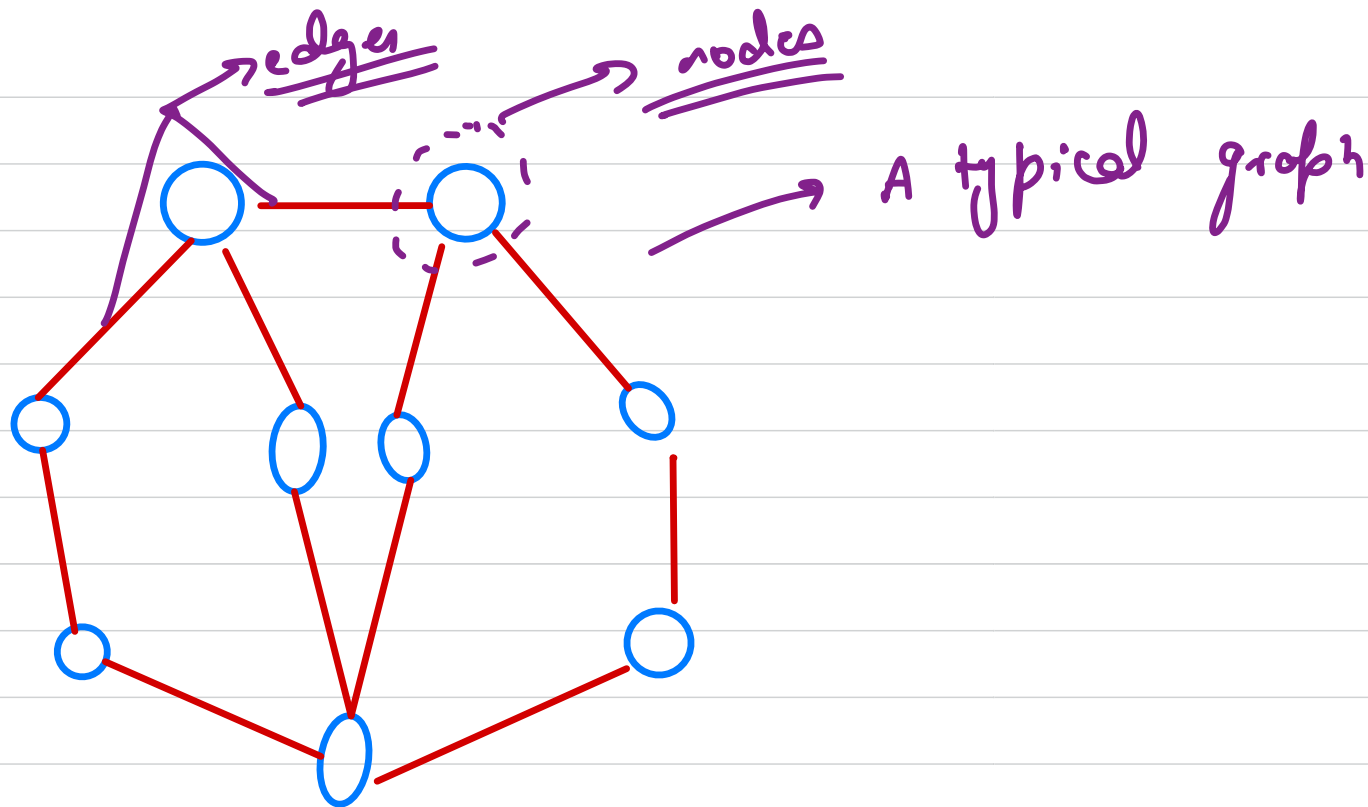


Graphs

Graph is a collection of nodes and edges, where each node might point to / connected to other nodes. The nodes represent real life entities and are connected by edges representing relationship between the nodes.



Biology → metabolic networks
→ PPI (Protein Protein Interactions)

Electrical → Circuit Organization

CSE → shortest path
→ Routing algo
→ graph dbs

Uber
Ola
Sunggy
Zomato
Foodpanda

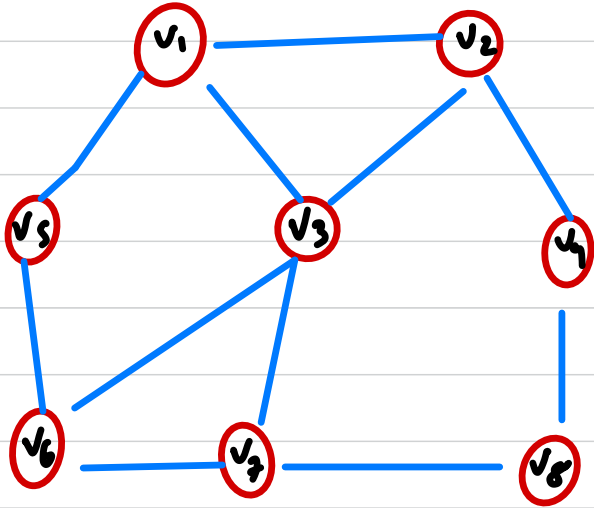
Map → gmail
→ her
→ apple us

(twitter
instagram
fb
:
)

formal definition of graph : $V = \{v_1, v_2, v_3, v_4, \dots, v_n\}$

$E = \{ \{v_1, v_2\}, \{v_2, v_4\}, \{v_2, v_3\}, \dots \}$
edges pairs are unordered

8 vertices
11 edges



$G = (V, E) \rightarrow$

mathematical notation of a
graph

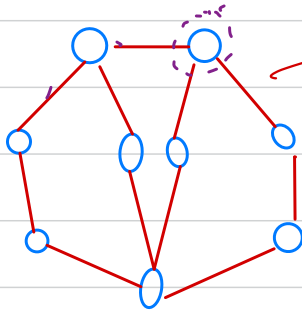
A graph G is an ordered pair of a set V vertices &
 E , a set of edges.

Type of graph:

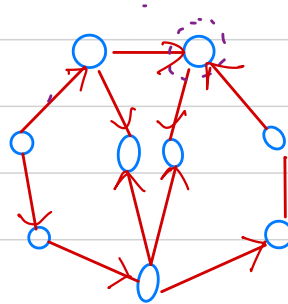


① Undirected → facebook

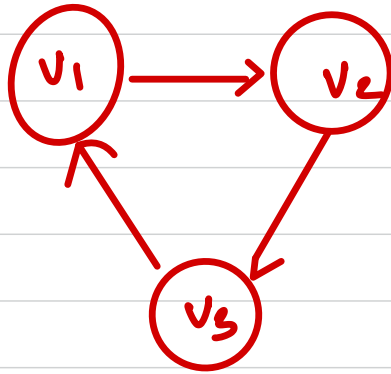
② Directed → Instagram, Twitter



→ undirected graph



→ Directed graph



$$G = (V, E)$$

$$E = \{ \{v_1, v_2\}$$

$$\{v_2, v_3\}$$

$$\{v_3, v_1\} \}$$

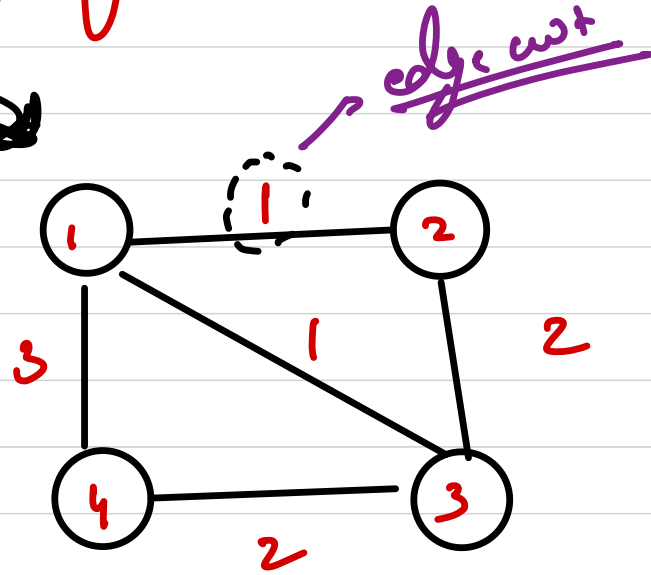
Set of
ordered
pairs

Based on edge property

① weighted



② Unweighted



Based on no. of edge

(1) Sparse

(2) Dense

Graph data structure →

↳ It is a non-linear data structure,
Graph is a collection of nodes and edges,
where each node might point to / connected to
other nodes. The nodes represent real life
entities and are connected by edges
representing relationship between the nodes.

