Revision on Conditional Probability and intro to inferential stats + probability distributions 21oct22

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Set theory elements:

1. Union ✓
2. Intersection ✓
3. Difference ✓
4. Complement ✓
5. P(AUB) formula ✓
6. And = x or = +

If A & B are 2 events from a sample space or
A & B are 2 events from an experiment

- $P[A \text{ or } B \text{ occurs}] = P[A \cup B]$
- $P[A \text{ and } B \text{ both occur}] = P[A \cap B]$

$A - B$, $B - A$ in sets are different

$A \Delta B$ or symmetric difference $\rightarrow (A \cup B) - (A \cap B)$

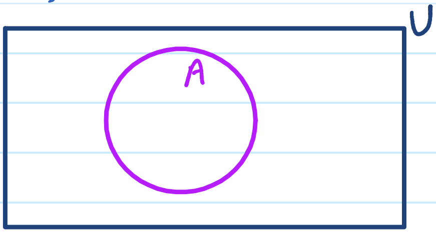
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- complement of a set :

Universal set (U) : A set which contains all the elements under consideration for my experiment.

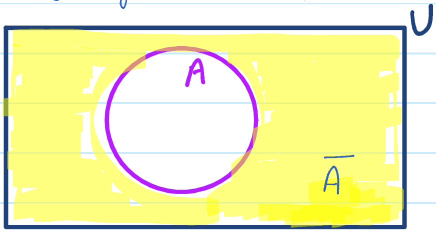
If $A \subset U$ then A^c or $\bar{A} = U - A$

subset

(everything other than A)



then



1. $P(A)$ lies b/w :

A -1 to 1 ✓ C 0 to 1

B $-\infty$ to ∞ D -1 to 0

2. $P(U) = 1$

Extra information about what has already happened thereby reducing the sample space

$P(A|B)$: Prob. of event A occuring when event B has already occured.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

For example, given that you drew a red card, what's the probability that it's a four (p(four|red))=2/26=1/13.

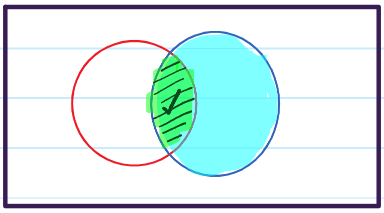
$$P(\text{four} \mid \text{Red}) = \frac{P(\text{red \& four})}{P(\text{red})} = \frac{2/52}{1/2}$$

$$= \frac{2}{52} \times 2 = \frac{4}{52} = \frac{2}{26}$$

$$= \frac{1}{13}$$

Show on venn

Show on venn



$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

DEFINITION

For any two events A and B with $P(B) > 0$, the **conditional probability of A given that B has occurred** is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad (2.3)$$

★ Hint $P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(A\cap C) - P(B\cap C) + P(A\cap B\cap C)$

Example 2.26 A news magazine publishes three columns entitled "Art" (A), "Books" (B), and "Cinema" (C). Reading habits of a randomly selected reader with respect to these columns are

	only	only	only	A&B	A&C	B&C	A&B&C
Read regularly	A	B	C	$A\cap B$	$A\cap C$	$B\cap C$	$A\cap B\cap C$
Probability	.14	.23	.37	.08	.09	.13	.05

Figure 2.9 illustrates relevant probabilities.

calc. $P(A\cup B|C)$ $\colon 33$

$P(A\cup B) = 0.14+0.23 - 0.08 = 0.29$

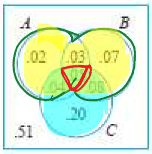


Figure 2.9 Venn diagram for Example 2.26

$P(A\cup B|C) = \dfrac{P[(A\cup B)\cap C]}{P(C)} = \dfrac{0.04+0.05+0.08}{0.37} = 0.459$

