# Physics of the Qubit

## Practical Quantum Computing using Qiskit and IBMQ

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October 1, 2020

# Outline

# Quantum Mechanics: Mathematical framework

- The state of a quantum system is defined as a vector $|\psi\rangle$ in an appropriately defined Hilbert space.
- Measurable physical quantities are represented by Hermitian operators ($\hat{A} = \hat{A}^{\dagger}$) and the expectation value of the quantity represented by $\hat{A}$ is given by $\left\langle \hat{A} \right\rangle = \left\langle \psi \middle| \hat{A} \middle| \psi \right\rangle$.
- The time evolution of the state (dynamics) is governed by another Hermitian operator, The Hamiltonian, $\hat{H}$, which in most cases represents the total energy of the system.
- The dynamics of the state is described by the Schrödinger equation:

$$\mathrm{i}\hbar \frac{\mathrm{d}}{\mathrm{d}t} |\psi(t)\rangle \; = \; \hat{H} |\psi\rangle$$

  where, $\mathrm{i}^2 = -1$ and $\hbar = \frac{h}{2\pi}$, $h$ is the Planck constant.
- It should be noted that this is only one approach to studying quantum systems, there are other equivalent frameworks.

# A word about magnets

- When a classical magnetic dipole (bar magnet, current carrying loop) is placed in a steady external magnetic field $\vec{\mathrm{B}}$, the energy $U$ of the dipole is given by:

  $$U = -\vec{\mu} \cdot \vec{\mathrm{B}} = -\mu\,\mathrm{B}\cos\theta$$

  here, $\vec{\mu}$ is the magnetic dipole moment.
- Therefore, the energy is lowest when the angle between the moment and the applied field is 0 and highest when the angle is $180^\circ$.
- If the direction of the magnetic field is taken to be the $z$-axis, then the energy is given in terms of the components of $\vec{\mu}$ as follows:

  $$U = -\mu_z \mathrm{B}$$

# The spin-$\frac{1}{2}$ particle

- It is known that in the quantum mechanical description of particles, one encounters the quantity of spin. Spin is a microscopic version of the magnetic moment.
- It is experimentally observed that certain particles (like an electron or proton) when measured have, their spins either perfectly along the magnetic field or perfectly against the field.
- The energies corresponding to these outcome states are given by $U = \pm\gamma\frac{\hbar}{2}\mathrm{B}$ for the aligned and non-aligned case respectively.
- Such particles are referred to as spin $\frac{1}{2}$ particles. The Hamiltonian for such a system is given by:

$$\hat{H} = -\gamma \mathrm{B}\,\frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = -\gamma \mathrm{B}\,\frac{\hbar}{2}\,\sigma_z$$

# An orthonormal basis

- It can be seen that the vector, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is an eigenstate of the Hamiltonian with eigenvalue (energy value) $E_0 = -\gamma\frac{\hbar}{2}\mathrm{B}$. This is labelled as $|0\rangle$.
- The vector, $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is an eigenstate of the Hamiltonian with eigenvalue $E_1 = \gamma\frac{\hbar}{2}\mathrm{B}$. This is labelled as $|1\rangle$.
- These vectors are orthonormal and may form a basis for a suitably defined Hilbert space.
- It is also possible to define the a linear combination state of these basis vectors $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$. The form of $a, b$ will define the Hilbert space.

# The dynamic state

- Solving the Schrödinger equation using the Hamiltonian defined above for the state $|\psi\rangle$ gives the following.

$$|\psi(t)\rangle \;=\; a\, e^{-\mathrm{i}E_0 \cdot t/\hbar}\,|0\rangle + b\, e^{-\mathrm{i}E_1 \cdot t/\hbar}\,|1\rangle$$

- These phase factors indicate that allowing $a, b$ to be complex numbers would be the right choice. The Hilbert space for this system is therefore $\mathbb{C}^2 \equiv \mathbb{C} \times \mathbb{C}$.
- These vectors are sometimes referred to as *spinors*. All previously defined properties of quantum states apply to spinors.
- A system with only two observable energy levels is called a "two-state quantum system".

