Name : ……………………………………………………….

Roll No. : ………………………………………………………

Invigilator's Signature : ……………………………………

CS/BCA/SEM-3/BM-301/2009-10

2009

MATHEMATICS FOR COMPUTING

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be provided by the Institution.

GROUP – A

(Multiple Choice Type Questions)

1.   Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i)   The generating function of

{ 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1 } is

a) $\dfrac{z}{1 + 2z}$ b) $\dfrac{z}{1 - z^3}$

c) $\dfrac{1}{1 + z}$ d) $\dfrac{1}{(1 + z)(1 - z)}$.

ii)   How many bit strings of length 10 contain exactly four 1's ?

a)   130   b)   720

c)   386   d)   210.

iii) In how many ways can 5 questions be selected from 8 questions ?

a) 20   b) 40

c) 56   d) 336.

iv) If ^7P_x = 210, then the value of x is

a) 4   b) 3

c) 8   d) 2.

v) Determine the type of grammar which consists of the productions,

$S \to aAB, \quad AB \to bB, \quad B \to b, \quad A \to aB$

a) Type-1 grammar

b) Regular

c) Type-3 grammar

d) Type-2 grammar.

vi) In a bipartite graph we cannot find a triangle.

a) True   b) False.

vii) Express the following sentence in symbolic form :

it is raining but not cloudy.

a) $\sim p \wedge \sim q$ b) $-(\sim p \vee q)$

c) $p \wedge -q$ d) none of these.

viii) Let $f : z \to z$ be a mapping defined by $f(x) = 2x - 3$. Then the mapping f is

a) one to one

b) onto

c) neither one-one nor onto

d) both.

ix) The minimum possible height of an n vertex binary tree is

 a) $[\log_2 (n - 1) + 1]$

 b) $[\log_2 (n - 1) - 1]$

 c) $[\log_2 (n + 1) - 1]$

 d) $[\log_e (n + 1) - 1]$.

x) Let L be a Language given by $L = \{ a^n b^n : n \geq 0 \}$, then L^2 is equal to

 a) $\{ a^n b^n a^m b^m : n \geq 0, \ m \geq 0 \}$

 b) $\{ a^n b^n : n \geq 0 \}$

 c) $\{ a^n b^n a^m b^m : n \geq 0 \}$

 d) none of these.

xi) Does there exist a simple graph with 5 vertices of the given degrees ?

 1, 2, 3, 4, 5.

 a) No

 b) Yes

 c) Somtime it exists.

xii) In how many ways can the letters of the word UTECH be arranged ?

 a) 60   b) 5

 c) 120   d) 10.

GROUP – B

(Short Answer Type Questions)

Answer any *three* from the following.   3 × 5 = 15

2.   A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is permitted to attempt not more than 4 from any group. In how many different ways can he make up his choice ?

3.   Find the incidence matrix of the graph :

dia

4.   Examine whether the graphs G and G_1 are isomorphic or not.

dia

5.   Show that $(2n)! = 2^n . n! \{ 1, 3, 5 \ldots (2n - 1) \}$.
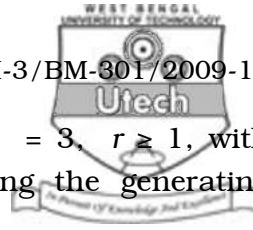
6. Solve the recurrence relation $a_r - 5a_{r-1} = 3$, $r \geq 1$, with the boundary conditions $a_0 = 1$ using the generating function.

7. In how many ways can three prizes be distributed among 4 boys when

   i) no one gets more than one prize ?

   ii) a boy can get any number of prizes ?

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Solve the following equivalence recurrence relation using generating functions :

   $a_n - 2a_{n-1} + a_{n-2} = 2^{n-2}$ for $n \geq 2$ & $a_0 = 1$, $a_1 = 5$.

   b) Let Z be the set of all integers and a binary relation ρ is defined on Z by the rule, $m\rho n$ means $m - n$ is divisible by 5 such that ρ is an equivalence relation on Z and identify all equivalent classes.

9. a) Draw the graph represented by the given adjacency matrix.

   $$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

   b) Show that a tree of n vertices has $n - 1$ edges.
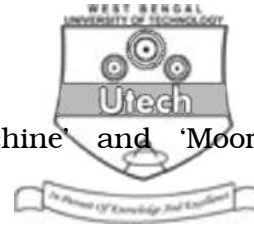
c) Write short notes on 'Mealy Machine' and 'Moore Machine'.

10. a) Show that a simple graph with n vertices and k components has at most

$(n - k) (n - k + 1) / 2$ edges.

b) Using Kruskal's algorithm find a spanning tree with minimum weight from the graph given below. Also calculate the total weight of spanning tree.

dia

11. a) Examine if the following two graphs are isomorphic :

dia

b) Construct the state diagram for finite state machine with state tables as under :

State	f		G	
	Input		Output	
	0	1	0	1
S_0	S_1	S_0	1	0
S_1	S_3	S_0	1	0
S_2	S_1	S_0	0	1
S_3	S_2	S_1	0	0

c) Write short notes on any *two* of the following :

i) Spanning Graph

ii) Hamiltonian Graph

iii) Digraph.

12. a) Solve the equation $2x^3 - x^2 - 22x - 24 = 0$, two of the roots being in the ratio 3 : 4.

b) Let $R = \left\{ \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} ; \ \alpha \in R \right\}$, show that R is an Abelian group with respect to usual matrix multiplication.

c) In a survey of entertainment habits of 1000 persons, it was found that 400 persons do not see cinema or see TV. 500 persons go to cinema and 200 persons see TV. Find out the number of persons who see cinema as well as TV. Find also the number of persons who see only TV but not cinema and those who go to cinema but not see TV. 5 + 5 + 5

13. a) Evaluate :

i) $\int \dfrac{1 + \sin x}{\sin x\,(\,1 + \cos x\,)}\ \mathrm{d}x$

ii) $\int\limits_{0}^{\pi} x \log \sin x\ \mathrm{d}x.$

b) Find the point of intersection of the straight lines

$\dfrac{K}{r}\ =\ \cos\theta - \cos(\,\theta - \alpha\,)$ and

$\dfrac{K}{r}\ =\ \cos\theta - \cos(\,\theta - \beta\,).$ (5 + 5) + 5

14. a) Discuss the nature of the conic represented by

$x^2 + 2xy + y^2 - 4x - 4y + 3 = 0$

by reducing to its canonical form.

b) If PSQ be a focal chord of a conic with focus S and semi-latus rectum l then prove that

$\dfrac{1}{SP} + \dfrac{1}{SQ}\ =\ \dfrac{2}{l}\,.$ 9 + 6

15. a) Find the point on the conic $\dfrac{9}{r}\ =\ 3 - 5\cos\theta$ which has the smallest radius vector.

b) If $u = \dfrac{x^2 + y^2}{\sqrt{xy}}$, $(\,x,\,y\,) \neq (\,0,\,0\,)$ and

$x\,\dfrac{\partial u}{\partial x} + y\,\dfrac{\partial u}{\partial y}\ =\ ku,$ find the value of k.

