

Assignment 2

AI1110: Probability and Random Variables

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11.16.1.12: One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

- Write the sample space showing all possible outcomes
- What is the probability that two black balls are chosen?
- What is the probability that two balls of opposite colour are chosen?

Solution:

Probability of an event E , written as $\Pr(E)$

$$\Pr(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total Number of possible outcomes}} \quad (1)$$

Let Z be a Bernoulli random variable

$$Z = \begin{cases} 0, & \text{if Urn 1 chosen} \\ 1, & \text{if Urn 2 chosen} \end{cases} \quad (2)$$

Since both events are equally likely

$$\Pr(Z = 0) = \Pr(Z = 1) \quad (3)$$

$$= \frac{1}{2} \quad (4)$$

Let X be a random variable denoting first ball is chosen and Y be random variable denoting second ball is chosen .

- $(X + Y = 1, Z = 0)$ represents 2 black balls are chosen.
- $(X + Y = 1, Z = 1)$ represents 2 white balls are chosen.
- $(X + Y) > 1$ represents 2 balls of opposite colors are chosen.

$X Z = 0$	Description	$Y Z = 0$	Description
0	B_1 chosen	0	B_1 chosen
1	B_2 chosen	1	B_2 chosen
2	W chosen	2	W chosen

TABLE 1

$X Z = 1$	Description	$Y Z = 1$	Description
0	W_1 chosen	0	W_1 chosen
1	W_2 chosen	1	W_2 chosen
2	B chosen	2	B chosen

TABLE 2

- (a) Sample Space S :

$$\{001, 010, 002, 020, 021, 012, 101, \quad (5)$$

$$110, 102, 120, 121, 112\} \quad (6)$$

$$\therefore n(S) = 12 \quad (7)$$

- (b) Let E be event that 2 black balls are chosen.
Required Probability:

$$\Pr(X + Y = 1, Z = 0) = \quad (8)$$

$$\Pr((X = 0, Y = 1)|Z = 0) \Pr(Z = 0) +$$

$$\Pr((X = 1, Y = 0)|Z = 0) \Pr(Z = 0)$$

$$= 1/3 \times 1/2 \times 1/2 + 1/3 \times 1/2 \times 1/2 \quad (9)$$

$$= 1/6 \quad (10)$$

$$\therefore \Pr(E) = \frac{1}{6} \quad (11)$$

- (c) Let E be event that balls of opposite colours are chosen.

From the axioms of probability ,
Required Probability:

$$\Pr((X + Y) > 1) = \quad (12)$$

$$1 - \Pr((X + Y) = 1, Z = 0) - \Pr((X + Y) = 1, Z = 1)$$

$$\Pr(X + Y = 1, Z = 1) = \quad (13)$$

$$\Pr((X = 0, Y = 1)|Z = 1) \Pr(Z = 1) +$$

$$\Pr((X = 1, Y = 0)|Z = 1) \Pr(Z = 1)$$

$$= 1/3 \times 1/2 \times 1/2 + 1/3 \times 1/2 \times 1/2 \quad (14)$$

$$= 1/6 \quad (15)$$

Using (8) and (13)

$$\Pr((X + Y) > 1) = 1 - 2 \times \frac{1}{6} \quad (16)$$

$$= 2/3 \quad (17)$$

$$\therefore \Pr(E) = \frac{2}{3} \quad (18)$$