



DOI:10.1145/3418290

Synthesizing the emerging directions of research at the intersection of differential privacy and cryptography.

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DP-Cryptography: Marrying Differential Privacy and Cryptography in Emerging Applications

ON FEB 15, 2019, John Abowd, chief scientist at the U.S. Census Bureau, announced the results of a *reconstruction attack* that they proactively launched using data released under the 2010 Decennial Census.¹⁹ The decennial census released billions of statistics about individuals like “how many people of the age 10–20 live in New York City” or “how many people live in four-person households.” Using only the data publicly released in 2010, an internal team was able to correctly reconstruct records of address (by census block), age, gender, race, and ethnicity for 142 million

people (about 46% of the U.S. population), and correctly match these data to commercial datasets circa 2010 to associate personal-identifying information such as names for 52 million people (17% of the population).

This is not specific to the U.S. Census Bureau—such attacks can occur in any setting where statistical information in the form of deidentified data, statistics, or even machine learning models are released. That such attacks are possible was predicted over 15 years ago by a seminal paper by Irit Dinur and Kobbi Nissim¹²—releasing a sufficiently large number of aggregate statistics with sufficiently high accuracy provides sufficient information to reconstruct the underlying database with high accuracy. The practicality of such a large-scale reconstruction by the U.S. Census Bureau underscores the grand challenge that public organizations, industry, and scientific research faces: How can we safely disseminate results of data analysis on sensitive databases?

An emerging answer is *differential privacy*. An algorithm satisfies differential privacy (DP) if its output is insensitive to adding, removing or changing one record in its input database. DP is considered the “gold standard” for privacy for a number of reasons. It provides a persuasive mathematical proof of privacy to individuals with several rigorous interpretations.^{25,26} The DP guarantee is composable and repeating

» key insights

- **Local Differential Privacy is increasingly being embraced as the primary model of deployment of differential privacy, albeit at a heavy accuracy cost.**
- **Cryptographic primitives can help bridge the utility gap between systems deployed in the local differential privacy model and standard differential privacy model, but the increased utility may come at the cost of performance.**
- **DP-cryptographic primitives, which are relaxed notions of cryptographic primitives that leak differentially private outputs, permit implementations that are orders of magnitude faster than the regular primitives.**

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invocations of differentially private algorithms lead to a graceful degradation of privacy. The U.S. Census Bureau was the first big organization to adopt DP in 2008 for a product called OnTheMap,²⁹ and subsequently there have been deployments by Google, Apple, Microsoft, Facebook, and Uber.^{2,11,17,18,36}

DP is typically implemented by collecting data from individuals in the clear at a trusted data collector, then applying one or more differentially private algorithms, and finally releasing the outputs. This approach, which we call *standard differential privacy (SDP)*, works in cases like the U.S. Census Bureau where there is a natural trusted data curator. However, when Google wanted to monitor and analyze the Chrome browser properties of its user base to detect security vulnerabilities, they chose a different model called *local differential privacy (LDP)*. In LDP, individuals perturb their records *before* sending them to the server, obviating the need for a trusted data curator. Since the server only sees perturbed records, there is no centralized database of sensitive information that is susceptible to an attack or subpoena requests from governments. The data that Google was collecting—browser fingerprints—uniquely identify individuals. By using LDP, Google was not liable to storing these highly identifying user properties. Due to these attractive security properties, a number of real-world applications of DP in the industry—Google’s RAPPOR,¹⁷ Apple Diagnostics² and Microsoft Telemetry¹¹—embrace the LDP model.

However, the improved security properties of LDP come at a cost in terms of utility. DP algorithms hide the presence or absence of an individual by adding noise. Under the SDP model, counts over the sensitive data, for example, “number of individuals who use the bing.com search engine,” can be released by adding a noise independent of the data size. In the LDP model, noise is added to *each individual record*. Thus, answering the same count query requires adding $O(\sqrt{N})$ error (Theorem 2.1 from Chen et al.¹⁰) for the same level of privacy, where N is the number of individuals participating in the statistic. In other words, under the LDP model, for a database of a billion people, one can only learn properties that



When used in practice, practical trust assumptions are made that enable the deployment of differential privacy-based systems.



are common to at least 30,000 people $O(\sqrt{N})$. In contrast, under SDP, one can learn properties shared by as few as a 100 people ($O(1)$ including constants¹⁵). Thus, the LDP model operates under more practical trust assumptions than SDP, but as a result incurs a significant loss in data utility. In this work, we review literature in this domain under two categories:

► **Cryptography for DP:** We review a growing line of research that aims to use cryptographic primitives to bridge the gap between SDP and LDP. In these solutions, the trusted data curator in SDP is replaced by cryptographic primitives that result in more practical trust assumptions than the SDP model, and better utility than under the LDP model. Cryptographic primitives such as anonymous communication and secure computation have shown significant promise in improving the utility DP implementations while continuing to operate under the practical trust assumptions that are accepted by the security community.

