

# Assignment 2

## AI1110: Probability and Random Variables

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**11.16.1.12:** One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

- Write the sample space showing all possible outcomes
- What is the probability that two black balls are chosen?
- What is the probability that two balls of opposite colour are chosen?

**Solution:**

Probability of an event  $E$ , written as  $\Pr(E)$

$$\Pr(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total Number of possible outcomes}} \quad (1)$$

Let  $Z$  be a Bernoulli random variable

$$Z = \begin{cases} 0, & \text{if Urn 1 chosen} \\ 1, & \text{if Urn 2 chosen} \end{cases} \quad (2)$$

Since both events are equally likely

$$\Pr(Z = 0) = \Pr(Z = 1) \quad (3)$$

$$= \frac{1}{2} \quad (4)$$

Let  $X$  be a random variable denoting first ball is chosen and  $Y$  be random variable denoting second ball is chosen .

- $(X + Y = 1, Z = 0)$  represents 2 black balls are chosen.
- $(X + Y = 1, Z = 1)$  represents 2 white balls are chosen.
- $(X + Y) > 1$  represents 2 balls of opposite colors are chosen.

$X Z = 0$	Description	$Y Z = 0$	Description
0	$B_1$ chosen	0	$B_1$ chosen
1	$B_2$ chosen	1	$B_2$ chosen
2	$W$ chosen	2	$W$ chosen

TABLE 1

$X Z = 1$	Description	$Y Z = 1$	Description
0	$W_1$ chosen	0	$W_1$ chosen
1	$W_2$ chosen	1	$W_2$ chosen
2	$B$ chosen	2	$B$ chosen

TABLE 2

- (a) Sample Space  $S$ :

$$\{001, 010, 002, 020, 021, 012, 101, \quad (5)$$

$$110, 102, 120, 121, 112\} \quad (6)$$

$$\therefore n(S) = 12 \quad (7)$$

- (b) Let  $E$  be event that 2 black balls are chosen.  
Required Probability:

$$\Pr(X + Y = 1, Z = 0) = \quad (8)$$

$$\Pr((X + Y = 1)|Z = 0) \Pr(Z = 0)$$

$$= \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} \\ = 1/6 \quad (9)$$

$$\therefore \Pr(E) = \frac{1}{6} \quad (10)$$

- (c) Let  $E$  be event that balls of opposite colours are chosen.

From the axioms of probability ,  
Required Probability:

$$\begin{aligned}\Pr((X + Y) > 1) &= 1 - \Pr((X + Y) = 1) \\ \Pr(X + Y = 1, Z = 0) &= \Pr(X + Y = 1, Z = 1) \\ &= \frac{1}{6}\end{aligned}\tag{11}$$

Using (8) and (11)

$$\Pr((X + Y) > 1) = 1 - 2 \times \frac{1}{6} \tag{12}$$

$$= \frac{2}{3} \tag{13}$$

$$\therefore \Pr(E) = \frac{2}{3} \tag{14}$$