Differential Calculus for Engineers

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Introduction to calculus

Calculus is a branch of mathematics involving or leading to calculations dealing with continuously varying functions. Calculus is a subject that falls into two parts:

(i) **differential calculus** (or **differentiation**) and
(ii) **integral calculus** (or **integration**).

Differentiation is used in calculations involving velocity and acceleration, rates of change and maximum and minimum values of curves.

Functional notation

In an equation such as $y = 3x^2 + 2x - 5$, y is said to be a function of x and may be written as $y = f(x)$.
An equation written in the form $f(x)= 3x^2 + 2x - 5$ termed **functional notation**. The value of f(x)when x = 0 is denoted by f(0), and the value of f(x) when x = 2 is denoted by f(2) and so on.
Thus when $f(x)= 3x^2 + 2x - 5$, then

$$f(0)= 3(0)^2 + 2(0) - 5 = -5$$

and $f(2)= 3(2)^2 + 2(2) - 5 = 11$ and so on.

Problem 1. If $f(x) = 4x^2 - 3x + 2$ find

f(0), f(3), f(-1) an= f(3) – f(-1)

$$f(x) = 4x^2 - 3x + 2$$

0

$$f(0) = 4(0)^2 - 3(0) + 2 = \mathbf{2}$$

$$f(3) = 4(3)^2 - 3(3) + 2$$
$$= 36 \_ 9 + 2 = \mathbf{29}$$

$$f(-1) = 4(-1)^2 - 3(-1) + 2$$
$$= 4 + 3 + 2 = \mathbf{9}$$

$$f(3)- f(-1)= 29 - 9 = 20$$

Problem 2. Given that $f(x) = 5x^2+ x - 7$ determine:

a) **f(2) ÷ f(1)** b) **f(3+a)** c) **f(3+a) - f(3)** d) $\dfrac{f(3+a)- f(3)}{a}$

$f(x) = 5x^2 + x - 7$

(i) $f(2) = 5(2)^2 + 2 - 7 = 15$

$f(1) = 5(1)^2 + 1 - 7 = -1$

$f(2) \div f(1) = \frac{15}{-1} = -15$

(ii) $f(3+a) = 5(3+a)^2 + (3+a) - 7$

$= 5(9 + a^2 + 6a) + (3+a) - 7$

$= 45 + 5a^2 + 30a + 3 + a - 7$

$= 5a^2 + 31a + 41$

(iii) $f(3) = 5(3)^2 + 3 - 7$

$= 45 + 3 - 7$

$= 41$

$f(3+a) - f(3) = 5a^2 + 31a + 41 - 41 = 5a^2 + 31a$

(iv) $\frac{f(3+a) - f(3)}{a} = \frac{5a^2 + 31a}{a} = 5a + 31$

Problem 3. Diffrentiate from first principles f(x) = x^2 and determine the value of the gradient of the curve at x =2.

To diffrentiate from first principles means to find f'(x) by using the expression

$$f'(x) = \lim_{\delta x \to 0} \left\{ \frac{f(x + \delta x) - f(x)}{\delta x} \right\}$$

$$f(x) = x^2$$

Substituting (x+δx) for x gives

$$f(x + \delta x) = (x + \delta x)^2 = x^2 + 2x\delta x + \delta x^2, \text{ hence}$$

$$f'(x) = \lim_{\delta x \to 0} \left\{ \frac{x^2 + 2x\delta x + \delta x^2 - x^2}{\delta x} \right\} = \lim_{\delta x \to 0} \left\{ \frac{2x\delta x + \delta x^2}{\delta x} \right\} = \lim_{\delta x \to 0} \{2x + \delta x\}$$

As $\delta x \to 0$, $[2x + \delta x] \to [2x + 0]$. Thus f'(x) = 2x, i.e., the differential coefficient of x^2 is 2x. At x =2, the gradient of the curve, f'(x) = 2(2) = 4.

Problem 4. Differentiate from first principles $2f(x) = 2x^3$

Solution: Substituting (x+δx) for x gives

$$2f(x + \delta x) = 2(x + \delta x)^3 = 2(x + \delta x)(x^2 + 2x\delta x + \delta x^2)$$

$$= 2(x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3)$$

$$= 2x^3 + 6x^2\delta x + 6x\delta x^2 + 2\delta x^3$$

$$\frac{dy}{dx} = f'(x) = \lim_{\delta x \to 0} \left\{\frac{f(x+\delta x) - f(x)}{\delta x}\right\}$$

$$= \lim_{\delta x \to 0} \left\{\frac{2x^3 + 6x^2\delta x + 6x\delta x^2 + 2\delta x^3 - 2x^3}{\delta x}\right\}$$

$$= \lim_{\delta x \to 0} \left\{\frac{6x^2\delta x + 6x\delta x^2 + 2\delta x^3}{\delta x}\right\}$$

$$= \lim_{\delta x \to 0} \{6x^2 + 6x\delta x + 2\delta x^2\}$$

Hence $\frac{dy}{dx} = f'(x) = 6x^2$

Problem 5. Find the differential coefficient of $y = 4x^2 + 5x - 3$ and determine the gradient of the curve at x= -3

Solution:

$$y = f(x) = 4x^2 + 5x - 3$$

$$f(x+\delta x) = 4(x+\delta x)^2 + 5(x+\delta x) - 3)$$

$$= 4(x^2 + 2x\delta x + \delta x^2) + 5x + 5\delta x - 3$$

$$= 4x^2 + 8x\delta x + 4\delta x^2 + 5x + 5\delta x - 3$$

$$\frac{dy}{dx} = f'(x) = \lim_{\delta x \to 0} \left\{\frac{4x^2 + 8x\delta x + 4\delta x^2 + 5x + 5\delta x - 3 - (4x^2 + 5x - 3)}{\delta x}\right\}$$

$$= \lim_{\delta x \to 0} \left\{\frac{8x\delta x + 4\delta x^2 + 5\delta x}{\delta x}\right\}$$

$$= \lim_{\delta x \to 0} \{8x + 4\delta x + 5\}$$

$$\frac{dy}{dx} = f'(x) = 8x + 5$$

At x = - 3, the gradient of the curve $\frac{dy}{dx} = f'(x) = 8(-3) + 5 = -19$

Problem 6. Using the general rule, differenciate the following with respect to x:

a) $y = 5x^7$ $b)$ $y = 3\sqrt{x}$ $c) y = \frac{4}{x^2}$

$$\boldsymbol{\frac{d}{dx}(ax^n) = anx^{n-1}}$$

$$y = 5x^7 \qquad \frac{dy}{dx} = 5 \times 7 \times x^{7-1} = \mathbf{35x^6}$$

$$y = 3\sqrt{x} = 3x^{1/2} \qquad \frac{dy}{dx} = 3 \times \frac{1}{2} \times x^{1/2-1} = \boldsymbol{\frac{3}{2}x^{-1/2}}$$

$$y = \frac{4}{x^2} = 4x^{-2} \qquad \frac{dy}{dx} = 4 \times (-2)x^{-2-1} = -8x^{-3} = \frac{\mathbf{-8}}{\boldsymbol{x^3}}$$

	Common Functions	Function(y)	Derivative(dy/dx)
1	Constant	C	0
2		X	1
3	Square	x^2	2x
4	Square root	$\sqrt{x}$	$1/2\sqrt{x}$
5	Exponential	e^x	e^x
6	Logarithms	ln(x)	$1/x$
7	Trigonometric functions	sin(x)	cos(x)
8		cos(x)	- sin(x)
9		tan(x)	$sec^2(x)$
10		cot(x)	$-cosec^2(x)$
11		sec(x)	$secx\ tanx$
12		cosec(x)	$-cosecx\ \ cotx$
13	Inverse Trigonometric functions	$sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
14		$cos^{-1}x$	$-\frac{1}{\sqrt{1-x^2}}$
15		$tan^{-1}x$	$\frac{1}{1+x^2}$
16		$cot^{-1}x$	$-\frac{1}{1+x^2}$
17		$sec^{-1}x$	$\frac{1}{x\sqrt{x^2-1}}$
18		$cosec^{-1}x$	$-\frac{1}{x\sqrt{x^2-1}}$
19	Rules	Function	Derivative
20	Multiplication by constant	Cf	cf'
21	Power Rule	x^n	nx^{n-1}
22	Sum Rule	$u+v$	$u'+v'$
23	Difference Rule	$u-v$	$u'-v'$
24	Product Rule	uv	$uv'+u'v$
25	Quotient Rule	$\frac{u}{v}$	$\frac{vdu-udv}{v^2}$

Problem 7.Find the differential coefficient of $y = \frac{2}{5}x^3 - \frac{4}{x^3} + 4\sqrt{x^5} + 7$

Solution:

$$y = \frac{2}{5}x^3 - \frac{4}{x^3} + 4\sqrt{x^5} + 7 = \frac{2}{5}x^3 - 4x^{-3} + 4x^{5/2} + 7$$

$$\frac{dy}{dx} = \frac{2}{5} \times 3x^{3-1} - 4(-3)x^{-3-1} + 4\left(\frac{5}{2}\right)x^{(5/2)-1} + 0$$

$$= \frac{6}{5}x^2 + 12x^{-4} + 10x^{3/2} = \boldsymbol{\frac{6}{5}x^2 + \frac{12}{x^4} + 10\sqrt{x^3}}$$

Problem 8. If $f(t) = 5t + \frac{1}{\sqrt{t^3}}$ $\quad find\ f'(t)$

$$f(t) = 5t + \frac{1}{\sqrt{t^3}} = 5t + t^{-3/2}$$

$$f'(t) = 5(1)t^{1-1} + \left(-\frac{3}{2}\right)t^{-3/2-1} = 5t^0 - \frac{3}{2}t^{-5/2} = 5 - \frac{3}{2t^{5/2}} = 5 - \frac{3}{2\sqrt{t^5}}$$

Power Rule

Problem 9. What is the derivative of x^3?

We can use the Power Rule, where n=3:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(x^3) = 3x^{3-1} = \boldsymbol{3x^2}$$

Problem 10. What is the derivative of (1/x)?

We can use the Power Rule, where n= -1:

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = -1x^{-1-1} = -x^{-2} = \frac{-1}{x^2}$$

Product Rule

Problem11. What is the derivative of **cosx sinx**?

$$d(uv) = uv' + u'v$$

Let u = cos(x) and v = sin(x)

$$u' = \frac{d}{dx}\cos(x) = -\sin(x) \qquad and \qquad v' = \frac{d}{dx}\sin(x) = co\,s(x)$$

$$\frac{d}{dx}\{cos(x)sin(x)\} = cos(x)cos(x) - sin(x)sin(x) = \boldsymbol{cos^2(x) - sin^2(x)}$$

$$= \frac{1 + cos2x}{2} - \left(\frac{1 - cos2x}{2}\right) = \frac{1 + cos2x - 1 + cos2x}{2} = \frac{2cos2x}{2} = \boldsymbol{cos2x}$$

Chain Rule

Problem 12. What is the derivative of $sin(x^2)$?

The Chain Rule says: the derivative of f(g(x)) = **f'(g(x)) g'(x)**

Let u= x^2 and y= sinu

$\frac{du}{dx}$ = 2x and $\frac{dy}{du}$ = cosu

So:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = cos(u) \times 2x = 2x\, cos(x^2)$$

Problem 13. What is the derivative of $\frac{\mathbf{1}}{\mathbf{sin}(\boldsymbol{x})}$?

Let u= sinx and y= $\frac{1}{u}$

$\frac{du}{dx}$ = cosx and $\frac{dy}{du} = -\frac{1}{u^2}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -\frac{1}{u^2} \times cosx = -\frac{\cos(x)}{sin^2(x)}$$

Problem 14. What is the derivative of $(\mathbf{5}\boldsymbol{x} - \mathbf{2})^{\mathbf{3}}$?

