↪ problem solving

↪ Fermat's little theorem.

Fermat's little theorem.

Assume 'p' is a prime no. and 'a' is an integer.

acc. to the theorem,

$$(a^p) \% p = a$$

$a = 2$
$p = 7$ $\Big] \rightarrow$ $a^p \rightarrow 2^7 \rightarrow 128$

$a^p \% p = 128 \% 7 \rightarrow 2 \rightarrow \underline{\underline{a}}$

$(a^p) \bmod p = a$ where

$p \to$ prime no

$$a^p \equiv a \pmod{p}$$

$$a^{p-1} \equiv 1 \pmod{p}$$

$$\boxed{(a^{p-1}) \% p = 1}$$

$(a^{p-1} - 1)$ is an integer multiple of p.

→ fermat's little theorem is used to calc modulo inverse.

$$a^p \equiv a \pmod p$$

$$a^{p-1} \equiv 1 \pmod p$$

multiple a^{-1} on both sides

$$a^{-1} \times (a^{p-1}) \bmod p = a^{-1}$$

taking mod both sides.

$$(\underbrace{a^{-1} \times a^{p-1} \% p}_{1}) \% p = (a^{-1}) \% p$$

$$(a^{p-2} \% p) \% p = \underbrace{a^{-1} \% p}_{1}$$

$$\boxed{(a^{p-2}) \% p = \text{Inverse modulo over } p}$$

modular exponentiation

$a^b \% c$

$p \rightarrow$ prime no

1 → ✓

2 → ✓

3 → ✓

⋮

Max → ✓

Product → ans

$5 \to 3$

$7 \to 2$

→ $5 \times 5 \times 5 \times 7 \times 7$

$5^3 \times 7^2$

Product of gcd

i^{th} gcd $\longrightarrow$ x

$\bar{\lambda}$ i^{x}

Easy

for any no. n, how to calc the no. of

Subsets for which it is a gcd.

(Repitition allowed)

1 2 3 4 - - - - - - - - - - - Max

$\{2, 2, 2\} \longrightarrow \gcd(2)$

$\{2\}$

Subsets having only multiple of 2, and

$\{4, 6\} \rightarrow \gcd(2)$

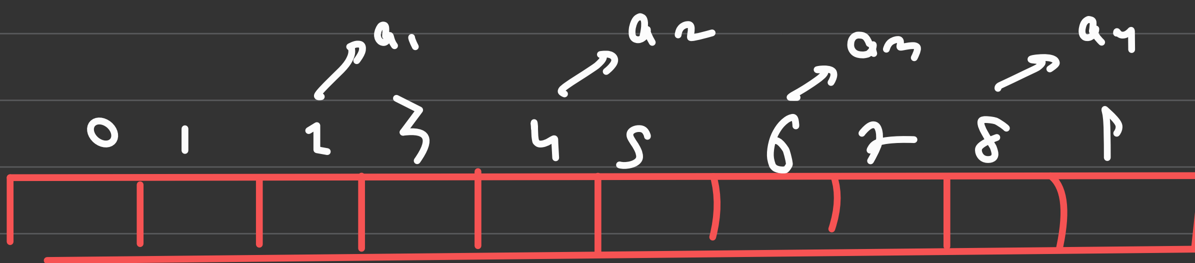
Except $\rightarrow$ $\{4, 4\}$ × × ×

$\{4, 8\}$ × × × × ×

↪ x → how many subsets possibly have gcd 2

y → how many subset have gcd as 2m

mod



$x = \underbrace{(a_1 + a_2 + a_3 + a_4)}$

$S_i \leftarrow \left(2^x - 1 - \left(\sum y\right)\right) \% \; 10^9+6$ → mod−1

non-prime

modulo expo

$[2, 4, 6, 8]$

$2-1$
$4-1$
$6-1$
$8-1$

2×2

0	1	2	3	4	5	6	7	8	9
		11		2		1	0	1	0

$((2^{11} \times 4^{2} \times 6^{1} \times 8^{1}))$ % (10^9+7) mod

ans

$O(n)$

$O(n \log n)$

$[1, +1+1+1+1]$ → 4

$[1+1]$ → for 2

$2^4-1-1-1-2$

11

$2^2-1 \Rightarrow 3-1 \Rightarrow 2$

$\{4\}$
$\{4, 8\}$

$\{2\}, \{2, 4\}$ $\{2, 6\}$ $\{2, 8\}$
$\{2, 4, 6\}$ $\{2, 4, 8\}$ $\{2, 4, 6, 8\}$ $\{4, 6\}$ $\{6, 8\}$
$\{2, 6, 8\}$ $\{4, 6, 8\}$

