AI1110 - Assignment2

Aditya Prashant Gawande - EE22BTECH11202

12.13.6.16 distribution Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Solution: Let X be the random variable for ball taken from Bag I.

$$X = \begin{cases} 0 & \text{Ball is black} \\ 1 & \text{Ball is red} \end{cases} \tag{1}$$

$$\Pr(X = n) = \begin{cases} 1-p & n = 0 \\ p & n = 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$p = \frac{3}{7} \tag{3}$$

Let A be the event - Ball taken from Bag II is red.

$$\Pr(A\,(X=0)) = \frac{16}{70} \tag{4}$$

$$\Pr(A|\,(X=0)) = \frac{\Pr(A\,(X=0))}{\Pr(X=0)} \tag{5}$$

$$\Pr(A|\,(X=0)) = \frac{4}{10} \tag{6}$$

$$\Pr(A\,(X=1)) = \frac{15}{70} \tag{7}$$

$$\Pr(A|\,(X=1)) = \frac{\Pr(A\,(X=1))}{\Pr(X=1)} \tag{8}$$

$$\Pr(A|\,(X=1)) = \frac{5}{10} \tag{9}$$

$$\Pr(X = n) = \begin{cases} \frac{4}{7} & n = 0 \\ \frac{3}{7} & n = 1 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$\Pr(A|\,(X=n))\Pr(X=n) = \begin{cases} \frac{16}{70} & n = 0 \\ \frac{15}{70} & n = 1 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Calculating $\Pr(A)$ here -

$$0 = (X=0)(X=1) \tag{12}$$

$$1 = (X=0) + (X=1) \tag{13}$$

$$A = A[(X=0) + (X=1)] \tag{14}$$

$$A = A\,(X=0) + A\,(X=1) \tag{15}$$

$$\Pr(A) = \Pr(A\,(X=0) + A\,(X=1)) \tag{16}$$

$$\Pr(A) = \Pr(A\,(X=0)) + \Pr(A\,(X=1)) - \Pr(A\,(X=0)\,(X=1) \tag{17}$$

$$\Pr(A) = \frac{16}{70} + \frac{15}{70} \tag{18}$$

$$\Pr(A) = \frac{31}{70} \tag{19}$$

$$\tag{20}$$

Using this result, we get

$$\Pr((X=0)|A) = \frac{\Pr(A\,(X=0))}{\Pr(A)} \tag{21}$$

$$= \frac{\frac{16}{70}}{\frac{31}{70}} \tag{22}$$

$$= \frac{16}{31} \tag{23}$$