Probability Distributions 12oct22

12 October 2022 06:46 PM

Pprobability distribution table

3 Coins example no of heads

And graph

Sample space	Prob.	
✓ HHH	1/8	X=3
✓ HTH	"	X=2
✓ HHT	"	X=2
✓ HTT	"	X=1
✓ THH	"	X=2
✓ TTH	"	X=1
✓ THT	"	X=1
✓ TTT	"	X=0

"equally likely events."

random exp. → outcome is not known before conducting it but set of possible outcomes is known prior.

set of possible outcomes is sample space.

equally likely outcomes.

Prob. distrⁿ table is a tabular format listing occurances and their attached probability.

now let X measure the no. of heads we can possibly get by tossing 3 coins at once.

X values	prob.	
1	3/8	HTT, THT, TTH
2	3/8	HHT, HTH, THH
3	1/8	HHH
0	1/8	TTT

X here is a random variable.

algebra
x, y, z
FIXED VALUES

X
values change w/ each trial

Y measures the difference of |heads − tails|

Y	Prob.	
3	2/8	HHH, TTT
1	6/8	HTT, THT, TTH, HHT, HTH, THH

$\Sigma = 1$

Random variable

A function which maps a number to a point or a combination of points in the sample space is known as a random variable.

Discrete vs continuous data and random variables

X, Y are discrete random variables

discrete: countable values or countably infinite

$A = \{0, 1, 2, 3, 4, 5, 6\}$ $\boxed{\text{countable values in set } A}$

B = set of natural nos. $B = \{1, 2, 3, 4, \ldots \infty\}$ $\boxed{\text{countably } \infty \text{ values in set } B}$

C = set which contains values b/w 1 & 2

$[1, 4]$ C_1

$(1, 4)$ C_2

$[1, 4)$ C_3

$(1, 4]$ C_4

$\in \rightarrow$ belongs to

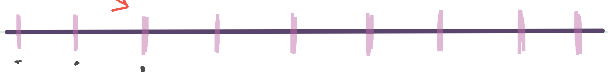
$1.67 \notin A$

$1.67 \in C_1, C_2, C_3, C_4$

$\boxed{\text{uncountable values in } (C_1, C_2, C_3, C_4)}$

A discrete r.v. takes values which are countable or countably infinite.

discrete



A continuous random variable can take any possible value in a defined range

cont.

Pmf and pdf - what are these functions?

- PMF (probability mass function)
- PDF (probability density function)

] similar functions but one is used for discrete & other for cont.

X is a random variable we denote PMF/PDF

f(x) this function

continuous PDF

Takes in any number gives out a positive value.

density

discrete PMF

p(x)

takes in any no. & gives out value b/w 0 & 1 [0,1]

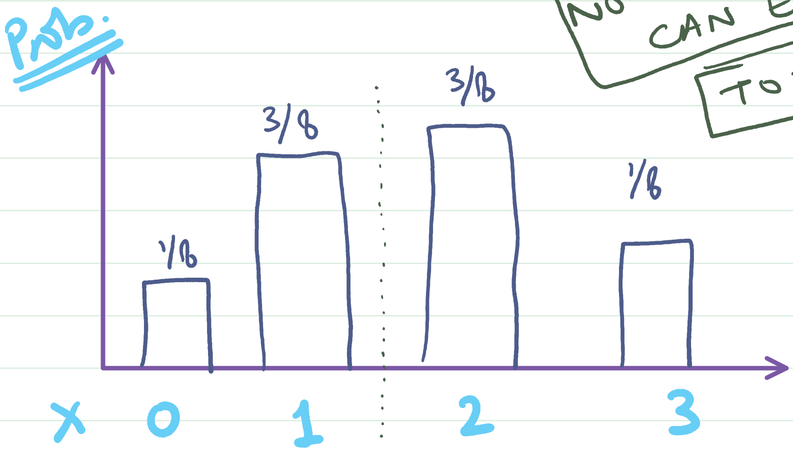
prob.

pmf

PMF is only defined for a discrete random var. and it tells us the probability attached to the particular value of rand. variable.

$p(X=2) = 3/8$ in above eg.

