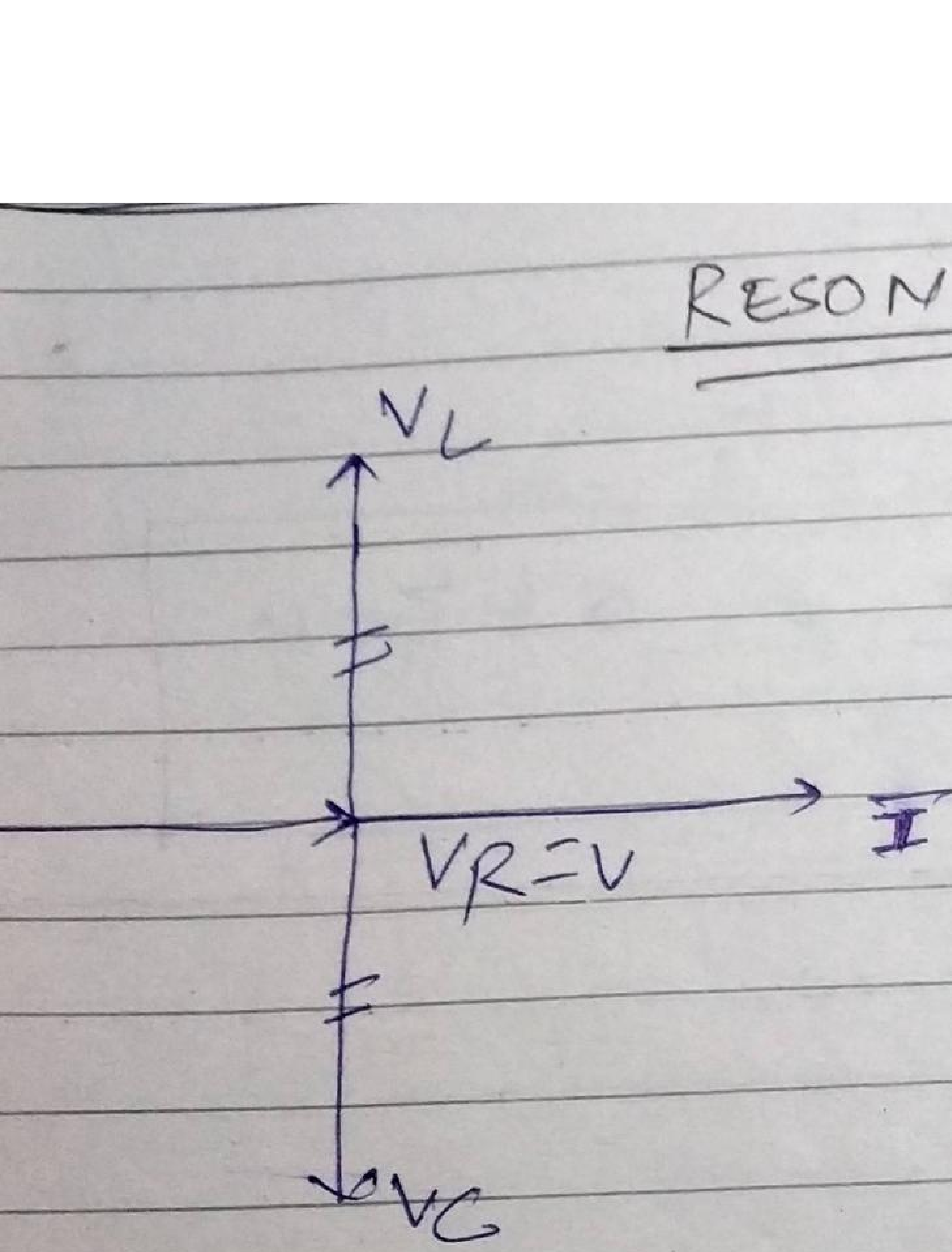
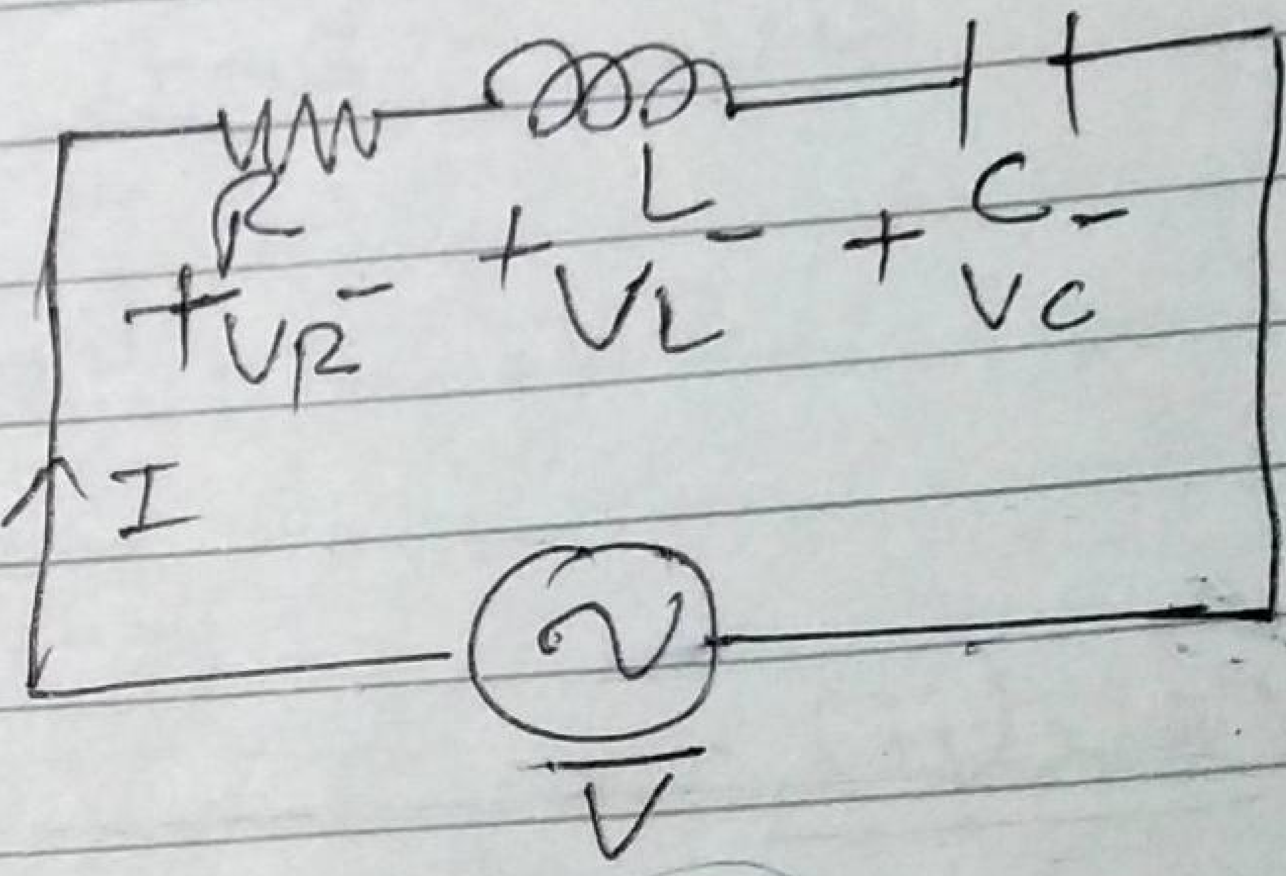
RESONANCE :-





