UNIT V

ELECTROMAGNETISM

17

Maxwell's Equations and Electromagnetic Waves

17.1 INTRODUCTION

The science of electricity has its roots in observation. It was known to Thales of Miletus in 600 B.C. that a rubbed piece of amber will attract bits of straw. The study of magnetism goes back to the observation that naturally occurring "stones" (i.e., magnetite) will attract iron. These two sciences developed quite separately until 1820, when Hans Christian Oersted (1777–1851) observed a connection between them, namely, that an electric current in a wire can affect a magnetic compass needle.

The new science of electromagnetism was developed further by many workers. One of the most important was Michael Faraday (1791–1867). Another worker, James Clerk Maxwell (1831–1879), put the laws of electromagnetism in the form in which we know them today. These laws are called *Maxwell's equations* and play the same role in electromagnetism that Newton's laws of motion do in mechanics.

Maxwell deduced that light is electromagnetic in nature and that its speed can be found by making purely electric and magnetic measurements. Thus, the science of optics was intimately connected with those of electricity and magnetism. The scope of Maxwell's equations is remarkable, as it includes the fundamental principles of large-scale electromagnetic and optical devices, such as motors, cyclotrons, electronic computers, radio, television, microwave radar, microscopes, and telescopes.

The development of classical electromagnetism did not end with Maxwell. Oliver Heariside (1850–1925) and H.A. Lorentz (1853–1928) contributed substantially to the clarification of Maxwell's theory. Hertz (1857–1894) took a great step forward when, more than twenty years after Maxwell set up his theory, he produced in the laboratory electromagnetic "Maxwellian waves" of a kind that we now call *short radio waves*. It remained for Marconi and others to exploit this practical application of the electromagnetic waves of Maxwell and Hertz.

Present interest in electromagnetism takes two forms. At the level of engineering applications, Maxwell's equations are used constantly and universally in the solution of a wide variety of practical problems. At the level of the foundations of the theory, there is a continuing effort to extend its scope in such a way that electromagnetism is revealed as a special case of a more general theory.

17.1.1 Scalar and Vector Fields

A continuous function of the position of a point in a region of space is called a *point function*. The region of space in which it specifies a physical quantity is known as a *field*. These fields are classified into two groups:

(i) *Scalar field*: A scalar field is defined as that region of space, whose each point is associated with a *scalar point function*, i.e., a continuous function which gives the value of a physical quantity as flux, potential, temperature, etc. In a scalar field, all the points having the same scalar physical quantity are connected by the means of surfaces called *equal* or *level surfaces*.

(ii) *Vector field*: A vector field is specified by a continuous vector point function having magnitude and direction, both of which change from point to point, in the given region of field. The method of presentation of a vector field is called *vector lines*, or *lines of surfaces*. The tangent at a vector line gives the direction of the vector at the point.

17.1.2 Gradient, Divergence, and Curl

In vector calculus, we study about the rate of change of scalar and vector fields. For this purpose, a common operator called *del*, or *nabla*, is used, which is written as

$$\vec{\nabla} = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

If $\phi\,(x, y, z)$ is a differentiable scalar function, its gradient is defined as

$$\text{grad}\,\phi = \nabla\phi = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\phi$$

or

$$\vec{\nabla}\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

Physically, grad ϕ is a vector whose magnitude at any point is equal to the rate of change of ϕ at a point along the normal to the surface at that point.

If $\vec{F}$ is a vector point function ($\vec{F} = F_1 i + F_2 j + F_3 k$), where F_1, F_2, and F_3 are functions of x, y, and z, then its divergence written as div F, or $\vec{\nabla} \cdot \vec{F}$, is given by

$$\vec{\nabla}\cdot\vec{F} = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot(F_1 i + F_2 j + F_3 k)$$

$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} \qquad \begin{bmatrix} i\cdot i = j\cdot j = k\cdot k = 1 \\ i\cdot j = j\cdot k = k\cdot i = 0 \end{bmatrix}$$

Divergence of a vector point function, physically signifies the outward normal flux of vector field from a closed surface.

If the divergence of any vector function is zero, then the flux of vector function entering into a region must be equal to that leaving it. This vector function is called *solenoidal*.

If $\vec{F}$ is a vector point function ($\vec{F} = F_1 i + F_2 j + F_3 k$), where F_1, F_2, and F_3 are functions of x, y, and z, then its curl is defined as

$$\text{Curl } \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= i\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) - j\left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right) + k\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$

A vector field $\vec{F}$ is called *irrotational* if curl $\vec{F} = 0$. Such fields are also known as *conservative fields*.

17.1.3 Gauss Divergence Theorem (Relation between Surface and Volume Integration)

This theorem states that the flux of a vector field $\vec{F}$, over any closed surface S, is equal to the volume integral of the divergence of that vector field over the volume V enclosed by the surface S.

$$\int_s \vec{F} \cdot d\vec{S} = \int_V \text{div } \vec{F}\, dV \tag{17.1}$$

17.1.4 Stokes Theorem (Relation between Line Integral and Surface Integration)

This theorem states that the surface integral of the curl of a vector field $\vec{A}$, taken over any surface S, is equal to the line integral of $\vec{A}$ around the closed curve forming the periphery of the surface.

$$\iint_s (\text{Curl } \vec{A}) \cdot d\vec{S} = \oint \vec{A} \cdot \vec{dl}$$

or

$$\iint_s (\vec{\nabla} \times \vec{A}) \cdot d\vec{S} = \oint \vec{A} \cdot \vec{dl} \tag{17.2}$$

17.1.5 Poisson's and Laplace's Equations

Poisson's and Laplace's equations are very useful mathematical relations for the calculations of electric fields and potentials that cannot be computed by using Coulomb's and Gauss's law in electrostatic problems. These equations can be derived as follows:

Gauss law in electrostatics is given by

$$\text{div } E = \frac{\rho}{\epsilon_0}$$

Electric field and potential are related as

$$\vec{E} = -\text{grad } V = -\vec{\nabla} V$$

Thus, we obtain

$$\text{div}\,(-\,\text{grad}\;V) = \frac{\rho}{\epsilon_0}$$

or $$\vec{\nabla}\cdot(-\vec{\nabla}V) = \frac{\rho}{\epsilon_0}$$

or $$\nabla^2 V = -\frac{\rho}{\epsilon_0} \tag{17.3}$$

This equation is known as *Poisson's equation* for a homogeneous region. For a charge-free region, i.e., $\rho = 0$, Poisson's equation becomes

$$\nabla^2 V = 0$$

This is called *Laplace's equation*. This equation is applicable to those electrostatic problems, where the entire charge resides on the surface of the conductor or is concentrated in the form of point charges, line charges, or surface charges at a single position. It is also applicable in the cases, where the region between two conductors is filled with one or more homogeneous dielectrics.

17.2 FUNDAMENTAL LAWS OF ELECTRICITY AND MAGNETISM

To understand Maxwell's equation, we must go through the basic laws of electricity and magnetism.

(i) *Gauss's law in electrostatics*: $\oint \vec{E}\cdot\vec{dS} = q/\epsilon_0$ (17.4)

i.e., the electric flux from a closed surface is equal to $1/\epsilon_0$ times the charge enclosed by the surface.

(ii) *Gauss's law in magnetostatics*: $\oint \vec{B}\cdot\vec{dS} = 0$ (17.5)

i.e., the rate of change of magnetic flux through a closed surface is always equal to zero. This also signifies that monopole cannot exist.

(iii) *Faraday's law of electromagnetic induction*: This law states that the rate of change of magnetic flux in a closed circuit induces an emf which opposes the cause, i.e.,

$$e = -\frac{d\phi}{dt} \tag{17.6}$$

(iv) *Ampere's law*: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ (17.7)

This law states that the line integral of magnetic flux is equal to μ_0 times the current enclosed by the current loop.

17.3 EQUATION OF CONTINUITY

Electric current is the rate of flow of charge. Therefore, we have

$$i = -\frac{dq}{dt} \tag{17.8}$$

If dq charge is enclosed in a volume element dV and is leaving a surface having area dS, we have

$$i = \int_s \vec{J} \cdot d\vec{S} \text{ and } q = \int_V \rho\, dV$$

where J is the current density and ρ is the volume charge density. Therefore, Eq. (17.8) becomes

$$\int_s \vec{J} \cdot d\vec{S} = -\frac{d}{dt}\int_V \rho\, dV$$

or

$$\int_s \vec{J} \cdot d\vec{S} = \int_V \frac{\partial \rho}{\partial t}\, dV \tag{17.9}$$

Using Gauss divergence theorem on LHS of Eq. (17.9), we get

$$\int_s \vec{J} \cdot d\vec{S} = \int_V \text{div } J\, dV$$

Therefore, Eq. (17.9) becomes

$$\int_V \text{div } \vec{J}\, dV = -\int_V \frac{\partial \rho}{\partial t}\, dV$$

or

$$\int_V \left(\text{div } \vec{J} + \frac{\partial \rho}{\partial t}\right) dV = 0$$

Therefore, for an arbitrary surface, we have

$$\text{div } \vec{J} + \frac{\partial \rho}{\partial t} = 0 \tag{17.10}$$

This expression is called *continuity equation*.

17.4 DISPLACEMENT CURRENT

According to Maxwell, it is not only the current in a conductor that produces a magnetic field. A changing electric field in vacuum or in a dielectric also produces a magnetic field. This implies that a changing electric field is equivalent to a current, which flows till the electric field is changing. This equivalent current produces the same magnetic effects as a conventional current in a conductor. This equivalent current is known as *displacement current*.

17.4.1 Modified Ampere's Law

The concept of displacement current due to the discharge of a condenser leads to the modification in Ampere's law. Consider the process of charging of a parallel plate capacitor through a series circuit as shown in Fig. 17.1.