Representation of graphs :

(1) Adjacency Matrix

(2) Adjacency List \longrightarrow Adjacency Map

(3) Incidence Matrix

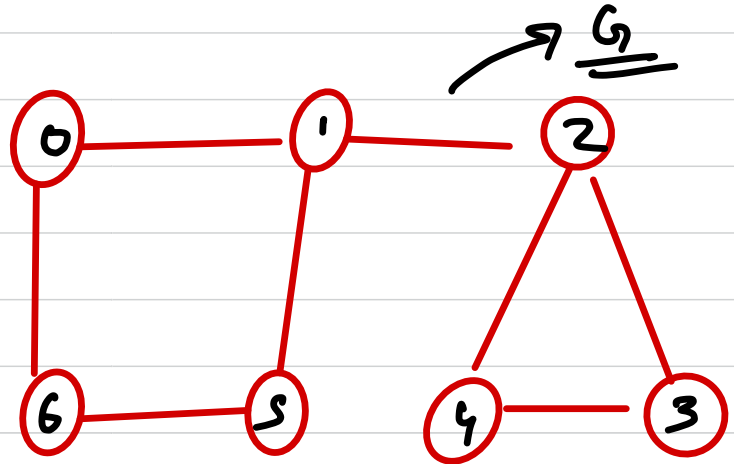
\longrightarrow edge List

Adjacency Matrix

$A_{ij} = \begin{cases} 1 & \text{if there is an edge from } i \text{ to } j \\ 0 & \text{else} \end{cases}$

$A =$

	0	1	2	3	4	5	6
0		1					1
1	1		1			1	
2		1		1	1		
3			1		1		
4			1	1			
5		1					1
6	1					1	



$\rightarrow \underline{\underline{O(V^2)}}$

$\underline{\underline{(V \times V)}} \rightarrow \underline{\underline{\text{dense}}}$

$\underline{\underline{|V|}} \Rightarrow \text{no of vertices}$

$V \times V$

Adjacency list

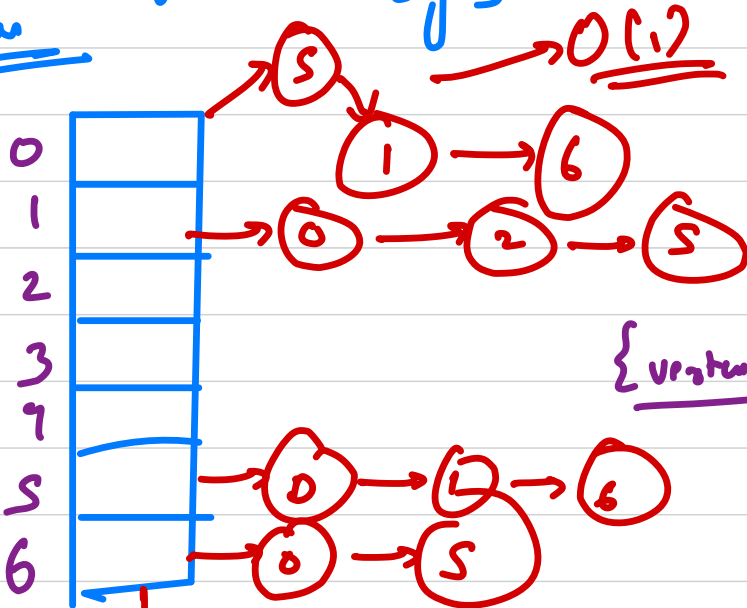
Array

of LL

→ space option

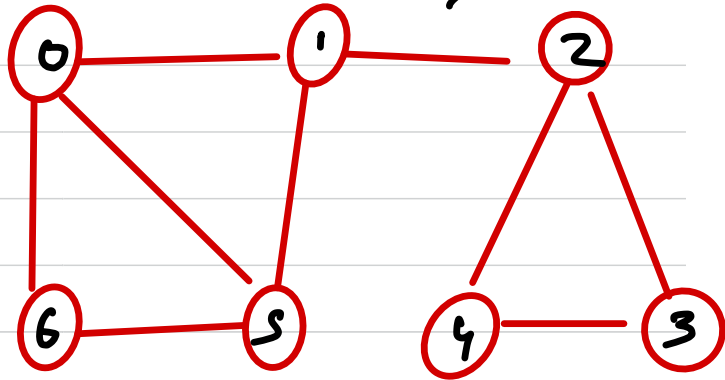
vertices

→ edges



{vertex, wt} → node

add at head
good for space in



$$(V + 2E) \approx \underline{\underline{O(V+E)}}$$

Bucket array → index stores info about its edges

adjacency map

↓
array of hashmap

hasedge (u,v) \rightarrow OG

Incidence Matrix

$V = \{1, 2, 3, 4\}$
 $E = \{a, b, c, d, e\}$

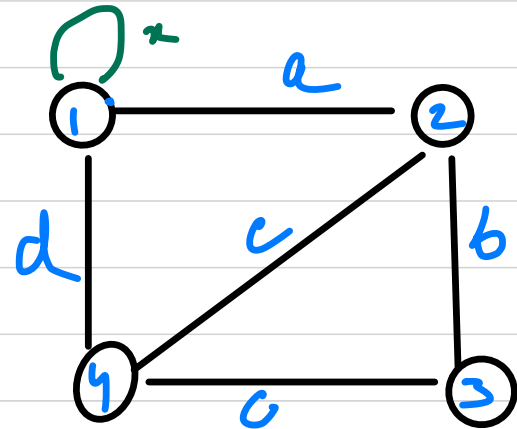
$M =$

	a	b	c	d	e
1	1	0	0	1	0
2	1	1	0	0	1
3	0	1	1	0	0
4	0	0	1	1	1

$(V \times E)$

rows \rightarrow vertex

column \rightarrow edge



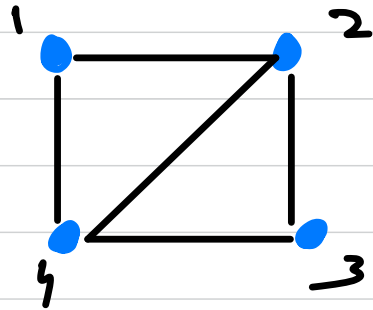
$(V \times E)$ dimension

$M_{ij} = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ vertex} \\ & \text{belongs to the } j^{\text{th}} \text{ edge.} \\ 0 & \text{else} \end{cases}$

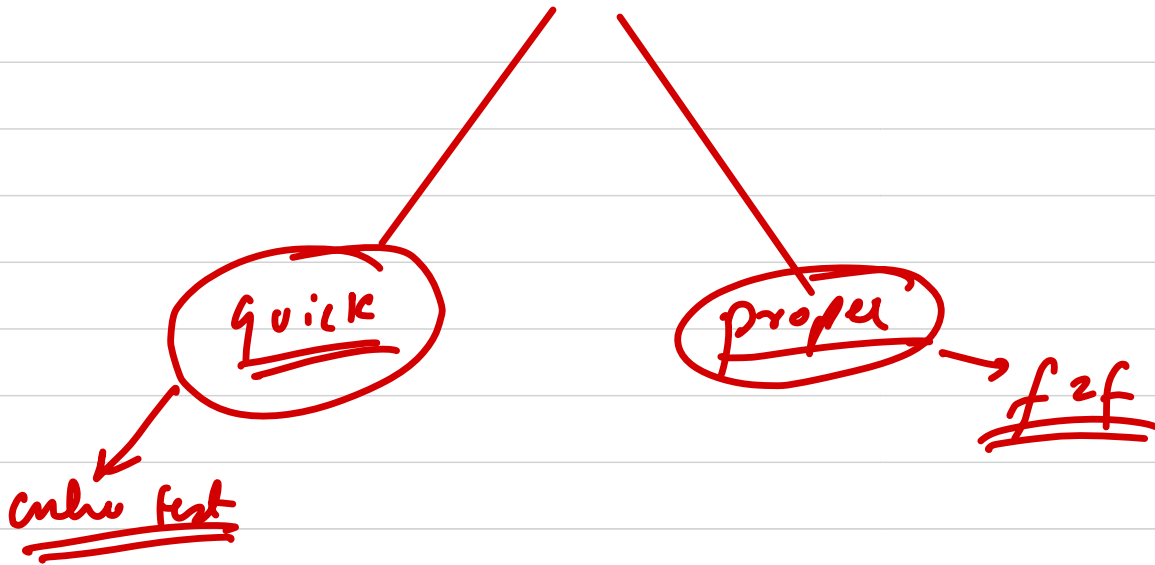
row-sum = deg
 column-sum = 2

Q₂ What is a degree??

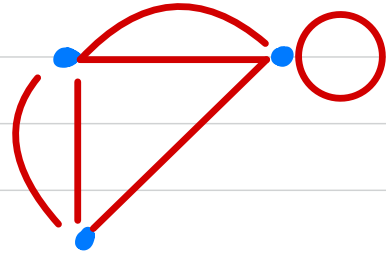
Degree of a vertex in a graph G , is the total no. of edges incident to it / associated with it (undirected graphs)



#note \rightarrow In a directed graph, the outdegree of a vertex is total no. of outgoing edges & indegree is total no. of incoming edges.

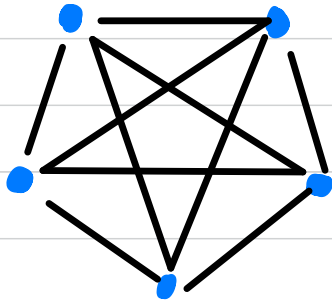


Multi graph \rightarrow an undirected graph with multiple edges and loops allowed.



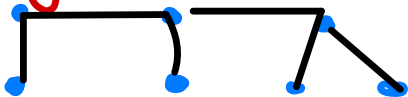
Simple Graph \rightarrow An undirected graph in which both multiple edge & loops are not allowed.

Complete Graph \rightarrow A graph in which every vertex is directly connected to every other vertex.

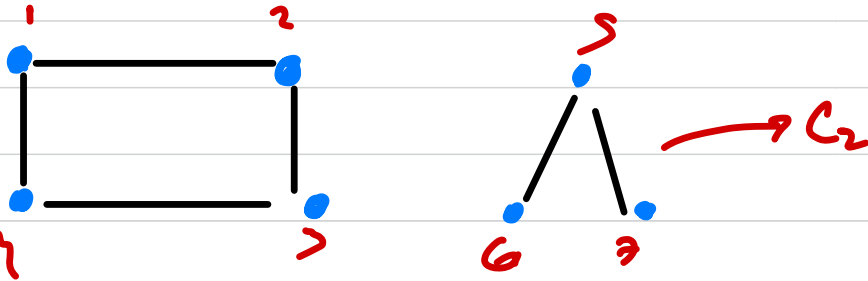


complete graph

Connected graph \rightarrow A connected graph has a path from every vertex to other vertices, not necessarily direct.