$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}}$$

V_L and V_C are equal and opposite so $V = V_R$

Resonance happens when reactive component is zero.

At Resonance

$V_L = V_C$

$IX_L = IX_C$

$X_L = X_C$

$\omega_0 L = \frac{1}{\omega_0 C}$

$\omega_0^2 = \frac{1}{LC}$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$$

→ For the occurance of Resonance in any system two energies are required.

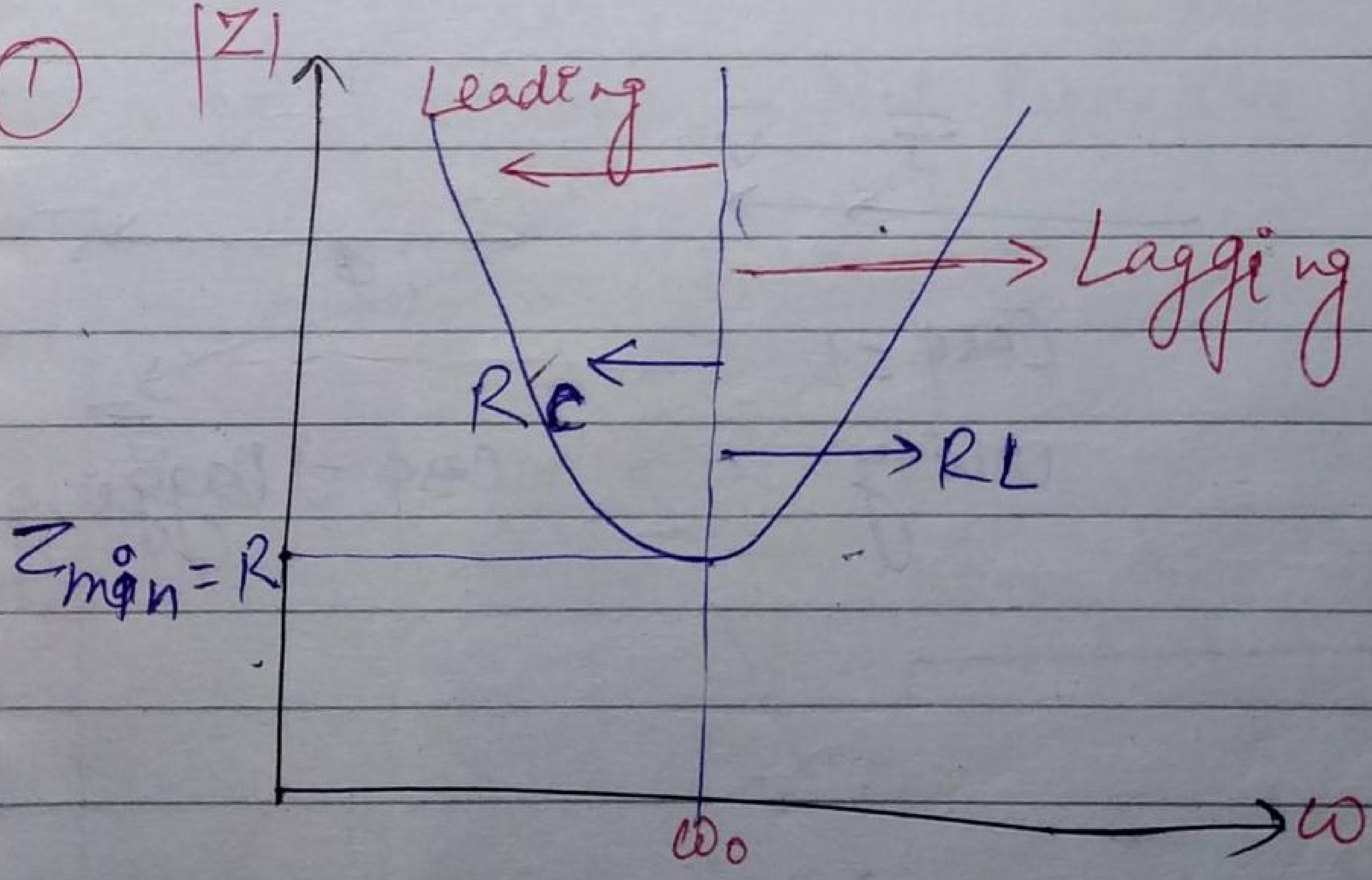
→ In an electrical system these two energies are provided by inductor and capacitor in the form of magnetic and electric field respectively.

→ The ckt is said to be in Resonance when source voltage and source current are in phase.

→ The Resonant frequency indicates the rate at which energy transformation takes place b/w the inductor and capacitor.

$$Z = R + j(X_L - X_C) \quad Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z| = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

