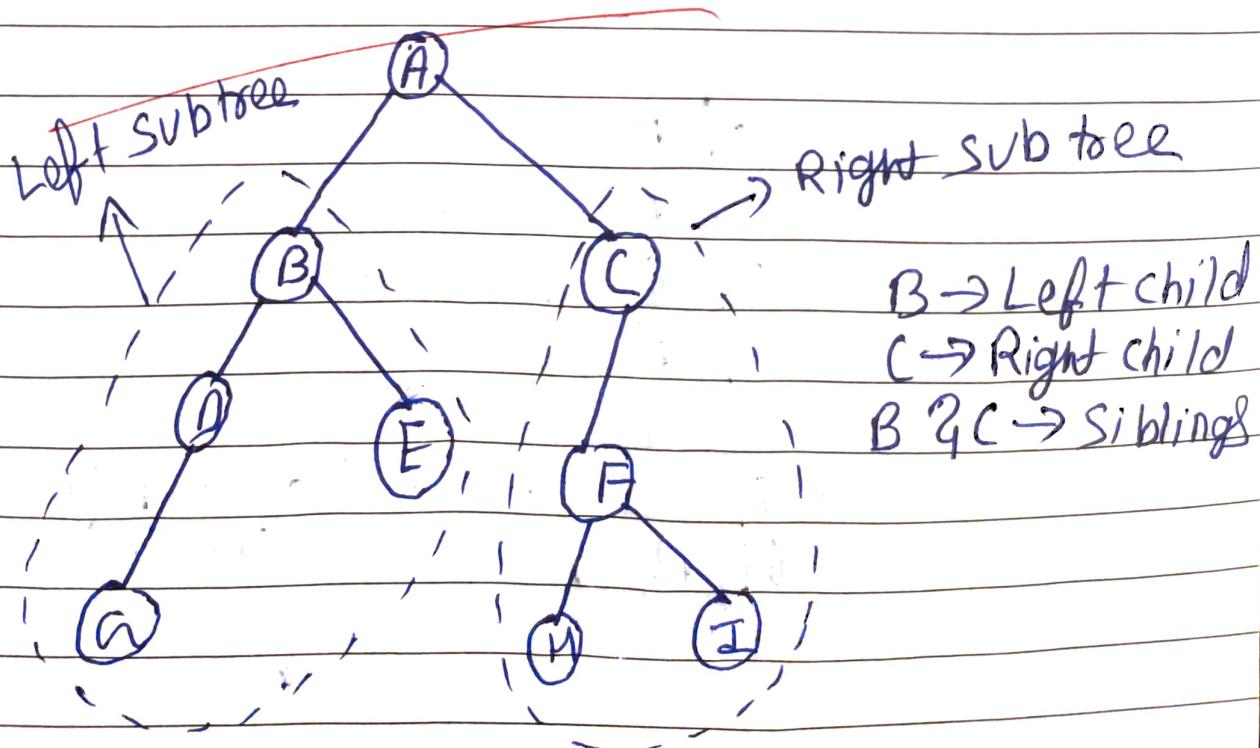


## Assignment = 04

(Q1) What is binary Tree? Describe representation of binary tree using arrays and linked list.

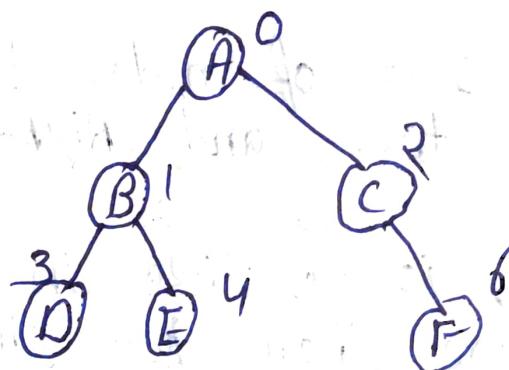
- A tree is a binary tree if each node of it can have at most two branches.
- A binary tree 'T' is either empty or consists of special node called root node.
- Tree has two sets of nodes  $L_T$  and  $R_T$  called the left sub tree and Right sub tree.
- A binary tree, is a tree in which no node can have more than two children (0-1-2).



## Representation of Binary tree :-

### 1) Sequential or Array Representation :-

- Useful when tree is complete or nearly complete
- used 1-D array
- In array the root node is always numbered by 0 and then it's left child and right child is stored at 1 and 2 respectively.

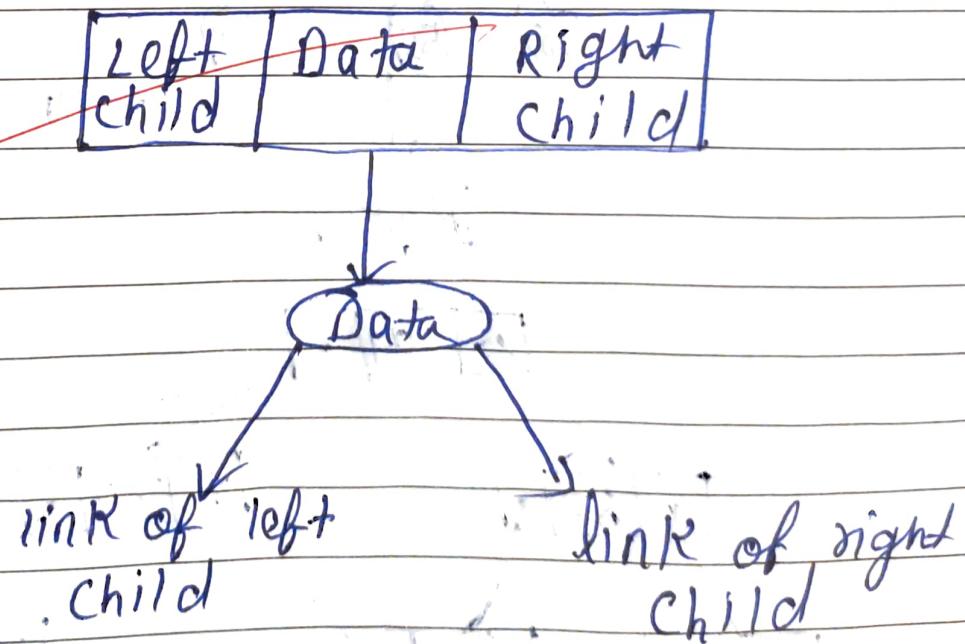


0	m
1	n
2	o
3	p
4	q
5	-
6	r

← NULL or empty left child

## 2) Linked List Representation

- Each binary tree node is represented as an object whose data type is Binary tree node.
- Basic component in binary tree is node. Each node has three fields, one for the link of left child, second for the data and third for the link of right child.
- Left child contains the address of its left node and the right child contains the address of its right Node.



## Representation :-

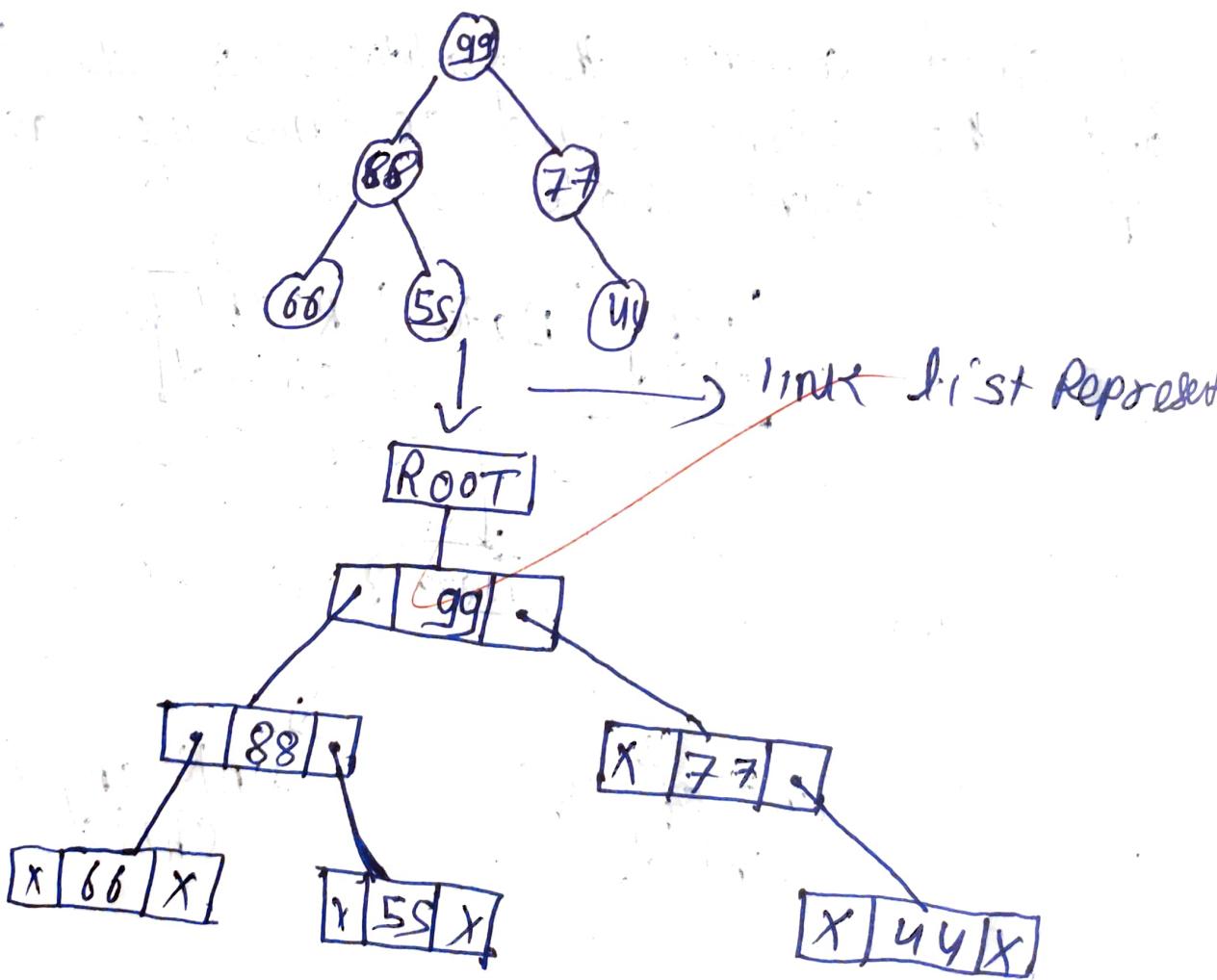
Struct Node

```
struct node * Lchild ;  
int Data ;
```

```
struct node * Rchild;
```

typed of start Node Bi Tree

zeg



Q2 Describe Operations and its Algorithm on Binary Tree ?

