

## Tutorial 1

Ans 1 - Asymptotic Notations : It is used to describe the running time of an algorithm - how much time an algorithm takes with a given input, 'n'. There are 3 different notations  $\text{big O}$ ,  $\text{big theta } (\Theta)$  &  $\text{big omega } (\Omega)$

i)  $\text{Big O}$  : It defines an upper bound of an algorithm. It counts how many iterations an algorithm will take in the worst-case scenario with an input N.

ii)  $\text{Big } \Omega$  notation :  $\text{Big Omega } (\Omega)$  notation describes best running time of algorithm. For eg :- Bubble sort algorithm

iii)  $\Theta$  notation : Theta notation enclose (f) from above & below. It represents upper & lower bound of algorithm. Used for calculating average-time complexity.

Ans 2 -

for ( $i = 1$  to  $n$ )

}

$i = i * 2$  ;

}

loop runs upto n time

i.e 1 . . . n

inside loop :  $i = i * 2$  i.e  $i = 1 * 2$  (for first value)

$i = 2$

$i = 4$  ( $2^{\text{nd}}$  value)

$i = 8$  ( $3^{\text{rd}}$  value)



$$L = 1, 2, 4, 8, 16, \dots, n$$

$$= 1 \cdot 2^1, 2^2, 2^3, 2^4, \dots, 2^n$$

$$\rightarrow 2^k = n \rightarrow k = \log^n$$

$$\therefore T(L) = O(n) \text{ i.e., } O(\log n)$$

Ans 3 -  $T(n) = \{ 3T(n-1) \text{ if } n > 0 \}$

$$n = n-1$$

$$\Rightarrow T(n-1) = 3T(n-1) - 1$$

$$T(n-1) = 3T(n-2)$$

$$T(n) = 3(3T(n-2))$$

$$T(n) = 3T(n-2)$$

$$\text{for } n = n-1$$

$$\Rightarrow T(n) = 9T((n-1)-2)$$

$$T(n-1) = 9T(n-3)$$

$$T(n) = 3(9T(n-3))$$

$$= 27T(n-3)$$

$$\Rightarrow 3T(n-1), 9T(n-2), 27T(n-3) \dots$$

$$\dots 3nT(n-n)$$

$$= 3nT(0)$$

$$= 3n$$

$$\Rightarrow T(L) = O(3n) \cdot O(3^n)$$

Ans 4 -  $T(n) = [2T(n-1) - 1 \text{ if } n > 0]$

$$n = n-1$$

$$\Rightarrow T(n-1) = 2T[(n-1)-1] - 1$$

$$T(n-1) = 2T(n-2) - 1$$

$$T(n) = 2(2T(n-2) - 1) - 1$$

$$T(n) = 4T(n-2) - 2 - 1$$

for  $n = n - 1$

$$\rightarrow T(n-1) = 4T(n-1-2) - 1$$

$$T(n-1) = 4T(n-3) - 1$$

$$T(n) = 2^2(2T(n-3) - 1) - 2 - 1$$

$$= 3T(n-3) - 4 - 2 - 1$$

$$2T(n-1) - 1, 4T(n-2) - 1 - 1, 8T(n-3) - 4 - 2 - 1$$

$$\dots 2nT(n-n) - 1$$

$$= 2^n - (2^n - 1)$$

$$T(n) = 1$$

$$\Rightarrow T(c) = O(1)$$

Ans 5 -  
 for  $i = 1, s = 1$ ;  
 while  $(s = n)$   
 }

$i++$ ;

$s = s + i$ ;

print f ("#");  
 }

$$1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2} > n$$

$$\Rightarrow k^2 = n$$

$$\Rightarrow k = \sqrt{n} \Rightarrow O(\sqrt{n}) = (c)$$



Ans 6 - void function (int n)  
{

int i; count = 0;

for (i = 1; i + i <= n; i++)

count++;

}

$$i = 1 \Rightarrow 1 + 1 <= n \Rightarrow 12 = n$$

$$\text{for } i = 2; 1 + 2 < n \Rightarrow 2 < n$$

$$1 * n \leq n \Rightarrow n \leq n$$

$$1, 2, 3 \dots n \Rightarrow A.P.$$

$$\text{Sum} \rightarrow \frac{n(n+1)}{2} = O(n^2)$$

Ans 7 - void function (int n)  
{

int i, j, k, count = 0;

for (i = n/2; i <= n; i++)  $\rightarrow n/2$

for (j = 1; j <= n; j = j \* 2)  $\Rightarrow \log_2 n$

for (k = 1; k <= n; k = k \* 2)  $\Rightarrow \log_2 n$

count++;

}

$$\Rightarrow T(c) = n \log_2^2 n$$

Ans 8 - function (int n)  
{

if (n == 1)

return;

```
for (i=1 to n) →  $O(n)$ 
{
```

```
for (j=1 to n) →  $O(n)$ 
{
```

```
    print ("*");
}
```

```
}
```

```
function (n-3);
}
```

$$\Rightarrow T(n) = O(n^2)$$

Ex 9 - void function (int n)

```
{
```

```
for (i=1 to n)
{
```

```
    for (j=1 ; j<=n ; j=j+i)
        print (" + ");
}
```

```
}
```

	1	2	3	...	n
i	1	2	3	...	(n-1)
j	2	3	4	...	n

$$T(n) = (n-1)(n) = O(n^2)$$



Ques 10 - function -  $n^k$ ,  $c^n$

⇒ Relation b/w the two pl

$$\therefore n^k = O(c^n) \Rightarrow$$