MATHEMATICS (SEMESTER - 2)

CS/BCA/SEM-2/BM-201/09

1. ....
Signature of Invigilator

2. ....
Signature of the Officer-in-Charge

Reg. No. | | | | | | | | | | | | | | | |

Roll No. of the Candidate | | | | | | | | | | | | | | | |

CS/BCA/SEM-2/BM-201/09

ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE – 2009

MATHEMATICS (SEMESTER - 2)

Time : 3 Hours] [Full Marks : 70

INSTRUCTIONS TO THE CANDIDATES :

1. This Booklet is a Question-cum-Answer Booklet The Booklet consists of **32 pages**. The questions of this concerned subject commence from Page No 3.
2. a) In **Group – A**, Questions are of Mult ple Choice type. You have to write the correct choice in the box provided **against each question**.
   b) For **Groups – B** & **C** you have to answer the questions in the space provided marked 'Answer Sheet'. Questions of **Group B** are Short answer type. Questions of **Group – C** are Long answer type. Write on both sides of the paper.
3. **Fill in your Roll No. in the box** provided as in your Admit Card before answering the questions.
4. Read the instructions given inside carefully before answering.
5. You should not forget to write the corresponding question numbers while answering.
6. Do not write your name or put any special mark in the booklet that may disclose your identity, which will render you liable to disqualification. Any candidate found copying will be subject to Disciplinary Action under the relevant rules.
7. **Use of Mobile Phone and Programmable Calculator is totally prohibited in the examination hall.**
8. You should return the booklet to the invigilator at the end of the examination and should not take any page of this booklet with you outside the examination hall, **which will lead to disqualification**.
9. Rough work, if necessary is to be done in this booklet only and cross it through.

No additional sheets are to be used and no loose paper will be provided

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FOR OFFICE USE / EVALUATION ONLY

Marks Obtained

Group – A **Group – B** **Group – C**

Question Number																	***Total Marks***	***Examiner's Signature***
Marks Obtained																		

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Head-Examiner/Co-Ordinator/Scrutineer

2324 (11/06)

ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE – 2009

MATHEMATICS

SEMESTER - 2

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $\alpha = (1, 0, 3)$ and $\beta = (-1, 2, 5)$, then $\alpha + 3\beta$ is equal to

a) $(-2, 6, 18)$ b) $(2, -6, -18)$

c) $(2, -6, 18)$ d) $(-1, -3, 5)$.

ii) The basis of a vector space on ains

a) linearly independent set of vectors

b) linearly dep ndent set of vectors

c) scalars only

d) none of these.

iii) Integrating factor of $x\,dx = -y\,dy$ is

a) $1/(xy)$ b) $1/(x^2 + y^2)$

c) $1/y^2$ d) none of these.

iv) The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if

a) $p = 1$ b) $p > 1$

c) $p < 1$ d) none of these.

v) The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{2/3} - 3\frac{dy}{dx} = 4$ are

a) 2, 2 b) $2, \frac{2}{3}$

c) 2, 1 d) 2, 3.

vi) If the three vectors (5, 2, 3), (7, 3, x) and (9, 4, 5) are linearly dependent, then x is

a) 1 b) 2

c) 3 d) 4.

vii) If $\lim_{n \to \infty} a_n = 0$, then the series $\Sigma (-1)^n a_n$ is

a) convergent b) divergent

c) oscillatory d) none of these.

viii) The family of curves $y = e^x (A \cos x + B \sin x)$ is represented by the differential equation

a) $\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - y$ b) $\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 2y$

c) $\frac{d^2y}{dx^2} = \frac{dy}{dx} - 2y$ d) $\frac{d^2y}{dx^2} = 2\frac{dy}{dx} + y$.

ix) The sequence $1, \frac{1}{2}, \frac{1}{3}, \ldots \frac{1}{n}, \ldots\ldots$ converges to

a) • b) 0

c) 1 d) $\frac{1}{2}$.

x) The four vectors (1, 1, 0, 0), (1, 0, 0, 1), (1, 0, a, 0,) and (0, 1, a, b) are linearly independent if

a) a π 0, b π 2 b) a π 2, b π 0

c) a π 0, b π – 2 d) a π – 2, b π 0.

xi) The solution of $\log\left(\frac{dy}{dx}\right) = ax + by$ is

a) $b\,e^{-ax} + a\,e^{-by} + k = 0$ b) $b\,e^{ax} + a\,e^{-by} + k = 0$

c) $b\,e^{-ax} + a\,e^{by} + k = 0$ d) $b\,e^{ax} + a\,e^{by} + k = 0$.

xii) $\lim_{n\, Ø\, •} \frac{x^n}{\lfloor n}$ is equal to

a) 0 b) 1

c) – 1 d) none of these.

xiii) The lower bound of the sequence $\left\{ \frac{(-1)^{n-1}}{n!} \right\}$) is

a) $-\frac{1}{2}$ b) $\frac{1}{2}$

c) 1 d) 0.

xiv) The value of $\lim_{n\, Ø\, •} \log(1/n)$ is equal to

a) 0 b) 1

c) – • d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find the equation of curve whose slope at any point (x, y) on it is $2y$ and which passes through the point (3, 1).

3. Test for the convergence of the series :

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \ldots\ldots, \ x > 0.$$

4. Examine whether the vectors (1, 2, 3, 0), (2, 1, 0, 3), (1, 1, 1, 1) and (2, 3, 4, 1) are linearly dependent or not. If yes, find among them which are independent.

5. Solve any three :

a) $x \frac{dy}{dx} + y = y^2 \log x$

b) $\cos^2 x \frac{dy}{dx} + y = \tan x$

c) $y = px - \frac{a}{p}$ where $p = \frac{dy}{dx}$

d) $\left(D^2 - 2D + 1 \right) y = x e^x$ where $D = \frac{d}{dx}$.

6. Define the limit of a sequence. Find

$$\lim_{n \to \infty} \left[\frac{1}{n^2} + \frac{2}{n^2} + \cdot \quad + \frac{n}{n} \right] .$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Define basis of a vector space V. Show that $\alpha_1 = (1, 0, 0)$, $\alpha_2 = (0, 1, 0)$ and $\alpha_3 = (0, 0, 1)$ form a basis of the vector space V_3. 5

b) If $\{ \alpha, \beta, \gamma \}$ be a basis of real vector space V and $c \neq 0$ be a real number, examine whether $\{ \alpha + c\beta, \beta + c\gamma, \gamma + c\alpha \}$ is a basis of V or not. 5

c) Find the value of k for which the vectors (1, 2, 1), (k, 1, 1) and (1, 1, 2) in R^3 are linearly dependent. 5

8. Test the convergence of any *three* of the following series : $3 \times 5 = 15$

a) $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^2}{10} + \ldots\ldots$

b) $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \ldots\ldots$

c) $\overset{\alpha}{\underset{n=1}{}} \left(1 + \frac{1}{\sqrt{n}}\right)^{-\sqrt{n}}$

d) $\overset{\alpha}{\underset{n=1}{}} \left(\frac{\cos nx}{n^2}\right)$.