Ans Operation on Binary tree

- ① Traversal of a binary tree
- ② Insertion of nodes
- ③ Deletion of nodes
- ④ Searching for the nodes

① Traversal of a binary tree

- Traversal is a process to visit all the nodes of a tree and may print their values too.
- All nodes are connected via edges (link) we always start from the root (head) node.
- In tree Creation we take three parameters node, left child, and Right child.
- There are three ways which we use to traverse tree.

- ① Pre-order Traversal
- ② In-order Traversal
- ③ Post-order Traversal.

1) Pre order Traversal :- (Root - left - Right)

- (a) Process the root
- (b) Traverse the left subtree in pre order
- (c) Traverse the Right subtree in pre order

Algo

1. Begin
2. IF tree Not equal to NULL then
3. write : Data [Tree]
4. call pre order [Left(Tree)]
5. call pre order [Right(Tree)]  
[End of it]
6. Return.

2) In-order Traversal :- (Left - Root - Right)

- (a) Process the left Subtree in in order
- (b) Process the Root
- (c) Traverse the Right Subtree in order

Algo

1. Begin
2. IF Tree Not equal to NULL then

3. call Inorder [Left [Tree]]
4. call Inorder [Right [Tree]]
- [End of IF]
5. Return.

### ① Post-order Traversal

1. Traverse the left subtree in post order
2. Traverse the Right subtree in post order
3. Process Root (r)

### ② Insertion of a node in Binary search Tree

- Insert fun<sup>n</sup> is used to add a new element in a binary search tree at appropriate location
- Insert fun<sup>n</sup> is to be designed in such a way that, it must not violate the property of BST at each value.

#### (1) Allocate the memory for tree

(2) Set the data part to the value & set the left and Right pointers of tree, point to NULL

(3) IF the item to be inserted, will be the first element of the tree, then left and Right of this node will point to NULL

(5) If this is false, then perform this operation  
recursing with the Right sub-tree of root.

Algo :-

1. Begin
2. IF Root = NULL then [Tree is empty]
  - (a) Create a Root node using malloc Fun'n
  - (b) Set Root  $\rightarrow$  Data = Num
  - (c) Set Root  $\rightarrow$  Left = NULL
  - (d) Set Root  $\rightarrow$  Right = NULL
3. Else if Num < Root  $\rightarrow$  Data then  
Root  $\rightarrow$  left = Inset(Root  $\rightarrow$  left, num)
4. Else  
Root  $\rightarrow$  Right = Inset(Root  $\rightarrow$  Right - Num)  
[End of IF-else]  
[End of IF-else]
5. Exit.

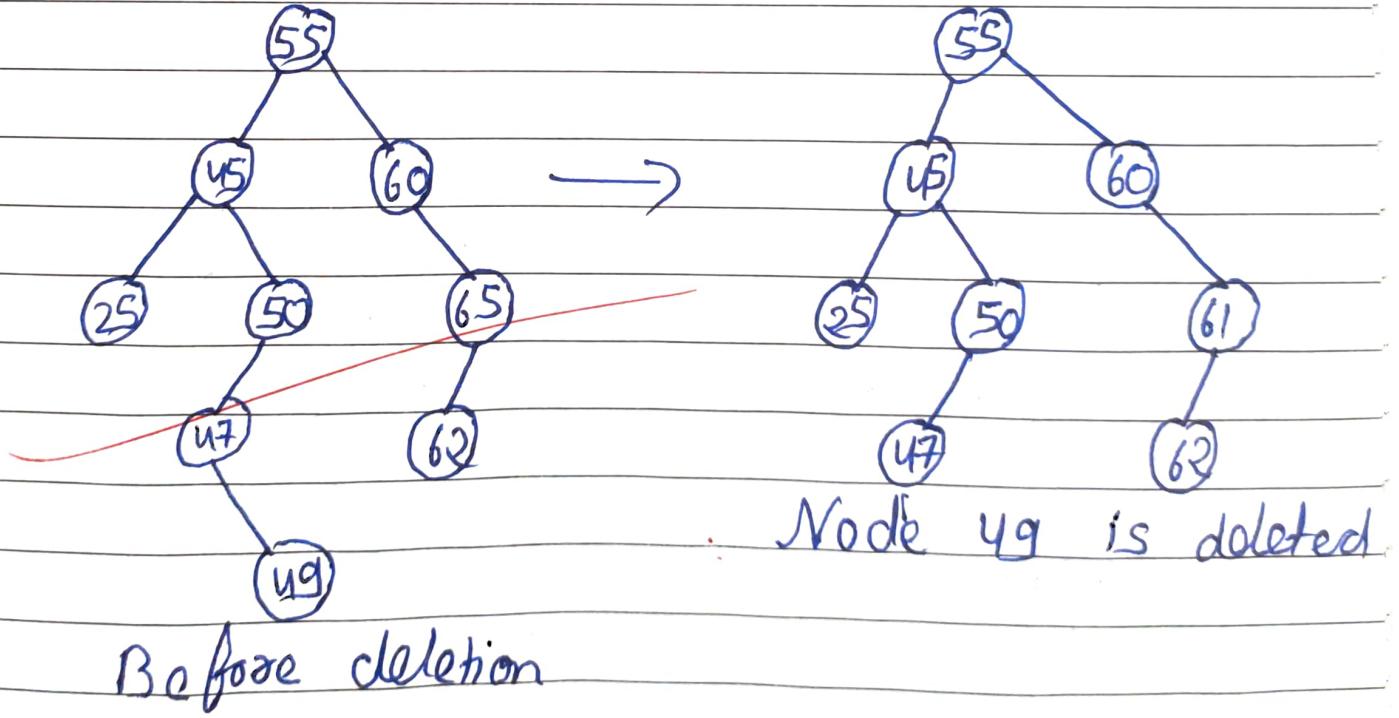
### 3) Deletion of a node in Binary Tree :-

These are four cases

Case 1) IF the node X to be deleted has no child

- In this case, node X is deleted from the tree by simply replacing the location of node in the parent node by the NULL pointer.

e.g.: -  $X = 49$



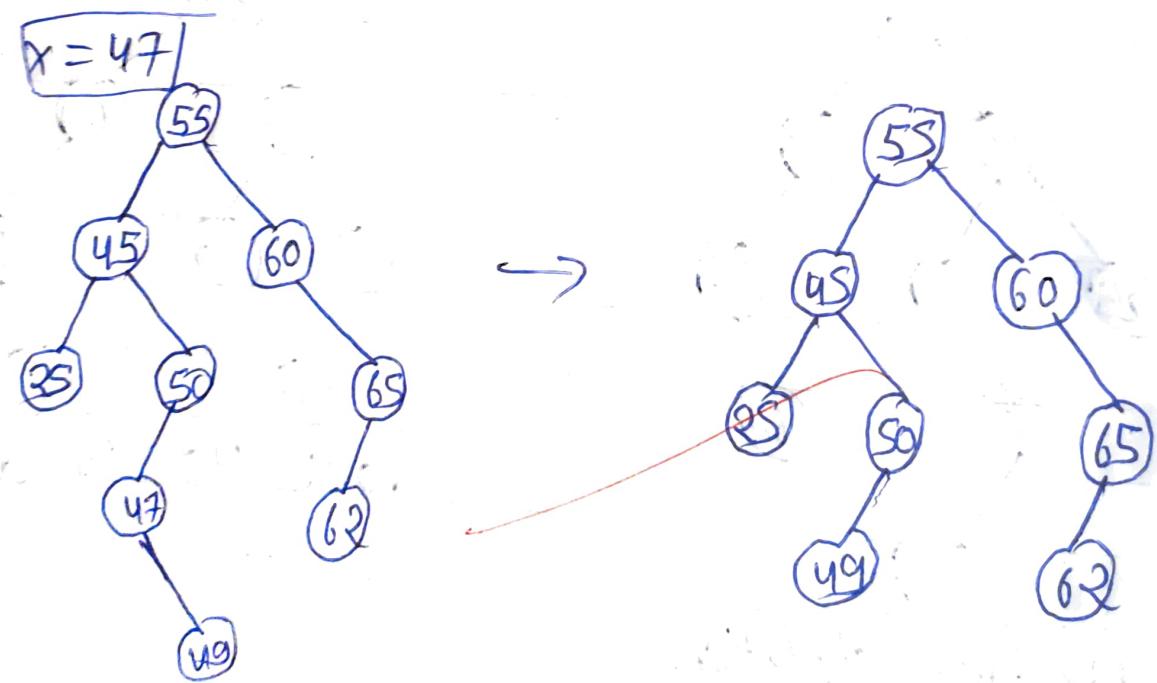
Algo

1. Begin

2. IF  $x \rightarrow \text{left} = \text{NULL}$  AND  $x \rightarrow \text{Right} = \text{NULL}$  then
  3.   IF  $\text{PARENT} \rightarrow \text{RIGHT} = 'x'$  then  
       Set  $\text{PARENT} \rightarrow \text{RIGHT} = \text{NULL}$   
       Else  
         Set  $\text{PARENT} \rightarrow \text{LEFT} = \text{NULL}$ ,  
         [End of IF-ELSE]
  4. Delete or Free nodes  
     [End of IF]
  5. Exit.

