Quick Sort → Divide N Conquer

Deterministic

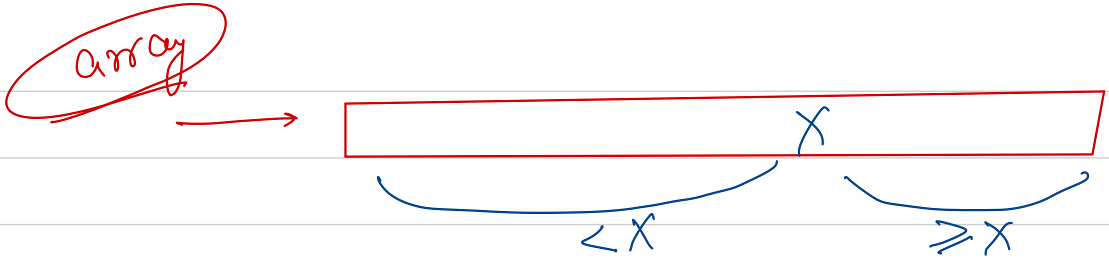
Randomized

i/p

Randomization

algo

o/p



1. quicksort partitions your array into 2 parts such that the left part has got all the elements lesser than the pivot element & the right side has got elements greater or equal to pivot element.

2. Apply same operation on left & right part.

50, 23, 9, 18, 61, 32

x → pivot

1 → 9, 18, $\boxed{23}$, 50, 61, 32

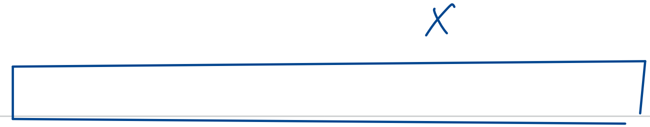
9, 18

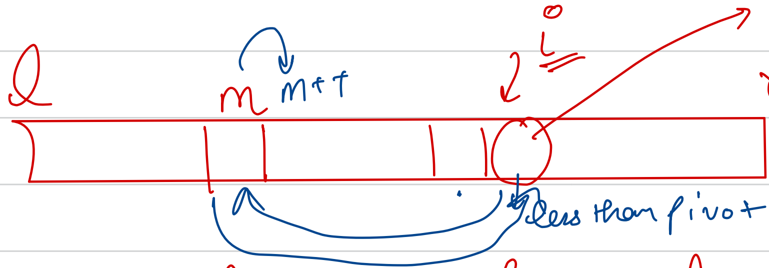
32, 61, 50

32, 50, 61

Partitioning

$O(n)$





compare with pivot

$[l, i] \rightarrow$ represent elements processed

$[l, m-1] \rightarrow$ all the elements less than pivot

$[m, i] \rightarrow$ all the elements greater than pivot.

$x = 23$

0	1	2	3	4	5
9	18	50	23	61	32

$l = 0$

$r = 5$

$m = \cancel{0}\ \cancel{1}\ 2$

$[l, m-1] \Rightarrow < x$

$[m - i] \longrightarrow \geq x$

→ Time Complexity of quicksort depends on the pivot

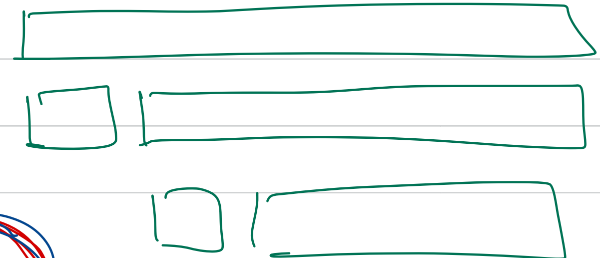
2 3 4 1 5 6 7

1 | 3 4 2 5 6 7

1 | 2 |

2x

n log n



n

2

n-1

n-2

TC → $O(n^2)$

$$T(n) = T(n-1) + T(1) + O(n)$$

worst case

Randomly pick pivot $\longrightarrow$ probability involved

Using probability analysis we can calc average time reqd for quicksort.