$p(Y=1) = 6/8$ $\quad$ $f(Y=5) = 0$

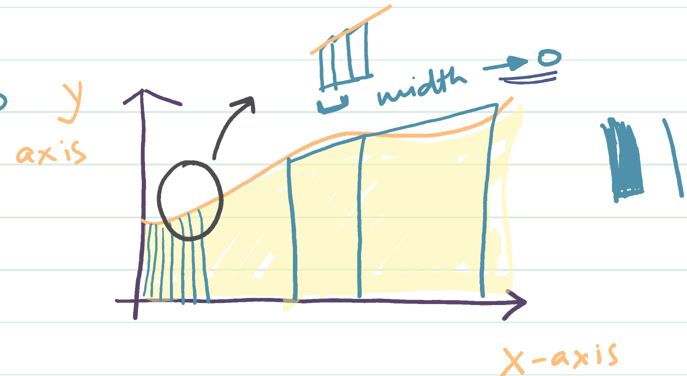


NO INDIVIDUAL BAR CAN EXCEED 1

TOTAL MUST BE 1

pdf

area of one trapezium = 0
when width of trap = 0



When we are talking about cont. var.

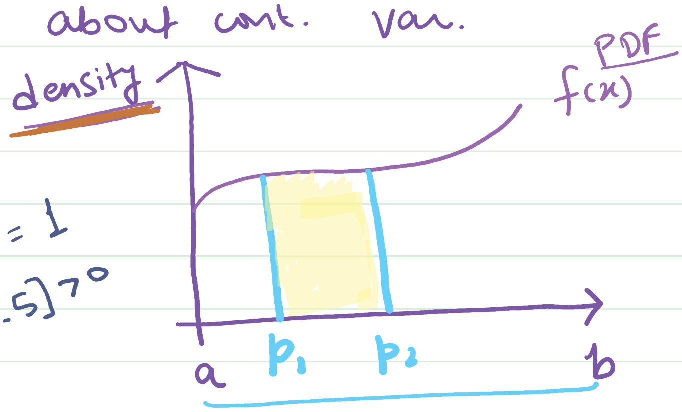
$X \in [a, b]$

eg $X \in [5, 6]$

$P[X = 5.25] = 0$

$P[5 \le X \le 6] = 1$

$P[5 \le X \le 5.5] > 0$



$$\text{Prob.}[p_1 \le X \le p_2] = \int_{p_1}^{p_2} f(x)\,dx =$$

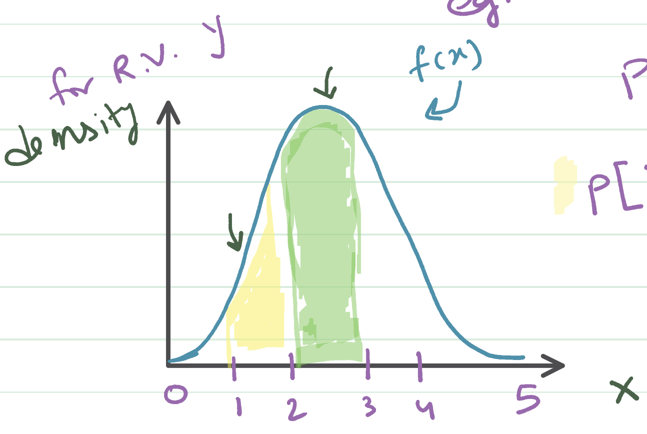
$$\int^{max} f(x)\,dx = 1 \quad \text{ALWAYS}$$

$$\int_{min} f(x)\,dx = 1 \quad \text{ALWAYS}$$

PMF directly gives Probability

PDF needs integration to give Probability

eg.



$P[0<y<5] = 1$

$P[1<y<2] < P[2<y<3]$

$\boxed{\begin{array}{l} 1.\ \text{density can be } \underline{\underline{>1}} \\ 2.\ \text{density always +ve} \end{array}}$

$\because$ sum of prob. has to be one $\int_{min}^{max} f(x)\,dx = 1$ (total prob.)

but $f(x)$ itself can be >1. only constraint is that it's positive and

$$\int_{\text{any point}}^{\text{any other point}} f(x)\,dx$$

CANNOT EXCEED 1

Probability Density Function and Probability Mass Function

- PDF is a statistical term that describes the probability distribution of the **Continuous** random variable

Probability Density Function

$F(x) = P(a \le x \le b) = \int_a^b f(x)dx \ge 0$

F(x) denotes the integrated answer

f(x) denotes density

- PMF is a statistical term that describes the probability distribution of the **Discrete** random variable

$p(x) = P(X = x)$

The probability of x = The probability (X = one specific x)

- People often get confused between PDF and PMF. The PDF is applicable for continuous random variables while PMF is applicable for discrete random variables

Discrete Probability Distribution

The role of a die or the toss of a coin are examples of events whose outcomes can only be predicted using discrete probability distributions.

