

Percentages, Ratio and Proportion

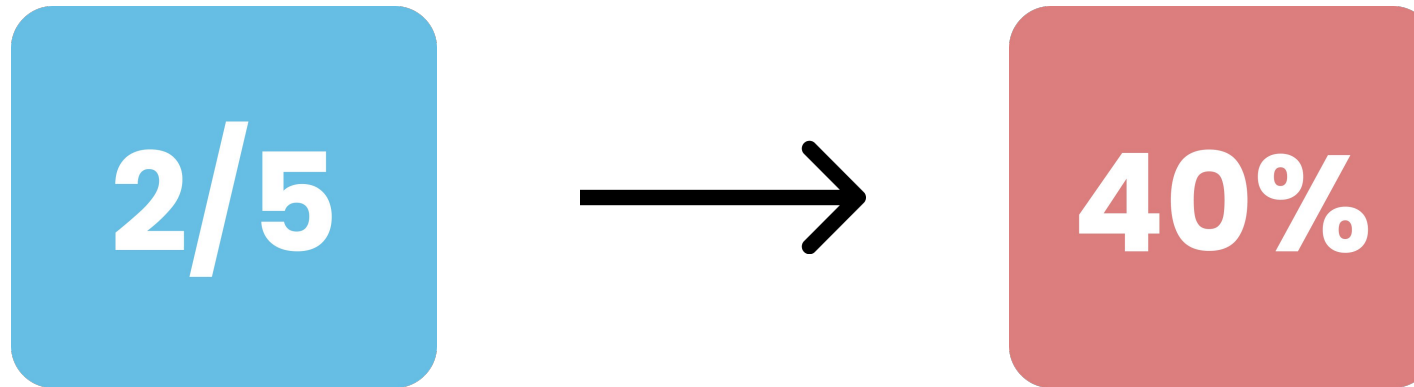
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Terms and Concepts Related to Percentages

1. To convert a fraction or decimal into percent, multiply by 100.

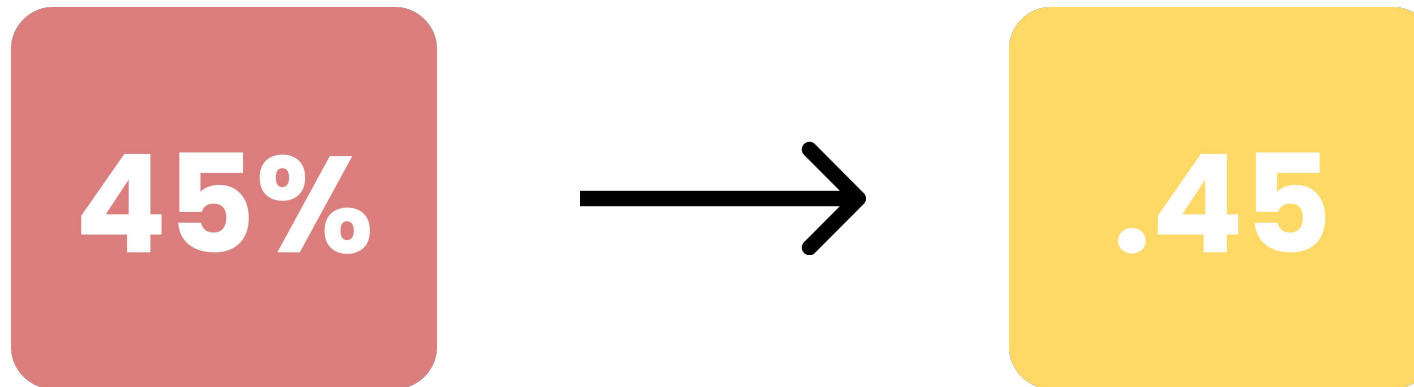
- $(2/5) \rightarrow (2/5) \times 100 = 40\%$
- $(3/8) \rightarrow (3/8) \times 100 = 37.5\%$
- $(4/9) \rightarrow (4/9) \times 100 = 44.44\%$
- $0.27 \rightarrow 0.27 \times 100 = 27\%$
- $0.5625 \rightarrow 0.5625 \times 100 = 56.25\%$