$$E(T(n)) = \sum p(T(n) = x)\, x$$

$$E(T(n)) = \sum_{k=0}^{n-1} \Big(\underbrace{E(T(k)) + E(T(n-k))}_{\text{Event}} + n \Big) \times \frac{1}{n}$$

estimated TC or avg TC

probability

(n - pivots)

$[0, n-1]$

if we pick from this it is a Bad split

if we pick from this it is a good split

available pivots in order

$$\sum_{k=0}^{n-1} \left(E(T(k)) + E(T(n-k)) + O(n) \right) \times \frac{1}{n} \leq$$

$$E(T(n)) \leq \frac{2}{3} \times \left(E(T(n-1)) + E(T(1)) + n \right) + \frac{1}{3} \times \left(E\left(T\left(\frac{n}{3}\right)\right) + E\left(T\left(\frac{2n}{3}\right)\right) + n \right)$$

$$E(T(n)) \leq \frac{2}{3} \left(E(T(n)) + n \right) + \frac{1}{3} \left(E\left(T\left(\frac{n}{3}\right)\right) + E\left(T\left(\frac{2n}{3}\right)\right) + n \right)$$

$$\frac{1}{3} E(T(n)) \leq n + \frac{1}{3} E\left(T\left(\frac{n}{3}\right)\right) + \frac{1}{3} E\left(T\left(\frac{2n}{3}\right)\right) +$$

$$E(T(n)) \leq 3n + E\left(T\left(\frac{n}{3}\right)\right) + E\left(T\left(\frac{2n}{3}\right)\right)$$

$$E(T(n)) \le c \cdot n \log n$$

$$E(T(n)) \le 3n + \frac{cn}{3} \log \frac{n}{3} + \frac{c2n}{3} \times \log \frac{2n}{3}$$

$$\le n\left(3 + \frac{c}{3} \log \frac{n}{3} + \frac{2c}{3} \log \frac{2n}{3}\right)$$

$$\le n\left(3 + \frac{c}{3}(\log n - \log 3) + \frac{2c}{3}(\log 2n - \log 3)\right)$$