Discrete probability distribution calculates the probabilities of discrete outcomes.

A discrete probability distribution gives each discrete result of a random variable a probability.

$x \rightarrow P(x)$

They can also simulate more complicated phenomena, like the volume of website visitors on a particular day. In general, discrete probability distributions are categorised based on the kind of random variable they express.

The most prevalent type of discrete probability distribution is the binomial distribution used to model events with two possible outcomes, such as success and failure when tossing a coin.

Now let's examine the various categories of probability distributions.

We need : definition | range | pmf/pdf | mean | Variance | graph

1

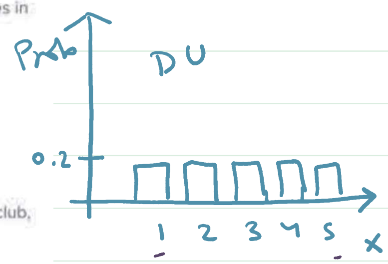
Uniform Distribution

— discrete unif.
— cont. uniform

equally likely events.

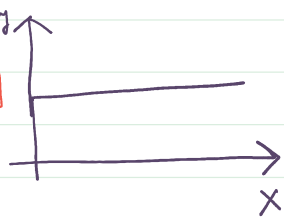
$x \in \{1,2,3,4,5\}$

It is a distribution where each outcome has an equal probability of occurrence. If there are n possible outcomes in a uniform distribution, the probability of occurrence of each outcome is 1/n. **It exists for both discrete and continuous variables.**



Examples of Uniform Distribution:

- Roll of a fair dice: It has six possible outcomes – 1,2,3,4,5,6. Each outcome has an equal probability.
- Deck of cards: If you draw a card randomly from a fair deck of playing cards, the likelihood that it will be a club, heart, diamond, or spade is uniformly distributed.
- Contests with a lucky draw: Everyone who enters a contest with a lucky draw has the same chance of winning. Because each person has an equal chance of winning, this distribution is known as the uniform probability distribution.



r.v. Parameters.

$X \sim \text{Uniform}(min, max)$; $X \in [min, max]$

follows

Range = [min, max]

PMF or $p(x) = \frac{1}{N}$ ← no. of points in the range

Mean of a random variable – exp. value $\{\mathbb{E}(X)\}$

mean is basically the expected value of r.v.

	X		prob.
1/n	x_1	⇢	p_1
1/n	x_2	⇢	p_2
1/n	x_3	⇢	p_3
⋮	⋮		⋮
1/n	x_n	⇢	p_n

$$E(X) = \sum_i x_i p_i$$

X (coins)	Prob.
0	1/8
1	3/8

$$\mathbb{E}(X) = 0 \times 1/8 = 0$$
$$+ 1 \times 3/8 = 3/8$$
$$+ 2 \times 3/8 = 6/8$$

0	1/8	$+ 1 \times 3/8 = 3/8$
1	3/8	$+ 2 \times 3/8 = 6/8$
2	3/8	$+ 3 \times 1/8 \quad 3/8$
3	1/8	$+ \; 12/8 = 6/4 = 3/2 = \boxed{1.5}$

CONTINUOUS UNIFORM — $E(X) = \frac{1}{b-a}$ (a, b)

variance $V(X) = \frac{(b-a)^2}{12}$

Question: The number of bouquets sold daily at a flower shop is uniformly distributed with a maximum of 40 and a minimum of 10. Find the probability that sales will be greater than 20.