We thus have

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{.08}{.23} = .348$$

$$P(A|B\cup C) = \frac{P(A\cap(B\cup C))}{P(B\cup C)} = \frac{.04+.05+.03}{.47} = \frac{.12}{.47} = .255$$

$$P(A|\text{reads at least one}) = P(A|A\cup B\cup C) = \frac{P(A\cap(A\cup B\cup C))}{P(A\cup B\cup C)}$$

$$= \frac{P(A)}{P(A\cup B\cup C)} = \frac{.14}{.49} = .286$$

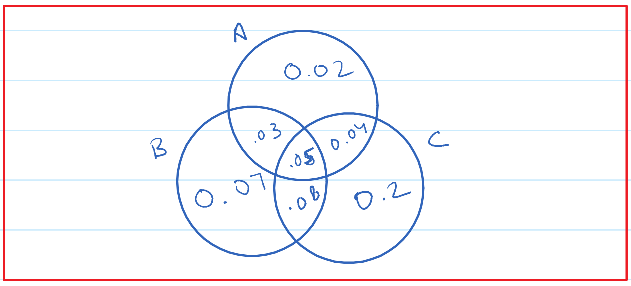
and

$$P(A\cup B|C) = \frac{P((A\cup B)\cap C)}{P(C)} = \frac{.04+.05+.08}{.37} = .459$$

$P((A\cup B)\cap C) = P[(A\cap C)\cup(B\cap C)]$

$= P[A\cap C] + P[B\cap C] - P[(A\cap C)\cap(B\cap C)]$

	only	only	only	A&B	A&C	B&C	A&B&C
Read regularly	A	B	C	$A\cap B$	$A\cap C$	$B\cap C$	$A\cap B\cap C$
Probability	.14	.23	.37	.08	.09	.13	.05



Example 1: The table below shows the occurrence of diabetes in 100 people. Let D and N be the events where a randomly selected person "has diabetes" and "not overweight". Then find P(D | N).

	Diabetes (D)	No Diabetes (D')
Not overweight (N)	$\boxed{5}$ $D \cap N$	45
Overweight (N')	17	33

N

D → diabetes

N → not overweight

$P(D|N)$

$$= \frac{P(D \cap N)}{P(N)} = \frac{5/(5+45+17+33)}{(5+45)/(5+45+17+33)}$$

$$= \frac{5/100}{50/100} = \frac{1}{10}$$

Solution:

From the given table, P(N) = (5+45) / 100 = 50/100.

P(D ∩ N) = 5/100.

By the conditional probability formula,

P(D | N) = P(D ∩ N) / P(N)

= (5/100) / (50/100)

= 5/50

= 1/10

HW

Example 3: If a fair die is rolled twice, observe the numbers that face up. Find the conditional probability that the sum of the numbers is 7, given that the first number is 2.

Solution:

Let us determine the sample space of rolling a die twice. S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6)(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

Considering events A and B as given: we have

A : the sum of the numbers is 7. Thus set A = {(1,6),(2,5), (3,4), (4,3), (5,2),(6,1) }

B: the first number is 2. Thus set B = {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)}

A ∩ B: {(2,5)}

By the conditional probability, we know that

P(A) = P(A ∩ B) / P(B)

$$P(A) = \frac{\frac{1}{36}}{\frac{6}{36}}$$

P(A) = 1/6

Answer: The conditional probability that the sum of the numbers is 7, given that the first number is 2 is 1/6

HW

Example: Two dies are thrown simultaneously and the sum of the numbers obtained is found to be 7. What is the probability that the number 3 has appeared at least once?

Solution: The sample space S would consist of all the numbers possible by the combination of two dies. Therefore S consists of 6 × 6 i.e. 36 events.

Event A indicates the combination in which 3 has appeared at least once.

Event B indicates the combination of the numbers which sum up to 7.

A = {(3, 1), (3, 2), (3, 3)(3, 4)(3, 5)(3, 6)(1, 3)(2, 3)(4, 3)(5, 3)(6, 3)}

B = {(1, 6)(2, 5)(3, 4)(4, 3)(5, 2)(6, 1)}

P(A) = 11/36

P(B) = 6/36

A ∩ B = 2

P(A ∩ B) = 2/36

Applying the conditional probability formula we get,

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P(A) = 11/36

P(B) = 6/36

$A \cap B = 2$

$P(A \cap B) = 2/36$

Applying the conditional probability formula we get,

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$$

HW

Example 3 Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

Solution Let A be the event 'the number on the card drawn is even' and B be the event 'the number on the card drawn is greater than 3'. We have to find P(A|B).