c) Use mean-value theorem to prove the following inequality :

$0 < \dfrac{1}{x}\ \log\dfrac{e^x - 1}{x}\ < 1.$ 4 + 5 + 6

CS / BCA / SEM-3 / BM (BCA)-301 / 2010-11

2010-11

MATHEMATICS FOR COMPUTING

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words
as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *one* of the following :

$$10 \times 1 = 10$$

i)   Maximum number of edges with n vertices in a completely connected graph is

   a) $(n-1)$ b) $\dfrac{n}{2}$

   c) $\dfrac{(n-1)}{2}$ d) $\dfrac{n(n-1)}{2}$

3017   [Turn over]

ii) A square matrix is said to be symmetric iff

    a) $A = -(A)$ b) $A^T = A$

    c) $A^T = -A$ d) $A = A$.

iii) If R_1 and R_2 are two Regular expressions (R.E.) then

$R_1 + R_2$ is

    a) R.E.   b) CFG

    c) CSG   d) Regular Grammar.

iv) Prim's Algorithm is used to find the minimal spanning tree of a

    a) Dense graph   b) Sparse graph

    c) Null graph   d) Normal graph.

v) A simple graph has

    a) no self loop   b) no parallel edges

    c) both (a) and (b)   d) none of these.

vi) The generating function of { 1, 1, 1, 1, ... } is

a) $\dfrac{1}{1-x}$

b) $\dfrac{1}{1+x}$

c) $\dfrac{1}{(1+x)(1-x)}$

d) none of these.

vii) A grammar is said to be regular if it is of

a) Type-0

b) Type-I

c) Type-2

d) Type-3.

viii) How many bit strings of length 10 contain exactly four 1's ?

a) 120

b) 720

c) 386

d) 210.

ix) Solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is

a) $1 - 2^n$

b) $2^n - 2$

c) $2^{n-1} - 1$

d) $2^n - 1$.

3017   3   [Turn over]

x) Number of four-digit number formed by the digits 3, 1, 3, 1 is

a) 5   b) 10

c) 20   d) 6.

xi) How many permutations of the letters '$A\ B\ C\ D\ E\ F\ G$' contain the string '$B\ C\ D$' ?

a) 24   b) $\lfloor 6$

c) 120   d) 0.

xii) If a binary tree has 20 pendant vertices, then the number of internal vertices of the tree is

a) 20   b) 21

c) 23   d) 19.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Let $A = \{\ 1, 2, 3, 4\ \}$ and $R = \{\ (1, 1), (2, 2), (3, 3), (4, 4), (1, 2),$ $(1, 3), (3, 2)\ \}$. Is R is equivalence relation ? Explain.

3. Prove that $((P \wedge \to Q) \to R) \to (P \to (Q \vee R))$ is a tautology.

4. Find out the characteristic roots for $a_n + 4a_{n-1} + 3a_{n-2} = 0$ and hence solve it.

5. Prove that for a graph $G = (V, E)$, there can be even number of odd vertices.

6. Show that there exists no simple graph with five vertices having degrees 4, 4, 4, 2, 2.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7.   a)   Solve the following recurrence relation using generating function $a_n - 2a_{n-1} + a_{n-2} = 2^{n-2}$ for $n \ge 2$ & $a_0 = 1, a_1 = 5$.

b)   Show that a simple graph with n vertices and k components has at most $\dfrac{(n-k)(n-k+1)}{2}$ edges.   7 + 8

8.   a)   Find the Grammar on the set of terminals { a, b } that generates the language $L = \{ a, ab, ab^2, ab^3, ... \}$.

b)   Draw the transition diagram for the FSA with $I = \{ a, b \}$, $Q = \{ q_0, q_1, q_2 \}$, $F = \{ q_0, q_1 \}$ and δ is given by

| δ	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_2	q_2

7 + 8

9. a) Find the CNF of the following statement :

$\neg (p \vee q) \leftrightarrow (p \wedge q)$

b) There are 50 students in each of the senior or junior classes. Each class has 25 male and 25 female students. In how many ways can an eight-student committee be formed so that there are four females and three seniors in the committee ?

10. a) Find by Kruskal's Algorithm a minimal spanning tree from the following graph G.
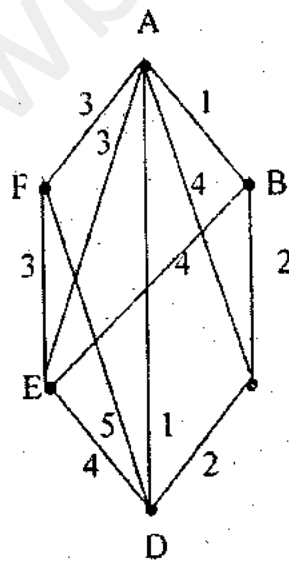
b) Draw the graph having the following matrices as their adjacency matrices.

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

8 + 7

11. a) How many selections any number at a time, may be made from 3 white balls, 4 green balls, 1 red ball and 1 black ball, if at least one must be chosen ?

b) How many integral solutions are there of

$x_1 + x_2 + x_3 + x_4 + x_5 = 30$

where $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x \geq 2, x_5 \geq 0$.

c) Solve the following recurrence relation :

$a_r - 6a_{r-1} + 8a_{r-2} = r.4^r$ where $a_0 = 8$ and $a_1 = 22$.

d) Find the characteristic roots of the following recurrence relation :

$a_n - 3a_{n-1} - 4a_{n-2} = 0$.

3 + 4 + 5 + 3

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Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-3/BM-301/2011-12

2011

MATHEMATICS FOR COMPUTING

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words

as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1.   Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

   i)   The mathematical model of a Mealy Machine is a

   a)   5-tuple   b)   4-tuple

   c)   6-tuple   d)   none of these.

   ii)   A regular language is accepted by

   a)   every DFA   b)   every NFA

   c)   no DFA   d)   at least one DFA.

   iii)   How many bit strings of length 10 contain exactly four 1's ?

   a)   120   b)   720

   c)   210   d)   386.

3022   [Turn over

iv) If a graph has 5 vertices and 7 edges then the size of its adjacency matrix is

    a) 5×5 b) 7×7

    c) 5×7 d) 7×5.

v) Number of elements contained in an incidence matrix of a digraph is

    a)   1   b)   2

    c)   3   d)   none of these.

vi) Solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is

    a) $1 - 2^n$ b) $2^n - 2$

    c) $2^{n-1} - 1$ d) $2^n - 1$.

vii) If δ is transition function of a mealy machine then for any state q and input $a, \delta(q, a) =$

    a)   an input   b)   a state

    c)   an output   d)   none of these.

viii) The generating function of the sequence $< 0, 1, 0, 0, 1, 0, 0, 1 \ldots >$ is

    a) $\dfrac{x}{1-x}$ b) $\dfrac{x}{1-x^3}$

    c) $\dfrac{1}{1-x}$ d) $\dfrac{1}{1-x^2}$.

ix) A spanning tree has

    a)   only one circuit   b)   two circuits

    c)   no circuit   d)   none of these.

