Quiz Assignment 3
Solutions

Question 1

Both sets, a: $\{|0,0\rangle, |1,1\rangle\}$ and b: $\{|+,+\rangle, |-,-\rangle\}$ are not bases as two qubits cannot span a 4 dimensional space.

Therefore, the answer is **D**.

Question 2

Both sets, a: $\{|0,+\rangle, |1,-\rangle\}$ and b: $\{|0,\mathrm{i}\rangle, |1,-\mathrm{i}\rangle\}$ are not bases as two qubits cannot span a 4 dimensional space.

Therefore, the answer is **D**.

Question 3

$(X \otimes I) \circ (I \otimes X)$ is multiplied as follows:

$$\begin{aligned}(X \otimes I) \circ (I \otimes X) &= (X \circ I) \otimes (I \circ X) \\ &= X \otimes X\end{aligned}$$

The statement in the question is therefore **TRUE**.

Question 4

The adjoint of an operator $A \otimes B$ is given by $(A \otimes B)^\dagger = A^\dagger \otimes B^\dagger$. The adjoint of $H \otimes H$ is:

$$\begin{aligned}(H \otimes H)^\dagger &= H^\dagger \otimes H^\dagger \\ &= H \otimes H\end{aligned}$$

The statement in the question is therefore **TRUE**, this can alternatively be verified from the matrix form given in the slides.

Question 5

As illustrated in the programming component of the lecture, entangled states cannot be represented on even two or more Bloch spheres.

The statement is therefore **FALSE**.

Question 6

As shown in the lecture, the answer to this question is option **B**, six real numbers.

Question 7

When $CNOT_2^1 \circ (I \otimes H)$ is applied to $|00\rangle$, the resultant state can be evaluated using the actions:

$$\begin{aligned}
CNOT_2^1 \circ (I \otimes H)\,|00\rangle &= CNOT_2^1\,|0+\rangle \\
CNOT_2^1\,|0+\rangle &= CNOT_2^1 \frac{1}{\sqrt{2}}\,(|00\rangle + |01\rangle) \\
CNOT_2^1 \frac{1}{\sqrt{2}}\,(|00\rangle + |01\rangle) &= \frac{1}{\sqrt{2}}\,(|00\rangle + |01\rangle) \\
&= |0+\rangle = |0\rangle \otimes |+\rangle
\end{aligned}$$

The above resultant state is **separable**, therefore the answer is **B**.

Question 8

The expression given is the definition of Big Endian, therefore the answer is **A**.

Question 9

When $CNOT \circ (H \otimes I)$ is applied to $|B_{00}\rangle = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$, the resultant state can be evaluated as follows:

$$\begin{aligned}
CNOT \circ (H \otimes I)\,|B_{00}\rangle &= CNOT\ \frac{1}{\sqrt{2}}\,(|{+0}\rangle + |{-1}\rangle) \\
&= CNOT\ \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\,(|00\rangle + |10\rangle) + \frac{1}{\sqrt{2}}\,(|01\rangle - |11\rangle)\right) \\
&= \frac{1}{2}\,(|00\rangle + |11\rangle + |01\rangle - |10\rangle) = \frac{1}{2}\,(|00\rangle + |01\rangle - |10\rangle + |11\rangle)
\end{aligned}$$

Assuming that the above state can be separated into single qubit states,

$$
\begin{aligned}
(a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle) &= \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle + |11\rangle) \\
\Rightarrow ac &= 1 \\
ad &= 1 \\
bc &= -1 \\
bd &= 1
\end{aligned}
$$

dividing the first and the second relations gives:

$$\frac{c}{d} = 1$$

dividing the third and the fourth relations gives:

$$\frac{c}{d} = -1$$

this is a contradiction, therefore the above state is **entangled** and the answer is **A**.