Now, the sample space of the experiment is S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Then $A = \{2, 4, 6, 8, 10\}$, $B = \{4, 5, 6, 7, 8, 9, 10\}$

and $A \cap B = \{4, 6, 8, 10\}$

Also $P(A) = \frac{5}{10}$, $P(B) = \frac{7}{10}$ and $P(A\cap B) = \frac{4}{10}$

Then $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{4}{10}}{\frac{7}{10}} = \frac{4}{7}$

Dependent and Independent events

Statistical definition: Two events A & B are said to be statistically independent if and only if $\boxed{P(A) \times P(B) = P(A\cap B)}$

Example 10 A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even' then find whether E and F are independent?

E → mult of 3
F → no. is even.

Solution We know that the sample space is S = {1, 2, 3, 4, 5, 6}

Now $E = \{3, 6\}$, $F = \{2, 4, 6\}$ and $E \cap F = \{6\}$

Then $P(E) = \frac{2}{6} = \frac{1}{3}$, $P(F) = \frac{3}{6} = \frac{1}{2}$ and $P(E\cap F) = \frac{1}{6}$

Clearly $P(E \cap F) = P(E).\ P(F)$

Hence E and F are independent events.

$P(E) = 2/6$
$P(F) = 3/6$
$P(E\cap F) = 1/6$
$P(E) \cdot P(F) = \frac{2}{6} \times \frac{3}{6} = \frac{1}{6}$
$\because P(E) \cdot P(F) = P(E\cap F)$
$\Rightarrow$ independent.

HW

Example 11 An unbiased die is thrown twice. Let the event A be 'odd number on the first throw' and B the event 'odd number on the second throw'. Check the independence of the events A and B.

Solution If all the 36 elementary events of the experiment are considered to be equally likely, we have

$$P(A) = \frac{18}{36} = \frac{1}{2} \text{ and } P(B) = \frac{18}{36} = \frac{1}{2}$$

Also $P(A \cap B) = P$ (odd number on both throws)

$$= \frac{9}{36} = \frac{1}{4}$$

Now $P(A)\ P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

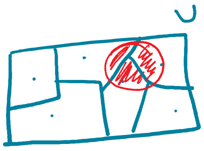
Clearly $P(A \cap B) = P(A) \times P(B)$

Thus, A and B are independent events

Bayes Theorem

The Bayes theorem establishes the likelihood of an event occurring given any condition. It is considered for the case of conditional probability. Also, this is known as the formula for the likelihood of "causes".

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$



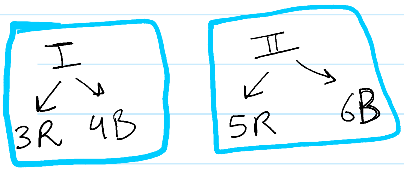
A,B = Events
P(A|B) = Probability of A given B is true
P(B|A) = Probability of B given A is true
P(A), P(B) = The independent probabilities of A and B

If $A_1, A_2, \dots, A_n$ is the partition of U and B is any other event.

$$\boxed{P(A_1|B) = \frac{P(B|A_1)\,P(A_1)}{P(B|A_1)\,P(A_1) + P(B|A_2)P(A_2) + \cdots + P(B|A_n)\,P(A_n)}}$$

Example 16 Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.

given info



1 ball is drawn
→ ball is red

Solution Let E_1 be the event of choosing the bag I, E_2 the event of choosing the bag II and A be the event of drawing a red ball.

