C$

$= -\frac{1}{3}\left(\frac{1}{x^2} - 1\right)^{\frac{3}{2}} + C$

$= -\frac{1}{3}\cdot\frac{1}{x^3}\cdot\left(1-x^2\right)^{\frac{3}{2}} + C$

$= \frac{-1}{3x^3}\left(\sqrt{1-x^2}\right)^3 + C$

Compare both sides,

$\Rightarrow A(x) = -\frac{1}{3x^3}$ and $m = 3$

$\Rightarrow (A(x))^3 = \frac{-1}{27x^9}$

**5.** **(2)**

Let $I = \int \frac{1}{a^2\sin^2 x + b^2\cos^2 x}dx$

$\Rightarrow I = \int\left[\frac{\sec^2 x}{a^2\tan^2 x + b^2}\right]dx$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\Rightarrow I = \int\left[\frac{1}{a^2t^2+b^2}\right]dt$

$\Rightarrow I = \frac{1}{a^2}\int\left[\frac{1}{t^2+\left(\frac{b}{a}\right)^2}\right]dt$

$\Rightarrow I = \frac{1}{ab}\tan^{-1}\left(\frac{at}{b}\right) + C$

$\Rightarrow I = \frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right) + C$

Therefore, we get

$\frac{1}{ab} = \frac{1}{12} \Rightarrow ab = 12$ and $\frac{a}{b} = 3 \Rightarrow a = 3b$

Hence, $a = \pm 6,\ b \pm 2$

Now, we know that

$-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$

$\Rightarrow -\sqrt{40} \le a\sin x + b\cos x \le \sqrt{40}$

Hence, maximum value of $a\sin x + b\cos x$ is $\sqrt{40}$.

**6.** **(1)**

Let $I = \int \frac{1}{7+5\cos x}dx$

$\Rightarrow I = \int \frac{1}{7+5\left[\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right]}dx$

$\Rightarrow I = \int\left[\frac{\left\{1+\tan^2\left(\frac{x}{2}\right)\right\}}{7\left\{1+\tan^2\left(\frac{x}{2}\right)\right\}+5\left\{1-\tan^2\left(\frac{x}{2}\right)\right\}}\right]dx$

$\Rightarrow I = \int\left[\frac{\sec^2\left(\frac{x}{2}\right)}{12+2\tan^2\left(\frac{x}{2}\right)}\right]dx$

$\Rightarrow I = \frac{1}{2}\int\left[\frac{\sec^2\left(\frac{x}{2}\right)}{\left(\sqrt{6}\right)^2+\tan^2\left(\frac{x}{2}\right)}\right]dx$

Put $\tan\left(\frac{x}{2}\right) = t \Rightarrow \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx = dt$

$\Rightarrow I = \int\left(\frac{1}{\left(\sqrt{6}\right)^2+t^2}\right)dt$

$\Rightarrow I = \frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{t}{\sqrt{6}}\right) + C$

$\Rightarrow I = \frac{1}{\sqrt{6}}\tan^{-1}\left[\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{6}}\right] + C$

$\left[\because \int\frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

**7. (2)**

$\int \frac{1-\cos x-x\sin x}{(x-\sin x)^2+\cos^2 x}dx=\int \frac{1}{1+\left(\frac{\cos x}{x-\sin x}\right)^2}\times\frac{1-\cos x-x\sin x}{(\mathrm{x}-\sin x)^2}dx$

Let $\frac{\mathrm{cosx}}{x-\sin x}=\mathrm{t}\Rightarrow\int \frac{1}{1+\mathrm{t}^2}dt=\tan^{-1}t+c=\tan^{-1}\left(\frac{\cos x}{x-\sin x}\right)+c$

**8. (1)**

$\Rightarrow\int\left(\frac{x-3}{x+4}\right)^{\frac{-6}{7}}\frac{1}{(x+4)^2}dx\ \ldots(\mathrm{i})$

Let $\frac{x-3}{x+4}=t^7$,

$\frac{7}{(x+4)^2}dx=7t^6dt\ldots$ (ii)

In (i) from (ii) $\int t^{-6}t^6dt=t+C$

**9. (2)**

We have,

$\int\frac{2\cos x-\sin x+\lambda}{\cos x+\sin x-2}dx=A\log_e(|\cos x+\sin x-2|)+Bx+C$

Differentiating R. H. S. w.r.t. $x$, we get

$\frac{d}{dx}[A\log_e(|\cos x+\sin x-2|)+Bx+C]=A\frac{\cos x-\sin x}{\cos x+\sin x-2}+B$

$=\frac{A\cos x-A\sin x+B\cos x+B\sin x-2B}{\cos x+\sin x-2}$

$\therefore\frac{2\cos x-\sin x+\lambda}{\cos x+\sin x-2}=\frac{A\cos x-A\sin x+B\cos x+B\sin x-2B}{\cos x+\sin x-2}$

So, $A+B=2,\ B-A=-1,\ \lambda=-2B$

$\Rightarrow A=\frac{3}{2},\ B=\frac{1}{2},\ \lambda=-1$

**10. (2)** Let, $\mathrm{f}(\mathrm{x})=a\mathrm{x}^3+b\mathrm{x}^2+c\mathrm{x}+d$. Then,

Given, $\mathrm{f}(0)=-1$ and $\mathrm{f}(1)=0$

$\Rightarrow\ \mathrm{d}=-1$ and $a+b+c+d=0$

$\Rightarrow\ \mathrm{d}=-1$ and $a+b+c=1$ ... (1)

It is given that $\mathrm{x}=0$, is a stationary point of $\mathrm{f}(\mathrm{x})$ but, it is not a point of extrema. Therefore,

$\mathrm{f}'(0)=0,\ \mathrm{f}''(0)=0\ \&\ \mathrm{f}'''(0)\neq 0$

Now, $\mathrm{f}(\mathrm{x})=ax^3+bx^2+\mathrm{cx}+\mathrm{d}$

$\Rightarrow\ \mathrm{f}'(\mathrm{x})=3ax^2+2b\mathrm{x}+\mathrm{c},\ \mathrm{f}''(\mathrm{x})=6ax+b$ and $\mathrm{f}'''(\mathrm{x})=6a$

$\therefore\ \mathrm{f}'(0)=0,\ \mathrm{f}''(0)=0\ \&\ \mathrm{f}'''(0)\neq 0$

$\Rightarrow c=0, b=0 \& a\neq 0 \ldots(2)$

From equations (1) and (2), we get

$\mathrm{a}=1,\ \mathrm{b}=\mathrm{c}=0$ and $\mathrm{d}=-1$.

$\therefore\ \mathrm{f}(\mathrm{x})=\mathrm{x}^3-\ 1$

Hence, $\int\frac{\mathrm{f}(\mathrm{x})}{\mathrm{x}^3-1}\mathrm{dx}=\int 1\mathrm{dx}=\mathrm{x}+\mathrm{c}$, where $c$ is the arbitrary constant.