► **DP for cryptography:** Differential privacy is typically applied to settings that involve complex analytics over large datasets. Introducing cryptographic primitives results in concerns about the feasibility of practical implementations at that scale. This has given rise to a second line of work that employs differential privacy as a tool to speed up cryptographic primitives, thereby pushing the frontiers of their practical deployments. While the original cryptographic primitives are defined with respect to perfect privacy, under differential privacy, it is OK to learn distributional information about the underlying dataset. We explore in depth the following cryptographic primitives: secure computation and secure communication and show how in the context of differential privacy one can build “leaky” but efficient implementations of these primitives.

These lines of work both reflect exciting directions for the computer science community. We begin by giving a brief technical introduction to DP. We then discuss the “Cryptography for DP” and “DP for cryptography” paradigms. Finally, we provide concrete ideas for future work as well as open problems in the field through the lens of combining differential privacy and cryptography.

Differential Privacy

Differential privacy¹³ is a state-of-the-art privacy metric for answering queries from statistical databases while protecting individual privacy. Since its inception, there has been considerable research in both the theoretical foundations^{12,14} as well as some real-world DP deployments.^{2,17} The rigorous mathematical foundation and the useful properties of DP have led to an emerging consensus about its use among the security and privacy community.

DP definition. Informally, the privacy guarantees of differential privacy can be understood as follows: Given any two databases, otherwise identical except one of them contains random data in place of data corresponding to any *single* user, differential privacy requires that the response mechanism will behave approximately the same on the two databases. Formally,

Definition 1. Let M be a randomized mechanism that takes a database instance D and has a range O . We say M is (ϵ, δ) -differentially private, if for any neighboring databases (D_1, D_2) that differ in the data of a single user, and for any $S \subseteq O$, we have

$$\Pr[M(D_1) \in S] \leq e^\epsilon \Pr[M(D_2) \in S] + \delta$$

DP enjoys some important properties that make it a useful privacy metric. First, the privacy guarantees of DP have been thoroughly studied using various metrics from statistics and information theory such as hypothesis testing and Bayesian inference.^{25,26} Thus, the semantic meaning of its privacy guarantees is well understood. DP also has a number of composition properties which enable the analysis of privacy leakage for complex algorithms. In particular, sequential composition addresses the impossibility result by Dinur and Nissim¹² and quantifies the degradation of privacy as the number of sequential accesses to the data increases. The post-processing theorem (a special case of sequential composition) ensures the adversary cannot weaken the privacy guarantees of a mechanism by transforming the received response. The end-to-end privacy guarantee of an algorithm over the entire database can thus be established using the above composition theorems and more advanced theorems.¹⁵

Differentially private mechanisms.

Next, we review two classic differentially private mechanisms—the Laplace mechanism and the Randomized Response mechanism—with the following scenario: A data analyst would like to find out how many users use drugs illegally. Such a question would not elicit any truthful answers from users and hence we require a mechanism that guarantees (a) response privacy for the users and (b) good utility extraction for the data analyst.

Laplace mechanism: The Laplace mechanism¹³ considers a trusted data curator (SDP model) who owns a table of N truthful records of users, for example, each record indicates whether a user uses drugs illegally. If a data analyst would like to learn how many users use drugs illegally, the data curator (trusted) computes the true answer of this query and then perturbs it with a random (Laplace distributed) noise that is sufficient to provide privacy. The magnitude of this noise depends on the largest possible change on the query output—also known as the sensitivity of the query—if the data corresponding to a single user is changed.

Randomized response mechanism: Randomized response was first introduced by Warner in 1965 as a research technique for survey interviews. It enabled respondents to answer sensitive questions (about topics such as

sexuality, drug consumption) while maintaining the confidentiality of their responses. An analyst interested in learning aggregate information about sensitive user behavior would like to query this function on a database that is *distributed* across N clients with each client having its own private response x_1, \dots, x_N . Instead of releasing x_i directly, the clients release a perturbed version of their response y_i , thus maintaining response privacy. The analyst collects these perturbed responses and recovers meaningful statistics using reconstruction techniques.

