

# A Universal Active Power Filter for Single-Phase Reactive Power and Harmonic Compensation

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**Abstract :** In the modern power distribution system, majority of loads draw reactive power and/or harmonic currents from ac source along with main active power currents. These non-unity power-factor linear and non-linear loads cause low efficiency of supply system, poor power-factor, destruction of other equipments due to excessive stresses and EMI problems. Active filters have been considered an effective solution to reduce these problems. This paper presents an Active Power Filter (APF) based on simple control technique to provide reactive power and harmonics compensation for linear and non-linear single-phase loads. A voltage source inverter with carrierless hysteresis PWM current control is used to form an APF. A simple P-I (proportional-integral) dc bus voltage controller with reduced energy storage capacitor is employed in the APF. A set of lagging/leading power-factor linear loads and a diode rectifier fed capacitive load and ac voltage regulator fed inductive load as the non-linear loads are taken on ac mains to demonstrate the effectiveness of the proposed APF for reactive power and harmonic compensation. A detailed steady state and dynamic performance of the APF is presented and discussed in brief.

## I. INTRODUCTION

Solid state power conversion equipments are widely employed to control and convert ac power to feed large number of electrical active and passive loads. Some examples of these loads are variable speed drives, temperature controllers, electric furnaces, light controllers, solid state ac voltage regulators, SMPS and UPS. These non-linear loads also draw reactive power and harmonic currents along with active power from ac mains. The reactive power and harmonic components of load current cause poor power-factor, poor utilization of distribution system, overheating which deteriorate life expectancy of other equipments and cause low efficiency, disturbance to other consumers and interference to communication network. Conventionally, passive L-C filters were employed to reduce harmonics and capacitors were used to compensate the lagging power-factor of the linear and non-linear loads. But they have many demerits like fixed compensation, large size and weight, resonance, noise, and increased losses. These problems of reactive power and harmonic pollution are well recognized and concept of active power filter was introduced a couple of decade ago by Gyugyi and Stryculla [1] to provide an effective solution for elimination of harmonics. A large number of attempts [1-7] were made in the last two decades but most of them were on 3-phase active filters. However, there are many applications of single-phase non-unity power-factor linear and non-linear loads which draw reactive power and harmonic currents from ac mains. Recently some attempts [8-17] have

been made on single-phase active filters. Most of them are either on steady state performance of the APF [8-11, 14-16] and/or use of complex control schemes [12, 13, 17]. This paper is aimed to investigate and design a universal type single-phase active filter with a simple control scheme for reactive power and harmonic compensation of linear and non-linear loads.

The proposed universal APF consists of a single-phase voltage source inverter with an energy storage capacitor at dc bus as shown in Fig. 1. This APF is connected in shunt mode with load through a filtering inductor. A simple P-I (proportional-integral) controller is employed to regulate an averaged dc bus voltage to derive the reference supply current peak value in phase with supply voltage. The carrierless hysteresis PWM current control over the supply current is used to generate the gating signals for the devices of the APF. A variety of leading/lagging power-factor linear loads, uncontrolled rectifier with capacitive and solid state ac regulator non-linear loads is considered to be compensated for reactive power and harmonics by the proposed APF. The steady state and transient performance along with harmonic analysis of the APF is given and described in brief.

## II. SYSTEM DESCRIPTION

Fig. 1 shows the fundamental building block of the proposed parallel APF. It is comprised of a standard single-phase voltage source MOSFET based bridge inverter with dc bus capacitor and dc boost voltage for an effective

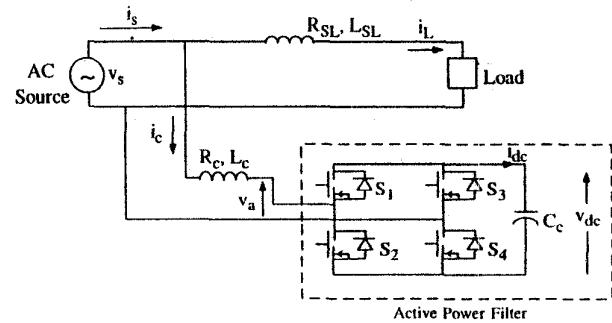


Fig. 1. Basic Circuit of the Active Power Filter

current control. A hysteresis rule based carrierless PWM current control technique is used to provide fast dynamic response of the APF. To demonstrate reactive power compensation capability of the APF, linear loads of lagging

