Sr.No.	Chapter Name	Pg. No.

Mathematical Tools.

(I) FUNCTION & TRIGONOMETRY

1. length of arc $(\hat{AB}) = r\theta$ where $r \rightarrow$ radius
$\theta \rightarrow$ angle in radian.

2. Standard values :-

$30^\circ = \pi/6$ rad	$45^\circ = \pi/4$ rad	$60^\circ = \pi/3$ rad
$90^\circ = \pi/2$ rad	$120^\circ = 2\pi/3$ rad	$135^\circ = 3\pi/4$ rad
$150^\circ = 5\pi/6$ rad	$180^\circ = \pi$ rad	$360^\circ = 2\pi$ rad
$270^\circ = 3\pi/2$ rad	$225^\circ = 5\pi/4$ rad	$300 = 5\pi/3$ rad.

3. a) IF angle $= (n\pi \pm \theta)$ where n is integer then trigonometric function of $(n\pi \pm \theta)$ = same trigonometric function of θ and sign will be decided by ASTC rule.

b) IF angle $= [(2n+1)\pi/2 \pm \theta]$ where n is integer then trigonometric function of $[(2n+1)\pi/2 \pm \theta]$ = complementary function of θ and sign will be decided by ASTC rule.

4. $\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \text{cosec}^2\theta.$

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5. $\cos(A+B) = \cos A \cdot \cos B - \cancel{\cos} \sin A \cdot \sin B.$

$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B.$

6. $\sin(A+B) = \sin A \cos B + \sin B \cos A$

$\sin(A-B) = \sin A \cdot \cos B - \sin B \cos A.$

7. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}.$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

8. $\cot(A+B) = \dfrac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$

$\cot(A-B) = \dfrac{\cot A \cdot \cot B + 1}{\cot A - \cot B}.$

9. $\sin 2\theta = 2\sin\theta \cdot \cos\theta = 2\tan\theta / (1+\tan^2\theta)$

$\cos 2\theta = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}.$

$\tan 2\theta = 2\tan\theta / (1 - \tan^2\theta)$

10. $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

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11. $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$

12. When the given triangle is not a right angled triangle then $c^2 = a^2 + b^2 - 2ab\cos\theta$.

13. If given angles are A, B and C then

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

14. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

15. Some important values

$\sin 37^\circ = 3/5$	$\sin 53^\circ = 4/5$	$\tan 37^\circ = 3/4$
$\cos 37^\circ = 4/5$	$\cos 53^\circ = 3/5$	$\tan 53^\circ = 4/3$
$\pi = 3.14 = 22/7$	$e = 2.71$	$i = \sqrt{-1}$

16. $y = e^x \longrightarrow$ Exponential Function.

17. Properties of Logarithm

$\log mn = \log m + \log n$.

$\log(m/n) = \log m - \log n$

$\log m^n = n\log m$

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$a^{\log_a x} = x^{\log_a a} = x.$

(I) DIFFERENTIATION

1. slope (m) = $\tan\theta$
   $\theta < 90°$, $m = +ve$
   $\theta > 90°$, $m = -ve$

2. Differentiation of

- $y = x^n \implies n = \text{constant}$
  $\frac{dy}{dx} = n x^{n-1}$

- $y = \text{constant}$
  $y' = 0$
- $y = f(x) \pm g(x) \rightarrow$ y = sum of two functions.
  $y' = f'(x) \pm g'(x)$
- $y = \log x$
  $y' = 1/x$
- $y = e^x$
  $y' = e^x$
- $y = \sin x$
  $y' = \cos x$
- $y = \cos x$
  $y' = -\sin x$
- $y = \tan x$
  $y' = \sec^2 x.$

- $y = \sec x$
  $y' = \sec x \cdot \tan x.$
- $y = \cot x$
  $y' = -\operatorname{cosec} x$
- $y = \operatorname{cosec} x$
  $y' = -\operatorname{cosec} x.\cot x.$
- $y = x + \sin x$
  $y' = 1 + \cos x$
- $y = f(x)\,g(x)$
  $y' = f'(x)\,g(x) + f(x).g'(x)$

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- $y = \tan x \cdot \sec x$

  $y' = \sec^3 x + \tan^2 x \cdot \sec x$
- $y = f(x)/g(x)$

  $$y' = \frac{g(x) \cdot f'(x) - f(x)\, g'(x)}{[g(x)]^2}$$
- $y = f[g(x)]$

  $y' = f'[g(x)] \cdot g'(x)$
- $x^2 y + y^2 x = 1$

$$x^2 \cdot \frac{dy}{dx} + y(2x) + y^2 \times 1 + x\left[2y \cdot \frac{dy}{dx}\right] = 0$$

$$\therefore \frac{dy}{dx} = -\left[\frac{2xy + y^2}{x^2 + 2xy}\right]$$

3. $y = f(x)$

$f'(x) = 0 \longrightarrow x = x_1$ and x_2

$f''(x) < 0$

$\quad \hookrightarrow x_1 \rightarrow$ Point of maxima

$f''(x) > 0$

$\quad \hookrightarrow x_2 \rightarrow$ Point of minima.

(III) INTEGRATION

1. Integration of.

- $\int e^x \cdot dx = e^x + c$
- $\int \cos x \cdot dx = \sin x + c$
- $\int -\sin x \cdot dx = \cos x + c$
- $\int \sec^2 x \cdot dx = \tan x + c$
- $\int \operatorname{cosec}^2 x \cdot dx = -\cot x + c$
- $\int \sec x \cdot \tan x \cdot dx = \sec x + c$
- $\int \operatorname{cosec} x \cdot \cot x \cdot dx = -\operatorname{cosec} x + c$
- $\int [f(x) \pm g(x)] \cdot dx = \int f(x) \cdot dx \pm \int g(x) \cdot dx$
- $\int k\, f(x) \cdot dx = k \int f(x) \cdot dx$ where $k \rightarrow$ constant.
- $\int x^n \cdot dx = \dfrac{x^{n+1}}{n+1} + c$

2. Definite integration.

$$A = \int_{x=a}^{x=b} f(x) \cdot dx$$

$$= [f(x)]_{x=a}^{x=b}$$

$$A = f(b) - f(a)$$

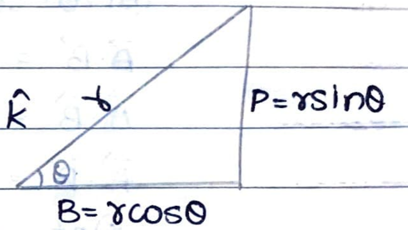
(IV) VECTOR

1. $\vec{A} = |A|\hat{A}$ where $\hat{A}$ is unit vector in direction of $\vec{A}$.

2. Rectangular component.

$\vec{r} = B\hat{i} + P\hat{j} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$

$|\vec{r}| = \sqrt{x^2+y^2+z^2}$ when $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$



3. Addition of vector is maximum when $\theta = 0°$

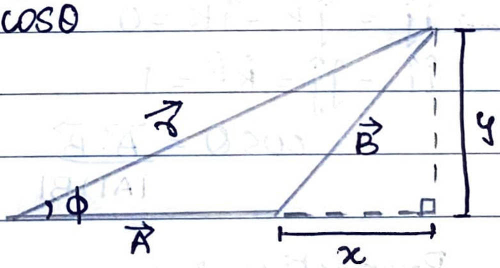
4. Triangle law of addition

$r = \sqrt{A^2+B^2+2|A||B|\cos\theta}$

Here, $A \rightarrow \vec{A}$

$B \rightarrow \vec{B}$

$\tan\phi = \dfrac{B\sin\theta}{A+B\cos\theta}$.



5. Subtraction of two vectors.

$|\vec{A}+(-\vec{B})| = \sqrt{|A|^2+|-B|^2+2|A||B|\cos\theta'}$

$= \sqrt{|A|^2+|B|^2+2|A||B|\cos(180-\theta)}$

$= \sqrt{|A|^2+|B|^2-2|A||B|\cos\theta}$

Here, θ' = angle between $\vec{A}$ and $(-\vec{B})$

θ = angle between $\vec{A}$ and $\vec{B}$.

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6. Multiplication of vector

• DOT PRODUCT (SCALAR PRODUCT) →

Angle between $\vec{A}$ and $\vec{B}$ = θ

$|A||B|\cos\theta = \vec{A}\cdot\vec{B}$

$\vec{A}\cdot\vec{B}$ = +ve when $\cos\theta$ = +ve i.e $\theta < 90°$

$\vec{A}\cdot\vec{B}$ = -ve when $\cos\theta$ = -ve i.e $\theta > 90°$

$\vec{A}\cdot\vec{B}$ = 0 when $|A| = 0$, $|B| = 0$ or $\cos\theta = 0$ i.e $\theta = 90°$

Dot product of $\perp$ vector = 0

and dot product of $\parallel$ vectors = maximum.

$(\vec{A}\cdot\vec{B})_{max} = |A||B|$

Here, $\cos\theta = 1$ i.e $\theta = 0°$

$\hat{i}\hat{j} = \hat{j}\hat{k} = \hat{i}\hat{k} = 0$

$\hat{i}\hat{i} = \hat{j}\hat{j} = \hat{k}\hat{k} = 1$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A||B|}$$

Properties :-

① $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

② $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$.

- CROSS PRODUCT (VECTOR PRODUCT)

$\vec{A} \times \vec{B} = |A||B| \sin\theta \, (\hat{n})$

Here, θ is angle between $\vec{A}$ and $\vec{B}$

$\hat{n}$ is the unit vector along $\vec{A} \times \vec{B}$

$|\gamma| = |A||B| \sin\theta$

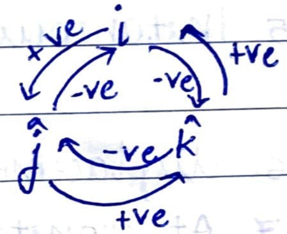
$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

$\hat{i}\hat{i} = \hat{j}\hat{j} = \hat{k}\hat{k} = 0$

$\hat{i}\hat{j} = k \; ; \; \hat{j}\hat{k} = \hat{i} \; ; \; \hat{k}\hat{i} = \hat{j}$

$\hat{j}\hat{i} = -\hat{k} \; ; \; \hat{k}\hat{j} = -\hat{i} \; ; \; \hat{i}\hat{k} = -\hat{j}$



7. Area of parallelogram $= |\vec{A} \times \vec{B}|$

$= |A||B| \sin\theta.$

Rectilinear Motion

1. $\vec{V}_{avg}$ = Total displacement / time.
2. Instaneous velocity ($\vec{v}$) = ds/dt
3. Avg. speed = Total distance / time.
4. $\vec{a}_{avg} = \langle a \rangle = \dfrac{\text{change in velocity}}{\text{time lapsed.}} = \dfrac{\Delta v}{\Delta t}$
5. Instaneous acceleration $= \dfrac{d\vec{v}}{dt}$
6. displacement (s) = velocity (v) × time (t)
7. At constant acceleration,

$$s = ut + \tfrac{1}{2}at^2$$

$$v = u + at$$

$$\vec{a} = \vec{v} \cdot \frac{d\vec{v}}{d\vec{s}}$$

$$v^2 = u^2 + 2as$$

$$s = vt - \tfrac{1}{2}at^2$$

$$s = t/2(u+v)$$

Here, $s \rightarrow$ displacement
$u \rightarrow$ initial velocity
$v \rightarrow$ final velocity
$a \rightarrow$ acceleration
$t \rightarrow$ time.

8. $S_{n^{th}} = u + \frac{a(2n-1)}{2}$ Here, $S_{n^{th}} \rightarrow$ displacement in n^{th} second

9. If a particle moves with constant acceleration along a straight line for same time interval then ratio of their displacement is 1:3:5:7

10. If displacement - time graph is given then find slope to plot velocity - time graph.

11. If velocity - time graph is given then find slope to plot acceleration - time graph.

12. If velocity - time graph is given then find ~~slope~~ area under curve to plot displacement - time graph.

13. Slope is not defined at the peak.

14. When acceleration is variable

$$\int a.dt = v$$

15. Dimensional Formula

Velocity $= [LT^{-1}]$

speed $= [LT^{-1}]$

acceleration $= [LT^{-2}]$

Force = $[MLT^{-2}]$

Work = $[ML^2T^{-2}]$

Power = $[ML^2T^{-3}]$

Impulse = $[MLT^{-1}]$

Projectile Motion

(I) Ground to Ground Projectile

1. Time of flight (T) = $\frac{2u\sin\theta}{g}$

2. Horizontal Range (R) = $\frac{u^2\sin 2\theta}{g}$

3. Maximum height (H_{max}) = $\frac{u^2\sin^2\theta}{2g}$

(II) Equation of Trajectory

1. $y = x\tan\theta - \frac{gx^2(1+\tan^2\theta)}{2u^2}$

2. $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

3. $y = x\tan\theta\left[1 - \frac{gx}{2u^2\cos^2\theta \cdot \tan\theta}\right]$

4. $y = x\tan\theta\left[1 - \frac{gx}{2u^2\cos\theta . \sin\theta}\right]$

5. $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

Here, x is the displacement
y is the height.

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(III) Projectile thrown parallel to horizontal from some height.

1. Time of flight (T) = $\sqrt{\frac{2h}{g}}$

2. Horizontal Range (R) = $u\sqrt{\frac{2h}{g}}$

3. Velocity at general point $(x, y) = \sqrt{u_x^2 + u_y^2}$

$= \sqrt{u^2 + g^2t^2}$

4. Final velocity (v) = $\sqrt{u^2+2gh}$
5. Trajector equation $= -\frac{gx^2}{2u^2}$

(IV) Projection on the inclined plane

	Up the incline	Down the incline
Range	$\frac{2u^2 \sin\alpha \cos(\alpha+\beta)}{g\cos^2\beta}$	$\frac{2u^2 \sin\alpha \cos(\alpha-\beta)}{g\cos^2\beta}$
Time of flight	$\frac{2u\sin\alpha}{g\cos\beta}$	$\frac{2u\sin\alpha}{g\cos\beta}$
Angle of projection for R_{max}	$\frac{\pi}{4} - \frac{\beta}{2}$	$\frac{\pi}{4} + \frac{\beta}{2}$
Maximum Range	$\frac{u^2}{g(1+\sin\beta)}$	$\frac{u^2}{g(1-\sin\beta)}$

Here $\alpha \rightarrow$ angle of projection
$\beta \rightarrow$ angle of inclination

(V) Elastic collision of a projectile with a wall

1. If $x \geq R/2$ then distance of landing place of projectile from its point of projection $= 2x - R$

2. If $x < R/2$ then distance of landing place of projectile from its point of projection $= R - 2x$.

Relative Motion

1. $x_{AB} = x_A - x_B$

Position of A wrt B = Position of A wrt origin − Position of B wrt origin

2. $a_{AB} = a_A - a_B$

Acceleration of A wrt B = Acceleration of A wrt origin − Acceleration of B wrt origin

3. $V_{AB} = V_A - V_B$

velocity of A wrt B = velocity of A wrt origin − velocity of B wrt origin

4. $t = \dfrac{\text{Separation b/w two particles (distance)}}{\text{velocity of approach b/w the particle}}$

5. For constant relative acceleration

$$V_{rel} = u_{rel} + a_{rel} t$$

$$S_{rel} = u_{rel} t + \frac{a_{rel} t^2}{2}$$

$$V_{rel}^2 = u_{rel}^2 + 2a_{rel} S_{rel}.$$

6. Projectile motion in a lift with acceleration 'a'.

$$T = \frac{2u\sin\theta}{g_{eff}}$$

$$H = \frac{u^2\sin^2\theta}{g_{eff}}$$

$$R = \frac{u^2\sin 2\theta}{g_{eff}}$$

7. River Problems.

$$\text{drift}(x) = \frac{(V_{mR}\cos\theta + V_R)\,d}{V_{mR}\sin\theta}$$

$$\text{shortest time } (t) = \frac{d}{V_{mR}\sin\theta}$$

shortest path i.e drift = 0

$\therefore X = 0$

i.e $V_{mR}\cos\theta + V_R = 0$

$$\cos\theta = \frac{-V_R}{V_{mR}}$$

time to cross river along the shortest path

$$= \frac{d}{V_{mR}\sin\theta} = \frac{d}{\sqrt{V_{mR}^2 - V_R^2}}$$

Here, $V_{mR} \rightarrow$ velocity of man wrt river

$V_R \rightarrow$ Velocity of river

$d \rightarrow$ distance b/w two banks.

8. Rain Problems

$$\vec{V}_{RM} = \vec{V}_R - \vec{V}_m$$

$$\vec{V}_{mR} = \sqrt{V_R^2 + V_m^2}$$

$$\theta = \tan^{-1}\frac{V_m}{V_R}$$

Here, $\vec{V}_R \rightarrow$ velocity of rain

$\vec{V}_m \rightarrow$ velocity of man

$V_{RM} \rightarrow$ Velocity of rain wrt man.

Newton's Law of Motion

1. $\vec{F} = m\vec{a}$ where $\vec{a}$ ~~is~~ and m are constant.

2. 1 Newton $= 10^5$ dyne

3. $\vec{F} = m\vec{g}$ → Gravitational Force.

4. $\vec{P} = m\vec{v}$ where $\vec{P}$ → linear momentum.

5. $\vec{F}_{spring} = kx$ where k → spring constant
   x → displacement from natural length.

6. Spring constrained
   Identify a single string and divide it into different linear sections
   i.e $l_1 + l_2 + l_3 + l_4 =$ constant
   Differentiate this to find velocity and double differentiate to find acceleration.

7. For two block problems.
   $$\vec{a} = \frac{\text{difference of mass}}{\text{sum of mass}} \times g.$$

$$T = \frac{2m_1m_2}{m_1+m_2} \times g.$$

8. Series combination of springs
IF multiple springs are replaced with one spring then $K = \frac{K_1K_2}{K_1+K_2}$.

9. Parallel combination of springs
IF multiple springs are replaced with one spring then $K = K_1 + K_2$.

10. If the given spring of natural length (L) and spring constant (k) is cut into two equal halves then if new natural length $= L/2$ then new spring constant $= 2k$.

11. Tension in the spring cannot be zero instaneously.

12. $\vec{F}_{real} + \vec{F}_{pseudo} = m\vec{a}$

$\vec{F}_{pseudo} = -m\vec{a}_{frame}$.

Pseudo force is always directed opposite to the direction of the acceleration of the frame.

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Friction

1. Magnitude of kinetic friction $= \mu N$
where $\mu \rightarrow$ coefficient of friction
$N \rightarrow$ Normal.

2. $(F_{static})_{max} = \mu N$

3. Magnitude of static friction lies between
$0 \leq F_{static} \leq \mu N$

4. $\mu_{static} > \mu_{kinetic}$
If μ is not specified then its common for both.

