Q: You've an array of distinct integers. A set of nos is called special if we can rearrange the elements such that any element a_i divides a_{i+1} where $a_i \in$ set

find no. of distinct special sets. (A set should have all values unique. S_1 and S_2 will be diff if & only if when there is a change of atleast 1 element)

↳ $\{6, 2, 3\} \rightarrow \underline{5}$ $\{2\}$ $\{3\}$ $\{6\}$ $\{2, 6\}$ $\{3, 6\}$

Cur

$N \le 5 \times 10^5$

$0 \le a_i \le 5 \times 10^5$

Brute force → generate all possible subsets $O(2^n)$ → TLE

Hint → LIS

$\{2, 3, 6, 12\}$

$\{2\} \rightarrow \{2, 6\} \rightarrow \{2, 6, 12\}$

$\{2\} \rightarrow \{2, 12\}$

$\{3\} \rightarrow \{3, 6\} \rightarrow \{3, 6, 12\}$

$\{3\} \rightarrow \{2, 12\}$

$\{6\} \rightarrow \{6, 12\}$

$\{12\}$

LIS

$\{2, 3, 6, 12\}$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	0	1	1	0	0	3	0	0	0	0	0	6	0	0	0	0

$O(N^2)$

TLE

Sieve

$\{2, 3, 6, 12\}$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	0	1	1	0	0	3	0	0	0	0	0	6	0	0	0	0

