Lower Tail Test of Population Mean with Population Standard Deviation (σ known)

The null hypothesis of the **lower tail test of the population mean** can be expressed as follows:-

$$H_0 : \mu = \mu_0$$
$$H_a : \mu < \mu_0$$

where μ_0 is a hypothesized lower bound of the true population mean μ.

Let us define the test statistic z in terms of the sample mean, the sample size and the population standard deviation σ:-

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Problem:-

Suppose the manufacturer claims that the mean lifetime of a light bulb is **less than 10,000** hours. In a **sample** of **30** light bulbs, it was found that they only last **9,900** hours on **average**. Assume the **population standard deviation** is **120** hours. At **0.05** significance level, can we reject the claim by the manufacturer?

Solution:-

#The null hypothesis is that $\mu \leq 10000$. We begin with computing the test statistic.

```
> xbar = 9900              # sample mean
> mu0 = 10000              # hypothesized value
> sigma = 120              # population standard deviation
> n = 30                   # sample size
> z = (xbar−mu0)/(sigma/sqrt(n))
> z                        # test statistic
```

Output:- `-4.564355`

#We then compute the **critical value** at **0.05** significance level.

```
> alpha = 0.05
> z.alpha = qnorm(1−alpha)
> −z.alpha                 # critical value
```

Output:- `-1.644854`

Answer:-

The test statistic **–4.564355** is **less than** the critical value of **–1.644854**. Hence, at 0.05 significance level, we **reject the claim** that mean lifetime of a light bulb is above 10,000 hours.

p-value Method:-

Instead of using the critical value, we apply the **pnorm()** function to compute the lower tail **p-value** of the test statistic. As it turns out **to be less than the 0.05** significance level, **we reject the null hypothesis** that $\mu \leq 10000$.

```
> pval = pnorm(z)
> pval                     # lower tail p−value
```

Output:- `2.505166e-06`

Upper Tail Test of Population Mean with Population Standard Deviation (σ known)

The null hypothesis of the **upper tail test of the population mean** can be expressed as follows:-

$$H_0 : \mu = \mu_0$$
$$H_a : \mu > \mu_0$$

where μ_0 is a hypothesized upper bound of the true population mean μ.
Let us define the test statistic z in terms of the sample mean, the sample size and the population standard deviation σ:-

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Problem:-

Suppose the food label on a cookie bag states that there is **at least 2** grams of saturated fat in a single cookie. In a **sample** of **35** cookies, it is found that the **mean** amount of saturated fat per cookie is **2.1** grams. Assume that the **population standard deviation** is **0.25** grams. At **0.05** significance level, can we reject the claim on food label?

Solution:-

#The null hypothesis is that $\mu \geq 2$. We begin with computing the test statistic.

```
> xbar = 2.1              # sample mean
> mu0 = 2                 # hypothesized value
> sigma = 0.25            # population standard deviation
> n = 35                  # sample size
> z = (xbar−mu0)/(sigma/sqrt(n))
> z                       # test statistic
```

Output:- `2.366432`

#We then compute the **critical value** at **0.05** significance level.

```
> alpha = 0.05
> z.alpha = qnorm(1−alpha)
> z.alpha                 # critical value
```

Output:- `1.644854`

Answer:-

The test statistic **2.366432** is **greater than** the critical value of **1.644854**. Hence, at 0.05 significance level, we **reject the claim** that there is at most 2 grams of saturated fat in a cookie.

p-value Method:-

Instead of using the critical value, we apply the **pnorm()** function to compute the upper tail **p-value** of the test statistic. As it turns out **to be less than the 0.05** significance level, **we reject the null hypothesis** that $\mu \geq 2$.

```
> pval = pnorm(z, lower.tail=FALSE)
> pval                    # upper tail p−value
```

Output:- `0.008980239`

Two-Tailed Test of Population Mean with Population Standard Deviation (σ known)

The null hypothesis of the **two-tailed test of the population mean** can be expressed as follows:-

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

where μ_0 is a hypothesized value of the true population mean μ.

Let us define the test statistic z in terms of the sample mean, the sample size and the population standard deviation σ:-

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Problem:-

Suppose the **mean** weight of King Penguins found in an Antarctic colony last year was **15.4** kg. In a **sample** of **35** penguins same time this year in the same colony, the **mean** penguin weight is **14.6** kg. Assume the **population standard deviation** is **2.5** kg. At **0.05** significance level, can we reject the null hypothesis that the mean penguin weight does not differ from last year?

