

Number System, Decimals and Fractions

Relevel
by Unacademy



Various Types of Numbers

1,2,..

1. Natural Numbers

0,1,..

2. Whole Numbers

-5,-4,..

3. Integers

0,1/8,..

4. Rational Numbers

5, π ,..

5. Real Numbers

7,11,..

6. Prime Numbers

2*7=14

7. Composite Numbers

2,4..

8. Even Numbers

1,3..

9. Odd Numbers

Consecutive Integers

Let's try to solve the following questions:

1. The sum of 5 consecutive even numbers is 80. What is least number among them?
2. 250 is the sum of 5 consecutive even numbers. Find the greatest among them.

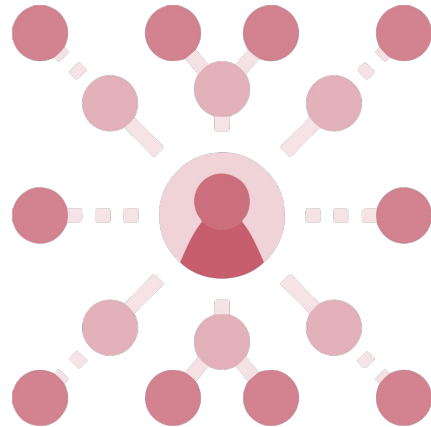


Number of Factors of a Composite Number

(i) First, write the number in the form of prime factors

$N = p^a * q^b * r^c \dots\dots\dots$, where $p, q, r \dots\dots$ are prime numbers.

(ii) Once above step is completed, number of factors of $N = (a+1)*(b+1)*(c+1)\dots\dots$



Factors and Multiples- Practice Questions

Find the number of factors of:

1. 240
2. 3720
3. 2370
4. 520



Fractions and Decimals

Types of Fractions:

$$\frac{5}{12}$$

1. Proper Fraction

$$\frac{12}{5}$$

2. Improper Fraction

$$\frac{1}{2}, \frac{3}{2}$$

3. Like Fraction

$$\frac{1}{2}, \frac{2}{3}$$

4. Unlike Fraction

$$8\frac{1}{2}$$

5. Mixed Fraction

$$\frac{14}{13 + \frac{14}{65 + \frac{2}{3}}}$$

6. Continued Fraction

$$\frac{7}{10}, \frac{25}{100}$$

7. Decimal Fraction

Comparison of Fractions

Comparison of Fractions:

- 1) If given fractions have same denominator then the one having greatest numerator will be greatest fraction and the one having lowest numerator will be lowest fraction.
- 2) If given fractions have same numerator then the one having lowest denominator will be the greatest fraction and the one having greatest denominator will be the lowest fraction.
- 3) If neither numerator nor the denominator of given fractions are equal then:
 - (i) Equalize denominators of the fractions by finding the LCM of denominators and then compare the numerator.
 - (ii) We convert them into decimal fractions. It will clarify the lowest and greatest fractions.

$$8\frac{1}{2}$$

$$7/10, 25/100$$

Comparison of Fractions (Examples)

- $\frac{2}{15}$, $\frac{4}{15}$, $\frac{7}{15}$, $\frac{11}{15}$ is ascending order of fractions
- $\frac{11}{15}$, $\frac{7}{15}$, $\frac{4}{15}$, $\frac{2}{15}$ is descending order of fractions.
- Among $\frac{3}{5}$, $\frac{3}{7}$, $\frac{3}{11}$, $\frac{3}{13}$; $\frac{3}{5}$ is greatest and $\frac{3}{13}$ is lowest

$\frac{2}{15}, \frac{4}{15}...$

$\frac{11}{15}, \frac{7}{15}...$

Comparison of Fractions- Questions

Q 1. Which one of the given fractions is smallest?

$\frac{5}{13}$, $\frac{3}{13}$, $\frac{7}{13}$ and $\frac{9}{13}$

Q 2. Which one of the given fractions is greatest?

$\frac{6}{17}$, $\frac{6}{13}$, $\frac{6}{11}$ and $\frac{6}{19}$

Q 3. Arrange the given fractions in ascending order.

$\frac{2}{3}$, $\frac{4}{7}$, $\frac{5}{8}$ and $\frac{3}{4}$

Q 4. Re-write the given fractions in descending order?

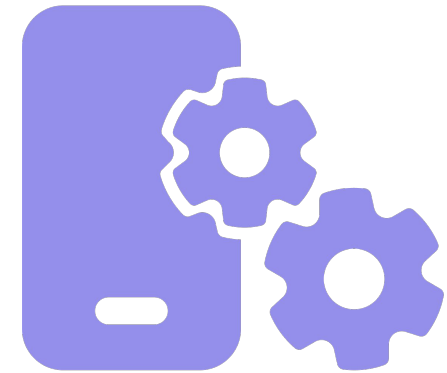
$\frac{3}{5}$, $\frac{4}{9}$, $\frac{2}{7}$ and $\frac{5}{8}$



Simplification of Fractions- Question

Q 1. Find the simplest value of

$$1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{2}}}}$$



Decimal Value of a Fraction

0.125

Terminating Decimals

0.33..

