Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

CS22BTECH11061
Waghmare Aditya Abhaykumar

12.13.4.2 An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable?

Solution: Possible values of X are as follows -

$$X = \{0, 1, 2\} \tag{1}$$

A random variable is an assignment of real values to each outcome of the experiment. Therefore, X is an random variable.

Probability Mass Distribution of X :-

Let $N = R + B$ and $n = r + b$

where R, B and r, b represent the number of red and black marbles/balls respectively within N and n. Then

$$\Pr(r, b) = \frac{\binom{R}{r}\binom{B}{b}}{\binom{R+B}{r+b}} \tag{2}$$

In our case ,
$R = 5$
$B = 2$
$N = 5 + 2 = 7$
and $n = 2$

Now,
$n = r + b$
$\therefore 2 = r + b$
$\therefore r = 2 - b$

Now as $X = b$
and

$$\Pr(r, b) = \frac{\binom{R}{r}\binom{B}{b}}{\binom{R+B}{r+b}} \tag{3}$$

$$\therefore \Pr(X = b) = \frac{\binom{5}{2-b}\binom{2}{b}}{\binom{7}{2}} \tag{4}$$

So,
Probability Distribution of X can be given as

$$p_X(b) = \frac{\binom{5}{2-b}\binom{2}{b}}{21} \tag{5}$$

With this we can find probabilities for X = {0, 1, 2} as follows

1) $\Pr(X = 0) = \frac{\binom{5}{2}}{\binom{7}{2}} = \frac{10}{21}$
2) $\Pr(X = 1) = \frac{\binom{5}{1}\binom{2}{1}}{\binom{7}{2}} = \frac{10}{21}$
3) $\Pr(X = 2) = \frac{\binom{2}{2}}{\binom{7}{2}} = \frac{1}{21}$