ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2008

MATHEMATICS – I

SEMESTER – 1

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $A = \{ 1, 2, 3, 4, 8 \}$, $B = \{ 2, 4, 6, 7 \}$, then $A \Delta B$ is

a) $\{ 2, 4 \}$

b) $\{ 1, 2, 3, 4, 6, 7, 8 \}$

c) ϕ

d) $\{ 1, 3, 6, 7, 8 \}$.

ii) $\lim_{x \to 0} (1 + x)^{1/x}$ is equal to

a) 1 b) e

c) 0 d) ∞ .

iii) $\frac{d}{dx} \left(\log_a x \right)$ is equal to

a) $\frac{1}{x}$ b) $\log (1/x)$

c) $(1/x) \log_a e$ d) $x \log e$.

iv) If $y = \log x^2$, the value of $\frac{d^2 y}{dx^2}$ is

a) $\frac{2}{x^2}$ b) $-\frac{2}{x^2}$

c) $\frac{2}{x}$ d) $2x$.

v) The matrix $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is an

a) orthogonal matrix b) idempotent matrix

c) identity matrix d) none of these.

vi) Derivative of x^4 with respect to x^2 is

a) $4x^3$ b) $2x^2$

c) $2x$ d) 4.

vii) If the roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$) are real and unequal, then its discriminant D satisfies

a) $D > 0$ and D = a perfect square

b) $D = 0$

c) $D > 0$ and $D \neq$ a perfect square

d) $D < 0$.

viii) If $A = \{ 1, 2, 3 \}$, $B = \{ 2, 3, 6 \}$, then $A \cup B$ is

a) $\{ 1, 2, 3 \}$ b) $\{ 2, 3 \}$

c) $\{ 1, 2, 3, 6 \}$ d) none of these.

ix) If α, β, γ be the roots of $x^3 - 3x^2 + 6x - 2 = 0$, then $\sum \alpha\beta$ is

a) -3 b) 6

c) 2 d) none of these.

x) If $f(x) = 3 + 2x$; when $x \geq 0$

$= -3 - 2x$; when $x < 0$,

then $\lim_{x \to 0} f(x)$ is

a) 3 b) -3

c) 0 d) none of these.

xi) If $f(x) = \frac{x}{|x|}$; when $x \neq 0$

$= 1$; when $x = 0$, then

a) $f(x)$ is continuous at $x = 0$

b) $f(x)$ is continuous, but not differentiable at $x = 0$

c) $f(x)$ is discontinuous at $x = 0$

d) none of these.

xii) The value of $\int_{-1}^{2} |x| dx$ is

a) 3 b) 5

c) 5/2 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If α, β, γ be the roots of the equation, $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma \alpha^3$.

3. If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$.

4. Prove that the set of even integers (including zero) forms an additive group.

5. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$.

6. If $P = \begin{pmatrix} 9 & 1 \\ 4 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 5 \\ 7 & 12 \end{pmatrix}$, find the matrix R so that $5P + 3Q + 2R$ is a null matrix.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions. $3 \times 15 = 45$

7. a) State Rolle's Theorem.

b) Differentiate n times the following equation :

$$\left(1 + x^2\right) y_2 + (2x - 1) y_1 = 0.$$

c) If $y = \sin\left(m \sin^{-1} x\right)$, show that

$$\left(1 - x^2\right) y_{n+2} - (2n + 1) x y_{n+1} + \left(m^2 - n^2\right) y_n = 0.$$ 4 + 5 + 6

8. a) If pth, qth and rth terms of an A.P. are P, Q and R respectively, show that $p(Q - R) + q(R - P) + r(P - Q) = 0$.

b) Show that the centroid of a triangle with vertices $\left(x_1, y_1\right)$, $\left(x_2, y_2\right)$ and $\left(x_3, y_3\right)$ is $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$.

c) Find the equation of a straight line through the point of intersection of lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ and that is perpendicular to the line $6x - 7y + 8 = 0$. 5 + 5 + 5

9. a) Show that $\cos x > 1 - \dfrac{x^2}{2}$ if $0 < x < \dfrac{\pi}{2}$.

b) If $f(x, y) = \begin{cases} \dfrac{x^2 y^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0, \end{cases}$

then show that $f_{xy}(0, 0) = f_{yx}(0, 0)$.

c) Evaluate $\displaystyle\int \frac{x^2}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)} \, dx$. 4 + 6 + 5

10. a) Reduce the following equation to its canonical form and determine the nature of the conic represented by it :

$$6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0.$$

b) Find the equation of the ellipse one of whose foci is $(-1, 1)$, eccentricity is $0 \cdot 5$ and the corresponding directrix is $y = x + 3$. 9 + 6

11. a) Solve the equation by Cardan's method, $2x^3 + 3x^2 + 3x + 1 = 0$.

b) Let $G = \{ a \in R / -1 < a < 1 \}$. Define a binary operation $\otimes$ on G by $a \otimes b = \frac{a+b}{1+ab}$ $\forall$ $a, b \in G$. Show that $(G, \otimes)$ is a group.

c) Find the nature of the roots $x^4 + qx^2 + rx - s = 0$ by Descartes' rule of signs (where q, r, s, being positive). 15

12. a) If by a trnasformation of one rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$, prove that $a^2 + b^2 = a'^2 + b'^2$.

b) Show that $\int_0^\infty \frac{dx}{(x+1)(x+2)} = \log 2$.

c) Use the method of integration to evaluate $\underset{n \to \infty}{Lt} \frac{1^k + 2^k + \ldots + n^k}{n^{k+1}}$; $k > 0$.