Solution:-

#The null hypothesis is that μ **= 15.4**. We begin with computing the test statistic.

```
> xbar = 14.6              # sample mean
> mu0 = 15.4               # hypothesized value
> sigma = 2.5              # population standard deviation
> n = 35                   # sample size
> z = (xbar−mu0)/(sigma/sqrt(n))
> z                        # test statistic
```

Output:- `-1.893146`

#We then compute the **critical values** at **0.05** significance level.

```
> alpha = 0.05
> z.half.alpha = qnorm(1−alpha/2)
> c(−z.half.alpha, z.half.alpha)
```

Output:- `-1.959964  1.959964`

Answer:-

The test statistic **–1.893146** lies **between** the critical values **–1.959964** and **1.959964**. Hence, at 0.05 significance level, we **do *not* reject the null hypothesis** that the mean penguin weight does not differ from last year.

p-value Method:-

Instead of using the critical value, we apply the **pnorm()** function to compute the two-tailed **p-value** of the test statistic. It doubles the *lower* tail p-value as the sample mean is *less* than the hypothesized value. Since **it turns out to be greater than the 0.05** significance level, we **do *not* reject the null hypothesis** that $\mu = 15.4$

```
> pval = 2 ∗ pnorm(z)          # lower tail
> pval                         # two−tailed p−value
```

Output:- `0.05833852`

Lower Tail Test of Population Mean with Population Standard Deviation (σ unknown means s known)

The null hypothesis of the **lower tail test of the population mean** can be expressed as follows:-

$$H_0 : \mu = \mu_0$$
$$H_a : \mu < \mu_0$$

where μ_0 is a hypothesized lower bound of the true population mean μ.
Let us define the test statistic t in terms of the sample mean, the sample size and the sample standard deviation s:-

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Problem:-

Suppose the manufacturer claims that the **mean** lifetime of a light bulb is **less than 10,000** hours. In a **sample** of **30** light bulbs, it was found that they only last **9,900** hours on **average**. Assume the **sample standard deviation** is **125** hours. At **0.05** significance level, can we reject the claim by the manufacturer?

Solution:-

#The null hypothesis is that $\mu \leq$ **10000**. We begin with computing the test statistic.

```
> xbar = 9900           # sample mean
> mu0 = 10000           # hypothesized value
> s = 125               # sample standard deviation
> n = 30                # sample size
> t = (xbar−mu0)/(s/sqrt(n))
> t                     # test statistic
```

Output:- `-4.38178`

#We then compute the **critical value** at **0.05** significance level.

```
> alpha = 0.05
> t.alpha = qt(1−alpha, df=n−1)
> −t.alpha              # critical value
```

Output:- `-1.699127`

Answer:-

The test statistic **–4.38178** is **less than** the critical value of **–1.699127**. Hence, at 0.05 significance level, **we can reject the claim** that mean lifetime of a light bulb is above 10,000 hours.

p-value Method:-

Instead of using the critical value, we apply the **pt()** function to compute the lower tail **p-value** of the test statistic. As **it turns out to be less than the 0.05** significance level, **we reject the null hypothesis** that $\mu \leq 10000$.

```
> pval = pt(t, df=n−1)
> pval                  # lower tail p−value
```

Output:- `7.035026e-05`

Upper Tail Test of Population Mean with Population Standard Deviation (σ unknown means s known)

The null hypothesis of the **upper tail test of the population mean** can be expressed as follows:-

$$H_0 : \mu = \mu_0$$
$$H_a : \mu > \mu_0$$

where μ_0 is a hypothesized upper bound of the true population mean μ.

Let us define the test statistic t in terms of the sample mean, the sample size and the sample standard deviation s:-

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Problem:-

Suppose the food label on a cookie bag states that there is **at least 2** grams of saturated fat in a single cookie. In a **sample** of **35** cookies, it is found that the **mean** amount of saturated fat per cookie is **2.1** grams. Assume that the **sample standard deviation** is **0.3** gram. At **0.05** significance level, can we reject the claim on food label?