Disconnected graph \rightarrow at least 2 vertices not have a path to any other vertex.

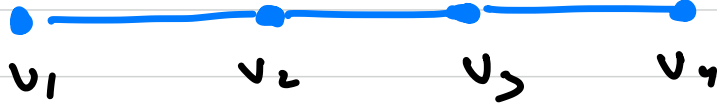


C_1

$$C_1 = (\{1, 2, 3, 4, 5, 6, 7\}, \{\{1, 2\} \{2, 3\} \{3, 4\} \{4, 1\} \{5, 6\} \{6, 7\} \{7, 5\}\})$$

component \rightarrow a subset of a disconnected / connected graph which is connected.

path \rightarrow A path P_n is a graph whose vertices can be arranged in some sequence



$$V = \{v_1, v_2, v_3, v_4\}$$

such that edge set of a graph is

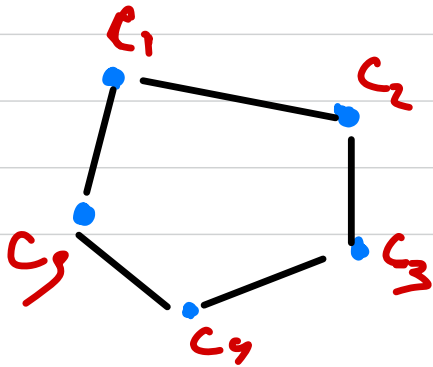
$$E = \{v_i v_{i+1} \mid \forall i \in \underline{1, n-1}\}$$

Cycle \rightarrow A cycle C_n is a graph whose vertices can be arranged in a cycle sequence

$$V = \{v_1, v_2, v_3, v_4, \dots, v_n\} \text{ such}$$

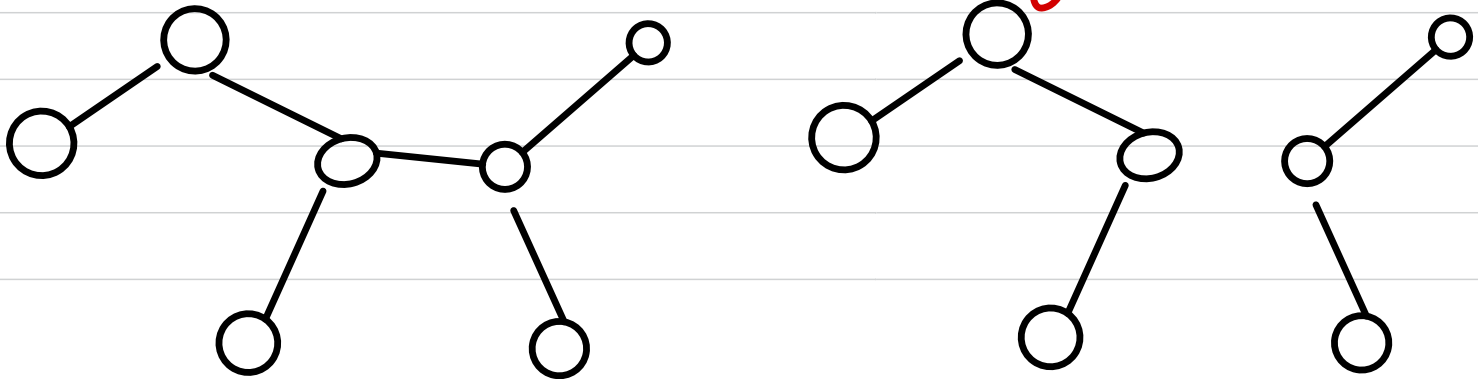
that edge set is

$$E = \{v_i, v_{i+1} \mid \forall i \in [1, n-1]\} \cup \{v_1, v_n\}$$

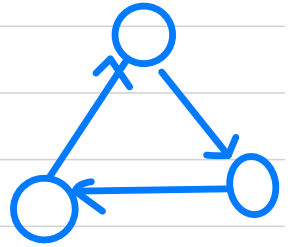
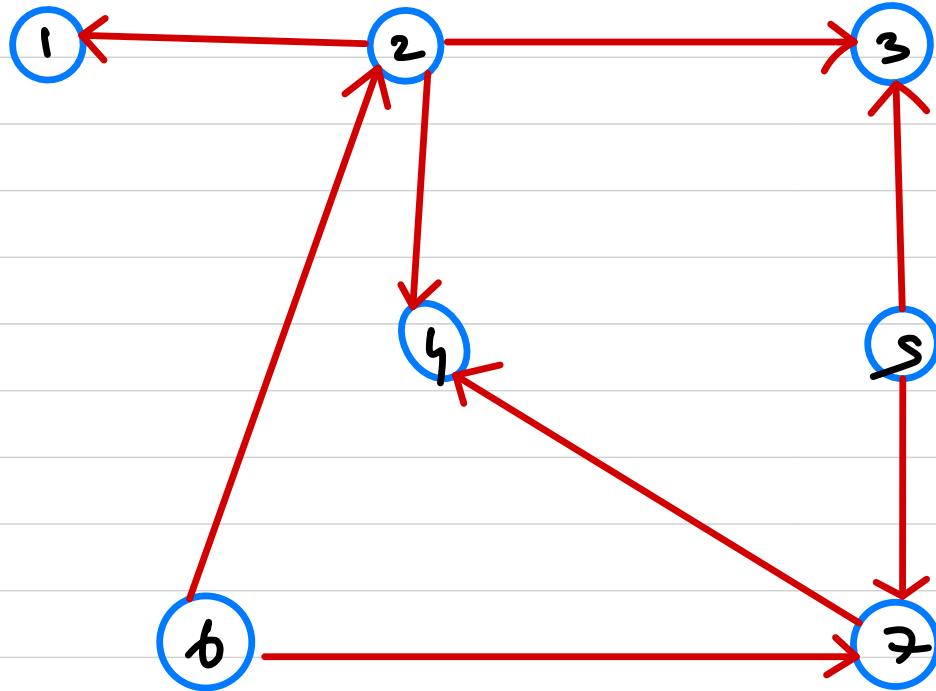


TREE \rightarrow Tree is a connected graph with no cycles.

forest \rightarrow If we remove an edge from tree, we get a forest viz collection of trees.



DAG (Directed acyclic graph)



not a
dag

How to Read Graphs

As graphs are non-linear, we need some mechanism to read graphs.

Graph Traversals

- ① Depth first search
- ② Breadth first search

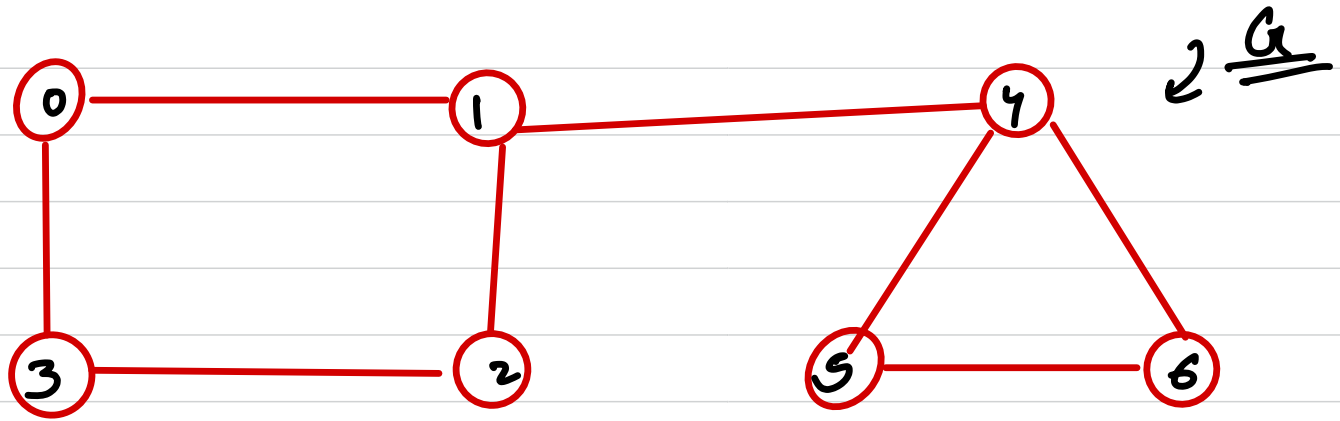
Depth first Search

Motivation problem:

Given a graph calculate all paths between 2 vertices

OR

Given a graph check whether there is a path between any 2 vertices.



is there a path from 0 to 5 ?

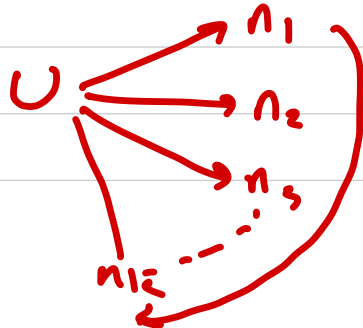
What is the most correct query for this ?

