

$x_i \rightarrow$

|    |
|----|
| -1 |
| 10 |
| 4  |
| 7  |
| 8  |

$v_i \rightarrow$

|    |
|----|
| 5  |
| 3  |
| 2  |
| 10 |
| 1  |

ans  $\rightarrow$  1.5

precise to  
the 6 decimal  
pts.

Q For there are  $n$  people on a st. line, & want to gather at one point. Each person knows his current position  $x_i$  & his speed  $v_i$ . find the min time in which they can gather at the point they choose.

$$n \leq 10^5$$

$$-10^9 \leq x_i \leq 10^9$$

$$1 \leq v_i \leq 10^9$$

-1

5

Ans  $\rightarrow$  1.5  $\rightarrow$  +

10

3

4

2

$\int \rightarrow$

$$s = 2$$

$$t = 1.5$$

$$s = \frac{d}{t}$$

$$d = s \times t \rightarrow 2 \times 1.5$$

3

$$1 \times 1.5$$

1.5

2

10

8

1



There are  $n$  persons.

→  $i^{th}$  person →  $x_i$   $v_i$   
↓  
current speed (any one)  
pos

Min time in which everyone meets at some point

assume the point of meet is x.

$$t_1 \rightarrow \frac{|x(t_1) - x|}{v(t_1)}$$

$$t_2 \rightarrow \frac{|x(t_2) - x|}{v(t_2)}$$

⋮

$$t_i \rightarrow \frac{|x(t_i) - x|}{v(t_i)}$$

all of them  
will parallel

line segd such that everyone reaches x.

$\nearrow$  den reqd to reach  $x$   
 $T(x) = \max \left( \frac{|x(i) - x|}{v(i)} \right) \quad \underline{\underline{H^0}}$

$\min(T(x))$   $\rightarrow$   $\min \left( \max \left( \frac{|x(i) - x|}{v(i)} \right) \right)$   
 $\downarrow$   
Binary Search

$\min(T(x)) \rightarrow$   $t$  is the min value &  
our ans

$\min(\tau(x)) \leq t$  → we need to find value of  $t$ .  
to get min of  $\tau(x)$  we use BS.  
Because we need to minimize the  $t$ .

lo → 0      mid      hi → 16  
Search space represents  $t$  only.

By choosing any mid:

$$\tau(x) \leq \underline{\text{mid}}$$

$$\max \left( \frac{|x_i - x|}{v_i} \right) \leq \text{mid} \quad \forall i$$

$$\rightarrow |x_i - x| \leq \text{mid} \times v_i$$

$$x_i - \text{mid} \times v_i \leq x \leq x_i + \text{mid} \times v_i$$

↻ ↻ ↻
where we can't reach in mid too

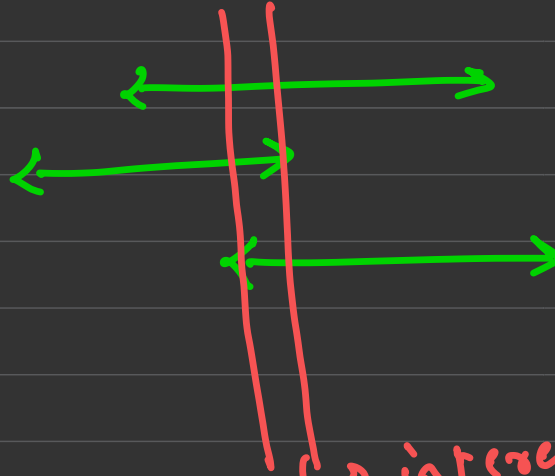
$$x_i - \text{mid} \leq x \leq x_i + \text{mid}$$

if intersection exist, then for all  $i$ , we have a range  
 for each  $i$ , we will get some range of  $x$ .

$i=1$

$i=2$

$i=3$



intersection of this range  
 has any value or not.



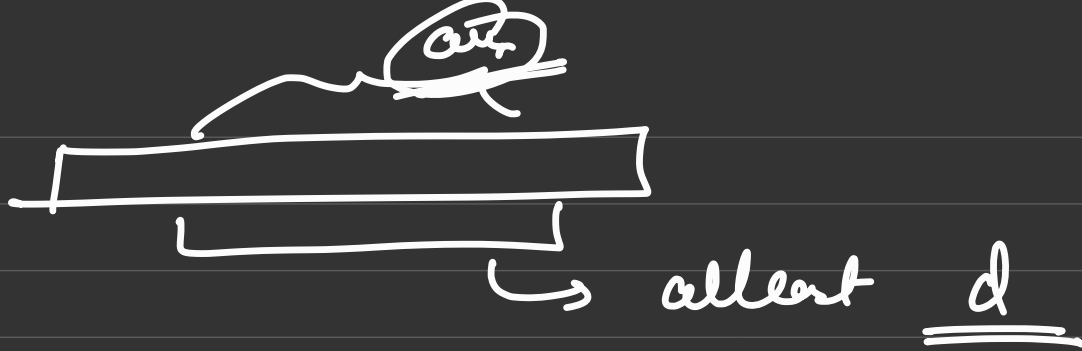
Q Given an array of size  $n$  and an integer  $d$ . find a subarray with length atleast  $d$ , such that average of elements of this subarray is max possible.