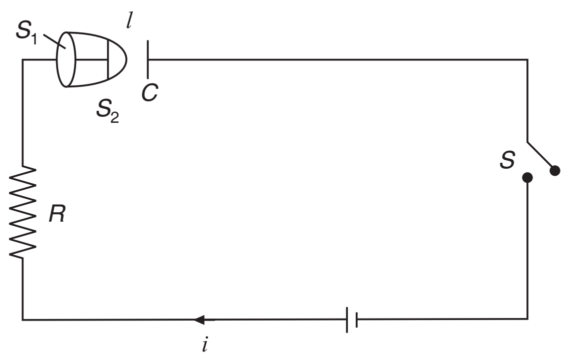

Fig. 17.1 Charging of a capacitor

Let us consider a plane surface S_1 and a hemispherical surface S_2 around the condenser plate as shown in Fig. 17.1. Let both surfaces be bounded by the same closed path l, and applying Ampere's law to the surface S_1, we get

$$\oint_{s_1} \vec{B} \cdot d\vec{l} = \mu_0 i \tag{17.11}$$

Now, during the process of charging, current i has been flowing through the plane surface S_1. If it is applied to the hemispherical surface S_2, we get

$$\oint_{s_2} \vec{B} \cdot d\vec{l} = 0 \tag{17.12}$$

(because no current is enclosed by the surface S_2).

But Eqs. (17.11) and (17.12) show contradiction to each other. Hence, Maxwell introduced the idea that a changing electric field is a source of magnetic field in the gap between the capacitor plates (during charging) and is equivalent to the displacement current devalued by i_d. If ϕ_E is the electric flux, then from equation of continuity, i_d should be equal to $\in_0 \, d\phi_E/dt$. Therefore, if along with an electric current, there exists a magnetic field, the modified Ampere's law becomes

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i + \in_0 \frac{d\phi_E}{dt} \right) = \mu_0 (i + i_d) \tag{17.13}$$

Now, the electric field for the charge q developed at the plates of a parallel plate capacitor, each having an area A is given by

$$E = \frac{q}{\in_0 A}$$

or $$\frac{dE}{dt} = \frac{1}{\in_0 A}\frac{dq}{dt} = \frac{i}{\in_0 A}$$

or $$i = \in_0 A \frac{dE}{dt} \tag{17.14}$$

$$= \in_0 \frac{d(EA)}{dt} = \in_0 \frac{d\phi_E}{dt} = i_d \qquad (\phi_E = EA)$$

Thus, the displacement current in the gap is identical to the conduction current in the connecting wires.

From Eq. (17.14), we can write

$$i_d = A\frac{d(\in_0 E)}{dt} = A\frac{dD}{dt} \qquad (\because \vec{D} = \in_0 \vec{E})$$

or $$\frac{i_d}{A} = \frac{dD}{dt}$$

or $$J_d = \frac{dD}{dt}$$

Hence, modified Ampere's law becomes

$$\oint \bar{B}\cdot d\vec{l} = \mu_0\left(i + \in_0 \frac{d\phi_E}{dt}\right) \tag{17.15}$$

17.5 MAXWELL'S ELECTROMAGNETIC EQUATIONS

Maxwell's equations are based on the fundamental laws of physics, which we have already discussed in previous articles. With the help of these equations, one can analyse time-varying fields.

17.5.1 Maxwell's Equations in Differential Form

(i) $\vec{\nabla}\cdot\vec{D} = \rho$ or Div $\vec{D} = \rho$

(ii) $\vec{\nabla}\cdot\vec{B} = 0$ or Div $\vec{B} = 0$

(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ or Curl $\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(iv) $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ or Curl $\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

where

$\vec{D}$ = Electric displacement vector, $\vec{B}$ = Magnetic flux density

$\vec{E}$ = Electric field intensity, $\vec{H}$ = Magnetic field intensity

$\vec{J}$ = Current density (conventional)

ρ = Charge density

17.5.2 Maxwell's Equations in Integral Form

(i) $\int_s \vec{D} \cdot \vec{dS} = \int_V \rho dV$ or $\oint_s \vec{E} \cdot \vec{dS} = q$

(ii) $\oint_s \vec{B} \cdot \vec{dS} = 0$

(iii) $\oint \vec{E} \cdot \vec{dl} = -\dfrac{\partial}{\partial t} \int \vec{B} \cdot \vec{dS}$

(iv) $\oint \vec{H} \cdot d\vec{l} = \int_s \left(\vec{J} + \dfrac{\partial \vec{D}}{\partial t} \right) \cdot \vec{dS}$

Symbols used have the same meaning, as given in Section 17.5.1.

17.5.3 Derivation of Maxwell's Equations

1. *Maxwell's first equation, div D* = ρ *or* $\vec{\nabla} \cdot \vec{D} = \rho$:

When a dielectric is placed in a uniform electric field, its molecules get polarised. Thus, a dielectric in an electric field contains two types of charges—free charges, which are embedded, and polarisation charges or bound charges. If ρ and ρ_P are the free and bound charge densities, respectively, at a point in a small volume element dV, then for such a medium, Gauss's law may be expressed as

$$\int_s \vec{E} \cdot \vec{dS} = \frac{1}{\epsilon_0} \int_V (\rho + \rho_P)\, dV \tag{17.16}$$

where ϵ_0 is the permittivity of the free space.

Now, the bound charge density

$$\rho_P = -\text{div } \vec{P}, \text{ where } \vec{P} \text{ is electric polarisation.}$$

Therefore, $$\int_s \vec{E} \cdot \vec{dS} = \frac{1}{\epsilon_0} \int_V (\rho - \text{div} \vec{P})\, dV$$

Using Gauss divergence theorem on left-hand side of the above expression, we get

$$\int_s \vec{E} \cdot d\vec{S} = \int_v \text{div}\, E\, dV = \frac{1}{\epsilon_0}\int_v \rho\, dV - \frac{1}{\epsilon_0}\int_v \text{div}\, \vec{P}\, dV$$

or
$$\int_v \epsilon_0\, \text{div}\, \vec{E}\, dV + \int_v \text{div}\, \vec{P}\, dV = \int_v \rho\, dV$$

$$\int_v \text{div} \epsilon_0\, \vec{E}\, dV + \int_v \text{div}\, \vec{P}\, dV = \int_v \rho\, dV$$

$$\int_v \text{div}(\epsilon_0\, \vec{E} + \vec{P})\, dV = \int_v \rho\, dV$$

But $\epsilon_0 \vec{E} + \vec{P} = \vec{D}$ is the electric displacement vector.

Thus,
$$\int_v \text{div}\, \vec{D}\, dV = \int_v \rho\, dV$$

or
$$\int_v (\text{div}\, \vec{D} - \rho)\, dV = 0$$

Therefore, for an arbitrary surface, we have

$$\text{div}\, \vec{D} - \rho = 0$$

or
$$\text{div}\, \vec{D} = \rho$$

or
$$\vec{\nabla} \cdot \vec{D} = \rho$$

This is the required Maxwell's first equation.

In free space, volume charge density $\rho = 0$.

Therefore, Maxwell's first equation in free space is reduced to

$$\text{div}\, \vec{D} = 0 \text{ or } \vec{\nabla} \cdot \vec{D} = 0$$

or
$$\text{div}\, \epsilon_0\, \vec{E} = 0$$

or
$$\epsilon_0\, \text{div}\, \vec{E} = 0$$

or
$$\text{div}\, \vec{E} = 0 \text{ or } \vec{\nabla} \cdot \vec{E} = 0$$

2. *Maxwell's second equation, div $\vec{B} = 0$ or $\vec{\nabla} \cdot \vec{B} = 0$:*

It has been experimentally observed that the number of magnetic lines of force entering any closed surface enclosing a volume is exactly the same as that leaving it, i.e., the net magnetic flux through any closed surface is always zero.

Hence,

$$\phi_B = \oint_s \vec{B} \cdot d\vec{S} = 0 \tag{17.17}$$

The above expression implies that a monopole or an isolated magnetic pole cannot exist to serve as a source or sink for the line of magnetic induction $\vec{B}$. This expression is also known as *Gauss's law in magnetostatics*.

Using Gauss divergence theorem in Eq. (17.6), we have

$$\oint \vec{B} \cdot d\vec{S} = \int_V \text{div}\, \vec{B}\, dV = 0$$

where V is the volume enclosed by surface S.

Hence, for an arbitrary surface,

$$\text{div}\ \vec{B} = 0$$

or

$$\vec{\nabla} \cdot \vec{B} = 0$$

3. *Maxwell's third equation (Faraday's law of electromagnetic induction)*:

According to Faraday's law of electromagnetic induction, the induced emf around a closed circuit is equal to the negative time rate of change of magnetic flux linked with the circuit, i.e.,

$$e = -\frac{d\phi_B}{dt} \tag{17.18}$$

If $\vec{B}$ is the magnetic induction, then the magnetic flux linked with an area $d\vec{S}$ is

$$\phi_B = \int_s \vec{B} \cdot d\vec{S} \tag{17.19}$$

On combining Eqs. (17.18) and (17.19), we get

$$e = -\frac{d}{dt}\int_s (\vec{B} \cdot d\vec{S})$$

or

$$e = \int_s \frac{\partial}{\partial t}(\vec{B} \cdot d\vec{S}) \tag{17.20}$$

According to definition, the induced emf is related to the corresponding electric field as

$$e = \int_c \vec{E} \cdot d\vec{l} \tag{17.21}$$

Equations (17.20) and (17.21) will give

$$\int_c \vec{E} \cdot d\vec{l} = -\int_s \frac{\partial}{\partial t}(\vec{B} \cdot d\vec{S})$$

$$= -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

Now, using Stoke's theorem on left-hand side, we get

$$\int_c \vec{E} \cdot d\vec{l} = \int_s \text{curl}\, \vec{E} \cdot d\vec{S}$$

Thus, we have

$$\int_s \text{curl}\, \vec{E} \cdot d\vec{S} = \int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

or
$$\int_s \left(\text{curl}\, \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{S} = 0$$

For any arbitrary surface *dS*, we will have

$$\text{curl}\, \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

or
$$\text{curl}\, \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

i.e.,
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

4. ***Maxwell's fourth equation (modified Ampere's law)*:**
In Section 17.2, Ampere's law is given as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Using formula $I = \oint \vec{J} \cdot d\vec{S}$ $\left(\text{using } J = \frac{I}{A} \right)$

we get

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \oint \vec{J} \cdot d\vec{S}$$