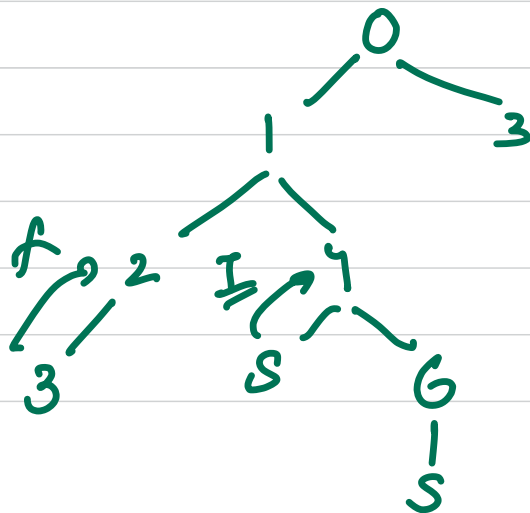
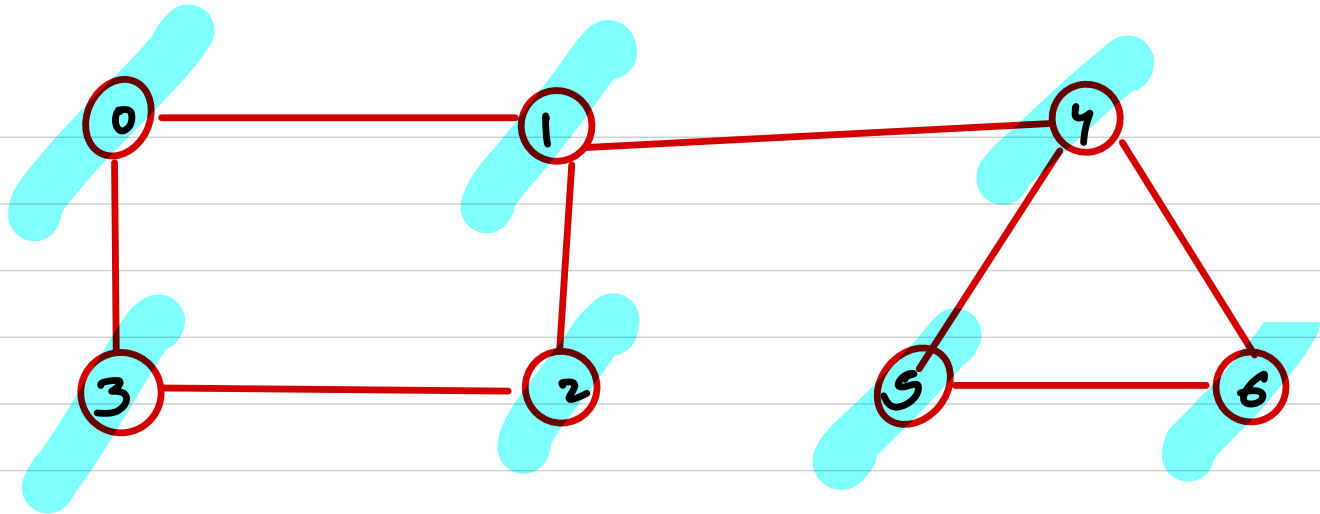
→ neighbors

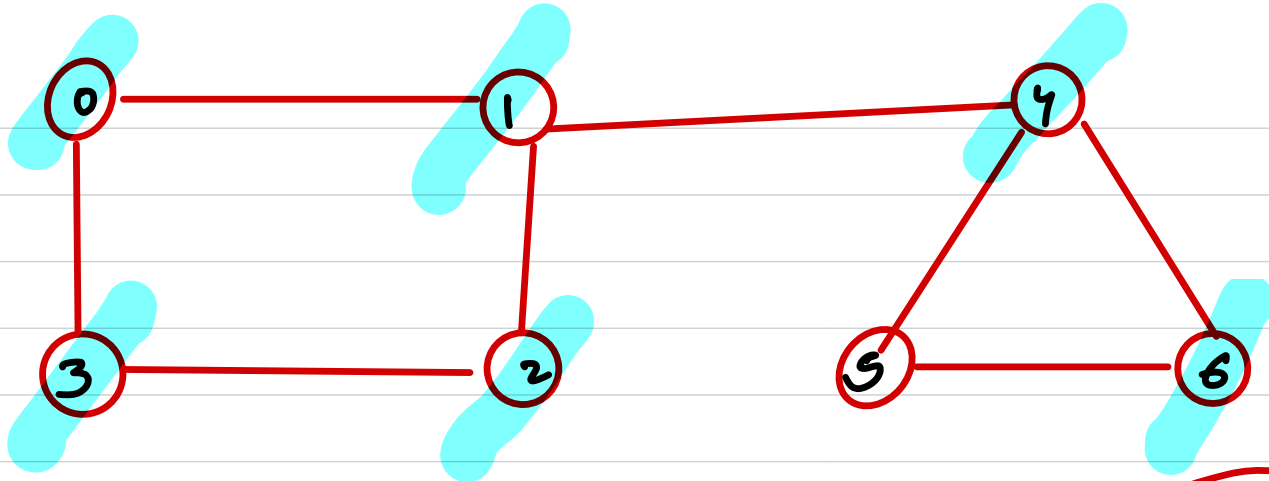
path always exist for invalide
reply

$$f(u, v) = \begin{cases} f(n_1, v) \\ \text{or} \\ f(n_2, v) \\ \text{or} \\ f(n_3, v) \\ \vdots \\ f(n_k, v) \end{cases}$$

whether there is
a path from
 u to v .

