

Assignment 2

AI1110: Probability and Random Variables

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11.16.1.12: One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

- What is the probability that two black balls are chosen?
- What is the probability that two balls of opposite colour are chosen?

Solution:

Let Z be a Bernoulli random variable

$$Z = \begin{cases} 0, & \text{if Urn 1 chosen} \\ 1, & \text{if Urn 2 chosen} \end{cases} \quad (1)$$

Since both events are equally likely

$$\Pr(Z = 0) = \Pr(Z = 1) \quad (2)$$

$$= \frac{1}{2} \quad (3)$$

Let X_i be a random variable where i denotes the turn

$$X_i = \begin{cases} 0, & \text{if Black ball chosen} \\ 1, & \text{if White ball chosen} \end{cases} \quad (4)$$

Let X_1 be a random variable denoting first ball is chosen and X_2 be random variable denoting second ball is chosen .

X_1	X_2	Description
0	0	Both Black chosen
1	1	Both White chosen
0	1	Black,White chosen
1	0	White,Black chosen

TABLE 1

- Let E be event that 2 black balls are chosen.

$$E = (X_1 + X_2)' \quad (5)$$

$$= X_1' X_2' \quad (6)$$

Required Probability:

$$\Pr(X_1' X_2') \quad (7)$$

$$= \Pr(X_1' X_2' | Z') \Pr(Z')$$

$$= \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} \quad (8)$$

$$\therefore \Pr(E) = \frac{1}{6} \quad (9)$$

- Let E be event that balls of opposite colours are chosen.

$$E = X_1 \oplus X_2 \quad (10)$$

$$= X_1 X_2' + X_1' X_2 \quad (11)$$

Required Probability :

$$\Pr(X_1 X_2' + X_1' X_2) \quad (12)$$

$$= \Pr(X_1 X_2') + \Pr(X_1' X_2) \quad (13)$$

$$\therefore \Pr(X_1 X_1' X_2 X_2') = 0 \quad (14)$$

$$= \left(\frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}\right) \times 2 \quad (15)$$

$$= \frac{2}{3} \quad (16)$$

$$\therefore \Pr(E) = \frac{2}{3} \quad (17)$$