$X \sim \text{Uniform}(10, 40)$

$P[X > 20] = \frac{40-20}{40-10} = \frac{20}{30} = \frac{2}{3}$

21 22...40

Solution:
The probability that daily sales will be greater than 20 is (40-20)*(1/(40-10)) = 2/3

Bernoulli Distribution

RESTRICTED TO 1 TRIAL

How do you choose who will bat or bowl first in a cricket game? A toss! The outcome of the coin toss will determine everything. Let's say you win if the coin comes up the head. If not, you lose. No halfway point exists.

The only two possible outcomes in a **Bernoulli distribution** are 1 (success) and 0 (failure), with only $\boxed{\text{one trial}}$. As a result, the random variable X, which has a Bernoulli distribution, can have a value of 1 for a success probability of p and a value of 0 for a failure probability of q or 1-p.
In this case, the presence of a head indicates success, while the presence of a tail indicates failure.
Given that there are only two possible outcomes, the probability of getting a head is equal to 0.5 of the probability of getting a tail.

$$P(x) = \begin{cases} 1-p, & x=0 \\ p, & x=1 \end{cases}$$

Success → p
or
Failure → 1-p

$$p(x) = \begin{cases} 1-p \; ; & x=0 \\ p \; ; & x=1 \end{cases}$$

X	Prob
0	1-p
1	p

6 success
other failure

Conditions for Bernoulli Distribution

Only if an event or experiment satisfies the following requirements can it be classified as a Bernoulli trial and thus be relevant to the Bernoulli distribution:

1 - There are only two outcomes that could occur during the trial. Consider this in terms of "success" or "failure," or, more specifically, does your experiment pose a "yes" or "no" question? Remember the examples from earlier, like "Will student X pass their math test?" or "Will taking this medication cure Patient Y?"

2 - Each of the two possibilities has a fixed likelihood of happening. ← prob. of success is fixed. In other words, the likelihood that a coin will land on its head is fixed, regardless of how often it is flipped. According to mathematics, the likelihood of success is always p, while the likelihood of failure is always 1 - p.

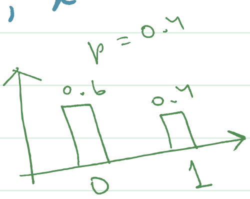
- Trials stand completely apart from one another. The outcome of one trial, such as the first coin flip, has no bearing on the results of any additional coin flips.

definition | range | pmf/pdf | mean | Variance | graph

In throw of a die I say getting 6 is a success
then P(success) = 1/6
P(failure) = 5/6

$$p(x) = \begin{cases} 1/6 \; ; & x = 1 \\ 5/6 \; ; & x = 0 \end{cases}$$

X = 1 for success
X = 0 for failure



$X = \{0, 1\}$ range

X   Prob.

$X = \{0, 1\}$ range

mean or $E(X)$: p

$\boxed{\text{Variance} : p(1-p)}$

X	prob.
$1 \longrightarrow$	p
$0 \longrightarrow$	$1-p$

$E(x) = \sum p_i x_i$

$p(1) + (1-p) \times 0$

$= \boxed{p}$

HOW TO CALCULATE VARIANCE

calculate

$E(X)$ & $E(X^2)$

$E(\text{anything}) = \sum \text{anything} \times p_i$

$E(x^2) = \sum p_i x_i^2$

Variance is

$V(x) = E(X^2) - [E(X)]^2$

eg. $1^2 \cdot p + 0^2 \cdot (1-p) - (p)^2$

$= p - p^2 = \boxed{p(1-p)}$

Question: Find the variance of the Bernoulli distribution with p=0.6.

Solution:
Variance = p * (1-p) = 0.6 *0.4 =0.24

$V(X) = p(1-p)$
$= 0.6(0.4) = 0.24$

Binomial Distribution

Only 2 outcomes
1 success
1 failure

but w/ n-trials taken together

Returning to cricket now. Consider the scenario where you won today's coin toss, and the event was a success. You try again, but this time you fail. Even if you win the toss today, you won't necessarily win it tomorrow. Let's give the number of times you won the toss a random variable, let's say X. What might X's possible value be? Depending on how many times you tossed a coin, any number could be the answer.

There are only two outcomes that could occur. Success is symbolised by the head and failure by the tail. As a result, the likelihood of getting a head is 0.5, and the likelihood of failure is easily calculated using the q = 1- p = 0.5.