9. Solve any *three* of the following : $3 \times 5 = 15$

a) $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y\,(\log y)^2}{x^2}$

b) $y = 2px - p^2$ where $p = \frac{dy}{dx}$

c) $e^x \sin y \, dx + (e^x + 1) \cos y \, dy = 0$

d) $(D^2 - 2D)\,y = e^x \sin x$.

10. a) Prove that $s = \{(0, 1, 1), (1\ 0, 1), (1, 1, 0)$ is a basis of R^3 .

b) Show that $w = \{(x, y\ z) \blacklozenge R^3 / x + y + z = 0\}$ is a sub-space of R^3 and find a basis of w.

c) Determine K so that the set S is linearly dependent in R^3

$$S = \{(1, 2, 1), (k, 3, 1), (2, k, 0)\}.$$ 5 + 5 + 5

11. a) Define the linear sum of two sets of vectors S and T.

b) If S and T are two sub-spaces of a vector space V, obtain a relation between rank (S), rank (T) and rank (V) .

c) Let $T : R^2 \varnothing R^2$ be a linear transformation such that $T(1, 1) = (2, -3)$ and $T(1, -1) = (4, 7)$. Find the matrix of T. 3 + 6 + 6

12. a) Using D' Alembert's ratio test, show that the following series is convergent :

$$x^2 + (2^2 / 3.4)\, x^4 + (2^2 . 4^2 / 3.4.5.6)\, x^6 + \ldots\ldots \bullet$$

b) Prove that every absolutely convergent series is convergent.

c) Show that the following series is convergent :

$$u_n = \sqrt{n^3 + 1} - \sqrt{n^3} \text{ in } n \ \square\ [\, 1, \bullet\,].$$

END

Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-2/BM-201/2010

2010

MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The basis of a vector space contains

      a) linearly independent set of vectors

      b) linearly dependent set of vectors

      c) scalars only

      d) none of these.

   ii) The solution of $\frac{d^2y}{dx^2} = 0$ is

      a) $y = e^x$ b) $y = 0$

      c) $y = \sin x$ d) $y = \log_e x$.

iii) If $f(3, 1) = x(1, 2) + y(0, 3)$ then the values of x and y are respectively

a) $(3, -5)$ b) $(3, 1)$

c) $(3, -5/3)$ d) $(3, -5/2)$.

iv) $\lim_{n \to \infty} (3n + 1) / (2n - 3)$ is

a) $\frac{1}{2}$ b) $\frac{3}{2}$

c) 1 d) $-\frac{1}{3}$.

v) The value of $\left(1/D^2\right)\left(x^3\right)$ is

a) x^5 b) $\frac{1}{20}$

c) 20 d) $\frac{1}{20}x^5$.

vi) $\sum 1/n^p$ is divergent if

a) $p \le 1$ b) $p > 1$

c) $p < 1$ d) $p = 1$.

vii) If $P = \{2, 4, 6, 7, 8, 9\}$, $Q = \{1, 2, 6, 9\}$, then $P - Q$ is

a) $\{4, 7, 8\}$

b) $\{4, 6, 8, 9\}$

c) $\{1\}$

d) $\{2, 4, 6, 7, 8, 9\}$.

viii) $\frac{1}{(D-2)(D-3)}e^{2x}$ is

a) $-e^{2x}$ b) xe^{2x}

c) $-xe^{2x}$ d) $-xe^{3x}$.

ix) Integrating factor of $\frac{dy}{dx}+y=x$ is

a) e^{-x} b) e^{x}

c) x^2 d) none of these.

x) The differential equation $\left(\frac{dy}{dx}\right)^2+ay^{\frac{1}{2}}=x$ is

a) linear of degree 2

b) non-linear of order one and degree 4

c) non-linear of order one and degree 2

d) none of these.

xi) If vectors (a, 0, 1), (0, 1, 0), (1, a, 1) of a vector space $\mathbb{R}^3$ over $\mathbb{R}$ be linearly dependent, then the value of a is

a) 2

b) 3

c) 1

d) none of these.

xii) Auxiliary equation of the differential equation $\frac{d^2y}{dx^2} + 4y = \sin x$ is

a) $y = \cos 2x + \sin 2x$

b) $y = c_1 \cos 2x + c_2 \sin 2x$

c) $y = c_1 \cos x + \sin 2x$

d) none of these.

xiii) The general solution of $\log \frac{dy}{dx} = x - y$ is

a) $e^y - e^x = c$

b) $e^x + e^y = c$

c) $e^{x+y} = c$

d) $e^{x-y} = c$.

xiv) If S and T be two subspaces of a vector space V, then which of the following is also a subspace of V ?

a) $S \cup T$

b) $S - T$

c) $T - S$

d) $S \cap T$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that the sequence $\left\{2 + (-1)^n \, 1/n\right\}$ is convergent.

3. Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 + e^{3x}$

4. Find the value of x for which the vectors $(1, 2, 1)$, $(x, 3, 1)$ and $(2, x, 0)$ become linearly independent.

5. Find the value of the limit $\lim_{n \to \infty} \left(4n^3 + 6n - 7\right)/\left(n^3 - 2n^2 + 1\right)$.

6. Find a basis and the dimension of $S \cap T$, where S and T are subspaces of R^3 defined by

$$S = \{ (x, y, z) \in R^3 : 2x + y + 3z = 0 \}$$

and $T = \{ (x, y, z) \in R^3 : x + 2y + z = 0 \}$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that $\left\{ \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}} \right\}$ is convergent and converges to 1.

b) Show that the sequence $\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \dots$ converges to 2. 8 + 7

8. Solve the following equations : 3×5

a) $\left(D^2 - 2D + 1\right)y = x \sin x$

b) $\frac{d^2y}{dx^2} + \frac{1}{x} \cdot \frac{dy}{dx} = \frac{12 \log x}{x^2}$

c) $3\frac{dy}{dx}+2\frac{y}{x+1}=\frac{x^3}{y^2}$

9. a) Prove that a subset S of a vector space V over $\mathbb{R}$ is a subspace if and only if $\alpha x+\beta y \in S$ for all $\alpha, \beta \in \mathbb{R}$ and $x, y \in S$.

b) Prove that the vectors $\{(1, 2, 2), (2, 1, 2), (2, 2, 1)\}$ are linearly independent in $\mathbb{R}^3$.

c) Find the basis and the dimension of the subspace W of $\mathbb{R}^3$ where

$W=\{(x, y, z) \in \mathbb{R}^3 : x+y+z=0\}$ 5 + 5 + 5

10. a) Solve $(px-y)(py+x)=a^2p$, by using the substitution $x^2=u$, $y^2=v$; where $p=\frac{dy}{dx}$.