Let $u = \mathbf{5}\boldsymbol{x} - \mathbf{2}$ and y= u^3

$\frac{du}{dx} = 5$ and $\frac{dy}{du} = 3u^2$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 3u^2 \times 5 = 3(5x-2)^2 \times 5 = \mathbf{15}(\mathbf{5}\boldsymbol{x} - \mathbf{2})^{\mathbf{2}}$$

Problem 15. Find the derivative of y = 2 x^3 - 4 x^2 + 3 x – 5

$$\frac{dy}{dx} = \frac{d}{dx}(2\,x^3) - \frac{d}{dx}(4\,x^2) + \frac{d}{dx}(3\,x) - \frac{d}{dx}(5) = 6x^2 + 8x + 3 + 0$$
$$= \mathbf{6}\boldsymbol{x}^{\mathbf{2}} + \mathbf{8}\boldsymbol{x} + \mathbf{3}$$

Problem 16. $Find\ \frac{dy}{dx}, if\ y = x^2 e^x$

$Let\, u = x^2$ and $v = e^x$ $\frac{du}{dx} = 2x$ and $\frac{dv}{dx} = e^x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = x^2.\ e^x + e^x.2x = \boldsymbol{x}\mathbf{e}^{\boldsymbol{x}}(\boldsymbol{x} + \mathbf{2})$$

Problem 17. $What\ is\ \frac{d}{dx}\left(\frac{lnx}{x}\right)$

By the quotient rule, $d\left(\frac{u}{v}\right) = \frac{vdu-udv}{v^2}$

$$\frac{d}{dx}\left(\frac{lnx}{x}\right) = \frac{x.\frac{1}{x} - lnx.1}{x^2} = \frac{1-lnx}{x^2}$$

Problem 18. Find $\frac{dy}{dx}, if\ y = \sqrt{x^2+1}$

$$y = (x^2+1)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(x^2+1)^{1/2-1}.2x = \frac{x}{\sqrt{x^2+1}}$$

Problem 19. Find $\frac{dy}{dx}$, $If\ \ y = \mathrm{e}^{\tan x}$

$$\frac{dy}{dx} = e^{tanx}sec^2x$$

Problem 20. Find the derivative of y = 3x + sin(x) - 4cos(x)

Solution

$$\frac{dy}{dx} = \frac{d}{dx}(3x) + \frac{d}{dx}(sinx) - 4\frac{d}{dx}(cos\ x) = 3 + cos\ x - 4(-sinx)$$

$$= \mathbf{3 + cos\ x + 4sinx}$$

Problem 21. Find the derivative of y = 3ln(x) - 4e^x

$$\frac{dy}{dx} = \frac{d}{dx}3ln(x) - \frac{d}{dx}(4e^x) = \frac{\mathbf{3}}{\boldsymbol{x}} - \mathbf{4}\boldsymbol{e^x}$$

Problem 22. Find $\frac{dy}{dx}$, $, if\ y = \sin(x^2)$

$$\frac{dy}{dx} = 2xcos(x^2)$$

Problem 23. Find $\frac{dy}{dx}$, $if, y = \sin^2 x$

$$y = (sinx)^2 \quad \Rightarrow \quad \frac{dy}{dx} = 2\sin x\cos x = \mathbf{\sin 2}\ \boldsymbol{x}$$

Problem 24. Find $\frac{dy}{dx}$, $if, y = (x^3 + x - 1)^4$

$$y = (x^3+x-1)^4 \quad \Rightarrow \quad \frac{dy}{dx} = 4(x^3+x-1)^3.(3x^2+1)$$

Problem 25. What is the equation of the tangent line to the curve $y = e^x ln(x)$ at the point (1, 0)?

Solution:

The first step is to find the slope of the tangent line at *x* = 1, which is the value of the derivative of *y* at this point:

slope at point (1,0)

$$= \left|\frac{dy}{dx}\right|_{x=1} = \left[e^x.\frac{1}{x} + lnx.e^x\right]_{x=1} = e$$

Since the point-slope formula says that the straight line with slope m which passes through the point (x_0, y_0) has the equation

$$y - y_o = m(x - x_o)$$

the equation of the desired tangent line is

$$y = e(x - 1)$$

Problem 26. Consider the curve given implicitly by the equation $3x^2y - y^3 = x + 1$ What is the slope of this curve at the point where it crosses the *x* axis?

Solution:

To find the slope of a curve defined implicitly (as is the case here), the technique of **implicit differentiation** is used: Differentiate both sides of the equation with respect to *x*; then solve the resulting equation for *y*′.

$$3x^2y - y^3 = x + 1$$
$$3x^2y' + 6xy - 3y^2y' = 1$$
$$y'(3x^2 - 3y^2) = 1 - 6xy$$
$$y' = \frac{1 - 6xy}{3(x^2 - y^2)}$$

The curve crosses the *x* axis when *y* = 0, and the given equation clearly implies that *x* = − 1 at *y* = 0. From the expression directly above, the slope of the curve at the point (−1, 0) is

$$|y'|_{(-1,0)} = \left|\frac{1 - 6xy}{3(x^2 - y^2)}\right|_{(-1,0)} = \mathbf{\frac{1}{3}}$$

Exercise

$y = 7x^4$	$28x^3$
$y = \sqrt{x}$	$1/2\sqrt{x}$
$y = \sqrt{t^3}$	$3\sqrt{t}/2$
$y = 6 + \frac{1}{x^3}$	$-3/x^4$

$y = 3x - \frac{1}{\sqrt{x}} + \frac{1}{x}$	$3 + \frac{1}{2\sqrt{x^3}} - \frac{1}{x^2}$
$y = \frac{5}{x^2} - \frac{1}{\sqrt{x^7}} + 2$	$-\frac{10}{x^3} + \frac{7}{2\sqrt{x^9}}$
$y = 3(t-2)^2$	$6t - 12$
$y = (x+1)^3$	$3x^2 + 6x + 3$

27. Differentiate $f(x) = 6x^2 - 3x + 5$ and find the gradient of the curve at a) x= -1, and b) x=2

$$12x - 3 \quad a) - 15 \qquad b) \ \ 21$$

28. Find the differential coefficient of $y = 2x^3 + 3x^2 - 4x - 1$ and determine the gradient of the curve at x = -2

$$6x^2 + 6x - 4, \qquad 8$$

29. Determine the derivative of $y = -2x^3 + 4x + 7$ and determine the gradient of the curve at x = -1.5

$$-6x^2 + 4, \quad - 9.5$$

29. $If\ y = cosa\theta, \ \ \frac{dy}{d\theta} = -asina\theta \qquad where\ a\ is\ a\ constant$

