$$f_n = f_{n-1} + f_{N-2} + f_{N-1} * f_{N-2}$$

$$f_n = f_{n-1}(1 + f_{n-2}) + f_{n-2} + 1 - 1$$

$$f_n = (1 + f_{n-1})(1 + f_{n-2}) - 1$$

$$1 + f_n = (1 + f_{n-1})(1 + f_{n-2})$$

$$1 + f_x = \underline{\underline{G_x}}$$

$$G_n = (G_{n-1})(G_{n-2})$$

$G_0 = a$ $\quad$ $G_1 = b$

$G_2 = ab$

$G_3 = ab^2$

$$\boxed{G_4 = a^2 b^3}$$

$G_5 = a^3 b^5$

$G_n = a^{f_{n-1}} b^{f_n}$

$a^2 b^3$

$a = 1+1 = 2$

$b = 2+1 = 3$

3

$2^{\times 3}$

$2^{\times 3}$

$= 7 \quad 4 \; 6 \; 2 \; 7$

10 $\quad$ 8-1

$$G_n = \left(a^{f_{n-1}} b^{f_n}\right) \% m$$

$G_n \ \% \ (10^9+7)$

prime no

$$G_n = \left(a^{f_{n-1}} \% m \times b^{f_n} \% m\right) \% m$$

$f_n \rightarrow$ can be big

$\hookrightarrow f_n \rightarrow$ matrix expo

$\rightarrow O(\log n)$

$$\boxed{a^{p-1} \% p = 1}$$

$\rightarrow$ By fermat theorem.

$$f_{n-1} = k \times (p-1) + f_{n-1} \% (p-1)$$

$$\boxed{14 = 2 \times 5 + 4}$$

$$a^{f_{n-1}} \bmod p = \left(a^{k \times (p-1) + \overbrace{f_{n-1} \bmod (p-1)}^{r}} \right) \bmod p$$

$$= \left(a^{k(p-1) + r} \right) \bmod p$$

$$= \underbrace{\left(a^{k(p-1)} \bmod p \right)}_{1} \times \left(a^{r} \bmod p \right)$$

→ fermat thm

$$a^{f_{n-1}} \bmod p = a^{r} \bmod p$$

→ modular expo

$$r = f_{n-1} \bmod (p-1)$$

Euler Totient Function

→ $\phi(n)$ → No. of m such that

phi n $\quad$ $n > 1$

$1 \le m < n$ && $\gcd(m, n) = 1$

m, n are co-prime.

$\phi(3) = 2 \rightarrow (1, 2)$

$\phi(4) = 2 \quad (1, 3)$

$\phi(5) = 4 \quad (1, 2, 3, 4)$

$$\phi(a \times b) = \phi(a) \times \phi(b) \quad \text{if } \gcd(a,b)=1$$

$\phi(n)$

$$n = p_1^{a} p_2^{b} p_3^{c} \cdots\cdots p_x^{z}$$

$$10 = 2^1 \times 5^1$$

$$\phi(n) = \phi(p_1^{a} p_2^{b} p_3^{c} \cdots\cdots)$$

$$\phi(p_1^{x} p_2^{y}) = \phi(p_1^{x})\, \phi(p_2^{y})$$

becoz $\gcd(p_1^{x}, p_2^{y}) = 1$

$$\phi(n) = \phi(p_1^{a})\,\phi(p_2^{b})\,\phi(p_3^{c})\cdots$$

$$\phi(p^a)$$

$p^a = 1, 2, 3, \ldots p, \ldots 2p \ldots 3p \cdots\cdots p^a$

Total elmt $= p^a$

$p^a - (\text{Total not coprime to } p^a)$

all multiple of p

$$\underline{\underline{\phi(p^a)}} = p^a - p^{a-1}$$

$$\Rightarrow p^a\left(1 - \frac{1}{p}\right)$$

$$\phi(n) = p_1^a\left(1 - \frac{1}{p_1}\right) p_2^b\left(1 - \frac{1}{p_2}\right) \cdots\cdots$$

$$\phi(n) = \underbrace{p_1^a \times p_2^b \times p_3^c \cdots\cdots}_{} \times \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\cdots$$

$$\phi(n) = n \times \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \times \left(1 - \frac{1}{p_3}\right)\cdots\cdots$$

1 2 3 4 5 6 7 8 9

Siev → 1 2 3 $\cancel{4}_2$ 5 $\cancel{6}_2$ 7 $\cancel{8}_4$ $\cancel{9}_6$

$4 \times \left(1 - \frac{1}{2}\right)$

1, 2, 4, 5, 7, 8

$2 \times \left(1 - \frac{1}{2}\right)$

$2 \times \frac{1}{2}$ → 1

$3 \times \left(1 - \frac{1}{3}\right)$

→ 2

$$\boxed{\phi(p) = p-1}$$

$$\phi(p) = p-1 \quad \text{if } p \to \text{prime}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$s \to$	1	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~	~~9~~	~~10~~	11	~~12~~	13	~~14~~
		1	2	2	4	~~3~~	6	4	6	~~5~~		~~6~~		~~7~~
						2				4		4		6