15

END

Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-1/BM-101/2009-10

2009

MATHEMATICS

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The value of $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ is

a) 1 b) 4

c) 0 d) 2.

ii) The value of $\int_1^2 dx$ is equal to

a) 1 b) 2

c) 3 d) 0.

iii) If $x = -1$ is a root of the equation $x^2 - x - k = 0$, then the value of k is

a) 1 b) 0

c) $\sqrt{2}$ d) 2.

iv) If α, β, γ be the roots of the equation

$x^3 - 3x^2 + 6x - 2 = 0$, then $\alpha + \beta + \gamma$ is

a) 2 b) 1

c) 3 d) none of these.

v) If $A \{ 1, 2, 3, 4 \}$ and $B = \{ 2, 4, 6 \}$, then $A \Delta B$ is

a) $\{ 1, 2 \}$ b) $\{ 1, 2, 3, 6 \}$

c) $\{ 1, 3, 6 \}$ d) $\{ 6 \}$.

vi) What is the order of the matrix B, if $[\ 3 \ \ 4 \ \ 2\ ]$

$B = [\ 2 \ \ 10 \ \ 3 \ \ 6 \ \ 9\ ]$?

a) 1×5 b) 1×3

c) 3×5 d) 5×3.

vii) The degree of the polynomial

$f(x) = (x^2 + x - 2) / (x - 1)$ is

a) 0 b) 1

c) 2 d) 3.

viii) If $y = \log x^2$, the value of $\frac{d^2y}{dx^2}$ is

a) $\frac{2}{x^3}$ b) $-\frac{2}{x^2}$

c) $\frac{2}{x}$ d) $2x$.

ix) The value of t for which the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{bmatrix}$ is singular, is

a) $-\frac{3}{2}$ b) 2

c) $\frac{3}{2}$ d) -2.

x) $\lim_{x \to 0} (1 + x)^{1/x}$ is equal to

a) 1 b) e

c) ∞ d) 0.

xi) If α, β, γ be the roots of the equation

$x^3 - 3x^2 + 6x - 2 = 0$, then $\sum \alpha\beta$ is

a) 3 b) 6

c) 2 d) none of these.

xii) If $A \{1, 2, 3\}$ and $B = \{2, 3, 6\}$, then $A \cup B$ is

a) $\{1, 2, 3\}$ b) $\{2, 3\}$

c) $\{1, 2, 3, 6\}$ d) none of these.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Evaluate the integral $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} \, dx$.

3. If $u = \log r$ and $r^2 = x^2 + y^2 + z^2$, prove that

$$r^2\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] = 1.$$

4. In a survey of 320 persons, number of persons taking tea is 210, taking milk is 100 and coffee is 70. Number of persons who take tea and milk is 50, milk and coffee is 30, tea and coffee is 50. The number of persons taking all three together is 20. Find the number of people who take neither tea nor coffee nor milk.

5. Express $\begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix}$ as a sum of a symmetric and a skew-symmetric matrix.

6. If α, β, γ be the roots of the equation $x^3 + 2x^2 + 3x + 4 = 0$, then find the equation whose roots are

$$1 + \frac{1}{\alpha},\ 1 + \frac{1}{\beta} \text{ and } 1 + \frac{1}{\gamma}.$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Verify whether the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

b) Solve the following system of linear equations by using Cramer's Rule :

$$2x + 5y + 3z = 9$$

$$3x + y + 2z = 3$$

$$x + 2y - z = 6$$

c) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ 3 & 3 \end{bmatrix}$, find AB.

d) Show that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \sin\theta \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \sin\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos 2\theta & -\sin\theta(\sin\theta + \cos\theta) \\ \sin\theta(\sin\theta + \cos\theta) & 0 \end{bmatrix}$$

2 + 5 + 4 + 4

8. a) Evaluate any *two* :

i) $\lim\limits_{x \to 0} \dfrac{\tan 2x - x}{3x - \sin x}$

ii) $\lim\limits_{x \to 0} \dfrac{x \log \sqrt{1 + x}}{\sin^2 x}$

iii) $\lim\limits_{x \to a} \dfrac{1 - \cos(x - a)}{(x - a)^2}$

b) Evaluate $\displaystyle\int_0^{\pi/2} x^2 \sin x \, dx$.

c) Differentiate $\dfrac{x^3}{(1 + x^3)}$ with respect to x^4.

5 + 5 + 5

9. a) If $A = \{ a, b, c, d, e \}$, $B = \{ c, a, e, g \}$ and $C = \{ b, e, f, g \}$, then show that

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C).$$

b) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then find A^2 and show that $A^2 = A^{-1}$.

c) Find the maxima and minima of $x^3 - 6x^2 + 9x - 8$.

5 + 5 + 5

10. a) Determine whether the function

$$f(x, y) = \frac{xy}{x^2 + y^2} \quad \text{if } (x, y) \neq (0, 0)$$
$$= 0 \quad \text{if } (x, y) = (0, 0)$$

is continuous at the origin.

b) Apply Descartes' rule of signs to find the nature of roots of the equation

$$x^4 + 2x^2 + x - 12 = 0$$

c) State Cauchy's mean value theorem. 5 + 5 + 5

11. a) Find the value of 'a' and 'b' for which the system of equations

$$x + 2y + z = 1$$
$$2x + y + 3z = b$$
$$x + ay + 3z = b + 1$$

has (i) unique solution, (ii) many solutions.

b) Solve the following system of equations by matrix inversion method

$$x + y + z = 6$$

$$x - 2y + z = 0$$

$$2x - y + z = 3$$

c) Find out the rank of the matrix $\begin{bmatrix} 2 & -4 & 6 \\ 2 & 3 & -1 \\ 3 & 1 & 2 \end{bmatrix}$.