A binomial distribution is one in which there are only two possible outcomes: success or failure, gain or loss, win or loss, and where the probability of success and failure is the same across all trials.

The results do not have to be equally likely. Do you still recall the incident involving Undertaker and me? As a result, it is simple to calculate the probability of failure in an experiment if the probability of success is 0.2: q = 1 - 0.2 = 0.8.

Each trial is independent of the others because the result of the previous toss has no bearing on or influence over the result of the present toss. Binomial experiments are run n times with only two possible outcomes. A binomial distribution has two variables, n and p, where n is the total number of trials and p is the likelihood that each trial will be successful.

Conditions for Binomial Distribution

Relate to strt of class

Binomial distribution properties:

1. Every trial stands alone.
2. There are only two possible outcomes in a trial: success or failure.
3. There are n identical trials performed in total.
4. For all trials, the odds of success and failure are equal. Trials are similar.

In n trials give me range of successes $\{0, 1, 2, \dots, n\}$

$P(x) = nCx * p^x * q^{(n-x)}$

in n trials
successes
$\{0, 1, 2, \dots, n\}$

$P(x) = nCx * p^x * q^{(n-x)}$

0 to n

definition | range | pmf/pdf | mean | Variance | graph

X is a binomial r.v. which measures the no. of successes in n-trials of an experiment

eg. If getting heads is success & I toss a coin 10 times
X measure total no. of heads we get in these 10 tosses.

no. of trials

$X \sim Bin(n, p)$

prob. of success

PMF : $^nC_x \, p^x (1-p)^{n-x}$; $x \in \{0 \text{ to } n\}$

eg. $^5C_1 \, p^1 (1-p)^{5-1}$

Question: A coin is tossed 12 times. What is the probability of getting exactly 7 heads?

$n = 12$
$x = 7$

$(^{12}C_7) \cdot p^7 (1-p)^5$

Solution:
Given that a coin is tossed 12 times. (i.e) n= 12
Thus, a probability of getting head in single toss = ½. (i.e) p = ½.
So, 1-p = 1-½ = ½.
We know that the binomial probability distribution is $P(r) = {}^nC_r \cdot p^r (1-p)^{n-r}$.
Now, we have to find the probability of getting exactly 7 heads.(i.e) r = 7.
Substituting the values in the binomial distribution formula, we get
$P(7) = {}^{12}C_7 \cdot (½)^7 (½)^{12-7}$
$P(7) = 792 \cdot (½)^7 (½)^5$
$P(7) = 792.(½)^{12}$
P(7) = 792 (1/4096)
P(7) = 0.193
Therefore, the probability of getting exactly 7 heads is 0.193.

Poisson Distribution

When events happen at random times and places, and all that is of interest is how many times they happen, the Poisson distribution is appropriate.

Assuming you work in a call centre, how many calls do you typically receive each day? Any number may be used. The Poisson distribution is now used to model the total number of calls a call centre receives per day. Additional instances include:

1. The daily total of emergency calls received by a hospital
2. The total number of thefts reported in a location on a given day
3. The number of clients who enter a salon in a given hour
4. The number of suicides that have been reported in a specific city
5. The percentage of printing mistakes on each page of the book

Range is $\{0,1,2,3,4,.....$

used to describe # of arrivals or occurances of a timed event

Assumptions for Poisson Distribution

When the presumptions are true, a distribution is considered a Poisson distribution.

1. One successful event shouldn't impact another successful event outcome; for example, one customer calling the customer centre shouldn't impact another customer's call.

3 buses/hr

6 buses for 2 hrs.

$$\boxed{P(X=x) = e^{-\mu}\frac{\mu^x}{x!}}$$

Lesson Plan: Probability Distribution — Relevel by Unacademy

2. The likelihood of success over a brief period must be equal to the likelihood of success over a longer period.

3. As an interval gets smaller, the chance of success decreases to zero. The likelihood of someone calling a call centre will be high if the period is one hour, low if it is ten minutes, even lower if it is one minute, and will continue to decline as the period decreases.

eg. if X measures # buses arriving at a stn in say 1 hour.

definition | range | pmf/pdf | mean | Variance | graph

# of printing error in a [chapter] of a book.

$0,1,2,\ldots$ $X \sim \text{Poisson}(\lambda)$

P.M.F. $p(x) = \dfrac{e^{-\lambda}\cdot\lambda^x}{x!}$ $\lambda \to$ mean of process.

long term avg.

eg. On an average 10 trains arrive at Pune Station per hour. Using a Poisson distrⁿ calculate the prob. that 12 trains will arrive in the next hour.

