The equivalent circuit is shown in Fig. 1.29

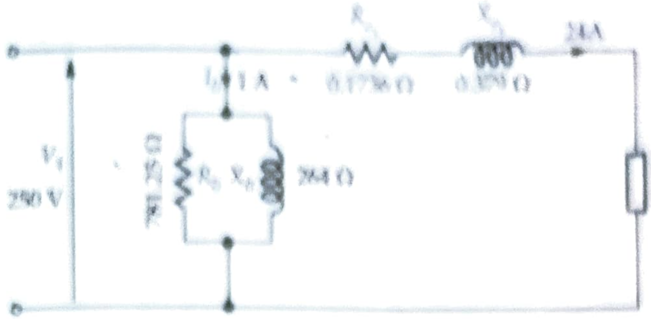

Fig. 1.29 Equivalent circuit of Example 1.19

EXAMPLE 1.20 *A 50-Hz, 1-phase transformer has a turn-ratio of 6. The resistances are 0.90 Ω and 0.03 Ω and the reactances 5 Ω and 0.13 Ω for high-voltage and low-voltage windings respectively. Find (a) the voltage to be applied to the high-voltage side to obtain full-load current of 200 A in the low-voltage winding on short-circuit, (b) the power factor on short circuit.*

SOLUTION. $\dfrac{T_1}{T_2} = 6, \; R_1 = 0.9\,\Omega, \; R_2 = 0.03\,\Omega$

$$X_1 = 5\,\Omega, \; X_2 = 0.13\,\Omega$$

$$R_{e_1} = R_1 + R_2\left(\frac{T_1}{T_2}\right)^2 = 0.9 + 0.03(6)^2 = 1.98\,\Omega$$

$$X_{e_1} = X_1 + X_2\left(\frac{T_1}{T_2}\right)^2 = 5 + 0.13(6)^2 = 9.68\,\Omega$$

$$Z_{e_1} = \sqrt{R_{e_1}^2 + X_{e_1}^2} = \sqrt{(1.98)^2 + (9.68)^2} = 9.88\,\Omega$$

$$\cos\phi_{sc} = \frac{R_{e_1}}{Z_{e_1}} = \frac{1.98}{9.88} = 0.2$$

$$I_{sc_2} T_2 = I_{sc_1} T_1$$

$$I_{sc_1} = I_{sc_2}\frac{T_2}{T_1} = 200 \times \frac{1}{6} = 33.33 \text{ A}$$

$$V_{sc} = I_{sc_1} Z_{e_1} = \frac{200}{6} \times 9.88 = 329.3 \text{ V}$$

1.36 TRANSFORMER EFFICIENCY

The ratio of the output power to the input power in a transformer is known as transformer efficiency (η).

$$\eta \triangleq \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \text{copper loss} + \text{iron loss}} \text{ pu}$$

Thus, the per unit efficiency at load current I_2 and power factor $\cos\phi_2$

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{e_2} + P_i} \text{ pu}$$

The per-unit efficiency at full load is

$$\eta_{fl} = \frac{V_2 I_{2fl} \cos\phi_2}{V_2 I_{2fl} \cos\phi_2 + I_{2fl}^2 R_{e_2} + P_i}$$

If $\quad S_{2fl} = (VA)_{2fl} = V_2 I_{2fl} = \text{full-load VA} = \text{rated VA}$

then $\quad \eta_{fl} = \dfrac{S_2 \cos\phi_2}{S_2 \cos\phi_2 + P_{cfl} + P_i}$

Since the transformer is a static device, there are no rotational losses such as windage and frictional losses in a rotating machine. In a well-designed transformer the efficiency can be as high as 99%.

1.37 CONDITION FOR MAXIMUM EFFICIENCY

The per-unit (pu) efficiency at load current I_2 is

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{e_2} + P_i} \tag{1.37.1}$$

$$= \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + I_2 R_{e_2} + (P_i / I_2)} \tag{1.37.2}$$

Equation (1.37.2) shows that the efficiency varies with the load. The plot of efficiency η versus load (or load current) is shown in Fig. 1.30.