and leading power-factors are considered with the step change. Non-linear loads comprising diode rectifier with capacitive loading and solid state ac regulator with inductive loading, are taken on the APF system to illustrate its capability for harmonic and reactive power compensation. Fig. 2 shows all four kinds of above mentioned loads to be compensated by the APF system. The main role of the proposed APF is to eliminate harmonics and to provide reactive power requirement of the load locally so that ac source feeds only fundamental sinusoidal active component of unity power-factor current. Since this APF system is connected in shunt with load, it improves the system efficiency because it does not process the active power delivered to the load.

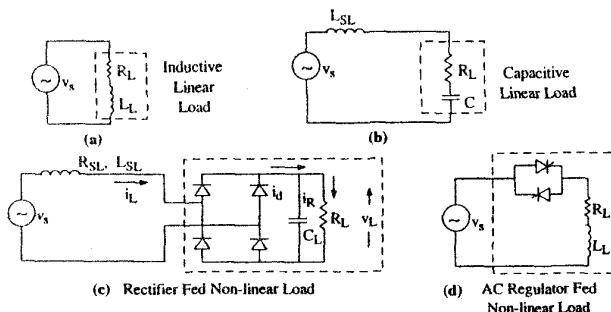


Fig. 2 Different Types of Loads

### III. CONTROL SCHEME

Fig. 3 shows the block diagram of an overall control scheme for the APF system. DC bus voltage and supply voltage and current are sensed to control the APF. AC source supplies fundamental active power component of load current and a fundamental component of a current to maintain average dc bus voltage to a constant value. The later component of source current is to supply losses in VSI such as switching loss, capacitor leakage current etc. in steady state and to recover stored energy on the dc bus capacitor during dynamic conditions such as addition or removal of the loads.

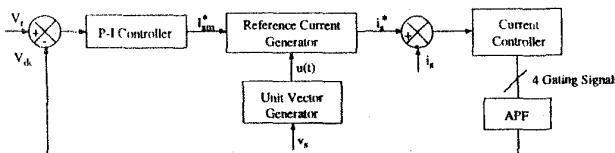


Fig. 3. Control Scheme of the APF

The sensed dc bus voltage of the APF along with its reference value are processed in the P-I voltage controller. The truncated output of the P-I controller is taken as peak of source current. A unit vector in phase with the source voltage is derived using its sensed value. The peak source current is multiplied with the unit vector to generate a reference sinusoidal unity power factor source current. The reference source current and sensed source current are processed in hysteresis carrierless PWM current controller to derive gating signals for the MOSFETs of the APF. In response to these gating pulses, the APF impresses a PWM voltage to flow a current through filter inductor to meet the harmonic and reactive components of the load current. Since all the

quantities such as dc bus voltage etc. are symmetric and periodic corresponding to the half cycle of the ac source, a corrective action is taken in each half cycle of the ac source resulting in fast dynamic response of the APF.

### IV. ANALYSIS AND MODEL EQUATIONS

The proposed APF system is comprised of a voltage controller, a current controller, a voltage source inverter bridge with dc bus, a non-linear load with input impedance and a filter inductance at the input of the APF. All parts are modeled separately and then joined together in order to simulate the APF system.

#### A) Voltage Controller

A P-I (proportional-integral) controller is used to regulate the dc bus capacitor voltage of the APF. The dc bus capacitor voltage  $V_{dc}$  is sensed using a voltage sensor and compared with set reference voltage ( $V_r$ ). The resulting voltage error ( $V_{e(n)}$ ) at nth sample instant is expressed as :

$$V_{e(n)} = V_r(n) - V_{dc}(n) \quad (1)$$

The output of the P-I voltage controller  $V_{0(n)}$  at the nth sampling interval is expressed as :

$$V_{0(n)} = V_{0(n-1)} + K_p \{ V_{e(n)} - V_{e(n-1)} \} + K_i V_{e(n)} \quad (2)$$

Where  $K_p$  and  $K_i$  are proportional and integral gain constants of the voltage regulator.  $V_{0(n-1)}$  and  $V_{e(n-1)}$  are the output of controller and voltage error at (n-1) th sampling instant. This output  $V_{0(n)}$  of the voltage controller is limited to safe permissible value and resulting limited output is taken as peak value of supply current  $I_{sm}^*$ .