Case 2 :- IF the node  $x$  to be deleted has one right child

$$\text{eg} \therefore \boxed{x = 47}$$



Node is deleted  
47

Algo

① Begin

② IF  $X \rightarrow \text{left} = \text{NULL}$  and  $X \rightarrow \text{Right} \neq \text{NULL}$

③ IF PARENT  $\rightarrow \text{Left} = X$  then

Set PARENT  $\rightarrow \text{left} = X \rightarrow \text{RIGHT}$

Else

Set PARENT  $\rightarrow \text{RIGHT} = X \rightarrow \text{RIGHT}$

[End of IF ALSO]

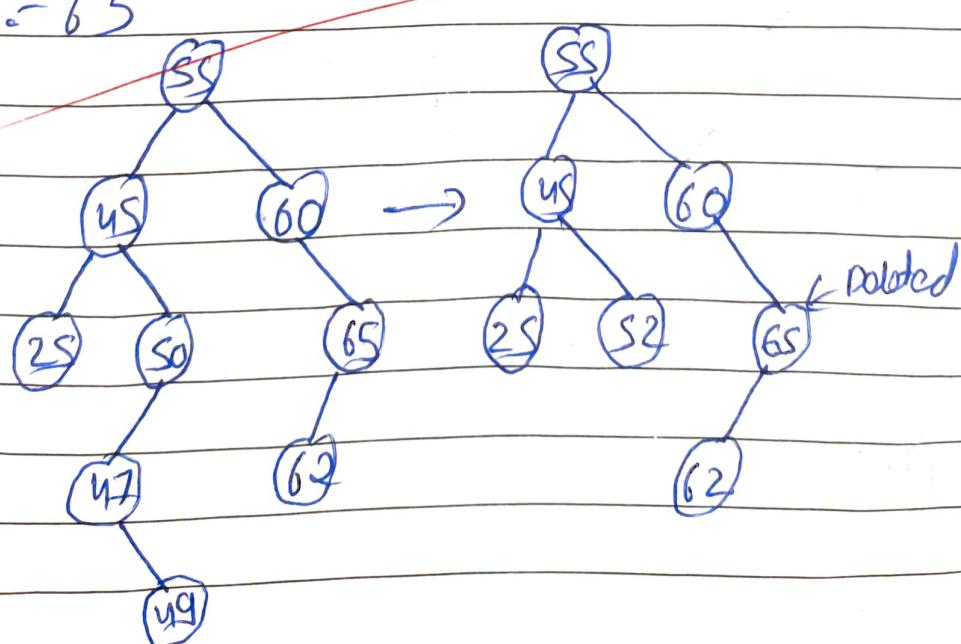
④ Delete or Free node X

[End of IF]

⑤ Exit

Case 3 IF the node X to be deleted has only left child

e.g. 65

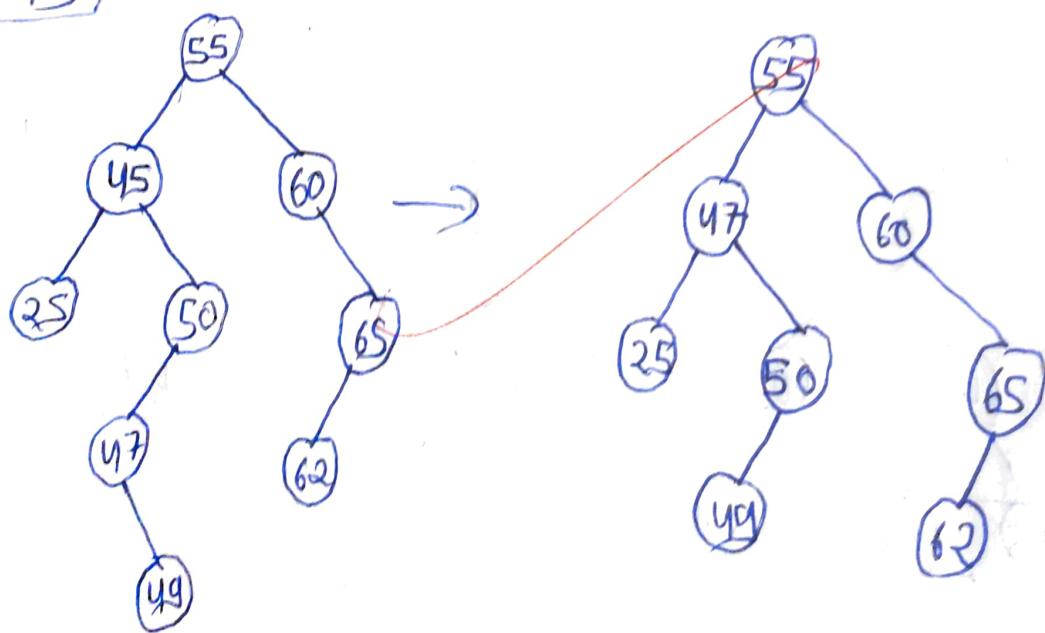


Algo

1. Begin
2. IF  $x \rightarrow \text{left} \neq \text{NULL}$  and  $x \rightarrow \text{Right} = \text{NULL}$  then
3.   IF PARENT  $\rightarrow \text{left} = x$  then  
        set PARENT  $\rightarrow \text{left} = x \rightarrow \text{left} +$   
    else  
        set PARENT  $\rightarrow \text{Right} = x \rightarrow \text{left}$
4. [End of IF else]
4. Deleted or Free node X
5. Exit

Case 4 :- IF the node X to be deleted has two children

$$x = 45$$



Algo :-

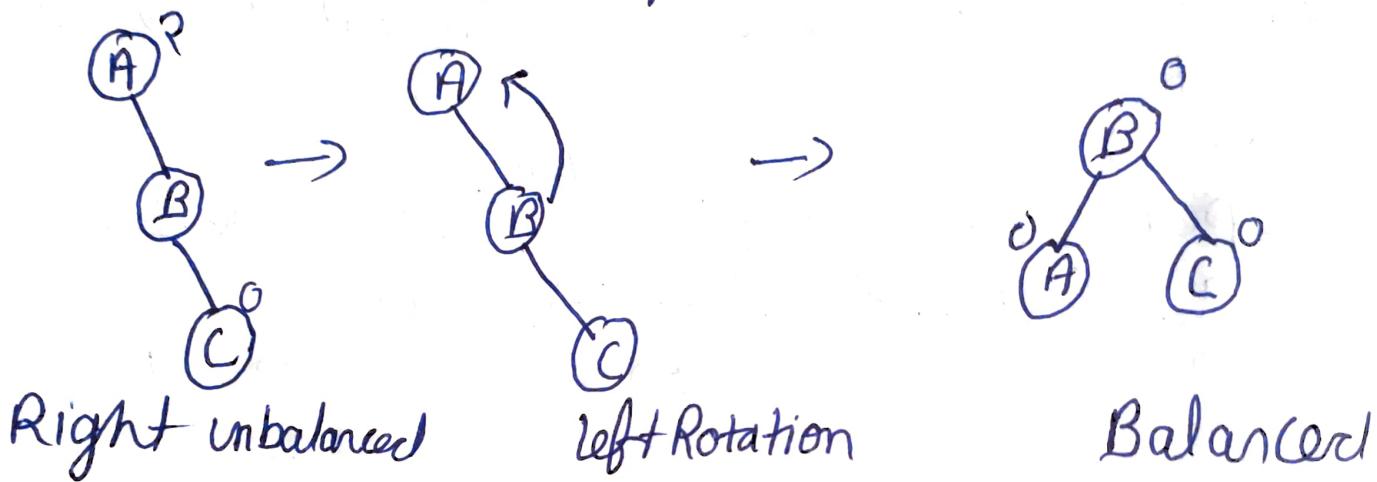
1. Begin
2. IF  $X \rightarrow \text{Left} \neq \text{NULL}$  &  $X \rightarrow \text{Right} \neq \text{NULL}$  then
3. Set Parent =  $X$
4. Set  $X_{\text{SUCC}} = X \rightarrow \text{Right}$
5. Repeat while  $X_{\text{SUCC}} \rightarrow \text{Left} \neq \text{NULL}$ 
  - (a) Set PARENT =  $X_{\text{SUCC}}$
  - (b) set  $X \rightarrow \text{Info} = X_{\text{SUCC}} \rightarrow \text{INFO}$
6. Set  $X \rightarrow \text{INFO} = X_{\text{SUCC}} \rightarrow \text{INFO}$
7. Set  $X = X_{\text{SUCC}}$ .

Q3 What is AVL Tree its Applications ? Create an AVL tree or Height balanced tree?  
Inserted Data :- F, V, E, W, D, X, Y, B, Z.

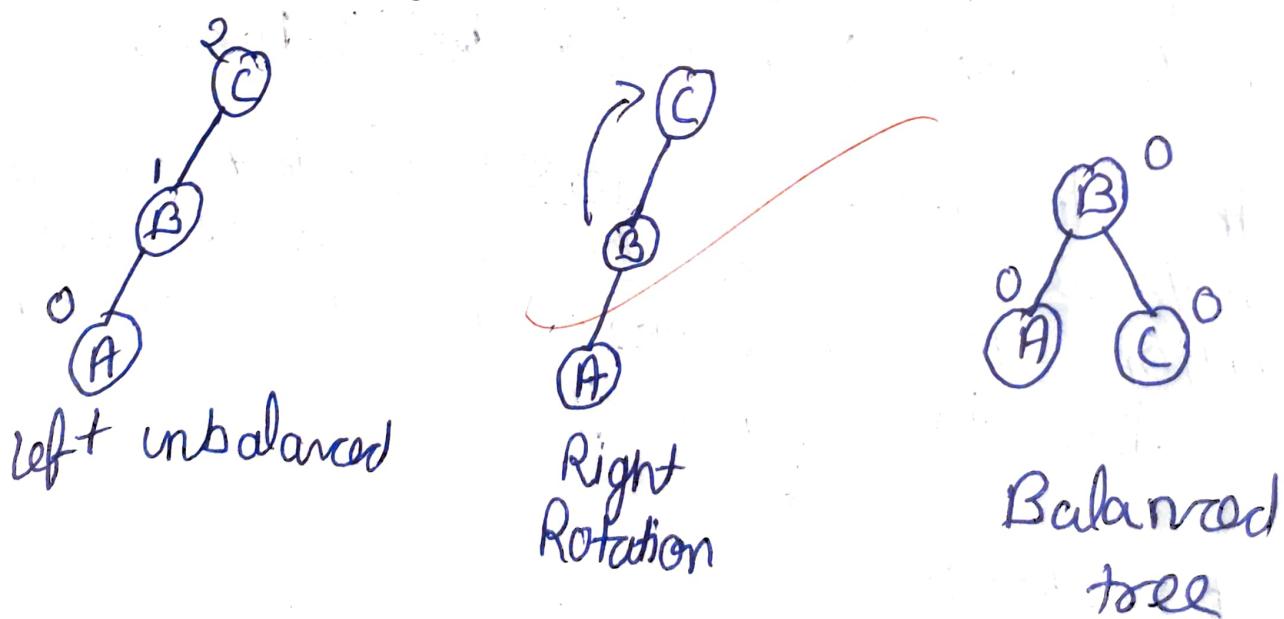
~~Ans~~ If the difference in height of left and right sub-trees is more than 1. There are four rotation techniques that we use to balance the AVL trees namely

- Left rotation
- Right rotation
- Left - Right rotation
- Right - Left rotation

1. Left Rotation: Here we can see that when a tree becomes unbalanced when a node is inserted in the right subtree of the right subtree, then perform a single left rotation.