x) The type-3 grammar in relation to the automata theory is known as

   a) context sensitive grammar

   b) regular grammar

   c) context free grammar

   d) none of these.

xi) If $n! = x(n-2)!$ then $x =$

   a) n b) $n-1$

   c) $n(n-2)$ d) $n(n-1)$.

xii) The automata
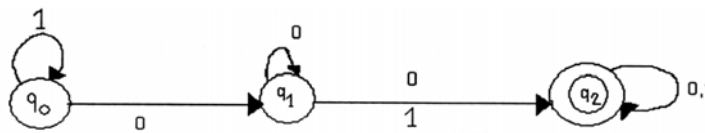
accepts the string

   a) 111   b) 01

   c) 111   d) none of these.

xiii) The generation function for the sequence 1, 2, 3, 4 ... is

   a) $(1+2x)^{-1}$ b) $(1-2x)^{-1}$

   c) $(1-x)^{-2}$ d) $(1+x)^{-2}$.

xiv) $2n < n\,!$ is true for

    a) $n < 4$ b) $n > 4$

    c) $n \geq 4$ d) $n = 4$.

xv) No. of four digit numbers formed by digits 3, 1, 3, 1 is

    a)   5   b)   10

    c)   20   d)   6.

xvi) The proposition $p \wedge (q \wedge \sim q)$ is a

    a)   contradiction   b)   tautology

    c)   both (a) and (b)   d)   none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following $3 \times 5 = 15$

2. Prove that a tree with n vertices has ($n - 1$) edges.

3. Write short notes on Euler path and Hamilton path.

4. There are 50 students in each of the senior or junior classes. Each class has 25 male and 25 female students. In how many ways can an eight student committee be formed so that there are four females and three juniors in the committee ?

5. Find the generating function corresponding to the sequence { 1, –1, 1, –1, 1, –1 ....}.

6. Prove that $^{n+1}C_r = {}^{n}C_{r-1} + {}^{n}C_r$.

7. Construct the graph or digraph corresponding to the following incidence matrix :

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

GROUP – C
(Long Answer Type Questions)
Answer any *three* of the following.   3 × 15 = 45

8. a) Write Kruskal's Algorithm for minimal spanning tree.

   b) Find the minimal spanning tree of the following labelled connected graph by Kruskal's Algorithm.
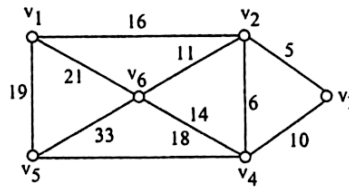
   c) How many permutations can be made out of the letters of the word 'Basic' that

   i)   begin with B ?

   ii) end with C ?   5 + 5 + 5

9. a) Write CNF & DNF of the following statement :

   $p \to (p \wedge (q \to p))$

   b) Verify whether the argument given below is valid or not :

   If I like Mathematics then I will study.

   Either I do not study or I pass the examination.

   If I do not graduate then I did not pass Mathematics.

   ... If I like Mathematics, then I will graduate.

c) Prove the following equivalence :

$$p \Leftrightarrow (p \wedge q) \vee (p \wedge \sim q)$$

5 + 5 + 5

10. a) Draw the graph represented by the given adjacency matrix :

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

b) Examine if the following two graphs are isomorphic :
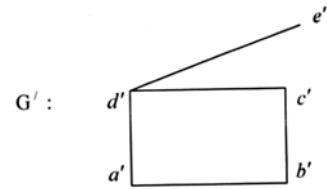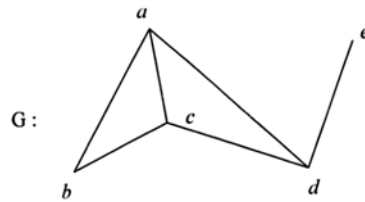
8 + 7

11. a) Prove that the number of vertices of odd degree in a graph is always even.

b) Show that $(2n) ! = 2^n . n ! \{ 1, 3, 5, ..., (2n-1) \}$.

c) In how many ways can three prizes be distributed among 4 boys when,

i) no one gets more than one prize ?

ii) a boy can get any number of prizes ? 5 + 5 + 5
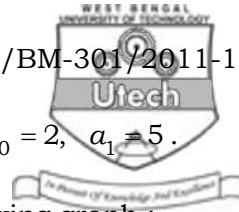
12. a) Solve $a_n - 5a_{n-1} + 6a_{n-2} = 0$, where $a_0 = 2$, $a_1 = 5$.

b) Find the incidence matrix of the following graph :

7 + 8

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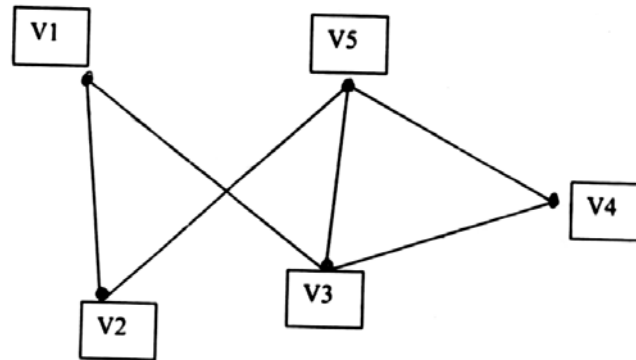
2012

MATHEMATICS FOR COMPUTING

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words
as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

   i)   The proposition $p \wedge (q \wedge \sim p)$ is a

   a)   contradiction   b)   tautology

   c)   both (a) and (b)   d)   none of these.

   ii) The type of the grammar G which consists of

   productions $P = \{S \rightarrow bAB, A \rightarrow aB, abAbb \rightarrow abbb\}$ is

   a)   Type-0   b)   Type-1

   c)   Type-2   d)   Type-3.

3022   [Turn over

iii) ρ is a relation on the set $R \times R$ of ordered pairs of real numbers as follows :

For all $(a, b), (c, d) \; \varepsilon \; R \times R \; (a, b) (c, d) \Leftrightarrow a = c$

Then ρ is

a) symmetric only

b) symmetric but not reflective

c) equivalence relation

d) none of these.

iv) Let $A = R - \{3\}$ and $B = R - \{1\}$.