#### B) Reference Current Generation

From sensed supply voltage ( $V_{sm} \sin \omega t$ ), a unit vector template is estimated by computing its peak value ( $V_{sm}$ ). The unit vector is as :

$$u(t) = v_s / V_{sm} = \sin \omega t \quad (3)$$

This unit vector is multiplied to peak estimated value of source current  $I_{sm}^*$ . The resulting signal is taken as reference source current as :

$$i_s^*(t) = I_{sm}^* u(t) = I_{sm}^* \sin \omega t \quad (4)$$

#### C) Current Controller

The carrierless PWM hysteresis current controller contributes the switching pattern of the APF devices. The

input reflected PWM voltage of the APF,  $v_a(t)$  is expressed in terms of switching functions.

$$v_a = V_{dc} (SA - SB) \quad (5)$$

Where SA and SB are switching functions of the APF devices. SA is taken 1 if  $S_1$  is ON and SA is considered zero if  $S_2$  is ON. Similarly SB is taken one or zero when  $S_3$  or  $S_4$  are ON. Switch 'S' is comprised with a MOSFET with an antiparallel diode. Therefore, either diode or MOSFET may be conducting when switch is considered in ON state.

#### D) Active Power Filter

The active power filter is modeled in terms of its two basic volt-current equations on ac as well as dc side. The ac side volt-ampere equation is as follows.

$$R_c i_c + L_c p i_c + v_a = v_s \quad (6)$$

Where  $i_c$  is current flowing into the APF,  $v_s$  and  $v_a$  are the supply and APF voltages, respectively.  $R_c$  and  $L_c$  are the resistance and inductance of the APF inductor.  $p$  is the differential operator ( $d/dt$ ).

Equation (6) may be expressed in state space derivative form as :

$$p i_c = \{v_s - v_a - R_c i_c\} / L_c \quad (7)$$

Similarly, dc side basic electrical equation may be written as :

$$p v_{dc} = i_{dc}/C_c \quad (8)$$

Where  $v_{dc}$  is the dc bus capacitor ( $C_c$ ) voltage.  $i_{dc}$  is the charging current and expressed as :

$$i_{dc} = i_c (SA - SB) \quad (9)$$

#### E) Loads on the APF System

In practice there are many kinds of linear and non-linear loads which draw reactive power and harmonic currents from ac source and must be compensated by the APF. Four types of typical load shown in Fig. 2 are considered to demonstrate compensation capability of the APF.

**Lagging Power-Factor (Inductive) Linear Load :** Most of the normal loads without any solid state control such as small electric motors etc. behave as inductive loads. They draw reactive power current component along with active power current component and result in poor power-factor of the supply. It may be represented by a simple series resistive-inductive circuit as shown in Fig. 2(a). The basic volt-ampere equation may be written as:

$$R_L i_L + L_L p i_L = v_s$$

Which may be modified in state space derivative form as :

$$p i_L = (v_s - R_L i_L) / L_L \quad (10)$$

Where  $R_L$  and  $L_L$  are the resistance and the inductance of the load including source impedance and  $i_L$  is the load current to be compensated for unity power-factor by the APF.

**Leading Power-Factor (Capacitive) Linear Load :** Some electric loads behave as leading power-factor loads and draw reactive power component of current along with active power component of current. It may be represented as series resistive-capacitive and inductive circuit as shown in Fig. 2(b). The basic volt-ampere equation may be written as :

$$R_L i_L + L_{SL} p i_L + \frac{1}{C} \int i_L dt = v_s$$

It may be expressed into state space derivative equations as :

$$p i_L = (v_s - R_L i_L - qL/C) / L_{SL} \quad (11)$$

and  $p qL = i_L$  (12)

Where  $R_L$  is resistance of the load inclusive source resistance  $R_{SL}$  and  $L_{SL}$  is the source inductance.  $C$  is the equivalent capacitance of the load.

