Assignment 3

AI1110: Probability and Random Variables
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Question[12.13.5.9]: On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Solution: Let X be the number of correct answers that the candidate gets just by guessing.

$$X = bin(n, p) \tag{1}$$

Where,

Parameter	Value	Description
n	5	total no of questions
p	$\frac{1}{3}$	probability of getting correct answer by guessing

TABLE I: The binomial random variable parameters and their values

The probability mass function for candidate getting k correct answers just by guessing:

$$\begin{aligned} p_X(k) &= \Pr(X = k) & (2)\\ &= {}^nC_k \times p^k \times (1-p)^{(n-k)} & (3)\\ &= {}^5C_k \times \left(\frac{1}{3}\right)^k \times \left(1 - \frac{1}{3}\right)^{(5-k)} & (4)\\ &= {}^5C_k \times \left(\frac{1}{3}\right)^k \times \left(\frac{2}{3}\right)^{(5-k)} & (5)\\ &= {}^5C_k \times \left(\frac{2}{3}\right)^5 \times \frac{1}{2^k} & (6) \end{aligned}$$

The Cdf for the following pmf:

$$\begin{aligned} F_X(k) &= p_X(0) + p_X(1) + \ldots + p_X(k) & (7)\\ &= {}^5C_0 \times \left(\frac{2}{3}\right)^5 \times \frac{1}{2^0} + {}^5C_1 \times \left(\frac{2}{3}\right)^5 \times \frac{1}{2^1} + \ldots + {}^5C_k \times \left(\frac{2}{3}\right)^5 \times \frac{1}{2^k} & (8)\\ &= \left(\frac{2}{3}\right)^5 \times \left(\sum_{i=0}^{k} {}^5C_i \times \frac{1}{2^i}\right) & (9) \end{aligned}$$

Knuth Lamport

Then, the probability that the candidate gets four or more correct answers just by guessing is:

$$\begin{aligned}
\Pr(X \geq 4) &= 1 - \Pr(X \leq 3) && (10)\\
&= 1 - F_X(3) && (11)\\
&= 1 - \left(\frac{2}{3}\right)^5 \times \left(\sum_{i=0}^{3} {}^5C_i \times \frac{1}{2^i}\right) && (12)\\
&= 1 - \left(\frac{2}{3}\right)^5 \times \left({}^5C_0 \times \frac{1}{2^0} + {}^5C_1 \times \frac{1}{2^1} + {}^5C_2 \times \frac{1}{2^2} + {}^5C_3 \times \frac{1}{2^3}\right) && (13)\\
&= 1 - \left(\frac{2}{3}\right)^5 \times \left(1 + \frac{5}{2} + \frac{5}{2} + \frac{5}{4}\right) && (14)\\
&= 1 - \frac{32}{243} \times \frac{29}{4} && (15)\\
&= \frac{11}{243} && (16)
\end{aligned}$$

$\therefore$ The probability that a candidate would get four or more correct answers just by guessing is $\frac{11}{243}$.