Terms and Concepts Related to Percentages

2. To convert a percent into fraction or decimal, divide by 100.

- $35\% \rightarrow (35/100) = 0.35$
- $87.5\% \rightarrow (87.5/100) = 0.875$
- $45 \rightarrow (45/100) = 0.45$



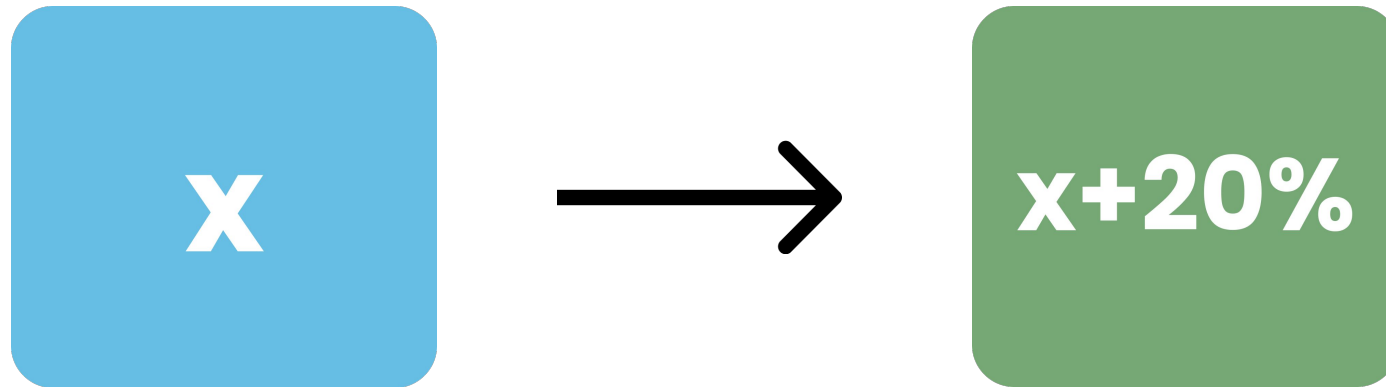
Terms and Concepts Related to Percentages

3 . Multiplying factor for Percentage increase: To increase a number by a given %, multiply the number by $[(100+\text{Rate})/ 100]$

Solve the below questions:

(1) Increase 20 by 15%

(2) The price of a commodity is Rs. 80. If price increases by 35%, find the new price.



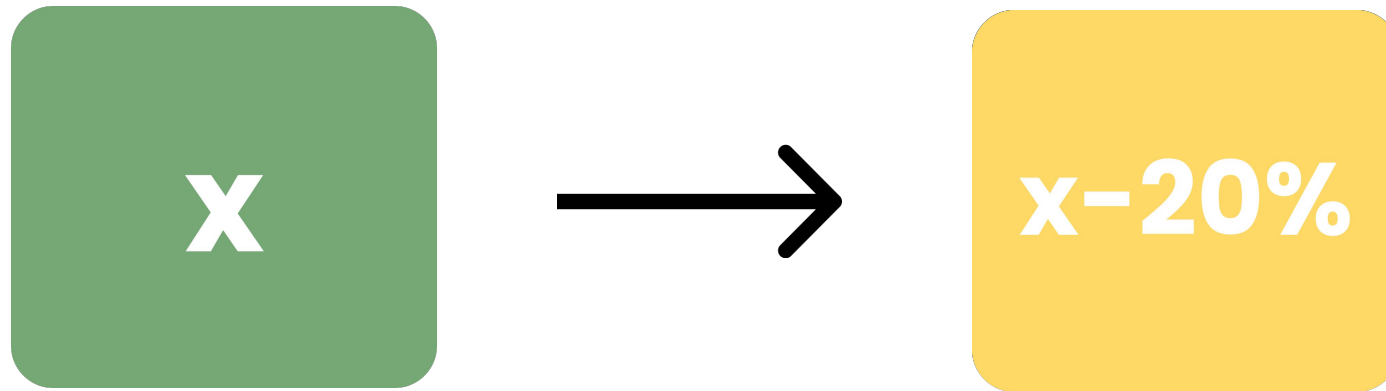
Terms and Concepts Related to Percentages

4 . Multiplying factor for Percentage decrease: To decrease a number by a given %, multiply the number by $[(100-\text{Rate})/ 100]$

Solve the following questions:

(1) Decrease 30 by 20%

(2) The sale of Tata motors in 2020 was 15000 Cr. Due to fuel price hike, sale decreased by 24% in 2021. Find current sale of Tata motors.