# From spinors to qubits

- Neglecting a global phase of $e^{-\mathrm{i}E_0 \cdot t/\hbar}$ from $|\psi(t)\rangle$ gives:

$$\begin{aligned} |\psi(t)\rangle &\equiv a\,|0\rangle + b\,e^{-\mathrm{i}(E_1 - E_0)\cdot t/\hbar}\,|1\rangle \\ &= a\,|0\rangle + b\,e^{-\mathrm{i}\gamma \mathrm{B}\cdot t}\,|1\rangle \end{aligned}$$

- If, $a, b$ are constrained to be real, then $|\psi(t)\rangle$ corresponds to a vector on the Bloch sphere rotating about the $z$-axis.

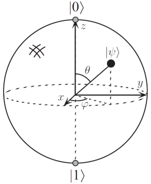


Figure 1: The Bloch Sphere [Source: *Quantum Computation and Quantum Information, Nielsen, M. and Chuamg I.*, $10^{th}$ *Anniversary edition, Cambridge University Press*]

- This phenomenon is called Larmor precession and $\omega = \gamma \mathrm{B}$ is called the Larmor frequency. $\gamma = \omega / \mathrm{B}$ is called the gyromagnetic ratio.

# Defining quantum gates

- It can be seen that the dynamics of a spin $\frac{1}{2}$ particle placed in a steady magnetic field is described by a quantum state with a time dependent relative phase.
- Therefore it is possible to rotate the state vector by applying this steady magnetic field for a limited amount of time.
- This is indeed how $Z$ and Phase $(P_\phi)$ gates are applied in the case of spin based qubits.
- These systems were first realized in Nuclear Magnetic Resonance experiments and they were the first physically demonstrated quantum computers.
- Other single qubit transformations are applied through magnetic fields (usually time-varying) along the $xy$ plane. These transformations can be understood by the concept Rabi oscillation.

# Practical Qubit systems

- In reality many two-state systems, are not easily available for use as qubits.
- In such cases, a quantum system with many discrete energy levels are utilized.
- The effect of Rabi oscillation is used to identify two energy states and ensure that the system exists in states that are linear combinations of these two energy levels.
- This approach has lead to the development of a wide variety of qubit systems and proposals to build quantum computers out of them.

# The problem multiple qubits

- Multi-qubit systems are are defined by having several single qubits that interact with each other.
- It is however not possible to control the interaction of these qubits to an arbitrary degree.
- This places limitations on the type of multi-qubit gates that can be applied and therefore, different platforms have different representations for multi-qubit operations.
- It is also not possible to make all qubits in a quantum computer interact with each other. This leads to that fact that different quantum computers have different connectivities for their qubits.
- This layout of qubits for a computer defined the *topology* of the quantum computer.

# Theoretical Prescriptions

- The theoretical physicist David P. Divincenzo identified the following criteria that a quantum system should fulfil for performing of quantum computations.
  - A scalable physical system with well-characterized qubits.
  - The ability to initialize the state of the qubits to a simple fiducial state, such as $|000\ldots\rangle$.
  - A "universal" set of quantum gates.
  - Long relevant decoherence times, much longer than the gate-operation time.
  - A qubit-specific measurement capability.
- In the present-day there are no systems that fulfil all of the above criteria.
- The American theoretical physicist John Preskill refers to the current phase of quantum computation as the "Noisy Intermediate Scale Quantum" computation era or the NISQ era.

# Examples of Quantum Computing Systems

Over the last few years, several companies have developed hardware capable of performing quantum computation. The following are some examples.

- IBM has developed several chip based quantum computing devices based on *Superconducting Circuits*. Access to some of these devices is available through the IBM Quantum Experience (IBMQ), `https://www.ibm.com/quantum-computing`.
- Rigetti Computing is a start up that aims to develop commercially viable quantum computers based on Superconducting Circuits, `https://www.rigetti.com`.
- Xanadu has developed *Photonics* based quantum computing chips and an associated computing library, Strawberry Fields, `https://strawberryfields.ai`.
- IonQ has developed quantum computers based on *Trapped Ion Qubits*, `https://ionq.com`.