**Rectifier with Capacitive Loading as a Non-linear Load :** Rectifier-inverter fed variable speed drive is a quite common load on ac source. There uncontrolled rectifier feeding resistive-capacitive load is considered a non-linear load as shown in Fig. 2(c). It has two operating modes i.e. when diodes are in conducting or non-conducting states.

When diodes are in conducting state, the ac source is connected to the load and basic equations are as :

$$R_{SL} i_L + L_{SL} p i_L + v_L = v_s$$

Which may be modified in space derivative form as :

$$p i_L = (v_s - v_L - R_{SL} i_L) / L_{SL} \quad (13)$$

And charging equation is as :

$$p v_L = (i_d - i_R) / C_L \quad (14)$$

Where  $R_{SL}$  and  $L_{SL}$  are source impedance elements and  $v_L$  is the voltage across load capacitor  $C_L$ .  $i_L$  is the load current drawn from ac source.  $i_d$  is the magnitude of  $i_L$  and  $i_R$  is resistive dc load current ( $v_L/R_L$ ).

When diodes are not conducting,  $i_L$  and  $i_d$  will be zero and charged capacitor ( $C_L$ ) with voltage  $v_L$  will feed the

load ( $R_L$ ). Equation (14) will be modified for this discharging mode of operation.

\* **AC Regulator fed Inductive Load as a Non-linear Load :** Back to back connected thyristors are used to control temperature, light and speed of small ac motors. This type of controller acts on the principle of phase control on ac wave to control the current in the load. Fig. 2(d) shows such a series resistive-inductive load with back to back connected thyristors. It has two operating modes when thyristors are in conducting state or in non-conducting states.

When thyristors are conducting the load is connected to ac source and basic volt-ampere equation is as :

$$R_L i_L + L_L \frac{di_L}{dt} = v_s \quad \text{for } \alpha < \omega t < \beta$$

and  $\pi + \alpha < \omega t < \pi + \beta$

It may be expressed in space derivative form as :

$$\frac{di_L}{dt} = (v_s - R_L i_L) / L_L \quad (15)$$

When thyristors are not conducting, current  $i_L$  will be zero and its derivative will also be zero. Output power and current is controlled by controlling the firing angle,  $\alpha$ .

The set of first order differential equations given in (7), (8) along with either (10), or (11), (12) or (13), (14) or (15) defines the dynamic model of the APF system for different kinds of loads. These equations are solved along with other equations using fourth order Runge-Kutta method to analyze transient and steady state behavior of the proposed APF system. A standard FFT package is used to compute harmonic spectrum and THD of the load and supply currents.

## V. PERFORMANCE OF THE APF SYSTEM

Performance of the APF system is demonstrated through Figs. 4-9 and Table I with different kinds of load. The parameters of the system are given in the Appendix. From these results the following observations are made.

Fig. 4 shows the reactive power compensation for an inductive load. Load draws lagging power-factor current from ac bus. APF injects a capacitive current at ac bus resulting in unity power of the source current exactly in phase with source voltage. An excellent dynamic performance for addition and removal of load may be observed from Fig. 4. Supply current settles smoothly to a new steady state value within a cycle of change in load. There is a small change in dc bus voltage at the instant of disturbance in the load to balance extra energy due to increased or decreased level of compensation. DC bus voltage settles to its steady state value within few cycles. P-I controller causes zero steady state error and provides effective control of APF without any overshoot and transient in source current. The source current remains less than load current because source feeds only active power current component of the load. While the reactive power component of load current is supplied by the APF.