X	Prob.
0	$e^{-10}\cdot 10^0/0! = e^{-10} = 0.0000454$
1	$e^{-10}\cdot 10^1/1! = 10e^{-10} = 0.000454$
2	$e^{-10}\cdot 10^2/2! = 50e^{-10} = 0.00227$
3	
4	
⋮	

$$\boxed{P(X=12) = \frac{e^{-10}\cdot 10^{12}}{12!}}$$

$\dfrac{e^{-\lambda}\cdot\lambda^x}{x!} \le 1$

$\boxed{e \simeq 2.718}$

In Poisson distribution $E(X) = V(X) = \lambda$ ALWAYS

Question: If a random variable X obeys the Poisson distribution with mean 2, what is its variance Var(X)?

Solution:
Variance = Mean = 2

Continuous Probability Distribution

eg.

Variance = Mean = λ

Continuous Probability Distribution

An example of a continuous probability distribution deals with random variables that can have any continuous value within a specific range. Contrary to discrete random variables, which can only have discrete, specific values, continuous random variables can take on various values.

Like height, weight, and volume, continuous random variables are frequently used in mathematics.

The radioactive decay rate or sound waves' speed are two examples of physical phenomena frequently modelled using continuous probability distributions. Continuous probability distributions come in various forms, each with its shape.

The bell-shaped normal distribution is the most prevalent. The exponential distribution and the uniform distribution are additional examples. Continuous probability distributions can describe a wide range of real-world phenomena. For instance:
A continuous probability distribution might be used to describe the height distribution of students in a classroom. A continuous probability distribution may also be used to describe the weight of newborn infants. This would allow for the possibility of any weight being observed and the fact that some babies are born heavier than others.

eg.
X measures the weight of all athletes.
X will take continous values so to model this we will continuous distributions.

search golden ratio?

Normal Distribution

Most situations in the universe behave according to the normal distribution, which is why it is called a "normal" distribution. Its widespread use is partly because the large sum of (small) random variables frequently turns out to be normally distributed.

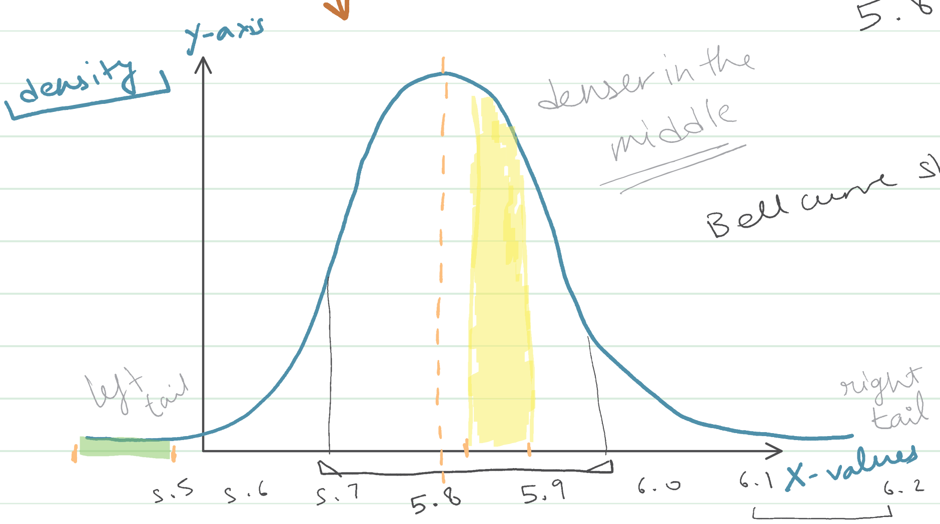
Any distribution that possesses the following traits is known as a normal distribution:
- The distribution's mean, median, and mode are all equal.
- The distribution curve is bell-like and symmetrically around the line x=.
- The curve has a total area under it of one.
- The values are split in half, half to the left and half to the right of the centre.