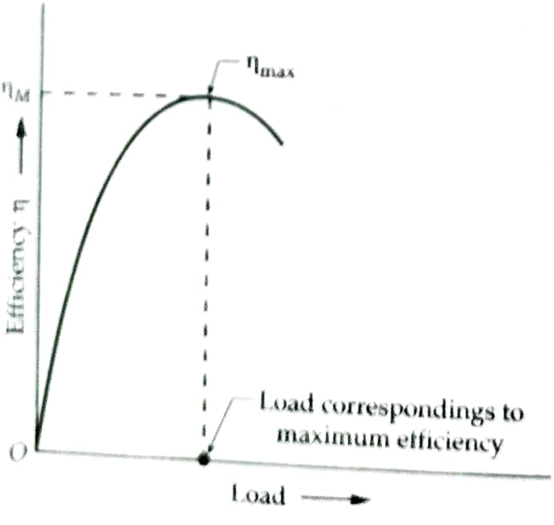

Fig. 1.30 Efficiency versus load curve.

CHAPTER

2

Transformer – II

2.1 SINGLE-PHASE AUTOTRANSFORMER

A *single-phase autotransformer* is a one-winding transformer in which a part of the winding is common to both high-voltage and low-voltage sides.

Consider a single winding *abc* of Fig. 2.1. The terminals *a* and *c* are the high-voltage terminals. The low-voltage terminals are *b* and *c* where *b* is a suitable tapping point. The portion *bc* of the full winding *abc* is common to both high-voltage and low-voltage sides. The winding *bc* is called the ***common winding*** and the smaller winding *ab* is called the ***series winding*** because it is connected in series with the common winding.

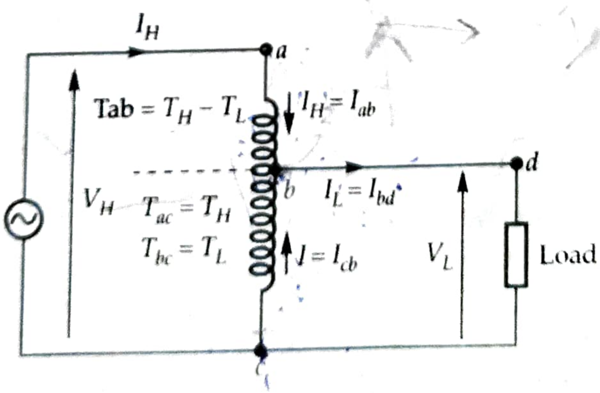

Fig. 2.1 Step-down autotransformer.

A *step-down autotransformer* is one in which the primary voltage is greater than the secondary voltage. The source voltage V_H is applied to the full winding *abc* and the load is connected across the secondary terminals *bc*. This arrangement is called the *step-down autotransformer* as shown in Fig. 2.1.

Since the transformer windings are physically connected, a different terminology is used for the autotransformer than for other types of transformers. Let

$T_H = T_{ac}$ = number of turns of full winding *abc*

= number of turns of *hv* side

$T_1 = T_{bc}$ = number of turns of the common winding *bc*

= number of turns of the *lv* side

$T_{ab} = T_H - T_L$ = number of turns of the series winding *ab*

V_H = input voltage on the *hv* side

V_l = output voltage on the *lv* side

I_H = input current in the *hv* side

I_L = output current in the *lv* side

Current in the series winding = $I_{ab} = I_H$

Current in the common winding $bc = I_{cb} = I$

In an autotransformer there are two voltage ratios namely circuit voltage ratio and winding voltage ratio. The ***circuit-voltage ratio***

$$\frac{V_H}{V_L} = \frac{T_H}{T_L} = a_A \tag{2.1.1}$$

The quantity a_A is called the ***transformation ratio*** of the autotransformer. It is seen from Eq. (2.1.1) that a_A is always greater than 1.

When the load is connected across the secondary terminals a current I flows in the common winding *bc*. It has a tendency to reduce the main flux but the primary current I_H increases to such a value that the mmf in winding *ab* neutralizes the mmf in winding *bc*. That is,

$$\mathbf{I}_{ab} T_{ab} = \mathbf{I}_{bc} T_{bc}$$

or

$$\mathbf{I}_H (T_H - T_L) = \mathbf{I} T_L \tag{2.1.2}$$

$$\frac{\mathbf{I}}{\mathbf{I}_H} = \frac{T_H - T_L}{T_L} = \frac{T_H}{T_L} - 1 = a_A - 1 \tag{2.1.3}$$

The winding-voltage ratio is

$$a = \frac{V_{ab}}{V_{bc}} = \frac{T_{ab}}{T_{bc}} = \frac{T_H - T_L}{T_L}$$

or

$$a = a_A - 1 \tag{2.1.4}$$

Since the right-hand side of Eq. (2.1.3) is a pure number, it follows that the current in windings *ab* and *cb* are in phase.