2. Right Rotation: AVL tree may also become unbalanced, if a node is inserted in left subtree of left subtree. The tree then needs a right rotation



3. Left - Right Rotation : Double rotations are slightly complex version of already explained version of rotations. To understand them better, we should take note of each action performed while rotation. To put it simply LR rotation is when we perform a left rotation followed by a Right rotation.
4. Right - left Rotation : This one is similar as of the above but the only difference is that this has a combination of right rotation followed by left rotation

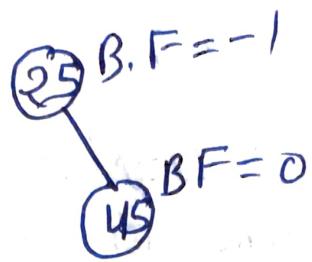
~~Create Height balanced tree in AVL tree~~

~~AVL Tree nodes 25, 45, 50, 55, 60~~

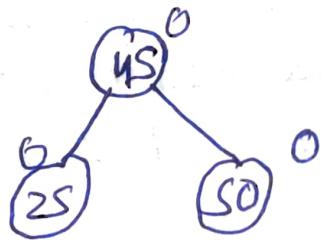
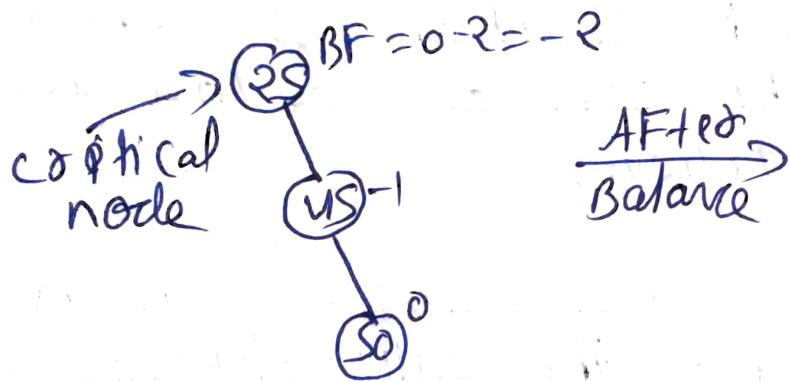
1. Start from first node 25

$$\textcircled{25} \text{ B.F} = 0$$

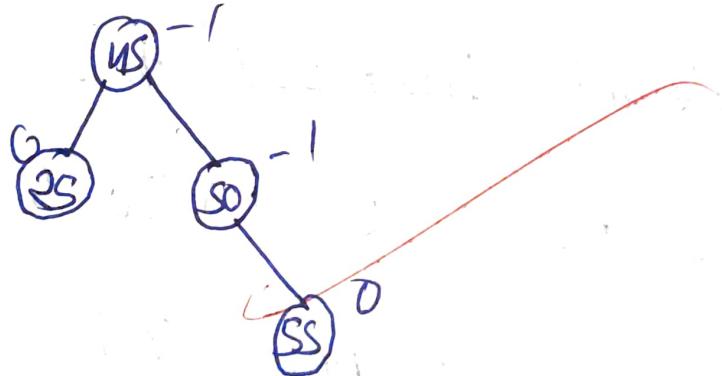
2. Insert 45 Node.  $45 > 25$ , so it will come Right side of 25.



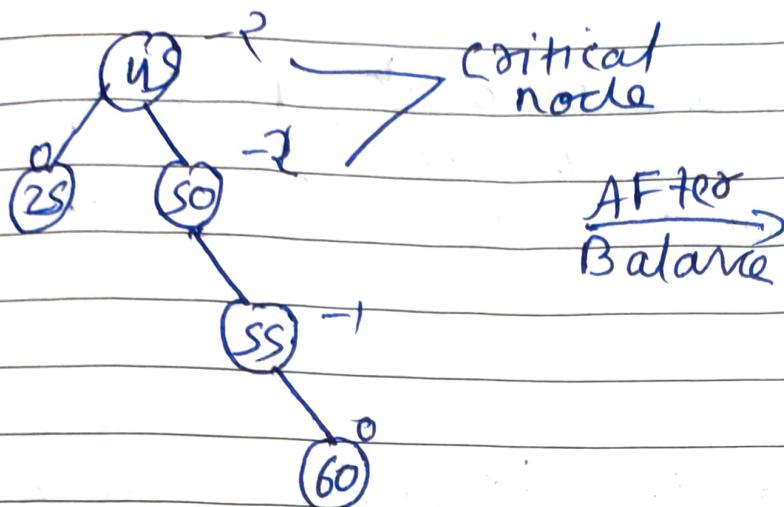
3. Insert 50, since  $50 > 45$ , it will come right side of node 45



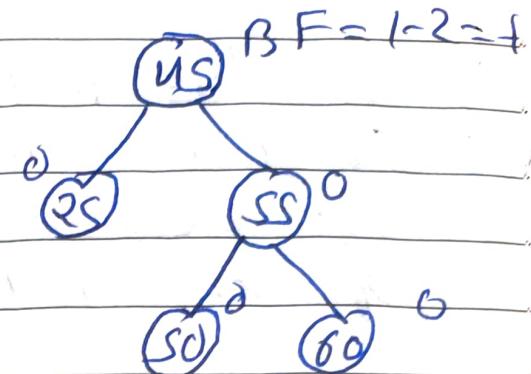
4. Insert 55 in the right side of 50 (since  $55 > 50$ ) Now find B.F of every node



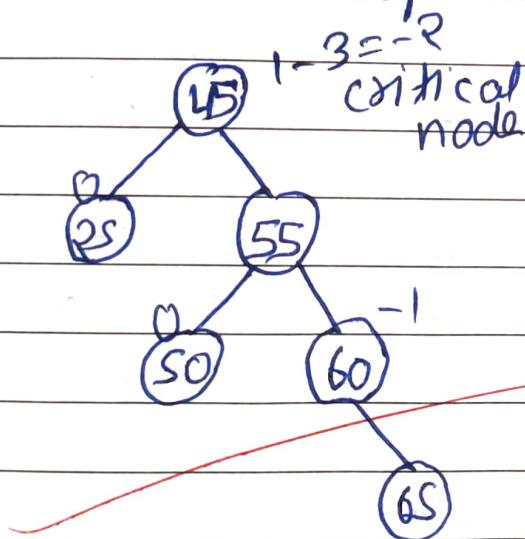
5. Insert 60 in the right side of 55 (since  $60 > 55$ )



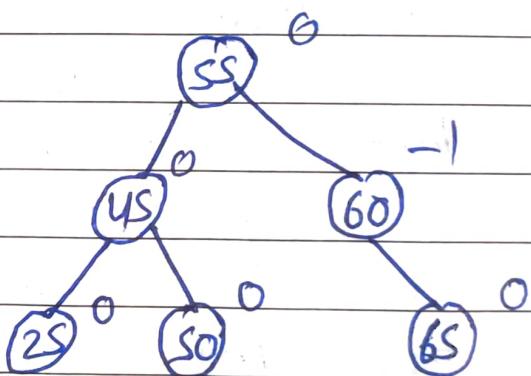
After Balance



6. Insert 65 in right side of 60 and now  
Find B.F of every node



After Balance



## Insertion in AVL Tree :-

- a) Insertion of a node in an AVL tree is similar to that in a binary search tree.
- b) Insertion may disturb the balance factor of an AVL tree and therefore.

Q1  
(d)  
(e) For this purpose, we need to perform insertion. Every new node will be at leaf node. At every insertion we will calculate the B.F. of each node.

Q4 Define tree and its terminology with example

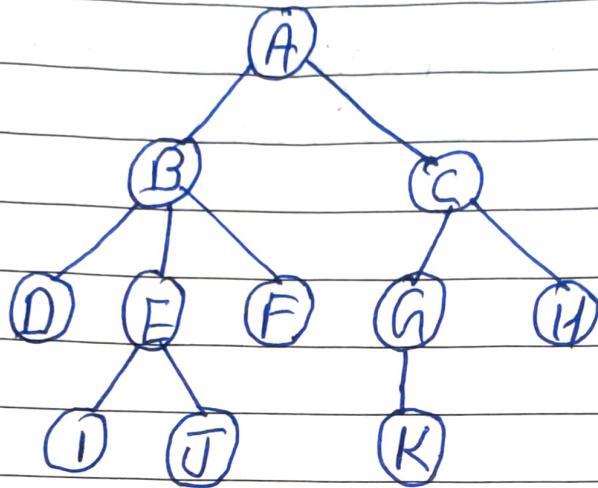
Ans Tree - Terminology

In linear data structure data is organized in sequential order and in non-linear data structure data is organized in random order. A tree is a very popular non-linear data structure used in a wide range of application. A tree data structure can be defined

Tree is a non-linear data structure which organizes data in hierarchical structure and this is a recursive definition

Tree data structure is a collection of nodes which is organized in hierarchical structure recursively.

## Example



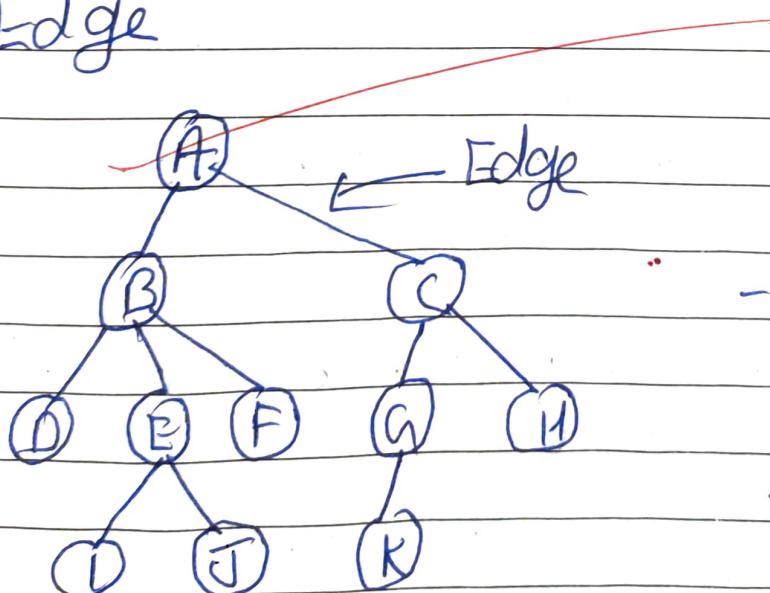
TREE with 11 nodes and 10 edges

1) Root

(A)

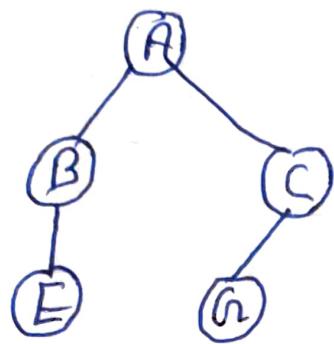
Here 'A' is the 'root' node

2) Edge



- In any tree 'Edge' is a connecting link between two nodes

### 3. Parent

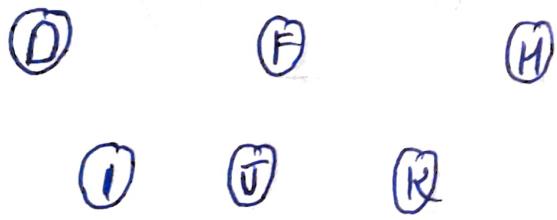


Here A, B, C, E are Parent.

