CS/BCA/SEM-1/BM-101/2012-13

2012
MATHEMATICS

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words
as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

    $10 \times 1 = 10$

    i)   The value of $\lim\limits_{x \to 0} \dfrac{x^2 - 4}{x - 2}$ is

    a)   1   b)   4

    c)   0   d)   none of these.

    ii) If $\phi\,(x,\, y) = 0$ then $\dfrac{dy}{dx} =$

    a) $\dfrac{\phi_x}{\phi_y}$ b) $\dfrac{\phi_y}{\phi_x}$

    c) ϕ_y d)   none of these.

    iii) The value of y_n , when $y = e^{-x}$ is

    a) e^{-x} b) $(-1)^n$

    c) $(-1)^n\, e^{-x}$ d)   none of these.

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iv) If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $A^2 =$

     a)   A   b)   3A

     c)   unit matrix   d)   2A.

v) The diagonal of real skew-symmetric matrix is

     a)   1   b)   − 1

     c)   0   d)   2.

vi) If in a group (G, o), x o $x = x$, then

     a) x = null element

     b)   this relation is not valid

     c) $x = e$

     d) $x \neq x^{-1}$.

vii) The value of $\int (\cos^2 x - \sin^2 x)\, dx$ is

     a)   secxtan$x$ b) $\sin 2x$

     c) $\dfrac{1}{2}\sin 2x$ d) $-\dfrac{1}{2}\sin 2x$.

viii) The polar equation $r = 4 \sin \theta$ represents a

     a)   circle   b)   ellipse

     c)   straight line   d)   none of these.

ix) If a, b and c are roots of

    $x^3 - 3x + 9 = 0$ then $a^2 + b^2 + c^2$ is

     a)   6   b)   − 6

     c)   0   d)   1.

x) Solution of the equation $x^3 + 2x + 3 = 0$ will give

     a)   no real positive roots but one real negative root

     b)   two real positive roots and one real negative root

     c)   one real positive root and two imaginary roots

     d)   two real negative roots and only one imaginary root.

xi) $\int_{-1}^{2} |x| \, dx$ is

    a)   3   b)   5

    c)   5/2   d)   3/2.

xii) Which of the following does not satisfy Rolles theorem in $[-2, 2]$?

    a) x b) $1/x$

    c) $1/(x-5)$ d) $x^2 - 5$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Out of 440 students, 112 students read German, 120 students read French and 168 read Spanish. Of these 32 read French and Spanish, 40 read German and Spanish, 20 read German and French, while 12 read all the three subjects. How many students

    a)   do not read any of the three languages

    b)   read just one language ?

3. Evalute $\int_{0}^{\pi} \log \tan x \, dx$.

4. If $f(x, y) = \begin{cases} \dfrac{x^2 - xy}{x + y}, & (x, y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$,

    what is the value of $f_x(0,0)$, $f_y(0,0)$

5. Obtain a relation between p, q and r so that $x^3 + px^2 + qx + r = 0$ has 3 roots that are in A.P.

6. Evaluate $\displaystyle\lim_{x \to 0} \frac{\operatorname{cosec} x - \cot x}{x}$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that $\{ 1, \omega, \omega^2 \}$, where $\omega^3 = 1$ forms a commutative group in respect of multiplication.

b) If $y = \sin^{-1} x$, then prove that
$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0$

c) In the mean value theorem $f(x + h) = f(x) + h f'(x + \theta h)$,
if $f(x) = px^2 + qx + r (p \neq 0)$, then show that $\theta = \frac{1}{2}$.

8. a) Reduce the equation $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$ into canonical form and hence determine the nature of the conic.

b) Find the nature of the conic $\dfrac{8}{r} = 4 - 5 \cos \theta$.

c) Expand e^x in ascending powers of x by Taylor's series.

9. a) Solve using Carden's method : $x^3 - 9x + 28 = 0$.

b) If by a transformation of motion of co-ordinate axes, the expression $ax^2 + 2hxy + by^2$ changes into $a'x^2 + 2h'x'y' + b'y'^2$, then show that $ab - h^2 = a'b' - h'^2$.

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10. a) Solve the equations by matrix inversion method :
$x + y + z = 4$
$2x - y + 3z = 1$
$3x + 2y - z = 1$

b) If $u = \tan^{-1} \dfrac{x^2 + y^2}{x + y}$, show that $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y} = \dfrac{1}{2} \sin 2u$.

c) Evaluate : $\displaystyle\int \dfrac{x - 1}{(x - 2)(x - 3)} dx$

11. a) Give the definition of a ring with two binary composition. Let H be the set of all matrices $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = 1 \right\}$. Prove that H forms a non-commutative group.

b) Apply Descerte's rule of sign to find the nature of the roots of the given equation : $x^4 + qx^2 + rx - s = 0$ (where q, r, s being positive).

c) Evaluate : $\displaystyle\lim_{n \to \infty} \left[\dfrac{1}{n^2 + 1^2} + \dfrac{1}{n^2 + 2^2} + \dots\dots + \dfrac{1}{n^2 + n^2} \right]$