b) Obtain the general solution and singular solution of the equation $y=px+\sqrt{a^2p^2+b^2}$. 7 + 8

11. a) Define basis of a vector space.

b) Show that the vectors $\alpha_1=(1,0,-1), \alpha_2=(1,2,1)$ and $\alpha_3=(0,-3,2)$ form a basis for $\mathbb{R}^3$. Express $(1, 0, 0)$ as a linear combination of α_1, α_2 and α_3.

c) Find the matrix of the linear transformation T on $V_3(\mathbb{R})$ defined as

$T(a, b, c) = (2b + c, a - 4b, 3a)$ with respect to the ordered basis B where

$B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$. 3 + 6 + 6

12. a) Prove that the sequence $\{a_n\}$ is monotonically increasing and bounded when

$a_n = (3n+1)/(n+2)$

b) State D' Alembert's Ratio Test.

c) If α, β, γ form a basis of a vector space V, then prove that $\alpha + \gamma$, $2\alpha + 3\beta + 4\gamma$ and $\alpha + 2\beta + 3\gamma$ also form a basis of the vector space V. 8 + 2 + 5

Name : ....

Roll No. : ....

Invigilator's Signature : ....

2011

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full mark
Candidates are required to give their answers in their own words as far as practicable.

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives f r any *ten* of the following :

10 × 1 = 10

i) The degree and order of the differential equation $y = \frac{x\,d^2y}{dx^2} + r\sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}}$ are

a) 2, 2 b) 2, 1

c) 3, 2 d) none of these.

ii) The geometric series $1 + r + r^2 + \dots$ is convergent if

a) $-1 < r < 1$ b) $r > 1$

c) $r = 1$ d) none of these.

iii) The series 1 + 1 + 1 + ... is

a) convergent b) divergent

c) oscillatory d) none of these.

iv) An absolutely convergent series is

a) necessarily convergent

b) not necessarily convergent

c) conditionally convergent

d) none of these.

v) Leibnitz's test is applied to

a) a constant series

b) an alternating series

c) series of positive terms only

d) none of these.

vi) If W_1 and W_2 be two subspaces of a vector space $V(F)$ then $W_1 \cap W_2$ is

a) necessarily a subspace

b) not a subspace

c) is a ubspace only when one is contained within another

d) none of these.

vii) In the vector space R^3 over the field R the vectors (1, 0, 0), (0, 1, 0) and (0, 0, 1) are

a) linearly independent

b) linearly dependent

c) none of these.

viii) Integrating factor of differential equation $x \log x \frac{dy}{dx} + y = 2 \log x$ is

a) $\log x$ b) $\log (\log x)$

c) e^{x} d) none of these.

ix) The upper bound of the sequence $\left\{ (-3)^{n} \right\}$ is

a) 4 b) 0

c) -3 d) none of these.

x) If T is a linear mapping for V to V and $\alpha, \beta \in V$ and a, b are scalers, then

a) $T (a\alpha + b\beta) = aT (\alpha) + bT (\beta)$

b) $T (a\alpha + b\beta) = aT (\alpha) - bT (\beta)$

c) $T (a\alpha + b\beta) = T (\alpha)$

d) $T (a\alpha + b\beta) = bT (\beta)$

xi) The root of the auxiliary equation of the given differential equation $\frac{d^{2}y}{dx^{2}} - \frac{4dy}{dx} + 4y = 0$ are

a) 2, 4 b) 2, 2

c) 1, 1 d) none of these.

xii) Let $T : R^{2} \rightarrow R^{2}$ be a linear transformation defined by $T(x, y) = (2x - y, x + y)$. Then kernel of T is

a) $\{ (1, 2) \}$ b) $\{ (0, 0) \}$

c) $\{ (1, 2), (1, -1) \}$ d) none of these.

xiii) If $T : V_2 \rightarrow V_3$ be defined by

$T(v_2) = \{ (x,0,0) : x$ is a real number $\}$ then rank of T is

a) 3 b) 2

c) 1 d) 0.

xiv) If T is a linear transformation from vector space V into W, then

a) rank (T) + nullity (T) = dim (V)

b) rank (T) + nullity (T) = dim (W)

c) rank (T) – nullity (T) = dim (V)

d) none of these.

Group – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

3. Show that the mapping defined by $T : R^2 \rightarrow R^3$

$T(x_1, x_2) = (x_1 + x \ , \ x_1 - x_2, \ x_2)$ is linear. Find the value of $T(1 \ \ 2)$.

4. Solve :

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x .$$

5. The linear transformation $T : R^3 \to R^3$ is defined by $T(x, y, z) = (x - y,\ x + 2y,\ y + 3z)$. Show that T is invertible and determine T^{-1}.

6. Prove that the set of vectors { (1, –2, 3), (2, 3, 1), (–1, 3, 2) } is linearly independent. Also verify whether this set forms a basis of V_3 or not.

7. Let $S = \{ (x, y, z) : (x, y, z) \in R^3, x + y + z = 0 \}$. Prov that S is a subspace. Find the dimension of S.

Group – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

8. a) Find the order and degree of the differential equation

$$\frac{d^2y}{dx^2} = \left(y + \left(\frac{dy}{dx} \right)^2 \right)^{1/4}$$

b) Verify whether the differential equation $e^y dx + \left(xe \;\; + 2y \right) dy = 0$ is exact. If so, then solve it.

c) Solve the differential equation $x \frac{dy}{dx} - 2y = xy^4$.

3 + 5 + 7

9. a) Solve $y = px + \frac{a}{p}$ and also obtain the singular solution.

b) Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - 5y = \sin(\log x)$.

c) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$.

5 + 6 + 4

10. a) State D' Alembert's ratio test. Test the convergence of the series $\sum_{n=1}^{\infty}\frac{2^n n!}{n^n}$.

b) Prove that the series

$$\left(1+\frac{1}{2}\right)-\left(1+\frac{1}{4}\right)+\left(1+\frac{1}{8}\right)-\left(1+\frac{1}{16}\right)+\ldots$$ is an oscillating series.

c) Verify for the following example that the converse of the statement "If $\sum a_n$ be a convergent series then $\lim_{n\to\infty} a_n = 0$" is not true, where $\sum a_n = \sum_{n=1}^{\infty}\frac{1}{n}$.

5 + 6 + 4

11. a) Define basis of a vector space.

b) Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ form a basis for R^3.

c) Determine the value of k so that the set $S = \{(1, 2, 1), (k, 3, 1), (2, k, 0)\}$ is linearly dependent in R^3.