30. $If\ y = \cos(a\theta + \alpha), \ \ \frac{dy}{d\theta} = -asin(a\theta + \alpha) \quad where\ aand\ \alpha\ are\ constants$

31. Differentiate the following with respect to the variable:

$$a)\ y = 2sin5\theta \qquad and\ b)\ f(t) = 3\ cos2t$$

$a)\ y = 2sin5\theta$

$$\frac{dy}{d\theta} = 2 \times 5 \times cos5\theta = \mathbf{10\ cos5\theta}$$

$b)\ f(t) = 3\ cos2t$

$$\frac{dy}{d\theta} = 3(-2)sin2t = -\mathbf{6\ sin2t}$$

32. Find the differential coefficient of $y = 7\sin 2x - 3cos4x$

$y = 7\sin 2x - 3cos4x$

$$\frac{dy}{dx} = (7 \times 2)\cos 2x - (3 \times -4)sin4x = 14\cos 2x + 12\ sin4x$$

33. Differentiate the following with respect to the variable:

$a)\ f(\theta) = 5\sin(100\pi\theta - 0.40) \quad and\ b)\ f(t) = 2\cos(5t + 0.20)$

$a) \quad If \quad f(\theta) = 5\sin(100\pi\theta - 0.40)$

$$f'(\theta) = 5[100\pi\cos(100\pi\theta - 0.40)] = \mathbf{500\pi\cos(100\pi\theta - 0.40)}$$

$b) \quad If \quad f(t) = 2\cos(5t + 0.20)$

$$f'(t) = 2\,[-5\sin(5t + 0.20)] = \mathbf{-10\sin(5t + 0.20)}$$

34. An alternating voltage is given by $v = 100\sin 200t\ volts$, where t is the time in seconds. Calculate the rate of change of voltage when a) t= 0.005 s and b) t=0.01 s

35. Differentiate with respect to x: $a) \quad y = 4sin3x \qquad b)\ y = 2cos6x$

$a) \quad 12cos3x \qquad b)\ y = -12sin6x$

36. Given $f(\theta) = 2sin3\theta - 5cos2\theta,\ find\ f'(\theta)$

$6cos3\theta + 10sin2\theta$

37. An alternating current is given by $i = 5\sin 100t\ amperes$, where t is the time in seconds. Determine the rate of change of current when a) t= 0.01 seconds

[270.2 A/s]

38. If $f(t) = 3\sin(4t + 0.12) - 2\cos(3t - 0.72)\ determine\ f'(t)$

$[12\cos(4t + 0.12) + 6\sin(3t - 0.72)]$

39. $If\ \ y = e^{ax}, then\ \frac{dy}{dx} = ae^{ax}$

40. Differentiate the following with respect to the variable:

$$a)\ y = 3e^{2x} \qquad and\ b)\ f(t) = \frac{4}{3e^{5t}}$$

$$y = 3e^{2x} \qquad \Rightarrow \quad \frac{dy}{dx} = 3(2e^{2x}) = 6e^{2x}$$

$$f(t) = \frac{4}{3e^{5t}} = \frac{4}{3}e^{-5t} = \frac{4}{3}(-5)e^{-5t} = -\frac{20}{3e^{5t}}$$

41. $Differentiate\ y = 5ln3x$

$$y = 5ln3x \qquad \frac{dy}{dx} = 5 \times \left(\frac{1}{x}\right) = \frac{\mathbf{5}}{\boldsymbol{x}}$$

42. Differentiate with respect to x: $a)\ \ y = 5e^{3x}$ $\qquad b)\,\frac{2}{7e^{2x}}$

$$\boldsymbol{a})\ \mathbf{15}\boldsymbol{e}^{\mathbf{3}\boldsymbol{x}} \qquad \boldsymbol{b}) - \frac{\mathbf{4}}{\mathbf{7}\boldsymbol{e}^{\mathbf{2}\boldsymbol{x}}}$$

43. Find the differential coefficient of $a)\ \ y = 12x^3$ $\qquad b)\ \ y = \frac{12}{x^3}$

$$If\ y = ax^n\ then\ \frac{dy}{dx} = anx^{n-1}$$

a) $Since\ y = 12x^3, a = 12\ and\ n = 3\ thus\ \frac{dy}{dx} = (12)(3)x^{3-1} = \mathbf{36}\boldsymbol{x}^{\mathbf{2}}$

$$b)\ y = \frac{12}{x^3} = 12x^{-3}, \quad a = 12\ and\ n = \ -3$$

$$Thus\ \frac{dy}{dx} = (12)(-3)x^{-3-1} = -36x^{-4} = -\frac{\mathbf{36}}{\boldsymbol{x}^{\mathbf{4}}}$$

44. Find the derivative of $a)\ \ y = 3\sqrt{x}$ $\qquad b)\ \ y = \frac{5}{\sqrt[3]{x^4}}$

$$a)\ y = 3\sqrt{x} = 3x^{1/2} \qquad \frac{dy}{dx} = 3 \times \frac{1}{2} \times x^{1/2-1} = \frac{3}{2}\,x^{-1/2} = \frac{\mathbf{3}}{\mathbf{2}\sqrt{\boldsymbol{x}}}$$

b) $\quad y = \frac{5}{\sqrt[3]{x^4}} = \frac{5}{x^{4/3}} = 5x^{-4/3}$

$$\frac{dy}{dx} = 5 \times \left(\frac{-4}{3}\right) \times x^{-4/3-1} = \frac{-20}{3}\,x^{-7/3} = \frac{-20}{3x^{7/3}} = \frac{\mathbf{-20}}{\mathbf{3}\,\sqrt[3]{\boldsymbol{x}^{\mathbf{7}}}}$$