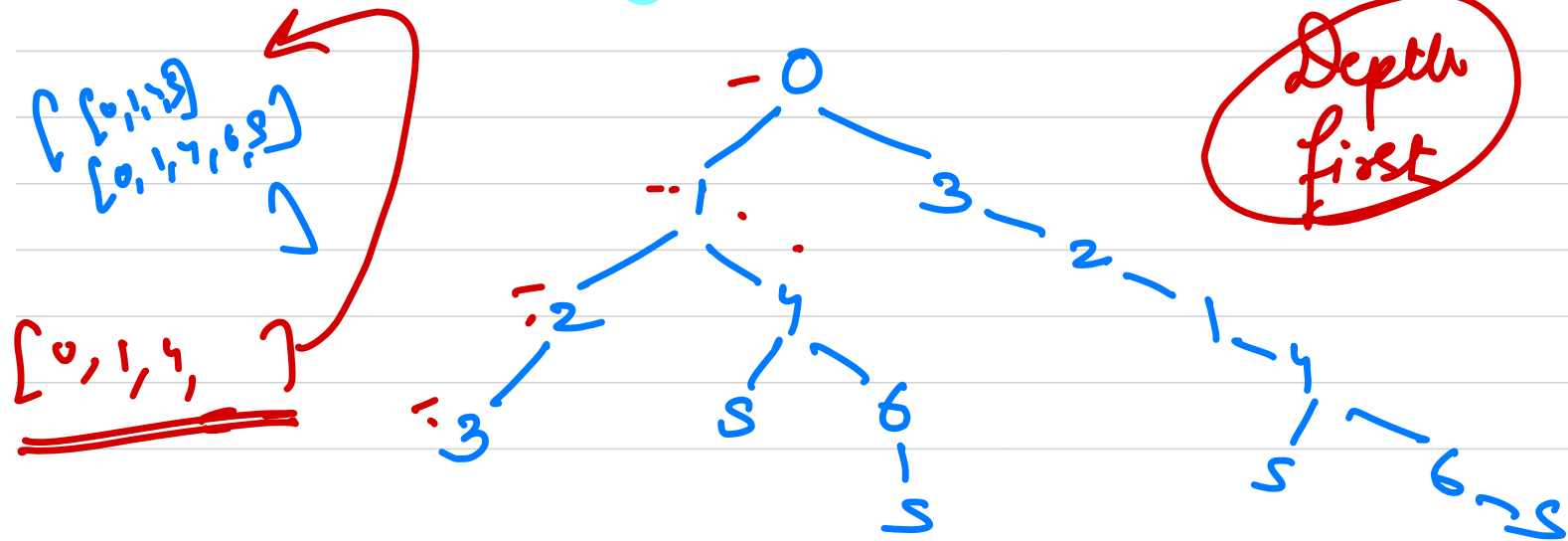


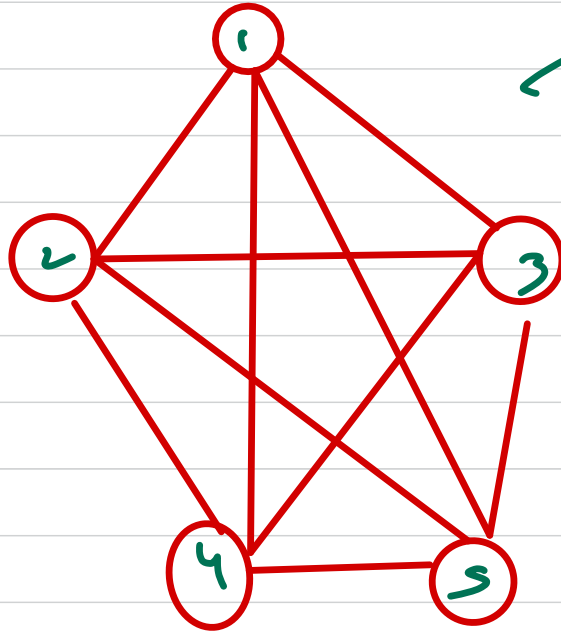




Any Path

Depth first





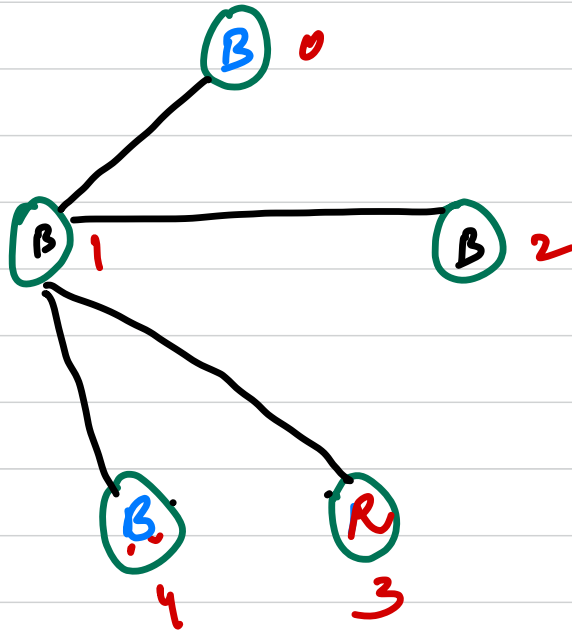
→ complete graph

1, 5

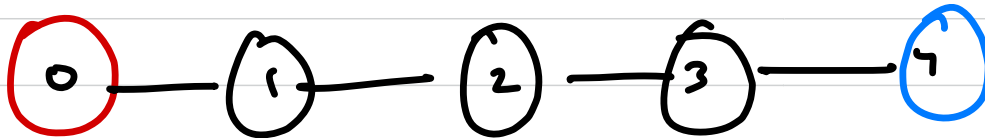
$O(v!)$

Q=1 You are given an undirected tree with v vertices. The vertices can be of any one of the following colors : Red, Blue, Black. Tree contains atleast one red and one blue node. You can remove an edge such that we get 2 trees where none of the trees has both red and blue color nodes together. Find how many such edges we can remove.

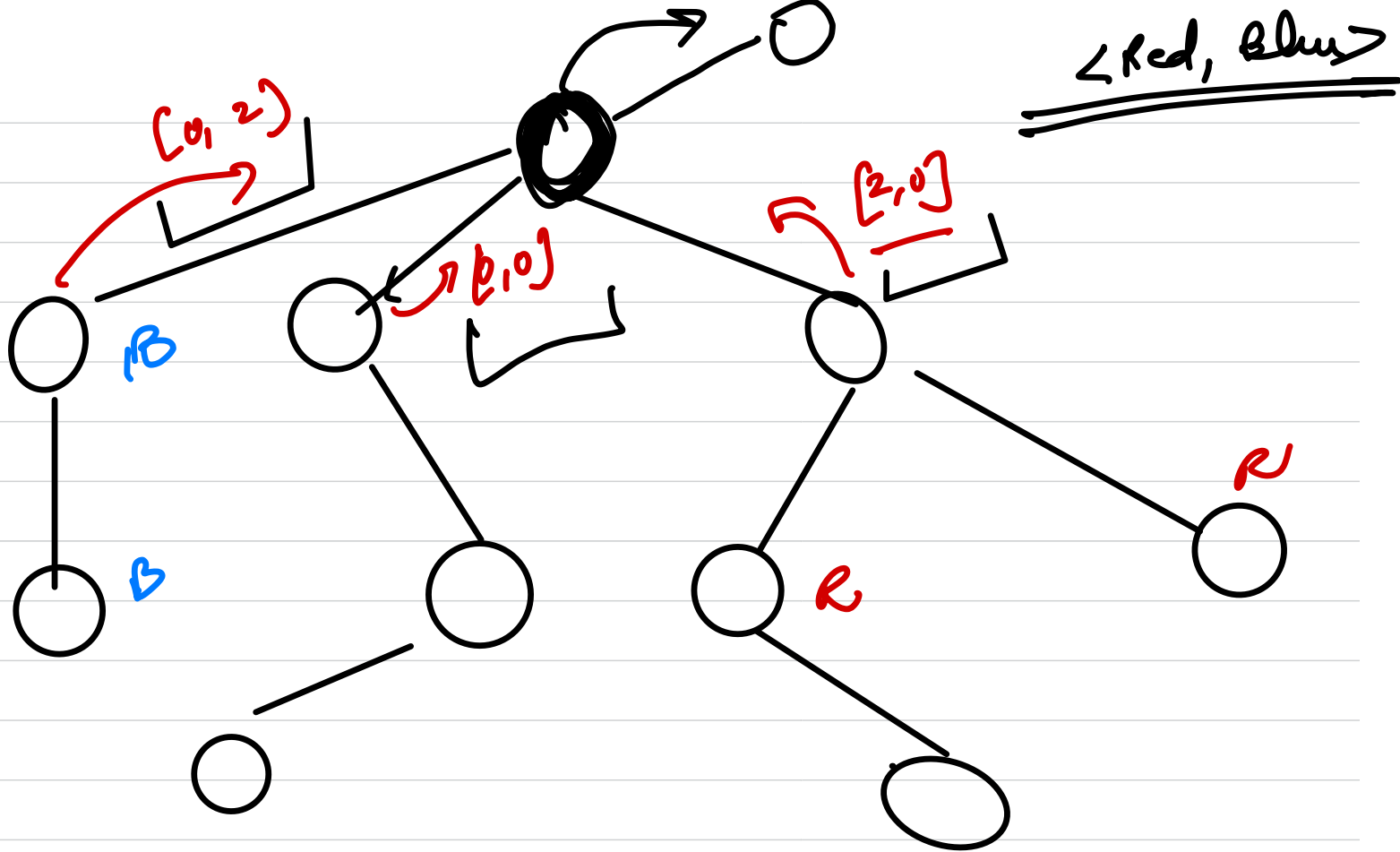
$$v \leq 3 \times 10^5$$



1 ans

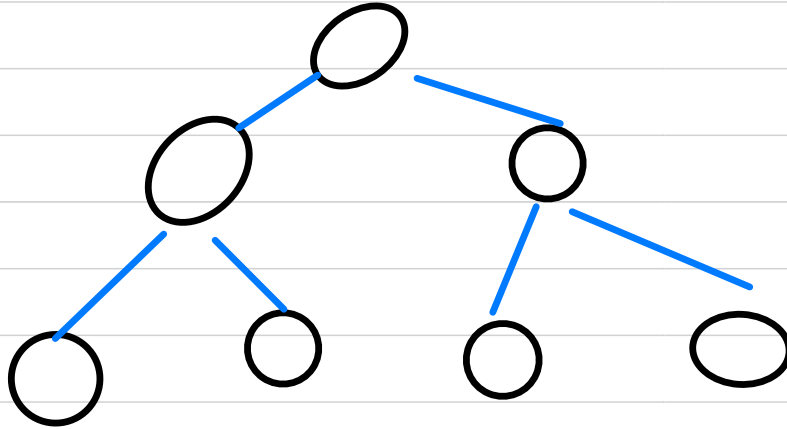


4



Tree \rightarrow V nodes

edges ???



Proove that a tree with n nodes has $n-1$

edges.

PM1 \rightarrow assume $f(n) \rightarrow n-1$

no. of edy for
 n nodes.

$$f(1) \rightarrow 0$$

$$f(2) \rightarrow 1$$

}

Base Case

assume $f(n) \rightarrow$ here

To prove $f(n+1)$

no. of edges will be $(n-1)$ + no. of edges
required for $(n+1)$ th node.

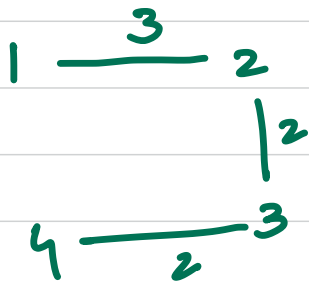
Every node that will be added to a tree
needs one edge.

$$\begin{aligned} f(\underline{n+1}) &= f(n) + 1 \\ &= (n-1) + 1 \Rightarrow \underline{(n+1) - 1} \end{aligned}$$

$$f(\underline{n}) = \underline{n-1} \quad \underline{\text{H.P.}}$$

→ Koli:

(Spoj)



1 \hookrightarrow 2

1 \hookrightarrow 3

1 \rightarrow 3

2 \rightarrow 3

1 $-$ 2

3 $-$ 2

10



1 \rightarrow 3 + 2 \rightarrow 5

3 \rightarrow 2 + 3 \rightarrow 5

4 \rightarrow 2 + 2 \rightarrow 4

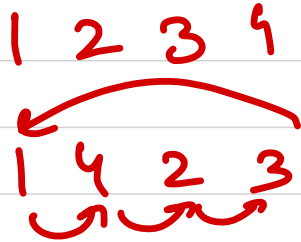
2 \rightarrow 2 + 2 \rightarrow 4

18 \leftarrow best

Brute force

1. 2. 3. N \rightarrow we can have $N!$

permutations



N nodes



$(N-1)$ edges

2-9

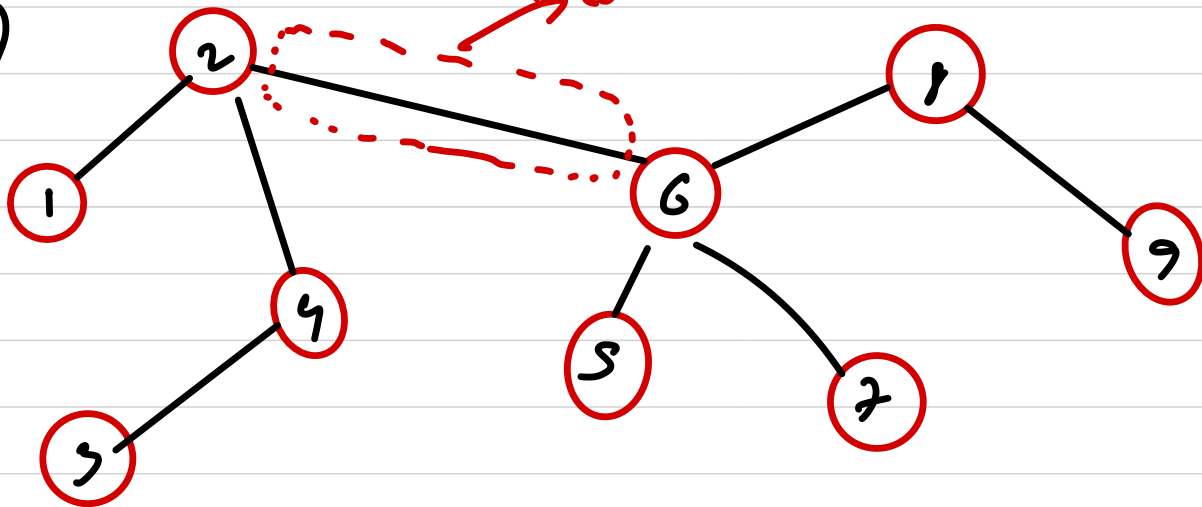
1-4

Tree (weighted)

