Revision on Measures of dispersion and probability + Probability - Expected Value and Set Theory 19oct22

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MAIN FORMULAS TO REMEMBER:

Range

It is simply the difference between the maximum value and the minimum value given in a data set.

Standard Deviation

The standard deviation is the average distance between each data point and the mean. Simply taking the variance's square root yields the standard deviation.
You have a reduced standard deviation when the values in a dataset are clustered more closely together. Conversely, when values are more dispersed, the standard deviation is higher because the standard deviation is higher.
The standard deviation conveniently uses the data's original units, simplifying interpretation. The standard deviation is, therefore, the most frequently employed measure of variability.

Take pizza delivery as an example; a standard deviation of 5 means that the delivery time will vary by 5 minutes from the mean. It's frequently stated alongside the average: 20 minutes (s.d. 5).

Simply taking the variance's square root yields the standard deviation. Remember that the variance is expressed in square units. The square root thereby converts the value to natural units.

Formula for Variance And Standard Deviation Ungrouped Data

N → popⁿ size

The formula for the variance of an entire population is the following:

$$\sigma = \sqrt{\frac{\Sigma(X-\mu)^2}{N}}$$

$$\sqrt{\frac{1}{N}\Sigma(x_i-\mu)^2} = \sigma$$

In the equation,
- σ2 is the population parameter for the variance
- σ is the population parameter for the standard deviation
- μ is the parameter for the population mean, and
- n is the number of data points, which should include the entire population

This is the same formula as before.

The formula for standard deviation becomes:

$$(\sigma) = 1/N\sqrt{N\sum_{i=1}^{n} f_i x_i^2 - \left(\sum_{i=1}^{n} f_i x_i\right)^2}$$

$$\frac{1}{N^2}\cdot\left\{N\Sigma f_i x_i^2 - (\Sigma f_i x_i)^2\right\}$$

Definition of probability

Probability is the measurement of an event's likelihood. Probability aids in determining the chance of an event occurring because many events cannot be predicted with 100% accuracy. It is the proportion of positive events to all of the events in an experiment.

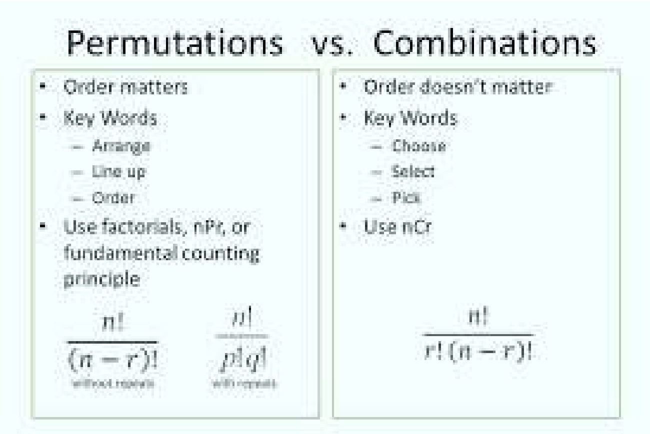
Probability(Event)=Favorable Outcomes/Total

Numerically the probability value always lies between 0 and 1.

$\sum p_i = 1$

0≤P(E)≤1

It is expressed in percentage, decimal, or fraction.



Expected Value

It relates to the outcome you can anticipate from a particular action, such as the number of items you might correctly estimate on a multiple-choice test.

It is calculated by multiplying each potential outcome by the probability that each possibility will materialise and then adding all of those values together.

For instance, you can anticipate receiving 25% of the answers correct on a 20-question multiple-choice examination with the answers A, B, C, and D. (5 out of 20). This form of anticipated value calculus is as follows:
The probability (P) that your guess will be correct on a question is ¼ = 0.25.
The number of questions on the test (n)*: 20
P x n = 0.25 x 20 = 5

Do not confuse expected value with probability: Expected value is in the same unit of measurement (here, number of questions) as the experiment.

Formula for Expected Value

The probability of an event multiplied by the number of times it occurs is the fundamental expected value formula: **(P(x)*n)**.

The expected value can be calculated for:

- Binomial (only two outcomes possible like the toss of a coin). Discrete Random Variable
- Multiple Discrete Random Variable
- Continuous Random Variable

Practice Questions: DISPERSION

Q find out the range & variance in each case

I

ungrouped data
51
42
14
53
40
56
58
45
10 ✓
37
46
62 ✓
28
20

range: 52

II

grouped data

LL	UL	freq
0	20	11
20	40	17
40	60	13
60	80	10
80	100	19
100	120	19
120	140	10
140	160	14
160	180	9
180	200	7

