Electrical Engineering 16A
Worksheet 0-2

1. Solutions of linear equations

(a) Consider the following set of linear equations:

$$2x+3y+5z=0$$
$$-1x-4y-10z=0$$
$$x-2y-8z=0$$

Place these equations into a matrix, and row reduce the matrix.

(b) Convert the row reduced matrix back into equation form.

(c) Intuitively, what does the last equation from the previous part tell us?

(d) Now that we've established that this system has infinite solutions, does every possible combination of $x, y, z \in \mathbb{R}$ solve these equations

(e) What is the general form (in the form of a constant vector multiplied by a variable t) of the infinite solutions to the system?

2. First Proof

Prove that a subset of a finite linear independent set of vectors is linearly independent.

3. Inverses!

(a) Find the inverse of:

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

(b) Find the inverse of:

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 6 & 8 \end{bmatrix}$$

using Gaussian Elimination.

A rotation matrix is a matrix that takes a vector and rotates it by some number of degrees. That matrix looks like:

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

for some angle θ. For example, if we had a rotation matrix with $\theta = 45°$, and we multiplied it with the vector [.5, .5], what would you expect?

(c) find the inverse of this matrix:

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

(d) i. Will a rotation matrix always have an inverse? Why or why not?

ii. Consider a matrix that mirrors a vector across the x-axis. Will it always have an inverse?

iii. Consider a matrix flattens a vector on to the x-axis (so for example $[3, 5]^\top$ becomes $[3, 0]^\top$). Will it have an inverse?

4. Invertibility and equations

(a) Consider the following system of equations

$$2x - 2y = -6$$

$$x - y + z = 1$$

$$3y - 2z = -5$$

Write these equations in matrix form. Then, write an expression for the solution to the equations using inverses, but don't compute the inverse.

(b) Let the system of equations be $\mathbf{A}\vec{x} = \vec{y}$. What does it mean if $\mathbf{A}$ is not invertible?
Hint: The solution to the previous part.

(c) Consider the matrix

$$\begin{bmatrix} -1 & 1 \\ -2 & 1 \\ 1 & -3 \end{bmatrix}$$

Is it invertible?

(d) Does the system of equations that is represented by the following have any solutions?

$$\underbrace{\begin{bmatrix} -1 & 1 \\ -2 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{B}\vec{x}} = \underbrace{\begin{bmatrix} 5 \\ 9 \\ -7 \end{bmatrix}}_{\vec{y}}$$

5. Are you linear?

(a) Consider a matrix $\mathbf{S}$ that transforms a vector $\vec{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to $\vec{y} = \begin{bmatrix} a-b-c \\ a-b-c \\ a-b+c \end{bmatrix}$. Note that a, b, c can take on any values in $\mathbb{R}$. In other words, $\mathbf{S}\vec{x} = \vec{y}$. Is this transformation linear?

(b) Now let's consider another matrix $\mathbf{Q}$ which takes a vector $\vec{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ to $\begin{bmatrix} a+5 \\ b \\ c \end{bmatrix}$. Is this matrix a linear operator?

(c) Let's dive deeper. Write out the matrix $\mathbf{S}$ and $\mathbf{Q}$. Are they invertible?