3 + 6 + 6

12. a) Let $T : R^3 \to R^2$ be a linear transformation defined by $T(x, y, z) = (x - 2y, y - 2z, z - 2x)$ for $(x, y, z) \in R^3$. Obtain a matrix representation for the linear transformation T.

b) Let V = set of all 2 × 2 matrices and $T : V \to V$ be defined by $T(X) = AX - XA$ where $X \in V$ and $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.

Find the basis of ker (T) and the nullity

c) Let $T : R^3 \to R^2$ be a linear m pping defined by $T(x, y, z) = (x + 2y, z)$. Verify that

dimension (kernel (T)) + dimension (Image (T))

= dim (R^3). 5 + 6 + 4

13. a) Define a subspace of a vector space.

b) State a necessary and sufficient condition for $W \subseteq V$ to be a subspace of $V(F)$.

c) Test whe her the series

$$\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \ldots (0 < x < 1)$$

is convergent or not. 3 + 3 + 9

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Name : ....

Roll No. : ....

Invigilator's Signature : ....

2012
MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their wn words as far as practicable

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for any *ten* the following: $10 \times 1 = 10$

i) Integrating factor of the differential equation

$x\,(1-x^2)\,dy + (2x^2y - y - ax^3)\,dx = 0$ is

a) $x/\sqrt{1-x^2}$ b) $x/\sqrt{x(x^2-1)}$

c) $x/\sqrt{x^2-1}$ d) $x^2/\sqrt{1-x^2}$.

ii) The order and degree of the differential equation

$\sqrt{d^2y\,/\,dx^2} + dy/dx = y$ are

a) 2, 1 b) 1, 2

c) 2, 2 d) 2, 3.

iii) $\frac{1}{(D-2)(D-3)} e^{2x}$ is

a) $-e^{2x}$ b) xe^{2x}

c) $-xe^{2x}$ d) $-xe^{3x}$.

iv) If for a sequence $\{U_n\}$, $\lim_{h \to \infty} U_n = 0$, then

a) $\{U_n\}$ is converget

b) $\{U_n\}$ is divergent

c) $\{U_n\}$ is convergent to 0

d) none of these.

v) The infinite series $\sum \frac{n}{n+1}$ is

a) divergent b) convergent

c) oscillatory d) none of these.

vi) The value of a for which { (1,2,3), (0,–1,9), (4,0,a) } is linearly dep ndent is

a) – 20 b) – 10

c) – 5 d) None of these.

vii) If the third order square matrix A is diagonalizable, then the number of independent eigenvectors of A is

a) two b) three

c) one d) none of these.

viii) If S and T be two subspaces of a vector space V, then which of the following is also a subspace of V ?

a) $S \cup T$ b) $S - T$

c) $T - S$ d) $S \cap T$.

ix) The dimension of the subspace $\{(x, 0, y, 0) \mid x, y \in R\}$ is

a) 1 b) 2

c) 3 d) 4.

x) Let V and W be two vector spaces over R nd $T : V \to W$ is a linear mapping. Then $Im\ T$ is a sub space of

a) V b) W

c) $V \cup W$ d) $V \cap W$.

xi) The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if

a) $p \geq 1$ b) $p > 1$

c) $p \leq 1$ d) none of these.

xii) The lower bound of the sequence $\left\{(-1)^{n-1} / n!\right\}$ is

a) $-1/2$ b) $1/2$

c) 0 d) none of these.

xiii) Eliminating A and B from $y = A \cos x + B \sin y$, the differential equation is

a) $\frac{d^2 y}{dx^2} = 0$ b) $\frac{d^2 y}{dx^2} - y = 0$

c) $\frac{d^2 y}{dx^2} + y = 0$ d) $\frac{d^2 y}{dx^2} = 1$.

xiv) The particular integral of $(D^2 + 1)\, y = \sin x$ is

a) $x \sin x$ b) $x \cos x$

c) $x \tan x$ d) $-\dfrac{x}{2} \cos x$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve : $(px - y)(py + x) = a^2 p$

   by using the substitution $x^2 = u,\ y^2 = v$ where $p = \dfrac{dy}{dx}$

3. Examine the convergence of the sequence

   $\left\{ \left(1 + \dfrac{2}{n} \right)^n \right\}$.

4. Examine the convergence o the series

   $\dfrac{1}{2} + \dfrac{2}{3} x + \left(\dfrac{3}{4} \right)^2 x^2 + \left(\dfrac{4}{5} \right)^3 x^3 + \ldots\ldots\ldots\ldots\ldots$

5. Show that $W \ \{ (x_1, x_2, x_3, x_4) \in R^4 \mid x_1 - x_2 + x_3 = x_4 \}$ is a subspace of R^4.

6. Find the representative matrix of the linear transformation $T : R^3 \to R^3$ defined by

   $T(x, y, z) = (3x + z,\ -2x + y,\ -x + 2y + 4z)$

7. Find the basis of

   $S = \{ (x, y, z) \in R^3 \mid x + 2y + z = 0,\ 2x + y + 3z = 0 \}$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Solve : $\dfrac{d^2y}{dx^2} + \dfrac{1}{x}\,\dfrac{dy}{dx} = 12\,\dfrac{\log x}{x^2}$

b) Obtain the general solution and singular solution of the equation $y = px + \sqrt{a^2\;p^2 + b^2}$

c) Solve : $3\dfrac{dy}{dx} + \dfrac{2y}{x+1} = \dfrac{x^3}{y^2}$ 5 + 6 + 4

9. a) State Leibnitz theorem for Alternating series and test the convergence of the series

$1 - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \ldots\ldots..$

b) Test the convergence of the following series

$\dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dfrac{1}{2^3} + \dfrac{1}{3^3} + \ldots\ldots\ldots\ldots$

c) Show that the sequence $\left\{ 2 + \dfrac{(-1)^n}{n} \right\}$ is convergent.

6 + 5 + 4

10. a) S lve $\dfrac{d^2y}{dx^2} + 4\,\dfrac{dy}{dx} + 29\,y = 0$

when $x = 0$, $y = 0$, $\dfrac{dy}{dx} = 15$

b) Show that the sequence

$\sqrt{2}$, $\sqrt{2+\sqrt{2}}$, $\sqrt{2+\sqrt{2+\sqrt{2}}}$, ... converges to 2.

c) Define basis and dimension of a vector space.