- In any tree the node which has child/children is called 'Parent'.

### 4. Leaf

Here D, I, J, F, K & H are Leaf.



- In any tree the node which does not have children is called 'Leaf'.

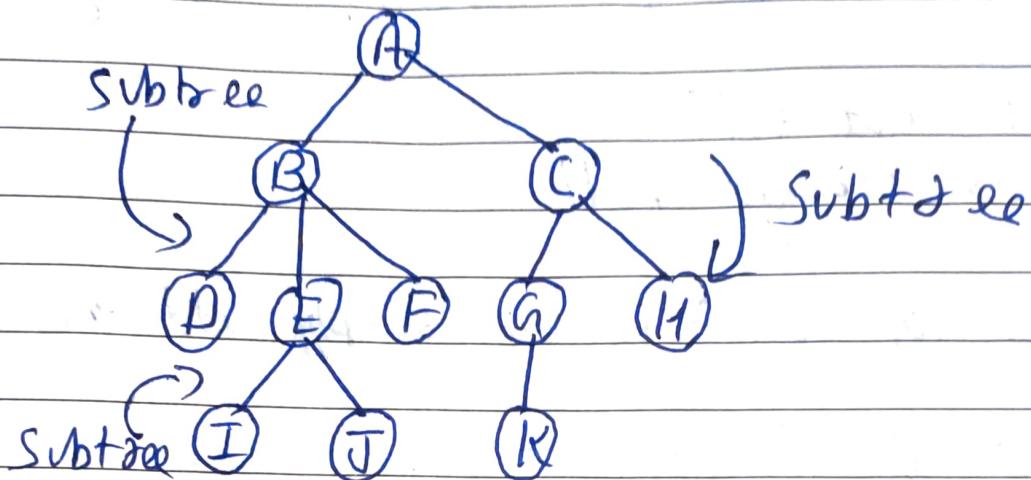
### 5. Path

- In any tree 'Path' is a sequence of nodes and edges between two nodes.

Here 'Path' between A & J is  
A - B - E - J

Here, 'Path' between C & K is  
C - G - K

## b. Sub Tree



Q5 What is B Tree? Describe insertion and Deletion with example ?

Ans B Tree :-

- A, B-tree is a balance m-way tree
- B - Tree is a self balancing search tree
- B-Tree of order M has following properties

- (a) It should be perfectly balanced i.e  $BF = 0$  and all leaf nodes must be at same level.
- (b) Each node (except root node) contain min.  $M/2$  children and max. M children
- (c) Each node has one key value (fewer value) than children, no node can contain more than  $M-1$  values.
- (d) Keys are arranged in defined order with in a node

## Insertion in B-Tree :-

We have two case for inserting. the Key

- ① Node is not full
- ② Node is already full

① Node is not full :- We can simply add the Key in

② Node is already Full :-

- We need to split the node into two nodes and median key will go to parent of that node.
- IF parent is also full then same thing will be repeated until non full parent node

Eg:- Let take a list of Keys and create a B-Tree of order 5.

10, 70, 60, 20, 110, 40, 80, 130, 100, 50, 190, 90, 180, 240, 120, 140, 160

Sol :- Step 1 :- Insert 10

10
----

Step 2 :- Insert 70

10	70
----	----

Step 3 :- Insert 60

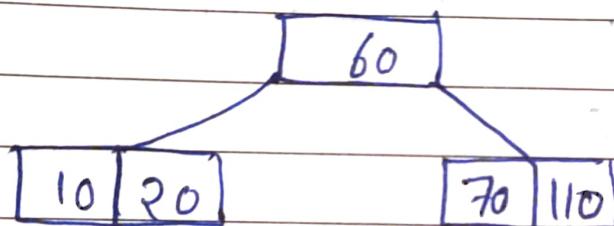
10	60	70
----	----	----

\* After 60 the Keys in node

Step 4 :- Insert 20

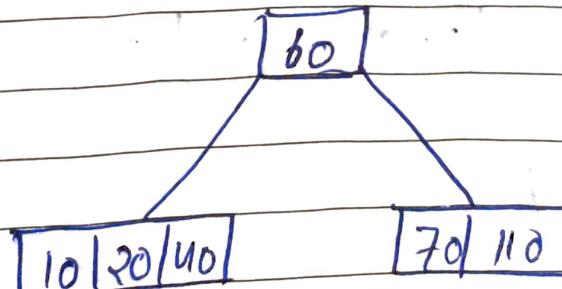
10	20	60	70
----	----	----	----

Step 5 :- Insert 110

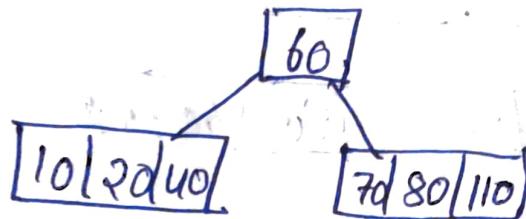


Note:- Here node was already full (5 nodes), so after insertion of 110, it should be splitted into two nodes. 60 is the median key so it will go in the parent node.

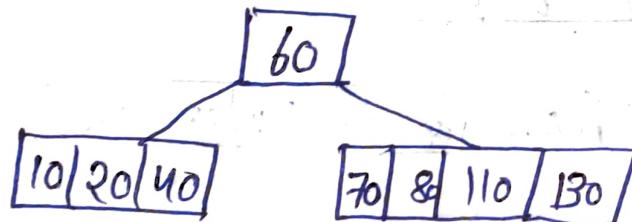
Step 6 :- Insert 40.



