Different types of functions

Types of Functions (Mapping ke Basis Par)

1. One-to-One (1:1) Functions / Injective Functions

Definition:

- Domain ke har element ki ek distinct (unique) image hoti hai codomain mein
- Koi bhi do different elements same image ko point nahi karte

Mathematical Definition:
Agar $f(x_1) = f(x_2)$ hai to $x_1 = x_2$ hona chahiye

Example:
$f(x) = 3x + 5$ (ye ek one-to-one function hai)

Checking Methods:

a) Graphical Method - Horizontal Line Test

- Graph par horizontal line draw karo
- Agar horizontal line curve ko sirf ek point par intersect kare to function one-to-one hai
- Agar multiple points par intersect kare to many-to-one hai

b) Analytical Method

- Do values x_1 aur x_2 lo (different)
- $f(x_1) = f(x_2)$ assume karo
- Agar $x_1 = x_2$ prove ho jaye to one-to-one hai
- Agar $x_1 \neq x_2$ to many-to-one hai

c) Derivative Method (for Continuous Functions)

- $f'(x)$ calculate karo
- Agar $f'(x) > 0$ (hamesha positive) ya $f'(x) < 0$ (hamesha negative) to one-to-one hai
- Agar $f'(x)$ kabhi positive kabhi negative to many-to-one hai

2. Many-to-One Functions / Non-Injective Functions

Definition:

- Domain ke multiple elements same image ko point karte hain
- Codomain ke kisi element ki multiple pre-images hoti hain

Example:

- $f(x) = x^2$ (when $x \in R$) - ye many-to-one hai
- $f(x) = \sin x$ (when $x \in R$) - ye bhi many-to-one hai

Important Examples aur Solutions

Example 1: $f(x) = \sin x$ ($x \in R$)

Analysis:

- sin x ka graph periodic hai
- Horizontal line test fail ho jata hai
- Multiple x values ke liye same sin value milti hai
- Result: Many-to-one function hai

Example 2: $f(x) = \sin x$ ($x \in N$ - Natural Numbers)

Special Case:

- Jab domain restrict kar diya natural numbers tak
- Tab ye one-to-one ban jata hai
- Kyunki natural numbers mein sin x ki values repeat nahi hoti same way mein

Example 3: $f(x) = x^3 + x + 1$

Derivative Method:

- $f'(x) = 3x^2 + 1$
- $3x^2 \geq 0$ hamesha (kyunki square hamesha non-negative)
- $3x^2 + 1 \geq 1 > 0$ hamesha
- $f'(x) > 0$ hamesha positive hai
- Result: Strictly increasing function hai, therefore one-to-one hai

Important Algebraic Identity

$a^3 - b^3$ Formula

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Pattern yaad rakhne ka tarika:

- Pehla bracket: (a - b) - same sign as original
- Dusra bracket: a^2 aur b^2 hamesha positive
- Middle term ab ka sign opposite hoga original ke

Similarly:
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Key Points to Remember

1. One-to-one functions ko Injective functions bhi kehte hain

2. Polynomial functions hamesha continuous hote hain
3. Horizontal line test sabse easy graphical method hai
4. Derivative method sirf continuous functions ke liye use karo
5. Domain restriction se many-to-one function ko one-to-one banaya ja sakta hai

Practice Tips

- Har function ko three methods se check karne ki practice karo
- Trigonometric functions ke behavior ko samjho
- Polynomial functions ke liye derivative method fastest hai
- Graph sketching ki practice karo horizontal line test ke liye

Ye notes JEE preparation ke liye bahut important hain aur exam mein direct questions aate hain functions ki nature check karne ke!