$O(n\log n)$

Q. → if a no. has n distinct prime factors then it is n-factorful.

$T \rightarrow 10^4$

a, b, n $\qquad [a, b] \rightarrow 10^6$

Sieve

0	1	2	3	4	5	6	7	8	9	10
		1	1	1	1	2	1	1	1	2

x -

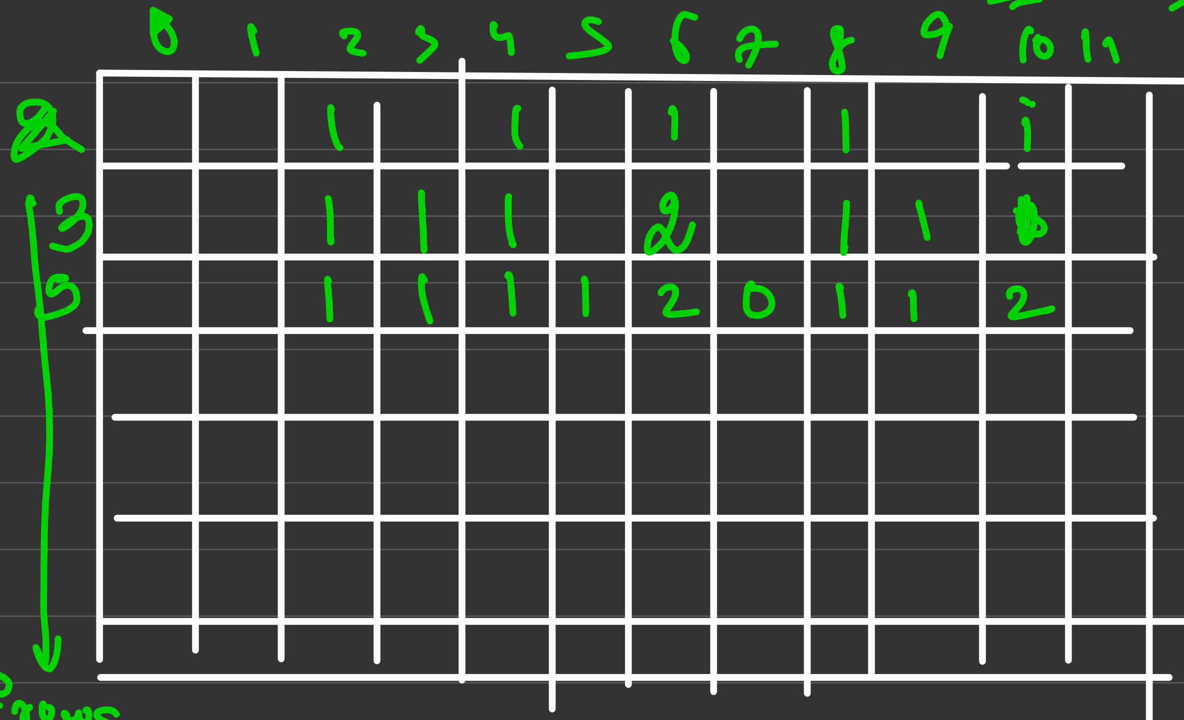
$\times 10^6$ $a - b$ n

$T \downarrow$ 10^4 10^7

$< O(b-a)$

$3 \times 5 \longrightarrow 10^6$

$4x$



10 rows

10^7

mat[i][j] →

$$mat[i][j] = mat[i][j-1] + (prime[j] == i)$$

$i \to$ n factor feelu

$j \to$ element

isPrime[j] $\to$ no. of distinct factors which are prime also for j.

$$\underline{mat[n][b]} - \underline{mat[n][a-1]}$$

~~int~~

Print # of distinct prime factors

~~0~~	~~0~~	1	1	1	1	2	~~0~~ 1	1	1	2	0	2	0	2
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

(prim[j] == i)

mat[i][j-1] + (prim[j]==i)

	0	1	2	3	4	5	6	7	8	9	10
0	$\boxed{}$	$\boxed{}$	0	0	0	0	0	0	0	0	0
1	0	0	1	2	3	4	4	5	6	7	7
2	0	0	0	0	0	0	1	1	1	1	2
3											
4											
5											
6											
7											
8											
9											

a = 2, b = 7, n = 1

mat[n][b] - mat[n][a-1]

Q: Given a no. n, find a set of three unique no. such that $2 \le A_i$ and $\prod_{i=0}^{2} A_i = n$

$n = 64 \longrightarrow 2, 4, 8$

There will be t ($t \le 10^2$) testcases.

For each case tell whether it is possible to create a set of not. If yes then print the set.

2

32 → No

2 → No

(For multiple ans, print any)

$\rightarrow$ $n \rightarrow \{a, b, c\}$

$a, b, c < n$

$a \ne b \ne c$

$2 \le a, b, c < n$

$\hookrightarrow n \rightarrow p_1^{a_1} \times p_2^{a_2} \text{ - - - - - } p_k^{a_k}$

factorization $\rightarrow$ atleast 3 prime no.

unique

$\hookrightarrow$ Yes

$42 \rightarrow 2^1 \times 3^1 \times 7^1$

$\hookrightarrow$ Yes

if you've only 1 distinct prime no.

$32 \rightarrow 2^5 \rightarrow$ NO

$64 \rightarrow 2^6$

$\underbrace{2}\times\underbrace{2\times2}\times\underbrace{2\times2\times2}$

the power should be atleast 6.

if you've 2 distinct prime factors

$\hookrightarrow n \rightarrow p_1^a \; p_2^b$

$a \geq 1$

$b \geq 1$

$12 \rightarrow 2^2 \times 3$

$\frac{12}{2 \times 3} \rightarrow 2$

NO

$\rightarrow \frac{n}{p_1 \times p_2} \rightarrow \underline{\underline{x}}$

$x \,!= p_1$ && $x \,!= p_2 \rightarrow \underline{\underline{\text{yes}}}$

$24 \rightarrow \underline{2^3} \times \underline{3}$

$\rightarrow \frac{24}{2 \times 3} \rightarrow \underline{\underline{4}}$ ✓

→ 1 → # of prime f ≥ 3 → Yes

2 # of " " == 1 → power should be atleast 6 for a Yes

spf

3 # of " " == 2 →

$\frac{n}{p_1 \times p_2} \to x$

$x \neq p_1$ && $x \neq p_2$ → Yes
else No

Q: Given 2 integers n and m. find maximum x such that

$$n! \% m^x = 0$$

$n = 5 \quad m = 2$

$$5! / 2^x = 0$$

$x \to \max$

$\to 3$ ans

$n! \% k^x == 0$

for sure power of x in k, $n!$ should be divisible

$z = n!$

$\frac{z}{k^x} = 0$ for max x

z will be divisible by k^2 if & only if, all the powers of prime factors of z are greater or equal to that of k^x

