Assignment 2

AI1110: Probability and Random Variables

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11.16.3.12: One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

(a) What is the probability that two black balls are chosen?
(b) What is the probability that two balls of opposite colour are chosen?

Solution:

Let Z be a Bernoulli random variable

$$Z = \begin{cases} 0, & \text{if Urn 1 chosen} \\ 1, & \text{if Urn 2 chosen} \end{cases} \tag{1}$$

Since both events are equally likely

$$\Pr(Z = 0) = \Pr(Z = 1) \tag{2}$$
$$= \frac{1}{2} \tag{3}$$

Let X_i be a random variable where i denotes the turn

$$X_i = \begin{cases} 0, & \text{if Black ball chosen} \\ 1, & \text{if White ball chosen} \end{cases} \tag{4}$$

Let X_1 be a random variable denoting first ball is chosen and X_2 be random variable denoting second ball is chosen .

X_1	X_2	**Description**
0	0	Both Black chosen
1	1	Both White chosen
0	1	Black,White chosen
1	0	White,Black chosen

TABLE 1

(a) Let E be event that 2 black balls are chosen.

$$E = (X_1 + X_2)' \tag{5}$$
$$= X_1' X_2' \tag{6}$$

Required Probability:

$$\Pr(X_1' X_2') \tag{7}$$
$$= \Pr(X_1' X_2' | Z') \Pr(Z')$$
$$= \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}$$
$$= 1/6 \tag{8}$$

$$\therefore \Pr(E) = \frac{1}{6} \tag{9}$$

(b) Let E be event that balls of opposite colours are chosen.

$$E = X_1 \oplus X_2 \tag{10}$$
$$= X_1 X_2' + X_1' X_2 \tag{11}$$

Required Probability :

$$\Pr(X_1 X_2' + X_1' X_2) \tag{12}$$
$$= \Pr(X_1 X_2') + \Pr(X_1' X_2) \tag{13}$$
$$\because \Pr(X_1 X_1' X_2 X_2') = 0 \tag{14}$$
$$= \left(\frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}\right) \times 2 \tag{15}$$
$$= \frac{2}{3} \tag{16}$$

$$\therefore \Pr(E) = \frac{2}{3} \tag{17}$$