What Is Absolute Percentage Change?

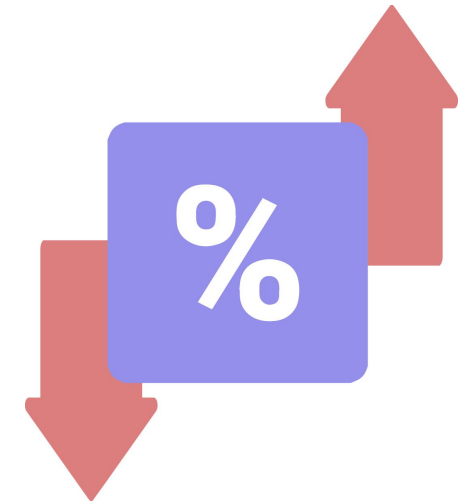
Absolute percentage change is concerned only with the actual quantity of percentage change between two terms and not the positive or negative value of percentage change.

$$\text{Percentage Change} = \frac{\text{Final value} - \text{Initial Value}}{\text{Initial Value}} \times 100$$

$$\text{Absolute Percentage Change} = \frac{|\text{Final value} - \text{Initial Value}|}{\text{Initial Value}} \times 100$$

Examples: -

- (1) A chair costs Rs 1200. After sometimes it becomes Rs 1500. Find percentage increase in the price of chair.
- (2) The sales of a company are Rs. 2500 Cr in the year 2020 and Rs. 2200 Cr in the year 2021. Find the absolute percentage change in the sales of company from 2020 to 2021.



Cases of Percentage Based Problems

1. If the price of a commodity increases by $x\%$, then reduction in consumption, at same level of expenditure =

$$\left(\frac{x}{100 + x} * 100 \right) \%$$

Questions to Solve:

- The price of a commodity increased by 20%. By what % will a consumer decrease its consumption so that expenditure on this commodity remains the same?
- The price of mustard oil is increased by 15%. By what percent should its selling price be decreased so that profit or loss remains unchanged?



Cases of Percentage Based Problems

2. If the price of a commodity decreases by $x\%$, then increase in consumption so as not to decrease expenditure is =

$$\left(\frac{x}{100 - x} * 100 \right) \%$$

Problem to solve: The price of petrol decreased by 10%. By what % will a consumer increase consumption so that expenditure on petrol remains the same.



Cases of Percentage Based Problems

3. If A's income is $x\%$ more than that of B, then B's income is less than that of A by

$$\left(\frac{x}{100 + x} * 100 \right) \%$$

Problem to solve: Ram's income is 25% more than Shyam's income. By what % Shyam's income is less than Ram's income?

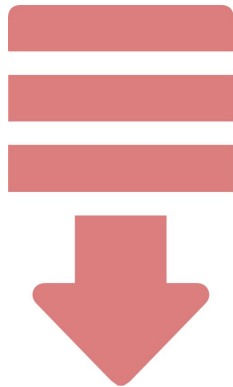


Cases of Percentage Based Problems

4. If A's income is $x\%$ less than that of B, then B's income is more than that of A by

$$\left(\frac{x}{100 - x} * 100 \right) \%$$

Example: A's income is 30% less than B's income. By what % B's income is more than A's income?



Successive Percentage Change

1. If a number is increased successively by $a\%$ and $b\%$, then net percentage increase is given by $r = a + b + \frac{ab}{100}$

2. If a number is decreased successively by $a\%$ and $b\%$, then net percentage decrease is given by $r = -a - b + \frac{ab}{100}$

Note: For increase/ mark-up/ profit \square use +ve sign and for decrease/ discount/ loss \square use -ve sign.

Question to solve: The price of a commodity is diminished by 15% and its sale increased by 10%.

- Find the effect on the revenue derived from it.
- With what increase percent in its sale would the revenue remains the same?