SPF

$\rightarrow K = p_1^{a_1} p_2^{a_2} p_3^{a_3} \cdots\cdots p_k^{a_k}$

$K^x = p_1^{a_1 x} p_2^{a_2 x} p_3^{a_3 x} \cdots\cdots p_k^{a_k x}$

$n! = 2 = p_1^{b_1} p_2^{b_2} p_3^{b_3} \cdots\cdots$

$$
\begin{aligned}
a_1 x &\le b_1 \\
a_2 x &\le b_2 \\
a_3 x &\le b_3 \\
&\vdots
\end{aligned}
\quad\longrightarrow\quad
\begin{aligned}
x &\le b_1/a_1 \\
x &\le b_2/a_2 \\
x &\le b_3/a_3 \\
&\vdots
\end{aligned}
\Rightarrow x \rightarrow \min\left(\frac{b_1}{a_1}, \frac{b_2}{a_2} \cdots\right)
$$

$$n! = p_1^{b_1} \times p_2^{b_2} \times p_3^{b_3} \cdots\cdots$$

$$\left\lfloor \frac{n}{p_1} \right\rfloor + \left\lfloor \frac{n}{p_1^2} \right\rfloor + \left\lfloor \frac{n}{p_1^3} \right\rfloor \cdots\cdots \longrightarrow b_1$$

$$\left\lfloor \frac{n}{p_2} \right\rfloor + \left\lfloor \frac{n}{p_2^2} \right\rfloor + \left\lfloor \frac{n}{p_2^3} \right\rfloor \cdots\cdots \longrightarrow b_2$$

$$\left\lfloor \frac{n}{p_3} \right\rfloor + \left\lfloor \frac{n}{p_3^2} \right\rfloor + \left\lfloor \frac{n}{p_3^3} \right\rfloor \cdots\cdots \longrightarrow b_3$$

$\vdots$

$n = 100$

$p_1 = 5$

$n! = 1 \times 2 \times 3 \times \cdots 5 \times \cdots \times 10 \cdots 15 \cdots \cdots 25 \cdots 50 \cdots 100$

5×5

5×5

$$\left\lfloor \frac{100}{5} \right\rfloor + \left\lfloor \frac{100}{5^2} \right\rfloor +$$

$$20 + 4 \rightarrow 24$$

$$100 \rightarrow 5^{24}$$

$$K \rightarrow p_1^{a_1}\, p_2^{a_2}\, p_3^{a_3} \cdots\cdots$$

$$K \le 25$$
$$\hookrightarrow 5^2$$

p_K

$$\frac{n!}{K^x}$$

$$n! = 7^{2x}$$

$$n! \rightarrow p_1^{b_1}\, p_2^{b_2}\, p_3^{b_3} \cdots\cdots p_n$$

for divisibility check we just need to check prime $\le p_K$

Q:- Given a array, count the no. of subarrays whose element product is divisible by k.

6	2	8

$k = 4$

[8]

[2, 8]

[6, 2]

[6, 2, 8]