Non terminating Decimals:

- Recurring
- Non-recurring
- Pure Recurring Decimal
- Mixed recurring Decimal

Practice Questions- Convert the Following into Fractions

1. $0.\overline{2}$
2. $0.\overline{13}$
3. $0.\overline{123}$
4. $0.12\overline{3}$
5. $0.12\overline{34}$
6. $0.\overline{123}$



Factorials

Factorial: A factorial is a number obtained by multiplying all the positive integers less than or equal to the given number. Factorial of a given integer 'n' is usually denoted by n!

- $0! = 1$ (By convention)
- $1! = 1$
- $2! = 1 \times 2 = 2$
- $3! = 1 \times 2 \times 3 = 6$
- $4! = 1 \times 2 \times 3 \times 4 = 24$
- $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$
- Factorial is defined only for whole numbers
- $N!$ is always an even number if $N \geq 2$
- $N!$ always ends with zero if $N \geq 5$
- $N! = N \times (N-1)!$
- Number of zeros at the end of $N!$ or product of numbers:
 $10 = 5 \times 2$, $100 = 5^2 \times 2^2$, $1000 = 5^3 \times 2^3$ etc
So, for n number of zeros at the end, we need n combinations of 5×2 .



Practice Question

Q1. Find the number of zeros at the end of $10!$

Sol: $10! = 1 * 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 * 10 = 1 * 2 * 3 * 2 * 2 * 5 * 2 * 3 * 7 * 2 * 2 * 2 * 3 * 3 * 2 * 5$

There are only two 5's and eight 2's. Hence, there are 2 combinations of $(5*2)$

So, there are two zeroes at the end of $10!$



Cyclicity

Unit digit of base	Unit digit if the power is of the form			
	$4n+1$	$4n+2$	$4n+3$	$4n$
0	0	0	0	0
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1

Cyclicity (Unit Digit)

Q1. Find the unit digit of 3^{40} .

Q2. Find the unit digit of $(73)^{98} + (39)^{87} + (76)^{99}$.

Q3. Find the unit digit of $(817)^{938} + (776)^{532} + (985)^{67} + (813)^{353}$.



Division

Dividend = Divisor * Quotient + Remainder

$(0 \leq \text{Remainder} < \text{Divisor})$

Sample Questions:

- Divide 37 by 5. What is the dividend, quotient, divisor and remainder?
- Divide 53 by 7. What is the dividend, quotient, divisor and remainder?
- Divide 60 by 8. What is the dividend, quotient, divisor and remainder?

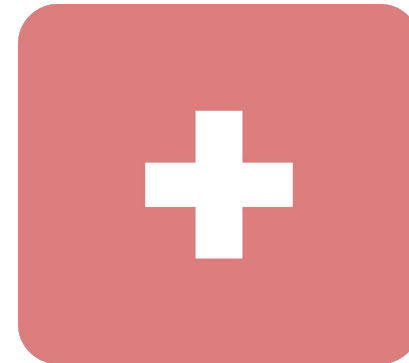


Divisibility

Divisibility by number	Divisibility Rule
Divisible by 2	A number which is even or a number whose last digit is an even number i.e. 0, 2, 4, 6, and 8.
Divisible by 3	The sum of all the digits of the number should be divisible by 3.
Divisible by 4	Number formed by the last two digits of the number should be divisible by 4 or should be 00.
Divisible by 5	Numbers having 0 or 5 as their ones place digit.
Divisible by 6	A number that is divisible by both 2 and 3.
Divisible by 7	Subtracting twice the last digit of the number from the remaining digits gives a multiple of 7.
Divisible by 8	Number formed by the last three digits of the number should be divisible by 8 or should be 000.
Divisible by 9	The sum of all the digits of the number should be divisible by 9.
Divisible by 10	Any number whose ones place digit is 0.
Divisible by 11	The difference of the sums of the alternative digits of a number is divisible by 11.
Divisible by 12	A number that is divisible by both 3 and 4.