11

$a^{r} \% p = a$

$10^{9} + 7 \rightarrow$ prime

$$\boxed{a^{p-1} \% p = 1}$$

$p \rightarrow$ prime

$b > p$

$$a^{b} \% p = \left(a^{(k)(p-1)+r}\right) \% p$$

$$= \left(a^{k(p-1)} \times a^{r}\right) \% p$$

$\rightarrow$ $a^{r} \% p$

$$(a^x \times b^y) \% m$$

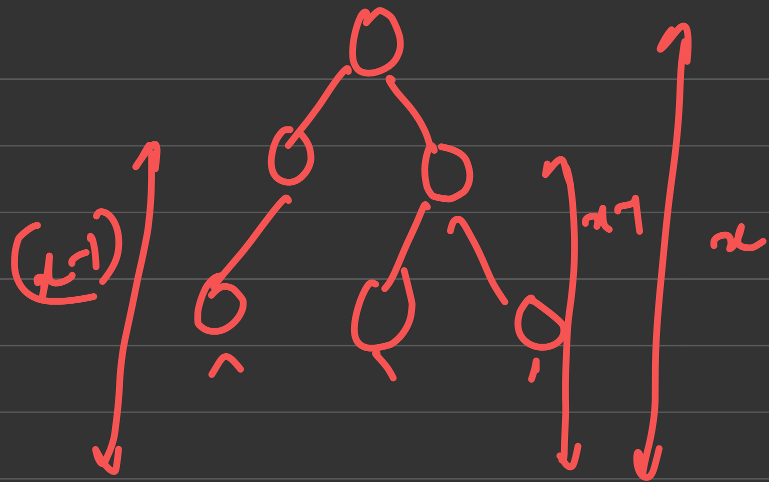
$$(a^x \% m \times b^y \% m) \% m$$

$$(a^b) \% f \rightarrow (a^{b \% f'}) \% \beta$$

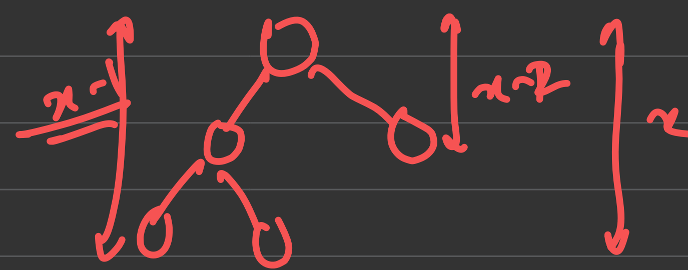
Q→ Given a number x, find the possible no. of balanced binary trees of height x. Return the count % 10^9+7