6 + 6 + 3

11. a) Prove that the vectors (x_1, y_1) and (x_2, y_2) are linearly dependent, if and only if $x_1 y_2 - x_2 y_1 = 0$.

b) Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$ and $\propto_3 = (0, -3, 2)$ form a basis of R^3. Express (1, 0, 0) as a linear combination of α_1, α_2 and α_3.

c) If $\alpha_1, \alpha_2, \alpha_3$ form a basis of a vector space V, then prove that $\alpha_1 + \alpha_3$, $2\alpha_1 + 3\alpha_2 + 4\alpha_3$ and $\alpha_1 + 2\alpha_2 + 3\alpha_3$ also form a basis of the vector space V. 4 + 6 + 5

12. a) Let T be defined by $T(x, y) = (x' \quad y')$ where

$x' = x \cos\theta - y \sin\theta$, $y' = x \sin\theta + y \cos\theta$

Prove that T is a linear transformation.

b) The linear transformation $T : R^3 \to R^3$ transforms the basis vectors (1, 2, 1), (2, 1, 0) & (1, –1, –2) to the basis vectors (1, 0 0), (0, 1, 0) & (0, 0, 1) respectively. Find T. Hence find T (3, –3, 3).

c) Find the Kernel, Image, Nullity and Rank of

$T : R^3 \to R^2$ where

T (1, 0, 0) = (2, 1)

T (0, 1, 0) = (0, 1)

T (0, 0, 1) = (1, 1) 4 + 7 + 4

13. a) Prove that a subset S of a vector space V over R is a subspace if and only if $\alpha x + \beta y \in S$ for all $\alpha, \beta \in R$ and $x, y \in S$.

b) Show that the family M_2 of all real square matrices of order 2 forms a vector space over reals, and find a basis for M_2.

c) Let $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a + b = 0 \text{ and } a, b, c, d \in R \right\}$

Prove that S is a subspace of M_2. 5 + 6 + 4

Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-2/BM-201/2013

2013

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) A m notoni and bounded sequence is

a) convergent b) divergent

c) oscillatory d) none of these.

ii) The sequence $\{r^n\}$ is oscillatory when

a) $r > 1$ b) $r < 1$

c) $-1 < r < 1$ d) none of these.

iii) Eliminating A and B from $y = A + Bx$, the differential equation is obtained as

a) $\frac{d^2y}{dx^2} + y = 0$ b) $\frac{d^2y}{dx^2} - y = 0$

c) $\frac{d^2y}{dx^2} = 0$ d) none of these.

iv) The order and degree of the equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = a\frac{dy}{dx}$ is

a) 2, 2 b) 2, 3

c) 3, 2 d) 3, 3.

v) The P.I. of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} \quad y = e^x$ is

a) $\frac{e^x}{3}$ b) $\frac{e^x}{2}$

c) $\frac{e^x}{6}$ d) none of these.

vi) The series $\sum_{n=1}^{\infty} n^{\frac{1}{p}}$ is convergent if

a) $p \geq 1$ b) $p < 1$

c) $p > 1$ d) $p \leq 1$.

vii) If the series $\sum_{n=1}^{\infty} u_n$ is convergent, then

a) $\lim\limits_{n \to \infty} u_n = 0$ b) $\lim\limits_{n \to \infty} u_n > 1$

c) $\lim\limits_{n \to \infty} u_n < 1$ d) none of these.

viii) The series 1 – 1 + 1 – 1 + ... is

a) convergent with sum 0

b) convergent with sum 1

c) divergent

d) oscillatory.

ix) The vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) in V_3 are

a) linearly dependent b) linearly independent

c) b th (a) and (b) d) none of these.

x) The basis of a vector space contains

a) linearly independent vectors

b) linearly dependent vectors

c) scalars only

d) none of these.

xi) The values of k for which the vectors (1, 2, 1), (k, 1, 1) & (1, 1, 2) in R^3 are linearly independent are

a) $k \neq -\frac{2}{3}$ b) $k \neq \frac{2}{3}$

c) $k \neq -\frac{3}{2}$ d) none of these.

xii) T is a transformation from R^2 to R^3 defined by $T(x_1, x_2) = (x_1,\ x_1^2 + 2,\ -x_1)$. Then the image of (1, 2) is

a) (1, 1, 1) b) (0, 3, –1)

c) (1, 3, –1) d) (0,0,0).

xiii) If (3, 1) = x (1, 2)+ y (0, 3) then the values of x and y are respectively

a) (3, –5) b) (3, 1)

c) $\left(3, -\frac{5}{3}\right)$ d) $\left(3, -\frac{5}{2}\right)$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve $(x + y)\,dy + (x - y)\,dx = 0$.

3. Find the general and singular solutions of

$$y - xp + p^2 = 0, \quad p = \frac{dy}{dx}.$$

4. Test the convergence of the series

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \ldots, \quad x > 0$$

5. Define monotone sequence. When is a monotone sequence convergent ? Is the following sequence convergent ?

$$\left\{ \frac{3n+1}{n+2} \right\}$$

6. Prove that the intersection of two subspac s of a vector space is a subspace.

7. Find the space generated by (1, 3, 0), (2, 1, –2). Examine whether (4, 7, –2) lies in this space.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

8. a) Find the basis and dimension of the subspace W of R^3 where $W = \{ (x, y, z) \in R^3 : x + y + z = 0 \}$. 5

b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n . n!}{n^n}$. 5

c) Solve $\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = x^2 e^{3x}$. 5

9. a) Determine the linear mapping $T : R^3 \rightarrow R^3$ which maps the basis vectors (0, 1, 1), (1, 0, 1), (1, 1, 0) of R^3 to the vectors (1, 2, 1), (1, 1, 2), (2, 1, 1) respectively. Find Ker (T) and Im (T). 8

b) Solve : $(x^2 D^2 - xD - 3) y = x^2 \log x$. 7

10. a) Define basis and dimension of a vector space. Find a basis and the dimension of $S \cap T$ where S and T are subspaces of R^3 defined by

$S = \{ (x, y, z) \in R^3 : 2x + y + 3z = 0 \}$

$T = \{ (x, y, z) \in R^3 : x + 2y + z = 0 \}$ 2 + 1 + 6

b) Examine whether the vectors (1, 2, 2), (2, 1, 2), (2, 2, 1) are linearly independent in R^3. 6

11. a) Test the convergence of the following series :

i) $\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \ldots$

ii) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}} \right)^{-n^{\frac{3}{2}}}$ 5 + 5

b) Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ converges conditionally. 5

12. Solve the following : 3×5

a) $(xy \sin xy + \cos xy)y\,dx + (xy \sin xy - \cos xy)x\,dy = 0$

b) $y = px + \sqrt{a^2 p^2 + b^2}\,,\ p = \frac{dy}{dx}$

c) $\frac{d^2 y}{dx^2} - y = \sin x$

13. a) Solve $(x^3 - 3xy^2)dx + (y^3 - 3x^2 y)dy = 0$ 5

b) Find the representative matrix of the linear transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (x - 2y, y - 2z, z - 2x)$. 5

c) Show that $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ .$ is a divergent series. 5

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CS/BCA/Even/2nd Sem/BM-201/2014

2014

Mathematics

Time Alloted : 3 Hours *Full Marks : 70*

The figure in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following: 10x1=10

i) If a=(1,0,3) and b=(-1,2,5) then a+3b is equal to

a) (-2,6,18) b) (2,-6,-18)
c) (2,-6,18) d) (I,3,5)

ii) If $\sum_n |a_n|$ is convergent, then $\sum_n a_n$ is

a) convergent b) divergent
c) oscillatory d) none of these.

