Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

CS22BTECH11061
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12.13.4.2 An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable?
Solution: Possible values of X are as follows -

$$X = \{0, 1, 2\} \tag{1}$$

A random variable is an assignment of real values to each outcome of the experiment. Therefore, X is an random variable.

Probability Mass Distribution of X :-

For any event E, probability of event E is represented as $\Pr(E)$ and defined as

$$\Pr(E) = \frac{\text{Number of possible outcomes in which } E \text{ occurs}}{\text{Number of total possible outcomes in Sample space}} \tag{2}$$

So we can find PMF of X as follows,

1) $\Pr(X = 0) = \frac{\binom{5}{2}}{\binom{7}{2}} = \frac{10}{21}$
2) $\Pr(X = 1) = \frac{\binom{5}{1}\binom{2}{1}}{\binom{7}{2}} = \frac{10}{21}$
3) $\Pr(X = 2) = \frac{\binom{2}{2}}{\binom{7}{2}} = \frac{1}{21}$

Therefore, PMF of X is

$$p_X(n) = \begin{cases} \frac{10}{21} & n = 0 \\ \frac{10}{21} & n = 1 \\ \frac{1}{21} & n = 2 \end{cases} \tag{3}$$