If $f : A \rightarrow B : f(x) = \dfrac{x-2}{x-3}$ then

a) f is into

b) f is surjective

c) f is bijective

d) none of these.

v) A pseudo graph

a) must has loops

b) does not have loop

c) must have parallel edges

d) none of these.

vi) Minimum height of a n vertex binary tree is

a) $\dfrac{n-1}{2}$

b) $\dfrac{n+1}{2}$

c) $\left\lfloor \log_2^{(n+1)} - 1 \right\rfloor$

d) $\left| \log_2^{(n+1)} - 1 \right|$.

vii) If the general term of the sequence $\{a^k\}$ be a^k which will be the generating function ?

     a) $\dfrac{1}{1-x}$ b) $\dfrac{a}{1-x}$

     c) $\dfrac{k}{1-x}$ d) $\dfrac{1}{1-ax}$.

viii) A simple graph with n vertices has maximum

     a) $\dfrac{n(n-1)}{2}$ edges   b) $(n-1)$ edges

     c) $\dfrac{n(n+1)}{2}$ edges   d) n^2 edges.

ix) If a language L is accepted by a automata M then

     a)   every string in L is accepted by M

     b)   at least one string in L is accepted by M

     c)   no string of L is accepted by M

     d)   only one string is accepted by M.

x) Number of elements contained in an incidence matrix of a digraph is

     a)   1   b)   2

     c)   3   d)   none of these.

xi) The degree of the origin of the longest path in a tree is

     a)   1   b)   2

     c)   3   d)   none of these.

xii) Choose the correct statement :

    a) Path is an open walk

    b) Every walk is trail

    c) Every trail is a path

    d) A vertex cannot appear twice in a walk.

xiii) How many permutations of the letters *ABCDEFG* contain in the string *BCF* ?

    a) 24   b) 6!

    c) 120   d) 252.

xiv) A spanning tree has

    a) one circuit   b) no circuit

    c) two circuits   d) none of these.

xv) You have five friends. In how many ways can you invite them ?

    a) 51   b) 36

    c) 25   d) none of these.

GROUP – B
(Short Answer Type Questions)
Answer any *three* of the following   3 × 5 = 15

2. Prove that $((P \wedge \sim Q) \to R) \to (P \to (Q \vee R))$ is a tautology.

3. In an examination a minimum is to be secured in each of the 5 subjects for a pass. In how many ways can a candidate fail ?

4. Find the sequence corresponding to the generating function $\dfrac{3 + 7x}{(1 - x)(1 + 4x)}$.

5. Suppose G is a non-directed graph with 12 edges. If G has 6 vertices each of degree 3 and rest have degree less than 3, find the minimum number of vertices in G.

6. What is Deterministic finite Automata (DFA) ? Explain with suitable example.

7. Write a short note on Moore Machine.

GROUP – C
(Long Answer Type Questions)
Answer any *three* of the following. $3 \times 15 = 45$

8. a) Find by Prim's algorithm a spanning tree with minimum weight from the graph given below. Also calculate total weight of spanning tree.
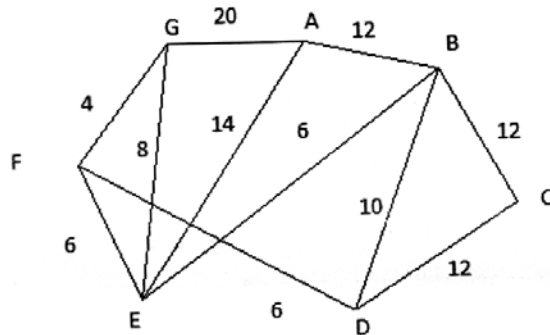
   b) Prove that a connected graph n with $n - 1$ vertices and edges is a tree.

   c) Determine the value of n if $4 \times {}^nP_3 = {}^{n+1}P_3$. $6 + 6 + 3$

9. a) Find the grammar on the set of terminals { a, b } that generates the language $L = \{a, ab, ab^2, ab^3, ...\}$.

b) Draw the transition diagram for the FSA with $I = \{a, b\}$, $Q = \{q_0, q_1, q_2\}$, $F = \{q_0, q_1\}$ and δ is given by

Δ	a	b
Q_0	Q_0	Q_1
Q_1	Q_0	Q_2
Q_2	Q_2	Q_2

7 + 8

10. a) Write DNF of the following statement :

$$\neg \{ \neg (p \leftrightarrow q) \wedge r \}$$

b) Verify whether the argument given below is valid or not :

All mammals are animals. Some mammals are two-legged. Therefore, some animals are two-legged.

c) Prove the following equivalence :

$$\neg p \wedge q \Leftrightarrow \neg (p \vee (\neg p \wedge q))$$

5 + 5 + 5

11. a) What is Grammar ?

b) Construct the sate diagram for finite state machine with state table as under :

State	Input		Output	
	0	1	0	1
$\rightarrow S0$	S1	S1	1	0
S1	S3	S0	1	0
S2	S1	S0	1	0
S3	S2	S1	0	0

5 + 10

12. a) Examine if the following two graphs are isomorphic :
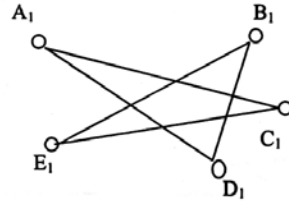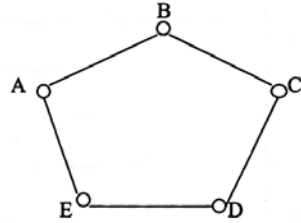
b) Solve the following recurrence relation using generating function :

$a_n - 2a_{n-1} + a_{n-2} = 2^{n-2}$ for $n \geq 2$ and $a_0 = 1, a_1 = 5$.

c) Write short notes on any *two* of the following :

   i) Spanning Graph

   ii) Hamiltonian Graph

   iii) Digraph. 5 + 5 + 5

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CS/BCA/SEM-3/BM-301/2013-14

2013

MATHEMATICS FOR COMPUTING

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words

as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i)   A statement *T* is called a tautology if

a)   *T* is true for all possible values of its variables

b)   *T* is false for all possible values of its variables

c)   *T* is true as well false for few possible values of its variables

d)   none of these.

ii)   The type-3 Grammar in relation to the automata theory is known as

a)   Context sensitive grammar

b)   Context free grammar

c)   Regular grammar

d)   none of these.

iii) Solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is

a) $1 - 2^n$

b) $2^n - 2$

c) $2^n - 1$

d)   none of these.

iv)   How many bit strings of length 10 contain exactly four's 1's ?

a)   130

b)   720

c)   210

d)   none of these.

v)   A spanning tree of a connected graph contains

a)   all the vertices of the graph

b)   all the vertices and edges of the graph

c)   a few vertices of the graph

d)   none of these.

vi)   If $^{16}C_r = {}^{16}C_{2r+1}$, then $r =$

    a)   6   b)   5

    c)   4   d)   3.

vii)   A binary tree has exactly

    a)   **one root**   b)   **two roots**

    c)   three roots   d)   none of these.

viii)   Let L be a language given by $L = \{a^n b^n : n \geq 0\}$, then L^2 is equal to

    a) $L = \{a^n b^n a^m b^m : n, m \geq 0\}$

    b) $L = \{a^{2n} b^{2n} : n \geq 0\}$

    c) $L = \{2a^n b^n : n \geq 0\}$

    d)   none of these.

ix)   If the length of input string processed in a Moore Machine is 15, then the length of the output string is

    a)   14   b)   15

    c)   16   d)   17.

x)   If a graph has 5 vertices and 7 edges then the size of its incidence matrix is

    a) 5×7 b) 7×7

    c) 5×5 d) 7×5.

xi) If a language L is accepted by an automata M then

    a) every string in L is accepted by M

    b) at least one string in L is accepted by M

    c) no string of L is accepted by M

    d) only one string of L is accepted by M.

xii) Suppose you have four friends; in how many ways can you invite them for dinner ?

    a) 15   b) 16

    c) 24   d) 10.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that $\sim (p \vee (\sim p \wedge q)) \equiv (\sim p \wedge \sim q)$.