iii) A bounded sequence is

a) Convergent b) divergent
c) Oscillatory d) none of these

iv) The series $\sum \frac{1}{n\sqrt{n+1}}$ is

a) convergent b) divergent
c) oscillatory d)none of these

v) The integrating factor of $\frac{dy}{dx}+2xy=x^3$ is

a) x^3 b) x^2

c) e^{x^2} d) e^{x^3}

vi) The infinite series $\sum_{n=1}^{\infty}\frac{n}{n+1}$ is

a) Convergent b) Divergent

c) Oscillatory d) None of these

vii) If the vectors (5, 2, 3), (7, 3, a), (9, 4, 5) of a vector space R^3 over R be linearly independent, then the value of a is not equal to

a) 2 b) 3

c) 1 d) 0

viii) The sequence 1, $\frac{1}{2}$, $\frac{1}{3}$ .... $\frac{1}{n}$ is converges to

a) ∞ b) 0

c) 1 d) $\frac{1}{2}$

ix) The order and degree of the differential equation

$$\frac{d^2y}{dx^2}=1+2\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^3$$ are

a) 2, 1 b) 1, 2

c) 1, 3 d) 3, 1

x) The sequence $\{(-1)^n\}$ is

a) Convergent b) Oscillatory

c) Divergent d) None of these

xi) The general solution of $\log\frac{dy}{dx}=x-y$ is

a) $e^y-e^x=c$ b) $e^y+e^x=c$

c) $e^{y+x}=c$ d) $e^{x-y}=c$

xii) Which of the following pair can form a basis of R^2?

a) {(1,2),(2,4)}　　b) {(0,0),(3,33)}

c) {(2,2),(3,3)}　　d) {(1,1),(1,2)}

xiii) The particular integral of $(d^2y/dx)^2 - 3(dy/dx) + 2y = \sin 3x$ is

a) 1/130 (9cos3x - 7sin 3x)

b) 1/130 (7cos3x - 9sin3x)

c) 1/130 sin3x

d) none of these

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. 3x5=15

2. Prove that the vectors {(1,2,2),(2,1,2),(2,2,1)} are linearly independent in R^3.
3. Test the convergence of the series: $1+\frac{2}{1!}+\frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^4}{4!}+......$
4. Solve: $e^y\left(1+x^2\right)\frac{dy}{dx}-2x\left(1+e^y\right)=0$
5. Define a subspace of a vector space. Show that the intersection of two subspaces of a vector space is a subspace.
6. Show that the sequence $\sqrt{2},\sqrt{2+\sqrt{2}},\sqrt{2+\sqrt{2+\sqrt{2}}}.....$ Converges to 2.

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. 3x15=45

7. a) Test the convergence of the following series: $\sum \frac{n^2-1}{n^2+1}x^n$

b) Examine whether the differential equation $(e^y+1)\cos\ x dx + e^y \sin\ y dy = 0$ is exact or not.

c) Find the basis and the dimension of the subspace W of R^3 where $W = \{(x,y,z) \in R^3 : 2x - y + 3z = 0\}$

8. a) Solve $\frac{dy}{dx} = \sin(x+y)$

b) Let $T: R^2 \to R^2$ be a linear transformation such that T(1, 1) = (2, -3) and T(1, -1) = (4,7). Find the matrix of T.

c) Prove that the sequence $\left\{\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \ldots\ldots \frac{1}{(2n)^2}\right\}$ is convergent. Find its limit.

9. a) Form a differential equation by eliminating A and B from the following:
y=A cos x + B sin x

b) Find whether the following vectors are linearly dependent or not {(1,2,3),(2,3,1),(3,2,1)}

c) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2+1}$.

10. a) Solve: $\frac{dy}{dx} + y\ \tan\ x = y^3 \cos\ x$

b) For what values of x the three vectors (1,1,2), (x,1,1), (1,2,1) are linearly independent.

c) Solve: $y = px + \sqrt{1+p^2}$

11. a) Prove that the vectors (x_1,y_1) and (x_2,y_2) are linearly dependent, if and only if $x_1y_2 - x_2y_1 = 0$

12. b) Test the convergence of the series $\sum \frac{x^n}{n\sqrt{n+1}}$

c) Find the linear transformations T, where $T: R^3 \to R^2$ such that T(1,0,0) = (1,2,) T(0,1,0)=(1,-1) and T(0,0,1)=(1,0).

WEST BENGAL UNIVERSITY OF TECHNOLOGY

BM-201

MATHEMATICS - II

Time Allotted: 3 Hours | Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10 \times 1 = 10$

(i) The differential coefficient of χ^6 with respect to χ^3 is

(A) $2x^3$ (B) $2x$ (C) $2x^2$ (D) 2

(ii) The degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} - 3\frac{dy}{dx} = 4$ are

(A) degree $= \frac{2}{3}$, order = 2 (B) degree = 2, order = 2

(C) degree = 2, order =1 (D) degree = 3, order =2

(iii) The series $1 - 1 + 1 - 1 + \ldots\ldots$ is

(A) convergent with sum 0 (B) convergent with sum 1

(C) divergent (D) oscillatory

(iv) Let T be a linear transformation from R^2 to R^3 defined by $T(x, y) = (x + 2y, x - y, y)$. Then the image of (1, 2) is

(A) (2, 1, –1) (B) (5, –1, 2) (C) (1, 1, 1) (D) (2, 2, 3)

(v) In R^3, the vectors (1, 0, 1), (1, 1, 0) and (0, 1, 1)are

(A) linearly dependent (B) linearly independent

(C) both (A) and (B) (D) none of these

(vi) If (5,7) = a(1, 1) + b(1, 2)the values of a and b are respectively

(A) 1, 2 (B) 2, 3 (C) 3, 2 (D) 3, 3

(vii) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if

(A) $p \geq 1$ (B) $p = 1$ (C) $p \leq 1$ (D) $p = 0$

(viii) $\frac{1}{(D-2)(D-3)} e^x is$

(A) $\frac{e^x}{2}$ (B) $\frac{xe^{2x}}{2}$ (C) $-\frac{xe^x}{6}$ (D) $-\chi e^{3x}$

(ix) If for a sequence $\{u_n\}$, $\lim_{n \to \infty} u_n = 0$ them

(A) $\{u_n\}$ is convergent to 1 (B) $\{u_n\}$ is divergent

(C) $\{u_n\}$ is convergent to 0 (D) none of these

(x) If S and T be two subspaces of a vector space V, then which of the following is also a subspace of V?