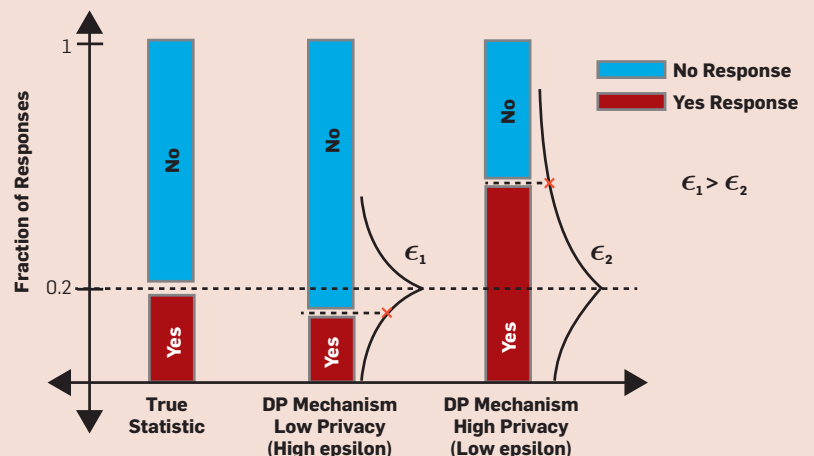
Both these approaches have gained popularity in many applications of differential privacy due to their simplicity as well as the rigorous privacy guarantee on user data. Figure 1 shows the behavior of DP mechanisms for two different privacy values in reference to the true statistic. A less private response results in a more accurate query result while a more private response results in a less accurate query result.

Cryptography for Differential Privacy

By itself, DP is a guarantee on a mechanism and hence is “independent” of the deployment scenario. However, when used in practice, practical trust assumptions are made that enable the deployment of differential privacy-based systems. Here,

Figure 1. Differentially private mechanisms randomize query response to achieve privacy.

If the true response to a query such as “What fraction of users use drugs illegally?” was 20%, then a high privacy response mechanism (low ϵ value) will add a lot of noise yielding low utility. On the contrary, if a low privacy response mechanism was used (high ϵ value), the response will be very close to 20% yielding high utility.



we consider two popular deployment scenarios for differential privacy—Standard Differential Privacy (SDP, graphically represented in Figure 2D) and Local Differential Privacy (LDP, graphically represented in Figure 2A). SDP relies on the need for a trusted data aggregator who follows the protocol. However, in practice, a trusted data aggregator may not always exist. LDP, on the other hand, does not require a trusted data aggregator.^a With the advent of privacy regulations, such as GDPR and FERPA, large organizations such as Google increasingly embrace the LDP model thereby avoiding the liability of storing such sensitive user data. This approach also insures data collectors from potential theft or subpoenas from the government. For these reasons, LDP is frequently a more attractive deployment scenario. However, the utility of the statistics released in LDP is poorer than that in SDP. Consequently, there is a gap in the trust assumptions and the utility achieved by mechanisms in SDP and LDP: high trust assumptions, high utility in SDP and lower trust assumptions, lower utility in LDP. We ask the following question:

a Differentially private federated learning is simply a special case of the LDP deployment scenario.

Can cryptographic primitives help in bridging the gap that exists between mechanisms in the SDP model and the LDP model?

An emerging direction of research has been to explore the use of cryptography to bridge the trust-accuracy gap and obtain the best of both worlds: high accuracy without assuming trusted data aggregator. We explore in depth two concrete examples of the role of cryptography in bridging this gap—anonymous communication, and secure computation and encryption.

Key challenges. There exists a big gap in the accuracy and trust achieved by known mechanisms in the SDP setting with a trusted data curator (Figure 2D) and LDP without such a trusted curator (Figure 2A). Achieving the utility as in the SDP setting while operating under practical trust assumptions such as those in LDP has proven to be a tough challenge. Cryptographic primitives show promise in solving this challenge.

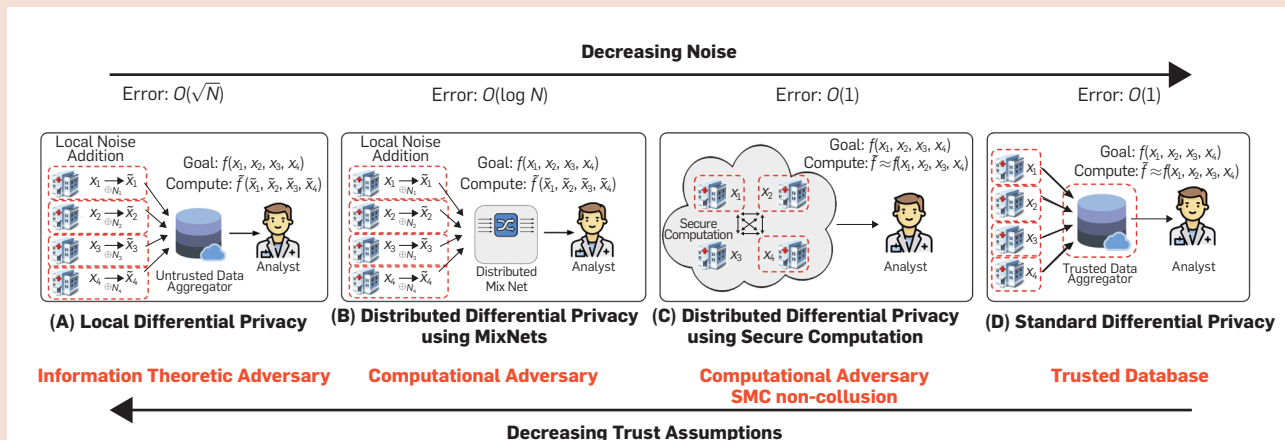
Improve accuracy via anonymous communication. In LDP, each data owner independently perturbs their own input (for example, using the randomized response mechanism) before the aggregation on an untrusted server. This results in a large noise in the final output, $O(\sqrt{N})$ for the case of statistical