Then $P(E_1) = P(E_2) = \frac{1}{2}$

Also $P(A|E_1) = P(\text{drawing a red ball from Bag I}) = \frac{3}{7}$

and $P(A|E_2) = P(\text{drawing a red ball from Bag II}) = \frac{5}{11}$

Now, the probability of drawing a ball from Bag II, being given that it is red, is $P(E_2|A)$

By using Bayes' theorem, we have

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)} = \frac{\frac{1}{2}\times\frac{5}{11}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{5}{11}} = \frac{35}{68}$$

$P(\text{II} | R)$

$$\frac{P(\text{II} \cap R)}{P(R)}$$

using Bayes thm here:

$$P(\text{I}|R) = \frac{P(R|\text{II})\,P(\text{II})}{P(R|\text{II})P(\text{II}) + P(R|\text{I})\,P(\text{I})}$$

$P(\text{I}) = 1/2$ $\quad$ $P(R|\text{II}) = 5/11$

$P(\text{II}) = 1/2$ $\quad$ $P(R|\text{I}) = 3/7$

$$P(\text{II}|R) = \frac{5/11 \times 1/2}{(5/11 \times 1/2) + (3/7 \times 1/2)} = \frac{5/11}{\frac{5}{11} + \frac{3}{7}} = \frac{\frac{5}{11}}{\frac{68}{77}} = \boxed{\frac{35}{68}}$$

HW

Example 18 Suppose that the reliability of a HIV test is specified as follows:

Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV–ive but 1% are diagnosed as showing HIV+ive. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ive. What is the probability that the person actually has HIV?

Solution Let E denote the event that the person selected is actually having HIV and A the event that the person's HIV test is diagnosed as +ive. We need to find P(E|A).

Also E′ denotes the event that the person selected is actually not having HIV.

Clearly, {E, E′} is a partition of the sample space of all people in the population. We are given that

$$P(E) = 0.1\% = \frac{0.1}{100} = 0.001$$

2022-23

$$P(E') = 1 - P(E) = 0.999$$

$P(A|E)$ = P(Person tested as HIV+ive given that he/she is actually having HIV)

$$= 90\% = \frac{90}{100} = 0.9$$

and $P(A|E')$ = P(Person tested as HIV +ive given that he/she is actually not having HIV)

$$= 1\% = \frac{1}{100} = 0.01$$

Now, by Bayes' theorem

$$P(E|A) = \frac{P(E)P(A|E)}{P(E)P(A|E) + P(E')P(A|E')}$$

$$= \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.01} = \frac{90}{1089}$$

$$= 0.083 \text{ approx.}$$

Thus, the probability that a person selected at random is actually having HIV given that he/she is tested HIV+ive is 0.083.

HW

Example 21 A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Solution Let E be the event that the man reports that six occurs in the throwing of the die and let S_1 be the event that six occurs and S_2 be the event that six does not occur.

Then $P(S_1)$ = Probability that *six* occurs $= \frac{1}{6}$

$P(S_2)$ = Probability that *six* does not occur $= \frac{5}{6}$

$P(E|S_1)$ = Probability that the man reports that *six* occurs when *six* has actually occurred on the die

= Probability that the man speaks the truth $= \frac{3}{4}$

$P(E|S_2)$ = Probability that the man reports that *six* occurs when *six* has not actually occurred on the die

= Probability that the man does not speak the truth $= 1 - \frac{3}{4} = \frac{1}{4}$

Thus, by Bayes' theorem, we get

$P(S_1|E)$ = Probability that the report of the man that *six* has occurred is actually a *six*

$$= \frac{P(S_1)P(E|S_1)}{P(S_1)P(E|S_1) + P(S_2)P(E|S_2)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{1}{8} \times \frac{24}{8} = \frac{3}{8}$$

Hence, the required probability is $\frac{3}{8}$

For intro to inf stats see old class deck <theoretical concepts are present only>