3. Prove by mathematical induction $3^{2n} - 8n - 1$ is divisible by 64.

4. There are 50 students in each of the senior or junior classes. Each class has 25 male and 25 female students. In how many ways can an eight student committee be formed so that there are four females and three juniors in the committee ?

5. Write short notes on Mealy Machine.

6. Draw the graph represented by the given incidence matrix :

$$\begin{array}{c} & \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 1 & -1 & 2 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{array}\right] \end{array}$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Find by Kruskal's algorithm, a minimal spanning tree with minimum weight of the following graph given below :
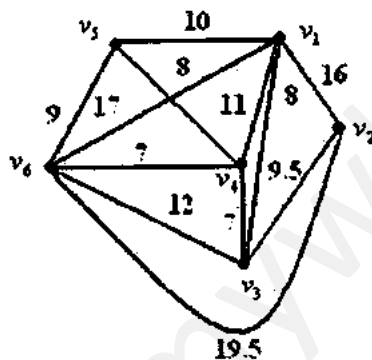
b) Prove that a complete graph with n vertices consists of $\dfrac{n(n-1)}{2}$ number of edges.

c) Prove that $^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$.   6 + 6 + 3

8. a) Convert the given Moore Machine to its equivalent Mealy Machine :

Present state	Next state		Output
	Input $a = 0$	Input $a = 1$	
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

b) Construct the state diagram for finite state machine with state table as under :

State	Input		Output	
	0	1	0	1
$\rightarrow S_0$	S_1	S_0	1	0
S_1	S_3	S_0	1	0
S_2	S_1	S_0	0	1
S_3	S_2	S_1	0	0

7 + 8

9. a) Write the DNF & CNF for $p \rightarrow (p \wedge (q \rightarrow p))$.

b) A graph has 21 edges, 3 vertices each of degree 4 and rest of the vertices are of degree 3. Find out the total number of vertices.

c) In how many ways can the letter of the word ALGEBRA be arranged such that the two A's are never come together. 5 + 5 + 5

10. a) Apply the Generating function technique to solve the recurrence relation :

$a_n = 4a_{n-1} + 3, a_0 = 2$

b) Solve the following recurrence relation using generating function :

$a_n - 7a_{n-1} + 10a_{n-2} = 2, \forall n > 1 \, \& \, a_0 = 3, a_1 = 3$.

c) Prove the following equivalence :

$p \Leftrightarrow (p \wedge q) \vee (p \wedge \sim p)$.   5 + 5 + 5

11. a) Write short notes on any *two* of the following :   2 × 5

    i) Euler Graph

    ii) CNF

    iii) Planar Graph.

b) Examine if the following *two* graphs are isomorphic :
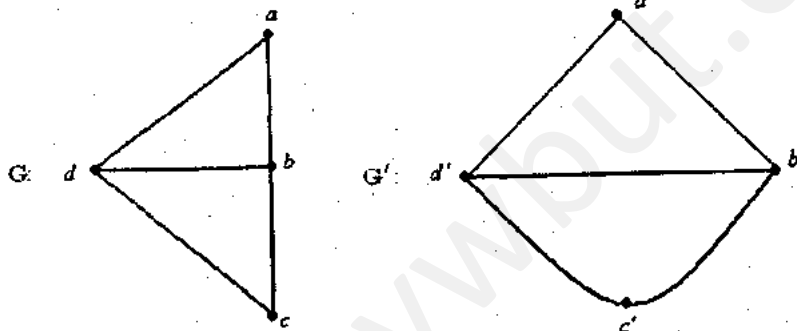
5

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BM-301

MATHEMATICS FOR COMPUTING

Time Allotted: 3 Hours   Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10×1 = 10$

   (i) Maximum number of edges with n vertices in a completely connected graph is

   (A) $(n-1)$ (B) $\frac{(h-1)}{2}$

   (C) $\frac{n}{2}$ (D) $\frac{n(n-1)}{2}$

   (ii) Prim's algorithm is used to find the minimal spanning tree of a

   (A) dense graph   (B) sparse graph
   (C) null graph   (D) normal graph

   (iii) A Simple graph has

   (A) no self loops   (B) no parallel edges
   (C) both (A) and (B)   (D) none of these

   (iv) The generating function of $\{1, 1, 1, ...\}$ is

   (A) $\frac{1}{(1-x)}$ (B) $\frac{1}{(1+x)}$

   (C) $\frac{1}{(1-x)(1+x)}$ (D) none of these

(v) A Grammar is said to be regular if it is of

(A) type 0   (B) type 1

(C) type 2   (D) type 4

(vi) The mathematical model of Mealy machine is a

(A) 5 tuple   (B) 4 tuple

(C) 6 tuple   (D) none of these

(vii) If $n! = x(n-2)!$ Then $x =$

(A) n (B) $n-1$

(C) $n(n-2)$ (D) $n(n-1)$

(viii) Number of four digit numbers formed by digits 3, 1, 3, 1 is

(A) 5   (B) 10

(C) 20   (D) 6

(ix) The proposition $P \wedge (Q \wedge -Q)$ is a

(A) contradiction   (B) tautology

(C) both (A) and (B)   (D) none of these

(x) Number of elements contained in an incidence matrix of a digraph is

(A) 1   (B) 2

(C) 3   (D) none of these

(xi) A pseudo graph

(A) Must have loops   (B) Does not have loop

(C) Must have parallel edges   (D) None of these

(xii) Choose the correct statement:

(A) path is an open walk

(B) every walk is a trail

(C) every trail is a path

(D) a vertex cannot appear twice in a walk

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3 \times 5 = 15$

2. In an examination a minimum is to be secured in each of the 5 subjects for a pass. In how many ways can a student fail?

3. Find the sequence from the generating function $\dfrac{(3 + 7x)}{(1 - x)(1 + 4x)}$

4. Prove that $((P \wedge \neg Q) \to R) \to (P \to (Q \vee R))$ is a tautology.

5. Write a short note on Moore machine.

6. Show that a tree with n vertices has $(n - 1)$ edges.

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

7. (a) Solve the following recurrence relation by generating function method: 5

$a^n - 2a_{n-1} + a_{n-2} = 2^{n-2}$ for $n \geq 2$ and $a_0 = 1, a_1 = 5$

   (b) What is a Deterministic Finite Automata (DFA)? Explain with suitable example. 5

   (c) Write CNF and DNF of the following statement: $p \vee \{\neg p \to (q \vee (q \to \neg r))\}$ 5

8. (a) Find the grammar of the set of the terminals $\{a, b\}$ that generates the language $L = \{a, ab, ab^2, ab^3, ...\}$. 6

   (b) Let Z be a set of all integers and a binary relation ρ is defined on Z by the rule, $m\rho n$ means $m - n$ is divisible by 5 such that ρ is an equivalence relation on Z and identify the equivalent classes. 6

   (c) Show that $\lfloor 2n = 2n\{1, 3, 5 \cdots (2n - 1)\}\lfloor n$ 3

3 [Turn over]

9.(a) Find out the characteristic roots of the recurrence relation $a_n + 4a_{n-1} + 3a_{n-2} = 0$ and hence solve it.   6