45. Differentiate $y = 5x^4 + 4x - \frac{1}{2x^2} + \frac{1}{\sqrt{x}} - 3$ with respect to x

$$y = 5x^4 + 4x - \frac{1}{2x^2} + \frac{1}{\sqrt{x}} - 3 = 5x^4 + 4x - \frac{1}{2}x^{-2} + x^{-1/2} - 3$$

$$\frac{dy}{dx} = 20x^3 + 4 + \frac{1}{2} \times 2x^{-2-1} - \frac{1}{2}\, x^{-1/2-1}$$

$$\frac{dy}{dx} = 20x^3 + 4 + x^{-3} - \frac{1}{2}\, x^{-3/2}$$

$$\frac{dy}{dx} = 20x^3 + 4 + \frac{1}{x^3} - \frac{1}{2x^{3/2}} = 20x^3 + 4 + \frac{1}{x^3} - \frac{1}{2\sqrt{x^3}}$$

46. Find the differential coefficients of a) $y = 3\sin 4x$ and b) $f(t) = 2cos3t$ with respect to the variable

a) $y = 3\sin 4x$; $\frac{dy}{dx} = 3 \times 4\cos 4x = \mathbf{12\cos 4x}$

b) $f(t) = 2cos3t$; $f'(t) = 2 \times (-3)sin3t = \boldsymbol{-6sin3t}$

47. Determine the derivative of a) $y = 3e^{5x}$ b) $f(\theta) = \frac{2}{e^{3\theta}}$ c) $y = 6\ln 2x$

$y = 3e^{5x}$; $\frac{dy}{dx} = 3 \times 5e^{5x} = \boldsymbol{15e^{5x}}$

$f(\theta) = \frac{2}{e^{3\theta}} = 2e^{-3\theta}$; $f'(\theta) = 2(-3)e^{-3\theta} = -6e^{-3\theta} = \boldsymbol{\frac{-6}{e^{3\theta}}}$

$y = 6\ln 2x$; $\frac{dy}{dx} = \boldsymbol{\frac{6}{x}}$

48. Determine the co-ordinates of the point on the graph $y = 3x^2 - 7x + 2$ where the gradient is -1.

Ans: The gradient of the curve is given by the derivative

$$y = 3x^2 - 7x + 2 \qquad ; \frac{dy}{dx} = 6x - 7 \quad then$$

$$6x - 7 \;\; = -1 \qquad \Rightarrow \;\; 6x = -1 + 7 = \;\; 6 \quad \Rightarrow \quad x = \mathbf{1}$$

When x = 1, $y = 3(1)^2 - 7(1) + 2 = 3 - 7 + 2 = \; -\mathbf{2}$

Hence the gradient is -1, at the point (1, -2)

49. Find the gradient of the curve $y = 3x^4 - 2x^2 + 5x - 2$ at the points (0,-2) and (1,4)

$$y = 3x^4 - 2x^2 + 5x - 2$$

$$\frac{dy}{dx} = 12x^3 - 4x + 5$$

At the point (0,-2), x = 0. Thus

$$\frac{dy}{dx} = \; 12(0)^3 - 4(0) + 5 = \mathbf{5}$$

At the point (1,4), x = 1. Thus

$$\frac{dy}{dx} = \; 12(1)^3 - 4(1) + 5 = 12 - 4 + 5 = \mathbf{13}$$

50. Differentiate $a)\; y = 5x^5 \quad ; \quad \frac{dy}{dx} = 25x^4$

$$b)\; y = 2.4x^{3.5} \quad ; \quad \frac{dy}{dx} = (2.4 \times 3.5)x^{3.5-1} = 8.4x^{2.5}$$

$$c)\;\; y = \frac{1}{x} \qquad ; \quad \frac{dy}{dx} = \; -\frac{1}{x^2}$$

$$d)\;\; y = \frac{-4}{x^2} \;\; = -4x^{-2} \qquad ; \quad \frac{dy}{dx} = 8x^{-3} \;\; = \frac{8}{x^3}$$

$e) \quad y = 2x \qquad ; \quad \frac{dy}{dx} = 2$

$f) \quad y = 2\sqrt{x} \qquad ; \quad \frac{dy}{dx} = 2 \times \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$

$g) \quad y = 3\sqrt[3]{x^5} = 3x^{5/3} \qquad ; \quad \frac{dy}{dx} = 3 \times \frac{5}{3}\, x^{\frac{5}{3}-1} = 5x^{\frac{2}{3}} = 5\sqrt[3]{x^2}$

$h) \; y = \frac{4}{\sqrt{x}} = 4x^{-1/2} \qquad ; \quad \frac{dy}{dx} = 4 \times \frac{-1}{2} x^{-\frac{1}{2}-1} = -2x^{-3/2} = \frac{-2}{\sqrt{x^3}}$

$i) \; y = \frac{-3}{\sqrt[3]{x}} = -3x^{-1/3} \qquad ; \frac{dy}{dx} = -3 \times \frac{-1}{3} x^{-\frac{1}{3}-1} = -x^{-\frac{4}{3}} = \frac{-1}{x^{\frac{4}{3}}} = \frac{-1}{\sqrt[3]{x^4}}$

$j) \quad y = (x-1)^2 \qquad ; \frac{dy}{dx} = 2(x-1)$

$k) \quad y = 2sin3x \qquad ; \frac{dy}{dx} = 2 \times 3cos3x = 6cos3x$

Differentiation of a product

When y = u v, and u and v are both functions of x, then

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

This is known as product rule.

