| Name: | | |
|---------------------------|--|--|
| Roll No.: | | |
| Invigilator's Signature : | | |
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2011

STATISTICS, NUMERICAL METHODS AND ALGORITHMS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

| 1. | Cho | ose | the | correct | alternativ | es | for | any | ten | of | the |
|----|-------|------|---------------|------------|-------------|-------|-------|---------|-------|-------|------|
| | follo | wing | : | | | | | | 10 | × 1 = | : 10 |
| | i) | The | num | ber of sig | nificant di | git i | n 0·0 | 0303 | is | | |
| | | a) | 6 | | | b) | 5 | | | | |
| | | c) | 3 | | | d) | non | e of th | nese. | | |
| | ii) | | en ro omes | ounded o | off after 4 | de | cima | l plac | ces 0 | 0.003 | 256 |
| | | a) | 0.00 |)32 | | b) | 0.00 | 033 | | | |
| | | c) | 0.03 | 326 | | d) | non | e of th | iese. | | |
| | | | | | | | | | | | |

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a)

iii) Divided difference formula is used for

equispaced point

| | b) | unequally spaced points | | | | | | | | |
|------|------|--|----------------------|-------------------------------------|--|--|--|--|--|--|
| | c) | both (a) & (b) | both (a) & (b) | | | | | | | |
| | d) | none of these. | | | | | | | | |
| iv) | | wton's forward formula is used for interpolating the u e of y near the | | | | | | | | |
| | a) | beginning of a set | | | | | | | | |
| | b) | end of a set | | | | | | | | |
| | c) | central of the set | | | | | | | | |
| | d) | none of these. | | | | | | | | |
| v) | In b | eackward difference V | $\nabla^2 f(x)$ is | S | | | | | | |
| | | | | $\nabla^2 f(x) - \nabla^2 f(x - h)$ | | | | | | |
| | c) | $\Delta f(x) - \Delta f(x+h)$ | d) | none of these. | | | | | | |
| vi) | The | iterative method to s | solve a s | ystem of equation is | | | | | | |
| | a) | Gauss-elimination | b) | Gauss-Jordan | | | | | | |
| | c) | Gauss-Seidel | d) | None of these. | | | | | | |
| vii) | The | error in the Simpson | n's $\frac{1}{3}$ rd | method is of order | | | | | | |
| | a) | h | b) | h^2 | | | | | | |
| | c) | h^3 | d) | h^4 . | | | | | | |
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| | | $f^{\prime}(x) = 0$ | b) | f'(x) > 0 |
|------|---------------------|---|--------|--|
| | c) | f'(x) < 0 | d) | none of these. |
| ix) | Diag | gonal dominance is mus | st for | |
| | a) | Gauss-Seidel method | | |
| | b) | Gauss-Jordan's matrix | k inve | ersion method |
| | c) | Gauss elimination met | thod | |
| | d) | none of these. | | |
| x) | | second order Runge-Kur which is of order of | ıtta f | ormula has a truncation |
| | a) | h^2 | b) | h^3 |
| | c) | h^4 | d) | none of these. |
| xi) | | order of h in the error | expre | ssion of trapezoidal rule |
| | is | | | |
| | a) | 1 | b) | 2 |
| | c) | 3 | d) | 4. |
| xii) | Rela x_{Γ} : | | | ical method where $x_A = Approximate value$ |
| | of so | olution is | | |
| | a) | $ x_{\Gamma}-x_A $ | b) | $\frac{\left x_{\Gamma}-x_{A}\right }{x_{\Gamma}}$ |
| | c) | $\frac{\left x_{\Gamma}-x_{A}\right }{x_{\Gamma}}\times100$ | d) | none of these. |
| | | | | |

viii) Newton-Raphson method fails when

xiii) Which is the direct method?

- a) Gauss-elimination method
- b) Gauss-Jacobi method
- c) Gauss-Seidel method
- d) none of these.

xiv) Newton-Raphson method is also known as

- a) chord method
- b) tangent method
- c) secant method
- d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any three of the following.