Summation:

1. Sum of first n natural numbers: $\frac{n(n+1)}{2}$
2. Sum of squares of first n natural numbers: $\frac{n(n+1)(2n+1)}{6}$
3. Sum of cubes of first n natural numbers: $\left[\frac{n(n+1)}{2}\right]^2$
4. Sum of first n even numbers: $n(n+1)$
5. Sum of first n odd numbers: n^2



Practice Questions

Q1. Find the value of

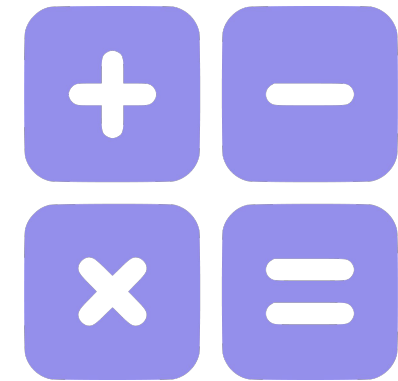
$$1 + 2 + 3 + 4 + 5 + 6 + 7 + \dots + 50$$

Q2. Find the value of

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + \dots + 50^2$$

Q3. Find the value of

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + \dots + 50^3.$$



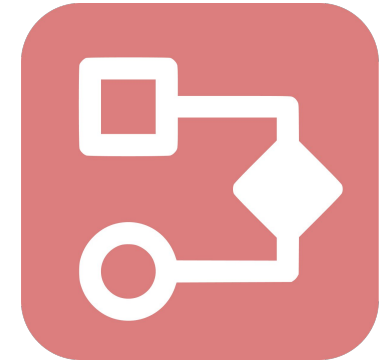
Practice Questions

1. Convert following decimal numbers into fractions

- A. $0.6\overline{785}$
- B. $0.0\overline{909}$
- C. $17.\overline{23}$
- D. $5.\overline{23}$

2. Sum of 2 consecutive prime numbers is 152. Their product is

- A. 5767
- B. 439
- C. 6027
- D. Can't be determined



Practice Questions

3. Classify the following numbers into rational & irrational numbers

A. $5.\overline{32}$ Rational

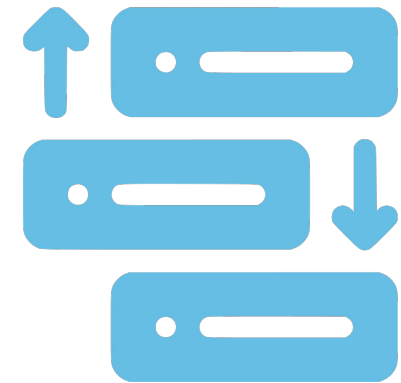
B. $\sqrt[3]{30}$ Irrational

C. $\sqrt[2]{2}$ Irrational

D. $\sqrt{\sqrt{64}}$ Irrational

E. $\frac{\sqrt[3]{3}}{4}$ Irrational

F. π Irrational



Practice Questions

4. Choose true or false:

- An irrational number multiplied by another different irrational number can never be a rational number (True/False)
- Product of first 7 prime numbers will always be odd. (True/False)
- $37^8 - 43^4 + 29^3$ will be odd. (True/False)
- Sum of 7 positive odd numbers & 8 even positive numbers will be even. (True/False)
- An even number divided by an even number will always be even. (True/False)
- Sum of an irrational number and a rational number can be an integer. (True/False)
- Product of a recurring decimal and a terminating decimal can never be a terminating decimal. (True/False)
- $22/7$ is a rational number (True/False)
- Difference of 2 natural numbers will be either a natural number or whole number (True/False)
- Sum of 2 positive prime numbers will invariably be a composite number (True/False)



Practice Questions

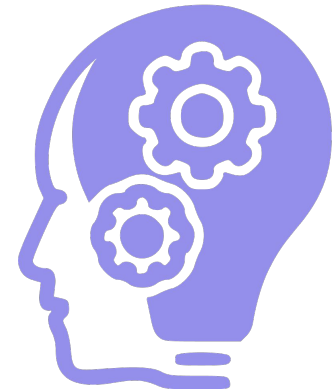
5. Simplify: $6.\overline{63} \times 1.\overline{571428}$

A. $9\frac{3}{7}$

B. $10\frac{3}{7}$

C. $10\frac{3}{11}$

D. $10\frac{7}{11}$



Practice Questions

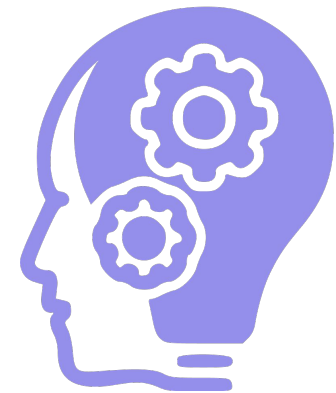
6. Simplify: $15.\overline{66} - 8.\overline{16} - 2.\overline{33}$

A. $6\frac{1}{6}$

B. $5\frac{1}{2}$

C. $5\frac{1}{3}$

D. None of these



Practice Questions

7. How many prime numbers exist between 40 & 100
- A. 10
 - B. 12
 - C. 14
 - D. None of these
8. If product of 5 consecutive odd integers is 45, find middle number.
- A. 1
 - B. 3
 - C. 5
 - D. No such no. exists
9. If $0.4\overline{39}$ is converted in the simplest fraction P/Q , then $P + Q =$
- A. 65
 - B. 75
 - C. 85
 - D. 95



Practice Questions

10. $(\text{Odd}^{\text{even}} + \text{Even}^{\text{odd}}) + \text{Even}$ is odd?

