

Assignment 3

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

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Question[12.13.5.9]: On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Solution: Let X be the number of correct answers that the candidate gets just by guessing.

$$X = \text{bin}(n, p) \quad (1)$$

Where,

n	5(total no of questions)
p	$\frac{1}{3}$ (probability of getting correct answer by guessing)

The probability mass function for candidate getting k correct answers just by guessing:

$$p_X(k) = \Pr(X = k) \quad (2)$$

$$= {}^nC_k \times p^k \times (1 - p)^{(n-k)} \quad (3)$$

$$= {}^5C_k \times \left(\frac{1}{3}\right)^k \times \left(1 - \frac{1}{3}\right)^{(5-k)} \quad (4)$$

$$= {}^5C_k \times \left(\frac{1}{3}\right)^k \times \left(\frac{2}{3}\right)^{(5-k)} \quad (5)$$

Then, the probability that the candidate gets four or more correct answers just by guessing is:

$$\Pr(X \geq 4) = \Pr(3 < X \leq 5) \quad (6)$$

$$= F_X(5) - F_X(3) \quad (7)$$

$$= p_X(4) + p_X(5) \quad (8)$$

$$= {}^5C_4 \times \left(\frac{1}{3}\right)^4 \times \left(\frac{2}{3}\right)^{(5-4)} + {}^5C_5 \times \left(\frac{1}{3}\right)^5 \times \left(\frac{2}{3}\right)^{(5-5)} \quad (9)$$

$$= 5 \times \left(\frac{1}{3}\right)^4 \times \left(\frac{2}{3}\right)^1 + \left(\frac{1}{3}\right)^5 \times \left(\frac{2}{3}\right)^0 \quad (10)$$

$$= \frac{11}{243} \quad (11)$$

\therefore The probability that a candidate would get four or more correct answers just by guessing is $\frac{11}{243}$.