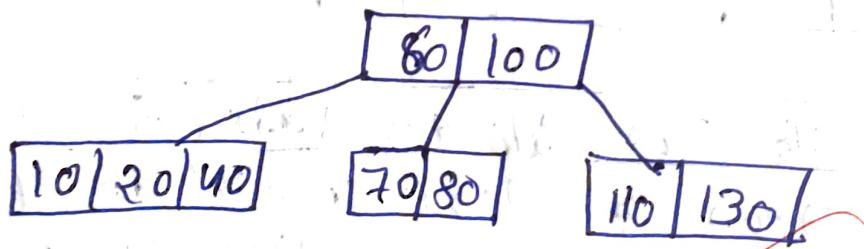
Step 7 :- Insert 80



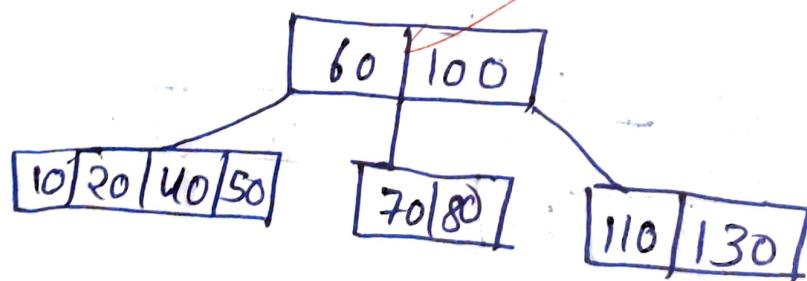
Step 8 :- Insert 130



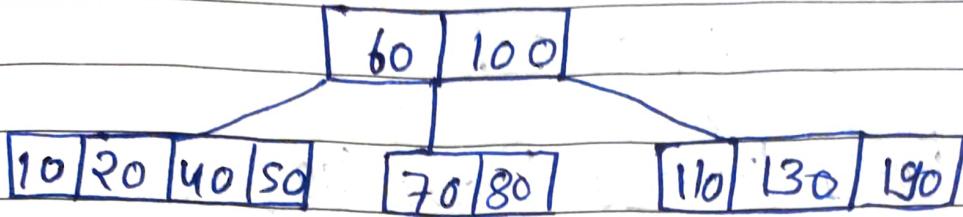
Step 9 :- Insert 100



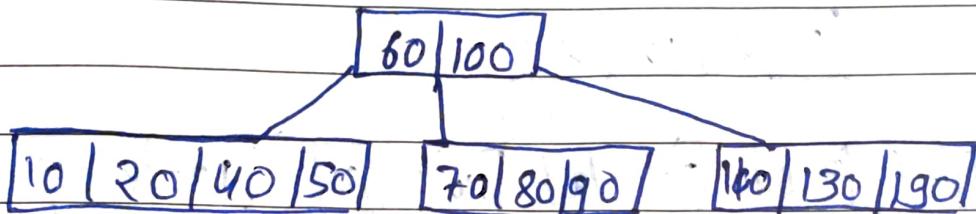
Step 10 :- Insert 50



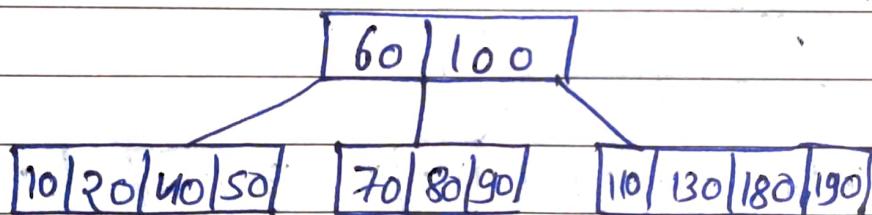
Step 11:- Insert 190



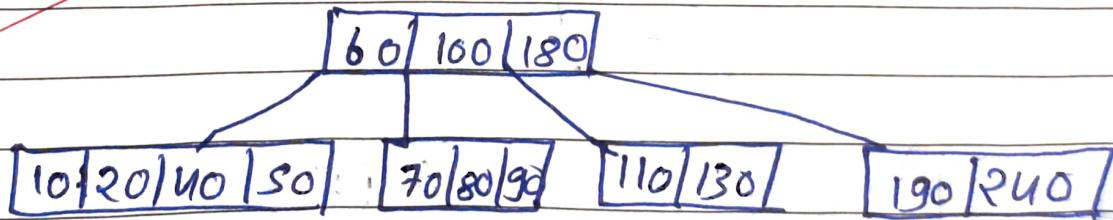
Step 12 :- Insert 90



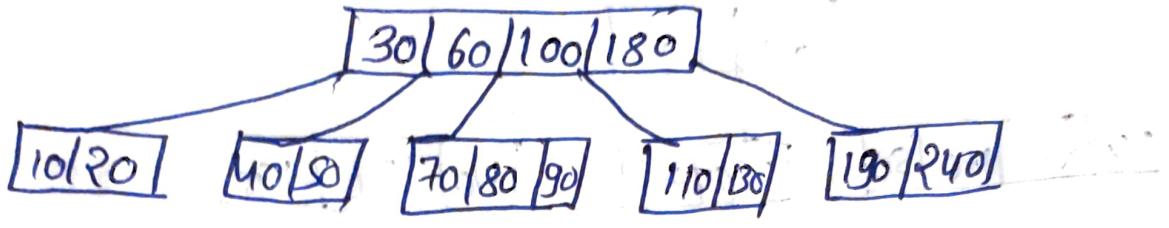
Step 13 :- Insert 180



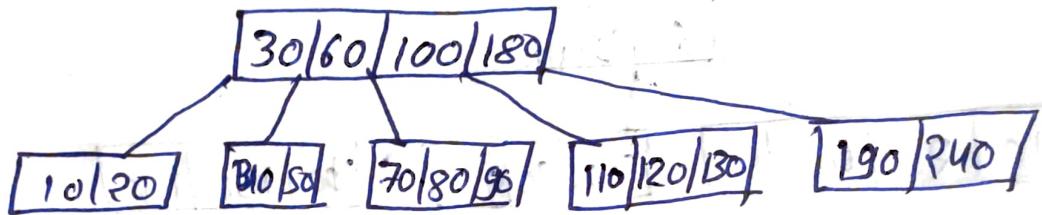
Step 14 :- 240



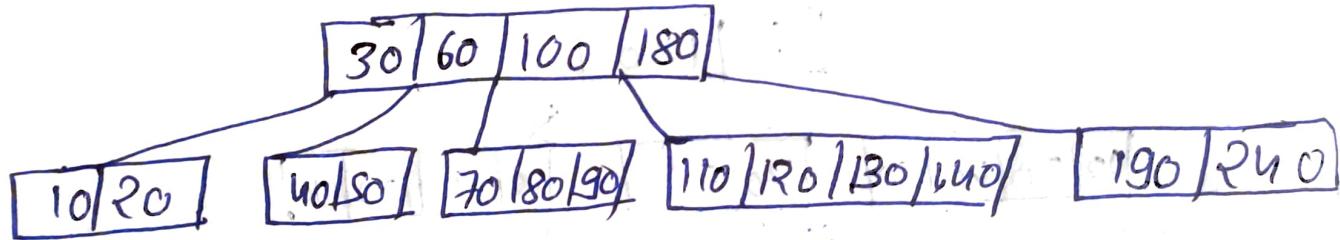
Step 15 :- 30



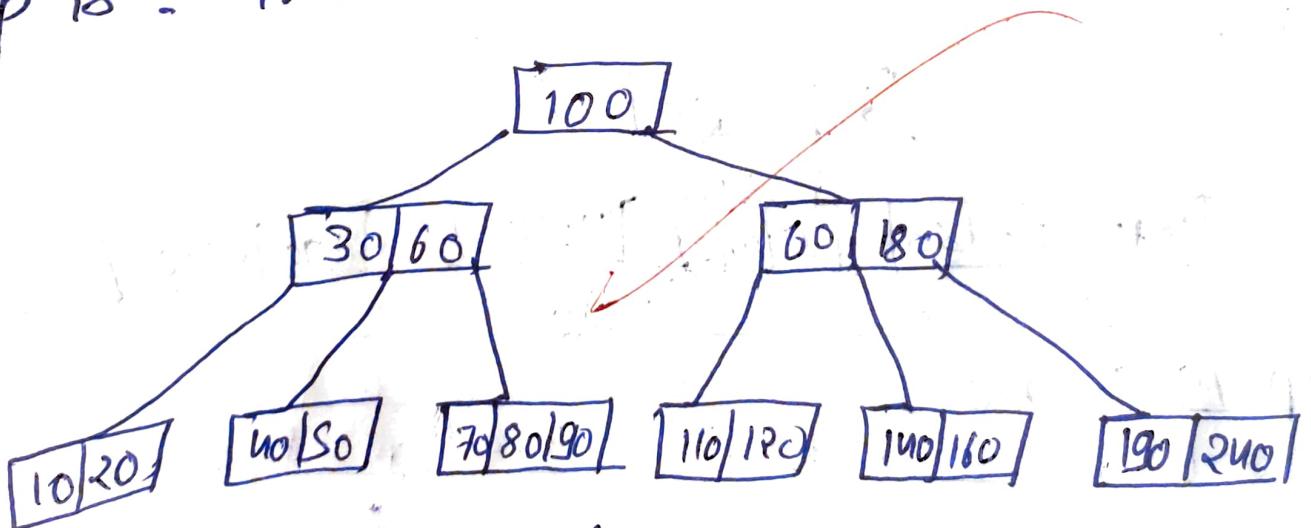
Step 16 :- 120



Step 17 :- 140



Step 18 :- 160



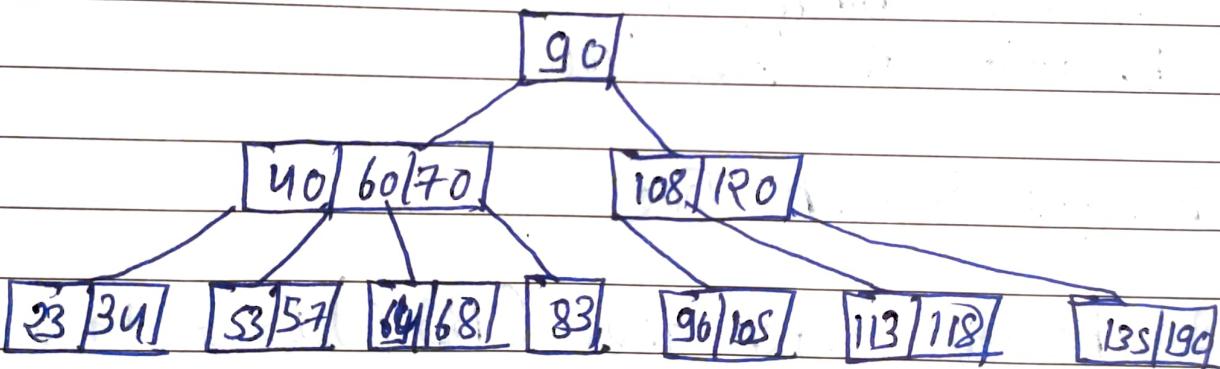
final B-tree

## Deletion in B. Tree

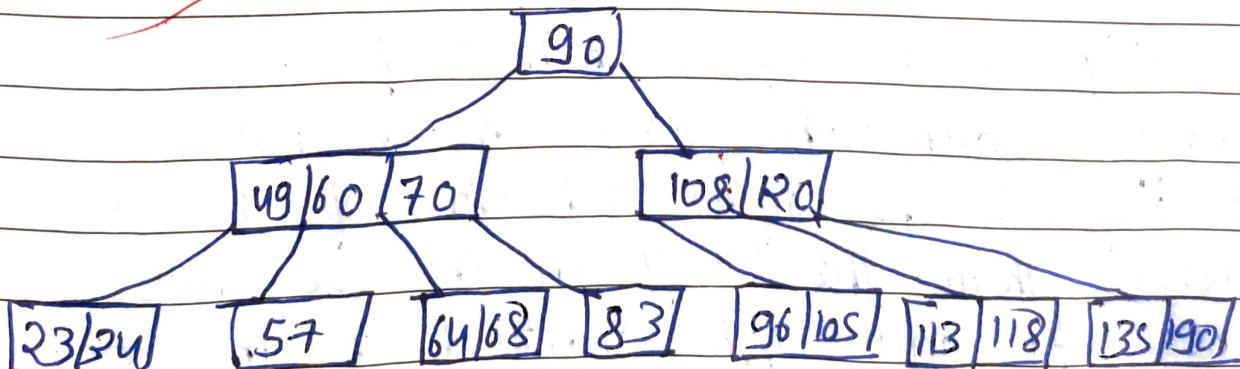
Deletion of key also requires first traversal in B-tree. After reading on particular node two case may occur :-

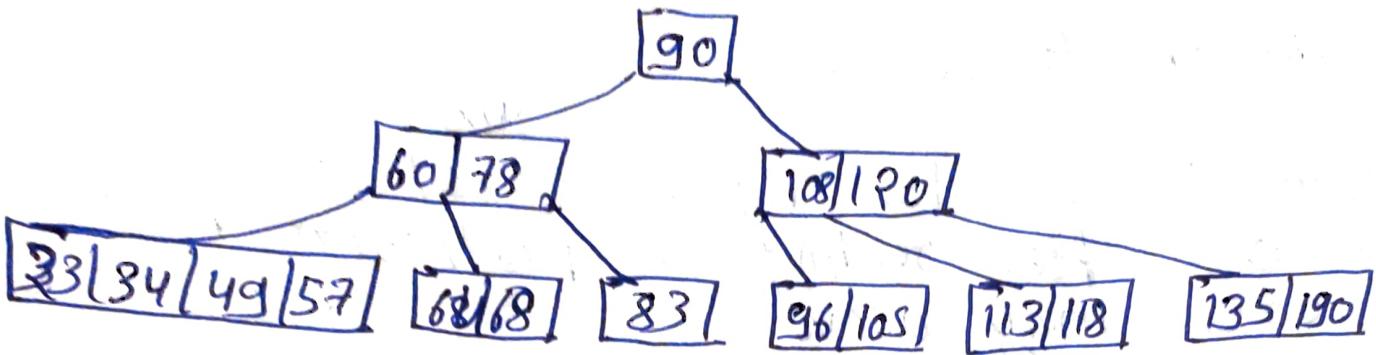
- Node is leaf node
- Node is non leaf node

### B. Tree of order 5



Delete 53





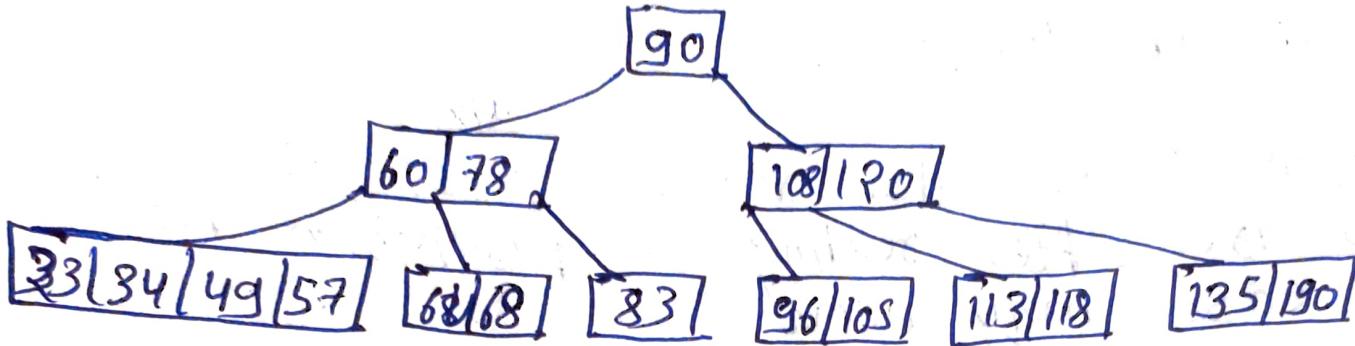
## Q6 Short Notes with example

- a] Binary Search Tree
- b] B' tree
- c] Threaded Binary Tree
- d] Deletion in AVL Tree

### a] Binary Search Tree

A binary Search tree (BST) is a tree that either empty or in which every node contain a Key (value) and satisfies the following condition

- All Key in the left sub-tree of the root are smaller than the Key in the root node.
- All Key in the right sub-tree of the root are greater than the Key in the root node
- The left and right subtrees of the root are again binary search tree.



