Scilab for Chemical Engineers

TSEC - Online Certificate Course

CLASS PROJECT 2

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1 Temperature profile: Laplace Equation in 2D

Solve the Laplace equation for the temperature distribution $T(x, y)$ in a 2D rectangular domain using the finite difference method in Scilab.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{1}$$

The domain is defined as $0 \leq x \leq 1$ and $0 \leq y \leq 1$. The boundary conditions are as follows:

- $T(0, y) = 100$ °C for $0 \leq y \leq 1$ - $T(1, y) = 50$ °C for $0 \leq y \leq 1$ - $T(x, 0) = 75$ °C for $0 \leq x \leq 1$ - $T(x, 1) = 0$ °C for $0 \leq x \leq 1$

Use the finite difference method to solve for the temperature distribution and visualize the temperature map using colors in Scilab.

1.1 Finite difference method

To solve Laplace equation numerically, we approximate the second derivatives using finite difference formulas.

Grid and Discretization: - Divide the 2D domain into a grid of points with spacing Δx in the x-direction and Δy in the y-direction. - The grid points are $(i\Delta x, j\Delta y)$, where i and j are integers.

Approximating the Second Derivatives: Using central difference formulas, we approximate the second derivatives at grid point (i, j) as follows:

- For the x-direction:

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2}$$

- For the y-direction:

$$\frac{\partial^2 T}{\partial y^2} \approx \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y)^2}$$

Combining the Approximations: Substitute the finite difference approximations into the Laplace equation:

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y)^2} = 0$$

For simplicity, we assume a uniform grid where $\Delta x = \Delta y = h$:

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{h^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{h^2} = 0$$

Simplifying the Equation: Combine the terms:

$$T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j} = 0$$

Rearrange to solve for $T_{i,j}$:

$$T_{i,j} = \frac{1}{4}(T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1})$$

Implementing the Finite Difference Method in Scilab: Using the above equation, we iteratively update the temperature at each grid point until the solution converges.