important examples

Onto and Into Functions ke Saare Examples - Detailed Solutions (Hinglish)

Example 1: $f(x) = x^2/(x^2 + 1)$ - Surjection ke liye Codomain find karna

Question: Agar function f: R → A given by $f(x) = x^2/(x^2 + 1)$ surjection hai, to A kya hoga?

Step-by-Step Solution:

Step 1: Function ko samjho

- Function $f(x) = x^2/(x^2 + 1)$ hai
- Domain = R (sabhi real numbers)
- Codomain A = ? (ye find karna hai)

Step 2: Range निकालना

Range निकालने ke liye, $y = f(x)$ assume करते हैं:

$y = x^2/(x^2 + 1)$

Step 3: Equation rearrange करना

```text
y = x²/(x² + 1)
y(x² + 1) = x²
yx² + y = x²
y = x² - yx²
y = x²(1 - y)
```

Step 4: x^2 के लिए solve करना

```text
x² = y/(1 - y)
```

Step 5: Condition लगाना

Since $x^2 \geq 0$ (square हमेशा non-negative), इसलिए:

```text
y/(1 - y) ≥ 0
```

Step 6: Inequality solve करना

$y/(1 - y) \geq 0$ का मतलब है:

- Case 1: $y \geq 0$ और $(1 - y) > 0$
    - य भाव: $0 \leq y < 1$
- Case 2: $y \leq 0$ और $(1 - y) < 0$
    - य भाव: $y \leq 0$ और $y > 1$ (कोई solution नहीं)

Step 7: Additional boundary check

जब $x = 0$: $f(0) = 0^2/(0^2 + 1) = 0$
जब $x \to \pm\infty$: $f(x) \to 1$ (लेकिन कभी 1 तक नहीं पहुंचता)

Step 8: Final Range

Range of $f(x) = [0, 1)$

Answer: A = [0, 1)

क्यों? Surjection के लिए range = codomain होना चाहिए, इसलिए $A = [0, 1)$

Example 2: $f(x) = e^{(x^2)} + \cos(x)$ - Function type identify करना

Question: Function $f: R \to R$, $f(x) = e^{(x^2)} + \cos(x)$ का type identify करो।

Step-by-Step Solution:

Step 1: Function components analyze करना

- $e^{(x^2)}$: हमेशा ≥ 1 (क्योंकि $x^2 \geq 0$)
- $\cos(x)$: range $[-1, 1]$ में vary करता है

Step 2: Range निकालना

Minimum value:

- जब $\cos(x) = -1$: $f(x) = e^{(x^2)} - 1 \geq 1 - 1 = 0$
- Actually minimum: $f(x) = e^0 + (-1) = 1 - 1 = 0$ (जब $x = 0$ और $\cos(0) = 1$)

Wait, let me recalculate:

- जब $x = 0$: $e^0 = 1$, $\cos(0) = 1$, so $f(0) = 2$
- Minimum होगा जब $\cos(x) = -1$: $f(x) = e^{(x^2)} - 1$

Actually, minimum value होगा जब x ऐसा हो कि $\cos(x) = -1$ और $e^{(x^2)}$ minimum हो।
Since $e^{(x^2)} \geq 1$ always, minimum value = $1 - 1 = 0$

Maximum value: $+\infty$ (क्योंकि $e^{(x^2)} \to \infty$ as $x \to \pm\infty$)

Step 3: Range determine करना

Range = $[0, +\infty)$

Step 4: Codomain के साथ compare करना

- Codomain = $R = (-\infty, +\infty)$

- Range = [0, +∞)
- Range ≠ Codomain

Step 5: One-to-one check करना

Derivative निकालते हैं:

```text
f'(x) = d/dx[e^(x²) + cos(x)]
f'(x) = 2x·e^(x²) - sin(x)
```

Analysis:

- $f'(x) = 2x \cdot e^{(x^2)} - \sin(x)$
- ye expression कभी positive कभी negative हो सकता है
- इसलिए function neither strictly increasing nor strictly decreasing

Step 6: Function behavior

- $x = 0$ पर: $f'(0) = 0 - 0 = 0$
- $x > 0$ पर: $2x \cdot e^{(x^2)}$ term dominant हो जाता है (positive)
- $x < 0$ पर: $2x \cdot e^{(x^2)}$ negative होता है

Answer:

$f(x) = e^{(x^2)} + \cos(x)$ है:

1. Into function (क्योंकि range ≠ codomain)
2. Many-to-one function (क्योंकि strictly monotonic नहीं है)

Example 3: Number of Onto Functions - Counting Problem

Question: A = {1, 2, 3} से B = {a, b} में कितने onto functions possible हैं?

Step-by-Step Solution:

Step 1: Formula apply करना

Onto functions का formula:

```text
Number of onto functions = Σ(r=1 t
```

जहाँ $m = |A| = 3$, $n = |B| = 2$

Step 2: Calculation

```text
```

```
= Σ(r=1 to 2) [(-1)^(2-r) × ²Cᵣ ×
= [(-1)¹ × ²C₁ × 1³] + [(-1)⁰ × ²C₂
= [-1 × 2 × 1] + [1 × 1 × 8]
= -2 + 8 = 6
```

Step 3: Verification by listing

Onto functions:

1. f = {(1,a), (2,a), (3,b)}
2. f = {(1,a), (2,b), (3,a)}
3. f = {(1,b), (2,a), (3,a)}
4. f = {(1,a), (2,b), (3,b)}
5. f = {(1,b), (2,a), (3,b)}
6. f = {(1,b), (2,b), (3,a)}

Answer: 6 onto functions possible हैं

Example 4: Composite Function Analysis

Question: f: R → R, f(x) = 2x + 1 और g: R → R, g(x) = x^2 हैं। (g∘f) onto है या into?

Step-by-Step Solution:

Step 1: Composite function निकालना

```text
(g∘f)(x) = g(f(x)) = g(2x + 1) = (
```

Step 2: Range निकालना

```text
(g∘f)(x) = (2x + 1)²
```

Since $(2x + 1)^2 \geq 0$ for all $x \in R$

जब 2x + 1 = 0, i.e., x = -1/2: minimum value = 0
जब x → ±∞: (g∘f)(x) → +∞

Step 3: Range vs Codomain

- Range = [0, +∞)
- Codomain = R = (-∞, +∞)
- Range ≠ Codomain

Answer: (g∘f) एक into function है

Key Takeaways:

1. Range निकालना सबसे important step है
2. Boundary conditions check करना जरूरी है
3. Derivative analysis one-to-one check के लिए use करें
4. Composite functions में individual functions के properties combine होते हैं
5. Counting formulas याद रखें onto functions के लिए

Ye सारे examples JEE mein high-frequency questions हैं और proper understanding se आसानी से solve हो जाते हैं!