5 + 5 + 5

12. a) If $u = \cos^{-1}\left\{ (x + y) / \sqrt{x} + \sqrt{y} \right\}$, then show that

$$x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$

b) If PSQ be a focal chord of a conic with focus S and semi latus rectum L, then prove that

$$1/SP + 2/SQ = 2/L.$$

c) Find the point on the conic $6/r = 1 + 4\cos\theta$ whose vertical angle is $\pi/3$. 8 + 4 + 3

Name : ....
Roll No. : ....
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CS/BCA/SEM-1/BM-101/2010-11

2010-11

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) $\lim_{x \to 0} (1+x)^{1/x} = ?$

a) 1

b) 0

c) $\frac{2}{3}$

d) *e*.

ii) If α, β, χ be the roots of the equation $x + yn = 2$ then $\Sigma x^2 =$

a) 0 b) 14

c) – 14 d) 4.

iii) An element x in a ring R is zero divisor if

a) $x \cdot b = 0$

b) $x \cdot b = 0$, for some non zero element b in R

c) $x \cdot b \neq 0$, for all element b in R

d) none of these.

iv) The value of $\int_{-1}^{2} |x| dx$ is

a) 3

b) 5

c) $\frac{5}{2}$

d) 0.

v) The value of $\frac{d}{dx}(\log_e x)$ is equals to

a) $\frac{1}{x}$

b) $\log\left(\frac{1}{x}\right)$

c) $\left(\frac{1}{n}\right)\log_a e$

d) $a \log e$.

vi) If $A = \{2, 4, 6\}$ and $B = \{1, 3, 5, 7\}$, then $A \cup B$ is

a) $\{0\}$

b) $\{1, 2, 3, 4, 5, 6, 7\}$

c) $\{1, 2, 4, 5, 6, 7\}$

d) $\{0, 2\}$.

vii) If A is a square matrix then

a) $A + A^T$ is symmetric

b) $A + A^T$ is skew symmetric

c) $A - A^T$ is symmetric

d) $A - A^T$ is skew symmetric.

viii) The matrix $A = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ is on

a) orthogonal matrix b) idempotent matrix

c) identity matrix d) none of these.

ix) If $y = 2$ at and $x = at^2$, then $\frac{dy}{dx}$ at $t = 1$ is

a) 1 b) $2a$

c) -1 d) $2a^2$.

x) The polar form of the equation $x^2 + y^2 - 8y = 0$ is

a) $r = 8 \cos \theta$ b) $r = 8 \sin \theta$

c) $r^2 = 8 \cos \theta$ d) none of these.

xi) If $A = \{ 1, 2, 3, 4, 8 \}$, $B = \{ 2, 4, 6, 7 \}$ then $A \Delta B$ is

a) $\{ 2, 4 \}$

b) $\{ 1, 2, 3, 4, 6, 7, 8 \}$

c) ϕ

d) $\{ 1, 3, 6, 7, 8 \}$.

xii) The diagonal elements of a real skew-symmetric matrix are

a) 1 b) -1

c) 2 d) 0.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. A function $f(x)$ is defined as follows

$$f(x) = x^2 \quad \text{when } 0 < x < 1$$
$$= x \quad \text{when } 1 \le x < 2$$
$$= 2 - x \quad \text{when } 2 \le x < 3$$

Show that the $f(x)$ is continuous at $x = 2$.

3. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$.

4. If α, β, γ be the roots of the cubic $x^3 + px + q = 0$, then find the equation whose roots are

$\frac{\beta+\gamma}{\alpha^2}, \frac{\gamma+\alpha}{\beta^2}, \frac{\alpha+\beta}{\gamma^2}$.

5. Prove that the ring of matrices of the form $\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$ of real number is a field.

6. In a survey concerning the smoking habits of consumers it was found that 55% smoke cigarette-A, 50% smoke cigarette-B, 42% smoke cigarette-C, 28% smoke cigarette-A & B, 20% smoke cigarette-A' & C, 12% smoke cigarette-B & C and 10% smoke all the three cigarette. What percentage do not smoke ?

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) If $y = \sin\left(m \sin^{-1} x\right)$, then show that $\left(1-x^2\right) y_{n+2} - (2n+1) x y_{n+1} + \left(m^2 - n^2\right) y_n = 0$.

   b) If α, β, γ are the 3 roots of $x^3 + px^2 + qx + r = 0$ obtain the value of $\sum(\alpha - \beta)^2$.

   c) Evaluate $\int \frac{1}{x^2} e^{1/x}\, dx$.

8. a) If $u = \frac{y}{z} + \frac{z}{x} + \frac{z}{y}$ then prove that, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

   b) If by a rotation of rectangular co-ordinate axes without change of origin expressions $ax + by$ and $cx + dy$ are transformed into $a' x' + b' y'$ and $c' x' + d' y'$. Show that $a' d' - b' c' = ad - bc$.

   c) Reduce the following equation to its canonical form and determine the nature of the conic represented by it :

   $$3x^2 - 8xy - 3y^2 + 10x - 13y + 18 = 0$$

9. a) Evaluate

$$\lim_{n \to \infty} \left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \ldots\ldots + \frac{n}{n^2 + n^2} \right].$$

b) Using mean value theorem prove the following inequality :

$$x \langle \sin^{-1} x \langle \frac{x}{\sqrt{1 - x^2}}, \text{ if } 0 < x < 1$$

c) Expand $\sin x$ in power of x in infinite series.

10. a) Solve the equation by Cardan's method :

$$2x^3 + 3x^2 + 3x + 1$$

b) Evaluate

$$\int \frac{x^2 \, dx}{(x^2 + a^2)(x^2 + b^2)}$$

c) If $y = x^{x-1} \log x$, show that $y_x = \frac{(x-1)!}{x}$.

11. a) Prove that $|A \cup B| = |A| + |B| - |A \cap B|$ where A and B are two non-empty sets.

b) If $A = \{ a, b, c, d \}$ $B = \{ b, c, p, q \}$, then find out $A \times B$, $B \times A$ and $A \Delta B$.

c) Define power set. Find the power set of $\{ a, b, c \}$.

Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-1/BM-101/2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

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Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The degree of the polynomial $(x^2 + x - 2) / (x - 1)$ is

a) 0 b) 1

c) 2 d) 3.

ii) If G be a group and $a, b \in G$. Then $\left(a^{-1} b \right)^{-1}$ is equal to

a) ab^{-1} b) $b^{-1}a$

c) $a^{-1} b^{-1}$ d) $b^{-1}a^{-1}$.

iii) $\frac{\partial}{\partial x}\left(x^{y}\right) =$

a) 1 b) yx^{y}

c) $x^{y} \log x$ d) yx^{y-1}.

iv) If $P = \{2, 4, 6, 7, 8, 9\}$, $Q = \{1, 2, 6, 9\}$ then $P \cap Q$ is

a) $\{1, 2, 6\}$ b) $\{2, 6, 9\}$

c) $\{1, 6, 9\}$ d) $\{4, 6, 9\}$.

v) The value of $\underset{x \to 3}{Lt} \frac{x^{3} - 3^{3}}{x - 3}$ is

a) -12 b) 12

c) 27 d) -27.

vi) If A be a matrix whose inverse exists then which of the following is not true ?

a) $\left(A^{T}\right)^{-1} = \left(A^{-1}\right)^{T}$

b) $A^{-1} = (\det(A))^{-1}$

c) $\left(A^{2}\right)^{-1} = \left(A^{-1}\right)^{2}$

d) none of these.

vii) The equation $x^4 + 2x^2 - 7x - 5 = 0$ has

a) one real roots and three complex roots

b) one complex roots and three real roots

c) two real roots and two complex roots

d) four real roots.

viii) Cardan's method is used for solving equation of degree

a) 2 b) 3

c) 4 d) none of these.

ix) If α, β, γ be the roots of $x^3 - 3x^2 + 6x - 2 = 0$, then $\sum \alpha\beta$ is

a) 3 b) 6

c) 2 d) none of these.

x) $f(x, y) = \sqrt{x} + \sqrt{y}$ is a function of degree

a) $\frac{1}{2}$ b) $\frac{1}{3}$

c) 0 d) $\frac{1}{4}$.

xi) The equation $r = 3 \sin \theta + 4 \cos \theta$ represents

a) a parabola b) an ellipse

c) a straight line d) a circle.

xii) The inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ is

a) $\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ -\frac{3}{2} & 3 \end{bmatrix}$

c) $\begin{bmatrix} -2 & 4 \\ -3 & 6 \end{bmatrix}$ d) does not exist.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that the set of real numbers of the form $a + b\sqrt{2}$ where a and b are rational numbers, forms a field under addition and multiplication.

3. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$, two of whose roots are in the ratio 3 : 2.

4. Prove that, any square matrix can be expressed assume of a symmetric matrix and a skew-symmetric matrix.

5. If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u.$$

6. A function $f(x)$ is defined as follows

$$f(x) = 1 + x \text{ when } x \le 2,$$
$$= 5 - x \text{ when } x > 2.$$

Show that $f(x)$ is continuous at $x = 2$ but $f'(2)$ does not exist.

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) State Descart's rule of sign. Using this rule find the nature of the roots of the equation

$$x^4 - 7x^3 + 21x^2 - 9x + 21 = 0.$$

b) Solve the following system of linear equations by Cramer's rule

$$x - y + 2z = 1$$
$$x + y + z = 2$$
$$2x - y + z = 5.$$

c) If by a transformation of one rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$, Prove that $a^2 + b^2 = a'^2 + b'^2$.

8. a) Show that the equation $20x^2 + 15xy + 9x + 3y + 1 = 0$ represents a pair of intersecting straight lines which are equidistant from the origin.

b) Show that $\cos x > 1 - \frac{x^2}{2}$ if $0 < x < \frac{\pi}{2}$.

c) If α, β, γ be the roots of the equation

$x^3 - px^2 + qx - r = 0$, then find the value of $\sum \frac{1}{\alpha}$.

9. a) If $A = \{ a, b, c, d, e \}$, $B = \{ c, a, e, g \}$ and $C = \{ b, e, f, g \}$,

then show that $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.

b) Reduce the following equation to the canonical form and determine the nature of the conic represented by it

$x^2 - 4xy + 4y^2 - 12x - 6y - 39 = 0$.

c) Evaluate $\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$.

10. a) Evaluate $\int \frac{dx}{(1+x)\sqrt{1-x^2}}$.

b) If PSQ be a focal chord of a conic with focus S and semi-latus rectum l, then prove that $\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{l}$.

c) If $A - 2B = \begin{bmatrix} 0 & 6 & 26 \\ 6 & -9 & 12 \\ 2 & 9 & -10 \end{bmatrix}$ and

$2A + B = \begin{bmatrix} 10 & -3 & 4 \\ 12 & -3 & 4 \\ 4 & 3 & 0 \end{bmatrix}$, find A and B.

11. a) If G be a group such that $(ab)^2 = a^2b^2 \ \forall \ a, b \in G$, show that the group G is abelian.

b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\log 2$.

c) If $y = e^{-x}\sin x$, then show that $y_4 + 4y = 0$.

Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-1/BM-101/2012-13

2012

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The value of $\lim\limits_{x \to 0} \dfrac{x^2 - 4}{x - 2}$ is

a) 1 b) 4

c) 0 d) none of these.

ii) If $\phi(x, y) = 0$ then $\dfrac{dy}{dx}$ =

a) $\dfrac{\phi_x}{\phi_y}$ b) $\dfrac{\phi_y}{\phi_x}$

c) ϕ_y d) none of these.

iii) The value of y_n, when $y = e^{-x}$ is

a) e^{-x} b) $(-1)^n$

c) $(-1)^n e^{-x}$ d) none of these.

iv) If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $A^2 =$

a) A b) 3A

c) unit matrix d) 2A.

v) The diagonal of real skew-symmetric matrix is

a) 1 b) – 1

c) 0 d) 2.

vi) If in a group (G, o), $x \text{ o } x = x$, then

a) x = null element

b) this relation is not valid

c) $x = e$

d) $x \neq x^{-1}$.

vii) The value of $\int(\cos^2 x - \sin^2 x)\, dx$ is

a) $\sec x \tan x$ b) $\sin 2x$

c) $\frac{1}{2}\sin 2x$ d) $-\frac{1}{2}\sin 2x$.

viii) The polar equation $r = 4 \sin\theta$ represents a

a) circle b) ellipse

c) straight line d) none of these.

ix) If a, b and c are roots of
$x^3 - 3x + 9 = 0$ then $a^2 + b^2 + c^2$ is

a) 6 b) – 6

c) 0 d) 1.

x) Solution of the equation $x^3 + 2x + 3 = 0$ will give

a) no real positive roots but one real negative root

b) two real positive roots and one real negative root

c) one real positive root and two imaginary roots

d) two real negative roots and only one imaginary root.

xi) $\int_{-1}^{2} |x| dx$ is

a) 3 b) 5

c) 5/2 d) 3/2.

xii) Which of the following does not satisfy Rolles theorem in $[-2, 2]$?

a) x b) $1/x$

c) $1/(x-5)$ d) $x^2 - 5$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Out of 440 students, 112 students read German, 120 students read French and 168 read Spanish. Of these 32 read French and Spanish, 40 read German and Spanish, 20 read German and French, while 12 read all the three subjects. How many students

a) do not read any of the three languages

b) read just one language ?

3. Evalute $\int_{0}^{\pi} \log \tan x \, dx$.

4. If $f(x, y) = \begin{cases} \dfrac{x^2 - xy}{x + y}, (x, y) \neq (0,0) \\ 0, (x, y) = (0,0) \end{cases}$,

what is the value of $f_x(0,0)$, $f_y(0,0)$

5. Obtain a relation between p, q and r so that $x^3 + px^2 + qx + r = 0$ has 3 roots that are in A.P.

6. Evaluate $\lim_{x \to 0} \dfrac{\operatorname{cosec} x - \cot x}{x}$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that $\{ 1, \omega, \omega^2 \}$, where $\omega^3 = 1$ forms a commutative group in respect of multiplication.

b) If $y = \sin^{-1} x$, then prove that
$(1 - x^2)\, y_{n+2} - (2n + 1)\, x\, y_{n+1} - n^2\, y_n = 0$

c) In the mean value theorem $f(x + h) = f(x) + h f^{/}(x + \theta h)$, if $f(x) = px^2 + qx + r\ (p \neq 0)$, then show that $\theta = ½$.

8. a) Reduce the equation $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$ into canonical form and hence determine the nature of the conic.

b) Find the nature of the conic $\frac{8}{r} = 4 - 5\cos\theta$.

c) Expand e^x in ascending powers of x by Taylor's series.

9. a) Solve using Carden's method : $x^3 - 9x + 28 = 0$.

b) If by a transformation of motion of co-ordinate axes, the expression $ax^2 + 2hxy + by^2$ changes into $a'x'^2 + 2h'x'y' + b'y'^2$, then show that $ab - h^2 = a'b' - h'^2$.

8 + 7

10. a) Solve the equations by matrix inversion method :

$x + y + z = 4$

$2x - y + 3z = 1$

$3x + 2y - z = 1$

b) If $u = \tan^{-1} \frac{x^2 + y^2}{x + y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$.

c) Evaluate : $\int \frac{x - 1}{(x - 2)(x - 3)}\,dx$

11. a) Give the definition of a ring with two binary composition. Let H be the set of all matrices $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = 1 \right\}$. Prove that H forms a non-commutative group.

b) Apply Descerte's rule of sign to find the nature of the roots of the given equation : $x^4 + qx^2 + rx - s = 0$ (where q, r, s being positive).

c) Evaluate : $\lim_{n \to \infty} \left[\frac{1}{n^2 + 1^2} + \frac{1}{n^2 + 2^2} + \ldots\ldots + \frac{1}{n^2 + n^2} \right]$

Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-1/BM-101/2013-14

2013

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) If $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$, then the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if

      a) $\Delta > 0$ b) $\Delta < 0$

      c) $\Delta = 0$ d) none of these.

ii) If the matrix $\begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{pmatrix}$ is singular then the value of λ is

a) 0 b) 4

c) 2 d) -1.

iii) If A be a matrix whose inverse exists then which of the following is not true ?

a) $(A^T)^{-1} = (A^{-1})^T$ b) $A^{-1} = (\det A)^{-1}$

c) $(A^2)^{-1} = (A^{-1})^2$ d) None of these.

iv) $\frac{\partial}{\partial x}(e^{xy}) =$

a) e^{xy} b) xe^{xy}

c) ye^{xy} d) none of these.

v) The degree of the function $f(x,y) = \tan^{-1}\frac{y}{x}$ is

a) 1 b) 0

c) 2 d) none of these.

vi) The inverse of the matrix $\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$ is

a) $\frac{1}{3}\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$

c) $\frac{1}{3}\begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$

d) none of these.

vii) The value of $\int \frac{dx}{x \log x}$ is

a) $\log |x| + c$

b) $\log |\log x| + c$

c) $x \log |x| + c$

d) none of these.

viii) If α, β and γ be the roots of the equation $x^3 + 7x - 2 = 0$ then $\sum \alpha^2 =$

a) 0

b) 14

c) −14

d) 4.

ix) Which of the following is a null set ?

a) $A = \{0\}$

b) $A = \{\phi\}$

c) $A = \{x : x \text{ is an integer } \& \ 1 < x < 2\}$

d) None of these.

x) The value of $\lim_{x \to 0} \frac{\sin x}{x}$ (where x is radian) is

a) 1 b) 0

c) ∞ d) -1.

xi) The conic $\frac{l}{r} = 1 - e\cos\theta$ represents a parabola if

a) $e = 1$ b) $e > 1$

c) $e < 1$ d) none of these.

xii) What is the value of the following limit ?

$$\lim_{x \to 0} (1+x)^{1/x}$$

a) 1 b) e

c) 0 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Evaluate the integral $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$.