Using Stoke's theorem on the left-hand side of the above expression, we get

$$\oint_s \text{curl}\, \vec{B} \cdot d\vec{S} = \mu_0 \oint_s \vec{J} \cdot d\vec{S}$$

$$\frac{1}{\mu_0}\oint_s \text{curl}\,\vec{B}\cdot d\vec{S} = \oint_s \vec{J}\cdot d\vec{S}$$

$$\oint_s \text{curl}\,\frac{\vec{B}}{\mu_0}\cdot d\vec{S} = \oint_s \vec{J}\cdot d\vec{S}$$

Now, from dielectric properties, we have

$$\frac{B}{\mu_0} = H$$

$$\therefore \qquad \int_s \text{curl}\,\vec{H}\cdot d\vec{S} = \int_s \vec{J}\cdot d\vec{S}$$

or

$$\int_s (\text{curl}\,\vec{H} - \vec{J})\cdot d\vec{S} = 0$$

For an arbitrary surface, we have

$$\text{curl}\,\vec{H} - \vec{J} = 0$$

or

$$\text{curl}\,\vec{H} = \vec{J} \tag{17.22}$$

Taking divergence on both sides, we get

$$\text{div curl}\,\vec{H} = \text{div}\,\vec{J}$$

But div curl $\vec{H}$ = 0 (From vector calculus)

$$\therefore \qquad \text{div}\,\vec{J} = 0$$

From continuity equation, we have

$$\text{div}\,\vec{J} + \frac{\partial\rho}{\partial t} = 0$$

Hence,

$$\frac{\partial\rho}{\partial t} = 0$$

or

$$\rho = \text{constant (static)}$$

This implies that Ampere's law is applicable only for static charges. However, for time-varying fields, Maxwell suggested that Ampere's law must be modified by adding a quantity having dimension as that of current and produced due to polarisation of charges. This physical quantity is called *displacement current* (J_d). Thus, modified Ampere's law now becomes

$$\text{curl}\,\vec{H} = \vec{J} + \vec{J}_d$$

Taking divergence on both sides, we get

$$\text{div curl}\,\vec{H} = \text{div}\left(\vec{J} + \vec{J}_d\right) \qquad (\text{div Curl}\,\vec{H} = 0)$$

$$0 = \text{div } \vec{J} + \text{div } \vec{J}_d$$

or $$\text{div } \vec{J} = -\text{div } \vec{J}_d$$

But $$\text{div } \vec{J} = -\frac{\partial \rho}{\partial t} \text{ (Continuity equation)}$$

∴ $$\text{div } \vec{J}_d = \frac{\partial \rho}{\partial t}$$

But $$\rho = \text{div } \vec{D}\,*$$

∴ $$\text{div } \vec{J}_d = \frac{\partial}{\partial t}(\text{div } \vec{D})$$

$$= \text{div}\left(\frac{\partial \vec{D}}{\partial t}\right)$$

or $$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

Therefore, modified Ampere's law now becomes

$$\text{curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

17.6 PHYSICAL SIGNIFICANCE OF MAXWELL'S EQUATIONS

(i) Maxwell's first equation $\oint \vec{E}\cdot d\vec{S} = q/\epsilon_0$ or div $\vec{E} = \rho/\epsilon_0$ represents the Gauss' law in electrostatics for the static charges, which states that the electric flux through any closed surface is equal to $1/\epsilon_0$ times the total charge enclosed by the surface.

(ii) Maxwell's second equation $\oint \vec{B}\cdot d\vec{S} = 0$ or div $\vec{B} = 0$ expresses Gauss's law in magnetostatics, which states that the net magnetic flux through any closed surface is zero. Since a magnetic monopole does not exist, any closed volume always contains equal and opposite magnetic poles (north and south poles), resulting in the net magnetic pole strength becoming zero. It also signifies that magnetic lines of flux are continuous, i.e., the number of magnetic lines of flux entering into a region is equal to the lines of flux leaving it.

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* $\oint \vec{E}\cdot d\vec{S} = \frac{q}{\epsilon_0}$ $\oint \vec{E}\cdot d\vec{S} = \frac{1}{\epsilon_0}\int \rho\, dV$ $\oint \epsilon_0 \cdot \vec{E}\cdot d\vec{S} = \int \rho\, dV$

$\oint \vec{D}\cdot d\vec{S} = \int \rho dV \int_V \text{div}\,\vec{D}\, dV = \int_V \rho\, dV$ div $\vec{D} = \rho$ (Gauss law)

(iii) Maxwell's third equation is the Faraday's law of electromagnetic induction, i.e., $\left(\text{curl}\ \vec{E} = -\partial \vec{B}\big/\partial t\right)$. It states that the induced electromotive force around any closed surface is equal to the negative time rate of change of the magnetic flux through the path enclosing the surface. This signifies that an electric field can also be produced by a changing magnetic flux.

(iv) Maxwell's fourth equation curl $\vec{B} = \mu_0 \vec{J} + \mu_0 \in_0 \partial \vec{E}\big/\partial t$ represents the generalised form of Ampere's law as extended by Maxwell to account for the time-varying magnetic fields. It is valid for both steady (electrostatic) and non-steady (charges are in motion) states. It states that the magnetomotive force around a closed path is equal to the sum of conduction current ($\mu_0 i$) and displacement current $(\in_0 \partial\phi_E/\partial t)$ through the surface bounded by that path. This signifies that a conduction current or a changing electric flux produces a magnetic field.

A close scrutiny of Maxwell's third and fourth equations reveals that the modified form of Ampere's law contains a term $\mu_0 i$ while Faraday's law does not. The absence of this term in Faraday's law signifies the absence of magnetic monopole. The term $\in_0 \partial\phi_E/\partial t$, i.e., the displacement current, signifies that magnetic field can also be produced by a changing electric field. Since the quantity $\mu_0 \in_0 \approx 10^{-17}$, therefore, the term $\mu_0 \in_0 \partial\phi_E/\partial t$ will not contribute significantly unless $\partial\phi_E/\partial t$ is extremely large, i.e., the displacement current can be detectable only when electric flux changes very rapidly. Hence, Maxwell's fourth equation gives the generation of magnetic field by displacement current.

17.7 ELECTROMAGNETIC ENERGY (POYNTING THEOREM)

This theorem analyses the transportation of energy in the medium from one place to another due to the propagation of electromagnetic waves.

Maxwell's third and fourth equations in differential form are as follows:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{17.23}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{17.24}$$

Taking scalar product of Eq. (17.23) with $\vec{H}$ and of Eq. (17.24) with $\vec{E}$, we get

$$\vec{H} \cdot (\nabla \times \vec{E}) = -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} \tag{17.25}$$

and

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \tag{17.26}$$

Subtraction of Eq. (17.26) from Eq. (17.25) gives

$$\vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H}) = -\vec{H} \cdot \frac{\partial \vec{B}}{dt} - \vec{E} \cdot \vec{J} - E \cdot \frac{\partial \vec{D}}{dt}$$

Now, using the vector identity $\nabla\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{H})$, the above equation becomes

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\left(\vec{H}\cdot\frac{\partial\vec{B}}{dt} + \vec{E}\cdot\frac{\partial\vec{D}}{dt}\right) - \vec{E}\cdot\vec{J} \tag{17.27}$$

For a linear medium, $\vec{B} = \mu\vec{H}$ and $\vec{D} = \in\vec{E}$, so that Eq. (17.27) becomes

$$\nabla\cdot(\vec{E}\times\vec{H}) = -\left(\vec{H}\cdot\frac{\partial(\mu\vec{H})}{\partial t} + \vec{E}\cdot\frac{\partial(\in\vec{E})}{\partial t}\right) - \vec{E}\cdot\vec{J}$$

$$= -\left\{\frac{1}{2}\frac{\mu\partial(H)^2}{\partial t} + \frac{1}{2}\frac{\in\partial(E)^2}{\partial t}\right\} - \vec{E}\cdot\vec{J}$$

$$= -\left\{\frac{1}{2}\frac{\partial(\vec{H}\cdot\mu\vec{H})}{\partial t} + \frac{1}{2}\frac{\partial(\vec{E}\cdot\in\vec{E})}{\partial t}\right\} - \vec{E}\cdot\vec{J}$$

$$= -\left\{\frac{1}{2}\frac{\partial(\vec{H}\cdot\vec{B})}{\partial t} + \frac{1}{2}\frac{\partial(\vec{E}\cdot\vec{D})}{\partial t}\right\} - \vec{E}\cdot\vec{J}$$

$$= -\frac{\partial}{\partial t}\left\{\frac{1}{2}(\vec{E}\cdot\vec{D}) + \frac{1}{2}(\vec{B}\cdot\vec{H})\right\} - \vec{E}\cdot\vec{J}$$

Integrating over a volume V bounded by a surface S, we get

$$\int_V \nabla\cdot(\vec{E}\times\vec{H})\,dV = -\int_V\left\{\frac{\partial}{\partial t}\frac{1}{2}(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H})\right\}dV - \int_V \vec{E}\cdot\vec{J}\,dV$$

Using Gauss's divergence theorem on the left-hand side, we get

$$\int_s(\vec{E}\times\vec{H})\cdot d\vec{S} = -\frac{\partial}{\partial t}\int_V\frac{1}{2}(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H})dV - \int_V \vec{E}\cdot\vec{J}\,dV$$

or

$$\int_V(\vec{E}\cdot\vec{J})\,dV = \frac{\partial}{\partial t}\int_V\frac{1}{2}(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H})\,dV - \int_s(\vec{E}\times\vec{H})\cdot d\vec{S} \tag{17.28}$$

Equation (17.28) is known as *Poynting theorem*. Each term of this expression has its own physical significance, which can be explained as follows:

(i) The term $\int_V \vec{E}\cdot\vec{J}\,dV$ is the generalised statement of Joule's law and represents the total power dissipated in volume V.

(ii) The first term on the right-hand side of the equation is the sum of energy stored in electric field $\frac{1}{2}(E.D)$ and in magnetic field $\frac{1}{2}(B.H)$ or the total energy stored in electromagnetic field. Therefore, this term represents the rate of change in energy stored in volume V.

(iii) The term is in accordance with the law of conservation of energy, and hence, represents the rate at which the energy is carried out of volume V across its boundary surface by electromagnetic waves.