(A) $S \cup T$ (B) S-T (C) T-S (D) $S \cap T$

(xi) Integrating factor of $ydx - xdy = y^2 \cos y dy$ is

(A) $\frac{1}{y^2}$, (B) y (C) $\frac{1}{y}$ (D) 1

(xii) Leibnitz's test is applied to

(A) a constant series (B) a series of positive terms

(C) an alternating series (D) a series of negative terms

(xiii) Let T be a linear transformation from R^2 to R^3 defined by $T(x, y) = (x + y, 0, 0)$.Then rank of T is

(A) 3 (B) 2 (C) 1 (D) 0

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3 \times 5 = 15$

2. Solve any two of the following :

   (a) $y = px + \dfrac{a}{p}$.

   (b) $(D^2 - 4)y = e^{2x} + e^{-4x}$.

   (c) $(D^2 + 9)y = \cos 3x$.

3. Test the convergence of the series

$$\chi + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \ldots\ldots\ldots \quad X > 0.$$

4. Let $S = \{(x, y, z) | x + y + z = 0, x, y, z \in R^3\}$. Prove that S is a subspace of R^3. Find the dimension of S.

5. Find the representative matrix of the linear transformation T : $R^3 \longrightarrow R^3$ defined by
$T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$.

6. Define monotone sequence. When is a monotone sequence convergent? Is the following sequence $\{u_n\}$ convergent?

$$u_n = \frac{3n+1}{n+2}.$$

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

7. (a) Verify whether the differential equation $e^y dx + (xe^y + 2y)dy = 0$ is exact. 3+7+5

   (b) Solve: $x\dfrac{dy}{dx} - 2y = xy^4$.

   (c) Find the general and singular solutions of $y = px - p^2$.

8. (a) Discuss the convergency of the sequence $\left\{\frac{1}{n}\sin\frac{n\pi}{2}\right\}$. 4

(b) Let $S=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a+b=0, a,b,c,d\in R\right\}$. Find a basis and dimension of S. 6

(c) Show that $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots$ Is a divergent series. 5

9.(a) Solve: $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-5y=\sin\log x$. 5

(b) If $\{\alpha,\beta,\gamma\}$ is basis of a real vector space V, show that $\{\alpha+\beta, \beta+\gamma, \gamma+\alpha\}$ is also a basis of V 4

(c) Determine the linear mapping $T: R^3 \longrightarrow R^3$ which maps the basis vectors (0, 1, 1), (1, 0, 1), (1, 1, 0) of R^3 to the vectors (1, 2, 1), (1, 1, 2), (2, 1, 1) respectively. find dim(kerT). 6

10. (a) State D' Alembert's ratio test. Test the convergence of the series $\sum_{n=1}^{\infty}\frac{2^n n!}{n^n}$. 5

(b) Show that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ converges conditionally. 5

(c) Show that the sequence $\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}},\ldots$ converges to 2. 5

11.(a) Find the differential equation of all circles touching the axis of x at the origin. 5

(b) Show that the vectors (1, –2, 3), (2, 3, 1) and (–1, 3, 2) form a basis of R^3. 5

(c) Give an example to show that union of two sub spaces need not be a sub space of v. 5

MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BM-201

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by
   $T(x, y) = (x + 2y, x - y, y)$. Then the image of $(2, 2)$ is

   a) $(2, 1, -1)$ b) $(6, 1, -1)$

   c) $(6, 0, 2)$ d) none of these.

   ii) An integrating factor of $x\frac{dy}{dx} - y = 1$ is

   a) x b) $\frac{1}{x}$

   c) $-x$ d) $-\frac{1}{x}$.

iii) If for a sequence $\{ u_n \}$, $\lim_{n \to \infty} u_n = 0$ then

a) $\{ u_n \}$ converges to 1

b) $\{ u_n \}$ converges to 0

c) $\{ u_n \}$ is divergent

d) none of these.

iv) The order and degree of the differential equation

$$\frac{d^2y}{dx^2} = \sqrt{\left(\frac{dy}{dx}\right)^3 - 2\frac{dy}{dx}}$$ are

a) (1, 3)
b) (2, 3)
c) (2, 2)
d) (1, 2).

v) α be linear combination of the vectors β and γ in a vector space V over a field F. Then the set

$S \equiv \{ \alpha, \beta, \gamma \}$ is

a) linearly independent

b) linearly dependent

c) S forms a basis of V

d) none of these.

vi) If $(2, 1) = x (1, 2) + y (0, 3)$ then the values of x and y are respectively

a) 3, 1
b) 2, − 1
c) 2, 0
d) 1, − 1.

vii) The value of k for which the vectors (1, 2, 1), (k, 1, 1) and (1, 1, 2) in $\mathbb{R}^3$ are linearly independent is

a) $k \neq -\frac{2}{3}$

b) $k \neq \frac{2}{3}$

c) $k \neq -\frac{3}{2}$

d) none of these.

viii) The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ is

a) convergent

b) divergent

c) oscillatory

d) none of these.

ix) The order of the differential equation whose general solution is $y = a(x-a)^2$, where a is an arbitrary constant is

a) 1

b) 2

c) 3

d) none of these.

x) $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ is convergent if

a) sequence $\{a_n\}$ is monotonic decreasing

b) $\lim_{n \to \infty} a_n = 0$

c) both (a) and (b)

d) none of these.

xi) The vectors (1, 0, 0), (1, 1, 0) and (1, 1, 1) are

a) linearly dependent

b) linearly independent

c) a generating set of $\mathbb{R}^3$

d) none of these.

xii) In a linear mapping $T : V \to W$, which of the following is true ?

a) $Ker(T)$ may not be a vector space

b) $Ker(T)$ is a subset of W

c) $\theta \in Ker(T)$ where θ being the null vector in V

d) $\theta' \in Ker(T)$ where θ' being the null vector in W.

xiii) A differential equation $M\,dx + N\,dy = 0$ is exact when

a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

b) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$

c) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

d) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find whether the following set of vectors is linearly independent or not :

$$\{ (1, 2, 3),\ (2, 3, 1),\ (3, 2, 1) \}$$

3. Solve : $\dfrac{dy}{dx} = \dfrac{y}{x} + \tan \dfrac{y}{x}$.

4. Solve : $(D^2 - 2D + 1) y = x e^{2x}$, $D \equiv \dfrac{d}{dx}$.

5. State D'Alembert's ratio test. Test the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$.

6. Find the representative matrix of the linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(x, y, z) = (x - 2y,\ y - 2z,\ z - 2x).$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Let $S = \{ (x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0,\ x, y, z \in \mathbb{R} \}$. Prove that S is a subspace of $\mathbb{R}^3$.

   b) Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}} \right)^{-n^{3/2}}$$

   c) Find the basis of M_2 (the family of all real square matrices of order 2). 5 + 5 + 5

8. a) Show that the set $\{ (2, 1, 1), (1, 2, 1), (1, 1, 2) \}$ is a basis of $\mathbb{R}^3$.

b) Solve : $(xy^2 - e^{1/x^3}) dx - x^2 y \, dy = 0$.

c) Solve : $y = px + \sqrt{a^2 p^2 + b^2}$, where $p \equiv \dfrac{dy}{dx}$.