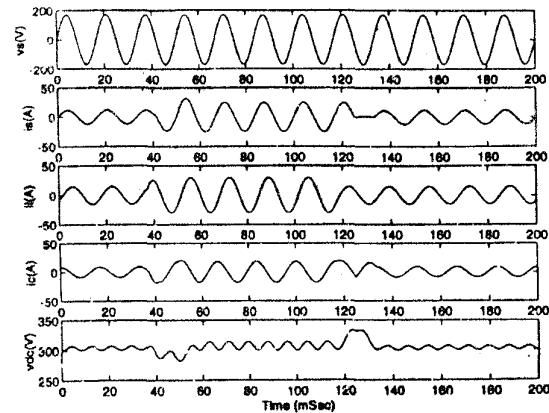


Fig. 4. Performance of the APF System under Linear Inductive Load for Peak Load Current Change from 15 A to 30 A to 15 A

Fig. 5 shows the reactive power compensation for a leading power-factor (capacitive) load. Load draws leading current from the ac bus. APF injects pure lagging current to the ac bus with a desired magnitude required by the load. This results in source current exactly in phase with source voltage. Transient behavior of the APF system may be observed from Fig. 5 for addition and removal of the load. The source current has attained new steady state value without any undesired disturbance. This confirms that system offers a very good transient performance. A small change is observed in dc bus voltage because of immediate need of energy stored in the dc bus of the APF. However, P-I voltage controller maintains steady state average voltage of dc bus to a desired reference value. From Figs. 4 and 5 an excellent performance of APF system is observed as a universal power-factor controller and ideal reactive power compensator.

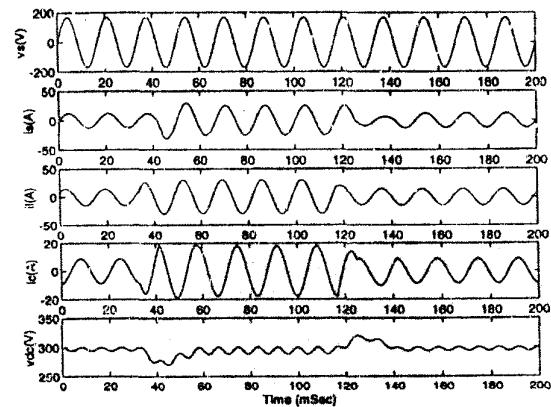


Fig. 5. Performance of the APF System under Linear Capacitive Load for Peak Load Current Change from 15 A to 30 A to 15 A

Fig. 6 shows performance of APF system with non-linear rectifier load. Load draws discontinuous peaky current from ac bus. APF feeds harmonic and reactive power components of load current. AC source feeds only fundamental active power component of load current resulting in source current exactly in phase with source voltage. An interesting

dynamic performance for increased and decreased load may be observed from Fig. 6. Source current remains sinusoidal during transient conditions and attains new steady state value without any overshoot and oscillation. A drop and a rise is observed in dc bus voltage at application and removal of additional load. However, during steady state the P-I voltage controller maintains a constant average voltage at the dc bus of the AF. Harmonic spectrum of load and supply currents at 15 A and 30 A peak values of load current is shown in Fig. 7 for rectifier type non-linear load. It may be observed that the harmonics are eliminated from source current. The THD in supply current is reduced to 1.316 % from 117.77 % of the load current (15 A peak) and 0.598 % from 95.10 % of the load current (30 A peak). Therefore, the APF is able to reduce THD of source current well below the permitted value of 5 % by IEEE-519 standard.

Table I shows the effect of different values of dc bus capacitor  $C_c$  on the performance of the APF for rectifier fed non-linear load. The value of dc bus capacitor of the APF affects the dc bus voltage ripple, dip and rise during addition and removal of loads, P-I controller parameters ( $K_p$  and  $K_i$ ) for satisfactory performance of the APF. The ripple and fluctuations in dc bus voltage of the APF increase with increased load and decreased capacitor value. The THD of supply current is not much affected by dc bus capacitor values. From these results it may be concluded that for the proper selection of ratings of dc bus capacitor, controller parameters and device specifications, the proposed modelling is essential at design stage the APF system.