5. Condition for sliding :- $\tan\theta > \mu$
If $\tan\theta < \mu$ then block will not slide unless an external force is applied.

Work, Power, Energy

1. $d\vec{w} = \vec{F}.d\vec{s}$

$w = F.s\cos\theta$

where $w \rightarrow$ work

$F \rightarrow$ force

$s \rightarrow$ displacement

$\theta \rightarrow$ angle b/w $\vec{F}$ and $\vec{s}$

2. $\vec{w}_{net} = \vec{w}_1 + \vec{w}_2 + \vec{w}_3 + \ldots$

$= d\vec{s}(\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots)$

$= ds(F_1\cos\theta + F_2\cos\alpha + F_3\cos\phi + \ldots)$

3. Special Cases.

(i) When angle between $\vec{F}$ and $d\vec{s}$ is constant.

$\therefore \quad w = F\vec{s}\cos\theta.$

If F is the kinetic energy, then

$w = -f_k \times$ (length of path)

(ii) When both magnitude ~~of~~ and direction of $\vec{F}$ is constant

$\therefore \quad w = f \times$ height.

4. $W = \int F.ds$

↳ Area under F-s curve.

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5. $U = \frac{1}{2}kx^2$

$W_{\text{spring Force}} = -\Delta U = U_i - U_f$

$= \frac{k(x_1^2 - x_2^2)}{2}$

$\therefore (W_{\text{external}})_{\text{on spring force}} = \Delta U.$

6. $K.E = \frac{1}{2}mv^2 = P^2/2m$

where, $m \rightarrow$ mass

$P \rightarrow$ Momentum

$V \rightarrow$ velocity

$\therefore (K.E)_{\text{system}} = K_1 + K_2 + K_3$

$= \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + \frac{P_3^2}{2m_3}$

Velocity of block is maximum when it is in equilibrium.

7. $W_{\text{conservative Forces}} = -\Delta U$

$= U_i - U_f$

8. $W_{net} = \Delta K$ $\longrightarrow$ By work energy theorem.

9. If there is no external force acting on the system then $P_i = P_f$ $\longrightarrow$ Momentum conservation.

10. $P.E = mgh$

$\hookrightarrow$ work done against gravitational force.

11. $W_{NC} + W_{PS} = \Delta K + \Delta U$

where, $W_{NC} \rightarrow$ work done by all non-conservative forces

$W_{PS} \rightarrow$ work done by all pseudo forces.

$\Delta K \rightarrow$ change in kinetic energy.

$\Delta U \rightarrow$ change in potential energy.

12. Power $= \frac{\text{Work}}{\text{time}} = Fv\cos\theta$.

13. Equilibrium

- Stable equilibrium

condition :- $-\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} = +ve$

- Unstable equilibrium

condition :- $-\frac{dU}{dx} = 0$ potential energy is maximum

i.e $\frac{d^2U}{dx^2} = -ve$.

14. $E = K + U$

where, $K \rightarrow$ Kinetic energy

$U \rightarrow$ Potential energy

$E \rightarrow$ Mechanical Energy.

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Circular Motion.

1. $\omega = \frac{2\pi}{T} = 2\pi f$ where $\omega \rightarrow$ angular velocity
$T \rightarrow$ Time period
$f \rightarrow$ frequency.

2. If $\alpha = 0$, circular motion is said to be uniform and $\omega t = \theta$ where $\alpha \rightarrow$ angular acceleration.

3. $\vec{\alpha}_{avg} = \frac{\Delta\vec{\omega}}{\Delta t}$

4.
$$\left.\begin{aligned} \omega &= \omega_0 + \alpha t \\ \theta &= \omega_0 t + \alpha t^2/2 \\ \omega^2 &= \omega_0^2 + 2\alpha\theta \\ \theta &= (\omega + \omega_0)t/2 \\ \theta_{n^{th}} &= \omega_0 + \frac{\alpha(\theta_n - \theta_{n-1})}{2} \end{aligned}\right\}$$
For α = constant

Here, $\omega_0 \rightarrow$ angular velocity (initial)
$\omega \rightarrow$ angular velocity (final)
θ ~~θ~~ $\rightarrow$ angular displacement
α ~~θ~~ $\rightarrow$ angular acceleration.

5. $\vec{v} = \vec{\omega} \times \vec{r}$ where, $v \rightarrow$ velocity
$\omega \rightarrow$ angular velocity
$r \rightarrow$ position of particle wrt centre of circle.

6. $\omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}}$

7. Tangential acceleration $(\vec{a_t}) = \vec{\alpha}\vec{r}$
Centripetal acceleration $(\vec{a_c}) = \vec{\omega}.\vec{v}$
Total acceleration $(\vec{a}) = \sqrt{(\vec{a_t})^2 + (\vec{a_c})^2}$
$\tan\theta = \frac{a_c}{a_t}$.

8. Centripetal force $= m\vec{a_c} = \frac{mv^2}{r} = m\omega^2 r$
Tangential force $= m\vec{a_t} = m\alpha r$.

9.

	A	B, D	C	P (general point)
Velocity	$\sqrt{5gl}$	$\sqrt{3gl}$	$\sqrt{gl}$	$\sqrt{gl(3+2\cos\theta)}$
Tension	6mg	3mg	0	$3mg(1+\cos\theta)$
Potential Energy	0	mgl	2mgl	$mgl(1-\cos\theta)$
Radial acceleration	5g	3g	g	$g(3+2\cos\theta)$
Tangential acceleration	0	g	0	$g\sin\theta$

Here, A is the lowest point
B and D are midpoints
C ~~is~~ is the topmost point.

10. Condition for oscillation or leaving the circle

(i) For oscillation,

$$0 < V_L < \sqrt{2gl}$$

$$0 < \theta < 90^\circ$$

Here, speed becomes zero before the tension.

(ii) For leaving the circular path after which motion converts into projectile.

$$\sqrt{2gl} < V < \sqrt{5gl}$$

$$90^\circ < \theta < 180^\circ$$

Here, tension becomes zero before speed.

11. Condition for completing the loop in some other cases

(i) A mass moving on a smooth vertical circular track.

Mass moving along a smooth vertical circular loop. Condition for just completing the loop, normal at highest point $= 0$

Minimum horizontal velocity at lowest point $= \sqrt{5gl}$

(ii) A particle attached to a light rod rotated in verticle circle.

condition for just completing the loop, velocity $(v) = 0$ at highest point

$$v_{min} = \sqrt{4gl}$$

(iii) A bead attached to a ring and rotated
Condition for completing the loop, velocity = 0 at highest point

$$V_{min} = \sqrt{4gl}$$

(iv) A block rotated between smooth surfaces of a pipe.
condition for just completing the loop, Velocity = 0 at the highest point.

$$V_{min} = \sqrt{4gl}$$

12. Circular turning on the road.

(i) By friction only.

$$v \leq \sqrt{\mu r g}$$

(ii) By banking of roads only.

$$v = \sqrt{rg\tan\theta}$$

(iii) By both friction and banking of road.

$$V_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{1 + \mu\tan\theta}}$$

$$V_{max} = \sqrt{\frac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}$$

$$\tan\theta = v^2/rg.$$

13. $W_{apparent} = mg - m\omega^2 R$

where $W_{apparent} \rightarrow$ apparent weight

$R \rightarrow$ Radius of earth.

$\omega \rightarrow$ Angular velocity of earth.

Centre of Mass

1. $$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + m_3 + \ldots + m_n}$$

2. $$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

$$r_1 = \frac{m_2}{m_1 + m_2} \times r$$

$$r_2 = \frac{m_1}{m_1 + m_2} \times r.$$

3. Centre of mass of a continous mass distribution

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}.dm$$ Here, dm → mass of body.

4. Centre of mass of uniform rod.

$$x_{cm} = \frac{\int_0^L x.dm}{\int dm} = \frac{\int_0^L x\left(\frac{M}{L}.dx\right)}{M} = \frac{1}{L}\int_0^L x.dx = L/2$$

5. Centre of mass of various objects.

- semicircular ring

  $y_{cm} = 2R/\pi$

- semicircular disc

  $y_{cm} = 4R/3\pi$

- solid ~~semi~~ hemisphere

  $y_{cm} = 3R/8.$

- Hollow hemisphere
  $y_{cm} = R/2$
- Solid cone
  $y_{cm} = 3H/4$
- Rectangular plate.
  $y_{cm} = L/2$ and $x_{cm} = b/2$.
- Triangular plate
  $y_{cm} = H/3$
- A solid circular cone
  $y_{cm} = H/4$
- A hollow circular cone
  $y_{cm} = H/3$

6. $$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \dots + m_n\vec{v}_n}{m_1 + m_2 + \dots + m_n}$$

7. $$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \dots + m_n\vec{a}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$= \frac{F_{net}}{M}$$

8. $\vec{P}_{system} = M\vec{V}_{cm}$

9. $\vec{F}_{ext} = M\vec{a}_{cm}$

10. If $F_{ext} = 0$ and $V_{cm} = 0$ then com remains at rest.

11. If $F_{ext} = 0$ then V_{cm} remains constant, therefore net momentum of the system also remains conserved.

12. If an external force is present then com continues its original motion as if the ~~tx~~ external force is acting on it, irrespective of internal forces.
Example :- Projectile and circular motion.

13. By momentum conservation,
$$\vec{P_1} + \vec{P_2} + \dots\dots + \vec{P_n} = \text{constant}.$$

14. Impulse is area under the F-t graph.
$$\vec{I} = \int_{t_1}^{t_2} \vec{F}.dt = \int m.d\vec{v}$$
$$\vec{I} = M(\vec{V_2} - \vec{V_1}) = \Delta\vec{P} \longrightarrow \text{change in momentum.}$$

15. COM of object with cavity
- When mass is distributed in area
$$x_{cm} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$
- When mass is distributed in volume
$$x_{cm} = \frac{V_1x_1 - V_2x_2}{V_1 - V_2}$$

where, $A_1 \Rightarrow$ area of complete objectect without cavity.

$A_2 \Rightarrow$ area of cavity.

$V_1 \Rightarrow$ volume of complete object without cavity

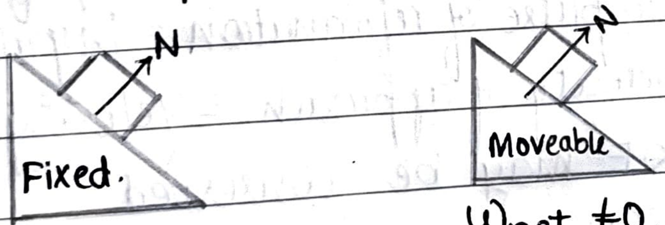
$V_2 \Rightarrow$ volume of cavity.

$x_1 \Rightarrow$ x-co-ordinate of COM of complete object.

$x_2 \rightarrow$ x-co-ordinate of COM of cavity.

16. Conserve energy of system only when forces are doing work on system and whose potential energy is defined.

Example → Spring force, mg, electrostatic force, etc.



$W_{net} = 0$ $W_{net} \neq 0$.

17. Spring-mass system

At max elongation and compression, velocity of both end of spring becomes equal and in same direction.

18. A force of relatively higher magnitude and acting for relatively shorter time, is called impulsive force.

19. Gravitational force and spring force are always non-impulsive.
Normal, tension and friction are case dependent.
An impulsive force can only be balanced by another impulsive force only.

20. $e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$

- $e = 1 \rightarrow$ Impulse of reformation = Impulse of deformation
  velocity of approach = velocity of separation
  K.E may be conserved.
  Elastic collision.
- $e = 0 \rightarrow$ Impulse of reformation = 0
  velocity of separation = 0
  K.E is not conserved
  Perfectly Inelastic collision
- $0 < e < 1 \rightarrow$ Impulse of reformation < Impulse of deformation
  velocity of separation < velocity of approach
  K.E is not conserved
  Inelastic collision.

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21. Line passing through common normal to the surface during collision is called line of Impact (LOI)
Impulse force act along LOI during the collision.

22 Classification of collisions :-

- Head - on collision.
  If the velocities of the colliding particles are along the same line before and after the collision.

- Oblique collision
  If the velocities of the colliding particles are along different lines before and after collision.

23. Variable mass system
If a mass is added or ejected from a system, at rate μ kg/s and relative velocity V_{rel} then the force exerted by this mass on the system has magnitude $\mu|v_{rel}|$

24. $F_t = V_{rel} \cdot \frac{dm}{dt}$

where, F_t = Thrust force.

Doubt *

$v = \frac{Ft}{M_0 \mu t}$

25. Rocket propulsion

If gravity is ignored and initial velocity of the rocket $u=0$,

$$v = v_r \ln\left(\frac{m_0}{m}\right)$$

$v_r \Rightarrow$ relative velocity.

Without any condition, general formula is

$$v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$$

Simple Harmonic Motion.

1. $F = -kx$

where, $k \Rightarrow$ spring constant

$x \Rightarrow$ displacement.

2. General equation of S.H.M is

$x = A \sin(\omega t + \phi)$

where, $A \Rightarrow$ Amplitude

$(\omega t + \phi) \Rightarrow$ Phase of motion

$\phi \Rightarrow$ Initial phase.

$\omega \Rightarrow$ Angular frequency.

3. $\omega = \frac{2\pi}{T} = 2\pi F$

where, $T \Rightarrow$ Time period

$F \Rightarrow$ Frequency.

4. $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{M}{K}}$

5. $V = \omega \sqrt{A^2 - x^2}$

where, $V \Rightarrow$ speed.

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6. $a = -\omega^2 x$

where, $a \Rightarrow$ acceleration.

7. $K.E = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} K (A^2 - x^2)$

$K = m\omega^2$

8. $P.E = \frac{1}{2} Kx^2$

9. $T.E = K.E + P.E = \frac{1}{2} K(A^2 - x^2) + \frac{1}{2} Kx^2 = \frac{1}{2} KA^2$

10. $T = 2\pi \sqrt{\frac{\mu}{K}}$

where, $\mu = \frac{m_1 m_2}{(m_1 + m_2)}$ → reduced mass.

11. Combination of springs.

- Series combination.

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$

- Parallel combination

$$K_{eq} = K_1 + K_2$$

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12. For simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

where, $l \Rightarrow$ length of string.

$g_{eff} \Rightarrow$ net acceleration

13. For compound pendulum or physical pendulum,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

where, $I = I_{cm} + ml^2$

$I_{cm} \Rightarrow$ moment of inertia of com which passes through com & parallel to the axis of oscillation.

$$T = 2\pi\sqrt{\frac{I_{cm} + ml^2}{mgl}}$$

$$I_{cm} = mk^2$$

$$T = 2\pi\sqrt{\frac{K^2 + l^2}{gl}}$$

$$T_{min} = 2\pi\sqrt{\frac{2K}{g}}$$

14. Torsional Pendulum

$$T = 2\pi\sqrt{\frac{I}{C}}$$

where, $C \Rightarrow$ Torsional Strength.

15. Superposition of SHM along the same direction.

$x_1 = A_1 \sin\omega t$ and $x_2 = A_2 \sin(\omega t + \theta)$

If equation of resultant SHM is taken as

$x = A\sin(\omega t + \phi)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$$

$$\tan\phi = \frac{A_2 \sin\theta}{A_1 + A_2\cos\theta}.$$

$\theta \Rightarrow$ angle b/w A_1 and A_2

$\phi \Rightarrow$ angle b/w A_1 and A.

16. At mean position,

$F_{net} = 0$

$a = 0$

$V = \text{maximum} = A\omega$

At extreme position.

$V = 0$

$F = \text{maximum} = -KA$

$a = \text{maximum} = -\omega^2 A$

Sound Waves

1. Equation of Sound Wave.

- Displacement wave equation

$$S = S_0 \sin(\omega t - kx + \phi_1)$$

where, $S \Rightarrow$ displacement

$S_0 \Rightarrow$ maximum displacement.

- Pressure wave equation

$$P = P_0 \sin(\omega t - kx + \phi_2)$$

where, $P \rightarrow$ excess pressure

$P_0 \Rightarrow$ maximum excess pressure.

2. $$P = -B \cdot \frac{ds}{dx}$$

$$P_0 = BkS_0$$

where, $B \Rightarrow$ Bulk modulus.

3. Phase difference b/w displacement wave and pressure wave is $\pi/2$.

4. Velocity of Sound waves.

- In solid, $v = \sqrt{\frac{Y}{\rho}}$

- In liquid, $v = \sqrt{\frac{B}{\rho}}$

where, Y = Young's Modulus of Elasticity
B = Bulk Modulus of medium
ρ = Density of medium.

5. Newton's Formula :

$$PV = \text{constant} \Rightarrow \text{Isothermal Process}$$

$$\frac{dP}{dV} = -\frac{P}{V} \Rightarrow B = P$$

$$\therefore v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{RT}{M}}$$

where, P = Pressure
R = Gas constant
T = Temperature
M = Molar mass.

6. Laplace's correction :

$$PV^{\gamma} = \text{constant} \Rightarrow \text{Adiabatic Process}$$

$$\frac{dP}{dV} = -\gamma\frac{P}{V} \Rightarrow B = \gamma P$$

$$\therefore v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

7. Factors affecting speed of sound. :

- Temperature

  $\Delta V = 0.6\ \Delta T$

- Pressure

  Pressure does not effect velocity of sound as long as temperature is constant.

- Humidity

  Humidity $\uparrow \Rightarrow \downarrow$ density.

8. $$P_{avg.} = \frac{P_0^2 A}{2\rho v} = \frac{\rho A v \omega^2 s_0^2}{2}$$

$$P_{max} = \rho A v \omega^2 s_0^2 = \frac{P_0^2 A}{\rho v}$$

$$T.E = P_{avg.} \times t$$

$$\langle I \rangle = \frac{P_0^2 v}{2B}$$

$$I = \frac{P_0^2}{2\rho v}$$

9. If the source is point source then

$I \propto \frac{1}{r^2}$ \$ $S_0 \propto \frac{1}{r}$

$$\therefore\ S = \frac{a}{r} \sin(\omega t - kr + \theta)$$

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If a sound source is a line source then

$I \propto \frac{1}{r}$ and $S_0 \propto \frac{1}{\sqrt{r}}$

$\therefore S = \frac{a}{\sqrt{r}} \sin(\omega t - kr + \theta)$

10. $\beta = 10 \log\left(\frac{I}{I_0}\right)$ dB

where, β = intensity level.