 $3 \times 5 = 15$

2. By means of Newton's divided differential interpolation formula find the value of f(8) from the following table :

| <i>x</i> = | 4 | 5 | 7 | 10 | 11 | 13 |
|------------|----|-----|-----|-----|------|------|
| f(x) = | 48 | 100 | 294 | 900 | 1210 | 2028 |

- 3. Prove that for small values of 'h' $\Delta^{n+1} f(x_0) \approx h^{n+1} f^{n+1}(x_0)$.
- 4. Evaluate $\int_{0}^{1} \cos x \, dx$ taking five equal intervals. Explain the reason behind your choice of integration formula used.
- 5. Compute f(1.42) from the following data:

| х | 1.1 | 1.2 | 1.3 | 1.4 |
|------|-------|-------|-------|--------|
| f(x) | 7.831 | 8.728 | 9.697 | 10.744 |

- 6. Solve $\frac{dy}{dx} = x^2y 1$, where y (0) = 1 by Taylor's series method. Also find y (0.1) correct to seven significant digits.
- 7. How many digits are to be taken in computing $\sqrt{13}$ so that error does not exceed 0.1%?

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Compute f(1.16) from the following table:

| х | 1.11 | 1.12 | 1.13 | 1.14 | 1.15 | 1·16 |
|------|--------|--------|--------|--------|--------|--------|
| f(x) | 6.2321 | 6.2544 | 6.2769 | 6.2996 | 6.3225 | 6.3456 |

- b) Find the positive root of the equation $x^2 + 2x 2 = 0$, correct up to 2 significant figures by Newton-Raphson method.
- c) Estimate the missing term from the table :

| x | 2 | 4 | 6 | 8 | 10 |
|---|---|----|---|----|----|
| у | 5 | 13 | * | 53 | 85 |

9. a) Solve the following system of linear equations by Gauss-Seidel method :

$$6x + 15y + 2z = 72$$

$$27x + 6y - z = 85$$

$$x + y + 54z = 110$$
.

- b) Evaluate $\int_{0}^{1} \frac{dx}{1+x^2}$ using Simpson's one-third rule taking $h = \frac{1}{6}$ correct up to 3 decimal places.
- c) Find the root of the equation $x \log_{10} x = 1 \cdot 2$, correct to 2 decimal places by Bisection method. 6 + 4 + 5
- 10. a) Solve by Gauss elimination method:

$$x + 2y + 3z = 10$$

$$x + 3y - 2z = 7$$

$$2x - y + z = 5.$$

b) Evaluate $\int_{0}^{1} (4x - 3x^{2}) dx$ taking 10 intervals by

Trapezoidal rule and then find the absolute error.

c) Prove that $E = e^{hD}$, $D = \frac{d}{dx}$ and E is the shift operator.

7 + 5 + 3

- 11. a) Use Euler's method to find the solution of $\frac{dy}{dx} = x y$ with y(0) = 1, h = 0.2 at x = 0.4.
 - b) Find the value of y (0.2) by 4th order Runge-Kutta method which is correct to *four* decimal places, where $\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 x^2, y(0) = 1 \text{ taking } h = 0.1.$ 7 + 8

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- 12. a) Compute a root of the equation $x^2e^{-x/2} = 1$ in the interval [0, 2] by secant method correct to 3 decimal places.
 - b) Find the inverse of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ by Gauss' method. 7+8
- 13. a) Solve the following equation by Jacobi's iteration method:

$$10x - 2y - z - w = 3$$
$$-2x + 10y - z - w = 15$$
$$-x - y + 10z - 2w = 27$$
$$-x - y - 2z + 10w = -9$$

b) Solve by *LV* factorization method :

$$2x - 3y + z = -1$$

$$x + 2y - 3z = -4$$

$$x - 4y + z = -6$$

$$7 + 8$$