(b) Show that a simple graph with n vertices and k components has at most $\dfrac{(n-k)(n-k+1)}{2}$ edges.   6

(c) Prove the following equivalence: $\neg p \wedge q \Leftrightarrow \neg(p \vee (\neg p \wedge q))$ 3

10.(a) Find whether the following two graphs are isomorphic or not: 5
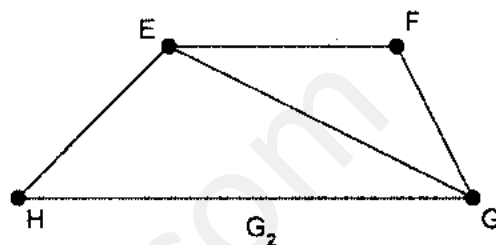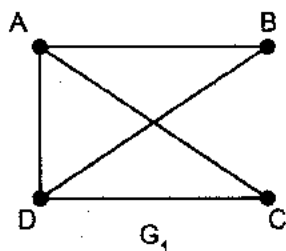
(b) How many integral solutions are there of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ where $x_1 \geq 2$, $x_2 \geq 3$, $x_3 \geq 4$, $x_4 \geq 2$, $x_5 \geq 0$ 6

(c) Determine the value of n if $4 \times {}^nP_3 = {}^{n-1}P_3$.   4

11.(a) Using Kruskal's algorithm find the minimal spanning tree from the following graph from S to Z.   6
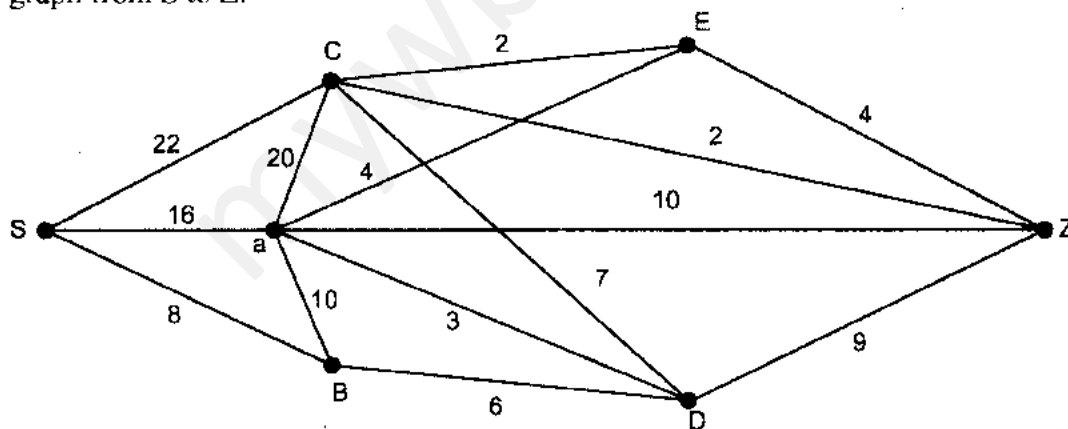
(b) Write short note on Hamiltonian Graph.   3

(c) Using the principle of mathematical induction prove that   6

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

CS/BCA/ODD SEM/SEM-3/BM-301/2016-17

Maulana Abul Kalam Azad University
of Technology, West Bengal

Utech

MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BM-301

MATHEMATICS FOR COMPUTING

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

    i) Solution of the recurrence relation $a_n = 2a_{n-1}$ with $a_0 = 1$ is

        a) 2^n

        b) 2^{n-1}

        c) 2^{n+1}

        d) 2^{n-2}.

3/30207   [Turn over

ii) If the truth value of *p* and *q* are *F* and *T* respectively then the truth value of $p \leftrightarrow q$ is

a)   *T*

b)   *F*

c)   both *T* and *F*

d)   none of these.

iii) A grammar is said to be regular if it is of

a)   Type-0

b)   Type-1

c)   Type-2

d)   Type-3.

iv) $p \vee (p \wedge q) =$

a)   *p*

b)   *q*

c) $p \wedge q$

d) $p \vee q$.

v) How many ways can be letters of the word "LEADER" be arranged ?

a)   72

b)   144

c)   360

d)   none of these.

vi) The generating function for the sequence $\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3} \dots\dots\dots$ is

a) $\dfrac{1/3}{(1+x)}$

b) $\dfrac{1}{3(1-x)}$

c) $\dfrac{1}{\frac{1}{3}(1-x)}$

d) $\dfrac{-3}{(1-x)}$.

2

vii) What is the minimum no. of vertices necessary for a graph with 6 edges ?

a) 6   b) 5

c) 7   d) none of these.

viii) A simple graph has

a) no parallel edges   b) no loops

c) both (a) and (b)   d) no isolated vertex.

ix) The difference between Mealy and Moore Machine lies on

a) state transition   b) output function

c) input function   d) none of these.

x) Maximum number of edge with n vertices in a completely connected graph is

a) $(n-1)$ b) $n/2$

c) $(n-1)/2$ d) $n(n-1)/2$.

xi) If a binary tree has 20 pendant vertices, then the number of internal vertices of the tree is

a) 20   b) 21

c) 23   d) 19.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. a) How may words can be made using all the letters in the word MONDAY ?

    b) In how many ways can the letters of the word ALGEBRA be arranged, such that two As are never together. $2 + 3$

3. Find the sequence for following generating function :

    $3x(1 - x)^5$.

4. Construct Incidence matrix from the following graph :
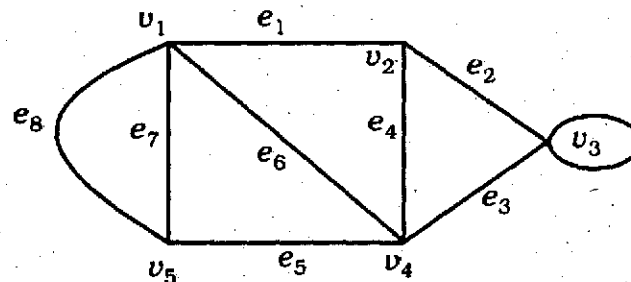
5.   Write short notes on Moore Machine.