$I_0 = 1 dB = 10^{-12}$ watt/m^2

11. If $P_1 = P_{m_1} \sin(\omega t - kx_1 + \theta_1)$

$\$ P_2 = P_{m_2} \sin(\omega t - kx_2 + \theta_2)$

$P = P_1 + P_2$

$P = P_0 \sin(\omega t - kx + \theta)$

$P_0 = \sqrt{P_{m_1}^2 + P_{m_2}^2 + 2P_{m_1}P_{m_2}\cos\phi}$

$\phi = |K(x_1 - x_2) + (\theta_2 - \theta_1)|$

12. For constructive interference

$\phi = 2n\pi \Rightarrow P_0 = P_{m_1} + P_{m_2}$

If ϕ is only due to path difference then,

$\Delta x = n\lambda \quad n \in [-2, 2]$

For destructive interference

$\phi = (2n+1)\pi \Rightarrow P_0 = |P_{m_1} - P_{m_2}|$

If ϕ only depends on path difference,

$\Delta x = (2n+1)\frac{\lambda}{2}$ $n \in [-2, 2]$

13. $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

14. For closed organ pipe,

$f = \frac{v}{4l}, \frac{3v}{4l}, \dots \frac{(2n+1)v}{4l}$ $n =$ overtone.

For open organ pipe,

$f = \frac{v}{2l}, \frac{v}{l}, \dots \frac{nv}{2l}$ $n =$ overtone

15. end correction $(e) = 0.6r$

For effective length of closed organ pipe,

$l' = l + e$

For effective length of open organ pipe,

$l' = l + 2e$

$\therefore f_c = \frac{v}{4(l+0.6r)}$

$f_o = \frac{v}{2(l+1.2r)}$

16. Frequency of beats $= f_1 - f_2$

beat time period $= \frac{1}{f_1 - f_2}$.

17. If the arm of tuning fork is waxed or loaded, then its frequency decreases.
If the arm of tuning fork is filed, then its frequency increases.

18. General Formula.

$$F' = \left(\frac{V - V_o}{V - V_s}\right) F$$

where, F' = observed frequency.
V = Velocity of sound wrt ground
V_o = velocity of observer.
V_s = velocity of source.
f = frequency

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String Waves.

1. General equation of wave

$$y = A \sin(\omega t \pm kx + \phi)$$

where, $y \rightarrow$ displacement
$A \rightarrow$ amplitude.
$\phi \rightarrow$ initial phase.

2. $$K = \frac{2\pi}{\lambda} = \frac{\omega}{v}$$

where, $K \rightarrow$ wave number
$\lambda \rightarrow$ wavelength

3. $$\Delta\phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{T} \times \Delta T$$

where, $\Delta\phi \rightarrow$ Phase difference.

4. $$V = \sqrt{\frac{T}{\mu}}$$ where, $T \rightarrow$ Tension
$\mu \rightarrow$ mass per unit length
$V \rightarrow$ speed of transverse wave.

5. $$P_{avg} = 2\pi^2 f^2 A^2 \mu v$$

6. $$\text{Intensity} = 2\pi^2 f^2 A^2 \rho v$$

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7. Reflection and refraction of wave.

(A) From rarer to denser

$y_t = A_t \sin(\omega t - k_2 x)$ $\quad V_2 < V_1$

$y_r = -A_r \sin(\omega t + k_1 x)$

(B) From denser to rarer

$y_t = A_t \sin(\omega t - k_2 x)$ $\quad V_2 > V_1$

$y_r = A_r \sin(\omega t + k_1 x)$

8. $A_r = \frac{|k_1 - k_2|}{k_1 + k_2} A_i$

$A_t = \frac{2k_1}{k_1 + k_2} A_i$

9. Standing waves.

$y_1 = A \sin(\omega t - kx + \theta_1)$

$y_2 = A \sin(\omega t + kx + \theta_2)$

$y_r = y_1 + y_2 = \left[2A \cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)\right] \sin\left(\omega t + \frac{\theta_1 + \theta_2}{2}\right)$

$y_r = A_r \sin\left(\omega t + \frac{\theta_1 + \theta_2}{2}\right)$

where, $A_r = 2A \cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)$

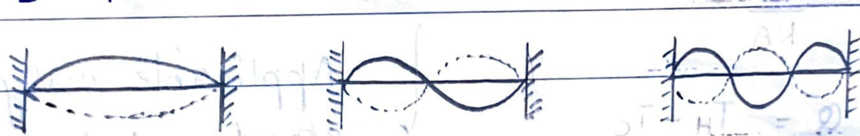
10. Points at which amplitude becomes zero are known as nodes whereas the points at which amplitude is maximum are known as antinodes.
distance b/w two successive nodes or antinodes $= \lambda/2$
distance b/w successive node and antinode $= \lambda/4$.

11. Vibration of strings

(a) Fixed at both ends

Fixed ends will be node. So waves for which

$L = \lambda/2$ $\quad$ $L = 2\lambda/2$ $\quad$ $L = 3\lambda/2$



are possible giving

$L = \frac{n\lambda}{2}$ or $\lambda = \frac{2L}{n}$ where, $n = 1, 2, 3 \ldots$

as $v = \sqrt{\frac{T}{\mu}}$ $\quad$ $f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$ where n = no. of loops.

(b) String free at one end

For fundamental mode $L = \frac{\lambda}{4}$ or $\lambda = 4L$

First overtone $L = \frac{3\lambda}{4}$ or $\lambda = \frac{4L}{3}$

So. $f_1 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$

Second overtone, $f_2 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$

$\therefore f_n = \frac{(2n+1)}{4L}\sqrt{\frac{T}{\mu}}$

Heat Transfer

1. $\frac{Q}{t} = KA\left(\frac{T_H - T_C}{L}\right)$ → Applicable for steady state only.

where, $Q \rightarrow$ heat

$t \rightarrow$ time

$K \rightarrow$ conductivity of the material.

$A \rightarrow$ face area.

$L \rightarrow$ Lateral thickness.

2. $R = \frac{L}{KA}$

$\therefore \frac{Q}{t} = \frac{T_H - T_C}{R}$

$\frac{Q}{T} = i_T = \frac{T_H - T_c}{R}$

Applicable only for steady state.

where, $R \rightarrow$ Resistance

$i_T \rightarrow$ thermal current.

3. For any state,

$\frac{Q}{t} = -KA\frac{dT}{dx}$

where, $\frac{dT}{dx} \rightarrow$ temperature gradient.

$\Delta T \rightarrow$ -ve quantity.

4. Slabs in series (in steady state)

$$i = \frac{T_H - T_C}{R_1 + R_2} = \frac{T_H - T_C}{R_{eq}}$$

$$R_{eq} = R_1 + R_2 + \dots$$

5. Slabs in parallel (in steady state)

$$i = \frac{T_H - T_C}{R_{eq}}$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots\right)^{-1}$$

6. intensity $\alpha \frac{1}{r^2}$ → For radiation from point source.

7. $Q = Q_r + Q_t + Q_a$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$$

$$1 = r + t + a$$

where, $r \Rightarrow$ reflecting power

$a \Rightarrow$ absorptive power

$t \Rightarrow$ transmission power.

(i) $r = 0, t = 0, a = 1$ → perfect black body

(ii) $r = 1, t = 0, a = 0$ → perfect reflector

(iii) $r = 0, t = 1, a = 0$ → perfect transmitter.

8. $a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$

For black body, Energy absorbed = Energy incident.

9. $E = \frac{Q}{\Delta A . \Delta t}$

$E \rightarrow$ Emissive power

$Q \rightarrow$ energy radiated.

$A \rightarrow$ area

$t \rightarrow$ time.

10. $\frac{dE}{d\lambda} = E_\lambda$

where, $E_\lambda \Rightarrow$ Spectral emissive power

$E \Rightarrow$ emissive power

$\lambda \Rightarrow$ wavelength

11. $e = \frac{\text{Emissive power of body at temperature T}}{\text{Emissive power of black body at same temperature}}$

$= \frac{E}{E_b}$

12. $E(\text{black body}) = \frac{E(\text{body})}{a(\text{body})} \longrightarrow$ Kirchoff's Law

13. $\lambda_m T = b \longrightarrow$ wien's displacement law.

where, $\lambda_m \rightarrow$ wavelength corresponding to maximum intensity

$T \rightarrow$ temperature

$b \rightarrow$ ~~0.2~~ wien's constant

$b = 0.282$ cm-K

14. For black body,

$u = \sigma A T^4$ → Stefan-Boltzmann's Law.

where, u → radiation emitted ~~per unit area~~.

A → area

T → absolute temperature

σ → Stefan's constant

$\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$

For any body,

$u = e\sigma A T^4$

where, e → emissivity.

15. $\theta_f = \theta_0 + (\theta_i - \theta_0)e^{-kt}$ → Newton's Law of cooling.

where, θ_f → Final temperature of object

θ_0 → room temperature

θ_i → initial temperature of object

t → time.

$$K = \frac{4e\sigma\theta_0^3 A}{ms}$$

where, m → mass of water

s → specific heat.

16. $\left\langle\frac{d\theta}{dt}\right\rangle = -k(\langle\theta\rangle - \theta_0)$ → Approximate method for applying Newton's Law of cooling.

$$\langle\theta\rangle = \frac{\theta_i + \theta_f}{2}$$

17. $$\frac{(w+m_1 s_1)(\theta_1-\theta_2)}{t_1} = \frac{(w+m_2 s_2)(\theta_1-\theta_2)}{t_2}.$$

$$\therefore \frac{w+m_1 s_1}{t_1} = \frac{w+m_2 s_2}{t_2}$$

If water equivalent (w) is negligible then,

$$\frac{m_1 s_1}{t_1} = \frac{m_2 s_2}{t_2} \quad \text{or} \quad \frac{\rho_1 s_1}{\rho_2 s_2} = \frac{t_1}{t_2} \quad (V_1 = V_2)$$

Here, $w \rightarrow$ water equivalent of calorimeter.
$m_1 \rightarrow$ mass of liquid
$s_1 \rightarrow$ specific heat of liquid
$m_2 \rightarrow$ mass of water
$s_2 \rightarrow$ specific heat of water.

18 Rate of heat loss $= \sigma e A (T^4 - T_0^4)$
where, $T_0 \Rightarrow$ absolute room temperature.

Elasticity & Vicosity.

1. Stress = $\frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$

tensile stress = $\frac{F \sin\theta}{A}$ = normal stress

Shear stress = $\frac{F \cos\theta}{A}$ = tangential stress.

2. Strain (e) = $\frac{\text{change in configuration}}{\text{original configuration}}$

Longitudinal strain $(\epsilon_l) = \frac{\Delta L}{L}$

Volume strain $(\epsilon_v) = \frac{\Delta V}{V}$

Shear strain :-

$\tan\phi = \frac{x}{L}$

3. Modulus of elasticity = $\frac{\text{stress}}{\text{strain}}$

∴ Young's modulus (Y) = $\frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{FL}{A\Delta L}$

Bulk modulus (B) = $\frac{\text{Pressure}}{\text{volume strain}} = -\frac{PV}{\Delta V}$

Adiabatic bulk modulus of gas (B) = $\gamma \times P = \frac{C_P}{C_V} \times P$.

Modulus of rigidity (η) = $\dfrac{\text{Tangential Stress}}{\text{Shear strain}}$ = $\dfrac{F}{A\phi}$

4. Elongation of rod under it's self weight.

$\therefore$ Total elongation = $\dfrac{WL}{2AY}$

where, $W \rightarrow$ self weight.

$L \rightarrow$ Length

$A \rightarrow$ cross sectional area

$Y \rightarrow$ Young's modulus.

$T = \dfrac{W}{L} x.$

$x \rightarrow$ displacement

$T \rightarrow$ tension.

5. $U = \frac{1}{2} \times$ stress $\times$ strain $\times$ volume.

$= \frac{1}{2} \dfrac{F^2}{A^2Y^2} A l Y = \frac{1}{2} Y (\text{strain})^2 \times \text{volume}.$

Strain energy density $= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$

6. maximum extension $= \dfrac{mg}{K_{eq}} + \sqrt{\dfrac{m^2g^2 + 2mgh}{K_{eq}}}$

7. thermal stress $= Y \alpha \Delta T$

$\therefore F = AY\alpha\Delta T.$

8. $C = \frac{\pi \eta r^4}{2l}$ where, $C \rightarrow$ torsion constant
$\eta \rightarrow$ modulus of rigidity
$r \rightarrow$ radius
$l \rightarrow$ length.

9. $\tau = C\theta$ where, $\tau \rightarrow$ torque required for twisting by angle θ.
$\theta \rightarrow$ angle.

10. $W = \frac{C\theta^2}{2}$ where, $W \rightarrow$ work done in twisting by angle θ

11. $h < \frac{\text{stress}}{\rho g}$

stress $= 3\times10^8 \, N/m^2$, $\rho = 3\times10^3 \, kg/m^3$, $g = 10 \, m/s^2$

$\therefore h_{max} = 10 \, km$
$\rightarrow$ maximum height of mountain.

12. $F = \eta A \frac{dv}{dx}$ $\rightarrow$ Newton's Law viscosity.

13. $F = 6\pi\eta r v$

$\therefore$ Terminal velocity $= \frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta}$

Surface Tension

1. $F = T(l)$

where, $F \rightarrow$ surface tension force
$l \rightarrow$ contact length
$T \rightarrow$ surface tension.

2. $U = T(A)$

where, $U \rightarrow$ surface energy
$A \rightarrow$ surface area

3. Inside a bubble,

$$P_{excess} = \frac{4T}{r}$$

4. Inside the water drop,

$$P_{excess} = \frac{2T}{r}$$

5. Inside air bubble in a liquid

$$P_{excess} = \frac{2T}{r}$$

6. Capillary rise,

$$h = \frac{2T\cos\theta}{\rho g R}.$$

Fluid Mechanics.

1. ρ = mass/volume

R.D (Relative density) = $\frac{\text{density of substance}}{\text{density of water at } 4°C}$

density of water at 4°C = $1 gm/cm^3$

2. Pressure = $\frac{F_\perp}{A}$

3. $P = P_0 + \rho g h$

where, $P_0 \rightarrow$ atmospheric pressure

$\rho \rightarrow$ density

$h \rightarrow$ depth

4. $W = \rho g h A$

where, $W \rightarrow$ weight of fluid element.

$A \rightarrow$ Area.

The pressure is same at any two points at the same level in the fluid. The shape of container does not matter.

5. $P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$ $\quad F_2 > F_1$ and $A_2 > A_1$

6. (i) Liquid placed in elevator :

When elevator accelerates upwards with acceleration a_0 then pressure in the fluid at depth h may be given by,

$$P = \rho h (g + a_0)$$

and Force of buoyancy, $B = m(g+a_0)$

(ii) Free surface of liquid in horizontal acceleration.

$$\tan\theta = \frac{a_0}{g}$$

$\therefore P = \rho l a_0$

$$h = \frac{l a_0}{g}$$

(iii) Free surface of liquid in case of rotating cylinder

$$h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$

7 Equation of continuity :-

$$a_1 v_1 = a_2 v_2$$

where, a_1 and a_2 → area of holes

v_1 and v_2 → velocity.

8. $\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant} \; (J/m^3)$ → Bernoulli's Theorem.

9. $v = \sqrt{\frac{2gh A_1^2}{A_1^2 - A_2^2}}$

where, v → speed of efflux

A_1 → area of vessel

A_2 → area of hole.

10. Buoyant Force $(F) = V_i \rho_L g_{eff}$

where, V_i → immersed volume of solid

ρ_L → density of liquid.

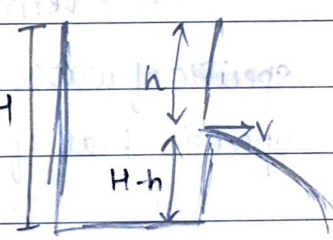
$$\frac{V_i}{V} = \frac{\rho_S}{\rho_L}$$

11. $W_{app} = W_{actual} - V\rho_L g$
$= (\rho_s - \rho_L)Vg$

where, $W_{app} \rightarrow$ apparent weight of ~~liquid~~ body in liquid
$\rho_s \rightarrow$ density of body
$\rho_L \rightarrow$ density of liquid
$V \rightarrow$ total volume of body.

12. Pressure energy $= P(Al)$
Kinetic energy $= \frac{\rho v^2}{2}$
Potential energy $= \rho g h$

13. $t = \sqrt{\frac{2(H-h)}{g}}$



$R = 2\sqrt{h(H-h)}$

$R_{max} = H$ when $h = H/2$.

$\cancel{t = \frac{A}{a}}$

14. $t = \frac{A}{a}\sqrt{\frac{2H}{g}}$

where, $t \rightarrow$ time to empty the tank
$A \rightarrow$ Cross sectional area of tank
$a \rightarrow$ cross sectional area of small hole.

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Calorimetry & Thermal Expansion

1. $W = JH$

where, $W \rightarrow$ ~~work~~ water equivalent.

$J \rightarrow$ mechanical equivalent of heat

$H \rightarrow$ Heat capacity.

2. $Q = ms\Delta T$

where, $Q \rightarrow$ Heat / Energy.

$m \rightarrow$ mass

$s \rightarrow$ Specific heat

$T \rightarrow$ temperature.

specific of water = 4200 J/kg°C = 1000 cal/kg°C = 1 cal/gm°C

specific heat of steam = specific heat of ice = $\frac{1}{2}$ specific heat of water

3. Heat capacity = ms Also, $W = ms$

where, $m \rightarrow$ mass

$s \rightarrow$ Specific heat.

$W \rightarrow$ water equivalent.

4. $Q = mL$

where, $Q \rightarrow$ heat

$L \rightarrow$ Latent energy

Latent heat of ice = 80 cal/gm

Latent heat of steam = 540 cal/gm.

5. Heat taken by one substance = Heat given by another substance
$m_1 s_1 (T_1 - T_m) = m_2 s_2 (T_m - T_2)$ → Law of mixture.
where, T_m → mixture's temperature.

6. Zeroth law of thermodynamics →
If objects A and B are separately in thermal equilibrium with a third object C, then objects A and B are in thermal equilibrium with each other.

7. $L = L_0 (1 + \alpha \Delta T)$
where, L_0 ⇒ original length
α ⇒ coefficient of linear expansion
L ⇒ length after heating the rod.

8.(i) If α varies with distance, $ax + b = \alpha$
then, total expansion $= \int (ax+b) \Delta T \cdot dx$.

(ii) If α varies with temperature, $\alpha = f(T)$
then, $\Delta L = \int \alpha L_0 \, dT$

9. Thermal strain $= \alpha \Delta T$

10. $T = T_0 \left(1 \pm \frac{\alpha \Delta \theta}{2}\right)$ where, θ → Temperature
T → new time period.
T_0 → original time period.