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\left(-1/2\left(\frac{x-\mu}{\sigma}\right)^2\right)} \quad \text{for } -\infty < x < \infty$$

$(-\infty, \infty)$

definition | range | pmf/pdf | mean | Variance | graph

say avg is. 5.8 ft



Bell curve shape

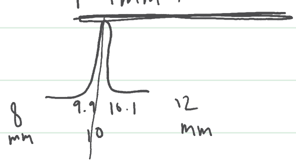
pop^n mean

pop^n variance

$$X \sim N(\mu, \sigma^2)$$

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}}$$

eg. make 10mm bolts
9.9mm to 10.1mm



to get prob. we need to integrate f(x) over the

required values $\int_0^5 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}\,dx$

$\mu, \sigma \rightarrow$ parameter (given in Q) are fixed constants

Question: Calculate the probability density function of normal distribution using the following data. x = 3, μ = 4 and σ = 2.

Solution:
Given, variable, x = 3
Mean = 4 and
Standard deviation = 2
By the formula of the probability density of normal distribution, we can write:

b/w a & b

$f(3) = \frac{1}{2\sqrt{2\pi}} e^{\frac{-(3-2)^2}{2\times 2^2}}$

Hence, f(3,4,2) = ~~1.106~~

$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$

$= \frac{1}{2\sqrt{2\pi}} e^{-\frac{1}{2\cdot 2^2}(3-4)^2} = e^{-\frac{1}{8}(1)} =$

$= \frac{1}{2\sqrt{2\pi}}$

$f(x)$ or $f(3) = \frac{1}{2\sqrt{2\pi}} e^{-1/8} = 0.17603$

$\hookrightarrow$ DENSITY NOT PROBABILITY

Exponential Distribution

Poisson → no. of occurance

Let's consider the call centre example one more time. What about the interval of time between the calls? Here, the exponential distribution comes to our rescue. Exponential distribution models the interval of time between the calls.
Other examples are:

1. Length of time between metro arrivals,
2. Length of time between arrivals at a gas station
3. The life of an Air Conditioner

time measurement or age measurement.

It is used to predict the expected life of the machine to the expected life of a human.

f(x) = { λe-λx, x ≥ 0

eg. time b/w 2 occurances

definition | range | pmf/pdf | mean | Variance | graph

time
10.01am, 10.15am 10.22 am, ... 10.58
1 * 2 * 3 * 16

Poisson will model this (16) → i.e. no. of occurances
while * Exponential will model time b/w 2 arrivals.

$X \sim exp(\theta)$ when $1/\theta$ is the mean of process.

X lies b/w 0 to ∞ $X \geq 0$ RANGE

$f(x) = \theta e^{-\theta x}$ mean = $1/\theta$ variance = $1/\theta^2$

Question: Assume that, you usually get two phone calls per hour. Calculate the probability that a phone call will come within the next hour.

2 calls per hour
(0.5) hrs b/w 2 calls

Solution:
It is given that, two phone calls per hour come by. So, it would be expected that one phone call at every half-an-hour comes by. So, we can take:
λ = 0.5
So, the computation is as follows:

$p(0 \le X \le 1) = \int_{x=0}^{1} 0.5e^{-0.5x}$

= 0.393469 ✓
Therefore, the probability of the phone calls arriving within the next hour is 0.393469.

P[0 ≤ x < 1] = ∫¹ θe^(−θx) ∫¹ −0.5x . . . 3934

= 0.393469

Therefore, the probability of the phone calls arriving within the next hour is 0.393469.

$$P[0 \le x \le 1] = \int_0^1 \theta e^{-\theta x} = \int_0^1 0.5\, e^{-0.5x}\, dx = 0.3934$$

Relationship Between Probability Distribution

1) - Bernoulli and Binomial Distribution
2) - Poisson and Binomial Distribution
3) - Normal and Binomial Distribution

1) # successes is measured

Bernoulli → 1 trial

Binomial → n trials

2) Binomial becomes poisson when $\underset{\text{\# trials}}{n} \to \infty$ $\underset{\text{prob. of success}}{p} \to 0$ & $(n \cdot p)$ is finite

3) When $n \to \infty$ np becomes mean and binomial → normal

$np(1-p)$ becomes variance