3. Express $\begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

4. If $u = \tan^{-1} \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{4}\sin 2u$.

5. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ two of whose roots are in the ratio 3 : 2.

6. Prove that the set of real numbers of the form $a + b\sqrt{2}$ where a and b are rational numbers, forms a field under addition and multiplication.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) State Decartes' rule of sign. Using this rule find the nature of the root of the equation $x^4 - 7x^3 + 21x^2 - 9x + 21 = 0$. 5

   b) Solve the following system of linear equations by Cramer's rule : 5

   $$x - y + 2z = 1, \quad x + y + z = 2, \quad 2x - y + z = 5.$$

   c) If by a transformation of rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$, prove that $a^2 + b^2 = a'^2 + b'^2$. 5

8. a) If G be a group such that $(ab)^2 = a^2b^2 \; \forall a, b \in G$, show that the group G is Abelian. 5

b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8}\log 2$. 5

c) If $y = e^{-x}\sin x$, then show that $y_4 + 4y = 0$. 5

9. a) Show that the matrix $A = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ is orthogonal and hence find A^{-1}. 4

b) If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, then show that $A^2 - 2A + I_2 = O_2$. Hence obtain A^{-1} and also find A^{100}. 4

c) Reduce the following equation to the canonical form and determine the nature of the conic represented by it : $8x^2 - 12xy + 17y^2 + 16x - 12y + 3 = 0$. 7

10. a) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method. 6

b) Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$. 4

c) If α, β, γ are the three roots of $x^3 + px^2 + qx + r = 0$, obtain the value of $\sum (\alpha - \beta)^2$. 5

11. a) State Rolle's theorem. Examine whether Rolle's theorem is applicable or not for the function $f(x) = 1 - |x - 1|, \forall x \in [0,2]$.

b) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

c) Find for what values of x, the following expression is maximum and minimum respectively :

$2x^3 - 21x^2 + 36x - 20$ 5 + 5 + 5

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BM-101

MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10\times 1 = 10$

(i) An element x in a ring R is a zero divisor if

(A) $x\cdot b = 0$

(B) $x\cdot b = 0$, for some non zero element in R

(C) $x\cdot b \neq 0$, for all elements b in R

(D) none of these

(ii) If A = {2, 4, 6} and B = {1, 3, 5, 7} then $A \cup B$ is

(A) {0} (B) {1, 2, 3, 4, 5, 6, 7}

(C) {1, 2, 4, 5, 6, 7} (D) {0, 2}

(iii) The polar form of the equation $x^2 + y^2 - 8y = 0$ is

(A) $r = 8\cos\theta$ (B) $r = 8\sin\theta$ (C) $r^2 = 8\cos\theta$ (D) none of these

(iv) The diagonal elements of a real skew symmetric matrix are

(A) 1 (B) –1 (C) 2 (D) 0

(v) If α, β, γ are the roots of the equation $x^3 - 3x^2 + 6x - 2 = 0$ then , $\alpha + \beta + \gamma$ is

(A) 0 (B) 1 (C) 3 (D) –2

(vi) The value of t for which the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{bmatrix}$ is singular is

(A) $\frac{-3}{2}$ (B) $\frac{3}{2}$ (C) 2 (D) –2

(vii) The function $f(x) = |x|$ then

(A) continuous and differentiable at $x = 0$

(B) continuous everywhere but differentiable at $x = 0$

(C) discontinuous and not differentiable at $x = 0$

(D) none of these

(viii) Which of the following function obeys Rolle's theorem in $[0,\pi]$

(A) x (B) $\sin x$ (C) $\cos x$ (D) $\tan x$

(ix) By 3rd order Maclaurin's theorem we have $\sin x = f(x) - \frac{x^3}{6}\cos\theta x$, then $f(x)$ equal to

(A) x^2 (B) $-x^2$ (C) x (D) $-x$

(x) If $f(x,y) = x^2y$ then df equal to

(A) $2x^2\,dx + dy$ (B) $x - 2\,dy$ (C) $x + dy$ (D) $2xy\,dx + x^2\,dy$

(xi) $f(x, y) = |x| + |y|$ then $f_x(0, 0)$ equal to

(A) 0 (B) 1 (C) does not exist (D) none of these

(xii) The value of $\int_1^2 \frac{e^{\log x}}{x} dx$

(A) 1 (B) – 1 (C) 2 (D) 0

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3 \times 5 = 15$

2. If α, β are the roots of the equation $x^2 - px + q = 0$ then find the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

3. Solve the following equations by matrix inversion method
$x + y + z = 2,\ x - y + 2z = 6,\ 3x + 5y + 7z = 14$

4. Give the definition of commutative group and show that $\{1, \omega, \omega^2\}$ where $\omega^3 = 1$ forms a commutative group w.r.t. multiplication.