If $\theta < \theta_0$ and $T \ngtr T_0$ clock becomes fast and gain time.
If $\theta > \theta_0$ and $T \nless T_0$ clock becomes slow and loose time

11. $A = A_0(1+\beta\Delta T)$

where, $A \rightarrow$ area of plate after heating

$A_0 \rightarrow$ original area of plate

$\beta \rightarrow$ coefficient of superficial expansion.

12. $V = V_0(1+\gamma\Delta T)$

where, $V \rightarrow$ volume of body after heating

$V_0 \rightarrow$ original volume of body

$\gamma \rightarrow$ coefficient of volume expansion.

13. $\alpha : \beta : \gamma = 1:2:3$ or $\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$ → For isotropic solids.

$\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$ → For non-isotropic solids

where, α_1, α_2 and α_3 are coefficient of linear expansion in X, Y and Z direction.

14. $d = d_0(1-\gamma\Delta T)$

where, $d_0 \rightarrow$ original density

$d \rightarrow$ density after heating.

15. $h = h_0\{1+(\gamma_L - 2\alpha_s)\Delta T\}$

where, $h_0 \rightarrow$ original height of liquid in container

$h \rightarrow$ height of liquid in container after heating

$\gamma_L \rightarrow$ volume expansion of liquid

$\alpha_s \rightarrow$ linear coefficient of expansion of container.

16. $\frac{F_B'}{F_B} = \frac{1+\gamma_s\Delta\theta}{1+\gamma_L\Delta\theta}$

where, $\gamma_s \rightarrow$ coefficient of volume expansion of solid

$\gamma_L \rightarrow$ coefficient of volume expansion of liquid

17. For bimetallic strip,

$$R = \frac{d}{(\alpha_2-\alpha_1)(\theta_2-\theta_1)}$$

where, α_1 and α_2 is coefficient of linear expansion of both metallic strip.

$d \rightarrow$ width of metallic strip.

$R \rightarrow$ radius of arc.

18. The formula for the conversion of temperature from one scale to another :

$$\frac{K-273}{100} = \frac{C}{100} = \frac{F-32}{180} = \frac{R}{80}$$

where, $C \rightarrow$ celcius scale

$K \rightarrow$ Kelvin scale

$F \rightarrow$ Fahrenheit Scale

$R \rightarrow$ Reaumer scale.

19. $$T = (273.16\,K)\left(\lim_{gas \to 0} \frac{P}{P_3}\right)$$

where, $P \rightarrow$ Pressure at temperature being measured.

$P_3 \rightarrow$ Pressure when bulb in a triple point cell.

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Rigid Body Dynamics.

1. Moment of Inertia about an axis :

(a) Moment of inertial of an system of n particles

$$I = \sum_{i=1}^{n} m_i r_i^2$$

(b) For a continous system

$$I = \int r^2 . dm$$

(c) Moment of inertia of a large object.

$$I = \int dI_{element}$$

2. Two Important Theorems :

- Perpendicular Axis Theorem

$$I_z = I_x + I_y$$

* only applicable for 2-D objects.

- Parallel Axis Theorem

$$I_{AB} = I_{CM} + Md^2$$

where, d → distance between CM and AB.

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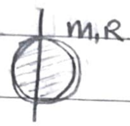
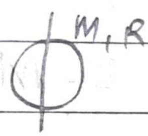
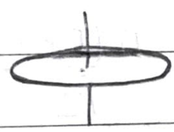
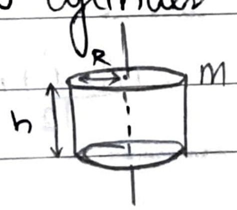
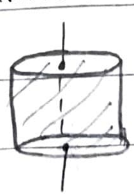
3. List of some useful formula.

	Object	Moment of Inertia.
(a)	Solid sphere. 	$\frac{2}{5} MR^2$ (uniform)
(b)	Hollow sphere. 	$\frac{2}{3} MR^2$ (Uniform)
(c)	Disc 	$\frac{MR^2}{2}$ (Uniform)
(d)	Ring 	MR^2 (Uniform or Non uniform)
(e)	Hollow cylinder 	MR^2 (Uniform or Non-uniform)
(F)	Solid cylinder 	$\frac{MR^2}{2}$ (Uniform)

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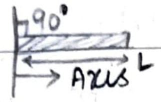
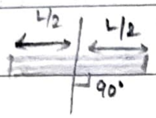
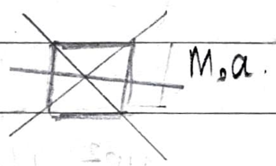
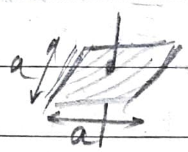
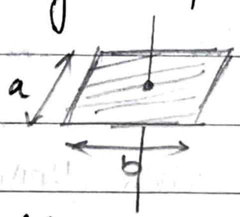
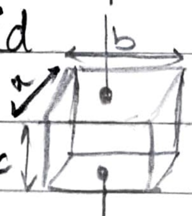
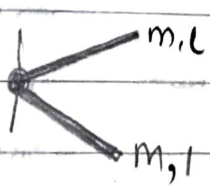
	Object	Moment of Inertia.
(g)	Rod.	
	(i) 	$\frac{ML^2}{3}$ (Uniform)
	(ii) 	$\frac{ML^2}{12}$ (Uniform)
(h)	Square Plate.	
	(i) 	$I = Ma^2/12$ (uniform)
	(ii) 	$Ma^2/6$ (uniform)
(i)	Rectangular plate. 	$\frac{M(a^2+b^2)}{12}$ (Uniform)
(j)	Cuboid 	$\frac{M(a^2+b^2)}{12}$ (Uniform)
(K)		$\frac{2ml^2}{3}$ (Uniform)

4. $K = \sqrt{\frac{I}{M}}$ where, $K \to$ Radius of gyration
$I \to$ Moment of inertia
$M \to$ Mass.

5. $\vec{\tau} = \vec{r} \times \vec{F} = r_{\perp} F = r F_{\perp}$

6. Torque due to couple $= F(2d)$

If net force acting on a system is zero, torque is same about any point.

7. $(\tau)_{external} = I\alpha$.

$K.E = \frac{I\omega^2}{2}$

$F_c = m\omega^2 r_{cm}$

$F_t = m\alpha r_{cm}$

8. $\vec{F}_{net} = 0$ and $\vec{\tau}_{net} = 0$ (about every point) for a system to be in mechanical equilibrium.

9. $L_H = I_H \omega$, where, $L \to$ angular momentum.

10. $\vec{\tau} = \frac{d\vec{L}}{dt}$

11. $\int \tau . dt = \Delta J$ $\to$ Impulse of momentum.

$\Delta J \to$ Change in angular momentum.

12. For pure rolling, $V = \omega r$ and $a = \alpha r$.

13. If a force is applied at com, then Normal shifts right so that torque of N can counterbalance torque of friction.

$\therefore F_{max} = f_r$

$N = mg$

$f_r . \frac{b}{2} = N . \frac{a}{2} \Rightarrow f_r = \frac{Na}{b} = \frac{mga}{b}$

$$F_{max} = \frac{mga}{b}.$$

14. IF surface is not sufficiently rough and the body slides before F is increased to $F_{max} = mga/b$ then body will slide before toppling. Once body starts sliding friction becomes constant and hence no toppling. This is the case if

$F_{max} > F_{limit}$

$mga/b > \mu mg$

$\mu < a/b.$

condition for toppling when $\mu \geq a/b$ in this case body will topple if $F > mga/b$.
but if $\mu < a/b$, body will not topple for any value of F applied at com.

Gravitation

1. $F = \frac{G m_1 m_2}{r^2}$ where, $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

2. $E = \frac{GM}{r^2}$ where, $E \rightarrow$ intensity of gravitational field

3. $V = \frac{-GM}{r}$ where, $V \rightarrow$ gravitational potential.

4. $E = -\frac{dV}{dr}$

5.

Object.	Gravitation Field	Gravitation Potential
• Ring	$-\frac{GMr}{(a^2+r^2)^{3/2}}\hat{r}$	$\frac{-GM}{(a^2+r^2)^{1/2}}$
• Thin circular disc	$\frac{2GM}{a^2}[1-\cos\theta]$	$-\frac{2GM}{a^2}(a^2+r^2)^{1/2}-r$
• Solid sphere		
(i) $r \leq a$	$\frac{GMr}{a^3}$	$\frac{-GM(3a^2-r^2)}{2a^3}$
(ii) $r \geq a$	$\frac{GM}{r^2}$	$-\frac{GM}{r}$
• Uniform thin spherical shell		
(i) $r \leq a$	0	$\frac{-GM}{a}$
(ii) $r \geq a$	$\frac{GM}{r^2}$	$-\frac{GM}{r}$

Object	Gravitation Field	Gravitation Potential
• Uniform thick spherical shell.		
(i) ~~$r \leq a$~~ Point outside the shell.	$\frac{-GM}{r^2}$	$\frac{-GM}{r}$
(ii) Point inside shell	0	$-\frac{3}{2} GM \left[\frac{R_2+R_1}{R_2^2+R_1R_2+R_1^2}\right]$
(iii) Point b/w 2 surfaces	$\frac{-GM}{r^2}\left[\frac{r^3-R_1^3}{R_2^3-R_1^3}\right]$	$\frac{-GM}{2r}\left(\frac{3rR_2^2-r^3-2R_1^3}{R_2^3-R_1^3}\right)$

6. $U = -\frac{GMm}{r}$

7. $W_{ext} = \Delta U = mgh$

* Valid only when $h << R_e$

8. Gravitation self energy of uniform sphere :

$$U_{sphere} = -\frac{3GM^2}{5R}$$

9. Acceleration variation due to gravity.

• Effect of altitude

$$g_h = g\left[1+\frac{h}{R_e}\right]^{-2}$$

when $h << R$,

$$\therefore g_h = \left(1-\frac{2h}{R}\right)g$$

- Effect of depth

$$g_d = g\left[1 - \frac{d}{R_e}\right]$$

- Effect of the rotation of the Earth

$$g_{pole} = g$$

$$g_{equator} = g\left[1 - \frac{R_e\omega^2}{g}\right]$$

$$\therefore g_{pole} > g_{equator}.$$

10. $V_e = \sqrt{\frac{2GM_e}{R}}$ where, $V_e \rightarrow$ escape speed.

$V_e = 11.2\, km/s$.

11. $V_o = \sqrt{\frac{GM_e}{r}}$ where, $V_o \rightarrow$ orbital velocity.

12. $K.E = \frac{GM_e m}{2r}$

$P.E = -\frac{GM_e m}{r}$

$T.E = -\frac{GM_e m}{2r}$

13. $T^2 = \frac{4\pi^2}{GM_e} \times r^3$

$r = 6.6 R_e$

$h = 5.6 R_e$

14. Areal velocity $= \frac{\text{area swept}}{\text{time}}$

KTG and Thermodynamics

1. Rate of change of momentum due to collision = $\frac{\text{change in momentum}}{\text{time taken}}$

2. $P = \frac{M}{3V}\langle v^2 \rangle = \frac{1}{3}\rho\langle v^2 \rangle$

where, $\langle v^2 \rangle \Rightarrow$ mean square speed of molecules
$M \rightarrow$ Total mass of gas.
$V \rightarrow$ volume of the container
$\rho \rightarrow$ density of gas.

3. Total translational K.E of gas $= \frac{1}{2}M\langle v^2 \rangle = \frac{3}{2}PV = \frac{3}{2}nRT$

4. $\langle V^2 \rangle = \frac{3P}{\rho} = \frac{3RT}{M_{mole}} = \frac{3KT}{m}$

$V_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_{mole}}} = \sqrt{\frac{3KT}{m}}$

5. $\langle l \rangle = \frac{1}{\sqrt{2}nd^2}$ where, $\langle l \rangle \rightarrow$ mean free path.
$n \rightarrow$ no. density ~~density of molecules~~
$d \rightarrow$ diameter of molecule

6. $V_{avg.} = \sqrt{\frac{8KT}{\pi m}}$

7. $V_p = \sqrt{\frac{2KT}{m}}$ where, $V_p \rightarrow$ most probable speed

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8. Dregree of freedom for various molecules.

- monoatomic :- $f = 3$
- ~~diam~~ diatomic :- $f = 5$

but if temp $< 70K$ then $f = 3$

if temp. in b/w $250K$ to $5000K$ then $f = 5$

if temp $> 5000K$ then $F = 7$.

9. $U = \frac{1}{2} FKT$ where, $f \Rightarrow$ degree of freedom

$U \rightarrow$ total kinetic energy of molecule

10. Internal energy for an ideal gas $(U) = \frac{FnRT}{2}$.

11. $W = \int P.dV \rightarrow$ i.e work done is area under P-V curve

12. Isothermal Process :-

$T = \text{constant}$ $\therefore PV = \text{constant}$

$W = nRT \ln\left(\frac{V_f}{V_i}\right)$

Internal energy $(\Delta U) = 0$

13. Iso-choric Process :-

$V = \text{constant}$ $\therefore \frac{P}{T} = \text{constant}$.

$W = 0$

Internal energy $(\Delta U) = \frac{FnR\Delta T}{2}$ = Heat given.

14. Isobaric Process :-

P = constant $\therefore \frac{V}{T}$ = constant.

$$\Delta W = P(V_f - V_i) = nR(T_f - T_i)$$

$$\Delta U = nC_v \Delta T$$

Heat : $\Delta Q = \Delta U + \Delta W$

$$\Delta Q = \frac{nf}{2}R\Delta T + P(V_f - V_i) = \frac{nf}{2}R\Delta T + nR\Delta T$$

15. Cyclic Process :

$\Delta U = 0$

$\therefore \Delta Q = \Delta W$.

16. $dQ = dU + PdV$

17. $\eta = 1 - \frac{Q_2}{Q_1}$ → Heat Engine

$\eta = 1 - \frac{T_2}{T_1}$ → Carnot Cycle.

18. $C_v = \left(\frac{\Delta Q}{n\Delta T}\right)_{\text{constant volume}} = \frac{f}{2}R.$

19. $C_p = \left(\frac{\Delta Q}{n\Delta T}\right)_{\text{constant pressure}} = \left(\frac{f}{2} + 1\right)R$

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20. Molar heat capacity of ideal gas in terms of R.

(i) For monoatomic gas, $f=3$

$C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$ and $\gamma = \frac{C_P}{C_V} = \frac{5}{3}$

(ii) For diatomic gas, $f=5$

$C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$ and $\gamma = \frac{C_P}{C_V} = \frac{7}{5}$

(iii) For triatomic gas, $f=6$

$C_V = 3R$, $C_P = 4R$ and $\gamma = \frac{C_P}{C_V} = \frac{4}{3}$

In general, if f is the degree of freedom then

$C_V = \frac{f}{2}R$, $C_P = \left(\frac{f}{2}+1\right)R$ and $\gamma = \left(1+\frac{2}{f}\right)$

21. $C_P - C_V = R$ → For ideal gases only.

22. Adiabatic Process :-

PV^{γ} = constant → Poisson Law

$T^{\gamma}P^{1-\gamma}$ = constant

$TV^{\gamma-1}$ = constant.

$$\Delta W = \frac{nR(T_i - T_f)}{\gamma - 1}$$

23. For free expansion; $\Delta Q = 0$, $\Delta U = 0$ and $\Delta W = 0$.

T = Constant.

24. For ~~non~~ mixture of non-reacting gases.

- Molecular weight $= \dfrac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$

- Specific heat: $C_v = \dfrac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$

$$C_P = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2}.$$

- $\gamma = \dfrac{n_1 C_{P_1} + n_2 C_{P_2} + \dots}{n_1 C_{V_1} + n_2 C_{V_2} + \dots}$

25. $W_{isothermal} > W_{adiabatic}$. → expansion to same volume.

$|W_{adiabatic}| > |W_{isothermal}|$ → compression to same volume.

Electrostatics

1. $F = \frac{kq_1q_2}{r^2}$ where, $K = \frac{1}{4\pi\varepsilon_0}$

2. $E = \frac{F}{q_0}$ where, $E \rightarrow$ Electric field intensity

3. $F = qE$

4. $V_P = \frac{(W_P)_{ext}}{q}$

where, $V_P \rightarrow$ Electric potential
$W_P \rightarrow$ work required to move a point from infinity to point P.

5. Formula for E and V of various objects.

- Point charge.

$E = \frac{Kq}{r^2}$ and $V = \frac{Kq}{r}$

- Infinitely long line charge.

$V =$ not defined , $V_B - V_A = 2K\lambda \ln\left(\frac{r_B}{r_A}\right)$

$E = \frac{2k\lambda}{r}$

- Infinite non conducting thin sheet.

$E = \frac{\sigma}{2\varepsilon_0}\hat{n}$

$V =$ not defined , $V_B - V_A = \frac{-\sigma}{2\varepsilon_0}(r_B - r_A)$

- Uniformly charged ring

$E_{axis} = \frac{kQx}{(R^2+x^2)^{3/2}}$ $E_{centre} = 0.$

$V_{axis} = \frac{kQ}{\sqrt{R^2+x^2}}$ $V_{centre} = \frac{kQ}{R}.$

- Infinitely charged conducting sheet.

$E = \frac{\sigma}{\varepsilon_0}\hat{n}$

$V =$ not defined , $V_B - V_A = -\frac{\sigma}{\varepsilon_0}(r_B - r_A)$

- Uniformly charged sphere (valid for all except non-conducting solid sphere)

* If $r \geq R$, then

$E = \frac{kQ}{r^2}$ and $V = \frac{kQ}{r}$

* If $r \leq R$, then

$E = 0$ and $V = \frac{kQ}{R}$

- Uniformly charged solid non-conducting sphere.

* If $r \geq R$, then

$E = \frac{kQ}{r^2}$ and $V = \frac{kQ}{r}$

* If $r \leq R$, then

$\vec{E} = \frac{kQ\vec{r}}{R^3} = \frac{\rho\vec{r}}{3\varepsilon_0}$ and $V = \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$

- Thin uniformly charged disc

$E_{axis} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$ and $V_{axis} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+x^2} - x\right]$

6. $W_{AB} = q(V_B - V_A)$

where, $W_{AB} \Rightarrow$ work done by external agent in taking charge A to B.

7. $U = qV$

where, $U \rightarrow$ electrostatic potential energy of point charg

8. $U = $ P.E of system $= \frac{U_1 + U_2 + U_3 + \cdots}{2}$.