→ Sample & Population : sample ⊂ Pop^n

→ Statistic & Parameter

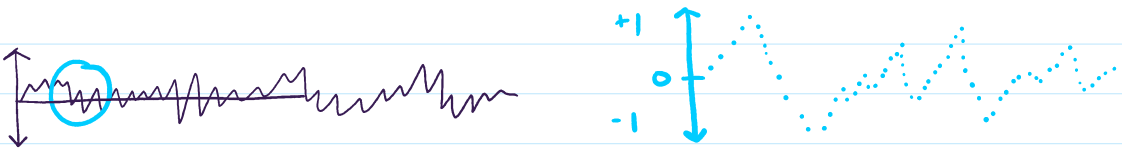
- changes w/ samp. · fixed
- function of samp. · function of pop

→ sample mean $(\bar{x})$: $\sum_{i=1}^{n} x_i / n$

pop^n (μ) $= \sum_{i=1}^{N} x_i / N$

→ sample var. $(s^2) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

pop^n var. $\sigma^2 = \frac{1}{N} \sum (x_i - \mu)^2$



Types of Sampling.

X → random variable : X - represents a population which follows a probability distribution. Since X can take various values at different trials, there is a probability attached to each value or a range of values that X can take, this combination of the values that X takes and the attached probability / probability density make up a probability distribution.

Probability distributions

We will be looking upon some useful p.d. today.

Revision of random variable and properties of probability distribution like pdf, range, definition, E(X), V(X)

→ Definition, Range, PDF/PMF, E(X), V(X)

(PDF: cont. ; PMF: discrete)

Discrete vs continuous

Probability Density Function and Probability Mass Function

- PDF is a statistical term that describes the probability distribution of the **Continuous** random variable

Probability Density Function

$F(x) = P(a \le x \le b) = \int_a^b f(x)dx \ge 0$

\# to calculate prob. we need to integrate pdf over the required range.

- PMF is a statistical term that describes the probability distribution of the **Discrete** random variable

$p(x) = P(X = x)$

The probability of x = The probability (X = one specific x)

- People often get confused between PDF and PMF. The PDF is applicable for continuous random variables while PMF is applicable for discrete random variables

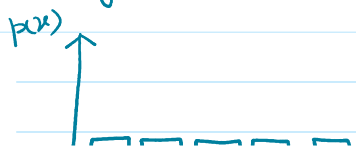
Examples:

DISCRETE

1. uniform → It has uniformity of probability at each value of X

7·8. Discrete Uniform Distribution. A random variable X is said to have uniform distribution on n points $\{x_1, x_2, ..., x_n\}$ if its p.m.f. is given by :

$$P(X = x_i) = \frac{1}{n};\ i = 1, 2, ..., n \qquad ...(7{\cdot}31)$$

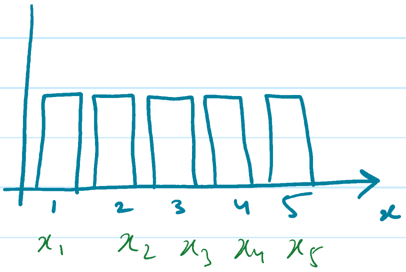


7·8. **Discrete Uniform Distribution.** A random variable X is said to have uniform distribution on n points $\{x_1, x_2, ..., x_n\}$ if its p.m.f. is given by:

$$P(X = x_i) = \frac{1}{n};\ i = 1, 2, ..., n \qquad ...(7\cdot31)$$

For example, if X has a uniform distribution on the points $\{0, 1, 2, ..., n\}$, then $P(X = i) = \frac{1}{n+1};\ i = 0, 1, 2, ..., n.$ ...(7·31 *a*)

Such distributions can be conceived in practice if under the given experimental conditions, the different values of the random variable become eugally likely. Thus for a die experiment, and for an experiment with a deck of cards such distribution is appropriate.



eg. taking a simple random sample.

If X~U(5, 100) find P[20<X<35]

eg: $X \sim U(5, 100)$ find $P[20 < X < 35]$

$= \frac{14}{96}$

range: $5 \le x \le 100$

$E(X) = \frac{\sum_5^{100} i}{96} = \sum p_i x_i$ $\quad \frac{\sum x_i}{n}$ $\quad \frac{5+6+7+\dots+100}{96} = \frac{5040}{96} = 52.5$

$E(X) = \frac{a+b}{2} = \frac{5+100}{2} = \frac{105}{2} = \boxed{52.5}$

2. Bernoulli

7·1. **Bernoulli Distribution.** A random variable X which takes two values 0 and 1, with probabilities q and p respectively, *i.e.*, $P(X = 1) = p$, $P(X = 0) = q$, $q = 1 - p$ is called a *Bernoulli variate* and is said to have a Bernoulli distribution.

