H: Derivation

we have

$$a_{ij} = \left[\sum_{k=1}^{n} l_{ik} u_{kj} \right]$$

if $i > j$,

$$\cancel{a_{ij} = \sum_{k=1}^{j} l_{ik} u_{kj} = l_{ij} u_{jj}}$$

$$a_{ij} = \sum_{k=1}^{j} l_{ik} u_{kj} = \sum_{k=1}^{j-1} l_{ik} u_{ij} + l_{ij} u_{jj}$$

if $i < j$

$$a_{ij} = \sum_{k=1}^{i} l_{ik} u_{kj} = \sum_{k=1}^{i-1} l_{ik} u_{kj} + l_{ii} u_{ij}$$

then $u_{jj} = 1$

$$\Rightarrow l_{ij} = a_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj}$$

$$\boxed{u_{ij} = \frac{1}{l_{ii}} \left(a_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj} \right)}$$

For Cholesky

$$l_{ij} = \frac{1}{l_{jj}} \left(a_{ij} - \sum_{k=1}^{j-1} l_{ik} l_{kj} \right)$$

$i < j$

$$l_{ij} = \frac{1}{l_{ii}}\left(a_{ij} - \sum_{k=1}^{i-1} l_{ik} l_{kj}\right)$$

$i < j$

$$l_{ij} = \frac{1}{l_{ii}}\left(a_{ij} - \sum_{k=1}^{i-1} l_{ik} l_{kj}\right)$$

if $i = j$

$$a_{ii} = \sum_{k=1}^{i} l_{ik} u_{ii} = \sum_{k=1}^{i-1} l_{ik}^2 + l_{ii}^2$$

$$l_{ii} = \pm\sqrt{a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2}$$