counting queries.¹⁰ Applications such as Google's RAPPOR,¹⁷ Apple Diagnostics,² and Microsoft Telemetry,¹¹ which use this LDP deployment model operate under more practical trust assumptions yet suffer from poor accuracy/utility. Recent works^{8,10,16} show the use of an anonymous communication channel can help improve the accuracy of statistical counting query for LDP and thereby eliminate the need for a trusted data curator. We will use one of these systems called Prochlo^{8,16} to illustrate the key idea of how anonymous communication can help improve the accuracy of such applications.

Case Study: Prochlo. Anonymous communication channels, first proposed by Chaum in 1981,⁹ are systems that enable a user to remain unidentifiable from a set of other users (called the anonymity set). A larger anonymity set corresponds to a greater privacy guarantee. Examples of such systems include Mixnets, which use proxies to mix communications from various users. In order to circumvent the limitations of LDP, Google explored the use of an anonymous communication channel to improve the accuracy of queries under DP. The proposed technique is called Prochlo^{8,16} and it consists of three steps as shown in Figure 2B: Encode, Shuffle, and Analyze (ESA).

Figure 2. Various deployment scenarios of differential privacy and the underlying trust assumptions in each of them.


(D) Standard Differential Privacy (SDP) assumes a trusted database, and is thus able to achieve high accuracy, such as, $O(1)$ error. (A) Local Differential Privacy (LDP) on the other hand, does not rely on the use of a trusted database but achieves lower accuracy, that is, $O(\sqrt{N})$ error. The goal is to achieve utility of the SDP setting while operating under more practical assumptions such as the LDP setting (that is, no trusted database). (B) and (C) show how different cryptographic primitives can be used to improve the utility of DP deployments under such practical assumptions.




The first encoding step is similar to LDP where data owners randomize their input data independently. The second step uses an anonymous communication channel to collect encoded data into batches during a lengthy time interval and shuffles this data to remove the linkability between the output of the communication channel and the data owners. Last, the anonymous, shuffled data is analyzed by a data analyst.

The shuffling step is the crucial link in achieving anonymous communication by breaking linkability between the user and their data. This step strips user-specific metadata such as time stamps or source IP addresses, and batches a large number of reports before forwarding them to data analysts. Additional thresholding in this step will discard highly unique reports (for example, a long API bit-vector) to prevent attackers with sufficient background information from linking a report with its data owner. Hence, attacks based on traffic analysis and longitudinal analysis can be prevented, even if a user contributes to multiple reports. Prochlo implements this shuffling step using trusted hardware as proxy servers to avoid reliance on external anonymity channel. Furthermore, this shuffling step can amplify the privacy guarantee of LDP and hence improves the accuracy of the analysis, even when there is a single invocation from a user. We will next show the intuition for this use case.

Accuracy improvement. To illustrate how anonymous communication can help improve accuracy, let us look at a simple example of computing the sum of boolean values from N data owners, $f: \Sigma_{i=1}^N x_i$, where $x_i \in \{0, 1\}$. In LDP, each data owner reports a random bit with probability p or reports the true bit with probability $1 - p$ to achieve ϵ -LDP. When using additional anonymous communication channels, the data owners can enhance their privacy by hiding in a large set of N values, since the attackers (aggregator and analyst) see only the anonymized set of reports $\{\tilde{x}_1, \dots, \tilde{x}_N\}$. The improved privacy guarantee can be shown equivalent to a simulated algorithm that first samples a value s from a binomial distribution $B(N, p)$ to simulate the number of data owners who report a random bit, and then samples a subset



Cryptographic primitives provide strong privacy guarantees. However, deployment of certain cryptographic primitives in practical systems is limited due to the large overhead of these primitives.



