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CSE-18

ASSIGNMENT - 2

Q.1) pmf: The probability mass function of binomial distribution X with parameters n, p is defined as

$$P(X=x) = \begin{cases} \binom{n}{x} p^{x} q^{1-x} ; & x = 0,1,2,3,\ldots,n \\ 0 & \text{otherwise} \end{cases}$$

p = Probability of success
q = Probability of failure

cdf: The cumulative distribution function of a binomial distribution X with parameters n, p is defined as:

$$p(X \le x) = \sum_{X=0}^{x} p(X=x) = p(x=0) + p(X=1) + \ldots + p(x=x)$$

(b)

$$b(1,6,0.35) = {}^{6}_{1}C\,(0.35)\,(1-0.35)^{5}$$
$$= 0.243$$

$$b(2,6,0.35) = {}^{6}_{2}C\,(0.35)^{2}\,(1-0.35)^{4}$$
$$= 0.328$$

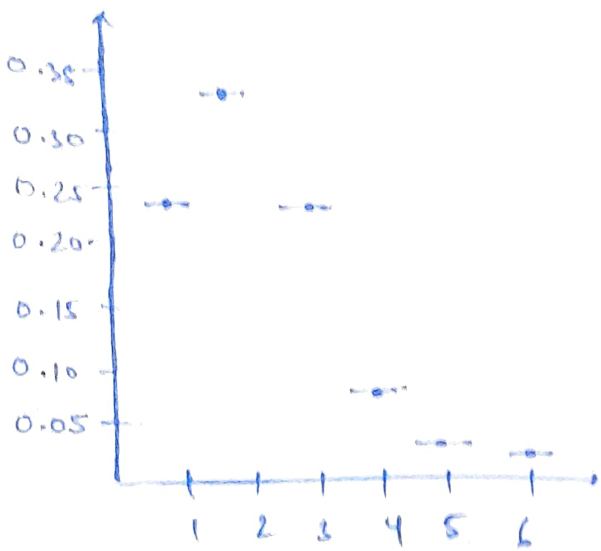
$$b(3,6,0.35) = {}^{6}_{3}C\,(0.35)^{3}(1-0.35)^{3}$$

$$= 0.235$$

$$b(4, 6, 0.35) = {}^{6}_{4}C\,(0.35)^4(1-0.35)^2$$
$$= 0.095$$

$$b(5, 6, 0.35) = {}^{6}_{5}C\,(0.35)^5(1-0.35)^1$$
$$= 0.020$$

$$b(6, 6, 0.35) = {}^{6}_{6}C\,(0.35)^6$$
$$= 0.018$$



(c) To compute:-

$$P(|X-\mu| \geq k\sigma), \quad k = 1, 2$$

$$\mu = np = 6 \times 0.35 = 2.1$$

$$\sigma = \sqrt{np(1-p)} = 1.17$$

For $k=1$;

$P(|X-2.1| \geq 1.17) = P(|X| \geq 3.27)$
$= 0.1168$

For $k=2$;

$P(|X-2.1| \geq 2.34) = P(|X| \geq 4.44)$

$= 0.2$

2(a) The pmf of rv x which has geometric distribution

$$P(X=x) = \begin{cases} p(1-p)^{x-1} & x = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

The cdf of rv x which has geometric distribution

$$P(X \leq x) = \sum_{y=1}^{x} p(1-p)^{y-1}$$

(b) $x \leq 6$; $n=6$; $p = 0.35$

$P(1) = p(1-p)^{x-1}$ $\qquad$ $P(5) = 0.062$

$= 0.35$ $\qquad$ $P(6) = 0.040$

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$$P(2) = p(1-p)^{x-1}$$
$$= (0.35)(0.65)$$
$$= 0.2275$$

$$P(3) = P(1-p)^{x-1}$$
$$= 0.35(0.65)^2$$
$$= 0.147$$

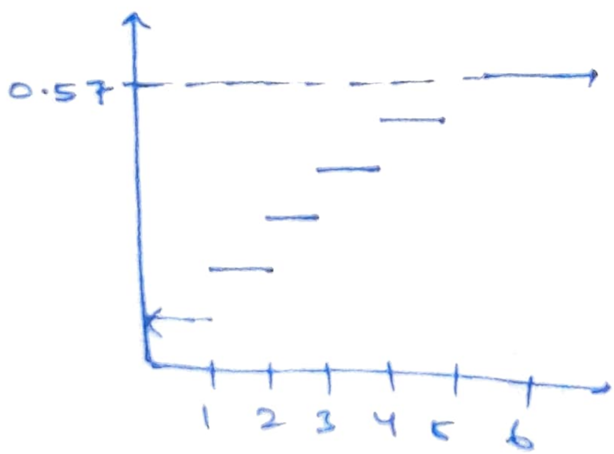
$$P(4) = P(1-p)^{x-1}$$
$$= 0.0966$$

$$f(x) = F(6) = P(x \leq 6) = 0.57$$

$$E(x) = \mu = \frac{1}{p} = 2.85$$

$$\sigma^2 = 5.3$$

$$\sigma = 2.3$$



(c) Given:

$\mu = 2.85$, $\sigma = 2.3$

for $k = 1$

$P(|X-\mu| \geq k\sigma)$

$= P(|X| \geq 5.15)$

$= P(6) = 0.040$

for $k = 2$

$P(|X-\mu| \geq k\sigma)$

$= P(|X| \geq 7.45)$

$= P(8) = 0$

3) The pmf of X having a hypergeometric distribution with parameter, n, M, N is

$$h(x; n, M, N) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

Given, $n = 6$, $M = 10$, $N = 15$

$$h(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} = \frac{{}^{10}_{1}C \; {}^{5}_{1}C}{{}^{15}_{6}C} = 0.0019$$

$h(1) = 0.0019$

$$h(2) = \frac{{}^{10}C_{2}\;{}^{5}C_{4}}{{}^{15}C_{6}} = 0.044$$

$$h(3) = \frac{{}^{10}C_{3}\;{}^{5}C_{3}}{{}^{15}C_{6}} = 0.239$$

$$H(3) = 0.2849$$

$$h(4) = \frac{{}^{10}C_{4}\;{}^{5}C_{2}}{{}^{15}C_{6}} = 0.419$$

$$H(4) = 0.7039$$

$$h(5) = \frac{{}^{10}C_{5}\;{}^{5}C_{1}}{{}^{15}C_{6}} = 0.251$$

$$H(5) = 0.9549$$

$$h(6) = \frac{{}^{10}C_{6}\;{}^{5}C_{0}}{{}^{15}C_{6}} = 0.041$$

$$H(6) = 0.99 \approx 1$$

(c) $H = n, \frac{M}{N} = 4$

$$\sigma^2 = \left(\frac{N-n}{N-1}\right)\frac{M}{N}\left(1-\frac{M}{N}\right)$$

$$= \frac{9}{14} \times 6 \times \frac{10}{15}\left(1-\frac{10}{15}\right)$$

$$= 0.86$$

$$\sigma = 0.92$$

$$P = (|X-M| \geq 0.92k)$$

$$P(5) = h(5) = 0.251$$

For $k = 2$

$$h(6) = 0.041$$

(4) PMF

$$nb(X=x) = \begin{cases} \binom{x+r-1}{r-1} p^r (1-p)^{y^r} & x = 0, 1, 2 \ldots \\ 0 & \text{otherwise} \end{cases}$$

cdf

$$NB(x; y, p) = \sum_{y \leq x} \binom{y+r-1}{r-1} p^r (1-p)^y$$

(b)

$nb(0; 3, 0.35) = 0.42$

$nb(1; 3, 0.35) = 0.083$

$nb(2; 3, 0.35) = 0.108$

$nb(3; 3, 0.35) = 0.117$

$nb(4; 3, 0.35) = 0.114$

$nb(5; 3, 0.35) = 0.104$

$F(0) = 0.042$
$F(1) = 0.125$
$F(2) = 0.233$
$F(3) = 0.35$
$F(4) = 0.464$
$F(5) = 0.568$

(c) $\mu = \dfrac{r(1-p)}{p} = \dfrac{3 \times 0.65}{0.35} = 5.51$

$\sigma^2 = \dfrac{r(1-p)}{p^2} = 15.74$

$\sigma = 3.98$

$P(|X - 5.51| \geq 3.98) = 0.125$

5 (a) pmf:

$$P(x;\mu) = \frac{e^{-\mu}\mu^{x}}{x!} \qquad x = 0, 1, 2, 3, \ldots$$

cdf:

$$F(x) = \sum_{y \le x} P(y, \mu)$$

(b) $\mu = 5$

$$P(1,5) = \frac{e^{-5}\,5}{1!} = 0.03$$

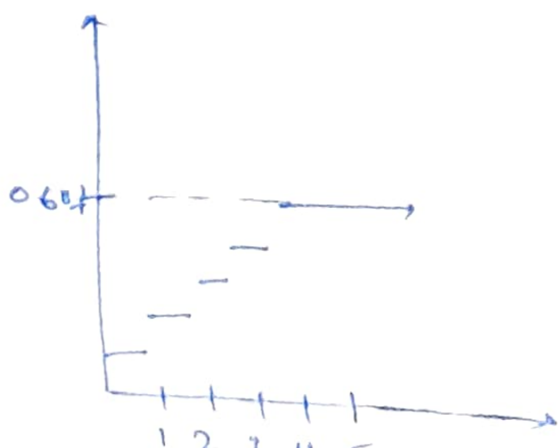
$$P(2,5) = \frac{e^{-5}\,5^{2}}{2!} = 0.084$$

$$P(3,5) = \frac{e^{-5}\,5^{3}}{3!} = 0.143$$

$$P(4,5) = \frac{e^{-5}\,5^{4}}{4!} = 0.175$$

$$P(5.5) = \frac{e^{-5}\,5^{5}}{5!} = 0.175$$

$F(1) = 0.03$
$F(2) = 0.114$
$F(3) = 0.257$
$F(4) = 0.432$
$F(5) = 0.607$



c) $\mu = 5$

$\sigma^2 = 5$

$\sigma = 2.23$

$P(|x-\mu| \geq 2.23) = P(x > 7.3) + P(x \leq 2.7)$

$= 0.03 + 0.114$

$= 0.144$