max contribution

of each edge

$\epsilon \times O(v \times \epsilon)$
 $v \times \epsilon \times 2$
 $O(v^2)$
 $\sum \epsilon$

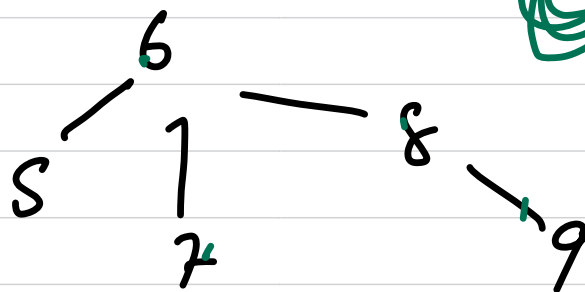
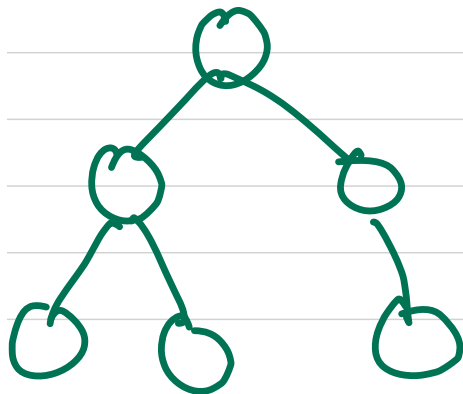
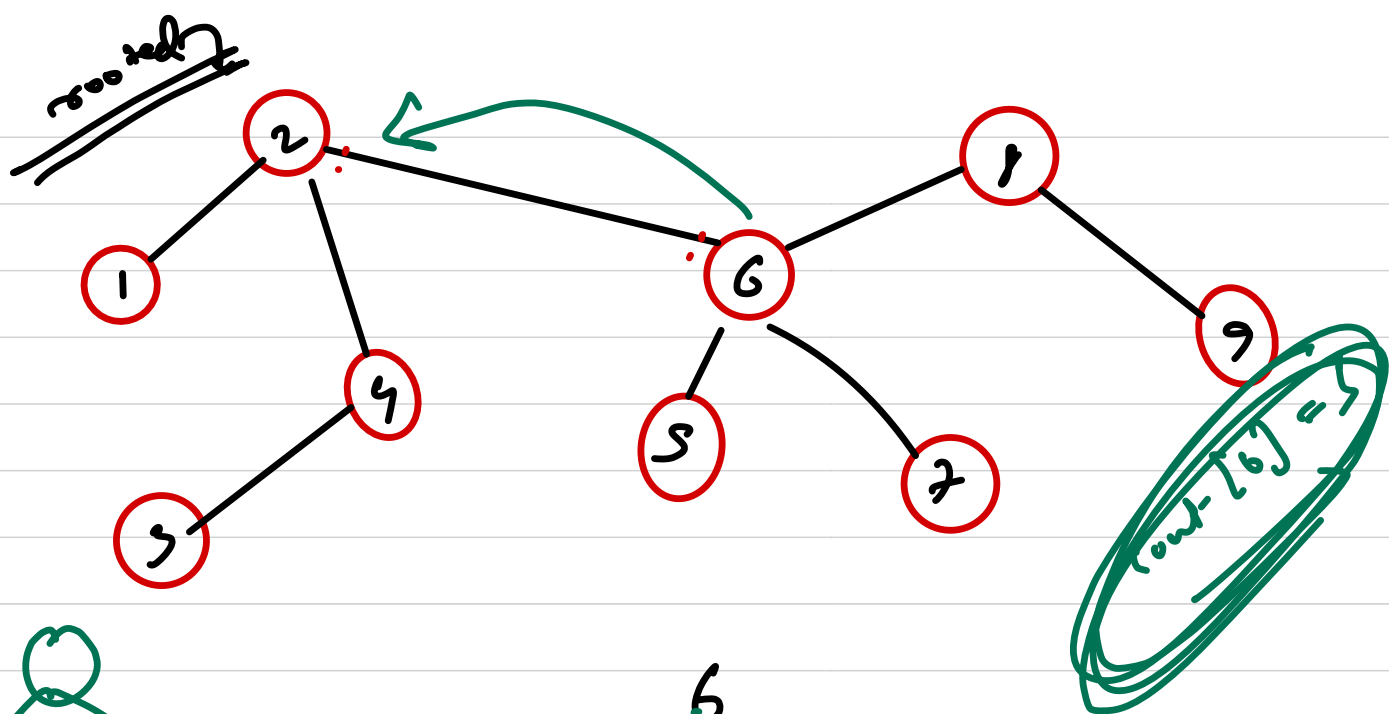