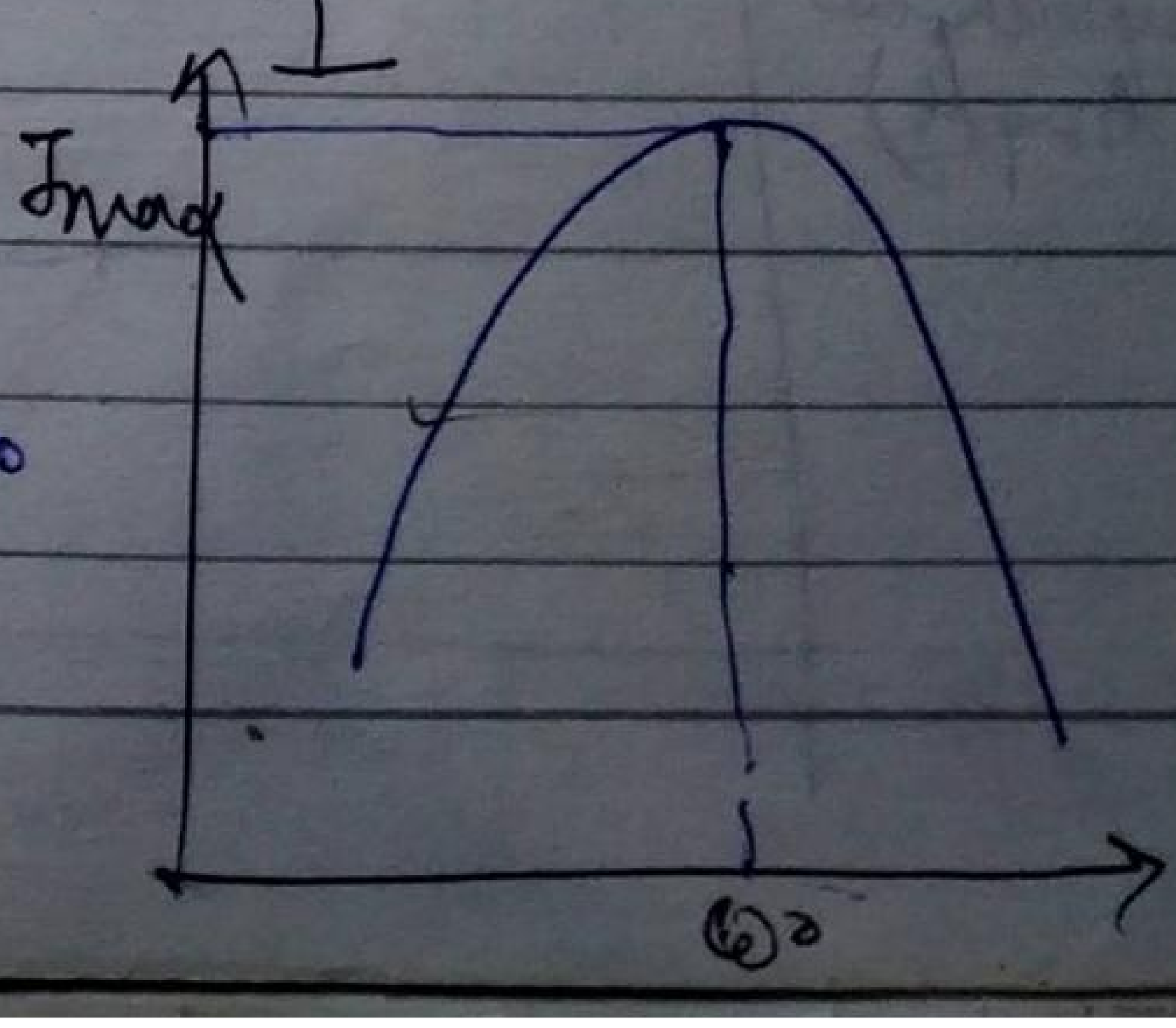
① 

at ω_0 $|Z| = min$

② $I = \frac{V}{|Z|}$

means at ω_0

at Z_{min} current will be max



→ From the graphs we can conclude the following points–

→ for $\omega < \omega_0$ the capacitive Reactance > Inductive Reactance $[X_C > X_L]$ and therefore the net Reactance capacitive in nature. And Hence the Current $\bar{I}$ leads the applied voltage $\bar{V}$ and gives the leading pf.

→ At $\omega = \omega_0$ both the reactances are equal and hence the impedance is min i.e. $Z = R$, the Current $\bar{I}$ and $\bar{V}$ are in phase and hence the pf is unity.

→ At $\omega > \omega_0$ the inductive Reactance > Capacitive Reactance $[X_L > X_C]$ and ∴ the net reactance is inductive in nature and the current '$\bar{I}$' lags the applied voltage $\bar{V}$. Giving lagging pf.

→ At Resonance Net Reactive voltage = 0

$$I = \frac{V}{R + j(X_L - X_C)} \qquad |I| = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Voltage across capacitor —

$$V_C = I \cdot X_C$$

$$V_C = \frac{V}{\omega C \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}} \Rightarrow \frac{dV_C}{d\omega} = 0$$

$$\boxed{f_C = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R^2 C}{2L}}}$$ → The frequency at which V_C is max.

Voltage across inductor —

$$V_L = I X_L = \frac{V}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}}$$

$$\frac{dV_L}{d\omega} = 0 \qquad \boxed{f_L = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(\frac{1}{1 - \frac{R^2 C}{2L}}\right)}}$$

→ The frequency at which V_L is max.

Quality Factor :

→ QF is a Ratio of the max energy stored in the ckt to the energy dissipated in the ckt in one time period.

$$Q = 2\pi \times \frac{\text{Max energy stored in the ckt}}{(\text{Energy dessipated per cycle of oscillation})}$$

$$i(t) = I_m \sin\omega t$$

$$W_g = \frac{1}{2} L i^2 + \frac{1}{2} C V_C^2$$

$$V_C = \frac{1}{C}\int i\,dt$$

$$V_C = \frac{1}{C}\int I_m \sin\omega t\,dt = -\frac{I_m}{\omega C}\cos\omega t$$

$$W_g = \frac{1}{2} L [I_m \sin\omega t]^2 + \frac{1}{2} C\left[-\frac{I_m}{\omega C}\cos\omega t\right]^2$$

$$W_g = \frac{1}{2} L I_m^2 \sin^2\omega t + \frac{1}{2}\cancel{C}\frac{I_m^2}{\omega^2 C^{\cancel{2}}}\cos^2\omega t$$

$$\omega = \frac{1}{\sqrt{LC}} \rightarrow \omega^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega^2 C}$$

$$W_g = \frac{1}{2} L I_m^2 \sin^2\omega t + \frac{1}{2} L I_m^2 \cos^2\omega t$$

$$W_g = \frac{1}{2} L I_m^2 [\sin^2\omega t + \cos^2\omega t]$$

$$\boxed{W_g = \frac{1}{2} L I_m^2}$$

$$Q = 2\pi \frac{\frac{1}{2} L I_m^2}{\left(\frac{I_m}{\sqrt{2}}\right)^2 R \cdot \frac{1}{f}} \Rightarrow \boxed{Q = \frac{\omega L}{R}}$$

$$Q = \frac{\frac{1}{\sqrt{LC}} \cdot L}{R} \qquad \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Scanned by TapScanner

$$Q = \frac{\omega L}{R} = \frac{X_L}{R}$$

$$Q = \frac{I X_L}{I \times R} = \frac{V_L}{V_R} = \frac{V_C}{V_R}$$ (At Resonance $V_L = V_C$)

$$Q = \frac{I X_C}{I R} = \frac{X_C}{R} = \frac{1}{\omega R C}$$

$$\boxed{Q = \frac{\omega L}{R} = \frac{1}{\omega R C}}$$

$$\boxed{Q = \frac{\text{Reactive Component of the voltage}}{\text{Active}}}$$

$$\boxed{Q = \frac{I^2 X_L}{I^2 R} = \frac{Q_L}{P}}$$

At $\omega = \omega_0$

1) $V_R = I_R \cdot R = IR = \frac{V}{R} \cdot R \Rightarrow V$ $\boxed{V_R = V}$

