WEST BENGAL UNIVERSITY OF TECHNOLOGY

BM-401

STATISTICS, NUMERICAL METHODS & ALGORITHMS

Time Allotted: 3 Hours | Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10\times1 = 10$

(i) The degree of precision of Simpson's $1/3^{rd}$ rule is

(A) 1 (B) 2 (C) 3 (D) 4

(ii) The rate of convergence of bisection method is

(A) linear (B) quadratic (C) cubic (D) none of these

(iii) If $f(x)=\frac{1}{x^2}$, then divided difference $f(a, b)$ is

(A) $\frac{a+b}{(ab)^2}$ (B) $-\frac{a+b}{(ab)^2}$ (C) $\frac{1}{a^2-b^2}$ (D) $\frac{1}{a^2}-\frac{1}{b^2}$

(iv) Which of the following relation is true?

(A) $\Delta=E-1$ (B) $\Delta\cdot\nabla=\Delta-\nabla$ (C) $\Delta\cdot\nabla=\Delta+\nabla$ (D) $\Delta=1-E$

(v) If $\frac{5}{3}$ is approximated to 1.6667, then absolute error is

(A) 0.000033 (B) 0.000043 (C) 0.000045 (D) 0.000051

(vi) When the Guass elimination method is used to solve **BX** = A, **B** is transformed into

(A) a lower triangular matrix (B) an upper triangular matrix
(C) zero matrix (D) none of these

(vii) Order of h in the error expression of Simpson's $1/3^{rd}$ rule is

(A) 2 (B) 4 (C) 3 (D) 5

(viii) If $f(x) = b.e^{ax}$, then $\Delta f(x)$ is

(A) $be^{ax}(e^{ah}-1)$ (B) $be^{ax}(a-1)$
(C) $be^{ax}(1-e^{ah})$ (D) none of these

(ix) The number of significant digits in 0.00303 is

(A) 6 (B) 5 (C) 3 (D) done of these

(x) Newton's forward interpolation formula uses

(A) the front part of the table (B) the end part of the table
(C) any part of the table (D) middle of the table

(xi) Diagonal Dominance is must for

(A) Gauss Seidal method of iteration
(B) Gauss Jordan matrix inversion method
(C) Gauss elimination method
(D) None of these

(xii) If $f(x) = 101$ and $h = 3$, then $\Delta f(x)$ is equal to

(A) 100 (B) 99 (C) 1 (D) 0

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3\times5 = 15$

2. Evaluate $\Delta^2 \cos 2x$.

3. Evaluate $\sqrt{12}$ to three places of decimals by Newton–Raphson method.

4. Evaluate $\int_0^1 \frac{dx}{1+x}$ by Simpson's 1/3rd rule taking 11 ordinates and hence find the value of $\ln 2$. Correct up to five significant figures.

5. Use Euler's Method to solve the differential equation $\frac{dy}{dx} = xy$, for $x = 1$. Given that when $x = 0$, $y = 1$. Take $h = 0.2$.

6. Find by Lagrange's interpolation formula for the polynomial which corresponds to the following data.

x :	−1	0	2	5
$F(x)$:	9	5	3	15

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. 3×15 = 45

7. (a) Evaluate $\int_0^2 \frac{dx}{1+x^2}$ correct up to 4 places of decimal by using Weddle's rule, taking 12 intervals. 8+7

(b) Estimate the missing term

x :	2	4	6	8	10
$F(x)$:	5	13	*	53	85

8. (a) Find the roots of the equation $x^2 - 3x - 2 = 0$ by using Newton-Raphson Method. 7+8

(b) Solve the following system of equations by using Jacobi Iteration Method:

$8x + 2y - 2z = 08$
$x - 8y + 3z = -4$
$2x + y + 9z = 12.$

9. (a) Starting from Gauss–Legendre Quadrature formulas establish the trapezoidal rule of integration. 8+7

(b) Evaluate $\int_0^1 (4x+3x^2)dx$, by using the trapezoidal rule taking 10 intervals. Compute the exact value and the absolute and relative errors in your results.

10.(a) Find the smallest positive root of the equation $3x^3 - 9x^2 + 8 = 0$, correct up to four places of decimal, using Newton-Raphson method. 5+5+5

(b) Find $y(1.1)$ using Runge-Kutta method of fourth order, given that $\frac{dy}{dx} = y^2 + xy,\ y(1) = 1$.

(c) Use the method of bisection to compute a root of the equation $x^3 - 4x - 1 = 0$ lying between 2 and 3 up to four significant figures.

11.(a) Solve $\frac{dy}{dx} = x^2y - 1, y(0) = 1$ by Taylor's series. Also find $y(0.1)$ correct up to seven significant digits. 5+5+5

(b) Solve the equation $\frac{dy}{dx} = x + y, y(0) = 1$ at $x = 0.2$ by Picard's method (take only three integrations).

(c) Use Euler's method to find the numerical solution of the following differential equation. $\frac{dy}{dx} = 1 + x - x^2, y(0) = 1$. Taking $h = 0.02$ find $y(0.1)$.

12(a) Use Regula-Falsi method to evaluate the smallest real root of the equation $x^3 + x^2 - 1 = 0$. 6+4+5

(b) Solve the following system of equations by LU method.

$$3x + 4y + 7z = 8$$
$$x + 2y + 3z = 6$$
$$x + 5y + 9z = 9.$$

(c) Use Newton's divided difference formula to find $f(8)$ and $f(15)$ from the following table :

x :	1	5	7	9
$f(x)$:	89	178	278	321