1 trial

S or F

p $\quad (1-p) = q$

X	P(x)	
1	p	success
0	q	failure

$E(X) = \sum p_i x_i = 1(p) + 0(q)$

$= \boxed{p}$

$\boxed{V(X) = pq}$

$V(X) = E(X^2) - [E(X)]^2$

$= (1^2 \cdot p + 0^2 \cdot q) - p^2$

$= p - p^2 = p(1-p) = pq.$

3. Binomial

used a lot

If $X \sim Bin(n, p)$ then X measures the number of successes in 'n' trials of a bernoulli experiment w/ prob of success as 'p' in each trial.

7·2. **Binomial Distribution.** Binomial distribution was discovered by James Bernoulli (1654-1705) in the year 1700 and was first published posthumously in 1713, eight years after his death). Let a random experiment be performed repeatedly and let the occurrence of an event in a trial be called a success and its non-occurrence a failure. Consider a set of n independent Bernoullian trials (n

RANGE: {0 to n}

7·2 **Fundamentals of Mathematical Statistics**

being finite), in which the probability 'p' of success in any trial is constant for each trial. Then $q = 1 - p$, is the probability of failure in any trial.

$$\boxed{p(x) = {}^nC_x \times p^x q^{n-x}}$$

eg. $n = 5$ $\quad \boxed{p = \left(\frac{1}{4}\right)}$

$x = 3)$

H H H T T $\rightarrow \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$

$p^x q^{n-x}$

T H T H H $\rightarrow \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$

$\frac{3}{4} \times \frac{1}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4}$

eg. n=3 p=(1/4)

$p(x=3)$

T H T H ... $\rightarrow \left(\frac{1}{4}\right)(\frac{3}{4})$

$\frac{3}{4} \times \frac{1}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4}$

$$E(X) = \sum p_i x_i = \sum_{x=0}^{n} {}^nC_x \, p^x q^{n-x} \cdot x = np$$

$$V(X) = npq$$

Example 7·11. *Determine the binomial distribution for which the mean is 4 and variance 3* ~~and find its mode~~*.* **(Madurai Kamraj Univ B.Sc. 1993)**

Solution, Let $X \sim B\ (n, p)$, then we are given that

$$E(X) = np = 4 \qquad ..(*)$$

and $$\text{Var}(X) = npq = 3 \qquad ..(**)$$

Dividing (**) by (*), we get

$$q = \tfrac{3}{4} \quad \Rightarrow \quad p = 1 - q = \tfrac{1}{4}$$

Hence from (*), $n = \frac{4}{p} = 16$

Thus the given binomial distribution has parameters $n = 16$ and $p = 1/4$.

$E(X) = 4$

$V(X) = 3$

$np = 4 \Rightarrow \frac{npq}{np} = \frac{3}{4}$

$npq = 3 \Rightarrow q = \frac{3}{4}$

$\Rightarrow \boxed{p = \frac{1}{4}}$

$np = 4 \Rightarrow n\left(\frac{1}{4}\right) = 4$

$\Rightarrow \boxed{n = 16}$

hint: Range of $Y = \{0,1,2,3,4,5\}$

eg. $Y \sim Bin\,(n=5,\ p=1/3)$ find $P[Y \le 1]$

prob. of atmost 1 success

$p(y) = {}^nC_y\, p^y q^{n-y}$

$$P[Y \le 1] = P[Y=0] + P[Y=1]$$

$$= {}^5C_0\, (1/3)^0 (2/3)^5 + {}^5C_1\, (1/3)^1 (2/3)^4$$

$$= 1 \times 1 \times \left(\frac{2}{3}\right)^5 + 5 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^4$$

$$= 0.131 + \frac{5}{3} \times \frac{16}{81}$$

$$= 0.131 + 0.329$$

$$= 0.46$$

4. Poisson

mostly it is used when we are measuring occurences of an event.