5. If $y = \cos(m \sin^{-1}x)$ then prove that $(1 - x^2)y_{n+2} - (2n + 1)\, xy_{n+1} + (m^2 - n^2)y_n = 0$

6. If $u = \tan^{-1}\frac{x^2 - y^2}{x - y}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$.

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

7. (a) In a class of 50 students, 15 read Physics, 20 read Chemistry and 20 read Mathematics, 3 read Physics and Chemistry, 6 read Chemistry and Mathematics and 5 read Physics and Mathematics, 7 read none of the subjects. How many students read all the subjects? 7

   (b) Discuss the nature of the conic represented by $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$ by reducing to its canonical form. 8

8. (a) Apply Descarte's rule of sign to show that the equation $x^4 + 2x^2 - 7x - 5 = 0$ has two real roots and two non real roots. 5

   (b) Verify Rolle's theorem for the function $f(x) = |x|, -1 \leq x \leq 1$ 5

(c) Discuss the continuity of the following function $f(x) = x - [x]$, where $[x]$ denotes the greatest integer not greater than x. 5

9. (a) Using the mean value theorem prove the following inequalities $x < \sin^{-1} x < \dfrac{x}{\sqrt{1 - x^2}}$ if $0 < x < 1$ 6

(b) Show that $z = \log\{(x-a)^2 + (y-b)^2\}$ satisfies the relation $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$ except at (a, b) 6

(c) Evaluate $\displaystyle\int \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$ 3

10. (a) Solve $x^3 - 9x + 28 = 0$ using Cardan's method. 6

(b) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$ 4

(c) Find the nature of the conic $\dfrac{8}{r} = 4 - 5\cos\theta$ 5

11. (a) Find $\dfrac{dy}{dx}$ when $x = y\log(xy)$ 5

(b) Give the definition of a ring with two binary composition. Let H be the set of all matrices $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = 1\right\}$ 5

Prove that H forms a non-commutative group with respect to matrix multiplication.

(c) If by a transformation of motion of co-ordinate axis, the expression $ax^2 + 2hxy + by^2$ changes into $a'x'^2 + 2h'x'y' + b'y'^2$ then show that $a + b = a' + b'$ 5

MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BM-101

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The value of $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ is

a) 1 b) 4

c) 0 d) 2.

ii) The value of $\int_0^1 x^3 \, dx$ is

a) $\frac{3}{4}$ b) 3

c) $\frac{1}{4}$ d) 1.

iii) $\frac{\partial}{\partial x}(x^y) =$

a) 1 b) yx^y

c) $x^y \log x$ d) yx^{y-1}.

iv) $A = \{2, 4, 6\}$, $B = \{1, 3, 5, 7\}$ then $A \cup B$ is

a) $\{0\}$ b) $\{1, 2, 3, 4, 5, 6, 7\}$

c) $\{1, 2, 4, 5, 6, 7\}$ d) $\{0, 2\}$.

v) The value of $\lim_{x \to 0} (1 + x)^{1/x}$ is

a) 1 b) e

c) ∞ d) 0.

vi) The inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ is

a) $\begin{bmatrix} 2 & -3 \\ 4 & 6 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 \\ -\frac{3}{2} & 3 \end{bmatrix}$

c) $\begin{bmatrix} -2 & 4 \\ -3 & 6 \end{bmatrix}$

d) Does not exit.

vii) If α, β, γ be the roots of the equation

$x^3 - 3x^2 + 6x - 2 = 0$, then $\sum \alpha \beta$ is

a) 3 b) 6

c) 2 d) none of these.

viii) The conic $\frac{l}{r} = 1 - e \cos \theta$ represents a parabola if

a) $e = 1$ b) $e > 1$

c) $e < 1$ d) none of these.

ix) If $x = at^2$, $y = 2at$, then $\frac{dy}{dx}$ at $t = 1$ is

a) 1 b) $2a$

c) -1 d) $2a^2$.

x) The degree of the polynomial $f(x) = x^2 + x - 2$ is

a) 0 b) 1

c) 2 d) 3.

xi) If $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$ then the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if

a) $\Delta > 0$ b) $\Delta < 0$

c) $\Delta = 0$ d) none of these.

xii) The polar form of the equation $x^2 + y^2 - 8y = 0$ is

a) $r = 8 \cos \theta$ b) $r = 8 \sin \theta$

c) $r^2 = 8 \cos \theta$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Express $\begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

3. Evaluate the integral $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$.

4. If $u = \cos^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then show that

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + \dfrac{1}{2}\cot u = 0.$

5. If α, β, γ be the roots of the equation $x^3 + px + q = 0$ then find the equation whose roots are $\dfrac{\beta+\gamma}{\alpha^2}, \dfrac{\gamma+\alpha}{\beta^2}, \dfrac{\alpha+\beta}{\gamma^2}$.

6. Prove that $G = \{1, -1, i, -i\}$ forms a commutative group under multiplication, where ω be the cube root of unity.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that the matrix $A = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ is orthogonal and hence find A^{-1}. 5

b) If $y = \sin(m \sin^{-1} x)$ then show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2 y = 0.$$ 5

c) Using mean value theorem prove that

$$\frac{x}{1+x} < \log(1+x) < x \text{ if } x > 0.$$ 5

8. a) Solve the following equations by matrix method : 5

$$x + y + z = 4$$

$$2x - y - 3z = 1$$

$$3x + 2y - z = 1$$

b) Solve $x^3 - 9x + 28 = 0$ using Carden's method. 5

c) Evaluate

$$\lim_{n \to \infty}\left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \ldots\ldots + \frac{n}{n^2+n^2}\right].$$ 5

9. a) State Descartes' rule of sign. Using this rule find the nature of the roots of the equation

$$x^4 - 7x^3 + 21x^2 - 9x + 21 = 0.$$ 7

b) Reduce the following equation is the canonical form and determine the nature of conic represented by it :

$$8x^2 - 12xy + 17y^2 + 16x - 12y + 3 = 0.$$ 8

10. a) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0.$$ 5

b) A function $f(x)$ is defined as follows :

$$\begin{aligned} f(x) &= -x^2 && \text{when } x \le 0 \\ &= 5x^2 && \text{when } 0 < x < 1 \\ &= 4 + x^2 && \text{when } x \ge 1. \end{aligned}$$

Show that $f(x)$ is continuous at $x = 0$ and $x = 1$.