Thus, the Poynting theorem states that the work done on the charge by an electromagnetic force is equal to the decrease in energy stored in the field, less than the energy which flowed out through the surface. It is also called the *energy conservation law in electrodynamics*. The energy per unit time per unit area transported by the electromagnetic field is called the *Poynting vector* and is given by

$$\vec{S} = \vec{E}\times\vec{H}$$

$$= \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

17.8 CHARACTERISTIC IMPEDANCE OR INTRINSIC IMPEDANCE, OR WAVE IMPEDANCE OF THE FREE SPACE

In free space, Maxwell's third and fourth equations are as follows:

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{17.29}$$

and

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t} \tag{17.30}$$

We know that the solution of equation of electromagnetic wave is given by

$$\vec{E}(r, t) = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(r, t) = \vec{B}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

Curl of $\vec{E}(r, t)$ is given by

$$\text{curl } \vec{E} = \vec{\nabla}\times\vec{E} = i(\vec{K}\times\vec{E}) \tag{17.31}$$

Comparison of Eqs. (17.29) and (17.31) will give

$$i(\vec{K}\times\vec{E}) = -\frac{\partial\vec{B}}{\partial t} = -\frac{\partial}{\partial t}\left[B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right]$$

$$= i\omega\vec{B}$$

or $$\vec{K} \times \vec{E} = \omega \vec{B} \tag{17.32}$$

Similarly, for Eq. (17.30), we can have

$$i(\vec{K} \times \vec{B}) = \mu_0 \in_0 (-i\omega \vec{E})$$

$$\vec{K} \times \vec{B} = -\omega \mu_0 \in_0 \vec{E} \tag{17.33}$$

Thus, it is clear from Eqs. (17.31) and (17.33) that electric and magnetic vectors $\vec{E}$ and $\vec{B}$, respectively, are normal or mutually perpendicular to each other and also to the direction of propagation of the vector $\vec{K}$.

Now, from Eq. (17.32), we have

$$\omega \vec{B} = \vec{K} \times \vec{E}$$

$$= (\hat{n} K \times \vec{E}) \qquad (\vec{K} = \hat{n} K)$$

$$= K(\hat{n} \times \vec{E})$$

or $$\vec{B} = \frac{K}{\omega}(\hat{n} \times \vec{E})$$

$$= \frac{1}{c}(\hat{n} \times \vec{E}) \qquad \left(\because \frac{K}{\omega} = \frac{1}{c}\right)$$

$$\vec{H} = \frac{1}{\mu_0 c}(\hat{n} \times \vec{E}) \qquad (\vec{B} = \mu_0 \vec{H})$$

In terms of magnitude, we have

$$H = \frac{1}{\mu_0 c} E \qquad (\hat{n} \times \vec{E} = E \sin 90° = E)$$

or $$\frac{E}{H} = Z_0 = \frac{E_0}{H_0} = \mu_0 c = \sqrt{\frac{\mu_0}{\in_0}} \left(c = \frac{1}{\sqrt{\mu_0 \in_0}} \right) \tag{17.34}$$

or $$\frac{E_0}{H_0} = \sqrt{\frac{4\pi \times 10^{-7} \text{ Wb / A m}}{8.85 \times 10^{-12} \text{C}^2 \text{Nm}^2}} = 376.72 = 377\ \Omega$$

The ratio E_0 / H_0 has the dimensions of electric resistance and a universal constant called *characteristic impedance*, or *intrinsic impedance*, or *wave impedance* of free space. Moreover, the ratio E_0 / H_0 is real and positive, which shows that $\vec{E}$ and $\vec{B}$ are in the same phase.

17.9 ENERGY FLOW IN PLANE ELECTROMAGNETIC WAVE

For a plane electromagnetic wave, the energy flow per unit time per unit area is given by Poynting vector as

$$\vec{S} = \vec{E} \times \vec{H}$$

From Eq. (17.34), we have

$$\vec{S} = \vec{E} \times \frac{1}{\mu_0 c}(\hat{n} \times \vec{E})$$

$$= \frac{1}{\mu_0 c}(\vec{E} \times \hat{n} \times \vec{E})$$

Using vector identity $\vec{A} \times \vec{B} \times \vec{C} = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$, we get

$$\vec{S} = \frac{1}{\mu_0 c}[(\vec{E} \cdot \vec{E})\hat{n} - (\hat{n} \cdot \vec{E})\vec{E}]$$

Now, $\hat{n} \cdot \vec{E} = 0$, because $\vec{E}$ is perpendicular to the direction of propagation. Therefore,

$$\vec{S} = \frac{E^2}{\mu_0 c}\hat{n} \qquad (\vec{E} \cdot \vec{E} = E^2) \tag{17.35}$$

$$\vec{S} = \frac{E^2}{Z_0}\hat{n} \qquad \left(C = \frac{Z_0}{\mu_0}\right) \tag{17.36}$$

For a complete cycle, the average value of $\vec{S}$ is

$$<\vec{S}> = \frac{1}{Z_0} < E^2 > \hat{n}$$

$$= \frac{1}{Z_0} R_e \left[E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}\right]^2 \hat{n}$$

$$= \frac{1}{Z_0} < \left[E_0 \cos(\vec{k} \cdot \vec{r} - \omega t)\right]^2 > \hat{n} \qquad (e^{i\theta} = \cos\theta + i \sin\theta)$$

$$= \frac{1}{Z_0} E_0^2 \cdot \frac{1}{2}\hat{n} \qquad \left(< \cos^2\theta > = \frac{1}{2}\right)$$

$$<\vec{S}> = \frac{E_{\text{rms}}^2}{Z_0}\hat{n} \tag{17.37}$$

Thus, the flow of energy in a plane electromagnetic wave in free space is along the direction of propagation of wave.

17.10 ENERGY DENSITY IN PLANE ELECTROMAGNETIC WAVE IN FREE SPACE

The energy per unit volume, or energy density in an electric field E, is given by

$$U_E = \frac{1}{2}\in_0 E^2$$

and the energy per unit volume, or energy density in a magnetic field B, is given by

$$U_B = \frac{1}{2}\mu_0 H^2$$

In an electromagnetic field, both E and B are present.

Therefore, the electromagnetic energy density is given as

$$U = U_A + U_B = \frac{1}{2}\in_0 E^2 + \frac{1}{2}\mu_0 H^2 \tag{17.38}$$

But in free space, we have

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\in_0}}$$

or $$H = E\sqrt{\frac{\in_0}{\mu_0}}$$

$\therefore$ $$U = \frac{1}{2}\in_0 E^2 + \frac{1}{2}\in_0 E^2$$

Hence, the total electromagnetic energy in free space is

$$U = \in_0 E^2 \tag{17.39}$$

Therefore, the average energy density per unit time is

$$< U > = <\in_0 E^2> = \in_0 < \left(E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}\right)_{\text{real}}^2 >$$

$$= \in_0 E_0^2 < \cos^2(\vec{k}\cdot\vec{r}-\omega t) >$$

$$= \frac{\in_0 E_0^2}{2} \qquad \left(< \cos^2\theta > = \frac{1}{2}\right)$$

or $$< U > = \in_0 E^2_{\text{rms}} \tag{17.40}$$

Dividing Eq. (17.37) by Eq. (17.40), we get

$$\frac{<\vec{S}>}{<\vec{U}>} = \frac{\hat{n}}{\in_0 Z_0} = \frac{\hat{n}}{\in_0 \sqrt{\mu_0/\in_0}} = \frac{\hat{n}}{\sqrt{\mu_0/\in_0}} = c\hat{n}$$

$$<\vec{S}> = \text{c} < U > \hat{n} \tag{17.41}$$

or energy flux = velocity of light × energy density.

17.11 ELECTROMAGNETIC WAVE IN FREE SPACE AND ITS SOLUTION

For free space or vacuum, Maxwell's equations are as follows:

$$\vec{\nabla} \cdot \vec{E} = 0 \tag{17.42}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{17.43}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{17.44}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \in_0 \frac{\partial \vec{E}}{\partial t} \tag{17.45}$$

Taking the curl of Eq. (17.44), we get

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t} \right)$$

Using the identity $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\,\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$, we get

$$(\vec{\nabla} \cdot \vec{E})\vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla})\vec{E} = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t} \right)$$

$$0 - \nabla^2 \vec{E} = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) \qquad \text{[using Eq. (17.42)]}$$

or $$\nabla^2 \vec{E} = \frac{\partial}{\partial t} \left(\mu_0 \in_0 \frac{\partial \vec{E}}{\partial t} \right) \qquad \text{[using Eq. (17.45)]}$$

or $$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad (17.46)$$

Equation (17.46) is a wave equation for electric field in free space. Similarly, for magnetic field, we can have

$$\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \qquad (17.47)$$

Now, the equation of wave propagating with a velocity υ is given as

$$\nabla^2 y = \frac{1}{\upsilon^2} \frac{\partial^2 y}{\partial t^2} \qquad (17.48)$$

Comparison of Eqs. (17.46) or (17.47) with Eq. (17.48) gives the velocity of propagation of electric and magnetic vectors in free space.

i.e., $$\frac{1}{\upsilon^2} = \mu_0 \epsilon_0$$

or $$\upsilon = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

or $$\upsilon = \frac{1}{\sqrt{\frac{\mu_0}{4\pi} \times 4\pi\epsilon_0}} = \frac{1}{\sqrt{\frac{\mu_0}{4\pi}}} \times \sqrt{\frac{1}{4\pi\epsilon_0}}$$

Now, $$\frac{\mu_0}{4\pi} = 10^{-7} \text{ Wb/A-m and } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$$

$\therefore$ $$\upsilon = \sqrt{\frac{1}{10^{-7}}} \times \sqrt{9 \times 10^9}$$

$$\upsilon = 3 \times 10^8 \text{ m/s} = c \text{ (velocity of light)}$$

Hence, the electromagnetic waves propagate in free space with the velocity of light.