III HW

grouped data

data	frequency
11	9
12	1
13	5
14	8
15	6
16	5
17	6
20	8
22	8
25	4
27	4

LL	UL	freq	M	fimi	fimi2
0	20	11	10	110	1,100
20	40	17	30	510	15,300
40	60	13	50	650	32,500
60	80	10	70	700	49,000
80	100	19	90	1,710	1,53,900
100	120	19	110	2,090	2,29,900
120	140	10	130	1,300	1,69,000
140	160	14	150	2,100	3,15,000
160	180	9	170	1,530	2,60,100
180	200	7	190	1,330	2,52,700
			total	12,030	14,78,500

HW-hint

data	frequency	fixi	fixi2
11	9	99	1089
12	1	12	144
13	5	65	845
14	8	112	1568
15	6	90	1350
16	5	80	1280
17	6	102	1734
20	8	160	3200
22	8	176	3872
25	4	100	2500
27	4	108	2916
total	64	1104	20498

5.6 = 5

HW

many vehicles for which users have reported their own values of fuel efficiency (mpg). Consider the following sample of $n = 11$ efficiencies for the 2009 Ford Focus equipped with an automatic transmission (for this model, EPA reports an overall rating of 27 mpg–24 mpg for city driving and 33 mpg for highway driving):

Car	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	
1	27.3	−5.96	35.522	
2	27.9	−5.36	28.730	
3	32.9	−0.36	0.130	
4	35.2	1.94	3.764	
5	44.9	11.64	135.490	
6	39.9	6.64	44.090	
7	30.0	−3.26	10.628	
8	29.7	−3.56	12.674	
9	28.5	−4.76	22.658	
10	32.0	−1.26	1.588	
11	37.6	4.34	18.836	
	$\Sigma x_i = 365.9$	$\Sigma(x_i - \bar{x}) = .04$	$\Sigma(x_i - \bar{x})^2 = 314.106$	$\bar{x} = 33.26$

PROBABILITY

Probability Example - 2

Question: Tom's teacher wrote each English alphabet on a different piece of paper and jumbled all those in a box. He asked Tom to randomly pick a piece of paper. What is the probability of having a vowel written on that piece of paper?

Solution:

Total number of English alphabet = 26

Number of vowels = 5

Probability of getting a vowel =Number of vowels/Total number of alphabets

=5/26

Probability Example - 4

Question: Mary had a jar containing 8 red balls, 5 blue balls, and 7 green balls. She called one of her friends and asked them to pick a ball from the jar. What is the probability that the ball which is picked is either a red or a blue ball?

Solution:

Let us take a count of the number of balls in the jar.
Number of red balls = 8
Number of blue balls = 5
Number of green balls = 7
Total number of balls = 20
P(red or a blue ball) = (No of red balls + No of blue balls)/Total number of balls
=(8+5)/20
=13/20 ✓

In a certain residential suburb, 60% of all households get Internet service from the local cable company, 80% get television service from that company, and 50% get both services from that company. If a household is randomly selected, what is the probability that it gets at least one of these two services from the company, and what is the probability that it gets exactly one of these services from the company?

With A = {gets Internet service} and B = {gets TV service}, the given information implies that $P(A) = .6$, $P(B) = .8$, and $P(A \cap B) = .5$. The foregoing proposition now yields

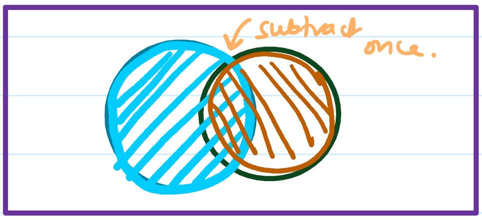
P(subscribes to at least one of the two services)

A : Internet service B : TV service

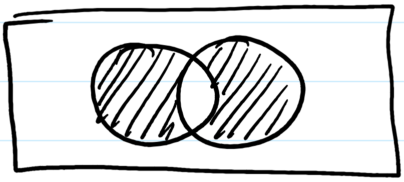
$P(A) : 0.6$ $P(B) : 0.8$ $P(A \cap B) = 0.5$

(a)

$P(A \cup B) =$
$P(A) + P(B) - P(A \cap B)$



$0.6 + 0.8 - 0.5 = \underline{\underline{0.9}}$ ✓



(b) $P(A \Delta B) = P(A) + P(B) - 2P(A \cap B)$

$= 0.6 + 0.8 - 1 = \underline{\underline{0.4}}$

Q (A particular iPod playlist contains 100 songs, 10 of which are by the Beatles. Suppose the shuffle feature is used to play the songs in random order) (the randomness of the shuffling process is investigated in "Does Your iPod *Really* Play Favorites?" (*The Amer. Statistician*, 2009: 263–268). (What is the probability that the first Beatles song heard is the fifth song played?)

[_ _ _ _ | 10]

Ans: In order for this event to occur, it must be the case that the first four songs played are not Beatles' songs (NBs) and that the fifth song is by the Beatles (B). The

$90 \times 89 \times 88 \times 87 \times 10$

first Beatles song heard is the fifth song played?)