Question 10

When $CNOT_2^1 \circ (H \otimes I)$ is applied to $|0+\rangle = \frac{|00\rangle+|01\rangle}{\sqrt{2}}$, the resultant state can be evaluated using the actions:

$$
\begin{aligned}
CNOT \circ (H \otimes I)\,|0+\rangle &= CNOT\,|++\rangle \\
&= CNOT\,\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \\
&= \frac{1}{2}(|00\rangle + |01\rangle + |11\rangle + |10\rangle) \\
&= |++\rangle
\end{aligned}
$$

The above resultant state is **separable**, therefore the answer is **B**.

Question 11

The state $\frac{1}{\sqrt{2}}\left(\left|+\right\rangle\left|+\right\rangle+\left|-\right\rangle\left|-\right\rangle\right)$ can be expanded in the computational basis as follows:

$$\begin{aligned}\frac{1}{\sqrt{2}}\left(\left|+\right\rangle\left|+\right\rangle+\left|-\right\rangle\left|-\right\rangle\right) &= \frac{1}{\sqrt{2}}\times\frac{1}{2}\left(\left|00\right\rangle+\left|01\right\rangle+\left|10\right\rangle+\left|11\right\rangle\right)\\ &\quad+\frac{1}{\sqrt{2}}\times\frac{1}{2}\left(\left|00\right\rangle-\left|01\right\rangle-\left|10\right\rangle+\left|11\right\rangle\right)\\ &= \frac{1}{\sqrt{2}}\left(\left|00\right\rangle+\left|11\right\rangle\right)\end{aligned}$$

therefore, the answer is **A**.

Question 12

The state $\frac{1}{\sqrt{2}}\left(\left|\mathrm{i}\right\rangle\left|\mathrm{i}\right\rangle+\left|-\mathrm{i}\right\rangle\left|-\mathrm{i}\right\rangle\right)$ can be expanded in the computational basis as follows:

$$\begin{aligned}\frac{1}{\sqrt{2}}\left(\left|\mathrm{i}\right\rangle\left|\mathrm{i}\right\rangle+\left|-\mathrm{i}\right\rangle\left|-\mathrm{i}\right\rangle\right) &= \frac{1}{\sqrt{2}}\times\frac{1}{2}\left(\left|00\right\rangle+\mathrm{i}\left|01\right\rangle+\mathrm{i}\left|10\right\rangle-\left|11\right\rangle\right)\\ &\quad+\frac{1}{\sqrt{2}}\times\frac{1}{2}\left(\left|00\right\rangle-\mathrm{i}\left|01\right\rangle-\mathrm{i}\left|10\right\rangle-\left|11\right\rangle\right)\\ &= \frac{1}{\sqrt{2}}\left(\left|00\right\rangle-\left|11\right\rangle\right)\end{aligned}$$

therefore, the answer is **A**.

Question 13

The state $\frac{1}{\sqrt{2}}\left(\left|0\right\rangle\left|+\right\rangle+\left|1\right\rangle\left|-\right\rangle\right)$ can be expanded in the computational basis as follows:

$$\begin{aligned}\frac{1}{\sqrt{2}}\left(\left|0\right\rangle\left|+\right\rangle+\left|1\right\rangle\left|-\right\rangle\right) &= \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\left(\left|00\right\rangle+\left|01\right\rangle\right)+\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\left(\left|10\right\rangle-\left|11\right\rangle\right)\\ &= \frac{1}{\sqrt{2}}\left(\left|00\right\rangle+\left|01\right\rangle+\left|10\right\rangle-\left|11\right\rangle\right)\end{aligned}$$

using the same method as question 9, it can be seen that this state is **entangled** and so, the answer is **A**.

Question 14

The state $|00\rangle$ can be written as a linear combination of the Bell states that have the $|00\rangle$ state in their definition.

$$\begin{aligned} |00\rangle &= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle\right) + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \left(|00\rangle - |11\rangle\right) \\ &= \frac{1}{\sqrt{2}} \left(|B_{00}\rangle + |B_{10}\rangle\right) \end{aligned}$$

therefore, the answer is **C**.