51. Find the differential coefficient of $y = 3x^2 sin2x$

$3x^2 sin2x$ is a product of two terms. Let $u = 3x^2 and\ v = sin2x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = 3x^2(2cos2x) + sin2x(6x) = 6x^2cos2x + 6xsin2x = \boldsymbol{6x(xcos2x + sin2x)}$$

52. Find the rate of change of y with respect to x : $y = 3\sqrt{x}\,ln2x$

Let $u = 3\sqrt{x}$ and $v = ln2x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = 3\sqrt{x} \times \frac{1}{x} + \ ln2x\frac{3}{2\sqrt{x}} = \frac{3}{\sqrt{x}} + \frac{3}{2\sqrt{x}}ln2x = \boldsymbol{\frac{3}{\sqrt{x}}\left(1 + \frac{1}{2}ln2x\right)}$$

53. Differentiate $y = x^3cos3xlnx$

Let $u = x^3cos3x \qquad and\ v = lnx$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\boldsymbol{\frac{du}{dx}} = x^3(-3sin3x) + cos3x(3x^2) \qquad \frac{dv}{dx} = \frac{1}{x}$$

$$\boldsymbol{\frac{dy}{dx}} = \ x^3cos3x \times \left(\frac{1}{x}\right) + lnx\ (-3x^3sin3x + 3x^2cos3x)$$

$$= x^2cos3x + 3x^2lnx\ (cos3x - xsin3x)$$

$$\frac{dy}{dx} = x^2(cos3x + 3lnx\ [cos3x - xsin3x])$$

HOME WORK y	$\frac{dy}{dx}$
$2x^3cos3x$	$6x^2(cos3x - xsin3x)$
$\sqrt{x^3}ln\ 3x$	$\sqrt{x}\left(1 + \frac{3}{2}ln\ 3x\right)$
$e^{3t}sin4t$	$e^{3t}(4cos4t + 3sin4t)$
$e^{4\theta}ln3\theta$	$e^{4\theta}\left(\frac{1}{\theta} + 4ln3\theta\right)$

DIFFERENTIATION OF A QUOTIENT

When$y = \frac{u}{v}$, and u and v are both functions of x, then

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

This is known as quotient rule.

54. Find the differential coefficient of $y = \frac{4\sin 5x}{5x^4}$

$$y = \frac{4\sin 5x}{5x^4}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Let $u = 4\ sin5x \quad and\ v = 5x^4$

$$\frac{du}{dx} = 4 \times 5cos5x = 20cos5x$$

$$\frac{dv}{dx} = 5 \times 4x^3 = 20x^3$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{5x^4 \times 20cos5x - 4\ sin5x \times 20x^3}{(5x^4)^2}$$

$$= \frac{100x^4cos5x - 80x^3\ sin5x}{25x^8}$$

$$= \frac{80x^3(5xcos5x - 4\ sin5x)}{25x^8}$$

$$= \frac{20x^3(5xcos5x - 4\ sin5x)}{25x^8}$$

$$\frac{dy}{dx} = \frac{4}{5x^5}(5xcos5x - 4\ sin5x)$$

55. Differentiate $y = \frac{te^{2t}}{2cost}$ Let $u = te^{2t} \quad and\ v = 2cost$

$$\frac{du}{dt} = 2t\ e^{2t} + e^{2t}(1)\ \ and\ \frac{dv}{dt} = -2sint$$

$$\frac{dy}{dt}=\frac{v\frac{du}{dt}-u\frac{dv}{dt}}{v^2}$$

$$\frac{dy}{dt}=\frac{2cost(2t\,e^{2t}+e^{2t})-te^{2t}(-2sint)}{(2cost)^2}$$

$$=\frac{4t\,e^{2t}cost+2\,e^{2t}cost+2te^{2t}sint}{4cos^2t}$$

$$=\frac{2\,e^{2t}(2t\,cost+cost+tsint)}{4cos^2t}$$

$$\frac{dy}{dt}=\frac{e^{2t}}{2cos^2t}(2t\,cost+cost+tsint)$$

56. Determine the gradient of the curve $y=\frac{5x}{2x^2+4}$ at the point $\left(\sqrt{3},\frac{\sqrt{3}}{2}\right)$

Let $u=5x\ and\ v=2x^2+4$

$$\frac{du}{dx}=5\ \ and\ \frac{dv}{dx}=4x$$

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}=\frac{(2x^2+4)5-5x\times 4x}{(2x^2+4)^2}$$

$$=\frac{10x^2+20-20x^2}{(2x^2+4)^2}=\frac{20-10x^2}{(2x^2+4)^2}$$

At the point $\left(\sqrt{3},\frac{\sqrt{3}}{2}\right)$, $x=\sqrt{3}$

Hence the gradient

$$\frac{dy}{dx}=\frac{20-10(\sqrt{3})^2}{\left[2(\sqrt{3})^2+4\right]^2}=\frac{20-30}{(6+4)^2}=-\frac{10}{100}=-\mathbf{\frac{1}{10}}$$

HOME WORK y	$\frac{dy}{dx}$
$2cos3x/_{x^3}$	$\frac{-6}{x^4}(xsin3x+cos3x)$

$\frac{2x}{x^2+1}$	$\frac{2(1-x^2)}{(x^2+1)^2}$
$\frac{3\sqrt{\theta^3}}{2sin2\theta}$	$\frac{3\sqrt{\theta}(3sin2\theta-4\theta cos2\theta)}{4sin^2\theta}$
$\frac{ln2t}{\sqrt{t}}$	$\frac{\left(1-\frac{1}{2}ln2t\right)}{\sqrt{t^3}}$
$\frac{2xe^{4x}}{sinx}$	$\frac{2e^{4x}}{sin^2x}\{(1+4x)sinx-xcosx\}$
Find the gradient of the curve $y=\frac{2x}{x^2-5}$ $at\ the\ point\ (2,-4)$	-18

Function of a function

If y is a function of x then $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$

This is known as 'function of a function' rule (or some times the chain rule)

57. For example $y=(3x-1)^9$ then

Let

$$u=3x-1,\quad y=u^9$$

$$\frac{du}{dx}=3\qquad ,\frac{dy}{du}=9u^8$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=9u^8\times 3=27u^8=\mathbf{27(3x-1)^8}$$

58. $Differentiate\quad y=3\cos(5x^2+2)$

$$\text{Let } u=5x^2+2\quad then\ y=3\ cosu$$

$$\frac{du}{dx}=10\ x\qquad \frac{dy}{du}=-3sinu$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=-3sinu\ \times 10x=\ -30xsinu$$

$$\frac{dy}{dx}=\mathbf{-30xsin(5x^2+2)}$$

59. Find the derivative of: $y=(4t^3-3t)^6$

Let $u=4t^3-3t, then\ y=u^6$

$$Hence\ \frac{du}{dt} = 12t^2 - 3\ and\ \frac{dy}{dt} = 6u^5$$

Using the function of a function rule

$$\frac{dy}{dt} = \frac{dy}{du} \times \frac{du}{dt} = (6u^5)(12t^2 - 3) = 6(4t^3 - 3t)^5(12t^2 - 3)$$

$$= \mathbf{18(4t^2 - 1)(4t^3 - 3t)^5}$$

60. Determine the differential coefficient of: $y = \sqrt{3x^2 + 4x - 1}$

$$y = \sqrt{3x^2 + 4x - 1} = (3x^2 + 4x - 1)^{1/2}$$

$$Let\ u = 3x^2 + 4x - 1 \Rightarrow \qquad y = u^{1/2}$$

$$Hence\ \frac{du}{dx} = 6x + 4\ and\ \frac{dy}{du} = \frac{1}{2}u^{-1/2} = \frac{1}{2\sqrt{u}}$$