Definition. *A random variable X is said to follow a Poisson distribution if it assumes only non-negative values and its probability mass function is given by*

$$p(x, \lambda) = P(X = x) = \frac{e^{-\lambda}\lambda^x}{x\,!}\,;\ x = 0, 1, 2, \ldots\,;\ \lambda > 0$$
$$= 0,\ otherwise \qquad ...(7\cdot 14)$$

$$p(x, \lambda) = P(X = x) = \frac{\quad}{x!}; \; x = 0, 1, 2, \ldots ; \lambda > 0$$
$$= 0, \; otherwise \qquad \ldots(7{\cdot}14)$$

Here λ is known as the parameter of the distribution.

We shall use the notation $X \sim P(\lambda)$ to denote that X is a Poisson variate with paramter λ.

$X \sim P(\lambda)$

PMF : $\boxed{p(x) = \frac{e^{-\lambda} \lambda^x}{x!}}$

$E(X) = \lambda$ $\qquad V(X) = \lambda$

Range : $0, 1, 2, 3, 4, \ldots$

mathematical constant $e \approx 2.7182$

Example 7·24. *A car hire firm has two cars which it fires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1·5. Calculate the proportion of days on which (i) neither car is used, and (ii) some demand is refused.* [Meerut Univ. B.Sc. 1993]

Solution. The proportion of days on which there are x demands for a car

$$= P\{\text{of } x \text{ demands in a day}\}$$
$$= \frac{e^{-1\cdot5}(1\cdot5)^x}{x!},$$

since the number of demands for a car on any day is a Poisson variate with mean 1·5. Thus

$$P(X = x) = \frac{e^{-1\cdot5}(1\cdot5)^x}{x!}; \quad x = 0, 1, 2, \ldots$$

(*i*) Proportion of days on which neither car is used is given by

$$P(X = 0) = e^{-1\cdot5}$$
$$= \left[1 - 1\cdot5 + \frac{(1\cdot5)^2}{2!} - \frac{(1\cdot5)^3}{3!} + \frac{(1\cdot5)^4}{4!} - \ldots\right]$$
$$= 0\cdot2231$$

(*ii*) Proportion of days on which some demand is refused is

$$P(X > 2) = 1 - P(X \le 2)$$
$$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$
$$= 1 - e^{-1\cdot5}\left[1 + 1\cdot5 + \frac{(1\cdot5)^2}{2!}\right]$$
$$= 1 - 0\cdot2231 \times 3\cdot625 = 0\cdot19126$$

mean = 1.5

$\lambda = 1.5 \qquad X \sim P(1.5)$

(i) $P(X=0)$

$$\frac{e^{-1.5}(1.5)^0}{0!} = e^{-1.5} = \boxed{0.2231}$$

$$= (2.7182)^{-1.5} = 0.2231$$

(ii) $P(X>2)$

$0, 1, 2, \boxed{3, 4, 5, \ldots}$

needed

since total prob. = 1

$$P(X>2) = 1 - P(X \le 2) = 1 - \left[\frac{e^{-1.5}\,1.5^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$$

complete as HW

CONTINUOUS

5. Normal

Definition. *A random variable X is said to ahve a normal distribution with parameters μ (called "mean") and σ^2 (called "variance") if its density function is given by the probability law :*

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left\{\frac{x-\mu}{\sigma}\right\}^2\right]$$

or

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

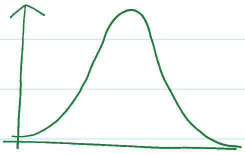
$$-\infty < x < \infty, \; -\infty < \mu < \infty, \sigma > 0 \qquad \ldots(8{\cdot}3)$$

imp ** RANGE ALWAYS $(-\infty, \infty)$

or $f(x\,;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\,e^{-(x-\mu)^2/2\sigma^2}$