6.   What is Deterministic finite automata (DFA) ? Explain with suitable example.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7.   a)   Examine Graphs H and G are isomorphic or not :
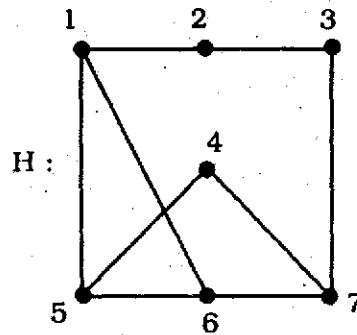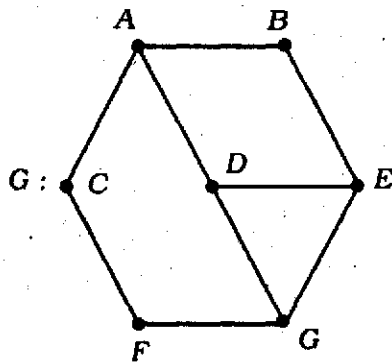
6

b)   Prove that

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots\ldots + \frac{1}{n(n+1)} = \frac{n}{(n+1)} \text{ by using}$$

mathematical induction.   5

c)   A graph has 21 edges, 3 vertices each of degree 4 and rest of the vertices are of degree 3. Find out the total number of vertices.   4

8. a) Using Kruskal's algorithm find minimal spanning tree of the following graph : 7
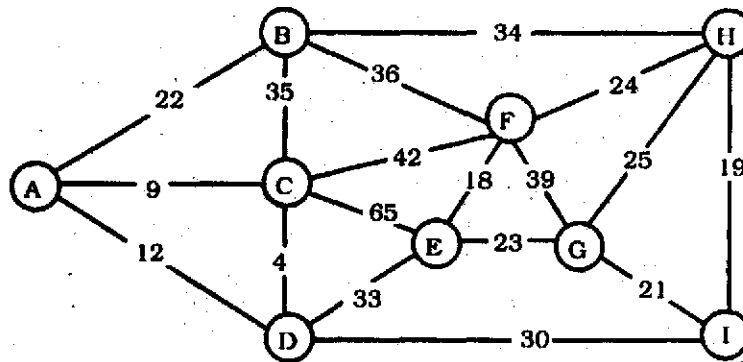
b) Prove that a simple graph with n vertices and k components has at most $\dfrac{(n-k)\ (n-k+1)}{2}$ edges. 8

9. a) Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 4a_n = 0$, $(n \geq 0)$ with $a_0 = 2$ and $a_1 = 1$ using generating function. 7

b) Convert the given Moore Machine to its equivalent Mealy Machine. 8

Present state	Next state		Output
	Input $a = 0$	Input $a = 1$	
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

10. a) Construct truth table and determine whether the following proposition is tautology or contradiction.

$\{ (p \wedge \sim q) \rightarrow r \} \rightarrow \{ p \rightarrow (q \vee r) \}.$ 7

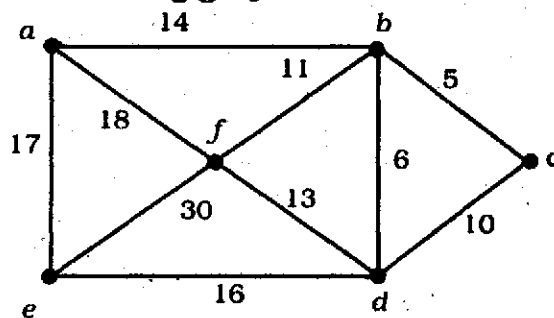b) Find all spanning trees from the following graph G :
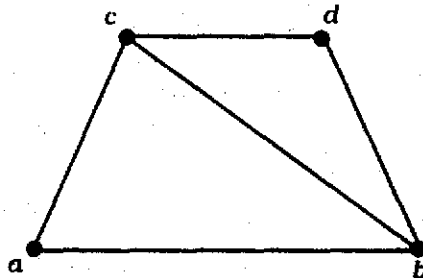
8

11. a) Draw the graph whose incidence matrix is given below :

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

7

b) By Prim's Algorithm find a minimal spanning tree and the corresponding weight of the spanning tree in the following graph : 8

MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BM-301

MATHEMATICS FOR COMPUTING

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

    i) In how many ways can 5 questions be selected from 8 questions ?

    a) 20   b) 40

    c) 56   d) 336.

    ii) Determine the type of grammar which consists of the productions,

    S → aAB, AB → bB, B → b, A → aB

    a) Type-1 grammar   b) Regular

    c) Type-3 grammar   d) Type-2 grammar.

iii) Express the following sentence in symbolic form :

It is raining but not cloudy.

a) ~p ∧ ~q

b) ~ (~ p V q)

c) p ∧ ~ q

d) none of these.

iv) Does there exist a simple graph with 5 vertices of the given degrees ?

1, 2, 3, 4, 5.

a) No

b) Yes

c) Sometime it exists

d) Never exists.

v) If R_1 and R_2 are two regular expressions (R.E.) then $R_1 + R_2$ is

a) R.E.

b) CFG

c) CSG

d) Regular grammar.

vi) Prim's Algorithm is used to find the minimal spanning tree of a

a) Dense graph

b) Sparse graph

c) Null graph

d) Normal graph.

vii) How many bit strings of length 10 contain exactly four 1's ?

a) 120

b) 720

c) 386

d) 210.

viii) The mathematical model of a Mealy Machine is a

a) 5-tuple

b) 4-tuple

c) 6-tuple

d) none of these.

ix) A regular language is accepted by

a) every DFA

b) every NFA

c) no DFA

d) at least one DFA.

x) Number of elements contained in an incidence matrix of a digraph is

a) 1

b) 2

c) 3

d) None of these.

xi) If n! = $x\,(n\text{-}2)!$ then $x =$

a) n

b) $n - 1$

c) $n\,(n - 2)$

d) $n\,(n - 1)$.

xii) The proposition $p \wedge (q \wedge \sim q)$ is a

a) contradiction

b) tautology

c) both (a) and (b)

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that $((P \wedge \sim Q) \to R) \to (P \to (Q \vee R))$ is a tautology.

3. There are 50 students in each of the senior or junior classes. Each class has 25 male and 25 female students. In how many ways can an eight student committee by formed so that there are four females and three juniors in the committee ?

4. Find out the characteristic roots for $a_n + 4a_{n-1} + 3a_{n-2} = 0$ and hence solve it.

5. Find the generating function corresponding to the sequence $\{1, -1, 1, -1, 1, -1, \ldots\}$.

6. Prove that for a graph $G = (V, E)$, there can be even number of odd vertices.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Write CNF & DNF of the following statement :

   $p \to (p \wedge (q \to p))$

b) Verify whether the argument given below is valid or not :

If I like Mathematics then I will study.

Either I do not study or I pass the examination.

If I do not graduate then I did not pass Mathematics.

....

... If I like Mathematics, then I will graduate.

c) Prove the following equivalence :

$p \equiv (p \wedge q) \vee (p \wedge \sim q)$.   5 + 5 + 5

8. a) Prove that the number of vertices of odd degree in a graph is always even.

b) Show that $(2n)! = 2^n \cdot n! \{1,3,5,......,(2n-1)\}$.

c) In how many ways can three prizes be distributed among 4 boys when,

   i) no one gets more than one prize ?

   ii) a boy can get any number of prizes ?   5 + 5 + 5

9. a) Find the Grammar on the set of terminals $\{a,b\}$ that generates the language $L = \{a,ab,ab^2,ab^3,...\}$.

b) Draw the transition diagram for the FSA with $I = \{a,b\}$, $Q = \{q_0,q_1,q_2\}$, $F = \{q_0,q_1\}$ and δ is given by

Δ	A	B
Q_0	Q_0	Q_1
Q_1	Q_0	Q_2
Q_2	Q_2	Q_2

                7 + 8

10. a) Write Kruskal's Algorithm for minimal spanning tree.