Question 15

The state $|\psi\rangle_1 = |+\rangle$ can be written in column vector form as $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and similarly, the state $|\phi\rangle_2 = |-\rangle$ can be written as $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$, therefore, $|\psi\rangle\,|\phi\rangle$ can be expressed as:

$$\begin{aligned} |\psi\rangle\,|\phi\rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \times \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \\ 1 \times \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix}1\\-1\\1\\-1\end{pmatrix} \end{aligned}$$

therefore, the answer is **A**.

Question 16

The operator $X \otimes H$ can be expanded using the matrix forms of the operators, $X = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ and $H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$

$$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \begin{pmatrix} 0 \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} & 1 \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \\ 1 \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} & 0 \times \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 0 & 1 & 1\\ 0 & 0 & 1 & -1\\ 1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\end{pmatrix}$$

therefore, the answer is **A**.

Question 17

The operator being applied applied is $X \otimes Y$, the *action* of this operator on $|00\rangle$ is given as:

$$X \otimes Y\,|00\rangle \;=\; X\,|0\rangle \otimes Y\,|0\rangle \;=\; \mathrm{i}\,|11\rangle$$

the global phase is not neglected in this case.

The resultant state when this operator is applied on $|01\rangle$ is obtained after removing the global phase factors.

$$X \otimes Y\,|01\rangle \;=\; X\,|0\rangle \otimes Y\,|1\rangle \;=\; -\mathrm{i}\,|10\rangle \;\equiv\; |10\rangle$$

therefore, the answer is **A**.

Question 18

The operator $Z \otimes Y$ can be expanded using the matrix forms of the operators, $Z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ and $Y = \begin{pmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{pmatrix}$

$$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \otimes \begin{pmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{pmatrix} = \begin{pmatrix} 1 \times \begin{pmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{pmatrix} & 0 \times \begin{pmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{pmatrix} \\ 0 \times \begin{pmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{pmatrix} & -1 \times \begin{pmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{pmatrix}\end{pmatrix}$$

$$= \begin{pmatrix}0 & -\mathrm{i} & 0 & 0\\ \mathrm{i} & 0 & 0 & 0\\ 0 & 0 & 0 & \mathrm{i}\\ 0 & 0 & -\mathrm{i} & 0\end{pmatrix}$$

therefore, the answer is **A**.

Question 19

When $(H \otimes X) \circ CNOT$ is applied to the basis element $|11\rangle$, the action is evaluated as follows:

$$\begin{aligned}(H \otimes X) \circ CNOT\,|11\rangle &= CNOT\ \ (H \otimes X)\,|10\rangle \\ &= (H \otimes X)\,|10\rangle \ = \ |-\rangle\,|1\rangle \\ &= \frac{1}{\sqrt{2}}\,(|01\rangle - |11\rangle)\end{aligned}$$

therefore, the answer is **A**.

Question 20

The state $\frac{1}{\sqrt{2}}\,(|+\rangle\,|-\rangle + |-\rangle\,|+\rangle)$ can be expanded in the computational basis as follows:

$$\begin{aligned}\frac{1}{\sqrt{2}}\,(|+\rangle\,|-\rangle + |-\rangle\,|+\rangle) \ &= \ \frac{1}{\sqrt{2}} \times \frac{1}{2}\,(|00\rangle - |01\rangle + |10\rangle - |11\rangle) \\ &\quad + \frac{1}{\sqrt{2}} \times \frac{1}{2}\,(|00\rangle + |01\rangle - |10\rangle - |11\rangle) \\ &= \frac{1}{\sqrt{2}}\,(|00\rangle - |11\rangle)\end{aligned}$$

This state is an element of the Bell basis ($|B_{10}\rangle$), with a global phase. It can also be reasoned that the above state is not separable in the basis $\{|+\rangle\,|+\rangle\,, |+\rangle\,|-\rangle\,, |-\rangle\,|+\rangle\,, |-\rangle\,|-\rangle\}$ and so the statement is **TRUE** the answer is **A**.