1-5
2-6

5-2
6-3 - - -

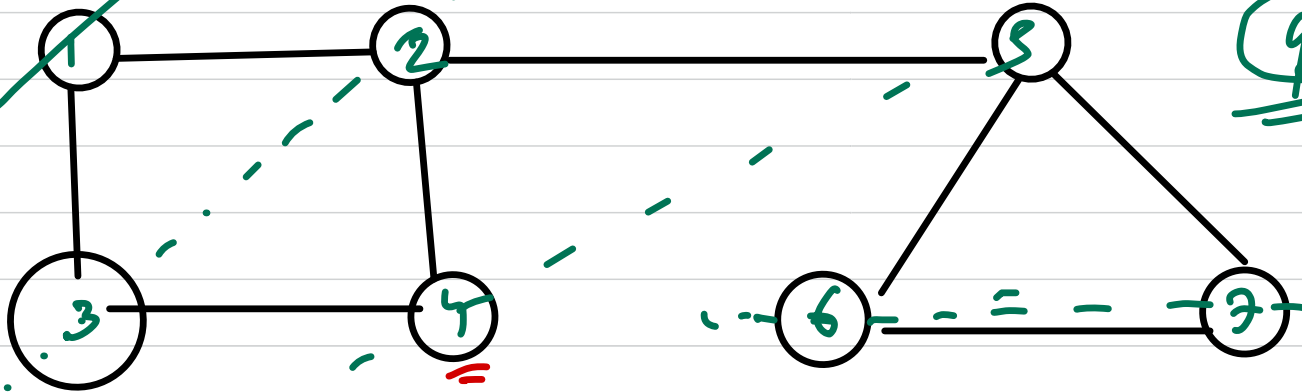
for any edge e ,

$$\text{contribution} = 2xw + x \min(s_2 - c_1, s_2 - c_2)$$



Breadth first Search

FIFO
Queue



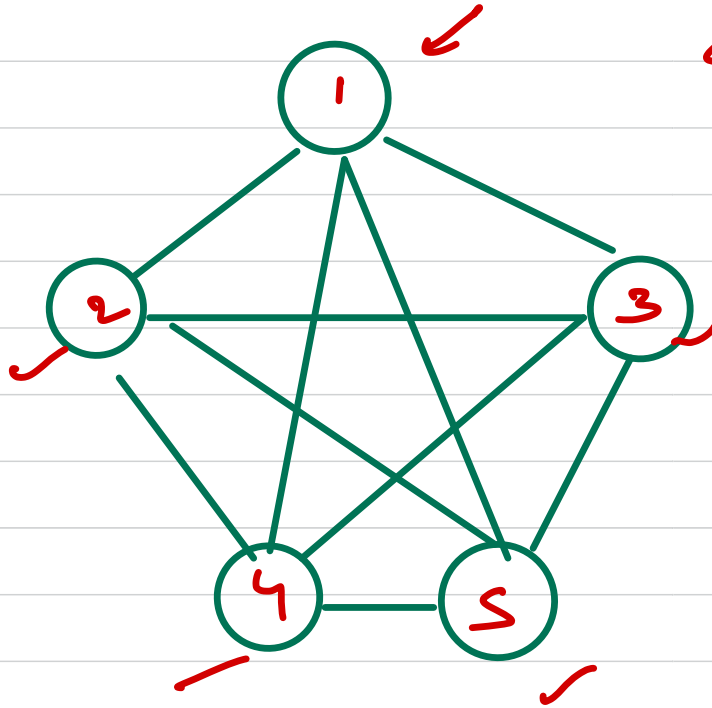
Queue

1, 2, 3, 4, 5, 6, 7

1, 2, 3, 4, 5, 6, 7

level order traversal

TC $\rightarrow O(V+E)$



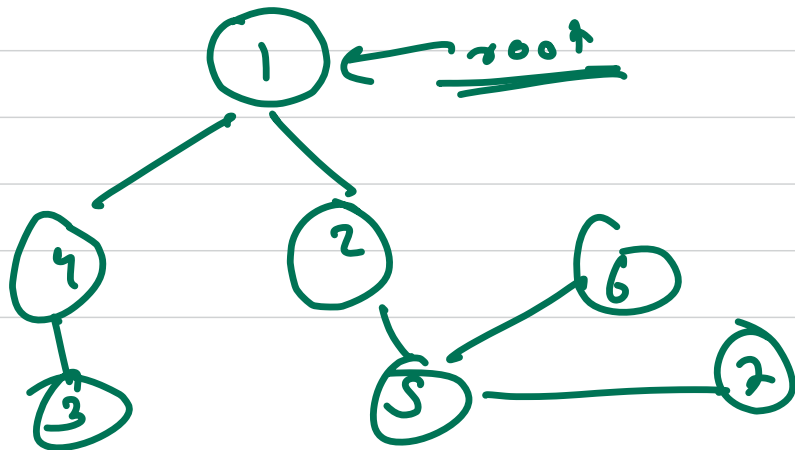
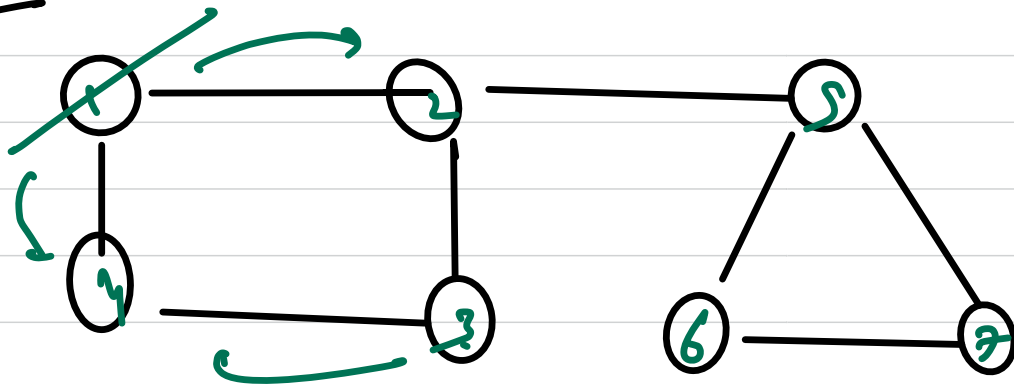
1	2	3	4	5
x	x	x	x	x

1 2 3 4 5

BFS Tree

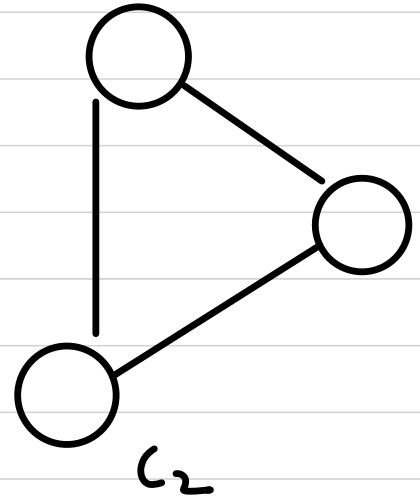
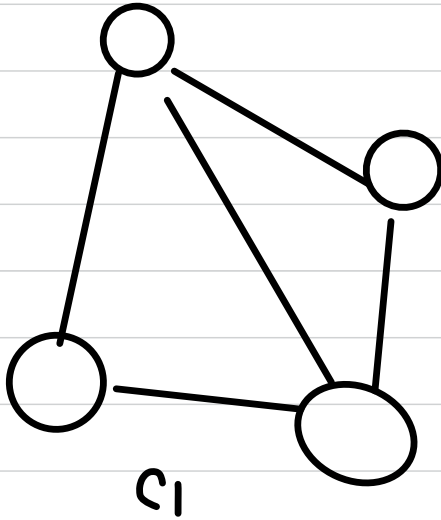
1, 4, 2, 3, 5
X X X X

parent 1 1 1



BFS Tree

* Connected Components



5/1

Journey to the moon

$$\underline{\underline{nC_2}} \quad \textcircled{1} \textcircled{2} \rightarrow \underline{\underline{\text{Total}}}$$

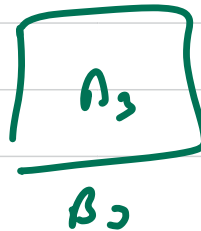
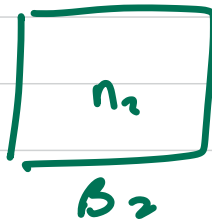
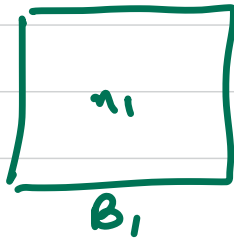
total - Σ pairs of sum neighbors

dfs



N2

of sum of calls
dfs = # of cc



...

Q You have a permutation of N integers $1, 2, 3, \dots, N$.

You want to rearrange this permutation to some other permutation P . You can swap only M given pairs, whatever no. of times you want. Figure

out if this is possible or not?_?

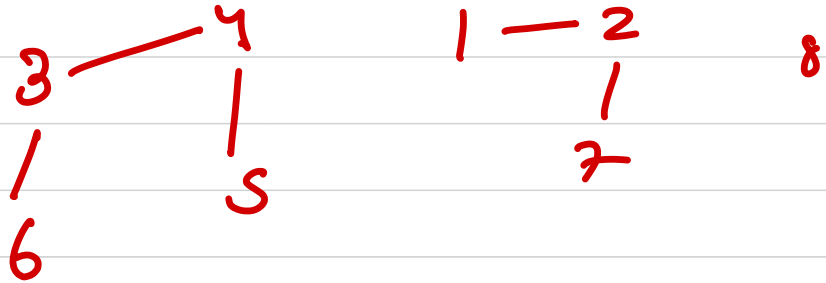
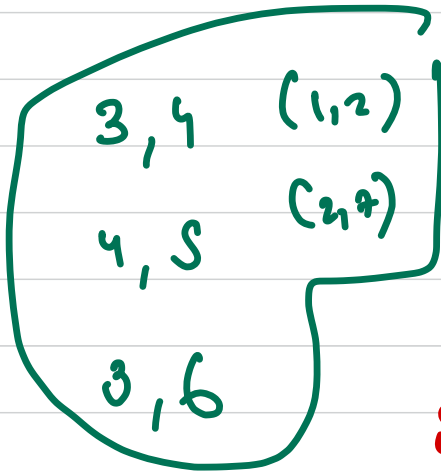
$G \rightarrow 1, 3, 2, 4 \quad (3, 4)$

$P \rightarrow 1, 4, 2, 3$

$\rightarrow \underline{\underline{\text{Yes}}}$

→ connected component ← direct application

index cc



Set → G → [3, 4, 5, 6]

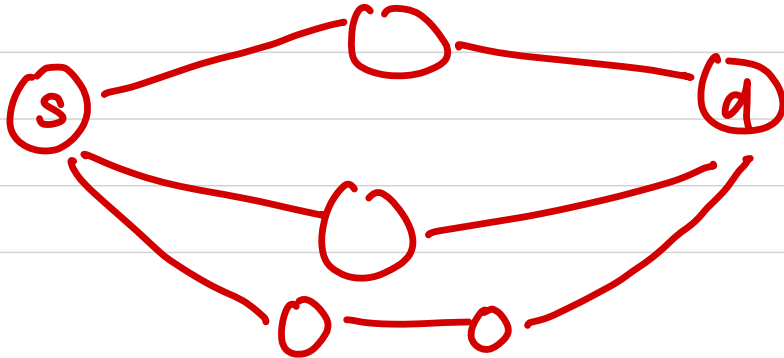
G → 1 2 3 4 5 6 7 8

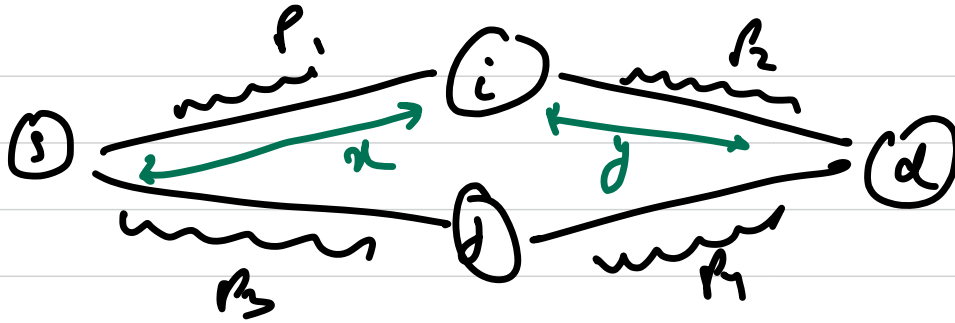
P → 7 1 4 6 5 3 2 8 //

all the elements in one connected component
can be arranged in any permutation under
some swaps

→ BFS → graph → Shortest path

Q ⇒ Given a graph, give src & dest nodes,
find all the nodes which are part of
atleast 1 Shortest path.





$$\underline{s[i] + d[i] = d}$$

$$\underline{O(V+E)}$$

$$\underline{y = d - x}$$

$$\underline{x + y = d}$$

Disjoint Set Union

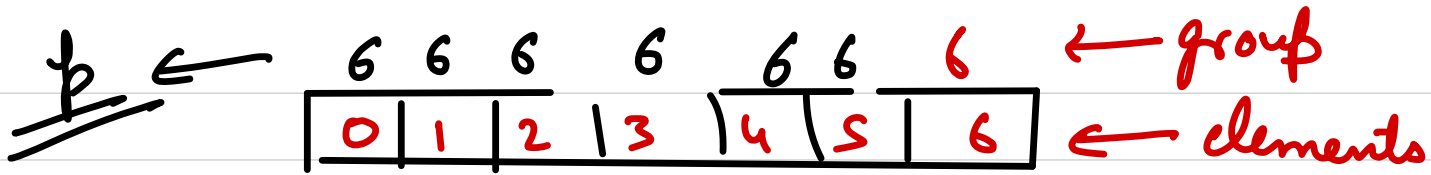
↳ We want to create clusters //

We have a set of elements & we need to group them. Some time you might be asked to return the group any element belongs to.