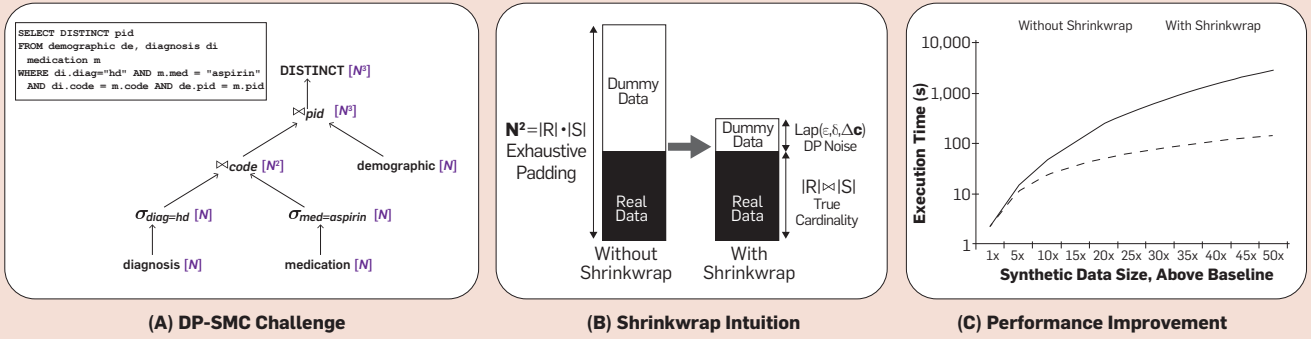
of responses for these s data owners from $\{\tilde{x}_1, \dots, \tilde{x}_N\}$. The randomness of these sampling processes can amplify the privacy parameter based on a well-studied sub-sampling argument.^{3,23} Therefore, given the value of the privacy parameter, the required noise parameter can be scaled down and hence the corresponding error can be reduced to $O(\sqrt{\log N})$. However, these bounds depend on the specific deployment scenarios. For instance, it is shown in Balle et al.⁴ that anonymous communication with a single message per data owner cannot yield expected error less than $O(N^{1/6})$. On the other hand, works such as Balle⁵ and Kasiviswanathan et al.²⁶ show that with a constant number of messages per data owner, it is possible to reduce the error for real-valued DP summation to $O(1)$. Note that these accuracy improvements assume that there is no collusion between the analyst and the anonymous communication, otherwise, the privacy guarantee will fall back to the same as LDP.

In reference to Figure 2, these works demonstrate the improvement in going from Figure 2A to Figure 2B showing a trade-off between accuracy and trust assumptions.

Improve trust via encryption and secure computation. SDP requires the use of a trusted data aggregator to achieve high accuracy. A number of works have explored the use of encryption and secure computation to eliminate the need for this trusted data aggregator.^{1,6,33} The key challenge here is to maintain the same level of accuracy as in SDP. We will use one of these proposed systems called DJoin to demonstrate the use of secure computation to enable high accuracy computation without the need for a trusted data aggregator. There is a complementary synergy between secure computation and DP and thus their combination achieves a strong privacy protection. For instance, secure computation ensures all parties learn only the output of the computation but nothing else while DP bounds the information leakage of individuals in the output of the computation, resulting in a system that is better than the use of secure computation or DP alone.

Case Study: DJoin. Consider a simple setting where two parties would like to compute the intersection size of their

Figure 3. (A) Exhaustive padding of intermediate results in an oblivious query evaluation; (B) Effect of Shrinkwrap on intermediate result sizes when joining tables R and S; (C) Aspirin count with synthetic data scaling. Executed using Circuit model. $\epsilon = 0.5$, $\epsilon = .00005$.



data while preserving DP for both datasets. If each party does not trust each other, how can we ensure a constant additive error as if they trust each other? It is well known that the lower bound for the error of this query is \sqrt{N} , where N is the data size of each party,³⁰ if we want to ensure the view of each party satisfies differential privacy. However, if we assume both parties are computationally bounded, a constant additive error can be achieved.

DJoin³³ offers a concrete protocol for achieving DP under this assumption. This protocol applies private set-intersection cardinality technique to privately compute the noisy intersection set of the two datasets. First, party A defines a polynomial over a finite field whose roots are the elements owned by A. Party A then sends the homomorphic encryptions of the coefficients to party B, along with its public key. Then the encrypted polynomial is evaluated at each of Party B's inputs, followed by a multiplication with a fresh random number. The number of zeros in the results is the true intersection size between A and B. To provide DP, party B adds a number of zeros (differentially private noise of $O(1)$ independent of data size) to the results and sends the randomly permuted results back to party A. Party A decrypts the results and counts the number of zeros. Party A also adds another copy of differentially private noise to the count and sends the result it back to party B. In other words, both parties add noise to their inputs to achieve privacy. However, the final protocol output has only an error of $O(1)$, which is the same as the SDP setting.

Trust improvement. Using secure computation and encryptions achieves

a constant additive error like SDP and prevents any party from seeing the other party's input in the clear. However, this requires an additional assumption of all parties being computationally bounded in the protocol. Hence, the type of DP guarantee achieved in DJoin is known as computational differential privacy.³² In addition, most of the existing protocols consider honest-but-curious adversaries who follow the protocol specification or consider malicious adversaries with an additional overhead to enforce honest behavior, that is, verify that the computation was performed correctly.

