"Number System, Decimals And Fractions"

Pre-read

Various Types of Numbers:

1. Natural numbers:
The numbers that are used for counting are known as natural numbers. The set of natural numbers is denoted by N = {1, 2, 3, 4, ------ ∞ }. Natural numbers are further categorised as- Prime Numbers and Composite numbers

2. Whole Numbers:
Whole numbers comprise natural numbers and the number zero. The set of whole numbers is usually denoted by W = {0, 1, 2, 3, 4....... ∞ }

3. Integers:
Integers comprise of natural numbers, their negatives and the number zero. The set of integers is denoted by I = {- ∞ .... -3, -2, -1, 0, 1, 2, 3, ... ∞ }

4. Rational Numbers:
Any number that can be written in the form of p/q where q ≠0 and p and q are integers, is called a rational number. Set of rational numbers include all the integers and fractions.

5. Real numbers:
Set of real numbers includes all rational and irrational numbers.

Prime Numbers:
Except 1 each natural number which is divisible by only 1 and itself is called a prime number.

Composite Numbers:
A number other than 1, which is not a prime number is called a composite number. It means it is divisible by some other number(s) other than 1 and the number itself. Lowest composite number is 4.

Even Numbers: All the numbers which are divisible by 2 are known as even numbers.

Odd numbers: All the numbers which are not divisible by 2 are known as odd numbers.
It is important to note that:
- Even $\pm$ Even = Even
- Even $*$ Even = Even
- Even/Even = Even or Odd
- Odd $\pm$ Odd = Even
- Odd * Odd = Odd
- Odd/Odd = Odd
- Even $\pm$ Odd = Odd
- Even * Odd = Even
- Even^{Even} = Even
- Odd$^{Even\ or\ Odd}$ = Odd

Co prime numbers: A set of 2 or more numbers which do not have any number in common that divides them other than 1.
eg: 21 and 16: 21 is divisible by 1,3, and 7; 16 is divisible by 1,2,4, and 8. The only common number that divides them is 1, hence they are co prime. 15 and 21: 15 is divisible by 1,3, and 5; 21 is divisible by 1,3 and 7. Along with 1, both numbers are also divisible by 3, and hence they are not co prime.

Factors and Multiples:

An integer 'n' is called a factor or a divisor of another integer 'm' if 'n' divides 'm' exactly, and in such a case 'm' is called a multiple of 'n'.
For example: 1, 2, 3, 4, 6 and 12 are the factors of 12 and 12, 24, 36, 48,......... are its multiples.

Number of factors of a composite number:
(i) First, write number in the form of prime factors
$N = p^a * q^b * r^c$ ...., where p, q, r ..... are prime numbers.
(ii) Number of factors of N = (a+1)(b+1)(c+1)......

Q 1. Find the number of factors of 240.
$240 = 2^4 * 3 * 5$
Number of factors = (4+1)(1+1)(1+1) = 20

Types of Fractions:

1) Proper Fraction: When the numerator is less than the denominator, then the fraction is called a proper fraction.
2/3 , 4/7 , 9/13 , 18/73 etc.

2) Improper Fractions: When the numerator is greater than the denominator, then the fraction is called an improper fraction.
12/7 , 9/5 , 18/13 , 7/4 etc.

3) Like fraction: Fractions having same denominator are called like fractions.
2/11 , 5/11 , 7/11 , 15/11 etc

4) Unlike Fraction: Fractions having different denominators are called unlike fractions. 18/73 , 15/11 , 53/19 etc.

5) Compound Fraction: Fraction of fraction is called a compound fraction.
1/2 of 2/7 , 5/12 of 7/19 , 7/9 of 61/53, etc.

6) Mixed Fraction: Those fractions which consists of a whole number and a proper fraction, are known as mixed fraction.
$5\frac{2}{3}$, $8\frac{7}{8}$, $12\frac{5}{11}$ etc.

Note: Every mixed fraction can be written as an improper fraction and every improper fraction can be written as a mixed fraction.

As $7\frac{2}{3} = \frac{23}{3}$

7) Continued Fraction: It contains an additional fraction in the numerator or denominator. Please see the example below to understand

$$12 + \cfrac{14}{13 + \cfrac{14}{65 + \cfrac{2}{3}}} \quad , \quad 5 + \cfrac{1}{2 + \cfrac{3}{4 + \cfrac{5}{6 + \cfrac{7}{8}}}} \quad \text{etc.}$$

8) Decimal Fraction: In such fractions, the denominator has any power of 10.
7/10 , 45/100 , 71/100000, etc.

Decimal value of a fraction:
Integers and fractions together constitute rational numbers. Decimal value of a fraction is of two types-

(i) Terminating Decimals: When denominator contains some power of 2 or 5 or both.
For example:
1/2 = 0.5, 1/4 = 0.25, 1/8 = 0.125, 1/5 = 0.2, 1/25 = 0.04, 1/20 = 0.05 etc

(ii) Non terminating Decimals: They are further categorised in two categories – recurring and non-recurring.
Recurring Decimals are further categories in two categories:
  • Pure Recurring Decimal: All the digits after decimal repeats. They can be converted into fractions using:
    $0.\overline{p}$ = p/9
    $0.\overline{pq}$ = (pq)/99
    $0.\overline{pqr}$ = (pqr)/999

  • Mixed Recurring Decimals: When some of the digits after decimal is not repeating. Mixed recurring decimals can be converted into simple fractions using
    $0.p\overline{q}$ = (pq-p)/90
    $0.pq\overline{r}$ = (pqr-pq)/900
    $0.p\overline{qr}$ = (pqr-p)/990
    $0.pq\overline{rs}$ = (pqrs-pq)/9900

Examples:

(a) $0.2222222 = 0.\overline{2}$
Let x = 0.2222222.....
10x = 2.22222222........ = 2 + x
9x = 2
X = 2/9

(b) $0.13131313.... = 0.\overline{13}$
Let x = 0.1313131.....
100x = 13.1313131313........ = 13 + x
99x = 13
X = 13/99

(c) 0.123123123…. = 0. $\overline{123}$

Let x = 0.123123123……..

1000x = 123.123123123123…… = 123 + x

999x = 123

X = 123/999

Pre Read Exercise:

1. Convert following decimal numbers into fractions

A. $5.\overline{23}$ B. $17.2\overline{3}$ C. $0.\overline{0909}$ D. $0.67\overline{85}$

2. Sum of 2 consecutive prime numbers is 152. Their product is

A. 5767   B. 439   C. 6027   D. Can't be determined

3. Classify the following numbers into rational & irrational numbers

A. $5.\overline{32}$ B. $\sqrt[3]{30}$ C. $\sqrt[2]{2}$ D. $\sqrt{\sqrt{64}}$ E. $\frac{3\sqrt{3}}{4}$ F. π

4. Choose true or false:
- An irrational number multiplied by another different irrational number can never be a rational number (True/False)
- Product of first 7 prime numbers will always be odd. (True/False)
- 378 – 434 + 293 will be odd. (True/False)
- Sum of 7 positive odd numbers & 8 even positive numbers will be even. (True/False)
- An even number divided by an even number will always be even. (True/False)
- Sum of an irrational number and a rational number can be an integer. (True/False)
- Product of a recurring decimal and a terminating decimal can never be a terminating decimal. (True/False)
- 22/7 is a rational number (True/False)
- Difference of 2 natural numbers will be either a natural number or whole number (True/False)
- Sum of 2 positive prime numbers will invariably be a composite number (True/False)