2) $V_L = j X_L I_L = j X_L I = j \omega L \frac{V}{R} = j\left(\frac{\omega L}{R}\right) V$

$V_L = +jQ\,V$ — $\underbrace{+jQ}_{\text{Amplification factor}}$

both are 180 out of phase
so when we add
$V = V_L + V_R + V_C$
$\boxed{V = V_R}$

3) $V_C = -jX_C I_C = -jX_C I$

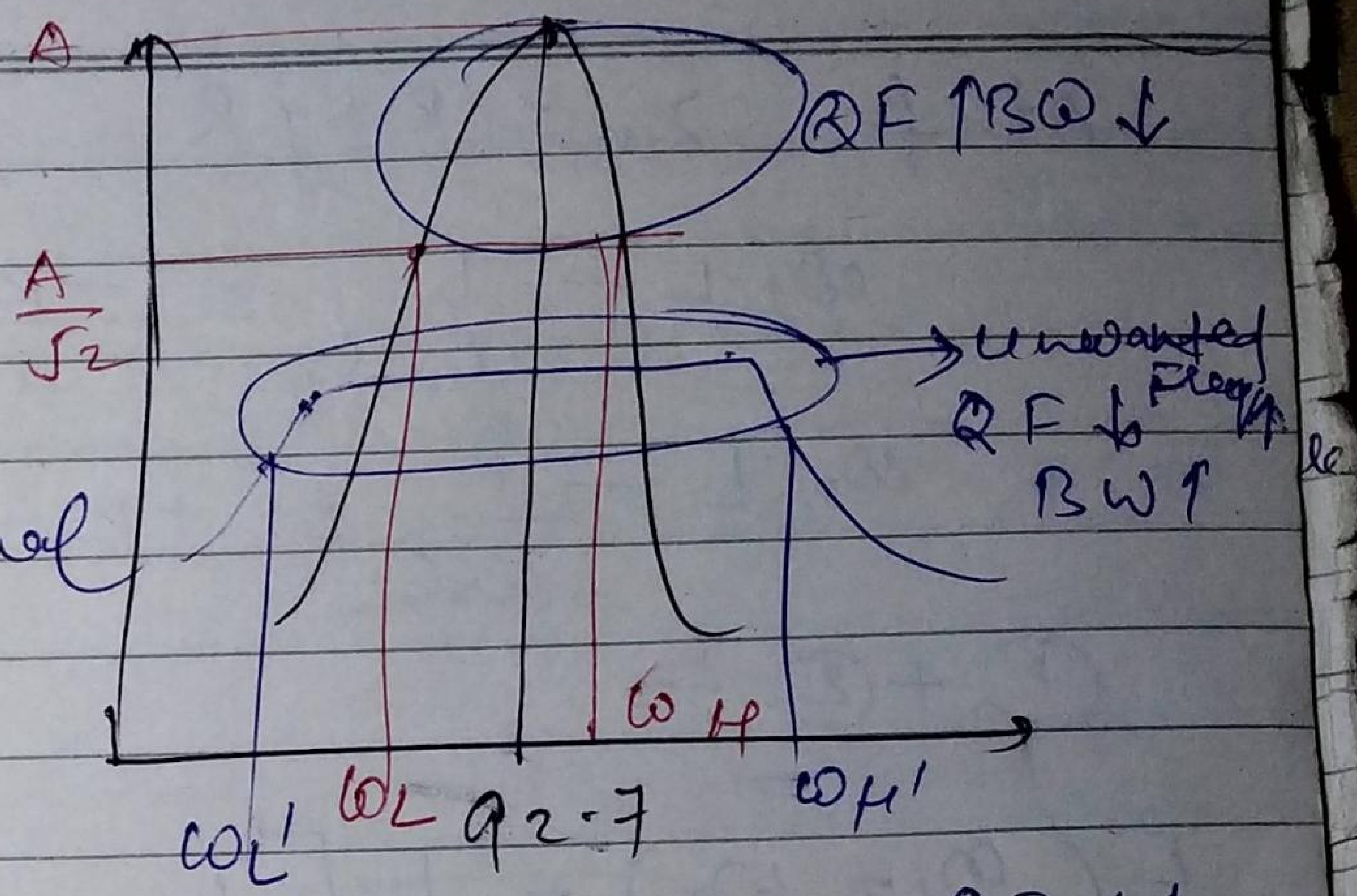
$= -jX_C \frac{V}{R} = -j\left(\frac{1}{\omega R C}\right) V$

$V_C = -jQ\,V$

Amplification factor

Bandwidth :-

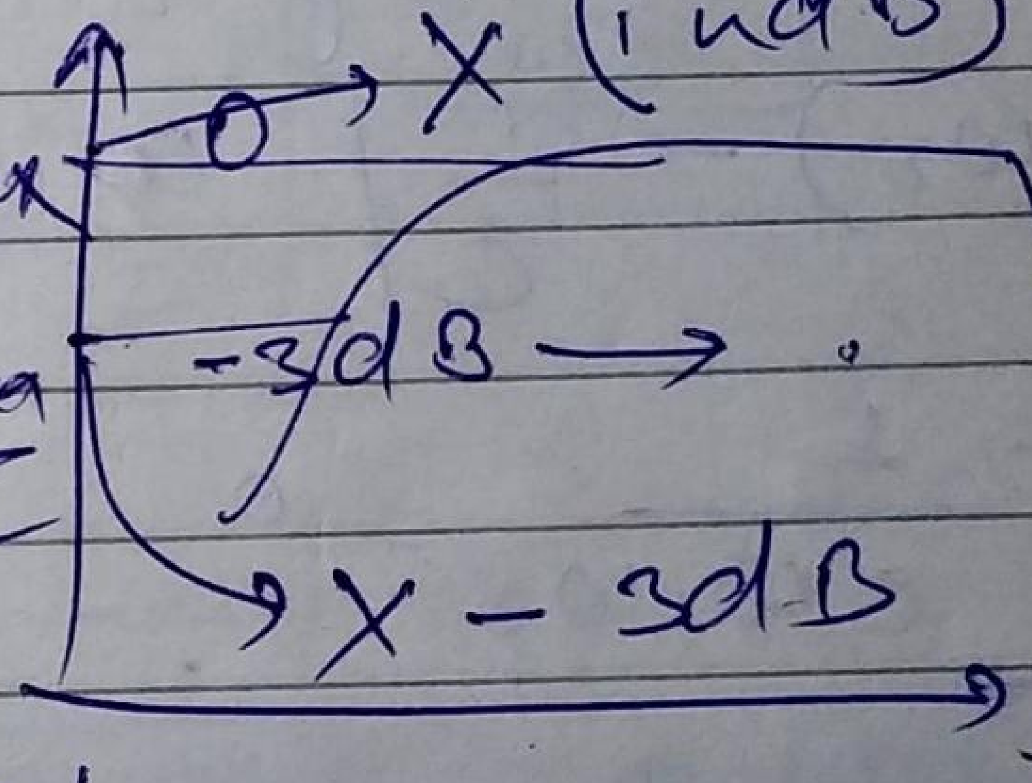
BW represents the range of frequencies for which power level in the signal is atleast half of the max. power