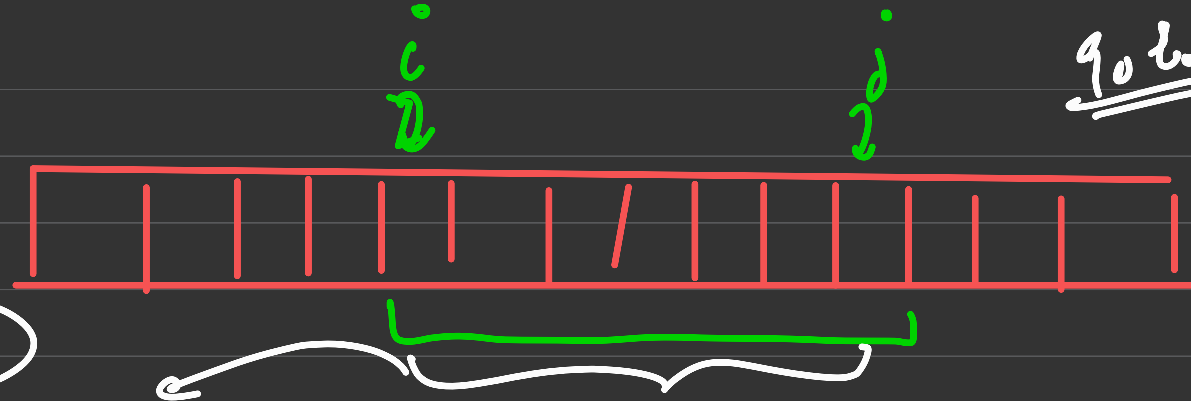
ans → 4

$n \le 10^5$

$k \le 10^9$

$a_i \le 10^9$

goto



anyhow

$$\left(\prod_{m=i}^{j} a_m\right) \% k == 0$$

$n-j$

$a_i \le 10^9$ $\quad$ $n \le 10^5$

i 2 $\quad$ 2j

0	1	2	3	4	5	6	7	8	9
2	2	2	2	2	x'	x	y	2	$2'$

$k = 32$

$n = 10$

$(2 \times 2 \times 2 \times 2 \times 2) \% 32 == 0$

$10 - 4 \rightarrow 6$

18

i 0

$2v$

18	10								

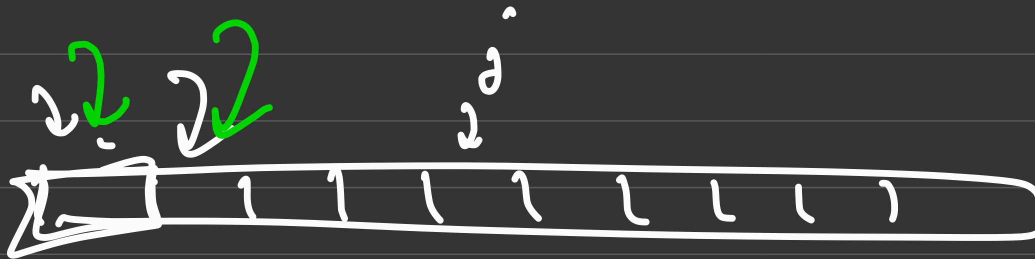
$2^1 3^2$

$2^2 3^2 5^1 \rightarrow 2^1 \times 5^1$

180

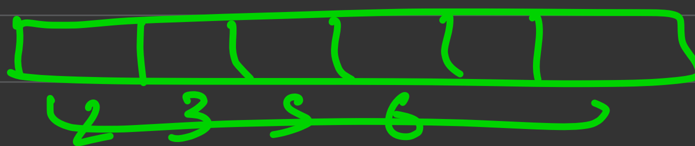
$x = p_1^a \, p_2^b \, p_3^c \, \ldots\ldots$

$\hookrightarrow [2^2, 3^2, 5^1]$

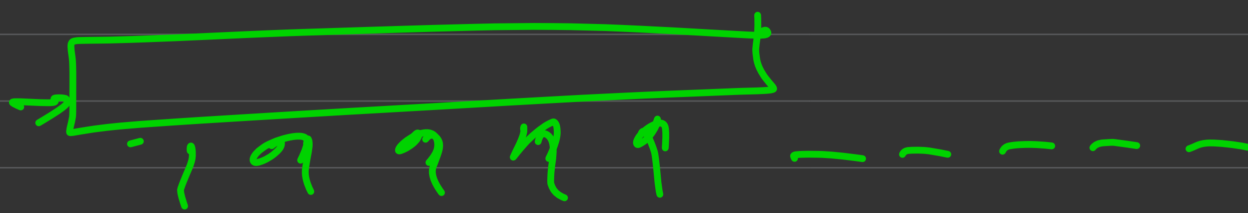


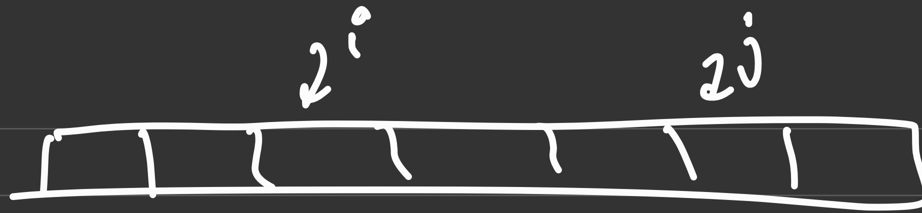
$+2(n-j)$

$k \rightarrow$



$\underline{a_i} \rightarrow$





$O(N \log n)$

→ 2 pointer → For every left starting point calc the first right for which subarray is durable. Add (n-right) in the ans, then increment left till the point you first arrive left for which [left, right] ~~is~~ <= k. then inc right again.

m