

Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

CS22BTECH11061

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12.13.4.2 An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X ? Is X a random variable?

Solution: Possible values of X are as follows -

$$X = \{0, 1, 2\} \quad (1)$$

A random variable is an assignment of real values to each outcome of the experiment. Therefore, X is an random variable.

Let $N = R + B$ and $n = r + b$ where,

Parameter	Value	Description
R	5	Red balls within N
B	2	Black balls within N
N	7	(R + G)
r	{0, 1, 2}	Red balls within n
b	{0, 1, 2}	Black balls within n
n	2	(r + g)

TABLE 0
PARAMETERS USED

then

$$\Pr(r, b) = \frac{{}^R C_r {}^B C_b}{{}^{R+B} C_{r+b}} \quad (2)$$

In our case ,

$$R = 5$$

$$B = 2$$

$$N = 5 + 2 = 7$$

$$\text{and } n = 2$$

Now,

$$n = r + b$$

$$\therefore 2 = r + b$$

$$\therefore r = 2 - b$$

Now as $X = b$

and

$$\Pr(r, b) = \frac{{}^R C_r {}^B C_b}{{}^{R+B} C_{r+b}} \quad (3)$$

$$\therefore \Pr(X = b) = \frac{{}^5 C_{2-b} {}^2 C_b}{{}^7 C_2} \quad (4)$$

So, Probability Distribution of X can be given as

$$p_X(b) = \frac{{}^5 C_{2-b} {}^2 C_b}{21} \quad (5)$$

where $b = \{0, 1, 2\}$