Practice Problem

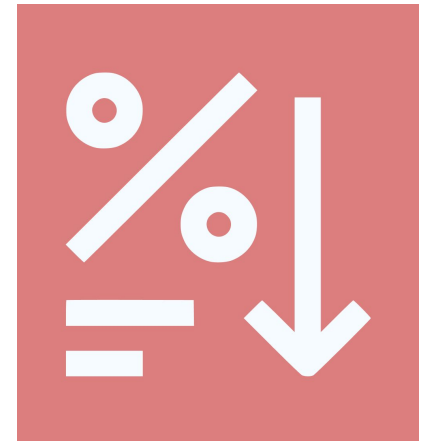
1. Two numbers x and y are 20% and 28% less than a third number z . Find by what percentage is the number y less than the number x ?

- A. 8%
- B. 12%
- C. 10%
- D. 9%

Questions 2 to 5: It is given that 5% increase in X always means 3% increase in Y and 5% increase in Y always implies 2.5% increase in Z .

2: Relationship between X and Z could be

- A. $1.05x = 1.025 Z$
- B. $0.5x = 0.25Z$
- C. $25 x = 7.5Z$
- D. $250 x = 213Z$



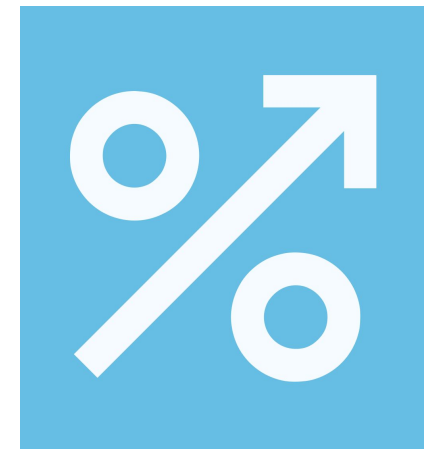
Practice Problem

3: If Y is increased by 30%, then Z^2 should increase by

- A. 32.25%
- B. 60%
- C. 69%
- D. 90%

4: If X is increased to 23 from 20, then Y should increase from 100 to

- A. 103
- B. 106
- C. 109
- D. 112



Practice Problem

5: If X is increased from 10 to 15, then Z must increase from 30 to

- A. 35
- B. 45
- C. 60
- D. 75

6: A is 50% more than B, C is $\frac{2}{3}$ of A and D is 60% more than C. Now each of A, B, C and D is increased by 10%. Find what percent of B is D after the increase?

- A. 150%
- B. 160%
- C. 175%
- D. 176%



Ratio and Proportion: Terms and Concepts:

The ratio of a and b is denoted by $a : b$ and measured by $\frac{a}{b}$

In the ratio $a : b$, a is called 'antecedent' and b is called 'consequent'

While comparing two quantities in terms of ratio:

- (i) the two quantities must be of the same kind
- (ii) the units of measurement of the two quantities must be the same.



a:b



a/b

Ratio and Proportion: Terms and Concepts:

Compounded Ratio:

Ratios are compounded by multiplying them.

Example: Compounded ratio of 3 : 4 and 5 : 7 is $(3/4) * (5/7)$ ie. $(15/28)$

Duplicate Ratio and Sub duplicate Ratio:

The duplicate ratio of $a : b$ is given by $a^2 : b^2$ and the sub duplicate ratio of $a : b$ is $\sqrt{a} : \sqrt{b}$

Example: The duplicate ratio of 4 : 9 is 16 : 81 and sub duplicate ratio of 4 : 9 is 2 : 3.



Ratio and Proportion: Terms and Concepts:

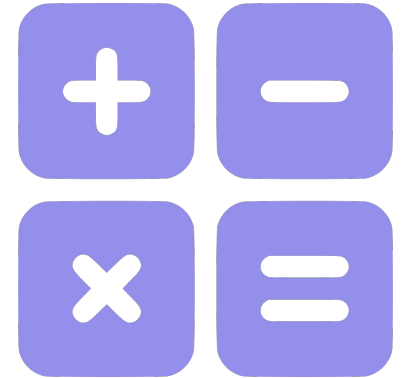
Triplicate and Sub Triplicate Ratio:

The triplicate ratio of $a : b$ is $a^3 : b^3$ and sub triplicate ratio of $a : b$ is $\sqrt[3]{a} : \sqrt[3]{b}$

Example: The triplicate ratio of $1 : 8$ is $1 : 512$ and sub duplicate ratio of $1 : 8$ is $1 : 2$.