In reference to Figure 2, these works demonstrate the improvement in going from Figure 2D to Figure 2C eliminating the need for a trusted data aggregator.

Differential Privacy for Cryptography

As we discussed earlier, cryptographic primitives show promise in bridging the utility gap between SDP and LDP. However, the large overhead of implementing these conventional cryptographic primitives forms a bottleneck for the deployment of such systems. This motivates the need to enhance the performance of such cryptographic primitives. We ask the following question:

“Can we formulate leaky versions of cryptographic primitives for enhancing system performance while rigorously quantifying the privacy loss using DP?”

DP-cryptographic primitives^{7,37,38} are significant for two reasons. First, since the final privacy guarantees of such systems are differential privacy, it is natural to relax the building blocks

such as cryptographic primitives to provide differentially private guarantees. Secondly, the composability properties of DP allow for rigorous quantification of the privacy of the end-to-end system. We showcase benefits of “DP-cryptographic” systems through two detailed case studies on secure computation and secure communication.

Key challenges. Cryptographic primitives provide strong privacy guarantees. However, deployment of certain cryptographic primitives in practical systems is limited due to the large overhead of these primitives. Relaxing the privacy guarantees in a manner that is amenable to rigorous quantification is difficult and differential privacy can be well utilized to provide a solution to this problem to improve performance overhead.

Improve performance of cryptographic computation primitives. Cryptographic computation primitives such as Fully Homomorphic Encryption (FHE) and secure Multi-Party Computation (MPC) enable private computation over data. Over the past few years, there has been tremendous progress in making these primitives practical—a promising direction is MPC, which allows a group of data owners to jointly compute a function while keeping their inputs secret. Here, we show the performance improvement on MPC based private computation, in particular, *differentially private query processing*.

Case Study: Shrinkwrap. Shrinkwrap⁷ is a system that applies DP throughout an SQL query execution to improve performance. In secure computation, the computation overheads depend on the largest possible data

size so that no additional information is leaked. For example, two parties would like to securely compute the answer for the SQL query shown in Figure 3A. This query asks for the number of patients with heart disease who have taken a dosage of “aspirin.” Figure 3A expresses this query as a directed acyclic graph of database operators. For example, the first filter operator takes N records from the two parties and outputs an intermediate result that has patients with heart disease (hd). To hide the selectivity (fraction of records selected) of this operator, the baseline system must pad the intermediate result to its maximum possible size, which is the same as the input size. Exhaustive padding will also be applied to the intermediate output of the two joins and result in an intermediate result cardinality of N^3 and a high-performance overhead. However, if the selectivity of the filter is 10^{-3} , cryptographic padding adds a $1000\times$ overhead. Is there a way to pad fewer dummies to the intermediate result while ensuring a provable privacy guarantee?

Shrinkwrap helps reduce this overhead by padding each intermediate output of the query plan to a differentially private cardinality rather than to the worst case. As shown in Figure 3B, without Shrinkwrap, the output of a join operator with two inputs, each of size N is padded to a size of N^2 . With Shrinkwrap, the output is first padded to the worst size and the output is

sorted such that all the dummies are at the end of the storage. This entire process is executed obliviously. Then Shrinkwrap draws a non-negative integer value with a general Laplace mechanism⁷ and truncates the storage at the end. This approach reduces the input size of the subsequent operators and thereby their I/O cost. We can see from Figure 3C that Shrinkwrap provides a significant improvement in performance over the baseline without DP padding for increasing database sizes.

The relaxed privacy in the secure computation of Shrinkwrap can be quantified rigorously⁷ using computational differential privacy. Assuming all parties are computationally bounded and work in the semi-honest setting, it can be shown that data owners have a computational differentially private view over the input of other data owners; when noisy answers are returned to the data analyst, the data analyst has a computational differentially private view over the input data of all the data owners.

Improve performance of cryptographic communication primitives. Anonymous communication systems aim to protect user identity from the communication recipient and third parties. Despite considerable research efforts in this domain, practical anonymous communication over current Internet architecture is proving to be a challenge. Even if the message contents are encrypted, the packet metadata is

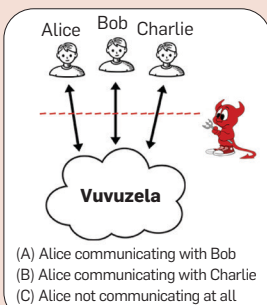
difficult to hide. On one end, systems such as Dissent³⁹ offer strong privacy guarantees yet can scale only to a limited number of participants. On the other end, practical deployed systems such as Tor are vulnerable to traffic analysis and other attacks, limiting their use due to the non-rigorous nature of their privacy guarantees. We will show a case study that uses DP to reduce the communication cost while offering rigorous privacy guarantee. We denote this primitive differentially private anonymous communication.