$-\infty<x<\infty,\ -\infty<\mu<\infty,\ \sigma>0$ ...(8·3)

$(-\infty,\ \infty)$



SNV

$\mathbb{E}(X)=\mu \qquad V(X)=\sigma^2$

Example 8·12. *X is normally distributed and the mean of X is 12 and S.D. is 4. (a) Find out the probability of the following :*

(i) $X\geq 20$, *(ii)* $X\leq 20$, and *(iii)* $0\leq X\leq 12$

HW HW

$X\sim N(12,4^2)$

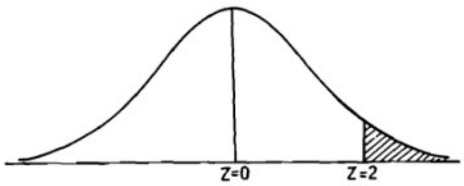
(i) $P(X\geq 20)=?$

When $X=20,\quad Z=\frac{20-12}{4}=2$

$\therefore\quad P(X\geq 20)=P(Z\geq 2)=0{\cdot}5-P(0\leq Z\leq 2)=0{\cdot}5-0{\cdot}4772=0{\cdot}0228$

(ii) $P(X\leq 20)=1-P(X\geq 20)$ $(\because$ Total probability $=1)$

$=1-0{\cdot}0228=0{\cdot}9772$

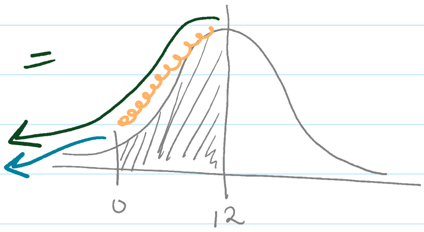


(iii) $P(0\leq X\leq 12)=P(-3\leq Z\leq 0)$ $\left(Z=\frac{X-12}{4}\right)$

$=P(0\leq Z\leq 3)=0{\cdot}49865$ (From symmetry)

$P(0\leq X\leq 12)$

$=$



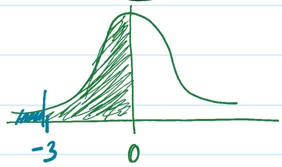
$P(X\leq 12)-P(X\leq 0)$

= reqd. prob.

converting to std. normal using 2 easy steps.

$$P\left[\frac{X-12}{4}\leq\frac{12-12}{4}\right]-P\left[\frac{X-12}{4}\leq\frac{0-12}{4}\right]$$

$$=P[Z\leq 0]-P[Z\leq -3]$$



$$=\frac{1}{2}-0.0013$$

$$=\underline{0.4987}$$

6. Exponential

8·6. The Exponential Distribution. A continuous random variable X assuming non-negative values is said to have an exponential distribution with parameter $\theta>0$, if its p.d.f. is given by

$$f(x)=\begin{cases}\theta\,.\,e^{-\theta x}, & x\geq 0\\ 0, & \text{otherwise}\end{cases}\qquad ...(8{\cdot}24)$$

$X\sim\exp(\theta)$

$X>0$ or $x\in[0,\infty)$

pdf : $\theta e^{-\theta x}$

$\mathbb{E}(X)=\frac{1}{\theta} \qquad V(X)=\frac{1}{\theta^2}$

Example: calculate P[X<5] if X ~ exp(0.25)

$X\sim\exp(0.25)\quad : \quad P(X<5)$

in continuous $P(X\leq 5)$ is also same.

-5

also same.

$$\int_0^5 \theta e^{-\theta x} dx = \int_0^5 \frac{1}{4} e^{-x/4} dx$$

$$= \frac{1}{4} \left[\frac{e^{-x/4}}{-1/4} \right]_0^5 = \frac{1}{4} \times \frac{-4}{1} \times \left[e^{-5/4} - e^{-0} \right]$$

$$= -1 \left\{ 0.2865 - 1 \right\}$$

$$= -1 \left\{ -0.713 \right\}$$

$$= \boxed{0.713}$$