Percentages, Ratio and Proportion

Pre-read

Relevel
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What is the Definition of Percentage?

The term per cent means “for every hundred”. For a fraction whose denominator is 100, the numerator of the fraction is called the rate or the percentage. It is denoted by the symbol %.

Exp. 15% = $\frac{15}{100}$, 23 % = $\frac{23}{100}$, 7.2% = $\frac{7.2}{100}$ etc.

(i) To convert a fraction or decimal into a percent, multiply by 100.
(ii) To convert a percent into a fraction or decimal, divide by 100.
(iii) To increase a number by a given %, multiply the number by a factor $\frac{100 + Rate}{100}$

(iv) To decrease a number by a given %, multiply the number by a factor $\frac{100 - Rate}{100}$

(v) Percentage Change = $\frac{Final\ Value - Initial\ Value}{Initial\ Value}$ x 100

(vi) Successive Percentage Change:
(a) If a number is increased successively by a% and b%, then net percentage increase is given by
$r = a + b + \frac{ab}{100}$
(b) If a number is decreased successively by a% and b%, then net percentage decrease is given by
$r = -a - b + \frac{ab}{100}$

Practice Exercise 1:

1. If $37\frac{1}{2}$ % of a number is 900, what is $62\frac{1}{2}$ % of the number?

2. i) a% of b + b% of a =? % of ab
   ii) 40% of 70 = 4 ×?
   iii) 0.025 in terms of percent will be?

3. My salary is increased from Rs 12000 to Rs. 15000 per month. What is the percentage change that occurred in my salary?

4. A number is multiplied by 2 instead of dividing it by 2. Find the percentage change in the result.

5. If the price of an article is raised by 20%. Find by how much percent a household must reduce consumption of that article so that expenditure remains the same.

6. Due to a fall of 10% in the rate of tea, 500 gm more tea can be purchased for Rs. 140. Find original and reduced price of tea.

7. If the annual increase in the population of a town is 4% and the present number of inhabitants is 15625; what will the population be in 3 years?

8. The population of a town is P. It increases by 5% during the first year, by 10% during the second year, and by 20% during the third year. Find the population after 3 years. Also, find net percentage change in population after 3 years.

9. The value of a machine depreciates at the rate of 10% after each year. Find its purchase price, if at the end of 3 years its value is only Rs. 8748.

10. The price of a commodity is diminished by 15% and its consumption increased by 10%.
    i) Find the effect on the revenue derived from it.
    ii) With what increase percent in its consumption, would the revenue remain the same?

11. A is increased by 7%, B is increased by 12% and C is decreased by 11%. Find the percentage change in A.B.C?

12. A shopkeeper marks his goods 12% higher than their original price. After that, he allows a discount of 12%. What is his percentage profit or loss?

13. Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight. The weight of dry grapes available from 20 kg of fresh grapes will be?

14. In an examination, 70% of the candidates passed in English, 65% in Mathematics, 27% failed in both the subjects and 248 passed in both the subjects. Find the total number of candidates.

15. 140 litres of an acid contain 90% of acid and the rest water. How much water must be added to make the water 12.5% of the resulting mixture?

Ratio and Proportion- Terms and Concepts:

The ratio of a and b is denoted by a : b and measured by $\frac{a}{b}$
In the ratio a : b, a is called 'antecedent' and b is called 'consequent'
While comparing two quantities in terms of ratio:

(i) the two quantities must be of the same kind
(ii) the units of measurement of the two quantities must be the same.

Note:
(i) Ratio is a pure number i.e. without any measurement.
(ii) The ratio would stay unaltered if both the antecedent and consequent are multiplied or divided by the same non-zero number.

Compounded Ratio:

Ratios are compounded by multiplying them
Example: Find the Compounded ratio of 3: 4 and 5: 7.

3: 4 can be rewritten as → $\frac{3}{4}$

5: 7 can be rewritten as → $\frac{5}{7}$

Compounded Ratio of 3: 4 and 5: 7 is given by $\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$ → 15: 28

Duplicate Ratio and Sub duplicate Ratio:
The duplicate ratio of a : b is given by $a^2 : b^2$ and the sub duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$

Triplicate and subtriplicate ratio:
The triplicate ratio of a : b is a^3 : b^3 and sub triplicate ratio of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$

Inverse Ratio or Reciprocal Ratio:
Inverse ratio of a : b is given by $\frac{1}{a} : \frac{1}{b}$

Proportion and continued proportion:
Proportion is a statement that tells two given ratios are equal.
i.e. If a:b is same as c:d, then a, b, c and d are in proportion.
OR
We can say if $\frac{a}{b} = \frac{c}{d}$ → ad = bc, then a, b, c and d are in proportion.
a is called first, b second, c third and d is called fourth proportional.
If a:b is same as b:c then we say a, b and c are in continued proportion.
In continued proportion, $b^2 = ac$ → b is called mean proportional, a is first and c is third.

Practice Exercise 2:

1. Find compounded ratio of
i) 2:3, 3:6 and 4:7
ii) 4:3, 9:13, 26:5 and 2:15
iii) 4a:3b, 7ab: 8c3 and c2: 7a3

2. Find
i) Duplicate ratio of 4: 3
ii) Sub duplicate ratio of 4: 3
iii) Triplicate ratio of 4: 3
iv) Sub triplicate ratio of 4: 3
v) Inverse ratio of 4: 3

3. Find three numbers in the ratio of 3: 2: 5, such that the sum of their squares is equal to 1862.

4. Divide Rs. 680 among A, B and C such that A gets $\frac{2}{3}$ of what B gets and B gets $\frac{1}{4}$ th of what C gets.

5. Compare the ratios 2: 3 and 4: 7.

6. If a: b = 2 :3, b : c = 4 : 5 find a : b : c ?

7. If a: b = 3: 4, b: c = 8: 9, c: d = 15: 16
i) Find the ratio for a to d
ii) Compare a, b, c & d

8. Given a: b: c = 2: 3: 4, find $\frac{1}{a} : \frac{1}{b} : \frac{1}{c}$

9. Calculate fourth proportional to the numbers.
(i) 1, 2, 3,
(ii) 500, 70, 69,
(iii) 2.5, 1.5, 1.5

10. Find third proportional to the numbers.
(i) 3,6
(ii) 1.2, 1.8
(iii) 225, 75