Qn Given a number N, $(N \le 10^{12})$

find the value of

$$\hookrightarrow \left\lfloor N \right\rfloor^3 + \left\lfloor \frac{N}{2} \right\rfloor^3 + \left\lfloor \frac{N}{3} \right\rfloor^3 \cdots\cdots + \left\lfloor 1 \right\rfloor^3$$

Brute force $\rightarrow$ $O(N)$ $\hookrightarrow$ $O(\sqrt{N})$

$\frac{12}{1} \Rightarrow 12$

$\frac{12}{2} \Rightarrow 6$

$\frac{12}{3} \Rightarrow 4$

$\frac{12}{4} \Rightarrow 3$

$\frac{12}{5} \Rightarrow 2$

$\frac{12}{6} = 2$

$\frac{12}{7} = 1$

$\frac{12}{8} = 1$

$\frac{12}{9} = 1$

$\frac{12}{10} = 1$

$\left\lfloor \frac{n}{i} \right\rfloor$

$\frac{12}{11} = 1$

$\frac{12}{12} = 1$

$N = 25$ $\qquad \left\lfloor \frac{N}{i} \right\rfloor \qquad 2 \times (12) - 9 + 1$

$\frac{25}{1} = 25$

$\frac{25}{2} = 12$

$\frac{25}{3} = 8$

$\frac{25}{4} = 6$

$\frac{25}{5} = 5$

$\frac{25}{6} = 4$

$\frac{25}{7} = 3$

$\frac{25}{8} = 3$

$\frac{25}{9} = 2$

$\frac{25}{10} = 2$

$\frac{25}{11} = 2$

$\frac{25}{12} = 2$

$\frac{25}{13} = 1$

$\frac{25}{14} = 1$

$\frac{25}{15} = 1$

$\frac{25}{16} = 1$

$\frac{25}{17} = 1$

$\frac{25}{18} = 1$

$\frac{25}{19} = 1$

$\frac{25}{20} = 1$

$\frac{25}{21} = 1$

$\frac{25}{22} = 1$

$\frac{25}{23} = 1$

$\frac{25}{24} = 1$

$\frac{25}{25} = 1$

How many distinct values in asymptotic term are there for $\frac{N}{i}$

$2\sqrt{N}$

$\frac{N}{i} \quad \forall\, i \in [1, N]$

$\hookrightarrow O(\sqrt{N})$ distinct values only.

$[1, N] \rightarrow [1, \sqrt{N}] + [\sqrt{N}, N]$

$\frac{N}{i} \rightarrow \sqrt{N} \qquad i \qquad \frac{N}{i} \qquad \left(\frac{N}{\sqrt{N}}, \frac{N}{N}\right) \quad (\sqrt{N}, 1)$

$\left\lfloor \frac{25}{2} \right\rfloor \longrightarrow 12$

$\downarrow$

1. There are 12 multiples of 2 ≤ 25

2. 2×12 is the last multiple of 2 ≤ 25

Sum = 0

for (i = 1 ; i ≤ N ;) {

$(x - i + 1)$

$$x = \left\lfloor \frac{N}{i} \right\rfloor$$

$$\text{Sum} += x^3 \times \left(\frac{N}{x} - i + 1 \right)$$

$$i = i + \left(\frac{N}{x} - i + 1 \right) \quad // \ i = \frac{N}{x} + 1$$

}

$N = 28$

$i=1 \rightarrow$	$x = 28$	$i=5$	$x=5$
$i=2 \rightarrow$	$x = 12$	$i=6$	$x=4$
$i=3 \rightarrow$	$x = 8$	$i=7$	$x=3$
$i=4 \rightarrow$	$x = 6$	$i=9$	$x=2$
		$i=13$	$x=1$

$\text{Sum} = 28^3 + 12^3 + 8^3 + 6^3 + 5^3 + 4^3 + 3^3 \times 2 + 2^3 \times 4 + 1^3 \times 13$

$O(\sqrt{n})$

$\left\lfloor \frac{25}{7} \right\rfloor \rightarrow 3$

$\hookrightarrow$ 1 $\rightarrow$ There are 3 multiple ≤ 25 for 7

$3 \times 7 = 21$

2 $\boxed{7} \times 3$ is the last multiple ≤ 25

$\forall j \in \left[\frac{N}{i} = 3 \right]$

$3 \times \boxed{8}$

$\frac{N}{3} = \frac{25}{3} \Rightarrow 8$

$\frac{N}{i}$

$i \mathrel{*}= x'$

x is the last value of i which results in same $\frac{N}{i}$

$\frac{N}{i} = x$

$i \mathrel{+}= \left\lfloor \frac{N}{x} \right\rfloor + 1$

Q. $G(i, j)$ for some given no.

$\downarrow$

sum of all divisors of (i, j)

$i, j \le n$ for some no.