3, 1, 8, 5, 7, 2  $d=2$

$d, n \leq 10^5$

$a_i \leq 10^2$

$(3, 5) \rightarrow$  index



$$\max \left( \frac{\sum_{i=l}^r a_i}{r-l} \right) \rightarrow \underline{\underline{avg}}$$

$$r-l \geq d$$

res no

BS  
 ↓  
 BS on res no  
100 - rsc

$$\frac{\sum_{i=1}^x a_i}{x-1} \geq \underline{\underline{mid}}$$



$$\sum_{i=l}^r a_i \geq \text{mid}(r-l)$$

$$\sum_{i=l}^r (a_i - \text{mid}) \geq 0$$

$$r-l \geq d$$

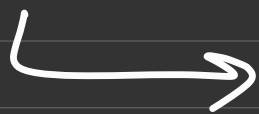
Prefix sum technique

$$\sum_{i=l}^r (a_i - \text{mid}) \geq 0$$

$$r - l \geq d$$

$$p[l] = \sum_{d=0}^{l-1} a_d - \text{mid} \rightarrow \text{let's defer proof}$$

$$\sum_{i=l}^r (a_i - \text{mid}) \geq 0$$



$$p[r] - p[l] \geq 0$$

$$r - l \geq d$$

$d \rightarrow$  use len

$$r \rightarrow [d+1, n-1]$$

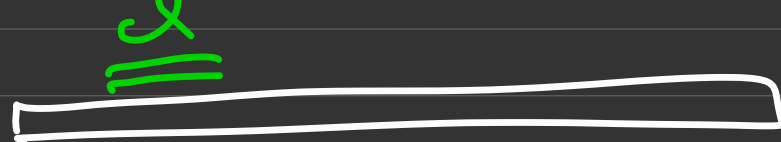
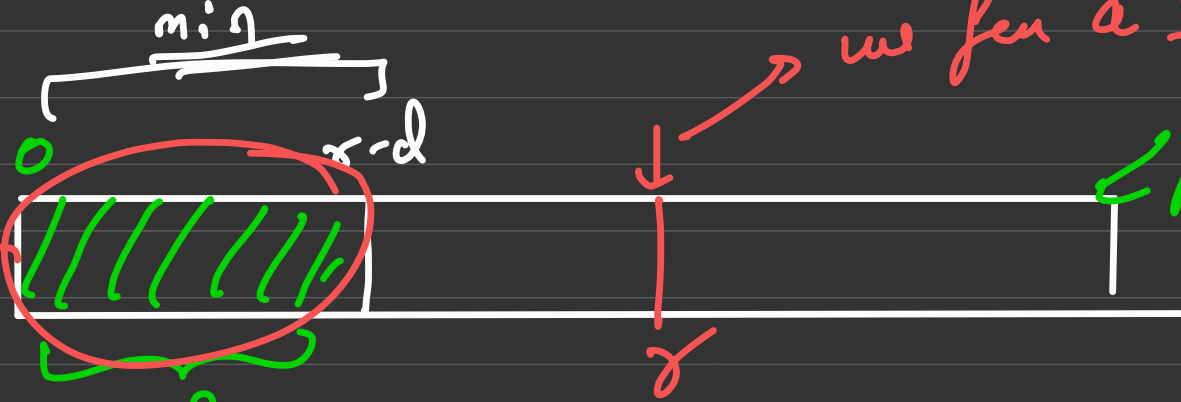
$$p[r] - p[l] \geq 0$$

$$r - l \geq d$$

$$l \leq r - d$$

use fen a r

$\min(p_i)$   
 $i \in [0, r-d]$



← prefer min on  
 $\min(p_i) \leq p_r$

$$p_r - p_l \geq 0 \rightarrow p_r \geq p_l$$

$$\min(r-d) \leq \underline{p(r)}$$

mid cuts & works so go on right

$$\underline{\min(0, r-d)} = \min(\min(0, r-d-1), p(r-d))$$

$p(i) \rightarrow$  cost for sum  
 $\Sigma(q_1 - mid)$

Q<sup>n</sup> You're getting 100% scholarship in a college by giving a test. To avoid any losses college came up with a scheme. College has  $N$  students &  $m$  discount coupon, A student gets 100% scholarship if he/she gets  $X$  coupons.

To have more coupons, in addition to the initial  $m$ , college says that those who perform badly in test need to pay additional  $y$  coupons. find max no. of students who get 100% scholarship.



$$1 \leq n, m, x, y \leq 10^9$$

$$N=5 \quad m=10 \quad x=2 \quad y=1 \quad \text{ans} \rightarrow \underline{\underline{5}}$$

$$N=3 \quad m=10 \quad x=4 \quad y=2 \quad \text{ans} \rightarrow \underline{\underline{2}}$$



mid  $\rightarrow$  guess

$O(\log n)$

Can we give 100 £ scholarship to  
mid students 3, 2

How many coupons reqd  $\rightarrow$  at least  
 $x \times \text{mid}$

BS on ans

$$\underbrace{m}_{\text{unclipped}} + (N - m) \times y \geq x \times \text{mid}$$

lower