Solution:-

#The null hypothesis is that $\boldsymbol{\mu \geq 2}$. We begin with computing the test statistic.

```
> xbar = 2.1          # sample mean
> mu0 = 2             # hypothesized value
> s = 0.3             # sample standard deviation
> n = 35              # sample size
> t = (xbar−mu0)/(s/sqrt(n))
> t                   # test statistic
```

Output:- `1.972027`

#We then compute the **critical value** at **0.05** significance level.

```
> alpha = 0.05
> t.alpha = qt(1−alpha, df=n−1)
> t.alpha          # critical value
```

Output:- `1.690924`

Answer:-

The test statistic **1.972027** is **greater than** the critical value of **1.690924**. Hence, at 0.05 significance level, **we can reject the claim** that there is at most 2 grams of saturated fat in a cookie.

p-value Method:-

Instead of using the critical value, we apply the **pt()** function to compute the upper tail **p-value** of the test statistic. As **it turns out to be less than the 0.05** significance level, **we reject the null hypothesis** that $\mu \geq 2$.

```
> pval = pt(t, df=n−1, lower.tail=FALSE)
> pval                         # upper tail p−value
```

Output:- `0.02839295`

Two-Tailed Test of Population Mean with Population Standard Deviation (σ unknown means s known)

The null hypothesis of the **two-tailed test of the population mean** can be expressed as follows:-

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

where μ_0 is a hypothesized value of the true population mean μ.
Let us define the test statistic t in terms of the sample mean, the sample size and the sample standard deviation s:-

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Problem:-

Suppose the **mean** weight of King Penguins found in an Antarctic colony last year was **15.4** kg. In a **sample** of **35** penguins same time this year in the same colony, the **mean** penguin weight is **14.6** kg. Assume the **sample standard deviation** is **2.5** kg. At **0.05** significance level, can we reject the null hypothesis that the mean penguin weight does not differ from last year?

Solution:-

#The null hypothesis is that $\mu = 15.4$. We begin with computing the test statistic.

```
> xbar = 14.6          # sample mean
> mu0 = 15.4           # hypothesized value
> s = 2.5              # sample standard deviation
> n = 35               # sample size
> t = (xbar−mu0)/(s/sqrt(n))
> t                    # test statistic
```

Output:- `-1.893146`

#We then compute the **critical values** at **0.05** significance level.

```
> alpha = 0.05
> t.half.alpha = qt(1−alpha/2, df=n−1)
> c(−t.half.alpha, t.half.alpha)
```

Output:- `-2.032245  2.032245`

Answer:-

The test statistic **–1.893146** lies **between** the critical values **–2.032245** and **2.032245**. Hence, at 0.05 significance level, we **do *not* reject the null hypothesis** that the mean penguin weight does not differ from last year.

Alternative Solution:-

Instead of using the critical value, we apply the **pt()** function to compute the two-tailed **p-value** of the test statistic. It doubles the *lower* tail p-value as the sample mean is *less* than the hypothesized value. Since **it turns out to be greater than the 0.05** significance level, **we do *not* reject the null hypothesis** that $\mu = 15.4$.

```
> pval = 2 ∗ pt(t, df=n−1)          # lower tail
> pval                              # two−tailed p−value
```

Output:- `0.06687552`

Lower Tail Test of Population Variance (Chi-Square Test):-

The null hypothesis of the **lower tail test of the population variance** can be expressed as follows:-

$$H_0: \sigma^2 = \sigma_0^{\,2}$$
$$H_a: \sigma^2 < \sigma_0^{\,2}$$

where σ_0 is a hypothesized upper bound of the true population variance σ^2.
Let us define the test statistic chi-square in terms of the sample variance, the sample size and the population variance σ^2:-

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2};\ \text{d.f.} = n - 1$$

Problem:-

Highway engineers in Ohio are painting white stripes on a highway. The stripes are supposed to be approximately **10 feet** long. However, because of the machine, the operator, and the motion of the vehicle carrying the equipment, considerable variation occurs among the stripe lengths. Engineers claim that the **variance** of stripes is **not more than 16 inches**. Use the **sample** lengths given here from **12** measured stripes to test the **variance** claim. Assume stripe length is normally distributed. Let α **= 0.05.** The **standard deviation** of the **12** stripes is **5.98544 inches**.

Solution:-

#The null hypothesis is that $\sigma^2 \le 16$. We begin with computing the test statistic.

```
> sigmasq = 16                # population variance(σ²)
> s = 5.98544                 # sample standard deviation
> ssq = s * s                 # sample variance(s²)
> n = 12                      # sample size
> chisq = ssq*(n−1)/sigmasq
> chisq                       # test statistic
```

Output:- `24.63003`

#We then compute the **critical value** at **0.05** significance level.

```
> alpha = 0.05
> chisq.alpha = qchisq(1−alpha, df=n−1)
> chisq.alpha            # critical value
```

Output:- `19.67514`

Answer:-

The test statistic **24.63003** is **greater than** the critical value of **19.67514**. Hence, at 0.05 significance level, **we can reject the null hypothesis** that there is Engineers claim that the variance of stripes is not more than 16 inches.