Fig. 8 shows dynamic performance of the APF system for ac regulator fed inductive load. A load of 0.8 lagging power-factor is taken and firing angle of thyristor is changed from 2.2 radians to 1.5 radians and back to 2.2 radians. APF is able to feed reactive power and harmonic current components of the load. Supply current remains sinusoidal even under transient conditions. This type of load results in maximum burden on the APF. It needs large reactive power because of poor power-factor of load caused by increased firing angle and substantial amount of harmonic currents. It may be observed

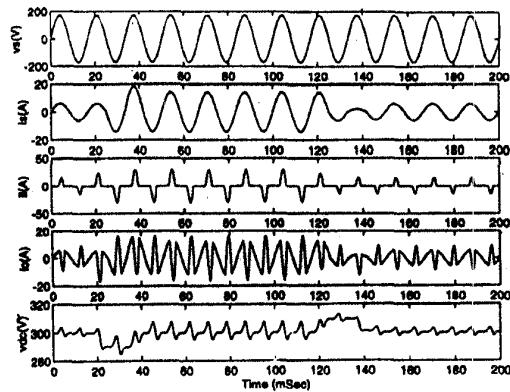


Fig. 6. Performance of the APF System under Non-linear Rectifier Fed Load for Peak Load Current Change from 15 A to 30 A to 15 A with 1000  $\mu\text{F}$  DC Bus Capacitor

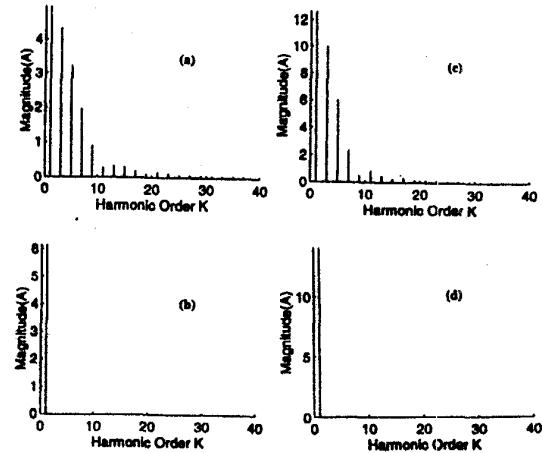


Fig. 7. Harmonic Spectrum with Nonlinear Rectifier Fed Load of  
(a) Load Current; (b) Supply Current (at 15A Peak Load Current);  
(c) Load Current; (d) Supply Current (at 30 A Peak Load Current)

TABLE I  
Performance Summary of APF System with Rectifier Fed Non-linear Load

dc bus capacitor $C_c$ ( $\mu\text{F}$ )	600	800	1000	1200	1400
% peak to peak $v_{dc}$ ripple at 15 A peak load current	3.193	2.397	1.907	1.603	1.373
% peak to peak $v_{dc}$ ripple at 30 A peak load current	6.870	5.167	4.140	3.461	3.010
$v_{dc}$ dip for peak load current change from 15 A to 30 A (volts)	269.50	279.94	284.96	285.28	286.05
$v_{dc}$ rise for peak load current change from 30 A to 15 A (volts)	323.47	316.68	312.02	311.82	311.02
% THD of supply current: 4.35 A (rms); load current: 15 A (peak)	1.616	1.232	1.316	1.616	1.427
% THD of supply current: 9.97 A (rms); load current: 30 A (peak)	0.622	0.671	0.598	0.602	0.689
$K_p$ : proportional gain of PI	0.225	0.325	0.425	0.425	0.425
$K_i$ : integral gain of PI	0.175	0.295	0.400	0.400	0.400
% THD of load current (5.41 A rms; 15 A peak)			117.772		
% THD of load current (12.29 A rms; 30 A peak)			95.100		

from dc bus voltage which has maximum ripple compared to all other three types of loads. However, around 10 % drop and rise in dc bus voltage are observed during dynamic conditions.

It may be reduced by using either higher value of dc bus capacitor or changing the parameters of P-I controller but at the expense of large transient currents in ac source in later

one. APF effectively meets the harmonic and reactive power requirements of the load and maintains supply current sinusoidal and exact in phase with the supply voltage. Fig. 9 shows the harmonic spectrum of load and supply currents at two values of firing angles. The THD of supply current is reduced from 30.33 % to 0.575 % at firing angle of 1.5 radians and from 68.37 % to 2.58 % at firing angle of 2.2 radians. Therefore, APF is quite effective to reduce the THD of supply current well below the permitted value of 5 % by IEEE-519 standard.