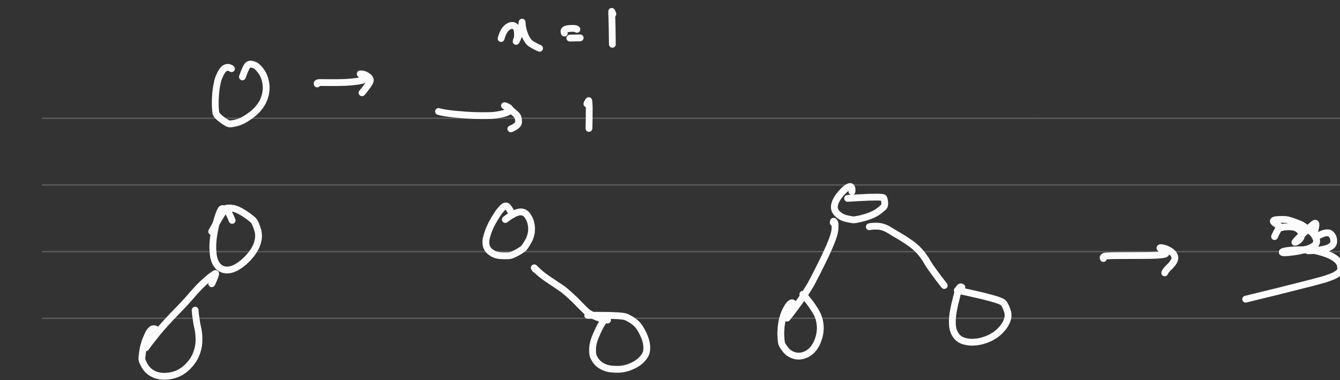
$h = 2$

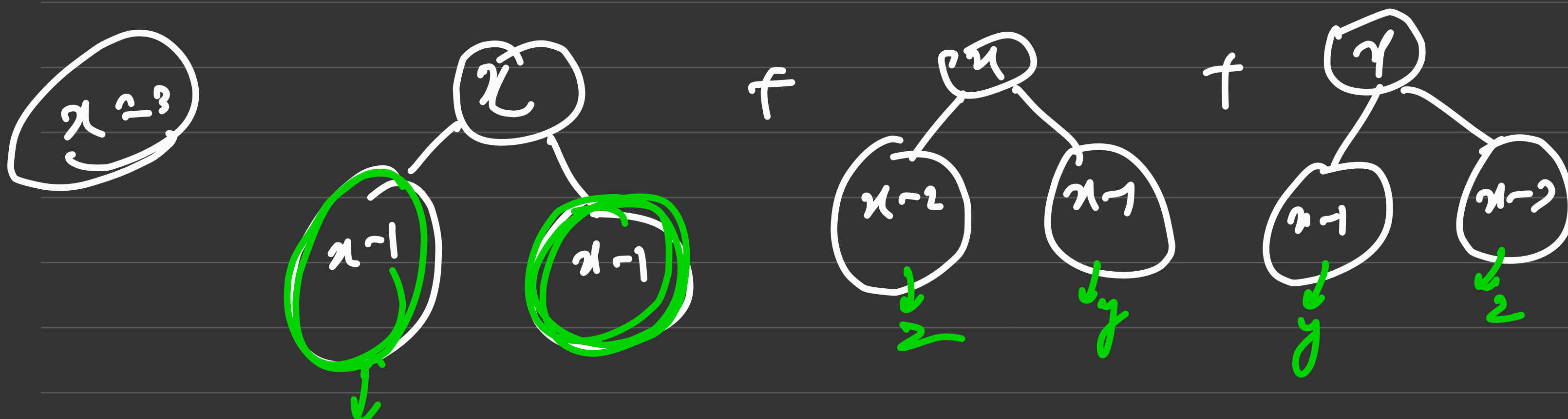
ans → 3


(x-1)
x-1
x


x-2
x-1
x


x-1
x-2
x





$y * y + 2y + 2y$

$\to y^2 + 4y = y \to 4$

$$\boxed{f(x) = f(x-1)^2 + 2x\, f(x-1)\, f(x-2)}$$

~~modulo~~ arithmetic

$\{10, 24, 40, 80\}$ → pairs → 3

$2^1 \times 5^1$ $2^3 \times 3$ $2^3 \times 5$ $2^4 \times 5$

→ greatest power of every prime factor

gcd → (3 as a prime factor) ✗ ✗ ✗ ✗

↳ 1 prime factor will occur in less than N-1 integers then it won't contri to the ans.

(2) if a prime factor is present in $(n-1)$ integers, then it will contribute its smallest power k as p^k to the ans.

(3) if a prime factor is present in all the no. then it will contribute it's 2nd smallest power k as p^k to the ans.

$(5, \ 125, \ 25)$

$\gcd(25, \ 125, \ 125) \rightarrow 25 \rightarrow 5^2$

2^{nd} smallest power

n_1 $\quad$ n_2 $\quad$ n_3 $\quad$ n_4 $\quad$ n_5

2^1 $\quad$ 2^2 $\quad$ 2^3 $\quad$ 2^4 $\quad$ 2^5

$$\gcd\left(l_1 \to 2^2 \quad l_2 \to 2^3 \quad l_3 \to 2^4 \quad l_4 \to 2^5 \right)$$

2^2