Upper Tail Test of Population Variance (Chi-Square Test):-

The null hypothesis of the **upper tail test of the population variance** can be expressed as follows:-

$$H_0: \sigma^2 = \sigma_0^{\ 2}$$
$$H_a: \sigma^2 > \sigma_0^{\ 2}$$

where σ_0 is a hypothesized upper bound of the true population variance σ^2.
Let us define the test statistic chi-square in terms of the sample variance, the sample size and the population variance σ^2:-

$$\boldsymbol{\chi^2 = \frac{(n-1)s^2}{\sigma^2}}\textbf{; d.f. = n − 1}$$

Problem:-

A company produces industrial wiring. One batch of wiring is specified to be **2.16** centimeters (cm) thick. A company inspects the wiring in **7** locations and determines that, on the **average**, the wiring is about **2.16** cm thick. However, the measurements vary. It is unacceptable for the **variance** of the wiring to be **more than 0.04** cm^2. The **standard deviation** of the **7** measurements on this batch of wiring is **0.34** cm. Use $\boldsymbol{\alpha}$ **= 0.01** to determine whether the **variance** on the sample wiring is too great to meet specifications. Assume wiring thickness is normally distributed.

Solution:-

#The null hypothesis is that $\boldsymbol{\sigma^2 \geq 0.04}$. We begin with computing the test statistic.

```
> xbar = 2.16                # sample mean
> sigmasq = 0.04             # population variance(σ²)
> s = 0.34                   # sample standard deviation
> ssq = s * s                # sample variance(s²)
> n = 7                      # sample size
> chisq = ssq*(n−1)/sigmasq
> chisq                      # test statistic
```

Output:- `17.34`

#We then compute the **critical value** at **0.05** significance level.

```
> alpha = 0.01
> chisq.alpha = qchisq(1−alpha, df=n−1)
> chisq.alpha            # critical value
```

Output:- `16.81189`

Answer:-

The test statistic **17.34** is **greater than** the critical value of **16.81189**. Hence, at 0.01 significance level, **we can reject the null hypothesis** that there is the variance on the sample wiring is too great to meet specifications.

Two Tail Test of Population Variance (Chi-Square Test):-

The null hypothesis of the **lower tail test of the population variance** can be expressed as follows:-

$$H_0: \sigma^2 = \sigma_0{}^2$$
$$H_a: \sigma^2 \neq \sigma_0{}^2$$

where σ_0 is a hypothesized upper bound of the true population variance σ^2.
Let us define the test statistic chi-square in terms of the sample variance, the sample size and the population variance σ^2:-

$$\boldsymbol{\chi^2 = \frac{(n-1)s^2}{\sigma^2}}\text{; } \mathbf{d.f. = n - 1}$$

Problem:-

A small business has 37 employees. Because of the uncertain demand for its product, the company usually pays overtime on any given week. The company assumed that about **50** total hours of overtime per week is required and that the **variance** on this figure is about **25**. Company officials want to know whether the variance of overtime hours has changed. Given here is a **sample** of **16** weeks of overtime data (in hours per week). Assume hours of overtime are normally distributed. Use these data to test the null hypothesis that the **variance** of overtime data is **25**. Let $\boldsymbol{\alpha}$ **= 0.10.** The **standard deviation** of the **16** weeks of overtime data is **5.30**.

Solution:-

#The null hypothesis is that $\boldsymbol{\sigma^2}$ **= 25**. We begin with computing the test statistic.

```
> sigmasq = 25                 # population variance(σ²)
> s = 5.30                     # sample standard deviation
> ssq = s * s                  # sample variance(s²)
> n = 16                       # sample size
> chisq = ssq*(n−1)/sigmasq
> chisq                        # test statistic
```

Output:- `16.854`

#We then compute the **critical value** at **0.10** significance level.

```
> alpha = 0.10
> chisq.alpha = qchisq(1−alpha/2, df=n−1)
> chisq.alpha                   # critical value
```

Output:- `24.99579`

Answer:-

The test statistic **16.854** is **less than** the critical value of **24.99579**. Hence, at 0.10 significance level, **we cannot reject the null hypothesis** that there is the population variance of overtime hours per week is 25.