Inverse Ratio or Reciprocal Ratio: Inverse ratio of $a : b$ is given by $\frac{1}{a} : \frac{1}{b}$

Example: Find the inverse ratio of $2 : 3 : 4$.



Practice Questions

1: If $a : b = 3 : 4$, $b : c = 5 : 6$, Find $a : b : c$

- A. 3:20:6
- B. 15:12:18
- C. 15:20:24
- D. 4:5:6

2: If $a : b = 2 : 3$, $b : c = 4 : 5$, $c : d = 3 : 5$, find $a : b : c : d$.

- A. 8:12:9:25
- B. 14:15:13:12
- C. 8:14:15:16
- D. 8:12:15:25



Practice Questions

3: If $2a = 3b = 4c$, find $a : b : c$.

- A. 2:3:4
- B. 4:9:16
- C. 6:4:3
- D. 3:4:6

4: If 4 times A's share = 3 times B's share = 5 times C's share, what is the ratio of A's share , B's Share and C's share?

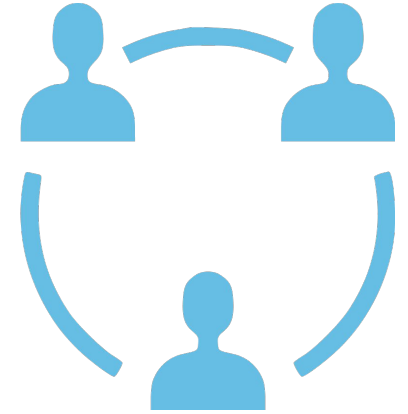
- A. 4:5:6
- B. 15:12:20
- C. 15:20:12
- D. 16:9:25



Practice Questions

5: A sum of Rs. 1400 is divided amongst A, B, C and D such that A's share : B's share = B's share : C's share = C's share : D's share = 3 : 4. How much is C's share

- A. Rs 72
- B. Rs 288
- C. Rs 216
- D. Rs 384



Proportion and Continued Proportion

Proportion is a statement that tells two given ratios are equal.

i.e. If $a:b$ is same as $c:d$, then a , b , c and d are in proportion.

OR

We can say if $\frac{a}{b} = \frac{c}{d} \rightarrow ad = bc$, then a , b , c and d are in proportion.

a is called first, b second, c third and d is called fourth proportional.

If $a:b$ is same as $b:c$ then we say a , b and c are in continued proportion.

In continued proportion, $b^2 = ac \rightarrow b$ is called mean proportional, a is first and c is third.



a:b



a/b

Practice Questions

1. Find compounded ratio of

i) 2: 3, 3: 6 and 4: 7

ii) 4: 3, 9: 13, 26: 5 and 2: 15

iii) $4a: 3b$, $7a: 8c^3$ and $c^2: 7a^3$

2. Find:

i) Duplicate ratio of 4: 3

ii) Sub duplicate ratio of 4: 3

iii) Triplicate ratio of 4: 3

iv) Sub triplicate ratio of 4: 3

v) Inverse ratio of 4: 3



4:3

Practice Questions

3. Find three numbers in the ratio of 3: 2: 5, such that the sum of their squares is equal to 1862.
4. Divide Rs. 680 among A, B and C such that A gets $\frac{2}{3}$ of what B gets and B gets $\frac{1}{4}$ of what C gets.
5. Compare the ratios 2: 3 and 4: 7.



