Assignment - Parameter Estimation

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Ans 1 Sample size 'n' is taken

$(x_1, x_2 \dots x_n)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

here $\mu = \theta_1$, $\sigma^2 = \theta_2$

$$f(x_i) = \frac{1}{\sqrt{2\pi\theta_2}} \cdot e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}} , \quad i = 1, 2, \dots n$$

joint density function

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \cdot e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$$

Taking log on both sides

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-1}{2} \log(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2} \right)$$

for θ_1

$$\frac{\partial \ln L(\theta_1, \theta_2)}{\partial\theta_1} = \frac{\partial}{\partial\theta_1} \left(\sum_{i=1}^{n} \left(\frac{-1}{2} \ln(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2} \right) \right)$$

$$= \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$\sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$n\theta_1 = \sum_{i=1}^{n} x_i$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

MLE of θ_1 = sample mean

for θ_2

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) = \frac{\partial}{\partial \theta_2} \left(\sum_{i=1}^{n} \left(\frac{-1}{2} \ln (2\pi\theta_2) - \frac{(x_i - \theta_1)^2}{2\theta_2} \right) \right) = 0$$

$$= \sum_{i=1}^{n} \left(\frac{-1}{2\theta_2} + \frac{(x_i - \theta_1)^2}{2(\theta_2)^2} \right) = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2(\theta_2)^2} \sum_{i=1}^{n} \left(x_i - \overset{\theta_1}{\cancel{\theta_2}}\right)^2$$

$$n\theta_2 = \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$\Rightarrow$ MLE of θ_2 is sample variance

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Q2 Sample $= (X\_1, X\_2 \ldots X\_n)$

$$f(x; M, \theta) = {}^{M}C_x \, \theta^{x} (1-\theta)^{M-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{M}C_{x_i} \, \theta^{x_i} (1-\theta)^{M-x_i}$$

taking ln on both sides

$$\ln L(\theta) = \ln \prod_{i=1}^{n} {}^{M}C_{x_i} \, \theta^{x_i} (1-\theta)^{M-x_i}$$

$$= \sum_{i=1}^{n} \left(\ln {}^{M}C_{x_i} + x_i \ln\theta + (M-x_i) \ln(1-\theta) \right)$$

for θ

$$\frac{\partial}{\partial\theta} \ln L(\theta) = \frac{\partial}{\partial\theta}\left(\sum_{i=1}^{n} \left(\ln {}^{M}C_{x_i} + x_i \ln\theta + (M-x_i) \ln(1-\theta) \right) \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{M-x_i}{1-\theta} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{x_i - \theta M}{\theta(1-\theta)} \right) = 0$$

$$\sum_{i=1}^{n} (x_i - \theta M) = 0$$

$$\sum_{i=1}^{n} x_i - n\theta M = 0$$

$$\theta = \frac{1}{nM} \sum_{i=1}^{n} x_i$$

$\Rightarrow$ MLE of $\theta =$ sample mean / M