

Assignment 3

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

Kudupudi D.V.Sai Aditya
AI22BTECH11013

Question[12.13.5.9]: On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Solution: Let X be the number of correct answers that the candidate gets just by guessing.

$$X = \text{bin}(n, p) \quad (1)$$

Where,

Parameter	Value	Description
X	$\text{bin}(n, p)$	no of correct answers that candidate gets by guessing
n	5	total no of questions
p	$\frac{1}{3}$	probability of getting correct answer by guessing

TABLE I: The binomial random variable, its parameters and their values

The probability mass function for candidate getting k correct answers just by guessing:

$$p_X(k) = \Pr(X = k) \quad (2)$$

$$= {}^nC_k \times p^k \times (1 - p)^{(n-k)} \quad (3)$$

$$= {}^5C_k \times \left(\frac{1}{3}\right)^k \times \left(1 - \frac{1}{3}\right)^{(5-k)} \quad (4)$$

$$= {}^5C_k \times \left(\frac{1}{3}\right)^k \times \left(\frac{2}{3}\right)^{(5-k)} \quad (5)$$

$$= {}^5C_k \times \left(\frac{2}{3}\right)^5 \times \frac{1}{2^k} \quad (6)$$

The Cdf for the following pmf:

$$F_X(k) = p_X(0) + p_X(1) + \dots + p_X(k) \quad (7)$$

$$= {}^5C_0 \times \left(\frac{2}{3}\right)^5 \times \frac{1}{2^0} + {}^5C_1 \times \left(\frac{2}{3}\right)^5 \times \frac{1}{2^1} + \dots + {}^5C_k \times \left(\frac{2}{3}\right)^5 \times \frac{1}{2^k} \quad (8)$$

$$= \left(\frac{2}{3}\right)^5 \times \left(\sum_{i=0}^k {}^5C_i \times \frac{1}{2^i}\right) \quad (9)$$

Then, the probability that the candidate gets four or more correct answers just by guessing is:

$$\Pr(X \geq 4) = 1 - \Pr(X \leq 3) \quad (10)$$

$$= 1 - F_X(3) \quad (11)$$

$$= 1 - \left(\frac{2}{3}\right)^5 \times \left(\sum_{i=0}^3 {}^5C_i \times \frac{1}{2^i}\right) \quad (12)$$

$$= 1 - \left(\frac{2}{3}\right)^5 \times \left({}^5C_0 \times \frac{1}{2^0} + {}^5C_1 \times \frac{1}{2^1} + {}^5C_2 \times \frac{1}{2^2} + {}^5C_3 \times \frac{1}{2^3}\right) \quad (13)$$

$$= 1 - \left(\frac{2}{3}\right)^5 \times \left(1 + \frac{5}{2} + \frac{5}{2} + \frac{5}{4}\right) \quad (14)$$

$$= 1 - \frac{32}{243} \times \frac{29}{4} \quad (15)$$

$$= \frac{11}{243} \quad (16)$$

\therefore The probability that a candidate would get four or more correct answers just by guessing is $\frac{11}{243}$.