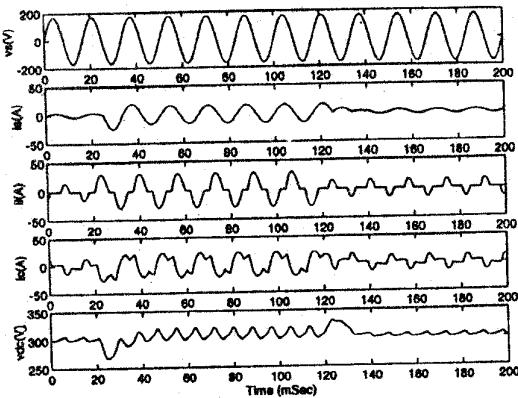


Fig. 8. Performance of the APF System under Nonlinear AC Regulator Fed Inductive Load for Change of Firing Angle from 2.2 Radians to 1.5 Radians to 2.2 Radians

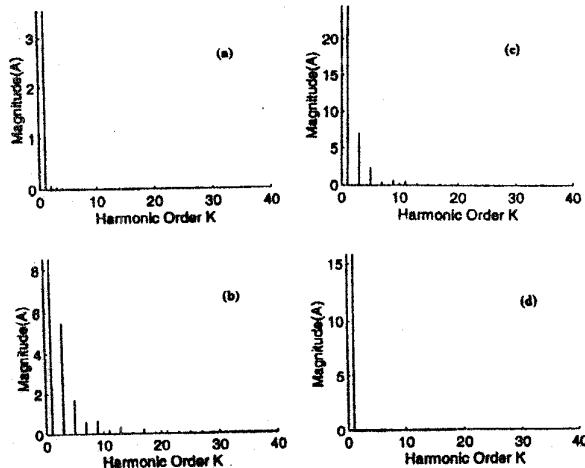


Fig. 9. Harmonic Spectrum with Non-linear AC Regulator Fed Inductive Load of (a) Load Current; (b) Supply Current at Firing Angle = 2.2 Radians; (c) Load Current; (d) Supply Current at Firing Angle = 1.5 Radians

## VI. CONCLUSIONS

A simple P-I controller based APF has been found effective to provide reactive power and harmonic compensation for the variety of loads. An excellent performance of APF system has been observed as a universal power-factor controller and an ideal reactive power compensator. APF is able to reduce the

harmonics well below 5 % in all the cases of extremely reactive and harmonic polluted loads. APF has maintained sinusoidal supply current in phase with the supply voltage resulting in unity power-factor of the supply both in steady state and transient conditions. It is concluded that the proper selection of value of dc bus capacitor and P-I controller parameters results in satisfactory performance of the APF system. Experimental verification of the proposed APF is being performed and test results will be reported in future.

## VII. ACKNOWLEDGMENTS

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## IX. APPENDIX

$V_s(\text{rms}) = 120 \text{ V}$ ,  $F = 60 \text{ Hz}$ ,  $R_c = 0.5 \text{ ohm}$ ,  $L_c = 2.4 \text{ mH}$ ,  $C_c = 1000 \mu\text{F}$ ,  
 $R_{SL} = 0.25 \text{ ohm}$ ,  $L_{SL} = 0.25 \text{ mH}$ ,  $K_p = 0.425$ ,  $K_i = 0.400$ , (a) Linear R-L

Load  $R_L = 4.525$  and  $9.051 \text{ ohms}$  and  $L_L = 9.003$  and  $18.006 \text{ mH}$ .  
(b) Linear R-C Load  $R_L = 4.525$  and  $9.051 \text{ ohms}$  and  $C = 390.75$  and  
 $781.51 \mu\text{F}$ , (c) Rectifier Fed Non-Linear Load  $R_L = 23.2$  and  $62.8 \text{ ohms}$   
and  $C_L = 2200 \mu\text{F}$ , (d) AC Regulator Fed Non-Linear Load  
 $R_L = 3.46 \text{ ohms}$ ,  $L_L = 6.86 \text{ mH}$ ,  $\alpha = 1.5$  and  $2.2 \text{ radians}$ .