9. Energy density $= \frac{1}{2}\varepsilon_0 E^2$

10. Self Energy of uniformly charged shell $(U_{self}) = \frac{KQ^2}{2R}$

Self energy of uniformly charged solid non-conducting sphere $(U_{self}) = \frac{3KQ^2}{5R}$

11. Electric Field Intensity due to dipole

- on the axis

$$\vec{E} = \frac{2K\vec{P}}{r^3}$$ where, dipole moment is P

- on equatorial ~~axis~~ position

$$\vec{E} = \frac{-K\vec{P}}{r^3}$$

- total electric field intensity at general point $O(r, \theta)$ is

$$E = \frac{KP}{r^3}\sqrt{1 + 3\cos^2\theta}$$

12. $U = -\vec{P}\cdot\vec{E}$

where, $U \rightarrow$ Potential Energy of dipole in external electric field.

13. Torque $= \vec{P}\times\vec{E}$ $\rightarrow$ For uniform electric field.

$\vec{F} = 0$

14. Torque $= \vec{P}\times\vec{E}$ $\rightarrow$ For non-uniform field.

$|F| = \left|P\frac{\partial E}{\partial r}\right|$

15. Electric potential due to dipole at general point (r,θ)

$$V = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{P}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

16. $\phi_E = \int \vec{E}\cdot d\vec{s}$ $\rightarrow$ Electric flux over the whole area.

$\phi_E = \frac{q_{in}}{\varepsilon_0}$ $\rightarrow$ Gauss Law.

17. Electric field near the conducting surface $= \frac{\sigma}{\varepsilon_0}\hat{n}$

18. $P = \frac{\sigma^2}{2\varepsilon_0}$ where, $P \rightarrow$ Electric Pressure

$\sigma \rightarrow$ surface charge density.

19. Potential difference between points A and B

$$V_B - V_A = \int_A^B \vec{E}\cdot d\vec{r}$$

$$\vec{E} = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V = -\text{grad } V.$$

Current Electricity

1. $I = \frac{q}{t} = \frac{ne}{t}$

where, $I \rightarrow$ current
$q \rightarrow$ charge
$t \rightarrow$ time
$e \rightarrow$ charge of electron
$n \rightarrow$ no. of electron per sec

2. $i = nqVA$

where, $i \rightarrow$ charge flow per unit volume
$n \rightarrow$ no. of free charge per unit volume
$v \rightarrow$ velocity
$q \rightarrow$ charge of each free particle.

3. $\vec{J} = \frac{di}{ds}\vec{n}$ where, $J \rightarrow$ current density
$\vec{n} \rightarrow$ direction vector

4. $I = nAeV_d$ where, $V_d \rightarrow$ drift velocity.

$V_d = \frac{\lambda}{\tau}$ where, $\lambda \rightarrow$ mean free path.
$\tau \rightarrow$ relaxation time.

5. $\rho = \frac{R \cdot A}{l} = \frac{2m}{ne^2\tau} = \frac{1}{\sigma}$ where, $\sigma \rightarrow$ conductivity
$\rho \rightarrow$ resistivity.

6. $\frac{R_2}{R_1} = \frac{l_2}{l_1} \times \frac{A_2}{A_1}$

7. For series combination of resistors

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

For parallel combination of resisters

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

8. $EQ = W$

where, $E \rightarrow$ Electromotive force

$Q \rightarrow$ Charge

$W \rightarrow$ work done

9. $I = \dfrac{E}{R+r}$ where, $r \longrightarrow$ Internal resistance.

$V = E - Ir.$

10. For series grouping of cell.

$I = \dfrac{nE}{R+nR}$ where, $n \rightarrow$ no. of identical cells.

For parallel grouping of cell

$$I = \frac{nE}{nR+r}$$

For mixed grouping of cell.

$$I = \frac{mnE}{(\sqrt{nR}-\sqrt{mR})^2 + 2\sqrt{nmrR}}$$

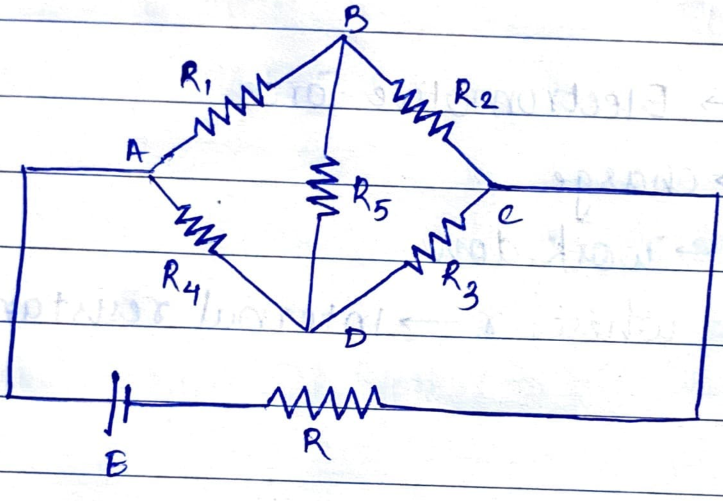
I will be max when $R = \dfrac{nR}{m}$

$$\therefore I_{max} = \frac{e\sqrt{N}}{2\sqrt{rR}}.$$

11. For maximum power transfer,
$R = X$.

12. $\Sigma i = 0 \longrightarrow$ Kirchoff's current law
$\Sigma iR - \Sigma emf = 0 \rightarrow$ Kirchoff's voltage law.

13.



If current through $R_5 = 0$ then $R_1 R_3 = R_2 R_4$

14. Ammeter :

$$S = \frac{I_G R_G}{I - I_G} = \frac{I_G R_G}{I} \quad \because I_m >> I_G$$

where, I is the maximum current
R_G is resistance of ammeter
S is sensitivity
I_G is current through ammeter.

15. Voltmeter :

For maximum potential difference,

$R_S = \frac{V}{I_G} - R_G$ where, R_S is shunt resistor.

If $R_G <<<< R_S$ then $R_S \approx \frac{V}{I_G}$.

16. $X = \frac{Rl}{100 - l}$

where, $X \rightarrow$ unknown resistor

$l \rightarrow$ balancing length.

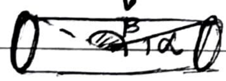
17. Potentiometer :

- Comparision of emf :- $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$

- Internal resistance of cell :- $r = \left(\frac{l_2}{l_1} - 1\right) R$

Magnetic Effect of Current and magnetic force on charge or current.

1. Magnetic field due to various objects.

Object	Magnetic Field.
• Point charge	$\frac{\mu_0}{4\pi} \times \frac{q(\vec{v} \times \vec{r})}{r^3}$
• Straight wire	$\frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$
• Infinite straight wire	$\frac{\mu_0 I}{2\pi r}$
• Circular loop.	
(i) At centre	$\frac{\mu_0 N I}{2r}$
(ii) At axis	$\frac{\mu_0}{2} \times \frac{N I R^2}{(R^2+x^2)^{3/2}}$
• Axis of solenoid 	$\frac{\mu_0 n I}{2}(\cos\alpha + \cos\beta)$
• Long cylindrical shell	
(i) $r < R$	0
(ii) $r \geq R$	$\frac{\mu_0 I}{2\pi r}$

2. $d\vec{B} = \frac{\mu_0 I}{4\pi} \times \frac{(\vec{dl} \times \vec{r})}{r^3}$ → Biot Savart's Law.

3. $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$ → Ampere's Law

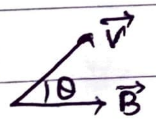
4. Magnetic force acting on a moving point charge.

- $\vec{F} = q(\vec{v} \times \vec{B})$

(i) $\vec{v} \perp \vec{B}$

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi m}{qB}$$

(ii) 

$$r = \frac{mv\sin\theta}{qB}$$

$$T = \frac{2\pi m}{qB}$$

$$\text{Pitch} = \frac{2\pi mv\cos\theta}{qB}$$

- $\vec{F} = q[(\vec{v} \times \vec{B}) + \vec{E}]$

5. Magnetic force acting on current carrying wire

$\vec{F} = i\vec{L} \times \vec{B}$

6. Magnetic moment on current carrying loop

$M = NIA.$

7. Torque acting on loop.

$\vec{\tau} = \vec{M} \times \vec{B}$

8. Magnetic field due to single pole.

$$\vec{B} = \frac{\mu_0 m}{4\pi r^2}$$

9. Magnetic field on axis of magnet.

$$B = \frac{\mu_0 2M}{4\pi r^3}$$

10. Magnetic field on equatorial axis of the magnet

$$B = \frac{\mu_0 M}{4\pi r^3}$$

11. Magnetic field at point P

$$B = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

Capacitance

1. $q = CV$ where, $q \rightarrow$ charge on conductor
$C \rightarrow$ capacitance of conductor
$V \rightarrow$ potential of conductor.

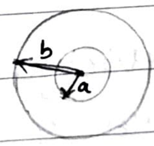
2. $W = U = \frac{q^2}{2C} = \frac{CV^2}{2} = \frac{qV}{2}$ where, $W \rightarrow$ work done
$U \rightarrow$ self energy.

3. $\frac{dU}{dV} = \frac{1}{2} \varepsilon_r \varepsilon_0 E^2$ where, $E \rightarrow$ Electric field
$\frac{dU}{dV} \rightarrow$ Energy density in a medium
$\varepsilon_r \rightarrow$ Relative permittivity of medium

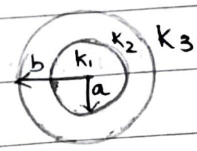
For vacuum, $\frac{dU}{dV} = \frac{1}{2} \varepsilon_0 E^2$

4. For parallel plate capacitor,
$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{K\varepsilon_0 A}{d}$ where, $A \rightarrow$ Area of plates
$d \rightarrow$ distance between plates.

5. For spherical capacitor,
- Capacitance of an isolated spherical conductor.
$C = 4\pi\varepsilon_0\varepsilon_r R$ where, $R \rightarrow$ Radius of spherical conductor
- Capacitance of spherical capacitor
$C = 4\pi\varepsilon_0 \frac{ab}{(b-a)}$



$C = \frac{4\pi\varepsilon_0 K_2 ab}{(b-a)}$



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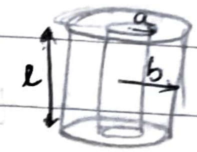
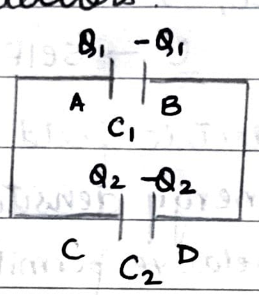
- Cylinderical capacitor $(l >> \{a, b\})$

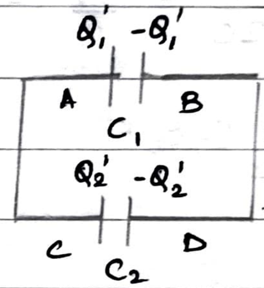
capacitance per unit length $= \dfrac{2\pi\varepsilon_0}{\ln(b/a)}$ F/m



6. Distribution of charges on connecting two charged capacitors



Initially



Finally.

Common potential (V) $= \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$$Q_1' = \frac{C_1}{C_1 + C_2}(Q_1 + Q_2)$$

$$Q_2' = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$$

$$\Delta H = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

7. Electric field intensity b/w plates of capacitor (air filled) $(E) = \dfrac{\sigma}{\varepsilon_0} = \dfrac{V}{d}$

8. Force experienced by plate of capacitor $(F) = \dfrac{q^2}{2A\varepsilon_0}$

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9. Combination of Capacitors :

- Series combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

- Parallel combination

$$C_{eq} = C_1 + C_2 + \cdots$$

10. Charging of capacitor

$$q = q_0 [1 - e^{-(t/RC)}]$$

$$V = V_0 [1 - e^{-(t/RC)}]$$

$$I = I_0 e^{-(t/RC)}$$

$$\text{Heat } (H) = \frac{V_0^2 C}{2}$$

11. Discharging of capacitor

$$q = q_0 e^{-(t/RC)}$$

$$V = V_0 e^{-(t/RC)}$$

$$I = -I_0 e^{-(t/RC)}$$

12. $C = \frac{K\varepsilon_0 A}{d} = KC_0$ where, $C_0 \rightarrow$ capacitance in absence of dielectric.

13. $E_{in} = E - E_{induced} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_b}{\varepsilon_0} = \frac{\sigma}{K\varepsilon_0} = \frac{V}{d}$

$E \rightarrow$ Electric Field in absence of dielectric.

$E_{induced} \rightarrow$ Induced charge density.

$$\sigma_b = \sigma\left(1 - \frac{1}{K}\right)$$

_/_/_

14. Force on dielectric

- When battery is connected

$$F = \frac{\varepsilon_0 b (K-1) V^2}{2d}$$

- When battery is not connected

$$F = \frac{Q^2}{2C^2} \cdot \frac{dC}{dx}$$

$x \rightarrow$ displacement.

Force on the dielectric will be zero when the dielectric is fully inside

Alternating Current

1. $F_{rms} = \sqrt{\dfrac{\int_{t_1}^{t_2} F^2 \, dt}{t_2 - t_1}}$

2. Average power consumed in a cycle $= \dfrac{\int_0^{2\pi/\omega} P \, dt}{\frac{2\pi}{\omega}} = \dfrac{V_m I_m \cos\phi}{2}$

Here, $\cos\phi \rightarrow$ power factor.

$I\sin\phi$ is called wattless current.

3. $\cos\phi = \dfrac{P_{ac}}{E_{rms} I_{rms}} = \dfrac{P_{av}}{P_v} = \dfrac{R}{Z}$

Here, $P_{av} \rightarrow$ average power.

$P_v \rightarrow$ virtual power

$R \rightarrow$ Resistance.

$Z \rightarrow$ Impedance.

4. $P_{av} = \dfrac{E_0 I_0}{2} = \dfrac{E_0^2}{2R} = E_{rms} I_{rms}$

5. $\tan\phi = \dfrac{X}{R} = \dfrac{(\omega L - \frac{1}{\omega C})}{R}$

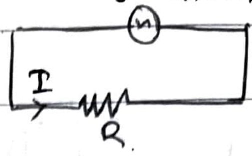
$\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\left[R^2 + (\omega L - \frac{1}{\omega C})^2\right]^{1/2}}$

6. $X_L = \omega L$ & $X_C = 1/\omega C$

where, $X_L \rightarrow$ inductive reactance

$X_C \rightarrow$ capacitive reactance

$V_s = V_m \sin\omega t$



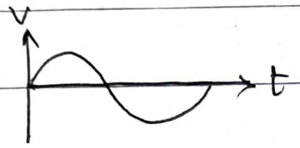
7. Purely resistive circuit

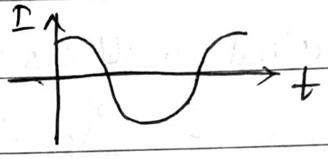
$I = I_m \sin\omega t = \frac{V_m}{R} \sin\omega t$

$\langle P \rangle = V_{rms} I_{rms} \cos\phi = \frac{V_{rms}^2}{R}$

8. Purely Capacitive Circuit

$I = \frac{V_m}{1/\omega C} \cos\omega t = \frac{V_m}{X_C} \cos\omega t = I_m \cos\omega t$

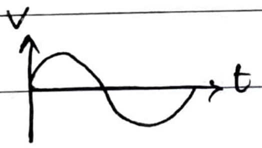


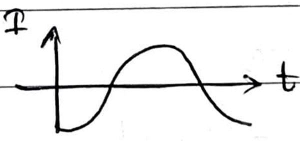


I_c leads V_c by $\pi/2$.

9. Purely inductive circuit

$I = \frac{V_m}{\omega L} \cos\omega t = \frac{V_m}{X_L} \cos\omega t = I_m \cos\omega t.$





I_L lags V_L by $\pi/2$.

10. RC series circuit

$V_m = \sqrt{(I_m R)^2 + (I_m X_c)^2 + 2(I_m)^2 R X_c \cos\frac{\pi}{2}}$

$I_m = \frac{V_m}{\sqrt{R^2 + X_c^2}}$

11. LR series circuit.

$V = I\sqrt{R^2+(X_L)^2}$

i.e $I = \frac{V}{\sqrt{R^2+X_L^2}}$

12. LC series circuit

$V = I|(X_L - X_C)|$ $\phi = 90°$

13. RLC series circuit

$I = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}}$

$\tan\phi = \frac{X_L - X_C}{R}$

14. Admittance $(Y) = \frac{1}{\text{Impedance}(Z)}$

15. Susceptance $(S) = \frac{1}{\text{Reactance}(X)}$

16. Conductance $(G) = \frac{1}{\text{Resistance}(R)}$

17. $P = \frac{P_{max}}{2}$

$I = \frac{I_{max}}{\sqrt{2}}$

Bandwidth $(\Delta f) = f_2 - F_1$

For series resonant $(\Delta f) = \frac{1}{2\pi}\left(\frac{R}{L}\right)$

$$Q = \frac{\text{Resonant Frequency}}{\text{Bandwidth}}$$

Here, $Q \rightarrow$ Quality factor.

18. Form factor = $\frac{\text{rms value of ac}}{\text{Avg. value of positive half cycle}}$

19. Transformer :

$$E_p I_p = E_s I_s \Rightarrow \text{Input power} = \text{Output power}$$

$$\frac{I_p}{I_s} = \frac{E_s}{E_p} = \frac{n_s}{n_p} = \sqrt{\frac{Z_s}{Z_p}}$$

Efficiency of transformer = $\frac{\text{Power obtained from secondary coil}}{\text{Power obtained from primary coil}} \times 100$

Electromagnetic Waves.

1. Maxwell's Equation

$\oint E.dA = Q/\varepsilon_0$ → Gauss's Law for electricity.

$\oint B.dA = 0$ → Gauss's Law for magnetism

$\oint E.dl = \frac{-d\phi_B}{dt}$ → Faraday's Law

$\oint B.dl = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$ → Ampere-Maxwell Law.

2. Oscillating electric and magnetic fields

$E = E_0 \sin(kz-\omega t) = E_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right]$

$E_0/B_0 = c$

$c = 1/\sqrt{\mu_0 \varepsilon_0}$ where, c → speed of light invacuum

$v = 1/\sqrt{\mu\varepsilon}$ v → speed of light in medium.

3. $p = \frac{U}{c}$ where, p → Total momentum

U → Total energy.

Electromagnetic Induction

1. $\phi = \int \vec{B} \cdot d\vec{A}$ where, $\vec{B} \rightarrow$ magnetic Field
$\vec{A} \rightarrow$ plane surface

Net magnetic Flux coming out of a closed surface is zero.