Using the function of a function rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{2\sqrt{u}}(6x + 4\ ) = \frac{3x + 2}{\sqrt{u}} = \frac{\mathbf{3x + 2}}{\sqrt{\mathbf{3x^2 + 4x - 1}}}$$

61. Differentiate $y = 3tan^4 3x$

$$Let\ u = tan3x \quad then\ y = 3u^4$$

$$\frac{du}{dx} = 3sec^2 3x \qquad and\ \frac{dy}{du} 12u^3$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 12u^3 \times 3sec^2 3x = 12(tan3x)^3 \times 3sec^2 3x$$

$$\frac{dy}{dx} = 36tan^3 3xsec^2 3x$$

62. Find the differential coefficient of $y = \dfrac{2}{(2t^3 - 5)^4}$

$$y = \frac{2}{(2t^3 - 5)^4} = 2(2t^3 - 5)^{-4}$$

$$Let\ u = 2t^3 - 5 \quad then\ y = 2(u)^{-4} \qquad \Rightarrow \frac{du}{dt} = 6t^2$$

$$\frac{dy}{du} = -8u^{-5} = \frac{-8}{u^5}$$

$$\frac{dy}{dt} = \frac{dy}{du} \times \frac{du}{dt} = \frac{-8}{u^5} \times 6t^2 = \frac{-48t^2}{(2t^3 - 5)^5}$$

	HOME WORK y	$\frac{dy}{dx}$
1	$(2x^3-5x)^5$	$5(6x^2-5)(2x^3-5x)^4$
2	$2\sin(3\theta-2)$	$6cos(3\theta-2)$
3	$2cos^5\alpha$	$-10cos^4\alpha\ sin\alpha$
4	$\frac{1}{(x^3-2x+1)^5}$	$\frac{5(2-3x^2)}{(x^3-2x+1)^6}$
5	$5e^{2t+1}$	$10e^{2t+1}$
6	$2\cot(5t^2+3)$	$-20t\ cosec^2(5t^2+3)$
7	$6\tan(3y+1)$	$18sec^2(3y+1)$
8	$2e^{tan\theta}$	$2sec^2\theta\ e^{tan\theta}$

Successive Differentiation

When a function $y=f(x)$ is differentiated with respect to x the differential coefficient is written as $\frac{dy}{dx}$ $or\ f'(x)$. If the expression is differentiated again, the second differential coefficient is obtained and is written as $\frac{d^2y}{dx^2}$ $or f''(x)$. By successive differentiation further higher derivative such as $\frac{d^3y}{dx^3}$ and $\frac{d^4y}{dx^4}$ may obtained.

63. Thus if $y=3x^4$

$$\frac{dy}{dx}=12x^3, \frac{d^2y}{dx^2}=36x^2, \frac{d^3y}{dx^3}=72x,\ \frac{d^4y}{dx^4}=72\ and\ \frac{d^5y}{dx^5}=0$$

$If \quad f(x)=2x^5-4x^3+3x-5, \quad find \quad f''(x)$

$$f(x)=2x^5-4x^3+3x-5$$
$$f'(x)=10x^4-12x^2+3$$
$$f''^{(x)}=40x^3-24x=4x(10x^2-6)$$

64. Given $y = 2xe^{-3x}$ *show that* $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 9y = 0$

$$y = 2xe^{-3x} \qquad \Rightarrow \frac{dy}{dx} = 2x(-3e^{-3x}) + e^{-3x}(2) = -6xe^{-3x} + 2e^{-3x}$$

$$\frac{d^2y}{dx^2} = -6x(-3e^{-3x}) + e^{-3x}(-6) + (-6e^{-3x}) = 18xe^{-3x} - 6e^{-3x} - 6e^{-3x}$$

$$\frac{d^2y}{dx^2} = 18xe^{-3x} - 12e^{-3x}$$

Substituting the values into $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 9y = 0$ *gives*:

$$18xe^{-3x} - 12e^{-3x} + 6(-6xe^{-3x} + 2e^{-3x}) + 9(2xe^{-3x})$$
$$= \mathbf{18xe^{-3x}} - \mathbf{12e^{-3x}} - 36xe^{-3x} + \mathbf{12e^{-3x}} + \mathbf{18xe^{-3x}} = \mathbf{0}$$

Thus when $y = 2xe^{-3x}$, $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 9y = 0$

1.

$$\boldsymbol{I = \int \frac{dx}{2x^2 + 20x + 51}}$$

$$2x^2 + 20x + 51 = 2(x^2 + 10x + 25) + 1 = 2(x+5)^2 + 1 = 2\left[(x+5)^2 + \frac{1}{2}\right]$$

$$I = \int \frac{dx}{2\left[(x+5)^2 + \frac{1}{2}\right]} = \frac{1}{2}\int \frac{dx}{(x+5)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

Let $u = x + 5 \Rightarrow du = dx$

$$I = \frac{1}{2}\int \frac{du}{(u)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{2}.\frac{1}{\frac{1}{\sqrt{2}}}tan^{-1}\left(\frac{u}{\frac{1}{\sqrt{2}}}\right) + c = \frac{\sqrt{2}}{2}tan^{-1}\left(\sqrt{2}(x+5)\right) + c$$
$$= \frac{1}{\sqrt{2}}tan^{-1}\left(\sqrt{2}(x+5)\right) + c$$

2.