17.12 SOLUTION OF PLANE ELECTROMAGNETIC WAVE: TRANSVERSE NATURE OF ELECTROMAGNETIC WAVES

From the above conclusion and from Eq. (17.48), it is clear that the equation of propagation for electromagnetic waves can be written as

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \qquad \text{(for electric field)}$$

and $$\nabla^2 B - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} = 0 \qquad \text{(for magnetic field)}$$

Solutions of these plane electromagnetic wave equations are as follows:

$$\vec{E}\,(r,t) = \vec{E}_0\, e^{i[(\vec{k}\cdot\vec{r}) - \omega t]} \tag{17.49}$$

and $$\vec{B}\,(r,t) = \vec{B}_0\, e^{i[(\vec{k}\cdot\vec{r}) - \omega t]} \tag{17.50}$$

where $\vec{E}_0$ and $\vec{B}_0$, respectively, are the complex amplitudes of the electric and magnetic fields, $\vec{r}$ $= xi + yi + zk$ is the position vector of the point under consideration, and $\vec{k}$ is the propagation vector defined as $\vec{k} = k\hat{n} = 2\pi/\lambda\,\hat{n} = 2\pi v/c\,\hat{n} = \omega/c\,\hat{n}$, where $\hat{n}$ is the unit vector in the direction of propagation of electromagnetic waves.

Now, $$\vec{\nabla}\cdot\vec{E} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

But $$\vec{k}\cdot\vec{r} = (\hat{i}k_x + \hat{j}k_y + \hat{k}k_z)\cdot(x\hat{i} + y\hat{j} + z\hat{k})$$

$$= k_x x + k_y y + k_z z$$

$$\vec{\nabla}\cdot\vec{E} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left[E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k}\right] e^{i[(k_x x + k_y y + k_z z) - \omega t]}$$

$$= \sum \frac{\partial}{\partial x}[E_{0x} e^{i[(k_x x + k_y y + k_z z) - \omega t]}]$$

$$= \sum [E_{0x}(ik_x)\, e^{i[(k_x x + k_y y + k_z z) - \omega t)]}]$$

$$= \sum iE_{0x}k_x e^{i[\vec{k}\cdot\vec{r} - \omega t]}$$

$$= i\,[E_{0x}k_x + E_{0y}k_y + E_{0z}k_z]\; e^{i[\vec{k}\cdot\vec{r} - \omega t]}$$

$$= i[\hat{i}E_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z}]\cdot[\hat{i}k_x + \hat{j}k_y + \hat{k}k_z]\, e^{i[\vec{k}\cdot\vec{r} - \omega t]}$$

$$= i\,\vec{E}_0\cdot\vec{K}\, e^{i[\vec{k}\cdot\vec{r} - \omega t]}$$

$$= i\,(\vec{K}\cdot\vec{E}) \qquad \left(\because \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} = \vec{E}\right)$$

Now, for free space, $\vec{\nabla} \cdot \vec{E} = 0$

Hence, $$i(\vec{K} \cdot \vec{E}) = 0$$

or $$\vec{K} \cdot \vec{E} = 0 \tag{17.51}$$

i.e., the direction of propagation is perpendicular to the electric field. Similar process can give

$$\vec{K} \cdot \vec{B} = 0 \tag{17.52}$$

Equations (17.51) and (17.52) on combination imply that the electric and the magnetic fields are perpendicular to the direction of propagation, i.e., the electromagnetic waves are transverse in nature.

17.13 DEPTH OF PENETRATION: SKIN DEPTH

It has been observed that an electromagnetic wave shows exponential damping $[\vec{E} = \vec{E}_0\, e^{i(\vec{k} \cdot \vec{r} - \omega t)}]$ with distance, due to various dissipative effects in the medium. It is found that in good conductors, the rate of attenuation (loss of amplitude with distance) is very high and the electromagnetic waves get almost attenuated after traversing quite a distance.

Skin depth describes the conducting behaviour in electromagnetic field, and in radio communication, it is defined as the depth for which the strength of electric field associated with the electromagnetic wave reduces to $1/e$ times of its initial value (Fig. 17.2).

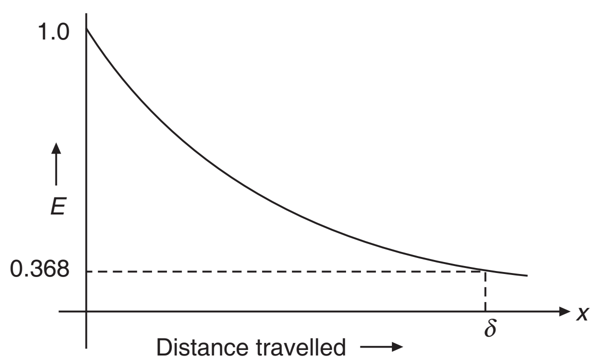

Fig. 17.2 Variation in skin depth with distance

In terms of attenuation constant, the amplitude of electric field of an electromagnetic wave is decreased by a factor $e^{-\alpha x}$. Therefore, according to the definition of skin depth, we should have

$$e^{-\alpha x} = \frac{1}{e} = e^{-1}$$

or $$\alpha x = 1$$

or $$x = \frac{1}{\alpha}$$

i.e., $$\text{skin depth} = \frac{1}{\text{attenuation constant}} \tag{17.53}$$

For good conductors, we have

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

where ω is the angular frequency, μ is the refractive index of the medium, and σ is the conductivity.

Therefore, the skin depth is

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} \tag{17.54}$$

$$= \sqrt{\frac{2}{2\pi f \mu\sigma}}$$

$$\delta = \sqrt{\frac{1}{f \mu\sigma\pi}} \tag{17.55}$$

Thus, the skin depth decreases with the increase in frequency. However, for poor conductors or good dielectrics, or insulators, the skin depth may be given as

$$\delta = \frac{2}{\sigma}\sqrt{\frac{\in}{\mu}} \tag{17.56}$$

where ε is the permittivity of the medium.

Thus, for dielectrics, the skin depth is independent of frequency.

Solved Examples

Example 17.1

Assuming that all the energy from a 1000 W lamp is radiated uniformly, calculate the average values of the intensities of electric and magnetic fields of radiation at a distance of 2 m from the lamp.

Solution

Energy of the lamp = 1000 W

= 1000 J/s

Area illuminated = $4\pi r^2 = 4\pi(2)^2$

$= 16\pi$ m^2

Therefore, the energy radiated per unit area per second = $\frac{1000}{16\pi}$

Hence, from Poynting theorem,

$$|S| = |E \times H| = EH = \frac{1000}{16\pi} \tag{1}$$

and
$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.72 \tag{2}$$

Multiplying Eq. (1) by Eq. (2), we get

$$E^2 = \sqrt{\frac{1000}{16\pi}} \times 376.72$$

$$E = 48.87 \text{ V/m}$$

Example 17.2

If the earth receives 2 cal/min cm^2 solar energy, what are the amplitudes of electric and magnetic fields of radiation?

Solution

The energy received by an electromagnetic wave per second per unit area is given by the Poynting vector as

$$|S| = |E \times H|$$
$$= EH \sin 90° = EH$$

($\therefore$ E is perpendicular to H)

As given, the energy received by earth's surface is

$$|S| = 2 \text{ cal/min cm}^2 = \frac{2 \times 4.2 \times 10^4}{60} \text{ J/m}^2\text{s}$$
$$= 1400$$

Hence,
$$EH = 1400 \tag{1}$$

But
$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}}$$

$$= 376.72\ \Omega \tag{2}$$

Multiplying Eq. (1) by Eq. (2), we get

$$E^2 = 1400 \times 376.72 = 5.274 \times 10^5$$

or
$$E = 726.2 \text{ V/m} \tag{3}$$

Substituting the value of E in Eq. (1), we get

$$726.2\, H = 1400 \text{ or } H = \frac{1400}{726.2}$$

or
$$H = 1.928 \text{ At/m}$$

Therefore, the amplitudes of electric and magnetic fields of radiation are

$$E_0 = E\sqrt{2} = 726.2\sqrt{2} = 1026.8 \text{ V/m}$$

and

$$H_0 = H\sqrt{2} = 1.928\sqrt{2} = 2.726 \text{ At/m}$$

Example 17.3

For silver, $\mu = \mu_0$ and $\sigma = 3 \times 10^7$ mhos/m. Calculate the skin depth at 10^8 Hz frequency.

Solution

The skin depth (δ) for good conductors is given by

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}} \tag{1}$$

Here, $f = 10^8$ Hz, $\sigma = 3 \times 10^7$ mhos/m, and $\mu = \mu_0 = 4\pi \times 10^{-7}$ N/A^2.

$$\therefore \quad \delta = \sqrt{\frac{2}{2\pi f\sigma\mu_0}} = \sqrt{\frac{1}{\pi f\sigma\mu_0}}$$

$$= \sqrt{\frac{1}{\pi \times 10^8 \times 3 \times 10^7 \times 4\pi \times 10^{-1}}}$$

$$= \sqrt{\frac{1}{12\pi^2 \times 10^8}} = 0.09 \times 10^{-4} \text{ m}$$

$$= 9 \times 10^{-4} \text{ cm}$$

Example 17.4

Determine the conduction current and displacement current densities in a material having conductivity of 10^{-3} mhos/m and relative permittivity $\epsilon_r = 2.45$. The electric field in the material is given by $E = 4 \times 10^{-6} \sin(9 \times 10^9 t)$ V/m.

Solution

The conduction current density (J) is given by

$$J = \sigma E \tag{1}$$

Here, $\sigma = 10^{-3}$ mhos/m and $E = 4 \times 10^{-6} \sin(9 \times 10^9 t)$ V/m.

$$\therefore \quad J = 10^{-3} \times 4 \times 10^{-6} \sin(9 \times 10^9 t)$$

$$= 4 \times 10^{-9} \sin(9 \times 10^9 t) \text{ A/m}$$

Further, the displacement current density (J_d) is given by

$$J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\epsilon_0 \epsilon_r E)$$

$$= \epsilon_0 \epsilon_r \frac{\partial E}{\partial t} \tag{2}$$

Here, $\in_0 = 8.85 \times 10^{-12}$ C^2/Nm^2 and $\in_r = 2.45$

$$J_d = 8.85 \times 10^{-12} \times 2.45 \frac{\partial}{\partial t} [4 \times 10^{-6} \sin (9 \times 10^9 t)]$$

$$= 8.85 \times 10^{-12} \times 2.45 \times 4 \times 10^{-6} \times 10^9 \cos (9 \times 10^9 t)$$

$$= 7.8 \times 10^{-7} \cos (9 \times 10^9 t) \text{ A/m}^2$$

Example 17.5

Ocean water can be assumed to be non-magnetic dielectric with $k\,[= \in/\in_0] = 80$ and $\sigma = 4.3$ mhos/m.