$QF \propto \frac{1}{BW}$

$P = A_{max}^2$ A_{max}

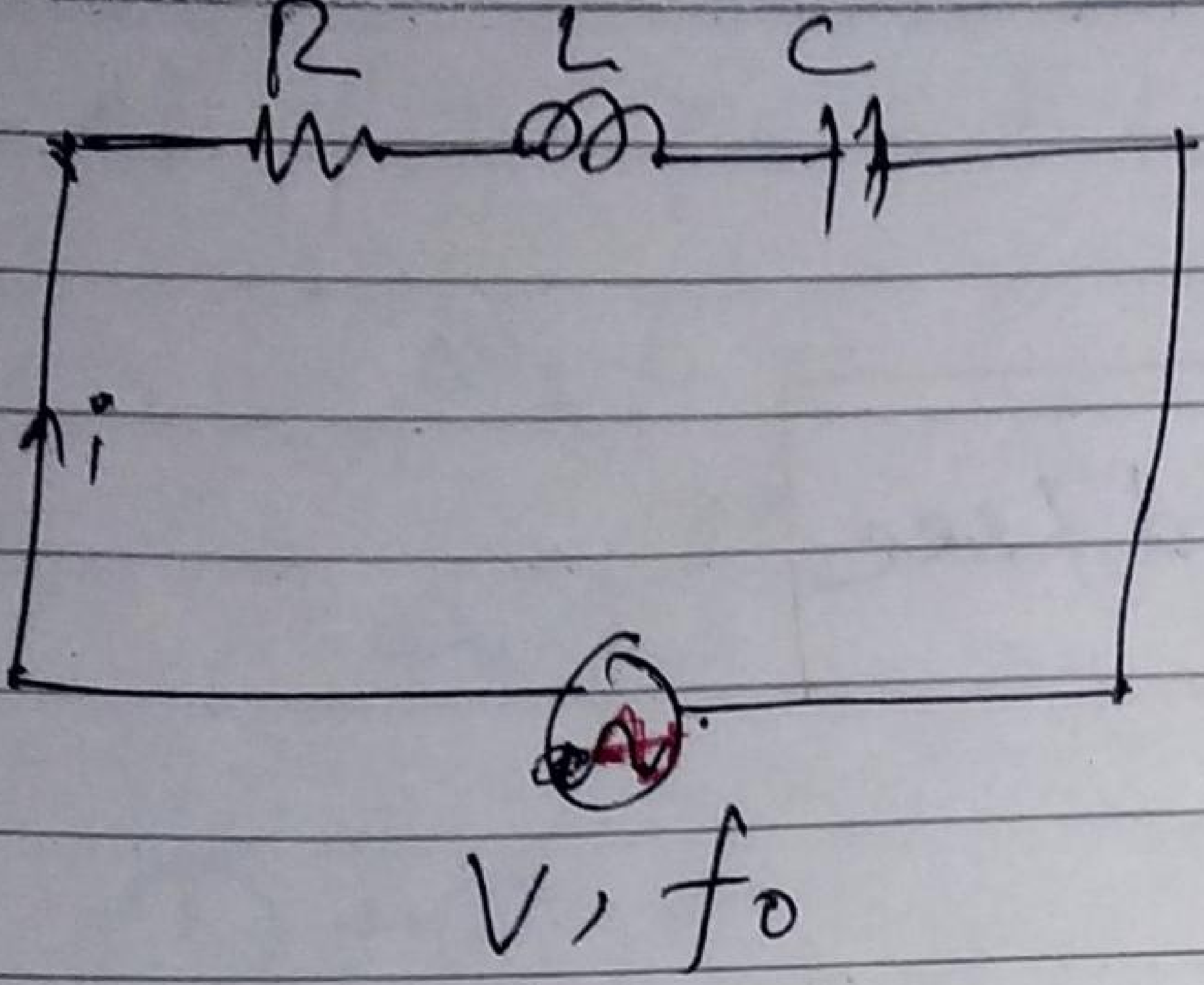
$P = \frac{A_{max}^2}{2}$ $\frac{A_{max}}{\sqrt{2}}$



$\boxed{QF = \frac{\omega_0}{BW}}$

$\boxed{BW \propto \frac{1}{Gain}}$

$= X - 20 \log \frac{1}{\sqrt{2}}$



$$v = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$\frac{dv}{dt} = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C}$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{1}{L}\frac{dv}{dt}$$

Characteristic eqn —

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$\frac{R}{L} \rightarrow BW$ $\quad$ $\frac{1}{LC} \rightarrow \omega_0^2$