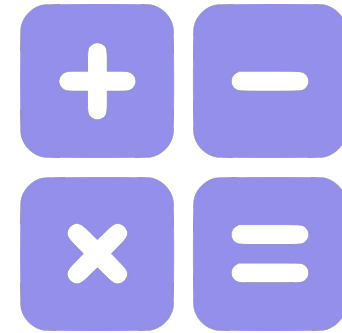
11. $z_1 = 2+3i$ and $z_2 = 4 - 6i$ then

(i) $z_1 + z_2 =$

(ii) $z_1 - z_2 =$

(iii) $z_1 \times z_2 =$

(iv) z_1/z_2

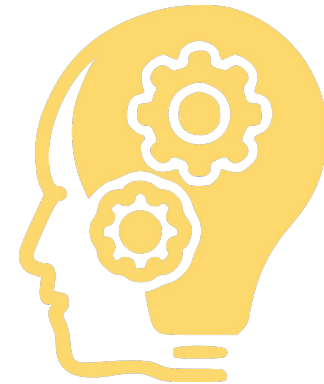


Practice Questions

12. Simplify

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{3}{4}}}}$$

- A. 34/21
- B. 1
- C. 47/29
- D. None



Practice Questions

13. Arrange the fractions in ascending order- ($\frac{5}{8}$, $\frac{7}{12}$, $\frac{13}{16}$, $\frac{16}{29}$, $\frac{3}{4}$)

- A. $\frac{16}{29} < \frac{7}{12} < \frac{5}{8} < \frac{3}{4} < \frac{13}{16}$
- B. $\frac{13}{16} > \frac{3}{4} > \frac{5}{8} > \frac{7}{12} > \frac{16}{29}$
- C. $\frac{16}{29} > \frac{13}{16} > \frac{7}{12} > \frac{5}{8} > \frac{3}{4}$
- D. None

14. Choose the largest among following fractions:
 $\frac{147}{203}$, $\frac{145}{264}$, $\frac{136}{270}$, $\frac{337}{628.9}$

- A. $\frac{147}{203}$
- B. $\frac{145}{264}$
- C. $\frac{136}{270}$
- D. $\frac{337}{628.9}$



Practice Questions

15. If r_1 and r_2 are rational numbers, then

- i) $r_1 + r_2$ is also rational
- ii) $r_1 - r_2$ is also rational
- iii) $r_1 \cdot r_2$ is also rational
- iv) r_1 / r_2 is also rational.

- A. I & II only
- B. II & III only
- C. All of these
- D. None

16. If a and b are two prime numbers, then $a + b$ can take values

- i) 78
- ii) 7
- iii) 23

- A. (i) only
- B (ii) only
- C. (i) & (ii) only
- D. (ii) & (iii) only



Practice Questions

17. If $x^2 = 212.5 \times 24 - 243 \div 1.52 - 23.75 \times 22 \times 7 + (3 \times 102 + 23) \div 22$. Then the value of x

A. $\sqrt{2600}$

B. $\sqrt{2625}$

C. $\sqrt{2681}$

D. None of these

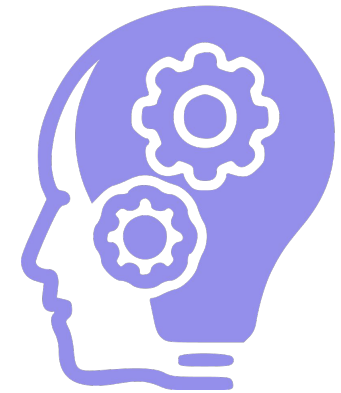
18. Simplify $23.25 * 76 - 32 * 24.625 - 48 \div 8 \div 4$

A. Less than 1000

B. Between 1001 and 1999

C. Greater than 2000 but less than 2010

D. Above 2010



Answers

1. A. $5\frac{23}{99}$ B. $17\frac{7}{30}$ C. 101/111 D. 3359/4950

2. A.

3. A. Rational, B. irrational , C. irrational , D. irrational, E. irrational F. irrational

4. 1. False, 2. False, 3. True, 4. False, 5. False, 6. False, 7. False, 8. True, 9. False, 10. False

5. B.

6. D.

7. D.

8. D.

9. D.

10. odd

11. A. $6-3i$, B. $-2 + 9i$, C. 26, D. i

12. C.

13. A.

14. A.

15. B.

16. C.

17. C.

18. A.



In the next class, we will study:

LCM

HCF

HCF

LCM



Simplification