2. Flux density $= \frac{\phi}{A}$

3. $E = -\frac{d\phi}{dt}$ where, $E \rightarrow$ induced e.m.f

4. Lenz's Law :

According to this law, emf will be induced in such a way that it will oppose the cause which has produced it.

5. $I = -\frac{N}{R}\left(\frac{d\phi}{dt}\right)$ where, $N \rightarrow$ no. of turns.

6. ~~Flux through~~ e.m.f of rotating rod $= Bvl\sin\theta$

7. Induced emf due to rotation

- Rotation of rod.

$$\mathcal{E} = \frac{1}{2} B\omega l^2$$

- Rotating disc

$$\mathcal{E} = \frac{B\omega r^2}{2}$$

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- Rotation of rectangular coil in uniform magnetic Field.

8. $E = \frac{r}{2}\frac{dB}{dt}$

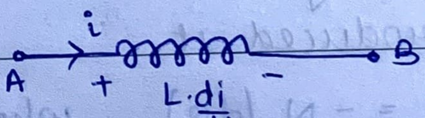
9. $\varepsilon = -L\frac{dI}{dt}$ where, $L \rightarrow$ self inductance. $\varepsilon \rightarrow$ instantaneous emf.

10. Solenoid :

$\phi = \mu_0 n i A$

$L = \mu_0 n^2 V$ where, $V \rightarrow$ volume.

11. Inductor :



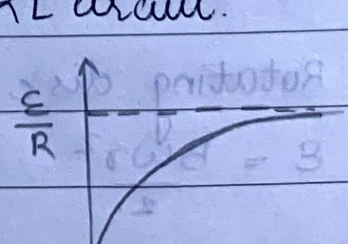
$V_A - L\frac{di}{dt} = V_B$

Energy density $\left(\frac{dU}{dV}\right) = \frac{B^2}{2\mu}$

$U = \frac{1}{2}LI^2$

12. Growth of current in series RL circuit.

$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$



$L_1 i_1 = L_2 i_2$

13. Decay of current in RL circuit

$$i = i_0 e^{-Rt/L}$$

14. Combination of Inductor.

- Series

$$L_{eq} = L_1 + L_2 + \dots$$

- Parallel

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots$$

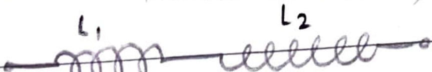
Above Formula is derived by neglecting mutual inductance.

15. Mutual Inductance.

(Coils wound in same direction)

$L_{eq} = L_1 + L_2 + 2M$

where, M → mutual inductance.



(coils wound in opp. direction)

$L_{eq} = L_1 + L_2 - 2M$.

$M = K\sqrt{L_1 L_2}$ where, K → coupling constant of coils.

16. Generator :

Frequency of AC by generator of multipoles $= \dfrac{\text{number of poles} \times \text{rotational frequency}}{2}$.

Efficiency $= \dfrac{\text{Electrical power generated}}{\text{Mechanical energy given}}$.

17. Motor :

Back emf $(e) = K\omega$ where, $K \rightarrow$ constant

$\omega \rightarrow$ angular velocity.

$$i = \frac{E - k\omega}{R}$$

$$\eta = \frac{\text{Back emf}}{\text{Applied emf}} \times 100$$ where, $E \rightarrow$ Applied emf

$\eta \rightarrow$ efficiency

Power of motor = ie

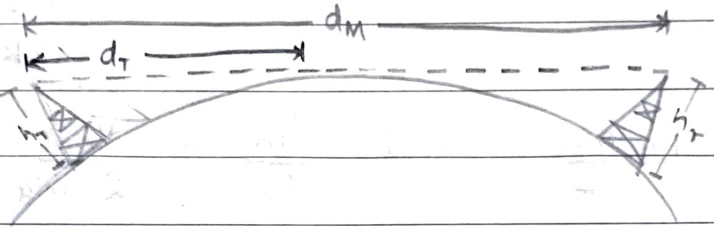
18. L-C oscillation.

$$\omega = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$f = \frac{1}{2\pi\sqrt{LC}} \text{ Hz.}$$

Principle of Communication.

1. Transmission from tower of height h.



the distance to the horizon $d_T = \sqrt{2Rh_T}$

$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

2. Amplitude Modulation

- The modulated signal $C_m(t)$ can be written as

$$C_m(t) = A_c \sin \omega_c t + \frac{\mu A_c}{2} \cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2} \cos(\omega_c + \omega_m)$$

- Modulation index $(m_a) = \frac{k(A_{max} - A_{min})}{A_{max} - A_{min}}$

- Total modulated index $(m_T) = \sqrt{m_1^2 + m_2^2 + m_3^2 + \cdots}$

- Side band frequencies

  $F_c + F_m \rightarrow$ Upper side band frequencies (USB)

  $F_c - F_m \rightarrow$ Lower side band frequencies (LSB)

- Band width = USB − LSB = $2F_m$

- Power in AM waves $(P) = \frac{V_{rms}^2}{R}$

carrier power $(P_c) = \frac{A_c^2}{2R}$

Total power of side bands (P_{sb}) = $\frac{m_a^2 A_c^2}{4R}$.

Total power of AM wave (P_{Total}) = $P_c + P_{sb}$

$$= \frac{A_c^2}{2R}\left[1 + \frac{m_a^2}{2}\right]$$

$$\frac{P_t}{P_c} = 1 + \frac{m_a^2}{2} \quad \text{and} \quad \frac{P_{sb}}{P_t} = \frac{m_a^2}{2 + m_a^2}$$

- Maximum power in AM (without distortion) will occur when $m_a = 1$ i.e $P_t = 1.5P = 3P_{sb}$
- $\frac{P_t}{P_c} = \frac{I_t^2}{I_c^2}$ i.e $\frac{I_t}{I_c} = \sqrt{1 + \frac{m_a^2}{2}}$

where, $I_t \rightarrow$ total or modulated current

$I_c \rightarrow$ Unmodulated current.

3. Frequency Modulation

- Frequency deviation (δ) = $f_{max} - f_c = f_c - f_{min} = \frac{K_f E_m}{2\pi}$
- Carrier swing (CS) = $2(\Delta F)$
- Frequency modulation index (m_f) = $\frac{\delta}{f_m} = \frac{f_{max} - f_c}{f_m} = \frac{K_f E_m}{2\pi f_m}$
- Frequency spectrum = FM side band modulated signal consists of infinite no. of sides bands whose frequencies are $(f_c \pm f_m), (f_c \pm 2f_m), \ldots$
- Deviation ratio = $(\Delta F)_{max} / (F_m)_{max}$
- % modulation (m) = $(\Delta F)_{actual} / (\Delta F)_{max}$.

Solid and Semiconductor Devices

1. resistivity of metals ⇒ 10^{-2} to 10^{-8} Ωm
   resistivity of semiconductors ⇒ 10^{-5} to 10^{6} Ωm
   resistivity of insulators ⇒ 10^{11} to 10^{19} Ωm
2. $i = i_e + i_h$ where, $i_e \rightarrow$ electron current
   $i_h \rightarrow$ hole current
3. $n_e n_h = n_i^2$ where, $n_i \rightarrow$ intrinsic concentration.
   $n_e \rightarrow$ electron density in conduction band
   $n_h \rightarrow$ hole density in valence band.
4. $\sigma = e(n_e \mu_e + n_h \mu_h)$ where, $\mu_e \rightarrow$ mobility of electrons
   $\mu_h \rightarrow$ mobility of holes
   $\sigma \rightarrow$ electrical conductivity.
5. $N = N_0 e^{-\Delta E/2KT}$ where, $T \rightarrow$ Temperature
   $\Delta E \rightarrow$ band gap
   $N \rightarrow$ no. of charge carriers ~~at~~ at T
   $N_0 \rightarrow$ total no. of charge carriers.
6. $R_d = \dfrac{\Delta V}{\Delta I}$ where, $R_d \rightarrow$ dyanamic resistance of P-N junction in forward biasing.
7. CB amplifier :

- $\alpha_{ac} = \dfrac{\Delta i_c}{\Delta i_e}$ where, $\Delta i_c \rightarrow$ small change in collector current
  $\Delta i_e \rightarrow$ small change in emitter current
  $\alpha_{ac} \rightarrow$ ac current gain.
- $\alpha_{dc} = \dfrac{i_c}{i_e}$ where, $i_c \rightarrow$ collector current
  $i_e \rightarrow$ emitter current
  $\alpha_{dc} \rightarrow$ dc current gain

- A_v (voltage gain) = $\frac{\text{Change in output voltage } (\Delta V_o)}{\text{Change in input voltage } (\Delta V_i)}$

$A_v = \alpha_{ac} \times$ Resistance gain

- Power gain = $\frac{\text{Change in output power } (\Delta P_o)}{\text{Change in input power } (\Delta P_i)}$

Power gain = $\alpha_{ac}^2 \times$ Resistance gain.

- Both in input and output are in same phase.

8. CE amplifier :

- ac current gain $(\beta_{ac}) = \left(\frac{\Delta i_c}{\Delta i_b}\right) V_{CE}$

- dc current gain $(\beta_{dc}) = \frac{i_c}{i_b}$

- voltage gain $(A_v) = \frac{\Delta V_o}{\Delta V_i} = \beta_{ac} \times$ Resistance gain

- Power gain $= \frac{\Delta P_o}{\Delta P_i} = \beta_{ac}^2 \times$ Resistance gain

- $g_m = \frac{\Delta i_c}{\Delta V_{EB}} = \frac{A_v}{R_L}$ where, $g_m \rightarrow$ trans conductance.

$V_{EB} \rightarrow$ emitter base voltage

$R_L \rightarrow$ Load resistance.

- Both input and output are in opposite phase.

9. Relation between α and β

- $\beta = \frac{\alpha}{1-\alpha}$

- $\alpha = \frac{\beta}{1+\beta}$

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Nuclear Physics.

1. $\Delta m = [Zm_p + (A-Z)m_n] - [M_{atom} - Zm_e]$

$\Delta m \rightarrow$ mass defect.

2. Binding energy $(B.E) = (\Delta m)c^2 = (\Delta m) \times 931$ MeV

3. Binding energy per nucleon $(B.E_n) = \frac{B.E}{A}$

where, $A \rightarrow$ Atomic mass number.

4. Q value $= [M_x - M_y - M_{He}]c^2$

where, Q value $\rightarrow$ energy released during decay process.

$M_x \rightarrow$ mass of atom ${}_ZX^A$

$M_y \rightarrow$ mass of atom ${}_{Z-2}Y^{A-4}$

$M_{He} \rightarrow$ mass of atom ${}_2He^4$

5. α - Decay :

- Theoritical

$T_\alpha = \frac{(A-4)Q}{A}$ and $T_y = \frac{4Q}{A}$

where, $T \rightarrow$ Kinetic Energy

- Experimental

$T_\alpha = \frac{m_y(Q-E)}{m_\alpha + m_y}$ and $T_y = \frac{m_\alpha(Q-E)}{m_\alpha + m_y}$

where, $E \rightarrow$ energy emitted by γ particles.

6. $r = \frac{mv}{qB} = \frac{mv}{2eB} = \frac{\sqrt{2KM}}{2eB}$

where, $v \rightarrow$ velocity

$B \rightarrow$ magnetic field

7. $N = N_0 e^{-\lambda t}$

where, $N_0 \rightarrow$ number of active nuclei at $t=0$

$N \rightarrow$ number of active nuclei at $t=t$

$N' = N_0 - N$ where, $N' \rightarrow$ number of nuclei formed of other atom.

$\therefore N' = N_0(1 - e^{-\lambda t})$

8. $t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$ where, $t_{1/2} \rightarrow$ half life

$\lambda \rightarrow$ decay constant

nuclei present after n half lives i.e after a time $t = n t_{1/2}$

no. of nuclei $= \frac{N_0}{2^n}$

9. Activity :

- $A = \lambda N = \lambda N_0 e^{-\lambda t}$
- 1 curie $= 3.7 \times 10^{10}$ dps.
- Activity after n half lives $= \frac{A_0}{2^n}$

10. $T_{avg} = \frac{1}{\lambda}$

11. $\lambda = \lambda_1 + \lambda_2 + \ldots$

$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} + \ldots$

Modern Physics.

1. $E = h\nu = \frac{hc}{\lambda} = mc^2$ where, $E \rightarrow$ Energy of photon
$\nu \rightarrow$ frequency
$\lambda \rightarrow$ wavelength
$h \rightarrow$ Planck's constant.

2. $m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c}$ where, $m \rightarrow$ effective mass of photon

3. $p = mc = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$ where, $p \rightarrow$ momentum of photon

4. $n = \frac{E}{h\nu} = \frac{E\lambda}{hc}$ where, $n \rightarrow$ no. of photons.

5. Intensity $(I_p) = nh\nu = \frac{NP}{4\pi r^2}$ where, $N \rightarrow$ no. of photons
$P \rightarrow$ Power of source.

For line source, $I_P = \frac{NP}{2\pi r l}$

6. $V_s = \frac{h(\nu - \nu_0)}{e}$ where, $V_s \rightarrow$ stopping potential

7. $\nu_{min} = \nu_{th}$ where, $\nu_{th} \rightarrow$ threshold frequency
$\lambda_{max} = \lambda_{th}$ $\lambda_{th} \rightarrow$ threshold wavelength.

8. $K_{max} = h(\nu - \nu_{th})$

9. $I = \frac{1}{2} \varepsilon_0 E_0^2 c$ where, $E_0 \rightarrow$ maximum electric field
$I \rightarrow$ Intensity.

10. $h\nu = W + K_{max}$ where, $W \rightarrow$ work function.

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11. Force due to radiation :

- When light is incident perpendicularly.

(a) $a = 1, r = 0$

$F = \frac{IA}{c}$ and Pressure $= \frac{I}{c}$

(b) $r = 1$ and $a = 0$

$F = \frac{2IA}{c}$ and Pressure $= \frac{2I}{c}$

(c) when $0 < r < 1$ and $a + r = 1$

$F = \frac{IA}{c}(1+r)$ and Pressure $= \frac{I}{c}(1+r)$

- When light is incident at ~~ano~~ angle θ.

(a) $a = 1, r = 0$

$F = \frac{IA\cos\theta}{c}$ and $P = \frac{F\cos\theta}{A} = \frac{I}{c}\cos^2\theta$

(b) $r = 1, a = 0$

$F = \frac{2IA\cos^2\theta}{c}$ and $P = \frac{2I\cos^2\theta}{c}$

(c) $0 < r < 1$, $a + r = 1$

$P = \frac{I\cos^2\theta}{c}(1+r)$

12. Debroglie wavelength

$\lambda = \frac{h}{mv} = \frac{h}{P} = \frac{h}{\sqrt{2mKE}}$

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13. $r_n = 0.529 \frac{n^2}{Z}$

14. $V_n = 2.19 \times 10^{16} \frac{Z}{n}$

15. $E_n = -13.6 \frac{Z^2}{n^2}$

16. $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For lymann series $n_1 = 1$ $n_2 = 2, 3, 4, \ldots$

Balmer series $n_1 = 2$ $n_2 = 3, 4, 5, \ldots$

Paschen series $n_1 = 3$ $n_2 = 4, 5, 6, \ldots$

17. Total number of possible transitions $= \frac{n(n-1)}{2}$

where, $n \rightarrow$ no. of state.

18. IF effect of nucleus motion is considered,

$$r_n = 0.529 \frac{mn^2}{\mu Z}$$

$$E_n = -13.6 \frac{Z^2 \mu}{mn^2}$$

Here, $\mu \rightarrow$ reduced mass

$\mu = \frac{Mm}{M+m}$ where, $M \rightarrow$ mass of nucleus.

19. $\lambda_{min} = \frac{hc}{eV_0}$ where, $\lambda_{min} \rightarrow$ minimum wavelength for X-rays

20. $\sqrt{\nu} = a(z-b) \rightarrow$ Moseley's Law

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Wave Optics

1. Inteference of waves of intensity I_1 and I_2 :

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$ where, $I \rightarrow$ resultant intenity

$\Delta\phi \rightarrow$ phase difference

- For constructive inteference

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

- For destructive inteference

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

2. YDSE :

Path difference $(\Delta p) = S_2P - S_1P = d\sin\theta$

Condition 1 :-

$d <<< D$ then $\Delta p = \frac{dy}{D}$

Condition 2 :-

$y << D$ then

- For maxima, $\Delta p = n\lambda \Rightarrow y = n\beta$ $n = 0, \pm 1, \pm 2, \dots$
- For minima,

$$\Delta p = \begin{cases} (2n-1)\frac{\lambda}{2} & \text{where, } n = 1, 2, 3, \dots \\ (2n+1)\frac{\lambda}{2} & \text{where, } n = -1, -2, -3, \dots \end{cases}$$

$$y = \begin{cases} (2n-1)\frac{\beta}{2} & \text{where, } n = 1, 2, 3, 4, \dots \\ (2n+1)\frac{\beta}{2} & \text{where, } n = -1, -2, -3, \dots \end{cases}$$

Here, $\beta = \frac{\lambda D}{d}$ where, $\beta \rightarrow$ fringe width

$\lambda \rightarrow$ wavelength in medium

3. Highest order maxima :

$$n_{max} = \left[\frac{d}{\lambda}\right]$$

total no. of maxima = $2n_{max} + 1$

4. Highest order minima :

$$n_{max} = \left[\frac{d}{\lambda} + \frac{1}{2}\right]$$

total no. of minima = $2n_{max}$.

5. $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi)$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta p$$

IF $I_1 = I_2$ then, $I = 4I_1\cos^2\left(\frac{\Delta\phi}{2}\right)$

6. YDSE with two wavelengths λ_1 and λ_2

- The nearest point to central maxima where the bright fringes coincide : $y = n_1\beta_1 = n_2\beta_2$ = LCM of β_1 and β_2
- The nearest point to central maxima where two dark fringes coincides : $y = \left(n_1 - \frac{1}{2}\right)\beta_1 = \left(n_2 - \frac{1}{2}\right)\beta_2$

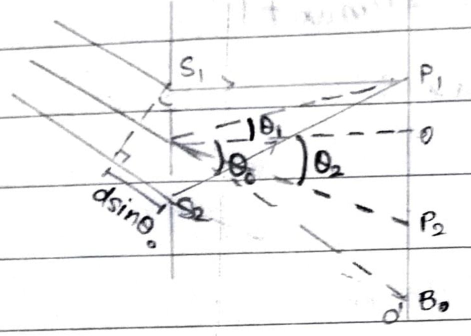
7. Optical path difference (ΔP_{opt}) = $\mu\,\Delta p$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta p = \frac{2\pi}{\lambda_{vacuum}} \times \Delta P_{opt}$$

$$\Delta = (\mu - 1)t\frac{D}{d} = (\mu - 1)t \cdot \frac{\beta}{\lambda}$$

8. ~~In~~ YDSE with oblique incidence :

In YDSE, ray is incident on the slit at an inclination of θ_0 to the axis of symmetry of the experimental set up.