↳ To uniquely identify a group, we will pick any one element of the group & name it leader/parent of group. This parent is the identifier

1) $\text{union}(a, b) \rightarrow$ adds b to the group of a or vice-a-versa.

2) $\text{get}(x) / \text{find}(x) \rightarrow$ to what group / cluster x belongs to.



union(0,1)

union(2,3)

union(2,4)

union(2,5)

union(2,6)

union(6,3)

```
int get(int x) { // O(1)  
    return p[x];  
}
```

```
void union(int a, int b) { // O(n)
```

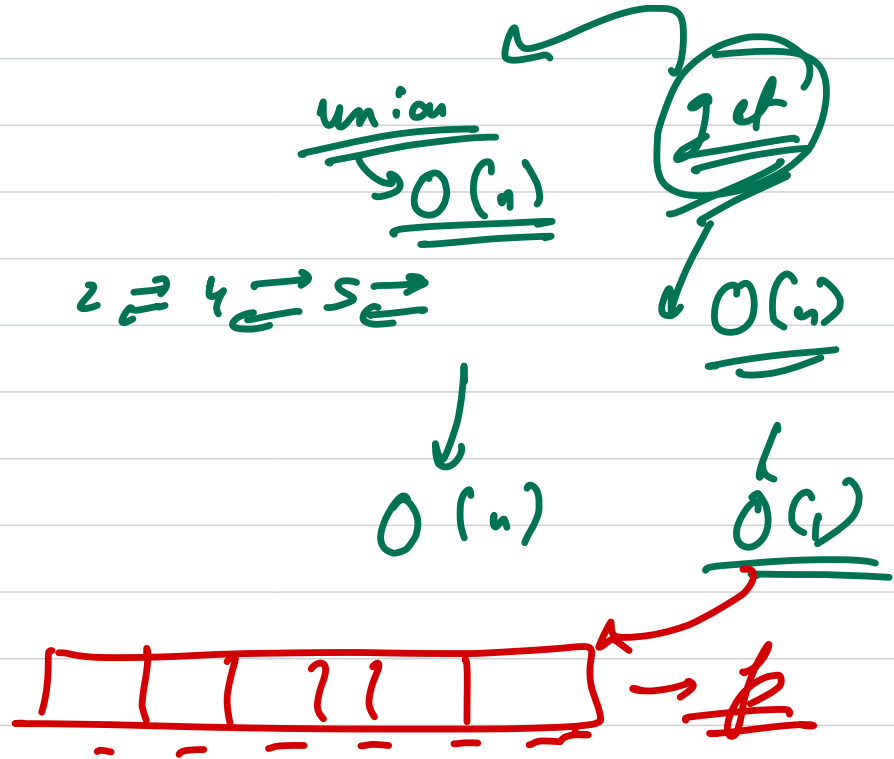
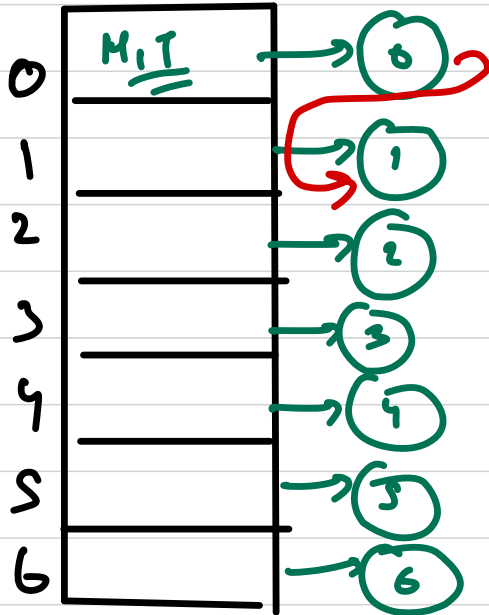
```
    a = get(a);  
    b = get(b);
```

```
    for i in 1..n { → linear search  
        if p[i] == b  
            p[i] = a;
```

```
}
```

```
}
```


// How about storing groups as ll. / del



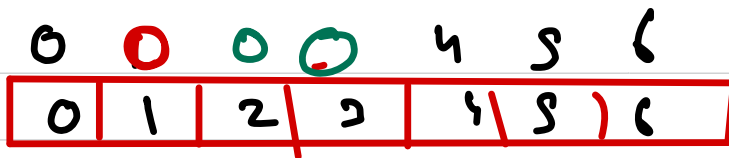
// So what we can see, if we add n elements then the operation of updating parent array for each element is $O(n)$

amortized

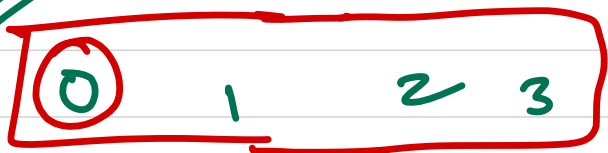
$$\frac{n \times O(n)}{n} \rightarrow \underline{\underline{O(n)}}$$

We will not arbitrarily add b to a instead
we can maintain group sizes. And we will
always add smaller group into bigger group.

union
by
size



groups
elements



1

union (0,1)

union (2,3)

union (4,5)

1 --- 1 → 2 --- 2 --- 4 ---

1
2
4
8
16
32
64
128
256
512
1024
2048
4096
8192
16384
32768
65536
131072
262144
524288
1048576
2097152
4194304
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amortized
cost

$$\frac{n \times (\log n)}{n}$$

$$\rightarrow \underline{\underline{O(\log n)}}$$

$$\frac{1}{2^k} = 1$$

$$\frac{1}{2^k} = 1 \rightarrow \frac{1}{2}$$

$n \rightarrow 16$
no ops / group
no ops / group

$\longleftrightarrow \longleftrightarrow \longleftrightarrow$

$$\frac{n}{2} \times C + \frac{n}{4} \times 2C + \frac{n}{8} \times 4C \dots 1 \times \frac{1}{2} C$$

n

$$\rightarrow \textcircled{K} C \frac{1}{2}$$

$$\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} \dots \frac{1}{2} \right)$$

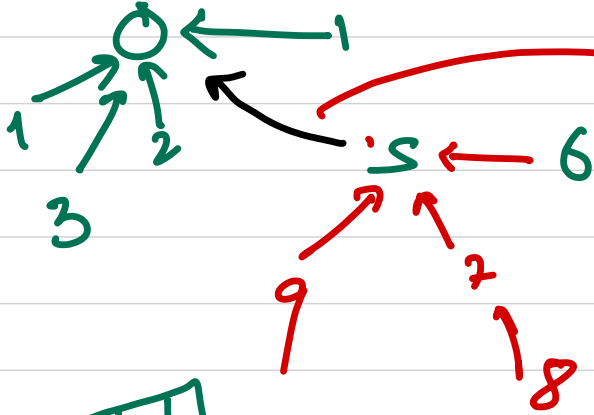
→ get → $O(1)$

→ unia → $O(\log n)$ ✓✓

Rank \rightarrow level

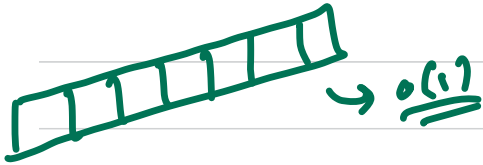
lower by π or $\frac{\pi}{2}$

same size

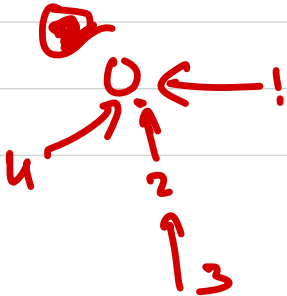


where is the new link added?!

an ngl of about
same size



8



2 ← 3

4 5 6

log 1

path compression \rightarrow inverse ackman \rightarrow complement

$O(\log^* n)$ \rightarrow extremely slow
growing funcⁿ

$\log^* n \rightarrow$ no. of steps we need to take

$\log_2 n$ as the value n to make it
smaller than one.

$$\log(2^{16}) \rightarrow \underline{\underline{16}}$$

$$\log \rightarrow \underline{\underline{2^{16}}} \rightarrow$$

$$\log 2^{16} \rightarrow \log 2^4 \rightarrow \log 2^1 \rightarrow \log 2^1 \rightarrow 0$$

(1)

connected component

$$\rightarrow \underline{\underline{O(V + E)}}$$
$$\hookrightarrow \underline{\underline{O(\log^* V)}} \leftarrow$$