$$I = \int_0^{\pi/6} 4sin(4t + {}^{\pi}/_{3})\, dt$$

Let $u = 4t + {}^{\pi}/_{3} \quad \Rightarrow du = 4dt$

$$I = \int_0^{\pi/6} sinu\, du = (-cosu)_0^{\pi/6} = \left[cos\left(4t + \frac{\pi}{3}\right)\right]_{\pi/6}^{0} = cos\left(\frac{\pi}{3}\right) - cos\left(\frac{4\pi}{6} + \frac{\pi}{3}\right)$$

$$= cos\, \pi/3 - cos\pi = 0.5 + 1 = \mathbf{1.5}$$

3.

$$I = \int \frac{x}{\sqrt{x+1}} dx$$

Let $\quad u = x + 1 \quad \Rightarrow \quad du = dx$

$$I = \int \frac{u-1}{\sqrt{u}} du = \int \left(\frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\right) du = \int \left(u^{1/2} - u^{-1/2}\right) du = \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} + c$$

$$I = \frac{2(x+1)^{3/2}}{3} - 2(x+1)^{1/2} + c = \frac{\mathbf{2}\left(\sqrt{\boldsymbol{x}+\mathbf{1}}\right)^{\mathbf{3}}}{\mathbf{3}} - \mathbf{2}\sqrt{\boldsymbol{x}+\mathbf{1}} + \boldsymbol{c}$$

4.

$$I = \int_{-1}^{5} (3x+1)\sqrt{3x^2 + 2x + 5}\, dx$$

Let $u = 3x^2 + 2x + 5 \quad \Rightarrow \quad du = (6x+2)dx \quad \Rightarrow \frac{du}{2} = (3x+1)dx$

$$I = \int_{-1}^{5} \sqrt{u}\, du = \int_{-1}^{5} u^{1/2}\, du = \left[\frac{u^{3/2}}{{}^{3}/_{2}}\right]_{-1}^{5} = \frac{2}{3}\left[(3x^2 + 2x + 5)^{3/2}\right]_{-1}^{5}$$

$$= \frac{2}{3}\left[(90)^{3/2} - (6)^{3/2}\right] = \mathbf{559.4}$$

5.

$$I = \int tan^2x\, sec^4x dx$$

sec²x = 1 + tan²x

$$I = \int (tan^2x)\ (sec^2x)(sec^2x)dx = \int (tan^2x)\ (1 + tan^2x)(sec^2x)dx$$

Let $u = tanx \qquad \Rightarrow du\ = sec^2x\, dx$

$$I = \int u^2(1+u^2)\, du = \frac{u^3}{3} + \frac{u^5}{5} + c = \frac{tan^3x}{3} + \frac{tan^5x}{5} + c$$

6.

$$I = \int x\sqrt{5+x^2}\ dx$$

Let $u = 5 + x^2 \qquad \Rightarrow \qquad du = 2xdx \qquad \Rightarrow xdx = \frac{du}{2}$

$$I = \int \sqrt{u}\ \frac{du}{2} = \frac{1}{2}\int u^{1/2}\, du = \frac{1}{2}\frac{u^{3/2}}{\left({}^{3}\!/_{2}\right)} + c = \frac{1}{2}.\frac{2}{3}u^{3/2} + c = \frac{1}{3}u^{3/2} + c$$

$$= \mathbf{\frac{1}{3}(5+x^2)^{3/2} + c}$$

7.

$$I = \int_0^1 3x\, e^{(2x^2-1)}\ dx$$

Let $u =\ 2x^2 - 1 \qquad \Rightarrow \qquad du = 4x\, dx \qquad \Rightarrow dx = \frac{du}{4x}$

$$I = \int_0^1 3x\, e^u\, \frac{du}{4x} = \frac{3}{4}\int_0^1\ e^u\ du = \frac{3}{4}[e^u]_0^1 = \frac{3}{4}\left[e^{(2x^2-1)}\right]_0^1 = \frac{3}{4}[e^1 - e^{-1}]$$

$$= \frac{3}{4}(2.71 - 0.3678) = 1.7628 = 1.763$$

8.

$$I = \int_{0}^{\pi/3} 3sin^2 3x\, dx$$

$$\because sin^2\theta = \frac{1 - cos2\theta}{2} \qquad \Rightarrow \qquad sin^2 3x = \frac{1 - cos6x}{2}$$

$$I = \int_{0}^{\pi/3} \frac{3}{2}\,(1 - cos6x)\;\; dx = \frac{3}{2}\left[x - \frac{sin6x}{6}\right]_0^{\pi/3} = \frac{3}{2}\left[\left(\frac{\pi}{3} - \frac{sin2\pi}{6}\right) - \left(0 - \frac{sin0}{6}\right)\right]$$

$$= \frac{3}{2} \times \frac{\pi}{3} = \frac{\pi}{2} = 1.571$$

9.

$$\int 2sin3x\; sinxdx$$

$$\sin A \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$$

$$\int 2sin3x\; sinxdx = -\int (cos4x - cos2x)\;\; dx = -\left[\frac{sin4x}{4} - \frac{sin2x}{2}\right] + c$$

$$= \left[\frac{sin2x}{2} - \frac{sin4x}{4}\right] + c$$

10.

$$\int_{1}^{2} 3cos8tsin3t\; dt$$

$$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$$

$$\int_{1}^{2} 3cos8tsin3t\,dt = \frac{3}{2}\int_{1}^{2}(\sin 11t + \sin 5t)\,dt = \frac{3}{2}\left(-\frac{\cos 11t}{11} + \frac{\cos 5t}{5}\right)_{1}^{2}$$

$$= \frac{3}{2}\left(\frac{\cos 5t}{5} - \frac{\cos 11t}{11}\right)_{1}^{2} = \left(\frac{3}{10}\cos 5t\right)_{1}^{2} - \left(\frac{3}{22}\cos 11t\right)_{1}^{2}$$

$$= \frac{3}{10}(\cos 10 - \cos 5) - \frac{3}{22}(\cos 22 - \cos 11)$$

$$= \frac{3}{10}(-0.8363 - 0.2836) - \frac{3}{22}(-0.9999 - 0.0044)$$

$$= -0.3359 — 0.1369 = \mathbf{-0.1990}$$