Ans · In order for this event to occur, it must be the case that the first four songs played are not Beatles' songs (NBs) and that the fifth song is by the Beatles (B). The number of ways to select the first five songs is 100(99)(98)(97)(96). The number of ways to select these five songs so that the first four are NBs and the next is a B is 90(89)(88)(87)(10). The random shuffle assumption implies that any particular set of 5 songs from amongst the 100 has the same chance of being selected as the first five played as does any other set of five songs; each outcome is equally likely. Therefore the desired probability is the ratio of the number of outcomes for which the event of interest occurs to the number of possible outcomes:

$$P(1^{\text{st}}\text{ B is the }5^{\text{th}}\text{ song played}) = \frac{90 \cdot 89 \cdot 88 \cdot 87 \cdot 10}{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96} = \frac{P_{4,90} \cdot (10)}{P_{5,100}} = .0679$$

$$\frac{\underbrace{90 \times 89 \times 88 \times 87}_{} \times \underbrace{10}_{}}{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96}$$

$$\boxed{\frac{{}^{90}P_4 \times {}^{10}P_1}{{}^{100}P_5}}$$

HW

HW

A university warehouse has received a shipment of 25 printers, of which 10 are laser printers and 15 are inkjet models. If 6 of these 25 are selected at random to be checked by a particular technician, what is the probability that exactly 3 of those selected are laser printers (so that the other 3 are inkjets)?

$$P(D_3) = \frac{N(D_3)}{N} = \frac{\binom{15}{3}\binom{10}{3}}{\binom{25}{6}} = \frac{\frac{15!}{3!12!} \cdot \frac{10!}{3!7!}}{\frac{25!}{6!19!}} = .3083$$

HW

In a certain state's lottery, 48 balls numbered 1 through 48 are placed in a machine and six of them are drawn at random. If the six numbers drawn match the numbers that a player had chosen, the player wins $1,000,000. In this lottery, the order the numbers are drawn in doesn't matter. Compute the probability that you win the million-dollar prize if you purchase a single lottery ticket.

In order to compute the probability, we need to count the total number of ways six numbers can be drawn, and the number of ways the six numbers on the player's ticket could match the six numbers drawn from the machine. Since there is no stipulation that the numbers be in any particular order, the number of possible outcomes of the lottery drawing is

${}_{48}C_6$ = 12,271,512. Of these possible outcomes, only one would match all six numbers on the player's ticket, so the probability of winning the grand prize is:

$$\frac{{}_6C_6}{{}_{48}C_6} = \frac{1}{12271512} \approx= 0.0000000815$$

Q Compute the probability of randomly drawing five cards from a deck and getting exactly one Ace.

Ans In many card games (such as poker) the order in which the cards are drawn is not important (since the player may rearrange the cards in his hand any way he chooses); in the problems that follow, we will assume that this is the case unless otherwise stated. Thus we use combinations to compute the possible number of 5-card hands,

${}_{52}C_5$. This number will go in the denominator of our probability formula, since it is the number of possible outcomes.

For the numerator, we need the number of ways to draw one Ace and four other cards (none of them Aces) from the deck. Since there are four Aces and we want exactly one of them, there will be

${}_4C_1$ ways to select one Ace; since there are 48 non-Aces and we want 4 of them, there will be ${}_{48}C_4$ ways to select the four non-Aces. Now we use the Basic Counting Rule to calculate that there will be ${}_4C_1 \times {}_{48}C_4$ ways to choose one ace and four non-Aces.

Putting this all together, we have

$$P(\text{one Ace}) = \frac{({}_4C_1)({}_{48}C_4)}{{}_{52}C_5} = \frac{778320}{2598960} \approx 0.299$$

4 48

favourable. $^{4}C_1 \times {}^{48}C_4$

total. $^{52}C_5$

EXPENCTED VALUE

Example: You work as a financial analyst for a development company, for instance. Your manager just instructed you to determine which future development initiatives are most likely to succeed and pick that one. Upon completion, Project A is predicted to have a 0.4 per cent chance of reaching a value of $2 million and a 0.6 per cent chance of reaching a value of $500,000. Upon completion, Project B has a 0.3 per cent chance of being valued at $3 million and a 0.7 per cent chance of being valued at $200,000.

Answer:
You must determine the expected value of each project and contrast the values to choose the best project. The following formula can be used to get the EV:
EV (Project A) equals [0.4 x $2,000,000] plus [0.6 x $500,000] to arrive at $1,100,000.
EV (Project B) equals [0.3 $3,000,000] plus [0.7 $200,000] for a total of $1,040,000.
Project A's EV is higher than Project B's EV. As a result, Project A should be chosen by your company.

Expected Value Problem - 2

Question: You toss a fair coin three times. X is the number of heads which appear. What is the Expected Value?

done already

SET THEORY

Question: Class XI is made up of 200 students in total. One hundred and twenty of them study mathematics, fifty study commerce, and thirty studies. Find the number of students who
i) Study mathematics but not commerce
ii) Study commerce but not mathematics
iii) Study mathematics or commerce