$$\le n\left(3 + c\log n + \frac{2c}{3} \log 2 - c\log 3\right)$$

$$E(T(n)) \le cn\log n + 3n + \frac{2nc}{3} \log 2 - nc\log 3$$

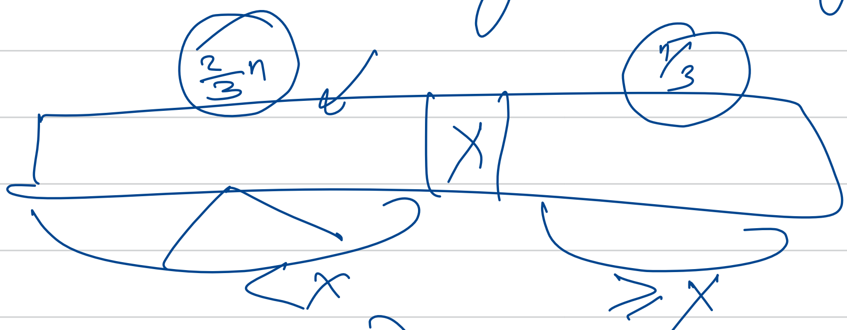
$$\boxed{T(n) = \Theta(n \log n)}$$

Selection algos

Order-Statistics → find the k^{th} smallest element in a given array

quickselect

heaps → $\Theta(n \log n)$



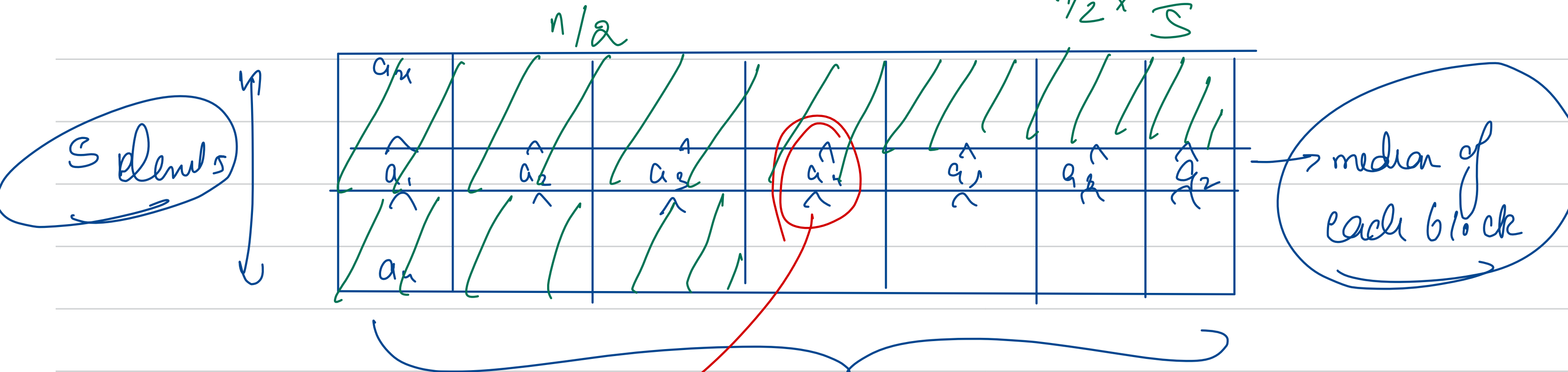
$n + \frac{2n}{3} + \frac{4n}{9} \cdots\cdots$

$n\left(1 + \frac{2}{3} + \frac{4}{9} \cdots\cdots\right)$

$= 3n$ → $\Theta(n)$

→ BFPRT algo → median of medians

given an array of n length → divide it into n/5 blocks



$a_x \le a_1 \le a_y$

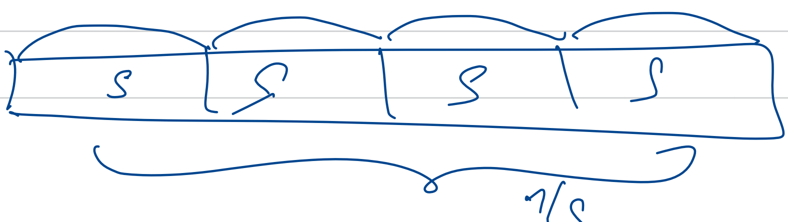
median

n/5 block

TC of quickselect with BFPRT

$\frac{n}{2} + \frac{2n}{10}$

$\frac{7n}{10}$



$$T(n) = \underbrace{O(n)}_{\text{BFPRT}} + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right)$$

$$T(n) = O(n) + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right)$$

$$T(n) \le c\, n$$

$$c \ge 10$$

$$T(n) \le n + c\frac{n}{5} + c \times \frac{7n}{10}$$

$$O(n)$$

$$n\left(1 + \frac{9c}{10}\right) \le cn$$

$$c - \frac{9c}{10}$$

$$\cancel{n}\left(1 + \frac{9c}{10}\right) \le c\,\cancel{n}$$

$$\frac{10c - 9c}{10} \to \frac{c}{10}$$

$$1 + \frac{9c}{10} \le c \to 1 \le \frac{c}{10} \to c \ge 10$$

$$\boxed{2,\ 2,\ 4,\ 5,\ 6}$$

$$\rightarrow \frac{19}{5} \rightarrow 3.\ldots$$

$$\rightarrow 2\times\frac{1}{5} + 2\times\frac{1}{5} + 4\times\frac{1}{5} + 5\times\frac{1}{5} + 6\times\frac{1}{5}$$

$$\rightarrow 2\left(\frac{2}{5}\right) + 4\times\frac{1}{5} + \cancel{5\times\frac{1}{5}} + 6\times\frac{1}{5}$$

$$\frac{4}{5} + 1 + \frac{4}{5} + \frac{6}{5}$$

$$\frac{4+5+4+6}{5} \rightarrow \frac{19}{5} \rightarrow 3.\ldots$$