onto and into functions

. Onto Function (Surjective Function)

Definition:

- Onto function wo hota hai jisme codomain ka har element kam se kam ek pre-image domain mein rakhta hai
- Matlab codomain ka koi bhi element unmapped nahi rehta
- Range = Codomain hota hai onto functions mein
- Ise Surjective Function bhi kehte hain

Mathematical Definition:

Function f: A → B onto hai agar:
∀ y ∈ B, ∃ x ∈ A such that f(x) = y

Visual Representation:

- Arrow diagram mein har codomain element par kam se kam ek arrow aata hai
- Koi element unmapped nahi rehta

Example:

```text
Domain A = {1, 2, 3}
Codomain B = {a, b}
f = {(1, a), (2, b), (3, a)}
```

Ye onto hai kyunki B ke dono elements (a, b) mapped hain

2. Into Function (Non-Surjective Function)

Definition:

- Into function mein codomain ka kam se kam ek element unmapped rehta hai
- Range ⊂ Codomain (proper subset)
- Range aur codomain kabhi equal nahi hote
- Ise Non-Surjective Function bhi kehte hain

Mathematical Condition:

Function f: A → B into hai agar:
∃ y ∈ B such that ∀ x ∈ A, f(x) ≠ y

Example:

```
text
Domain P = {1, 2, 3}
Codomain Q = {7, 8, 9, 10}
f = {(1, 7), (2, 9), (3, 8)}
```

Ye into hai kyunki element 10 unmapped hai

3. Onto vs Into Functions - Complete Comparison

Feature	Onto Function (Surjective)	Into Function (Non-Surjective)
Range vs Codomain	Range = Codomain	Range ⊂ Codomain
Mapping Coverage	Har element mapped	Kam se kam ek unmapped
Pre-image	Har y ka kam se kam ek x	Kuch y ka koi x nahi
Alternative Name	Surjective	Non-Surjective
Inverse Function	Possible (conditions apply)	Not possible

4. Important Checking Methods

For Onto Functions:

1. Range निकालो function ka
2. Check करो range = codomain hai ya nahi
3. Agar equal hai to onto, nahi to into

Graphical Method:

- Horizontal line test - har horizontal line graph को kam se kam ek point par cut करे
- Onto: हर y-value के लिए कम से कम एक x-value मिले

5. JEE Important Examples

Example 1: $f(x) = x^2/(x^2 + 1)$

Question: Agar ye function surjection hai to codomain A kya hoga?

Solution Process:

- Range निकालना होगा
- f'(x) से critical points find करना
- Range = (0, 1) aayega
- Answer: A = (0, 1) होगा onto function ke liye

Example 2: $f(x) = e^{x^2} + \cos x$

Function type identify करना:

- e^{x^2} हमेशा ≥ 1
- cos x range [-1, 1] मे
- Combined range होगी अगर codomain R है
- Into function होगा

6. Number of Onto Functions Formula

Agar A mein m elements aur B mein n elements hain ($1 \le n \le m$):

Number of onto functions = $\Sigma(r=1 \text{ to } n) [(-1)^{(n-r)} \times {}^nC_r \times r^m]$

7. Key Properties to Remember

Onto Functions:

- Range = Codomain hamesha
- Inverse function ka chance hai (agar one-to-one bhi ho)
- Composition of two onto functions is onto
- 3D graphics mein vector projection mein use hota hai

Into Functions:

- Range ≠ Codomain hamesha
- No inverse function possible
- Codomain mein unused elements hote hain
- Efficiency ke liye sometimes preferred

8. Common JEE Question Patterns

1. Function type identify करना
2. Range निकालकर onto/into decide करना
3. Codomain modify करके onto बनाना
4. Counting problems - kitne onto/into functions possible

5. Composite functions का behavior

9. Important Tips for JEE

- Range हमेशा calculate करो before concluding
- Domain restrictions को carefully देखो
- Trigonometric functions के standard ranges याद रखो
- Exponential aur logarithmic functions के behavior समझो
- Modulus functions के cases separately handle करो

10. Practice Strategy

1. Basic definitions strong रखो
2. Standard functions ke ranges याद करो
3. Graphical visualization practice करो
4. Previous year questions solve करते रहो
5. Composite functions पे focus करो

Ye concepts JEE mein direct questions के साथ-साथ function composition, inverse functions, aur advanced calculus mein bhi use होते hain!