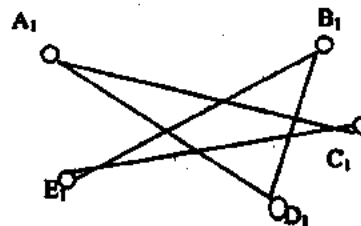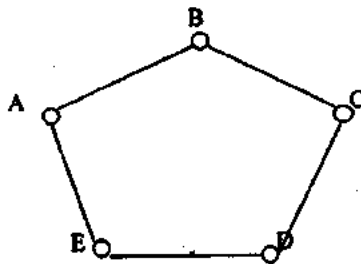   b) Find the minimal spanning tree of the following labeled connected graph by Kruskal's algorithm.
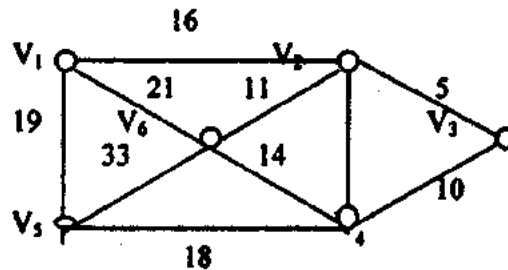
    c) How many permutations can be made out of the letters of the word 'Basic' that

    i) Begin with B ?

    ii) End with C ?   5 + 5 + 5

11. a) Examine if the following two graphs are isomorphic

    b) Using generating function solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ for all n > 1, and $a_0 = 3, a_1 = 3$.

c) Write short notes on any *two* of the following :

i) Spanning Graph.

ii) Hamiltonian Graph.

iii) Digraph. 5 + 5 + 5

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UNIVERSITY OF TECHNOLOGY,
WEST BENGAL

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Paper Code : BM-301

MATHEMATICS FOR COMPUTING

Time Allotted: 3 Hours   *Full Marks*: 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words
as far as practicable.

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternative for *any ten* of the following: 1×10=10

   (i) A grammar is said to be regular if it is of

   (a) Type–0   (b) Type–1

   (c) Type–2   (d) Type–3

   (ii) How many non-isomorphic trees are there with the number of vertices equal to 4?

   (a) 1   (b) 2

   (c) 3   (d) 4

   (iii) Choose the correct statement:

   (a) Euler line is a open path.   (b) Euler line is a open trial.

   (c) Euler line is a open walk.   (d) Euler line is a closed path.

   (iv) The solution of recurrence relation $a_n - a_{n-1} = 2^n, a_0 = 2$ is

   (a) $a_n = 2n$ (b) $a_n = 2^n$

   (c) $a_n = 2(n + 1)$ (d) $a_n = 2^{n+1}$

   (v) Out of the following statements the formula for tautology is

   (a) $(p \lor q) \to q$ (b) $p \lor (q \to p)$

   (c) $p \lor (p \to q)$ (d) $p \to (p \to q)$

(vi) The type of the grammar, which consists of the following productions:

$s \rightarrow aA, A \rightarrow aAB, B \rightarrow bb, A \rightarrow Aa$

(a) Type–0
(b) Type–1

(c) Type–2
(d) Type–3

(vii) The generating function of the sequence $\{2, 0, 2, 0, 2, 0, \ldots \ldots \ldots \}$ is

(a) $x^2(1 + x)^{-1}$
(b) $2(1 + x)^{-2}$

(c) $2(1 - x^2)^{-1}$
(d) $x^2(1 - x)^{-1}$

(viii) If a tree has 10 vertices, then number of its edges is

(a) 8
(b) 11

(c) 10
(d) None of these

(ix) Number of edges in a cubic graph on 4 vertices is

(a) 3
(b) 4

(c) 5
(d) 6

(x) Which one is true?

(a) Type–0 grammar is also Type–1 grammar.
(b) Type–1 grammar is also Type–0 grammar.

(c) Type–2 grammar is also Type–3 grammar.
(d) Type–1 grammar is also Type–3 grammar.

(xi) What is the minimum number of vertices necessary for a graph with 6 edges?

(a) 2
(b) 3

(c) 4
(d) 5

(xii) If a binary tree has 14 pendant vertices, then find the number of internal vertices.

(a) 2
(b) 3

(c) 4
(d) 5

Group – B

(Short Answer Type Questions)

Answer *any three* of the following.   5×3=15

2. Draw the graphs for the following incidence matrix I:

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

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3. Prove by mathematical induction $3^{2n} - 8n - 1$ is divisible by 64.

4. If you arrange all the letters of the word "*SENJUTI*". How many possible ways are there such that no two vowels come together?

5. Prove whether $[(P \to Q) \to R] \to [P \to (Q \vee R)]$ is a tautology or not.

6. Verify whether the argument given below is valid or not:

    All mammals are animals.

    <u>Some mammals are two-legged.</u>

    Therefore, some animals are two-legged.

Group – C

(Long Answer Type Questions)

Answer *any three* of the following.   15×3=45

7. (a) Define bi-partite and complete bi-partite graph with example.

    (b) Show that the number of vertices in a binary tree is always odd number.

    (c) Draw the three distinct connected graphs from the degree sequence {1, 3, 3, 4, 5} which are not isomorphic.   5+5+5=15

8. (a) Define Deterministic Finite Automata and Non-deterministic Finite Automata.

   (b) Draw the DFA for the following transition function (δ):

Present state	Input 0	Input 1
	Next state	Next state
A	B	C
B	A	C
C	B	A

    (c) Convert the NFA, $M = \langle \{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, q_2 \rangle$ to DFA, δ is given below:

Present state	Input 0	Input 1
	Next state	Next state
→ a	a	b
b	c	a, c
ⓒ	b	a

    5+5+5=15

9. (a) Use mathematical induction to prove that $n^3 + 2n$ is divisible by 3.

(b) Solve the recurrence relation $a_n - 6a_{n-1} + 5a_{n-2} = 0, n \geq 2, a_0 = 2, a_1 = 5$.

(c) Draw the transition diagram of a automation M that accepts all odd numbers. 5+5+5=15

10. (a) Prove that the number of odd vertices in a graph is always even.

(b) Define grammar.

(c) Find the minimal spanning tree of the following graph by Kruskal's algorithm (show intermediate steps): 5+3+7=15
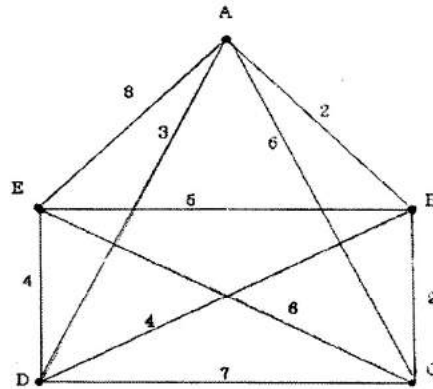
11. (a) Let $\Sigma = (a, b)$. Write the grammar for the language $L = \{a^n b^{n+1}; n \geq 0\}$.

(b) There are 15 shades of colour pencils in a shop. In how many ways can 5 pencils be purchased such that—

(i) all are of different shades, and

(ii) all are of the same shades

(c) Test whether the following two graphs are isomorphic to each other or not. 5+5+5=15

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