System's Natural frequency = Resonant Frequency

$$\boxed{\omega_n = \frac{1}{\sqrt{LC}}}$$

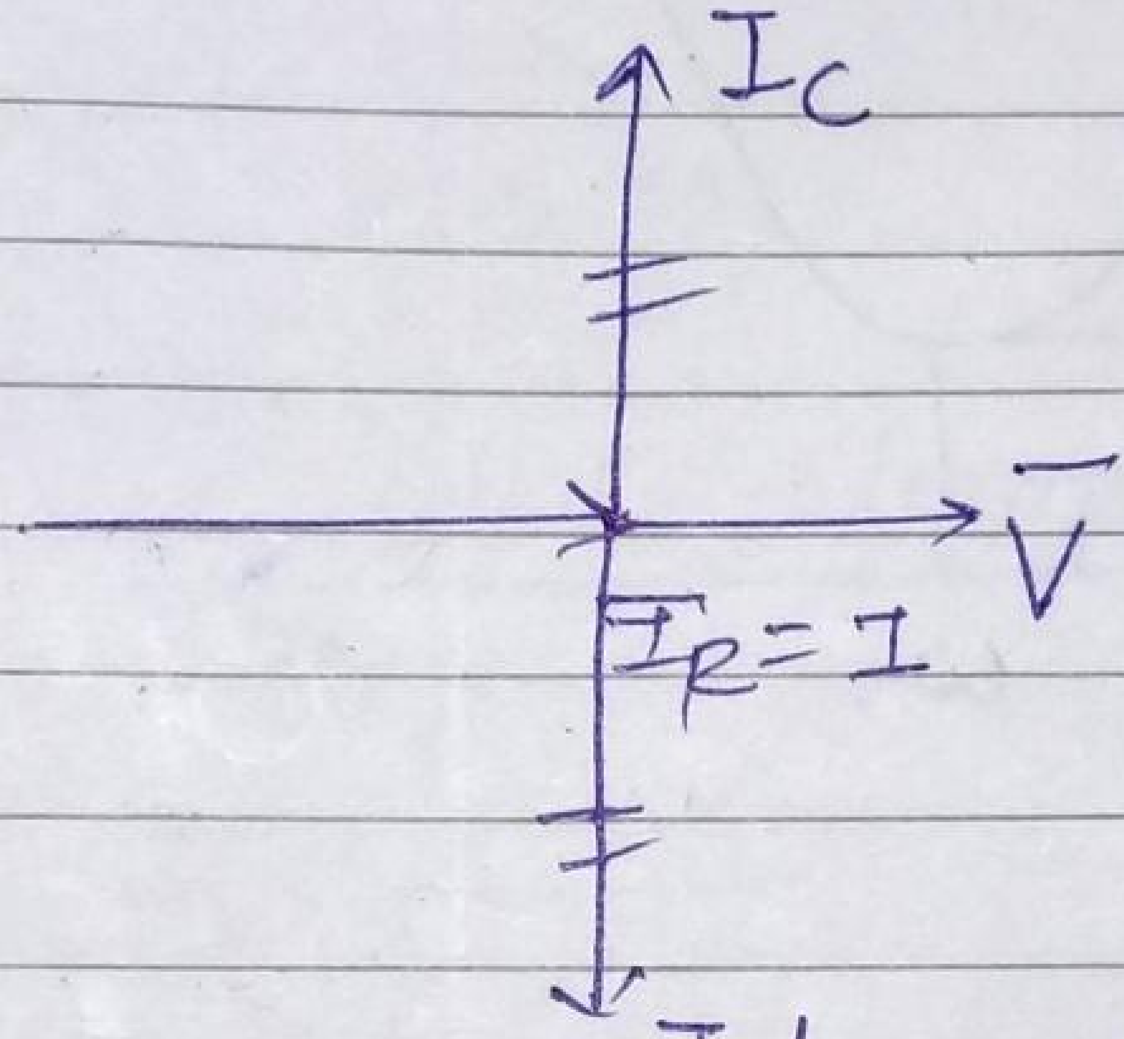
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

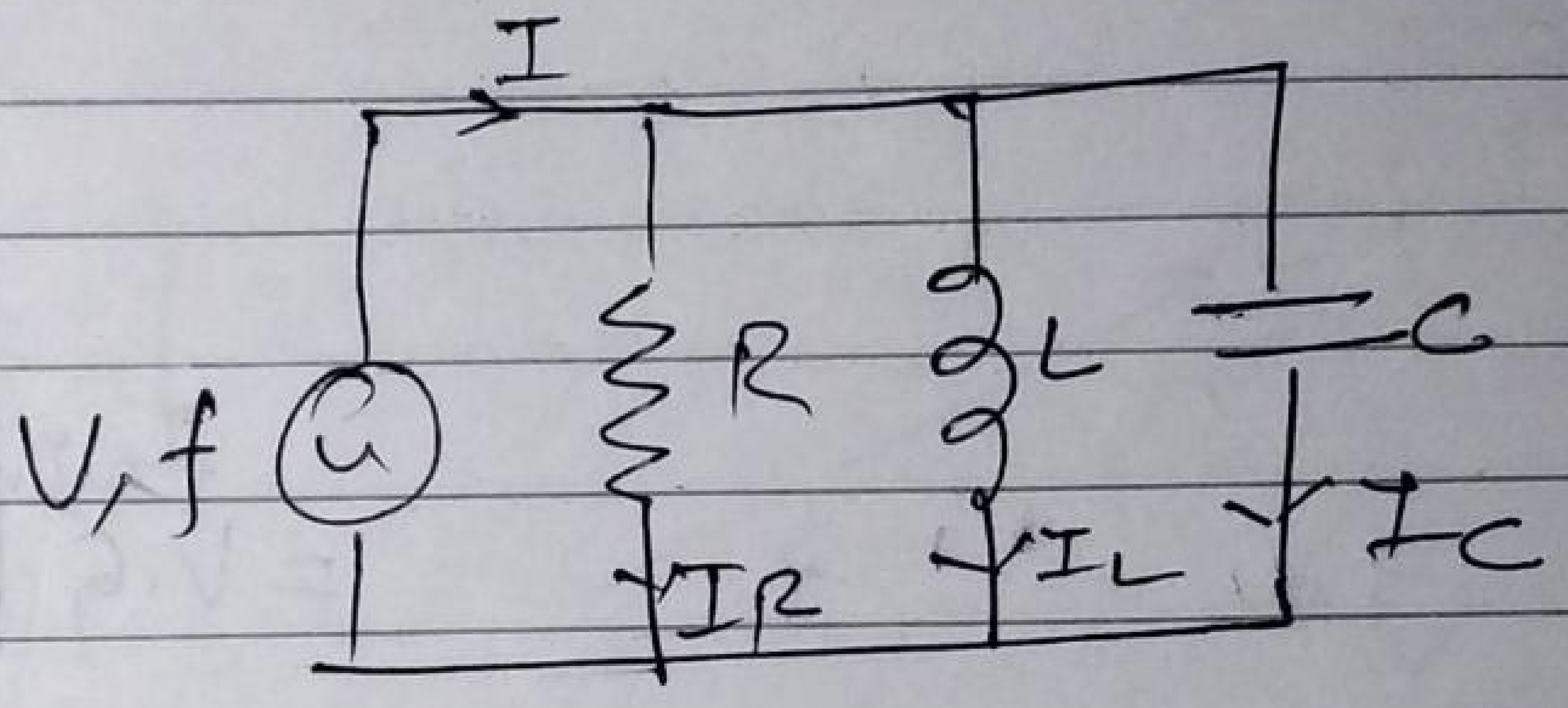
$$2\zeta\omega_n = \frac{R}{L}$$

$$\zeta = \frac{R}{2L}\cdot\frac{1}{\omega_n} = \frac{R}{2L}\cdot\frac{\sqrt{LC}}{1}$$

$$\boxed{\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}} \qquad \boxed{\zeta = \frac{1}{2Q}}$$