(i) Calculate the frequency at which the penetration depth will be 10 cm.

(ii) Show that for frequencies less than 10^8 Hz, the sea water can be considered as a good conductor.

Solution

(i) The skin depth (δ) is given by

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \qquad (1)$$

Here, $\omega = 2\pi f$, $\mu = \mu_0 = 4\pi \times 10^{-7}$ N/A^2, $\sigma = 4.3$ mhos/m, and $\delta = 10$ cm $= 0.1$ m.

$$\therefore \qquad 0.1 = \sqrt{\frac{2}{2\pi f \times \mu_0 \times \sigma}} = \sqrt{\frac{1}{3.14 f \times 4\pi \times 10^{-7} \times 4.3}}$$

Squaring both sides, we get

or
$$0.01 = \sqrt{\frac{1}{3.14 f \times 4 \times 3.14 \times 4.3 \times 10^{-7}}}$$

or
$$f = \sqrt{\frac{1}{3.14 \times 4 \times 3.14 \times 4.3 \times 10^{-7} \times 0.01}}$$

$$= 6 \times 10^6 \text{ Hz}$$

(ii) For the ocean water to be a good conductor, the value of $\sigma/\omega\in$ should be greater than 1.
Here, $\sigma = 4.3$, $\omega = 2\pi f = 2 \times 3.14 \times 10^8$ and $\in = 80 \in_0$ or $\in = 80 \in_0$.

Hence, for frequency less than 10^8 Hz, we have

$$\frac{\sigma}{\omega\in} > \frac{4.3}{2\pi \times 10^8 \times 80 \in_0}$$

or
$$> \frac{4.3 \times 2}{4\pi \in_0 \times 80 \times 10^8} > \frac{8.6 \times 9 \times 10^9}{80 \times 10^8} > \frac{8.6 \times 9}{8}$$

or
$$> 9.67$$

Since the value of $\sigma/\omega\in$ is >> 1, therefore, the ocean water is a good conductor at frequency less than 10^8 Hz.

Example 17.6

A plane electromagnetic wave propagating along the X-direction has a wavelength of 5.0 mm. The electric field is in the Y-direction and its maximum magnitude is 48 V/m. Write the equations of the electric and magnetic fields as a function of x and t.

Solution

The equations of electric and magnetic fields of a plane electromagnetic wave are given by

$$E = E_0 \sin\left[\frac{2\pi}{\lambda}(ct - x)\right] \text{ and } H = H_0 \sin\left[\frac{2\pi}{\lambda}(ct - x)\right]$$

Given: $E_0 = 38$ V/m and $\lambda = 5.0$ mm $= 5 \times 10^{-3}$ m

Hence, $$E = 38 \sin\left[\frac{2\pi}{5\times10^{-3}}(ct - x)\right]$$

or $$E = 38 \sin\left[\frac{2\pi\times10^{3}}{5}(ct - x)\right] \qquad (1)$$

The magnitude of the magnetic field is given by

$$H_0 = \frac{E_0}{c} = \frac{38}{3\times10^{8}} = 1.27 \times 10^{-7} \text{ Weber/m}^2$$

$\therefore$ $$H = 1.27 \times 10^{-7} \sin\left[\frac{2\pi\times10^{3}}{5}(ct - x)\right] \qquad (2)$$

The electric field is along the Y-axis and the magnetic field is along the Z-axis.

Example 17.7

Show that for frequency less than or equal to 10^9 Hz, a sample of silicon will act like a good conductor. For silicon, one may assume $\in/\in_0 = 12$ and $s = 2$ mhos/cm. Also, calculate the penetration depth for this sample at a frequency of 10^6 Hz.

Solution

For a material to be a conductor, the value of $\sigma/\omega\varepsilon$ should be greater than 1.

Here, $\sigma = 2$ mhos/cm $= 200$ mhos/m, $\omega = 2\pi f = 2 \times 3.14 \times 10^9$, $\in = 12\in_0$, and $1/4\pi\in_0 = 9 \times 10^9$.

$\therefore$ $$\frac{\sigma}{\omega\in} = \frac{200}{2\times\pi\times10^{9}\times12\in_0}$$

$$= \frac{200\times2}{4\pi\times\in_0\times12\times10^{9}} = \frac{400\times9\times10^{9}}{12\times10^{9}} = 300$$

Since $\sigma/\omega\epsilon$ is greater than 1, silicon is a good conductor at frequency $\leq 10^9$ Hz.

The penetration depth is given by

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Here, $f = 10^6$ Hz, $\mu = \mu_0 = 4\pi \times 10^{-7}$, and $\sigma = 200$.

$$\therefore \qquad \delta = \sqrt{\frac{2}{2\pi \times 10^6 \times 4\pi \times 10^{-7} \times 200}} = 0.036 \text{ m} = 3.6 \text{ cm}$$

Example 17.8

An electromagnetic wave absorption is studied using a copper sheet with $\mu = \mu_0$, $\epsilon_r = 1$, and conductivity (σ) = 5.8×10^7 S/m. Calculate the frequency of incident electromagnetic wave for which the skin depth is 0.1 mm. Also, tell the name of incident radiation.

Solution

The skin depth is given by the expression

$$\delta = \frac{1}{\pi\sigma f \mu}$$

Substituting values of δ, σ, and μ, we get

$$0.1 \times 10^{-3} = \sqrt{\frac{1}{3.14 \times 5.8 \times 10^7 \times f \times 4 \times 3.14 \times 10^{-7}}}$$

$$(3.14)^2 \times 4 \times 5.8 \times 10^7 \times 10^{-7} \times f = \left(\frac{1}{0.1 \times 10^{-3}}\right)^2$$

$$f = \frac{10^6}{0.1 \times 0.1 \times 9.86 \times 23.2}$$

$$= \frac{10^6}{2.29}$$

$$= 3.36 \times 10^5 \text{ Hz}$$

The incident electromagnetic wave is the radio part of the spectrum.

Example 17.9

A silver foil is exposed to microwave radiation at a frequency of 10^{10} Hz. Calculate the skin depth if its conductivity is 3×10^7 S/m (given $\mu = \mu_0$).

Solution

The skin depth is calculated by the expression

$$\delta = \sqrt{\frac{1}{\pi\sigma f \mu}}$$

Substituting parameters like f, μ, and σ, the skin depth is calculated as

$$\delta = \sqrt{\frac{1}{3.14\times 3\times 10^{7}\times 10^{10}\times 4\times 3.14\times 10^{-7}}}$$

$$= \frac{1}{\sqrt{118.3\times 10^{10}}}$$

$$= \frac{10^{-5}}{\sqrt{118.3}}\ \text{m}$$

$$= \frac{10^{-5}}{10.8}\ \text{m}$$

$$= 0.93\ \mu\text{m}$$

Example 17.10

A lamp radiates 500 W power uniformly in all directions. Calculate the electric and magnetic field intensities at 1 m distance from the lamp.

Solution

From Poynting theorem, the energy radiated per unit time is defined as electromagnetic power flow per unit area and is given as $|\vec{S}| = |\vec{E}\times\vec{H}|$.

The area of the spread of electromagnetic power recorded from the lamp is calculated as

Area = $4\pi r^2$ (as power is radiated in the form of a sphere-like unrealised light)

$$\text{Area of illumination} = 4\pi(1)^2$$
$$= 4\pi\ \text{m}^2$$

$$|\vec{S}| = \frac{500}{4\pi}\ \text{J/m}^2\text{s}$$

Therefore, $$EH = \frac{500}{4\pi} \tag{1}$$

For free space, impedance is shown by

$$Z_0 = \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$= 376.6$$

$$\frac{E}{H} = 376.6$$

$$E = 376.6\,H \tag{2}$$

Substituting the value of E in the expression of illuminated power region [Eq. (1)], we get

$$376.6 \times H^2 = \frac{500}{12.56}$$

$$H^2 = \frac{500}{376.6 \times 12.56}$$

$$= 0.105$$

$$H = 0.33 \text{ At/m}$$

Solving power expression for electric field, we get

$$EH = \frac{500}{12.56}$$

$$E = \frac{500}{12.56 \times 0.33}$$

$$= 120.63 \text{ V/m}$$

Example 17.11

An aluminum sheet with $\mu = \mu_0$ and $\sigma = 3.5 \times 10^7$ S/m is exposed to an electromagnetic wave. The skin depth was measured to be 0.03 mm. Calculate the frequency of incident radiation and write about its location in electromagnetic spectra.

Solution

The skin depth is calculated using the expression

$$\delta = \sqrt{\frac{1}{\pi\sigma f \mu}}$$

Substituting the values of parameters given like μ and σ, we get

$$0.3 \times 10^{-3} = \sqrt{\frac{1}{3.14 \times 3.5 \times 10^7 \times f \times 4 \times 3.14 \times 10^{-7}}}$$

$$f = \frac{10^6}{0.03 \times 0.03 \times 138}$$

$$= \frac{10^{10}}{1242}$$

$$= 8.05 \times 10^6 \text{ Hz}$$

$$= 8 \text{ MHz}$$

The incident radiation is radio part of the electromagnetic spectrum.

Example 17.12

Calculate the solar energy received by the moon during solar eclipse. Also, find the amplitude of electric and magnetic fields of radiation. Given that the distance between the sun and the earth ≈ 1.47×10^{11} m, the distance between the moon and the earth ≈ 3×10^8 m, and the power radiated from the sun is 3.8×10^{26} W.

Solution

The minimum effective distance between the sun and the moon during eclipse is 1.47×10^{11} m. The average solar energy recovered by the moon during eclipse is calculated as

$$S = \frac{\rho}{4\pi R^2}$$

$$= \frac{3.8\times10^{26}}{4\times3.14\times(1.47\times10^{11})^2}\ \text{W/m}^2$$

$$1\ \text{W} = \frac{1\ \text{J}}{1\text{s}}$$

$$= \frac{1}{4.2\times10^4}\ \text{cal/s}$$

$$= \frac{60}{4.2\times10^4}\ \text{cal/min}$$

$$\therefore \qquad S = \frac{3.8\times10^{26}\times60}{4\times3.14\times1.44\times1.44\times4.2\times10^{22}\times10^4}\ \text{cal/min m}^2$$

$$= \frac{228\times10^{26}\times10^{-26}}{113.99}$$

$$= 2.1\ \text{cal/min m}^2$$

Example 17.13

An aluminum sheet is exposed to laser radiation at a wavelength of 6328 Å. Calculate the skin depth if $\mu = \mu_0$ and $\sigma = 3.5 \times 10^7$ S/cm are the values for aluminum.