$N \le 10^{15}$

$\sqrt{n}$? set

$$\sum_{i=1}^{N}\sum_{j=1}^{N} f(i,j) = G(N)$$

$f(1,4) \rightarrow 1, \rightarrow 1$

$f(2,4) \rightarrow 1, 2 \rightarrow 3$

$f(3,4) \rightarrow 1 \rightarrow 1$

$f(4,4) \rightarrow 1, 2\ 4 \rightarrow 7$

$f(1,2) \rightarrow 1 \rightarrow 1$

$f(2,2) \rightarrow 1, 2 \rightarrow 3$

There is a
repetition
of terms
divisors

1 → How many numbers will have 1 as the contribution

2 → How many numbers will have 2 as the contribution

⋮

for How many pairs 2 will be a divisor ?:

$N = 25$

$\frac{N}{2}$

4, 4

4, 6   6, 6

4, 8   6, 8

4, 10

2, 2 → 2

2, 4 → ~~2~~

2, 6 → 2

2, 8 → 2

2, 10 → 2

2, 12 → 2

2, 14 → 2

2, 16 → 2

2, 18 → 2

2, 20 → 2

2, 22 → 2

2, 24 → 2

$\forall\, i \in [1, N]$

$\text{cnt}(i) = \dfrac{\left(\frac{N}{i}\right)\left(\frac{N}{i}+1\right)}{2}$

$$\sum_{i=1}^{N} i \times \text{cnt}(i)$$

$$\sum_{i=1}^{N} i \times$$

$O(N)$

$N \le 10^{15}$

TLE

2s

$\frac{N}{i} \rightarrow$ will have common values

$\hookrightarrow$ contiguous range.

$$\sum_{i=1}^{N} i \times \left(\frac{\frac{N}{i} \times \left(\frac{N}{i}+1\right)}{2} \right)$$

$i = 9$

$2 \times \left(\frac{3 \times 4}{2}\right) +$

$8 \times \left(\frac{3 \times 4}{2}\right) +$

$9 \times \left(\frac{2 \times 3}{2}\right) +$

$10 \times \left(\frac{2 \times 3}{2}\right) +$

$11 \times \left(\frac{2 \times 3}{2}\right) +$

$12 \times \left(\frac{2 \times 3}{2}\right) +$

$\rightarrow \left(\frac{3 \times 4}{2}\right)(7+8) + \left(\frac{2 \times 3}{2}\right)(9+10+11+12)$

$2S$

$\rightarrow$ distinct $O(\sqrt{n})$

$x = \frac{n}{i}$

$i = \frac{N}{x} \rightarrow$

Q→ Given a number check if it is a multiple of 3. But N is very huge.

N = "$d_{K-1} d_{K-2} \ldots\ldots d_0$"

$T \le 10^3$

$$\boxed{N = 10^{K-1} \times d_{K-1} \ldots\ldots - 10^1 \times d_1 + 10^0 \times d_0}$$

$2 \le K \le 10^{12}$

$1 \le d_0 \le 9$

$0 \le d_1 \le 9$

$\left.\begin{matrix} d_0 \\ d_1 \end{matrix}\right\} \rightarrow$ given $\qquad K \rightarrow$ given

$$d_i = \left(\sum_{j=0}^{i-1} d_j\right) \% \, 10 \qquad \boxed{2 \le i < K}$$

$$d_2 = (d_0 + d_1) \% 10$$

$$d_3 = (d_2 + d_0 + d_1) \% 10$$

$$= ((d_0 + d_1) \% 10 + d_0 + d_1) \% 10$$

$$= 2 \times (d_0 + d_1) \% 10$$

$$d_4 = 4 (d_0 + d_1) \% 10$$

$$d_5 = 8 (d_0 + d_1) \% 10$$

$$d_6 = 6 (d_0 + d_1) \% 10$$

$$d_7 = 2 (d_0 + d_1) \% 10$$