Parallel Resonance :-





$$I = I_R + I_L + I_C$$

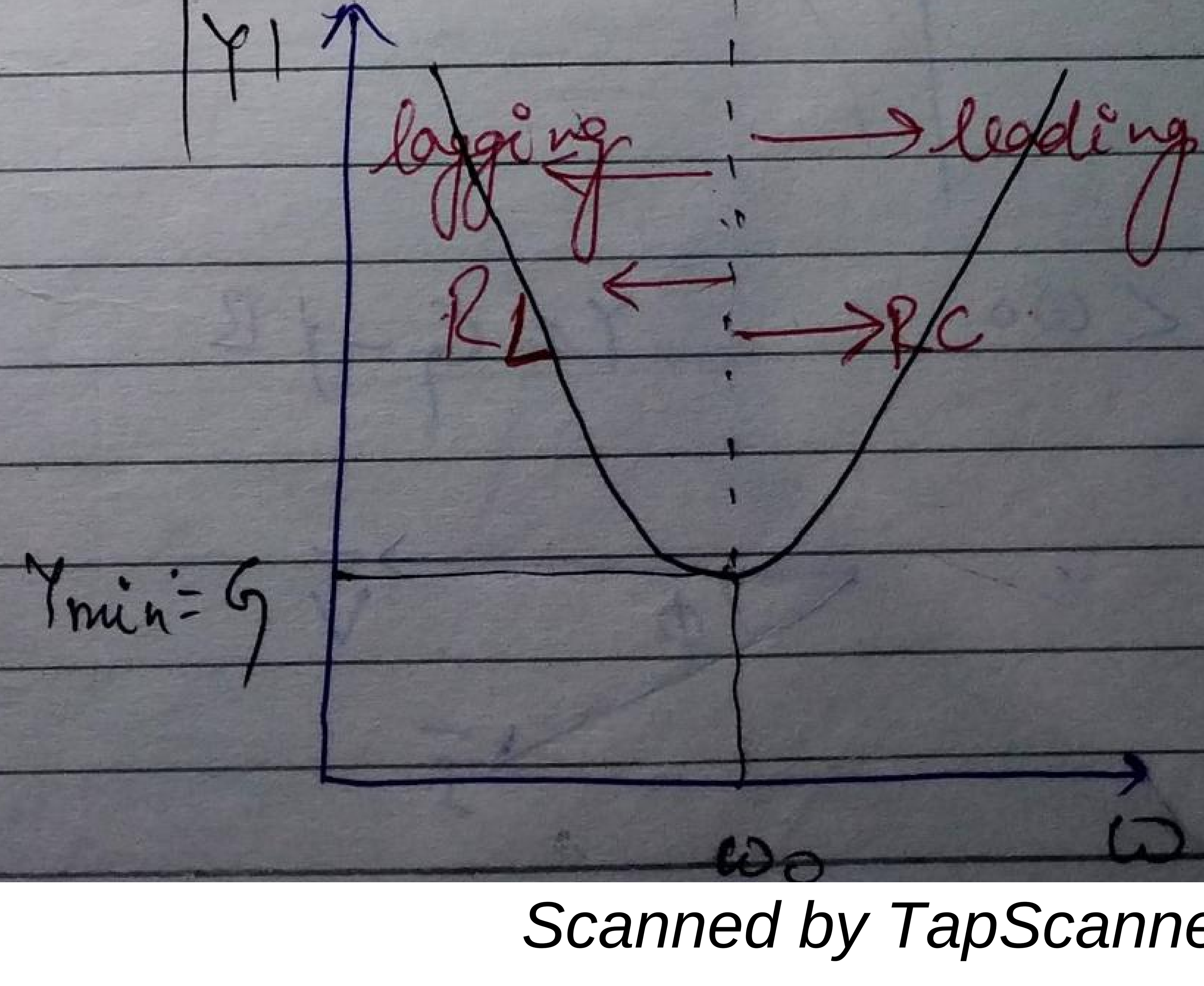
$$\frac{V}{Z} = \frac{V}{R} + \frac{V}{jX_L} - \frac{V}{jX_C}$$