Case Study: Vuvuzela. Vuvuzela³⁷ is an anonymous communication system that uses DP to enable a highly scalable system with relaxed yet rigorously quantified privacy guarantees. Vuvuzela provides indistinguishable traffic patterns to clients who are actively communicating with other clients, and clients who are not communicating with anyone. In reference to Figure 4, an adversary is unable to distinguish the following three scenarios: Alice not communicating; Alice communicating with Bob; and, Alice communicating with Charlie. In each of the scenarios, a Vuvuzela client’s network traffic appears indistinguishable from the other scenarios.

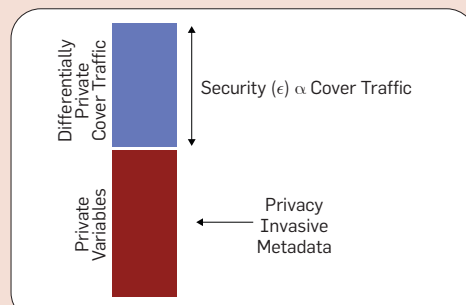
Vuvuzela employs a number of servers S_1, \dots, S_n where at least one of the servers is assumed to be honest. Clients send (and receive) messages to (and from) the first server, which in turn is connected to the second server and so

Figure 4. Vuvuzela is a secure messaging system.

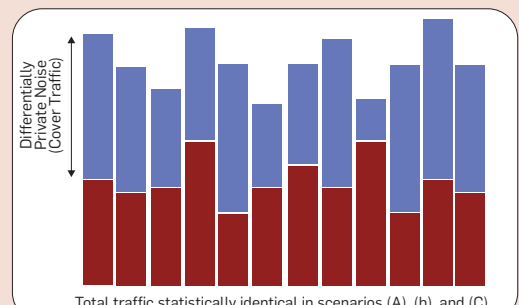
An adversary who can observe and tamper with all network traffic cannot distinguish whether Alice is messaging Bob, Charlie, or is simply not communicating. Vuvuzela uses differential privacy to add noise and mask the privacy invasive metadata, thereby provably hiding information about user communication patterns. Vuvuzela achieves a throughput of 68,000 messages per second for a million users scaling linearly with number of users.



(A) Schematic for Vuvuzela



(B) Performance improvement of Vuvuzela



(C) Security proportional to cover traffic

on. The client creates a layered encryption of its message m , that is, $\text{Enc}_{s_1}(\dots \text{Enc}_{s_n}(m))$, where $\text{Enc}_S(\cdot)$ is the encryption under the key of server S . The clients leave messages at virtual locations in a large space of final destinations (called dead drops), where the other legitimate client can receive it. To hide if a client is communicating or not, a client not in an active conversation makes fake requests to appear indistinguishable from a client in an active conversation. If two clients are in active conversation, they exchange messages via the same random dead drop.

Vuvuzela's threat model assumes at least one server is honest and the adversary is a powerful network level adversary (observing all network traffic) potentially corrupting all other servers.^b The only computation hidden from the adversary is the local computation performed by the honest server which unlinks users' identifiers from the dead drops and adds cover (dummy) traffic. As a consequence, the adversary can only observe the number of single or double exchange requests at the dead drop locations. Each Vuvuzela server adds cover traffic using a Laplace distribution to randomize the number of single dead drops and the number of double dead drops, which is observable by the adversary. Such random cover traffic addition along with the assumption of at least one honest server provides DP guarantees for the observed variables. In other words, Vuvuzela adds noise (cover network traffic) to the two observables (by the adversary) viz. the number of dead drops with one exchange request, and the number of dead drops with two exchange requests, thereby providing communication privacy to clients. This privacy relaxation enables Vuvuzela to scale to a large number of users—it can achieve a throughput of 68,000 messages per second for a million users. Systems such as Stadium,³⁵ and Karaoke²⁷ further improve upon Vuvuzela and scale to even larger sets of users.

Limitations of differentially private cryptography. We caution readers against careless combinations of differential privacy and cryptographic primitives. First, the limitations of both DP as well as cryptographic primitives apply to DP

cryptographic primitives. For instance, an open question is deciding an appropriate level for the privacy budget. Most applications that utilize DP to improve the performance of cryptographic systems involve a trade-off between the level of privacy achieved and the performance of the systems. More generally, differentially private cryptographic systems open up new trade-offs in a privacy-performance-utility space. For instance, in the case of Shrinkwrap, weaker privacy guarantee directly leads to lower performance overhead (privacy performance trade-off while keeping the accuracy level of the query answer constant). On the other hand, systems such as RAPPOR allow for approximate computation of statistics and primarily provide a privacy-utility trade-off. Second, designers need to carefully consider the suitability of these hybrid techniques in their applications as these combinations involve more complex trust assumptions and hence a more complicated security analysis. We remind the reader that while proposing newer DP systems for cryptography, it is imperative to understand the meaning of the privacy guarantees for the application in context. In other words, differential privacy for cryptography may not be the right thing to do in all cases; however, it is well motivated when the goal is to build a differentially private system. Finally, composition results, which bound the privacy loss for a sequence of operations need to be independently studied.

