Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

CS22BTECH11061
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12.13.4.2 An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable?

Solution: Possible values of X are as follows -

$$X = \{0, 1, 2\} \tag{1}$$

A random variable is an assignment of real values to each outcome of the experiment. Therefore, X is an random variable.

Let $N = R + B$ and $n = r + b$
where,

Parameter	Value	Description
R	5	Red balls within N
B	2	Black balls within N
N	7	(R + G)
r	{0, 1, 2}	Red balls within n
b	{0, 1, 2}	Black balls within n
n	2	(r + g)

then

$$\Pr(r, b) = \frac{\binom{R}{r}\binom{B}{b}}{\binom{R+B}{r+b}} \tag{2}$$

In our case ,
$R = 5$
$B = 2$
$N = 5 + 2 = 7$
and $n = 2$

Now,
$n = r + b$
$\therefore 2 = r + b$
$\therefore r = 2 - b$

Now as $X = b$

and

$$\Pr(r, b) = \frac{\binom{R}{r}\binom{B}{b}}{\binom{R+B}{r+b}} \tag{3}$$

$$\therefore \Pr(X = b) = \frac{\binom{5}{2-b}\binom{2}{b}}{\binom{7}{2}} \tag{4}$$

So, Probability Distribution of X can be given as

$$p_X(b) = \frac{\binom{5}{2-b}\binom{2}{b}}{21} \tag{5}$$

where b = {0, 1, 2}