$Y = G + j(B_C - B_L)$ for Resonant $B_C = B_L$

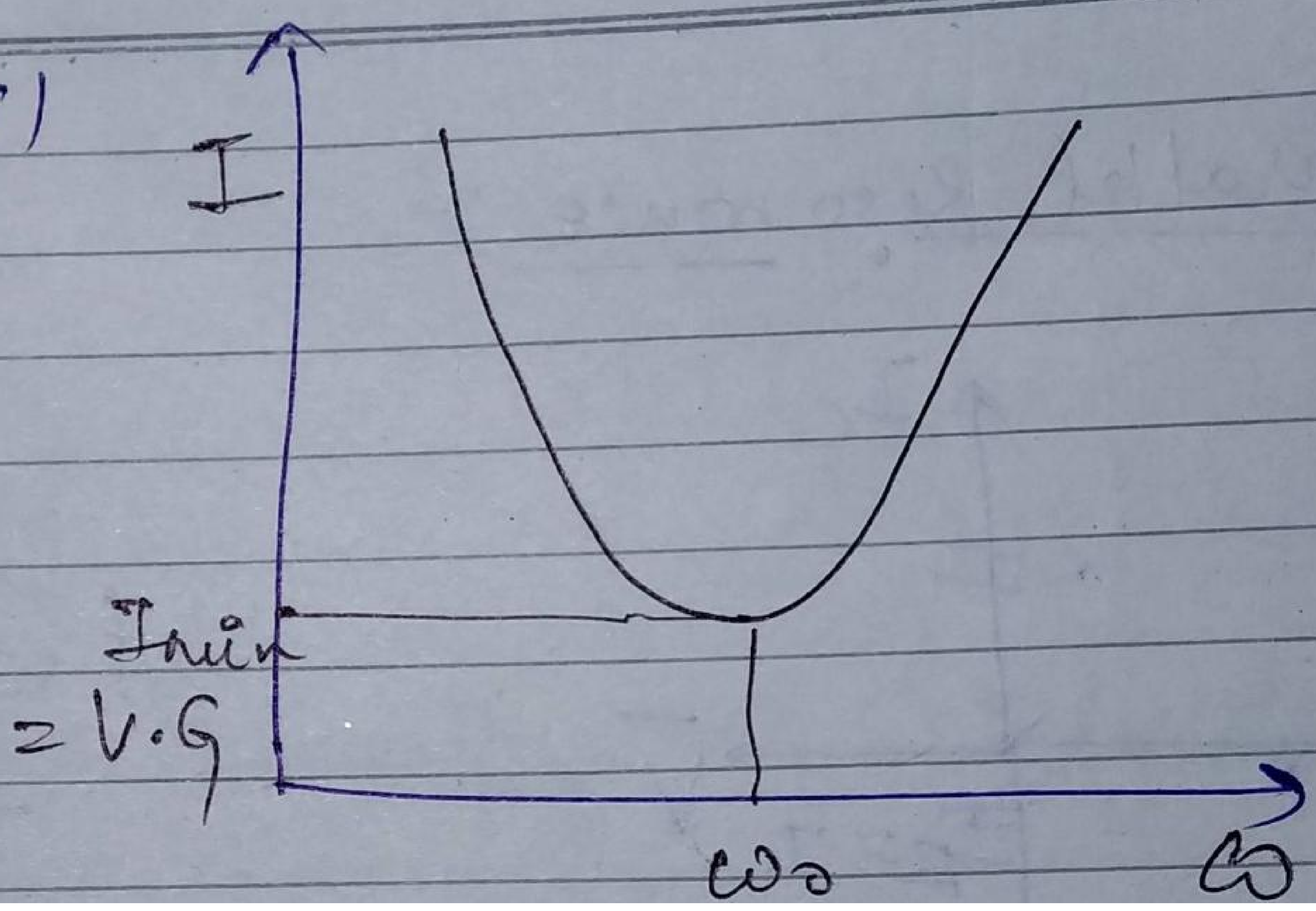
$\omega_0 C = \frac{1}{\omega_0 L}$ $\omega_0^2 = \frac{1}{LC}$ $\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$

(1) $Y = G + j(B_C - B_L)$

$Y = G + j\left(\omega C - \frac{1}{\omega L}\right)$ $|Y| = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$



(2) $I = \frac{V}{|Z|}$ $I = V|Y|$



Quality factor –

$$Q = \frac{\text{Reactive Component of current}}{\text{Active}}$$

$$Q = \frac{I_L \text{ or } I_C}{I_R} = \frac{\frac{V}{X_L} \text{ or } \frac{V}{X_C}}{V/R}$$

$$Q = \frac{R}{X_L} \text{ or } \frac{R}{X_C}$$

$$\boxed{\begin{array}{l} Q = \dfrac{R}{\omega L} \\ Q = \omega R C \end{array}}$$

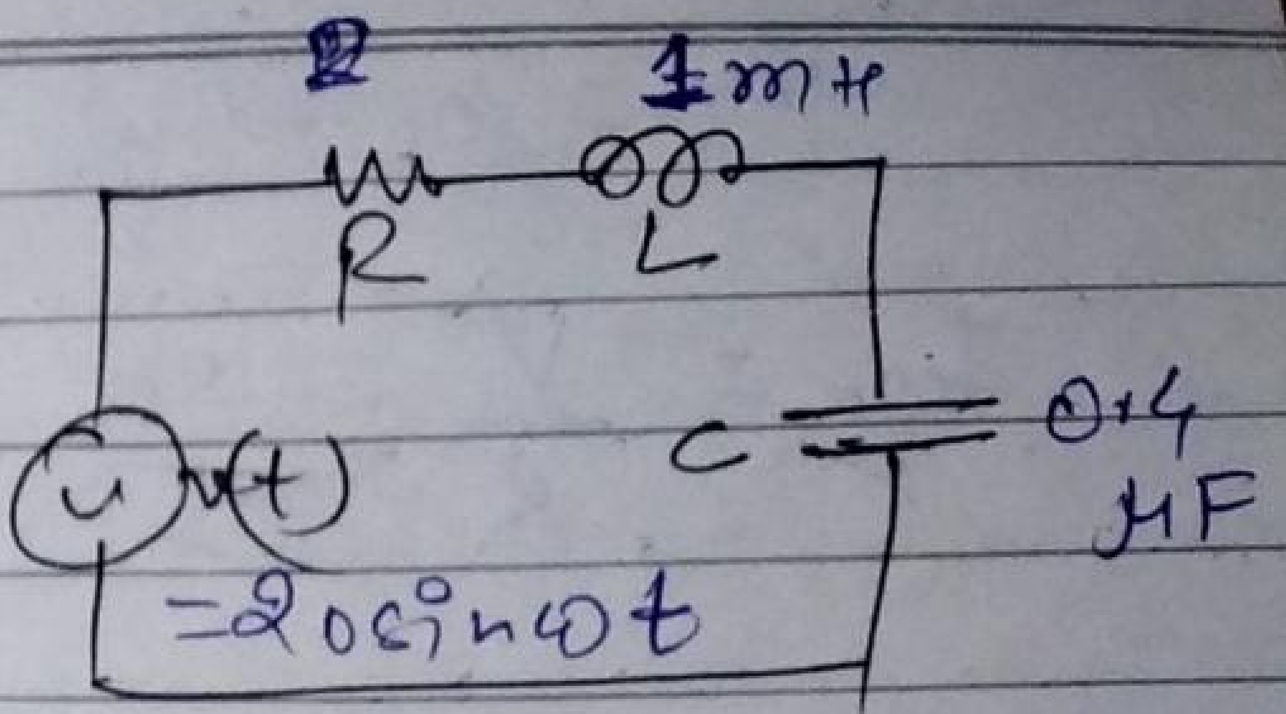
$$Q = \frac{R}{L} \cdot \frac{\sqrt{LC}}{1} \Rightarrow \boxed{Q = R\sqrt{\frac{C}{L}}}$$

Ques:- In the given ckt find

(a) Resonant Frequency at the half power frequencies.

(b) Q F and B.W.

(c) Amplitude of the current at resonant frequency and half power frequencies.



(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}}$$

$$\omega_0 = 50K \text{ rad/sec.}$$

At ω_1, ω_2 $Z = \sqrt{2}R$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Or

$$BW = \frac{R}{L} = \frac{2}{1\,mH} = 2K \text{ rad/sec}$$

$$Q = \frac{\omega_0}{BW} = \frac{50K}{2K} \quad \boxed{Q = 25}$$

$$\omega_1 = \omega_0 - \frac{BW}{2} = 49K \text{ rad/sec}$$