We obtain central maxima at a point where, $\Delta p = 0$ or $\theta_2 = \theta_0$. This corresponds to the point 'O'' in the diagram

$$\Delta p = \begin{cases} d(\sin\theta_0 + \sin\theta) & \rightarrow \text{for points above O} \\ d(\sin\theta_0 - \sin\theta) & \rightarrow \text{for points b/w O and O'} \\ d(\sin\theta - \sin\theta_0) & \rightarrow \text{for points below O'} \end{cases}$$

9. Thin Film Inteterence

- For interference in reflected light

$$2\mu d = \begin{cases} n\lambda & \text{for destructive interference} \\ (n+\frac{1}{2})\lambda & \text{for constructive interference} \end{cases}$$

- For interference in transmitted light

$$2\mu d = \begin{cases} (n+\frac{1}{2})\lambda & \text{for destructive interference} \\ n\lambda & \text{for constructive interference} \end{cases}$$

10. Polarisation :

- $\mu = \tan\theta_0$ → Brewster angle

$\theta_p + \theta_r = 90°$ ie reflectracted and reflected rays are mutually perpendicuar.

- Law of Malus -

$I = I_0 \cos^2\theta$ where, $\theta \rightarrow$ angle b/w the transmission axis of polaroid

$I = KA^2 \cos^2\theta$

11. Diffraction :

- $a\sin\theta = \frac{(2m+1)}{2}$ for maxima where, $m = 1, 2, 3, \ldots$

- $\sin\theta = \frac{m\lambda}{a}$, $m = \pm1, \pm2, \pm3, \ldots$ for minima.

- Linear width of central maxima $= \frac{2d\lambda}{a}$

- Angular width of central maxima $= \frac{2\lambda}{a}$

- $I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$ where, $\beta = \frac{\pi a \sin\theta}{\lambda}$

12. Resolving power :

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}$$

Here, $\lambda = \frac{\lambda_1 + \lambda_2}{2}$

Geometrical Optics

1. Reflection of light,

$\angle i = \angle r$

i.e angle of incidence = angle of reflection.

2. Relation between velocity ~~and~~ of image and object :

$(V_{im})_x = (-V_{ob})_x$

$(V_{im})_y = (V_{ob})_y$

$(V_{im})_z = (V_{ob})_z$

3. Spherical Mirror :

- $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$ where, v → image distance
  u → object distance
  f → focal length
  R → Radius of curvature.

- $m = \frac{h_2}{h_1} = \frac{-v}{u}$ where, h_2 → height of image
  ↳ transverse magnification h_1 → height of object

- $XY = f^2$ where, X → distance of object from principal focus
  Y → distance of image from principal focus

- Optical power of mirror (in Diopter) $= \frac{1}{f}$

- Longitudinal magnification $= \frac{v_2 - v_1}{u_2 - u_1}$

4. Refraction of Light :

- $\mu = \frac{c}{v} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$

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- $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

where, n or $\mu \rightarrow$ Refractive index

5. $\delta = |i - r|$ where, $\delta \rightarrow$ Deviation

6. Apparent depth and shift of submerged object :

- $d' = \frac{d}{n_{rel}}$ Here, $d' \rightarrow$ apparent shift.

- $n_{rel} = \frac{n_i \text{ (R.I of medium of incidence)}}{n_r \text{ (R.I of medium of refraction)}}$

$\therefore$ Apparent shift $= d\left(1 - \frac{1}{n_{rel}}\right)$

7. Refraction through composite slab :

- Apparent depth $= \frac{t_1}{(n_1)_{rel}} + \frac{t_2}{(n_2)_{rel}} + \ldots$

Apparent shift $= t_1\left[1 - \frac{1}{(n_1)_{rel}}\right] + t_2\left[1 - \frac{1}{(n_2)_{rel}}\right] + \ldots$

8. Critical Angle and Total Internal Reflection :

- $C = \sin^{-1} \frac{n_{r}}{n_{d}}$ $n_r \rightarrow$ rarer, $n_d \rightarrow$ denser

- Condition of TIR -

(i) light is incident on the interface from denser medium.

(ii) Angle of incidence should be greater than critical angle.

9. $\delta = i + e - A$ where, $A \rightarrow$ Apex angle of prism.
e $\rightarrow$ angle of emergence.

10. For small values of $\angle A$,

$\delta_{min} = (n-1)A$

$n_{rel} = \dfrac{\sin\left[\frac{A+\delta_{min}}{2}\right]}{\sin(A/2)} = \dfrac{n_{prism}}{n_{surrounding}}$

11. $n(\lambda) = a + \dfrac{b}{\lambda^2}$ where, a and b are (+)ve constants

12. For small A and i

$\theta = (n_v - n_r) A$

$\delta = \delta_y = (n_y - 1) A$ Here, $\delta \rightarrow$ mean deviation

$\theta \rightarrow$ angle of dispersion.

13. $\omega = \dfrac{n_v - n_r}{n_y - 1}$ where, $\omega \rightarrow$ dispersive power

For small A and i, $\omega = \dfrac{\theta}{\delta_y}$

$n_y = \dfrac{n_v + n_r}{2}$ Here, $n_y \rightarrow$ R.I of material for yellow colour

$n_r \rightarrow$ R.I of material for red

$n_v \rightarrow$ R.I of material for violet

14. Combination of Two Prisms :

- Direct vision combination (dispersion without deviation)

The condition of direct vision combination is

$\left(\dfrac{n_v + n_r}{2} - 1\right) A = \left[\dfrac{n_v' - n_r'}{2} - 1\right] A' \Rightarrow (n_y - 1) A = (n_y' - 1) A'$

- Achromatic combination (deviation without dispersion)

Condition of achromatic combination is

$(n_v - n_r) A = (n_v' - n_r') A'$

15. Refraction at Spherical Surfaces :

- $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

where light moves from medium of refractive index n_1 to medium of refractive index n_2.

- $m = \frac{v - R}{u - R} = \frac{v}{u} \times \frac{n_1}{n_2}$

16. Refraction at Spherical Thin Lens :

- $\frac{1}{v} - \frac{1}{u} = (n_{rel} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow n_{rel} = \frac{n_{lens}}{n_{surrounding}}$

$\frac{1}{F} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$ → Lens Maker's Formula

- $M = \frac{v}{u}$

- Combination of lens - $\frac{1}{F} = \frac{1}{F_1} + \frac{1}{f_2} + \dots$

17. Simple Microscope :

- Magnifying power $= \frac{D}{U_o}$

- when image is formed at ∞, $M_\infty = \frac{D}{F}$

- when image is formed at near point D, $M_D = 1 + \frac{D}{F}$

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18. Compound Microscope

Magnifying power

$$M = \frac{V_o D_o}{U_o U_e}$$

Length of microscope

$$L = V_o + U_e$$

$$M_\infty = \frac{V_o D}{U_o F_e}$$

$$L = V_o + F_e$$

$$M_D = \frac{V_o}{U_o}\left(1 + \frac{D}{F_e}\right)$$

$$L_D = V_o + \frac{D F_e}{D + F_e}$$

19. Astronomical Telescope :

Magnifying power

$$M = \frac{F_o}{U_e}$$

Length of microscope

$$L = F + U_e$$

$$M_\infty = \frac{F_o}{F_e}$$

$$L = f_o + f_e$$

$$M_D = \frac{F_o}{F_e}\left[1 + \frac{F_e}{D}\right]$$

$$L = f_o + \frac{D f_e}{D + f_e}$$

20. Terrestrial Telescope

Magnifying power

$$M = \frac{f_o}{U_e}$$

Length of microscope

$$L = F_o + 4f + U_e$$

$$M_\infty = \frac{f_o}{F_e}$$

$$L = f_o + 4f + f_e$$

$$M_D = \frac{f_o}{F_e}\left(1 + \frac{f_e}{D}\right)$$

$$L = f_o + 4f + \frac{D f_e}{D + f_e}$$

21. Galilean Telescope :

Magnifying power

$M = \frac{F_o}{U_e}$

$M_\infty = \frac{F_o}{F_e}$

$M_D = \frac{F_o}{F_e}\left(1 - \frac{F_e}{D}\right)$

Length of Microscope

$L = F_o - U_e$

$L = F_o - F_e$

$L = F_o - \frac{F_e D}{D - F_e}$

22. Resolving Power

- Microscope :- $R = \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda}$
- Telescope :- $R = \frac{1}{\Delta \theta} = \frac{a}{1.22\lambda}$

Circle

1. Equation of circle in various forms :

- The circle with centre as origin and radius 'r',
  Equation → $x^2+y^2=r^2$
- The circle with centre (h,k) and radius 'r',
  Equation → $(x-h)^2+(y-k)^2=r^2$
- General equation of circle is given as
  $x^2+y^2+2gx+2fy+c=0$
  Here, centre $\equiv (-g,-f)$
  radius $= \sqrt{g^2+f^2-c^2}$

  If $g^2+f^2-c>0$ → real circle
  $g^2+f^2-c=0$ → point circle
  $g^2+f^2-c<0$ → imaginary circle with real centre $(-g,-f)$
- The equation of circle with (x_1,y_1) and (x_2,y_2) as extremists of its diameter is
  $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

2. Intercepts made by the circle $x^2+y^2+2gx+2fy+c=0$ on the co-ordinate axes are $2\sqrt{g^2-c}$ on x-axis and $2\sqrt{f^2-c}$ on y-axis.

   If $g^2>c$ → circle cuts axis at two distinct points.
   $g^2=c$ → circle touches axis
   $g^2<c$ → circle completely lies above or below x-axis

3. Parametric equations of circle :

$(x-h)^2+(y-k)^2=r^2$

Here, $x=h+r\cos\theta$ and $y=k+r\sin\theta$

where, $(h,k) \rightarrow$ centre

$r \rightarrow$ radius

$\theta \rightarrow$ parameter.

4. If $S_1<0 \rightarrow$ inside circle

$S_1=0 \rightarrow$ on the circle

$S_1>0 \rightarrow$ outside the circle

$S_1 \equiv x_1^2+y_1^2+2gx_1+2fy_1+c$

5. Line and circle :

- $p>r \rightarrow$ line does not meet circle
- $p=r \rightarrow$ line touches the circle (tangent to the ci
- $p<r \rightarrow$ line is a secant of circle
- $p=0 \rightarrow$ line is diameter of circle.

Also if $y=mx+c$ is line and $x^2+y^2=a^2$ is circle then,

- $c^2<a^2(1+m^2) \rightarrow$ line is secant of circle
- $c^2=a^2(1+m^2) \rightarrow$ line is tangent of circle
- $c^2>a^2(1+m^2) \rightarrow$ does not meet circle.

Here, $p \rightarrow$ length of $\perp^{r}$ from centre on the line

$a, r \rightarrow$ radius

6. Pair of tangents from a point :

$SS_1 = T^2$

7. Tangent :

- Slope form –

$y = mx \pm a\sqrt{1+m^2}$

- Point form –

$xx_1 + yy_1 = a^2$ or $T = 0$

- Parametric form :–

$x\cos\alpha + y\sin\alpha = a.$

8. Length of tangent $= \sqrt{S_1}$

9. Director circle :

$x^2+y^2 = 2a^2$ for $x^2+y^2=a^2$

10. Chord of contact :

- $T=0$
- Length of chord of contact $= \dfrac{2LR}{\sqrt{R^2+L^2}}$
- Area of triangle formed by pair of tangents and their chord of contact $= \dfrac{RL^3}{R^2+L^2}$
- ~~Angle~~ Tangent of the angle b/w pair of tangents $= \dfrac{2LR}{L^2-R^2}$
- Equation of circle circumscribing the ΔPT_1T_2 is

$(x-x_1)(x+g) + (y-y_1)(y+f) = 0$

Here, $L \to$ Length of tangent

$R \to$ radius.

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11. Condition for orthogonality of two circle :

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

12. Equation of chord with a given midpoint :

$T = S_1$

13. Radical Axis :

$\cancel{S_1 - S_2}$ $S_1 - S_2 = 0$ ~~ie~~ $\cancel{2(g_1 - g_2)x + 2(f_1 - f_2) =}$

i.e $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$

14. Family of circles :

$S_1 + KS_2 = 0$ or $S + KL = 0$

Conic Section

1. PARABOLA :

(i) Equation of standard parabola

- $y^2 = 4ax$
- Vertex is $(0,0)$
- Focus is $(a,0)$
- Directrix is $x+a=0$
- Axis is $y=0$
- Length of latus rectum $= 4a$
- Ends of latus rectum are $(a, 2a)$ and $(a, -2a)$

(ii) Parametric equation of parabola

$x = at^2$ and $y = 2at$

(iii) Tangents to Parabola $y^2 = 4ax$

- Slope form -

  $y = mx + \frac{a}{m}$ $(m \neq 0)$

- Parametric form -

  $ty = x + at^2$

- Point form -

  $T = 0$

(iv) Normal to parabola $y^2 = 4ax$

- $y - y_1 = -\frac{y_1}{2a}(x - x_1)$ at (x_1, y_1)
- $y = mx - 2am - am^3$ at $(am^2, -2am)$
- $y + tx = 2at + at^3$ at $(at^2, 2at$.

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2. Ellipse :

(i) Standard equation:-

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where, $a > b$
- Eccentricity $(e) = \sqrt{1 - \frac{b^2}{a^2}}$ $(0 < e < 1)$
- Directrix is $x = \pm \frac{a}{e}$
- Focii is $(\pm ae, 0)$
- Length of major axis $= 2a$
- Length of minor axis $= 2b$
- vertices are $(-a, 0)$ and $(a, 0)$
- Latus Rectum $= \frac{2b^2}{a} = 2a(1-e^2)$

(ii) Auxiliary circle :-

$x^2 + y^2 = a^2$

(iii) Parametric representation :-

$x = a\cos\theta$, $y = b\sin\theta$

(iv) Position of point w.r.t. Ellipse :-

$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 > 0 \rightarrow$ outside

$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 = 0 \rightarrow$ on ellipse

$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 < 0 \rightarrow$ inside

(v) Line and an Ellipse :-

The line $y = mx + c$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in two points

IF $c^2 < 0 \rightarrow$ real points

$c^2 = 0 \rightarrow$ coincident

$c^2 > 0 \rightarrow$ imaginary.

(vi) Tangents :-

- Slope form

$y = mx \pm \sqrt{a^2m^2 + b^2}$

- Point form

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

- Parametric form

$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

(vii) Normal :-

- Slope form

$\frac{a^2x}{x_1} \cancel{-b}$ $\quad y = mx - \frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$

- Point form

$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

- Parametric form

$ax\sec\theta - by\operatorname{cosec}\theta = a^2 - b^2$

(viii) Director circle

$$x^2+y^2=a^2+b^2$$

3. HYPERBOLA

(i) Standard Equation

- $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
- $e=\sqrt{\frac{b^2}{a^2}+1}$
- Focii is $(\pm ae, 0)$
- Directrix are $x=\pm a/e$
- vertices are $(\pm a, 0)$
- Latus Rectum $=\frac{2b^2}{a}$

(ii) Conjugate Hyperbola.

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}+$

(iii) Auxillary circle

$$x^2+y^2=a^2$$

(iv) Parametric Representation

$x=a\sec\theta$ and $y=b\tan\theta$

(v) Position of point wrt Hyperbola

IF $\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1>0 \rightarrow$ inside

$\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 = 0 \rightarrow$ on the curve

$\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 < 0 \rightarrow$ outside

(vi) Tangents

- Slope form -
  $y = mx \pm \sqrt{a^2m^2 - b^2}$
- Point form -
  $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$
- Parametric form -
  $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$

(vii) Normals

- at the point $P(x_1, y_1)$ is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2+b^2 = a^2e^2$
- at the point $P(a\sec\theta, b\tan\theta)$ is $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2+b^2 = (ae)^2$
- $y = mx \pm \frac{(a^2+b^2)m}{\sqrt{a^2 - b^2m^2}}$

(viii) Asymptotes

$\frac{x}{a} + \frac{y}{b} = 0$ and $\frac{x}{a} - \frac{y}{b} = 0$ are ~~pair~~ asymptotes

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ are pair of asymptotes.

(xi) Rectangular or Equilateral Hyperbola -

- $xy = c^2$
- $e = \sqrt{2}$
- vertices are $(\pm c, \pm c)$
- Focii is $(\pm\sqrt{2}c, \pm\sqrt{2}c)$
- directrix is $x + y = \pm\sqrt{2}c$
- Latus Rectum $= 2\sqrt{2}c$
- Parametric equation
  $x = ct$ and $y = c/t$
- Equation of the tangent at $P(x_1, y_1)$ is $\frac{x}{x_1} + \frac{y}{y_1} = 2$ and at $P(t)$ is $\frac{x}{t} + ty = 2c$

- Equation of normal at $P(t)$ is $xt^3 - yt = c(t^4 - 1)$
- Chord with a given middle point as (h, k) is $kx + hy = 2hk$.

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Straight Line

1. Distance Formula

- Cartesian co-ordinate system -

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

- Polar co-ordinate system -

$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$

2. Section Formula

$x = \frac{mx_2 + nx_1}{m+n}$ and $y = \frac{my_2 + ny_1}{m+n}$

If $\frac{m}{n}$ is +ve then division is internal but if $\frac{m}{n}$ = +ve then division is external.

$\frac{m}{n} = -\left[\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}\right]$

3. Slope Formula

$m = \frac{y_1 - y_2}{x_1 - x_2}$

4. Condition of collinearity of three points :

A (x_1, y_1) B (x_2, y_2) and C (x_3, y_3). are collinear if

(i) $\frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_3}{x_2 - x_3}$

(ii) $\Delta ABC = 0$ i.e $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(iii) AC = AB + BC or AB ∩ BC

(iv) A divides line segment ~~in~~ BC in same ratio

5. Area of triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Provided the vertices are ~~the~~ considered in counter clockwise sense. The above formula will give (-ve) area if the vertices are placed in clockwise sense

6. Area of n-sided polygon $= \frac{1}{2}\left[\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \ldots \begin{vmatrix} x_n & x_1 \\ y_n & y_1 \end{vmatrix}\right]$

7. Equation of line in various forms

- Slope-point form

$y - y_1 = m(x - x_1) + c.$

- Two point form

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + c$

- Determinant form

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

- Intercept form :

$\frac{x}{a} + \frac{y}{b} = 1$

$a \rightarrow x$-intercept

$b \rightarrow y$-intercept

- Normal form

$x\cos\alpha + y\sin\alpha = p$

Here, $p \rightarrow$ length of perpendicular from origin to line

$\alpha \rightarrow$ angle of line with +ve x-axis.