Practice Questions

6. If $a : b = 2 : 3$, $b : c = 4 : 5$ find $a : b : c$?

7. If $a : b = 3 : 4$, $b : c = 8 : 9$, $c : d = 15 : 16$

i) Find the ratio for a to d

ii) Compare a, b, c & d

8. Given $a : b : c = 2 : 3 : 4$, find $\frac{1}{a} : \frac{1}{b} : \frac{1}{c}$



Practice Questions

9. Calculate a fourth proportional to the numbers.

(i) 1, 2, 3

(ii) 500, 70, 69

(iii) 2.5, 1.5, 1.5

10. Find a third proportional to the numbers.

(i) 3, 6

(ii) 1.2, 1.8

(iii) 225, 75



Practice Questions

11. Calculate mean proportional between

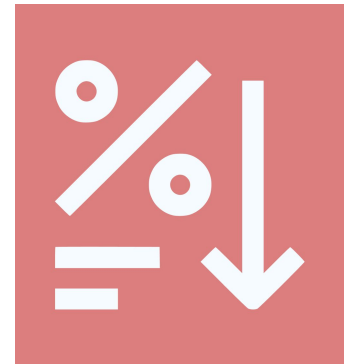
(i) 3 and 192

(ii) $\frac{14}{363}$ and $\frac{21}{128}$

(iii) 0.5 and 1922.

12. If $x = \frac{6ab}{a+b}$,

find the value of $\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b}$



Practice Questions

13. A tiger pursues a deer and takes 5 leaps for every 7 leaps of the deer, but 3 leaps of the tiger are equal to 5 leaps of the deer. What is the ratio of their speeds?

- A. 15: 35
- B. 25: 21
- C. 24: 21
- D. CBD

14. The content of two vessels containing water and milk in the ratio 1: 2 and 2: 5 are mixed in the ratio 1: 4. The resulting mixture will have water and milk in the ratio:

- A. 32: 55
- B. 3: 65
- C. 31: 74
- D. None of these



Practice Questions

15. Three glasses A, B and C with their capacities in the ratio 2: 3: 4 are filled with a mixture of spirit and water. The ratio of spirit to water in A, B and C is 1 : 5, 3 : 5 and 5 : 7 respectively. If the contents of these glasses are mixed together. Find the ratio of spirit to water in the mixture.

- A. 26: 48
- B. 47: 25
- C. 25: 47
- D. 46: 25

16. 465 coins consist of 1 rupee, 50 paise and 25 paise coins. Their values are in the ratio of 5: 3: 1. Find the number of each coin.

- A. 155, 187, 125
- B. 155, 186, 124
- C. 166, 186, 124
- D. None



Answers

(1). (i) 4 : 21, (ii) 16 : 25, (iii) 1: 6 ac

(2). (i) 16 : 9, (ii) $2 : \sqrt{3}$, (iii) 64 : 27, (iv) $\sqrt[3]{4} : \sqrt[3]{3}$, (v) 3 : 4

(3). 21, 14, 35

(4). A's share = Rs. 80, B's share = Rs. 120, C's share = Rs. 480

(5). $(2 : 3) > (4 : 7)$

(6). 8 : 12 : 15

(7). (i) $a : d = 5 : 8$ (ii) $a : b : c : d = 30 : 40 : 45 : 48$

(8). 6 : 4 : 3

(9). (i) 6, (ii) $9 \frac{33}{50}$, (iii) 0.9

(10). (i) 12, (ii) 2.7, (iii) 25



Answers

(11). (i) 24, (ii) $\frac{7}{88}$, (iii) 31

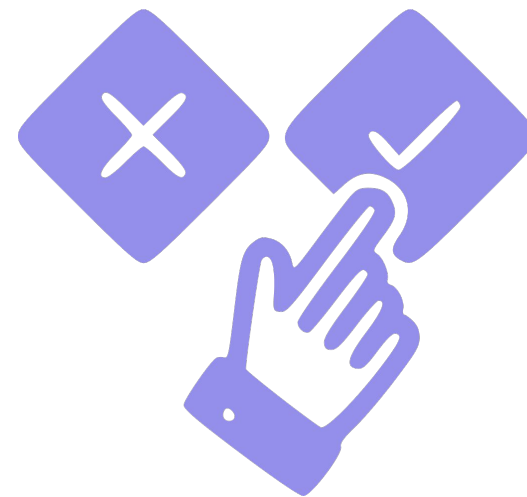
(12). 2

(13). B

(14). C

(15). C

(16). B



Practice Questions

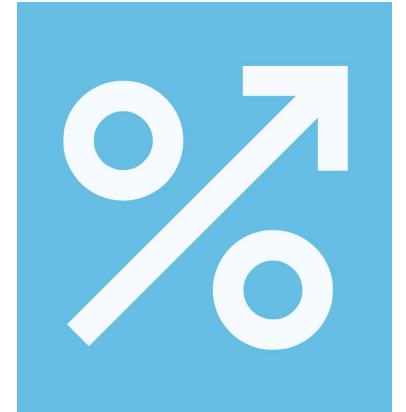
1. If $37\frac{1}{2}\%$ of a number is 900, what is $62\frac{1}{2}\%$ of the number?