## Q6 Shoot Notes with example

- a] Binary Search Tree
- b] B' tree
- c] Threaded Binary Tree
- d] Deletion in AVL Tree

### a] Binary Search Tree

A binary Search tree (BST) is a tree that either empty or in which every node contain a Key (value) and satisfies the following condition

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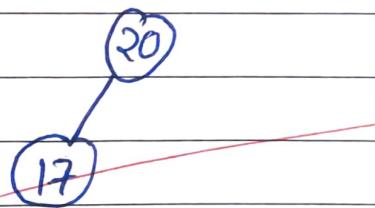
- If a binary search tree is traversed in order, then the contents of each node are printed in ascending order.
- In binary search tree we can search for an element easily an average running time

$$F(n) = O(\log_2 n)$$

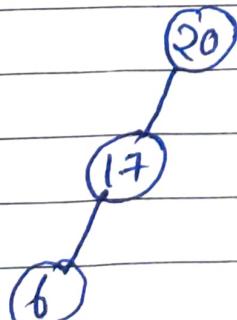
eg:- ~~50, 30, 60, 25, 40, 35, 20, 17, 6, 8, 25, 10, 23, 5, 27, 9~~

Step 1 :- 20

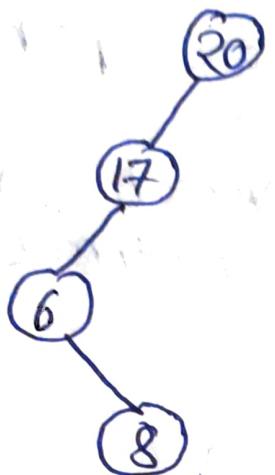
Step 2 :- Data = 17



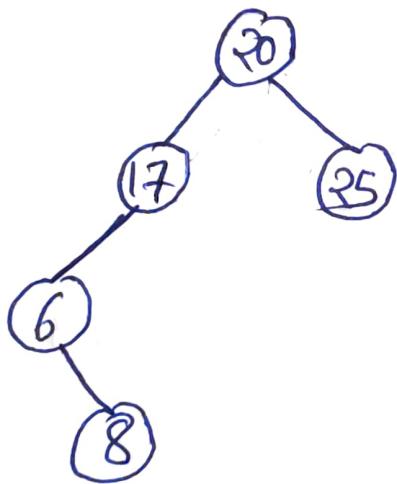
Step 3 :- Data = 6



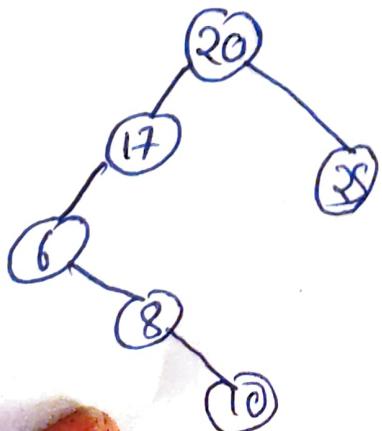
Step 4 :- Data = 8



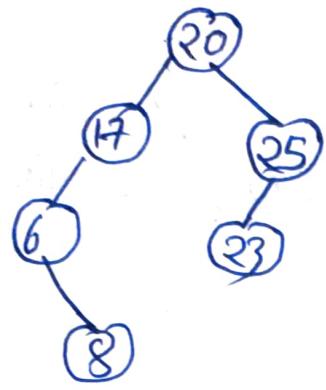
Step 5 :- Data = 25



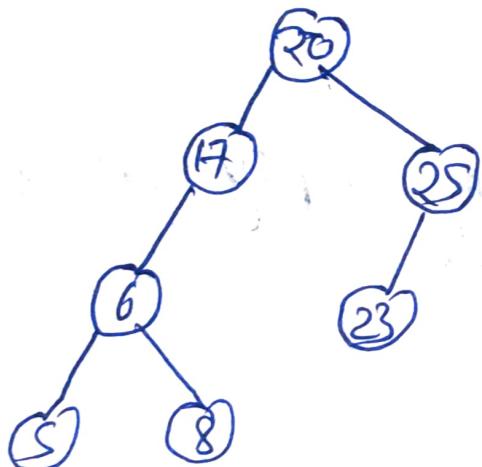
Step 6 :- Data := 10



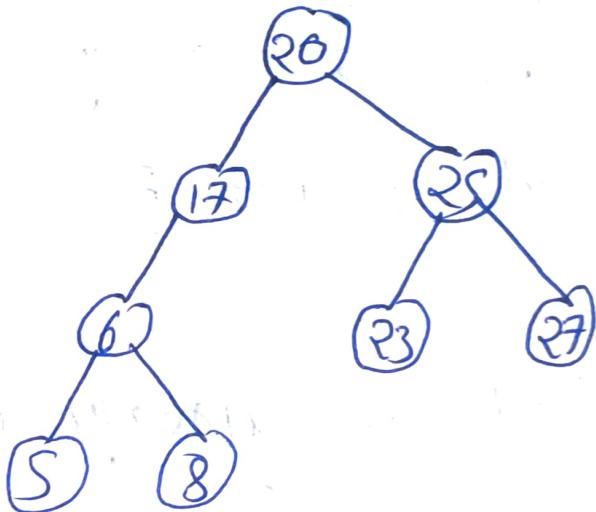
Step 7 :- Data = 23



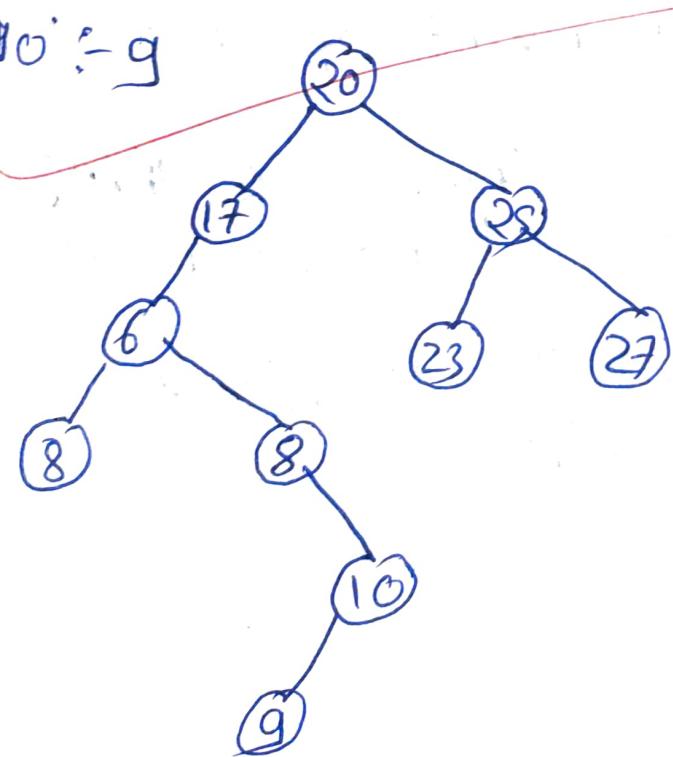
Step 8 :- Data :- 5



Step 9 :- 27



Step 10 :- g



## B Tree

→ B-tree is a balance M-way tree (5-way)  
B-Tree is a self balancing search tree

Insertion is B-Tree

① We have two case for inserting the Key

1) Node is not Full

2) Node is already full

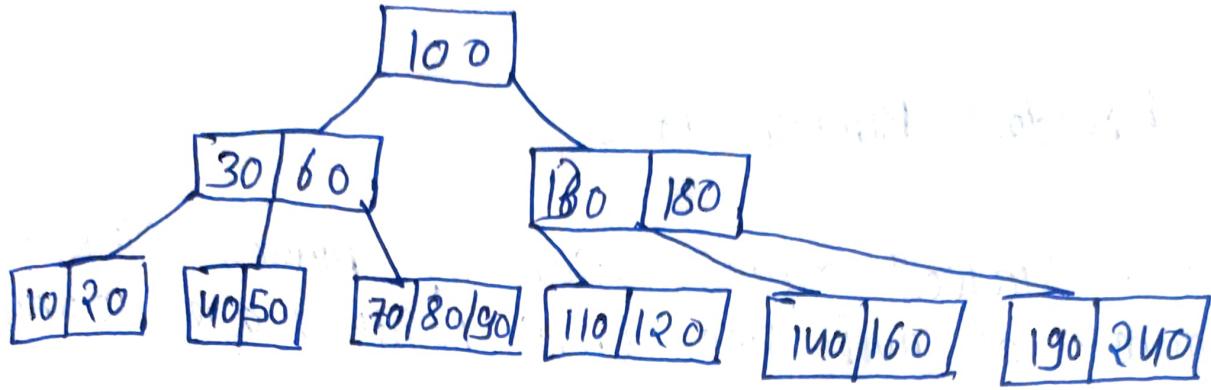
① Node is not Full :- We can simply add the key in node

② Node is already Full :-

• We need to split the node into two nodes and medium key will go to Parent of that node

• If Parent is also full then same thing will be repeated until non full Parent node