5 + 5 + 5

9. a) Examine the convergence of the following series for different values of x :

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$$

b) Test the convergence of the following series :

$$1 + \frac{1}{2} \cdot \frac{x^2}{4} + \frac{1.3.5}{2.4.6} \cdot \frac{x^4}{8} + \frac{1.3.5.7.9}{2.4.6.8.10} \cdot \frac{x^6}{12} + \ldots \infty .$$

c) Find a basis and the dimension of the subspace W of $\mathbb{R}^3$, where

$$W = \{ (x, y, z) \in \mathbb{R}^3 : x + 2y + z = 0, \ 2x + y + 3z = 0 \}.$$

5 + 5 + 5

10. a) Determine the linear transform $T : \mathbb{R}^3 \to \mathbb{R}^2$ which maps the basis vectors $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ of $\mathbb{R}^3$ to the vectors $(1, 1)$, $(2, 3)$, $(3, 2)$ respectively. Also find $Ker(T)$ and $Im(T)$.

b) Solve : $\dfrac{d^2y}{dx^2} + 9y = \cos 3x$.

c) Solve : $(x^2 D^2 - xD - 3) y = x^2 \log x, \ D \equiv \dfrac{d}{dx}$.

5 + 5 + 5

11. a) Solve : $y = 2x \dfrac{dy}{dx} - \left(\dfrac{dy}{dx} \right)^2$.

b) Find the coordinate vector of $(0, 3, 1) \in \mathbb{R}^3$ relative to the basis $\{ (1, 1, 0), (1, 0, 1), (0, 1, 1) \}$.

c) Discuss the convergence of the sequence $\left\{ \dfrac{n^2}{3^n} \right\}$.

OR

c) Find the differential equation of all circles touching the axis of x at the origin. 5 + 5 + 5

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Paper Code : BM-201

MATHEMATICS

Time Allotted: 3 Hours *Full Marks*: 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for *any ten* of the following: 1×10=10

(i) If $(3,1) = x(1,2) + y(0,3)$, then the values of x and y are respectively

(a) $3, -5$ (b) $3, 1$
(c) $3, -\frac{5}{3}$ (d) $3, -\frac{5}{2}$

(ii) The equation of the curve which satisfies the differential equation $\frac{dy}{dx} = 0$ is

(a) a straight line passing through the origin (b) a straight line parallel to x-axis
(c) a straight line parallel to y-axis (d) None of these

(iii) $\{x^n\}$ converges if

(a) $x = 1$ (b) $x > 1$
(c) $x < -1$ (d) $-1 < x < 1$

$1^n, 2^n, 3,$
x^n

(iv) The order and degree of the differential equation $\frac{d}{dx}\left(y.\frac{dy}{dx}\right) = 5$ are

(a) $1, 2$ (b) $2, 2$
(c) $1, 1$ (d) $2, 1$

(v) If $T(x,y,z) = (x,y,0)$ for all $(x,y,z) \in R^3$ is a linear transformation, then Kernel(T) is

(a) $\{(0,0,0)\}$ (b) x-axis
(c) y-axis (d) z-axis

Turn Over

(vi) The series $\sum \frac{1}{n^P}$ is convergent if

(a) $P \geq 1$ (b) $P > 1$

(c) $P < 1$ (d) $P \leq 1$

(vii) The sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is

(a) divergent (b) unbounded

(c) bounded (d) None of these

(viii) $\frac{1}{D^2}(x^4) = ?$

(a) x^5 (b) x^6

(c) $\frac{x^6}{30}$ (d) $\frac{x^4}{30}$

(ix) The I.F. of $\frac{dx}{dy} = x + y^2$ is

(a) e^x (b) e

(c) e^{2x} (d) e^{-x}

(x) T is a linear transformation by $T(x_1, x_2) = (-x_1, -x_2)$ then the dimension of $T(V_2)$ is

(a) 2 (b) 1

(c) 0 (d) None of these

(xi) A bounded monotonic increasing sequence converges to its

(a) any upper bound (b) greatest lower bound

(c) least upper bound (d) 0

(xii) The differential equation $(x\,e^{axy} + 2y)\frac{dy}{dx} + y.e^{xy} = 0$ is exact for $a = ?$

(a) 3 (b) 1

(c) 2 (d) 0

Group – B

Answer *any three* questions: **5x3=15**

2. Show that the differential equation of all parabolas with foci at the origin and axes along the x-axis is

$$y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$$

3. Find the value of x such that the vectors $(1,2,1)$, $(x,3,1)$ and $(2,x,0)$ are linearly dependent.

4. Examine the convergence of the sequence $\left\{\frac{n^n}{\underline{|n}}\right\}$.

5. Solve : $(D^2 - 4)y = e^{2x} + e^{-4x}$

6. Show that $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots$ to ∞ is a divergent series.

(15)

Group – C

Answer *any three* questions: 15×3=45

7. (a) Solve : $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ 5

(b) Let $W = \{(x, y, z) \in \mathbb{R}^3 : x + y - 2z = 0\}$. Show that W is a subspace of $\mathbb{R}^3$. Find a basis and dimension of W. 3+1+1=5

(c) From the definition, prove that, $\lim_{n\to\infty} x^n = 0$, when $-1 < x < 1$. 5

8. (a) Prove that the vectors $(1, -2, 3), (2, 3, 1)$ and $(-1, 3, 2)$ form a basis of V_3.

(b) Solve : $(x + y + 1)\frac{dy}{dx} = 1$

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

9. (a) Solve : $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2 e^{3x}$ 5

(b) Determine the linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ which maps the basis vectors $(0, 1, 1), (1, 0, 1), (1, 1, 0)$ of $\mathbb{R}^3$ to the vectors $(1, 2, 1), (1, 1, 2), (2, 1, 1)$ respectively. Find dim(ker(T)). 5+2=7

(c) Obtain a singular solution of the equation: $(y - px)(p - 1) = p; p = \frac{dy}{dx}$ 3

10. (a) If $T: \mathbb{R}^3 \to \mathbb{R}^3$ is given by $T(x, y, z) = (x - y, y - z, z - x)$; show that T is a linear transformation and obtain ker(T). 5

(b) Solve : $(D^2 + 1)y = \sin 2x$

(c) State Leibnitz's test. Using this show that $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots$ to ∞ is convergent. (5)

11. (a) Solve : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = \log x$ 5 (15)

(b) Solve : $(D^2 + 4)y = x \sin x$

(c) Test the convergence of the series $\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \ldots\ldots$ to ∞. (5)