2. i) $a\%$ of $b + b\%$ of $a = ?\%$ of ab

ii) 40% of $70 = 4 \times ?$

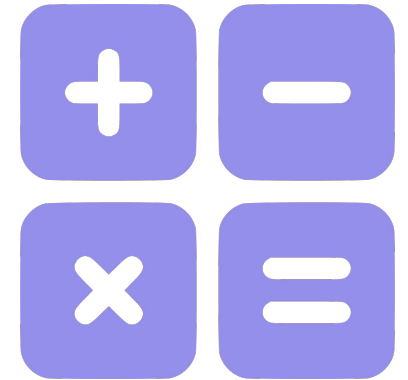
iii) 0.025 in terms of percent is

3. My salary is increased from Rs 12000 to Rs. 15000 per month. What is the percentage change in my salary?



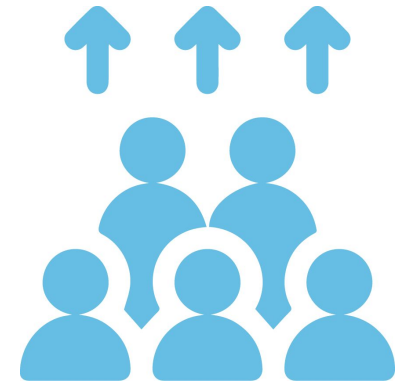
Practice Questions

4. A number is multiplied by 2 instead of dividing it by 2. Find the percentage change in the result.
5. If the price of an article is raised by 20%, by how much percent a household must reduce consumption of that article so that expenditure is same.
6. Due to a fall of 10% in the rate of tea, 500 gm more tea can be purchased for Rs. 140. Find original and reduced price of tea.



Practice Questions

7. If the annual increase in the population of a town is 4% and the present number of inhabitants is 15625; what will the population be in 3 years?
8. The population of a town is P. It increases by 5% during first year, by 10% during the second year and by 20% during the third year. Find the population after 3 years. Also find net percentage change in population after 3 years.
9. The value of a machine depreciates at the rate of 10% after each year. Find its purchase price, if at the end of 3 years its value is only Rs. 8748.



Practice Questions

10. The price of a commodity is diminished by 15% and its consumption increased by 10%.

i) Find the effect on the revenue derived from it.

ii) With what increase percent in its consumption would the revenue remain the same?

11. A is increased by 7%, B is increased by 12% and C is decreased by 11%. Find the percentage change in A.B.C?

12. A shopkeeper marks up his goods 12% higher than its original price. After that he allows a discount of 12%. What is his percentage profit or loss?



Practice Questions

13. Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight. The weight of dry grapes available from 20 kg of fresh grapes will be?

14. In an examination, 70% of the candidates passed in English, 65% in Mathematics, 27% failed in both the subjects and 248, passed in both the subjects. Find the total number of candidates.

15. 140 litres of an acid contain 90% of acid and the rest water. How much water must be added to make the water 12.5% of the resulting mixture?



Practice Questions

16. Shanta has some money with her. She gave 50% of it to Deepak and 30% to Bhupendra. 60% of the remaining was donated to a school. If Rs 8040 are still left with her, find the original amount of money she had.

17. If A's salary is 50% more than B's, then by what percent B's salary is less than A's salary?

18. The price of milk is increased by 10%. Find by what percent consumption of milk should be reduced, so as not to increase the expenditure.