- General form

$ax + by + c = 0$

- Parametric form

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

8 $\tan\theta = \left|\frac{m_1 - m_2}{1 - m_1 m_2}\right|$

9. $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are two lines

- parallel if $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$

- distance b/w two parallel lines $= \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

- perpendicular if $aa' + bb' = 0$

10. A point and a line

- distance b/w point and line $= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

- Reflection of a point about a line

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2\left[\frac{ax_1 + by_1 + c}{a^2 + b^2}\right]$$

- Foot of $\perp^{r}$ from a point on the line

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

11. Angle bisector b/w two lines

$$\frac{ax+by+c}{\sqrt{a^2+b^2}} = \pm \frac{a'x+b'y+c'}{\sqrt{(a')^2+(b')^2}}$$

12. Condition of concurrency of three straight lines $a_i x + b_i y + c = 0$ $i = 1, 2, 3$ is

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

13. Pair of straight line through origin

$$ax^2 + 2hxy + by^2 = 0$$

If θ is acute angle b/w pair of straight lines then

$$\tan\theta = \left| \frac{2\sqrt{h^2-ab}}{a+b} \right|$$

Limit, Continuity and Derivability

1. $\lim_{h\to 0^-} F(a-h) = \lim_{h\to 0^+} f(a+h) = \text{Finite}$

2. Fundamental Theorems on limits

Let $\lim_{x\to a} F(x) = l$ and $\lim_{x\to a} g(x) = m$. If l and m are infinite, then

- $\lim_{x\to a} \{F(x) \pm g(x)\} = l \pm m$
- $\lim_{x\to a} \{F(x) \cdot g(x)\} = lm$
- $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{l}{m}$, provided $m \neq 0$
- $\lim_{x\to a} KF(x) = Kl$ where k is a constant
- $\lim_{x\to a} f(g(x)) = F\left[\lim_{x\to a} g(x)\right] = f(m)$ provided f is continous at $g(x) = m$.

3. Intermediate Forms

$\frac{0}{0}, \frac{\infty}{\infty}, 0\times\infty, \infty-\infty, \infty^0, 0^0, 1^\infty.$

We have to solve $f(x)$ or limit if an intermediate form is obtained.

4. Standard limits

- $\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\tan x}{x} = 1$ where, x is measured in radian
- $\lim_{x\to 0} \frac{\sin^{-1} x}{x} = \lim_{x\to 0} \frac{\tan^{-1} x}{x} = 1$
- $\lim_{x\to 0} (1+x)^{1/x} = \lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$
- $\lim_{x\to 0} (1+ax)^{1/x} = \lim_{x\to\infty} \left(1+\frac{a}{x}\right)^x = e^a$

- $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x} = \lim\limits_{x \to 0} \dfrac{\ln(1+x)}{x} = 1$

- $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x} = \ln a \quad a > 0$

- $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a} = n a^{n-1}$

* ~~to~~ If $f(x) \to 0$ when $x \to a$ then

- $\lim\limits_{x \to a} \dfrac{\sin f(x)}{f(x)} = \lim\limits_{x \to a} \dfrac{\tan f(x)}{f(x)} = 1$

- $\lim\limits_{x \to a} \cos(f(x)) = 1$

- $\lim\limits_{x \to a} \dfrac{e^{f(x)} - 1}{f(x)} = 1$

- $\lim\limits_{x \to a} \dfrac{b^{f(x)} - 1}{f(x)} = \ln b \quad b > 0$

- $\lim\limits_{x \to a} \dfrac{\ln(1 + f(x))}{f(x)} = 1$

- $\lim\limits_{x \to a} (1 + f(x))^{1/f(x)} = e$

* $\lim\limits_{x \to a} f(x) = A > 0$ and $\lim\limits_{x \to a} \phi(x) = B$ (a finite quantity) then

$\lim\limits_{x \to a} [f(x)]^{\phi(x)} = A^B$

5. *Limits using expansion*

- $a^x = 1 + \frac{x \ln a}{1!} + \frac{x^2 \ln^2 a}{2!} + \frac{x^3 \ln^3 a}{3!} + \dots \quad a > 0$

- $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

- $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ for $-1 < x \le 1$

- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

- $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots$

- For $|x| < 1$, $n \in R$; $(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2} x^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} x^3 + \dots \infty$

6. Sandwich Theorem or squeeze play theorem

If $f(x) \le g(x) \le h(x) \ \forall x$ and $\lim_{x \to a} f(x) = l = \lim_{x \to a} h(x)$

then $\lim_{x \to a} g(x) = l$.

7. A function $f(x)$ is said to be continous at $x = c$ if $\lim_{x \to c} f(x) = f(c)$

i.e f is continous at $x = c$ if $\lim_{h \to 0} f(c-h) = \lim_{h \to 0} f(c+h) = f(c)$

8. Theorems on continuity

- IF f and g are two function which are continous at $x = c$ then

$f(x) \pm g(x)$ → continous at $x = c$
$f(x) . g(x)$ → continous at $x = c$
$kf(x)$ → continous at $x = c$
$f(x)/g(x)$ → continous at $x = c$ provided $g(c) \neq 0$

- IF $f(x)$ is continous and $g(x)$ is discontinous at $x = a$ then,

$f(x) . g(x)$ → may or may not be continous
$f(x) \pm g(x)$ → discontinous at $x = a$

- IF both are discontinous at $x = a$ then

$f(x) . g(x)$ → not necessarily discontinous
$f(x) \pm g(x)$ → atmost one of addition or substraction is continous.

9. Where to check ~~to~~ Continuity?

- continuity of a function should be checked at the points where definition of function changes
- continuity of $\{f(x)\}$ and $[f(x)]$ should be checked at all points where $f(x)$ becomes integer
- continuity of $sgn(f(x))$ should be checked at points where $f(x) = 0$ [if $f(x) = 0$ in any open interval containing a then $x = a$ is not a point of discontinuity]
- In case of composite function $f(g(x))$ continuity should be checked at all possible points of discontinuity of $g(x)$

and at points where $g(x) = c$ ~~an~~ where, $x = c$ is a possible point of discontinuity of $f(x)$

10. Slope of tangent $= f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

11. If $f'(a)$ exists then $f(x)$ is continous at $x = a$

If $f(x)$ is differentiable at every point of its domain of definition then its is continuous in that domain

Method of Differentiation

1. Differentiation of some elementary functions

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(a^x) = a^x \ln a$
- $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$
- $\frac{d(\ln|x|)}{dx} = \frac{1}{x}$
- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$
- $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x.$

2. Basic Theorems

- $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
- $\frac{d}{dx}[kf(x)] = k \cdot \frac{d(f(x))}{dx}$
- $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$
- $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$
- $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

3. Derivative of Inverse Trigonometric Functions

- $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$ $-1 < x < 1$

- $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$ $-1 < x < 1$

- $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$ $x \in R$

- $\frac{d(\cot^{-1}x)}{dx} = -\frac{1}{1+x^2}$ $x \in R$

- $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$ $x \in (-\infty,-1) \cup (1,\infty)$

- $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$ $x \in (-\infty,-1) \cup (1,\infty)$

4. Parametric Differentiation

If $y = f(\theta)$ and $x = g(\theta)$ where θ is a parameter, then

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

5. $$F(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}$$ where, $f, g, h, l, m, n, u, v, w$ are differentiable function.

$$\therefore F'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l'(x) & m'(x) & n'(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u'(x) & v'(x) & w'(x) \end{vmatrix}$$

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Application of Derivatives

1. Equation of tangent and normal

- Tangent at (x_1, y_1) is $(y - y_1) = f'(x_1)(x - x_1)$ when $f'(x_1)$ is real.
- Normal at (x_1, y_1) is $(y - y_1) = \frac{-1}{f'(x_1)}(x - x_1)$ when $f'(x_1)$ is nonzero real.

2. Tangent from an external point

Given a point $P(a, b)$ which does not lie on the curve $y = f(x)$, then the equation of possible tangents to the curve $y = f(x)$ passing ~~thr~~ through (a, b) can be found by solving for the point of contact Q

$f'(h) = \frac{f(h) - b}{h - a}$ and equation of tangent is

$$y - b = \frac{f(h) - b}{h - a}(x - a)$$

3. Length of tangent, normal, subtangent, subnormal

- $PT = |k|\sqrt{1 + \frac{1}{m^2}}$ → length of tangent
- $PN = |k|\sqrt{1 + m^2}$ → length of normal
- $TM = \left|\frac{k}{m}\right|$ → length of subtangent
- $MN = |km|$ → length of subnormal.

4. Angle between the curves

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

5. Rolle's Theorem

If function f defined on [a,b] is
- continous on [a,b]
- derivable on (a,b)
- $f(a) = f(b)$

then there exists at least one real number c b/w a and b such that $f'(c) = 0$.

6. Lagrange's Mean Value Theorem

If a function f defined on [a,b] is
- continous on [a,b]
- derivable on (a,b)

then there exists at least one real number b/w a and b such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

7. Useful formula of mensuration to remember
- volume of cuboid = lbh
- Surface area of cuboid = $2(lb + bh + hl)$
- volume of cube = a^3
- surface area of cube = $6a^2$
- volume of a cone = $\frac{1}{3}\pi r^2 h$
- curved surface area of cone = $\pi r l$
- curved surface area of cylinder = $2\pi r h$

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- Total surface area of cylinder = $2\pi r(r+h)$
- Volume of a sphere = $\frac{4}{3}\pi r^3$
- Surface area of sphere = $4\pi r^2$
- Area of circular sector = $\frac{1}{2}r^2\theta$
- Volume of prism = (area of base) × (height)
- Lateral surface area of prism = perimeter of base × height
- Total surface area of prism = lateral surface area
  2(area of base)
- Volume of pyramid = $\frac{1}{3}$ (area of bases) (height)
- Curved surface area of pyramid = $\frac{1}{2}$ × (perimeter of base) × (slant height)

Indefinite Integration

1. IF f and g are function of x such that $g'(x) = f(x)$ then
$\int f(x).dx = g(x)+c \Leftrightarrow \frac{d}{dx}\{g(x)+c\} = f(x)$ where, $c \rightarrow$ constant

2. Standard Formula

- $\int (ax+b)^n.dx = \frac{(ax+b)^{n+1}}{x(n+1)} + c \quad n \neq -1$
- $\int \frac{dx}{ax+b} = \frac{1}{a} \ln(ax+b) + c$
- $\int e^{ax+b}.dx = \frac{e^{ax+b}}{a} + c$
- $\int a^{px+q}.dx = \frac{1}{p} \frac{a^{px+q}}{\ln a} + c \quad a>0$
- $\int \sin(ax+b).dx = \frac{-\cos(ax+b)}{a} + c$
- $\int \cos(ax+b).dx = \frac{1}{a} \sin(ax+b) + c$
- $\int \tan(ax+b).dx = \frac{1}{a} \ln \sec(ax+b) + c$
- $\int \cot(ax+b).dx = \frac{1}{a} \ln \sin(ax+b) + c$
- $\int \sec^2(ax+b).dx = \frac{1}{a} \tan(ax+b) + c$
- $\int \operatorname{cosec}^2(ax+b).dx = \frac{-1}{a} \cot(ax+b) + c$

- $\int \sec x \cdot dx = \ln|\sec x + \tan x| + c = \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$

- $\int \operatorname{cosec} x \cdot dx = \ln|\operatorname{cosec} x - \cot x| + c = \ln\left|\tan\frac{x}{2}\right| + c =$
  $= -\ln|\operatorname{cosec} x + \cot x| + c$

- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

- $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

- $\int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$

- $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left[x + \sqrt{x^2 + a^2}\right] + c$

- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left[x + \sqrt{x^2 - a^2}\right] + c$

- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{a + x}{a - x}\right| + c$

- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x - a}{x + a}\right| + c$

- $\int \sqrt{a^2 - x^2} \cdot dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

- $\int \sqrt{x^2 + a^2} \cdot dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln\left(\frac{x + \sqrt{x^2 + a^2}}{a}\right) + c$

- $\int \sqrt{x^2 - a^2} \cdot dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left[\frac{x + \sqrt{x^2 - a^2}}{a}\right] + c$

3. Integration by substitution

If we substitute $f(x) = t$ then $f'(x).dx = dt$

4. Integration by part

$$\int f(x).g(x).dx = f(x)\int g(x).dx - \int\left[\frac{d}{dx}f(x)\int g(x).dx\right]dx$$

5. Integration of type $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$, $\int \sqrt{ax^2+bx+c}\,.dx$

Express ax^2+bx+c in the form of perfect square and then apply the standard rules

6. Integration of type $\int \frac{px+q}{ax^2+bx+c}.dx$, $\int \frac{px+q}{\sqrt{ax^2+bx+c}}.dx$, $\int (px+q)\sqrt{ax^2+bx+c}$

Express $px+q = A$ (differential coefficient of denominator) $+B$

$$= A\left[\frac{d}{dx}(ax^2+bx+c)\right] + B.$$

7. Integration of trigonometric functions

- $\int \frac{dx}{a+b\sin^2 x}$, $\int \frac{dx}{a+b\cos^2 x}$, $\int \frac{dx}{a\sin^2 x + b\sin x\cos x + c\cos^2 x}$

Multiply numerator and denominator by $\sec^2 x$ and put $\tan x = t$.

- $\int \frac{dx}{a+b\sin x}$, $\int \frac{dx}{a+b\cos x}$, $\int \frac{dx}{a+b\sin x + c\cos x}$

Convert sin and cos into their respective tan of half the angles and then put $\tan\left(\frac{x}{2}\right) = t$

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- $\int \frac{a\cos x + b\sin x + c}{l\cos x + m\sin x + n} \cdot dx$

Express numerator = A(Denominator) + B $\frac{d}{dx}$(denominator) + C

8. Integration of type $\int \sin^m x \cos^n x \cdot dx$

Case 1.

If m and n are even natural number then convert higher power into higher angle.

Case 2

If at least one of m or n is odd natural number then if m is odd put $\cos x = t$ and vice vessa.

Case 3

When m+n is negative even integer then put $\tan x = t$

9. Integration of type

$\frac{x^2 \mp 1}{x^4 + kx^4 + 1} \cdot dx$ where K is constant then divide numerator and denominator by x^2 and put $x \pm 1/x = t$.

10. Integration of type

- $\int \frac{dx}{(ax+b)\sqrt{px+q}}$ OR $\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$ Put $px+q = t^2$

- $\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$ Put $ax+b = \frac{1}{t}$

- $\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}}$ Put $x = \frac{1}{t}$

11. Integration of type

- $\int \sqrt{\frac{x-\alpha}{\beta-x}}\,.dx$ OR $\int \sqrt{(x-\alpha)(\beta-x)}\,.dx$ ~~OR~~ Put $x = \alpha\cos^2\theta + \beta\sin^2\theta$

- $\int \sqrt{\frac{x-\alpha}{x-\beta}}\,.dx$ OR $\int \sqrt{(x-\alpha)(x-\beta)}$ Put $x = \alpha\sec^2\theta - \beta\tan^2\theta$

- $\int \frac{dx}{(x-\alpha)(x-\beta)}$ Put $x-\alpha = t^2$ or $x-\beta = t^2$

12. Reduction formula.

- $\int \tan^n x\,.dx = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$ $n \geq 2$

- $\int \cot^n x\,.dx = -\frac{\cot^{n-1}x}{n-1} - I_{n-2}$ $n \geq 2$

- $\int \sec^n x\,.dx = \frac{\tan x.\sec^{n-2}x}{n-1} + \frac{n-2}{n-1} I_{n-2}$

- $\int \mathrm{cosec}^n x\,.dx = \frac{\cot x\ \mathrm{cosec}^{n-2}x}{-(n-1)} + \frac{n-2}{n-1} I_{n-2}$

- $\int \sin^n x\,.dx = -\frac{\cos x\,(\sin x)^{n-1}}{n} + \frac{(n-1)}{n} I_{n-2}$ $n \geq 2$

Definite Integration

1. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

i.e definite integral is independent of variable integration

2. $\int_a^b f(x).dx = -\int_b^a f(x)\,dx$

3. $\int_a^b f(x).dx = \int_a^c f(x).dx + \int_c^b f(x).dx$ where $c \in [a,b]$

4. $\int_{-a}^{a} f(x).dx = \int_0^a [f(x) + f(-x)].dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(-x)=f(x) \\ 0 & \text{if } f(-x)=-f(x) \end{cases}$

5. $\int_a^b f(x).dx = \int_a^b f(a+b-x).dx$

6. $\int_0^{2a} f(x).dx = \int_0^a [f(x)+f(2a-x)].dx = \begin{cases} 2\int_0^a f(x) & \text{if } f(2a-x)=f(x) \\ 0 & \text{if } f(2a-x)=-f(x) \end{cases}$

7. Integration of periodic function

IF $f(x)$ is a periodic function with time period T, then

- $\int_0^{nT} f(x).dx = n\int_0^T f(x).dx$
- $\int_a^{a+nT} f(x).dx = n\int_0^T f(x).dx$
- $\int_{mT}^{nT} f(x).dx = (n-m)\int f(x).dx$
- $\int_{nT}^{a+nT} f(x).dx = \int_0^a f(x).dx$
- $\int_{a+nT}^{b+nT} f(x).dx = \int_a^b f(x).dx$

8. If $\psi(x) \le f(x) \le \phi(x)$ for $a \le x \le b$ then

$$\int_a^b \psi(x).dx \le \int_a^b f(x).dx \le \int_a^b \phi(x).dx$$

9. If $m \le f(x) \le M$ for $a \le x \le b$ then $m(b-a) \le \int_a^b f(x).dx \le M(b-a)$

10. $$\left| \int_a^b f(x).dx \right| \le \int_a^b |f(x)|.dx$$

11. $f(x) \ge 0$ on $[a,b]$ then $\int_a^b f(x).dx \ge 0$

12. Leibnitz Theorem

If $F(x) = \int_{g(x)}^{h(x)} f(t).dt$ then $\frac{d}{dx}[F(x)] = h'(x) f(h(x)) - g'(x) f(g(x))$

13. Definite integral as a limit of sum.

$$\int_a^b f(x).dx = \lim_{n \to \infty} \sum_{r=0}^{n-1} h.f(a+hr)$$

where, $h = \frac{b-a}{n}$