Discussion and Open Questions

Here, we provide directions for future work highlighting important and emerging open questions in the field. We discuss open challenges in deploying differential privacy in the real world—realistic datasets, alternative models and trust assumptions, and other DP-cryptographic primitives. Finally, we caution readers against callous combinations of differential privacy and cryptography.

Differential privacy frameworks—SDP, LDP, and beyond. Over the past decade, there has been significant progress in enabling applications in the standard differential privacy model. For instance, there have been research efforts in attuning DP to handle realistic challenges such as multi-dimensional and complex data—involving graphs,

time series, correlated data.^{24,28} Similarly, there has been work in designing a tailored DP mechanism that is optimized for particular application setting to achieve good accuracy.^{22,31} Prior work has explored combinations of sequential and parallel composition, dimensionality reduction, and sensitivity bound approximations to achieve good accuracy in the SDP model. However, much work needs to be done in adapting state-of-the-art techniques in SDP to more complex deployment scenarios such as LDP. For instance, an open question is the following:

“Is there an algorithm that can efficiently search the space of DP algorithms in the LDP setting for the one that answers the input query with the best accuracy?”

Research advances have demonstrated such mechanisms for the SDP model,^{22,31} however, the discovery of such mechanisms in the LDP setting remains an open question. On a similar note, it is unclear how nuanced variants of DP that have been proposed to handle these more complex databases^{24,28} in the SDP setting translate into LDP or more complex deployment settings.

Differential privacy in practice—Trust assumptions vs accuracy gap.

We have seen how deployments of DP that differ in the trust assumptions provide approximately the same privacy guarantee, but with varying levels of accuracy. In particular, we looked at two popular deployment scenarios: SDP and LDP. There exist other trust assumptions that we have not covered in this article in detail. For instance, Google's recently proposed Prochlo system⁸ uses trusted hardware assumptions to optimize utility of data analytics. On a similar note, Groce et. al.²¹ consider yet another model—where the users participating are malicious. This is the first work to explore a malicious adversarial model in the context of DP and the development of better accuracy mechanisms for such a model is an open research question. More concretely, we can ask:

“What other models of deployment of differential privacy exist and how

^b Even Tor, a practical anonymous communication system, does not protect against such network level adversaries.³⁴

do we design mechanisms for them? Can other technologies such as MPC, FHE, trusted hardware opens up new opportunities in mechanism design?"

An interesting theoretical question is to characterize the separation between different trust models in terms of the best accuracy achievable by a DP algorithm under that model. For instance, McGregor et. al.³⁰ provide separation theorems, that is, gaps in achievable accuracy between (information-theoretic) differential privacy and computational differential privacy for two-party protocols. We ask:

In the Mixnets model (Figure 2B), what is the lower bound on the error for aggregate queries over relational transformations (like joins and group-by) over the data records? An example of such an aggregate is the degree distribution of a graph that reports the number of nodes with a certain degree.

Relaxing cryptographic security via DP: The emerging paradigm of leaky yet differentially private cryptography leads to a number of open questions for the research community. So far, the research community has explored the intersection of differential privacy and cryptographic primitives in limited contexts such as ORAM, MPC, and anonymous communication. However, there exists a broader opportunity to explore the trade-offs of DP cryptographic primitives in contexts such as program obfuscation, zero-knowledge proofs, encrypted databases, and even traffic/protocol morphing. Here, we can ask:

"What other cryptographic primitives can benefit in performance from a privacy relaxation quantified rigorously using differential privacy? How can we design such relaxed primitives?"

In the context of differentially private data analysis, there is a trade-off between privacy and utility. In the context of differentially private cryptographic primitives and resulting applications, there is a broader trade-off space between privacy, utility, and performance. Another open question is:

"What lower bounds exist for overhead of cryptographic

primitives when the privacy guarantees are relaxed using DP?"

Another challenge is how to design optimized protocols that achieve desired trade-offs in the new design space of differentially private cryptography. The trade-off space between privacy, utility, and performance is non-trivial, especially for complex systems. An interesting research question is:

"How to correctly model the trade-off space of real systems so that system designers can decide whether it is worth sacrificing some privacy or utility for a better performance?"

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