

## Q21: Skewness

**Skewness** is a measure of the asymmetry of the probability distribution of a real-valued random variable about its mean. It indicates whether the observations in a dataset are skewed to the left (negative skewness) or to the right (positive skewness).

- **Types of Skewness:**
  1. **Positive Skewness** (Right Skewness):
    - The right tail (higher values) is longer or fatter than the left tail.
    - Most data points are concentrated on the left.
    - $\text{Mean} > \text{Median} > \text{Mode}$ .
  2. **Negative Skewness** (Left Skewness):
    - The left tail (lower values) is longer or fatter than the right tail.
    - Most data points are concentrated on the right.
    - $\text{Mean} < \text{Median} < \text{Mode}$ .
  3. **Zero Skewness** (Symmetrical Distribution):
    - The left and right tails are equally balanced.
    - $\text{Mean} = \text{Median} = \text{Mode}$ .

### Graphical Representation:

- **Positive Skewness:**
- **Negative Skewness:**

## Q22: Probability Mass Function (PMF) and Probability Density Function (PDF)

- **Probability Mass Function (PMF):**
  - Defines the probability distribution for discrete random variables.
  - The PMF gives the probability that a discrete random variable is exactly equal to some value.
  - Mathematically:  $P(X=x)$
- **Probability Density Function (PDF):**
  - Defines the probability distribution for continuous random variables.
  - The PDF describes the relative likelihood for this random variable to take on a given value.
  - The area under the PDF curve represents the probability.
  - Mathematically:  $f(x)$

### Difference:

- PMF is used for discrete variables, while PDF is used for continuous variables.
- PMF gives probabilities directly, while the PDF must be integrated over an interval to obtain probabilities.

## Q23: Correlation

**Correlation** measures the strength and direction of a linear relationship between two variables.

- **Types of Correlation:**
  1. **Positive Correlation:**
    - Both variables move in the same direction.
    - As one variable increases, the other also increases.
  2. **Negative Correlation:**
    - Variables move in opposite directions.
    - As one variable increases, the other decreases.
  3. **No Correlation:**
    - No linear relationship between the variables.

### Methods of Determining Correlation:

1. **Pearson's Correlation Coefficient:**
  - Measures the linear relationship between two continuous variables.
  - Values range from -1 to 1.
  - Formula:  $r = \frac{\sum (X - \overline{X})(Y - \overline{Y})}{\sqrt{\sum (X - \overline{X})^2 \sum (Y - \overline{Y})^2}}$
2. **Spearman's Rank Correlation:**
  - Measures the relationship between two ranked variables.
  - Values range from -1 to 1.
  - Formula:  $r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$
3. **Kendall's Tau:**
  - Measures the strength of dependence between two variables.
  - Values range from -1 to 1.

## Q24: Calculate the Coefficient of Correlation

To calculate the Pearson correlation coefficient between the marks obtained by students in Accountancy and Statistics, we can use the following formula:

$$r = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{(\sum (X_i - \bar{X})^2 \sum (Y_i - \bar{Y})^2)^{1/2}}}$$

Where:

- $X_i$  and  $Y_i$  are the individual sample points indexed with  $i$ .
- $\bar{X}$  and  $\bar{Y}$  are the means of the  $X$  and  $Y$  datasets, respectively.

Let's calculate it using Python:

```
import numpy as np
```

```
# Marks obtained by students
accountancy_marks = [45, 70, 65, 30, 90, 40, 50, 75, 85, 60]
statistics_marks = [35, 90, 70, 40, 95, 40, 60, 80, 80, 50]

# Calculate Pearson's correlation coefficient
correlation_coefficient = np.corrcoef(accountancy_marks,
statistics_marks)[0, 1]
Correlation_coefficient
```

## Q25: Differences between Correlation and Regression

### 1. Definition:

- **Correlation** measures the strength and direction of a linear relationship between two variables.
- **Regression** analyzes the nature of the relationship between two variables, specifically how one variable affects another.

### 2. Symmetry:

- **Correlation** is symmetric, meaning the correlation between XXX and YYY is the same as between YYY and XXX.
- **Regression** is asymmetric; the regression of YYY on XXX is generally different from the regression of XXX on YYY.

### 3. Purpose:

- **Correlation** aims to quantify the degree of relationship between variables.
- **Regression** aims to predict the value of one variable based on the value of another variable.

### 4. Units:

- **Correlation** is a unitless measure, ranging from -1 to 1.
- **Regression** coefficients have units and depend on the units of the variables involved.

## Q26: Finding the Most Likely Price at Delhi

Given:

- Correlation coefficient ( $r_{rr}$ ) = +0.8
- Price at Agra (XXX) = Rs. 70

Let's assume:

- Mean price at Agra ( $\bar{X}$ ) = 60

- Mean price at Delhi ( $\bar{Y}$ ) = 55
- Standard deviation at Agra ( $\sigma_X$ ) = 10
- Standard deviation at Delhi ( $\sigma_Y$ ) = 8

Using the regression equation

$$Y - \bar{Y} = r \left( \frac{\sigma_Y}{\sigma_X} \right) (X - \bar{X})$$

Let's calculate:

$$Y - 55 = 0.8 \left( \frac{8}{10} \right) (70 - 60)$$

$$Y - 55 = 0.8 \times 0.8 \times 10$$

$$Y - 55 = 6.4$$

$$Y = 55 + 6.4$$

$$Y = 61.4$$

Therefore, the most likely price at Delhi corresponding to the price of Rs. 70 at Agra is Rs. 61.4.

## Q28: Normal Distribution

**Normal Distribution** is a continuous probability distribution characterized by a bell-shaped curve, symmetric about the mean.

**Assumptions:**

1. **Independence:**
  - Each observation is independent of the others.
2. **Sample Size:**
  - Large sample sizes tend to produce more accurate approximations of normality.
3. **Random Sampling:**
  - Data is collected through random sampling methods.
4. **Measurement Scale:**
  - Data should be measured on an interval or ratio scale.

## Q29: Characteristics of Normal Distribution Curve

1. **Bell-shaped and Symmetric:**
  - The curve is symmetric about the mean.
2. **Mean, Median, Mode:**
  - All are equal and located at the center of the distribution.
3. **Asymptotic:**

- The tails approach the horizontal axis but never touch it.

**4. Empirical Rule:**

- About 68% of data within 1 standard deviation ( $\sigma$ ), 95% within  $2\sigma$ , and 99.7% within  $3\sigma$ .

**5. Total Area:**

- The total area under the curve is 1.

**Q30: Correct Options about Normal Distribution Curve**

- (a): Incorrect. The middle 50% area is covered by mean  $\pm 0.6745\sigma$ .
- (b): Correct.
- (c): Correct.
- (d): Correct.
- (e): Correct.