Solution

The incident radiation wavelength can be written in terms of frequency as

$$c = f\lambda$$

$$f = \frac{c}{\lambda}$$

For skin depth, we have

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$$

$$= \sqrt{\frac{1}{3.14 \times \frac{3\times 10^8}{6328\times 10^{-10}} \times 4 \times 3.14 \times 10^{-7} \times 3.5 \times 10^7}}$$

$$= \frac{1}{\sqrt{0.065\times 10^8}} \text{ m}$$

$$= \frac{1}{\sqrt{650\times 10^{14}}} \text{ m}$$

$$= \frac{10^{-7}}{25.5} \text{ m}$$

$$= 3.9 \text{ Å}$$

17.14 FORMULAE AND HIGHLIGHTS

1. Poisson's and Laplace's equations: $\nabla^2 V = \frac{\rho}{\epsilon_0}$, $\nabla^2 V = 0$
2. Equation of continuity: div $\vec{J} + \frac{\partial \rho}{\partial t} = 0$
3. Maxwell's electromagnetic equations:
   (i) Differential form

   (a) $\vec{\nabla} \cdot \vec{D} = \rho$ (b) $\vec{\nabla} \cdot \vec{B} = 0$

   (c) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (d) $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

   (ii) Integral form

   (a) $\int_s \vec{D} \cdot \vec{dS} = \int_V \rho dV$ (b) $\int_s \vec{B} \cdot \vec{dS} = 0$

   (c) $\oint \vec{B} \cdot \vec{dl} = -\frac{\partial}{\partial t} \int_s \vec{B} \cdot \vec{dS}$ (d) $\oint \vec{H} \cdot \vec{dl} = \int_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot \vec{dS}$

(iii) In free space

(a) $\vec{\nabla} \cdot \vec{D} = 0$ or $\vec{\nabla} \cdot \vec{E} = 0$ (b) $\vec{\nabla} \cdot \vec{B} = 0$ or $\vec{\nabla} \cdot \vec{H} = 0$

(c) $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} = -\mu_0 \dfrac{\partial \vec{H}}{\partial t}$ (d) $\vec{\nabla} \times \vec{H} = \dfrac{\partial \vec{D}}{\partial t} = \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

(iv) Poynting theorem

$$-\int_V \vec{J} \cdot \vec{E}\, dV = \frac{\partial}{\partial t} \int_V \frac{1}{2}\left(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}\right) dV + \int_s \left(\vec{E} \times \vec{H}\right) \cdot \vec{dS}$$

This equation represents the law of conservation of energy.

The term $-\int_V \vec{J} \cdot \vec{E}\, dV$ represents the rate of transfer of energy into the electromagnetic field due to the motion of the charges.

The term $\dfrac{\partial}{\partial t} \int_V \dfrac{1}{2}\left(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}\right) dV$ represents the rate of electromagnetic energy stored.

The term $\int_s \left(\vec{E} \times \vec{H}\right) \cdot \vec{dS}$ represents the amount of energy crossing per second through the closed surface.

The factor $\vec{E} \times \vec{H} = \vec{S}$ is called Poynting vector.

5. For a free space:
   $\mu = 1$, $\sigma = 0$, $\epsilon = 1$, $J = 0$, $\rho = 0$, $D = \epsilon_0 E$, and $B = \mu_0 H$
6. The speed of light in free space, $\upsilon = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8$ m/s
7. For good conductors, $\dfrac{\sigma}{\omega \epsilon} >> 1$
8. Skin depth or depth of penetration

   (i) For good conductors, $\delta = \sqrt{\dfrac{2}{\omega \mu \sigma}} = \sqrt{\dfrac{1}{\pi f \mu \sigma}}$

   (ii) For good dielectrics, $\delta = \dfrac{2}{\sigma}\sqrt{\dfrac{\epsilon}{\mu}}$

   Electromagnetic waves are transverse in nature.

Exercises

Section A
Theoretical Questions

1. Write the Maxwell's equation in integral and differential forms. Explain the physical significance of each equation.
2. Explain the concept of Maxwell's displacement current and show how it led to the modification of the Ampere's law.
3. State Ampere's law in differential and integral forms. Discuss the modification of Ampere's law in terms of displacement current. Explain the term displacement current and give its implications.
4. Deduce Maxwell's four equations in free space. Explain the concept of Maxwell's displacement current and show how it led to the modification of Ampere's law.
5. Write the Maxwell's equation in integral as well as in differential forms and explain their physical significance. Show that the velocity of plane electromagnetic waves in the free space is given by $c = 1/\sqrt{\mu_0 \in_0}$.
6. Prove that electromagnetic waves are transverse in nature.
7. Deduce the equation for the propagation of plane electromagnetic waves in the free space. Show that the electric and magnetic vectors are normal to each other and to the direction of the propagation of the wave.
8. State and explain Poynting theorem for the flow of energy in electromagnetic waves.
9. What is Poynting vector? Discuss the work–energy theorem for the flow of energy in an electromagnetic field.
10. Deduce Poynting theorem for the flow of energy in an electromagnetic field.
11. Define Poynting vector. Derive an expression for it and explain its physical significance for electromagnetic waves in free space.
12. Show that the wave equation for electric field $\vec{E}$ is given by

$$\nabla^2 \vec{E} = \mu_0 \in_0 \left(\frac{\partial^2 \vec{E}}{\partial t^2} \right)$$

13. Derive electromagnetic wave equations in conducting medium and discuss its solutions.
14. Define skin depth or depth of penetration. Derive expression for it for good conductors.
15. Derive an expression for intrinsic impedance of an electromagnetic wave in good conductors.

Section B
Numerical Problems

1. If the magnitude of $\vec{H}$ in a plane wave is 1 A/m, find the magnitude of $\vec{E}$ for plane wave in free space. (*Ans*. 376.72 V/m)

2. The maximum electric field in a plane electromagnetic wave is 10^2 N/C. The wave is going in the X-direction and the electric field is in the Y-direction. Find the maximum magnetic field in the wave in its direction. (*Ans*. 3.33×10^{-7} T)
3. If the earth receives 2 cal/min cm^2 solar energy, what are the amplitudes of electric and magnetic fields of radiation? (*Ans*. $E_0 = 1026.8$ V/m, $H_0 = 2.726$ At/m)
4. Assuming that all the energy from a 1000 W lamp is radiated uniformly, calculate the average values of the intensities of electric and magnetic fields of radiation at a distance of 2 m from the lamp. (*Ans*. $E = 86.59$ V/m, $H = 0.23$ At/m)
5. The sunlight strikes the upper atmosphere of the earth with energy flux of 1.38 kW/m^2. What will be the peak values of electric and magnetic fields at the points?
(*Ans*. $E_0 = 1.02$ KV/m, $B_0 = 3.4 \times 10^{-6}$ Wb/m^2)
6. The relative permittivity of distilled water is 81. Calculate the refractive index and the velocity of light in it. (*Ans*. $\upsilon = 3.33 \times 10^7$ m/s, $\mu = 9$)
7. The electric field intensity of a uniform plane electromagnetic wave in air is 7.5 kV/m in the Y-direction. The wave is propagating in the X-direction at a frequency of 2×10^9 rad/s. Determine
   (i) the wavelength of electromagnetic wave,
   (ii) the frequency,
   (iii) the time period, and
   (iv) the amplitude of magnetic field intensity.
   ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2 and $\in_0 = 8.854 \times 10^{-12}$ C^2/Nm^2)
   (*Ans*. $\lambda = 0.942$ m, $f = 318.5$ MHz, $T = 3.14 \times 10^{-9}$ s, $H_0 = 14.91$ A/m)
8. Calculate the depth of penetration δ at the frequency 71.6 MHz in aluminum. The related parameters for aluminium are $\mu = \mu_0 = 4\pi \times 10^{-7}$ N/A^2, $\sigma = 3.54 \times 10^7$ S/m. (*Ans*. 10 μm)
9. Find the skin depth δ at a frequency of 3.0×10^6 Hz in aluminium where $\sigma = 38.0 \times 10^6$ S/m and $\mu_r = 1$. Also, find out the propagation constant and the wave velocity.
(*Ans*. $\delta = .04716$ mm, $Y = 29.986 \times 10^3 < 45°$/m, $\upsilon = 888.51$ m/s)
10. For sea water, $\mu = \mu_0 = 4\pi \times 10^{-7}$ N/A^2, $\in = 70 \in_0$, and the conductivity $\sigma = 5$ S/m. Find the skin depth and the attenuation constant for sea water. (*Ans*. $\delta = 0.0089$ m, $\alpha = 112.36$ NP/m)

Section C
Multiple Choice Questions

1. Gauss's law in electrostatics is represented by

(a) $\iint \vec{E} \cdot d\vec{S} = 0$ (b) $\iint \vec{E} \cdot d\vec{l} = q$

(c) $\iint \vec{E} \cdot d\vec{S} = \dfrac{q}{\in_0}$ (d) $\iint \vec{E} \cdot d\vec{S} = q$

