Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/BCA/SEM-3/BM-301/2013-14

2013

MATHEMATICS FOR COMPUTING

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) A statement T is called a tautology if

a) T is true for all possible values of its variables

b) T is false for all possible values of its variables

c) T is true as well false for few possible values of its variables

d) none of these.

ii) The type-3 Grammar in relation to the automata theory is known as

a) Context sensitive grammar

b) Context free grammar

c) Regular grammar

d) none of these.

iii) Solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is

a) $1 - 2^n$ b) $2^n - 2$

c) $2^n - 1$ d) none of these.

iv) How many bit strings of length 10 contain exactly four's 1's ?

a) 130 b) 720

c) 210 d) none of these.

v) A spanning tree of a connected graph contains

a) all the vertices of the graph

b) all the vertices and edges of the graph

c) a few vertices of the graph

d) none of these.

vi) If ${}^{16}C_r = {}^{16}C_{2r+1}$, then $r =$

a) 6 b) 5

c) 4 d) 3.

vii) A binary tree has exactly

a) **one root** b) **two roots**

c) three roots d) none of these.

viii) Let L be a language given by $L = \{a^n b^n : n \geq 0\}$, then L^2 is equal to

a) $L = \{a^n b^n a^m b^m : n, m \geq 0\}$

b) $L = \{a^{2n} b^{2n} : n \geq 0\}$

c) $L = \{2a^n b^n : n \geq 0\}$

d) none of these.

ix) If the length of input string processed in a Moore Machine is 15, then the length of the output string is

a) 14 b) 15

c) 16 d) 17.

x) If a graph has 5 vertices and 7 edges then the size of its incidence matrix is

a) 5×7 b) 7×7

c) 5×5 d) 7×5.

xi) If a language L is accepted by an automata M then

a) every string in L is accepted by M

b) at least one string in L is accepted by M

c) no string of L is accepted by M

d) only one string of L is accepted by M.

xii) Suppose you have four friends; in how many ways can you invite them for dinner ?

a) 15 b) 16

c) 24 d) 10.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that $\sim (p \vee (\sim p \wedge q)) \equiv (\sim p \wedge \sim q)$.

3. Prove by mathematical induction $3^{2n} - 8n - 1$ is divisible by 64.

4. There are 50 students in each of the senior or junior classes. Each class has 25 male and 25 female students. In how many ways can an eight student committee be formed so that there are four females and three juniors in the committee ?

5. Write short notes on Mealy Machine.

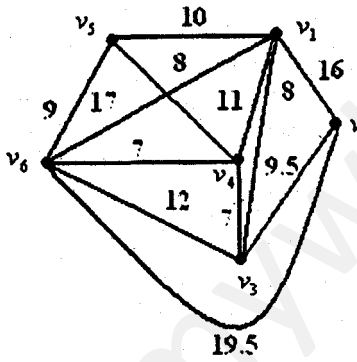
6. Draw the graph represented by the given incidence matrix :

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix} \end{array}$$

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Find by Kruskal's algorithm, a minimal spanning tree with minimum weight of the following graph given below :



b) Prove that a complete graph with n vertices consists of $\frac{n(n-1)}{2}$ number of edges.

c) Prove that ${}^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$. 6 + 6 + 3

8. a) Convert the given Moore Machine to its equivalent Mealy Machine :

Present state	Next state		Output
	Input $a = 0$	Input $a = 1$	
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

b) Construct the state diagram for finite state machine with state table as under :

State	Input		Output	
	0	1	0	1
$\rightarrow S_0$	S_1	S_0	1	0
S_1	S_3	S_0	1	0
S_2	S_1	S_0	0	1
S_3	S_2	S_1	0	0

7 + 8

9. a) Write the DNF & CNF for $p \rightarrow (p \wedge (q \rightarrow p))$.

b) A graph has 21 edges, 3 vertices each of degree 4 and rest of the vertices are of degree 3. Find out the total number of vertices.

c) In how many ways can the letter of the word ALGEBRA be arranged such that the two *A*'s are never come together.

5 + 5 + 5

10. a) Apply the Generating function technique to solve the recurrence relation :

$a_n = 4a_{n-1} + 3, a_0 = 2$

b) Solve the following recurrence relation using generating function :

$a_n - 7a_{n-1} + 10a_{n-2} = 2, \forall n > 1 \& a_0 = 3, a_1 = 3$.

c) Prove the following equivalence :

$p \Leftrightarrow (p \wedge q) \vee (p \wedge \sim p)$. 5 + 5 + 5

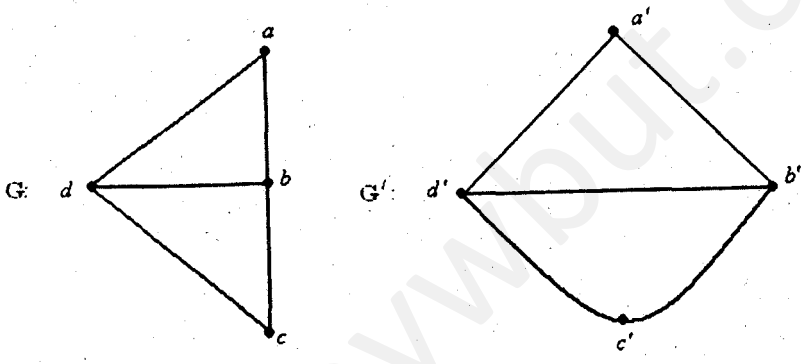
11. a) **Write short notes on any *two* of the following :** **2 × 5**

i) **Euler Graph**

ii) CNF

iii) Planar Graph.

b) Examine if the following *two* graphs are isomorphic :



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