

Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

CS22BTECH11061

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12.13.4.2 An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X ? Is X a random variable?

Solution: Possible values of X are as follows -

$$X = \{0, 1, 2\} \quad (1)$$

A random variable is an assignment of real values to each outcome of the experiment. Therefore, X is an random variable.

Probability Mass Distribution of X :-

Let $N = R + B + G$ and $n = r + b + g$
where R, B, G and r, b, g represent the number of red, black and green marbles/balls respectively within N and n . Then

$$\Pr(r, b, g) = \frac{\binom{R}{r}\binom{B}{b}\binom{G}{g}}{\binom{R+B+G}{r+b+g}} \quad (2)$$

In our case ,

$$R = 5$$

$$B = 2$$

$$G = 0$$

$$N = 5 + 2 + 0 = 7$$

$$\text{and } n = 2$$

also,

$$\text{here } g = 0 \text{ as } G = 0$$

Now,

$$n = r + b + g$$

$$\therefore 2 = r + b + 0$$

$$\therefore r = 2 - b$$

Now as $X = b$

and

$$\Pr(r, b, g) = \frac{\binom{R}{r}\binom{B}{b}\binom{G}{g}}{\binom{R+B+G}{r+b+g}} \quad (3)$$

$$\therefore \Pr(X = b) = \frac{\binom{5}{2-b}\binom{2}{b}\binom{0}{0}}{\binom{7}{2}} \quad (4)$$

So,

Probability Distribution of X can be given as

$$p_X(b) = \frac{\binom{5}{2-b}\binom{2}{b}}{21} \quad (5)$$

With this we can find probabilities for $X = \{0, 1, 2\}$ as follows

$$1) \Pr(X = 0) = \frac{\binom{5}{2}\binom{2}{0}}{\binom{7}{2}} = \frac{10}{21}$$

$$2) \Pr(X = 1) = \frac{\binom{5}{1}\binom{2}{1}}{\binom{7}{2}} = \frac{10}{21}$$

$$3) \Pr(X = 2) = \frac{\binom{5}{0}\binom{2}{2}}{\binom{7}{2}} = \frac{1}{21}$$