5

c) If by a transformation of one rectangular axes to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$, prove that

$$a^2 + b^2 = a'^2 + b'^2.$$ 5

11. a) Find $\frac{dy}{dx}$ when $x = y \log(xy)$. 5

b) Find for what values of x, the following expression is maximum and minimum respectively :

$$2x^3 - 21x^2 + 36x - 20.$$ 5

c) Show that the set of rational numbers other than 1, Q' forms a group under the binary operation * defined by $a * b = a + b - ab : a, b \in Q$. 5

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Paper Code : BMN-101

BASIC MATHEMATICAL COMPUTATION

Time Allotted: 3 Hours *Full Marks*: 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group – A

(Multiple Choice Type Questions)

1. Choose the correct alternative for *any ten* of the following: 1×10=10

(i) The number of permutations that can be made out of the letters "COTTON" is

(a) 720 (b) 180
(c) 120 (d) 30

(ii) The value of $\int \frac{log x}{x^2} dx$ is

(a) $\log(x+1)+c$ (b) $-\frac{1}{x}\log(x+1)+c$
(c) $\log(x-1)+c$ (d) $\frac{1}{2}\log(x+1)+c$

(iii) The function $f(x)=|x|$ is

(a) continuous and differentiable at $x=0$
(b) continuous everywhere but differentiable at $x=0$
(c) discontinuous and not differentiable at $x=0$
(d) None of the above

(iv) $f(x,y) = |x| + |y|$ then $f_x(0,0)$ equal to

(a) 1 (b) 0
(c) does not exist (d) None of these

(v) The value of y_3, when $y = \left(\frac{x}{2}+1\right)^8$ is

(a) $42\left(\frac{x}{2}+1\right)^3$

(b) $336\left(\frac{x}{2}+1\right)^3$

(c) $42\left(\frac{x}{2}+1\right)^5$

(d) $336\left(\frac{x}{2}+1\right)^5$

(vi) The equation of the straight line passing through the point (4,3) and making intercepts on the coordinate axes whose sum is –1

(a) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

(b) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

(c) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

(d) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

(vii) If $y = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$, then the value of $\frac{dy}{dx}$ at $x=-1$ is

(a) 1

(b) 0

(c) e

(d) $1/e$

(viii) If A be a matrix whose inverse exists then which of the following is not true?

(a) $(A^T)^{-1} = \left(A^{-1}\right)^T$

(b) $(A^2)^{-1} = \left(A^{-1}\right)^2$

(c) $A^{-1} = (detA)^{-1}$

(d) None of these

(ix) If $y = 2\,at$ and $x = at^2$ then $\frac{dy}{dx}$ at t=2 is

(a) 1

(b) 2

(c) $2a^2$

(d) $1/2$

(x) Which of the following does not satisfy Rolle's theorem in [–2,2]?

(a) x

(b) $\frac{1}{x'}$

(c) $\frac{1}{x-5'}$

(d) x^2-5

(xi) The value of $\lim_{x\to 2}[x]$ is

(a) 2

(b) 1

(c) 3

(d) Does not exist

(xii) The angle between the lines $y=x-3$ and $y=\left(2-\sqrt{3}\right)x$ is

(a) 30°

(b) 45°

(c) 60°

(d) 90°

Group – B

(Short Answer Type Questions)

Answer *any three* of the following. 5×3=15

2. How many license plates can be formed involving 3 English letters and 4 digits, if the letters must appear either in the beginning or in the end?

3. If $y = cos\left(m \sin^{-1} x\right)$ then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n = 0$.

4. If $u=\tan^{-1}\frac{x^2-y^2}{x-y}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin 2u$.

5. Solve the following system of linear equations by Cramer's Rule or Matrix Inversion method

$2x + 5y + 3z = 5$

$3x + y + 2z = 5$

$x + 2y - z = 0$

6. Verify Rolle's theorem for the function $f(x) = |x-2|, 0 \le x \le 4$.

Group – C

(Long Answer Type Questions)

Answer *any three* of the following. 15×3=45

7. (a) Find the 3rd term from end in the expansion of $\left(x^2+\frac{1}{2x}\right)^{13}$

(b) Find A^{-1} where $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 2 & 1 \\ -2 & 1 & 5 \end{bmatrix}$

(c) Evaluate $\int \frac{x^5\, dx}{x^2+1}$

8. (a) If $\lim_{x\to 0}\frac{ae^x-b}{x}=2$ then find the value of a, b.

(b) Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(c) Find the area bounded by $y = 2 - x^2$ and $x + y = 0$

9. (a) Expand $\log(1+2x)$ using Maclaurin's series.

(b) The parabola $y^2 = 2ax$ passes through the center of the circle $4x^2 + 4y^2 - 8x + 12y - 7 = 0$. Find the focus and length of the latus-rectum of the parabola.

(c) If $y = \frac{3x-1}{(x+3)(x-1)}$, find $\frac{dy}{dx}$.

10. (a) A straight line passes through the point (2,3) and the sum of its intercepts on X-axis and Y-axis is 10. Find the equation of the straight line.

(b) A function $f(x)$ is defined as follows:

$$f(x) = x + 1, \text{when } x \geq 1$$
$$= \frac{3}{2}, \text{when } x = 1$$
$$= x, \text{when } x < 1$$

Draw the graph of $f(x)$ and examine the continuity of $f(x)$ at $x = \frac{1}{2}$.

(c) Find the equation of the circle whose center $\left(\frac{5}{3}, -3\right)$ is and which touches the line $3x + 2y + 5 = 0$.

11. (a) Evaluate $\int_{-2}^{2} \left|1 - x^2\right| dx$

(b) Using MVT prove that $1 + \frac{x}{2\sqrt{1+x}} < \sqrt{1+x} < 1 + \frac{x}{2}$

(c) Evaluate $\lim_{x \to \infty} \left[\frac{1}{2n+1} + \frac{1}{2n+2} + \dots \dots \frac{1}{2n+n}\right]$