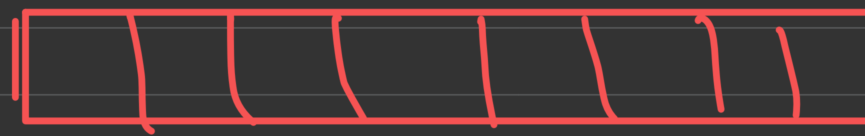
Given N days $\rightarrow$ happy for exact K days

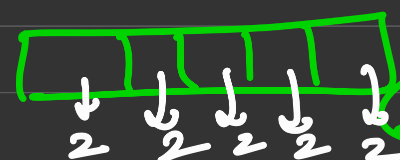
choose some k days



N

$^{N}C_{K}$

play / sleep



K

2^{K}

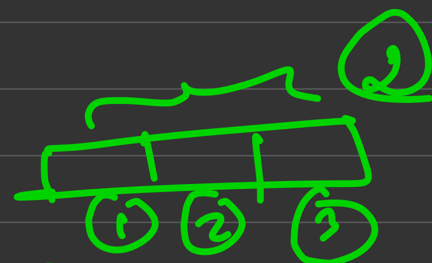
K = 2

PP
PS
SP
SS
→ 4

N = 3

$^{3}C_{2}$

$\frac{3!}{2!\,1!} \to 3$



ans $\rightarrow \left({}^{N}C_{K} \times 2^{K}\right) \% \left(10^{9}+7\right)$

$\left({}^{N}C_{K} \times 2^{k}\right) \% m$

$\left(\left({}^{n}C_{K} \% m\right) \times \underbrace{\left(2^{k}\right) \% m}_{\text{modular exponentiation}}\right) \% m$

$^nC_k \% m$? ?

$n \le 10^6$
$k \le 10^6$

$$^nC_k \% m = \left(\frac{n!}{k!\,(n-k)!}\right) \% m$$

$$= \left((n!) \% m \times \left(\frac{1}{(n-k)!}\right) \% m \times \left(\frac{1}{k!}\right) \% m \right) \% m$$

① $N! \% m$ → wilson theorem
→ precomputation

$1 \quad \ldots \quad 10^6$

$$fact[i] = \big((fact[i-1] \% m) \times (i \% m)\big) \% m$$

1	2	3	4	5
1	2	6	24	120

$(24 \% m \times 5 \% m) \% m$

$(2 \% m \times 3 \% m) \% m = 6$

$(6 \% m \times 4 \% m) \% m = 24$

$\rightarrow \left(\frac{1}{(n-k)!}\right) \% m$

$\rightarrow \left((n-k)!\right)^{-1} \% m$

By fermat's theorem

$a^{-1} \% m \rightarrow a^{m-2} \% m$

prime

$\underline{\left((n-k)! \% m\right)^{m-2} \% m}$

$$\left(\frac{1}{k!}\right)^{\varphi_0 m}$$

$$(k!)^{-1}\varphi_0 m$$

$$\left((k!)\varphi_0 m\right)^{n-2}\varphi_0 m$$

$$a^{n-2}\varphi_0 m \longrightarrow$$

mod
exp

Q:- Evaluate the following expression for a given value of n

$$S = \operatorname{lcm}(1, n) + \operatorname{lcm}(2, n) + \operatorname{lcm}(3, n) \cdots \cdots \operatorname{lcm}(n, n)$$

$S = \text{lcm}(1,n) + \text{lcm}(2,n) + \text{lcm}(3,n) \text{ ------- }$

$\text{---------- } \text{lcm}(n,n) \rightarrow \text{lcm}(n,n) = n$

$$\boxed{\text{lcm}(a,b) = \frac{ab}{\gcd(a,b)}}$$

we know

$S - n = \text{lcm}(1,n) + \text{lcm}(2,n) \text{ ...... } \text{lcm}(n-1,n)$

we know $\gcd(a,n) = \gcd(n-a,n)$

① $S - n = lcm(1, n) + lcm(2, n) - - - - - - - - - - lcm(n-1, n)$

② $S - n = lcm(n-1, n) + lcm(n-2, n) - - - - - - - lcm(1, n)$

① + ②

$$2S - 2n = (lcm(1, n) + lcm(n-1, n)) + (lcm(2, n) + lcm(n-2, n)) - - - - -$$

$$lcm(a, n) + lcm(n-a, n)$$

$$\rightarrow \frac{an}{gcd(a, n)} + \frac{(n-a)(n)}{gcd(n-a, n)} \rightarrow \frac{\cancel{an} + n^2 - \cancel{an}}{gcd(a, n)}$$

$$= n^2 / gcd(a, n)$$

$$2S - 2n = \sum_{i=1}^{n-1} \frac{n^2}{\gcd(i, n)} \longrightarrow \underline{\underline{(n \log n)}}$$

Total oppr → $\underline{\underline{n \text{ values}}}$

$$\gcd(a, b) = \underline{\underline{O(\log)}}$$

$$2S - 2n = n^2 \sum_{i=1}^{n-1} \frac{1}{\gcd(i, n)}$$

$\gcd(a, n)$, $\gcd(m, n)$, $\gcd(y, n)$

d'

$i < n$ → we can have many gcds but some can be same also

$\gcd(2, 15) \to 1$

$\gcd(4, 15) \to 1$

$$\left[\; \begin{array}{ccc} d_1 & d_2 & d_3 \\ \downarrow & \downarrow & \downarrow \\ 4 & 3 & 7 \end{array} \;\right] \qquad \text{odd}$$

$$\underbrace{\frac{n^2}{d_1} + \frac{n^2}{d_1} + \frac{n^2}{d_1} + \frac{n^2}{d_1}} + \frac{n^2}{d_2} + \frac{n^2}{d_2} + \frac{n^2}{d^2}$$

$$- - - - -$$

$$\frac{4n^2}{d_1}$$

~~for how many i's~~

$\gcd(i, n) = \underline{\underline{d}}$ ✓

$$\boxed{\gcd\left(\frac{i}{d}, \frac{n}{d}\right) = 1}$$

all no.s smaller than $\frac{n}{d}$, coprime

to $\underline{\underline{n/d}}$

$\underline{\underline{\phi\left(\frac{n}{d}\right)}}$ Euler totient

$$2S-2n = \sum_{d \neq n} \frac{n^2}{d} \times \varphi\left(\frac{n}{d}\right)$$

$d \to$ diviseur

$\downarrow$

$n/d \to$ diviseur

$d \to n/d$

$$2s - 2n = \sum_{d} n \times d \times \phi(d) - 1$$

$d = 1 \rightarrow \phi(1) \times 1 \rightarrow 1$

$$2S - 2n = n \times \left(\sum_{d} \phi(d) \times d \quad -1 \right)$$

$$2S - 2n = n \sum_{d} \phi(d) \times d \quad - n$$

$$2S = n \sum_{d} \phi(d) \times d + n$$

$$\boxed{S = \frac{n}{2} \left(\sum_{d} \phi(d) \times d + 1 \right)}$$

$$\forall\, d < n$$