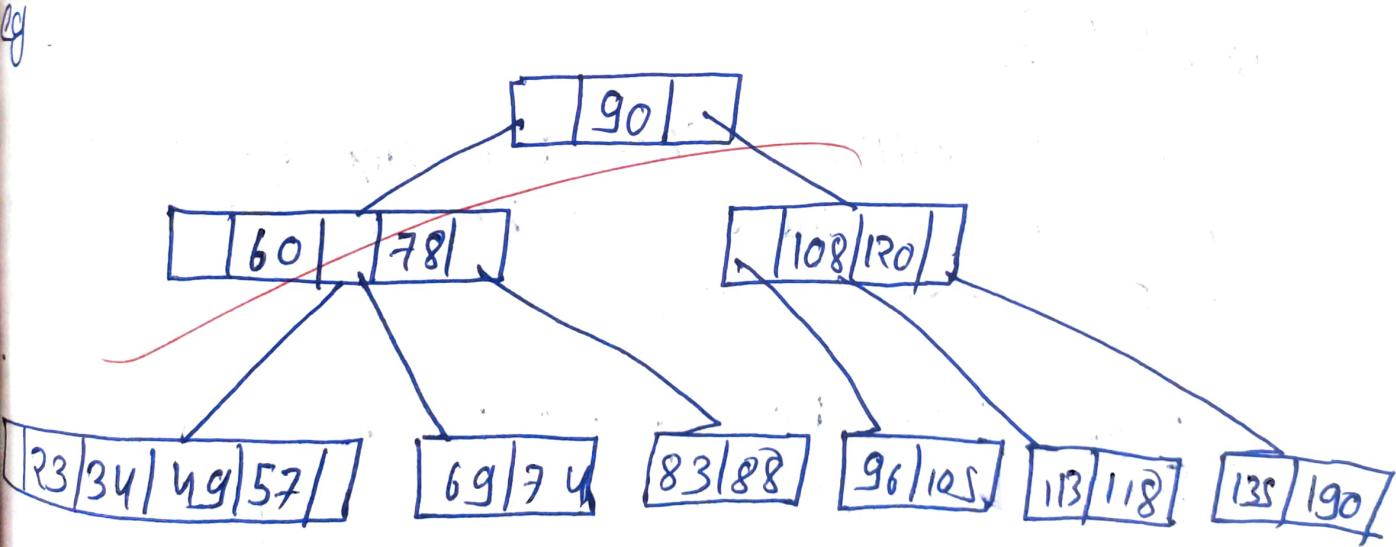
Eg:- 10, 70, 60, 20, 110, 40, 80, 130, 100, 50, 90, 190, 180, 240, 30, 120, 140, 160



## Deletion in B Tree

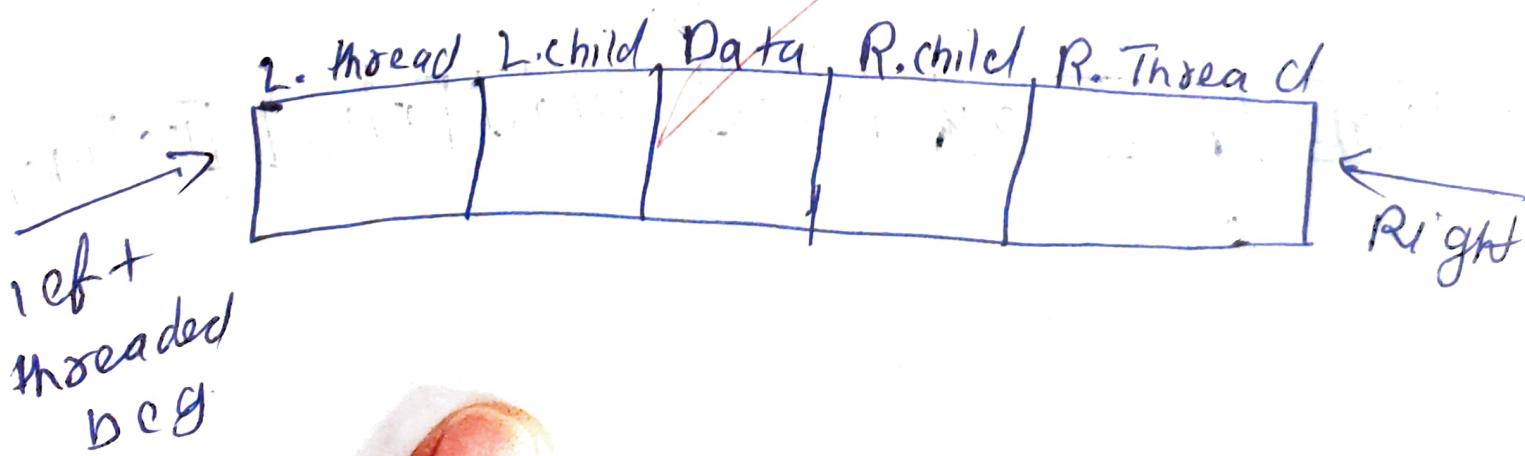
Deletion of key also requires first traversal in B-tree. After reading on particular node two case may occur

- a) Node is leaf node
- b) Node is non leaf node



## Q) Threaded Binary Tree.

- In a linked representation of binary tree, the number of null links (null pointers) are actually more than non-null pointers.
- In above binary tree, there are 8 null pointers and actual 6 pointers.
- In all there are 14 pointers.
- The objective here to make effective use of those null pointers.
- And binary tree with such pointers are called Threaded tree.
- We can generalize it that for any binary tree with  $n$  nodes, there will be  $(n+1)$  null pointers and  $2n$  total pointers.

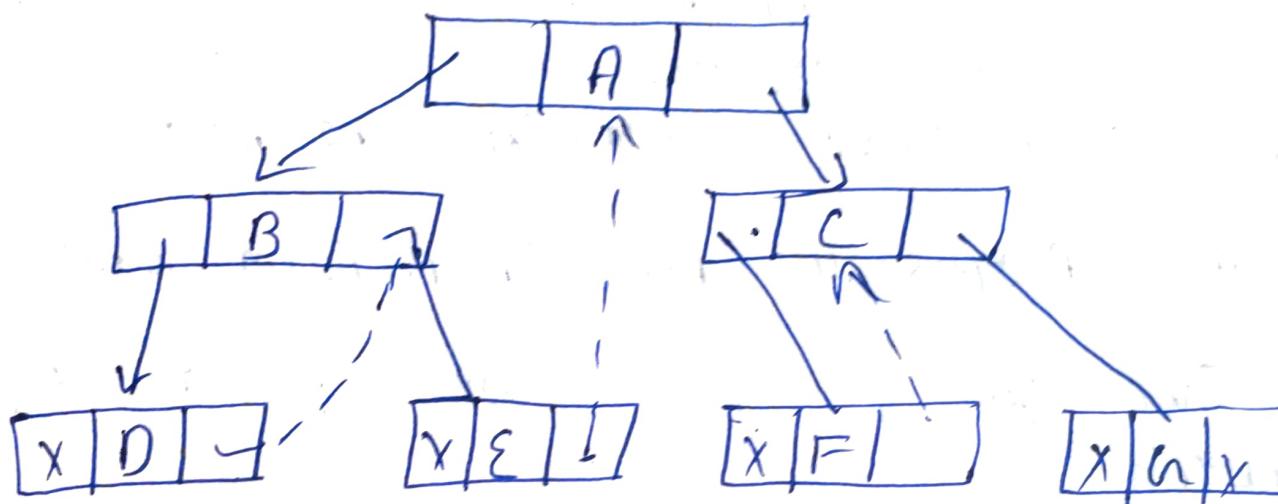


Types of Threaded Binary Tree :-

one way Threading

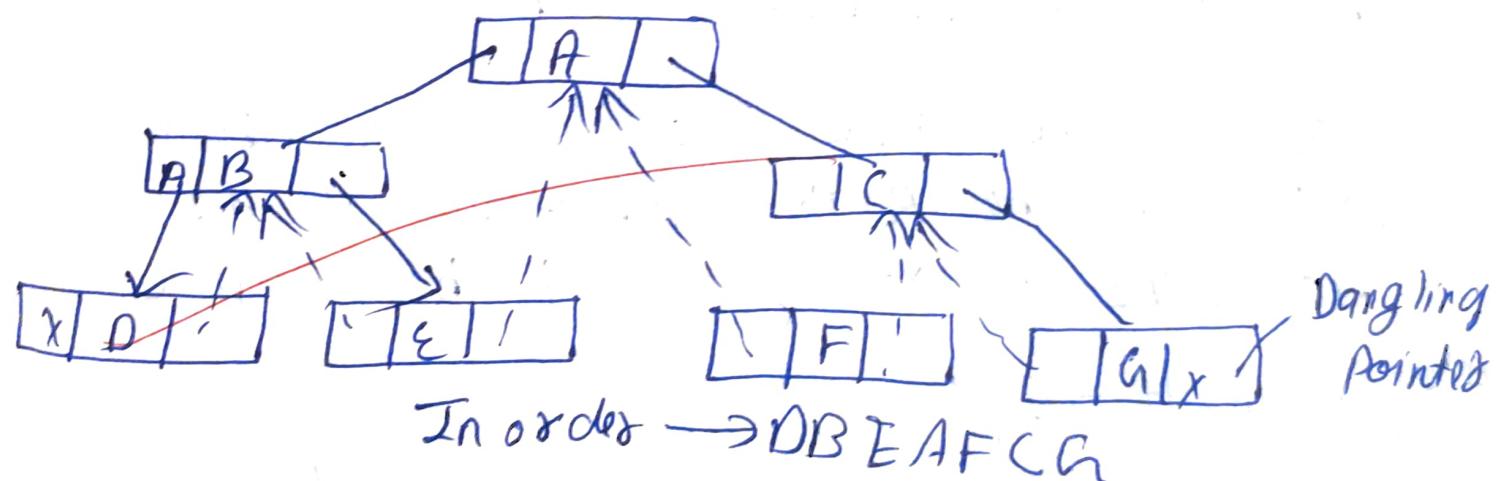
Two way Threading

one way Threading



Right Pointer = next node of in order

of Two way Threading.



In order  $\rightarrow$  DBEAFCA

Dangling  
Pointers

Right Pointer = next node in -order traversal.

Left Pointer = previous node of in order traversal

- 4) Deletion in AVL Tree
- 5) Deleting a node from an AVL tree is similar to that in a binary search tree
- 6) Deletion may disturb the balance factor of an AVL tree and therefore the tree needs to be rebalanced in order to maintain the balance.
- 7) For this purpose, we need to perform rotation
- 8) The two types of rotations are left and right rotation.
- 9) If the node which is to be deleted is present in the left subtree of the critical node then L rotation needs to be applied
- 10) Else if the node which is to be deleted is present in the right subtree of the critical node, the R rotation will be applied

