Assignment 2

AI1110: Probability and Random Variables

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11.16.1.12: One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

(a) Write the sample space showing all possible outcomes
(b) What is the probability that two black balls are chosen?
(c) What is the probability that two balls of opposite colour are chosen?

Solution:

Let X be a random variable denoting first ball is chosen and Y be random variable denoting second ball is chosen .

X	**Description**
0	B_1 chosen
1	B_2 chosen
2	W chosen
3	W_1 chosen
4	W_2 chosen
5	B chosen

TABLE 1

Y	**Description**
0	B_1 chosen
1	B_2 chosen
2	W chosen
3	W_1 chosen
4	W_2 chosen
5	B chosen

TABLE 2

Since X is a uniformly distributed random variable, For x: $\{0, 1, 2, 3, 4, 5\}$

$$\Pr(X = x) = \frac{1}{6} \tag{1}$$

(a) Sample Space S:

$$\{01, 10, 02, 20, 12, 21, 34, 43, 35, 53, 45, 54\} \tag{2}$$

$$\therefore n(S) = 12 \tag{3}$$

(b) Let E be event that 2 black balls are chosen. Required Probability:

$$\Pr(X = 0, Y = 1) + \Pr(X = 1, Y = 0) \tag{4}$$
$$= \frac{1}{6} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{2} \tag{5}$$
$$= 1/6 \tag{6}$$

$$\therefore \Pr(E) = \frac{1}{6} \tag{7}$$

(c) Let E be event that balls of opposite colours are chosen.
From the axioms of probability ,
Required Probability:

$$1 - (\Pr(X + Y = 1) + \Pr(X + Y = 7)) \tag{8}$$

By Symmetry,

$$\Pr(X + Y = 1) = \Pr(X + Y = 7) \tag{9}$$
$$\because \Pr(X = 2, Y = 5) = 0 \tag{10}$$
$$\therefore \Pr(X + Y = 7) = \frac{1}{6} \tag{11}$$
$$= 1 - 2 \times \frac{1}{6} \tag{12}$$
$$= \frac{2}{3} \tag{13}$$

$$\therefore \Pr(E) = \frac{2}{3} \tag{14}$$