WEST BENGAL UNIVERSITY OF TECHNOLOGY

BM-201

MATHEMATICS - II

Time Allotted: 3 Hours   Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10 \times 1 = 10$

(i) The differential coefficient of χ^6 with respect to χ^3 is

(A) $2x^3$ (B) $2x$ (C) $2x^2$ (D) 2

(ii) The degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} - 3\frac{dy}{dx} = 4$ are

(A) degree $= \frac{2}{3}$, order = 2   (B) degree = 2, order = 2

(C) degree = 2, order =1   (D) degree = 3, order =2

(iii) The series $1 - 1 + 1 - 1 + \ldots\ldots$ is

(A) convergent with sum 0   (B) convergent with sum 1

(C) divergent   (D) oscillatory

(iv) Let T be a linear transformation from R^2 to R^3 defined by T(x, y) = (x + 2y, x − y, y). Then the image of (1, 2) is

(A) (2, 1, −1) (B) (5, −1, 2) (C) (1, 1, 1) (D) (2, 2, 3)

(v) In R^3, the vectors (1, 0, 1), (1, 1, 0) and (0, 1, 1)are

(A) linearly dependent (B) linearly independent

(C) both (A) and (B) (D) none of these

(vi) If (5,7) = a(1, 1) + b(1, 2)the values of a and b are respectively

(A) 1, 2 (B) 2, 3 (C) 3, 2 (D) 3, 3

(vii) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if

(A) $p \geq 1$ (B) $p = 1$ (C) $p \leq 1$ (D) $p = 0$

(viii) $\frac{1}{(D-2)(D-3)} e^x$ is

(A) $\frac{e^x}{2}$ (B) $\frac{xe^{2x}}{2}$ (C) $-\frac{xe^x}{6}$ (D) $-\chi e^{3x}$

(ix) If for a sequence $\{u_n\}$, $\lim_{n\to\infty} u_n = 0$ them

(A) $\{u_n\}$ is convergent to 1 (B) $\{u_n\}$ is divergent

(C) $\{u_n\}$ is convergent to 0 (D) none of these

(x) If S and T be two subspaces of a vector space V, then which of the following is also a subspace of V?

(A) $S \cup T$ (B) S-T (C) T-S (D) $S \cap T$

(xi) Integrating factor of $ydx - xdy = y^2 \cos y\, dy$ is

(A) $\frac{1}{y^2}$ (B) y (C) $\frac{1}{y}$ (D) 1

(xii) Leibnitz's test is applied to

(A) a constant series (B) a series of positive terms

(C) an alternating series (D) a series of negative terms

(xiii) Let T be a linear transformation from R^2 to R^3 defined by $T(x, y) = (x + y, 0, 0)$.Then rank of T is

(A) 3 (B) 2 (C) 1 (D) 0

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3\times5=15$

2. Solve any two of the following :

   (a) $y = px + \dfrac{a}{p}$.

   (b) $(D^2 - 4)y = e^{2x} + e^{-4x}$.

   (c) $(D^2 + 9)y = \cos 3x$.

3. Test the convergence of the series

   $$\chi + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \ldots\ldots\ldots \quad X > 0.$$

4. Let $S = \{(x, y, z) | x + y + z = 0, x, y, z \in R^3\}$. Prove that S is a subspace of R^3. Find the dimension of S.

5. Find the representative matrix of the linear transformation T : $R^3 \longrightarrow R^3$ defined by
   $T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$.

6. Define monotone sequence. When is a monotone sequence convergent? Is the following sequence $\{u_n\}$ convergent?

   $$u_n = \frac{3n+1}{n+2}.$$

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3\times15=45$

7. (a) Verify whether the differential equation $e^y dx + (xe^y + 2y)dy = 0$ is exact. 3+7+5

   (b) Solve: $x\dfrac{dy}{dx} - 2y = xy^4$.

   (c) Find the general and singular solutions of $y = px - p^2$.

8. (a) Discuss the convergency of the sequence $\left\{\frac{1}{n}\sin\frac{n\pi}{2}\right\}$. 4

(b) Let $S=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a+b=0, a,b,c,d\in R\right\}$. Find a basis and dimension of S. 6

(c) Show that $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots$ Is a divergent series. 5

9.(a) Solve: $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-5y=\sin\log x$. 5

(b) If $\{\alpha,\beta,\gamma\}$ is basis of a real vector space V, show that $\{\alpha+\beta, \beta+\gamma, \gamma+\alpha\}$ is also a basis of V 4

(c) Determine the linear mapping $T: R^3 \longrightarrow R^3$ which maps the basis vectors (0, 1, 1), (1, 0, 1), (1, 1, 0) of R^3 to the vectors (1, 2, 1), (1, 1, 2), (2, 1, 1) respectively. find dim(kerT). 6

10. (a) State D' Alembert's ratio test. Test the convergence of the series $\sum_{n=1}^{\infty}\frac{2^n n!}{n^n}$. 5

(b) Show that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ converges conditionally. 5

(c) Show that the sequence $\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots$ converges to 2. 5

11.(a) Find the differential equation of all circles touching the axis of x at the origin. 5

(b) Show that the vectors (1, –2, 3), (2, 3, 1) and (–1, 3, 2) form a basis of R^3. 5

(c) Give an example to show that union of two sub spaces need not be a sub space of v. 5