By KCL at point *b*

$$\mathbf{I}_L = \mathbf{I}_H + \mathbf{I} \tag{2.1.5}$$

∴

$$\mathbf{I} = \mathbf{I}_L - \mathbf{I}_H \tag{2.1.6}$$

Since $\mathbf{I}_H$ and $\mathbf{I}$ are in phase

$$\frac{I}{I_H} = \frac{I_L - I_H}{I_H} \tag{2.1.7}$$

From Eq. (2.1.3) $\quad \frac{I}{I_H} = a_A - 1$

From Eq. (2.1.7) $\quad \frac{I_L - I_H}{I_H} = a_A - 1$

or

$$\frac{I_L}{I_H} = a_A \tag{2.1.8}$$

and

$$I_H = \frac{I_L}{a_A} \tag{2.1.9}$$

From Eqs. (2.1.3) and (2.1.9)

$$\frac{I}{I_L} = \frac{a_A - 1}{a_A} \tag{2.1.10}$$

The induced voltages in windings *ab* and *bc* are in time phase because they are induced by the same flux.

Let $\quad E_1 = E_{ac} = E_H\ ;\ E_2 = E_{bc} = E_L$

$$E_{ab} = E_{ac} - E_{bc}$$

$$\frac{E_{ab}}{E_{bc}} = \frac{E_{ac} - E_{bc}}{E_{bc}} = \frac{E_{ac}}{E_{bc}} - 1 = \frac{E_H}{E_L} - 1$$

$\therefore$

$$\frac{E_{ab}}{E_{bc}} = a_A - 1 \tag{2.1.11}$$

and

$$a_A = \frac{E_{ab}}{E_{bc}} + 1$$

or

$$a_A = a + 1 \tag{2.1.12}$$

where $a = \frac{E_{ab}}{E_{bc}}$ = two-winding transformation ratio.

Equation (2.1.12) shows that the transformation ratio of an autotransformer is greater than that of the same set of windings were connected as a 2-winding transformer.

Equations (2.1.4) and (2.1.8) show that the ratio of voltages and currents in the windings *ab* and *bc* are the same as if turns T_{ab} formed the *primary* and the turns T_{bc} formed the *secondary* of an ordinary transformer having a ratio of transformation of $(a_A - 1)$. Thus, an autotransformer may be considered as an ordinary transformer, treating winding *ab* as the *primary* and the winding *bc* as the *secondary*. In other words, the transformer action present in the autotransformer takes place in the windings *ab* and *bc*.

autotransformer at the top. Fig. 2.3(*d*) shows the same circuit with common terminal at the bottom. Since the polarity is additive, $V_H = 2300 + 230 = 2530$ V and $V_L = 2300$ V, the transformer acts as a step-up autotransformer.

Subtractive polarity

Figure 2.3(*e*) shows the series connections of the windings with subtractive polarity. The circuit is redrawn in Fig. 2.3(*f*) with common terminal at the top. Figure 2.3(*g*) shows the same circuit with common terminal at the bottom. Since the polarity is subtractive $V_H = 2300$ V and $V_L = 2300 - 230 = 2070$ V, the transformer acts as a step-down autotransformer.

2.7 ADVANTAGES OF AUTOTRANSFORMERS

1. An autotransformer uses less winding material than a 2-winding transformer. The saving is large if the transformation ratio is small.
2. An autotransformer is smaller in size and cheaper than the two-winding transformer of the same output.
3. Since there is a reduction in conductor and core materials the ohmic losses in conductor and the core losses are smaller, an autotransformer has higher efficiency than the equivalent 2-winding transformer.
4. Since one winding has been completely eliminated, the resistance and leakage flux of this winding are zero. Hence the voltage regulation of the autotransformer is superior because of reduced voltage drops in the resistance and reactance.
5. An autotransformer has variable output voltage when a sliding contact is used for the secondary.

2.8 DISADVANTAGES OF AUTOTRANSFORMERS

1. There is a direct connection between the high-voltage and low-voltage sides. In case of an open circuit in the common winding *bc* (Fig. 2.1), the full primary voltage would be applied to the load on the secondary. This high voltage may burn out or seriously damage the equipment connected to the secondary side.
2. The effective per-unit impedance of an autotransformer is smaller compared to a 2-winding transformer. The reduced internal impedance results in a larger short-circuit (fault) current.
3. In an autotransformer there is a loss of isolation between input and output circuits. This is particularly important in three-phase transformers where one may wish to use a different winding and earthing arrangement on each side of the transformer.