2. Ampere's circuital law for direct current is shown by
   (a) $\iint \vec{B} \cdot d\vec{l} = \vec{J}$
   (b) $\iint \vec{B} \times d\vec{l} = \mu \vec{J}$
   (c) $\oint \vec{B} \cdot d\vec{l} = \mu \vec{J}$
   (d) $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$
3. Gauss's law in magnetostatics is expressed by the relation
   (a) $\vec{\nabla} \times \vec{B} = \mu \vec{J}$
   (b) $\vec{\nabla} \cdot \vec{B} = 0$
   (c) $\vec{\nabla} \cdot \vec{B} = 0$
   (d) $\vec{\nabla} \cdot \vec{B} = \mu_0 \vec{J}$
4. Faraday's law of electromagnetic induction is expressed by the relation
   (a) $\vec{\nabla} \cdot \vec{E} = 0$
   (b) $\vec{\nabla} \cdot \vec{E} = \rho$
   (c) $\vec{\nabla} \cdot \vec{E} = \dfrac{\partial \vec{B}}{\partial t}$
   (d) $\vec{\nabla} \cdot \vec{E} = 0$
5. Which of the following is not the part of Maxwell's equations?
   (a) div $\vec{E} = 0$
   (b) div $\vec{B} = 0$
   (c) curl $\vec{F} = 0$
   (d) curl $\vec{B} = \mu_0 \vec{J}$
6. The equation of continuity states about
   (a) conservation of charge and its flow
   (b) flow of electrostatic lines or forces
   (c) conservation of magnetic field
   (d) electromagnetic power flow
7. Which of the following is not from the set of Maxwell's equations written for conducting medium?
   (a) div $\vec{D} = \rho$
   (b) div $\vec{B} = 0$
   (c) curl $\vec{E} = \dfrac{-\partial \vec{B}}{\partial t}$
   (d) curl $\vec{B} = \mu \left(\vec{J} + \dfrac{-\partial \vec{D}}{\partial t} \right)$
8. If the curl of a vector field is zero, say curl $\vec{F} = 0$, then it shows that
   (a) $\vec{F}$ is a rotational field
   (b) $\vec{F}$ is a diverging field
   (c) $\vec{F}$ is a parabolic field
   (d) $\vec{F}$ is a conservative field
9. If the divergence of velocity of water is zero between two points, the water is flowing in
   (a) streamline motion
   (b) turbulent motion
   (c) velocity gradient
   (d) none of these

10. The Gauss's divergence theorem connects
    (a) line integral to volume integral (b) surface integral to volume integral
    (c) volume integral to line integral (d) line integral to surface integral
11. The Stoke's theorem converts
    (a) line integral to volume integral (b) surface integral to volume integral
    (c) volume integral to line integral (d) line integral to surface integral
12. Which of the following is not the part of set of Maxwell's equation in free space?
    (a) div $\vec{E} = \rho/E$ (b) div $\vec{B} = 0$
    (c) curl $\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$ (d) curl $\vec{B} = -\mu_0 \in_0 \frac{\partial \vec{E}}{\partial t}$
13. The velocity of an electromagnetic wave in free space is expressed by the relation
    (a) $\upsilon = \frac{1}{\sqrt{\mu \in}}$ (b) $\upsilon = \frac{1}{\sqrt{\mu_0 \in_0}}$
    (c) $\upsilon = \sqrt{\frac{\mu_0}{\in_0}}$ (d) $\upsilon = \sqrt{\mu_0 \in_0}$
14. The transverse nature of an electromagnetic wave is justified by the option
    (a) $\vec{K} \cdot \vec{E} = 0$ and $\vec{K} \cdot \vec{B} \neq 0$ (b) $\vec{K} \cdot \vec{E} \neq 0$ and $\vec{K} \cdot \vec{B} = 0$
    (c) $\vec{K} \cdot \vec{E} = 0$ and $\vec{K} \cdot \vec{B} = 0$ (d) $\vec{K} \cdot \vec{E} \neq 0$ and $\vec{K} \cdot \vec{B} \neq 0$
15. The impedance offered by free space to the propagation of an electromagnetic wave is given by
    (a) $Z_0 = \sqrt{\frac{\mu_0}{\in_0}}$ (b) $Z_0 = \sqrt{\frac{\in_0}{\mu_0}}$
    (c) $Z_0 = \sqrt{\mu_0 \in_0}$ (d) $Z_0 = \frac{1}{\sqrt{\mu_0 \in_0}}$
16. The Poynting vector signifies
    (a) the flow of magnetostatic lines of force
    (b) the flow of electromagnetic power
    (c) the flow of electrostatic lines of force
    (d) none of these
17. Poynting vector is expressed by the relation
    (a) $\vec{S} = \vec{E} \cdot \vec{H}$ (b) $\vec{S} = \vec{K} \cdot \vec{H}$
    (c) $\vec{S} = \vec{K} \cdot \vec{E}$ (d) $\vec{S} = \vec{E} \times \vec{H}$

18. The electrostatic energy component of Poynting vector is shown by
    (a) $\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{E}\cdot\vec{B}\right)$
    (b) $\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{E}\cdot\vec{D}\right)$
    (c) $\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{H}\cdot\vec{B}\right)$
    (d) $\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{E}\times\vec{D}\right)$
19. The electromagnetic power density due to moving charge is shown by
    (a) $\iiint \vec{J}\cdot\vec{E}\,dV$
    (b) $\iint (\vec{J}\times\vec{E})\,dV$
    (c) $\iint (\vec{J}\cdot\vec{E})\,dV$
    (d) $\iiint (\vec{J}\times\vec{E})\,dV$
20. The free space impedance value is
    (a) 300 Ω
    (b) 350 Ω
    (c) 370 Ω
    (d) 376.6 Ω
21. A transfer of Maxwell's equation from free space to conductor medium does not involve change in
    (a) div $\vec{E} = \rho$
    (b) div $\vec{B} = 0$
    (c) curl $\vec{E} = -\frac{\partial \vec{B}}{\partial t}$
    (d) curl $\vec{B} = \mu\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$
22. The conducting medium characteristics are specified by
    (a) $\rho = 0, \sigma \neq 0$
    (b) $\rho = 0, \sigma = 0$
    (c) $\rho \neq 0, \sigma \neq 0$
    (d) $\rho \neq 0, \sigma = 0$
23. From the solution of electromagnetic wave equation $|\vec{k}| = \alpha + i\beta$, what forms the attenuation factor?
    (a) $e^{i(\alpha + i\beta)\, r}$
    (b) $e - \beta r$
    (c) $e^{i\alpha|\hat{k}|\cdot\vec{r}}$
    (d) $|\vec{k}|$
24. The penetration depth is defined as the
    (a) depth up to which electric field intensity decreases to 37% of its value on surface
    (b) depth up to which magnetic field intensity decreases to 37% of its value on surface
    (c) depth up to which electromagnetic power reduces to 37% of its value on surface
    (d) depth up to which electromagnetic power reduces to 63% of its value on surface

25. The good conductors are specified by the condition

(a) $\left(\frac{\sigma}{\omega\epsilon}\right) = 1$ (b) $\left(\frac{\sigma}{\omega\epsilon}\right) << 1$

(c) $\left(\frac{\sigma}{\omega\epsilon}\right)^2 = 1$ (d) $\left(\frac{\sigma}{\omega\epsilon}\right) >> 1$

26. The skin depth is expressed by the relation

(a) $\delta = \sqrt{\frac{2}{\mu\epsilon\,\omega}}$ (b) $\delta = \sqrt{\frac{1}{\mu\pi\sigma f}}$

(c) $\delta = \sqrt{\frac{1}{\mu\pi\epsilon f}}$ (d) $\delta = \sqrt{\frac{\mu\sigma\omega}{2}}$

Answers

1. (c)	2. (d)	3. (b)	4. (c)	5. (c)	6. (a)
7. (d)	8. (d)	9. (a)	10. (b)	11. (d)	12. (a)
13. (b)	14. (c)	15. (a)	16. (b)	17. (d)	18. (b)
19. (a)	20. (d)	21. (d)	22. (a)	23. (b)	24. (c)
25. (d)	26. (b)				

Section D
Fill in the Blanks

1. The change in any variable with respect to time is defined as ....
2. The change in any variable with respect to space coordinate is defined as ....
3. The electrostatic potential gradient is defined as .... field.
4. The del operator is expressed as ....
5. The divergence of vector field is defined as .... product of this field with the del operator.
6. If a vector field has its lines of force more at source but less at sink, its divergence will be ....
7. Streamline motion for water flowing with a velocity υ is expressed by ....
8. A solenoidal vector $(\vec{A})$ is one for which ....
9. The curl of a vector field depicts its .... behaviour.
10. If a charged particle is moving in a circle, the curl of its velocity will be ....
11. A vector field with its curl zero is described as .... and irrotational.
12. The Stoke's theorem is used to convert .... integral to .... integral.

13. The skin depth for a bad conductor is expressed by ....
14. The Gauss divergence theorem is used to convert .... integral to .... integral.
15. Gauss law in electrostatics states that the net electric flux enclosed by a surface is $1/\varepsilon_0$ times the .... enclosed by the surface.
16. Gauss law in magnetostatics gives an indication about non-existence of ....
17. Displacement current is one which satisfies .... equation for charge.
18. The concept of displacement current was proposed due to the contradiction offered by the equation .... for all types of current (either dc or ac).
19. The velocity of an electromagnetic wave in free space is shown by ....
20. Transverse nature of electromagnetic wave is expressed by .... and ....
21. The free space impedance (Z_0) is calculated equal to 376.6 Ω using the expression ....
22. A conducting medium is characterised by .... and ....
23. Free space medium is characterised by .... and ....
24. Skin depth is defined as the depth at which the .... of an electromagnetic wave reduces to 37% of its surface value.
25. The skin depth for good conductors is .... at higher frequency.
26. The sheets of insulating medium are not preferred as shield for electromagnetic wave because of the frequency .... nature of skin depth.
27. Poynting theorem is defined for .... power flow per unit area.
28. Poynting vector $\vec{S}$ is expressed as ....
29. Poynting vector is used to express electromagnetic power flow as a sum of .... and .... field energies along with power density due to many charges.
30. The average electromagnetic power flow is given by .... in free space.

Answers

1. rate	2. gradient	3. electric
4. $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$	5. dot	6. negative
7. div $\vec{V} = 0$	8. $\vec{V} \cdot \vec{A} = 0$	9. spiral
10. zero	11. conservative	12. line, surface
13. $\delta = \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$	14. surface, volume	15. total charge

16. monopoles
17. continuity
18. div $\vec{J} = 0$
19. $V = 1/\sqrt{\mu_0 \in_0}$
20. $\vec{K} \cdot \vec{E} = 0, \vec{K} \cdot \vec{B} = 0$
21. $Z_0 = \sqrt{\mu_0 / \in_0}$
22. $\rho = 0, \sigma \neq 0$
23. $\rho = 0, \sigma = 0$
24. intensity
25. lower
26. independent
27. electromagnetic
28. $\vec{S} = \vec{E} \times \vec{H}$
29. electrostatic, magnetostatic
30. $S_{\text{avg}} = \dfrac{E_{rms}^2}{Z_0}$