- A. 9%
- B. 10.1%
- C. 9.09%
- D. 10.01%



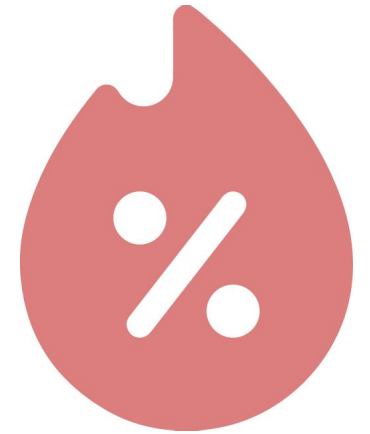
Practice Questions

19. A no. is increased by 10%. Again, it is increased by 15%, what is net % increase in the number?

- A. 25%
- B. 26.5%
- C. 25.5%
- D. none

20. A shopkeeper marks the price of his goods at 20% higher than the original price. After that he allows a discount of 10%. What profit or loss did he get?

- A. 8% profit
- B. 8% loss
- C. 10% profit
- D. 10% loss



Practice Questions

21. If the price is increased by 10% and the sale is decreased by 5%, then what will be the effect on income?

- A. 5% decrease
- B. 5% increase
- C. 4.5% increase
- D. 4.5 % decrease

22. After receiving two successive raises, Harsh's salary becomes equal to $\frac{15}{8}$ times of his initial salary. By how much percentage was the salary raised the first time if the second raise was twice as high (in percent) as the first

- A. 20%
- B. 25%
- C. 30%
- D. 35%



Practice Questions

23. What is 20% of 50% of 75% of 70?

- A. 7.25
- B. 5.00
- C. 5.25
- D. 4.25

24. 379 is what percentage of 45?

- A. 841.2%
- B. 841.8%
- C. 842.2%
- D. 842.6%

20%

50%

75%

↓
70

Practice Questions

25. Which of the following is/are true?

- a) 5% of x added to x is equal to multiplying x by 1.05
 - b) 20% of x subtracted from x is equal to dividing x by 1.25
 - c) 23% of x added to 17% of x is same as dividing x by 2.5
- A. a & b only
B. a & c only
C. b & c only
D. a, b & c



Answers

(1). 1500

(2). 2, 7, 2.5%

(3). 25%

(4). 300%

(5). $16 \frac{2}{3}$

(6). Rs 31.11 & Rs. 28

(7). 17576

(8). 1.386 P and 38.60%

(9). Rs 12000

(10). $6 \frac{1}{2}$ % decrease, $17 \frac{11}{17}$ %



Answers

(11). 6.6%

(12). 1.44% loss

(13). 2.5kg

(14). 400

(15). 4 litres

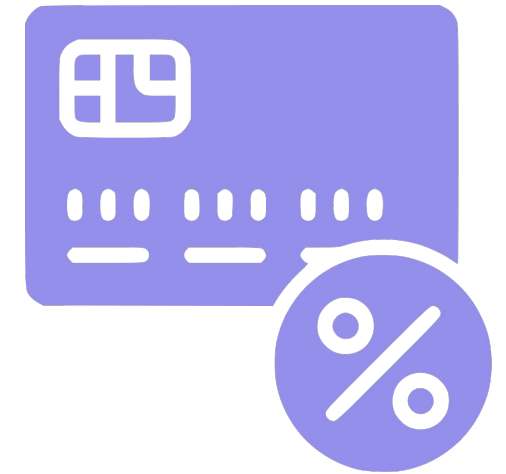
(16). Rs 100500

(17). 33.33%

(18). C

(19). B

(20). A



Answers

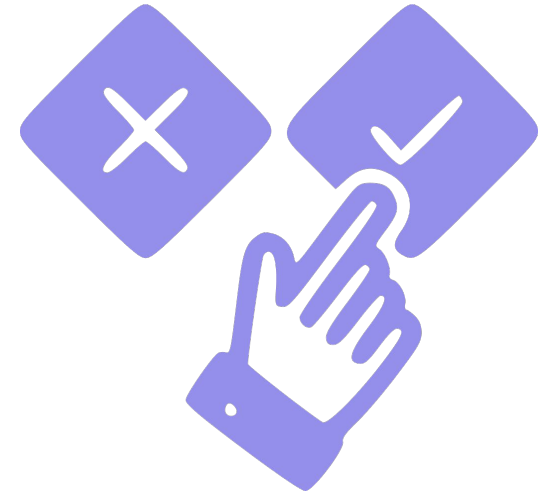
(21). C

(22). B

(23). C

(24). D

(25). C



In the next class, we will study



Caselets



Tables